Testable Flipped $SU(5) \times U(1)_X$ Models

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Abstract

The little hierarchy between the GUT scale and the string scale may give us some hints that can be tested at the LHC. To achieve string-scale gauge coupling unification, we introduce additional vector-like particles. We require that these vector-like particles be standard, form complete GUT multiplets, and have masses around the TeV scale or close to the string scale. Interestingly, only the flipped $SU(5) \times U(1)_X$ models can work elegantly. We consider all possible sets of vector-like particles with masses around the TeV scale. And we introduce vector-like particles with masses close to the string scale which can mimic the string-scale threshold corrections. We emphasize that all of these vector-like particles can be obtained in the interesting flipped $SU(5) \times U(1)_X$ string models from the four-dimensional free fermionic string construction. Assuming the low-energy supersymmetry, high-scale supersymmetry, and split supersymmetry, we show that the string-scale gauge coupling unification can indeed be achieved in the flipped $SU(5) \times U(1)_X$ models. These models can be tested at the LHC by observing simple sets of vector-like particles at the TeV scale. Moreover, we discuss a simple flipped $SU(5) \times U(1)_X$ model with string-scale gauge coupling unification and high-scale supersymmetry by introducing only one pair of the vector-like particles at the TeV scale, and we predict the corresponding Higgs boson masses. Also, we briefly comment on the string-scale gauge coupling unification in the model with low-energy supersymmetry by introducing only one pair of the vector-like particles at the intermediate scale. And we briefly comment on the mixings among the SM fermions and the corresponding extra vector-like particles.

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I. INTRODUCTION

Supersymmetry provides an elegant solution to the gauge hierarchy problem, and Grand Unified Theories (GUTs) gives us a simple understanding of the quantum numbers of the Standard Model (SM) fermions. In particular, the success of gauge coupling unification in the Minimal Supersymmetric Standard Model (MSSM) strongly supports the possibility of the supersymmetric GUTs [1]. Moreover, the electroweak gauge symmetry can be broken by the radiative corrections due to the large top quark Yukawa coupling [2], and the tiny neutrino masses can be generated naturally via the see-saw mechanism. Therefore, supersymmetric GUTs may describe all the known fundamental interactions in nature except gravity.

The interesting question is whether we can test supersymmetric GUTs at future colliders and experiments. The major prediction of supersymmetric GUTs is the dimension-5 proton decay from the colored Higgsino exchange [3]. This kind of proton decay is suppressed due to the Yukawa couplings. However, we can introduce the non-renormalizable operators to mimic such proton decay, *i. e.*, generic dimensional-5 proton decay opertors with Planck scale suppression and without Yukawa coupling suppression [4]. So, even if we observe such proton decay at future experiments, we can not confirm the possibility of supersymmetric GUTs.

If string theory is correct, it seems to us that one new clue is the little hierarchy between the GUT scale M_{GUT} and the string scale M_{string}. It is well-know that the gauge coupling unification scale in the MSSM, which is called the GUT scale in the literature, is around 2×10^{16} GeV [1]. The gauge coupling unification in the MSSM is based on two implicit assumptions: (1) the $U(1)_Y$ normalization is canonical; (2) there are no intermediate-scale threshold corrections. On the other hand, the string scale M_{string} in the weakly coupled heterotic string theory is [5]

$$M_{\mathrm{string}} = g_{\mathrm{string}} \times 5.27 \times 10^{17} \mathrm{~GeV} , \tag{1}$$

where g_{string} is the string coupling constant. Note that since $g_{\mathrm{string}} \sim \mathcal{O}(1)$, we have

$$M_{\mathrm{string}} \approx 5 \times 10^{17} \mathrm{~GeV} . \tag{2}$$

Thus, there exists a factor of approximately 20 to 25 between the MSSM unification scale and the string scale (In the strong coupled heterotic string theory or M-theory on S^1/Z_2 [6],

the eleven-dimensional Planck scale can be the MSSM unification scale due to the large eleventh dimension [7]. But in this paper we concentrate on the weakly coupled heterotic string theory.). The discrepancy between the scales M_{GUT} and M_{string} implies that the weakly coupled heterotic string theory naively predicts wrong values for the electroweak mixing angle ($\sin^2 \theta_W$) and strong coupling (α_3) at the weak scale. Because the weakly coupled heterotic string theory is one of the leading candidates for a unified theory of the fundamental particles and interactions in the nature, how to achieve the string-scale gauge coupling unification is an important question in string phenomenology [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

In general, we can always achieve the string-scale gauge coupling unification by introducing additional vector-like particles with arbitrary masses or arbitrary SM quantum numbers. Therefore, in order to have interesting and natural models, we make the following three requirements for the additional vector-like particles:

(1) All the vector-like particles are standard. We define the standard particles as the particles that can decay into the MSSM particles via Yukawa couplings. So, the extra vector-like particles are not stable, and then there are no strong cosmological constraints on them.

(2) All the vector-like particles must form the complete GUT multiplets. So, we do not need to split the multiplets, which is a generic problem in the GUTs.

(3) All the vector-like particles must have masses around the TeV scale or close to the string scale, and the string-scale threshold corrections can not be very large which is unnatural. So, the TeV-scale vector-like particles can be produced and tested at the Large Hadron Collider (LHC), and the superheavy vector-like particles can be considered as the string-scale threshold corrections. Also, there are no intermediate-scale threshold corrections.

From requirement (2), we can not achieve the string-scale gauge coupling unification in all the GUTs with simple GUT groups, for example, $SU(5)$, $SO(10)$ and E_6. Also, in the MSSM or standard-like supersymmetric Standard Models that can be constructed

from the weakly coupled heterotic string theory directly, to achieve the $SU(2)_L$ and $SU(3)_C$ gauge coupling unification at the string scale, we either need intermediate-scale (10^{13} GeV) threshold corrections or very large string-scale threshold corrections [20], which violates the requirement (3). In addition, to achieve string-scale gauge coupling unification, we should introduce at the TeV scale sets of vector-like particles in GUT multiplets, whose contribution to the one-loop beta functions of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ gauge symmetry, Δb_1, Δb_2 and Δb_3 respectively, satisfy $\Delta b_1 < \Delta b_2 = \Delta b_3$. Since there is no such set of vector-like particles forming complete GUT representations in the Pati-Salam models with symmetric $SU(2)_L$ and $SU(2)_R$, we can not achieve the string-scale gauge coupling unification with the above requirements. Furthermore, in the Pati-Salam models with asymmetric $SU(2)_L$ and $SU(2)_R$, similar to the MSSM or standard-like supersymmetric Standard Models we can not achieve the string-scale gauge coupling unification for $SU(2)_L \times SU(3)_C$ or $SU(2)_L \times SU(4)$ unless there exist intermediate-scale or very large string-scale threshold corrections. Thus, in this paper we will not consider the MSSM, standard-like supersymmetric Standard Models, and Pati-Salam models with asymmetric $SU(2)_L$ and $SU(2)_R$, although these models can be constructed in the weakly coupled heterotic string theory [21, 22, 23, 24, 25, 26].

