CERN-TH/96-200
King's College/kcl-th-96-??
Texas A & M/CTP TAMU-28/96
ICTP/IC/96/126
hepth/9607227

Superbranes

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Abstract

We consider the doubly supersymmetric formulation of various p-branes, that is, we replace the worldsurface of a super p-brane by a super worldsurface and consider the embedding of the latter into a target superspace. The number of worldsurface fermionic coordinates is taken to be half as many as those of the target superspace. We show that a simple geometrical constraint and its integrability condition lead to manifestly worldsurface supersymmetric field equations for a large class of super p-branes. We illustrate this procedure in some detail in the case of the $D = 11$ superfivebrane. We also describe a class of super p-branes in which a worldsurface linear supermultiplet arises. In some cases we show that an additional constraint involving the curvature of an appropriate worldsurface antisymmetric tensor potential is needed to put the theory on-shell.

CERN-TH/96-159
September 5, 2018

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1 Introduction

With the emergence of super p-branes as important ingredients in non-perturbative string physics, it has become more urgent to develop a better understanding of their properties. For example, still not much is known about the manifestly supersymmetric actions that govern the dynamics of these objects, with the exception of Type I branes, by which we mean those for which the worldsurface degrees of freedom are described by a scalar supermultiplet [1, 2, 3]. It is now known that there are a number of other super p-branes, called Type II branes [4], in which the worldsurface degrees of freedom form either Maxwell multiplets (the D-branes in $D = 10$ [5]), or the tensor multiplet (the $11D$ superfivebrane [6, 7]).

In all these cases it would be very useful to have manifestly supersymmetric actions or, at least, equations of motion. Among other things, these would help us to gain a better understanding of the duality symmetries of M-theory. Moreover, they might help in uncovering various relationships among the diversity of branes that exist through the examination of the properties of the relevant brane actions or equations of motion in appropriate limits.

In the case of a superparticle or a superstring, it is well known that one can either formulate the action with manifest worldline/worldsheet supersymmetry (NSR formalism), or with manifest target space supersymmetry (GS formalism). In some cases, there is also a formulation in which both the worldline/worldsheet and target space supersymmetry is manifest. Variants of such formulations are known as twistor, twistor-like or doubly supersymmetric formulations. See, for example, [8] for an extensive list of references. In fact, the doubly-supersymmetric approach is the main theme of this paper. However, before we present our results, it is useful to recall a few more facts about super p-brane actions.

To begin with, we recall that an NSR type formulation of p-branes beyond $p = 1$ does not seem to be possible, while a GS type formulation does exist, at least for Type I branes. The GS type formulation has a fermionic worldsurface symmetry, known as κ-symmetry, which plays a crucial rôle in obtaining worldsurface supersymmetry after gauge fixing. It should be noted, however, that proving worldsurface supersymmetry is a very nontrivial task which has been carried out fully only for superstrings (see, for example, [9]) and the supermembrane in $D = 4$ [10].

As for D-branes, it is now well-known that they can be described as surfaces on which an open superstring ends [5]. This description makes it possible to understand various properties of their actions, in particular, the necessity of replacing the usual Nambu-Goto action by the Dirac-Born-Infeld generalization due to the presence of vector fields. However, it should be noted that, so far, a κ-symmetric D-brane action is known only for the Dirichlet 2-brane, and it is obtained from the κ-symmetric $D = 11$ supermembrane action by dimensional reduction to $D = 10$, followed by the dualization of the 11th scalar to a world volume vector [11].

In the case of the eleven dimensional superfivebrane, since an open string description is not available, even the bosonic action is not known, although some partial results have been obtained [11, 12, 13]. This is one of the most interesting cases, and the case which we will study in most detail in this paper.

The fact that Type II branes involve worldsurface supermultiplets other than scalar multiplets makes it difficult to apply the usual GS formalism. The requirement of κ-symmetry is very restrictive, and we recall that worldsurface supersymmetry is to be expected only after gauge fixing. This state of affairs strongly suggests that we should make much greater use of worldsurface supersymmetry from the very beginning while maintaining manifest spacetime supersymmetry.

