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The String Universe: High T_c Superconductor or Quantum Hall Conductor?

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Abstract

Our answer is the latter. Space-time singularities, including the initial one, are described by world-sheet topological Abelian gauge theories with a Chern-Simons term. Their effective $N = 2$ supersymmetry provides an initial fixed point where the Bogomolny bound is saturated on the world-sheet, corresponding to an extreme Reissner-Nordstrom solution in space-time. Away from the singularity the gauge theory has world-sheet matter fields, bosons and fermions, associated with the generation of target space-time. Because the fermions are complex (cf the Quantum Hall Effect) rather than real (cf high-T_c superconductors) the energetically-preferred vacuum is not parity or time-reversal invariant, and the associated renormalization group flow explains the cosmological arrow of time, as well as the decay of real or virtual black holes, with a monotonic increase in entropy.

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1 Introduction and Summary

Condensed matter physicists have recently been fascinated with conduction effects in two planar systems: Hall conductors [1] and high-T_c superconductors [2]. Both of these are described by QED in 2+1 dimensions interacting with matter that may be fermionic, bosonic or in general anyonic. At the other end of physics, it has been realized that Robertson-Walker-Friedmann cosmology [3] and black holes in string theory [4] are described by coset Wess-Zumino models with, in the interesting cases of 2-dimensional or spherically-symmetric 4-dimensional systems, an Abelian U(1) or O(1,1) gauge theory on the 2-dimensional world-sheet. For reasons which we explained recently [5] it is useful to make a homotopic extension of the world-sheet Abelian gauge theory action to a third dimension, an idea familiar from the Wess-Zumino term. Confronted with this convergence between the field-theoretical descriptions of string cosmology and condensed matter systems, it is natural to ask in more detail whether the string Universe bears a closer similarity to a Hall conductor or to a high-T_c superconductor.

The answer is a Hall conductor, as we have already suggested previously [5]. Early theories of high-T_c superconductivity suggested violations of parity P and time-reversal invariance T, and a zero-magnetic-field Hall effect [6, 7]. However, physical high-T_c superconductors exhibit neither feature: in particular, they conserve both P and T [8]. On the other hand, the stringy description of the neighbourhood of a space-time singularity involves in an essential way an Abelian Chern-Simons term that violates both P and T, as in the Hall effect at non-zero magnetic field [1]. This similarity between space-time singularities and Hall conductors extends to the underlying infinite-dimensional W-algebras [9]. In the case of singular stringy space-times, a $W_{1+\infty}$ algebra ensures the maintenance of quantum coherence [10], whilst a wedge subalgebra has been exhibited in connection with the non-singular Laughlin wave-functions of the Quantum Hall Effect [11]. As we show later, this can be elevated to the full $W_{1+\infty}$ algebra if Hall conductors with non-trivial topology are considered, such as annular or doped conductors. The convergence of these physical descriptions is supported by the observation that the $c = 1$ string model, which represents the spatially or temporally asymptotic limit of a full stringy singularity, can be regarded as an incompressible quantum fluid [12], as can Hall charge carriers [1].

Specifically, stringy singularities are described on the world-sheet by an effective Chern-Simons-Higgs theory which exhibits $N = 2$ supersymmetry. This means that they satisfy the Bogomolny bound [13] and correspond (in target space-time) to extreme Reissner-Nordstrom black holes [14]. It also means that they correspond to the zero-field Hall effect, which can be realized on a honeycomb lattice [6], but is not seen in the known high-T_c superconductors, as we have already commented. The enhanced symmetry at the black hole core contains a $W_{1+\infty} \otimes W_{1+\infty}$ algebra, whose breakdown away from the singularity to the coherence-preserving $W_{1+\infty}$ algebra is accompanied by the appearance of discrete massless modes [15]. Similar states should appear in analogous Hall conductors.

The $N = 2$ zero-field Hall effect theory is a fixed point of the renormalization group flow. The T- and P-violation in the generic Hall conductor correspond to the irreversibility of black hole decay and the arrow of cosmological time. We identify the Hall fraction with the level parameter $k = 9/4$ of the stringy black hole in the absence of additional matter. Hall conductors with different fractions can be understood as conductivity plateaux in the Laughlin hierarchy [16], and the expanding Universe can be regarded as sliding down this hierarchy [3], with the corresponding creation of matter degrees of freedom.

Corresponding to the above paragraphs, we discuss planar electron systems in section 2, the correspondence of a Hall conductor to a Minkowski black hole in section 3, and the rôle of $N = 2$ supersymmetry and the renormalization flow in section 4. Section 5 develops further the isomorphism between Hall conductors and Minkowski black holes, and finally section 6 correlates the irreversibility of the renormalization group flow, P and T violation in the Hall vacuum, the cosmological arrow of time, black hole decay and microscopic entropy growth.

