ON THE APPLICATION OF EXTREME VALUE THEORY TO THE PERFORMANCE OF K-TH BEST LINK SELECTION IN WIRELESS COMMUNICATION SYSTEMS

A Dissertation

by

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ABSTRACT

In this dissertation we investigate the performance of various wireless communication systems that feature selection of the k-th best link from a number of links. We first consider the best link selection (i.e., $k = 1$) and analyze the effective throughput of a transmit antenna selection scheme for multiple antenna systems subject to Rayleigh fading. We derive an analytical expression for the effective throughput and closed form expressions for the effective throughput in asymptotically high and low signal-to-noise ratio (SNR) regimes.

Next, we consider the k-th best link selection scheme over various channel models that are widely used to characterize fading in wireless communication systems such as, Weibull, Gamma, α -μ and Gamma-Gamma. Assuming a large number of links, we use extreme value theory to show that the k-th highest SNR converges uniformly in distribution to a log-gamma random variable for a fixed k and large number of links. We derive simple closed-form asymptotic expressions for the average throughput, effective throughput and average bit error probability of the k-th best link. The derived results cover many practical systems of interest in radio frequency and free space optical systems.

Furthermore, we analyze the asymptotic performance of a multiuser diversity scheme for an interference limited secondary multiuser network of underlay cognitive radio systems. Assuming a large number of secondary users and that the noise at each secondary user's receiver is negligible compared to the interference from the primary transmitter, the secondary transmitter transmits information to the secondary user with the k-th best signal-to-interference ratio (SIR). We use extreme value theory to show that the k-th highest SIR converges uniformly in distribution to an inverse gamma random variable for a fixed k and large number of secondary users. We use this result to analyze the asymptotic average throughput, effective throughput, average bit error rate and outage probability for the k-th best secondary user under continuous power adaptation at the secondary transmitter, which ensures satisfaction of an instantaneous interference constraint at the primary receiver caused by the secondary transmitter.

Finally, we investigate the secrecy performance of a multiuser diversity scheme for an interference limited wireless network with a base-station (BS), multiple legitimate users and an eavesdropper, in the presence of a single dominant interferer. Assuming interference dominates noise power at the eavesdropper and at each legitimate user's receiver, the BS transmits information to the legitimate user with the k-th best SIR. We derive a closed-form expression for the secrecy outage probability for an arbitrary number of users and an asymptotic expression for a fixed k and large number of users. Furthermore, we derive a closed form asymptotic expression for the ergodic secrecy capacity of the k-th best user and show that it scales logarithmically with the number of users.

DEDICATION

To the memory of my father. To my beloved mother. To my lovely wife and daughter. To my brother and sisters.

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NOMENCLATURE

MRC	Maximal ratio combining
EGC	Equal gain combining
SD	Selection diversity
RF	Radio frequency
CR	Eognitive radios
FSO	Free space optical
EVT	Extreme value theory
SNR	Signal-to-noise ratio
BER	Bit error rate
CDF	Cumulative distribution function
CSI	Channel state information
TDMA	Time-division multiple access
SISO	Single-input single-output
MISO	Multiple-input single-output
MIMO	Multiple-input multiple-output
SIMO	Single-input multiple-output
TAS	Transmit antenna selection
PT	Primary transmitter
SU	Secondary user
ST	Secondary transmitter
SIR	Signal-to-interference ratio
SOP	Secrecy outage probability

ESC	Ergodic secrecy capacity
QoS	Quality of service
MGF	Moment generating function
RAS	Receive antenna selection
LTD	Limiting throughput distribution
BEP	Bit Error Probability
SER	Symbol error rate
PHY	Physical layer
BS	Base-station

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1. INTRODUCTION AND LITERATURE REVIEW

1.1 Diversity in Wireless Communications

The basic idea of diversity in wireless systems is to exploit information from several signals transmitted over independent paths. Diversity can significantly improve the performance of wireless systems since it exploits the low probability of concurrence of deep fades in all diversity paths to increase the transmit-receive link quality [1]. The most commonly employed diversity schemes in wireless systems are:

1. Space diversity, which employs multiple antennas spaced enough from each other to ensure independent fading.

2. Frequency diversity, which is very advantageous in frequency selective fading environment. In this case the information signal is modulated over multiple sub-channels separated from each other by at least the coherence bandwidth.

3. Time diversity, which features transmitting the information signal over multiple time slots separated from each other by at least the coherence time of the channel.

1.1.1 Diversity Combining Techniques

Maximal ratio combining (MRC) diversity, equal gain combining (EGC) and selection diversity (SD) are among the most common diversity combining techniques employed in wireless communications. In MRC technique, all diversity branches are used simultaneously. Each branch is multiplied by an optimal weight coefficient to maximize the received signal strength at the receiver. In some cases, it is not possible to find the optimal weight coefficients to perform MRC. Hence, all weight coefficient are set to unity and each signal is rotated in phase and added simultaneously to provide EGC. The SD scheme is a very simple combining technique, it basically selects only one diversity branch which has the strongest (best) signal-to-noise ratio (SNR).

Due to its low complexity and simple implementation, SD is an attractive scheme for multiple antennas configuration, multiuser networks and relay networks in many practical communication systems, such as radio frequency (RF), cognitive radios (CR) and free space optical communication (FSO) systems. The main focus of this dissertation is to explore the performance of many practical wireless communication systems under a general SD scheme in which the k-th best link (k-th highest SNR or k-th largest order statistics) is selected instead of the best link. We base our performance analysis on the extreme value theory (EVT).

1.1.2 An Overview of EVT

EVT investigates the asymptotic behavior of the extremes (maximum or minimum) of a set of random variables [2]. EVT is a very useful tool to analyze SD schemes in wireless systems since it provides simple asymptotic expressions for the commonly used performance metrics such as, ergodic throughput (capacity) and bit error rate (BER). In particular, EVT was used to analyze the asymptotic average throughput (in the limit of large number of links) of the conventional SD scheme, in which the link with the highest SNR is selected for transmission or reception [3], [4]. EVT was also used to evaluate the average BER of the conventional SD scheme [5], [6]. In Theorem 1.1 and Theorem 1.2 below, we provide a brief overview of the main results of EVT.

Theorem 1.1. [2] Let $X_1, X_2, ..., X_N$ be independent identically distributed (i.i.d) random variables with common cumulative distribution function (CDF) of $F(x)$ and let $X_{(N)} = \max_{i=1,...,N} X_i$. Then, the limiting CDF of the sequence $\frac{X_{(N)}-a_N}{b_N}$, where a_N and $b_N > 0$ are normalizing constants, must belong to one of the three extreme value distributions $G_i(x)$, namely,

1. Fréchet:

$$G_1(x) = \begin{cases} 0, & x \leq 0, \\ \exp(-x^{-\alpha}), & x > 0,\ \alpha > 0 \end{cases}, \tag{1.1}$$

2. Weibull:

$$G_2(x) = \begin{cases} \exp[-(-x)^{-\alpha}], & x > 0,\ \alpha > 0 \\ 1, & x > 0, \end{cases} , \tag{1.2}$$

3. Gumbel:

$$G_3(x) = \exp(-e^{-x}), \ \ -\infty < x < \infty. \tag{1.3}$$

In the following Theorem we summarize the sufficient conditions for absolutely continuous $F(x)$ to belong to the domain of attraction of $G_i(x)$, namely, $F \in D\left(G_i\right)$.

Theorem 1.2. [2] Sufficient conditions for $F(x)$ to belong to $D\left(G_i\right)$ are as follows:

1. $F \in D\left(G_1\right)$ if $F^{'}(x) = f(x) > 0$ for all large x and for some α,

$$\lim_{x\to\infty} \frac{xf(x)}{1 - F(x)} = \alpha. \tag{1.4}$$

The normalizing constant b_N can be obtained as

$$b_N = F^{-1}\left(1 - \frac{1}{N}\right). \tag{1.5}$$

2. $F(x) \in D\left(G_2\right)$ if $\zeta_1 < \infty$ and for some α,

$$\lim_{x\to\zeta_1} \frac{(\zeta_1 - x)f(x)}{1 - F(x)} = \alpha, \tag{1.6}$$

where $f(x) = F^{'}(x)$. The normalizing constant b_N can be obtained as

$$b_N = \zeta_1 - F^{-1}\left(1 - \frac{1}{N}\right). \tag{1.7}$$

3. $F(x) \in D(G_3)$ if $F^{'}(x) = f(x) > 0$ and is differentiable for all $x \in (x_1, \zeta_1)$ for some x_1, and

$$\lim_{x \to \zeta_1} \frac{d}{dx} \left[\frac{1 - F(x)}{f(x)} \right] = 0. \tag{1.8}$$

The normalizing constants a_N and b_N can be obtained as

$$a_N = F^{-1} \left(1 - \frac{1}{N} \right), \tag{1.9}$$

$$b_N = F^{-1} \left(1 - \frac{1}{Ne} \right) - F^{-1} \left(1 - \frac{1}{N} \right). \tag{1.10}$$

In what follows, we combine the results from Sections 10.5 and 10.6 (distribution of the k-th extreme) of [2] in Theorem 1.3 below.

Theorem 1.3. [2] Let $X_{(N)}$ denote the largest order statistic of N i.i.d random variables with a common CDF $F(x)$, where $X_{(1)} \leq X_{(2)} \leq .... \leq X_{(N)}$. If $\frac{X_{(N)} - a_N}{b_N}$, where a_N and b_N are normalizing constants, has a limiting CDF, $G(x) = G_i(x)$, then, for a fixed k and $N \to \infty$, the limiting CDF of $\frac{X_{(N-k+1)} - a_N}{b_N}$ is of the form

$$G^{(k)}(x) = G(x) \sum_{j=0}^{k-1} \frac{\left[-\log\left(G(x) \right) \right]^j}{j!}. \tag{1.11}$$

Equivalently, for a fixed k and $N \to \infty$, the sequence $\frac{X_{(N-k+1)} - a_N}{b_N}$ converges uniformly in distribution to a random variable X, where the CDF of X is as in (1.11). We will use the result from Theorem 1.3 to analyze the performance of the k-th best link selection in the following chapters.

1.2 The k-th Best Link Selection Scheme

1.2.1 Overview

Motivated by its simple implementation and low complexity, SD is an important diversity technique to enhance the performance of wireless communication systems. The theory of order statistics [2] is considered a powerful tool to analyze the performance of SD techniques. As a classical example, the theory of order statistics was used to analyze the performance of conventional SD schemes in which the link with the highest SNR is selected for transmission or reception from independent and identically distributed (i.i.d) links [1]. However, in practical communication systems the link with highest SNR may not be available for transmission or reception under given traffic conditions. Therefore, a more general SD scheme that features selection of the k-th best link (k-th highest SNR or k-th largest order statistics) is of practical interest in wireless communication systems. In what follows, we discuss the motivation behind addressing the k-th best link selection problem for wireless systems.

1.2.2 Motivation

The motivation behind considering the k-th best link selection scheme can be summarized in the view of the following communication systems configurations:

1. Relay networks: In cooperative-diversity or cognitive relay networks, the best relay may not be selected due to imperfect channel state information (CSI), some scheduling or load-balancing conditions [7], [8]. In such cases, the second best relay or in general the k-th best relay can be selected.

2. Multiple antenna cognitive radio systems: Controlling the amount of interference caused by the secondary transmitter to the primary receiver is the major challenge in the design of cognitive radio systems. Considering a secondary transmitter with transmit antennas selection scheme, the antenna that maximizes the signal to noise ratio (best antenna) may not be selected if the induced interference at the primary receiver exceeds the acceptable threshold [9].

3. Generalized multiuser diversity in wireless networks: Consider a generalized multiuser diversity scheme in a wireless network where the first M best users are selected, based on their SNRs, from a total number of N users. The M users are allowed to access the channel simultaneously in time-division multiple access (TDMA) fashion [10].

4. Fairer multiuser diversity in wireless networks: Selecting the best user is not always beneficial since it prevents other users with good channel quality from transmission or reception although it can achieve maximum diversity gain. One might improve the performance from a fairness standpoint by selecting the k-th best user to achieve some fairness to multiple users with good channels conditions [11]. Fairer multiuser diversity is also beneficial in secondary multiuser cognitive radio networks where only one secondary user can access the channel at any time instant.

5. Priority aware multiuser diversity in wireless networks: In this scenario, we select the first M best users from a total number of N users. Then only one user is selected which is the one with the most important data (higher priority) among the M users. This scenario is of practical interests in secondary multiuser cognitive radio networks where the secondary users are competing for the channel in opportunistic fashion. If the second, third best or in general the k-th best secondary user has more important data then it will be granted access instead of the best user.

1.3 Dissertation Outline and Contributions

The rest of this dissertation is divided into five chapters. In Chapter 2 we provide a detailed analysis of the achievable effective throughput of multiple-input single-output (MISO) with transmit antenna selection (TAS). More specifically, we present a novel integral expression for the effective throughput of MISO/TAS systems in terms of the cumulative distribution function (CDF) of the channel gain assuming a generalized fading environment. Then, we analyze the effective throughput specifically in Rayleigh fading and derive closed-form expressions for it in asymptotically high and low SNR regimes. Furthermore, we consider a MISO/TAS system with large number of transmit antennas and generalized antenna selection and derive an asymptotic analytical expression for the effective throughput.

The exact results for the effective throughput and average throughput derived in Chapter 2 and [12], respectively, are valid only for the conventional SD scheme in Rayleigh fading and it is hard to extend them for general SD schemes and different fading channels, such as Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma. In Chapter 3, another approach based on EVT or extreme order statistics is used to analyze the effective throughput, average throughput and average BER of the k-th best link over different fading channels. We utilize EVT to derive simple closed-form asymptotic and more intuitive expressions for the average throughput, effective throughput and average BER of the k-th best link over various fading channel models, such as Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma. To the best of our knowledge, such analysis has not been considered in the literature before.

In Chapter 4, we consider an interference-limited secondary multiuser network, where the noise at each secondary user receiver is negligible compared to the interference from the primary transmitter (PT). We assume that the secondary transmitter transmits information to the k-th best secondary user (SU), namely, the SU with the k-th highest signal-to-interference ratio (SIR). Meanwhile, the ST adjusts its transmit power to satisfy the instantaneous interference constraint at the primary receiver (PR). We use EVT to analyze the performance of the k-th best SU in underlay CR systems.. More specifically, we show that the SIR of the k-th best user converges uniformly in distribution to an inverse gamma random variable for a fixed k and large number of secondary users. Then, we derive novel closed-form asymptotic expressions for the average and effective throughputs of the k-th best SU employing continuous power adaptation at the ST with both limited and unlimited transmit power. Furthermore, novel closed-form asymptotic expressions for the average BER and outage probability with continuous power adaptation and unlimited ST power are derived.

Chapter 5 is devoted to study the secrecy performance of the k-th best user selection scheme in multiuser wireless network. In particular, we analyze the the secrecy outage probability (SOP) and the ergodic secrecy capacity (ESC) of the k-th best user selection scheme of a multiuser wireless network in the presence of a single dominant interferer. Assuming that the noise power at each user's receiver and Eve's receiver are negligible compared to the interference power, and the user with the k-th best signal to interference ratio (SIR) is selected from a total number of users N, we derive a closed-form expression for the secrecy outage probability for an arbitrary N and an asymptotic expression for a fixed k and large N. Furthermore, we derive an asymptotic closed form expression for the ESC of the k-th best user and show that the ESC scales like $O\left(\log(N)\right)$ for a fixed k and large N.

Finally, Chapter 6 concludes the dissertation and discuss future work orientations and extensions.

2. EFFECTIVE THROUGHPUT OF MISO SYSTEMS WITH TRANSMIT ANTENNA SELECTION IN RAYLEIGH FADING *

2.1 Introduction

Motivated by the fact that the most important emerging real time applications (such as voice over IP, smart grid applications, interactive and multimedia streaming) impose stringent quality of service (QoS) constraints, the concept of effective throughput (capacity) was introduced by Wu and Negi [13] to address the impact of statistical delay QoS on system performance. Recently, there has been an increased interest in the theory of effective throughput to quantify the achievable rate of various wireless systems under delay QoS limitations. In particular, the effective capacity of multiple-antenna wireless systems in Rayleigh fading is investigated in [14]. Effective throughput analysis of multiple-input single-output (MISO) systems was extensively studied over different fading channel models. In [15], the effective throughput of Nakagami-m, Rician and generalized-K MISO fading channels were investigated. In [16]-[18], the authors study the effective throughput of, $\eta - \mu$, $k - \mu$ and $\alpha - \mu$ fading channels, respectively. In [19], [20] the authors analyze the effective throughput of correlated MISO in Rayleigh and Nakagami-m fading, respectively. The effective throughput over independent but not identical (i.n.i.d) MISO Weibull fading channels is considered in [21].

Antenna selection schemes in multiple-input multiple-output (MIMO) wireless systems provide significant advantages in practical communication since they can reduce complexity and substantially increase capacity [22]. Motivated by these advantages, the effective capacity analysis of wireless systems with receive/transmit antenna selection schemes has been recently considered in [23]. The authors analyze the effective capacity of a multiple-input multiple-output (MIMO) system in two different cases with receive antenna selection (RAS) and transmit antenna selection (TAS) schemes. For the TAS scheme, the authors consider an optimal power-control policy in-

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vestigated in [24] that can maximize the effective capacity; they also investigated the maximum effective capacity in two asymptotic cases with loose and strict QoS requirements. While the work in [23] has improved our knowledge on the effective capacity of multiple antenna systems with antennas selection, some implications on the effective achievable throughput have not yet been investigated. Motivated by [15], in this chapter we provide a detailed analysis of the achievable effective throughput of MISO/TAS to draw some conclusions on the impact of system parameters on the effective throughput. More specifically, we present a novel integral expression for the effective throughput of MISO/TAS systems in terms of the cumulative distribution function (CDF) of the channel gain assuming a generalized fading environment. Then, we analyze the effective throughput specifically in Rayleigh fading and derive closed-form expressions for it in asymptotically high and low signal-to-noise ratio (SNR) regimes. Furthermore, we consider a MISO/TAS system with large number of transmit antennas and generalized antenna selection and derive an asymptotic analytical expression for the effective throughput.

The rest of this chapter is organized as follows. In Section II we discuss the system and channel models. In Section III we discuss the effective throughput of the MISO/TAS system. Section IV includes numerical results. Finally, Section V concludes.

2.2 System Model

Consider a MISO system in which the transmitter selects the best transmit antenna from N available ones to transmit information to the receiver. Let h_i denote the complex channel gain between the i-th transmit antenna and the receiver and X_i the squared magnitude of the channel gain, where $i = 1, 2, ..., N$. The received signal can be expressed as

$$y = h_{(N)} s + w, \tag{2.1}$$

where s is the complex valued symbol transmitted with (normalized) power ρ, w is the circularly symmetric additive white Gaussian noise (AWGN) with zero mean and unit variance and $h_{(N)}$ is the flat fading channel gain of the best best transmit antenna. Then, the instantaneous SNR at the receiver side is $\rho X_{(N)}$ where $X_{(N)} = |h_{(N)}|^2$ and we assume the ordering $X_{(1)} \leq X_{(2)} \leq .... \leq X_{(N)}$. According to [2], the probability density function (PDF) of $X_{(N)}$ can be expressed in terms of the PDF, $f(x)$, and cumulative distribution function (CDF), $F(x)$, of X_i as

$$f_{X_{(N)}}(x) = Nf(x)[F(x)]^{N-1}. \tag{2.2}$$

The effective throughput is defined in [13] as the maximum constant arrival rate that can be supported by a time-varying wireless system under a statistical QoS constraint described by the delay QoS exponent θ. Assuming block fading channels, the effective throughput of the service process can be formulated as

$$\alpha(\zeta) = -\frac{1}{\zeta T} \log\left(E\left\{e^{-\zeta T R}\right\}\right),\ \zeta > 0, \tag{2.3}$$

where R is a random variable which represents the instantaneous throughput during a single block and T is the block length. The delay QoS exponent ζ captures the so-called asymptotic decay-rate of the buffer occupancy and is given by [14]

$$\theta = -\lim_{x \to \infty} \frac{\log \Pr\{L > x\}}{x}, \tag{2.4}$$

where L is the equilibrium queue-length of the buffer at the transmitter. $\zeta = 0$ implies there is no delay constraint and the effective throughput is then the ergodic throughput of the corresponding wireless channel.

2.3 Effective Throughput of MISO/TAS Systems

Considering a MISO/TAS scheme described in Section I, the effective throughput can be expressed as

$$\alpha(\rho, \theta, N) = -\frac{1}{A} \log_2\left(\int_0^{\infty} e^{-A\log(1+\rho x)} f_{X_{(N)}}(x)dx\right) \text{ bit/s/Hz}, \tag{2.5}$$

where $A \triangleq \frac{\theta T B}{\ln(2)}$, with B denoting the bandwidth of the system and $f_{X_{(N)}}(x)$ as defined in (2.2). An alternative representation of (2.5) using integration by parts is

$$\alpha(\rho, \theta, N) = -\frac{1}{A}\log_2(\rho A) - \frac{1}{A}\log_2\left(\int_0^\infty (1+\rho x)^{-A-1}[F(x)]^N dx\right), \tag{2.6}$$

where we assume that $\lim_{x\to\infty}(1+\rho x)^{-A}[F(x)]^N = 0$, and $\lim_{x\to 0}(1+\rho x)^{-A}[F(x)]^N = 0$, for all $N \geq 1$. These assumptions hold for the family of exponential distributions that are widely used to characterize fading in wireless communication. In the analysis that follows, we focus on the effective throughput of a MISO/TAS system in Rayleigh fading and derive an analytical expression for it. The asymptotically high and low-SNR regimes are also considered to provide physical insight of system and channel parameters on the effective throughput.

2.3.1 Exact Analysis

Proposition 2.1. The effective throughput of MISO/TAS in Rayleigh fading can be expressed as

$$\alpha(\rho, \theta, N) = -\frac{1}{A}\log_2\left(1 + A\sum_{n=1}^{N}\binom{N}{n}(-1)^n \exp\left(\frac{n}{\rho}\right)\Gamma\left(-A, \frac{n}{\rho}\right)\left(\frac{n}{\rho}\right)^A\right), \tag{2.7}$$

where $\Gamma(s, x) = \int_x^\infty u^{s-1}e^{-u}du$ is the upper incomplete gamma function.

Proof. Starting from the fact that in Rayleigh fading, $F(x) = 1 - e^{-x}$, and applying binomial expansion, we have

$$\begin{aligned}\int_0^\infty (1+\rho x)^{-A-1}[F(x)]^N dx &= \int_0^\infty (1+\rho x)^{-A-1}\left[1-e^{-x}\right]^N dx \\ &= \sum_{n=0}^{N}\binom{N}{n}(-1)^n I_n(\rho, A),\end{aligned} \tag{2.8}$$

where $I_n(\rho, A)$ is given by

$$I_n(\rho, A) = \int_0^\infty (1+\rho x)^{-A-1} e^{-nx} dx = \begin{cases} \frac{1}{\rho A}, n = 0 \\ \frac{1}{\rho} \exp\left(\frac{n}{\rho}\right) \Gamma\left(-A, \frac{n}{\rho}\right) \left(\frac{n}{\rho}\right)^A, n = 1, 2, ...N \end{cases} . \tag{2.9}$$

Making use of (2.8) and (2.9) in (2.6), after some basic algebraic manipulation, the effective throughput can be expressed as in (2.7). □

2.3.2 Asymptotic SNR Analysis

In this section we study the effective throughput for asymptotically high and low SNR regimes. In the high-SNR regime, i.e., ($\rho \to \infty$) in (2.6), we have

$$\alpha(\rho, \theta, N) \approx \log_2(\rho) - \frac{1}{A} \log_2 \left(A \int_0^\infty x^{-A-1} (1 - e^{-x})^N dx \right), \tag{2.10}$$

as $\rho \to \infty$, where $N \geq 1$ and $0 < A < N$ are required for the above integral to converge. The first condition results from (2.6) while the second condition results from the convergence behavior of the integrand in (2.10) as $x \to 0$ and $x \to \infty$. As $x \to 0$ the integrand behaves like $O\left(x^{N-A-1}\right)$ and therefore $A < N$ is required for convergence. On the other hand, as $x \to \infty$, the integrand behaves like $O\left(x^{-A-1}\right)$ and therefore $A > 0$ is required for convergence.

Proposition 2.2. For $N \geq A + 1 \geq 2$ and A an integer, the effective throughput in (2.10) can be expressed as

$$\alpha(\rho, \theta, N) \approx \log_2(\rho) - \frac{1}{A} \log_2 \left(\frac{(-1)^{A+1}}{(A-1)!} \sum_{n=1}^{N} \binom{N}{n} (-1)^n n^A \log(n) \right), \tag{2.11}$$

as $\rho \to \infty$.

