

AdS Wormholes

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ABSTRACT

We obtain a large class of smooth Lorentzian p -brane wormholes in supergravities in various dimensions. They connect two asymptotically flat spacetimes. In cases where there is no dilaton involved in the solution, the wormhole can connect an $\text{AdS}_n \times S^m$ in one asymptotic region to a flat spacetime in the other. We obtain explicit examples for $(n, m) = (4, 7), (7, 4), (5, 5), (3, 3), (3, 2)$. These geometries correspond to field theories with UV conformal fixed points, and they undergo decompactification in the IR region. In the case of AdS_3 , we compute the central charge of the corresponding conformal field theory.

1 Introduction

Asymptotically AdS solutions in supergravities play an important rôle in the AdS/CFT correspondence [1–3], since they provide supergravity duals to quantum field theories with conformal fixed points in the UV region. In the bulk of such a solution, there are limited possibilities. There can be a black hole horizon with non-zero (or zero) temperature, or there can be an AdS horizon of different AdS radius, corresponding to a conformal field fixed point in the IR region [4]. A third possibility is that the solution is solitonic, such as an R-charged AdS bubble solution in an AdS gauged supergravity [5–7]. Most likely, the solution will have a naked singularity. Examples include the large class of AdS domain wall solutions with naked singularities constructed in [8,9], which are dual to the Coulomb branch of the dual gauge theories.

A more intriguing situation is when there exists a wormhole in the bulk that connects smoothly to different AdS boundaries. In Lorentzian signature such a geometry appears unlikely, and disconnected boundaries can only be separated by horizons [10]. Thus the recent studies of wormholes in string theory and in the context of the AdS/CFT correspondence have so far concentrated on Euclidean-signature spaces [11–15].

In [16], Ricci-flat and charged Lorentzian wormholes in higher dimensions were obtained. These include the previously-known $D = 5$ Ricci-flat case [17]. The wormholes are smooth everywhere, and connect two asymptotically flat Minkowski spacetimes. Although these wormholes are not traversable geodesically (see [18,19] and [16]), it was demonstrated in [16] that there exist traversable accelerated timelike trajectories across the wormholes.

A class of magnetically-charged wormholes in $D = 5$ supergravity was also obtained in [16]. It was shown that for appropriate choices of the parameters, the wormhole can connect an $\text{AdS}_3 \times S^2$ in one asymptotic region to a Minkowski spacetime in the other. This geometry then provides a supergravity dual of a two-dimensional field theory at the boundary of the AdS_3 .

In this paper, we begin in section 2 with a review of the Ricci-flat wormhole solutions that were obtained in [16]. We then construct p -brane wormhole solutions in section 3, supported by a dilaton and n -form field strength. In non-dilatonic cases, these p -brane wormholes connect an $\text{AdS}_n \times S^m$ in one asymptotic region to a flat spacetime in the other. We obtain explicit examples for $(n, m) = (4, 7), (7, 4), (5, 5), (3, 3), (3, 2)$. These geometries correspond to field theories with UV conformal fixed points, which undergo decompactification in the IR region.

In section 4, we study the AdS_3 wormhole obtained in [16] in detail and compute the

central charge of the corresponding dual conformal field theory.

In sections 5 and 6, we examine AdS₅, AdS₄, AdS₇ and another AdS₃ wormhole in detail. Included in these discussions is a calculation of the mass and momentum of the configurations, as measured from the asymptotically AdS region. To do this, we make use of a construction of conserved charges in asymptotically AdS spacetimes, which we summarise in an appendix.

We conclude the paper in section 7.

2 Ricci-Flat Wormholes in $D \geq 5$ Dimensions.

In this section, we review the Ricci-flat wormhole solutions in general dimensions obtained in [16]; they are given by

$$ds_D^2 = (r^2 + a^2)d\Omega_{D-3}^2 + \frac{r^2 dr^2}{(r^2 + a^2)\sin^2 u} + \cos v(-dt^2 + dz^2) + 2 \sin v dt dz, \quad (2.1)$$

where v and u are functions of r , given by

$$v = \sqrt{\frac{D-3}{2D-8}}(\pi - 2u), \quad (2.2)$$

and

$$u = \arctan \sqrt{\left(1 + \frac{r^2}{a^2}\right)^{D-4} - 1}. \quad (2.3)$$

One can also rewrite the relation between u and r in the simpler form

$$\cos^2 u = \left(1 + \frac{r^2}{a^2}\right)^{4-D}. \quad (2.4)$$

Note that $u = 0$ when $r = 0$. Neither (2.3) nor (2.4) satisfactorily exhibits the fact that as r passes through zero, the *sign* of u should be correlated with the sign of r . Instead, we can make this explicit by expanding the expression in (2.3) and writing

$$u = \arctan \left[\frac{r}{a} \sqrt{\sum_{n=0}^{D-5} \binom{D-4}{n+1} \left(\frac{r}{a}\right)^{2n}} \right]. \quad (2.5)$$

Thus we see that as r ranges from $-\infty$ to $+\infty$, u ranges from $-\frac{1}{2}\pi$ to $+\frac{1}{2}\pi$. For specific values of D , there are sometimes simpler expressions for the relation between u and r , as we shall see later.

It is sometimes useful to use u , rather than r , as the radial coordinate. The solution is then given by

$$ds_D^2 = \frac{a^2}{(\cos u)^{\frac{2}{D-4}}} \left(\frac{du^2}{(D-4)^2 \cos^2 u} + d\Omega_{D-3}^2 \right) + \cos v(-dt^2 + dz^2) + 2 \sin v dt dz, \quad (2.6)$$

As was discussed in [16], the metric describes a smooth wormhole in $D \geq 5$ dimensions that connects two flat asymptotic spacetimes at $r \rightarrow \pm\infty$. Note that the general Ricci-flat wormhole metric (2.1) is related to that in [16] by a coordinate rotation in the (t, z) plane.