2 Planar Electron Systems

Let us review the general features of such systems, emphasizing those characteristics relevant for our purposes. The action for QED_3 with fermionic matter is

$$S = \int d^3x[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ + eA\!\!\!/)\psi + \kappa\epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho + ...] \tag{1}$$

where we have allowed for the possibility of a Chern-Simons term which violates P and T but not PT. Such a term is absent from the fundamental Lagrangian, but can be generated by matter loops, for example non-relativistic electrons via a Pauli magnetic moment term

$$S_{Pauli} = \frac{ge\hbar}{4Mc}\sigma B(x) \qquad ; \qquad \sigma = spin - matrix \tag{2}$$

calculated in the adiabatic limit. Note that the effective action is supersymmetric if the gyromagnetic ratio g is 2 [17], as expected for elementary Dirac charge carriers.

The fermionic current obtained from (1) using the classical equations of motion is transverse to the direction of the applied electromagnetic field,

$$J_\mu = 2\kappa\epsilon_{\mu\nu\rho}\partial_\nu A_\rho \tag{3}$$

so the charge density is proportional to the applied magnetic field, and the transverse Hall conductivity is

$$\sigma_{xy} = \frac{\partial \rho}{\partial B} \tag{4}$$

The P- and T-violation inherent in the action (1) can show up as a non-zero current in the presence of a non-zero magnetic field - the usual Hall effect which occurs within separated Landau levels. However, there can also be a zero-field Hall effect in the form of a non-zero transverse conductivity (4) even in the limit $B \to 0$, when the Landau levels become degenerate. This is in principle a possibility in planar superconductors, thanks to the Meissner effect.

High-T_c superconductors are planar, and it is natural to enquire whether they exhibit such P- and T-violation. Many theories of high-T_c superconductivity exploit the existence of excitations with fractional statistics, anyons, in planar systems [18]. When starting from fermions or bosons, the description of anyons requires the introduction of a statistical Abelian gauge field a, which will in general have its own Chern-Simons term, as well as a mixed $a - A$ Chern-Simons term in the effective action. Such anyonic theories of high-T_c superconductivity do indeed violate P and T in general, and predict a zero-field Hall effect. Neither of these features are seen in known high-T_c superconductors [8], and such anyonic theories are therefore ruled out. P and T are, however, conserved if the charge-carriers are real fermions, which is predicted in one microscopic theory of high-T_c superconductivity [19]. This theory also predicts successfully the ratio of the gap Δ to T_c [19, 20].

Stringy singularities clearly resemble more closely the Hall conductor mentioned in the next-to-last paragraph: they are described (see the next Section) by Abelian world-sheet gauge theories with a (dimensionally-reduced) Chern-Simons term interacting with complex fermions. Anyons play no apparent role in the known singular string solutions, and there is no analogue of the statistical gauge field on the world-sheet. Moreover, just like the Hall electrons in each Landau level, which form an incompressible quantum fluid, there is an equivalent structure in the $c = 1$ string model [12], which describes the spatially- or temporally-asymptotic form of the singular stringy solution.

This physical similarity is reflected in the underlying group-theoretical structure. It has been known for some time that the $c = 1$ model and stringy black holes possess a global $W_{1+\infty}$ symmetry on the world-sheet [21, 22], which is elevated to a local symmetry in target space-time [10, 23]. This W-algebra contains an infinite Cartan subalgebra, whose associated charges constitute W-hair for the black hole which maintains quantum coherence [10]. A wedge subalgebra of a similar W-algebra has recently been identified in the integer Quantum Hall Effect [11]. However, because of physical regularity conditions on the Laughlin wave functions, the full W-algebra has not been obtained. The requirement of regularity would be relaxed in planar Hall conductors with non-trivial topology, such as an annulus. Indeed, as we shall see in more detail later on, an annular Hall conductor is an accurate model of a black hole in string theory. The latter is described by a spike-antispike configuration on the world-sheet, and these defects correspond to the inside and outside of the annulus. A more general multi-spike-antispike configuration, which describes a generic foamy state of target space-time with many microscopic black holes, can in principle be realized by doping a Hall conductor. The corresponding generalization of the Laughlin wave function need not be regular at any of the doping sites, and hence would realize a similar mathematical structure as well as underlying quantum fluid picture.

3 Correspondence to Minkowski Black Hole

We now demonstrate in more detail the connection of a Hall conductor to a Minkowski Black hole in string theory. We first recall that a Minkowski black hole is described on the world-sheet by an $SL(2,R)/O(1,1)$ Wess-Zumino coset model [4], which in the neighbourhood of the space-time singularity becomes a $O(1,1)$ topological gauge theory coupled to matter fields [4, 24] :

$$S_{eff} = -\frac{k}{4\pi}\int d^2z\sqrt{h}h^{ij}D_iaD_jb + i\frac{k}{2\pi}\int d^2zw\epsilon^{ij}(F(A))_{ij} + ... \tag{5}$$

Here w is the degree of freedom describing the singularity, the D_i, $i = 1{,}2$ are gauge covariant derivatives, $(F(A))_{ij}$ is the field strength of the Abelian $O(1,1)$ gauge potential, the bosonic fields a, b are $SL(2,R)$ coordinates, and the dots represent higher-order terms in the expansion around the singularity, which is represented by a spike on the world-sheet [5].

