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World-Sheet Duality, Space-Time Foam, and the Quantum Fate of a Stringy Black Hole

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Abstract

We interpret Minkowski black holes as world-sheet *spikes* which are related by world-sheet *duality* to *vortices* that correspond to Euclidean black holes. These world-sheet defects induce defects in the gauge fields of the corresponding coset Wess-Zumino descriptions of spherically-symmetric black holes. The low-temperature target space-time foam is a Minkowski black hole (spike) plasma with confined Euclidean black holes (vortices). The high-temperature phase is a *dense* vortex plasma described by a topological gauge field theory on the world-sheet, which possesses enhanced symmetry as in the target space-time singularity at the core of a black hole. Quantum decay via higher-genus effects induces a back-reaction which causes a Minkowski black hole to lose mass until it is indistinguishable from intrinsic fluctuations in the space-time foam.

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1 Introduction and Summary

The reconciliation of gravity with quantum mechanics raises problems at several different levels. One is the meaningful calculation of quantum gravitational corrections to scattering processes in a flat space-time background. Another is the treatment of quantum effects in a topologically non-trivial background, such as a black hole, with an event horizon and a singularity. Then there is the problem of treating microscopic quantum gravitational fluctuations in the space-time vacuum, i.e., space-time foam. Next there is the phase structure of space-time, and the possibility that a new, more symmetric phase may appear at high temperatures and/or close to singularities. And so on ....

String theory is sufficiently ambitious to claim to solve these and all other problems of quantum gravity. So far, progress has been more modest. However, it has been shown that string theory gives finite results in each order of perturbation theory [1], so that the quantum corrections to scattering in flat space-time are indeed tamed. The discovery of a black hole solution of string theory in a two-dimensional space-time [2], which can equally well be regarded as a spherically-symmetric solution in a four-dimensional space-time [3], has opened the way to the study of quantum effects in a topologically non-trivial space-time background. We have shown that quantum coherence is maintained in a stringy black hole background [4], thanks to an infinite set of gauged [4, 5] W-symmetries [6] and associated conserved charges [4], which we have called W-hair. We have also shown [7] how a higher double W-symmetry is realized at the singularity at the core of the stringy black hole, and is broken down to the remaining exact W-symmetry by an infinite generalization of the Berezinsky-Kosterlitz-Thouless [8] vortex condensation mechanism. The symmetric phase at the singular core of the black hole is described by a topological gauge field theory on the world-sheet [2, 9], whilst the symmetry-breaking corresponding to the generation of a non-singular space-time metric is akin [7] to the condensation of matter fields such as quarks in QCD [10].

In this paper we build on the above results to discuss the nature of space-time foam and the phase structure of space-time. We show that whereas world-sheet vortices (topological solitons) correspond to Euclidean thermal black holes, the dual world-sheet "spikes", which are non-topological solitons correspond to Minkowski black holes. We discuss in both cases the corresponding "monopole" configurations of the associated gauge fields in their respective Wess-Zumino coset model representations. We then show that space-time has a low-temperature phase in which the vortices are confined and there is a plasma of spikes on the world-sheet, i.e., Minkowski black holes in target space. Any macroscopic black hole decays via the quantum effects associated with higher genera [11], its mass decreasing via back-reaction until it is indistinguishable from a quantum-mechanical fluctuation in the low-temperature vacuum. There is a high-temperature phase characterized by spike confinement and a high-density plasma of vortices, in which the higher stringy symmetry is restored as at the core of a black hole. These results are derived explicitly in a two-dimensional target space-time, but we give naive entropy arguments suggesting a similar phase structure for four-dimensional target space-time.

2 Dual Defects on the World-Sheet

In this section we introduce the dual pair of types of defect that can appear on the world-sheet, namely vortices and "spikes" associated with "monopoles" in different auxiliary gauge fields, and display their respective interpretations as Euclidean and Minkowski black holes in target space-time.

Vortices: These are solutions X_v of Green function equations of the following type [12, 13]:

$$\partial_z \overline{\partial}_z X_v = \frac{i\pi q_v}{2}[\delta(z-z_1) - \delta(z-z_2)] \tag{1}$$

which corresponds to a vortex centred at z_1 and an antivortex centred at z_2, and we have made a stereographic projection of the lowest-genus world-sheet sphere onto the complex z-plane. There is always zero net vorticity on the compact world-sheet of the closed string, so such defects always appear in vortex/antivortex pairs. If the vortex is located at the origin (South Pole) and the antivortex at infinity (North Pole), the corresponding σ-model coordinate solution of (1) is

$$X_v = q_v Imlnz \tag{2}$$

It is clear from this representation that in order for X_v to be single-valued, it must have period 2π, and hence the vortex charge q_v must be an integer.