Proof. Let $g(x) = (1 - e^{-x})^N$; using binomial expansion, the j-th derivative of $g(x)$ is $g^j(x) = \sum_{n=0}^{N} (-1)^{n+j} \binom{N}{n} n^j e^{-nx}$. Note that $g^j(x) = O(x^{N-k})$ as $x \to 0$ for $0 \leq k \leq N$. Furthermore,

$g^j(x) = O(e^{-x})$ as $x \to \infty$ for $1 \le k \le N$. Using these observations and applying integration by parts m times, we can write

$$\begin{aligned}\int_0^\infty x^{-m} g(x) dx &= \frac{-1}{(m-1)!} \int_0^\infty g^m(x) \log(x) dx \\ &= \frac{(-1)^{m-1}}{(m-1)!} \sum_{n=0}^{N} (-1)^n \binom{N}{n} n^m \int_0^\infty e^{-nx} \log(x) dx,\end{aligned} \tag{2.12}$$

for $N \ge m \ge 2$. Invoking Eq. (4.352.1) of [25], we have $\int_0^\infty e^{-nx} \log(x) dx = \frac{-E_0 - \log(n)}{n}$, where E_0 is the Euler constant. Substituting this and $m = A + 1$ in (2.12) after some straightforward algebraic manipulations we reach (2.11). $\square$

At low SNR, the effective throughput can be evaluated via a second-order Taylor expansion of $\alpha(\rho, \theta, N)$ at $\rho = 0$; hence we can write [26]

$$\alpha(\rho, \theta, N) = \alpha'(0, \theta, N)\rho + \alpha''(0, \theta, N)\frac{\rho^2}{2} + o(\rho^2), \tag{2.13}$$

where $\alpha'(0, \theta, N)$ and $\alpha''(0, \theta, N)$ denote, respectively, the first and second derivatives of the effective throughput with respect to ρ at $\rho = 0$. In order to investigate the effective throughput in the low SNR regime, two important parameters of interest, the minimum transmit energy per information bit normalized by the noise spectral density, $\frac{E_b}{N_0}_{\min}$, and the wideband slope S_0, have been introduced in [27]. These two parameters can be obtained as

$$\frac{E_b}{N_0}_{\min} = \lim_{\rho \to 0} \frac{\rho}{\alpha(\rho, \theta, N)} = \frac{1}{\alpha'(0, \theta, N)}, \tag{2.14}$$

$$S_0 = -\frac{2\ln(2)\left[\alpha'(0, \theta, N)\right]^2}{\alpha''(0, \theta, N)}. \tag{2.15}$$

Hence, the effective throughput at low SNRs can be expressed as [27]

$$\alpha\left(\frac{E_b}{N_0},\theta,N\right)=S_0\log_2\left(\frac{E_b}{N_0}/\frac{E_b}{N_0}_{\min}\right). \tag{2.16}$$

Proposition 2.3. For a MISO/TAS scheme in Rayleigh fading, $\frac{E_b}{N_0}_{\min}$ and S_0 are given by

$$\frac{E_b}{N_0}_{\min}=\frac{\ln(2)}{\psi(N+1)+E_0}, \tag{2.17}$$

$$S_0=\frac{2\left(\psi(N+1)+E_0\right)^2}{\left(\psi(N+1)+E_0\right)^2+(A+1)\left(\frac{\pi^2}{6}-\psi(1,N+1)\right)}, \tag{2.18}$$

where, $\psi(x)$ is the digamma function and $\psi(m,x)$ is the m-th derivative of $\psi(x)$.

Proof. Invoking (2.5) and following the same line of reasoning as in Appendix I of [19], the first and second derivatives in (2.13) are given by

$$\alpha'(0,\theta,N)=\frac{E\left[X_{(N)}\right]}{\ln(2)}, \tag{2.19}$$

$$\alpha''(0,\theta,N)=\frac{A}{\ln(2)}\left(E\left[X_{(N)}\right]\right)^2-\frac{A+1}{\ln(2)}E\left[(X_{(N)})^2\right], \tag{2.20}$$

where $E\left[X_{(N)}\right]$ and $E\left[(X_{(N)})^2\right]$ are given by

$$E\left[X_{(N)}\right]=\psi(N+1)+E_0, \tag{2.21}$$

$$E\left[(X_{(N)})^2\right]=\left(\psi(N+1)+E_0\right)^2+\frac{\pi^2}{6}-\psi(1,N+1). \tag{2.22}$$

We provide a detailed proof for (2.21) and (2.22) in Appendix A. Making use of (2.19) - (2.22) in (2.14) and (2.15), after some straightforward algebraic manipulations, we get (2.17) and (2.18).

$\square$

Note that the minimum bit energy is a function of the number of transmit antennas N. It is interesting to observe that for $N = 1$, it follows that $\frac{E_b}{N_0}_{\min} = \ln(2)$ and $S_0 = \frac{2}{A+2}$. These results are in agreement with what was obtained for MISO Nakagami-m fading by setting $m = 1, N_t = 1, \Omega = 1$ in Eq. (20)-(21) of [15]. From (2.18), it can be also shown that S_0 is a decreasing function of A.

For large number of transmit antennas ($N \to \infty$), the minimum $\frac{E_b}{N_0}$ and the wideband slope S_0 are respectively given by

$$\frac{E_b}{N_0}_{\min} = \frac{\ln(2)}{\log(N)}, \tag{2.23}$$

$$S_0 = 2. \tag{2.24}$$

In order to prove (2.23) and (2.24), we invoke Eq. (6.13.18) and Eq. (6.4.12) of [28], then we have $\psi(N) = O\left(\log(N)\right)$ and $\psi(1, N+1) = O\left(1/N\right)$, respectively as $N \to \infty$. Substituting these in (2.17) and (2.18) we get (2.23) and (2.24). Note that S_0 is a monotonically increasing function in the number of transmit antennas with maximum value of $S_0 = 2$.

2.3.3 Asymptotic Effective Throughput for Large N

In this section we analyze the effective throughput of a MISO system with large N where generalized transmit antenna selection is performed such that we select the best L from all available N antennas, where $1 \leq L \leq N$. With perfect CSI and transmit beamforming, the achievable throughput of this system can be expressed as [29]

$$I = \log_2\left(1 + \rho \sum_{i=1}^{L} X_{(N-i+1)}\right) \text{ bit/s/Hz}. \tag{2.25}$$

The effective throughput of this system is given in the following proposition.

Proposition 2.4. For fixed ρ, $1 \leq L \leq N$, and $N \to \infty$, the effective throughput of MISO systems with generalized transmit antenna selection and transmit beamforming can be expressed as

$$\alpha(\rho, \theta, N, L) = -\frac{1}{A} \log_2 \left[\mathcal{L}_I \left(A \ln(2) \right) \right] \text{ bit /s/Hz}, \tag{2.26}$$

where $\mathcal{L}_I(\cdot)$ is the Laplace transform of the PDF of the random variable I, given by

$$\mathcal{L}_I(t) = e^{\frac{r^2 t^2}{2} - \mu t} \left[1 - \Phi \left(-\frac{\mu}{r} + rt \right) \right] + e^{\frac{r^2 t^2}{2} + \mu t} \left[1 - \Phi \left(\frac{\mu}{r} + rt \right) \right], \tag{2.27}$$

where $\Phi(\cdot)$ is the CDF of standard normal random variable. The terms μ and r are given by

$$\mu = \log_2 \left(1 + \left(1 + \ln \frac{N}{L} \right) \rho L \right), \tag{2.28}$$

$$r = \sqrt{\frac{\left(\log_2(e) \right)^2 \rho^2 L \left(2 - \frac{L}{N} \right)}{\left(1 + \left(1 + \ln \frac{N}{L} \right) \rho L \right)^2}}. \tag{2.29}$$

Proof. According to Theorem 1 of [29], for large N, the CDF of the random variables I can be approximated by a folded normal distribution with parameters of μ and r^2, where μ and r as given in (2.28) and (2.29), respectively. Then the effective throughput can be expressed as

$$\alpha(\rho, \theta, N, L) = -\frac{1}{A} \log_2 \left(E \left\{ e^{-A \ln(2) I} \right\} \right) \text{ bit /s/Hz}. \tag{2.30}$$

$\square$

Using the fact that the term $E\left\{e^{-A\ln(2)I}\right\}$ represents the Laplace transform of the PDF of the random variable I, $\mathcal{L}_I(t)$ at $t = A\ln(2)$, the effective throughput of this system can be expressed as in (2.26). As a special case of $L = 1$, the expression in (2.26) can be used to approximate the effective throughput derived in (2.7) for asymptotically large N.

Note that $\mu \to \infty$ and $r \to 0$ as $N \to \infty$, then $\mathcal{L}_I(A\ln(2))$ behaves like $e^{-A\ln(2)\mu}$ for a fixed A.

Therefore the effective throughput in (2.26) converges asymptotically almost surely to the mean μ. This implies that employing transmit antenna selection in the presence of large number of antennas is asymptotically optimal in the sense that the effective throughput tends to the ergodic (Shannon) throughput and the delay requirement vanishes asymptotically when N grows large.

2.4 Numerical Results

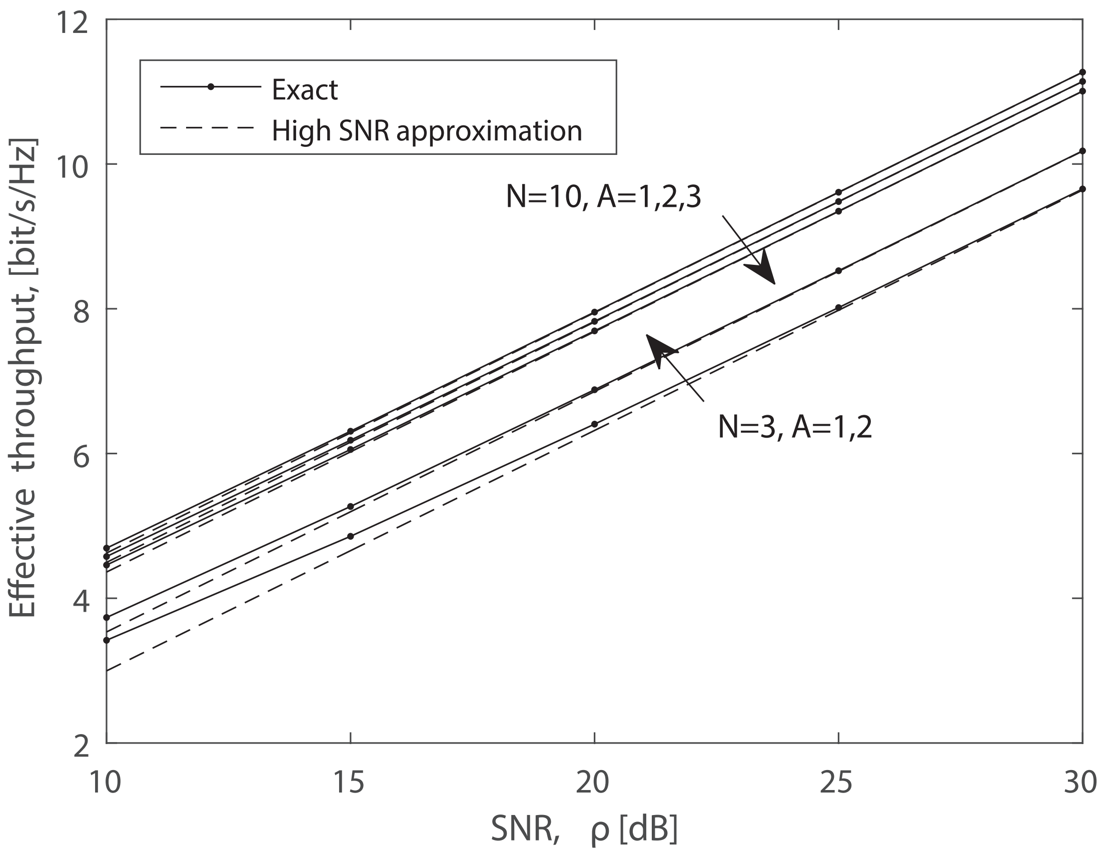

Figure 2.1: Exact analytical effective throughput and high-SNR approximation versus SNR of MISO/TAS systems.

In this section, we numerically illustrate and verify the obtained analytical results in the previous section. In Fig. 2.1, we compare the exact effective throughput of a MISO/TAS system in

(2.7) with the high-SNR approximation of Proposition 2.2. As shown, the high-SNR expression in (2.11) is quite accurate even at moderate SNR values. In Fig. 2.2, we plot the effective throughput as a function of the normalized bit energy for $N = 10$, we note that the minimum bit energy for all values of A is -6.259 dB. In Fig. 2.3, we plot the effective throughput as a function of the transmit antennas, N, for different values of selected antennas L. We validate the obtained analytical results using Monte Carlo simulations. We observe that the asymptotic expression is accurate even for small N. We also observe that asymptotic expression becomes more accurate as L increases.

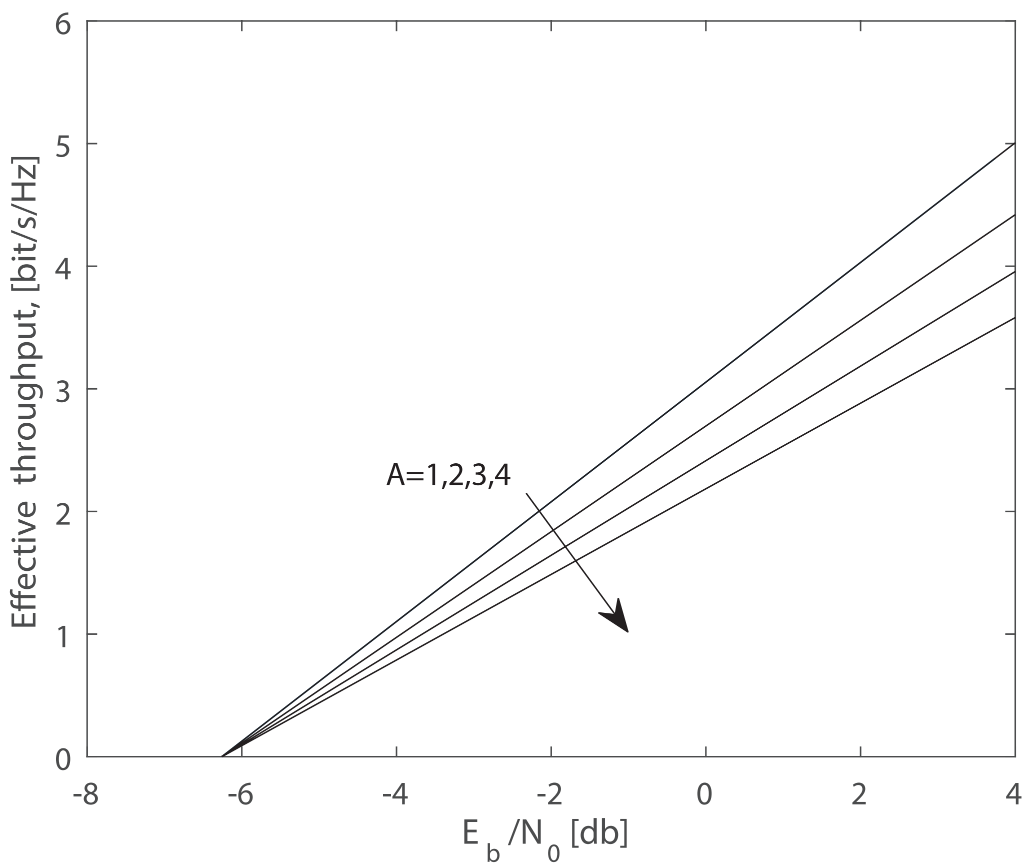

Figure 2.2: Low-SNR effective throughput of MISO/TAS systems versus bit energy for $N = 10$ and A=1, 2, 3, 4.

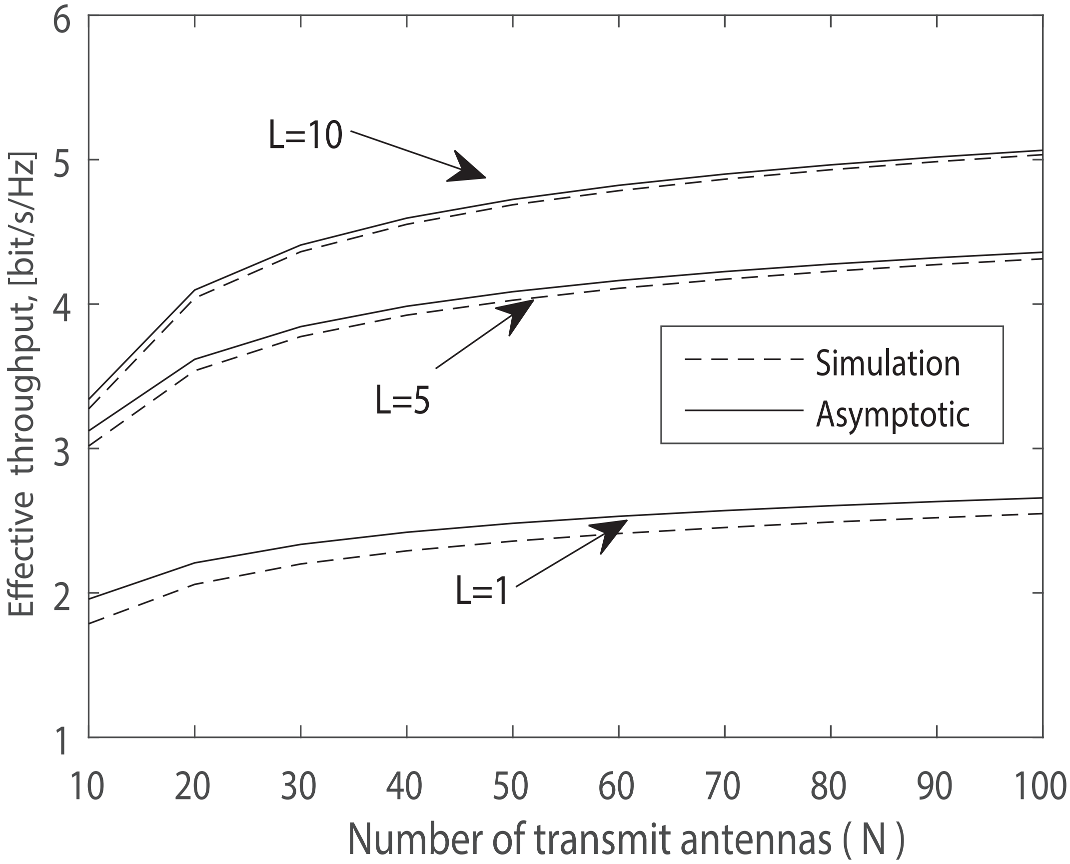

Figure 2.3: Asymptotic effective throughput of MISO/TAS systems versus the number of transmit antennas for $L = 1, 5, 10$ and $A = 2$, at $\rho = 0$ dB.

2.5 Summary

A detailed effective throughput analysis of MISO/TAS systems was considered. An analytical expression for the effective throughput of the considered system in Rayleigh fading is obtained. Moreover, we analyzed the effective throughput in the asymptotically low and high-SNR regimes and closed form expressions were derived. At the low-SNR regime, we showed that the minimum bit energy depends on the number of transmit antennas but not on the delay constraint. However, we observed that the wideband slope is a decreasing function of the delay constraint and monotonically increasing in the number of transmit antennas. Finally, we derived an asymptotic analytical expression for the effective throughput of MISO/TAS systems with large number of transmit antennas and showed that effective throughput tends to the ergodic (Shannon) throughput and the delay requirement vanishes asymptotically as the number of transmit antennas grows large.

3. ASYMPTOTIC PERFORMANCE ANALYSIS OF THE K-TH BEST LINK SELECTION OVER WIRELESS FADING CHANNELS: AN EXTREME VALUE THEORY APPROACH *

3.1 Introduction

Motivated by its simple implementation and low complexity, selection-diversity (SD) is an important diversity technique to enhance the performance of wireless communication systems. The theory of order statistics [2] is considered a powerful tool to analyze the performance of SD techniques. As a classical example, the theory of order statistics was used to analyze the performance of conventional SD schemes in which the link with the highest signal-to-noise ratio (SNR) is selected for transmission or reception from independent and identically distributed (i.i.d) links [1]. However, in practical communication systems the link with highest SNR may not be available for transmission or reception under given traffic c onditions. Therefore, a more general SD scheme that features selection of the k-th best link (k-th highest SNR or k-th largest order statistics) is of practical interest in wireless communication systems. Selection of the k-th best link has been considered in relay networks [7], [8] and cognitive radio networks [9].

Other applications of ordered statistics in wireless communication systems include analyzing the performance of generalized multiuser diversity schemes [10], minimum-selection generalized selection combining (MS-GSC) [30] and modeling the aggregate interference for centralized and decentralized selection schemes for the radio environment map (REM) approach in cognitive radio networks [31].

The effective throughput is defined as the maximum constant arrival rate that can be supported by a time-varying wireless channel under a statistical delay quality of service (QoS) constraint [13]. If no delay constraint is imposed, the effective throughput is the average (ergodic) throughput (capacity) of the corresponding wireless channel. In general, it is difficult to express in closed form

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the exact effective and average throughputs of SD schemes for various fading channel models. This is due to the complicated nature of the distribution of the SNR of the selected link. For example, the exact effective throughput of the conventional SD scheme in Rayleigh fading can be expressed as a logarithmic function of a finite sum of weighted Tricomi hypergeometric functions [32]. Furthermore, the average throughput of the conventional SD scheme in Rayleigh fading was expressed as a finite sum of weighted exponential integral functions [12].

The exact results for the effective throughput and average throughput derived in [32] and [12], respectively, are valid only for the conventional SD scheme in Rayleigh fading and it is hard to extend them for general SD schemes and different fading channels, such as Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma. A recent attempt to analyze the average throughput of the k-th best link selection over a generalized Gamma ($\alpha - \mu$) fading channel was considered in [33], where the authors derived lower and upper bounds on the average throughput. However they did not investigate the effective throughput and average bit error probability (BEP).

In this Chapter, another approach based on extreme value theory (EVT) or extreme order statistics is used to analyze the effective throughput, average throughput and average BEP of the k-th best link over different fading channels. EVT was used to analyze the asymptotic average throughput (in the limit of large number of links) of the conventional SD scheme [3], [4]. EVT was also used to evaluate the average BEP of the conventional SD scheme [5], [6]. Our contribution is to utilize EVT to derive simple closed-form asymptotic and more intuitive expressions for the average throughput, effective throughput and average BEP of the k-th best link over various fading channel models, such as Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma. To the best of our knowledge, such analysis has not been considered in the literature before.

The rest of this Chapter is organized as follows. In Section 3.2 we discuss the system model. In Section 3.3 we discuss the average and effective throughputs of the k-th best link over Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma fading channels. In Section 3.4 we analyze the average BEP. Section 3.5 includes numerical results and Section 3.6 concludes.

3.2 System Model

Consider a system in which the transmitter selects the k-th best link from N i.i.d links to transmit information to the receiver. Let h_i denote the complex channel gain of the i-th link between the transmitter and the receiver and X_i the squared magnitude of the channel gain of the i-th link, where $i = 1, 2, ..., N$. The received signal can be expressed as

$$y = h_{(N-k+1)}s + w, \tag{3.1}$$

where s is the complex valued symbol transmitted with (normalized) power ρ, w is the circularly symmetric additive white Gaussian noise (AWGN) with zero mean and unit variance and $h_{(N-k+1)}$ is the flat fading channel gain of the k-th best link for $k = 1, 2, ..., N$. Then, the instantaneous SNR at the receiver side is $\rho X_{(N-k+1)}$ where $X_{(N-k+1)} = |h_{(N-k+1)}|^2$ and we assume the ordering $X_{(1)} \leq X_{(2)} \leq .... \leq X_{(N)}$. According to [2], the probability density function (PDF) of $X_{(N-k+1)}$ can be expressed in terms of the PDF, $f(x)$, and cumulative distribution function (CDF), $F(x)$, of X_i as

$$f_{X_{(N-k+1)}}(x) = k\binom{N}{k} f(x)F(x)^{N-k} \left(1 - F(x)\right)^{k-1}. \tag{3.2}$$

Let $R_i = B \log_2(1 + \rho X_i)$ denote the instantaneous throughput of the i-th link; then $R_{(N-k+1)}$ represents the instantaneous throughput of the selected link and can be evaluated as

$$R_{(N-k+1)} = B \log_2 \left(1 + \rho X_{(N-k+1)}\right), \tag{3.3}$$

where $R_{(1)} \leq R_{(2)} \leq .... \leq R_{(N)}$ and B is the system bandwidth. Therefore, the average throughput of the selected link, $E\left[R_{(N-k+1)}\right]$, can be evaluated as

$$E\left[R_{(N-k+1)}\right] = B \int_0^\infty \log_2(1 + \rho x) f_{X_{(N-k+1)}}(x)dx. \tag{3.4}$$

Considering the k-th best link selection scheme, the effective throughput of the selected link,

$\alpha(\theta, k, N)$, can be expressed as

$$\alpha(\theta, k, N) = -\frac{1}{\theta}\log_2\left(E\left[e^{-\theta\ln(2)R_{(N-k+1)}}\right]\right) \tag{3.5}$$

where $\theta = \zeta T$ and the expectation is taken over the distribution of $R_{(N-k+1)}$. Applying L'hospital's rule, one can show that $\lim_{\theta\to 0}\alpha(\theta, k, N) = E\left[R_{(N-k+1)}\right]$, as stated before.