The correspondence to the planar electron systems of the previous section, which are described by Abelian (electromagnetic) gauge models coupled to fermions, becomes clearer if we regard the effective action (5) as the adiabatic approximation to a three-dimensional gauge theory. This limit is particularly relevant in view of the relation given in ref. [5] of space-time foam to multi-defect configurations on the world-sheet. The present-day physical phase described by a plasma of Minkowski black holes arises at low "temperature", defined to be the homotopic parameter of the third dimension compactified on a circle of radius $\beta = T^{-1}$. The three-dimensional effective gauge theory whose constant-"temperature" sections define the conformal WZ model corresponding to a Minkowski black hole is a scalar Higgs-Chern-Simons gauge theory [25]. Defining

$$\Phi \equiv \begin{pmatrix} a+b \\ a-b \end{pmatrix} \tag{6}$$

it can be written in the form

$$\frac{\tilde{k}}{4\pi}\int_{\Sigma^2\times S^1} d\tau d^2z\sqrt{h}h^{ij}D_i\overline{\Phi}D_j\Phi - i\frac{\tilde{k}}{2\pi}\int d\tau d^2z\epsilon_{\mu\nu\rho}A(\Sigma,\tau)_\mu\partial_\nu A(\Sigma,\tau)_\rho \tag{7}$$

where $\tilde{k} \equiv \frac{k}{\beta}$, and $\overline{\Phi} \equiv \Phi\sigma_3$, with σ_3 a 2×2 Pauli matrix. One can easily obtain equation (5) by making a dimensional reduction with respect to τ and eliminating the A_τ component of the gauge potential via its equation of motion. The theory (7) can then be written in terms of a single complex field by identifying $\overline{\Phi}$ with the complex conjugate of Φ.

To complete the model (7) we must specify the effective scalar potential. Since the approach to the singularity is equivalent to a symmetry-restoration process, as described in ref. [15], the effective potential should vanish when $\Phi = 0$. It should also have a symmetry-breaking minimum at some non-zero $|\Phi| = V$. On the other hand, if we want the three-dimensional quantum theory to be well-defined, we should require it to be renormalizable. The most general potential obeying all these requirements is

$$V(\Phi) = \frac{\alpha e^4}{\tilde{k}^2}(|\Phi|^2 - V^2)^2|\Phi|^2 \tag{8}$$

This corresponds to the Bogomolny limit of interest to us if $\alpha = 1/4\pi^2$ [25]. As already mentioned, in the case of black holes, the singularity corresponds to the symmetric state $\Phi = 0$, whilst the horizon corresponds to $\Phi = 1$, and hence $V = 1$ in the effective potential (8).

We now observe that the Minkowski black hole is a static solitonic configuration of the model (7,8) which satisfies the classical Euler-Lagrange equations:

$$\begin{aligned} D_1\Phi &= \mp iD_2\Phi \\ e\epsilon^{ij}\partial_i A_j \equiv eB &= \pm\frac{(8\pi e^2)^2}{\tilde{k}^2}|\Phi|^2(1-|\Phi|^2) \end{aligned} \tag{9}$$

It was shown in ref. [25] that the equations (9) possess topologically stable solutions for which $|\Phi| \to 1$ at large distances, and the corresponding magnetic flux is quantized. However, these solutions are not consistent with the stereographic embedding

$$|z|^2 = -uv \tag{10}$$

of the world-sheet into space-time, which leads to the target-space Minkowski black hole metric

$$ds^2_{target} = \frac{1}{1-uv}dudv \tag{11}$$

We see in (10), (11) that the origin of the stereographically-projected world-sheet corresponds to the horizon $uv = 0$ of the black hole, where $|\Phi| = 1$, since the $SL(2,R)$ coordinates obey $ab + uv = 1$. The embedding (10) actually describes only the interior of the horizon of the Minkowski black hole. Thus we can without loss of generality place the anti-spike describing the space-time singularity at the spatial world-sheet point at infinity. The Minkowski black hole is therefore described by a solution of the Higgs-Chern-Simons equations (9) in which the Higgs field Φ approaches the symmetric vacuum asymptotically at large distances. In this case, the equations (9) possess non-topological soliton solutions, for which the magnetic flux is not quantized, but continuously varying. At the singularity, the flux vanishes as argued in ref. [5] on the basis of a trivial w-integration.

