CTP-TAMU-2/98
RU98-2-B
SU-ITP-98/01
hep-th/9801072

g=1 for Dirichlet 0-branes

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ABSTRACT

Dirichlet 0-branes, considered as extreme Type IIA black holes with spin carried by fermionic hair, are shown to have the anomalous gyromagnetic ratio $g = 1$, consistent with their interpretation as Kaluza-Klein modes.

[1]Research supported in part by NSF Grant PHY-9722090.
[2]Research supported in part by the U. S. Department of Energy under grant no. DOE-91ER40651-TASKB.
[3]Research supported by NSF Grant PHY-9219345.

1 Dirichlet 0-branes

Dirichlet 0-branes enjoy a multiple personality. They are believed to be (in historical order):

1) **The massive Kaluza-Klein (KK) modes associated with compactifying D=11 supergravity on a circle [1]**. As such they carry mass $M_n = |n|/R$ and charge $Q_n = n$ where R is the S^1 radius. Consequently, they saturate the BPS bound and belong to short massive multiplets of $D = 10, N = 2$ supersymmetry belonging to the $(44 + 128 + 84)$-dimensional representation of the $SO(9)$ little group.

2) **Extreme black holes of Type IIA string theory [2]**. Since these $D = 10$ extreme black holes preserve one half of the supersymmetry [3], they also belong to the same short supermultiplets as the Kaluza-Klein modes. Moreover, they also have the same mass and charge quantum numbers as the KK modes and may tentatively be identified with them [4], in much the same way as KK string states may be identified with extreme black holes in $D = 4$ [5, 6]. There is a curious difference, however. In $D = 4$, spherically symmetric black holes may represent spin-0 members of a KK supermultiplet. In $D = 10$, the $(44+128+84)$ multiplet has no $SO(9)$ singlet, even though the Horowitz-Strominger black hole solution [2] is spherically symmetric and preserves half the supersymmetry in the extreme limit [3]. We shall return to this puzzle below.

3) **The p=0 special case of surfaces of dimension p on which open strings can end [7]**. These strings obey Dirichlet boundary conditions, hence the name. Their Type I origin also implies half of the available supersymmetry.

4) **The partons of the Matrix Model [8]**. In this picture, a charge n Dirichlet 0-brane is a zero energy bound state of n singly charged 0-branes [9].

In the absence of a complete formulation of some overarching M-theory, each of the above pictures is necessary and useful, and it is still worthwhile to perform consistency checks on their presumed equivalence. One such check concerns the gyromagnetic ratios, similar to the check performed for black holes and string states in $D = 4$ [10]. In this paper, we compare the g-factors of Dirichlet 0-branes from the perspectives of (1) and (2). The black hole calculation relies on generating the spin from the fermionic zero-modes as in [11, 10] and yields the anomalous value $g = 1$, consistent with the KK result from compactifying on a circle[4].

2 g=1 from Kaluza-Klein arguments

Recall that for a Kaluza-Klein compactification from five to four dimensions, it was shown in [15] that all massive Kaluza-Klein states share a common gyromagnetic ratio of $g = 1$. At first sight, this is somewhat of a surprise, as tree-level unitarity ordinarily demands that the gyromagnetic ratio takes on the "natural" value of $g = 2$ [16, 17, 18]. Consequently, the value $g = 2$ is required in QED, in the Standard Model and indeed in the perturbative sector of open string theory [19]. However this consideration is only important in the energy range $M_{\rm Pl} > E > M/Q$. Since Kaluza-Klein states have $M \sim Q$ (which is also the BPS

[4]In this paper we assume for simplicity a direct product $M^{10} \times S^1$ but, as discussed in [12], the $D = 11$ origin of Type IIA string theory [13, 4, 14] and the Dirichlet 0-brane interpretation work equally well when the compactifying manifold is a $U(1)$ fibration.

condition), we see that this range is essentially empty, and hence there is no conflict with tree-level unitarity, gauge invariance or any other principle.

Although the gyromagnetic ratio is traditionally defined in $D = 4$ as a proportionality relation between spin and magnetic moment 3-vectors, it turns out that there is a natural generalization to arbitrary dimensions. To see this, we note that for $D > 4$ the angular momentum is more properly represented not as a vector, $\vec{J}$, but rather as the adjoint representation of the $SO(D-1)$ rotation group, J^{ij} $(i, j = 1, 2, \ldots, D-1)$. Similarly, both the magnetic field strength, F_{ij}, and the magnetic dipole moment, μ^{ij}, are in the 2-index antisymmetric representation as well, so that the magnetic dipole interaction in any dimension may be written as

$$\Delta E = -\frac{1}{2}\mu^{ij} F_{ij} \ . \tag{1}$$

The gyromagnetic ratio g may then be defined in general by

$$\mu^{ij} = \frac{gQ}{2M} J^{ij} \ , \tag{2}$$

which reduces to the standard expression in $D = 4$.