This brings us to the main point of this paper, namely the description of the dynamics of all super p-branes, both Type I and Type II, by simple geometrical considerations involving the embedding of the world supersurface into the target superspace. The basic idea is that once we elevate the target superspace coordinates into worldsurface superfields, we can see immediately that there is room for the worldsurface supermultiplets in their θ-expansions. In this approach, κ-symmetry is traded for a more encompassing *local* worldsurface supersymmetry. The key issue is how to impose the right constraints on these superfields. In this paper we will show that, if we do not insist on an action to begin with, but demand only the right covariant equations of motion, the answer to this question is very simple and is given by the imposition of very natural embedding conditions that have clear and simple geometrical meanings. Once these constraints are well understood, one can of course look for an action principle from which they can be derived.

As mentioned earlier, the doubly supersymmetric aproach to super p-branes, with emphasis on $p = 0, 1, 2$, has been investigated previously in many papers. The basic idea was introduced in the context of superparticles in $D = 3, 4$ [14, 15], and extended to other superparticles in [16, 17, 18, 19], heterotic superstrings in [20, 21], supermembranes in [22], and higher super p-branes in [23]. The equations of motion of the supermembrane in $D = 11$ were studied from the point of view of superspace embeddings in [8] and other Type I super p-branes were studied in [24] where a generalised action principle was used as a means of deriving the corresponding GS equations, although this is not an action principle in the usual sense of the term. Moreover, for $p \geq 2$, the worldsurface supermultiplets involved have not been discussed in detail. One of the main results of this paper is that we clarify the precise worldsurface multiplet structures that arise due to the geometrical constraints in all possible cases. The basic embedding equation (1) (or its linearised version (13)) in fact determines various types of worldsurface supermultiplet as a result of which we are able to say whether the type of Lagrange multiplier action first proposed in [19] is likely to succeed for the brane in question or not.

We shall begin by specifying this basic embedding condition and discussing its geometrical meaning. We shall than fix a physical gauge to obtain a "master constraint equation" that encodes the equation of motion for the super p-brane in some cases. We will illustrate the power of the master constraint by considering the interesting case of the superfivebrane in $D = 11$, and show explicitly, at the linearised level, how the equations of motion for the worldsurface $d = 6, N = (2, 0)$ tensor multiplet emerge. We also mention the worldsurface three-form field strength and its relation to the target space four-form. This is the superspace version of the phenomenon observed in [11].

We then go on to discuss the master equation for Type I branes in arbitrary dimensions, D-branes in $D = 10$ and what we call L-branes, which involve worldsurface linear multiplets. The L-branes are new, but easy to understand: they correspond to dimensional reductions of Type I branes by one dimension, followed by dualization of the scalar corresponding to that dimension to a suitable worldsurface p-form potential. These are analogous to the Dirichlet twobrane mentioned earlier, but they do not seem to have been considered before.

We shall also discuss the possibility of a superfivebrane in $D = 7$ which involves a worldsurface $(1, 0)$ tensor multiplet and comment on a possible ninebrane in $D = 11$. In the conclusions we will comment on the nonlinear aspects of the master equation as well as a number of other aspects of our results. Throughout the paper we restrict our attention to target spaces with $N = 1$ supersymmetry, except in the case of $D = 10$ where we will consider both types of $N = 2$ as well as $N = 1$.

2 The Embedding Equations

The basic equation describing all branes as embedded subsupermanifolds is simple to state and has a simple geometrical meaning. Let M denote the world-surface, T its tangent bundle, T_0 and T_1 its even and odd tangent bundles, let T^* etc denote the corresponding dual bundles, and use the same notation for the target superspace, but with underlining. Let $M \xrightarrow{f} \underline{M}$ denote the embedding of the worldsurface in the target superspace. The basic condition to be imposed on all embeddings we shall consider is

$$T_1 = T \cap \underline{T_1} \ , \tag{1}$$

where on the RHS $\underline{T_1}$ is restricted to M. Dually one has

$$T_0^* = T^* \cap \underline{T_0}^* \ . \tag{2}$$

The geometrical meaning of these statements is clear: the odd tangent space of the worldsurface at any point is a subspace of the odd tangent space of the target space at the same point, and the even cotangent space of the worldsurface at any point is a subspace of the even cotangent space of the target space at the same point. As we have mentioned, these equations imply the structure of the worldsurface multiplet as we shall show below. Several cases can arise: the world surface multiplet can be either on- or off-shell and one can have either Type I or Type II branes. If the representation is off-shell one can presumably construct actions of the type studied in [19, 20], whereas if it is on-shell the condition (1) above determines the worldsurface theory completely at the level of the equations of motion. In the former case the equations of motion are equaivalent to some additional geometrical conditions which can be stated using various p-forms on the target space and on the world-surface.