An important aspect of this Bogomolny limit is that the mass of the elementary excitations of the system equals their electric charge. This can be seen as follows: due to the Chern-Simons term, an object carrying a magnetic flux Φ_M also carries a non-zero electric charge Q:

$$Q = -\frac{\tilde{k}}{2\pi}\Phi_M \tag{12}$$

The energy E of a non-topological soliton is then given by

$$E = e|\Phi_M| = \frac{2\pi e}{\tilde{k}}|Q| \tag{13}$$

which is the lower bound of the general energy relation[25]

$$E \geq e|\Phi_M| \tag{14}$$

which follows from the general expression for the energy in the model (7,8) without using its equations of motion (9). Hence the Minkowski black hole is of extreme Reissner-Nordstrom type [14].

As already mentioned, in a previous paper [5] we showed that the magnetic flux Φ_M vanishes at the singularity, as the result of a trivial integration over the w variable. However, that argument does not apply away from the singularity, where the action is not of pure Chern-Simons type, but has extra terms that can be described collectively as the effective potential discussed above. This is why the flux can in general be non-zero away from the singularity, leading to non-topological solitons with continously-varying non-zero fluxes, interpretable as possible masses of Minkowski black holes.

4 Supersymmetry and Renormalization Group Flow

We have already recalled that the region around the singularity can be described by a topological field theory (TFT). This can be put in a form with twisted N=2 supersymmetry, in which the supersymmetries become fermionic BRST gauge symmetries F, and the fermions become ghosts. As was pointed out in ref. [24], fixed points in the action of F dominate the path integral. An example is the region around a singularity in two-dimensional space-time, which is described by a twisted N=2 supersymmetric Wess-Zumino (SWZ) model. This leads to a super W-symmetry at the singularity, which contains an enhanced $W_{1+\infty} \otimes W_{1+\infty}$ bosonic symmetry at the core of the black hole [15]. This enhanced symmetry is broken as one moves away from the singularity, and the symmetry breaking is accompanied by the appearance of discrete massless modes [15]. However, although the topological nature of the theory is lost away from the singularity, its supersymmetry is maintained by undoing the twist present at the core. The fermionic fields that are ghosts at the singularity become the usual fermionic partners of N=2 supersymmetry when the model is untwisted, and the transformations F have no fixed points away form the singularity.

We now reconsider these ideas from a three-dimensional point of view. Consider the region close to the core of a two-dimensional black hole, where the physics is described by an SWZ model [24, 26]

$$S_{SWZ} = S_{WZ} + \frac{i}{2\pi}\int d^2z[\Psi D_{\bar{z}}\Psi + \overline{\Psi} D_z \Psi] \tag{15}$$

and the fermions of the coset model have been written in the matrix form $\Psi = \begin{pmatrix} 0 & \psi \\ \psi^* & 0 \end{pmatrix}$ and $\overline{\Psi} = \begin{pmatrix} 0 & \overline{\psi} \\ \overline{\psi}^* & 0 \end{pmatrix}$. In the case of the Minkowski black hole, the explicit N=1 supersymmetry can be enhanced to a higher N=2 supersymmetry by imposing a GSO projection [26]. Since the effective Higgs-Chern-Simons theory is known to have such an extended N=2 supersymmetry [27], we assume that this is done before the homotopic extension to the three-dimensional manifold.

Under this assumption, we rewrite the action (15) using a Dirac-like notation for the fermions: $\chi \equiv \begin{pmatrix} \psi \\ \overline{\psi} \end{pmatrix}$ so that :

$$S_{SWZ}^{fermion} = \frac{i}{2\pi}\int d^2z \overline{\chi} \not{D}(A)\chi \tag{16}$$

As in the purely bosonic case, the two-dimensional fermion-scalar theory can be interpreted as the adiabatic approximation to a three-dimensional model, after using the equation of motion for the time component of the gauge field. The homotopically-extended model is then an N=2 supersymmetric Abelian Chern-Simons-Higgs model with a symmetry-breaking vacuum. This we can interpret as a renormalization group fixed point of a more general model of a charged scalar field coupled to a Chern-Simons gauge field [28]:

$$\frac{1}{2}\epsilon^{\mu\nu\rho}A_\mu \partial_\nu A_\rho + |D(A)_\mu \phi|^2 + \overline{\chi}\not{D}(A)\chi + \alpha\overline{\chi}\chi\phi^*\phi - h(\phi^*\phi)^3 + ... \tag{17}$$

where the dots denote terms that are irrelevant in the renormalization group sense, and play no role in our arguments. Also, we did not write explicitly a quartic interaction because, although dominant in the infrared limit, it does not move the fixed point [28].

