APPLYING UNCERTAINTY QUANTIFICATION AND VALUE-OF-INFORMATION CONCEPTS IN UNCONVENTIONAL RESERVOIR DEVELOPMENT

A Thesis

by

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Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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May 2018

Major Subject: Petroleum Engineering

ABSTRACT

The oil and gas industry has a long history of underperformance relative to forecasts. Underperformance in the industry has been directly linked to poor assessment of uncertainty. Uncertainty is large in the context of unconventional reservoir development. Therefore, reliable assessment of uncertainty is necessary for the optimization of decision making in unconventional reservoir development. Once uncertainty has been reliably assessed, the financial benefit of reducing each uncertainty should be estimated. Not all uncertainties are worth reducing; in fact, the value driven by the reduction of some uncertainties may be less than the cost of acquiring the relevant data – meaning that the data acquisition hurts financial performance. Yet, a well-established method of quantifying financial support for data-acquisition decisions in a multiple-variable context is largely absent from the literature related to unconventional reservoir development. Value-of-information analysis quantifies the financial benefit of reducing the uncertainty of variables within specific decision contexts. In this work, multi-variable value-of-perfect-information analysis was applied to a well-spacing decision model in the context of unconventional reservoir development.

The application of multi-variable value-of-perfect-information analysis to an Eagle Ford well-spacing decision context indicated that the parameters for which uncertainty reduction would provide the most value are commodity price, created-fracture propagation, and matrix porosity. This analysis also indicated that reducing the

uncertainty related to matrix permeability and natural fracture density would provide little value in the analyzed well-spacing decision context.

The effect of biases in uncertainty quantification on multi-variable value-of-information calculations was investigated, and it was demonstrated that biased uncertainty assessment for one variable can skew value-of-perfect-information calculations for all uncertain variables and can change value-of-perfect-information rankings.

A rational approach for data-acquisition decisions is achievable through creation of a reliable decision model and multi-variable value-of-information analysis. Widespread awareness of the power of multiple-variable value-of-information analysis to justify data acquisition and focus research efforts could lead to increased application of value-of-information analysis. Increased application should lead to improved decision making and financial performance in unconventional reservoir development.

ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Duane McVay, for his patience, guidance, and support throughout the course of this research. Thanks also are extended to Dr. Martin Wortman and Dr. John Lee for their tips along the way.

In addition, I would like to say thank you to my friends and colleagues in the Petroleum Engineering department. They, along with the department faculty and staff, have shown lots of support and made my time at Texas A&M University a great experience.

Thanks to my mother, Allison, and my father, Mark, for their interest and encouragement. Finally (and most importantly), thanks to my wife, Annie, for her patience and unconditional love.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

This work was supervised by a thesis committee consisting of Dr. Duane A. McVay [advisor] and Dr. W. John Lee of the Department of Petroleum Engineering and Dr. Martin Wortman of the Department of Industrial Engineering.

The data analyzed in Section 3 was obtained from a survey conducted by Dr. Duane A. McVay of industry members of the Crisman Institute for Petroleum Research and the Berg-Hughes Center for Petroleum and Sedimentary Systems. The method of assigning biases to continuous distributions depicted in Section 6 was developed by Mubarak Dossary of the Department of Petroleum Engineering and was published in 2012. The embedded-discrete-fracture-model employed to simulate various cases for a well-spacing decision in Section 5 was developed by Chai Zhi of the Department of Petroleum Engineering.

All other work conducted for the thesis was completed by the student independently.

Funding Sources

Graduate study was supported by a fellowship from Texas A&M University and a research assistantship from the joint-venture between the Crisman Institute for Petroleum Research and the Berg-Hughes Center for Petroleum and Sedimentary Systems.

NOMENCLATURE

bbl	Barrel, a unit of volume equivalent to 5.615 cubic feet
bbl/day	Barrels per day of oil production
CAPEX	Capital expenditure
DB	Directional bias
E&P	Exploration and Production
EIA	Energy Information Administration
EOL	Expected opportunity loss
EDFM	Embedded-discrete-fracture-model
EV	Expected value
EVWPI	Expected value with perfect information
ft	feet
IntSpacing	Interference spacing
Mcf	Thousand cubic feet
mD	millidarcy
mm	millimeter
nD	nanodarcy
NPV	Net present value
Oil Pr.	Oil Price
OL	Opportunity loss
OPEX	Operational expenditures

P10	10^{th} percentile
P50	50^{th} percentile
P90	90^{th} percentile
q_i	Initial rate of oil production (bbl/day)
SD	Standard deviation
VOI	Value of information
VOII	Value of imperfect information
VOPI	Value of perfect information

TABLE OF CONTENTS

Page

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

TABLE Page

1. INTRODUCTION

1.1 Background

Though the petroleum industry has existed for well over a century, development of unconventional (shale oil/gas, tight oil/gas, and coal-bed methane) reservoirs is a relatively new practice. It was not until the 2000's that the practices of horizontal drilling and hydraulic fracturing were paired together, with the result of commercial oil and gas production from unconventional reservoirs. Since unconventional-reservoir development is such a new practice, the physics of fluid flow in unconventional reservoirs is not well understood in comparison to conventional reservoirs. For this reason, reliable deterministic predictive models have not yet been developed for unconventional reservoirs. Until reliable deterministic models are developed, probabilistic models can be very helpful in making sound decisions under large uncertainty.

The petroleum industry has a long history of poor performance relative to forecasts (Capen 1976, Brashear 2001). This poor performance was largely due to poor uncertainty assessment in the development of conventional reservoirs. The introduction of unconventional-reservoir development, which involves a larger amount of uncertainty, calls for a focus on improved quantification of uncertainty. If quantification of uncertainty is improved, forecasts of project financial performance and, thus, decision making will generally be improved (McVay and Dossary 2012). To truly optimize the decision-making process in the context of exploration and production (E&P), it is important to not only accurately assess each uncertainty but also to assess the financial benefit of reducing each uncertainty through measurement. Value-of-information (VOI) concepts can be used to prioritize uncertainties that would benefit from reduction and, thus, guide data acquisition and research recommendations. For the purposes of my research, information is defined as data acquired or research efforts undertaken pertaining to a particular variable by an uncertainty assessor that lead to a decrease in the standard deviation of the assessed probability mass function for that variable.

1.2 Status of the Question

It is well established in the oil and gas literature that poor uncertainty assessment leads directly to poor decision making (Capen 1976, Brashear et al. 2001). In the context of petroleum-reservoir development, improved decision making leads to improved financial performance. McVay and Dossary (2012) quantified the impact that biased uncertainty assessment has on oil and gas portfolio performance. They assumed that all human biases present in the assessment of the value of individual petroleum projects can be summarized in two overall biases: overconfidence bias and directional bias. Overconfidence bias is associated with the range of possible outcomes while directional bias is associated with the central tendency of the assessed uncertainty. They created a model that quantified the relationship between unbiased and biased distributions in terms of numerical overconfidence and directional-bias values. These biases were then applied to a portfolio of projects with pre-defined distributions of true project value so that project selection under biased uncertainty assessment could be compared to project selection under unbiased uncertainty assessment. This study showed that even a moderate amount of overconfidence and directional bias can lead to an expected portfolio disappointment (estimated value minus realized value) of over 30%. These results show that accurate uncertainty assessment leads to significantly improved financial performance in the context of oil and gas projects. This should be especially true in the context of unconventional reservoir development, which generally has more uncertainty than conventional reservoirs.

Although McVay and Dossary provided a thorough assessment of the cost of biased uncertainty assessment, they only minimally addressed how to improve uncertainty assessment. It has been suggested by Capen (1976) and other authors that the best way to improve uncertainty assessment is through look-backs and calibration. Look-backs refers to comparing actual results to previous probabilistic predictions and calibration is the measure of how well the probabilistic predictions compared to actual results. Fondren et al. (2013) developed a relational database application demonstrating that use of look-backs and subsequent calibration of probabilistic predictions improves uncertainty assessment significantly. They presented three experiments, each showing that calibrating probabilistic forecasts based on the degree to which historical probabilistic forecasts have been incorrect significantly improves uncertainty assessment. The point is that assessing your uncertainty about a given quantity is an entirely different skill than assessing the quantity itself, and this skill is sharpened through look-backs and calibration. Assessing uncertainty is a foundational element of VOI calculations. Therefore, uncertainty assessment must be calibrated and reliable for VOI calculations to be meaningful.

There is a wealth of research establishing value-of-information analysis as a useful tool for creating value in the oil and gas industry. Koninx (2001) gives a thorough review of VOI methodology. He explains that VOI analysis involves identifying uncertain variables for which data is gatherable, assessing the impact that the data could have, and valuing the impact that the data could have. Koninx then gives two example scenarios. The first example involves a case of proposed 3D-seismic acquisition where the increase in expected value is greater than the cost of the data acquisition, and the second example involves a proposed appraisal well where the increase in expected value is less than the expected cost of drilling the appraisal well. In the first example, the VOI analysis shows that the data acquisition is justified, but in the second example the VOI analysis shows that the data acquisition is not justified. This shows that "VOI analysis is a powerful tool for short-term rationalization of data-acquisition costs." Koninx then connects rationalization of data acquisition in the short term to value creation in the long term. That is, a focused approach to data acquisition leads to an effective use of data. With very little data being gathered and unused, data-acquisition costs drop significantly. Other authors have shown the power of VOI analysis in similar fashion. Leach, Brown, and Haskett (2007) demonstrate the power of VOI analysis to guide data-acquisition decisions in a similar manner to Koninx's paper, but in the context of unconventional resource development.

Bratvold (2007) has written on the history and future of the use of VOI analysis in the oil and gas industry in hopes of making VOI analysis more accessible and widely used. He explains the power of VOI analysis and shows how it has been underutilized since Grayson (1960) introduced the concept to the industry. Bratvold's paper concludes that VOI analysis has not become an integral part of the decision-making process in industry due to a lack of decision-analysis skills among petroleum engineers and geoscientists, inexperience applying VOI methodology, and misconceptions about information value. However, Bratvold remains optimistic about the future of the use of VOI methodology in the industry due to its power as a decision-analysis tool. A key takeaway from his research of the history of VOI analysis in the oil and gas industry is that almost all of the applications of VOI in an oil and gas context published from 1960-2006 focus on valuing a single information source. In fact, only two papers published in that time frame consider multiple sources of information (Dougherty 1971; Wills and Graves 2004). Dougherty's 1971 paper is a review of statistical decision theory in an exploration context that gives an example of VOI analysis considering two data sources. Wills and Graves (2004) present a true multi-variable VOI analysis. The decision context of their multi-variable VOI model is conventional reservoir development and production volumes are calculated volumetrically. The variables considered in their analysis were the inputs to the volumetric reserves equation (reservoir area, reservoir thickness, porosity, water saturation, formation volume factor, and recovery factor). VOIs calculated by Wills and Graves were based on data quality, quantified as the probability that the acquired data related to each variable represents perfect information.

Because horizontal drilling and hydraulic fracturing are necessary for economically feasible unconventional-reservoir development, but not for conventional-reservoir development, key uncertainties affecting development decisions for unconventional reservoirs and conventional reservoirs are quite different. Also, the uncertainty facing decision makers is likely much greater in unconventional than conventional reservoir development, because development of unconventional reservoirs is relatively new and these reservoirs are less well understood. For these reasons, multi-variable VOI analysis in the context of conventional reservoir development is not necessarily translatable to unconventional reservoir development decisions. Also, Wills and Graves consider subsurface uncertainty but not economic uncertainty.

In the years since Bratvold's 2007 paper, there have been more papers published addressing use of VOI analysis in the oil and gas industry. However, there has not, as far as I know, been a serious attempt to apply multi-variable VOI analysis in the decision context of unconventional reservoir development in order to guide data acquisition.

Though multi-variable VOI analysis has not been applied to unconventional reservoir development in a way that considers the value of information for all uncertain variables within a particular decision context, such an approach to decision analysis can be found outside of the oil and gas industry. Hubbard (2014) gives a review of an approach he has used to determine the information value of as many as 90 uncertain variables within a single decision context in industries such as information technology security, water management, and healthcare. In Hubbard's approach there is no upper limit to the number of uncertain variables for which the information value can be found within a particular decision context. His approach involves building a model that connects all uncertain variables to expected project value, then running a series of Monte Carlo simulations where it is successively assumed that each one of the uncertain variables is known exactly. At the end of the series of simulations, the degree to which perfect information for each individual variable increases the expected value of the project is known. The value-of-perfect-information (VOPI) values can be ranked to determine which variables should be targeted for uncertainty reduction through measurement. Though perfect information is usually unattainable in the real world, this process still provides valuable insight to guide data-acquisition decisions. If the value of perfect information (VOPI) for an uncertain variable is high, it may indicate that the value of imperfect information (VOII) for that uncertain variable is large enough to merit further data acquisition. Hubbard suggests that VOII is approximately 10% of VOPI as a general rule of thumb. Also, VOPI gives an indication of relative information values, which may be more important to a decision maker than absolute information values because relative information values indicate the variables for which the most value is associated with uncertainty reduction through data acquisition. Hubbard's multi-variable VOI approach can be applied to key decision contexts within unconventional reservoir development to provide insight on the variables for which value is driven by uncertainty reduction through data acquisition.

It has been established that accurate uncertainty quantification is a crucial component of the decision-making process in unconventional-reservoir development. Biases that creep into probabilistic estimations can be systematically reduced through look-backs and calibration. It has also been well established is that VOI analysis can be a powerful tool when applied to data-acquisition decisions for oil and gas projects. However, almost every VOI application in the current petroleum literature is limited to analysis of the value created by reducing the uncertainty of one variable. There are no applications of multiple-variable VOI analysis to unconventional-reservoir development. In addition, the effect that biases in uncertainty quantification have on VOI calculations has not been established. Thus, there is a need for further research in these areas.

1.3 Research Objectives

1. Create a generalized multiple-variable VOPI model that can be applied to unconventional reservoir development.
2. Determine the parameters for which additional information can provide the most value in the context of a typical well-spacing decision in the Eagle Ford shale.
3. Assess the impact of biases on VOI calculations.

1.4 Research Methodology

1. Single-variable VOI methodology was reviewed.
2. Crisman/Berg-Hughes Center members were surveyed about the relative importance of different decisions and uncertainties in unconventional reservoir development.
3. A VOPI workflow was developed based on Hubbard's (2014) description of multiple-variable VOPI models. A generalized multiple-variable VOPI model that can be applied to different decision contexts was created.
4. The VOPI model was applied to an Eagle Ford gas reservoir well-spacing decision.

a. Uncertainties relevant to the decision context were identified.

b. Key uncertainties were quantified for an example scenario.

c. A model that relates the decision context, relevant uncertain parameters, and expected value of the project was created.

d. The VOPI model was used to determine the parameters for which additional information can provide the most value.

5. Overconfidence and directional biases were applied to a simple, theoretical VOPI problem to determine the effect of biases on VOPI. The impact of uncertainty quantification biases on the well-spacing VOPI analysis was assessed.

2. SINGLE-VARIABLE VOPI OVERVIEW

E&P companies operating in unconventional plays face large uncertainty when making development decisions. Because uncertainty is large, they acquire and analyze large amounts of data for improved understanding of their development areas. If understanding of the development area is improved, more informed and better decisions can be made. Thus, the ultimate aim of data acquisition is improved decision making. However, there is almost always a cost associated with data acquisition. The prudent decision maker should investigate the anticipated benefit of acquiring any particular data relative to the data-acquisition cost. The goal of VOI analysis is to quantify the anticipated financial benefit of acquiring a particular set of new information.

