

TYPE II P-BRANES: THE BRANE-SCAN REVISITEDM. J. Duff[†] and J. X. Lu[‡]

Theory Division, CERN, Geneva

and

Center for Theoretical Physics

Physics Department

Texas A&M University

College Station, TX 77843

ABSTRACT

We re-examine the classification of supersymmetric extended objects in the light of the recently discovered Type II p -branes, previously thought not to exist for $p > 1$. We find new points on the brane-scan only in $D = 10$ and then only for $p = 3$ (Type IIB), $p = 4$ (Type IIA), $p = 5$ (Types IIA and IIB) and $p = 6$ (Type IIA). The case $D = 10$, $p = 2$ (Type IIA) also exists but is equivalent to the previously classified $D = 11$ supermembrane.

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1. Introduction

In 1986, Hughes et al [1] discovered a superthreebrane with worldvolume dimension $d = 4$ in $D = 6$ spacetime. Their superthreebrane action was a generalization of the Green-Schwarz action for superstrings in that it exhibited spacetime supersymmetry and worldvolume fermionic κ -symmetry. Shortly afterwards, Bergshoeff, Sezgin and Townsend [2] found corresponding actions for other values of d and D , called super “ p -branes” where $p = d - 1$ is the number of spatial dimensions of the worldvolume. Moreover, Duff, Howe, Inami and Stelle [3] showed how the action for a super $(p - 1)$ -brane in $(D - 1)$ dimensions could be derived from that of a p -brane in D dimensions by the process of simultaneous dimensional reduction. A complete classification of all supersymmetric extended objects, incorporating all of the previous observations, was then attempted by Achucarro, Evans, Townsend and Wiltshire [4]. Their results, which we shall discuss in section 2, can be summarized by the “brane-scan” of Figure 1. According to this classification, Type II p -branes, i.e those with $N = 2$ spacetime supersymmetry, do not exist for $p > 1$.

Recently, however, it has been discovered that, in $D = 10$, super p -branes exist not only for Type IIA and IIB strings ($p = 1$) but also for Type IIA and IIB fivebranes ($p = 5$) [5] and Type IIB threebranes ($p = 3$) [6]. The no-go theorem is circumvented because in addition to the superspace coordinates X^M and θ^α there are also higher spin fields on the world volume: vectors or antisymmetric tensors. This raises the question: are there other Type II super p -branes, and if so, for what p and D ? The purpose of the present paper will be to attempt to classify these new supersymmetric extended objects.

We begin in section 3 by asking what new points on the brane-scan are permitted by bose-fermi matching alone. There are surprisingly few: $p = 3, 4, \dots, 9$ in $D = 10$; $p = 3, 4, 5$ in $D = 6$ and $p = 3$ in $D = 4$. The much harder task is to narrow down these possibilities to objects that actually exist. One obvious handicap is that, unlike the p -branes discussed by Achucarro et al [4], no-one has yet succeeded in writing down the action for these new Type II p -branes. The existence of the $p = 3$ and $p = 5$ objects mentioned above was established indirectly: by showing that they emerge as soliton solutions of either Type IIA or Type IIB supergravity. The nature of the worldvolume fields is then established by studying the zero modes of the soliton. In particular, a super p -brane requires that the

soliton solution preserves some unbroken supersymmetry and hence that the zero modes form a supermultiplet. Although we know of no general proof that all supersymmetric extended objects correspond to a soliton, this is true of all those on the old brane-scan and thus seems a good guide to constructing the new one. Following this route we shall conclude that of all the possible $D = 10$ Type II super p -branes permitted by bose-fermi matching alone, only those with $p = 0$ (Type IIA), $p = 1$ (Type IIA and IIB), $p = 3$ (Type IIB), $p = 4$ (Type IIA) $p = 5$ (Type IIA and IIB) and $p = 6$ (Type IIA) actually exist. [The reader may wonder why there seems to be a gap at $p = 2$. Indeed, duality would seem to demand that in $D = 10$ a Type IIA superfourbrane should imply a Type IIA supermembrane. This object does indeed exist but it should not be counted as a new theory since vectors are dual to scalars in $d = 3$ and so its worldvolume action is simply obtained by dualizing one of the 11 X^M of the $D = 11$ supermembrane.] Our results thus confirm the conjecture of Horowitz and Strominger [8] that super Type II p -brane solitons in $D = 10$ exist for all $0 \leq p \leq 6$. Moreover, we find that no new fundamental Type II p -branes emerge in $D < 10$.