The ϕ and χ fields can develop dynamical masses in the model (17), via a three-dimensional Berezinsky-Kosterlitz-Thouless (BKT) phase transition [29] which can lead to superfluidity. However, we will argue that this is not the case for the Minkowski black hole, which resembles a Hall conductor rather than an anyon superfluid. To see this, we recall the mass generation mechanism in the model (17): the nonlinear interactions among the ϕ fields can be linearized in the Hartree-Fock approximation by replacing pairs $\phi^*\phi$ by their vacuum expectation value $< \phi^*\phi >$. This produces a theory with a dynamical effective potential of the form (8) without a local order parameter. Dynamical mass generation can be studied in this model with the aid of the large-N expansion for the number of scalar fields [19], although there is just one scalar field in the physical case. The large-N expansion gives a correct qualitative description of the gap, but one must go beyond this approximation to see the BKT nature of the transition. The result of the analysis depends on the values of the respective couplings. Among the leading-order renormalization group fixed points there is one with N=2 supersymmetry given by $\alpha = 3e^2, h = e^4$. This fixed point is infrared stable [28], implying that supersymmetry is an asymptotic symmetry of the system. This observation confirms our physical intuition about the importance of the Bogomolny limit for black hole physics. Viewed from three dimensions, the underlying theory has a cut-off parameter whose variation drives it to a non-trivial N=2 SWZ model describing matter interacting with a Reissner-Nordstrom black hole. Its extremality is intimately connected with supersymmetry [30]. In view of this supersymmetry, this fixed point separates phases in which either both fermions and scalars have identical non-zero masses, or both are massless. Clearly, the space-time singularity corresponds to the massless phase, whilst the non-singular part of the space-time, including the horizon, corresponds to the massive phase.

The three-dimensional system violates both time-reversal T and parity P in the massive phase. This is because when one integrates out a Dirac fermion with mass

$$m_f = \langle\phi^*\phi\rangle 3e^2 \tag{18}$$

one generates a Chern-Simons term with coefficient $sgn(m_f)\frac{1}{8\pi}$, which is independent of the magnitude of the mass [31]. This is unable to cancel the bare Chern-Simons coefficient in general, and certainly not in the adiabatic limit where the bare coefficient vanishes. The induced $T-$ and $P-$ violation are physical motivations for our identification of black holes with Hall conductors rather than high-T_c superconductors. The latter do not show any experimental signature of T and P violation [8], a fact attributed by one of us (N.E.M.) and N. Dorey [19] to the observation that defects in high-T_c superconductors correspond to an even number of species of real fermions, for which the induced Chern-Simons coefficient vanishes. In our case, the non-compact nature of the coset and supersymmetry give us a complex fermion representation that induces a non-zero coefficient. The consequent T violation implies irreversibility in the flow of the homotopic scale or cut-off parameter, corresponding to the irreversibility of black hole decay. Moreover, in view of the embedding (10) of the world-sheet into space-time, the induced $P-$ violation corresponds to an "arrow" in the target time direction. This observation leads to the cosmological arrow of time, when applied to a space-time with a cosmological singularity, as we discuss in more detail in section 6.

The appearance of parity violation in the string vacuum does not contradict any no-go theorems [32]. Spontaneous parity violation is forbidden in vector-like local field theories [33], and in three-dimensional gauge theories with fermions in real representations of the Lorentz group SO(2,1) spontaneous or dynamical parity violation is energetically disfavoured if there is an even number of fermion flavours. However, these arguments do not apply when fermions belong to complex representations of SO(2,1), and spontaneous parity violation can indeed be energetically preferred in theories with an odd number of complex fermions [31], as is the case in the SWZ model of interest here.

It is instructive to make connections with the description of target-space black holes as world-sheet spikes and vortices. At the horizon, the composite scalar field in the effective gauge theory description acquires a non-trivial vacuum expectation value $< \phi^* \phi > \neq 0$. Small fluctuations about this non-trivial minimum in the effective potential can be fermionized as follows. As already mentioned, the horizon corresponds to the limit $uv \to 0$, where the coefficient of the Chern-Simons term vanishes. Due to supersymmetry, there is an induced fermion mass that becomes infinite in this limit. Integrating out the gauge field in this non-trivial vacuum, one easily obtains

$$L_{eff} = -(\phi^* \partial_\mu \phi + \overline{\chi} \gamma_\mu \chi)^2 + i \overline{\chi} \partial\!\!\!/ \chi + (scalar - sector) \tag{19}$$

Redefining the fermion fields by [34]

$$\lambda \equiv \phi \chi \qquad : \qquad \overline{\lambda} \equiv \phi^* \overline{\chi} \tag{20}$$

and using the non-zero vacuum expectation value $< \phi^* \phi > \neq 0$, we find a massive 2+1-dimensional Thirring model with an attractive four-fermion interaction:

$$i \overline{\lambda} \partial\!\!\!/ \lambda - \frac{1}{4} (\overline{\lambda} \gamma_\mu \lambda)^2 \tag{21}$$

Upon the dimensional reduction corresponding to the adiabatic limit, this model becomes after bosonization [35] a 2-dimensional sine-Gordon model which describes spikes on the world-sheet.

