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# Assessment of 2-D resistivity structures using 1-D inversion

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## ABSTRACT

Schlumberger and Wenner array resistivity soundings over 2-D resistivity structures are interpreted using apparent resistivity pseudosections and cross-sections constructed from 1-D inversions in order to determine the effectiveness of 1-D interpretations over such structures. Cross-sections contoured from resistivities of inverted "layers" show distinct differences from the apparent resistivity pseudosections and may be used as interpretational aids. Contour lines in the cross-sections locate the horizontal interfaces of the 2-D structures quite well. The vertically oriented segments of the cross-section contours are relatively undistorted in the inversion process and are similar to the vertically oriented portions of contours in the apparent resistivity pseudosection. A simple, empirically determined formula is used to separate the sections into resistive and conductive zones and helps to define the geometry of the anomaly. In order to apply the formula, it is necessary to know whether the target is a relative conductor or a relative resistor. Except for the case of a square prism, the Schlumberger array appears to hold advantages over the Wenner in qualitatively assessing an anomaly. The primary drawback of the Wenner array is that its expanding potential electrodes create false anomalous zones and complicate interpretation. As might be expected, structures with long horizontal interfaces, i.e. those more nearly 1-D, yield the most accurate interpretations.


## INTRODUCTION

Geophysical data are often interpreted using a horizontally layered earth model (1-D model) when a 2-D or a 3-D approach might better approximate the local subsurface geometry, particularly in areas where horizontal layering does not predominate. It is important to understand how and to what extent 1-D approximations represent the earth's true subsurface structure. In this paper, we examine the effect of 1-D inversion of resistivity sounding data from simple models of 2-D subsurface resistivity structures. The structures modeled are a vertical dike, a buried fault block, a tabular prism, a square prism, and a ramp, each infinite along one horizontal axis. All computations were performed on a VAX 11/780 computer. The forward model results were generated using a 2-D finite-difference program. Five to seven different soundings were computed at various locations over each structure. These soundings were then assumed to have been taken from a horizontally layered earth, and were interpreted using an iterative least-squares inversion approach described by Johanssen (1977). The forward program for the 1-D inversion procedure used the linear digital filters of Ghosh (1971), Johanssen (1975), and O'Neill (1975). Contoured cross-sections for Wenner and Schlumberger arrays were compared based on: (1) the apparent resistivity sounding data and (2) the resistivities of the layers predicted from inversion. We found that the Schlumberger array was usually superior to the Wenner array in analyzing 2-D resistivity structures of low to moderate (2:1 to 10:1) resistivity contrast. The square prism model was an important exception. We found that certain 2-D structures lend themselves more readily to interpretation using 1-D inversion than do others. We developed rules of thumb that could serve as guidelines in field investigations. Since a growing number of companies use resistivity as a tool in environmental, engineering, and groundwater investigations, such guidelines are vital, especially since small companies may not have the computational resources necessary for 2-D or 3-D inversion.

We recognize that in a 2-D or 3-D environment it is best to use interpretational tools which assume 2-D or 3-D models. We do not advocate the use of 1-D interpretation in a 2-D or 3-D environment. Even so, it is a fact that such analyses are performed, either inadvertently, by necessity, or by choice, so it is important to assess the extent to which such an analysis can be trusted and to understand the key factors affecting such an interpretation.

## PROCEDURE

The resistivity sounding data were generated by a 2-D foward modeling program, RESIS2D, developed by Dey and Morrison (1979). The program uses a finite-difference approach to solve

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for the potential distribution due to point sources of current, and the potential distribution is converted into apparent resistivity values. The grid used by Dey and Morrison consists of 113 horizontal nodes and 16 vertical nodes. The grid coarsens away from the center of the air-earth interface, allowing for accurate results with fewer nodes. Each node on the grid could be assigned a particular resistivity. We used the 113 × 16 grid for our vertical dike and buried fault block models, but expanded the grid to 173 × 20 nodes for the other models. RESIS2D was tested for the case of a two-layer earth for both Wenner and Schlumberger arrays, and we found it yielded apparent resistivities to within 5 percent of our own layered forward modeling programs. The 5 percent error appeared to be systematic, so we implemented a small correction factor that randomized the error and reduced it by half. This proved sufficient for our purposes. A more elaborate approach to correcting mesh errors in the finite difference method is described in Lowry et al. (1989). We also tested RESIS2D for scale change effects. Scale changes usually influenced the apparent resistivity values less than 1 percent though in some cases up to 3 percent. Scaling was necessary to obtain the appropriate number of sounding data for interpretation and inversion.

After establishing the reliability of the 2-D forward program, we proceeded to create sounding data sets for Wenner and Schlumberger arrays. With each model, a set of five to seven sounding curves were generated. For soundings near the anomalous zone, 15 to 25 apparent resistivity values were computed. For soundings farther away, 10 to 15 values were sufficient. The model was scaled up or down as necessary to build a data set over 2.5 to 3 decades.

Once a sounding set was generated, these data had to be interpolated to yield apparent resistivity values for specific arrays. This was achieved using Lagrange's interpolation formula (Abramowitz and Stegun, 1970, p. 878).

After the interpolated apparent resistivity values were obtained, the data were input into a 1-D inverse program. An initial guess was made as to the number of "layers" constituting the sounding data, as well as each layer's thickness and resistivity. The program was then allowed to iterate to a solution. If too few layers were chosen, a large rms error would result. If an unnecessarily large number of layers was chosen, the thickness of one or more layers approached zero or the resistivity of consecutive layers was made approximately the same. If the appropriate choice was still not obvious, a resolution matrix (Wiggins, 1972) was employed to aid in choosing the appropriate number of layers. If the diagonal terms of the resolution matrix were all near 1 and the off-diagonal terms were near zero, good resolution of all parameters was assumed to have been obtained.