VOPI analysis is a type of VOI analysis in which it is assumed that the information acquired reduces the uncertainty of the associated parameter to zero. Though the perfect information assumption is unrealistic in real-world data-acquisition scenarios, VOPI analysis sheds light on the impact that an uncertain variable has on the optimal decision alternative within a particular decision context. Consider the following example scenario: An E&P company is considering an infill program in a section of previously developed unconventional reservoir. If the infill program is a success, the expected value (EV) of the project is $100 million. However, if the program is a failure it is anticipated that the E&P company will lose $10 million. Data currently available to the decision maker indicate that the probability of a successful project is 25%. For the purposes of this example, it is assumed the possible outcomes of this project are binary—success or failure, and no possible outcomes in between. The following steps can be followed to calculate the value of perfect information regarding the outcome of the project.

1. Define the decision context. In this example, the decision maker is faced with a binary decision: proceed with the infill development project or do not.
    a. It is helpful to use a decision tree to chart the decision alternatives and their possible consequences (**Fig. 1**). Typical decision trees contain decision nodes, chance nodes, and end nodes. Decision nodes represent points at which the decision maker must make a decision and are typically represented with a square (Fig. 1). Chance nodes represent points at which the consequences of a particular decision are uncertain and are typically represented with a circle (Fig. 1). End nodes represent payoffs and are typically represented with a triangle (Fig. 1).
2. Calculate the EV of the decision alternatives at the current level of uncertainty. This step is shown in the top decision tree in Fig. 1. The optimal decision is to proceed with the infill project and the EV of the project is $17.5 million without perfect information.
3. Calculate the EV of the infill project if perfect information regarding the project outcome is acquired prior to making a development decision. This step is shown in the bottom decision tree in Fig. 1. Because the decision to acquire perfect information is made before the perfect information is obtained, a new chance node (representing the nature of the perfect information) is introduced in the

“acquire perfect information” part of the decision tree (Fig. 1). The calculation of EV with perfect information is accomplished in the following manner:

   a. Calculate the EV of the infill project if the perfect information indicates that the project will be a success. This step is shown in the top branch of the bottom decision tree in Fig. 1. The EV of the project is $100 million if success is indicated

   b. Calculate the EV of the infill project if the perfect information indicates that the project will be a failure. This step is shown in the bottom branch of the bottom decision tree in Fig. 1. The EV of the project is $0 if failure is indicated

   c. Multiply both EVs by their respective probabilities and sum the results. This calculation is shown in the bottom decision tree in Fig. 1. The EV of the project is $25 million with perfect information.

4. The VOPI is calculated as the difference between the EV of the infill project if perfect information is acquired and the EV of the project if perfect information is not acquired (Fig. 1).

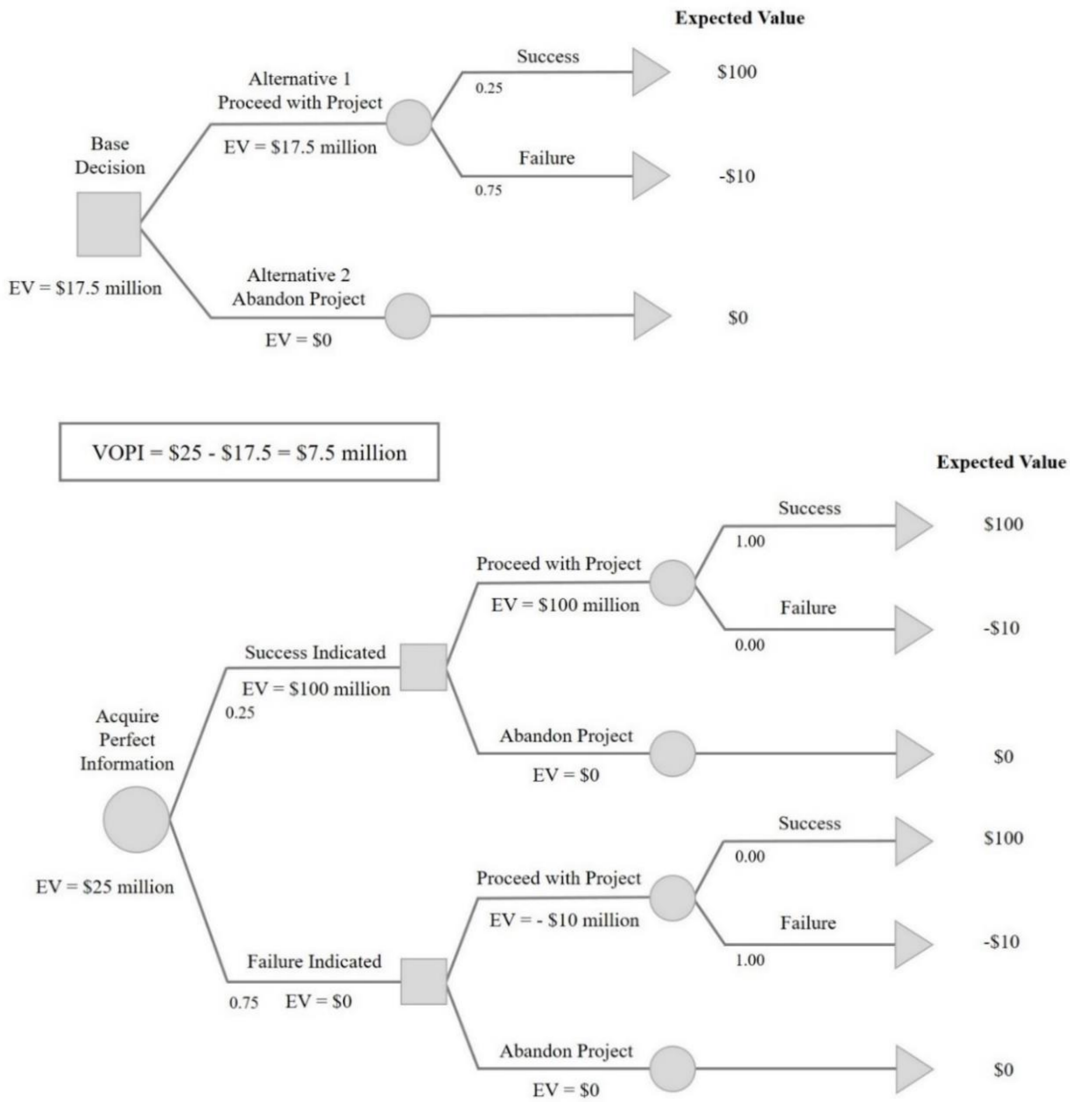

Fig. 1—Single-variable VOPI calculation scenario.

In this example scenario, the VOPI is $7.5 million. This means that a prudent decision maker should not pay more than $7.5 million for information regarding the development outcome. It should be noted that the VOI in this scenario would be less, possibly much less, than the VOPI because no information is truly perfect. VOI/VOPI calculations are functions of both decisions and uncertainties. Key decisions and uncertainties in the context of unconventional reservoir development are addressed in the Section 3.

3. CRISMAN INSTITUTE/BERG-HUGHES MEMBER SURVEY

3.1 Survey Background and Format

The power of value-of-information analysis lies in its ability to quantify the amount of money that one should be willing to spend to reduce specific uncertainties within a particular decision context. To apply VOI analysis to decisions in unconventional reservoir development, it was important that steps be taken to identify the key uncertainties and decisions in that context. Also, my research is one part of uncertainty quantification/decision analysis research being conducted in unconventional reservoir development by the joint venture between the Crisman Institute for Petroleum Research and the Berg-Hughes Center for Petroleum and Sedimentary Systems. For these reasons, Crisman/Berg-Hughes industry members were surveyed at the October 2016 Crisman/Berg-Hughes meeting at Texas A&M University to determine what they consider the key decisions and uncertainties to be in the context of unconventional reservoir development. For both decisions and uncertainties, survey takers were presented with a list of approximately 15 decisions/uncertainties that are widely acknowledged to be present during unconventional reservoir development. They also were given the opportunity to write in other decisions/uncertainties if so desired. Survey takers were asked to rank the five decisions/uncertainties that they believe to be the most important, in order of decreasing importance. When the survey was complete, each participant had ranked what they believed to be the five most important decisions and five most important uncertainties. To compile the results of the survey, each decision and uncertainty was given a score. The scoring system was as follows: five points for each “1” ranking (most important), four points for each “2” ranking, three points for each “3” ranking, two points for each “4” ranking, and one point for each “5” ranking. Each survey had a total of 15 points to be distributed to different decisions and 15 points to be distributed to each uncertainty. When survey takers ranked their answers in a way that was inconsistent with the instructions, the 15 points were distributed across their responses consistent with the survey response, thus ensuring that all participants were weighted equally. Scores from all participants were combined to yield an aggregate score for each decision and uncertainty.

3.2 Survey Results

They key uncertainties are summarized below in **Fig. 2**, and the key decisions are summarized below in **Fig. 3**.

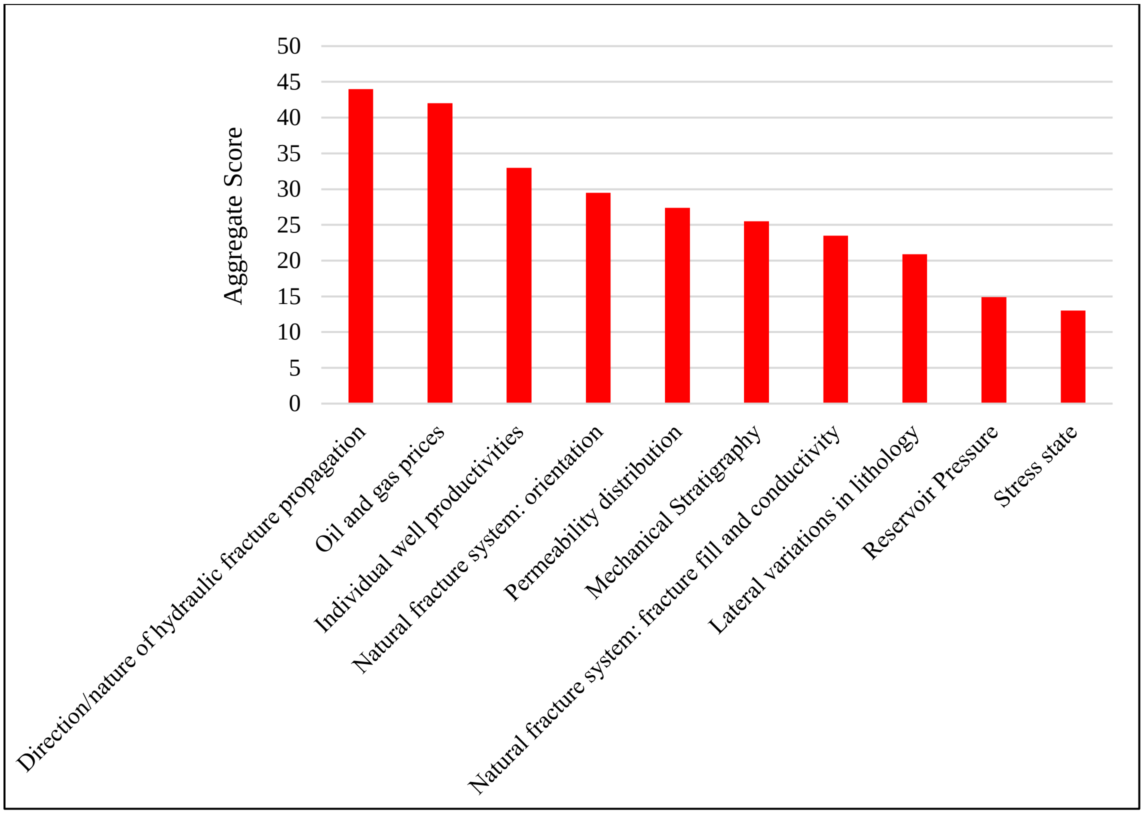

Fig. 2—10 most important uncertainties in unconventional reservoir development according to survey of Crisman/Berg-Hughes industry members.

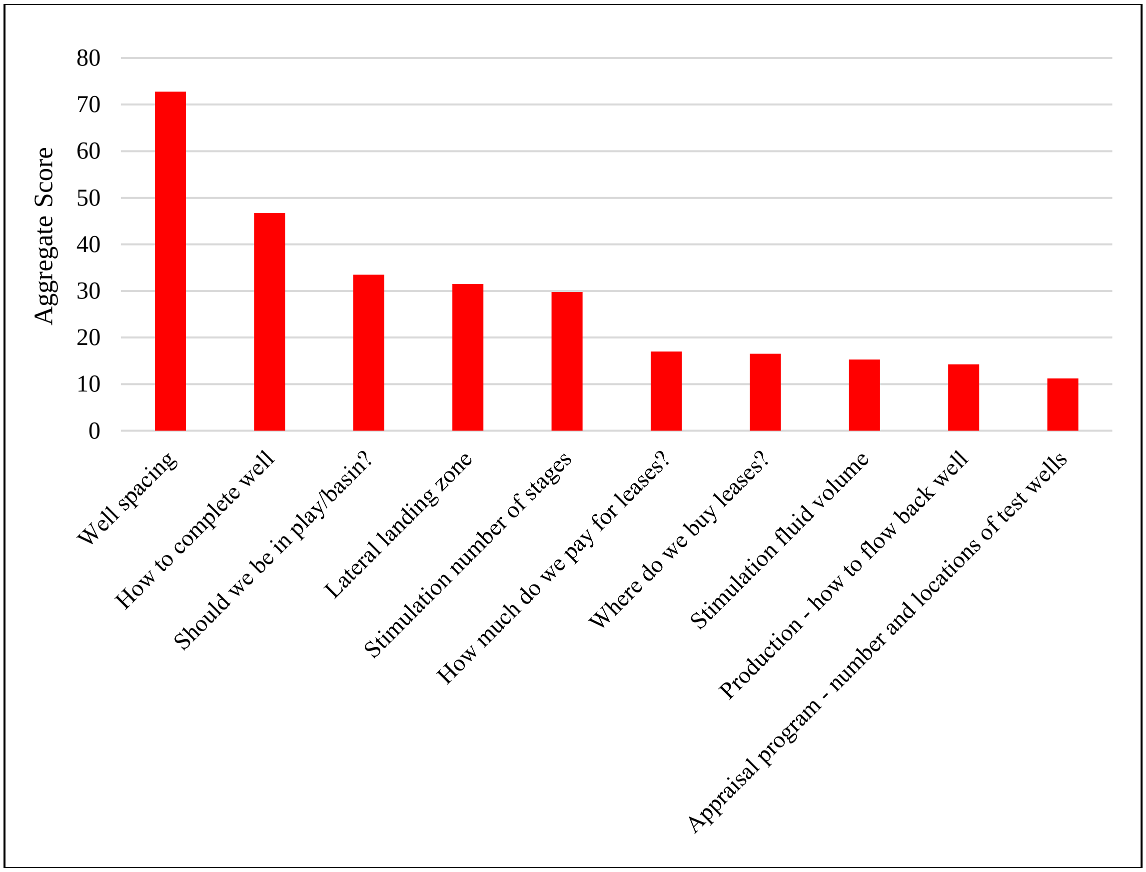

Fig. 3—10 most important decisions in unconventional reservoir development according to survey of Crisman/Berg-Hughes industry members.

Crisman/Berg-Hughes industry members most commonly considered the well-spacing decision to be the most important decision in unconventional reservoir development (Fig. 3). Its aggregate score was about 55% higher than the decision voted second-most important. For this reason, the well-spacing decision was chosen as the decision context for the application of multi-variable VOPI methodology later in this thesis. The key uncertainties (Fig. 2) were helpful in deciding which uncertainties to model in the application of VOPI methodology. A full list of responses to the Crisman/Berg-Hughes survey and their aggregate scores are shown in **Tables A-1 and A-2**.