A similar mechanism operates at the singularity, which corresponds to an antispike in the world-sheet picture. The bare Chern-Simons term is non-vanishing at the singularity, and is the dominant term in a derivative expansion, because of the short-distance nature of the problem. It can therefore be fermionized by infinite-mass fermions, which add an extra effective flavour to the Thirring model (21), that yield after bosonization [35] sine-Gordon terms that describe the world-sheet antispike corresponding to the singularity [5].

5 Isomorphism with the Quantum Hall Effect

We now develop the isomorphism of the above effective gauge model description of a Minkowski black hole with the formalism for a Hall conductor. We recall that the Hall effect has long been described using a σ-model [36] with a Chern-Simons term, in which the longitudinal and transverse conductivities σ_{xx}, σ_{xy}, are treated as couplings subject to renormalisation, the conductivity plateaux corresponding to vanishing β-functions and hence conformal invariance of the σ-model. This approach has recently been extended from the integer to the fractional quantum Hall effect [37], exploiting better the complex $SL(2,Z)$ duality symmetry of the effective σ-model.

We have already touched on similarities of the effective gauge theory description of the black hole WZ coset model to the physics underlying the quantum Hall effect. The bosonic fields (a,b) in the Lagrangian (7) have charges coupled to the gauge field A_μ, and the corresponding current $\delta S_{eff}/\delta A_\mu$ can easily be found by integrating out the scalar fields in the massive phase which corresponds to space-time [1]. It is evident from the effective potential (8) that the scalar fields acquire masses of order $\mu = \frac{e^2}{\tilde{k}}$ near the symmetric vacuum, where $\tilde{k}$ is the coefficient of the Chern-Simons term. In the adiabatic limit, the homotopic scale $\beta \to \infty$, hence $\tilde{k} \propto \frac{1}{\beta} \to 0$ and the scalar mass $\mu \to 0$. Thus the effective theory is well approximated by one-loop graphs that yield Maxwell terms for the gauge field [38]

$$S_{gauge} = \int d^3x [\frac{1}{24\pi|\mu|}(F_{\mu\nu}(A))^2 + \kappa A \wedge F(A) + \overline{\chi}(i\partial\!\!\!/ + eA\!\!\!/)\chi + ...] \qquad (22)$$

Notice the opposite sign of the induced Maxwell term as opposed to the standard electrodynamics. The fermion current then exhibits a transverse Hall form. Summing over all "times" to obtain the two-dimensional current

$$\hat{J}_i = \int_0^\beta j_i^\chi = \sigma_{ij}^{xy}\partial_j w \qquad (23)$$

[1]We recall that the massless phase corresponds to the singularity, where the gauge field-theory is purely topological.

we find

$$\sigma_{ij}^{xy} = -\epsilon_{ij}\frac{ke^2}{2\pi} \tag{24}$$

for the transverse conductivity.

We see immediately from (24) that *the WZ coefficient k is the Hall fraction*. The Minkowski black hole case therefore corresponds to a *fractional charge* $\frac{9}{4}e$ in the language of the Hall effect.

As we have mentioned in section 2, a wedge subalgebra of $W_{1+\infty}$ has been found as a symmetry of the quantum Hall system [11]. This symmetry is generated by the magnetic translation operators in the $x-y$ plane, i.e. translations in y up to x-dependent gauge transformations (in a gauge where the electromagnetic potential is $A_0 = 0, A_x = -By, A_y = A_z = 0$) :

$$\begin{aligned} b &\equiv -\partial_y + i\partial_x + ieBx \\ b^\dagger &\equiv \partial_y + i\partial_x - ieBx \end{aligned} \tag{25}$$

where B denotes the external magnetic field in the direction perpendicular to the plane. Integer powers of the operators (25)

$$V_{n,m} = (b^\dagger)^{n+1}(b)^{m+1} \qquad : \qquad n,m \geq -1 \tag{26}$$

generate quantum deformations of W-algebras [11] that include area-preserving diffeomorphisms of the type recently argued [10] to be responsible for quantum coherence in systems with space-time singularities. As we mentioned earlier, the difference of the Hall systems discussed in [11] from the ordinary W_∞-case is the requirement of regularity of the wavefunction at the position of the electron. This implies a *truncation* of the algebra to the positive modes (26). On the other hand the KP-hierarchy

$$\Lambda_{KP} = \partial_z + \sum_{i=0}^{\infty} u_i(\partial_z)^{-i-1} \tag{27}$$

that generates the full W_∞ algebras [22], contains modes corresponding to the negative integer powers of b and $b^\dagger$ in (26), which lead to divergences of the electron wavefunction at the origin. Such divergences are associated with topological defects on the plane, as is the case of annular or doped Hall conductors [2]. The existence of the full W_∞ algebra can be verified directly in those cases using an effective gauge field theory description of the lowest Landau level by a pure Chern-Simons gauge theory on a topologically non-trivial space, which is equivalent to the infinite topological-mass limit of a Maxwell-Chern-Simons theory [39]. The generators