Interestingly enough, in the flipped $SU(5) \times U(1)_X$ models [27, 28, 29, 30, 31, 32, 33], we do have such kind of vector-like particles, for instance, the XF and $\overline{XF}$ with respectively the quantum numbers $(\mathbf{10}, \mathbf{1})$ and $(\overline{\mathbf{10}}, -\mathbf{1})$ under the $SU(5) \times U(1)_X$ gauge symmetry. Especially, flipped $SU(5) \times U(1)_X$ models can be constructed naturally in the weakly coupled heterotic string theory at Kac-Moody level one [30, 31, 33]. Therefore, with the requirements above, we can only achieve the string-scale gauge coupling unification in the flipped $SU(5) \times U(1)_X$ models elegantly. In fact, introducing such vector-like particles with masses in the intermediate scale, the string-scale gauge coupling unification has been realized previously [34, 35].

We also require that all the gauge couplings have no Landau pole problem below the string scale. To systematically study the string-scale gauge coupling unification, we consider all the possible sets of vector-like particles with masses around the TeV scale. And we introduce vector-like particles with masses close to the string scale which can mimic the string-scale threshold corrections. We emphasize that all of these vector-like particles can be obtained in the interesting flipped $SU(5) \times U(1)_X$ string models from the four-dimensional free fermionic formulation of the weakly coupled heterotic string theory [31]. Moreover, for

the supersymmetry breaking scenarios, we will consider the low energy supersymmetry, high-scale supersymmetry [36, 37], and split supersymmetry [38, 39]. We will show that the string-scale gauge coupling unification can indeed be realized. For the high-scale supersymmetry and split supersymmetry, we also calculate the corresponding Higgs boson masses.

Furthermore, we briefly discuss a simple flipped $SU(5) \times U(1)_X$ model with string-scale gauge coupling unification and high-scale supersymmetry breaking by introducing only one pair of the vector-like particles at the TeV scale, and we predict the Higgs boson masses. Also, we briefly comment on a simple model with low-energy supersymmetry and one pair of intermediate-scale vector-like particles. And we briefly comment on the mixings among the SM fermions and the corresponding extra vector-like particles.

This paper is organized as follows: in Section II, we briefly review the flipped $SU(5) \times U(1)_X$ models and the calculations of Higgs boson mass in the high-scale supersymmetry and split supersymmetry. We study the string-scale gauge coupling unification in the flipped $SU(5) \times U(1)_X$ models in Section III. We discuss simple flipped $SU(5) \times U(1)_X$ models with string-scale gauge coupling unification in Section IV. We comment on the mixings among the SM fermions and the corresponding extra vector-like particles in Section V. Our discussion and conclusions are in Section VI. The renormalization group equations (RGEs) and beta functions for the non-supersymmetric and supersymmetric Standard Models with additional vector-like particles are given in Appendix A, and for the flipped $SU(5) \times U(1)_X$ models with additional vector-like particles are given in Appendix B.

II. BRIEF REVIEW

A. Flipped $SU(5)$ Model

In this subsection, we would like to briefly review the flipped $SU(5)$ model [27, 28, 29]. The gauge group for flipped $SU(5)$ model is $SU(5) \times U(1)_X$, which can be embedded into $SO(10)$ model. We define the generator $U(1)_{Y'}$ in $SU(5)$ as

$$T_{U(1)_{Y'}} = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right).$$ (3)

The hypercharge is given by

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right).$$ (4)

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There are three families of the SM fermions whose quantum numbers under $SU(5) \times U(1)_X$ are

$$F_i = (\mathbf{10}, \mathbf{1}), \ \bar{f}_i = (\bar{\mathbf{5}}, -\mathbf{3}), \ \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{5}$$

where $i = 1, 2, 3$. As an example, the particle assignments for the first family are

$$F_1 = (Q_1, D_1^c, N_1^c), \ \bar{f}_1 = (U_1^c, L_1), \ \bar{l}_1 = E_1^c, \tag{6}$$

where Q and L are respectively the superfields of the left-handed quark and lepton doublets, U^c, D^c, E^c and N^c are the CP conjugated superfields for the right-handed up-type quark, down-type quark, lepton and neutrino, respectively, To generate the heavy right-handed neutrino masses, we introduce three SM singlets ϕ_i.

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs representations

$$H = (\mathbf{10}, \mathbf{1}), \ \overline{H} = (\overline{\mathbf{10}}, -\mathbf{1}), \ h = (\mathbf{5}, -\mathbf{2}), \ \bar{h} = (\bar{\mathbf{5}}, \mathbf{2}). \tag{7}$$

We label the states in the H multiplet by the same symbols as in the F multiplet, and for $\overline{H}$ we just add "bar" above the fields. Explicitly, the Higgs particles are

$$H = (Q_H, D_H^c, N_H^c), \ \overline{H} = (\overline{Q}_{\overline{H}}, \overline{D}_{\overline{H}}^c, \overline{N}_{\overline{H}}^c), \tag{8}$$

$$h = (D_h, D_h, D_h, H_d), \ \bar{h} = (\overline{D}_{\bar{h}}, \overline{D}_{\bar{h}}, \overline{D}_{\bar{h}}, H_u), \tag{9}$$

where H_d and H_u are one pair of Higgs doublets in the MSSM. We also add one singlet S.

To break the $SU(5) \times U(1)_X$ gauge symmetry down to the SM gauge symmetry, we introduce the following Higgs superpotential at the GUT scale

$$W_{\text{GUT}} = \lambda_1 HHh + \lambda_2 \overline{H}\overline{H}\bar{h} + S(\overline{H}H - M_{\text{H}}^2). \tag{10}$$

There is only one F-flat and D-flat direction, which can always be rotated along the N_H^c and $\overline{N}_{\overline{H}}^c$ directions. So, we obtain that $< N_H^c >=< \overline{N}_{\overline{H}}^c >= M_{\text{H}}$. In addition, the superfields H and $\overline{H}$ are eaten and acquire large masses via the supersymmetric Higgs mechanism, except for D_H^c and $\overline{D}_{\overline{H}}^c$. And the superpotential $\lambda_1 HHh$ and $\lambda_2 \overline{H}\overline{H}\bar{h}$ couple the D_H^c and $\overline{D}_{\overline{H}}^c$ with the D_h and $\overline{D}_{\bar{h}}$, respectively, to form the massive eigenstates with masses $2\lambda_1 < N_H^c >$ and $2\lambda_2 < \overline{N}_{\overline{H}}^c >$. So, we naturally have the doublet-triplet splitting due to the missing partner

mechanism [29]. Because the triplets in h and $\overline{h}$ only have small mixing through the μ term, the Higgsino-exchange mediated proton decay are negligible, *i.e.*, we do not have the dimension-5 proton decay problem.

The SM fermion masses are from the following superpotential

$$W_{\text{Yukawa}} = y_{ij}^D F_i F_j h + y_{ij}^{U\nu} F_i \overline{f}_j \overline{h} + y_{ij}^E \overline{l}_i \overline{f}_j h + \mu h \overline{h} + y_{ij}^N \phi_i \overline{H} F_j \ , \tag{11}$$

where y_{ij}^D, $y_{ij}^{U\nu}$, y_{ij}^E and y_{ij}^N are Yukawa couplings, and μ is the bilinear Higgs mass term.