There are two asymptotic regions. In the $r \rightarrow +\infty$ region, we have $u \rightarrow \frac{1}{2}\pi$ and hence $v \rightarrow 0$. It follows that the metric becomes

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\Omega_{D-3}^2, \quad (2.7)$$

which is a flat Minkowskian spacetime in D dimensions. In the $r \rightarrow -\infty$ region, we have $u \rightarrow -\frac{1}{2}\pi$ and

$$v \rightarrow 2\theta_0 \equiv 2\pi \sqrt{\frac{D-3}{2D-8}}. \quad (2.8)$$

The metric becomes

$$\begin{aligned} ds^2 &= -\cos(2\theta_0)(dt^2 - dz^2) + 2\sin(2\theta_0) dt dz + dr^2 + r^2 d\Omega_{D-3}^2, \\ &= -d\tilde{t}^2 + d\tilde{z}^2 + dr^2 + r^2 d\Omega_{D-3}^2, \end{aligned} \quad (2.9)$$

where

$$\begin{pmatrix} \tilde{t} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} t \\ z \end{pmatrix}. \quad (2.10)$$

Thus the solutions connect smoothly two flat asymptotic spacetimes. However, only for the case of $D = 5$ do the two asymptotic regions have the same time coordinate. In higher dimensions, the notion of asymptotic time is different in the two regions.

3 General p -Brane Wormholes

We construct charged wormholes as solutions to \hat{D} -dimensional Einstein gravity coupled to an n -form field strength, together with a dilaton. The Lagrangian has the following general form

$$\mathcal{L}_{\hat{D}} = \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2n!} e^{\alpha\phi} F_{(n)}^2 \right). \quad (3.1)$$

where $F_{(n)} = dA_{(n-1)}$. The constant α can be parameterised as

$$\alpha^2 = \Delta - \frac{2(n-1)(\hat{D}-n-1)}{\hat{D}-2}. \quad (3.2)$$

The Lagrangian (3.1) is of the form that typically arises as a truncation of the full Lagrangian in many supergravities, with Δ being given by

$$\Delta = \frac{4}{N}, \quad (3.3)$$

for integer N . The values of N that can arise depends on the spacetime dimensions; they are classified in [20].

We may consider an electric “ $(n-2)$ -brane wormhole” in \hat{D} spacetime dimensions, based on the D -dimensional wormhole solution (2.6) with

$$\hat{D} = D + n - 3. \quad (3.4)$$

We therefore make as an ansatz for the \hat{D} -dimensional metric and the dilaton field

$$\begin{aligned} ds_{\hat{D}}^2 &= \frac{H^{\frac{(n-1)N}{(\hat{D}-2)}}}{(\cos u)^{\frac{2}{\hat{D}-4}}} \left(\frac{a^2 du^2}{(D-4)^2 \cos^2 u} + a^2 d\Omega_{D-3}^2 \right) \\ &\quad + H^{\frac{-(D-4)N}{(\hat{D}-2)}} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx^i dx^i \right), \\ e^{\alpha\phi} &= H^{2 - \frac{(n-1)(D-4)N}{(\hat{D}-2)}}, \end{aligned} \quad (3.5)$$

where the coordinates of the D -dimensional wormhole (2.6) have been augmented by $(n-3)$ additional world-volume coordinates x^i . The function H is assumed to depend only on the radial coordinate u . For the field strength $F_{(n)}$, we make the ansatz

$$F_{(n)} = \sqrt{N} dt \wedge dz \wedge d^{n-3}x \wedge dH^{-1}, \quad (3.6)$$

Substituting into the equations of motion following from (3.1), we find that they are all satisfied provided $H'' = 0$, and hence H is given by

$$H = c_0 - \frac{q}{a^{D-4}} u, \quad (3.7)$$

where c_0 and q are integration constants. Without loss of generality, let us take q to be non-negative. (Note that taking a limit of $a \rightarrow 0$ leads to BPS p -brane solutions to the Lagrangian (3.1), obtained in general in [21].) It should again be emphasised that $r \leftrightarrow -r$ is not a symmetry, since the expression for u in terms of r is defined by (2.5), showing that the signs of u and r are correlated. The function H in general approaches a constant when $r \rightarrow \pm\infty$, given by

$$\begin{aligned} r \rightarrow +\infty : \quad H &\sim c_0 - \frac{\pi q}{2a^{D-4}} + \frac{q}{r^{D-4}} + \dots, \\ r \rightarrow -\infty : \quad H &\sim c_0 + \frac{\pi q}{2a^{D-4}} - (-1)^{[D/2]} \frac{q}{r^{D-4}}. \end{aligned} \quad (3.8)$$

Thus provided that $c_0 > \pi q/(2a^{D-4})$, the p -brane wormholes link two asymptotically flat spacetimes. When $c_0 = \frac{1}{2}\pi q a^{4-D}$, for the non-dilatonic case $\alpha = 0$, AdS wormholes can

arise that link AdS×Sphere in the $r \rightarrow +\infty$ asymptotic region to flat spacetime in the $r \rightarrow -\infty$ region. We shall discuss these solutions case by case in the following sections.

Note that we can also consider “magnetic p -brane wormholes,” which are equivalent to the previously-discussed electric cases, but constructed using the $(\hat{D} - n)$ -form dual of the n -form field strength $F_{(n)}$. In other words, we can introduce the dual field strength

$$\tilde{F}_{(\tilde{n})} = e^{\alpha\phi} *F_{(n)}, \quad (3.9)$$

where $\tilde{n} = \hat{D} - n$, in terms of which the Lagrangian (3.1) can be rewritten as

$$\mathcal{L}_{\hat{D}} = \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\tilde{n}!} e^{-\alpha\phi} \tilde{F}_{(\tilde{n})}^2 \right). \quad (3.10)$$

where $\tilde{F}_{(\tilde{n})} = d\tilde{A}_{(\tilde{n}-1)}$. The electric solution (3.5) of (3.1), with $F_{(n)}$ given by (3.6), can then be reinterpreted as a magnetic solution of (3.10), with $\tilde{F}_{(\tilde{n})}$ given by

$$\tilde{F}_{(\tilde{n})} = \sqrt{N} (D - 4) q \Omega_{D-3}. \quad (3.11)$$

4 Magnetic String and AdS₃ Wormholes

In this section, we consider the magnetically-charged wormhole solution in five-dimensional $U(1)^3$ supergravity that was obtained in [16]. It was observed that for appropriate choice of the parameters, the solution smoothly connect AdS₃ × S^2 in one asymptotic region to the flat spacetime in the other.