Once a layered interpretation had been obtained for each of the soundings, the apparent resistivities were entered on a grid and contoured to create a cross-section. A pseudosection was created by contouring the apparent resistivity data generated from RESIS2D, and the sections were compared. Often additional information regarding the anomaly's geometry could be obtained from the inverted cross-section, particularly the locations of horizontal interfaces. (Note that the term "pseudosection" has been reserved for contours of apparent resistivity data taken from resistivity soundings, in keeping with ordinary usage of the term. We use the term cross-section to describe resistivities which have been contoured after 1-D inversion.)

To rely only on apparent resistivity pseudosections or inverted cross-sections to estimate the extent and geometry of an anomaly can often be misleading. In an effort to better establish an anomaly's true shape, we attempted to find a simple method that would use the contours to predict the anomaly's geometry. We were only partially successful, but we feel the method employed could aid in field investigations, provided its limitations are understood. Our method aimed at determining a resistivity cutoff value on the pseudosection or contoured cross-section. Any resistivities less than the cutoff value were shaded as relative conductors. All resistivities greater than the cutoff were left unshaded and considered to be relatively resistive. To establish the resistivity cutoff, the highest and lowest resistivity values were located on the pseudosection. Denote the logs of these values $x_h$ and $x_l$ respectively. Let $\Delta x = (x_h - x_l)/3$. If the anomalous body was judged to be resistive, the cutoff resistivity was calculated by

$$\rho_c = 10^{x_h - \Delta x}. \tag{1}$$

If the anomaly was judged conductive, $\rho_c$ was given by

$$\rho_c = 10^{x_\ell + \Delta x}. \tag{2}$$

Shading resistivity values defined by $\rho_c$ had the effect of approximately delineating the anomaly's geometry in most cases, particularly if the Schlumberger array was used.

## MODELS

Figure 1 illustrates the five 2-D geometries considered: (1) a buried, vertically faulted block, (2) a vertical dike, (3) a tabular prism, (4) a square prism, and (5) a ramp. In the first four models, both the Schlumberger and Wenner arrays were examined. In the fifth case, only the Schlumberger array was considered.

### Buried fault block

Figure 1a shows the buried fault block. The block's vertical edge is located at zero on the $x$-axis and is assumed to extend downward to infinity. The horizontal interface is 10 units deep. The block is resistive (1000 ohm-m) with respect to the surrounding unit (200 ohm-m).

Figure 2a shows the results of five separate Schlumberger soundings after 1-D inversion. Note that each of the soundings inverted to two layers. In the two soundings centered directly over the block ($x = -40, -80$), the horizontal interface is predicted near a depth of 10 units. The upper layer resistivity in each case is near the true value of 200 ohm-m. The lower layer shows a somewhat lower resistivity (688 ohm-m, 680 ohm-m) than the true value due to the influence of the 200 ohm-m material to the right of the block. The $x = 0$ sounding follows a similar pattern. The $x = 40$ and $x = 80$ soundings invert to two layers with the lower layer showing a higher than true resistivity due to the influence of the resistive block.

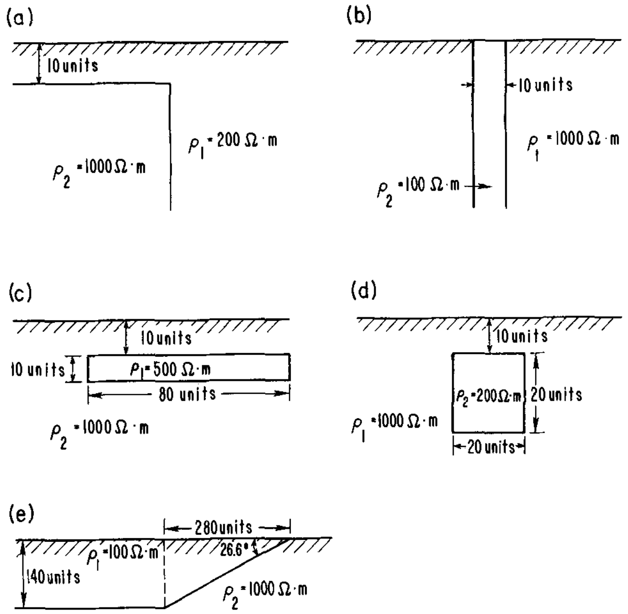


FIG. 1. 2-D models from which Schlumberger and Wenner array resistivity soundings were computed. (a) Resistive, vertically faulted block. (b) Conductive vertical dike extending to the earth's surface. (c) Conductive tabular prism. (d) Conductive square prism. (e) Ramp structure. All structures are infinite along strike.

The inverted results of five Wenner soundings are shown in Figure 2b. The inverted Wenner data results in a two-layer pattern very similar to that of the Schlumberger soundings, except for the $x = 0$ sounding over the vertical edge of the block. Here a four-layer inversion results, due to a more complicated sounding curve resulting from the potential electrodes expanding outward with increasing array width.

Figures 3a and 3c show apparent resistivity pseudosections for Schlumberger and Wenner cases, respectively. The four-layer Wenner sounding at $x = 0$ is reflected in the more complicated contour pattern. The gray region in each figure represents the more resistive area as calculated from the resistivity cutoff filter previously mentioned.

Contoured cross-sections based on the inverted layers of Figures 2a and 2b are shown in Figures 3b and 3d for the Schlumberger and Wenner arrays. The Schlumberger inversion has the effect of concentrating the horizontally oriented contours near the horizontal interface while the vertically oriented contours are less obviously affected. The resistivity cutoff shows a block-like resistive structure with interface at $z = 10$ units. The vertical interface is less accurately predicted. The contoured cross-section based on the inverted Wenner soundings shows an accentuation of the slight stairstep pattern of Figure 3c. The many closed contours below $x = 0$ reveal the effects of the four-layer inversion.