2. The old brane-scan

As the p -brane moves through spacetime, its trajectory is described by the functions $X^M(\xi)$ where X^M are the spacetime coordinates ($M = 0, 1, \dots, D - 1$) and ξ^i are the worldvolume coordinates ($i = 0, 1, \dots, d - 1$). It is often convenient to make the so-called “static gauge choice” by making the $D = d + (D - d)$ split

$$X^M(\xi) = (X^\mu(\xi), Y^m(\xi)) \tag{2.1}$$

where $\mu = 0, 1, \dots, d - 1$ and $m = d, \dots, D - 1$, and then setting

$$X^\mu(\xi) = \xi^\mu \tag{2.2}$$

Thus the only physical worldvolume degrees of freedom are given by the $(D - d)$ $Y^m(\xi)$. So the number of on-shell bosonic degrees of freedom is

$$N_B = D - d \tag{2.3}$$

To describe the super p -brane we augment the D bosonic coordinates $X^M(\xi)$ with anticommuting fermionic coordinates $\theta^\alpha(\xi)$. Depending on D , this spinor could be Dirac, Weyl, Majorana or Majorana-Weyl. The fermionic κ -symmetry means that half of the spinor degrees of freedom are redundant and may be eliminated by a physical gauge choice. The net result is that the theory exhibits a *d-dimensional worldvolume supersymmetry* where the number of fermionic generators is exactly half of the generators in the original spacetime supersymmetry. This partial breaking of supersymmetry is a key idea. Let M be the number of real components of the minimal spinor and N the number of supersymmetries in D spacetime dimensions and let m and n be the corresponding quantities in d worldvolume dimensions. Let us first consider $d > 2$. Since κ -symmetry always halves the number of fermionic degrees of freedom and going on-shell halves it again, the number of on-shell fermionic degrees of freedom is

$$N_F = \frac{1}{2} mn = \frac{1}{4} MN \quad (2.4)$$

Worldvolume supersymmetry demands $N_B = N_F$ and hence

$$D - d = \frac{1}{2} mn = \frac{1}{4} MN \quad (2.5)$$

A list of dimensions, number of real components of the minimal spinor and possible supersymmetries is given in Table 1, from which we see that there are only 8 solutions to (2.5) all with $N = 1$, as shown in Fig. 1. We note in particular that $D_{\max} = 11$ since $M \geq 64$ for $D \geq 12$ and hence (2.5) cannot be satisfied. Similarly $d_{\max} = 6$ since $m \geq 16$ for $d \geq 7$. The case $d = 2$ is special because of the ability to treat left and right moving modes independently. If we require the sum of both left and right moving bosons and fermions to be equal, then we again find the condition (2.5). This provides a further 4 solutions all with $N = 2$, corresponding to Type II superstrings in $D = 3, 4, 6$ and 10 (or 8 solutions in all if we treat Type IIA and Type IIB separately. The gauge-fixed Type IIB superstring will display (8, 8) supersymmetry on the worldsheet and the Type IIA will display (16, 0), the opposite [5] of what one might naively expect). If we require only left (or right) matching, then (2.5) is replaced by

$$D - 2 = n = \frac{1}{2} MN \tag{2.6}$$

which allows another 4 solutions in $D = 3, 4, 6$ and 10, all with $N = 1$. The gauge-fixed theory will display (8,0) worldsheet supersymmetry. The heterotic string falls into this category. The results are also shown in Fig. 1.

An equivalent way to arrive at the above conclusions is to list all scalar supermultiplets in $d \geq 2$ dimensions and to interpret the dimension of the target space, D , by

$$D - d = \text{number of scalars} \tag{2.7}$$

A useful reference is Strathdee [9] who provides an exhaustive classification of all unitary representations of supersymmetry with maximum spin 2. In particular, we can understand $d_{\text{max}} = 6$ from this point of view since this is the upper limit for scalar supermultiplets. In summary, according to the above classification, Type II p -branes do not exist for $p > 1$.