We begin by reviewing the solution. For simplicity, let us consider the special case where all the three charges are equal. The corresponding minimal supergravity solution is given by

$$\begin{aligned} ds_5^2 &= -H^{-1} \left(\frac{r^2 - 2asr - a^2}{r^2 + a^2} dt^2 + \frac{4acr}{r^2 + a^2} dt dz - \frac{r^2 + 2asr - a^2}{r^2 + a^2} dz^2 \right) \\ &\quad + H^2 (dr^2 + (r^2 + a^2) d\Omega_2^2), \\ F_{(2)} &= \sqrt{3} q \Omega_{(2)}, \quad H = c_0 - \frac{q}{a} \arctan\left(\frac{r}{a}\right). \end{aligned} \quad (4.1)$$

It is straightforward to verify that this solution is contained in the general form of p -brane wormhole (3.5) with $\hat{D} = D = 5$, $n = 3$ and $N = 3$. Compared with (3.5), a boost parameter $s = \sinh \beta$ ($c = \cosh \beta$) is also introduced, as in [16]. The solution describes a smooth charged wormhole as long as

$$c_0 \geq \frac{\pi q}{2a}. \quad (4.2)$$

The coordinate r runs from $-\infty$ to $+\infty$, corresponding to two flat spacetimes when the above inequality holds. Interesting things happen when $c_0 = \pi q/(2a)$. In this case, we have

$$\begin{aligned} r \rightarrow +\infty : \quad H &\sim \frac{q}{r} - \frac{q a^2}{3r^3} + \frac{q a^4}{5r^5} - \frac{q a^6}{7r^7} + \dots, \\ r \rightarrow -\infty : \quad H &\sim \frac{q}{a} \left(\pi + \frac{a}{r} - \frac{a^3}{3r^3} + \frac{a^5}{5r^5} - \frac{a^7}{7r^7} + \dots \right). \end{aligned} \quad (4.3)$$

Thus asymptotically as $r \rightarrow -\infty$, the spacetime is flat, whilst as $r \rightarrow +\infty$, the spacetime is a direct product of $\text{AdS}_3 \times S^2$.

Since the size of the S^2 never vanishes, we can reduce the solution on the S^2 and obtain a smooth solution in $D = 3$. Such a breathing mode reduction was obtained in general dimensions in [22]. The reduction ansatz is given by

$$ds_5^2 = e^{2\alpha\varphi} ds_3^2 + e^{-\alpha\varphi} q^2 d\Omega_2^2, \quad F_{(2)} = \sqrt{3} q \Omega_{(2)}, \quad (4.4)$$

with $\alpha = 1/\sqrt{3}$. The $D = 3$ system contains the metric and a scalar, with the Lagrangian given by

$$\mathcal{L}_3 = \sqrt{-g}(R - \frac{1}{2}(\partial\phi)^2 - V), \quad V = -\frac{1}{2q^2}(4e^{3\alpha\varphi} - 3e^{4\alpha\varphi}). \quad (4.5)$$

The scalar potential contains an AdS fixed point $\varphi = 0$. The resulting three-dimensional solution in the Einstein frame is given by

$$\begin{aligned} ds_3^2 &= q^{-4} \left[(r^2 + a^2)^2 H^6 dr^2 \right. \\ &\quad \left. - (r^2 + a^2) H^3 \left((r^2 - 2asr - a^2) dt^2 + 4acr dt dz - (r^2 + 2asr - a^2) dz^2 \right) \right], \\ e^{-\alpha\phi} &= q^{-2} (r^2 + a^2) H^2. \end{aligned} \quad (4.6)$$

In the asymptotic region $r \rightarrow -\infty$, we have that H is constant, and the solutions becomes

$$ds_3^2 \sim r^4 (dr^2 - dt^2 + dz^2), \quad \varphi \rightarrow -\infty. \quad (4.7)$$

The metric is locally flat¹ for $r \rightarrow -\infty$; however, the scalar describing the breathing mode of the internal S^2 diverges in this limit. This breathing mode singularity is just a lower-dimensional artifact, reflecting the fact that the radius of the S^2 becomes infinite. The system should really be lifted to five dimensions in this limit.

¹If we let $x^\mu = (t, z)$ and define $y^\mu = r^2 x^\mu$ and $w = r^3/3$, the metric (4.7) becomes

$$ds_3^2 = dw^2 + dy^\mu dy_\mu - \frac{4}{3w} y_\mu dy^\mu dw + \frac{4}{9w^2} y^\mu y_\mu dw^2,$$

which is asymptotically flat when $|w| \gg |y|$.

In the $r \rightarrow +\infty$ limit, the metric approaches AdS₃. We would like to compute the central charge as in the work of Brown and Henneaux [23]. As a first step, we make the following change of coordinates

$$r \rightarrow \frac{r^2}{2q}, \quad t \rightarrow \frac{t}{\sqrt{2}}, \quad z \rightarrow \frac{z}{\sqrt{2}}. \quad (4.8)$$

Including the subleading term at asymptotic $+\infty$, the metric (4.6) now has the following form:

$$ds_3^2 = \frac{\ell^2}{r^2} dr^2 - \left(1 - \frac{2a\ell s}{r^2}\right) \frac{r^2}{\ell^2} dt^2 + \left(1 + \frac{2a\ell s}{r^2}\right) \frac{r^2}{\ell^2} dz^2 - \frac{4ac}{\ell} dt dz, \quad (4.9)$$

where $\ell = 2q$. If we had considered the general three unequal charge solution, we would have $\ell = 2(q_1 q_2 q_3)^{1/3}$. The traditional form of the Poincaré patch of the AdS₃ metric is:

$$ds_3^2 = \frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2} (dz^2 - dt^2). \quad (4.10)$$

We can now directly compare with, say, [24, 25] and see that the central charge of the system is

$$C = \frac{3}{2}R = \frac{3}{2}\ell. \quad (4.11)$$

Here, we use C rather than the more conventional c to denote the central charge, to avoid confusion with the short-hand notation $c = \cosh \beta$ of this paper.