In general, it is difficult to obtain closed form expressions for $E\left[R_{(N-k+1)}\right]$ and $\alpha(k, \theta, N)$ over various fading distributions. Therefore, in this Chapter, we consider another approach based on extreme value theory to analyze the average throughput and effective throughput over Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma fading channels.

3.3 Throughput Analysis

In this section, we invoke Theorem 1.3. to derive the limiting distribution of the throughput of the k-th best link and to evaluate the average throughput and effective throughput.

Theorem 1.3. [2] Let $Z_{(N)}$ denote the largest order statistic of N i.i.d random variables with a common CDF $F(z)$, where $Z_{(1)} \leq Z_{(2)} \leq .... \leq Z_{(N)}$. If $\frac{Z_{(N)}-a_N}{b_N}$, where a_N and b_N are normalizing constants, has a limiting CDF, $G(z)$, then, for a fixed k and $N \to \infty$, the limiting CDF of $\frac{Z_{(N-k+1)}-a_N}{b_N}$ is of the form

$$G^{(k)}(z) = G(z)\sum_{j=0}^{k-1}\frac{\left[-\log\left(G(z)\right)\right]^j}{j!}. \tag{3.6}$$

Equivalently, for a fixed k and $N \to \infty$, the sequence $\frac{Z_{(N-k+1)}-a_N}{b_N}$ converges uniformly in distribution to a random variable Z, where the CDF of Z is as in (3.6).

Let X_i be i.i.d non-negative random variables for $i = 1, 2, ..N$, where X_i can be modeled as one of the following distributions: Exponential, Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma. In what follows we use Lemma 1 of [3] to obtain the limiting distribution of the largest order statistic, $X_{(N)}$. If the CDF of X_i satisfies the conditions of this lemma, then the limiting distribution of the largest order statistic, $X_{(N)}$, is such that the sequence $\frac{X_{(N)}-\tilde{a}_N}{\tilde{b}_N}$ converges uniformly in distribution, for large N, to a normalized Gumbel random variable whose CDF, $G(x)$, is given by

$$G(x) = e^{-e^{-x}}, \ \ -\infty < x < \infty. \tag{3.7}$$

Furthermore, the normalizing constants $\tilde{a}_N$ and $\tilde{b}_N$ can be determined from the CDF of X_i as [3]

$$\tilde{a}_N = F^{-1}\left(1 - \frac{1}{N}\right), \tag{3.8}$$

$$\tilde{b}_N = F^{-1}\left(1 - \frac{1}{Ne}\right) - F^{-1}\left(1 - \frac{1}{N}\right), \tag{3.9}$$

where $F^{-1}(x) = \inf\{y : F(y) \geq x\}$. It was shown that the CDFs of Exponential, Gamma and Weibull distributions [3], $\alpha - \mu$ [34] and Gamma-Gamma [35] satisfy the conditions of Lemma 1 of [3]. Consequently, $\frac{X_{(N)}-\tilde{a}_N}{\tilde{b}_N}$ has a limiting distribution as expressed in (3.7). The normalizing constants $\tilde{a}_N$ and $\tilde{b}_N$ for the distributions of interest are tabulated in Table 3.1.

Recall that $R_i = B\log_2(1+\rho X_i)$ is the instantaneous throughput of the i-th link, where X_i can be modeled as one of the distributions referred to above. The limiting throughput distribution (LTD) Theorem of [3] provides a simpler approach to investigate the instantaneous throughput of the best link, $R_{(N)}$. The LTD Theorem indicates that the CDF of $R_i = B\log_2(1+\rho X_i)$ also satisfies the conditions of Lemma 1 of [3] and states that the sequence $\frac{R_{(N)}-a_N}{b_N}$ converges uniformly in distribution to a normalized Gumbel random variable with CDF, $G(x)$, as in (3.7). The normalizing constants a_N and b_N associated with $R_{(N)}$ can be obtained from $\tilde{a}_N$ and $\tilde{b}_N$ as [3]

$$a_N = B\log_2(1+\rho\tilde{a}_N), \tag{3.10}$$

$$b_N = B\log_2\left(1 + \frac{\rho\tilde{b}_N}{1+\rho\tilde{a}_N}\right). \tag{3.11}$$

Table 3.1: Normalizing constants for common fading distributions

Distribution	PDF	$\tilde{a}_N$	$\tilde{b}_N$
Exponential	$e^{-x}u(x)$	$\log(N)$	1
Weibull	$\frac{\eta x^{\eta-1}}{\alpha^{\eta}} e^{-x^{\frac{\eta}{\alpha^{\eta}}}} u(x)$	$\alpha\left(\ln\left(N\right)\right)^{\frac{1}{\eta}}$	$\frac{\alpha}{\eta}\left(\ln\left(N\right)\right)^{-\frac{\eta-1}{\eta}}$
Gamma	$\frac{x^{m-1}e^{-x}}{\Gamma(m)}u(x)$	$\log(N) + (m-1)\log(\log(N)) + O(\log(\log(\log(N))))$	1
Gamma-Gamma [35]	$\frac{2(km)^{\frac{k+m}{2}}}{\Gamma(m)\Gamma(k)}(x)^{\frac{k+m}{2}-1} \times K_{k-m}\left[2\sqrt{kmx}\right]u(x)$	$\frac{(\log(N))^2}{4km} + O\left(\log(N)\log(\log(N))\right)$	$\frac{\log(N)}{2km}$
$\alpha-\mu$ [34]	$\frac{\alpha\mu^{\mu}x^{\frac{\alpha\mu}{2}-1}\exp\left(-\mu\frac{x^{\frac{\alpha}{2}}}{\zeta^{\frac{\alpha}{2}}}\right)}{2\zeta^{\frac{\alpha\mu}{2}}\Gamma(\mu)}u(x)$	$\zeta\left(\frac{\log(K)}{\mu}\right)^{\frac{2}{\alpha}} + O\left(\left(\frac{\log(N)}{\mu}\right)^{\frac{2}{\alpha}-1}\log(\log(N))\right)$	$\frac{2\zeta\left(\frac{\log(N)}{\mu}\right)^{-1+\frac{2}{\alpha}}}{\alpha\mu}$

While we focus next on the case where X_i can be modeled as Exponential, Weibull, Gamma, $\alpha-\mu$ and Gamma-Gamma to derive the average throughput, effective throughput and average BEP for the k-th best link, it should be noted that the following derived results are valid for any random variable X_i whose CDF satisfies the conditions of Lemma 1 of [3].

3.3.1 Average Throughput

The LTD Theorem states that the sequence $\frac{R_{(N)}-a_N}{b_N}$ converges in distribution to a normalized Gumbel random variable. Using this and Theorem 1.3, we characterize the limiting distribution of the throughput of the k-th best link, $R_{(N-k+1)}$, in the following proposition.

Proposition 3.1. For a fixed k and $N \to \infty$, the sequence $\frac{R_{(N-k+1)}-a_N}{b_N}$ converges uniformly in distribution to a random variable X with CDF given by

$$G^{(k)}(x) = e^{-e^{-x}} \sum_{j=0}^{k-1} \frac{e^{-jx}}{j!}, \quad -\infty < x < \infty. \tag{3.12}$$

Furthermore, the average throughput can be approximated as

$$E\left[R_{(N-k+1)}\right] \approx a_N - \psi(k) b_N, \tag{3.13}$$

where $\psi(x)$ is the digamma function.

Proof. The sequence $\frac{R_{(N)}-a_N}{b_N}$ converges uniformly in distribution to a normalized Gumbel random variable with CDF $G(x) = e^{-e^{-x}}, \ -\infty < x < \infty$, where a_N and b_N are as defined in (3.10) and (3.11), respectively. Using Theorem 1.3, it follows that for a fixed k and $N \to \infty$, the sequence $\frac{R_{(N-k+1)}-a_N}{b_N}$ converges uniformly in distribution to a random variable X with CDF $G^{(k)}(x)$ as expressed in (3.12).

To prove (3.13) of Proposition 3.1, we first prove the following Lemma in Appendix B.

Lemma 3.1 : If $\frac{R_{(N-k+1)}-a_N}{b_N}$ converges in distribution to a random variable X whose CDF as expressed in (3.12) then for any positive real number p, we have

$$E\left[\left(\frac{R_{(N-k+1)} - a_N}{b_N}\right)^p\right] \to E\left[X^p\right], \tag{3.14}$$

for a fixed k and $N \to \infty$. Furthermore, $E\left[X\right] = -\psi(k)$ and the moment generating function (MGF) associated with X, $\mathcal{M}_X(t) = E\left[e^{tX}\right] = \Gamma(k-t)/(k-1)!, \ k > t$.

It follows from this Lemma that for $p = 1$, $E\left[\left(\frac{R_{(N-k+1)}-a_N}{b_N}\right)\right] \to E\left[X\right] = -\psi(k)$, for a fixed k and $N \to \infty$. Therefore, the average throughput can be approximated as

$$E\left[R_{(N-k+1)}\right] \approx a_N - b_N\psi(k). \tag{3.15}$$

□

As special case, if $k = 1$, $\psi(1) = -\gamma$ (Euler's constant) and thus $E\left[R_{(N)}\right] \approx a_N + \gamma b_N$, which is exactly the same expression derived in [3]. Since $\psi(k) > 0$, for $k > 1$ and integer k, it follows that $E\left[R_{(N-k+1)}\right] < E\left[R_{(N)}\right]$, for $k > 1$. It should be noted that the result in Proposition 3.1 can be used to evaluate the outage throughput of the k-th best link. Given a rate R_0, $P_{out}(R_0)$ can be approximated as

$$
\begin{aligned}
P_{out}(R_0) &\triangleq \Pr\{R_{(N-k+1)} \leq R_0\} \\
&= \Pr\left\{\frac{R_{(N-k+1)} - a_N}{b_N} \leq \frac{R_0 - a_N}{b_N}\right\} \\
&\approx \Pr\left\{X \leq \frac{R_0 - a_N}{b_N}\right\} = G^{(k)}\left(\frac{R_0 - a_N}{b_N}\right),
\end{aligned} \tag{3.16}
$$

where $G^{(k)}(x)$ is as expressed in (3.12).

3.3.2 Effective Throughput

In this subsection, we use the result from Proposition 3.1 to analyze the effective throughput of the k-th best link as in the following proposition.

Proposition 3.2. The effective throughput of the k-th best link, $\alpha(\theta, k, N)$, can be approximated as

$$
\alpha(\theta, k, N) \approx a_N - \frac{1}{\theta}\log_2\left(\frac{\Gamma\left(\theta \ln(2) b_N + k\right)}{(k-1)!}\right) \tag{3.17}
$$

for fixed k, $\theta > 0$ and $N \to \infty$, where $\Gamma(\cdot)$ is the gamma function.

Proof. Invoking (3.5), the term $E\left[e^{-\theta \ln(2) R_{(N-k+1)}}\right]$ represents the MGF associated with the random variable $R_{(N-k+1)}$, $\mathcal{M}_{R_{(N-k+1)}}(t)$, at $t = -\theta \ln(2)$. Since $\theta > 0$, then we are always interested in the case $t \in (-\infty, 0)$. Applying the result from Theorem 2 of [36], which implies that if $\mathcal{M}_{R_{(N-k+1)}}(t)$ exists for all $t \in (-\infty, 0)$ and the sequence $Y_{N,k} = \frac{R_{(N-k+1)} - a_N}{b_N}$ converges uniformly in distribution to a random variable X with CDF as expressed in (3.12) and MGF of $\mathcal{M}_X(t)$ which exists for all $t \in (-\infty, 0)$, then for a fixed k, $\lim_{N\to\infty} \mathcal{M}_{Y_{N,k}}(t) = \mathcal{M}_X(t)$ for all $t \in (-\infty, 0)$, where $\mathcal{M}_{Y_{N,k}}(t)$ is the MGF associated with the random variable $Y_{N,k}$.

It is obvious that $\mathcal{M}_X(t) = \Gamma(k-t)/(k-1)!$ exists for all $t \in (-\infty, 0)$. However, to show that $\mathcal{M}_{R_{(N-k+1)}}(t)$ exists for all $t \in (-\infty, 0)$, we use Lemma 1.7.2. of [37], with g(x)= $(1+x)^t$,

$x \geq 0$ and $t \in (-\infty, 0)$; we have

$$\begin{aligned}
\mathcal{M}_{R_{(N-k+1)}}(t) &= E\left[e^{tR_{(N-k+1)}}\right] \\
&= E\left[e^{\frac{tB}{\ln(2)}\log\left(1+X_{(N-k+1)}\right)}\right] \\
&= E\left[\left(1+X_{(N-k+1)}\right)^{\frac{tB}{\ln(2)}}\right] \\
&\leq \frac{N!}{(k-1)!(N-k)!}E\left[(1+X_i)^{\frac{tB}{\ln(2)}}\right].
\end{aligned} \tag{3.18}$$

Since X_i is non-negative random variable and it can be modeled as Exponential, Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma, it follows that $E\left[(1+X_i)^{\frac{tB}{\ln(2)}}\right] < \infty$, for all $t \in (-\infty, 0)$. Using Theorem 2 of [36], we have

$$\lim_{N\to\infty} E\left[e^{t\left(\frac{R_{(N-k+1)}-a_N}{b_N}\right)}\right] = E\left[e^{tX}\right] = \frac{\Gamma(k-t)}{(k-1)!}, \tag{3.19}$$

for all $t \in (-\infty, 0)$. Therefore,

$$\begin{aligned}
&E\left[e^{-\theta\ln(2)R_{(N-k+1)}}\right] \\
&= E\left[e^{-\theta\ln(2)\left[b_N\left(\frac{R_{(N-k+1)}-a_N}{b_N}\right)+a_N\right]}\right] \\
&\approx e^{-\theta\ln(2)a_N}E\left[e^{-\theta\ln(2)b_N X}\right] \\
&= \frac{e^{-\theta\ln(2)a_N}\Gamma(k+\theta\ln(2)b_N)}{(k-1)!},
\end{aligned} \tag{3.20}$$

for a fixed k and $N \to \infty$. Substituting (3.20) in (3.5) we reach (3.17). □

It is interesting to observe that if $\theta \to 0$, the effective throughput in (3.17) becomes the average throughput in (3.14). In order to show this, we apply L'hospital's rule and use $\frac{\partial}{\partial x}\log(\Gamma(x)) = \psi(x)$; then we have

$$\begin{aligned}
\lim_{\theta\to 0}\alpha(\theta,k,N) &\approx a_N - b_N \lim_{\theta\to 0} \quad (\theta\ln(2)b_N + k) \\
&= a_N - b_N\psi(k) = E\left[R_{(N-k+1)}\right].
\end{aligned} \tag{3.21}$$

3.4 Asymptotic Average Bit Error Probability

We consider a general class of modulation schemes whose conditional BEP, P_e, is given by [1]

$$P_e = Ce^{-gY}, \tag{3.22}$$

where C and g are positive constants and Y is a random variable which represents the instantaneous received SNR. The average BEP, $\overline{P_e}$, can be expressed as

$$\overline{P_e} = CE\left[e^{-gY}\right]. \tag{3.23}$$

Considering the k-th best link selection scheme, the instantaneous received SNR is $Y = \rho X_{(N-k+1)}$, as stated earlier. Therefore, the average BEP of the k-th best link is $\overline{P_e} = CE\left[e^{-g\rho X_{(N-k+1)}}\right]$. Using the fact that the sequence $\frac{X_{(N)}-\tilde{a}_N}{\tilde{b}_N}$ converges in distribution to a normalized Gumbel random variable along with Theorem 1.3, we derive the average BEP of the k-th best link in the following proposition.

Proposition 3.3. The average BEP of the k-th best link, $\overline{P_e}$, can be approximated as

$$\overline{P_e} \approx Ce^{-g\rho\tilde{a}_N}\frac{\Gamma(k+g\rho\tilde{b}_N)}{(k-1)!} \tag{3.24}$$

for fixed k and $N \to \infty$.

Proof. Using moment generating function approach, the average BEP of the k-th best link can be expressed as [1]

$$\overline{P_e} = CE\left[e^{-g\rho X_{(N-k+1)}}\right] = C\mathcal{M}_{X_{(N-k+1)}}(-g\rho), \tag{3.25}$$

where $\mathcal{M}_{X_{(N-k+1)}}$ represents the MGF associated with the random variable $X_{(N-k+1)}$. Using the fact that the sequence $\frac{X_{(N)}-\tilde{a}_N}{\tilde{b}_N}$ converges uniformly in distribution for large N to a normalized Gumbel random variable, it follows that, from Theorem 1.3, the sequence $\frac{X_{(N-k+1)}-\tilde{a}_N}{\tilde{b}_N}$ converges uniformly in distribution to the random variable X with CDF as expressed in (3.12). Following similar analysis in the proof of the effective throughput, we infer that the MGF of $\frac{X_{(N-k+1)}-\tilde{a}_N}{\tilde{b}_N}$ converges to the MGF of X and therefore $\overline{P_e}$ can be approximated as

$$\begin{aligned}\overline{P_e} &= CE\left[e^{-g\rho X_{(N-k+1)}}\right] \\ &\approx Ce^{-g\rho\tilde{a}_N}E\left[e^{-g\rho\tilde{b}_N X}\right] = Ce^{-g\rho\tilde{a}_N}\frac{\Gamma(k+g\rho\tilde{b}_N)}{(k-1)!},\end{aligned} \tag{3.26}$$

for a fixed k and $N \to \infty$. □

3.5 Numerical Results

We consider a multiple-input single-output (MISO) channel with N transmit antennas in Weibull fading. The PDF of the Weibull distribution can be characterized by shape parameter η and scale parameter α[1]. According to [2], the normalizing constants for Weibull distribution are given by $\tilde{a}_N = \alpha\left(\ln\left(N\right)\right)^{\frac{1}{\eta}}$ and $\tilde{b}_N = \frac{\alpha}{\eta}\left(\ln\left(N\right)\right)^{-\frac{\eta-1}{\eta}}$, as $N \to \infty$. In Fig. 3.1, we plot the average throughput versus the number of transmit antennas, N, for $\eta = 2$ and $\alpha = 3$ and different values of k. We validate the obtained analytical results using Monte Carlo simulations. We observe that the asymptotic expression is accurate even for not so large N while for small values of N the asymptotic expression is less accurate compared to the simulations.

It is intractable to provide a mathematical characterization of the difference between the exact and the asymptotic results for the average throughput. However, numerical results show that this difference is not large even for small values of N as shown in Fig. 3.1. For example, if we consider the worst case in which there exist three transmit antennas and we select the third best channel (the worst channel) the difference in average throughput between the asymptotic and simulation results for the case of $N = 3$ and $k = 3$ is approximately 0.4 bit/s, while the difference for the case of $N = 100$ and $k = 3$ is approximately 0.025 bit/s, which emphasizes that the difference vanishes asymptotically as N grows large with respect to k.

Furthermore, we can observe from Fig. 3.1 that the gap between the asymptotic and simulation results increases as k increases. This is because the asymptotic analysis is more accurate for large

N relative to a fixed k. Consequently, if the value of k is close enough to N, it is expected that the asymptotic expression will be less accurate. For example, the gap between the asymptotic and simulation results for $N = 5$, and $k = 3$ is larger compared to the case when $N = 5, k = 2$ and $N = 5, k = 1$.

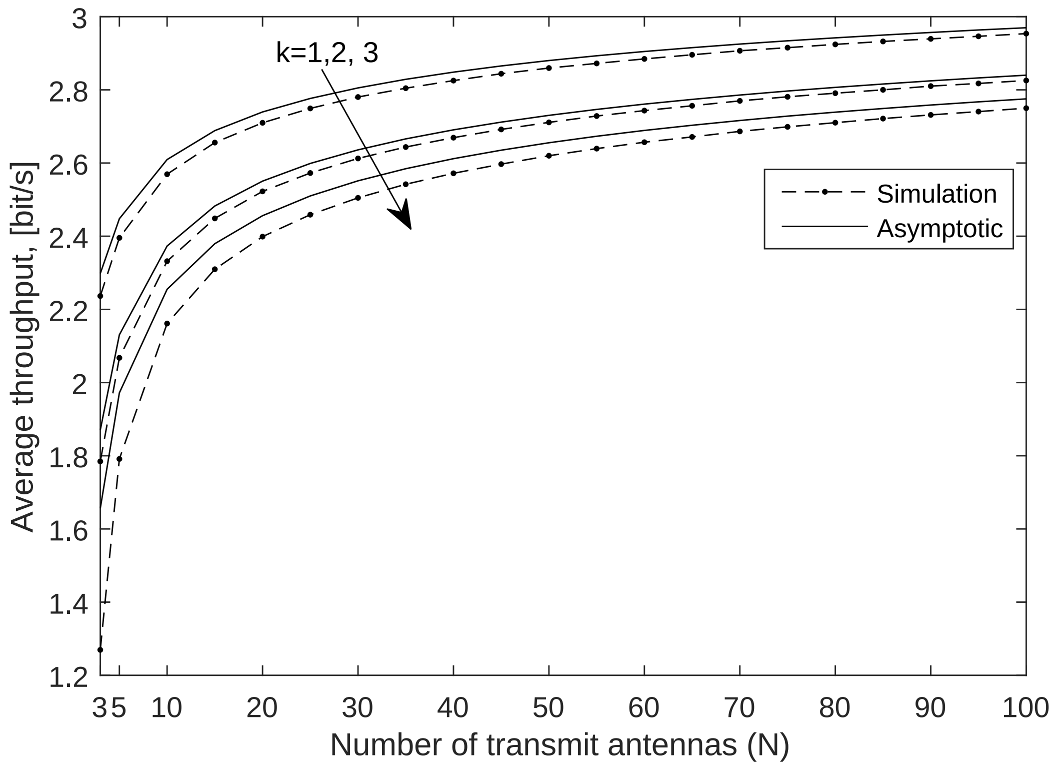

Figure 3.1: Average throughput versus the number of transmit antennas N, for $k = 1, 2, 3$, at $\rho = 0$ dB.

In Fig. 3.2, we plot the effective throughput as a function of delay exponent, θ, for $N = 5$ and $N = 50$, for $\eta = 2$ and $\alpha = 3$ and different values of k. We observe that for $N = 5$ the derived asymptotic result is less accurate as θ increases. However, for $N = 50$ the derived asymptotic results becomes very accurate and the effective throughput does not dramatically change as θ increases and it remains close to the average throughput ($\theta = 0$). This is because employing SD

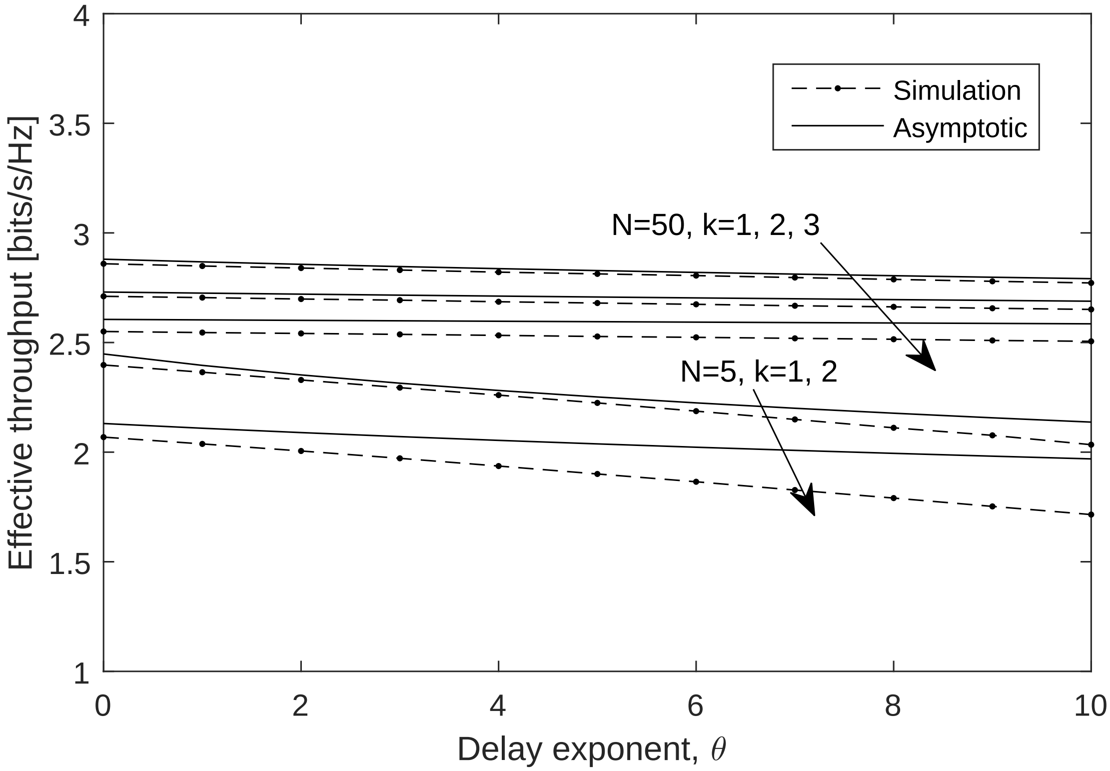

Figure 3.2: Effective throughput versus delay exponent, θ, for $N = 5, 50$ for different values of k at $\rho = 0$ dB.

schemes takes advantage of the tail behavior of the fading distribution as the number of antennas increases. This emphasizes that employing SD schemes in the presence of large number of transmit antennas will combat stringent delay QoS requirements. We also observe that the asymptotic expression is less accurate as k gets closer to N as previously observed for the average throughput.