After the $SU(5) \times U(1)_X$ gauge symmetry is broken down to the SM gauge symmetry, the above superpotential gives

$$
\begin{aligned}
W_{SSM} = {} & y_{ij}^D D_i^c Q_j H_d + y_{ji}^{U\nu} U_i^c Q_j H_u + y_{ij}^E E_i^c L_j H_d + y_{ij}^{U\nu} N_i^c L_j H_u \\
& + \mu H_d H_u + y_{ij}^N \langle \overline{N_{\overline{H}}^c} \rangle \phi_i N_j^c + \cdots \text{(decoupled below } M_{GUT}).
\end{aligned}
\tag{12}
$$

B. Higgs Boson Mass

Since we will consider the high-scale supersymmetry and split supersymmetry, let us briefly review how to calculate the Higgs boson masses in these scenarios.

We denote the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ as g_Y, g_2, and g_3, respectively, and define $g_1 \equiv \sqrt{5/3} g_Y$. The major prediction in the models with high-scale supersymmetry or split supersymmetry is the Higgs boson mass [36, 37, 38, 39]. We can calculate the Higgs boson quartic coupling λ at the supersymmetry breaking scale M_S [36, 38]

$$\lambda(M_S) = \frac{g_2^2(M_S) + 3g_1^2(M_S)/5}{4} \cos^2 2\beta \ , \tag{13}$$

where β is the mixing angle of one pair of Higgs doublets in the supersymmetric Standard Models [36, 37, 38, 39]. And then we evolve it down to the weak scale. The renormalization group equation for the Higgs quartic coupling is given in Appendix A. Using the one-loop effective Higgs potential with top quark radiative corrections, we calculate the Higgs boson mass by minimizing the one-loop effective potential

$$V_{eff} = m_h^2 H^\dagger H - \frac{\lambda}{2!}(H^\dagger H)^2 - \frac{3}{16\pi^2} h_t^4 (H^\dagger H)^2 \left[\log \frac{h_t^2(H^\dagger H)}{Q^2} - \frac{3}{2} \right] \ , \tag{14}$$

where m_h^2 is the bare Higgs mass square, h_t is the top quark Yukawa coupling, and the scale Q is chosen to be at the Higgs boson mass. For the $\overline{MS}$ top quark Yukawa coupling, we use

the one-loop corrected value [40], which is related to the top quark pole mass by

$$m_t = h_t v \left(1 + \frac{16}{3} \frac{g_3^2}{16\pi^2} - 2 \frac{h_t^2}{16\pi^2} \right) . \tag{15}$$

We define $\alpha_i = g_i^2/4\pi$ and denote the Z boson mass as M_Z. In the following numerical calculations, we vary $\tan\beta$ from 1.5 to 50. We choose the recent top quark pole mass measurement $m_t = 172.7 \pm 2.9$ GeV [41], the strong coupling constant $\alpha_3(M_Z) = 0.1182 \pm 0.0027$ [42], and the fine structure constant α_{EM}, weak mixing angle θ_W and Higgs vacuum expectation value (VEV) v at M_Z to be [43]

$$\alpha_{EM}^{-1}(M_Z) = 128.91 \pm 0.02 \,,$$
$$\sin^2\theta_W(M_Z) = 0.23120 \pm 0.00015 \,,$$
$$v = 174.10 \,\text{GeV} \,. \tag{16}$$

III. STRING-SCALE GAUGE COUPLING UNIFICATION

To achieve string-scale gauge coupling unification, we introduce vector-like particles which form complete flipped $SU(5) \times U(1)_X$ multiplets. The quantum numbers for these additional vector-like particles under the $SU(5) \times U(1)_X$ gauge symmetry are

$$XF = (\mathbf{10}, \mathbf{1}) \,, \quad \overline{XF} = (\overline{\mathbf{10}}, -\mathbf{1}) \,, \tag{17}$$
$$Xf = (\mathbf{5}, \mathbf{3}) \,, \quad \overline{Xf} = (\overline{\mathbf{5}}, -\mathbf{3}) \,, \tag{18}$$
$$Xl = (\mathbf{1}, -\mathbf{5}) \,, \quad \overline{Xl} = (\mathbf{1}, \mathbf{5}) \,, \tag{19}$$
$$Xh = (\mathbf{5}, -\mathbf{2}) \,, \quad \overline{Xh} = (\overline{\mathbf{5}}, \mathbf{2}) \,. \tag{20}$$

It is obvious that XF, $\overline{XF}$, Xf, $\overline{Xf}$, Xl, $\overline{Xl}$, Xh, and $\overline{Xh}$ are standard vector-like particles.

Moreover, the particle contents for XF, $\overline{XF}$, Xf, $\overline{Xf}$, Xl, $\overline{Xl}$, Xh, and $\overline{Xh}$ are

$$XF = (XQ, XD^c, XN^c) \,, \quad \overline{XF} = (XQ^c, XD, XN) \,, \tag{21}$$
$$Xf = (XU, XL^c) \,, \quad \overline{Xf} = (XU^c, XL) \,, \tag{22}$$
$$Xl = XE \,, \quad \overline{Xl} = XE^c \,, \tag{23}$$
$$Xh = (XD, XL) \,, \quad \overline{Xf} = (XD^c, XL^c) \,. \tag{24}$$

Under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the quantum numbers for the extra vector-like particles are

$$XQ = (\mathbf{3}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{6}}) \,, \quad XQ^c = (\bar{\mathbf{3}}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{6}}) \,, \tag{25}$$

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$$XU = (\mathbf{3}, \mathbf{1}, \frac{2}{3}) , \ XU^c = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}) , \tag{26}$$

$$XD = (\mathbf{3}, \mathbf{1}, -\frac{1}{3}) , \ XD^c = (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) , \tag{27}$$

$$XL = (\mathbf{1}, \mathbf{2}, -\frac{1}{2}) , \ XL^c = (\mathbf{1}, \mathbf{2}, \frac{1}{2}) , \tag{28}$$

$$XE = (\mathbf{1}, \mathbf{1}, -\mathbf{1}) , \ XE^c = (\mathbf{1}, \mathbf{1}, \mathbf{1}) , \tag{29}$$

$$XN = (\mathbf{1}, \mathbf{1}, \mathbf{0}) , \ XN^c = (\mathbf{1}, \mathbf{1}, \mathbf{0}) , \tag{30}$$

$$XY = (\mathbf{3}, \mathbf{2}, -\frac{5}{6}) , \ XY^c = (\bar{\mathbf{3}}, \mathbf{2}, \frac{5}{6}) . \tag{31}$$