Dimension (D or d)	Minimal Spinor (M or m)	Supersymmetry (N or n)
11	32	1
10	16	2, 1
9	16	2, 1
8	16	2, 1
7	16	2, 1
6	8	4, 3, 2, 1
5	8	4, 3, 2, 1
4	4	8, ..., 1
3	2	16, ..., 1
2	1	32, ..., 1

Table 1. Minimal spinor components and supersymmetries.

3. Bose-fermi matching: a necessary condition

Given that the gauge-fixed theories display worldvolume supersymmetry, and given that we now wish to include the possibility of vector (and/or antisymmetric tensor) fields, it is a relatively straightforward exercise to repeat the bose-fermi matching conditions for vector (and/or antisymmetric tensor) supermultiplets. Once again, we may proceed in one of two ways. First, given that a worldvolume vector has $(d - 2)$ degrees of freedom, the scalar multiplet condition (2.5) gets replaced by

$$D - 2 = \frac{1}{2} mn = \frac{1}{4} MN \quad (3.1)$$

Alternatively, we may simply list all the supermultiplets in Strathdee's classification and once again interpret D via (2.7). The results are shown in Fig. 2.

Several comments are now in order:

- 1) Vector supermultiplets exist only for $4 \leq d \leq 10$ [9]. In $d = 3$ vectors have only 1 degree of freedom and are dual to scalars. So these multiplets will already have been included as scalar multiplets in section 2. In $d = 2$, vectors have no degrees of freedom.
- 2) The number of scalars in a vector supermultiplet is such that, from (2.7), $D = 4, 6$ or 10 only, in accordance with (3.1).
- 3) Repeating the analysis for antisymmetric tensors does not introduce any new points on the scan. For example in $d = 6$ there is a chiral $(2, 0)$ tensor supermultiplet, with a second rank tensor whose field strength is self dual: $(B_{\mu\nu}^-, \lambda^I, \phi^{[IJ]})$, $I = 1, \dots, 4$, corresponding to the Type IIA fivebrane and a non-chiral $(1, 1)$ vector multiplet $(B_\mu, \chi^I, A^J_I, \xi)$, $I = 1, 2$, corresponding to the Type IIB fivebrane [5]. Both occupy the $(d = 6, D = 10)$ slot in Fig. 2.
- 4) We emphasize that Fig. 2 merely tells us what is allowed by bose/fermi matching. We must now try to establish which of these possibilities actually exists.

4. p -brane solutions

All of the circles on the brane-scan are known to correspond to soliton solutions of an underlying supersymmetric field theory [1, 7, 10, 11, 12, 13, 14]. As for the crosses,

supersymmetric soliton solutions of both Type IIA and Type IIB supergravity have been found for the case $(d = 6, D = 10)$ [5] and of Type IIB for $(d = 4, D = 10)$ [6]. What about the others? In this section we shall exhibit the solutions and then in section 5 ask whether they are supersymmetric.

To this end, consider the following generic $D = 10$ action

$$I_{10}(d) = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2(d+1)!} e^{-\alpha\phi} F_{d+1}^2 \right) \quad (4.1)$$

This describes the interaction of an antisymmetric tensor potential of rank d , $A_{M_1 M_2 \dots M_d}$ ($M = 0, 1, \dots, 9$), interacting with gravity g_{MN} , and the dilaton ϕ , where the rank $(d+1)$ field strength F_{d+1} is given by

$$F_{d+1} = dA_d \quad (4.2)$$

and the constant α is given by

$$\alpha = \frac{(4-d)}{2} (-1)^d. \quad (4.3)$$

To solve the corresponding field equations, we follow [12, 13] and make an ansatz corresponding to the most general $d/(10-d)$ split invariant under $P_d \times SO(10-d)$ where P_d is the d -dimensional Poincaré group. (The black $(d-1)$ branes discussed in section 5 exhibit P_d invariance only in the mass = charge limit [8], so this ansatz will automatically single out these extreme cases.) We split the indices

$$x^M = (x^\mu, y^m) \quad (4.4)$$

where $\mu = 0, 1, \dots, (d-1)$ and $m = d, d+1, \dots, 9$ and write the line-element as

$$ds^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B} \delta_{mn} dy^m dy^n \quad (4.5)$$