In Fig. 3.3, we plot the asymptotic average BEP as a function of the number of transmit antennas, N, for $\eta = 2$ and $\alpha = 5$ and different values of k. We validate the obtained analytical results using Monte Carlo simulations. We observe that the asymptotic expression is accurate even for not so large N.

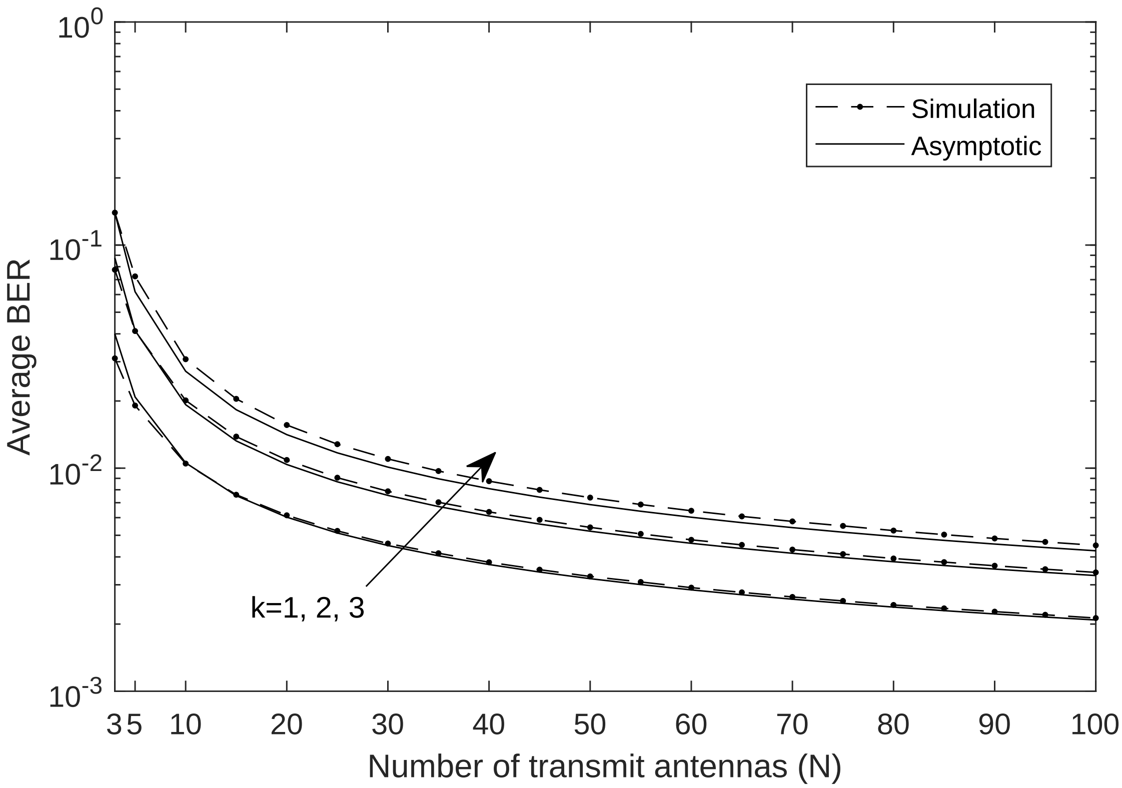

Figure 3.3: Asymptotic BER of BFSK ($C = g = 0.5$) versus the number of transmit antennas N, for $k = 1, 2, 3$, at $\rho = 0$ dB.

3.6 Summary

We used extreme value theory to derive the asymptotic distribution of the throughput of the k-th best link over Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma fading channels. Using this result, we derived simple closed-form asymptotic expressions for the average throughput and effective throughput. Furthermore, we analyzed the average BEP and derived closed-form asymptotic expression for it. As a special case, we considered the Weibull fading channel model and used Monte Carlo simulations to confirm the accuracy of the derived asymptotic expressions.

4. ASYMPTOTIC PERFORMANCE ANALYSIS OF THE K-TH BEST SECONDARY USER SELECTION FOR INTERFERENCE-LIMITED UNDERLAY COGNITIVE RADIO SYSTEMS *

4.1 Introduction

Cognitive radio (CR) is an important technology to maximize radio spectrum utilization efficiency [38]-[40]. In CR systems, the secondary network is allowed to share the spectrum allocated to the primary network provided that the interference caused by the secondary transmitter (ST) does not deteriorate the performance of the primary network. Consequently, the challenge is to maintain the interference caused by the ST to the primary receiver (PR) below a pre-determined threshold level. This can be achieved by adapting the ST transmit power that ensures satisfaction of the interference constraint at the PR [41].

Multiuser diversity is considered an important diversity technique to improve wireless communication systems performance [42]. Considering a multiuser network where the users experience independent fading conditions, the basic idea of multiuser diversity is to select the users with the best fading conditions for transmission or reception to obtain a specific p erformance gain. Multiuser diversity in CR systems has attracted much attention recently. Researchers analyze the performance of multiuser diversity techniques for uplink multiuser underlay CR systems without taking the interference from the primary network into consideration[43] -[48]. In particular, the ergodic capacity (throughput) of multiuser diversity gain of uplink multiuser underlay CR systems is investigated in [43]. In [44], the authors analyze the achievable capacity gain of uplink multiuser spectrum-sharing systems over dynamic fading environments. In [45], the outage probability and effective capacity are analyzed for opportunistic spectrum sharing in Rayleigh fading environment. In [46], the authors analyze the outage probability, average symbol error rate (SER) and ergodic capacity of an opportunistic multiuser cognitive network with multiple primary users assuming

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the channels in the secondary network are independent but not identical Nakagami-m fading. In [47], [48] the authors analyze the outage probability and average capacity of multiuser diversity in single-input multiple-output (SIMO) spectrum sharing systems. The ergodic capacity of multiuser diversity in CR systems with interference from the primary network is investigated in [49]. Recently, the ergodic capacity of various multiuser scheduling schemes in downlink cognitive radio networks with interference from the primary network is analyzed in [50]; here, the authors analyze the ergodic capacity of multiuser diversity scheduling under the outage constraint of multiple primary user receivers and the secondary user (SU) maximum transmit power limit. Interference-limited underlay CR systems are considered in [6], where the authors analyze the average BER and outage probability for receive antenna selection schemes under discrete power adaptation at the ST.

Related previous work has focused on conventional multiuser diversity in underlay CR systems where the user with the best link quality is selected. However, in practical underlay CR systems the secondary user with the best link quality may not be available for transmission or reception under given traffic conditions. Consequently for such systems, a more general multiuser diversity scheme that features selection of the secondary user with the k-th best link quality (k-th best secondary user) is of practical interest.

In this Chapter we consider an interference-limited secondary multiuser network, where the noise at each secondary user receiver is negligible compared to the interference from the primary transmitter (PT). We assume that the ST transmits information to the k-th best secondary user (SU), namely, the SU with the k-th highest signal-to-interference ratio (SIR). Meanwhile, the ST adjusts its transmit power to satisfy the instantaneous interference constraint at the primary receiver (PR). In general, it is hard to find exact and tractable expressions for the common performance measures of the k-th best SU, such as average and effective throughputs, average bit error rate (BER) and outage probability. This difficulty is due to the complicated nature of the distribution of the received SIR at the k-th best secondary user's receiver. Therefore, another approach based on extreme value theory (EVT) or extreme order statistics is used to analyze the performance of the

k-th best SU in underlay CR systems. EVT was considered for traditional wireless communication systems with no spectrum sharing. In particular, EVT was used to analyze the asymptotic average throughput of the conventional selection diversity scheme where the link with the highest signal to-noise ratio (SNR) is selected from independent and identically distributed (i.i.d) links [4], [3]. EVT was also used to analyze the average BER and outage probability for underlay CR systems with receive antenna selection under discrete power adaptation [6]. Recently, we used EVT to derive simple closed form asymptotic expressions for the average throughput, effective throughput and average BER of the k-th best link over different wireless fading channels[51].

Our contribution is to utilize EVT to analyze the performance of the k-th best SU for underlay CR systems. More specifically, we show that the SIR of the k-th best user converges uniformly in distribution to an inverse gamma random variable for a fixed k and large number of secondary users. Then, we derive novel closed-form asymptotic expressions for the average and effective throughputs of the k-th best SU employing continuous power adaptation at the ST with both limited and unlimited transmit power. Furthermore, novel closed-form asymptotic expressions for the average BER and outage probability with continuous power adaptation and unlimited ST power are derived.

The rest of this Chapter is organized as follows. In Section II we discuss the system model. In Section III we discuss the asymptotic average and effective throughputs of the k-th best SU. In Section IV we analyze the average BER. In Section V we analyze the outage probability. Section VI includes numerical results and Section VII concludes.

4.2 System Model

As described in Fig. 4.1, we consider an underlay secondary network consisting of one ST equipped with a single antenna, and N secondary users each equipped with a single antenna. The secondary network is sharing the spectrum of a primary network with one PT and one PR. The PT and PR are equipped with a single antenna each. Let g_i denote the channel gain from the PT to the i-th secondary user's receiver (SU-Rx), where $i = 1, 2, ..., N$. Let h_0 and h_i denote the channel gain from the ST to the PR and the i-th SU-Rx, respectively. We assume that the primary network is far away from the secondary network and therefore $|h_0|$ and $|g_i|$ are assumed to be independent Rayleigh distributed random variables. This implies that the channel power gains, $|h_0|^2$ and $|g_i|^2$ have a probability density functions (PDFs) $f_0(x) = \eta e^{-\eta x} u(x)$ and $g(x) = \lambda e^{-\lambda x} u(x)$, respectively, where $u(x)$ is the unit step function and the parameters η and λ are the fading parameters. The channel power gains in the secondary network, $|h_i|^2$, for $i = 1, 2, ..., N$, are assumed to be independent and identically distributed (i.i.d) Gamma RVs with PDF of

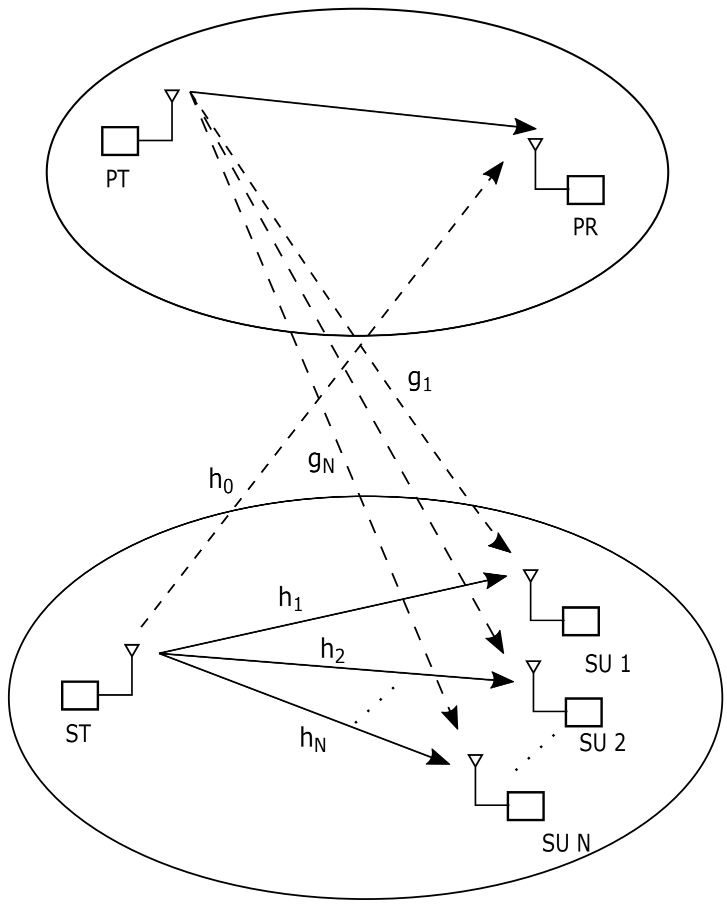

Figure 4.1: An underlay cognitive radio network with an ST serving N secondary users.

$$f(x) = \frac{x^{m-1}}{\beta^m \Gamma(m)} e^{-\frac{x}{\beta}} u(x). \tag{4.1}$$

where the parameters m and β are positive reals and $\Gamma(m)$ is the Gamma function.

Similar to [41], [44], [46], [6], [52] and [53], it is assumed that the ST has perfect channel state information (CSI) regarding the secondary transmitter to primary receiver channel, h_0. The ST can be informed about h_0 through a mediate band manager between PR and ST [54] or by considering proper signaling [55]. Analyzing the performance of the k-th best SU with imperfect CSI is left for future work; thus, the derived results throughout this Chapter are optimistic compared to the results with imperfect CSI.

With a perfect knowledge of $|h_0|^2$, we consider a continuous power adaptation policy at the ST to control its interference to the PR such that the instantaneous transmit power of the ST is $P = \min\left(P_S, \frac{T}{|h_0|^2}\right)$, where P_S is the maximum instantaneous power available at the ST and T is the maximum tolerable interference level at the PR. Assuming the noise at the i-th SU-Rx is negligible compared to the interference from the PT, then the ST will select the k-th best SU; namely, the SU with the k-th highest signal-to-interference ratio (SIR); i.e.,

$$i^* = \arg k\text{-th} \max_i \{PZ_i\}_{i=1}^N \tag{4.2}$$

where $Z_i = \frac{|h_i|^2}{P_M|g_i|^2}$, P_M is the transmit power of the PT and $P_M|g_i|^2$ is the PT interference power at the i-th SU-Rx.

Let $PZ_{(N-k+1)}$ denote the instantaneous SIR at the k-th best SU-Rx, where $Z_{(1)} \leq Z_{(2)} \leq \dots \leq Z_{(N)}$. According to [2], the PDF of $Z_{(N-k+1)}$ can be expressed in terms of the PDF, $f(z)$, and CDF, $F(z)$, of Z_i as

$$f_{Z_{(N-k+1)}}(x) = k\binom{N}{k} f(z)F(z)^{N-k}\left(1 - F(z)\right)^{k-1}, \tag{4.3}$$

where the CDF and PDF of Z_i are given by [6]

$$F(z) = \left(\frac{P_M z}{\lambda\beta + P_M z}\right)^m u(z), \tag{4.4}$$

$$f(z) = \frac{m\lambda\beta \left(P_M\right)^m z^{m-1}}{\left(\lambda\beta + P_M z\right)^{m+1}} u(z). \tag{4.5}$$

Let $R_{(N-k+1)} = B\log_2(1 + PZ_{(N-k+1)})$ denote the instantaneous throughput of the k-th best SU, where $R_{(1)} \leq R_{(2)} \leq .... \leq R_{(N)}$ and B is the system bandwidth. Then, the average throughput of the k-th best SU, $E\left[R_{(N-k+1)}\right]$, can be evaluated as

$$E\left[R_{(N-k+1)}\right] = E\left[\log_2(1 + PZ_{(N-k+1)})\right] \tag{4.6}$$

in bit/s/Hz. The expectation in (4.6) is taken over the joint distribution of random variables P and $Z_{(N-k+1)}$.

Considering the selection of the k-th best SU, the effective throughput of the k-th best SU, $\alpha(\theta, k, N)$, can be expressed as

$$\alpha(\theta, k, N) = -\frac{1}{A}\log_2\left(E\left[\left(1 + PZ_{(N-k+1)}\right)^{-A}\right]\right), \tag{4.7}$$

in bit/s/Hz, where $A = \theta TB/\ln(2)$ and the expectation is taken over the joint distribution of P and $Z_{(N-k+1)}$.

If we conisder a general class of modulation schemes whose conditional BER, P_e, is given by

$$P_e = c\, e^{-gY}, \tag{4.8}$$

where c and g are positive constants and Y is a random variable which represents the instantaneous received SIR, the average BER of the k-th best SU can be expressed as

$$\overline{P_e}(k, N) = c\, E\left[e^{-gPZ_{(N-k+1)}}\right], \tag{4.9}$$

where the expectation is taken over the joint distribution of P and $Z_{(N-k+1)}$.

Due to the complicated nature of the distribution of the instantaneous SIR at the k-th best SU-Rx, it is difficult to obtain exact expressions for $E\left[R_{(N-k+1)}\right]$, $\alpha(\theta, k, N)$ and $\overline{P_e}(k, N)$. Therefore, in this Chapter we consider another approach based on extreme value theory to analyze the performance of the k-th best SU in terms of average throughput, effective throughput, outage probability and average BER.

4.3 Asymptotic Throughput Analysis

In this section, we derive the limiting distribution of $Z_{(N-k+1)}$ based on Theorem 1.3 and use this result to analyze the average and effective throughputs of the k-th best SU.

4.3.1 The Limiting Distribution of $Z_{(N-k+1)}$

Using Theorem 1.3 and the following Proposition, we derive the limiting distribution of $Z_{(N-k+1)}$ in Proposition 4.2.

Proposition 4.1. Let $Z_{(N)}$ denote the largest order statistic of N i.i.d random variables with a common CDF $F(z)$, as expressed in (4.4), then for $N \to \infty$, the CDF of $\frac{Z_{(N)}-a}{b}$ converges to a unit Fréchet distribution; i.e.,

$$G(z) = e^{-z^{-1}} u(z), \tag{4.10}$$

where $a = 0$ and $b = \frac{\beta\lambda}{P_M\left(\left(1-\frac{1}{N}\right)^{-\frac{1}{m}}-1\right)} > 0$.

Proof. The proof is identical to that provided in Proposition 2 of [6]. □

Proposition 4.2. Let $Z_{(N-k+1)}$ denote the k-th largest order statistic of N i.i.d random variables with a common CDF of $F(z)$, as expressed in (4.4), then for a fixed k and $N \to \infty$, $\frac{Z_{(N-k+1)}-a}{b}$ converges in distribution to a random variable Z with CDF $G^{(k)}(z)$, which can be characterized by an inverse gamma distribution as

$$G^{(k)}(z) = \frac{\Gamma\left(k, \frac{1}{z}\right)}{(k-1)!} u(z), \tag{4.11}$$

where $a = 0$, $b = \frac{\beta\lambda}{P_M\left(\left(1-\frac{1}{N}\right)^{-\frac{1}{m}}-1\right)} > 0$ and $\Gamma(s,x) = \int_x^\infty u^{s-1}e^{-u}du$ is the upper incomplete gamma function. Furthermore, the PDF of Z, $f^{(k)}(z)$, can be obtained as

$$f^{(k)}(z) = \frac{e^{-z^{-1}}}{z^{k+1}(k-1)!}u(z). \tag{4.12}$$

Proof. From Proposition 4.1, $\frac{Z_{(N)}-a}{b}$ converges in distribution to a unit Fréchet distribution i.e., $G(z) = e^{-z^{-1}}u(z)$, where $a = 0$ and $b = F^{-1}\left(1-\frac{1}{N}\right) = \frac{\beta\lambda}{P_M\left(\left(1-\frac{1}{N}\right)^{-\frac{1}{m}}-1\right)}$. Using Theorem 1.3, it follows that for a fixed k and $N \to \infty$, the sequence $\frac{Z_{(N-k+1)}}{b}$ converges in distribution to a random variable Z with CDF of $G^{(k)}(z)$. From (3.6), $G^{(k)}(z)$ can be expressed as

$$G^{(k)}(z) = e^{-z^{-1}}\sum_{j=0}^{k-1}\frac{(z^{-1})^j}{j!}u(z). \tag{4.13}$$

Using the fact that $\Gamma(k,x) = (k-1)!\, e^{-x}\sum_{j=0}^{k-1}\frac{x^j}{j!}$ for an integer k, $G^{(k)}(z)$ can be finally expressed as in (4.11). By differentiating (4.11) we obtain (4.12). $\square$

4.3.2 The Distribution of the ST Transmit Power

We consider a continuous power adaptation scheme in which the transmit power of the ST can be adapted with a power limit of P_S; therefore, the instantaneous transmit power of the ST is $P = \min\left(P_S, \frac{T}{|h_0|^2}\right)$. Furthermore, we consider a continuous power adaptation scheme in which the transmit power of the ST can be adapted without any power limit, i.e. $P_S = \infty$ [52], [53]. In such case, the ST transmit power, P, can be written as $P = \frac{T}{|h_0|^2}$. We focus next on the PDF of the instantaneous transmit power of the ST, $P = \min\left(P_S, \frac{T}{|h_0|^2}\right)$, then we use this PDF and Proposition 4.2 to evaluate the average and effective throughputs of the k-th best SU.

Considering $P = \min(P_S, X)$ is a continuous random variable, where $X = \frac{T}{|h_0|^2}$ and P_S is constant, then the CDF of the random variable P, $F_P(t)$, can be given as

$$F_P(t) = F_X(t) + u(t-P_S) - u(t-P_S)F_X(t), \tag{4.14}$$

where $F_X(t)$ is the CDF of the random variable $X = \frac{T}{|h_0|^2}$ and $u(t - P_S)$ is the unit step function given by

$$u(t - P_S) = \begin{cases} 1, & t \geq P_S \\ 0, & t < P_S \end{cases} . \tag{4.15}$$

Then it follows that the PDF of P, $f_P(t)$, can be expressed as

$$f_P(t) = f_X(t) \left[1 - u(t - P_S)\right] + \delta(t - P_S) \left[1 - F_X(P_S)\right], \tag{4.16}$$

where $f_X(t)$ is the PDF of the random variable X and $\delta(t - P_S)$ is the Dirac delta function, the derivative of $u(t - P_S)$.

Using the PDF of $|h_0|^2$, $f_0(x) = \eta e^{-\eta x} u(x)$, and variable transformation then it follows that $F_X(t) = e^{-\frac{\eta T}{t}} u(t)$ and $f_X(t) = \frac{\eta T}{t^2} e^{-\frac{\eta T}{t}} u(t)$. Finally we can write

$$f_P(t) = \frac{\eta T}{t^2} e^{-\frac{\eta T}{t}} \left[1 - u(t - P_S)\right] + \delta(t - P_S) \left(1 - e^{-\frac{\eta T}{P_S}}\right). \tag{4.17}$$

4.3.3 Average and Effective Throughputs

Proposition 4.3. The average and effective throughputs of the k-th best SU for continuous power adaptation with limited ST power are respectively given by

$$E\left[R_{(N-k+1)}\right] \approx \frac{\ln\left(bP_S\right) - E1\left(\frac{\eta T}{P_S}\right) - \psi\left(k\right)}{\ln(2)}, \tag{4.18}$$

$$\alpha(\theta, k, N) \approx -\frac{1}{A} \log_2 \left(\frac{\Gamma\left(k + A\right) \Gamma\left(A + 1, \frac{\eta T}{P_S}\right)}{\left(b\eta T\right)^A \left(k - 1\right)!} + \frac{\Gamma\left(k + A\right) \left(1 - e^{-\frac{\eta T}{P_S}}\right)}{\left(bP_S\right)^A \left(k - 1\right)!} \right), \tag{4.19}$$

for fixed k and $N \to \infty$, where $E_1(x) = \int_x^\infty \frac{e^{-y}}{y} dy, x \geq 0$ is the exponential integral function and $\Gamma(s, x) = \int_x^\infty u^{s-1} e^{-u} du$ is the upper incomplete gamma function, $\psi(x)$ is the digamma function and γ is the Euler's constant.