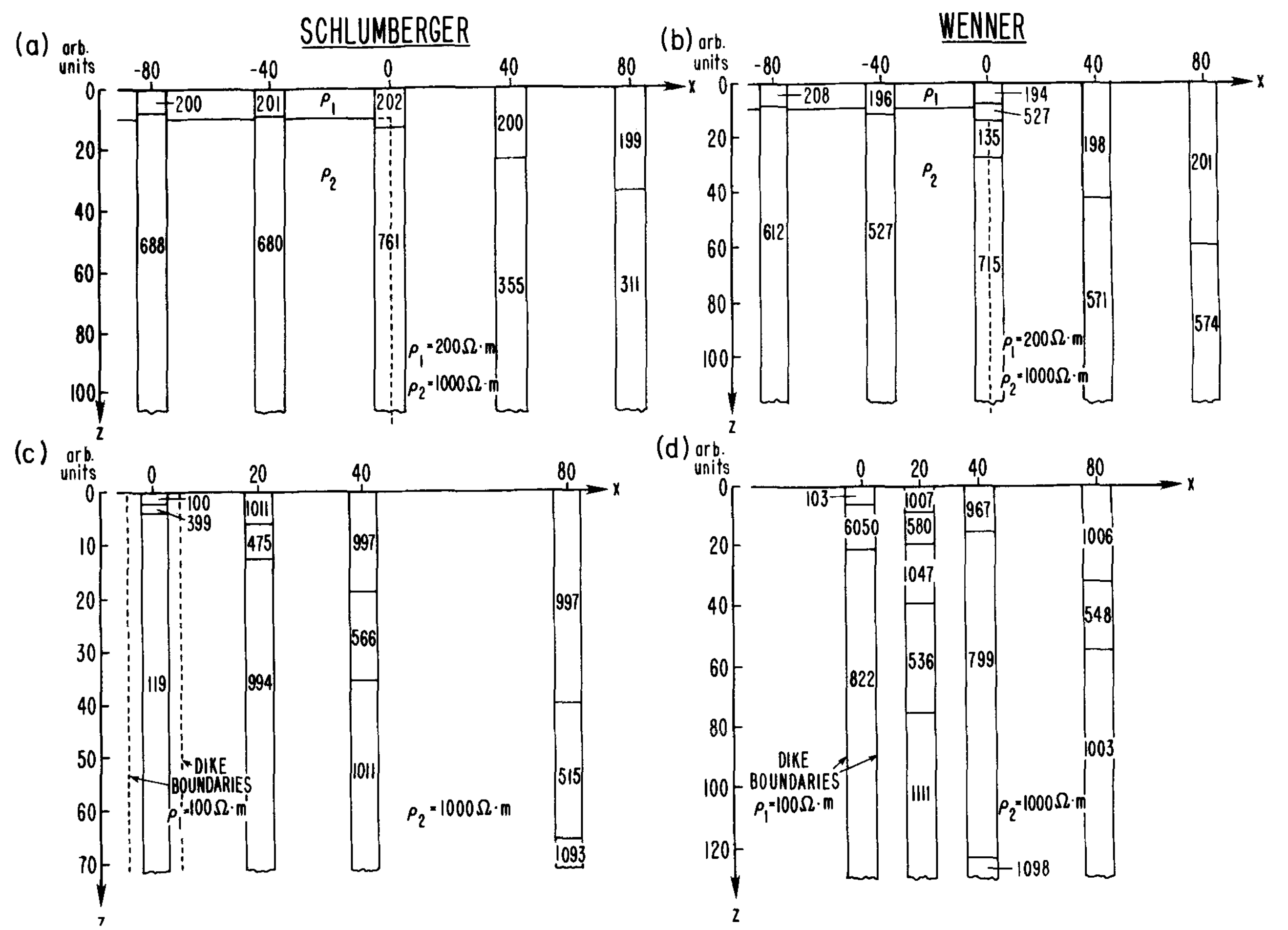


FIG. 2. 1-D inversions of Wenner and Schlumberger array resistivity soundings over buried fault block (Figure 1a) and a vertical dike (Figure 1b). Layer resistivities in ohm-m. (a) Inversions of Schlumberger array soundings over fault block. (b) Inversions of Wenner array soundings over fault block. (c) Inversions of Schlumberger array soundings over vertical dike. (d) Inversions of Wenner array soundings over vertical dike.