5 D3-brane and AdS₅ Wormhole

It is perhaps more interesting to obtain an AdS₅ wormhole in type IIB theory, which would be expected to be dual to certain four-dimensional Yang-Mills theory. Since AdS₅ appears naturally in the type IIB theory in AdS₅ \times S^5 , the near horizon geometry of the D3-brane, we consider the D3-brane wormhole solution. From (3.5) with $D = 8$, $n = 5$ and $N = 1$, we find that the D3-brane wormhole solution is given by

$$\begin{aligned} ds_{10}^2 &= \left(\frac{H}{\cos u}\right)^{1/2} \left[a^2 d\Omega_5^2 + \frac{a^2 du^2}{16 \cos^2 u} \right] \\ &\quad + H^{-1/2} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx_1^2 + dx_2^2 \right), \\ F_{(5)} &= G_{(5)} + *_{10} G_{(5)}, \quad G_{(5)} = dt \wedge dz \wedge dx_1 \wedge dx_2 \wedge dH^{-1}, \\ H &= c_0 - \frac{q}{a^4} u, \quad u = 2 \arcsin \left(\frac{r(r^2 + 2a^2)^{1/2}}{\sqrt{2}(r^2 + a^2)} \right), \quad v = \sqrt{\frac{5}{8}}(\pi - 2u). \end{aligned} \quad (5.1)$$

Choosing the integration constant $c_0 = \pi q/(2a^4)$, we have

$$\begin{aligned} r \rightarrow +\infty : \quad H &\sim \frac{q}{r^4} - \frac{2a^2 q}{r^6} + \dots, \\ r \rightarrow -\infty : \quad H &\sim \frac{\pi q}{a^4} - \frac{q}{r^4} + \dots. \end{aligned} \quad (5.2)$$

Thus we have constructed a wormhole solution that is asymptotically $\text{AdS}_5 \times S^5$ when $r \rightarrow +\infty$ and flat when $r \rightarrow -\infty$.

Since the solution obtained above is spherically symmetric, it can be dimensionally reduced on S^5 to give a solution in five dimensions. Using the results in [22], we can reduce to the five-dimensional metric ds_5^2 given by

$$\begin{aligned} ds_{10}^2 &= e^{2\alpha\varphi} ds_5^2 + e^{-\frac{6\alpha}{5}\varphi} \ell^2 d\Omega_5^2, \\ G_{(5)} &= 4\ell^4 \Omega_{(5)}, \end{aligned} \quad (5.3)$$

where $\alpha = \sqrt{5/48}$ and $\ell \equiv q^{1/4}$ is the AdS_5 length. The five-dimensional system is then Einstein gravity coupled to a dilaton with a scalar potential, namely

$$\mathcal{L}_5 = \sqrt{-g} \left(R - \frac{1}{2} (\partial\varphi)^2 - V \right), \quad (5.4)$$

with the scalar potential given by

$$V = -\frac{4}{\ell^2} (5e^{\frac{16}{5}\alpha\varphi} - 2e^{8\alpha\varphi}). \quad (5.5)$$

Performing the S^5 reduction on the solution, we find

$$\begin{aligned} ds_5^2 &= \left(\frac{a}{\ell}\right)^{\frac{10}{3}} \left\{ \left(\frac{H}{\cos u}\right)^{4/3} \frac{a^2 du^2}{16 \cos^2 u} \right. \\ &\quad \left. + \frac{H^{1/3}}{\cos^{5/6} u} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx_1^2 + dx_2^2 \right) \right\}. \\ e^{-\frac{6\alpha}{5}\varphi} &= \frac{a^2}{\ell^2} \sqrt{\frac{H}{\cos u}}. \end{aligned} \quad (5.6)$$

There is no metric singularity as u ranges from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$, corresponding to r ranging from $-\infty$ to $+\infty$. In the asymptotic region $r \rightarrow -\infty$, the solution becomes

$$\begin{aligned} ds_5^2 &\sim \pi^{4/3} \left(\frac{r^2}{\ell^2}\right)^{\frac{5}{3}} \left(\frac{\ell}{a}\right)^{16/3} \left[dr^2 + \frac{1}{\pi} \left(\frac{a}{\ell}\right)^4 (-d\tilde{t}^2 + d\tilde{z}^2 + dx_1^2 + dx_2^2) \right], \\ \varphi &\sim -\infty, \end{aligned} \quad (5.7)$$

where \tilde{t} and \tilde{z} are defined in (2.10). The metric is locally flat in this limit, in the sense that the Riemann tensor tends to zero as r approaches $-\infty$ (see footnote 1). The dilaton is singular in the limit, but the scalar potential (5.5) goes to zero. Such a singularity, in which the potential is bounded above, is called a ‘‘good singularity’’ in [26]. However, the situation considered in [26] is when such a singularity occurs at a finite $r = r_0$, where $g_{tt} \rightarrow 0$, corresponding to non-trivial infrared physics in the dual field theory. In our case, only the scalar becomes singular as $r \rightarrow -\infty$, with $g_{tt} \sim \infty$. From the supergravity point of

view, this is clearly a “good singularity,” and is nothing but an artifact of the dimensional reduction. It implies that the system decompactifies into ten dimensions.

In the asymptotic region $r \rightarrow +\infty$, the solution becomes,

$$\begin{aligned} ds_5^2 &\sim \frac{\ell^2}{r^2} dr^2 + \frac{r^2}{\ell^2} (-dt^2 + dz^2 + dx_1^2 + dx_2^2), \\ \varphi &\sim 0. \end{aligned} \tag{5.8}$$

(Note that in the limit of $a \rightarrow 0$, the solution becomes literally AdS₅ for all r .)

The scalar potential (5.5) tends to zero as φ goes $-\infty$, and diverges to $+\infty$ as φ goes to $+\infty$. There is a minimum, $V_{\min} = -12/\ell^2$, which occurs at $\varphi = 0$. The wormhole solutions corresponds to φ traversing from $\varphi = -\infty$ in the asymptotically locally flat region to $\varphi = 0$ in the asymptotically AdS region, with V_{\min} determining the cosmological constant of AdS₅.