Since they require periodicity of the world-sheet scalar field, vortices are topological solitons that appear only in models with compactified target space-time dimensions. The application that interests us is to Euclidean black holes in equilibrium with a heat-bath, that are described by a periodic imaginary time coordinate. In this application, the charge q_v of the vortex is proportional to the square root of the black hole mass. The vortex quantization condition implies that the Euclidean black hole cannot lose any mass, as should be expected from its equilibrium with a heat bath (reservoir).

To see formally the space-time interpretation of a world-sheet vortex as a Eucldidean black hole, we rewrite the vortex configuration θ as

$$e^{2i\theta} = \frac{z}{\bar{z}} \tag{3}$$

and complexify the phase by introducing a real part r, which is defined by the following embedding of the world-sheet in a two-dimensional target space spanned by r and θ [14],

$$z = (e^r - e^{-r})e^{i\theta} \tag{4}$$

The induced target-space metric is inferred from the world-sheet arc-length, which is $dl = \frac{dz}{1+z\bar{z}}$ after the stereographic projection, and the infinitesimal Euclidean displacement $d\bar{z}$ induced by the r-coordinate. The result is a Euclidean black hole with a coordinate singularity at $r = 0, \theta = 0$, which corresponds to the position of the world-sheet vortex, and

$$ds^2_{target} \equiv \frac{dzd\bar{z}}{1+z\bar{z}} = dr^2 + tanh^2 r d\theta^2 \tag{5}$$

is the target space-time line element.

Because of the representation of the Euclidean black hole as a coset $\frac{SL(2,R)}{U(1)}$ Wess-Zumino (WZ) model [2], the world-sheet vortices induce gauge defects of the underlying world-sheet gauge theory. The compactness of the gauged $U(1)$ subgroup and the non-triviality of its first homotopy class $\Pi_1(U(1)) = \mathbf{Z}$ suggest the identification of the vortex configurations with topologically non-trivial gauge field configurations of the gauged WZ model. Indeed, eliminating the non-propagating gauge field A of the gauged WZ model using its equations of motion yields

$$A_z = -\frac{u\partial_z(a-b) - (a-b)\partial_z u}{(a+b)^2} \tag{6}$$

and a similar expression for $-A_{\bar{z}}$ (with z replaced everywhere by $\bar{z}$). Here, $u = sinhrsin\theta$, $a = coshr + sinhrcos\theta$, and $b = coshr - sinhrcos\theta$. At the singularity $r \to \varepsilon$, whilst θ is free and non-zero, and hence $u \to \epsilon sin\theta$, $a - b \to 2\epsilon cos\theta$. Thus the gauge potential (6) becomes that of a *singular* gauge transformation,

$$A_z \to \epsilon^2 \partial_z \theta \tag{7}$$

which has the interpretation of an $U(1)$ *gauge monopole*. Its topologically non-trivial behaviour follows from its angular dependence, as is apparent from the embedding (4). The argument of z winds non-trivially around the compact dimension θ, thereby producing a cut in the two-dimensional manifold with the form of a Dirac string. Thus the interpretation of the world-sheet vortex configuration leading to a Euclidean black hole as a topologically non-trivial $U(1)$ gauge field configuration in the WZ model.

"Spikes": These are solutions X_m of Green function equations of the following type [15, 13]:

$$\partial_z \overline{\partial}_z X_m = -\frac{q_m \pi}{2}[\delta(z - z_1) - \delta(z - z_2)] \tag{8}$$

which corresponds to a "spike" at z_1 and an "antispike" at z_2. Once again, the compactness of the closed string world-sheet imposes zero net "spikiness", but in this case X_m is non-compact and hence aperiodic, so the spike charge q_m is not quantized *a priori* and the spike is a non-topological soliton. After stereographic projection of the South Pole on the sphere onto the origin and of the North Pole onto the point at infinity in the complex plane,

$$X_m = q_m Re ln z = q_m ln|z| \tag{9}$$

is the σ-model coordinate solution of (8).