To have the string-scale gauge coupling unification and avoid the Landau pole problem, we need to introduce sets of vector-like particles at the TeV scale whose contributions to the one-loop beta functions satisfy $\Delta b_1 < \Delta b_2 = \Delta b_3$. To avoid the Landau pole problem, we find that there are only four possible such sets of vector-like particles as follows due to the quantizations of the one-loop beta functions

$$Z0 : XF + \overline{XF} ; \tag{32}$$

$$Z1 : XF + \overline{XF} + Xf + \overline{Xf} ; \tag{33}$$

$$Z2 : XF + \overline{XF} + Xl + \overline{Xl} ; \tag{34}$$

$$Z3 : XF + \overline{XF} + Xl + \overline{Xl} + Xh + \overline{Xh} . \tag{35}$$

We assume the masses for each set of vector-like particles are the same, and denote them as M_V. In the interesting flipped $SU(5) \times U(1)_X$ string models from the four-dimensional fermionic formulation of the weakly coupled heterotic string theory [31], for example, the so called 5/2 model in Table 4 in Ref. [31], in addition to the particle content in the flipped $SU(5) \times U(1)_X$ model reviewed in the subsection IIA, we have one pair of vector-like particles XF and $\overline{XF}$, one pair Xf and $\overline{Xf}$, one pair Xl and $\overline{Xl}$, and two pairs Xh and $\overline{Xh}$. Moreover, some of these vector-like particles can be light and have masses around the TeV scale, while the others may have masses around the string scale. So, we can indeed obtain the above Zi sets of vector-like particles with masses around the TeV scale.

In addition, to obtain the exact string-scale gauge coupling unification, we may need to consider the string-scale (or GUT-scale) threshold corrections. Interestingly, in the same 5/2 model in Table 4 in Ref. [31], there exist additional gauge symmetries $SO(10) \times SU(4) \times U(1)^5$ in the hidden sector. Also, there are five pairs of vector-like particles XT_i and $\overline{XT_i}$ and one pair of vector-like particles XT' and $\overline{XT'}$ [31]. These particles are only charged under

$SU(4) \times U(1)_X$, and their quantum numbers are

$$XT_i = (\mathbf{4}, \frac{\mathbf{5}}{\mathbf{2}}) \; , \; \overline{XT}_i = (\overline{\mathbf{4}}, -\frac{\mathbf{5}}{\mathbf{2}}) \; , \tag{36}$$

$$XT' = (\mathbf{4}, -\frac{\mathbf{5}}{\mathbf{2}}) \; , \; \overline{XT'} = (\overline{\mathbf{4}}, \frac{\mathbf{5}}{\mathbf{2}}) \; . \tag{37}$$

To mimic the string-scale threshold corrections, we introduce this set of vector-like particles with masses close to the string scale. We denote this set of vector-like particles as ZT set. For simplicity, we assume that the masses for ZT set of vector-like particles are universal, and we denote this mass as $M_{V'}$.

In this paper, we assume that H, $\overline{H}$, and the triplets D_h and $\overline{D}_{\overline{h}}$ in h and $\overline{h}$ have masses around the $SU(2)_L \times SU(3)_C$ unification scale M_{23}. From the weak scale to M_{23}, we employ two-loop RGE running for the SM gauge couplings and one-loop running for the Yukawa couplings. For simplicity, we only consider the contributions to the gauge coupling RGE running from the Yukawa couplings of the third family of the SM fermions, $i.e.$, the top quark, bottom quark and τ lepton Yukawa couplings. And we neglect the contributions to the gauge coupling RGE running from the Yukawa couplings of the extra vector-like particles, and the threshold corrections at the supersymmetry breaking scale due to the scalar mass differences.

From M_{23} to M_{string}, we consider two-loop RGE running for the $SU(5) \times U(1)_X$ gauge couplings. For simplicity, we neglect the Yukawa coupling corrections. The RGEs and beta functions are given in the Appendicies A and B. Also, the beta functions for XT' and $\overline{XT'}$ are the same as these for XT_i and $\overline{XT}_i$, and then will not be presented there.

In addition, we would like to point out that the gauge coupling α'_1 of $U(1)_X$ is related to α_1 and α_5 at the scale M_{23} by

$$\alpha_1'^{-1}(M_{23}) = \frac{25}{24}\alpha_1^{-1}(M_{23}) - \frac{1}{24}\alpha_5^{-1}(M_{23}) \; . \tag{38}$$

A. Universal Supersymmetry Breaking

We consider the following cases for the vector-like particle mass scales M_V and supersymmetry breaking scales M_S

   (1) $M_V = 200$ GeV, $M_S = 360$ GeV, 1000 GeV, and 1.0×10^4 GeV;

   (2) $M_V = 600$ GeV, $M_S = 1000$ GeV;

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(3) $M_V = 1000$ GeV, $M_S = 1.0 \times 10^4$ GeV.

In our numerical calculations, the RGE running is performed for $\tan\beta = 10$. Varying its value will only generate negligible changes to the mass scales. However, the choice of $\tan\beta$ affects the Higgs boson mass range significantly if the supersymmetry breaking scale is high. The Higgs boson mass ranges are shown with the lower end calculated with $\alpha_3 = 0.1209$, $m_t = 169.8$ GeV and $\tan\beta = 1.5$, and the upper end with $\alpha_3 = 0.1155$, $m_t = 175.6$ GeV and $\tan\beta = 50$.

	M_V	M_S	M_{23}	$M_{V'}$	g_{string}	M_{string}	m_h
Z2	200	360	1.9×10^{16}	1.1×10^{17}	1.301	6.9×10^{17}	$-$
Z2	200	1000	1.9×10^{16}	1.5×10^{17}	1.207	6.4×10^{17}	$-$
Z2	200	1.0×10^4	2.1×10^{16}	2.7×10^{17}	1.064	5.6×10^{17}	$102 - 132$
Z2	600	1000	1.8×10^{16}	1.8×10^{17}	1.170	6.2×10^{17}	$-$
Z2	1000	1.0×10^4	2.0×10^{16}	3.5×10^{17}	1.031	5.4×10^{17}	$102 - 132$

TABLE I: Mass scales in the flipped $SU(5) \times U(1)_X$ models with string-scale gauge coupling unification and universal supersymmetry breaking.
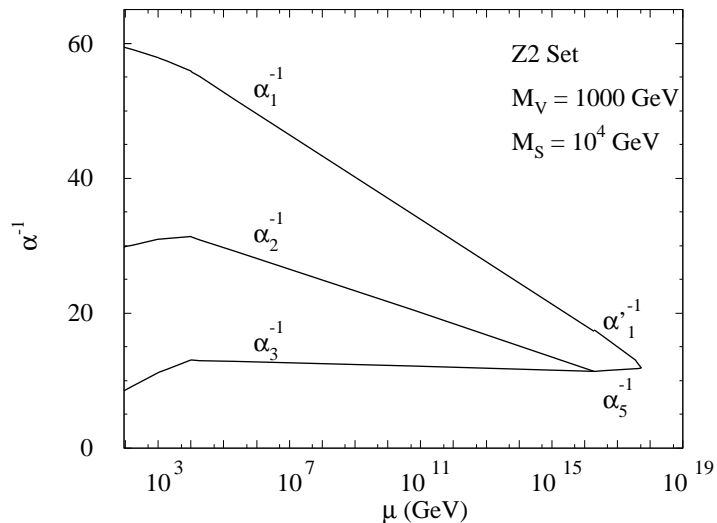

FIG. 1: Two-loop string-scale gauge coupling unification for universal supersymmetry breaking in the flipped $SU(5) \times U(1)_X$ model with Z2 set of vector-like particles, $M_V = 1000$ GeV, and $M_S = 1.0 \times 10^4$ GeV.