and the d -form gauge field as

$$A_{01\dots d} = -e^C \quad (4.6)$$

All other components of $A_{M_1\dots M_d}$ are set to zero. P_d invariance then requires that the arbitrary functions A, B, C depend only on y^m ; $SO(10-d)$ invariance requires that this dependence be only through $r = \sqrt{\delta_{mn}y^m y^n}$. Similarly our ansatz for the dilaton is

$$\phi = \phi(r) \tag{4.7}$$

Substituting these ansätze into the field equations leads to the following solutions, assuming that g_{MN} tends asymptotically to η_{MN} :

$$\begin{aligned} A &= \frac{\tilde{d}}{16} C \\ B &= \frac{-d}{16} C \\ \phi &= \frac{\alpha}{2} C \end{aligned} \tag{4.8}$$

where, for simplicity, we have set the vev of the dilaton equal to zero. C is given by

$$e^{-C} = 1 + \frac{k_d}{r^{\tilde{d}}} \quad \tilde{d} > 0 \tag{4.9}$$

where k_d is a constant. Here we have introduced \tilde{d} , the dimension of the extended object dual to the $(d-1)$ -brane in $D=10$,

$$\tilde{d} = 8 - d \tag{4.10}$$

We shall refer to these solutions as “elementary $(d-1)$ -branes”. They are characterized by a non-vanishing electric Noether charge

$$e_d = \frac{1}{\sqrt{2\kappa}} \int_{S^{\tilde{d}+1}} e^{-\alpha\phi} *F = \frac{1}{\sqrt{2\kappa}} \tilde{d}\Omega_{\tilde{d}+1} k_d \tag{4.11}$$

where $S^{\tilde{d}+1}$ is the $(\tilde{d}+1)$ sphere surrounding the elementary $(d-1)$ brane, and $\Omega_{\tilde{d}+1}$ is volume of the unit $S^{\tilde{d}+1}$. Strictly speaking, these configurations display $\delta(r)$ singularities and fail to solve the field equations at $r=0$ unless we augment the action (4.1) by the action for the $(d-1)$ -brane source. We need not dwell on this here; a full discussion may be found in [15].

However, we can also find non-singular “solitonic $(\tilde{d} - 1)$ -brane” solutions which are characterized by a non-vanishing topological magnetic charge

$$g_{\tilde{d}} = \frac{1}{\sqrt{2}\kappa} \int_{S^{d+1}} F \quad (4.12)$$

satisfying the Dirac quantization condition

$$e_d g_{\tilde{d}} = 2\pi n \quad n = \text{integer} \quad (4.13)$$

To obtain these solutions we now make an ansatz invariant under $P_{\tilde{d}} \times SO(10 - \tilde{d})$. Hence we write (4.4) and (4.5) as before where now $\mu = 0, 1 \dots (\tilde{d} - 1)$ and $m = \tilde{d}, \tilde{d} + 1, \dots 9$. The ansatz for the antisymmetric tensor, however, will be made on the field strength rather than the potential.

$$\frac{1}{\sqrt{2}\kappa} F_{d+1} = g_{\tilde{d}} \varepsilon_{d+1} / \Omega_{d+1} \quad (4.14)$$

where ε_{d+1} is the volume form on S^{d+1} . Since this is a harmonic form, F can no longer be written globally as the curl of A , but it satisfies the Bianchi identities. It is now not difficult to show that all the fields equations are satisfied simply by making the replacements $d \rightarrow \tilde{d}$ and hence $\alpha(d) \rightarrow \alpha(\tilde{d}) = -\alpha(d)$ in (4.8-10).

One may now consider the theory “dual” to (4.1) for which the roles of antisymmetric tensor field equations and Bianchi identities (and hence electric and magnetic charges) are interchanged. The action is given by

$$\tilde{I}_{10}(\tilde{d}) = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2(\tilde{d} + 1)!} e^{\alpha\phi} \tilde{F}_{\tilde{d}+1}^2 \right) \quad (4.15)$$

where the rank $(\tilde{d} + 1)$ field strength \tilde{F} is the curl of a \tilde{d} -form potential

$$\tilde{F}_{\tilde{d}+1} = d\tilde{A}_{\tilde{d}} \quad (4.16)$$

and is related to F via

$$\tilde{F}_{\tilde{d}+1} = e^{-\alpha\phi} F_{d+1} \quad (4.17)$$

α is the same constant appearing in (4.1) but occurs with the opposite sign. It should be clear that the system $\tilde{I}_{10}(\tilde{d})$ admits the same elementary and solitonic solutions as $I_{10}(d)$ provided we everywhere make the replacement $d \rightarrow \tilde{d}$ and hence $\alpha(d) \rightarrow \alpha(\tilde{d}) = -\alpha(d)$. In the dual theory, therefore, the roles of elementary and solitonic solutions are interchanged.