Although the charges q_m of the spikes are not quantized when they are in isolation, the q_m *are* quantized in the presence of vortices on the world-sheet [13]. This can be seen by considering the interaction of a spike at the point z with a vortex at the point z' at finite temperature $T = \beta^{-1}$:

$$-\beta S_{eff}(z, z')_{int} = 2i\beta\pi q_v q_m Im ln(z - z') \tag{10}$$

The imaginary nature of the effective action (10) can be understood from the observation that the vortex (1) and spike (8) equations are related by analytic continuation (Wick rotation) in the respective charges $q_{v,m}$. Single-valuedness of the partition function requires that the phase on the right-hand side of (10) also be single-valued, and therefore imposes the following quantization condition on the spike charge:

$$2\pi\beta q_v q_m = integer \tag{11}$$

The dual nature of vortices and spikes suggested in (11) is discussed in ref. [13] and in the next section. For the physical application to black holes that is of interest to us, the minimal vortex charge $q_v = 1$ is allowed, so the quantization condition (11) becomes

$$2\pi\beta q_m \equiv e = integer \tag{12}$$

at finite temperature $T = \beta^{-1} \neq 0$.

In our application, spikes are to be interpreted as Minkowski black holes. This is to be expected from the fact that spikes are non-compact configurations on the world-sheet. To see this one again has to consider an embedding of the world-sheet in target space-time which is consistent with conformal invariance. It turns out that the appropriate one is

$$|z|^2 = -uv \qquad : \qquad u = e^{R+t}, v = -e^{R-t} \tag{13}$$

where R is a spike configuration of unit charge , $R = ln|z|$. Taking into account the stereographic projection of the sphere onto the complex plane, R expresses a spike-antispike pair located at the origin (South Pole) and infinity (North Pole) respectively. The target space-metric is that of a Minkowski black hole . In terms of the embedding in R, t non-compact coordinates it reads:

$$ds^2 = \frac{dz d\bar{z}}{1 + z\bar{z}} = dr^2 - tanh^2 r dt^2 \qquad : \qquad R = r + ln(1 - e^{-2r}) \tag{14}$$

The coordinate singularity at $r = 0$, which corresponds to the position of the spike at $z = 0$ on the world-sheet, is the horizon of the black hole. In this way spikes on the world-sheet are responsible for generating event horizons in a Minkowski space-time. To pass the horizon one needs analytic continuation in the Kruskal-Szekeres sense [16]. This can be achieved with the embedding (13) but using the u, v coordinates, where the target metric takes the form

$$ds^2 = -\frac{dudv}{1 - uv} \tag{15}$$

These are actually the natural σ-model coordinates appearing in the WZ description, which makes use of gauging the non-compact subgroup $O(1,1)$ of $SL(2,R)$ [2].

The latter provides a consistent description of the singularity using a well-defined topological field theory model. Reparametrizing the singularity by $w = lnu = -lnv$ one ends up with a topological gauge theory on the world-sheet [2]

$$S_{CS} = i\int d^2z\sqrt{h}\frac{k}{2\pi}w\varepsilon^{ij}(F(A))_{ij} + ... \tag{16}$$

where h is the (non-trivial) world-sheet metric, $A_i, i = 1,2$ is the Abelian gauge field of the non-compact $O(1,1)$ subgroup, F_{ij} is its field strength, and the dots represent additional "matter" fields corresponding to space-time coordinates. Their generic form close to the singularity is [2]

$$-\frac{k}{4\pi}\int d^2z\sqrt{h}h^{ij}D_iaD_jb + (higher - order - terms) \tag{17}$$

where $D_i = \partial_i + iA_i, i = 1,2$ is a covariant derivative, and $ab + uv = 1$.

In the neighbourhood of the singularity it can be shown using the equations of motion that the gauge field A has the form

$$A_z = -A_{\bar{z}} = -\frac{\partial_z w}{2(1 - uv)} \tag{18}$$

Its analyticity properties depend therefore on the nature of the w field. We shall now argue that the relevant configuration is a *w-monopole* where the w field is singular, in which case the total flux of the A field through the compact world-sheet surface vanishes. To this end, we first note that the topological term (16) can be thought of as the dimensional reduction of a three-dimensional Abelian Chern-Simons term

$$i\int_{\Sigma\times S^1} d^3xA(z,\bar{z},\tau)\wedge F(A(z,\bar{z},\tau)) \tag{19}$$

where S^1 is the manifold of a homotopic extension variable τ which we will use later as a 'pseudo-temperature' to describe the phase structure of space-time, and to parametrize adiabatic flow in an ensemble of Minkowski black hole models. The system (19) becomes effectively two-dimensional when an adiabatic condition of periodicity in τ is imposed on the fields a, b and A and the circumference $2\pi\beta$ of the homotopic S^1 is allowed to become large [1]. The singular parameter w is related to $A(z,\bar{z},\tau)_\tau \equiv A(\Sigma,\tau)_\tau$ but there is no formal reason to assume periodicity in τ for the homotopically extended $A(\Sigma,\tau)_\tau$. The w field which appears in (16) is viewed as a β-independent ensemble average, $\frac{1}{\beta}\int_0^\beta d\tau A_\tau$. The A_τ field is allowed to have non-trivial boundary conditions of the monopole type $A(\Sigma,\beta) = A(\Sigma,0) + \int_0^\beta d\tau\partial_\tau\theta = A(\Sigma,0)+\Lambda$, and the coefficient of the three-dimensional Chern-Simons term is a β-dependent parameter, $\tilde{k}$, such that $\tilde{k}\beta$=const=k. This is allowed in Abelian gauge theories, since there is no *a priori* quantisation condition. Consequently, the Chern-Simons term (16) is not invariant under large gauge transformations but changes as