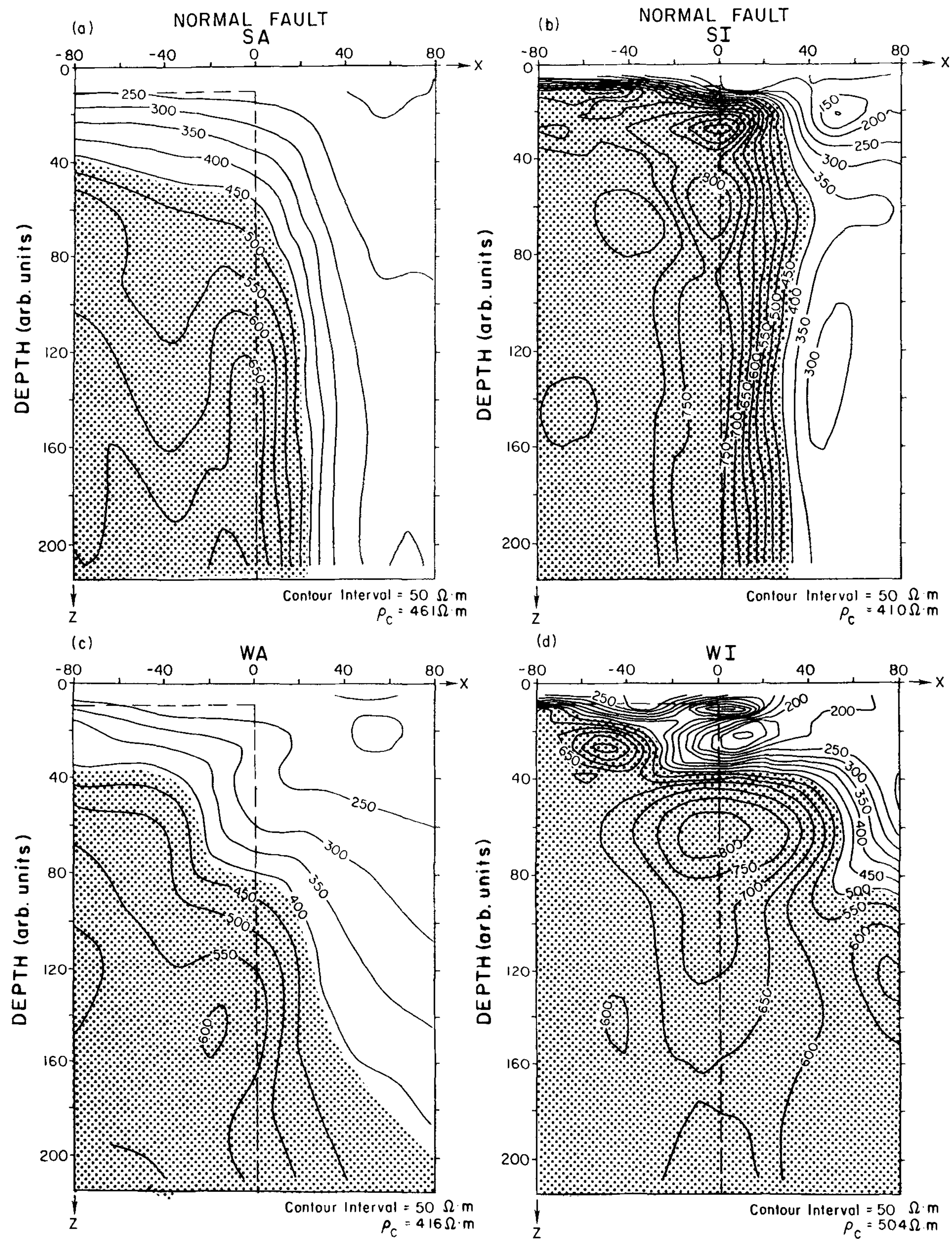


FIG. 3. Buried fault block. (a) Apparent resistivity pseudosection based on five Schlumberger soundings. (b) Cross-section obtained by contouring 1-D inversions of Schlumberger soundings. (c) Apparent resistivity pseudosection based on five Wenner soundings. (d) Cross-section obtained from contouring Wenner inversions. The following designations apply to this and to following figures: SA = Schlumberger apparent resistivity pseudosection, WA = Wenner apparent resistivity pseudosection, SI = Schlumberger cross-section based on 1-D inversions, WI = Wenner cross-section based on 1-D inversions.

Based on the four cross-sections, the fault block structure is most accurately predicted by the Schlumberger data, in particular from the inverted cross-section.

### Vertical dike

Figure 1b shows a vertical dike extending to the surface and downward to infinity. The dike has a 100 ohm-m resistivity and is surrounded by a 1000 ohm-m matrix. Schlumberger and Wenner soundings were centered at $x = 0$, 20, 40, and 80 units from the center of the dike. Using symmetry, the soundings at $x = 20$, 40, and 80 units were reflected about the dike axis to provide soundings at $x = -20$, $-40$, and $-80$. Thus, a total of seven soundings were contoured.

Figures 2c and 2d show the layered inversions of the apparent resistivity data. The Schlumberger soundings off the dike yield three-layer inversions with the upper and lower layers having a resistivity near the true value of 1000 ohm-m and the middle layer having a lower resistivity caused by the current electrode of the expanding array getting near the dike. Note that the thickness and depth of the conductive middle layer increases with increasing distance of the sounding center from the dike. The greater the distance the array center is from the dike, the farther the electrodes may expand before the effect of the conductive dike becomes apparent. Furthermore, the anomaly will appear broader and less pronounced with increasing distance from the dike. Upon inversion, the low-resistivity zones on the sounding curves will appear as a progressively deeper and possibly thicker conductive layer. Due to expanding potential electrodes, the Wenner data yielded more complicated inversions. At $x = 0$, the three-layer inversion yielded a middle layer of 6050 ohm-m, the effects of which can be clearly seen in the Wenner contoured cross-sections. At $x = 20$, a five-layer inversion results as both the current and potential electrodes expand over the dike.

Figures 4a and 4b represent a Schlumberger apparent resistivity pseudosection and a contoured cross-section based on inverted layer data, respectively. Both contours are very similar and the resistivity cutoff clearly indicates a dike-like feature. Little change occurred in the inverted contour because there were no horizontal resistivity interfaces for the inversion process to locate.

Figures 4c and 4d show the pseudosection and cross-section based on the Wenner soundings. Figure 4c, the apparent resistivity pseudosection, shows no clear indication of a dike. In the inverted contour, Figure 4d, the dike shows up in an unusual fashion below a high-resistivity plug which is itself beneath a low-resistivity zone. Due to the low-resistivity contrast below $z = 50$ units, the resistivity cutoff filter failed to predict a dike-like feature.

Based on the above figures, the Schlumberger array again appears superior to the Wenner in predicting 2-D geometries.

### Tabular prism

Of the 2-D structures we examined, the tabular prism was the one most nearly representative of a layered earth. As expected, the 1-D inversion approach yielded good results in both Schlumberger and Wenner cases. The tabular prism model is shown in Figure 1c. The 500 ohm-m prism is buried 10 units deep in a 1000 ohm-m half-space. The prism is 10 units thick and 80 wide. Figures 5a and 5b show the layered inversions based on Schlumberger and Wenner data respectively. Symmetry was used to reflect the $x = 30$, 60, and 120 soundings about the $x = 0$ axis to simulate seven soundings. With both array types, soundings over and near the prism inverted to three layers with a low-resistivity middle layer sandwiched between two nearly 1000 ohm-m layers. The $x = 120$ sounding inverted to one layer with a resistivity near 1000 ohm-m.