Now let us return to the question of supersymmetry. First of all, the generic action (4.1) correctly describes the bosonic sector of the 3-form field strength version of $N = 1$, $D = 10$ supergravity, the field theory limit of the superstring. We simply set $d = 2$ and hence $\tilde{d} = 6$ and $\alpha = 1$. The resulting elementary solution (4.8-9) is the Dabholkar et al string [10] and the soliton solution is the Duff-Lu fivebrane [13]. The action (4.1) also describes the bosonic sector of the 7-form field strength version of $N = 1$, $D = 10$ supergravity. We simply set $d = 6$ and hence $\tilde{d} = 2$ and $\alpha = -1$. The resulting elementary solution (4.8-9) is the Duff-Lu fivebrane [13], and the soliton solution is the Dabholkar et al [10] string. As shown in [10,13], both the string and the fivebrane break one half of the spacetime supersymmetries.

d	\tilde{d}	$\alpha(d)$	A	B	ϕ
1	7	$-3/2$	$7C/16$	$-C/16$	$-3C/4$
2	6	1	$3C/8$	$-C/8$	$C/2$
3	5	$-1/2$	$5C/16$	$-3C/16$	$-C/4$
4	4	0	$C/4$	$-C/4$	0
5	3	$1/2$	$3C/16$	$-5C/16$	$C/4$
6	2	-1	$C/8$	$-3C/8$	$-C/2$
7	1	$3/2$	$C/16$	$-7/16$	$3C/4$

Table 2. The functions A, B and ϕ in terms of C as demanded by supersymmetry.

Now let us turn to $D = 10$ Type IIA supergravity, whose bosonic action is given by

$$\begin{aligned}
I_{10}(IIA) = & \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2.3!} e^{-\phi} F_3^2 \right. \\
& \left. - \frac{1}{2.2!} e^{3\phi/2} F_2^2 - \frac{1}{2.4!} e^{\phi/2} F_4'^2 \right] \\
& - \frac{1}{8\kappa^2} \int F_4 \wedge F_4 \wedge A_2
\end{aligned} \tag{4.18}$$

where

$$F_4' = dA_3 + \kappa^{-1} A_1 \wedge F_3 \tag{4.19}$$

From (4.3) we see that the kinetic terms for gravity, dilaton and antisymmetric tensors are also correctly described by the generic action $I_{10}(d)$ with $d = 1, 2, 3$ (i.e $\tilde{d} = 7, 6, 5$). Both the elementary string ($d = 2$) and fivebrane ($d = 6$) solutions of $N = 1$ supergravity described above continue to provide solutions to Type IIA supergravity, as may be seen by setting $F_2 = F_4 = 0$. [This observation is not as obvious as it may seem in the case of the elementary fivebranes or solitonic strings, however, since it assumes that one may dualize F_3 . Now the Type IIA action follows by dimensional reduction from the action of $D = 11$ supergravity which contains F_4 . There exists no dual of this action in which F_4 is replaced by F_7 essentially because A_3 appears explicitly in the Chern-Simons term $F_4 \wedge F_4 \wedge A_3$ [16]. Since F_4 and F_3 in $D = 10$ originate from F_4 in $D = 11$, this means that we cannot *simultaneously* dualize F_3 and F_4 but one may do either *separately*.[†] By partial integration one may choose to have no explicit A_3 dependence in the Chern-Simons term of (4.18) or no explicit A_2 dependence, but not both.] Furthermore, by setting $F_2 = F_3 = 0$ we find elementary membrane ($d = 3$) and solitonic fourbrane ($\tilde{d} = 5$) solutions, and then by dualizing F_4 , elementary fourbrane ($d = 5$) and solitonic membrane ($\tilde{d} = 3$) solutions. Finally, by setting $F_3 = F_4 = 0$, we find elementary particle ($d = 1$) and solitonic sixbrane ($\tilde{d} = 7$) solutions and then by dualizing F_2 , elementary sixbrane ($d = 7$) and solitonic particle ($\tilde{d} = 1$) solutions.