Given this generalization of the gyromagnetic ratio, it is natural to wonder whether $g = 1$ is a universal property of Kaluza-Klein states in any dimension. In fact, since the results of [15] are mostly dimension independent, they are easily extended to arbitrary dimension, thus showing that $g = 1$ is indeed universally true. As an example, we consider the Kaluza-Klein reduction of a massless d-form potential C_d with field strength $K_{d+1} = dC_d$ from $D+1$ to D dimensions. We use the Kaluza-Klein decomposition

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} + e^{2\varphi} A_\mu A_\nu & e^{2\varphi} A_\mu \\ e^{2\varphi} A_\nu & e^{2\varphi} \end{pmatrix} \ , \tag{3}$$

where for simplicity $e^{2\varphi}$, which determines the length-scale of the compactification, is taken to be a constant. Furthermore, in order to examine the magnetic dipole interaction, it is sufficient to work to linear order in the Kaluza-Klein gauge field A_μ. In this case, the $(D+1)$-dimensional equation of motion, $\nabla^M K_{MN_1 \cdots N_d} = 0$, reduces to the following two equations in D dimensions:

$$\begin{aligned} 0 &= D^\lambda K_{\lambda\mu_1\cdots\mu_d} - (\nabla^\lambda A_\lambda + A_\lambda \nabla^\lambda - e^{-2\varphi}\partial_i) K_{i\mu_1\cdots\mu_d} + d\, \nabla^\lambda A_{\mu_1} K_{i\lambda\mu_2\cdots\mu_d} \ , \\ 0 &= D^\lambda K_{i\lambda\mu_1\cdots\mu_{d-1}} + \tfrac{1}{2} e^{2\varphi} F^{\rho\sigma} K_{\rho\sigma\mu_1\cdots\mu_{d-1}} \ , \end{aligned} \tag{4}$$

where $D_\mu = \nabla_\mu - A_\mu \partial_i$ and antisymmetrization of the $\{\mu_1, \mu_2, \mu_3, \ldots\}$ is always implied. Note that i denotes the compact coordinate and only takes on a single value in this case. For $z = z + 2\pi L$, the n-th Kaluza-Klein state has mass $M_n = \langle e^{-\varphi} \rangle |n|/L = |n|/R$ measured in the $(D+1)$-dimensional metric[5] and charge $Q_n = n/L$. Here R is the radius $R = L\langle e^{\varphi} \rangle$.

For massive states $(n \neq 0)$ we may employ the gauge condition $C_{i\mu_1\cdots\mu_{d-1}} = 0$ to finally arrive at the equation of motion

$$0 = (D^\lambda D_\lambda - M_n^2) C_{\mu_1\cdots\mu_d} + \tfrac{1}{2} Q_n F_{\rho\sigma} (\Sigma^{\rho\sigma} C)_{\mu_1\cdots\mu_d} - i \tfrac{d}{Q_n} e^{2\varphi} F^{\rho\sigma} \nabla_{\mu_1} \nabla_\rho C_{\sigma\mu_2\cdots\mu_d} \ , \tag{5}$$

[5] Note that for $D = 10$ the usual string dilaton ϕ is given by $\phi = 3\varphi/2$. This explains the unconventional factors in the expressions for the mass and radius.

where the second term contains the magnetic dipole interaction and the last term represents the Thomas precession. We have used the explicit form of the angular momentum generators in the d-fold antisymmetric representation:

$$(\Sigma^{\mu\nu})_{\{\alpha\}}{}^{\{\beta\}} = -2id\delta^{[\mu}{}_{[\alpha_1}\eta^{\nu][\beta_1}\delta^{\beta_2\cdots\beta_d]}_{\alpha_2\cdots\alpha_d]} , \tag{6}$$

where all symbols are antisymmetric with weight one. Combining (1) and (2), we finally obtain the result that $g = 1$ for the massive Kaluza-Klein modes of the d-form potential C_d. Similar arguments may be used to show that $g = 1$ holds for the reduction of arbitrary fields as well.