Proof. Average Throughput:

From Proposition 4.2, the CDF of $\frac{Z_{(N-k+1)}}{b}$ approaches the CDF of Z for a fixed k and $N \to \infty$, where the CDF of Z is as expressed in (4.11). Or equivalently, the PDF of $Z_{(N-k+1)}$ can be approximated by the PDF of bZ for a fixed k and $N \to \infty$, where the PDF of Z is as expressed in (4.12). Then for a fixed k, $N \to \infty$ and conditioning on the ST transmit power P, $E\left[R_{(N-k+1)}|P\right]$ can be approximated as

$$\begin{aligned} E\left[R_{(N-k+1)}|P\right] &\approx \frac{1}{\ln(2)} E\left[\log\left(1 + bPZ\right)|P\right] \\ &= \frac{1}{\ln(2)} \int_0^\infty \ln(1 + bPz) \frac{e^{-z^{-1}}}{z^{k+1}(k-1)!} dz. \end{aligned} \tag{4.20}$$

Noting that b is an increasing function of N, we have $\ln(1+bPz) \approx \ln(bPz)$ in (4.20) for large N. Using this and variable transformation of $u = (bPz)^{-1}$, $E\left[R_{(N-k+1)}|P\right]$ can be further approximated as

$$E\left[R_{(N-k+1)}|P\right] \approx \int_0^\infty \frac{-\ln(u)\,(bP)^k e^{-bPu} u^{k-1}}{\ln(2)(k-1)!} du = \frac{\ln(bP) - \psi(k)}{\ln(2)}, \tag{4.21}$$

where the above integral is evaluated with help of Eq. (4.352, 1) of [25]. Averaging $\ln(P)$ over the PDF of $f_P(t)$ yields

$$\int_0^\infty \ln(t) f_P(t) dt = \int_0^{P_S} \ln(t) \frac{\eta T}{t^2} e^{-\frac{\eta T}{t}} dt + \left(1 - e^{-\frac{\eta T}{P_S}}\right) \ln\left(P_S\right). \tag{4.22}$$

Using variable transformation of $u = P_S t^{-1}$ with help of Eq. (4.331, 2) of [25] and after some basic algebraic manipulation, we have

$$\int_0^{P_S} \ln(t)\frac{\eta T}{t^2}e^{-\frac{\eta T}{t}}dt = e^{-\frac{\eta T}{P_S}}\ln\left(P_S\right) - E_1\left(\frac{\eta T}{P_S}\right). \tag{4.23}$$

Combining (4.21), (4.22) and (4.23), it follows that $E\left[R_{(N-k+1)}\right]$ is as expressed in (4.18).

Effective Throughput:

Conditioning on the ST transmit power P in (4.7) and by exploiting Lemma 3 of Appendix C, we infer that $E\left[\left(1+PZ_{(N-k+1)}\right)^{-A}|P\right]$ can be approximated as

$$\begin{aligned} E\left[\left(1+PZ_{(N-k+1)}\right)^{-A}|P\right] &\approx E\left[\left(1+bPZ\right)^{-A}|P\right] \\ &= \int_0^\infty \frac{\left(1+bPz\right)^{-A}e^{-z^{-1}}}{z^{k+1}(k-1)!}dz, \end{aligned} \tag{4.24}$$

for fixed k and $N\to\infty$. Making use as above of $1+bPz \approx bPz$ for large N in (4.24) and variable transformation of $u=(bPz)^{-1}$, $E\left[\left(1+PZ_{(N-k+1)}\right)^{-A}|P\right]$ can be further approximated as

$$\begin{aligned} E\left[\left(1+PZ_{(N-k+1)}\right)^{-A}|P\right] &\approx \int_0^\infty \frac{(bP)^k u^{A+k-1}e^{-bPu}}{(k-1)!}dz \\ &= \frac{(bP)^{-A}\Gamma\left(A+k\right)}{(k-1)!}, \end{aligned} \tag{4.25}$$

where the above integral is evaluated with help of Eq. (3.351, 3) of [25]. Averaging (4.25) over the PDF of $f_P(t)$ yields

$$\begin{aligned} E\left[\left(1+PZ_{(N-k+1)}\right)^{-A}\right] &\approx \int_0^\infty \frac{(bt)^{-A}\Gamma\left(A+k\right)}{(k-1)!}f_P(t)dt \\ &= \int_0^{P_S}\frac{\Gamma\left(k+A\right)\eta Te^{-\frac{\eta T}{t}}}{b^A t^{A+2}\left(k-1\right)!}dt + \frac{\Gamma\left(k+A\right)\left(1-e^{-\frac{\eta T}{P_S}}\right)}{\left(bP_S\right)^A\left(k-1\right)!}. \end{aligned} \tag{4.26}$$

Using variable transformation of $u=\eta Tt^{-1}$ and using the definition of the upper incomplete gamma function, $\Gamma(s,x) = \int_x^\infty u^{s-1}e^{-u}du$, we have

$$\int_0^{P_S} \frac{\Gamma(k+A)\,\eta\,T e^{-\frac{\eta T}{t}}}{b^A t^{A+2}(k-1)!}dt = \frac{\Gamma(k+A)\,\Gamma\left(A+1, \frac{\eta T}{P_S}\right)}{(b\eta T)^A (k-1)!}. \tag{4.27}$$

Combining (4.26), (4.27) and (4.7), it follows that $\alpha(\theta, k, N)$ is as expressed in (4.19). □

Now we consider a continuous power adaptation with $P_S = \infty$ [53], [52]. In this case, the ST transmit power, P, can be written as $P = \frac{T}{|h_0|^2}$. Using the result from Proposition 4.3, we derive the average and effective throughputs of the k-th best SU with unlimited ST power in the following corollary.

Corollary 4.1. The average and effective throughputs of the k-th best SU for continuous power adaptation with unlimited ST power are respectively given by

$$E\left[R_{(N-k+1)}\right] \approx \frac{\ln(bT\eta) - \psi(k) + \gamma}{\ln(2)}, \tag{4.28}$$

$$\alpha(\theta, k, N) = \frac{\ln(bT\eta)}{\ln(2)} - \frac{1}{A}\log_2\left(\frac{\Gamma(A+k)\,\Gamma(A+1)}{(k-1)!}\right), \tag{4.29}$$

for fixed k and $N \to \infty$.

Proof. Average Throughput:

Using Puiseux series for the exponential integral function, $E_1(x)$, we have

$$E_1(x) = -\gamma - \ln(x) - \sum_{n=1}^{\infty} \frac{(-x)^n}{nn!},\ x > 0. \tag{4.30}$$

Invoking (4.18) and with the help of (4.30), one can show that $\lim_{P_S\to\infty}\left(\ln(bP_S) - E1\left(\frac{\eta T}{P_S}\right)\right) = \ln(bT\eta) + \gamma$. Therefore, as $P_S \to \infty$, the average throughput is as expressed in (4.28).

Effective Throughput:

Invoking (4.19) and $\lim_{P_S\to\infty}\Gamma(A+1,\frac{\eta T}{P_S}) = \Gamma(A+1)$ one can show that the effective throughput is as expressed in (4.29) as $P_S \to \infty$. $\square$

4.4 Average BER

We now derive the average BER for the limited and unlimited continuous ST power in the following proposition.

Proposition 4.4. The average BER of the k-th best SU for continuous limited power adapdation scheme can be approximated as

$$\begin{aligned}\overline{P_e}(k,N) \approx &\int_0^{P_S} 2\,\frac{(gbt)^{k/2}\,K_k\left(2\sqrt{gbt}\right)\eta\,T}{(k-1)!\,t^2} e^{-\frac{\eta T}{t}}\,\mathrm{d}t \\ &+ \frac{2\,(gbP_S)^{k/2}\,K_k\left(2\sqrt{gbP_S}\right)}{(k-1)!}\left(1-e^{-\frac{\eta T}{P_S}}\right),\end{aligned} \tag{4.31}$$

for fixed k and $N \to \infty$, where $K_\nu(\cdot)$ is the modified Bessel function of the second kind and order ν. Furthermore, for the unlimited ST transmit power, the average BER of the k-th best SU can be approximated as

$$\overline{P_e}(k,N) \approx \frac{c\,(\eta\,Tgb)^{k/2-1}\,G_{0,3}^{3,0}\left(\eta\,Tgb \,\middle|\, {}^{-}_{1+k/2,2-k/2,1-k/2}\right)}{(k-1)!}, \tag{4.32}$$

for fixed k and $N \to \infty$, where $G_{p,q}^{m,n}(.)$ is the Meijer G-function [56].

Proof: Conditioning on the ST transmit power P in (4.9) and by exploiting Lemma 2 of Appendix C, we infer that $E\left[e^{-gPZ_{(N-k+1)}}\middle| P\right]$ can be approximated as

$$\begin{aligned}E\left[e^{-gPZ_{(N-k+1)}}\middle| P\right] &\approx E\left[e^{-gbPZ}\middle| P\right] \\ &= \int_0^{\infty} \frac{e^{-gbPz}e^{-z^{-1}}}{z^{k+1}\,(k-1)!}dz \\ &= \frac{2\,(gbP)^{k/2}\,K_k\left(2\sqrt{gbP}\right)}{(k-1)!},\end{aligned} \tag{4.33}$$

for a fixed k and $N \to \infty$, where the above integral is evaluated with help of Eq. (2.11) of [57]. It is hard to find an analytical expression for the avearge BER for the limited ST transmit power case. Therefore, averaging (4.33) over $f_P(t)$ in (4.17) yields the average BER for limited ST power as in (4.31). For the unlimited ST transmit power, one can show that by letting $P_S \to \infty$ in (4.31), we have

$$\overline{P_e}(k,N) \approx \int_0^\infty \frac{2c\,(gbt)^{k/2}\, K_k\left(2\sqrt{gbt}\right)}{(k-1)!}\frac{\eta\, T}{t^2}e^{-\frac{\eta T}{t}}dt. \tag{4.34}$$

The above integral can be expressed in terms of the Meijer G-function as in (4.32)

4.5 Outage Probability

We now derive the outage probability for limited and unlimited continuous ST power in the following proposition.

Proposition 4.5. The outage probability of the k-th best SU for continuous limited power adapdation scheme can be approximated as

$$\begin{aligned} P_{out}(x_0) \approx & \int_0^{P_S} \Gamma\left(k, \frac{bt}{x_0}\right) \frac{\eta\, T}{(k-1)!\, t^2} e^{-\frac{\eta T}{t}}\, dt \\ & + \Gamma\left(k, \frac{bP_S}{x_0}\right) \frac{\left(1 - e^{-\frac{\eta T}{P_S}}\right)}{(k-1)!}, \end{aligned} \tag{4.35}$$

for fixed k and $N \to \infty$. Furthermore, for the unlimited ST transmit power, the outage probability of the k-th best SU can be approximated as

$$P_{out}(x_0) \approx \frac{2\left(\frac{\eta T b}{x_0}\right)^{\frac{k}{2}} K_k\left(2\sqrt{\frac{\eta T b}{x_0}}\right)}{(k-1)!}, \tag{4.36}$$

for fixed k and $N \to \infty$.

Proof. The outage probability of the k-th best SU, $P_{out}(x_0)$, can be expressed as

$$
\begin{aligned}
P_{out}(x_0) &= \int_0^\infty \Pr\{tZ_{(N-k+1)} \leq x_0\} f_P(t)dt. \\
&= \int_0^\infty \Pr\{tZ_{(N-k+1)} \leq x_0\} f_P(t)dt,
\end{aligned}
\tag{4.37}
$$

where $f_P(t)$ as given in (4.17). From Proposition 4.2, the CDF of $\frac{Z_{(N-k+1)}}{b}$ approaches the CDF of Z for fixed k and $N \to \infty$, where the CDF of Z is as expressed in (4.11). Then, we have

$$
\begin{aligned}
P_{out}(x_0) &= \int_0^\infty \Pr\{tZ_{(N-k+1)} \leq x_0\} f_P(t)dt. \\
&= \int_0^\infty \Pr\left\{\frac{Z_{(N-k+1)}}{b} \leq \frac{x_0}{bt}\right\} f_P(t)dt, \\
&\approx \int_0^\infty \Pr\left\{Z \leq \frac{x_0}{bt}\right\} f_P(t)dt, \\
&= \int_0^\infty \frac{\Gamma\left(k, \frac{bt}{x_0}\right)}{(k-1)!} f_P(t)dt.
\end{aligned}
\tag{4.38}
$$

Averaging (4.38) over $f_P(t)$ in (4.17) yields the outage probability for the limited ST power as in (4.35). For the unlimited ST transmit power, one can show that by letting $P_S \to \infty$ in (4.35), we have

$$
\begin{aligned}
P_{out}(x_0) &\approx \int_0^\infty \Gamma\left(k, \frac{bt}{x_0}\right) \frac{\eta T}{(k-1)!\, t^2} e^{-\frac{\eta T}{t}}\, dt \\
&= \frac{2\left(\frac{\eta T b}{x_0}\right)^{\frac{k}{2}} K_k\left(2\sqrt{\frac{\eta T b}{x_0}}\right)}{(k-1)!},
\end{aligned}
\tag{4.39}
$$

where the integral above is evaluated with help of Eq. (6.453) of [25] after variable transformation of $u = (\eta T t)^{-1}$. $\square$

4.6 Numerical Results

In this section, we numerically illustrate and verify the obtained asymptotic expressions in the previous sections. In Fig. 4.2, we plot the average throughput of the k-th best SU versus the number of secondary users, N, for unlimited ST power, $P_S = \infty$, and limited ST power with $P_S = 10$ dB, for $k = 1, 2, 3$. We validate the obtained asymptotic expressions for the average throughput using Monte Carlo simulations. We observe that the asymptotic expressions are accurate even for not so large N while for small values of N the asymptotic expressions are less accurate compared to the simulations. In Fig. 4.3, we plot the average throughput of the best SU versus the ST power, P_S, in dB. We observe that, compared to the simulations, the accuracy of the asymptotic average throughput increase as N increase from 6 to 30. We also observe that the accuracy of the asymptotic average throughput increases as P_S increases. Furthermore, for larger values of P_S the asymptotic average throughput approaches the one with unlimited ST power, $P_S = \infty$.

In Fig. 4.4, we plot the average throughput of the best SU versus the interference level, T, in dB for unlimited ST power, $P_S = \infty$, and limited ST power with $P_S = -20$ dB. Some interesting observations can be made from this figure. First, we observe that, compared to the simulations, the accuracy of the asymptotic average throughputs increase as N increases from 20 to 200. Second, as T or P_S increases, the accuracy of the asymptotic average throughputs also increases. Last, for the limited ST power, $P_S = -20$dB, the average throughput is saturated and it does not improve as $T \geq -30$ dB. This is due to the fact that for higher values of T, the ST will select P_S with a higher probability.

In Fig. 4.5, we plot the effective throughput of the k-th best SU versus the number of secondary users, N, with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = 5$ dB, for $k = 1, 2, 4$. We observe that the accuracy of the asymptotic effective throughput increases as N increases. However, it is shown that for $k = 4$, the asymptotic effective throughputs is less accurate for small to moderate values of N. This is because the asymptotic analysis is more accurate for large N relative to a fixed k. Consequently, if the value of k is close enough to N, it is expected that the asymptotic expression will be less accurate.

In Fig. 4.6, we plot the effective throughput of the best SU versus the delay exponent, A, for $N = 30$ and different values of P_S. We observe that the effective throughput significantly decreases for smaller values of P_S. On the other hand, for reasonably large values of P_S, the

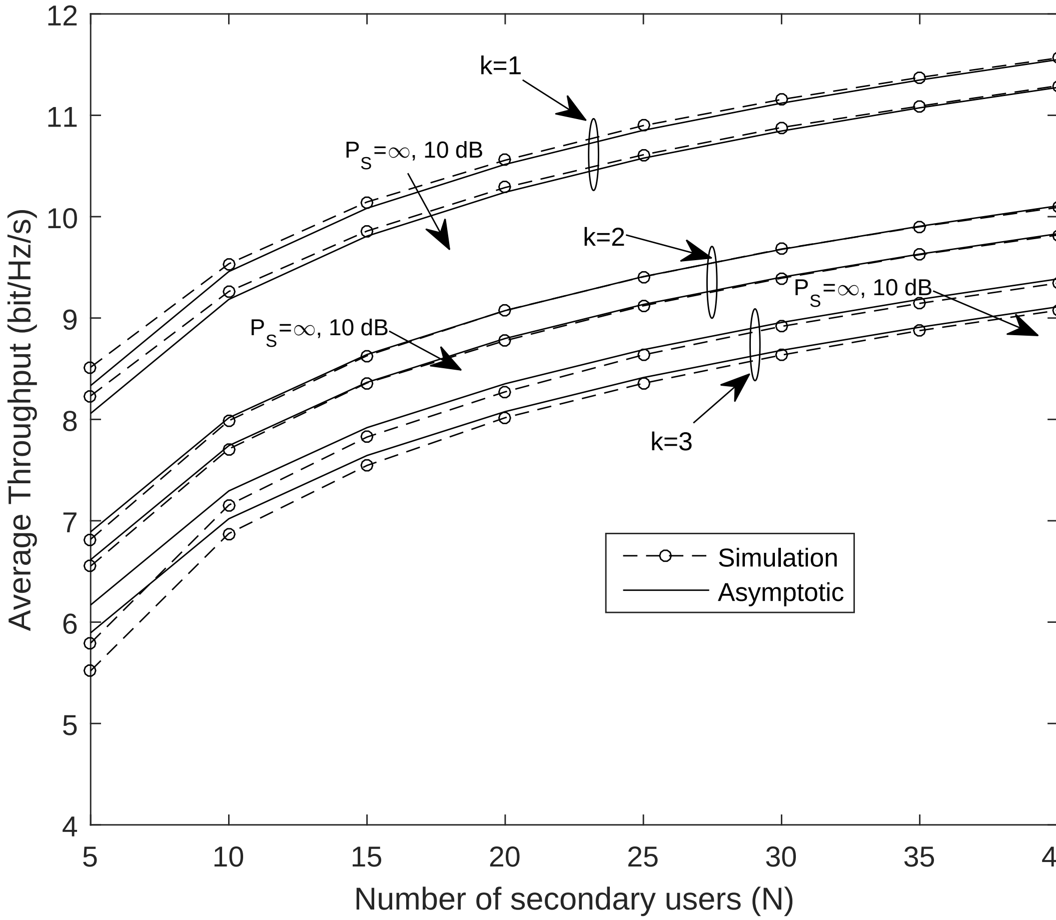

Figure 4.2: Average throughput of the k-th best SU versus the number of secondary users N with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = 10$ dB, for $k = 1, 2, 3$, $\lambda = 2$, $\beta = 3$, $\eta = 20$, $T = -10$ dB and $P_M = 0$ dB.

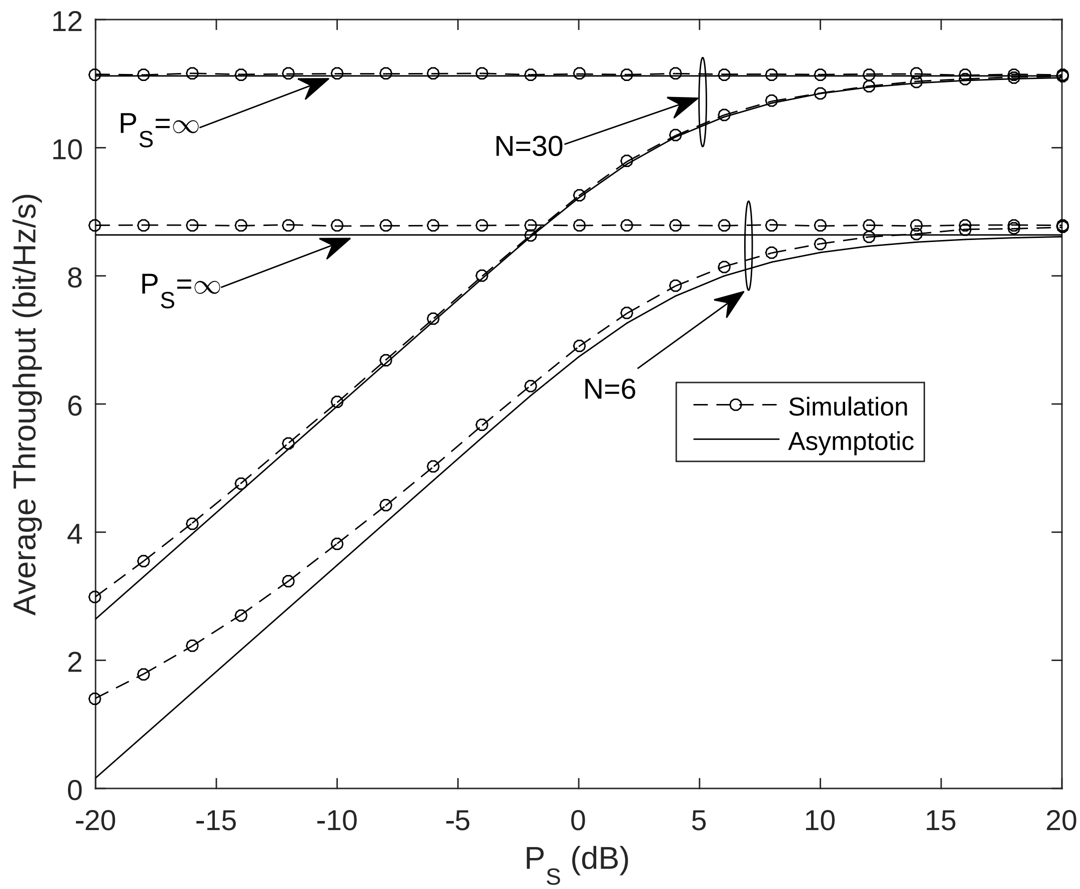

Figure 4.3: Average throughput of the best SU versus the ST power, $P_S(\mathrm{dB})$, for $N = 6, 30$, $\lambda = 2$, $\beta = 3$, $\eta = 20$, $T = -10$ dB, $P_M = 0$ dB.