$$\delta S_{CS} = i\frac{\tilde{k}}{\pi}\Lambda\int_\Sigma \epsilon_{ij}\partial_i A_j \tag{20}$$

In a path integral formalism, quantum fluctuations of monopoles are not consistent with fixed boundary conditions [18], so one has to integrate over the whole range of the gauge parameter Λ. In our case the gauge field A_τ might be considered as the *continuum* version of a corresponding phase variable defined on a discretized three-dimensional manifold. The circle S^1 is viewed as a discretized real line with spacing ϵ and periodic boundary conditions. Thus Λ takes values on the interval $[-\frac{\pi}{\epsilon},\frac{\pi}{\epsilon}]$, so in the limit $\epsilon\to 0$ it extends over the whole range of real numbers. Hence, one gets the following contribution to the path-integral

$$\int_{-\infty}^{+\infty} D\Lambda e^{i\frac{\tilde{k}}{2\pi}\Lambda\int_\Sigma \epsilon_{ij}F_{ij}} = \delta[\frac{\tilde{k}}{\pi}\int_\Sigma dA] \tag{21}$$

which implies that configurations of A with non-zero total flux through the compact world-sheet surface are suppressed.

This result has another elegant intepretation in terms of self-dual Chern-Simons Higgs models [19], to which the theory (16),(17) can be mapped. As shown in [20], the Minkowski black hole corresponds to a self-dual non-topological soliton in the Bogomolny limit [2]. In this limit, the magnetic field of the Chern-Simons gauge potential A vanishes [19, 20] in the symmetric vacuum which corresponds to the singularity [7]. Moreover, the Bogomolny limit corresponds to $N=2$ supersymmetry [22, 23] and hence to the interpretation [2, 9] of target-space singularities as (twisted) $N=2$ supersymmetric fixed points, which leads to an enhancement [7] of the symmetry of the target-space theory.

[1] As explained in the next section it is the low-temperature phase that allows for free Minkowski black holes which have the non-compact gauge description (16), so this adiabatic picture is appropriate. It should be noticed that dimensional reduction also occurs in effective lagrangians of *hot local* field theories. However this situation is not applicable to string theory due to the existence of winding modes [17]. Hence our adiabatic interpretation seems more appropriate in this context.

[2] This is consistent with its extreme Reissner Nordstrom character and justifies the 'no-net-force' condition[21] used later in our discussion of multi-defect solutions and space-time foam.

To recapitulate: there are descriptions of two dual types of world-sheet defects, interpreted as Euclidean and Minkowski black holes, as gauge monopoles. In the Euclidean case there are monopole-like configurations of the A field of the WZ model that describe coordinate singularities. In the Minkowski case, a topological gauge field theory description of the singularity is available in the form of a mixed Chern-Simons term. In this case there are no non-trivial configurations for the A field, but still there is a monopole-like gauge potential for the description of the singularity, that of the w field.

3 The Phase Structure of Space-Time

For the physical applications to the phase structure of target space-time and to the nature of space-time foam which we make in this and subsequent sections, we need to study the statistical distribution of multiple world-sheet defects. This can be derived from an action principle associated with the following partition function, where we have made a stereographic projection of the sphere of radius r onto the complex plane, inducing an effective metric $g(z)$ defined in [13]:

$$Z = \int D\tilde{X} exp(-\beta S_{eff}(\tilde{X})) \tag{22}$$

where $\tilde{X} \equiv \beta^{\frac{1}{2}} X$, and

$$\begin{aligned} \beta S_{eff} &= \int d^2z[2\partial\tilde{X}\overline{\partial}\tilde{X} + \frac{1}{4\pi}[\gamma_v \epsilon^{\frac{\alpha}{2}-2}(2\sqrt{|g(z)|})^{1-\frac{\alpha}{4}} : cos(\sqrt{2\pi\alpha}[\tilde{X}(z) + \tilde{X}(\bar{z})]) : \\ &+ (\gamma_v, \alpha, \tilde{X}(z) + \tilde{X}(\bar{z})) \rightarrow (\gamma_m, \alpha', \tilde{X}(z) - \tilde{X}(\bar{z}))]] \end{aligned} \tag{23}$$