The general result of $g = 1$ for Kaluza-Klein states can also be understood from the closed string point of view. While the left- and right-moving modes on the string world sheet naturally lead to the definition of separate g-factors [20, 21]

$$(g_L, g_R) = \left(\frac{2\langle S_R\rangle}{\langle S_L + S_R\rangle}, \frac{2\langle S_L\rangle}{\langle S_L + S_R\rangle}\right) \tag{7}$$

($\vec{S}_L$ and $\vec{S}_R$ are contributions to the total spin from worldsheet left- and right-movers), it is nevertheless possible to focus on a single Kaluza-Klein g_{KK}. This has in fact been calculated in [20] with the result

$$g_{KK} = 1 - \frac{W}{Q}\frac{\langle S_L - S_R\rangle}{\langle S_L + S_R\rangle} , \tag{8}$$

where Q and W are Kaluza-Klein and winding charge respectively. Therefore we see that $g_{KK} = 1$ for pure Kaluza-Klein states where $W = 0$. Note that although [20, 21] were interested in four dimensions, the resulting g-factors are in fact valid in arbitrary dimensions based on the definitions (1) and (2).

3 g=1 from black hole arguments

We now turn to the properties of the Dirichlet 0-brane, viewed as a soliton of ten-dimensional Type IIA supergravity, and proceed to calculate its gyromagnetic ratio. In this case, the relevant part of the supergravity action is given (in the Einstein frame) by

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{-g}[R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{4}e^{\frac{3}{2}\phi}F^2_{\mu\nu}] , \tag{9}$$

where $F = dA$ is the field strength of the RR gauge boson A_μ. Ignoring both $B_{\mu\nu}$ and $C_{\mu\nu\lambda}$, which do not play a role in the 0-brane solution, the relevant supersymmetry variations are given by [22, 1]

$$\begin{aligned} \delta e_\mu{}^a &= -i\bar{\epsilon}\gamma^a\psi_\mu , \\ \delta\phi &= -i\sqrt{2}\bar{\epsilon}\Gamma_{11}\lambda , \\ \delta A_\mu &= -ie^{-\frac{3}{4}\phi}\bar{\epsilon}(\Gamma_{11}\psi_\mu + \frac{3}{2\sqrt{2}}\gamma_\mu\lambda) , \end{aligned} \tag{10}$$

for the bosons, and

$$
\begin{aligned}
\delta\lambda &= -\frac{1}{2\sqrt{2}}[\gamma^\mu\Gamma_{11}\partial_\mu\phi + \frac{3}{8}e^{\frac{3}{4}\phi}\gamma^{\mu\nu}F_{\mu\nu}]\epsilon \ , \\
\delta\psi_\mu &= [\nabla_\mu - \frac{1}{64}e^{\frac{3}{4}\phi}(\gamma_\mu{}^{\nu\lambda} - 14\delta_\mu{}^\nu\gamma^\lambda)\Gamma_{11}F_{\nu\lambda}]\epsilon \ ,
\end{aligned} \tag{11}
$$

for the fermions.

The 0-brane soliton was originally constructed in [2] as a ten-dimensional black hole solution to the bosonic equations of motion of (9). In the extreme limit we obtain the field configuration

$$
\begin{aligned}
&ds^2 = -e^{-\frac{7}{6}(\phi-\phi_0)}dt^2 + e^{\frac{1}{6}(\phi-\phi_0)}d\vec{x}^2 \ , \\
&A_0 = e^{-\frac{4}{3}(\phi-\phi_0)-\frac{3}{4}\phi_0} \ , \\
&e^{\frac{4}{3}(\phi-\phi_0)} = \left(1 + \frac{ke^{-\frac{3}{4}\phi_0}}{r^7}\right) \ .
\end{aligned} \tag{12}
$$

The properties of this solution may be determined asymptotically through an ADM procedure. To read off the mass M, charge Q, angular momentum J^{ij}, electric dipole moment d^i and magnetic dipole moment μ^{ij}, we compare the 0-brane solution with the generic D-dimensional configuration:

$$
\begin{aligned}
g_{00} &\sim -\left(1 - \frac{2\kappa^2 M}{(D-2)\omega_{D-2}r^{D-3}}\right) \ , \\
g_{ij} &\sim \left(1 + \frac{2\kappa^2 M}{(D-2)(D-3)\omega_{D-2}r^{D-3}}\right)\delta_{ij} \ , \\
g_{0i} &\sim -\kappa^2 J^{ij}\frac{\hat{x}_j}{\omega_{D-2}r^{D-2}} \ ,
\end{aligned} \tag{13}
$$

for the metric (where $\omega_n = 2\pi^{(n+1)/2}/\Gamma((n+1)/2)$ is the volume of an n-sphere) [23], and

$$
\begin{aligned}
A_0 &\sim \frac{Q}{(D-3)r^{D-3}} + d^i\frac{\hat{x}_i}{r^{D-2}} \ , \\
A_i &\sim \qquad\qquad - \mu^{ij}\frac{\hat{x}_j}{r^{D-2}} \ ,
\end{aligned} \tag{14}
$$

for the gauge fields. Thus the 0-brane has mass $M = \frac{7\omega_8}{2\kappa^2}ke^{-\frac{3}{4}\phi_0}$ and electric charge $Q = -7ke^{-\frac{3}{2}\phi_0}$. The bosonic 0-brane solution is spherically symmetric and hence appears to have vanishing angular momentum and vanishing dipole moments. However, this is not necessarily the case when fermion zero modes are included, as we see below. Note that in $D = 10$, bosonic Kerr-type angular momentum associated with *rotating* black hole solutions will play no role in determining the g-factor of Dirichlet 0-branes. The role of Kerr angular momentum in four dimensions will, however, be discussed in section (4).