Figures 6a and 6c show the apparent resistivity pseudosections for the Schlumberger and Wenner arrays. Figures 6b and 6d are based on the inverted layers. All four figures indicate a prism-like structure, but the inverted contours clearly locate the upper interface, and, to a lesser degree, the lower interface. The resistivity cutoff, especially in the inverted Schlumberger contour, predicts prism geometry very accurately.

### Square prism

The square prism model is shown in Figure 1d. It is 20 units wide, 20 thick, and is buried 10 units deep. The prism has a resistivity of 200 ohm-m and the surrounding half-space is 1000 ohm-m. Figures 5c and 5d show the results of 1-D inversion of Schlumberger and Wenner soundings. Symmetry was used to simulate five soundings. For both arrays, the $x = 20$ and $x = 40$ soundings are similar. The $x = 0$ sounding over the prism center inverts to two layers in the Schlumberger case and to three layers in the Wenner case. This is due to the Schlumberger array's stationary potential electrodes not being able to "see through" the prism. The expanding Wenner electrodes were an advantage in this case.

Figures 7a and 7c show apparent resistivity pseudosections. The Wenner array's advantage is evident in comparing the figures. The Wenner contours and cutoff indicated a buried equidimensional body while the Schlumberger contours and cutoff seem to indicate a dike-like structure. A small advantage was gained by contouring the inverted Schlumberger data (Figure 7b). A thick plug is evidenced by the resistivity cutoff. In the Wenner case (Figure 7d), no clear interpretational advantage was gained by contouring the inverted data.

The square prism was the only model tested in which the Wenner array showed a decided advantage over the Schlumberger.

### Ramp

Figure 1e shows the ramp model and the 1-D inversions based on Schlumberger soundings. The ramp angle is 26.6 degrees to the horizontal. Its horizontal length is 280 units. Its vertical throw is 140 units. The unit above the ramp has a resistivity of 100 ohm-m while the unit below has 1000 ohm-m. Taking the point where the ramp intersects the surface as $x = 0$, soundings were centered at $x = -400$, $-200$, $-100$, 100, and 200 units from zero. Because of the poor results of the Wenner array in the case of the normal fault, only the Schlumberger array was examined. Figure 5e shows the results of five Schlumberger soundings over the ramp. All the soundings over the 100 ohm-m layer inverted to two layers with the upper layer somewhat shallower than the true interface depth. The soundings off the ramp inverted to three and four layers. As the left current electrode began to pass over the ramp, the apparent resistivity initially decreased, but as the array expanded the resistivity increased again as more 1000 ohm-m material was averaged in. Contours based on the apparent resistivity (Figure 8a) and on the inverted layers (Figure 8b) indicate these complications. However, note that in spite of