Here $\gamma_{v,m}$ are the fugacities for vortices and spikes respectively, and

$$\alpha \equiv 2\pi\beta q_v^2 \qquad \alpha' \equiv \frac{e^2}{2\pi\beta} \tag{24}$$

are related to the conformal dimensions $\Delta_{v,m}$ of the vortex and spike creation operators respectively, namely

$$\begin{aligned} \alpha = 4\Delta_v \qquad \alpha' &= 4\Delta_m \\ \Delta_m &= \frac{(eq_v)^2}{16\Delta_v} \end{aligned} \tag{25}$$

It is easily seen [13] that the effective action (23) is *invariant* under the simple duality transformation $q_v \longleftrightarrow e$, $\pi\beta \longleftrightarrow \frac{1}{4\pi\beta}$, which is thus an *exact* symmetry of the system. Indeed, systems of the form (23) are known to possess [24] extended (complex) duality symmetries $SL(2, \mathbf{Z})$ which could explain the quantum Hall effect hierarchy [25]. As discussed in [20], the Minkowski black hole case is field-theoretically equivalent to a Hall conductor, motivating further studies of the implications of such world-sheet symmetries for the physics of black holes. We also note the connection of the cosine terms in (23) with the gauge field theory description of the defects discussed in the previous section. For instance, in the case of Minkowski black holes, the cosine terms in (23) correspond after fermionization [26] to interactions of a massive Thirring model. A similar result holds for the fermionized Chern-Simons lagrangian coupled to scalar Higgs fields around the asymmetric vacuum [27], which in our case corresponds to the Minkowski black hole horizon (position of the spike in the embedding (13)). In this case, the Chern-Simons term can be represented by an infinity of higher order fermion interactions, of which the four-fermion one is a relevant operator in the renormalisation group sense [27, 20].

Since the relevant dynamics of the partition function (22) is that on the two-dimensional world-sheet, only generalized Berezinsky-Kosterlitz-Thouless phase transitions [8] are possible. These may be induced by either vortices or spikes (or both) when the corresponding anomalous dimension(s) become equal to unity. When $\Delta_{v,m} > 1$ the corresponding charges are confined and screened, when $\Delta_{v,m} = 1$ the corresponding operator is marginal and a phase transition occurs, and when $\Delta_{v,m} < 1$ it is relevant and the corresponding charges dissociate to form a plasma. Such phases have been discussed previously in other applications, but the physical interpretation here as the phase structure of space-time is new.

We see from the expression (25) for the conformal dimension Δ_m that free spikes occur when the temperature

$$T < T_m^c = (\beta_m^c)^{-1} = \frac{8\pi}{e^2} \tag{26}$$

and we interpret this phase as containing a plasma of free Minkowski black holes. The corresponding expression for the conformal dimension Δ_v tells us that free vortices occur when the temperature

$$T > T_v^c = (\beta_v^c)^{-1} = \frac{\pi q_v^2}{2} \tag{27}$$

and we interpret this phase as containing a plasma of free Euclidean black holes : T_v^c is conventionally known as the Hagedorn temperature [28]. On the other hand, we see that when $T > T_m^c$ the Minkowski black holes (spikes) are confined, whereas the Euclidean black holes (vortices) are confined when $T < T_v^c$. There are three possible phase diagrams depending on whether $eq_v >, =$ or < 4.

In each case there is a low-temperature phase, in which we live, in which Minkowski black holes form a free plasma and Euclidean black holes (vortices) are confined, and a high-temperature phase, possibly realized in the very early Universe, in which Euclidean black holes form a free plasma and Minkowski black holes (spikes) are confined. In this sense space-time could have seemed Euclidean in the early Universe. We know from standard results on the renormalization group behaviour of vortex/spike systems that whereas the vortex fugacity remains small in the low-temperature phase, it is driven to large values in the high-temperature phase. This means that the high-temperature phase is a *dense* vortex plasma. Since the core of a Euclidean black hole is characterized by a compact $U(1)$ topological gauge field theory on the world-sheet, which has a higher super-W symmetry with a bosonic $W_{1+\infty} \otimes W_{1+\infty}$ subsymmetry [7], we expect that this symmetry would have been present in this early dense high-temperature phase. This expectation is supported by the observation [17] that the number of effective degrees of freedom at high temperatures is characteristic of a two-dimensional field theory.