To justify these statements we introduce local bases $E_A = (E_a, E_\alpha)$ for T and $E^A = (E^a, E^\alpha)$ for T^* where we use the notation $A = (a, \alpha)$ for frame indices, $M = (m, \mu)$ for coordinate indices and where indices for M and $\underline{M}$ are distinguished by underlining the latter. We use latin and greek indices to refer to even and odd components respectively. We shall also use primed indices to denote quantities normal to M. We can set

$$E_A = E_A{}^{\underline{A}} E_{\underline{A}} \ , \tag{3}$$

and

$$E^{\underline{A}} = E^A E_A{}^{\underline{A}} \ , \tag{4}$$

where, in the latter equation, $E^{\underline{A}}$ is pulled back onto M. In terms of local coordinates z^M and $z^{\underline{M}}$ we have

$$E_A{}^{\underline{A}} = E_A{}^M \partial_M z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} \tag{5}$$

where $E_A{}^M$ is the inverse supervielbein on the worldsurface and $E_{\underline{M}}{}^{\underline{A}}$ is the supervielbein on the target space. The basic equation (1) translates to

$$E_\alpha{}^{\underline{a}} = 0 \ . \tag{6}$$

Dually, we have

$$E^{\underline{a}} = E^a E_a{}^{\underline{a}} \ . \tag{7}$$

The basic tensor on a supermanifold is the Frobenius tensor associated with the odd tangent bundle. Its components $T_{\alpha\beta}{}^c$ with respect to a given basis are defined by

$$\langle [E_\alpha, E_\beta], E^c \rangle = -T_{\alpha\beta}{}^c \ , \tag{8}$$

where $\langle,\rangle$ denotes the standard pairing between vectors and forms. From the above definitions it is easy to deduce that

$$E_{\alpha}{}^{\underline{\alpha}} E_{\beta}{}^{\underline{\beta}} T_{\underline{\alpha}\,\underline{\beta}}{}^{\underline{c}} = T_{\alpha\beta}{}^{c} E_{c}{}^{\underline{c}} \ . \tag{9}$$

This equation is similar in some respects to the equation defining the induced metric in (pseudo)-Riemannian geometry. In (almost) all cases we shall consider one can always find a basis in which the components of the target space Frobenius tensor take the form

$$T_{\underline{\alpha}\,\underline{\beta}}{}^{\underline{c}} = -i(\Gamma^{\underline{c}})_{\underline{\alpha}\,\underline{\beta}} \ . \tag{10}$$

The exceptional cases only differ from this form by the inclusion of invariant tensors associated with internal symmetry groups.

To study the implications of these equations we consider two simplifications. First, we shall suppose that the target space is flat, and second, we shall take the embedding to be infinitesimal. If we denote the coordinates of M by (x^a, θ^α) and those of $\underline{M}$ by $(X^{\underline{a}}, \Theta^{\underline{\alpha}})$, an infinitesimal embedding, in a standard gauge (often referred to as the physical gauge), is given by

$$\begin{aligned} X^a &= x^a \ , & X^{a'} &= X^{a'}(x,\theta) \ , \\ \Theta^\alpha &= \theta^\alpha \ , & \Theta^{\alpha'} &= \Theta^{\alpha'}(x,\theta) \ . \end{aligned} \tag{11}$$

(Recall that the primed indices refer to quantities normal to the world supersurface.) Substituting the above gauge choice into (9), and performing the shift

$$\tilde{X}^{a'} = X^{a'} - \frac{i}{2}\theta^\alpha (\Gamma^{a'})_{\alpha\beta'} \Theta^{\beta'} \ , \tag{12}$$

we find that the linearised version of (9) is simply

$$D_\alpha \tilde{X}^{a'} = i(\Gamma^{a'})_{\alpha\beta'} \Theta^{\beta'} \ , \tag{13}$$

where D_α is the flat superspace covariant derivative on the worldsurface. It satisfies

$$[D_\alpha, D_\beta] = i(\Gamma^a)_{\alpha\beta} \ , \tag{14}$$

and the Γ-matrices are those of the target space decomposed according to the embedding. The leading components in the θ-expansion of $\tilde{X}^{a'}$ and $\Theta^{\alpha'}$ can be thought of as the Goldstone fields associated with the partial breaking of translational symmetry and supersymmetry repectively. Indeed one could apply non-linear realisation theory using these fields, as has been done for some cases previously [25, 26, 27], but this is technically complicated for all but the simplest cases whereas the current approach builds in full covariance from the beginning.