Comparing the mass with the charge, we find that the 0-brane saturates the Bogomol'nyi bound, $M \geq |Z|$, where $Z = \frac{\omega_8}{2\kappa^2}Qe^{\frac{3}{4}\phi_0}$ is the central charge. As a result, this state preserves exactly half of the supersymmetries as may also be seen from the supersymmetry variations [3]. In fact, inserting the solution, (12), into the fermionic supersymmetry variations, we find

$$\begin{aligned}\delta\lambda &= -\frac{1}{\sqrt{2}}\gamma^i\Gamma_{11}\partial_i\phi P_+\epsilon \ ,\\ \delta\psi_0 &= -\frac{7}{12}{\gamma_0}^i\partial_i\phi P_+\epsilon \ ,\\ \delta\psi_i &= \frac{1}{12}({\gamma_i}^j - 7{\delta_i}^j)\partial_j\phi P_+\epsilon \ ,\end{aligned} \tag{15}$$

where $\epsilon = e^{-\frac{7}{24}\phi}\epsilon_0$ with ϵ_0 a constant spinor. These supersymmetry variations clearly indicate the splitting of the 32 (real) component spinor ϵ into a set of Killing spinors, $P_+\epsilon_- = 0$, and a set of fermion zero mode spinors, $P_+\epsilon_+ = \epsilon_+$, where the half-supersymmetry projection operator is given by $P_\pm = \frac{1}{2}(1 \pm \gamma^{\overline{0}}\Gamma_{11})$ and the overline indicates tangent space indices.

Before focusing our attention on the gyromagnetic ratio calculation, we note that it is the presence of the fermion zero modes that ensures that the 0-brane fills out a complete supermultiplet. In the absence of any fermion zero modes (*i.e.* in a hypothetical bosonic truncation of the supergravity theory), the bosonic solution of (12) would describe a single spinless particle. However since the 0-brane soliton preserves exactly half of the ten-dimensional $N = 2A$ supersymmetry, there are 16 fermion zero modes present, corresponding to the non-trivially realized supercharges Q_+. Recall that the supersymmetry algebra in the rest frame is of the form

$$\{Q_+, Q_+\} = M + |Z| \ , \qquad \{Q_-, Q_-\} = M - |Z| \ , \qquad \{Q_+, Q_-\} = 0 \ , \tag{16}$$

so that in an appropriate basis the 16 Q_+ form a $SO(16)$ Clifford algebra with two 128-dimensional spinor representations. Making note of the special embedding of $SO(9)$ in $SO(16)$ where $16 \to 16$, $128 \to 128$ and $128' \to 44 + 84$, this demonstrates that the D0-brane soliton indeed fills out a complete $(44 + 128 + 84)$-dimensional representation of the massive $SO(9)$ little group.

Perhaps somewhat surprisingly, the Clifford vacuum associated with the bosonic solution (12) must necessarily carry some non-trivial $SO(9)$ spin. This may be seen by noting that the 16 Q_+'s can be grouped to form 8 complex pairs of creation and annihilation operators on the Clifford vacuum. However there is no 8-dimensional representation of $SO(9)$. Hence any choice of the Clifford vacuum on which the creation/annihilation operators act would break $SO(9)$ invariance. The most natural choice allowing complex Q_+ appears to follow the decomposition $SO(9) \supset SU(2) \times SU(4)$ where $16 \to (2, 4) + (2, \overline{4})$. In this case, the Clifford vacuum corresponds to a $SU(2) \times SU(4)$ singlet, $(1, 1)$, while the complex creation operators $Q_+^\dagger$ transform as $(2, 4)$. With up to eight creation operators acting on the Clifford vacuum, the only $SU(2) \times SU(4)$ singlets appear as $|\Omega\rangle$ and $(Q_+^\dagger)^8|\Omega\rangle$, where $|\Omega\rangle$ corresponds to the bosonic solution (12). Since both the 44 and 84 dimensional representations of $SO(9)$ contain $SU(2) \times SU(4)$ singlets, we find that the bosonic solution falls in general in some combination of the 44 and 84 dimensional representations, and that it is not possible to determine where it truly belongs based purely from the superalgebra alone. Nevertheless, the complete resulting 0-brane supermultiplet is in precisely the correct $(44 + 128 + 84)$-dimensional representation necessary to correspond to the massive Kaluza-Klein states of the 11-dimensional supergraviton.