Including subleading terms, the metric in the asymptotically AdS region is given by

$$\begin{aligned} ds_5^2 &\sim \frac{\ell^2}{r^2} \left(1 - \frac{2a^2}{r^2}\right) dr^2 + \frac{r^2}{\ell^2} \left(1 + \frac{a^2}{r^2}\right) [-dt^2 + dz^2 + dx_1^2 + dx_2^2 + \frac{\sqrt{10}a^4}{r^4} dt dz], \\ \varphi &\sim -\frac{5\sqrt{\frac{5}{3}}a^8}{r^8}. \end{aligned} \tag{5.9}$$

If we express the metric near $r = +\infty$ in terms of a radial coordinate ρ , for which the metric in the radial direction is $\ell^2 d\rho^2/\rho^2$, then the solution looks like

$$\begin{aligned} ds_5^2 &= \frac{\ell^2}{\rho^2} d\rho^2 + \frac{\rho^2}{\ell^2} \left[\left(1 - \frac{25a^8}{24\rho^8}\right) (-dt^2 + dz^2) + \frac{\sqrt{10}a^4}{\rho^4} dt dz \right. \\ &\quad \left. + \left(1 + \frac{5a^8}{24\rho^8}\right) (dx_1^2 + dx_2^2) \right] + \dots \end{aligned} \tag{5.10}$$

Using the energy and momentum formulae obtained in appendix A, we can straightforwardly to obtain the mass and linear momentum per unit 3-volume spanned over (z, x_1, x_2) for the AdS₅ wormhole, given by

$$E = 0, \quad P = \frac{\sqrt{10}a^4}{8\pi\ell^5}. \tag{5.11}$$

Of course, we can boost the system along the (t, z) direction and obtain a non-zero mass; however, we shall always have $E^2 - P^2 = -25a^8/(16\pi^2\ell^{10}) < 0$.

6 Further AdS Wormholes

6.1 M2-brane and AdS₄ wormhole

We can obtain an M2-brane wormhole solution of eleven-dimensional supergravity, given by (3.5) with $D = 10$, $n = 4$, $N = 1$:

$$\begin{aligned} ds_{11}^2 &= \left(\frac{H}{\cos u}\right)^{1/3} \left[a^2 d\Omega_7^2 + \frac{a^2 du^2}{36 \cos^2 u} \right] \\ &\quad + H^{-2/3} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx^2 \right), \\ A &= \frac{1}{H} dt \wedge dz \wedge dx, \quad H = c_0 - \frac{q}{a^6} u, \quad v = \sqrt{\frac{7}{12}}(\pi - 2u). \end{aligned} \quad (6.1)$$

The coordinate u is related to the original r coordinate by

$$u = \arctan \sqrt{\left(1 + \frac{r^2}{a^2}\right)^6 - 1}, \quad (6.2)$$

or, in other words,

$$\cos u = \left(1 + \frac{r^2}{a^2}\right)^{-3}, \quad \sin \frac{1}{2}u = \frac{r(r^4 + 3r^2 a^2 + 3a^4)^{1/2}}{\sqrt{2}(r^2 + a^2)^{3/2}}. \quad (6.3)$$

As r ranges from $-\infty$ to $+\infty$, u ranges from $-\frac{1}{2}\pi$ to $+\frac{1}{2}\pi$.

Choosing $c_0 = \pi q / (2a^6)$, we find that H has the asymptotic forms

$$\begin{aligned} r \rightarrow +\infty : \quad H &= \frac{q}{r^6} - \frac{3a^2 q}{r^8} + \dots, \\ r \rightarrow -\infty : \quad H &= \frac{\pi q}{a^6} - \frac{q}{r^6} + \frac{3a^2 q}{r^8} + \dots. \end{aligned} \quad (6.4)$$

The metric approaches AdS₄ \times S^7 near $r = +\infty$, while it becomes flat as r approaches $-\infty$.

A breathing-mode reduction of eleven-dimensional supergravity on S^7 , in which the metric and 4-form are written as

$$\begin{aligned} ds_{11}^2 &= e^{2\alpha\varphi} ds_4^2 + (2\ell)^2 e^{-\frac{4\alpha}{7}\varphi} d\Omega_7^2, \\ *F_{(4)} &= 6(2\ell)^6 \Omega_7, \end{aligned} \quad (6.5)$$

where $\alpha = \sqrt{7}/6$, yields [22] the four-dimensional bosonic Lagrangian

$$\mathcal{L}_4 = \sqrt{-g} \left(R - \frac{1}{2}(\partial\varphi)^2 - V \right), \quad V = -\frac{3}{2\ell^2} \left(7e^{\frac{18\alpha}{7}\varphi} - 3e^{6\alpha\varphi} \right). \quad (6.6)$$

Reducing the solution (6.1), with $2\ell = q^{1/6}$, therefore gives the four-dimensional solution

$$\begin{aligned} ds_4^2 &= \left(\frac{a}{2\ell}\right)^7 \left\{ \left(\frac{H}{\cos u}\right)^{3/2} \frac{a^2 du^2}{36 \cos^2 u} \right. \\ &\quad \left. + \frac{H^{1/2}}{\cos^{7/6} u} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx^2 \right) \right\}, \\ e^{-\frac{4\alpha}{7}\varphi} &= \frac{a^2}{4\ell^2} \left(\frac{H}{\cos u}\right)^{1/3}. \end{aligned} \quad (6.7)$$

In the limit of $r \rightarrow \infty$, the metric including the subleading terms is given by

$$ds_4^2 = \frac{\ell^2}{\tilde{r}^2} \left(1 - \frac{a^2}{2\ell\tilde{r}}\right) d\tilde{r}^2 + \frac{\tilde{r}^2}{\ell^2} \left(1 + \frac{a^2}{2\ell\tilde{r}}\right) (-dt^2 + dz^2 + dx^2 + \frac{\sqrt{7/3} a^6}{\ell^3 \tilde{r}^3} dt dz) + \dots, \quad (6.8)$$

where $\tilde{r} = r^2/(4\ell)$. Expressed instead in terms of a radial variable ρ for which the radial term is exactly $\ell^2 d\rho^2/\rho^2$, the expansion at large r takes the form

$$ds_4^2 = \frac{\ell^2 d\rho^2}{\rho^2} + \frac{\rho^2}{\ell^2} \left[\left(1 - \frac{7a^{12}}{2^{15} \ell^6 \rho^6}\right) (-dt^2 + dz^2) + \left(1 + \frac{7a^{12}}{3 \cdot 2^{15} \ell^6 \rho^6}\right) dx^2 + \frac{\sqrt{7/3} a^6}{32 \ell^3 \rho^3} dt dz \right] + \dots. \quad (6.9)$$

Using the formulae in appendix A for the energy and z -momentum per unit area in the (z, x) plane, we find

$$E = 0, \quad P = \frac{\sqrt{21} a^6}{8\pi(2\ell)^7}. \quad (6.10)$$

6.2 M5-brane and AdS₇ wormhole

There is also an M5-brane wormhole solution in eleven-dimensional supergravity, given by taking $D = 7$, $n = 7$, $N = 1$ in (3.5):

$$\begin{aligned} ds_{11}^2 &= \left(\frac{H}{\cos u}\right)^{2/3} \left[a^2 d\Omega_4^2 + \frac{a^2 du^2}{9 \cos^2 u} \right] \\ &\quad + H^{-1/3} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx^i dx^i \right), \\ F_4 &= 3q \Omega_4, \quad H = c_0 - \frac{q}{a^3} u, \end{aligned} \quad (6.11)$$

where

$$v = \frac{2(\pi - 2u)}{\sqrt{6}}. \quad (6.12)$$

The index i ranges over $1 \leq i \leq 4$.