4. MULTI-VARIABLE VOPI CALCULATION WORKFLOW

4.1 Multi-Variable VOPI Calculation Methodology

The method of calculating VOPI in a multi-variable context—considering the value of reducing the uncertainty of certain parameters relative to the value of reducing the uncertainty of certain other parameters—was inspired by Hubbard (2014). A synthetic decision with two discrete uncertain variables was first considered. Because there are only two uncertain variables, it is easy to visualize manual calculations of the overall EOL and the change in overall EOL if either variable is known perfectly. This manual calculation served as both an establishment of the multi-parameter VOPI calculation methodology and a quality check when the methodology was automated with the generalized multi-parameter VOPI model discussed later.

The synthetic decision context used for the development of a two-variable VOPI calculation was a well-spacing decision for a 640-acre oil reservoir. A simple model was created that calculates the expected net present value of a discrete set of decision alternatives under every possible combination of the two discrete uncertain variables. In this model, economic parameters were held constant and it was assumed that each well in the field can be represented by the field's average decline-curve parameters. **Table 1** presents the set of physical and economic input parameters held constant in the synthetic two-variable model.

Table 1—Economic and physical parameters held constant in the synthetic well-spacing model.

Parameter	Value	Units
Oil Price	54	$/STB
Oil Marginal Production Cost	12	$/STB
CAPEX per Well	1,250,000	$
OPEX	2,100	$/well/month
D_i	35	/year
b	2	Unitless
Minimum Decline Rate	0.2	/year

This synthetic model assumes that there is a well spacing, known as the interference spacing, at or under which production interference occurs. The purpose of the interference spacing is to penalize the decision maker for placing wells too close together and it is one of the two uncertain variables in this decision scenario. It is assumed to be constant throughout the 640-acre section. The fraction of production for a well experiencing interference is assumed to be directly proportional to the ratio of the chosen well spacing and the field interference spacing. For example, if the interference spacing for the field is 40 acres and the decision maker chooses to develop the field at 20-acre well spacing, then each well produces 50% in each month of what it would have had there been no interference. Production decline curves representing the assumed production of a well in this field, if the chosen well spacing is 20 acres, under various values of interference spacing are shown in **Fig. A-1**. This model is not meant to forecast production from an actual oil reservoir, but rather to approximate interference effects for the synthetic well-spacing decision context. The methodology could have been established with equal legitimacy within any decision context, but a synthetic well-spacing context was used for the sake of consistency with the well-spacing decision context presented in the next section of this research. The purpose of the model is to tie combinations of uncertain parameters to expected project value in a consistent manner. Because this model was used only for the purposes of establishing the method of multi-variable VOPI calculations and validating the accuracy of the calculations made by the generalized model, the scientific legitimacy of the synthetic model is irrelevant.

The successive pages show the multi-variable VOPI calculation methodology step-by-step. The first step is to use probability distributions to describe uncertain input parameters. The two variables that were considered uncertain in this application of the synthetic model were the average initial production rate of the field (q_i) and the interference spacing (IntSpacing). q_i was assigned a normal distribution with a mean of 490 bbl/day and standard deviation of 150 bbl/day. IntSpacing was arbitrarily assigned a normal distribution with a mean of 40 acres and a standard deviation of 10 acres. Because it is commonly used in the oil & gas industry, Swanson's mean was used to find discrete approximations to these normal probability distributions (Bickel 2011; **Table 2**).

Table 2—Probability distributions for q_i and IntSpacing, the two uncertain variables in the synthetic model.

Parameter 1: q_i (bbl/day)		
Percentile	Value	Probability
P10	298	30%
P50	490	40%
P90	682	30%

Parameter 2: IntSpacing (acres)		
Percentile	Value	Probability
P10	27	30%
P50	40	40%
P90	53	30%

The joint probability space contains nine possible states. Correlation between uncertain variables should be considered in the development of a joint-probability matrix. For this example, q_i and IntSpacing are assumed to be independent of each other.

The probability that each combination of q_i and IntSpacing will be realized upon development of this synthetic field is shown in **Table 3**.

Table 3—Joint-probability matrix for the two-variable synthetic model.

	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	9%	12%	9%
40	12%	16%	12%
53	9%	12%	9%

The considered decision alternatives were as follows: development at 20-acre spacing, development at 40-acre spacing, development at 80-acre spacing, and rejecting the project resulting in a project value of $0. The expected value, defined as the 10-year discounted net present value (NPV), of the development project, for all 9 possible states was calculated for each decision alternative (**Table 4**).

Table 4—NPV for each possible realization of q_i and IntSpacing under each decision alternative.

	Decision = 20 Acre Spacing		
	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	($22,223,005)	($10,291,524)	$1,639,957
40	($28,144,346)	($20,035,567)	($11,926,788)
53	($31,192,098)	($25,050,888)	($18,909,678)

	Decision = 40 Acre Spacing		
	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	($7,791,950)	$316,829	$8,425,608
40	($7,791,950)	$316,829	$8,425,608
53	($10,839,702)	($4,698,492)	$1,442,718

Table 4—Continued

	Decision = 80 Acre Spacing		
	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	($3,895,975)	$158,415	$4,212,804
40	($3,895,975)	$158,415	$4,212,804
53	($3,895,975)	$158,415	$4,212,804

	Decision = Reject Project		
	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	$0	$0	$0
40	$0	$0	$0
53	$0	$0	$0

For each of the 9 possible states, a particular decision would have been the best choice (maximum NPV of the four choices). However, the optimal decision changes depending on which state is realized (**Table 5**). In this example, the only decision alternative that cannot possibly be the best choice is 20-acre spacing.

Table 5—Optimal decision matrix for the two-variable synthetic model.

	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	Reject Project	40 Acre Spacing	40 Acre Spacing
40	Reject Project	40 Acre Spacing	40 Acre Spacing
53	Reject Project	80 Acre Spacing	80 Acre Spacing

If the decision maker has perfect information for both q_i and IntSpacing, Table 5 shows the decision that they would make for each possible realized state within the probability space. Correspondingly, the value of the project if the decision maker has perfect information for both uncertain variables is shown for the entire probability space in **Table 6**.

Table 6—Project value matrix under perfect information for the two-variable synthetic model.

	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	$ 0	$ 316,829	$ 8,425,608
40	$ 0	$ 316,829	$ 8,425,608
53	$ 0	$ 158,415	$ 4,212,804

Now that the maximum value of the project has been determined for the entire probability space, the next step is to calculate the opportunity loss (OL) across the entire probability space for each decision alternative. Opportunity loss is the difference between project value under the optimal decision and project value under the actual decision. So the OL will be zero for the optimal decision alternative at each state in the probability space. In this example, the OL matrix is calculated by subtracting the values in Table 4 from the values in Table 6 for each point in the probability space (**Table 7**).

Table 7—Opportunity loss for each possible realization of q_i and IntSpacing under each decision alternative.

	Decision =	20 Acre Spacing	
	q_i (bbl/day)		
IntSpacing (acres)	**298**	**490**	**682**
27	$22,223,005	$10,608,353	$6,785,651
40	$28,144,346	$20,352,396	$20,352,396
53	$31,192,098	$25,209,303	$23,122,482

Table 7—Continued

	Decision =	**40 Acre Spacing**	
		q_i **(bbl/day)**	
IntSpacing (acres)	**298**	**490**	**682**
27	$7,791,950	$0	$0
40	$7,791,950	$0	$0
53	$10,839,702	$4,856,907	$2,770,086

	Decision =	**80 Acre Spacing**	
		q_i **(bbl/day)**	
IntSpacing (acres)	**298**	**490**	**682**
27	$3,895,975	$158,415	$4,212,804
40	$3,895,975	$158,415	$4,212,804
53	$3,895,975	$0	$0

	Decision =	**Reject Project**	
		q_i **(bbl/day)**	
IntSpacing (acres)	**298**	**490**	**682**
27	$0	$316,829	$8,425,608
40	$0	$316,829	$8,425,608
53	$0	$158,415	$4,212,804

The next step is to determine the expected opportunity loss (EOL) associated with each decision alternative at the current level of uncertainty. Since the joint-probability matrix has already been defined in Table 3, this is a simple calculation. The EOL of each decision alternative is calculated by performing an element-by-element multiplication of the OL matrix for each decision defined in Table 7 by the probability matrix defined in Table 3. The EOL for a particular decision is the sum of all elements in the resulting matrix. The EV of each decision alternative is calculated using the same methodology, but using the NPV matrix (Table 4) and the joint-probability matrix (Table 3). The EOL and EV at the current level of uncertainty for each decision alternative in this example are shown in **Table 8**.

Table 8—EOL for each decision alternative at current level of uncertainty.

Decision	20-acre Spacing	40-acre Spacing	80-acre Spacing	Reject Project
EOL	$ 20,873,202	$ 3,444,019	$ 2,097,838	$ 2,256,252
EV	($18,616,950)	($1,187,767)	$158,415	$0

The 80-Acre Spacing decision alternative is highlighted because it has the smallest EOL ($2,097,837) and largest EV ($158,415). At the current level of uncertainty, a risk-neutral decision maker would develop the field at 80-acre well spacing if they must make a decision with no additional information. This is because the decision alternative with the lowest EOL is always the decision alternative with the highest expected NPV. EOL and EV are two sides of the same coin. In other words, in any single-discrete-decision context, the approaches of minimizing expected opportunity loss and maximizing expected value lead to the same decision.

At this point in the multi-variable VOPI workflow, the initial analysis of the subject decision is complete. The decision that should be made at the current level of uncertainty and its associated EOL have been determined. The minimum EOL prior to uncertainty reduction is the overall VOPI, or in other terms, the value of knowing all uncertain variables perfectly. The next step is to re-analyze the decision context as many times as there are uncertain variables. In each re-analysis, perfect information for a different uncertain variable is assumed. The VOPI for a particular variable is the reduction in EOL from the original EOL that occurs when that variable is assumed to be known perfectly, and the decision maker is privy to no other new information. To calculate EOL under an assumption of perfect information for any particular variable, EOL must be separately calculated for every combination of decision alternatives and possible values of the perfectly known variable. EOLs specific to possible values of the known variable for particular decision alternatives are calculated by applying probabilities conditional on the known variable to the OL matrix (**Table 9**). Conditional probabilities are derived from the joint-probability matrix (Table 3). For the assumption that q_i is perfectly known in the two-variable example decision context, inputs and outputs of this calculation are found in Table 9.

Table 9—Opportunity loss and conditional probability for each possible realization of q_i and IntSpacing under each decision alternative if q_i is known perfectly. The EOL values are associated with each potential value q_i and each decision alternative.

	Decision =				**20 Acre Spacing**	
	Known Variable: q_i (bbl/day)					
Unknown Variable	**298**		**490**		**682**	
IntSpacing (acres)	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**
27	$22,223,005	30%	$10,608,353	30%	$6,785,651	30%
40	$28,144,346	40%	$20,352,396	40%	$20,352,396	40%
53	$31,192,098	30%	$25,209,303	30%	$23,122,482	30%
EOL	**$27,282,269**		**$18,886,255**		**$17,113,398**	

Table 9—Continued

	Decision = 40 Acre Spacing					
	Known Variable: q_i (bbl/day)					
Unknown Variable	**298**		**490**		**682**	
IntSpacing (acres)	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**
27	$7,791,950	30%	$0	30%	$0	30%
40	$7,791,950	40%	$0	40%	$0	40%
53	$10,839,702	30%	$4,856,907	30%	$2,770,086	30%
EOL	**$8,706,275**		**$1,457,072**		**$831,026**	

	Decision = 80 Acre Spacing					
	Known Variable: q_i (bbl/day)					
Unknown Variable	**298**		**490**		**682**	
IntSpacing (acres)	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**
27	$3,895,975	30%	$158,415	30%	$4,212,804	30%
40	$3,895,975	40%	$158,415	40%	$4,212,804	40%
53	$3,895,975	30%	$0	30%	$0	30%
EOL	**$3,895,975**		**$110,890**		**$2,948,963**	

	Decision = Reject Project					
	Known Variable: q_i (bbl/day)					
Unknown Variable	**298**		**490**		**682**	
IntSpacing (acres)	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**
27	$0	30%	$316,829	30%	$8,425,608	30%
40	$0	40%	$316,829	40%	$8,425,608	40%
53	$0	30%	$158,415	30%	$4,212,804	30%
EOL	**$0**		**$269,305**		**$7,161,767**	

Minimum EOLs for each potential q_i value and their associated decisions are shown below in **Table 10**. In this discrete two-variable decision context, q_i must have a value of either 298 bbl/day, 490 bbl/day, or 682 bbl/day. An assumption of perfect information for q_i is an assumption that, after the perfect information is obtained, the decision maker knows which of these three values will be realized before the section is developed. Thus, the decision maker will know the optimal spacing decision to make for each potential value of the q_i.

Table 10—EOL for each decision alternative if q_i is known perfectly. Minimum EOL for each potential value of q_i highlighted.

	Known Variable: q_i (bbl/day)		
Decision Alternative	**298**	**490**	**682**
20 Acre Spacing	$27,282,269	$18,886,255	$17,113,398
40 Acre Spacing	$8,706,275	$1,457,072	$831,026
80 Acre Spacing	$3,895,975	$110,890	$2,948,963
Reject Project	$0	$269,305	$7,161,767

If q_i is 298 bbl/day then rejecting the project is the decision that minimizes EOL; if q_i is 490 bbl/day then developing the field with 80-acre spacing is the decision that minimizes EOL; and if q_i is 682 bbl/day then developing the field with 40-acre spacing is the decision that minimizes EOL (Table 10). In two of the three cases, the optimal decision changes—from the best decision of 80-acre spacing without information—because q_i is known perfectly. In this scenario, the probability that the optimal decision changes based on knowing q_i perfectly is 60%. Perfect knowledge of q_i adds value (reduces EOL) because, and only because, it can potentially change the decision to be made.

However, the calculation of VOPI is made before the information is obtained and, thus, before the value of q_i is known. Thus, all possible values of q_i must be considered to calculate the overall project EOL with perfect knowledge of q_i (**Table 11)**. Each minimum EOL value is multiplied by the marginal probability of its associated q_i value, then the results are summed. In this example, the overall project EOL if q_i is known perfectly is $293,664. The VOPI for q_i can then be calculated by subtracting the overall project EOL if q_i is known perfectly from the overall project EOL with no new information ($2,097,838; Table 10). In this case, the VOPI for q_i is $1,804,174.

Table 11—Overall EOL calculation if q_i is known perfectly.

q_i (bbl/day)	298	490	682
Optimal Decision	Reject Project	80 Acre Spacing	40 Acre Spacing
Min EOL	$ 0	$ 110,890	$ 831,026
Marginal Probability	30%	40%	30%
EOL	**$293,664**		

The calculations shown above and tabulated in Tables 9, 10, and 11 must be repeated for each unknown variable, just one more time for this two-variable example. **Tables 12-14** below show the results of employing the same methodology used to determine overall EOL if q_i is known to determine overall EOL if IntSpacing is known.

Table 12—Opportunity loss and conditional probability for each possible realization of q_i and IntSpacing under each decision alternative if IntSpacing is known perfectly. The EOL are values associated with each potential value of IntSpacing and each decision alternative.