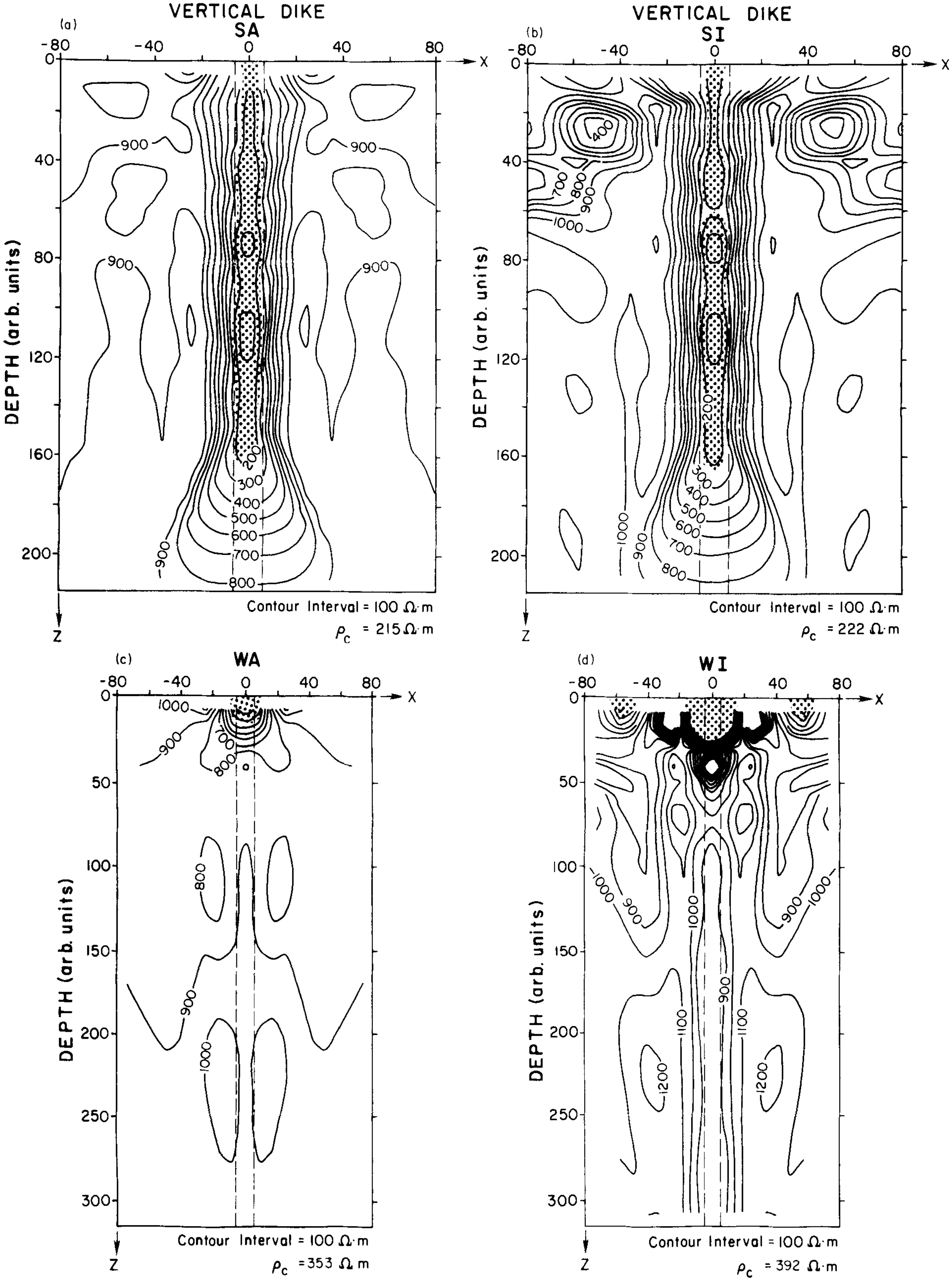


FIG. 4. Vertical dike. (a) Apparent resistivity pseudosection based on seven Schlumberger soundings. (b) Cross-section obtained from contouring 1-D inversions of Schlumberger soundings. (c) Apparent resistivity pseudosection based on seven Wenner soundings. (d) Cross-section obtained from Wenner inversions.

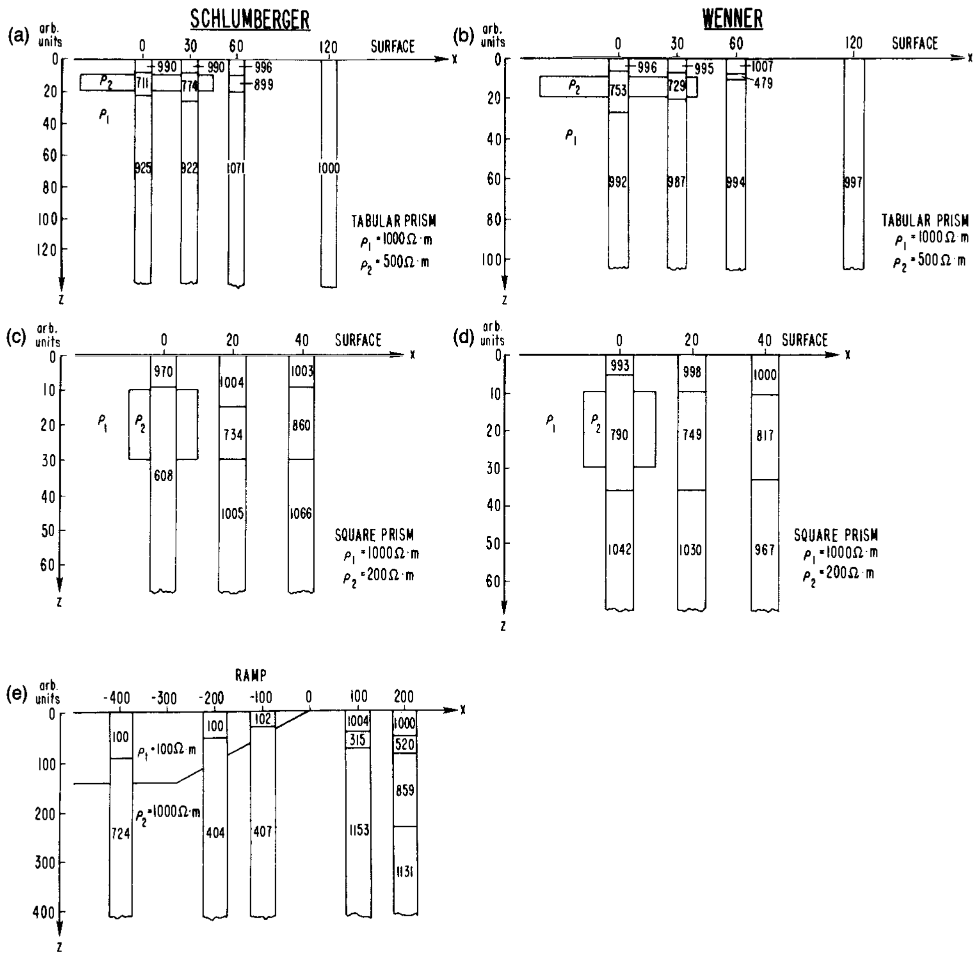


FIG. 5. 1-D inversions of Schlumberger and Wenner array resistivity soundings over tabular prism (Figure 1c), square prism (Figure 1d), and ramp (Figure 1e). Layer resistivities in ohm-m. (a) Inversions of Schlumberger array soundings over tabular prism. (b) Inversions of Wenner array soundings over tabular prism. (c) Inversions of Schlumberger array soundings over square prism. (d) Inversions of Wenner array soundings over square prism. (e) Inversions of Schlumberger array soundings over ramp.

the unusual contour patterns, the resistivity cutoff shows a conductive region in the vincinity of the ramp in both cases. The resistivity cutoff applied to the apparent resistivity data indicates a conductive zone extending somewhat too deep while the cutoff applied to the inverted cross-section makes the conductive zone appear too shallow. Resistivity interpretations based on layered earth models appear particularly ill-suited for assessing ramp-like or sloping structures.

## CONCLUSIONS AND RECOMMENDATIONS

The use of contoured cross-sections based on layered interpretations of inverted apparent resistivity data proved to be useful in determining the subsurface structure of simple 2-D structures. The inverse method was especially effective in locating horizontal resistivity interfaces while leaving information on vertical interfaces relatively unaffected. Contours based on the Schlumberger array were usually more easily interpretable than those of the Wenner array. The exception was the square prism where the expanding potential electrodes of the Wenner array enabled it to "see through" the structure. While it is true that contours based on inverted layers are more complicated than those based on apparent resistivity, the advantage of having horizontal interfaces more accurately determined, combined with the use of the resistivity cutoff to predict the geometry of the anomalous zone, makes this method a valuable one when more sophisticated interpretational methods are unavailable. We do not contend that contours based on 1-D inversions will yield a better subsurface picture than those based on 2-D or 3-D inversion. The results of this study indicate that 2-D subsurface geometry can be approximately delineated using contours based on apparent resistivity and 1-D inversion.