The coordinate u is related to the original radial coordinate r by

$$\cos u = \left(1 + \frac{r^2}{a^2}\right)^{-3/2}, \quad \sin u = \frac{r(r^4 + 3r^2 a^2 + 3a^4)^{1/2}}{(r^2 + a^2)^{3/2}}. \quad (6.13)$$

As usual, r ranges from $-\infty$ to $+\infty$, implying that u ranges from $-\frac{1}{2}\pi$ to $+\frac{1}{2}\pi$.

With the choice $c_0 = \pi q/(2a^3)$, the function H tends to zero at $r = +\infty$ and it tends to a constant at $r = -\infty$:

$$\begin{aligned} r \rightarrow +\infty : \quad H &= \frac{q}{r^3} - \frac{3qa^2}{2r^5} + \dots, \\ r \rightarrow -\infty : \quad H &= \frac{\pi q}{a^3} - \frac{q}{r^3} + \frac{3qa^2}{2r^5} + \dots. \end{aligned} \quad (6.14)$$

The metric approaches AdS₇ \times S^4 near $r = +\infty$, while it becomes flat as r approaches $-\infty$.

The breathing-mode reduction of eleven-dimensional supergravity on S^4 , using the ansatz

$$\begin{aligned} ds_{11}^2 &= e^{2\alpha\varphi} ds_7^2 + (\tfrac{1}{2}\ell)^2 e^{-\frac{5}{2}\alpha\varphi} d\Omega_4^2, \\ F_4 &= 3(\tfrac{1}{2}\ell)^3 \Omega_4, \end{aligned} \quad (6.15)$$

with $\alpha = 2/(3\sqrt{10})$, yields the seven-dimensional bosonic Lagrangian

$$\mathcal{L}_7 = \sqrt{-g} \left(R - \tfrac{1}{2}(\partial\varphi)^2 - V \right), \quad V = -\frac{6}{\ell^2} \left(8e^{\frac{9}{2}\alpha\varphi} - 3e^{12\alpha\varphi} \right). \quad (6.16)$$

The solution (6.11), with $\frac{1}{2}\ell = q^{1/3}$, reduces to give

$$\begin{aligned} ds_7^2 &= \left(\frac{2a}{\ell} \right)^{8/5} \left\{ \left(\frac{H}{\cos u} \right)^{6/5} \frac{a^2 du^2}{9 \cos^2 u} \right. \\ &\quad \left. + \frac{H^{1/5}}{\cos^{8/15} u} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz + dx^i dx^i \right) \right\}, \\ e^{-\frac{5\alpha}{2}\varphi} &= \frac{4a^2}{\ell^2} \left(\frac{H}{\cos u} \right)^{2/3}. \end{aligned} \quad (6.17)$$

Taking the limit $r \rightarrow \infty$, the metric up to sub-leading order terms is

$$ds_7^2 = \left(1 - \frac{8a^2\ell^2}{\tilde{r}^4} \right) \frac{\ell^2 d\tilde{r}^2}{\tilde{r}^2} + \frac{\tilde{r}^2}{\ell^2} \left(1 + \frac{2a^2\ell^2}{\tilde{r}^4} \right) \left(-dt^2 + dz^2 + dx^i dx^i + \frac{32\sqrt{2/3} a^3 \ell^3}{\tilde{r}^6} dt dz \right) + \dots, \quad (6.18)$$

where \tilde{r} is given by $r = \tilde{r}^2/(2\ell)$. Expressed in terms of a radial coordinate ρ for which the radial term in the metric is exactly $\ell^2 d\rho^2/\rho^2$, the expansion takes the form

$$\begin{aligned} ds_7^2 &= \frac{\ell^2 d\rho^2}{\rho^2} + \frac{\rho^2}{\ell^2} \left[\left(1 - \frac{384a^6\ell^6}{5\rho^{12}} \right) (-dt^2 + dz^2) + \left(1 + \frac{128a^6\ell^6}{15\rho^{12}} \right) dx^i dx^i \right. \\ &\quad \left. + \frac{32\sqrt{2/3} a^3 \ell^3}{\rho^6} dt dz \right] + \dots. \end{aligned} \quad (6.19)$$

The energy and z -momentum per unit 5-volume spanned by (z, x^i) , calculated using the results in appendix A, are given by

$$E = 0, \quad P = \frac{2\sqrt{6} a^3}{\pi\ell^4}. \quad (6.20)$$

6.3 Dyonic string and AdS₃ wormholes

The dyonic string is a six dimensional solution supported by a 3-form field strength, corresponding to the Lagrangian (3.1), but with $\hat{D} = 6$, $n = 3$ and $N = 1$. The solution can be lifted to $D = 10$ and viewed as either the D1/D5 system or the NS-NS-1/NS-NS-5 system.