Eq. (13) plays a central rôle in this paper and we shall refer to it as the "master equation" or "master constraint". One can now analyse the various possibilities that can arise according to the number of worldsurface scalars and fermions that are involved. We will enumerate these possibilities later. Examining these possibilities, we will learn that the master constraint (13) is sufficient to put the theory on-shell in a number of interesting cases, including the $D = 11$ superfivebrane, in that it describes correctly the equations of motion for the relevant worldsurface supermultiplet. In some other cases, however, we will find that the master constraint (13) is *not* sufficient to put the theory on-shell. In such cases, we will propose a further constraint involving appropriate super p-form potentials on the world supersurface, whose rôle will be to put the theory on-shell.

The geometrical interpretation of these results, for the on-shell case, is that, for Type I branes the basic constraint corresponds to an adapted frame, that is, one can consider $E_\alpha{}^{\underline{\alpha}}$ to be part of a $Spin(1, D-1)$ matrix (up to a conformal factor). In the Type II case, the frame can no longer be considered as adapted because of the appearance of extra degrees of freedom associated with the worldsurface vector or tensor multiplet. The appearance of these extra degrees of freedom is related to actions of the Dirac-Born-Infeld type which occur in Type II. In the off-shell cases one also does not have the adapted frame interpretation although this is recovered on-shell in Type I.

We shall come back later to the points made briefly above. We now turn to the analysis of the master equation (13) in detail for the interesting case of the $D = 11$ superfivebrane.

3 The Eleven Dimensional Superfivebrane

Let us consider equation (13) for the case of the superfivebrane in $D = 11$. It is convenient to introduce some notation which reflects the $Spin(1, 5) \times USp(4)$ symmetry of six-dimensional superspace. A $D = 11$ Majorana spinor $\psi_{\underline{\alpha}}$ decomposes as

$$\psi_{\underline{\alpha}} = (\psi_{\alpha i}, \psi_i^\alpha) \ , \tag{15}$$

where $i = 1, \ldots, 4$ is a $USp(4)$ index and $\alpha = 1, \ldots, 4$ is a six-dimensional Weyl spinor index with upper (lower) indices corresponding to anti-chiral (chiral) spinors respectively. The six-dimensional spinors satisfy a $USp(4)$ symplectic Majorana-Weyl reality condition. One can choose a representation for the $D = 11$ Γ-matrices in which, for example,

$$(\Gamma^a)_{\alpha i, \beta j} = \eta_{ij} (\sigma^a)_{\alpha\beta} \tag{16}$$

where η_{ij} is the $USp(4)$ antisymmetric invariant tensor and the σ^a the the six-dimensional chirally-projected gamma-matrices. Similarly the remaining components are given by products of six-dimensional and five-dimensional invariants.

The embedding equation (13) becomes

$$D_{\alpha i} \tilde{X}^{a'} = i(\gamma^{a'})_{ij} \Theta_\alpha^j \ , \tag{17}$$

where the $\gamma^{a'}$ are the five-dimensional (Euclidean) gamma-matrices which are antisymmetric. This equation defines the $d = 6,\ N = (2, 0)$ tensor multiplet introduced in [28]. Its components consist of a self-dual third-rank antisymmetric tensor, a set of chiral fermions transforming under the four-dimensional representation of $USp(4)$ and five scalars. The scalars and spinors are the leading components of $\tilde{X}^{a'}$ and Θ_α^i respectively, so it remains to locate the antisymmetric tensor and to show that there are no further components. To do this one applies a spinorial covariant derivative to (17) and uses the six-dimensional supersymmetry algebra. One finds

$$D_{\alpha i} \Theta_{\beta j} = \eta_{ij} L_{\alpha\beta} - \frac{1}{2} (\gamma^{a'})_{ij} (\sigma^a)_{\alpha\beta} \partial_a \tilde{X}_{a'} \ . \tag{18}$$

The symmetric bispinor $L_{\alpha\beta}$ defines a self-dual third-rank antisymmetric tensor as required. Continuing in this manner one finds by applying further spinorial covariant derivatives that the fermion field satisfies the Dirac equation, the scalar fields satisfy the Klein-Gordon equation and the tensor field satisfies the Bianchi identity and field equation for a third-rank antisymmetric field strength tensor. Furthermore, there are no other spacetime components, so that equation (13) defines an on-shell tensor multiplet as claimed.