[2]We remind the reader that a "missing" electron changes the Hilbert space of the problem in a topologically non-trivial way [19].

of the full W_∞ symmetry in that case can be expressed in terms of the magnetic translation operators (25) as

$$W_{n,\bar{n}} = exp(\frac{2\pi}{k} n b^\dagger) exp(-\frac{2\pi}{k} \bar{n} b) \tag{28}$$

where $n \equiv -i\tau n + im$, $\bar{n}$ is the complex conjugate, n, m are integers (not necessarily positive) and τ denotes a (complex) modular parameter of the Riemann surface on which the system is defined. The coefficient of the Chern-Simons term is normalized to $k/8\pi$ and $[b, b^\dagger] = k/2\pi Im\tau$. Compared with the smooth case of ref. [11], the Chern-Simons theory can be considered as describing excitations on the edge of a *large* disk which the Hall system lives on. In that case, since the relevant wave-functions are defined far away from the origin, any regularity requirements can be relaxed, leading to an enhancement of the symmetry to the full $W_{1+\infty}$.

We showed in ref. [15] that a higher $W_{1+\infty} \otimes W_{1+\infty}$ symmetry group is recovered at a space-time singularity, as a bosonic subgroup of $N = 2$ super $W_{1+\infty}$ symmetry [40]. The spontaneous breakdown of $W_{1+\infty} \otimes W_{1+\infty}$ to $W_{1+\infty}$ away from the singularity is accompanied by the appearance of discrete massless leg-poles. In view of the intimate connection of the Hall systems with $N = 2$ supersymmetry [17, 41], we expect a similar feature of the excitation spectrum in annular Hall conductors : discrete long-range excitations should appear whenever the planar topology of the Hall conductor is non-trivial. The following facts point in this direction: In Hall electron systems the $N = 2$ supersymmetry is generated by the *covariant derivative* operators [41]

$$\begin{aligned} a &\equiv \partial_x + i\partial_y - ieBy \\ a^\dagger &\equiv -\partial_x + i\partial_y + ieBy \end{aligned} \tag{29}$$

The latter *commute* with b and $b^\dagger$, but not with the Landau Hamiltonian $H \propto aa^\dagger + a^\dagger a$, since $[a, a^\dagger] = k/2\pi$. These operators generate [39] a W_∞ algebra in a way similar to (28), which is not a symmetry of the Landau Hamiltonian. Restriction to the first Landau level, make the effective gauge theory infinitely massive, and thus purely topological. The Landau Hamiltonian formally vanishes in that limit, and the W_∞ algebra generated by the a and $a^\dagger$ operators, which acts on this Landau level, may be promoted to a symmetry algebra of this reduced system. In the black hole case this corresponds to the limit close to the singularity, where there is an enhanced $W_\infty \otimes W_\infty$ [15].

This is just one example of the rich interplay that we expect between studies of Hall conductors and string black holes in view of the isomorphism we have developed. The above paragraph is one example of a theoretical discovery in black hole physics that may have implications for experiments on Hall conductors. Conversely, measurements on Hall conductors can be regarded as a laboratory for doing experimental black hole physics on a *table top* .

It would be interesting to study the renormalisation group flow that relates different fractions in the hierarchy of Hall systems, which has been associated with extended duality symmetries [37]. It would seem that there is only one value $k = \frac{9}{4}$ which admits a space-time interpretation in the two-dimensional black hole case. However, the W_∞-symmetry structure of the WZ is essentially unchanged for other values of k [21]. Moreover, we expect that it is possible to construct consistent models by tensoring a WZ model with other field theories on the world-sheet so as to obtain a total central charge of 26 (or 15 for supersymmetric theories), as required for the space-time interpretation. Such mixed WZ models have recently been considered in connection with higher-dimensional universes [42]. One would consider such a tensored WZ model as corresponding to some Hall system with a filling fraction falling in a hierarchy of values appearing in a flow of central charge from the other tensored components, interpreted as matter fields.

Looking at the form (9) of the magnetic field in the effective gauge theory description of the WZ model, we see that it vanishes close to the singularity, as well as at the horizon. Thus the system close to the singularity resembles that of a zero-field Hall conductor, as discussed in section 1. This is consistent with the spontaneous nature of $T-$ and $P-$ violation [6], which, we have earlier argued, explains the irreversibility of black hole decay and the arrow of time when applied to the cosmological singularity. It is natural to ask whether space-time is still a Hall liquid far away from the singularity, where the Chern-Simons description acquires higher-order corrections. The answer is yes, according to recent studies of $c = 1$ matrix models [12], which represent the spatially-asymptotic form of the black hole. The character of an incompressible Hall fluid and the associated $W_{1+\infty}$-symmetry structure are preserved [43, 12]. Thus space-time foam, as described by a system of multiple topological defects on the world-sheet, shares many common features with Hall fluids.