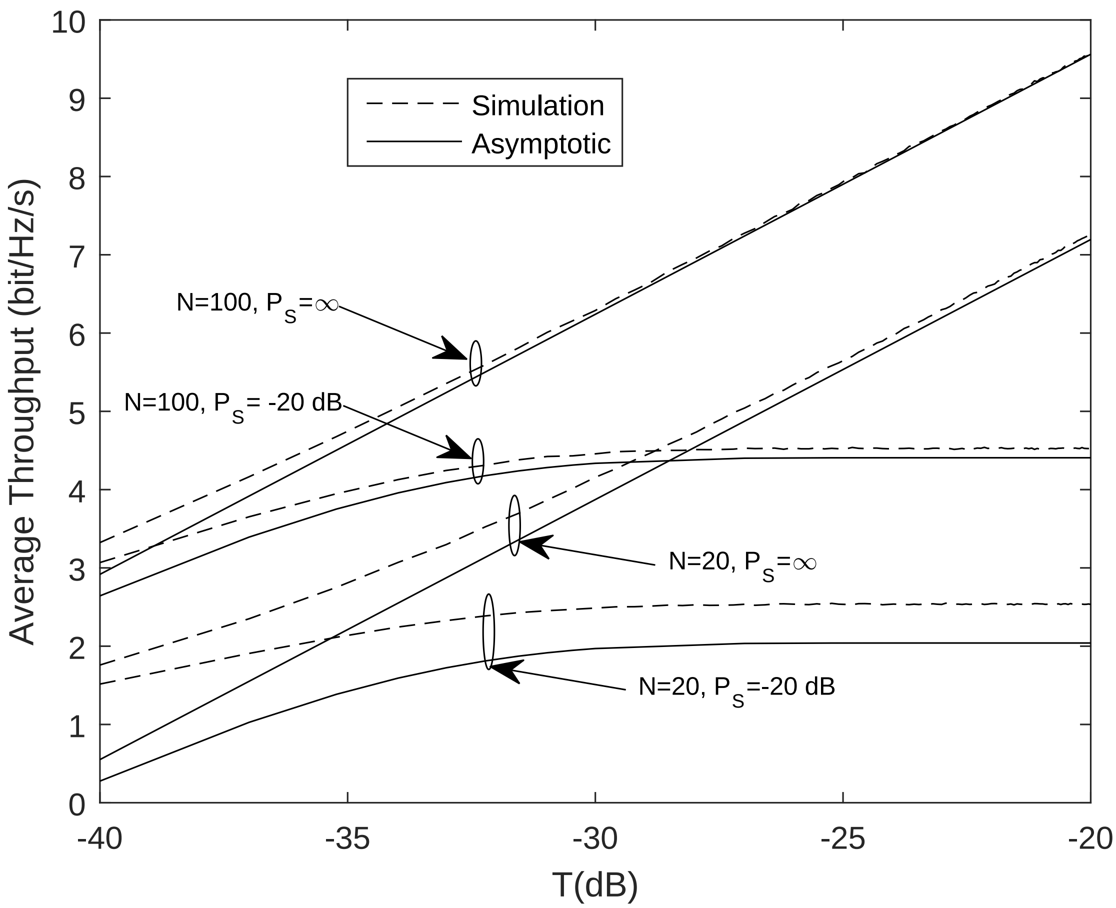

Figure 4.4: Average throughput of the best SU versus interference level, T, in (dB), for $N = 20, 100$, $\lambda = 2$, $\beta = 3$, $\eta = 20$ and $P_M = 0$ dB at $P_S = -20$ dB, $P_S = \infty$,

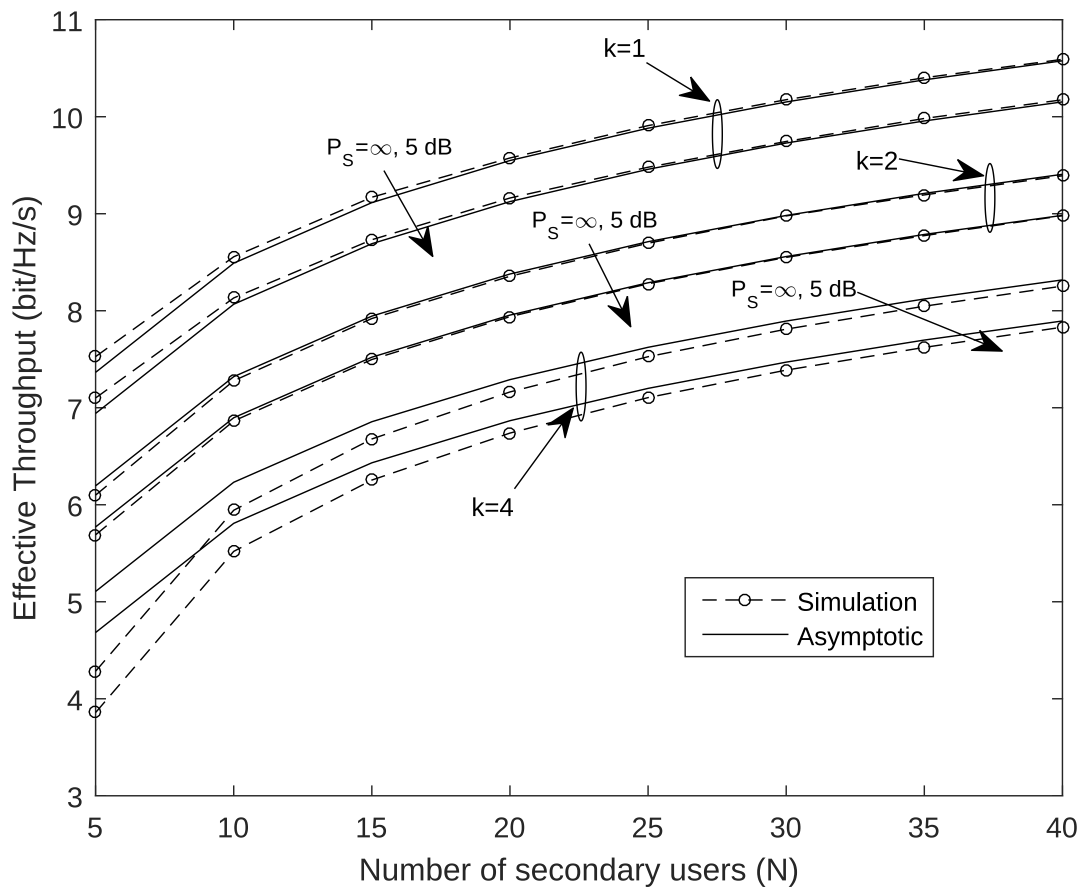

Figure 4.5: Effective throughput of the k-th best SU versus the number of secondary users (N) with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = 5$ dB, for $k = 1, 2, 4$, $A = 1/2$, $\lambda = 2$, $\beta = 3$, $\eta = 20$, $T = -10$ dB and $P_M = 0$ dB.

effective throughput does not significantly improve compared to the unlimited ST power case, $P_S = \infty$. This is due to the fact that for higher values of P_S the effective throughput is dominated by the interference level T.

In Fig. 4.7, we plot the outage probability of the k-th best SU versus the number of secondary users, N, for the unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = -10$ dB, for $k = 1, 2$ and $x_0 = 10$ dB . In Fig. 4.8, the outage probability of the k-th SU is plotted versus interference level, T, in (dB) at $N = 30$ with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = -10$ dB, for $k = 1, 2$ at $x_0 = 13$ dB. The saturation in the outage probability for the

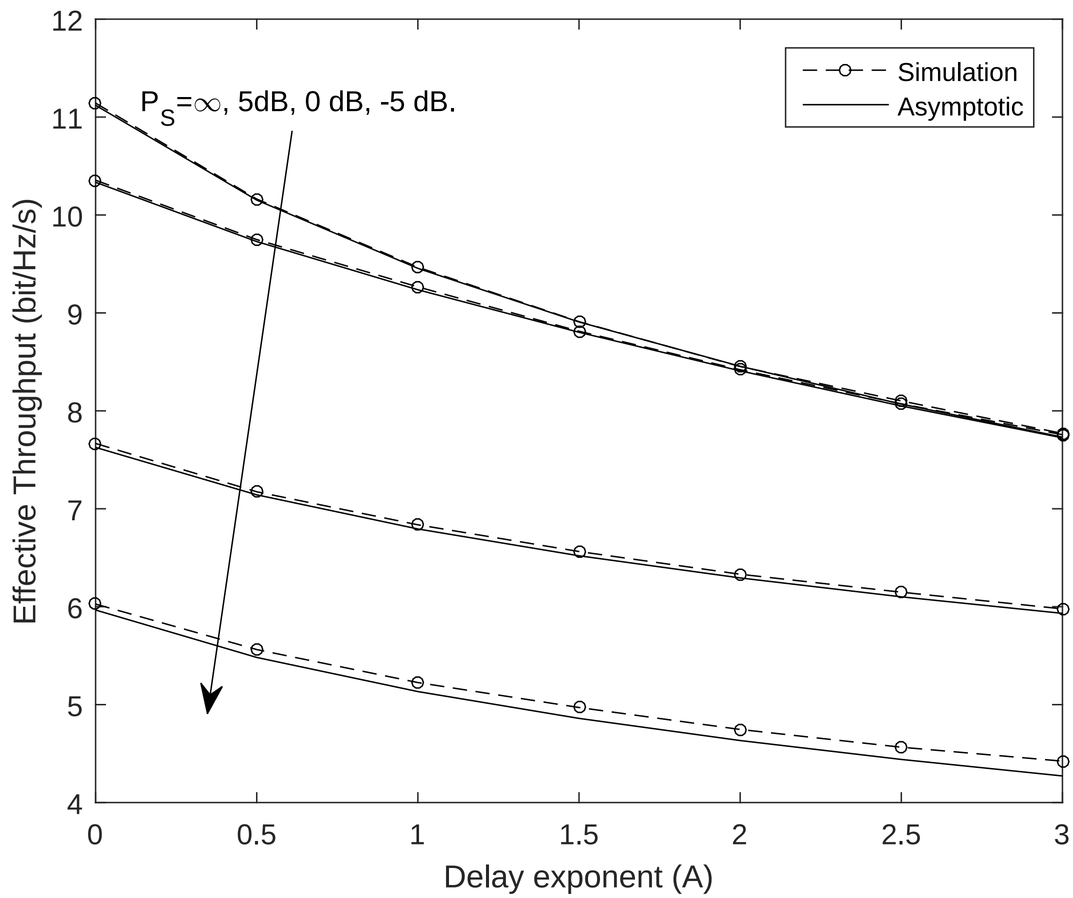

Figure 4.6: Effective throughput of the best SU versus delay exponent at $N = 30$, for $\lambda = 2$, $\beta = 3$, $\eta = 20$, $T = -10$ dB, $P_M = 0$ and different values of P_S.

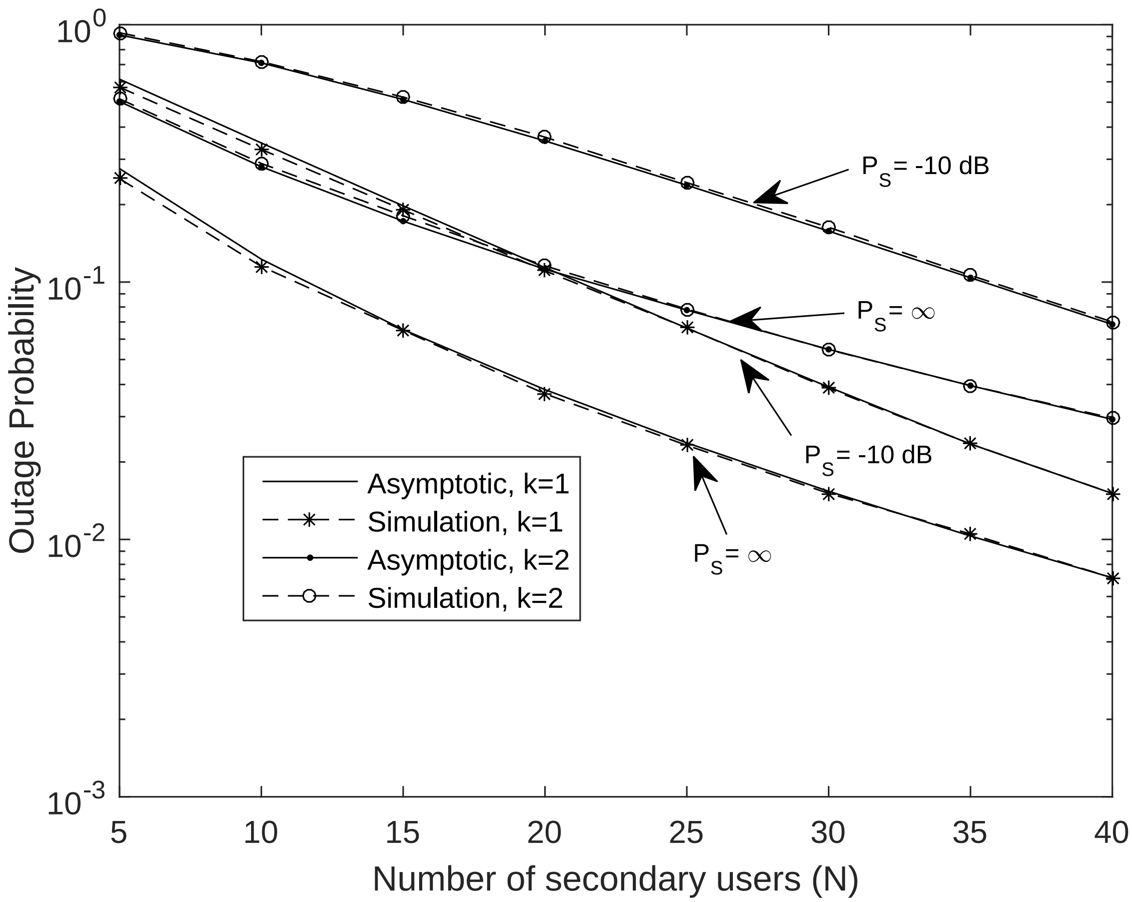

Figure 4.7: Outage probability of the k-th best SU versus the number of secondary users (N) with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = -10$ dB, for $k = 1, 2$, $\lambda = 2$, $\beta = 3$, $\eta = 20$, $T = -20$ dB, $P_M = 0$ dB and $x_0 = 10$ dB.

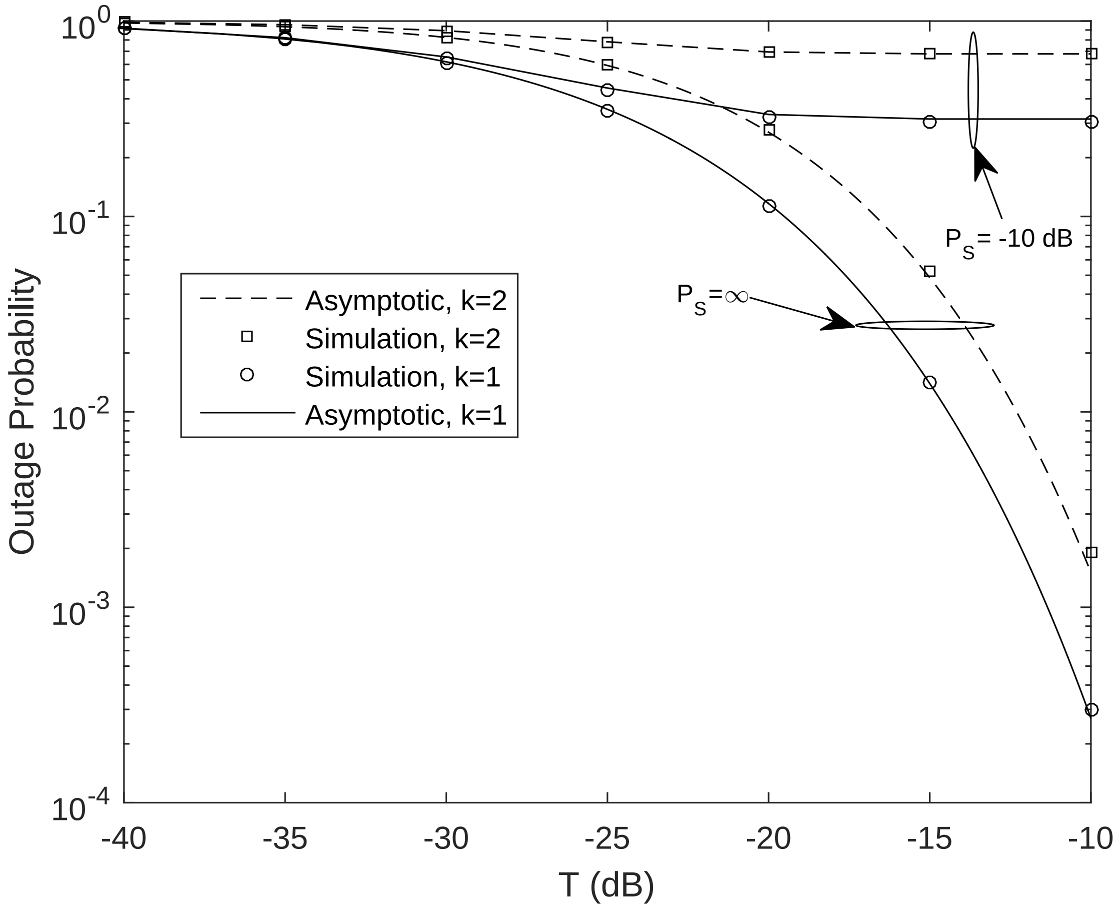

Figure 4.8: Outage probability of the k-th best SU versus interference level, T, in (dB) at $N = 30$ with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = -10$ dB, for $k = 1, 2$, $\lambda = 2$, $\beta = 3$, $\eta = 20$, $P_M = 0$ and $x_0 = 13$ dB.

limited ST power case is due to the fact that for higher values of T, the ST will select P_S for most of the time. In Fig. 4.9, we plot the asymptotic average BEP as a function of the number of secondary users, N, for the unlimited ST power with $P_S = \infty$ and limited ST power with $P_S = -5$ dB, for $k = 1$.

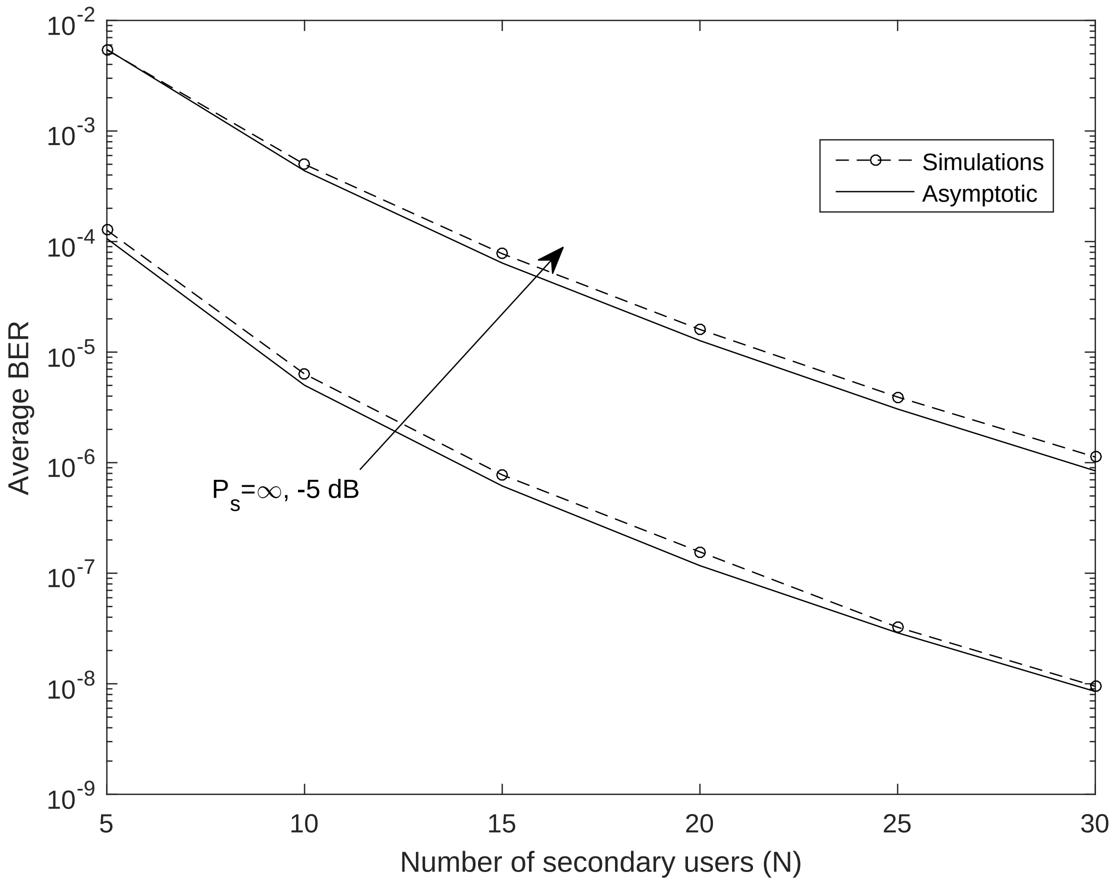

Figure 4.9: Average BER of BFSK ($C = g = 0.5$) versus the number of secondary users N with unlimited ST power, $P_S = \infty$ and limited ST power with $P_S = -5$ dB, for $k = 1$, $\lambda = 2$, $\beta = 3$, $\eta = 20$, $T = -10$ dB and $P_M = 0$ dB.

4.7 Conclusion

We analyzed the asymptotic performance of the k-th best SU for an interference-limited secondary multiuser network of underlay CR systems. We used extreme value theory to show that the k-th highest SIR converges in distribution to an inverse gamma random variable for a fixed k and large number of secondary users. We used this result to analyze the asymptotic average throughput, effective throughput, average BER and outage probability for the k-th best SU under continuous power adaptation at the ST. We verified the accuracy of the derived asymptotic expressions, for different system parameters, through Monte Carlo simulations.

5. ON THE SECRECY PERFORMANCE OF GENERALIZED USER SELECTION FOR INTERFERENCE-LIMITED MULTIUSER WIRELESS NETWORKS

5.1 Introduction

The notion of secure communication was first introduced by Shannon in [58]. Thereafter, Wyner introduced the wiretap channel in which Alice transmits confidential messages to Bob in the presence of an eavesdropper, Eve [59]. Physical layer (PHY) security was first investigated in [60], where the authors analyze the secrecy outage probability (SOP) and the ergodic secrecy capacity (ESC) for single-input single-output (SISO) systems subject to a quasi-static Rayleigh fading.

Multiuser diversity can improve PHY security. The impact of interference on the PHY security for multiuser diversity schemes is investigated in [61] where the authors analyze the SOP and secrecy diversity order of multiuser diversity scheduling in the presence of cochannel interference, but no closed form expression was derived for the SOP. In [62], the SOP of a multiuser diversity scheme of cognitive radio systems is investigated in the presence of interference from the primary transmitter. The fading statistics of the interference were modeled as complex Gaussian, assuming the primary signal is generated by a random Gaussian codebook. Most recently, the effect of fading of multiple interference channels is considered in [63] where the authors analyze the SOP for a single user (no multiuser diversity).

Related previous work focused on the secrecy performance of the conventional multiuser diversity scheme where the user with the best fading (the best user) is selected. However, in a practical wireless network the best user may not be available under given traffic conditions. Accordingly, the main contribution of this Chapter is to study the secrecy performance of a more general multiuser diversity scheme which selects the k-th best user. In particular, we analyze the SOP and ESC of a k-th best user selection scheme of a multiuser wireless network in the presence of a single dominant interferer. Assuming that the noise power at each user's receiver and Eve's receiver are negligible compared to the interference power, and the user with the k-th best signal to interference ratio (SIR) is selected from a total number of users N, we derive a closed-form expression for the secrecy outage probability for an arbitrary N and an asymptotic expression for a fixed k and large N. Furthermore, we derive an asymptotic closed form expression for the ESC of the k-th best user and show that the ESC scales like $O(\log(N))$ for a fixed k and large N.

In Section II we discuss the system model. In Section III we analyze the SOP of the k-th best user and in Section IV the ESC. Sections V and VI present numerical results and the conclusion, respectively.

5.2 System Model

As shown in Fig. 5.1, we consider a wireless network consisting of one BS (Alice), N legitimate users (Bobs) and an eavesdropper (Eve), in the presence of another interfering BS. We assume that Eve is equipped with L receive antennas and all other terminals with one antenna each. Let h_i and t_l denote the channel gain from Alice to the i-th user's receiver and Eve's l-th receive antenna, respectively. Let g_i denote the channel gain from the interfering BS to the i-th user's receiver and e_l the channel gain from the interfering BS to Eve's l-th receive antenna. The channel gains are modeled as independent Rayleigh distributed random variables. In particular, $|t_l|^2$ and $|e_l|^2$, for $l = 1, 2, ..., L$, are independent identically distributed (i.i.d) exponential random variables with parameters λ_E and β_E, respectively. Furthermore, $|h_i|^2$ and $|g_i|^2$, for $i = 1, 2, ..., N$, are i.i.d exponential random variables with parameters λ_M and β_M, respectively. Assuming the interference power from the BS is much larger than the noise power at the i-th user's receiver, the signal-to-interference ratio (SIR) at the i-th user's receiver is given by

$$Z_i = \frac{P|h_i|^2}{P_I|g_i|^2}, \tag{5.1}$$

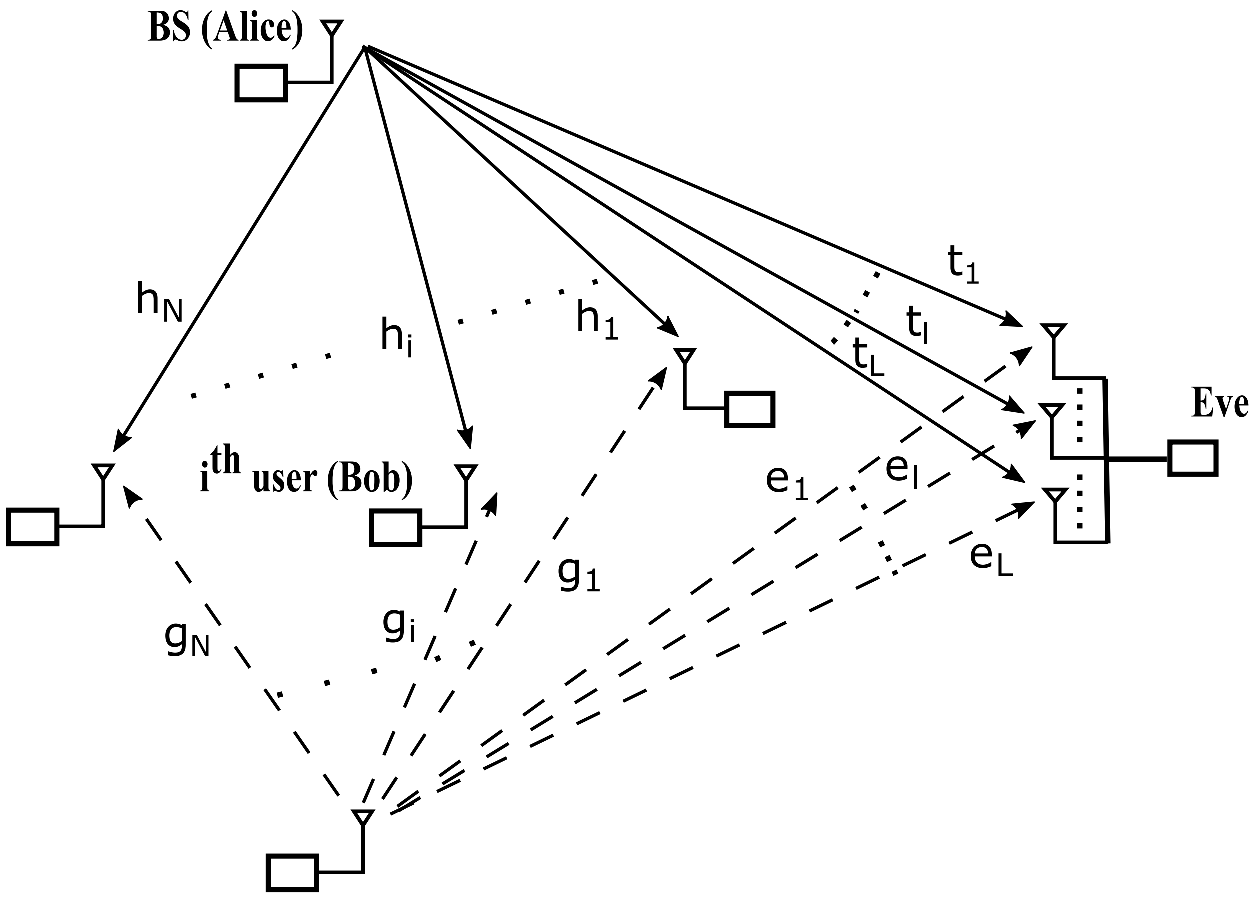

Figure 5.1: Multiuser wireless network with a BS (Alice), N legitimate users (Bobs), an eavesdropper (Eve) equipped with L antennas, in the presence of another interfering BS.

where P and P_I are the transmit power of Alice and the interfering BS, respectively. The cumulative distribution function (CDF) Z_i is given by [6]

$$F(z) = \frac{z}{C_M + z}\, u(z), \tag{5.2}$$

where $C_M = \frac{P\beta_M}{P_I\lambda_M}$ and $u(z)$ is the unit step function. We sort the random variables Z_i in an increasing order denoted as $Z_{(1)} \leq Z_{(2)} .... \leq Z_{(N-k+1)} \leq .... \leq Z_{(N)}$, such that Alice selects the user with the k-th highest SIR, $Z_{(N-k+1)}$. The CDF of $Z_{(N-k+1)}$ then is [2]

$$F_{Z_{(N-k+1)}}(x) = \sum_{v=N-k+1}^{N} \binom{N}{v} (F(z))^{v} (1-F(z))^{N-v}. \tag{5.3}$$

Assuming Eve is equipped with L receive antennas and the noise power at the l-th receive antenna is negligible compared to the interference power from the interfering BS, the instantaneous SIR at the Eve's l-th receive antenna is $X_l = \frac{P|t_l|^2}{P_I|e_l|^2}$. The CDF of X_l is similar to that in (5.2), but with parameter $C_E = \frac{P\beta_E}{P_I\lambda_E}$. Assuming that selection combining (SC) is employed at Eve, such that the best receive antenna is selected, the instantaneous SIR of the SC output is $X_{(L)} = \max\limits_{k=1,\ldots,L} X_l$. Using (5.3), the CDF and the probability density function (PDF) of $X_{(L)}$ are

$$F_{X_{(L)}}(z) = \left(\frac{z}{C_E + z}\right)^{L} u(z), \tag{5.4}$$

$$f_{X_{(L)}}(z) = \frac{d\left(F_{X_{(L)}}(z)\right)}{dz} = \frac{LC_E\, z^{L-1}}{(C_E+z)^{L+1}}\, u(z). \tag{5.5}$$

We focus next on analyzing the secrecy outage probability assuming that Alice has no knowledge about the eavesdropper's channel state information (CSI), similar to Scenario A in [64].