For this case we have also verified that equation (13) is consistent up to quadratic order, and that it also implies that

$$T_{\alpha i,\beta j}{}^{c} = -i\eta_{ij}(\sigma^{c})_{\alpha\beta} \ , \tag{19}$$

up to a choice of basis on the world-surface. Furthermore, one can verify that the geometry outlined above is consistent with the existence of a 3-form H_3 on M such that

$$dH_3 = \underline{H}_4 \ , \tag{20}$$

where $\underline{H}_4$ is the usual 4-form on $D = 11$ superspace pulled back on to the world supersurface. Note that it is not necessary to impose this equation in order to obtain the required worldsurface multiplet, as the embedding equation automatically ensures this. However, equation (20) is very useful in the non-linear case as it establishes the existence of a potential for the antisymmetric tensor and shows how the latter is related to the embedding tensor $E_{\alpha}{}^{\underline{\alpha}}$. A more detailed exposition of the fivebrane in eleven dimensions will be given in a forthcoming article.

4 The Superbrane Scan

Given an embedded p-brane world supersurface, the target space Lorentz group breaks down to $SO(1,p)\times SO(D-p-1)$. The transverse symmetry group $SO(D-p-1)\equiv G^t$ will be identified with the automorphism group G^a of the worldsurface Popincaré superalgebra, in all cases except the Type I, codimension 4 embeddings (see later), where the spin group corresponding to $G^t = Sp(1)\times Sp(1)'$, but $G^a = Sp(1)$. The Goldstone superfield $X^{a'}$ always transforms under the vector representation of G^t, while the Goldstino superfield $\Theta^{\alpha'}$ and the worldsurface fermionic coordinates transform under equivalent or inequivalent fundamental spinor representations of $SO(p,1)\times G^a$, depending on the type of the embedding.

In determining the embedded world supersurface, we shall always maintain the symmetries mentioned above. This means that the Dirac matrices $(\Gamma^{a'})_{\alpha\beta'}$ should exist as invariant tensors of these symmetry groups. While this requirement may sound rather innocent, it restricts the possible embeddings considerably, as we shall see below.

In our search for possible embeddings, we will restrict our attention to minimal target superspaces, with the exception of $D = 10$, by which we mean $N = 1$ superspaces in which the number of fermionic coordinates is minimal. In $D = 10$, we will consider the $N = 2$ cases as well. To specify the relevant superspace more precisely, let us use the notation

$$(D|D';\ SO(1,D-1) \text{ spinor type })\ , \tag{21}$$

where D' is the real dimension of target space fermionic coordinates. Thus, the minimal target superspaces we will consider are:

$$\begin{array}{lllll} (11|32;M) & (10|16;MW) & (9|16;PM) & (8|16;PM) & (7|16;D) \\ (6|8;W) & (5|8;D) & (4|4;M) & (3|2;M) & \end{array} \tag{22}$$

where M stands for Majorana, W for Weyl, D for Dirac, S for symplectic and P for pesudo. For example, SMW stands for symplectic-Majorana-Weyl, etc. In $D = 5,6,7$, we can equally well work with PSM, SMW and SM spinors, respectively, in which case the target Poincaré superalgebra acquires an $Sp(1)$ automorphism group. The precise definitions of these reality conditions can be found in [29]. See also [30] for a summary, in which the mostly positive spacetime signature $(-,+,+,...,+)$ adopted in this paper is used. The analysis of the supermultiplets which arise is done in each case using the method discussed in the previous section in the context of the $D=11$ fivebrane. We shall not give any further details here; we shall simply state the results.