From the concrete numerical calculations, we obtain that the string-scale gauge coupling

unification can not be achieved in all the cases with Z1 and Z3 sets of vector-like particles, because the strong coupling g_3 runs into Landau pole below the $SU(2)_L \times SU(3)_C$ unification scale M_{23}. The string-scale gauge coupling unification can be realized precisely in the cases with Z2 set of vector-like particles and the suitable mass $M_{V'}$ for the ZT set of vector-like particles. In the Table I, we present the corresponding M_{23}, $M_{V'}$, string coupling g_{string}, string-scale M_{string}, and the Higgs boson mass m_h if the supersymmetry breaking scale is high. We find that $M_{V'}$ is close to the string scale. Thus, the vector-like particles (XT_i, $\overline{XT}_i$), and (XT', $\overline{XT'}$) can indeed be considered as the string-scale threshold corrections. Moreover, if the supersymmetry breaking scale is about 1.0×10^4 GeV, the corresponding Higgs boson masses are from 102 GeV to 132 GeV. As an example, we present the two-loop string-scale gauge coupling unification in the model with Z2 set of vector-like particles, $M_V = 1000$ GeV, and $M_S = 1.0 \times 10^4$ GeV in Fig. 1.

B.   Split Supersymmetry

The RGEs and beta functions in the split supersymmetry can be found in Ref. [39]. For simplicity, we assume that the gaugino and Higgsino masses are the same and equal to the vector-like particle mass scale M_V. We consider the following cases for M_V and supersymmetry breaking scale M_S that is the universal scalar mass in split supersymmetry

(1) $M_V = 200$ GeV, $M_S = 1.0 \times 10^4$ GeV, and 1.0×10^{10} GeV;

(2) $M_V = 1000$ GeV, $M_S = 1.0 \times 10^4$ GeV, and 1.0×10^{10} GeV.

If supersymmetry breaking scale is 1.0×10^4 GeV, we find that the string-scale gauge coupling unification can not be achieved in all the cases with Z1 and Z3 sets of vector-like particles, because there exists the Landau pole problem for the strong coupling g_3 below M_{23}. The string-scale gauge coupling unification can be realized precisely in the rest cases with suitable $M_{V'}$. In Table II, we present the corresponding M_{23}, $M_{V'}$, g_{string}, M_{string}, and m_h. Because $M_{V'}$ is also close to the string scale (all within one order), the vector-like particles (XT_i, $\overline{XT}_i$), and (XT', $\overline{XT'}$) can be considered as string-scale threshold corrections, too. Moreover, if the supersymmetry breaking scale is about 1.0×10^4 GeV, the corresponding Higgs boson masses range from 102 GeV to 134 GeV. And if the supersymmetry breaking scale is 1.0×10^{10} GeV, the Higgs boson masses are from 122 GeV to 151 GeV. In Fig. 2, we present the two-loop string-scale gauge coupling unification in the model with Z2 set of

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	M_V	M_S	M_{23}	$M_{V'}$	g_{string}	M_{string}	m_h
Z1	200	1.0×10^{10}	1.2×10^{17}	3.9×10^{16}	1.179	6.2×10^{17}	$122 - 149$
Z1	1000	1.0×10^{10}	8.7×10^{16}	1.6×10^{17}	1.005	5.3×10^{17}	$126 - 151$
Z2	200	1.0×10^{4}	2.6×10^{16}	9.2×10^{16}	1.171	6.2×10^{17}	$104 - 134$
Z2	200	1.0×10^{10}	7.9×10^{16}	3.6×10^{16}	0.918	4.8×10^{17}	$123 - 149$
Z2	1000	1.0×10^{4}	2.2×10^{16}	1.9×10^{17}	1.081	5.7×10^{17}	$104 - 133$
Z2	1000	1.0×10^{10}	6.2×10^{16}	1.5×10^{17}	0.842	4.4×10^{17}	$127 - 151$
Z3	200	1.0×10^{10}	1.3×10^{17}	3.1×10^{16}	1.160	6.1×10^{17}	$122 - 149$
Z3	1000	1.0×10^{10}	8.6×10^{16}	1.4×10^{17}	1.005	5.3×10^{17}	$126 - 151$

TABLE II: Mass scales in the flipped $SU(5) \times U(1)_X$ models with string-scale gauge coupling unification and split supersymmetry.
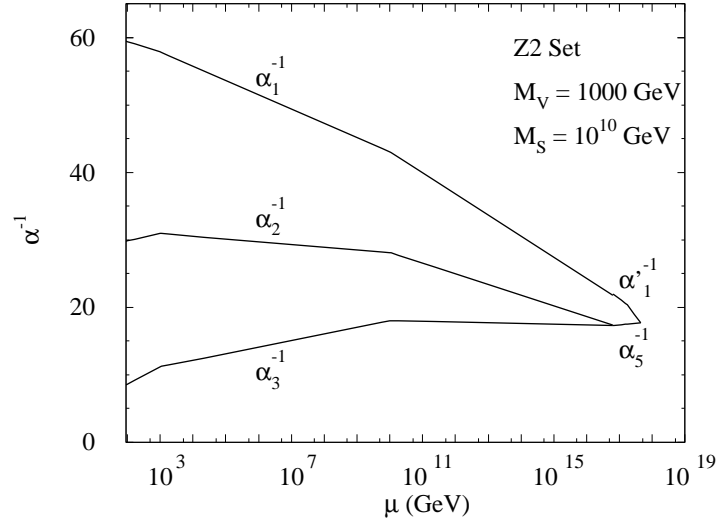

FIG. 2: Two-loop string-scale gauge coupling unification for split supersymmetry in the flipped $SU(5) \times U(1)_X$ model with Z2 set of vector-like particles, $M_V = 1000$ GeV, and $M_S = 10^{10}$ GeV.

vector-like particles, $M_V = 1000$ GeV, and $M_S = 10^{10}$ GeV.

IV. THE SIMPLE FLIPPED $SU(5) \times U(1)_X$ MODELS

First, we consider a simple flipped $SU(5) \times U(1)_X$ model with high-scale supersymmetry breaking. Interestingly, we can achieve the string-scale gauge coupling unification by intro-

ducing only one Z0 set of vector-like particles around the TeV scale and choosing suitable supersymmetry breaking scale.

	M_V	M_S	M_{23}	g_{string}	M_{string}	m_h
Z0	200	5.8×10^{12}	4.4×10^{16}	0.664	3.5×10^{17}	128 - 142
Z0	1000	1.4×10^{12}	3.9×10^{16}	0.670	3.5×10^{17}	131 - 144

TABLE III: Mass scales in the simple flipped $SU(5) \times U(1)_X$ model with string-scale gauge coupling unification and intermediate-scale supersymmetry breaking.