Properties of the Dirichlet 0-brane superpartners may be derived by performing successive supersymmetry transformations to the original bosonic solution. In particular, denoting the bosonic solution by Φ, complete information of its superpartners are encoded in the finite transformation [11]

$$\Phi \longrightarrow e^{\delta}\Phi = \Phi + \delta\Phi + \frac{1}{2}\delta\delta\Phi + \cdots \ . \tag{17}$$

With 16 zero modes, this series does not terminate until order δ^{16}. However, since each additional pair of supersymmetries brings in another power of the momentum, it is sufficient to work only up to second-order variations when considering properties of the dipole moments. This approach was used in [10] to calculate both electric and magnetic dipole moments of four-dimensional $N = 4$ black holes.

The double supersymmetry variation of the metric yields corrections to the mass and angular momentum of the superpartners. Using $\delta\delta g_{\mu\nu} = -2i\bar{\epsilon}\gamma_{(\mu}\delta\psi_{\nu)}$ and the gravitino transformations for the 0-brane background from (15), we find (for zero-mode spinors ϵ_+)

$$\delta\delta g_{00} = 0 \ , \qquad \delta\delta g_{ij} = 0 \ , \tag{18}$$

so in fact there is no mass shift among the different members of the supermultiplet. On the other hand, we may read off the angular momentum from the asymptotics of the mixed components of the metric

$$\begin{aligned} \delta\delta g_{0i} &= -\frac{2i}{3}(\bar{\epsilon}_+\gamma_{0i}{}^{j}\epsilon_+)\partial_j\phi \\ &\sim \frac{\kappa^2 M}{\omega_8}(i\bar{\epsilon}_+\gamma_{0i}{}^{j}\epsilon_+)\frac{\hat{x}_j}{r^8} \ . \end{aligned} \tag{19}$$

Using the ADM definition of (13), the resulting angular momentum is

$$J^{ij} = -M(i\bar{\epsilon}_+\gamma_0{}^{ij}\epsilon_+) \ . \tag{20}$$

More precisely, this expression may be used to determine the angular momentum carried by any member of the 0-brane supermultiplet by choosing the fermion zero mode spinors ϵ_+ appropriately to correspond to the given state.

For the RR gauge field A_μ, we find

$$\begin{aligned} \delta\delta A_0 &= 0 \ , \\ \delta\delta A_i &= \frac{2i}{3}e^{-\frac{3}{4}\phi}(\bar{\epsilon}_+[\gamma_i{}^{j} + 2\delta_i{}^{j}]\Gamma_{11}\epsilon_+)\partial_j\phi \\ &\sim \frac{Q}{2}(i\bar{\epsilon}_+\gamma_{0i}{}^{j}\epsilon_+)\frac{\hat{x}_j}{r^8} + \text{gauge} \ . \end{aligned} \tag{21}$$

The vanishing of $\delta\delta A_0$ indicates that no electric dipole moment is generated for the superpartners, while $\delta\delta A_i$ results in a magnetic dipole moment

$$\mu^{ij} = -\frac{Q}{2}(i\bar{\epsilon}_+\gamma_0{}^{ij}\epsilon_+) \ . \tag{22}$$

Finally, using the definition (2) for the g-factor, we see that indeed $g = 1$ for all states in the 0-brane supermultiplet, in agreement with the expected Kaluza-Klein result.

4 Lower dimensions and U-duality

While we have shown above that Kaluza-Klein solitons have $g = 1$, this statement is easily generalized to other 0-brane states as well through the inclusion of U-duality. In particular, any 0-brane (in any dimension) that is U-conjugate to a Kaluza-Klein state must necessarily have $g = 1$ as long as U-duality is valid. This result is most straightforward in the maximal supergravities where all gauge bosons are graviphotons so that U-duality indeed relates all 0-branes to Kaluza-Klein states.

The bosonic 0-brane solutions [2] are easily constructed in any dimension, and in fact are part of a large family of general p-brane solutions [24, 25]. To see this, we recall that p-branes may be constructed as solutions to a general bosonic action of a scalar field, φ, coupled to a $(p+2)$-form field strength, $F_{p+2} = dA_{p+1}$, in the presence of gravity. Correspondence to actual theories is then obtained by choosing the scalar coupling appropriately, and with proper identification of φ and F_{p+2} with the actual supergravity fields. While this approach is valid for the bosonic solutions, it does not appear possible to treat the fermions in an equally general manner. In particular, since we are interested in the supersymmetric properties of the 0-branes, each theory must be investigated individually.