There are two dyonic string wormhole solutions. The first one can be obtained by lifting the magnetic string solution of the $U(1)^3$ theory obtained in [16] to six dimensions. It is given by

$$\begin{aligned}
ds_6^2 &= (H_e H_m)^{-\frac{1}{2}} \left(\frac{r^2 - 2asr - a^2}{r^2 + a^2} dt^2 + \frac{4acr}{r^2 + a^2} dt dz - \frac{r^2 + 2asr - a^2}{r^2 + a^2} dz^2 \right) \\
&\quad + (H_e H_m)^{\frac{1}{2}} \left(f [dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2 \theta d\phi^2)] + f^{-1} a^2 (d\psi + n \cos \theta d\phi)^2 \right), \\
\varphi &= \frac{1}{\sqrt{2}} \log \left(\frac{H_e}{H_m} \right), \quad F_{(3)} = dA_{(2)}, \quad A_{(2)} = \frac{1}{H_e} dt \wedge dz + q_m a \cos \theta d\phi \wedge d\psi, \\
H_e &= \alpha_e - \frac{q_e}{a} \arctan\left(\frac{r}{a}\right), \quad H_m = \alpha_m - \frac{q_m}{a} \arctan\left(\frac{r}{a}\right), \quad f = \alpha - n \arctan\left(\frac{r}{a}\right). \quad (6.21)
\end{aligned}$$

In this solution, the level surface of the four-dimensional tranverse is not spherical symmetric, but a squashed S^3 . This solution is effectively the same as the one discussed earlier, and we shall not analyse it further.

There is another dyonic string wormhole solution that is spherically symmetric on the S^3 , given by

$$\begin{aligned}
ds_6^2 &= \frac{(H_e H_m)^{\frac{1}{2}}}{\cos u} \left[a^2 d\Omega_3^2 + \frac{a^2 du^2}{4 \cos^2 u} \right] \\
&\quad + (H_e H_m)^{-\frac{1}{2}} \left(\cos v (-dt^2 + dz^2) + 2 \sin v dt dz \right), \\
\varphi &= \frac{1}{\sqrt{2}} \log \left(\frac{H_e}{H_m} \right), \quad F_{(3)} = dt \wedge dz \wedge dH_e^{-1} + *_6(dt \wedge dz \wedge dH_m^{-1}), \\
H_e &= c_e - \frac{q_e}{a^2} u, \quad H_m = c_m - \frac{q_m}{a^2} u, \quad v = \frac{1}{2} \sqrt{3} (\pi - 2u). \quad (6.22)
\end{aligned}$$

The coordinate u is related to the original r coordinate by

$$\sin u = \frac{r \sqrt{r^2 + 2a^2}}{r^2 + a^2}. \quad (6.23)$$

In order for the metric to be asymptotic to AdS_3 for $r \rightarrow \infty$, it is necessary to choose $c_e = \pi q_e / (2a^2)$ and $c_m = \pi q_m / (2a^2)$. For simplicity, let us consider the case with $q_e = q = q_m$. We can reduce the solution on the three sphere, with the reduction ansatz given by

$$ds_6^2 = e^{2\alpha\varphi} ds_3^2 + e^{-\frac{2}{3}\alpha\varphi} q d\Omega_{(3)}^2, \quad F_3 = 2q(\Omega_{(3)} + q^{3/2} e^{4\alpha\varphi} \epsilon_{(3)}). \quad (6.24)$$

The resulting three-dimensional Langrangian is given by

$$\mathcal{L}_3 = \sqrt{-g} (R - \frac{1}{2} (\partial\varphi)^2 - V), \quad V = -q^{-1} (6e^{\frac{8}{3}\alpha\varphi} - 4e^{4\alpha\varphi}). \quad (6.25)$$

The corresponding $D = 3$ metric is given by

$$ds_3^2 = \frac{a^6}{\ell^6} \left[\frac{H^4 a^2 du^2}{4 \cos^6 u} + \frac{H^2}{\cos^3 u} (\cos v (-dt^2 + dz^2) + 2 \sin v dt dz) \right]. \quad (6.26)$$

We now take the limit of $r \rightarrow \infty$, and compare the solution with the metric in the Poincaré patch. We find that the metric becomes

$$ds_3^2 = \frac{\ell^2}{\rho^2} d\rho^2 + \left(1 - \frac{3a^2}{4\rho^4}\right) \left(-\frac{\rho^2}{\ell^2} dt^2 + \frac{\rho^2}{\ell^2} dz^2\right) - \frac{2\sqrt{3}a^2}{\ell^2} dt dz + \dots, \quad (6.27)$$

where $\ell^2 = q$.

Note that for general electric and magnetic charges (q_e, q_m) , we have $\ell = \sqrt{q_e q_m}$. Up to the order of $1/\rho^2$ in the cross-term and $1/\rho^4$ in the rest, this is precisely the metric given in (4.10).

7 Conclusions

In this paper, we have constructed various examples of smooth Lorentzian-signature wormholes in supergravity theories. In general, the solutions are supported by gravity, a dilatonic scalar, and a p -form field strength, and the resulting wormhole connects two asymptotic regions that are locally flat. Of particular interest are the cases where only the metric and the p -form are involved. In these non-dilatonic cases, the parameters in the solution can be adjusted so that one of the asymptotic regions approaches AdS.

Acknowledgements

We are grateful to Gary Gibbons and Don Page for helpful discussions. While this work was being completed, A.B. enjoyed the hospitality of the Simons Workshop on Mathematics and Physics, and the KITP Miniprogram on Gauge Theory and Langlands Duality, and thanks them for their support. C.N.P. is grateful to the Centre for Theoretical Cosmology, DAMTP, Cambridge, for hospitality during the course of this work. The research of A.B. was supported in part by DARPA and AFOSR through the grant FA9550-07-1-0543, by the National Science Foundation under Grants No. PHY05-51164 and PHY-0505757, and by Texas A&M University. The research of C.N.P. is supported in part by DOE grant DE-FG03-95ER40917, and the Cambridge-Mitchell Collaboration.

A Conformal Mass for Asymptotically AdS Geometries

In any spacetime that approaches AdS sufficiently rapidly at infinity, we can define conserved charges associated with each of the asymptotic Killing vectors. In particular, by taking the appropriate asymptotically timelike Killing vector, we can calculate the total mass,

or energy, of the spacetime. There are various ways in which this can be done (see, for example, [27] for a discussion), but the simplest and most straightforward is based on a procedure involving a calculation on the conformal boundary of the spacetime, developed by Ashtekar, Magnon and Das [28,29]. In this appendix, we show how this AMD approach may be used to calculate the energy and the momentum of the AdS wormhole solutions that we have constructed in this paper.

There are two types of boundary that are of particular interest when one considers an asymptotically AdS spacetime. One of these is the boundary at large radius in a global coordinate system, for which the boundary topology is $\mathbb{R} \times \text{Sphere}$. (It is assumed here that we are working in the universal covering space CAdS of AdS, in which time ranges over the entire real line rather than being periodically identified.) The other boundary of interest is the one that arises when one considers the Poincaré patch of AdS, for which the boundary is just Minkowski spacetime.