Let us now consider the embedding of Type I branes. In this class, both the number of bosonic as well as fermionic degrees of freedom are given by the codimension of the embedding, which for a p brane in D dimsnions is defined by $D-d=D-p-1$. The master constraint for codimension 8,4,1 embeddings takes the form

$$D_{\alpha i}\, X^{a'} = i(\sigma^{a'})_{ij'}\,\Theta^{j'}_{\alpha}\ , \qquad a'=1,...,D-p-1\ , \qquad \alpha=1,...,m\ , \qquad i,j'=1,...,n\ , \quad (23)$$

where (m,n) are the dimensions of the relevant $SO(1,p)\times G^a$ fundamental spinor, which we will specify case by case below. The σ-matrices are the van der Warden symbols, i.e. chirally projected γ-matrices, which obey the $SO(D-p-1)$ Clifford algebra. The spinor index α is also chirally projected when it labels a Weyl spinor. To list the possibilities, let us use the following notation:

$$(\ p,\ D,\ (m,n);\ \ SO(1,p)\times G^a \text{ spinor type }\ )\ . \quad (24)$$

The constraints (23) are possible for the following (codimension 8,4,1) embeddings:

$$\begin{array}{llll} \text{codimension } 8: & (2,\ 11,\ (2,8);\ M) & (1,\ 10,\ (1,8);\ MW) & \\ \text{codimension } 4: & (5,\ 10,\ (4,2);\ SMW) & (4,\ 9,\ (4,2);\ PSM) & (3,\ 8,\ (4,2);\ PSM) \\ & (2,\ 7,\ (2,2);\ D) & (1,\ 6,\ (1,2);\ W) & \\ \text{codimension } 1: & (2,\ 4,\ (2,1);\ M) & (1,\ 3,\ (1,1);\ MW) & \end{array} \quad (25)$$

The codimension 8 cases are the supermembrane in $D=11$ and the heterotic string in $D=1$; the corresponding worldsurface supermultiplets are the on-shell $d=3, N=8$ scalar multiplet and the off-shell $d=2,\ (8,0)$ scalar multiplet, respectively. The codimension 4 cases are the fivebrane in $D=10, N=1$, the fourbrane in $D=9$, the threebrane in $D=8$, the twobrane in $D=7$ and the heterotic string in $D=6$. The corresponding worldsurface multiplets are hypermultiplets in dimensions $d=6,5,4,3$ and 2 respectively, each such multiplet having four scalars. All of these multiplets are on-shell except for the string case. In codimension 1, we have the $D=4$ supermembrane and the $D=3$ heterotic string. The corresponding worldsurface supermultiplets are the $d=3, N=1$ and $d=2,\ (1,0)$ scalar multiplets, respectively. Both of these multiplets are off-shell.

The remaining Type I branes have codimension 2, and the master constraint for this case takes the form

$$D_\alpha X = \Theta_\alpha\ \ , \quad (26)$$

where we have defined the complex scalar $X=(X^{D-1}+iX^{D-2})/\sqrt{2}$. This constraint is possible for the following (codimension 2) embeddings:

$$\text{codimension } 2: \qquad (3,\ 6,\ (2,1);\ W) \qquad (2,\ 5,\ (2,1);\ D) \qquad (1,\ 4,\ (1,1);\ W) \quad (27)$$

These branes are the threebrane in $D=6$, a twobrane in $D=5$ and the heterotic string in $D=4$, with corresponding worldsurface multiplets which are chiral multiplets, with $d=4, N=1$, $d=3, N=2$ and $d=2,\ (2,0)$ supersymmetry, respectively.

Next, we list the master constraints for D-branes. As is well known, Type IIA supergravity has even branes with $p = 2, 4, 6, 8$, and Type IIB supergravity has odd branes with $p = 3, 5, 7$. The master constraints for the former case takes the form

$$D_{\alpha i}\, X^{a'} = i(\gamma^{a'})_{ij}\, \Theta^j_\alpha \ , \qquad a' = 1, ..., D-p-1 \ , \qquad \alpha = 1, ..., m \ , \qquad i, j = 1, ..., n \ , \tag{28}$$

where the γ-matrices are $SO(D-p-1)$ matrices with $D-p-1 = 1, 3, 5, 7$. More precisely, these constraints are possible for the following Type IIA branes:

$$(8,\ 10,\ (16,1);\ PM) \qquad (6,\ 10,\ (8,2);\ SM) \qquad (4,\ 10,\ (4,4);\ PSM) \qquad (2,\ 10,\ (2,8);\ M)\ . \tag{29}$$

Note that the transverse symmetry group G^t is identified with the automorphism group G^a, and that, unlike in the case of Type I branes, the world supersurface fermionic coordinates and the Goldstino fermions are in equivalent fundamental spinor representations of G^a.