Next we consider Type IIB supergravity in $D = 10$ whose bosonic sector consists of the graviton g_{MN} , a complex scalar ϕ , a complex 2-form A_2 (i.e with $d = 2$ or, by duality

[†] We are grateful to H. Nishino for this observation.

$d = 6$) and a real 4-form A_4 (i.e with $d = 4$ which in $D = 10$ is self-dual). Because of this self-duality of the 5-form field strength F_5 , there exists no covariant action principle of the kind (4.15) and, strictly speaking, our previous analysis ceases to apply. Nevertheless we can apply the same logic to the equations of motion and we find that the solution again falls into the generic category (4.8-9). First of all, by truncation it is easy to see that the same string ($d = 2$) and fivebrane ($d = 6$) solutions of $N = 1$ supergravity continue to solve the field equations of Type IIB. On the other hand, if we set to zero F_3 and solve the self-duality condition $F_5 = -*F_5$ then we find the special case of (4.8) with $d = \tilde{d} = 4$ and hence $\alpha = 0$ and $\phi = 0$. This is the self-dual superthreebrane [6].

All of the above elementary solutions saturate a Bogomoln'yi bound between the mass per unit p -volume \mathcal{M}_d and the electric charge

$$\mathcal{M}_d = \frac{1}{\sqrt{2}} |e_d| \quad (4.20)$$

It follows that the solitonic solutions obey

$$\mathcal{M}_{\tilde{d}} = \frac{1}{\sqrt{2}} |g_{\tilde{d}}| \quad (4.21)$$

We shall refer to these equations as the “mass = charge” conditions.

5. Supersymmetry

Horowitz and Strominger [8] have exhibited a two-parameter family of solutions of $D = 10$ Type IIA and Type IIB supergravity with event horizons for $d = 1, 2, 3, 4, 5, 6, 7$: the “black p -branes”. In some respects, these solutions resemble the Reissner-Nordstrom black-hole solutions of general relativity which are known to admit unbroken supersymmetry in the extreme mass = charge limit. Horowitz and Strominger then conjectured that, in this limit, their black p -branes would also be supersymmetric and hence that there exist Type II super $(d - 1)$ branes for all these values of d . As we shall now demonstrate, this is indeed the case.

We begin by making the same ansatz as in section 4, namely (4.5-7) but this time substitute into the supersymmetry transformation rules rather than the field equations,

and demand unbroken supersymmetry. This reduces the four unknown functions A, B, C and ϕ to one. We then compare the results with the known solutions.

For Type IIA supergravity with vanishing fermion background, the gravitino transformation rule is

$$\begin{aligned}\delta\psi_M &= D_M\varepsilon + \frac{1}{64} e^{3\phi/4}(\Gamma_M{}^{M_1M_2} - 14\delta_M{}^{M_1}\Gamma^{M_2})\Gamma^{11}\varepsilon F_{M_1M_2} \\ &+ \frac{1}{96} e^{-\phi/2}(\Gamma_M{}^{M_1M_2M_3} - 9\delta_M{}^{M_1}\Gamma^{M_2M_3})\Gamma^{11}\varepsilon F_{M_1M_2M_3} \\ &+ \frac{i}{256} e^{\phi/4}(\Gamma_M{}^{M_1M_2M_3M_4} - \frac{20}{3}\delta_M{}^{M_1}\Gamma^{M_2M_3M_4})\varepsilon F_{M_1M_2M_3M_4}\end{aligned}\quad (5.1)$$

and the dilatino rule is

$$\begin{aligned}\delta\lambda &= \frac{1}{4}\sqrt{2}D_M\phi\Gamma^M\Gamma^{11}\varepsilon + \frac{3}{16}\frac{1}{\sqrt{2}}e^{3\phi/4}\Gamma^{M_1M_2}\varepsilon F_{M_1M_2} \\ &+ \frac{1}{24}\frac{i}{\sqrt{2}}e^{-\phi/2}\Gamma^{M_1M_2M_3}\varepsilon F_{M_1M_2M_3} \\ &- \frac{1}{192}\frac{i}{\sqrt{2}}e^{\phi/4}\Gamma^{M_1M_2M_3M_4}\varepsilon F_{M_1M_2M_3M_4}\end{aligned}\quad (5.2)$$