	Decision =		20 Acre Spacing			
	Known Variable: IntSpacing (acres)					
Unknown Variable	27		40		53	
q_i (bbl/day)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	$ 22,223,005	30%	$ 28,144,346	30%	$ 31,192,098	30%
490	$ 10,608,353	40%	$ 20,352,396	40%	$ 25,209,303	40%
682	$ 6,785,651	30%	$ 20,352,396	30%	$ 23,122,482	30%
EOL	**$ 12,945,938**		**$ 22,689,981**		**$ 26,378,095**	

	Decision =		40 Acre Spacing			
	Known Variable: IntSpacing (acres)					
Unknown Variable	27		40		53	
q_i (bbl/day)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	$ 7,791,950	30%	$ 7,791,950	30%	$ 10,839,702	30%
490	$ 0	40%	$ 0	40%	$ 4,856,907	40%
682	$ 0	30%	$ 0	30%	$ 2,770,086	30%
EOL	**$ 2,337,585**		**$ 2,337,585**		**$ 6,025,699**	

	Decision =		80 Acre Spacing			
	Known Variable: IntSpacing (acres)					
Unknown Variable	27		40		53	
q_i (bbl/day)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	$ 3,895,975	30%	$ 3,895,975	30%	$ 3,895,975	30%
490	$ 158,415	40%	$ 158,415	40%	$ 0	40%
682	$ 4,212,804	30%	$ 4,212,804	30%	$ 0	30%
EOL	**$ 2,496,000**		**$ 2,496,000**		**$ 1,168,792**	

Table 12—Continued

	Decision = Reject Project					
	Known Variable: IntSpacing (acres)					
Unknown Variable	**27**		**40**		**53**	
***q_i* (bbl/day)**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**
298	$ 0	30%	$ 0	30%	$ 0	30%
490	$ 316,829	40%	$ 316,829	40%	$ 158,415	40%
682	$ 8,425,608	30%	$ 8,425,608	30%	$ 4,212,804	30%
EOL	**$ 2,654,414**		**$ 2,654,414**		**$ 1,327,207**	

Table 13—EOL for each decision alternative if IntSpacing is known perfectly. Minimum EOL for each potential value of IntSpacing is highlighted.

	Known Variable: IntSpacing (acres)		
Decision Alternative	**27**	**40**	**53**
20 Acre Spacing	$12,945,938	$22,689,981	$26,378,095
40 Acre Spacing	$2,337,585	$2,337,585	$6,025,699
80 Acre Spacing	$2,496,000	$2,496,000	$1,168,792
Reject Project	$2,654,414	$2,654,414	$1,327,207

Table 14—Overall EOL calculation if IntSpacing is known perfectly.

IntSpacing (acres)	27	40	53
Optimal Decision	40 Acre Spacing	40 Acre Spacing	80 Acre Spacing
Min EOL	$ 2,337,585	$ 2,337,585	$ 1,168,792
Marginal Probability	30%	40%	30%
EOL	**$1,986,947**		

For this two-variable example, the overall project EOL if IntSpacing is known perfectly is $1,986,947. The VOPI can then be calculated by subtracting the overall project EOL if IntSpacing is known perfectly from the overall project EOL with no new information ($2,097,837; Table 10). In this case, the VOPI for q_i is $110,890. A summary of key results from the multi-variable VOPI calculation shown in the preceding pages is found in **Table 15**. These results tell the decision maker that (a) information about q_i is much more valuable than information about IntSpacing, and (b) information about each unknown variable has value. However, before deciding whether to acquire information the decision maker must estimate the VOII from the VOPI for each variable and compare to the cost of acquiring the information. If the cost of information for any variable exceeds the value of information for that variable, then data acquisition is not justified.

Table 15—Overall EOL if each unknown variable is perfectly known and associated VOPI.

EOL: Current level of uncertainty		$2,097,838
Known Variable	**EOL**	**VOPI**
q_i	$ 293,664	$ 1,804,174
IntSpacing	$ 1,986,947	$ 110,891

The preceding pages show the step-by-step methodology of the multi-variable VOPI calculation for a simple, two-parameter discrete scenario. Before creating a model that generalizes this process in a way that can be applied to decision with any number of decision alternatives and uncertain parameters, the VOPI output for a three-variable discrete decision scenario was calculated manually. The purpose was to (a) highlight the intricacies of applying this methodology to a higher-dimensional problem and (b) have a second manual calculation to validate the output of the generalized model. The same synthetic model used for the two-variable problem was used to generate NPV values for a three-variable problem. The difference was that in the three-parameter problem, oil price was also considered to be an uncertain variable. The methodology was exactly the same as the two-parameter problem, but extended to three dimensions. Independence between the three parameters was assumed. The intermediate results and outputs from the three-parameter example are tabulated in **Tables A-3 to A-18**. See the commentary on the two-parameter model to understand the calculations applied.

Analysis of the three-parameter multi-variable VOPI model shown in *Appendix A* reveals that (a) information related to q_i is by far more valuable than information related to oil price and interference spacing and (b) perfect information about each of the three unknown variables has value. The two multi-variable VOPI manual calculation models serve multiple purposes, even though the cases analyzed were synthetic. These purposes are to:

- establish the methodology used to solve multi-parameter VOPI problems,
- provide a framework that was followed during construction of the generalized multi-parameter VOPI model, and
- validate the output of the generalized multi-parameter VOPI model.

4.2 Generalized Multi-Variable VOPI Model

A major deliverable of this research was the development of a generalized model to solve multi-variable VOPI problems. The model was developed in Microsoft Excel® utilizing Visual Basic®. It can be used to analyze any Excel-based decision model that calculates present value for a set of decision alternatives based on the given input parameters. Both the number of uncertain parameters to be considered and the number of decision alternatives to be considered are variable with no upper limit (other than practical memory storage and computational time limits). The generalized model is built to handle discrete uncertain-parameter distributions and decision alternatives. Any continuous uncertain-parameter or decision distribution has to be discretized to be compatible with the generalized multi-variable VOPI model. There is no upper limit to the discretization granularity that the model can handle (again, other than practical limits).

A flowchart describing the logic of the model is shown in **Fig. 4**. Each step shown in the flowchart is displayed with a manual calculation example in the preceding section. The model indexes all possible combinations of uncertain variables, interacts with an NPV model to fill an NPV array, applies joint-probabilities to calculate overall EOL, applies conditional probabilities to calculate EOL under perfect information for each particular variable if all other variables retain their original level of uncertainty, and finally calculates the VOPI for each uncertain variable (Fig. 4). A display of the generalized model interface without input or output data is shown in **Fig. 5**.

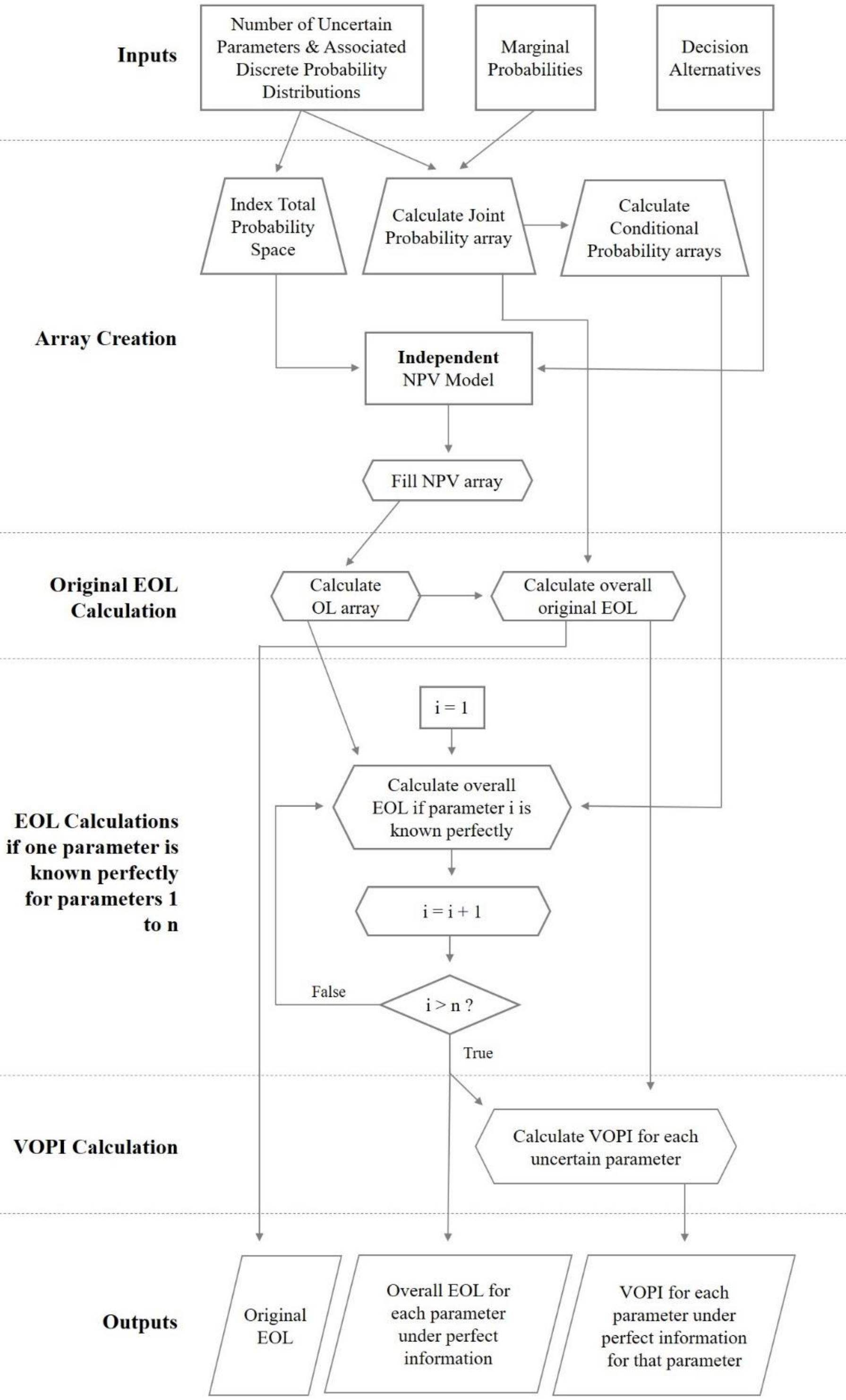

Fig. 4—Logic followed by the generalized multi-variable VOPI model.

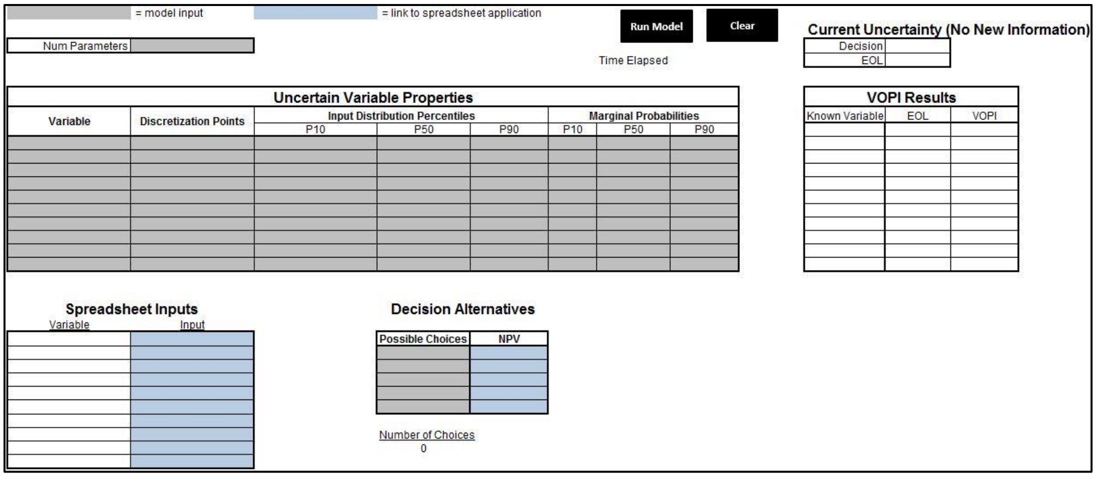

Fig. 5—Generalized multi-variable VOPI calculation model display without input or output data.

Each input required by the generalized multi-variable VOPI model is shaded in gray (Fig. 5). The user must define the number of uncertain variables, the names of each variable, the number of discretization points used to describe the distribution of uncertainty for each variable, the cumulative probabilities on the input distribution for each variable, the marginal probabilities of each point on the input distribution for each variable, and the name of each decision alternative. The display in Fig. 5 is configured for 3-point discretizations; however, the program can handle discretizations of variable granularity. There is no upper limit to how many point values can be used to describe an uncertain input distribution. The cumulative probabilities on the input distributions must be input to the generalized model in ascending order, and an equal number of marginal probabilities summing to 1 must be input to the right of the cumulative probability values on the same row.

The generalized multi-variable VOPI model requires a link to an NPV model for a set of uncertain parameters. The generalized model is compatible with NPV models that output the NPV of a discrete set of decision alternatives based on a set of defined inputs. In Fig. 5, cells that must be linked to the NPV model are highlighted in blue. The generalized multi-variable VOPI model and the NPV model it is being applied to must be linked manually by the user. Once the user has input the uncertain variable names under “Uncertain Variable Properties,” the list of variables under “Spreadsheet Inputs” automatically populates. The user must then link the uncertain variable input cells within the NPV model spreadsheet to the proper cell in the “Input” column underneath “Spreadsheet Inputs.” This allows the generalized model to calculate NPV outputs for the entire probability space. The user of the generalized model must also link the NPV values calculated in the NPV model for each decision alternative to the generalized model. The cells in the “NPV” column under “Decision Alternatives” must be linked to the proper calculated NPV values in the NPV model.

The EOL and VOPI calculations from the generalized multi-variable VOPI model are also displayed in the interface (Fig. 5). Under “Current Uncertainty,” the generalized model outputs the decision that a risk-neutral decision maker should make at the current level of uncertainty and the EOL of that decision. Under “VOPI Results” the generalized model lists each defined uncertain variable, the overall EOL if the variable is known perfectly (and the decision maker has no other new information), and the VOPI for the variable. The list is configured to rank itself in order of descending VOPI.

Both the two-variable and three-variable VOPI scenarios discussed in the methodology section and solved manually were also solved using the generalized model to establish the validity of the generalized model. The solution to the two-variable synthetic model scenario from the previous section is shown in **Fig. 6**. Individual VOPIs listed represent VOPI if only the corresponding variable is known perfectly and the decision maker is privy to no additional new information. The list of VOPI for each unknown variable does not show incremental VOPI as each variable becomes known perfectly in sequence.

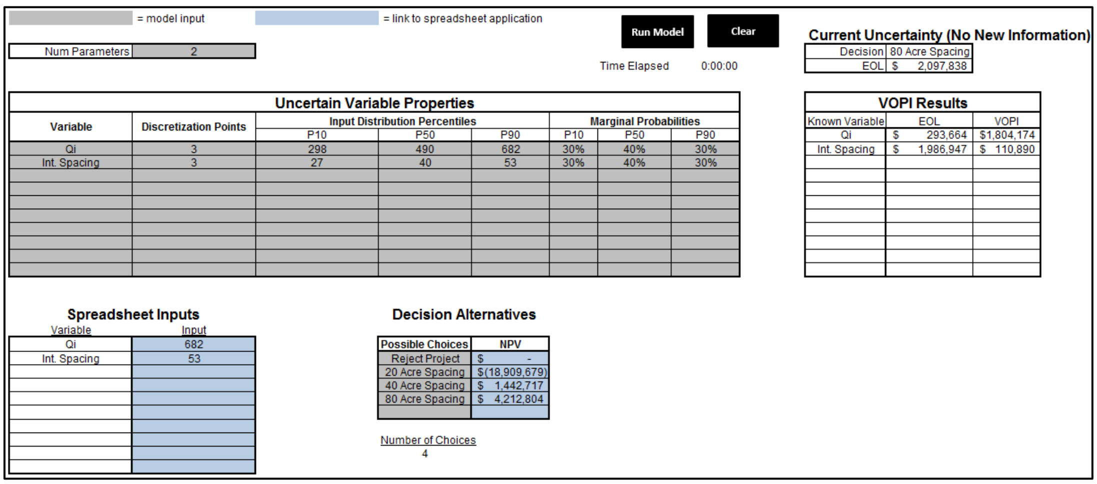

Fig. 6—Generalized multi-variable VOPI model input and calculations for the two-variable synthetic model scenario.