For the twobrane, the fourbrane and the sixbrane the worldsurface multiplet determined by the master equation is the maximally supersymmetric Maxwell multiplet in $d = 3, 5$ and 7 with $N = 8, 2$ and 1 supersymmetry respectively. These multiplets are all on-shell. For the eightbrane, which has codimension 1, the master equation gives an unconstrained scalar superfield. An additional constraint of the type of equation (20) will be necessary to put the theory on-shell in which case the worldsurface mutliplet will be the $d = 9$ Maxwell multiplet.

The master constraints for the Type IIB branes are one of the following three types:

(i) the constraint (23) for $(3,\ 10,\ (2,4);\ W)$;

(ii) the constraint (23) for $(5,\ 10,\ ((4_+, 2_+) + (4_-, 2_-)); SMW)$ and

(iii) the constraint (26) for $(7,\ 10,\ (8,1);\ W)$.

In case (ii), $4_\pm$ refer to the left and right handed spinors of $SO(5,1)$, and $2_\pm$ refer to the left and right handed spinors of $G^a = SO(4)$. In this case, the constraint (23) is to be written for $(4_+, 2_+)$ and $(4_-, 2_-)$ separately.

For the Type IIB branes, the threebrane and the fivebrane again have worldsurface multiplets which are maximally supersymmetric Maxwell multiplets in $d = 4$ and $d = 6$, that is, $N = 4$ and $N = (1,1)$, respectively. In the case of the sevenbrane, however, the brane has codimension 2 and so the worldsurface supermultiplet is a chiral scalar superfield. Moreover, unlike the case of Type I codimension 2, this chiral multiplet cannot be used to write an off-shell Lagrangian. A further constraint, presumably of the type of equation (20), but with a worldsurface twoform, is necessary to put the theory on-shell. After this constraint has been implemented, the worldsurface multiplet will be the on-shell Maxwell multiplet in $d = 8$.

We next examine the master constraints which correspond to a new class of branes, which we refer to as L-branes. The master constraint for these branes takes the form given in (28), with the following data:

$$(5,\ 9,\ (4,2);\ SMW) \qquad (4,\ 8,\ (4,2);\ PSM) \qquad (3,\ 7,\ (4,2);\ M) \qquad (3,\ 5,\ (4,1);\ M)\ . \tag{30}$$

Note that the first three are codimension 3, and the last one is codimension 1. Thus, the transverse group is $G^t = SO(3)$ for the first three cases, and the world supersurface fermions and the Goldstone spinor superfield carry the equivalent doublet representation of $SO(3)$. Note also that the L-branes, for which (p, D) is given by $(3,5)$, $(3,7)$, $(4,8)$ and $(5,9)$, correspond precisely to the double dimensional reductions by one dimension, followed by dualization of the scalar associated with this dimension, of the following Type I branes: $(3,6)$ $(3,8)$, $(4,9)$ and $(5,10)$.

For the L-branes, the master equation determines the worldsurface mutiplet to be the maximally supersymmetric linear multiplet in all cases with codimension 3. For the remaining case, which has codimension 1, (i.e. the threebrane in $D=5$), the worldsurface multiplet is an unconstrained scalar multiplet. An additional constraint, presumably involving antisymmetric tensors, is necessary to obtain the $d=4, N=1$ linear multiplet. Note that the linear multiplets are always off-shell. Since the linear multiplets involve antisymmetric tensor gauge fields it seems probable that we will again have equations such as (20).

Finally, we point out two more possible master constraints. The first one is a fivebrane in $D=7$, for which the resulting world supersurface multiplet is a $d=6$ tensor multiplet. This constraint is

$$D_{\alpha i}\ X = \Theta_{\alpha i}\ , \qquad \alpha = 1,...,4\ , \qquad i=1,2\ , \tag{31}$$

where the $\theta^{\alpha i}$ are SMW spinors in six dimensions. We expect that this constraint, which by itself gives rise to a scalar superfield, will, when supplemented by a suitable H-constraint, describe the $(1,0)$ tensor multiplet in $d=6$. This latter multiplet is on-shell and involves a self-dual third-rank antisymmetric tensor.