where Γ^M are the $D = 10$ Dirac matrices, where the covariant derivative is given by

$$D_M = \partial_M + \frac{1}{4}\omega_{MAB}\Gamma^{AB}\quad (5.3)$$

with ω_{MAB} the Lorentz spin connection, where

$$\Gamma^{M_1M_2\dots M_n} = \Gamma^{[M_1}\Gamma^{M_2}\dots\Gamma^{M_n]}\quad (5.4)$$

and where

$$\Gamma^{11} = i\Gamma^0\Gamma^1\dots\Gamma^9\quad (5.5)$$

Similarly the Type IIB rules are

$$\begin{aligned}\delta\psi_M &= D_M\varepsilon + \frac{i}{4\times 480}\Gamma^{M_1M_2M_3M_4}\Gamma_M\varepsilon F_{M_1M_2M_3M_4} \\ &+ \frac{1}{96}(\Gamma_M{}^{M_1M_2M_3} - 9\delta_M{}^{M_1}\Gamma^{M_2M_3})\varepsilon^* F_{M_1M_2M_3}\end{aligned}\quad (5.6)$$

and

$$\delta\lambda = i\Gamma^M \varepsilon^* P_M - \frac{1}{24} i\Gamma^{M_1 M_2 M_3} \varepsilon F_{M_1 M_2 M_3}$$

where

$$P_M = \partial_M \phi / (1 - \phi^* \phi). \quad (5.7)$$

In the Type IIB case, ε is chiral

$$\Gamma_{11} \varepsilon = \varepsilon \quad (5.8)$$

The requirement of unbroken supersymmetry is that there exist Killing spinors ε for which both $\delta\psi_M$ and $\delta\lambda$ vanish. Substituting our ansatz into the transformation rules we find that for every $1 \leq d \leq 7$ there exist field configurations which break exactly half the supersymmetries. This is just what one expects for supersymmetric extended object solutions [1,7,10,11,12,13] and is intimately related to the κ -symmetry discussed in section 2 and the Bogomoln'yi bounds of section 4. The corresponding values of A, B and ϕ in terms of C are given in Table 2. The important observation, from (4.8), is that the values required by supersymmetry also solve the field equations. Thus in addition to the $D = 10$ super $(d - 1)$ branes already known to exist for $d = 2$ (Heterotic, Type IIA and Type IIB), $d = 4$ (Type IIB only) and $d = 6$ (Heterotic, Type IIA and Type IIB), we have established the existence of a Type IIA superparticle ($d = 1$), a Type IIA supermembrane ($d = 3$), a Type IIA superfourbrane ($d = 5$) and a Type IIA supersixbrane ($d = 7$).

One may now repeat the $D = 10$ analysis of sections 4 and 5 for $N = 2$ supergravities in $D < 10$. Here we simply state the results. Details will be discussed elsewhere [17]. We find supersymmetric solitons for all $1 \leq \tilde{d} \leq 7$ where $\tilde{d} = D - 2 - d$, as shown in Fig. 3. At first sight, this seems to contradict Fig. 2 since solutions appear where no supermultiplet is allowed. The resolution is simply that only the cases $d = 1, 3, 4, 5, 6$ and 7 in $D = 10$ are fundamental. All the others are obtained by simply dimensional reduction of these or the $D = 11$ supermembrane, and are thus described by the same gauge-fixed action. In

summary the new brane-scan including (fundamental) Type II super p -branes is given in Fig. 4.

6. Conclusions

We have classified all supersymmetric extended objects that correspond to solitons of a Poincaré supersymmetric field theory in the usual spacetime signature which break half the spacetime supersymmetries, as shown in Fig. 4. We cannot at the present time rigorously rule out the existence of other super p -branes, denoted by the points in Fig. 2 not appearing in Fig. 4, which do not correspond to solitons. However, we regard their existence as unlikely. (Nor can we rule out the possibility of other super p -branes described by non-Poincaré supersymmetries in other signatures as discussed in [18] e.g a (2,2) worldvolume in a (10,2) spacetime). Further progress would require that we construct the spacetime Green-Schwarz supersymmetric and κ -symmetric actions for these new Type II p -branes and, to date, this has not been done. All we know is that, in a physical gauge, the worldvolume theory corresponding to the zero modes of the soliton is described by vector or antisymmetric tensor supermultiplet as in Table 3.