The VOPI scenario that the generalized model solved with the inputs and calculations displayed in Fig. 6 is the same that is solved manually in Tables 1-15. The generalized model output under "Current Uncertainty" matches the manual solution in Table 8 and the generalized model output under "VOPI Results" matches the manual solution in Table 15 within $1 for each VOPI (error is due to rounding within the VOPI model). To further validate the accuracy of the generalized model, intermediate results were compared to the manual calculation. Since the generalized model stores values of NPV and OL in two-dimensional arrays, the output format is different than the manual calculation. However, comparison of the generalized model intermediate results displayed below in **Tables 16 and 17** with the values calculated manually in Tables 4 and 7 confirms the accuracy of the generalized multi-variable VOPI model.

Table 16—NPV array from generalized model solution to the synthetic two-variable scenario. Results match the manual solution in Table 4.

Potential Outcome Num.	Reject Project	20 Acre Spacing	40 Acre Spacing	80 Acre Spacing
1	$ 0	$ (22,223,004)	$ (7,791,949)	$ (3,895,975)
2	$ 0	$ (10,291,524)	$ 316,829	$ 158,415
3	$ 0	$ 1,639,957	$ 8,425,608	$ 4,212,804
4	$ 0	$ (28,144,345)	$ (7,791,949)	$ (3,895,975)
5	$ 0	$ (20,035,567)	$ 316,829	$ 158,415
6	$ 0	$ (11,926,788)	$ 8,425,608	$ 4,212,804
7	$ 0	$ (31,192,098)	$ (10,839,702)	$ (3,895,975)
8	$ 0	$ (25,050,888)	$ (4,698,492)	$ 158,415
9	$ 0	$ (18,909,679)	$ 1,442,717	$ 4,212,804

Table 17—OL array from generalized model solution to the synthetic two-variable scenario. Results match the manual solution in Table 7.

Potential Outcome Num.	Reject Project	20 Acre Spacing	40 Acre Spacing	80 Acre Spacing
1	$ 0	$ 22,223,004	$ 7,791,950	$ 3,895,975
2	$ 316,829	$ 10,608,353	$ 0	$ 158,415
3	$ 8,425,608	$ 6,785,652	$ 0	$ 4,212,804
4	$ 0	$ 28,144,346	$ 7,791,950	$ 3,895,975
5	$ 316,829	$ 20,352,396	$ 0	$ 158,415
6	$ 8,425,608	$ 20,352,396	$ 0	$ 4,212,804
7	$ 0	$ 31,192,098	$ 10,839,702	$ 3,895,975
8	$ 158,415	$ 25,209,304	$ 4,856,907	$ 0
9	$ 4,212,804	$ 23,122,484	$ 2,770,087	$ 0

The generalized model match of the manual solution to the two-variable synthetic VOPI model establishes that the generalized multi-variable VOPI model generates valid results in a two-dimensional scenario. To validate the model's ability to extend the VOPI calculation methodology into higher dimensions, the manual calculations for the three-variable synthetic VOPI scenario were matched with the generalized model as well. The generalized model output can be found in **Fig. A-2**. Intermediate results were also verified. **Tables A-16 and A-17** contain the intermediate results generated by the generalized VOPI model for the three-variable synthetic VOPI scenario.

The generalized model match of the manual solution to the three-variable VOPI model establishes that the generalized multi-variable VOPI model soundly applies multi-variable VOPI methodology to higher-dimensional scenarios. Now that the generalized multi-variable VOPI model had been built and its soundness had been established, the next step was to apply the generalized model to a decision scenario representing the unconventional-reservoir well-spacing decision.

5. APPLICATION OF VOPI MODEL TO UNCONVENTIONAL-RESERVOIR WELL-SPACING DECISION

5.1 Decision Context

In the preceding section, a multi-variable VOPI workflow was established and a generalized model for the application of the workflow to higher-dimensional problems was developed. While in the previous section the generalized multi-variable VOPI model was applied to a synthetic decision context to establish its mathematical accuracy, in this section it is applied to a more realistic well-spacing decision in the context of unconventional reservoir development. A reservoir simulation model was developed based on data found in the industry literature describing the Eagle Ford Shale. Development of the Eagle Ford was chosen as the decision context due to the availability of data describing its properties found in the petroleum industry literature. The specific unconventional reservoir development decision scenario that was analyzed utilizing the reservoir simulation model was the choice of proximity for a series of parallel horizontal wells. Well length and completion/stimulation strategy were considered to be fixed decisions. The goal of the decision maker in this context was to space the parallel horizontal wells optimally for maximization of the expected net present value of the development project. Performing multi-variable VOPI analysis with consideration of certain variables deemed to be of particular interest in the described decision context reveals the power of value-of-information analysis to provide a rational approach for data-acquisition decisions in unconventional reservoir development.

5.2 Quantification of Uncertainty

The first step towards creating a meaningful well-spacing decision model in the context of unconventional reservoir development was deciding which variables to consider in the VOPI calculation. It is recognized that almost all input variables to an unconventional reservoir model have some amount of inherent uncertainty. Although for some variables the inherent uncertainty is small, for other variables the inherent uncertainty is large and is believed to have a significant impact on development decisions. Ideally, all variables with inherent uncertainty would be tested with VOPI analysis. However, due to constraints on time and computational power this is not feasible. The decision maker must choose which variables to consider uncertain and which variables to consider fixed when performing multi-variable VOPI analysis.

Considering the VOPI analysis presented in this section, the primary source of information utilized in the determination of variables to model as uncertain was the survey of Crisman-Berg Hughes members reviewed in Section 3. Opinions of Dr. Duane McVay and Dr. Steven Holditch of the Texas A&M Department of Petroleum Engineering were also taken into account. It is assumed that the only information the decision maker had access to regarding the choice of variables to consider uncertain are the Crisman/Berg Hughes survey presented in Section 3 and the opportunity to discuss the uncertainties with Drs. McVay and Holditch. Synthesizing information from these sources, it was decided to consider gas price, created-fracture propagation, matrix porosity, matrix permeability, and natural-fracture density as uncertain variables in the multi-variable VOPI analysis.

Accuracy in uncertainty assessment is of the utmost importance in the context of VOPI calculations. It is well established in the petroleum literature that accurate uncertainty assessment leads directly to improved decision making (Capen 1976, Brashear 2001). The driving force behind the connection between accurate uncertainty assessment and improved decision making is that accurate output calculations, in any context, are dependent on the accuracy of the input variables. In the context of multi-variable VOPI calculations, the calculated VOPI for each variable could be skewed if the uncertainty of one or more variables is quantified in an inaccurate or biased way. To ensure reliability in the output of VOPI calculations, it is of paramount importance to invest significant effort into the reliable assessment of uncertainty in inputs. The consequences of unreliable uncertainty quantification for VOPI calculation accuracy are addressed in further detail in Section 6.

The price of gas is a significant consideration for decision makers in unconventional-gas-reservoir development. Though a measurement to reduce the uncertainty of the gas price cannot be devised in the same way that a measurement to reduce the uncertainty of the reservoir variables can, a high information value for gas price may provide a financial justification for the hedging of gas sales price. To quantify gas price uncertainty, price data between November 2014 and November 2017 were obtained from the United States Energy Information Administration (EIA) website. The lowest average monthly gas price (1.73 $/Mcf) was considered to be the P10 value, the highest average monthly gas price (4.12 $/Mcf) was considered to be the P90 value, and the average gas price from November 2017 (2.88 \$/Mcf) was considered to be the P50 value (**Table 18**).

Table 18—Estimated distributions for uncertain variables in Eagle Ford reservoir development decision model.

Variable	**P10**	**P50**	**P90**
Gas Price	1.73 \$/Mcf	2.88 \$/Mcf	4.12 \$/Mcf
Matrix Porosity	5.7%	8.7%	11.7%
Matrix Permeability	50 nD	180 nD	480 nD

<table>
<tr><th></th><th>Case 1</th><th>Case 2</th></tr>
<tr><td>Natural-Fracture Density</td><td>No natural fractures</td><td>20 natural fractures per 2,500 square ft of reservoir area</td></tr>
<tr><td>Created-Fracture Propagation</td><td colspan="2">18 cases considered. See Figs. 7 to 12, A-3 to A-8.</td></tr>
</table>

Industry literature was surveyed for the purpose of estimating probability distributions to describe the uncertainty of the reservoir-property and fracture-property variables. A geological study, two reservoir models, and a case study pertaining to the Eagle Ford found in the petroleum literature were considered in the estimation of a representative distribution of uncertainty for matrix porosity. The case study estimated Eagle Ford porosity at 8% (Mullen 2010), the geological study estimated porosity to range from 3% to 10% with an average of 6% (Arguijo 2012), a reservoir model estimated porosity at 8% or 9% (Lalehrokh 2014), and another reservoir model consisted of 13 cases with porosity ranging from 6% to 12% (Gong 2013). P10, P50, and P90 values from the aggregate population made up of porosity data points from all listed sources were calculated to yield a P10 porosity of 5.7%, a P90 porosity of 11.7%, and a P50 value of 8.7% (Table 18).

A geological study and a reservoir model pertaining to the Eagle Ford found in the petroleum literature were considered in the estimation of a representative distribution for uncertainty in matrix permeability. The geological study estimated that the maximum permeability found in the Eagle Ford is 480 nD and the average is 180 nD (Arguijo 2012). The Eagle Ford reservoir model considers permeability to normally be in the tens or hundreds of nanodarcies (Gong 2013). Synthesizing these data, matrix permeability in the decision model was considered to have a P10 value of 50 nD, a P90 value of 480 nD, and a P50 value of 180 nD (Table 18).

The uncertainty in propagation of created fractures was modelled with a set of discrete cases. Three uncertain attributes of the created hydraulic-fracture network were considered: variability in created-fracture half-length, distribution of fracture lateral locations, and interconnectivity of hydraulic fractures from adjacent wells. Three discrete possibilities were considered to model variability in created-fracture half-length, three discrete possibilities were considered to model the distribution of created-fracture lateral locations, and two discrete possibilities were considered to model the interconnectivity of fractures from adjacent wells. The modelling approaches applied for each of these three attributes of created-fracture propagation is presented in the following subsection, "Application of Generalized VOPI Model to Unconventional Reservoir Well-Spacing Decision." These parameters related to created-fracture propagation were combined into one uncertain variable with 18 equally-likely discrete cases (Table 18). This approach was chosen due to the inseparability of obtainable information to describe created-fracture propagation. It is likely not feasible to acquire data that provides information on created-fracture lengths, spacing, or interconnectivity independently.

The petroleum industry literature is sparse on data pertaining to density of natural fractures in the Eagle Ford. This indicates that the uncertainty of the distribution and density of the natural fracture system is large. The Eagle Ford has regions in which natural fractures are present and regions in which they are largely absent (Kahn 2016). To measure the impact of natural fractures being either present or absent in the development area, two cases were considered in the decision model. In Case 1, no natural fractures are present. In Case 2, I assumed 20 natural fractures are present per 2,500 square ft of reservoir area (Table 20). In the absence of information in the industry literature describing the typical distribution and density of natural fractures in the Eagle Ford, these two cases serve to provide insight on the value of knowing whether or not the natural-fracture system will have a significant impact on reservoir performance.

The discretized probability distributions displayed in Table 20 were assigned marginal probabilities based on a 25-50-25 convention (Bickel 2011). Bickel contends that assigning a probability of 25% to the 10th percentile, 50% to the 50th percentile, and 25% to the 90th percentile is the most accurate method to discretize a continuous normal probability distribution using three discretization points. Marginal probabilities for each uncertain variable or case are displayed in **Table 19**.

Table 19—Marginal probabilities for uncertain variables in Eagle Ford reservoir development decision model.

Variable	P10	P50	P90
Gas Price	25%	50%	25%
Matrix Porosity	25%	50%	25%
Matrix Permeability	25%	50%	25%

	Case 1	Case 2
Natural-Fracture Density	50%	50%
Create-Fracture Propagation	Each of the 18 cases are equally likely (5.5% marginal probabilities)	

5.3 Application of Generalized VOPI Model to Unconventional-Reservoir Well-Spacing Decision

A model to describe the above scenario and calculate the NPV array across the total probability space was built using reservoir simulation. Because the natures of the created and natural-fracture networks were key uncertainties to be considered in the VOPI analysis, a reservoir simulation package with a robust ability to model fracture networks found in collaboration with Chai Zhi, a PhD candidate in the Texas A&M Department of Petroleum Engineering, was used. Chai has developed an embedded-discrete-fracture-model (EDFM) reservoir simulator (Chai et al. 2016). In this simulation package, a discrete fracture network is defined by the user. This discrete fracture network is embedded within a traditional grid-block-based reservoir description. Because of its robustness in modelling fracture networks, this EDFM reservoir simulator was used to model the unconventional well-spacing decision. NPV was determined by calculating the present value of discounted cash flow based on gas production forecasted by the EDFM simulator.

The context for the modelled decision scenario is the development of a rectangular section of unconventional dry gas reservoir that is 2,500 ft in width and 4,000 ft in length. Horizontal well length was fixed at 4,000 ft for each well, or the entire length of the modelled section. Well completion decisions were also considered to be fixed. Each horizontal well is designed to have bi-wing hydraulic fractures with half-lengths of 150 ft. Stage spacing was assumed to be 100 ft along the lateral length of the well. All other development decisions, other than the distance between parallel horizontal wells, were considered to be fixed decisions. Specific, discrete well-spacing alternatives were then defined as the decision context for the multi-variable VOPI analysis. Based on review of literature pertaining to well-spacing decisions that oil and gas operating companies are making in the Eagle Ford, 300–600 ft was found to encompass the majority of well-spacing decisions (Lalehrokh 2014). Through testing of different well-spacing alternatives within this range by application of the EDFM reservoir simulator and NPV analysis, it was discovered that the optimal well-spacing decision in the defined decision context is typically between 300 and 500 ft. Therefore, discrete well-spacing decision alternatives of 300, 400, and 500 ft were considered.

In this decision model, certain reservoir simulation and NPV calculation inputs were considered to be fixed, including reservoir thickness, initial reservoir pressure, reservoir depth, particular natural-fracture properties, particular created-fracture properties, and CAPEX per well (**Tables 20 and 21**). Reservoir thickness of 250 ft is typical in the Eagle Ford (Kahn 2016). Depth and pressure gradient maps, along with well production maps, were analyzed to determine reasonable reservoir depth and initial pressure assumptions for a heavily gas-bearing region of the Eagle Ford (Tian 2014). Discrete natural fractures with lengths of approximately 50-100 ft, permeability of 500 mD, and width of 0.1 mm were used to simulate the effects of Eagle Ford natural fractures (Wang 2015). Created-fracture permeability of 10,000 mD is typical in other Eagle Ford reservoir models, with a created fracture-width assumption of 0.01 ft (Gong 2013). Cost data from the United States Energy Information Administration (2017) was analyzed to estimate CAPEX per well. Other economic parameters were assumptions made by the author.

Table 20—Fixed reservoir simulation input assumptions applied to the model used to describe the unconventional well-spacing decision scenario.

Variable	Fixed Assumed Value
Reservoir thickness	250 ft
Initial reservoir pressure	10,000 psi
Depth of reservoir top	12,500 ft
Reference depth	12,500 ft
Created-fracture dip angle	90°
Created-fracture width	0.01 ft
Created-fracture porosity	100%
Natural-fracture width	0.1 mm
Natural-fracture permeability	500 mD
Natural-fracture porosity	100%

Table 21—Fixed economic assumptions applied in the unconventional well-spacing decision scenario.