In the case $eq_v > 4$, in addition to this high-temperature phase and the familiar low-temperature phase with free Minkowski black holes and confined vortices, there is an intermediate range of temperatures $\beta_v^c < \beta < \beta_m^c$ in which neither type of black hole, i.e., neither free vortices nor free spikes, exist, but both vortices and spikes are confined. This intermediate phase disappears when $eq_v = 4$. Finally, when $eq_v < 4$ a new intermediate phase appears when $\beta_m^c < \beta < \beta_v^c$, in which black holes of both Minkowski and Euclidean types coexist freely, i.e., neither vortices nor spikes are confined. In each of these cases, the appearance of a vortex condensate breaks the bosonic $W_{1+\infty} \otimes W_{1+\infty}$ subsymmetry of the initial super-$W_{1+\infty}$ down to a residual single $W_{1+\infty}$, responsible for the maintenance of quantum coherence [4], and discrete massless modes appear [7], like pions in QCD below the quark-hadron phase transition.

Which of eq_v >, =, < 4 occurs in Nature? By analogy with the XY model, one would expect the dominant vortex configuration to be that with $q_v = 1$. This expectation is confirmed by the following simple free energy argument. In the stringy space-time interpretation of the vortex, the Euclidean time variable is identified with a conformal field $\tilde{X}$ compactified on a circle, that describes the "matter" part of a Liouville-matter theory, with kinetic term normalized to

$$\frac{1}{2\pi} \int d^2 z \partial \tilde{X} \overline{\partial} \tilde{X} \tag{28}$$

The energy of an isolated vortex (Euclidean black hole) of charge q_v is then $\pi q_v^2 ln(\frac{R}{a})$, where R is the size of the system and a is an ultraviolet cutoff, and its entropy is $2ln(\frac{R}{a})$, where the prefactor is determined by the two-dimensionality of the world-sheet. Hence the free energy of an isolated vortex is

$$f_s = (\pi q_v^2 - 2) ln(R/a) \tag{29}$$

indicating that a vortex plasma is energetically preferred when $q_v = 1$. A similar result holds for the spike configurations of the Liouville field ϕ of quantum gravity. The dominant terms in the effective action for this field have the form

$$S_L = \frac{25 - D}{96\pi} \int d^2 z [(\nabla \phi)^2 + \phi R^{(2)} + ...] \tag{30}$$

where D is the central charge of the matter theory, i.e., the target space dimensionality, and the dots represent deviations from flat target space backgrounds whose detailed form does not affect our arguments. Note the unconventional D-dependent normalization of the kinetic term in (30). In the string case with $D > 25$, one should continue analytically ϕ to $i\phi$ and view the Wick-rotated ϕ as proportional to target time. Recalling that the Euler characteristic of the sphere is $\int R^{(2)} = 8\pi$, we observe that the action (30) is periodic for $\phi \to \frac{12n}{25-D}\phi$, with n an integer. Thus a convenient normalization to discuss spike solutions in Liouville theory, corresponding to a Wick rotation of vortices, is

$$\tilde{\phi} \equiv \frac{25-D}{12}\phi \tag{31}$$

One can then introduce a pseudo-temperature

$$T_{ps} = (\beta_{ps})^{-1} \qquad : \qquad \beta_{ps} = \frac{3}{\pi(D-25)} \tag{32}$$

in terms of which one has the following quantization condition analogous to (12) :

$$2\pi\beta_{ps}q_m = e = integer \tag{33}$$

Taking account of the unconventional normalization of the spike field, we see that the self-energy of such a spike is

$$U = \frac{3}{(25-D)}q_m^2 ln(R/a) \tag{34}$$

where R is the size and a the ultraviolet cutoff of the system as in the vortex case, and the entropy of an isolated spike is also $2ln(R/a)$. Using the quantization condition (33), we see that the free energy is again minimized for the physical case $D = 1$ when $e = 1$. Taking into account the normalization (31), we see that this corresponds to the charge-2 spikes of Cates [15]. We conclude that the lowest-lying vortex and spike configurations are

$$q_v = e = 1 \tag{35}$$

implying that the case $eq_v < 4$ describes the black hole problem. There is an intermediate phase where free Euclidean (vortex) and Minkowski (spike) black holes *both* occur.

It should be stressed that the above analysis applies to Minkowski black holes of arbitrary mass. This can be seen formally by considering the case when the background metric in (30) is flat but with an overall scale factor $e^{-2\phi_0}$, corresponding to a constant shift in the dilaton. This factor expresses the (dimensionless) ratio of the black hole mass renormalized by the pseudo-temperature to an initial scale set by the choice of the constant dilaton shift. The above arguments are easily modified by introducing factors of $e^{2\phi_0}$ into the definition of β_{ps}^{-1} (32) and q_m (9).