The resistivity contrasts used in this investigation were low to moderate — 2:1 to 10:1. Investigations of this nature using more extreme contrasts could indicate this method's reliability under such conditions. Furthermore, analagous work could be done in predicting 3-D geometries using 1-D inversion or 2-D inversion.

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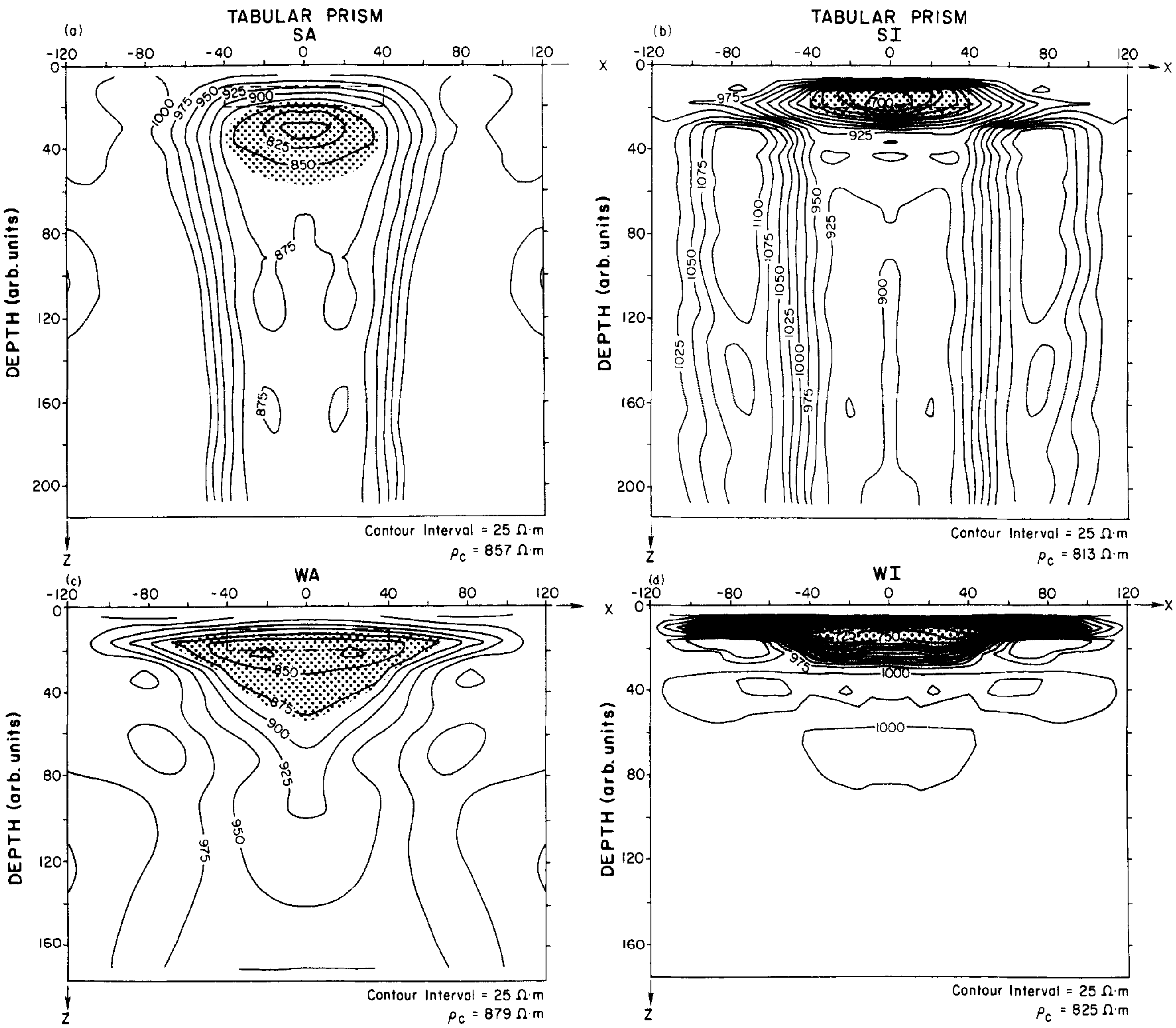


FIG. 6. Tabular prism. (a) Apparent resistivity pseudosection based on seven Schlumberger soundings. (b) Cross-section obtained from 1-D inversion of Schlumberger soundings. (c) Apparent resistivity pseudosection based on seven Wenner soundings. (d) Cross-section obtained from Wenner inversions.