Variable	Fixed Assumed Value
Water disposal cost	5 $/bbl
Gas transportation cost	0.25 $/Mcf
CAPEX per well	$3,922,500
OPEX per well	$2000 per month
Discount rate	10%

To set up a multi-variable VOPI calculation in the described decision scenario, the EDFM reservoir simulator was used to calculate a gas-production stream for each possible combination of discrete uncertain reservoir and fracture variables defined in Table 18. NPV was then calculated for each of the three potential gas prices defined in Table 18 for each calculated gas-production stream. Microsoft® Excel's Visual Basic code was written to automate interaction with the EDFM reservoir simulator.

To simulate unconventional reservoir performance in the context of the decision scenario described above using the EDFM reservoir simulation package, certain assumptions were made. Reservoir symmetry was assumed to reduce the modelled reservoir simulation area to the area between two parallel horizontal wells, reducing computational needs. Another assumption made to limit computational needs was that the performance of 1/8 of a well can be scaled by a factor of eight to represent the performance of the entire well. Under this assumption, the well length of both wells in each simulation was 1/8 of the true well length described in the decision scenario. Calculated NPV based on simulation output for each discrete point in the probability space of uncertain inputs was then normalized to the entire 4,000-ft by 2,500-ft area to ensure like-to-like comparison between cases. **Fig. 7** displays simulated areas and normalization formulas for each of the considered well-spacing alternatives.

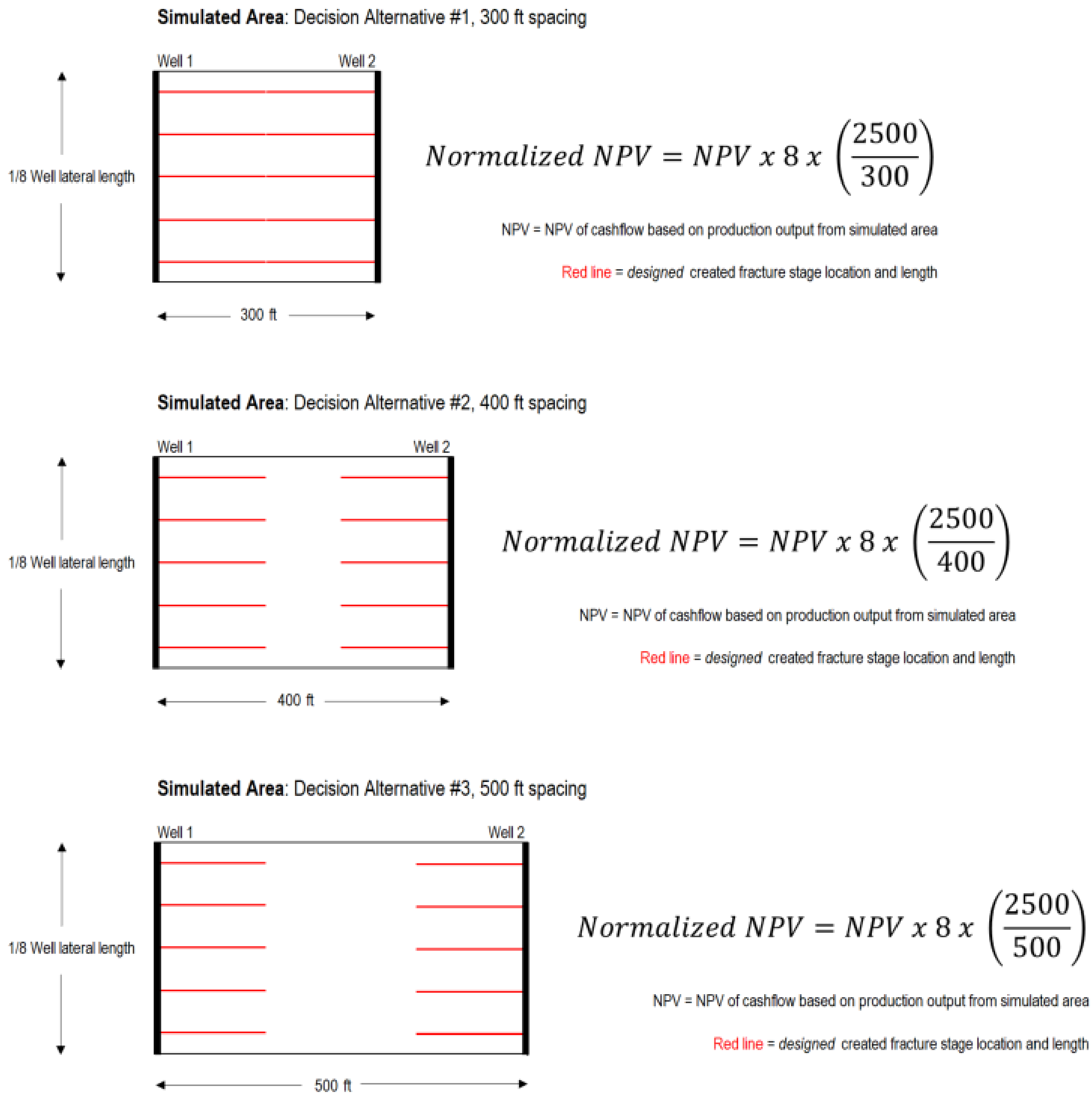

Fig. 7—Simulated areas and normalization formulas for each of the considered well-spacing alternatives.

Discrete cases were used to model the uncertainties related to propagation of created fractures and density of natural fractures. Eighteen cases were considered to represent the uncertainty related to propagation of created fractures (**Figs. 8 to 13**, **A-3 to A-8**). These eighteen cases represent all possible combinations of three sub-cases for variability in created-fracture half-length, three sub-cases for distribution of fracture lateral locations, and two sub-cases for interconnectivity of fractures in adjacent wells.

Variability in created-fracture half-length was modelled by sampling from distributions with different standard deviations and mean equal to the fixed designed fracture half-length. Microseismic measurements in the Eagle Ford have indicated that it is typical for fracture half-lengths to vary considerably from the designed length, up to 100 ft longer or shorter than the target (Centurion 2014). Another Eagle Ford model has indicated variability in created-fracture half-lengths of roughly 30–50 ft (Lalehrokh 2014). A qualitative synthesis of this information led to the creation of the following three sub-cases: standard deviation of 20 ft, standard deviation of 40 ft, and standard deviation of 80 ft (Figs. 8 to 13, A-3 to A-8).

To model the uncertainty in lateral distribution of created fractures, three sub-cases were considered in the decision model (Figs. 8 to 13, A-3 to A-8). In the first sub-case, the created fractures were uniformly distributed. In the second sub-case, the distribution of created fractures was partially uniform and partially random. In the third sub-case, the distribution of created fractures was fully random. The inspiration for modelling uncertainty related to the distribution of fracture lateral locations in this manner was an Eagle Ford case study in which a well core taken from a section of reservoir previously stimulated by hydraulic fracturing was analyzed (Raterman 2017). In this study, created fractures were found to be unevenly spaced in the lateral direction and not necessarily forming near perforation clusters. Also, the number of discrete created fractures identified was much greater than the number of fracture stages. Ten discrete created fractures were defined for the simulated length of both wells in each case to incorporate this observation (Figs. 8 to 13, A-3 to A-8).

Interconnectivity between fractures in adjacent wells was modelled by considering two sub-cases. In the first sub-case, specific lateral locations of the randomly located created fractures are the same in both wells (Figs. 8 to 13). This results in a simulation in which the created fractures of the two simulated wells mirror each other exactly. In the second sub-case, specific lateral locations of the randomly located created fractures are different between the two wells (Figs. A-3 to A-8). This results in a simulation in which the created-fracture networks of the two simulated wells do not mirror each other. Since parallel planes were used to represent created fractures, mirroring the created-fracture networks of the two simulated wells results in a model in which created fractures are more likely to intersect (Figs. 8 to 13). In the non-mirrored fracture network sub-case, parallel planar created fractures are much less likely to intersect (Figs. A-3 to A-8).

Two discrete natural-fracture-density cases were considered to represent the uncertainty associated with the natural-fracture system (Figs. 8 to 13, A-3 to A-8). The first case assumes that natural fractures do not have a significant effect on reservoir performance. Therefore, no natural fractures were included in EDFM reservoir simulations in which Case 1 was considered (Figs. 8, 10, 12, A-3, A-5, and A-7). Three separate discrete natural-fracture systems were generated to be utilized for natural-fracture-density Case 2 simulations (Figs. 9, 11, 13, A-4, A-6, and A-8). Each of the generated natural-fracture systems were designed to be utilized in simulations pertaining to one of the three well-spacing alternatives. The natural-fracture density was kept consistent, but a different realization was needed for each well-spacing alternative because of variance in the reservoir area being simulated. For each of the three natural-fracture systems generated, random variables were utilized to determine the length, orientation, and location of each natural fracture. All locations and orientations were considered equally likely, within the constraints of the defined density. In the absence of information about the length of natural fractures in the Eagle Ford available in the petroleum literature, natural fractures were assumed to vary from 50 to 150 ft on a uniform distribution (Wang 2015). Natural-fracture length, orientation, and location were kept constant across each simulation of each well-spacing alternative to ensure consistency (Figs. 9, 11, 13, A-4, A-6, and A-8). The three well-spacing alternatives, 18 created fracture-propagation cases, and two natural-fracture-density cases add up to 108 different possible combinations of reservoir area and discrete created/natural-fracture locations to be simulated using the EDFM reservoir simulator.

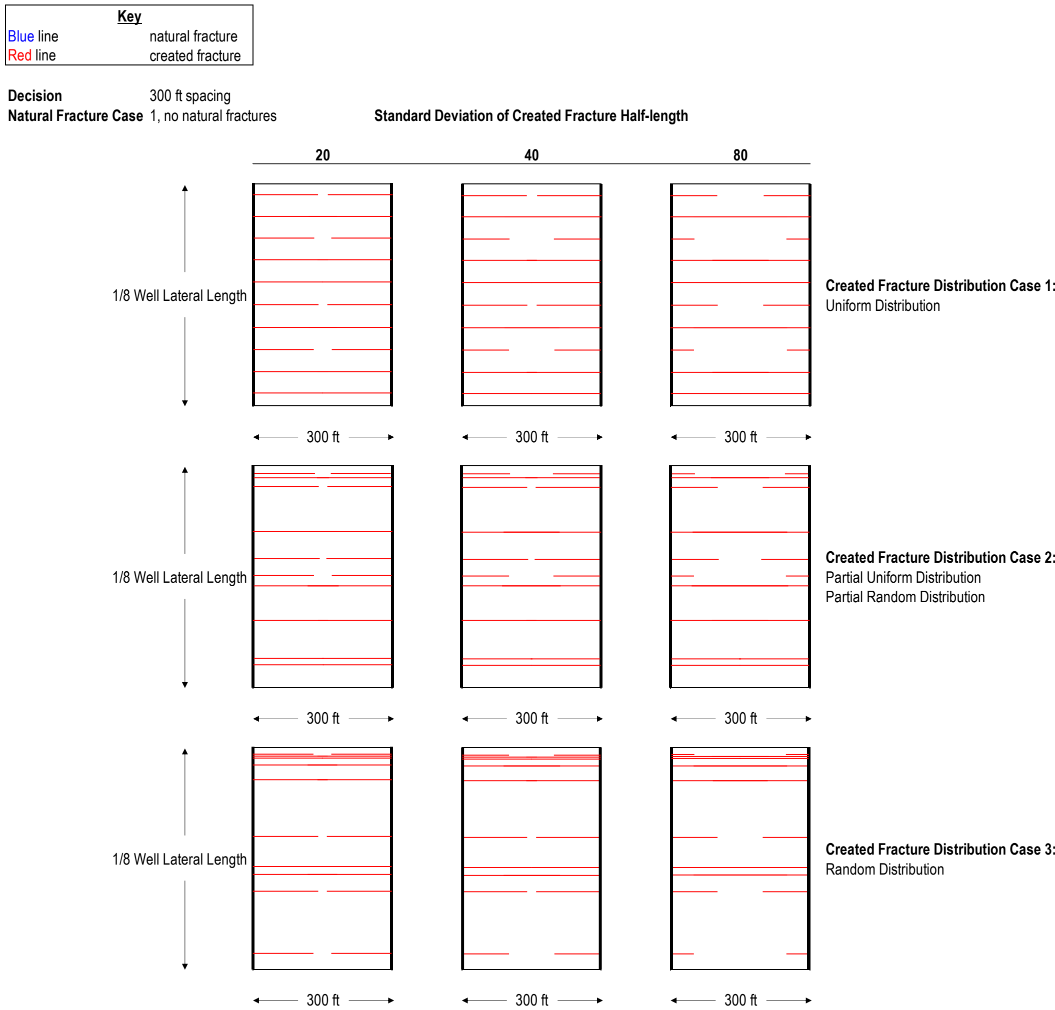

Fig. 8—Discrete fracture network cases considered under the no natural fractures case when the well-spacing decision is 300 ft: Mirror image wells.

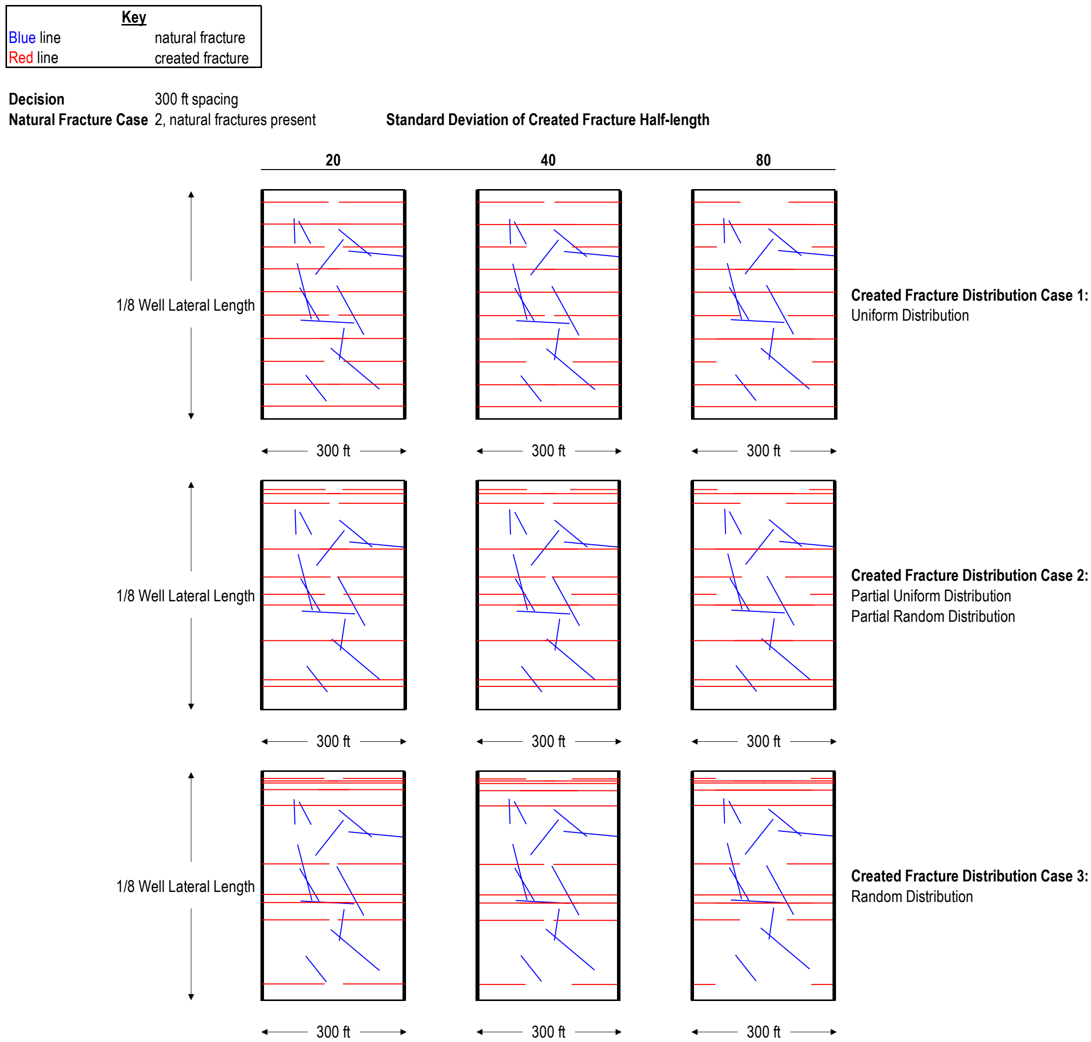

Fig. 9—Discrete fracture network cases considered under the natural fractures present case when the well-spacing decision is 300 ft: Mirror image wells.