We would also like to stress that the above arguments for the three phases of the $eq_v < 4$ case, a high-temperature phase with a dense plasma of Euclidean and confined Minkowski black holes, an intermediate-temperature phase with both types free, and a low-temperature phase with a plasma of Minkowski black holes and confined Euclidean black holes that breaks a higher symmetry, are likely to apply to the physical situation of three target space dimensions. The arguments that the dominant vortex and spike configurations would be those with $q_v = e = 1$ do not depend on the dimensionality of target space, but only on the fixed total central charge in the case of the spike. The self-energy of any individual vortex or spike is determined by properties of the world-sheet, notably its two-dimensionality. The numbers of embeddings of a vortex or spike configuration in the physical space would have constant (i.e., R- and a-independent) extra numerical factors, and so would not affect the dominant logarithmic behaviour in the entropy, which is also determined by the dimensionality of the world-sheet. Therefore the above naive free-energy arguments on the circumstances when vortex or spike condensation occurs go through for any dimension of target space. In such a case, the relevant expression for the free energy of an isolated spike would read

$$f_D = 2[\frac{e^2}{8\pi\beta_{ps}(D)} - 1]ln(R/a) \tag{36}$$

and obviously corresponds (c.f. (32) after the replacement $D - 25 \to 25 - D$) to a pseudo-temperature smaller than the critical one ($D = 1$), and, therefore, to the region of the phase-diagram where there is a proliferation of spikes as we discussed above. It should be stressed, though, that these arguments, although suggestive, do not prove the existence of exact conformal field theory models for the description of higher-dimensional strings ($1 < D < 25$) in singular space-time backgrounds.

4 The Quantum Fate of a Minkowski Black Hole

In the light of the above results, we are now able to discuss the quantum evolution of a Minkowski black hole induced by higher-genus corrections to the effective action, and its quantum fate, which we argue is to become lost among the quantum fluctuations intrinsic to space-time foam. We first recall that the mass of a Minkowski black hole is given by [2, 16]

$$M_{bh} = \sqrt{\frac{2}{k-2}}e^{\phi_0} \tag{37}$$

where k is the level of the $SL(2,R)/O(1,1)$ coset model, which is related to the central charge: $c = \frac{3k}{k-2}$ so that $c = 26$ corresponds to $k = 9/4$, ϕ_0 is a constant of integration that reflects the asymptotic value of the dilaton field

$$\Phi = 2lncosh(r) + \phi_0 \tag{38}$$

and the metric is

$$ds^2 = e^{-2\phi_0}[dr^2 - tanh^2 r d\theta^2] \tag{39}$$

We have argued previously [11] that a Minkowski black hole is classically stable for any value of ϕ_0. Particle emission from black hole is an intrinsically quantum phenomenon that appears in string theory only when higher-genus effects are taken into account, and we have displayed explicitly the imaginary part of the one-loop effective action of the $SL(2,R)/O(1,1)$ coset model that corresponds to non-thermal Minkowski black hole decay in any space-time dimension [11].

Since perturbative corrections cannot change the total central charge c, higher-genus decay effects cannot change the level k, but can renormalize the effective value of the integration constant ϕ_0. Physically, we would expect in the light of the mass formula (37) that they would tend to decrease ϕ_0. In the limit of large r at fixed ϕ_0, or at fixed r for $\phi_0 \to -\infty$, the dilaton field ϕ (38) becomes linear in r, and the metric (39) resembles that of flat space. Thus the Minkowski black hole looks asymptotically like the flat $c = 1$ string model.

The dynamics can be described using the three-dimensional homotopic extension of the effective gauge theory introduced in section 2. The rôle of the radius of the extra dimension, the homotopic scale β, provides us with a scale whose flow determines the black hole decay, in analogy with the finite-size scaling [29] of two-dimensional systems formulated on a strip of width L. Away from renormalisation group criticality [30], the free energy of the finite-size system is related to the "bulk" free energy (corresponding to $L \to \infty$) through the relation

$$F_L = F_{bulk} + \frac{V}{L^2}[C(g) + O(\epsilon ln L)] \tag{40}$$

where V is the strip volume, ϵ is a covariant ultraviolet cut-off, $\{ \text{ g } \}$ is a set of coupling constants, and $C(g)$ is related to the Zamolodchikov c-function [31], which at the fixed points becomes the central charge of the theory. Relation (40) defines an 'effective' central charge which determines the critical properties of the finite-size system [30]. In our case. the rôle of the scale L is played by the homotopic scale β leading to a definition of an effective central charge in the Liouville sector which varies with the homotopic scale β. This Liouville-sector central charge deficit is compensated by an equal and opposite variation in the matter sector of the theory at *higher genera*, so as to maintain the conformal invariance ($c_{total} = 26$) of the complete system. It should be stressed that although this balancing of the central charges in the Liouville and matter sectors is familiar from the theory of non-critical strings [32], in our case there is an important difference. In the generic non-critical string theory with $D > 25$, this balance occurs already at string tree level due to ordinary tachyonic instabilities related to relevant operators to drive the flow. In our case, it is a higher genus effect that drive the flow.