To give further evidence for $g = 1$, we now examine 0-branes in eight-dimensional $N = 2$ supergravity. This theory has $SL(3) \times SL(2)$ U-duality, and has six 1-form gauge fields transforming as $(3, 2)$ under the U-duality group. Ignoring the 2-form and 3-form potentials, the bosonic part of the action is

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{-g}[R - \mathrm{tr}P_\mu P^\mu - \frac{1}{2}(\partial_\mu\phi)^2 - e^{-2\phi}(\partial_\mu b)^2 - \frac{1}{4}e^{\phi}F^T_{\mu\nu}MF^{\mu\nu} - \frac{1}{4}e^{-\phi}G^T_{\mu\nu}MG^{\mu\nu}] \ , \quad (23)$$

where the seven scalars of the $D = 8$ $N = 2$ theory have been split into (b, e^ϕ) describing $SL(2)/SO(2)$ and $M = V^TV$ with V_{ai} being a coset representative of $SL(3)/SO(3)$. For the latter coset, the scalar kinetic terms, P_μ, and composite $SO(3)$ connection, Q_μ, are given by the decomposition of the Maurer-Cartan form, $(\partial_\mu VV^{-1})^{ab} = P_\mu^{(ab)} + Q_\mu^{[ab]}$. Note that from an M-theory point of view the six gauge fields split into three Kaluza-Klein fields, A^i_μ, and three fields coming from the reduction of the original 3-form potential, $B_{\mu\, i} \sim \epsilon_{ijk}C_{\mu jk}$. The field strengths are then given by $F^i = dA^i$ and $G_i = dB_i + 2bF^i$.

The relevant supersymmetries are [26]

$$\begin{aligned}
\delta\psi_\mu &= [\nabla_\mu + \frac{1}{4}Q_\mu^{ab}\Gamma^{ab} + \frac{1}{2}e^{-\phi}\partial_\mu b\Gamma^{\overline{123}}]\epsilon \\
&\quad + \frac{1}{48}[e^{\frac{1}{2}\phi}F_{\nu\lambda}{}^i\Gamma_i - \frac{1}{2}e^{-\frac{1}{2}\phi}\epsilon^{ijk}G_{\nu\lambda\, i}\Gamma_{jk}][\gamma_\mu{}^{\nu\lambda} - 10\delta_\mu{}^\nu\gamma^\lambda]\epsilon \ , \\
\delta\chi^a &= [\frac{1}{2}P_\mu^{ab}\Gamma^b + \frac{1}{6}\partial_\mu\phi\Gamma^a - \frac{1}{6}e^{-\phi}\partial_\mu b\epsilon^{abc}\Gamma^{bc}]\gamma^\mu\epsilon \\
&\quad - \frac{1}{8}[e^{\frac{1}{2}\phi}F_{\mu\nu}{}^a + \frac{1}{6}e^{-\frac{1}{2}\phi}\epsilon_{bcd}G_{\mu\nu\, d}(\Gamma_{abc} - 4\delta_{ab}\Gamma_c)]\gamma^{\mu\nu}\epsilon \ , \\
\delta e_\mu{}^a &= -\overline{\epsilon}\gamma^a\psi_\mu \ , \\
V_{ai}\delta A^i_\mu &= -e^{-\frac{1}{2}\phi}\overline{\epsilon}[\Gamma^a\psi_\mu + \frac{1}{6}\gamma_\mu(\Gamma^a\Gamma^b + 6\delta^{ab})\chi^b] \ , \\
(V^{-1})^{ia}\delta B_{\mu\, i} &= -4bV_{ai}\delta A^i_\mu - \frac{1}{2}e^{\frac{1}{2}\phi}\epsilon^{abc}\overline{\epsilon}[\Gamma^{bc}\psi_\mu - \frac{1}{6}\gamma_\mu\Gamma^b(\Gamma^c\Gamma^d - 12\delta^{cd})\chi^d] \ , \qquad (24)
\end{aligned}$$

where we have used an 11-dimensional notation for the Dirac matrices, $\Gamma^M = \{\gamma^\mu, \Gamma^i\}$.