5.3 SOP

For k-th best user selection, the secrecy capacity is given by

$$C_s(k,N,L) = \begin{cases} \log_2\left(\frac{1+Z_{(N-k+1)}}{1+X_{(L)}}\right), & Z_{(N-k+1)} > X_{(L)} \\ 0, & Z_{(N-k+1)} \le X_{(L)} \end{cases}. \tag{5.6}$$

The SOP for a target secrecy rate R_s is given by [60]

$$\begin{aligned} P_{out}(R_s) &= \Pr\{C_s(k,N,L) \le R_s\} \\ &= \int_0^{\infty} f_{X_{(L)}}(z) F_{Z_{(N-k+1)}}(\tau - 1 + \tau z)\, dz, \end{aligned} \tag{5.7}$$

where $\tau = 2^{R_s}$. Using (5.7), the probability of strictly positive secrecy capacity (SPSC) can be evaluated as $\Pr\{C_s(k,N,L) > 0\} = 1 - P_{out}(0)$.

We derive next an exact expression for the SOP of k-th best user selection for arbitrary values of N and L, an asymptotic expression for the SOP of the k-th best user for arbitrary L and large N compared to fixed k and τ, and obtain a simple asymptotic expression for the SOP for large N compared to fixed k and τ and for large L.

5.3.1 Exact SOP Analysis

In Proposition 5.1, we provide an exact analysis for the SOP of the k-th best user in (5.7). Then, we derive exact expressions for probability of SPSC of the k-th best user and the SOP when Alice and Eve are equipped with one antenna. in Corollary 5.1 and Corollary 5.2, respectively.

Proposition 5.1. For arbitrary N and L, the exact SOP of the k-th best user is

$$\begin{aligned} P_{out}(R_s) =& \frac{L}{\tau^L \left(C_E\right)^L} \sum_{v=N-k+1}^{N} \binom{N}{v} \left(C_M\right)^{N-v} \sum_{j=0}^{v} \binom{v}{j} \\ & \times \left(\tau - 1\right)^{v-j} B\left(L+j, N-j+1\right) \\ & \times \left(\tau - 1 + C_M\right)^{L+j-N} {}_2F_1\left(L+1, L+j;\right. \\ & \left. N+L+1; 1 - \frac{\tau - 1 + C_M}{\tau\, C_E}\right), \end{aligned} \tag{5.8}$$

where $\tau = 2^{R_s}$, ${}_2F_1\left(x, y; z; w\right)$ is the Gauss hypergeometric function and $B\left(x, y\right)$ is the Beta function.

Proof. Using (5.2), (5.3) and (5.5), after some basic algebraic manipulations, $P_{out}(R_s)$ in (5.7) can be expressed as

$$P_{out}(R_s) = LC_E \sum_{v=N-k+1}^{N} \binom{N}{v} \tau^{v-N} \left(C_M\right)^{N-v} \times \underbrace{\int_0^\infty \frac{z^{L-1}\left(z+\frac{\tau-1}{\tau}\right)^v}{\left(z+\frac{\tau-1+C_M}{\tau}\right)^N \left(z+C_E\right)^{L+1}} dz}_{I}. \tag{5.9}$$

Applying binomial expansion for the term $\left(z+\frac{\tau-1}{\tau}\right)^v$ and making use of Eq. (3.197.1) of [25], I can be expressed as

$$I = \sum_{j=0}^{v} \binom{v}{j} \left(\frac{\tau-1}{\tau}\right)^{v-j} \frac{B\left(L+j, N-j+1\right)}{{C_E}^{L+1}} \times \left(\frac{\tau-1+C_M}{\tau}\right)^{L+j-N} {}_2F_1\left(L+1, L+j; N+L+1; 1-\frac{\tau-1+C_M}{\tau\, C_E}\right). \tag{5.10}$$

Combining (5.9) and (5.10), $P_{out}(R_s)$ can be finally expressed as in (5.8). □

Corollary 5.1. Setting $R_s = 0$ (i.e., $\tau = 1$) in (5.8), the probability of SPSC is

$$\Pr\left\{C_s(k,N,L) > 0\right\} = 1 - L\left(\frac{C_M}{C_E}\right)^L \sum_{v=N-k+1}^{N} \binom{N}{v} \times B\left(L+v, N-v+1\right) {}_2F_1\left(L+1, L+v; N+L+1; 1-\frac{C_M}{C_E}\right). \tag{5.11}$$

Corollary 5.2. As a special case if Alice serves only one user (i.e., $N = 1$) and Eve is equipped with a single antenna (i.e., $L = 1$), $P_{out}(R_s)$ in (5.8) reduces to

$$P_{out}(R_s) = \frac{1}{\tau C_E} \sum_{j=0}^{1} \left(\tau-1\right)^{1-j} B\left(1+j, 2-j\right) \left(\tau-1+C_M\right)^j \times {}_2F_1\left(2, 1+j; 3; 1-\frac{\tau-1+C_M}{\tau\, C_E}\right). \tag{5.12}$$

5.3.2 Asymptotic SOP Analysis

In Proposition 5.2 below, we derive an asymptotic expression for the SOP of the k-th best user for an arbitrary L and large N relative to fixed k and τ. Then, we derive an asymptotic expression for the probability of SPSC in Corollary 5.3.

Proposition 5.2. For arbitrary L and large N, with respect to fixed k and τ, the SOP of the k-th best user can be approximated as

$$P_{out}(R_s) \approx 1 - \left(\frac{b_N}{\tau\, C_E}\right)^k U\left(k; k+1-L; \frac{b_N}{\tau\, C_E}\right), \tag{5.13}$$

where $b_N = C_M(N-1)$ and $U(a; d; z)$ is the Tricomi hypergeometric function [65].

Proof. As shown in Proposition 4.2, if the random variable Z_i has a CDF $F(z)$ as in (5.2), then for a fixed k and $N \to \infty$, $\frac{Z_{(N-k+1)}}{b_N}$ converges in distribution to a random variable Z whose CDF, $G^k(z)$, has an inverse gamma distribution.

$$G^k(z) = \frac{\Gamma\left(k, \frac{1}{z}\right)}{(k-1)!} u(z). \tag{5.14}$$

$\Gamma(s,x) = \int_x^\infty t^{s-1} e^{-t} dt$ is the upper incomplete gamma function and $b_N = C_M(N-1)$. Equivalently, for fixed k and $N \to \infty$, $F_{Z_{(N-k+1)}}(z)$ can be approximated as

$$F_{Z_{(N-k+1)}}(z) \approx \frac{\Gamma\left(k, \frac{b_N}{z}\right)}{(k-1)!} u(z). \tag{5.15}$$

Based on the asymptotic distribution of $\frac{Z_{(N-k+1)}}{b_N}$ above and noting that b_N is an increasing function of N, we derive next an asymptotic expression for $P_{out}(R_s)$. Invoking (5.7), we have

$$\begin{aligned} P_{out}(R_s) &= \int_0^{\infty} f_{X_{(L)}}(z) F_{Z_{(N-k+1)}}\left(\tau - 1 + \tau z\right) dz \\ &= \Pr\left\{Z_{(N-k+1)} \leq \tau - 1 + \tau X_{(L)}\right\} \\ &= \Pr\left\{\frac{Z_{(N-k+1)}}{b_N} \leq \frac{\tau - 1 + \tau X_{(L)}}{b_N}\right\} \\ &\approx \Pr\left\{Z \leq \frac{\tau X_{(L)}}{b_N}\right\}, \end{aligned} \tag{5.16}$$

for fixed k and τ and $N \to \infty$, where the CDF of Z is as in (5.14). Using (5.16), $P_{out}(R_s)$ can be expressed as

$$P_{out}(R_s) \approx \underbrace{\int_0^{\infty} \frac{L C_E z^{L-1}}{(C_E + z)^{L+1}} \frac{\Gamma\left(k, \frac{b_N}{\tau z}\right)}{(k-1)!} dz}_{I_1}, \tag{5.17}$$

as $N \to \infty$ and for fixed k and τ. Using integration by parts:

$$I_1 = 1 - \int_0^{\infty} \left(\frac{z}{z + C_E}\right)^L \frac{(b_N)^k \tau^{-k} z^{-1-k}}{(k-1)!} e^{-\frac{b_N}{\tau z}} dz. \tag{5.18}$$

Using $u = \frac{C_E}{z}$ and Eq. (39) of [65], I_1 can be finally expressed as in (5.13). □

Corollary 5.3. For arbitrary L and large N, with respect to fixed k and τ, the probability of SPSC is

$$\Pr\left\{C_s(k, N, L) > 0\right\} \approx \left(\frac{b_N}{C_E}\right)^k U\left(k; k+1-L; \frac{b_N}{C_E}\right). \tag{5.19}$$

In Proposition 5.2 above, we derive the SOP of the k-th best user for an arbitrary L and large N relative to fixed k and τ. If we further assume that L is large we derive a simpler and an accurate expression for the SOP as in Proposition 5.3 below.

Proposition 5.3. For large N compared to fixed k and τ and for large L, the SOP of the k-th best user can be approximated as

$$P_{out}(R_s) \approx 1 - \left(1 + \frac{\tau b_L}{b_N}\right)^{-k}, \tag{5.20}$$

where $b_N = C_M(N-1)$ and $b_L = C_E(L-1)$.

Proof. As discussed earlier, Z_i and Z_l have the same CDF with parameters C_M and C_E, respectively. Then the asymptotic distribution of $X_{(L)}$ for large L can be obtained by replacing N with L and setting $k = 1$ in (5.15). Hence, as $L \to \infty$, with $b_L = C_E(L-1)$, $F_{X_{(L)}}(z)$ can be expressed as

$$F_{X_{(L)}}(z) \approx e^{-\frac{b_L}{z}} u(z). \tag{5.21}$$

Making use of (5.21) in (5.16) we have

$$\begin{aligned} P_{out}(R_s) &\approx \Pr\left\{Z \leq \frac{\tau X_{(L)}}{b_N}\right\} \\ &= \int_0^\infty \frac{d\left(e^{-\frac{b_L}{z}}\right)}{dz} \frac{\Gamma\left(k, \frac{b_N}{\tau z}\right)}{(k-1)!} dz \\ &= \int_0^\infty \frac{b_L\, e^{-\frac{b_L}{z}}}{z^2} \frac{\Gamma\left(k, \frac{b_N}{\tau z}\right)}{(k-1)!} dz \\ &= 1 - \left(1 + \frac{\tau b_L}{b_N}\right)^{-k}, \end{aligned} \tag{5.22}$$

for large N relative to fixed k and τ and for large L. The above integral is evaluated using Eq. (6.451, 2) of [25]. □

Note that $P_{out}(R_s)$ in (5.22) is an increasing function of L and k, and a decreasing function of N. As a special case, if $N = L$ and L is large, $P_{out}(R_s)$ converges to a constant value, i.e,. $P_{out}(R_s) \approx 1 - \left(1 + \frac{\tau \beta_E \lambda_M}{\lambda_E \beta_M}\right)^{-k}$. This shows that if L is large and scales linearly with N, the SOP converges to a constant that only depends on the fading parameters and R_s. This can be intuitively explained by the fact that for large L and scaling linearly with N, the multiuser diversity effect on the SOP is eliminated due to the employment of the selection combining scheme at Eve.

5.4 ESC

For k-th best user selection, the ESC is given by [66]

$$\overline{C_s}(k,N,L) = \frac{1}{\ln(2)} \int_0^\infty \frac{F_{X_{(L)}}(z)}{1+z} \left(1 - F_{Z_{(N-k+1)}}(z)\right) dz, \tag{5.23}$$

which in general is intractable to express in closed form for arbitrary values of N and L. However, using the asymptotic approximation of $F_{Z_{(N-k+1)}}(z)$ in (5.15) yields a closed form asymptotic expression for the ESC when Eve has a single antenna (i.e., $L = 1$). In Proposition 5.4 below, we derive the ESC of the k-th best user for large N relative to a fixed k and $L = 1$. We use $\overline{C_s}(k,N)$ to denote $\overline{C_s}(k,N,L)$ at $L = 1$.

Proposition 5.4. For large N relative to fixed k, the ESC of the k-th best user can be approximated as

$$\overline{C_s}(k,N) \approx \begin{cases} \frac{-\psi(k) + \frac{C_E V\left(k;\frac{b_N}{C_E}\right) - V(k;b_N)}{C_E - 1}}{\ln(2)}, & C_E \neq 1 \\ \frac{-\psi(k) + V(k;b_N) - (b_N)^k e^{b_N} \Gamma(-k+1, b_N)}{\ln(2)}, & C_E = 1 \end{cases}, \tag{5.24}$$

where $\psi(k)$ is the digamma function, $b_N = C_M(N-1)$ and $V(k;a)$ is as expressed in (5.31).

Proof. Using (5.15) and the fact that $\frac{\Gamma(k,x)}{(k-1)!} = 1 - \frac{\gamma(k,x)}{(k-1)!}$, then

$$1 - F_{Z_{(N-k+1)}}(z) \approx \frac{\gamma\left(k, \frac{b_N}{z}\right)}{(k-1)!}, \tag{5.25}$$

where $\gamma(s,x) = \int_0^x t^{s-1} e^{-t} dt$ is the lower incomplete gamma function. Substituting (5.4) and

(5.25) in (5.23), yields

$$\overline{C_s}(k,N) \approx \frac{1}{\ln(2)} \underbrace{\int_0^\infty \frac{z}{(1+z)(C_E+z)} \frac{\gamma\left(k, \frac{b_N}{z}\right)}{(k-1)!} dz}_{I_2} . \tag{5.26}$$

Using $x = \frac{b_N}{z}$ and the integral representation of the lower incomplete gamma function, I_2 can be expressed as

$$I_2 = \int_0^\infty \frac{(b_N)^2}{x\,(x+b_N)\,(C_E x + b_N)} \int_0^x \frac{t^{k-1}e^{-t}}{(k-1)!} dt\, dx. \tag{5.27}$$

Changing the order of integration, I_2 can be rewritten as

$$I_2 = \int_0^\infty \frac{t^{k-1}e^{-t}}{(k-1)!} \underbrace{\left(\int_t^\infty \frac{(b_N)^2}{x\,(x+b_N)\,(C_E x + b_N)} dx \right)}_{I_3(t)} dt, \tag{5.28}$$

where $I_3(t)$ can be easily evaluated as

$$I_3(t) = \begin{cases} -\ln(t) + \frac{C_E \ln\left(t + \frac{b_N}{C_E}\right) - \ln(t+b_N)}{C_E - 1}, & C_E \neq 1 \\ -\ln(t) + \ln(t+b_N) - \frac{b_N}{t+b_N}, & C_E = 1 \end{cases} . \tag{5.29}$$

Combining (5.26) and (5.28), we have

$$\overline{C_s}(k,N) \approx \frac{1}{\ln(2)} \int_0^\infty \frac{t^{k-1}e^{-t}}{(k-1)!} I_3(t) dt. \tag{5.30}$$

To evaluate (5.30), let $V(k;a) = \int_0^\infty \frac{t^{k-1}e^{-t}}{(k-1)!} \ln(t+a) dt$. Using Eq. (4.337, 5) of [25], then we have

$$V(k;a) = \sum_{\mu=0}^{k-1} \frac{1}{(k-\mu-1)!} \Bigg((-1)^{k-\mu} a^{k-\mu-1} e^{a} E_i(-a) + \sum_{v=1}^{k-\mu-1} (v-1)!(-a)^{k-\mu-1-v} \Bigg) + \ln(a), \tag{5.31}$$

where $E_i(x) = -\int_{-x}^{\infty} \frac{e^{-y}}{y} dy$ is the exponential integral function. From Eq. (4.352, 1) and Eq. (3.383, 10) of [25], we have

$$\int_0^{\infty} \frac{t^{k-1} e^{-t}}{(k-1)!} \ln(t)\, dt = \psi(k) \tag{5.32}$$

$$\int_0^{\infty} \frac{b_N}{t+b_N} \frac{t^{k-1} e^{-t}}{(k-1)!} dt = (b_N)^k e^{b_N} \Gamma(-k+1, b_N), \tag{5.33}$$

respectively. Making use of (5.31)-(5.33) in (5.30), we finally obtain $\overline{C_s}(k,N)$ in (5.24). $\square$

In what follows, we use Proposition 5.4 to obtain a scaling law for the ESC in Corollary 5.4 below.

Corollary 5.4.: For large N relative to a fixed k, $\overline{C_s}(k,N)$ scales as

$$\overline{C_s}(k,N) \sim O(\log(N)). \tag{5.34}$$

Furthermore,

$$\overline{C_s}(1,N) - \overline{C_s}(k,N) \to \frac{H_{(k-1)}}{\ln(2)} \text{ bits/s/Hz}, \tag{5.35}$$

where $H_{(k-1)} = E_0 + \psi(k)$ is the harmonic number and $E_0 = -\psi(1) = 0.5772156649$ is the Euler constant.

Proof. Using the asymptotic behavior of $\Gamma(s,x) \approx x^{s-1}e^{-x}$ for large x, one can show that

$$(b_N)^k e^{b_N} \Gamma(-k+1, b_N) \approx 1, \tag{5.36}$$

for large N relative to a fixed k. Applying Jensen's inequality:

$$\begin{aligned} V(k; b_N) &= \int_0^\infty \frac{t^{k-1}e^{-t}}{(k-1)!} \ln(t+b_N)dt \\ &\leq \ln\left(\int_0^\infty \frac{t^k e^{-t}}{(k-1)!}dt + b_N\right) \\ &= \ln(k+b_N) \approx \ln(b_N) \end{aligned} \tag{5.37}$$

for large N relative to a fixed k. From (5.31) and (5.37), we have $V(k;b_N) \approx \ln(b_N)$ as $N \to \infty$. Using $V(k;b_N) \approx \ln(b_N)$ and (5.36) with $b_N = C_M(N-1)$, (5.24) can be rewritten as

$$\overline{C_s}(k,N) \approx \begin{cases} \frac{-\psi(k)+\ln[C_M(N-1)]-\frac{C_E \ln(C_E)}{C_E-1}}{\ln(2)}, & C_E \neq 1 \\ \frac{-\psi(k)+\ln[C_M(N-1)]-1}{\ln(2)}, & C_E = 1 \end{cases}. \tag{5.38}$$

From (5.38), we see that $\overline{C_s}(k,N) \sim O(\log(N))$. Furthermore, $\overline{C_s}(1,N) - \overline{C_s}(k,N) \to \frac{\psi(k)-\psi(1)}{\ln(2)} = \frac{H_{(k-1)}}{\ln(2)}$. □

5.5 Numerical Results

Fig. 5.2, plots the SOP of the k-th best user versus the number of users, N, for $k = 1, 2$, $R_s = 1, 4$ bit/s/Hz and $L = 2$. Some interesting observations can be made: First, we observe that the exact SOP is in good agreement with the simulation results, and the accuracy of the asymptotic SOP in (4.10) increases as N increases. Second, we see that the asymptotic SOP expression is less accurate for small to moderate values of N, as k or R_s increase. This is due to the fact that the asymptotic analysis holds with a high accuracy for large N compared to fixed k and R_s. Therefore, if the value of k or R_s is close enough to N then the accuracy of the asymptotic analysis decreases.

In Fig. 5.3, we plot the SOP of the k-th user as a function of the number of receive antennas,

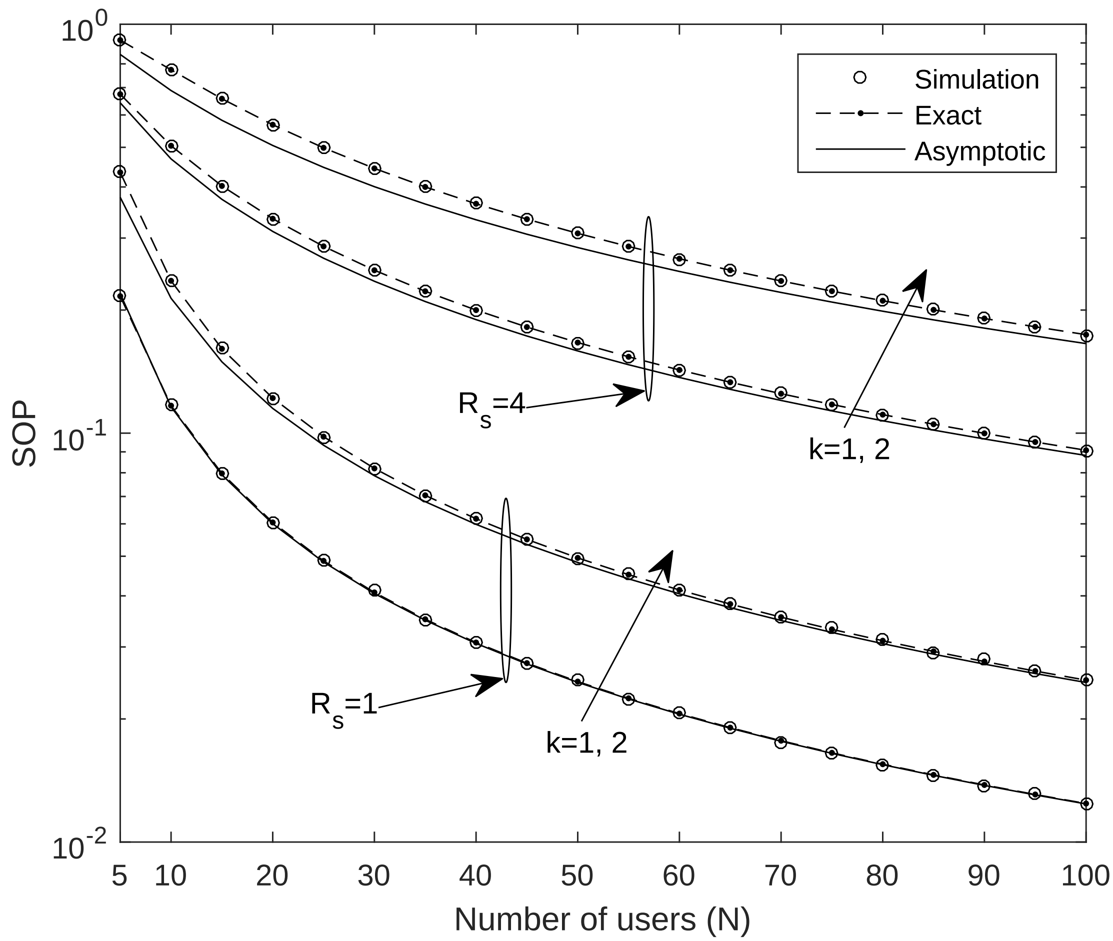

Figure 5.2: SOP of the k-th best user vs number of users for $k = 1, 2$, $R_s = 1, 4$ bit/s/Hz, $P/P_I = 2$, $\beta_M = 2$, $\lambda_M = 1/2$, $\beta_E = 5$, $\lambda_E = 4$ and $L = 2$.