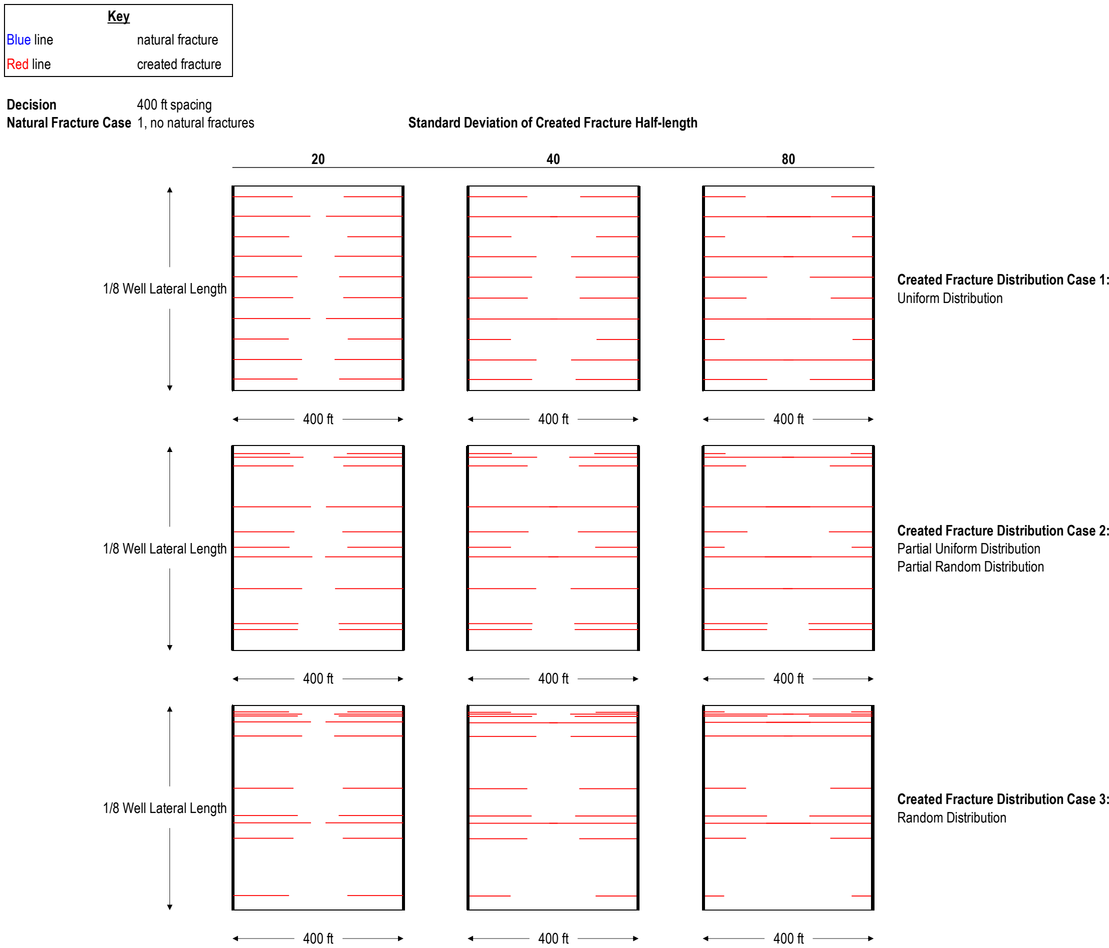

Fig. 10—Discrete fracture network cases considered under the no natural fractures case when the well-spacing decision is 400 ft: Mirror image wells.

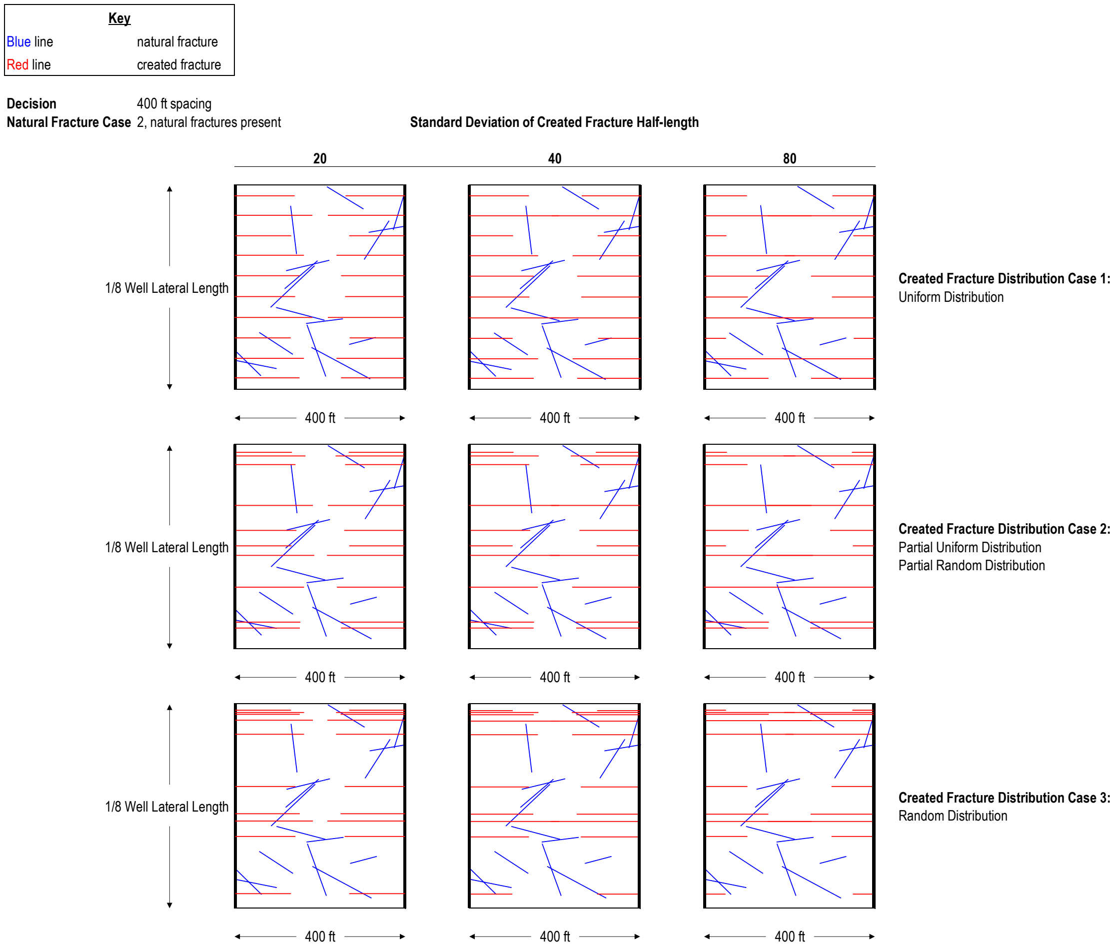

Fig. 11—Discrete fracture network cases considered under the natural fractures present case when the well-spacing decision is 400 ft: Mirror image wells.

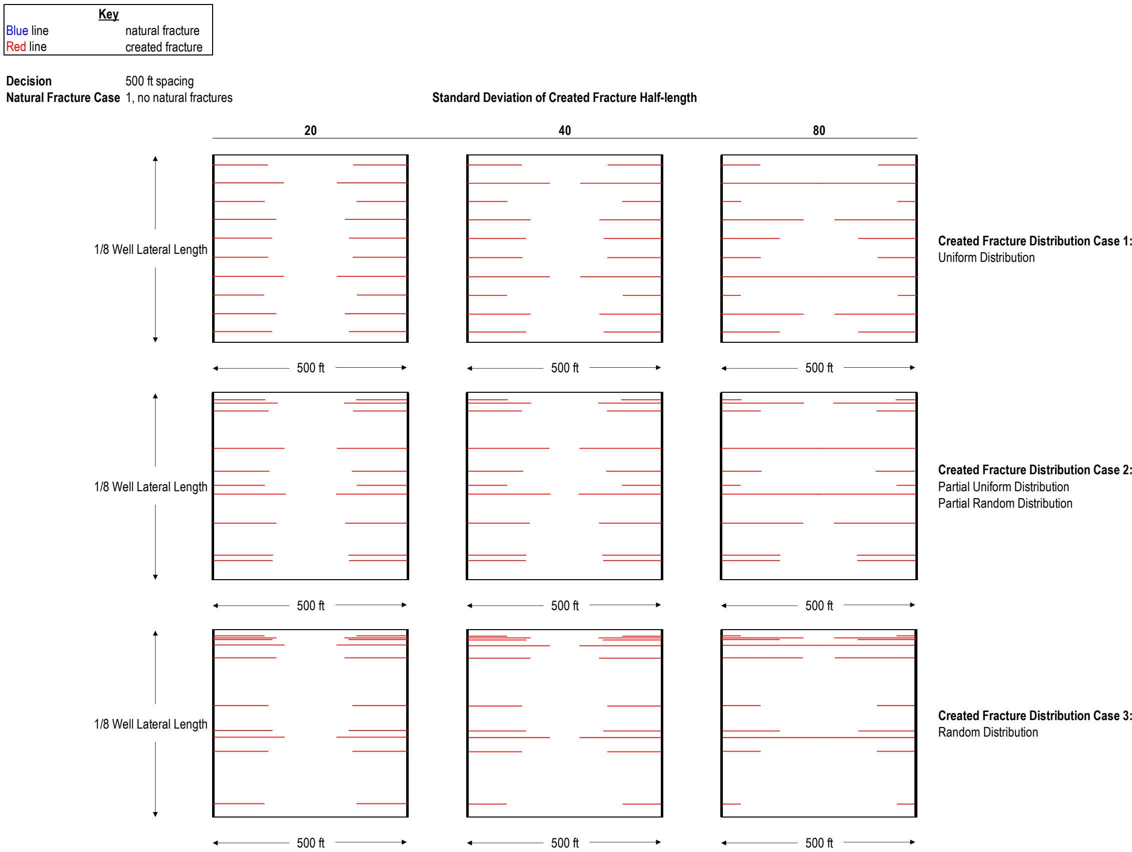

Fig. 12—Discrete fracture network cases considered under the no natural fractures case when the well-spacing decision is 500 ft: Mirror image wells.

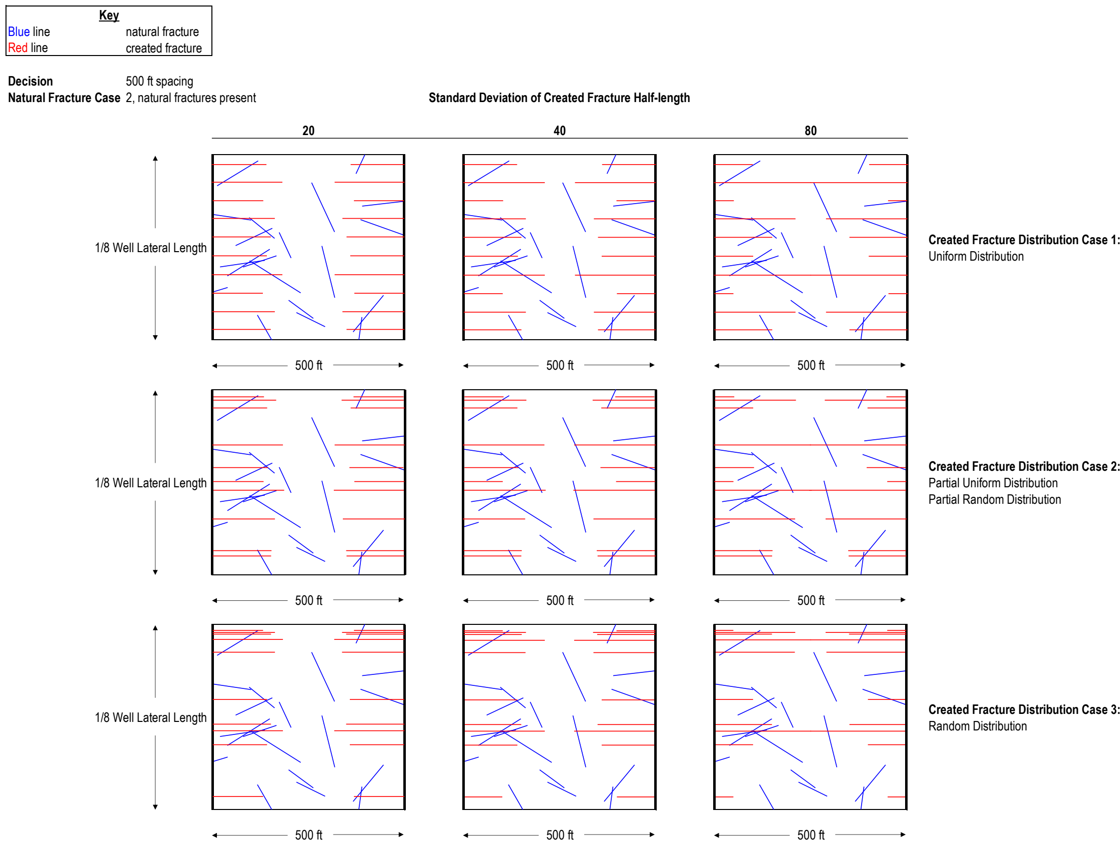

Fig. 13—Discrete fracture network cases considered under the natural fractures present case when the well-spacing decision is 500 ft: Mirror image wells.

To sample the total probability space, each of the 108 possible combinations of well spacing and created/natural-fracture locations were simulated with every combination of the three considered matrix porosities and three considered matrix permeabilities. In total, gas-production streams for 972 different, discrete scenarios were simulated. Including the three possible gas prices considered in this decision scenario, the total number of discrete outcomes was 2,916. Each of the three possible well-spacing alternatives have 972 potential discrete outcomes in their probability space. Rejecting the development project, resulting in an NPV of $0, was also considered as a decision alternative.

An indexing system was created so that the generalized multi-variable VOPI model could interact with the Eagle Ford well-spacing NPV model. Given any possible combination of discrete uncertain parameters, the indexing system returns the NPV of the development project under each considered decision alternative. The generalized multi-variable VOPI model then used the indexing system to build the NPV array for the total probability space. The generalized VOPI model calculated EOL for each decision alternative considered in the unconventional well-spacing decision scenario and found that 400-ft spacing is the optimal decision at the original level of uncertainty (**Table 22**).

Table 22—EOL values under the original level of uncertainty for each considered decision in the unconventional well-spacing decision scenario.

Decision Alternative	EOL
300 ft Spacing	$7,850,037
400 ft Spacing	$3,457,590
500 ft Spacing	$6,384,307
Reject Project	$48,198,140

The multi-variable VOPI model calculated VOPI for each uncertain variable, assuming in each case that the decision maker has no other new information and that each variable is independent of every other variable (**Table 23**).

Table 23—Multi-variable VOPI calculation outputs for unconventional well-spacing decision scenario.