For numerical calculations, we choose $M_V = 200$ GeV and 1000 GeV. We obtain that the supersymmetry breaking scale is about 10^{12} GeV, the $SU(2)_L \times SU(3)_C$ unification scale is about 4×10^{16} GeV, and the string scale is about 3.5×10^{17} GeV. The concrete results including the Higgs boson masses are given in Table III. For $M_V = 1000$ GeV, we present the two-loop string-scale gauge coupling unification in Fig 3.
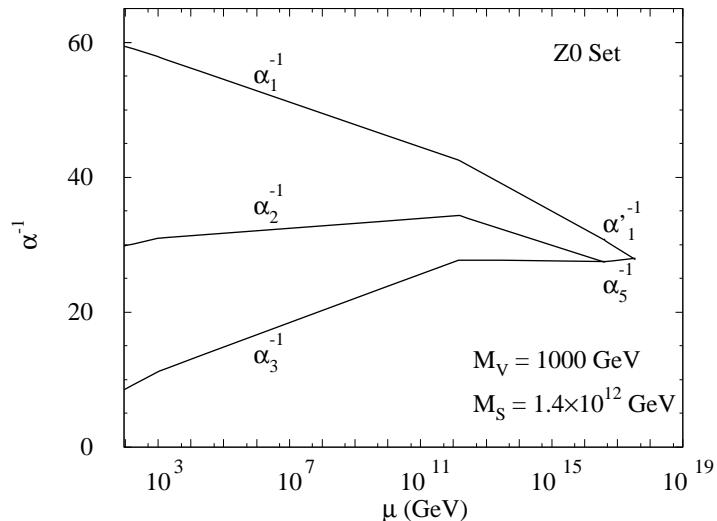
FIG. 3: Two-loop string-scale gauge coupling unification for the simple flipped $SU(5) \times U(1)_X$ model with $M_V = 1000$ GeV and intermediate-scale supersymmetry breaking.

As we know, the Peccei-Quinn mechanism provides an elegant solution to the strong CP problem [44]. However, the Peccei-Quinn mechanism may not be stable against the quantum gravity corrections. And the Peccei-Quinn mechanism may be stabilized if and only if the supersymmetry breaking scale is around 10^{11-12} GeV [36, 45]. Interestingly, our supersymmetry breaking scales are within this range.

	M_S	M_V	M_{23}	g_{string}	M_{string}
Z0	200	4.5×10^{11}	1.6×10^{16}	0.800	4.2×10^{17}
Z0	1000	1.6×10^{11}	1.6×10^{16}	0.789	4.2×10^{17}

TABLE IV: Mass scales in the simple flipped $SU(5) \times U(1)_X$ model with string-scale gauge coupling unification and intermediate-scale mass for the vector-like particles.

Second, the string-scale gauge coupling unification can also be achieved in the flipped $SU(5) \times U(1)_X$ models with low-energy supersymmetry by introducing only one Z0 set of vector-like particles at the intermediate scale, which has been studied previously [34, 35]. Using the updated gauge couplings at the weak scale, we are able to reproduce the string-scale gauge coupling unification in this case, too. With $M_S = 200$ GeV and 1000 GeV, we present the M_V, M_{23}, g_{string} and M_{string} in Table IV. We find that the masses for the vector-like particles are about 10^{11} GeV, which are about two orders higher than the previous results [34, 35]. For $M_S = 1000$ GeV, we present the two-loop string-scale gauge coupling unification in Fig 4.
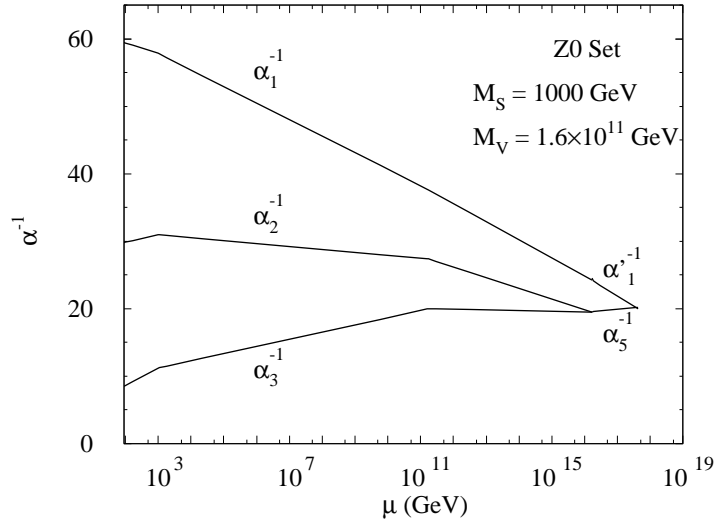

FIG. 4: Two-loop string-scale gauge coupling unification for the simple flipped $SU(5) \times U(1)_X$ model with intermediate-scale universal mass for the vector-like particles.

V. COMMENTS

In our models, the XN^c in XF and XN in $\overline{XF}$ can have masses around the scale M_{23}, and the SM fermions may mix with the additional vector-like particles. For example, we consider the model with Z_2 set of vector-like particles. The relevant Langrangian is

$$W_{\text{Yukawa}} = y_{ij}^D F_i F_j h + y_{ij}^{U\nu} F_i \overline{f}_j \overline{h} + y_{ij}^E \overline{l}_i \overline{f}_j h + \mu h \overline{h} + y_{kj}^N \phi_k \overline{H} F_j + y_j'^D X F F_j h$$
$$+ y_j'^{U\nu} X F \overline{f}_j \overline{h} + y_j'^E \overline{Xl} f_j h + y_k'^N \phi_k \overline{H} X F + y^{\overline{XF}} \phi' H \overline{XF} \,, \qquad (39)$$

where $k = 1, 2, 3, 4$, and ϕ_4 and ϕ' are additional SM singlets. Therefore, the XN^c in XF and XN in $\overline{XF}$ obtain masses around the scale M_{23}. And the SM fermions (including neutrinos) will mix with the corresponding vector-like particles. In particular, the additional vector-like partilces, for example, XF and $\overline{XF}$, and/or Xf and $\overline{Xf}$, will definitely affect the discussions of neutrino masses due to the extra mixing terms. The concerete discussions of the mixings among the SM fermions and the vector-like particles are beyond the scope of this paper.

VI. DISCUSSION AND CONCLUSIONS

Whether we can test GUTs in the future experiments is an interesting question. We pointed out that the little hierarchy between the GUT scale and the string scale may be tested at the LHC. To realize the precise string-scale gauge coupling unification, we introduce the additional vector-like particles. We require that these vector-like particles be standard, form complete GUT multiplets, and have masses around the TeV scale or close to the string scale. The vector-like particles with TeV-scale masses can be observed at the LHC, and the vector-like particles with masses close to the string-scale can be considered as the string-scale threshold corrections. We found that only the flipped $SU(5) \times U(1)_X$ models can work elegantly. Moreover, we listed all the possible sets of vector-like particles with masses around the TeV scale. And we introduce vector-like particles with masses close to the string scale which can mimic the string-scale threshold corrections. We emphasize that all of these vector-like particles can be obtained in the interesting flipped $SU(5) \times U(1)_X$ string models from the four-dimensional free fermionic formulation of the weakly coupled heterotic string theory [31]. Assuming the low-energy supersymmetry, high-scale supersymmetry, or

split supersymmetry, we show that the string-scale gauge coupling unification can indeed be achieved in the flipped $SU(5) \times U(1)_X$ models. These models can be tested at the LHC by observing the simple sets of vector-like particles with masses around the TeV scale. In addition, we discuss a simple flipped $SU(5) \times U(1)_X$ model with string-scale gauge coupling unification and high-scale supersymmetry by introducing only one pair of the vector-like particles at the TeV scale, and we predict the corresponding Higgs boson masses. Also, we briefly comment on the string-scale gauge coupling unification in the model with low-energy supersymmetry and only one pair of vector-like particles with masses at the intermediate scale. And we briefly comment on the mixings among the SM fermions and the corresponding extra vector-like particles.