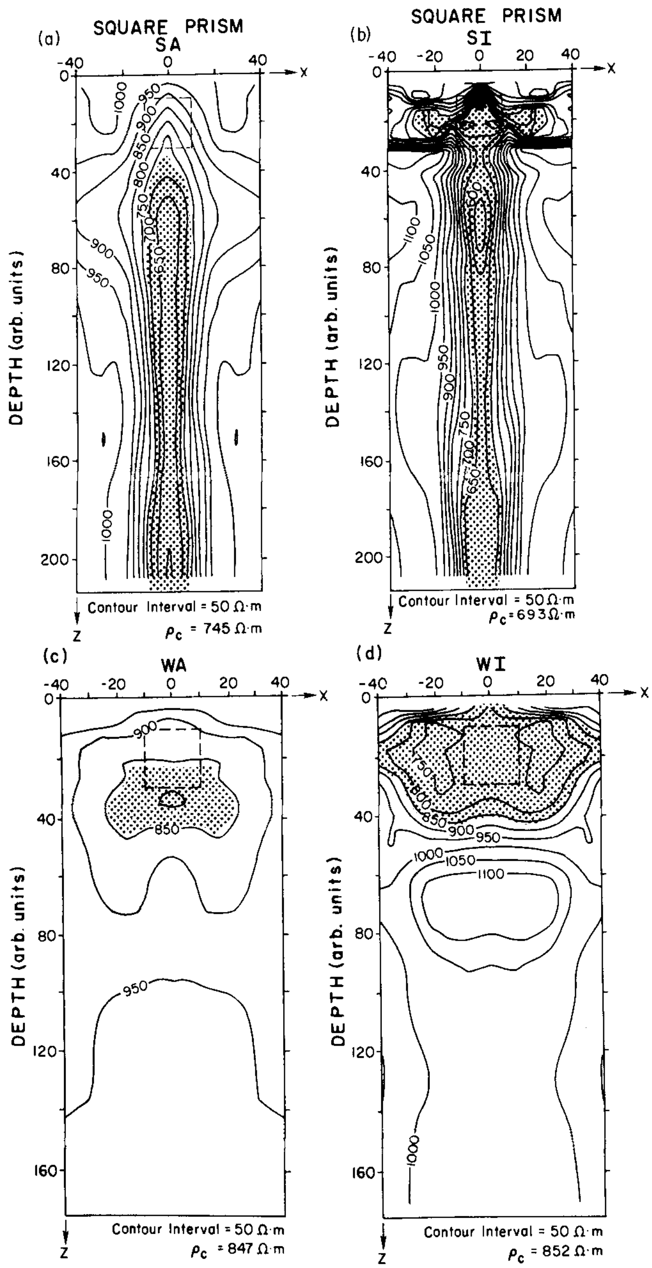


FIG. 7. Square prism. (a) Apparent resistivity pseudosection based on five Schlumberger soundings. (b) Cross-section obtained from 1-D inversion of Schlumberger soundings. (c) Apparent resistivity pseudosection based on five Wenner soundings. (d) Cross-section obtained from Wenner inversions.

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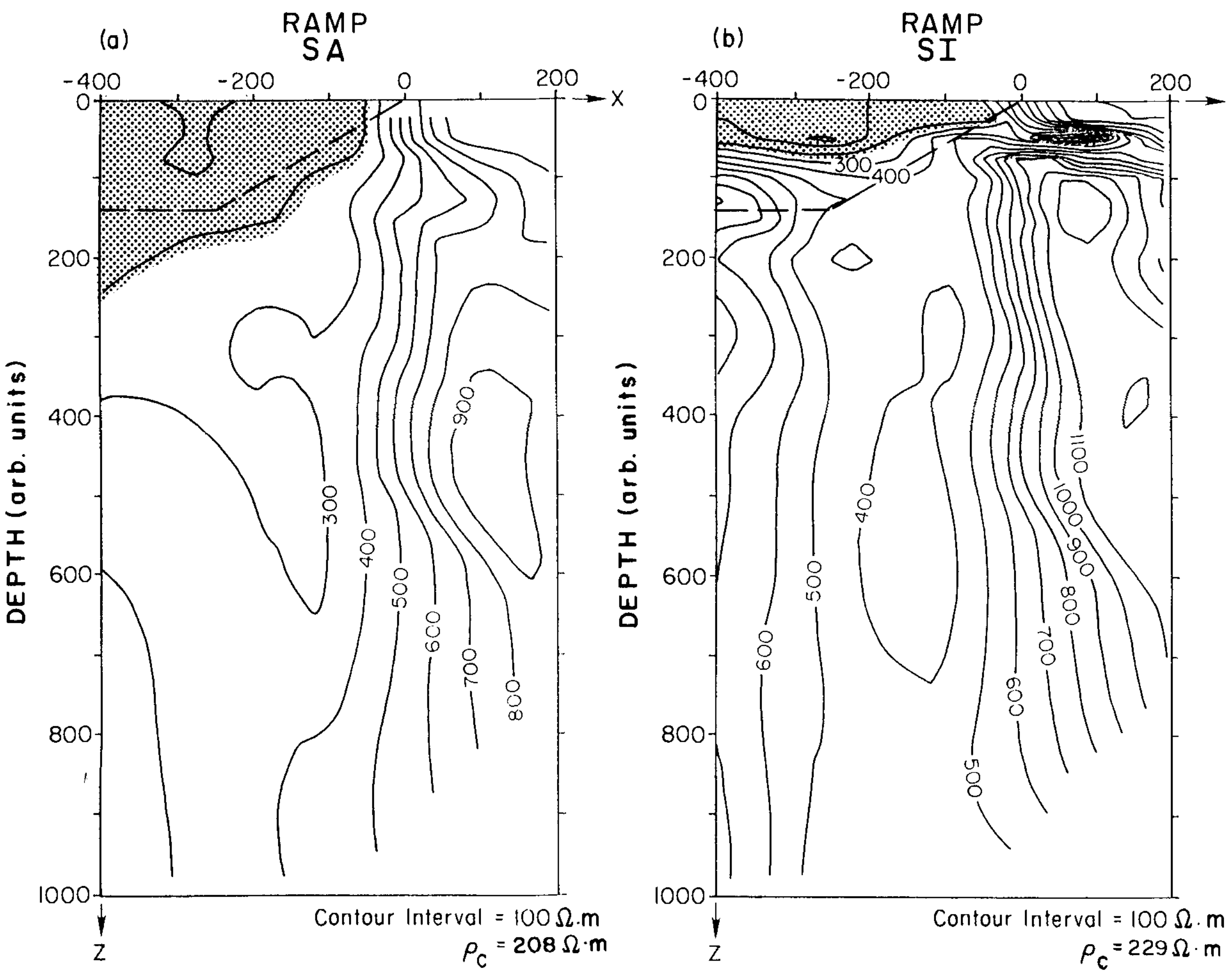


FIG. 8. Ramp. (a) Apparent resistivity pseudosection based on five Schlumberger soundings. (b) Cross-section obtained from Schlumberger inversions.