The second possibility is a ninebrane in $D=11$. The master equation takes the simple form $D_\alpha X = \Theta_\alpha$, where the spinor is 16 component Majorana-Weyl, thereby leading to a real scalar superfield X in $d=10, N=1$. This is reducible, and could lead to a spin 3/2 multiplet with an additional constraint, or perhaps to a spin 2 multiplet. However, it is not clear that either of these multiplets would make sense in the interacting case and neither is it clear which additional constraints would need to be imposed.

5 Concluding Remarks

To summarise the preceeding sections, we have shown that the embedding equation (6), or more precisely its linearised form (13), determines various worldsurface supermultiplets. These supermultiplets can be on-shell, off-shell irreducible or off-shell reducible. In the first case we arrive directly at the equations of motion while in the other two cases additional constraints are needed to get irreducibility and/or the equations of motion. We expect that these additional constraints should involve various forms on both the target space and the worldsurface. For Type II branes and L-branes these equations should be of the type given in equation (20) while in the Type I case an appropriately modified pull-back of the target space $(p+1)$-form field strength should vanish, as discussed for various cases in [20, 8, 24, 23].

The main focus of attention in this paper has been the linearised brane equation (13) and the resulting supermultiplets. However, the full non-linear theory can be studied using equations (6) and (9) as a basis. Moreover, non-trivial target spaces can be incorporated straightforwardly. The method should therefore be powerful enough to determine the full non-linear structure of the eleven-dimensional fivebrane, at least at the level of the equations of motion. This will be the subject of a forthcoming article. We note here that, in the case of Type II branes, the departure of the embedding from involving a straightforward adapted frame is a reflection of the Dirac-Born-Infeld structure of the bosonic action. In addition, the induced geometry on the worldsurface will play a more important rôle in the non-linear theory. Our expectation is that in many cases this geometry will correspond to off-shell conformal supergravity. This is not unreasonable since the induced supergravity multiplet is composite, and hence off-shell, even when the equations of motion of the brane are satisfied.

We have commented earlier on the difficulties that have been encountered in constructing doubly supersymmetric actions for various branes. The analysis we have given shows clearly that, for those branes for which the master constraint leads to on-shell multiplets, actions involving a Lagrange multiplier to impose this constraint will not work; one will inevitably encounter additional unwanted propagating degrees of freedom. For those branes which have irreducible off-shell worldsurface multiplets, on the other hand, one might hope that such actions might work. We note that our list includes all the examples of this type which have already been found but that there are several new possibilities[1]. We further remark that finding actions for many of the on-shell branes looks to be a difficult task. For example, the Type II branes in $D = 10$ have worldsurface multiplets which are maximally supersymmetric Maxwell multiplets for which no off-shell extensions (leading to Lagrangians) are known. In the case of fivebranes in $D = 11$ and $D = 7$ the world surface multiplets involve self-dual antisymmetric tensors, and it is well-known that there are severe problems in formulating Lorentz covariant actions for such objects even before one takes off-shell supersymmetry into account. On the other hand, we note that, leaving aside the $D = 6$ heterotic string for which the wordsurface supermultiplet is off-shell, the codimension 4, Type I branes all involve hypermultiplets in various guises. The hypermultiplet is an on-shell multiplet but in this case it is known how to go off-shell by using harmonic superspace. We might therefore conjecture that actions for these branes could be found by enlarging the worldsurface to include the harmonic variables.

The branes we have considered here correspond in many cases to soliton solutions of supergravity theories. It would therefore be interesting to see if there are solitons related to the proposed L-branes and the $D = 7$ fivebrane. (In the latter case, there is a candidate soliton [32], which arises in the gauged $D = 7, N = 1$ supergravity with a topological mass term.) In particular, it would be interesting to see if one could approach this topic from a superspace perspective as it might help one to understand the close relationship between the target space and worldsurface points of view. A related topic is the question of branes with less than one-half supersymmetry. In particular, it would be interesting to see how the intersecting branes [**?**] found in $D = 11$ would fit into the geometrical picture advocated here.

E.S. would like to thank the International Center for Theoretical Physics in Trieste for hospitality.

[1] The $D = 3, N = 2$ string considered in [31] is a double dimensional reduction of the $D = 4, N = 1$ supermembrane

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