Descriptions of the bulk AdS metrics in these two cases can be given in a unified form, by writing the D -dimensional metric, which satisfies $R_{\mu\nu} = -(D-1)\ell^{-2}g_{\mu\nu}$, as

$$ds_D^2 = -w^2 dt^2 + \frac{dr^2}{w^2} + r^2 d\omega_{D-2}^2, \quad (\text{A.1})$$

where

$$d\omega_{D-2}^2 = \frac{du^2}{1-k^2u^2} + u^2 d\Omega_{D-3}^2, \quad w^2 = k^2 + \frac{r^2}{\ell^2}. \quad (\text{A.2})$$

For any non-vanishing k , $d\omega_{D-2}^2$ is the metric on a round sphere S^{D-2} of radius $1/k$, and (A.1) is a metric on AdS_D in global coordinates. The scale size k can be absorbed by means of coordinate rescalings so that any non-zero k can be set equal to 1 without loss of generality. If $k = 0$, on the other hand, the metric (A.1) instead describes the Poincaré patch of AdS.²

The use of the AMD method to calculate the mass of various higher-dimensional asymptotically AdS black holes was described in [30] and [27]. In all these examples, the black hole metrics were asymptotic to global AdS. However, in the AdS wormhole solutions that we have constructed in this paper, the asymptotic form approaches the Poincaré patch of AdS. It is instructive, therefore, first to check how the AMD calculation of the mass works in a simple $k = 0$ example.

²One can also take k^2 to be negative, in which case the metric (A.1) describes de Sitter spacetime, and $d\omega_{D-2}^2$ is the metric on a hyperboloid of constant negative curvature. Including the possibility of negative k^2 , one can always, by means of coordinate scalings, set k^2 to be 0, 1 or -1 , depending on whether it is initially zero, positive or negative.

The conformal boundary of AdS in either case can be brought in from infinity by rescaling the metric with a conformal factor

$$\Omega = \frac{\ell}{r}, \quad (\text{A.3})$$

to give $\bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$. Let $\bar{C}^\mu{}_{\nu\rho\sigma}$ to be the Weyl tensor of the metric $\bar{g}_{\mu\nu}$ and $\bar{n}_\mu = \partial_\mu \Omega$. The conserved charge $Q[K]$ associated to the asymptotic Killing vector K is then given by

$$\begin{aligned} Q[K] &= \frac{\ell}{8\pi(D-3)} \oint_{\Sigma} \bar{\mathcal{E}}^\mu{}_\nu K^\nu d\bar{\Sigma}_\mu, \\ \bar{\mathcal{E}}^\mu{}_\nu &= \ell^2 \Omega^{3-D} \bar{n}^\rho \bar{n}^\sigma \bar{C}^\mu{}_{\rho\nu\sigma}. \end{aligned} \quad (\text{A.4})$$

In order to define the energy, one takes $K = \partial/\partial t$, to give

$$E = \frac{\ell}{8\pi(D-3)} \oint_{\Sigma} \bar{\mathcal{E}}^t{}_t d\bar{\Sigma}_t. \quad (\text{A.5})$$

In the case of our AdS wormhole solutions, we can also obtain the linear momentum along the z direction, by taking $K = \partial/\partial z$, giving

$$P = \frac{1}{8\pi(D-3)} \oint_{\Sigma} \bar{\mathcal{E}}^t{}_z d\bar{\Sigma}_t. \quad (\text{A.6})$$

A simple example that illustrates the calculation of the AMD mass for both the $k = 1$ and $k = 0$ cases is provided by charged non-rotating black holes in five-dimensional minimal gauged supergravity. The solution can be written as

$$\begin{aligned} ds_5^2 &= f H^{-2} dt^2 + H \left(f^{-1} dt^2 + r^2 ds_3^2 \right), \\ A &= \sqrt{3} (1 - H^{-1}) \sqrt{\frac{\mu + k^2 q}{q}} dt, \end{aligned} \quad (\text{A.7})$$

where

$$f = k^2 - \frac{\mu}{r^2} + g^2 r^2 H^3, \quad H = 1 + \frac{q}{r^2}, \quad (\text{A.8})$$

and

$$ds_3^2 = \frac{du^2}{1 - k^2 u^2} + u^2 d\Omega_2^2. \quad (\text{A.9})$$

Here $g = 1/\ell$. Note that ds_3^2 is a metric on a 3-sphere of radius k^{-1} . The solution is valid for any k , including $k = 0$.

Calculating the thermodynamic quantities, we find

$$\begin{aligned} S &= \frac{\pi^2 (q + r_+^2)^{3/2}}{2k^3}, \quad T = \frac{k^2 r_+^4 + g^2 (2r_+^2 - q)(q + r_+^2)^2}{2\pi r_+^2 (q + r_+^2)^{3/2}}, \\ Q &= \frac{\pi \sqrt{3q} \sqrt{q + r_+^2} \sqrt{k^2 r_+^2 + g^2 (q + r_+^2)^2}}{k^3 r_+}, \quad \Phi = \frac{\sqrt{3q} \sqrt{k^2 r_+^2 + g^2 (q + r_+^2)^2}}{4r_+ \sqrt{q + r_+^2}}, \\ E &= \frac{3\pi \left[k^2 r_+^2 (2q + r_+^2) + g^2 (q + r_+^2)^3 \right]}{8k^3 r_+^2}, \end{aligned} \quad (\text{A.10})$$

where r_+ is the largest root of $f(r) = 0$. E here is calculated using the AMD procedure. These quantities satisfy the first law of thermodynamics,

$$dE = TdS + \Phi dQ, \tag{A.11}$$

for any arbitrary constant value for k , including $k = 0$.

All of E , S and Q have a factor k^3 in the denominator. This is associated with the fact that ds_3^2 describes a 3-sphere of radius k^{-1} . In the limit where $k \rightarrow 0$, corresponding to the black hole with flat horizon, we should multiply E , S and Q by k^3 before taking the limit, and interpret the rescaled quantities as the energy, entropy and charge per unit 3-volume. (Or else, re-interpret the metric ds_3^2 as being defined on T^3 , and so take $\int \sqrt{g_3} d^3x$ to be the volume of the T^3 .)

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