The 0-branes may be constructed by solving the first-order Killing-spinor equations resulting from setting the fermion variations in (24) to zero [10]. For a 0-brane charged under the Kaluza-Klein fields A^i_μ, we choose a half-supersymmetry projection $P_\pm = \frac{1}{2}(1 \pm \gamma^{\overline{0}}\hat{n}\cdot\Gamma)$ where $\hat{n}$ is a unit 3-vector selecting the $U(1)$ component that the 0-brane is charged under. Setting $\phi_0 = 0$ for simplicity, the 0-brane solution is then given by

$$
\begin{aligned}
&ds^2 = -e^{-\frac{5}{3}\phi}dt^2 + e^{\frac{1}{3}\phi}d\vec{x}^2 \ ,\\
&Q_i^{ab} = 0 \ , \qquad P_i^{ab} = -\frac{1}{3}\partial_i\phi(\delta^{ab} - 3\hat{n}^a\hat{n}^b) \ , \qquad b = 0 \ ,\\
&E_i^a = \mp\frac{3}{2}\hat{n}^a\partial_i e^{-\frac{4}{3}\phi} \ ,
\end{aligned} \tag{25}
$$

with resulting Killing-spinor equations

$$
\begin{aligned}
\delta\psi_0 &= -\frac{5}{6}\gamma_0{}^i\partial_i\phi P_+\epsilon \ ,\\
\delta\psi_i &= \frac{1}{6}(\gamma_i{}^j - 5\delta_i{}^j)\partial_j\phi P_+\epsilon \ ,\\
\delta\chi^a &= \hat{n}^a\gamma^i\gamma^{\overline{0}}\partial_i\phi P_+\epsilon \ .
\end{aligned} \tag{26}
$$

In this case, $\epsilon = e^{-\frac{5}{12}\phi}\epsilon_0$ where ϵ_0 is a constant spinor. Note that the first order Killing-spinor equations are incomplete in the sense that they do not fully determine the behavior of $e^{2\phi}$. Only when the bosonic equations of motion are taken into account do we find the harmonic function condition, namely $e^{2\phi} = 1 + k/r^5$ for a spherically symmetric 0-brane solution. In terms of k, the 0-brane has mass $M = \frac{5\omega_6}{2\kappa^2}k$ and a six-dimensional charge vector $\mathcal{Q} = [\vec{Q}, \vec{0}] = [-5k\hat{n}, \vec{0}]$, where the first entry denotes the three Kaluza-Klein charges and the last one the three $B_{\mu\, i}$ charges.

Following the procedure carried out above for the Dirichlet 0-brane, we now examine double supersymmetry variations to determine the properties of superpartners. For the metric, we find

$$
\delta\delta g_{0i} = -(\overline{\epsilon}_+\gamma_{0i}{}^j\epsilon_+)\partial_j\phi \ , \tag{27}
$$

resulting in a supersymmetry generated spin $J^{ij} = -M(\overline{\epsilon}_+\gamma_0{}^{ij}\epsilon_+)$. The gauge field variations are somewhat more intricate. We find

$$
\begin{aligned}
\delta\delta A_0^a &= e^{-\frac{1}{2}\phi}(\overline{\epsilon}_+\Gamma^a\gamma_0{}^i\epsilon_+)\partial_i\phi \ ,\\
\delta\delta B_{0\,a} &= \tfrac{1}{2}e^{\frac{1}{2}\phi}\epsilon^{abc}(\overline{\epsilon}_+\Gamma^{bc}\gamma_0{}^i\epsilon_+)\partial_i\phi \ ,\\
\delta\delta A_i^a &= \hat{n}^a e^{\frac{1}{3}\phi}(\overline{\epsilon}_+\gamma_{0i}{}^j\epsilon_+)\partial_j\phi + \text{gauge} \ ,\\
\delta\delta B_{i\,a} &= \epsilon^{abc}\hat{n}^b e^{\frac{4}{3}\phi}(\overline{\epsilon}_+\Gamma^c\gamma_{0i}{}^j\epsilon_+)\partial_j\phi + \text{gauge} \ ,
\end{aligned} \tag{28}
$$

indicating that superpartners carry both electric and magnetic dipole moments. Using the asymptotics of (14), we find the complete electric and magnetic dipole moment vectors of the 0-brane to be

$$
\begin{aligned}
d^i &= [\frac{|Q|}{2M}(-M\overline{\epsilon}_+\Gamma^a\gamma_0{}^i\epsilon_+), \qquad \frac{Q^a}{2M}(M\overline{\epsilon}_+\tfrac{1}{2}\epsilon^{bcd}\hat{n}^b\Gamma^{cd}\gamma_0{}^i\epsilon_+)] \ ,\\
\mu^{ij} &= [\frac{Q^a}{2M}(-M\overline{\epsilon}_+\gamma_0{}^{ij}\epsilon_+), \qquad \epsilon^{abc}\frac{Q^b}{2M}(-M\overline{\epsilon}_+\Gamma^c\gamma_0{}^{ij}\epsilon_+)] \ .
\end{aligned} \tag{29}
$$

In particular, note the unavoidable presence of dipole moments that are not parallel to the charge vector.