$d = 7$	Type IIA	$(A_\mu, \lambda, 3\phi)$		$n = 1$
$d = 6$	Type IIA	$(B_{\mu\nu}^-, \lambda^I, \phi^{[IJ]})$	$I = 1, \dots, 4$	$(n_+, n_-) = (2, 0)$
	Type IIB	$(B_\mu, \chi^I, A^I{}_J, \xi)$	$I = 1, 2$	$(n_+, n_-) = (1, 1)$
$d = 5$	Type IIA	$(A_\mu, \lambda^I, \phi^{[IJ]})$	$I = 1, \dots, 4$	$n = 2$
$d = 4$	Type IIB	$(B_\mu, \chi^I, \phi^{[IJ]})$	$I = 1, \dots, 4$	$n = 4$
$d = 3$	Type IIA	(χ^I, ϕ^I)	$I = 1, \dots, 8$	$n = 8$
$d = 2$	Type IIA	(λ_L^I, ϕ_L^I)	$I = 1, \dots, 16$	$(n_+, n_-) = (16, 0)$
	Type IIB	$(\chi_L^I, \phi_L^I), (\chi_R^I, \phi_R^I)$	$I = 1, \dots, 8$	$(n_+, n_-) = (8, 8)$

Table 3: Gauge-fixed theories on the worldvolume, corresponding to the zero modes of the soliton, are described by the above supermultiplets.

The case of the Type IIA membrane in $D = 10$ is particularly interesting. It emerges as an elementary solution of the usual formulation of Type IIA supergravity with a 4-form field strength or else as a soliton solution of the dual formulation with a 6-form field

strength. Indeed, this solution is dual to the $D = 10$ superfourbrane solution. However, its zero-modes are on-shell equivalent to those of the $D = 11$ supermembrane and so does not occupy a separate slot on the brane-scan of Fig. 3. The reason is because in $d = 3$ the worldvolume vector has only 1 degree of freedom and is dual (in the three dimensional sense) to a scalar. Indeed, this provides the exception to the rule that we do not have an explicit expression for the Type II p -brane Green-Schwarz actions for $p > 1$. Its action is obtained by making a $10 + 1$ split of the Green-Schwarz action for the $D = 11$ supermembrane coordinates $\hat{X}^{\hat{M}} = (X^M, X^{10})$ where $\hat{M} = 0, 1, \dots, 10$ and $M = 0, 1, \dots, 9$, and then dualizing the X^{10} . Perhaps the most bizarre aspect of all this is that an object living in eleven dimensions should emerge as a soliton of a ten dimensional theory!

Of course, one might ask why the $D = 7$ membrane occupies a separate slot since it too can be viewed as a spacetime dimensional reduction of the $D = 11$ membrane. The answer is that when we reach $D = 7$ the multiplet becomes reducible and we can thus perform a consistent truncation to a smaller theory with half the supersymmetries. Similar remarks apply to the membranes in $D = 5$ and 4, and indeed to all the circles appearing on the brane-scan.

In our classification, we have also omitted supersymmetric solitons which break *more than half* the supersymmetries since these solutions presumably admit no κ -symmetric Green-Schwarz action (at least, not of the kind presently known). Examples of this are provided by the $D = 10$ octonionic string of Harvey and Strominger [19], (which breaks $7/8$), and the $D = 11$ extreme black fourbrane and extreme black sixbrane of Gven [20] (which break $3/4$ and $7/8$, respectively).

Finally, we ask what are the implications of our results for the idea of “duality”, in the sense that one theory is simply providing a dual description of the same physics of another theory [21] with the weak-coupling regime of one being the strong-coupling of the other [11]? At the classical level discussed in this paper, we see that supersymmetry has narrowed down the possibilities to just four, all in $D = 10$, namely particle/sixbrane duality (Type IIA only), string/fivebrane duality (Heterotic, Type IIA or Type IIB), membrane/fourbrane duality (Type IIA only) and threebrane self-duality (Type IIB only). The implications for quantum duality will be discussed elsewhere.

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