APPENDIX A: THE NON-SUPERSYMMETRIC AND SUPERSYMMETRIC STANDARD MODELS WITH VECTOR-LIKE PARTICLES

1. Renormalization Group Equations

We give the renormalization group equations in the SM and MSSM. The general formulae for the renormalization group equations in the SM are given in Refs. [46, 47], and these for the supersymmetric models are given in Refs. [48, 49, 50].

First, we summarize the renormalization group equations in the SM. The two-loop renormalization group equations for the gauge couplings are

$$(4\pi)^2 \frac{d}{dt} g_i = b_i g_i^3 + \frac{g_i^3}{(4\pi)^2} \left[\sum_{j=1}^{3} B_{ij} g_j^2 - \sum_{\alpha=u,d,e} d_i^\alpha \text{Tr} \left(h^{\alpha\dagger} h^\alpha \right) \right] , \tag{A1}$$

where $t = \ln \mu$ and μ is the renormalization scale. The g_1, g_2 and g_3 are the gauge couplings for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively, where we use the $SU(5)$ normalization $g_1^2 \equiv$

$(5/3)g_Y^2$. The beta-function coefficients are

$$b = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right) \; , \; B = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix} \; , \tag{A2}$$

$$d^u = \left(\frac{17}{10}, \frac{3}{2}, 2 \right) \; , \; d^d = \left(\frac{1}{2}, \frac{3}{2}, 2 \right) \; , \; d^e = \left(\frac{3}{2}, \frac{1}{2}, 0 \right) \; . \tag{A3}$$

Since the contributions in Eq. (A1) from the Yukawa couplings arise from the two-loop diagrams, we only need Yukawa coupling evolution at the one-loop order. The one-loop renormalization group equations for Yukawa couplings are

$$(4\pi)^2 \frac{d}{dt} h^u = h^u \left(-\sum_{i=1}^{3} c_i^u g_i^2 + \frac{3}{2} h^{u\dagger} h^u - \frac{3}{2} h^{d\dagger} h^d + \Delta_2 \right) , \tag{A4}$$

$$(4\pi)^2 \frac{d}{dt} h^d = h^d \left(-\sum_{i=1}^{3} c_i^d g_i^2 - \frac{3}{2} h^{u\dagger} h^u + \frac{3}{2} h^{d\dagger} h^d + \Delta_2 \right) , \tag{A5}$$

$$(4\pi)^2 \frac{d}{dt} h^e = h^e \left(-\sum_{i=1}^{3} c_i^e g_i^2 + \frac{3}{2} h^{e\dagger} h^e + \Delta_2 \right) , \tag{A6}$$

where h^u, h^d and h^e are the Yukawa couplings for the up-type quark, down-type quark, and lepton, respectively. Also, c^u, c^d, and c^e are given by

$$c^u = \left(\frac{17}{20}, \frac{9}{4}, 8 \right) \; , \; c^d = \left(\frac{1}{4}, \frac{9}{4}, 8 \right) \; , \; c^e = \left(\frac{9}{4}, \frac{9}{4}, 0 \right) \; , \tag{A7}$$

$$\Delta_2 = \mathrm{Tr}(3 h^{u\dagger} h^u + 3 h^{d\dagger} h^d + h^{e\dagger} h^e) \; . \tag{A8}$$

The one-loop renormalization group equation for the Higgs quartic coupling is

$$(4\pi)^2 \frac{d}{dt} \lambda = 12\lambda^2 - \left(\frac{9}{5} g_1^2 + 9 g_2^2 \right) \lambda + \frac{9}{4} \left(\frac{3}{25} g_1^4 + \frac{2}{5} g_1^2 g_2^2 + g_2^4 \right) + 4\Delta_2 \lambda - 4\Delta_4 \; , \tag{A9}$$

where

$$\Delta_4 = \mathrm{Tr} \left[3(h^{u\dagger} h^u)^2 + 3(h^{d\dagger} h^d)^2 + (h^{e\dagger} h^e)^2 \right] \; . \tag{A10}$$

Second, we summarize the renormalization group equations in the MSSM. The two-loop renormalization group equations for the gauge couplings are

$$(4\pi)^2 \frac{d}{dt} g_i = b_i g_i^3 + \frac{g_i^3}{(4\pi)^2} \left[\sum_{j=1}^{3} B_{ij} g_j^2 - \sum_{\alpha=u,d,e} d_i^\alpha \mathrm{Tr} \left(y^{\alpha\dagger} y^\alpha \right) \right] , \tag{A11}$$

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where the beta-function coefficients are

$$b = \left(\frac{33}{5}, 1, -3\right) , \quad B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} , \tag{A12}$$

$$d^u = \left(\frac{26}{5}, 6, 4\right) , \quad d^d = \left(\frac{14}{5}, 6, 4\right) , \quad d^e = \left(\frac{18}{5}, 2, 0\right) . \tag{A13}$$

$$\tag{A14}$$

The one-loop renormalization group equations for Yukawa couplings are

$$(4\pi)^2 \frac{d}{dt} y^u = y^u \left[3y^{u\dagger}y^u + y^{d\dagger}y^d + 3\mathrm{Tr}(y^{u\dagger}y^u) - \sum_{i=1}^{3} c_i^u g_i^2 \right] , \tag{A15}$$

$$(4\pi)^2 \frac{d}{dt} y^d = y^d \left[y^{u\dagger}y^u + 3y^{d\dagger}y^d + \mathrm{Tr}(3y^{d\dagger}y^d + y^{e\dagger}y^e) - \sum_{i=1}^{3} c_i^d g_i^2 \right] , \tag{A16}$$

$$(4\pi)^2 \frac{d}{dt} y^e = y^e \left[3y^{e\dagger}y^e + \mathrm{Tr}(3y^{d\dagger}y^d + y^{e\dagger}y^e) - \sum_{i=1}^{3} c_i^e g_i^2 \right] , \tag{A17}$$

where y^u, y^d and y^e are the Yukawa couplings for the up-type quark, down-type quark, and lepton, respectively. Also, c^u, c^d, and c^e are given by

$$c^u = \left(\frac{13}{15}, 3, \frac{16}{3}\right) , \quad c^d = \left(\frac{7}{15}, 3, \frac{16}{3}\right) , \quad c^e = \left(\frac{9}{5}, 3, 0\right) . \tag{A18}$$

2. Beta Functions for the Vector-Like Particles

We present one-loop and two-loop beta functions to the SM gauge couplings from the vector-like particles. The general formulae are also given in Refs. [46, 47, 48, 49, 50].