For the Kaluza-Klein gauge fields (the left-hand entries in (29)), we find that the magnetic dipole moment corresponds to $g = 1$ as expected. In addition to the magnetic dipole moment, which is in the same Kaluza-Klein direction as the charge, there is an electric dipole moment which is only non-vanishing in the two directions orthogonal to the charge (since $\hat{n}^a d_i^a[A_\mu] = 0$). This is indeed similar to the case of four-dimensional $N = 4$ BPS black holes, where graviphoton electric dipole moments were found for electric black holes [10] (also with the electric dipole moments orthogonal to the direction of the charge).

Turning to the three gauge fields $B_{\mu\, i}$, we see the interesting result that a magnetic dipole moment is generated for the superpartners that is not in the direction of the charge. Since $\hat{n}^a \mu_{ij}^a[B_\mu] = 0$, only two of the three possible magnetic moments are non-zero. Furthermore, since the zero-mode structure of $\mu_{ij}^a[B_\mu]$ indicates that it is no longer parallel to the spin J_{ij}, transition moments are necessarily present. Putting the electric charge in the first Kaluza-Klein field, the various dipole moments may be represented schematically as

$$\begin{aligned} \mathcal{Q} &= \quad [\,Q \quad 0 \quad 0, \quad 0 \quad 0 \quad 0\,]\,, \\ |d/\mathcal{S}| &= \frac{|Q|}{2M}[\,0 \quad 1 \quad 1, \quad 1 \quad 0 \quad 0\,]\,, \\ |\mu/J| &= \frac{|Q|}{2M}[\,1 \quad 0 \quad 0, \quad 0 \quad 1 \quad 1\,]\,. \end{aligned} \tag{30}$$

This shows that the full 0-brane supermultiplet, in addition to having $g = 1$, in fact has dipole moment couplings to all six graviphotons, regardless of which charge is originally turned on. The dipole moments are split to give three electric and three magnetic moments, which is suggestive of the $SL(2)$ part of the U-duality group having some identification with electric/magnetic duality.

Although we have focused on a Kaluza-Klein 0-brane in the eight-dimensional theory, the above results must hold for all 0-brane solitons as long as U-duality is valid. We have indeed verified this for the case of a 0-brane charged under $B_{\mu\, i}$. For such a solution we need to consider the "dual" projection $\widetilde{P}_\pm = \frac{1}{2}(1 \pm \gamma^{\overline{0}} \epsilon^{abc} \hat{n}^a \Gamma^{bc})$, resulting in a 0-brane with similar properties to the above, with the main difference being $\phi \to -\phi$ in this case.

Finally, the reader might be wondering what role is played by conventional *rotating* black hole solutions of the Kerr type, carrying bosonic angular momentum. This was discussed in [10, 27] in the context of identifying four-dimensional string states and extreme black holes. The answer is that the Kerr angular momentum provides the *superspin* L of the supermultiplet in question. Thus $N = 8$ KK states belong to an $L = 0$ supermultiplet and hence the spins of all members of the multiplet come purely from fermionic hair. This is consistent with their interpretation as the Dirichlet 0-branes of the present paper, in which we perform a further T^6 compactification from $D = 10$ to $D = 4$. $N = 4$ KK states, on the other hand, belong to an $L = 1$ supermultiplet and hence the spins of the members of the multiplet are a combination of Kerr angular momentum and fermionic hair. Similar remarks apply to $N = 2$, for which $L = 3/2$. For $N = 0$, all the KK modes derive their spin purely from Kerr angular momentum.

The gyromagnetic ratios of conventionally rotating black holes in Kaluza-Klein theory were calculated in [28], with the result $g = 2 - v^2$ where v is the boost velocity in the

x^5 (internal) direction. Since KK states saturating the BPS bound correspond to $v = 1$, it follows that Kerr angular momentum also contributes $g = 1$ to the gyromagnetic ratio. Thus the $g = 1$ result for KK states is a universal one, irrespective of the extreme black hole interpretation of the particle's spin.

5 Conclusions

In this paper we have calculated the gyromagnetic ratios of the Dirichlet 0-branes, regarding them as extreme Type IIA black holes whose spin is generated by the fermionic zero modes. We again find $g = 1$ consistent with their interpretation as Kaluza-Klein modes. We also note that, following the techniques discussed in [27], the Matrix model [8] parton interpretation, in which a charge n 0-brane is a zero-energy bound state of n singly charged 0-branes, is also (trivially) consistent with $g = 1$ since the tensor product of two short representations each with $g = 1$ contains another short representation with $g = 1$. To complete the picture it would be interesting to verify $g = 1$ also from the viewpoint of surfaces of dimension 0 on which open strings can end [7].

Acknowledgment. JTL wishes to thank S. Ferrara and W. Sabra for useful discussions.

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