L, for $k = 1, 2$, $R_s = 1/2$ bit/s/Hz and for different values of N. We verify the accuracy of the asymptotic SOP expressions derived in (5.13) and (5.20) by comparing them with the exact SOP result. As expected, we observe that for $N = 20$, the SOP increases as L increases. However, for $N = L$, the SOP remains constant as L grows large as we discussed earlier at the end of Section III.

In Fig. 5.4, we plot the ESC of the k-th best user versus the number of users, N, for $k = 1, 2, 3$. We validate the accuracy of the asymptotic ESC using Monte Carlo simulations. We observe that the asymptotic ESC is accurate for small to moderate values of N. We also observe that asymptotic

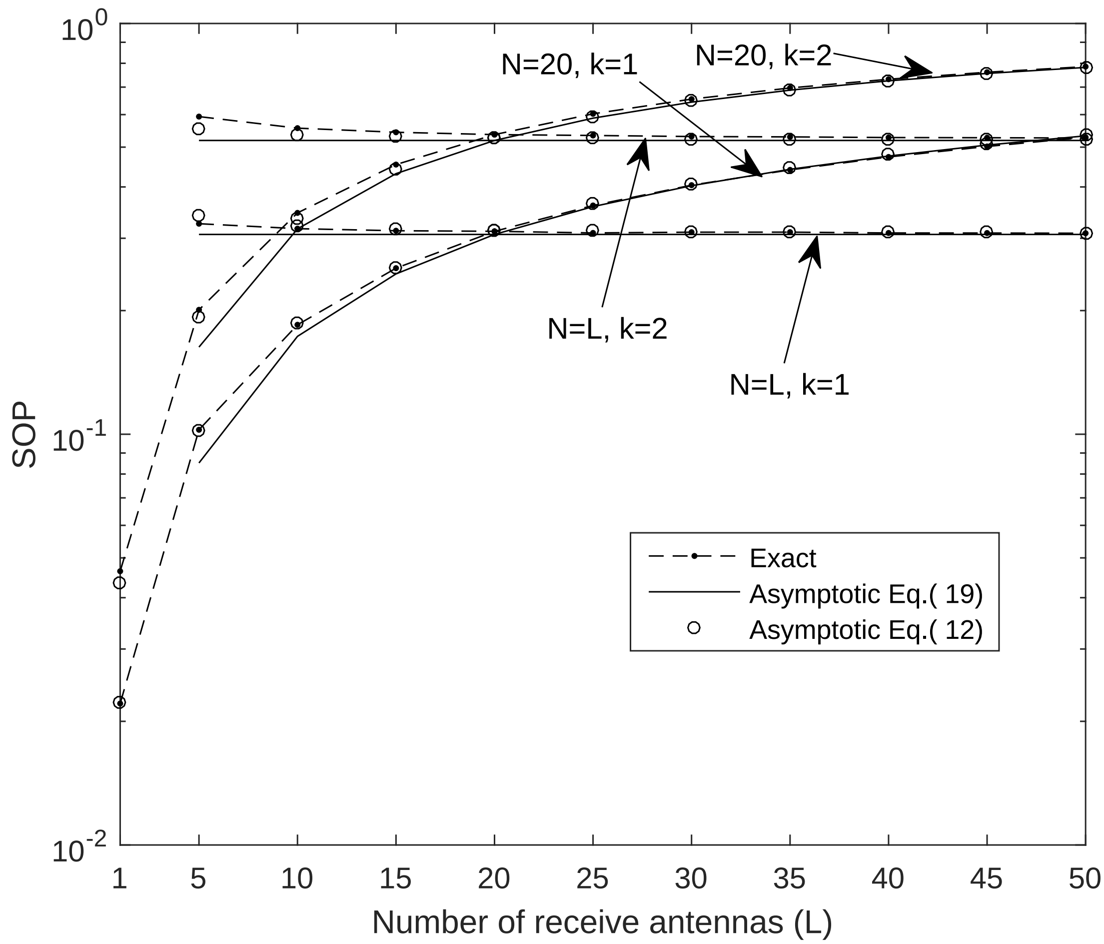

Figure 5.3: SOP of k-th best user vs number of receive antennas for $N = 20$ and $N = L$, $k = 1, 2$, $R_s = 1/2$ bit/s/Hz, $P/P_I = 2$, $\beta_M = 2$, $\lambda_M = 1/2$, $\beta_E = 5$ and $\lambda_E = 4$.

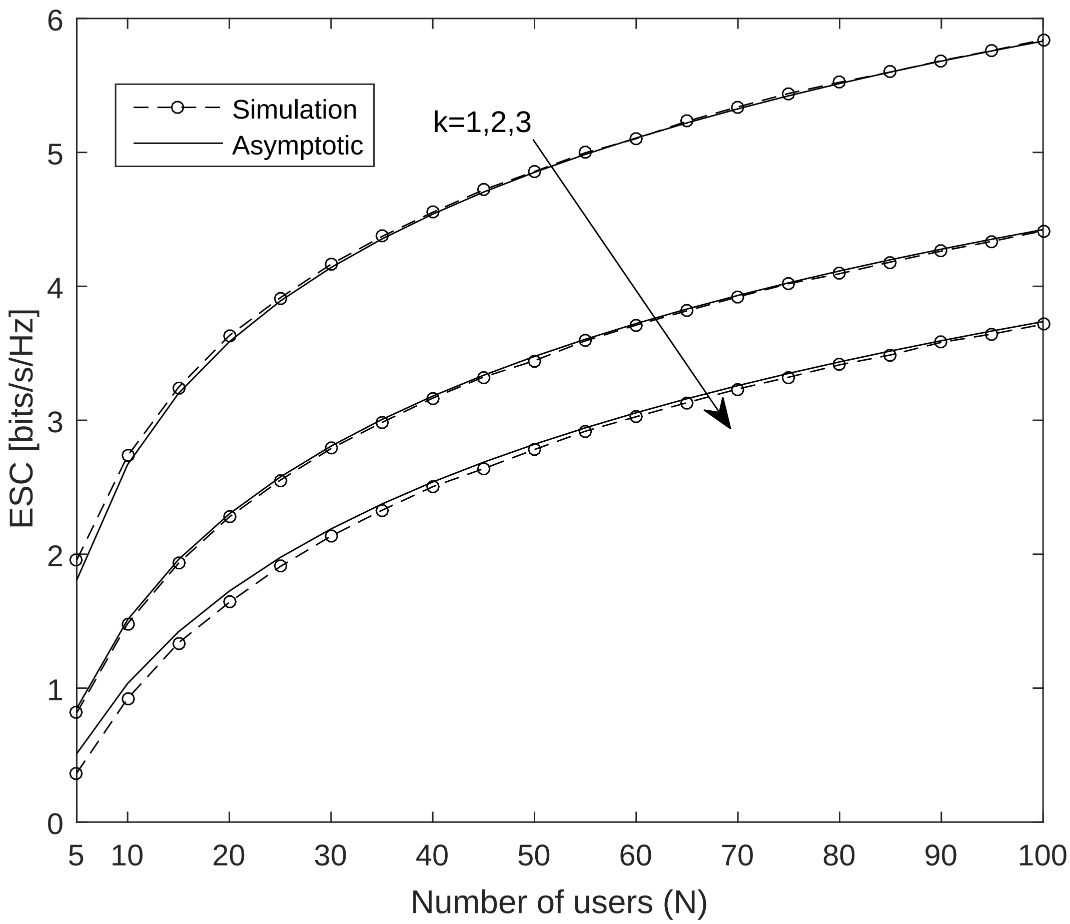

Figure 5.4: ESC of the k-th best user vs number of users N for $k = 1, 2, 3$, $P/P_I = 4$, $\beta_M = 2$, $\lambda_M = 4$, $\beta_E = 3$ and $\lambda_E = 3$ and $L = 1$.

ESC is less accurate as k approaches N.

6. SUMMARY AND CONCLUSIONS

In this dissertation we analyzed the performance of many practical wireless communication systems under a general SD scheme in which the k-th best link. We base our performance analysis on the extreme value theory (EVT). The derived results cover many practical systems of interest in RF, FSO and CR systems. We also analyzed the secrecy performance of the k-th best link in the context of multiuser wireless networks.

In Chapter 2, a detailed effective rate analysis of MISO/TAS systems was considered. An analytical expression for the effective rate of the considered system in Rayleigh fading is obtained. Moreover, we analyzed the effective rate in the asymptotically low and high-SNR regimes and closed form expressions were derived. At the low-SNR regime, we showed that the minimum bit energy depends on the number of transmit antennas but not on the delay constraint. However, we observed that the wideband slope is a decreasing function of the delay constraint and monotonically increasing in the number of transmit antennas. Finally, we analyzed the effective rate of MISO/TAS systems with large number of transmit antennas and generalized antenna selection and derived an asymptotic analytical expression for it.

In Chapter 3, we used extreme value theory to derive the asymptotic distribution of the throughput of the k-th best link over Weibull, Gamma, $\alpha - \mu$ and Gamma-Gamma fading channels. Using this result, we derived simple closed-form asymptotic expressions for the average throughput and effective throughput. Furthermore, we analyzed the average BEP and derived closed-form asymptotic expression for it. As a special case, we considered the Weibull fading channel model and used Monte Carlo simulations to confirm the accuracy of the derived asymptotic expressions.

In Chapter 4, we analyzed the asymptotic performance of the k-th best SU for an interference-limited secondary multiuser network of underlay CR systems. We used extreme value theory to show that the k-th highest SIR converges in distribution to an inverse gamma random variable for a fixed k and large number of secondary users. We used this result to analyze the asymptotic average throughput, effective throughput, average BER and outage probability for the k-th best SU under continuous power adaptation at the ST. We verified the accuracy of the derived asymptotic expressions, for different system parameters, through Monte Carlo simulations.

Finally, in Chapter 5 we analyzed the secrecy performance of the k-th best user for an interference-limited multiuser network consisting of N legitimate users. We derived closed form exact and asymptotic expressions for the SOP of the k-th best user assuming an arbitrary N and large N relative to a fixed k, respectively. Furthermore, we derived an asymptotic closed form expression for the ESC of k-th best user and showed that the ESC scales like $O(\log(N))$ when N grows large relative to a fixed k. We also showed that the loss in the ESC between the best user and the k-th best user selection converges to a fixed value and it can be quantified by the harmonic number $H_{(k-1)}$. The accuracy of the derived exact and asymptotic expressions were verified, for different system parameters, through Monte Carlo simulations.

6.1 Challenges and Further Work

In this section we address some challenges in our research and some the future extensions that account for more practical considerations. First, in our investigation of the k-th best selection scheme we assumed that the CSI is perfectly known at the transmitter. However, perfect CSI is an idealistic assumption that simplifies the analysis and yields tractable closed- form expressions in most scenarios. As a future endeavor, imperfect or outdated CSI can be considered to investigate their impact on the asymptotic analysis. In particular, considering the imperfect CSI in CR systems is of significant practical interest due to the difficulty of acquiring perfect CSI of the ST-PR cross link.

Second, in chapters 4 and 5 we considered a multiuser network with a determinist number of users. However, in practical multiuser network the number of active users competing to access the channel is a random variable. For example, one might consider a Poisson distributed number of users and analyze the impact of the number of users randomization on the performance. Furthermore, we only considered multiuser network under small-scale fading throughout this dissertation. As a future direction, one can consider the path loss effect which accounts for users locations.

Finally, we focused on analyzing the secrecy outage probability and the average secrecy capacity assuming that the transmitter has no knowledge about the eavesdropper's CSI. As part of future research, the analysis can be extended to consider the case when eavesdropper's CSI is accounted for at the transmitter

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APPENDIX A

THE FIRST AND SECOND MOMENT OF $X_{(N)}$

The n-th moment of the random variable $X_{(N)}$ can be expressed as

$$E\left[\left(X_{(N)}\right)^n\right] = N\int_0^\infty x^n\left(1-e^{-x}\right)^{N-1}e^{-x}dx. \tag{A.1}$$

Using change of variables $u = e^{-x}$, (A.1) can be expressed as

$$E\left[\left(X_{(N)}\right)^n\right] = N\int_0^1 \left[-\ln(u)\right]^n(1-u)^{N-1}du. \tag{A.2}$$

Invoking Proposition 9.1 of [67], we have

$$\int_0^1 u^{a-1}(1-u)^{b-1}\ln(u)du = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\left(\psi(a)-\psi(a+b)\right), \tag{A.3}$$

where a and b are positive real numbers. Making use of (A.2) with $n = 1$ and set $a = 1, b = N$ in (A.3), then the first moment can be expressed as

$$E\left[X_{(N)}\right] = \psi(N+1) + E_0. \tag{A.4}$$

In order to obtain the second moment, we differentiate (A.3) with respect to a, then we have

$$\int_0^1 u^{a-1}(1-u)^{b-1}\left(\ln(u)\right)^2du = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\Big(\left(\psi(a)-\psi(a+b)\right)^2+\psi(1,a)-\psi(1,a+b)\Big), \tag{A.5}$$

where a and b are positive real numbers. Making use of (A.2) with $n = 2$ and set $a = 1, b = N$ in (A.5), then the second moment can be expressed as

$$E\left[\left(X_{(N)}\right)^2\right] = \left(\psi(N+1)+E_0\right)^2+\frac{\pi^2}{6}-\psi(1,N+1). \tag{A.6}$$

APPENDIX B

PROOF OF LEMMA 3.1

To prove Lemma 3.1 in Chapter 3, we first need the following Lemma from [3].

Lemma: For any sequence Z_N converging in distribution to a random variable X that has a nondegenerate distribution function, if $E\left[[(Z_N)^-]^p\right] < \infty$ for any positive real number p, where $(x)^- = \max(-x, 0)$, then

$$\lim_{N\to\infty} E\left[Z_N^p\right] = E\left[X^p\right], \tag{B.1}$$

provided $E\left[|X|^p\right] < \infty$.

Since $R_{(N-k+1)}$ represents the k-th maximum of non-negative random variables, then it follows that $E\{[(R_{(N-k+1)})^-]^p\} < \infty$ is satisfied. However, for $E\left[\left(\frac{R_{(N-k+1)}-a_N}{b_N}\right)^p\right] \to E\left[X^p\right]$ to hold, $E\left[|X|^p\right] < \infty$ should be also satisfied. Let $Y = |X|$, where the CDF of X is as expressed in (3.12). Then the CDF of Y, $G(y)$, can be expressed as

$$G(y) = \sum_{j=0}^{k-1} \frac{e^{-e^{-y}}e^{-jy} - e^{-e^{y}}e^{jy}}{j!}, \quad y \geq 0, \tag{B.2}$$

and $E\left[Y^p\right]$ can be expressed as

$$\begin{aligned} E\left[Y^p\right] &= p\int_0^\infty y^{p-1}\left[1 - G(y)\right]dy \\ &= p\int_0^\infty y^{p-1}\left[1 + \sum_{j=0}^{k-1} \frac{e^{-e^{y}}e^{jy} - e^{-e^{-y}}e^{-jy}}{j!}\right]dy. \end{aligned} \tag{B.3}$$

As $y \to 0$, the integrand behaves like y^{p-1}, using p-test; the integral is convergent for all $p > 0$. As $y \to \infty$, the integrand behaves like $O\left(-y^{p-1}\sum_{j=1}^{k-1} e^{-jy}\right)$ and therefore the integral is convergent for all $p > 0$. Hence, we conclude that $E\left[|X|^p\right] < \infty$ for $p > 0$. To prove the rest of Lemma 3.1, we need the PDF of X, $f^{(k)}(x)$. Differentiating (3.12), after some basic algebraic manipulation, $f^{(k)}(x)$ can be simplified as

$$f^{(k)}(x) = \frac{e^{-kx}e^{-e^{-x}}}{(k-1)!},\ -\infty < x < \infty. \tag{B.4}$$

Then, $E\left[X^p\right]$ can be expressed as

$$E\left[X^p\right] = \int_{-\infty}^{\infty} \frac{x^p e^{-kx} e^{-e^{-x}}}{(k-1)!} dx. \tag{B.5}$$

Using $u = e^{-x}$, we have

$$E\left[X^p\right] = \int_{0}^{\infty} \frac{\left(-\ln\left(u\right)\right)^p e^{-u} u^{k-1}}{(k-1)!} du. \tag{B.6}$$

From the definition of gamma function, we have $\Gamma(k) = \int_0^\infty u^{k-1}e^{-u}du$. By differentiating both sides with respect to k, where k is an integer, we have $\Gamma(k)\psi(k) = \int_0^\infty u^{k-1}e^{-u}\ln(u)du$. Using this and setting $p = 1$ in (B.6), it follows that $E\left[X\right] = -\psi(k)$. Using (B.4) and the transformation of $u = e^{-x}$, $\mathcal{M}_X(t)$ can be expressed as

$$\begin{aligned} \mathcal{M}_X(t) = E\left[e^{tX}\right] &= \int_0^\infty \frac{e^{-u}u^{k-t-1}}{(k-1)!}du. \\ &= \frac{\Gamma(k-t)}{(k-1)!},\ k > t. \end{aligned} \tag{B.7}$$

APPENDIX C

MGF AND MOMENTS CONVERGENCE OF $Z_{(N-K+1)}$

Lemma 2 below establishes the connection between convergence in distribution and convergence of MGF. For a positive random variable X, the MGF of is $E\left[e^{tX}\right] = \int_0^\infty e^{tx} f(x)dx$, where $f(x)$ is the PDF of X.

Lemma 1: If $\frac{Z_{(N-k+1)}}{b}$ converges in distribution to a random variable Z whose CDF is as in (4.11), then for a fixed k we have

$$\lim_{N\to\infty} E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right] = E\left[e^{tZ}\right],\ t<0, \tag{C.1}$$

where $E\left[e^{tZ}\right] = \frac{2(-t)^{k/2}K_k\left(2\sqrt{-t}\right)}{(k-1)!},\ t<0$.

Proof: Theorem 2 of [36] implies that if $E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right]$ exists for all $t < 0$ and $\frac{Z_{(N-k+1)}}{b_N}$ converges uniformly in distribution to a random variable Z with CDF as in (4.11) where $E\left[e^{tZ}\right]$ exists for all $t<0$, then for a fixed k, $\lim_{N\to\infty} E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right] = E\left[e^{tZ}\right]$, for all $t<0$. We note that $E\left[e^{tZ}\right]$ for all $t<0$ exists and can be evaluated as

$$\begin{aligned} E\left[e^{tZ}\right] &= \int_0^\infty \frac{e^{tz}e^{-z^{-1}}}{z^{k+1}(k-1)!}dz \\ &= \frac{2(-t)^{k/2}K_k\left(2\sqrt{-t}\right)}{(k-1)!},\ t<0, \end{aligned} \tag{C.2}$$

where the above integral is evaluated with help of Eq. (2.11) of [57].

To show that $E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right]$ exists for all $t<0$, we use Lemma 1.7.2. of [37], with $g(x) = e^{tx}$, $x\ge 0$ and $t<0$; we have

$$E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right] \le \frac{N!}{(k-1)!(N-k)!}E\left[e^{\frac{tZ_i}{b}}\right], \tag{C.3}$$

where the CDF and PDF of Z_i are as in (4.4) and (4.5), respectively. We note that $E\left[e^{\frac{tZ_i}{b}}\right]$ exists for all $t<0$ and it can be expressed as

$$\begin{aligned} E\left[e^{\frac{tZ_i}{b}}\right] &= \int_0^{\infty} \frac{m\lambda\beta\left(P_M\right)^m z^{m-1} e^{\frac{tz}{b}}}{\left(\lambda\beta+P_M z\right)^{m+1}} dz \\ &= \Gamma\left(m+1\right) e^{-\frac{\lambda\beta t}{2bP_M}} W_{-m,-1/2}\left(\frac{-\lambda\beta t}{bP_M}\right),\ t<0, \end{aligned} \tag{C.4}$$

where $W_{l,n}\left(\cdot\right)$ is the Whittaker function[68]. Using (C.3) and (C.4), we infer that for all $t<0$, $E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right]<\infty$. Finally, since both $E\left[e^{t\left(\frac{Z_{(N-k+1)}}{b}\right)}\right]$ and $E\left[e^{tZ}\right]$ exist, based on Theorem 2 of [36], (C.1) holds.

Lemma 3 below establishes the connection between convergence in distribution and convergence of negative moments.

Lemma 3: If $\frac{Z_{(N-k+1)}}{b}$ converges in distribution to a random variable Z whose CDF is as in (4.11), then for a fixed k we have

$$\lim_{N\to\infty} E\left[\left(1+\frac{Z_{(N-k+1)}}{b}\right)^{-A}\right] = E\left[\left(1+Z\right)^{-A}\right],\ A>0. \tag{C.5}$$

Proof: To prove (C.5), it is equivalent to show that

$$\lim_{N\to\infty} E\left[e^{-A\ln\left(1+\frac{Z_{(N-k+1)}}{b}\right)}\right] = E\left[e^{-A\ln(1+Z)}\right],\ A>0. \tag{C.6}$$

Using continuous mapping theorem [69], we infer that if $\frac{Z_{(N-k+1)}}{b}$ converges in distribution to a random variable Z whose CDF is as in (4.11); then $\ln\left(1+\frac{Z_{(N-k+1)}}{b}\right)$ converges in distribution to $\ln(1+Z)$.

Theorem 2 of [36] implies that to show that (C.6) holds, it suffices to show that $E\left[\left(1+Z\right)^{-A}\right]$ and $E\left[\left(1+\frac{Z_{(N-k+1)}}{b}\right)^{-A}\right]$ exist, for $A>0$. We note that $E\left[\left(1+Z\right)^{-A}\right]$ exists and it can be evaluated as

$$E\left[(1+Z)^{-A}\right] = \int_0^\infty \frac{(1+z)^{-A}\, e^{-z^{-1}}}{z^{k+1}(k-1)!}dz$$
$$= \frac{U\left(A+k;k+1;1\right)\Gamma\left(A+k\right)}{(k-1)!},\ A>0, \tag{C.7}$$

where $U\left(a;b;z\right) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$, $a>0$ is the Tricomi hypergeometric function. The above integral is evaluated after variable transformation of $u = z^{-1}$ and with the help of Eq. (39) of [65].

To show that $E\left[\left(1+\frac{Z_{(N-k+1)}}{b}\right)^{-A}\right]$ exists for all $A>0$, we use Lemma 1.7.2. of [37], with $g(x) = (1+x)^{-A}$, $x \geq 0$ and $A>0$; we have

$$E\left[\left(1+\frac{Z_{(N-k+1)}}{b}\right)^{-A}\right] \leq \frac{N!}{(k-1)!(N-k)!}E\left[\left(1+\frac{Z_i}{b}\right)^{-A}\right], \tag{C.8}$$

where the CDF and PDF of Z_i are as in (4.4) and (4.5), respectively. Making use of Eq. (3.197.1) of [25], we note that $E\left[\left(1+\frac{Z_i}{b}\right)^{-A}\right]$ exists for $A>0$ and can be expressed as

$$E\left[\left(1+\frac{Z_i}{b}\right)^{-A}\right] = \frac{m\lambda\beta b^A}{P_M}\int_0^\infty \frac{z^{m-1}}{(b+z)^A\left(\frac{\lambda\beta}{P_M}+z\right)^{m+1}}dz$$
$$= mB\left(m,A+1\right){}_2F_1\left(A;m;A+m+1;1-\frac{\lambda\beta}{bP_M}\right),\ A>0, \tag{C.9}$$

where ${}_2F_1\left(x;y;z;w\right)$ is the Gauss hypergeometric function [25] and $B\left(x,y\right)$ is the Beta function.

Combining (C.8) and (C.9), we infer that $E\left[\left(1+\frac{Z_{(N-k+1)}}{b}\right)^{-A}\right] < \infty$ for all $A>0$. Finally, since both $E\left[\left(1+\frac{Z_{(N-k+1)}}{b}\right)^{-A}\right]$ and $E\left[(1+Z)^{-A}\right]$ exist for $A>0$, based on Theorem 2 of [36], (C.5) holds.