6 Renormalisation Group Flow and the Arrow of Time

We have argued recently [44] that space-time foam can be formulated as a renormalization flow problem on the world-sheet, through the identification of the renormalization group mass scale, that was introduced as a covariant world-sheet cut-off, with the target time in Planck units. This cut-off must be introduced to separate the light degrees of freedom measured in laboratory experiments from the massive string degrees of freedom which are not observed. The massless states are related to the massive states by W-symmetries which maintain quantum coherence [3] [10],

[3] It should be noticed that the W-symmetries pertain to the two-dimensional target space string theory, where the massive modes are solitonic non-propagating modes having definite energies and momenta [43, 10]. Two-dimensional strings have been argued to constitute the s-wave sector of higher-dimensional target spaces [45].

and quantum mechanics is modified in the effective theory of the light degrees of freedom, which behave like an open system with monotonically increasing entropy [44]. It is worth emphasizing the fact that the full string theory, with its massive degrees of freedom, is a conformally invariant theory in which there is no renormalization group flow. This in some sense defines a concept of "eternity" within which the light-mode subsystem (our world) interacts thermodynamically by exchanging energy with its environment (massive string states). This leads to an irreversible time flow for an observer who is part of this subsystem. Formally the above ideas can be expressed as follows [44]. Let $\{G^i\}$ be a collective notation for the heavy and light string modes, and t be the renormalisation group scale. The evolution equation of the density matrix ρ away from a (conformally invariant) fixed point reads [44]

$$\dot{\rho} \equiv \frac{\partial \rho}{\partial t} = i[\rho, H] + iG_{ij}[\rho, G^i]\beta^j \tag{30}$$

where H is the Hamiltonian and $\beta^i = dG^i/dt$ is the renormalization group β-function. The non-triviality of the friction non-commutator term in (30) *implies* the non triviality of the commutator term. Indeed, for systems interacting with a reservoir of particles at temperature β^{-1}, as is our light-mode system, the density matrix is expressed as $exp(\beta(F-H))$ where F is the free energy. In the case of strings F is the generating functional of connected string amplitudes and is associated [46] with the Zamolodchikov c-function [47]. At a conformally-invariant fixed point (fp) the latter becomes a c-number, the central charge of the theory, and so $[\rho, H]_{fp} = 0$, and the evolution equation (30) has trivial content. On the other hand, by considering deformations of the pertinent stringy σ-models by massless states only [44], one goes away from the conformally-invariant fixed point in an irreversible way, as implied by Zamolodchikov's c-theorem [47, 48]. The situation is similar to the conventional renormalization group flow in local field theories. Due to logarithmic infinities, the experimentalist in the laboratory measures "running" coupling constants correponding to light particles. Formally, the light mode infinities can be remedied by introducing heavy states which make the theory finite and are responsible for the renormalization group flow of the light states. This formal procedure of local field theories acquires, therefore, an important physical meaning in string theory propagating in singular backgrounds, such as black holes or cosmological singularities [4, 42]. Within the framework of string theory the observed flow of time is a consequence [44] of the existence of massive (solitonic) string modes which couple to the observed light states on account of the W-symmetries [10]. The irreversibility of the renormalization group flow implied by Zamolodchikov's c-theorem [47, 48] corresponds to the $P-$ and $T-$ violation in the effective Chern-Simons theory of the Hall fluid.

This $P-$ and $T-$ violation also determine the cosmological arrow of time, as we now argue. In the Hall analogue description of singular space-times, one introduces a third dimension τ, which plays the rôle of an adiabatic evolution parameter. As explained in [5], τ can be interpreted as an inverse pseudo-"temperature" which determines the phase structure of the universe through a BKT transition [29] from a hot Euclidean to a cold Minkowski space-time. The irreversibility of this cosmological evolution in pseudo-"temperature" is guaranteed by the $T-$violation induced by the three-dimensional Chern-Simons theory. The associated $P-$violation implies reflection non-invariance on the world-sheet which the embedding (10) elevates into target-time irreversibility. The latter is associated with the irreversibility of the renormalization group flow once the world-sheet cut-off mass-scale is identified with the target time.

Thus the renormalization group flow down the Hall hierarchy [37] and the $P-$ and $T-$ violation in the Hall vacuum correlate the cosmological arrow of time, the decay of black holes, and a monotonic increase in entropy at the microscopic and macroscopic levels. We plan to return in a future paper [49] to a more quantitative study of this and other cosmological issues in string theory.

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