This is easily seen by the observation [33] that non-critical string theory formulated on a world-sheet torus can be consistently regularised with respect to *both* modular and ultraviolet infinities at the cost of introducing "spiky" configurations in the dilaton field. The generic result of [33] is that a covariant heat-kernel regularization procedure with a cut-off ϵ yields for the free string partition function in d space-time dimensions

$$Z_{torus} \propto (\frac{V}{4\pi\epsilon})^{-\frac{1+d}{2}} exp[-(\frac{1}{2}d-1)\frac{V}{4\pi\epsilon}\Psi(\alpha_c)] \tag{41}$$

where $\Psi(\alpha_c)$ is a function evaluated at a certain saddle point α_c, whose detailed form is not important for our subsequent arguments. In two dimensions the tachyonic infinities are absent but there are logarithmic infinities ('spikes' $e^{(1+\frac{d}{2})ln\epsilon}$) in the torus partition function which cannot be absorbed by the usual renormalization of the torus background coupling constants of the σ-model. They are absorbed by renormalization of the σ-model couplings on the sphere [34]. They are divergent contributions to the dilaton field that couples to the Euler characteristic of the sphere. This produces a renormalization of the conformal 'anomaly' of the non-critical string in both the matter and the Liouville sectors [33]. This enables higher-genus instabilities of the black hole space-time to be incorporated consistently in a flow where the total cental charge remains zero, and we know this happens thanks to the exact conformal invariance of the matrix model. This discussion clearly applies only to the non-compact Minkowski case, which is the only one that can allow variation of the black hole mass. This picture is perfectly consistent with the one advocated in [11] from a two-dimensional string effective action (S-matrix amplitude) point of view.

This balance of central charges between the Liouville and matter sectors during the evaporation/decay of the stringy black hole constitutes precisely what in general relativity would be called the *back reaction* of quantum effects on the gravitational dynamics. The irreversibility of black hole decay can be understood [20] by analogy with the isomorphic theory of the Quantum Hall Effect as a manifestation of the violation of parity and time-reversal induced by the three-dimensional Chern-Simons term (19). When applied to the initial singularity, with the rôle of target time played by the Liouville mode, such a picture would provide an explanation of the arrow of target time as well.

5 Conclusions and Prospects

We have shown in this paper how the two dual species of defects on the world-sheet, vortices and spikes, are related to black holes of Euclidean and Minkowski signature respectively, and in both cases related to gauge defects in the WZ coset model descriptions. Taking over known results on Berezinsky-Kosterlitz-Thouless phase transitions, we have argued that space-time has three phases : a high-temperature phase with a Euclidean black hole (vortex) plasma and confined Minkowski black holes (spikes), an intermediate temperature phase with free black holes of both signatures, and a low-temperature phase with a plasma of Minkowski black holes (spikes) and confined Euclidean black holes (vortices). This last phase constitutes stringy space-time foam. Thermal Euclidean black holes have quantized winding number and cannot decay, whilst Minkowski black holes have a continuous mass spectrum and decay via coherent higher-genus quantum corrections which cause a back-reaction that reduces their masses until they become indistinguishable from the quantum-mechanical fluctuations intrinsic to space-time foam. Thus, in our view the quantum fate of a black hole is not to leave behind a stable remnant, nor to disappear completely, but "none of the above ", namely to become a nondescript quantum fluctuation in the vacuum.

The phase structure discussed above was derived in the case of a two-dimensional target space-time, but we have given arguments why it could also apply to the relevant four-dimensional case. Certainly, our understanding of black hole decay and back-reaction as coherent quantum phenomena applies to (at least) spherically-symmetric black holes in any number of dimensions. Nevertheless, it is desirable to acquire a deeper understanding of space-time foam in four dimensions, as well as the nature of the phase diagram in this case. A deeper understanding of the high-temperature phase with space-time symmetry restoration is also desirable, together with its embedding in a realistic cosmological context.

We are encouraged by the successes of string theory in providing solutions for many of the key problems in quantum gravity to believe that these outstanding questions now are also vulnerable to answers from string theory.

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