First, we present the one-loop beta functions $\Delta b \equiv (\Delta b_1, \Delta b_2, \Delta b_3)$ as complete supermultiplets from the extra particles

$$\Delta b^{XQ+XQ^c} = (\frac{1}{5}, 3, 2) , \quad \Delta b^{XU+XU^c} = (\frac{8}{5}, 0, 1) , \quad \Delta b^{XD+XD^c} = (\frac{2}{5}, 0, 1) , \tag{A19}$$

$$\Delta b^{XL+XL^c} = (\frac{3}{5}, 1, 0) , \quad \Delta b^{XE+XE^c} = (\frac{6}{5}, 0, 0) , \quad \Delta b^{XN+XN^c} = (0, 0, 0) , \tag{A20}$$

$$\Delta b^{XY+XY^c} = (5, 3, 2) , \quad \Delta b^{XT_i+\overline{XT}_i} = (\frac{6}{5}, 0, 0) , \tag{A21}$$

Second, we present the two-loop beta functions (ΔB_{ij}) from the extra particles in the non-supersymmetric models

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$$\Delta B^{XQ+XQ^c} = \begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{49}{2} & 8 \\ \frac{1}{15} & 3 & \frac{76}{3} \end{pmatrix} \ , \ \Delta B^{XU+XU^c} = \begin{pmatrix} \frac{64}{75} & 0 & \frac{64}{15} \\ 0 & 0 & 0 \\ \frac{8}{15} & 0 & \frac{38}{3} \end{pmatrix} \ , \tag{A22}$$

$$\Delta B^{XD+XD^c} = \begin{pmatrix} \frac{4}{75} & 0 & \frac{16}{15} \\ 0 & 0 & 0 \\ \frac{2}{15} & 0 & \frac{38}{3} \end{pmatrix} \ , \ \Delta B^{XL+XL^c} = \begin{pmatrix} \frac{9}{50} & \frac{9}{10} & 0 \\ \frac{3}{10} & \frac{49}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{A23}$$

$$\Delta B^{XE+XE^c} = \begin{pmatrix} \frac{36}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \ \Delta B^{XN+XN^c} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{A24}$$

$$\Delta B^{XY+XY^c} = \begin{pmatrix} \frac{25}{6} & \frac{15}{2} & \frac{40}{3} \\ \frac{5}{2} & \frac{49}{2} & 8 \\ \frac{5}{3} & 3 & \frac{76}{3} \end{pmatrix} \ , \ \Delta B^{XT_i+\overline{XT}_i} = \begin{pmatrix} \frac{9}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ . \tag{A25}$$

Third, we present the two-loop beta functions from the extra particles in the supersymmetric models

$$\Delta B^{XQ+XQ^c} = \begin{pmatrix} \frac{1}{75} & \frac{3}{5} & \frac{16}{15} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{15} & 6 & \frac{68}{3} \end{pmatrix} \ , \ \Delta B^{XU+XU^c} = \begin{pmatrix} \frac{128}{75} & 0 & \frac{128}{15} \\ 0 & 0 & 0 \\ \frac{16}{15} & 0 & \frac{34}{3} \end{pmatrix} \ , \tag{A26}$$

$$\Delta B^{XD+XD^c} = \begin{pmatrix} \frac{8}{75} & 0 & \frac{32}{15} \\ 0 & 0 & 0 \\ \frac{4}{15} & 0 & \frac{34}{3} \end{pmatrix} \ , \ \Delta B^{XL+XL^c} = \begin{pmatrix} \frac{9}{25} & \frac{9}{5} & 0 \\ \frac{3}{5} & 7 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{A27}$$

$$\Delta B^{XE+XE^c} = \begin{pmatrix} \frac{72}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \ \Delta B^{XN+XN^c} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{A28}$$

$$\Delta B^{XY+XY^c} = \begin{pmatrix} \frac{25}{3} & 15 & \frac{80}{3} \\ 5 & 21 & 16 \\ \frac{10}{3} & 6 & \frac{68}{3} \end{pmatrix} \ , \ \Delta B^{XT_i+\overline{XT}_i} = \begin{pmatrix} \frac{18}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ . \tag{A29}$$

APPENDIX B: THE FLIPPED $SU(5) \times U(1)_X$ MODELS WITH VECTOR-LIKE PARTICLES

1. Renormalization Group Equations

The two-loop renormalization group equations for the gauge couplings in the flipped $SU(5) \times U(1)_X$ models [51] in the subsection A in Section II are

$$(4\pi)^2 \frac{d}{dt} g_i = b_i g_i^3 + \frac{1}{(4\pi)^2} \sum_{j=1,5} B_{ij} g_i^3 g_j^2 , \tag{B1}$$

where $i = 1, 5$, and the beta-function coefficients are

$$b = \left(\frac{15}{2}, -5 \right) , \quad B = \begin{pmatrix} \frac{33}{4} & 60 \\ \frac{5}{2} & 82 \end{pmatrix} . \tag{B2}$$

$$\tag{B3}$$

2. Beta Functions for the Vector-Like Particles

First, we present the one-loop beta functions $\Delta b \equiv (\Delta b_1, \Delta b_5)$ as complete supermultiplets from the extra particles

$$\Delta b^{XF + \overline{XF}} = (\frac{1}{2}, 3) , \quad \Delta b^{Xf + \overline{Xf}} = (\frac{9}{4}, 1) , \tag{B4}$$

$$\Delta b^{Xl + \overline{Xl}} = (\frac{5}{4}, 0) , \quad \Delta b^{Xh + \overline{Xh}} = (1, 1) , \tag{B5}$$

$$\Delta b^{XT_i + \overline{XT}_i} = (\frac{5}{4}, 0) . \tag{B6}$$

Second, we present the two-loop beta functions from the vector-like extra particles

$$\Delta B^{XF + \overline{XF}} = \begin{pmatrix} \frac{1}{20} & \frac{36}{5} \\ \frac{3}{10} & \frac{366}{5} \end{pmatrix} , \quad \Delta B^{Xf + \overline{Xf}} = \begin{pmatrix} \frac{81}{40} & \frac{108}{5} \\ \frac{9}{10} & \frac{98}{5} \end{pmatrix} , \tag{B7}$$

$$\Delta b^{Xl + \overline{Xl}} = \begin{pmatrix} \frac{25}{8} & 0 \\ 0 & 0 \end{pmatrix} , \quad \Delta B^{Xh + \overline{Xh}} = \begin{pmatrix} \frac{2}{5} & \frac{48}{5} \\ \frac{2}{5} & \frac{98}{5} \end{pmatrix} , \tag{B8}$$

$$\Delta b^{XT_i + \overline{XT}_i} = \begin{pmatrix} \frac{25}{32} & 0 \\ 0 & 0 \end{pmatrix} . \tag{B9}$$

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