Decision at current uncertainty	400 ft spacing
EOL at current uncertainty	$3,457,590

VOPI Results		
Known Variable	**EOL**	**VOPI**
Gas Price	$2,494,165	$963,425
Created-fracture propagation	$2,911,388	$546,202
Matrix porosity	$2,927,996	$529,594
Matrix permeability	$3,457,592	$(2)
Natural-fracture density	$3,457,593	$(2)

The VOPI calculation results indicate that the uncertain variable for which perfect information has the most value, approximately $960,000, is the gas price (Table 23). This is unsurprising, given that gas prices ranged from 1.73 $/Mcf to 4.12 $/Mcf over the period of November 2014—November 2017 and commodity prices have an impact on the health of the oil and gas industry. There was a large step down in value between the VOPI of gas price and the variable with the second highest VOPI, which was created-fracture propagation (Table 23). The VOPI of the propagation of the created fractures was calculated to be approximately $550,000. This indicates that the uncertainty related to the propagation of the created-fracture network (fracture lengths, fracture spacing, and fracture interconnectivity) has significant information value. Matrix porosity was found to have a VOPI of near $530,000 (Table 23). This indicates that the porosity in an unconventional gas reservoir has a significant effect on the information value. The calculated VOPIs for matrix permeability and natural-fracture density are both zero. This indicates that further reduction of uncertainty related to these two parameters may not be necessary for optimization of the well-spacing decision presented in this research. This does not mean that information regarding matrix permeability or natural-fracture density has no value in any decision context within unconventional reservoir development. High matrix permeability and dense natural fractures have positive impacts on the EV of unconventional reservoir development projects. For decision contexts such as choice of resource play to develop or choice of lease locations, these variables may have high VOPI. They also may have high VOPIs in well-spacing decision contexts for which the probability that the development project will result in an economic loss is higher than the chance of economic loss in the decision context presented in this example. What is indicated by these results is that gas price, matrix porosity, and propagation of created fractures may be much more relevant considerations in the well-spacing development decision presented in this research than matrix permeability and natural-fracture density.

According to Hubbard (2014), VOII is closely tied with VOPI. Also, VOPI gives an indication of relative information values, which may be more important to a decision maker than absolute information values. Hubbard suggests estimating VOII as 10% of VOPI as a rule-of-thumb. However, one must be cautious when using this rule-of-thumb. It is not a scientifically rigorous calculation, but simply an indication of the likely order of magnitude of VOII. A rigorous VOII calculation would produce more accurate VOII calculation results. However, a multi-variable VOII calculation would be quite difficult due to the added dimension of assessing the uncertainty regarding the accuracy of acquired information. The data regarding the accuracy of the information acquired would be difficult, if not impossible, to obtain. Using Hubbard's rule of thumb, estimation of VOII based on calculated VOPI was determined for each uncertain variable (**Table 24**). Note that all value estimations represent value per development area under consideration in the NPV model. In this scenario, estimated VOIIs are indicative of VOII per 2,500 ft by 4,000 ft (approximately 230 acres) of reservoir area.

Table 24—Indication of VOII for each uncertain variable.

Variable	VOPI	VOII Estimation
Gas price	$ 963,425	~$ 96,000
Created-fracture propagation	$ 546,202	~$ 55,000
Matrix porosity	$ 529,594	~$ 53,000
Matrix permeability	$ 0	$ 0
Natural-fracture density	$ 0	$ 0

The variable with the highest information value is gas price. Though uncertainty regarding the future price of gas cannot be reduced through data acquisition, E&P companies developing unconventional reservoirs can reduce the uncertainty of the future price their gas will sell at through hedging. Since hedging guarantees the future sales price of gas for a period of time, the information that it provides is more precise than information typically obtained through data acquisition. Therefore, the value of reducing the uncertainty of gas price through hedging is likely closer to VOPI than indicated by Hubbard's rule-of-thumb. The VOII (in this case, the value of hedging) should always be considered in comparison with the cost of information. In this scenario, hedging is recommended if the cost of the hedging contract is significantly less than approximately $963,000.

The VOII indications for created-fracture propagation and matrix porosity are, respectively, $55,000 and $53,000 (Table 24). Certain types of data could potentially be acquired by the decision maker to reduce the uncertainty in these variables. For example, microseismic data or radioactive-proppant completion diagnostic data could be acquired to reduce the uncertainty regarding the propagation of the created hydraulic fractures. Core data could be acquired to reduce the uncertainty regarding the matrix porosity. In this decision context, the justified spending level for acquiring microseismic data and/or radioactive-proppant completion diagnostic data is approximately $55,000 per 230 acres. Similarly, the justified spending level for acquiring core data in the described decision context is approximately $53,000 per 230 acres.

The VOPI analysis indicates that gas price dwarfs other uncertain variables with regard to the impact on the optimal development decision. Considering the gas price to be fixed at 2.88 $/Mcf, the November 2017 price, the VOPI for the other four uncertain variables was re-calculated to test their sensitivity to the inclusion of gas price in the VOPI calculation (**Table 25**).

Table 25—Multi-variable VOPI calculation outputs for unconventional well-spacing decision scenario excluding consideration of gas price.

Decision at current uncertainty	400 ft spacing
EOL at current uncertainty	$1,711,026

VOPI Results		
Known Variable	**EOL**	**VOPI**
Created-fracture propagation	$1,146,215	$564,811
Matrix porosity	$1,204,499	$506,527
Matrix permeability	$1,711,026	$ -
Natural-fracture density	$1,711,026	$ -

The calculated VOPI for each of the four uncertain variables considered in the VOPI analysis that excludes the gas price is reasonably similar to their calculated VOPI when gas price is also considered. This is likely because reservoir and fracture properties are independent of the gas price. However, if the gas price is low enough, information on reservoir and fracture properties would lose its value because the development would not be economically feasible no matter what the reservoir and fracture properties were.

The results of the VOPI analysis for the Eagle Ford unconventional-reservoir well-spacing decision indicate that a rational approach for data-acquisition decisions is achievable through creation of a reliable decision model and multi-variable VOPI analysis. Application of similar VOPI analysis to unconventional reservoir development decision models could lead to improved data-acquisition decisions. If data-acquisition decisions were improved, financial performance would be improved because of better informed decision making and less capital spent on insignificant data. Research institutions could also benefit from utilizing similar VOPI analysis with the goal of focusing research efforts on reducing uncertainties that have high information values. However, it is critical that uncertainty be reliably quantified in the decision models used for multi-variable VOPI analysis. Consequences of biased uncertainty quantification on VOPI calculations are addressed in Section 6.

6. EFFECT OF OVERCONFIDENCE AND DIRECTIONAL BIAS ON VOPI ANALYSIS

6.1 Effect of Biases on VOPI in a Simple-Context Experiment

McVay and Dossary (2012) showed that biases in the quantification of uncertainty can lead to sub-optimal project selection and a negative impact on financial performance in the oil and gas industry. While McVay and Dossary's 2012 research focused on the negative effects that biases in uncertainty quantification have on project selection, the focus of the research presented in this thesis is VOPI calculations. For this reason, the effect that biases in uncertainty quantification have on VOPI calculations was investigated. By understanding how biases affect VOPI studies, we can quantify the extent to which information is overvalued or undervalued due to biases. Overvaluing or undervaluing information can, in some circumstances, lead to sub-optimal data acquisition decisions.

The methodology that McVay and Dossary used to assign biases to continuous distributions was applied in this research to analyze the effect that biased uncertainty quantification has on VOPI studies. McVay and Dossary's bias assignment model is based on the premise that human biases affecting decision making in the oil and gas industry can be summarized by two primary bias types: overconfidence bias and directional bias. These biases are in reference to what may be considered the "true" distribution of possible outcomes. What is meant by a "true" outcome distribution is that the assessed range of uncertainty is the exact same as it would have been if the assessor was perfectly calibrated (completely unbiased).

Overconfidence bias is concerned with the variance of the estimate. The bias-assignment model involves a quantity ranging between 0 and 1 given to specify the fraction of the “true” distribution of an uncertain parameter that is not considered in the biased assessment of uncertainty (**Fig. 14**). In this model, an overconfidence bias of 0 means that the entire “true” distribution is considered by the estimated distribution. At the other end of the spectrum, an overconfidence parameter of 1 represents a deterministic estimate. The overconfident assessment of uncertainty is generated by truncating the “true” distribution at its tails according to the overconfidence parameter (Fig. 14). For example, assuming no directional bias, an overconfidence parameter of 0.5 means that the estimated distribution is only considering the most likely 50% of possible outcomes. In this case, 25% of the total “true” probability is truncated from each end of the “true” distribution.

Directional bias is concerned with the central tendency of the estimate. The bias model involves a quantity ranging between -1 and 1 given to represent the extent to which the central tendency of an estimate is optimistic or pessimistic (Fig. 14). A directional bias parameter of -1 corresponds to complete pessimism, meaning that the estimated distribution considers only the most pessimistic outcomes. This means that the percent of the “true” outcome distribution that is not considered (specified by the overconfidence parameter) is truncated entirely from the high end of the distribution. If the directional bias parameter is 1, this corresponds to complete optimism and means that only the low end of the “true” distribution is truncated to generate the biased distribution (Fig. 14). These definitions of optimism and pessimism apply to value-based parameters; they would be reversed for cost-based parameters. A directional bias of 0 means that the high end and low end of the "true" distribution are truncated equally to generate the biased distribution. For other values of the directional bias parameter, linear interpolation is used to determine how much to truncate each end of the distribution (Fig. 14). In this model, there can be directional bias only if the overconfidence parameter is greater than 0. This is because if the entire "true" range of possible outcomes is being considered (no overconfidence), the location of the assessed distribution within the "true" distribution cannot be changed. Therefore, the directional bias parameter has no meaning when the overconfidence parameter = 0 (Fig. 14).

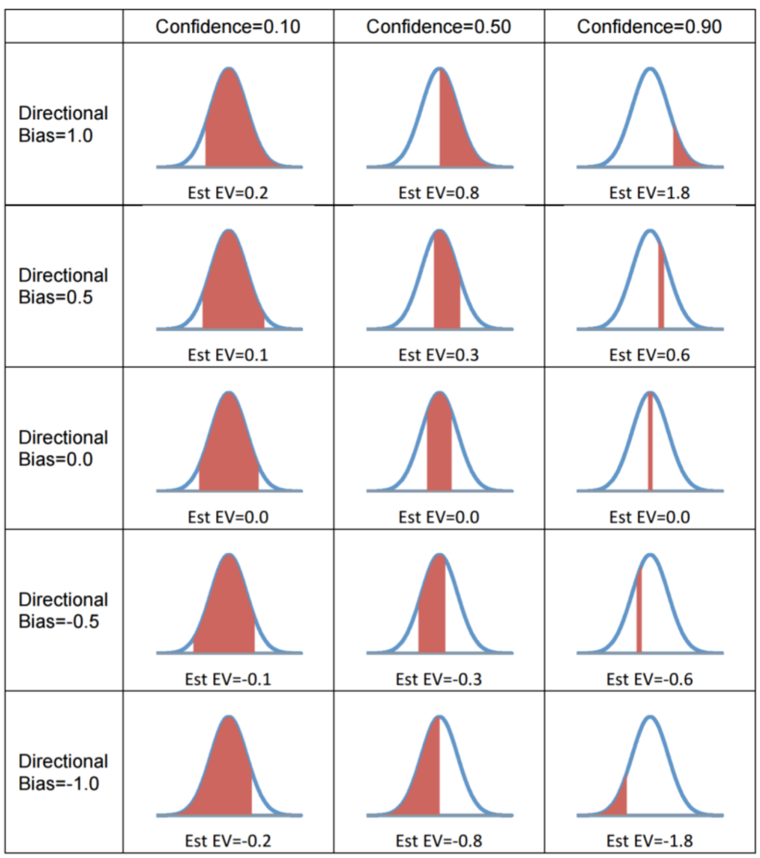

Fig. 14—Relationship between the estimated distribution (shaded) and the "true" distribution (unshaded) for a sample of overconfidence/directional bias combinations for a normally distributed, value-based parameter. Reprinted with permission (McVay and Dossary 2012).

The goal of this portion of the research was to compare the VOPI that would be calculated using the "true" distribution for an uncertain variable with the VOPIs that would be calculated using a series of biased distributions. For the sake of simplicity, an experiment in which the uncertain variable was the NPV of the project was designed. The scenario was a simple go/no-go decision where the threshold for expected NPV was $0 and the uncertainty about whether the project would meet that threshold was large (**Fig. 15**).

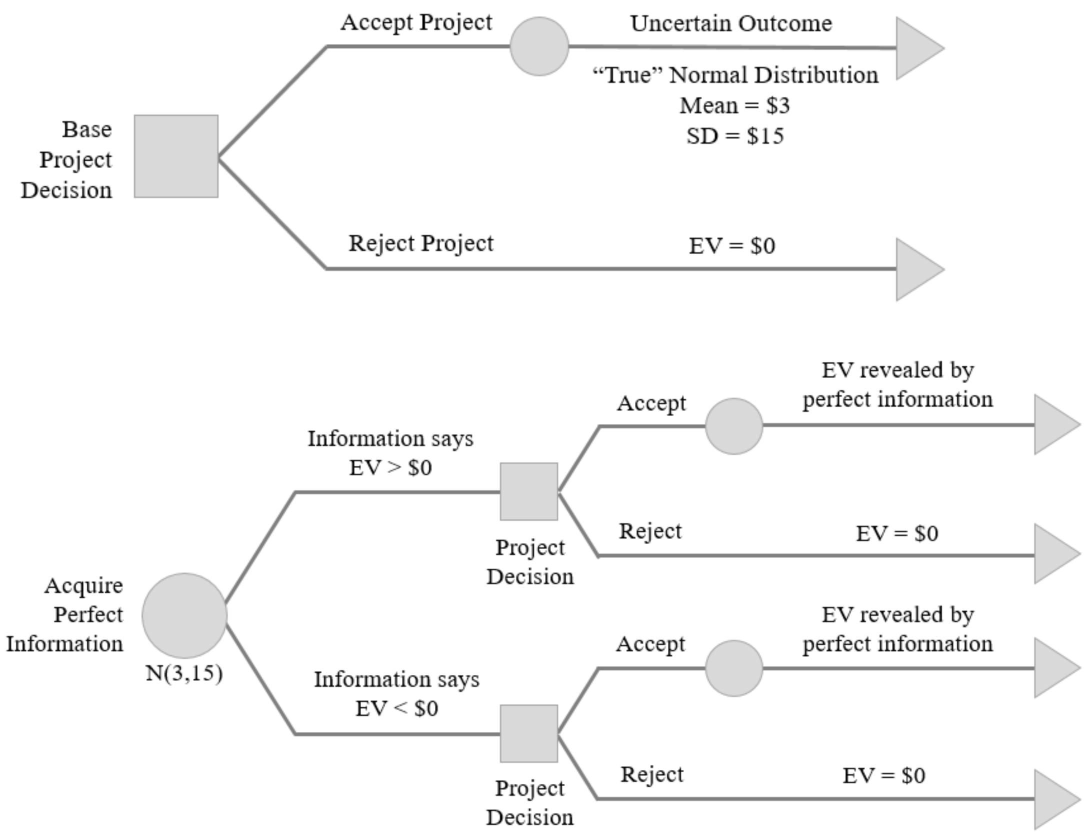

Fig. 15—Decision scenario for the simple-context effect-of-biases-on-VOPI-calculations experiment.

It is recognized that, in reality, there is no information that can predict the value of an oil and gas project with 100% accuracy. However, this simple scenario is helpful to understand the effects that uncertainty quantification biases have on value-of-information studies in general. The "true" distribution of project NPV was defined to be a normal distribution with a mean of \$3 and a standard deviation of \$15 (**Fig. 16**).

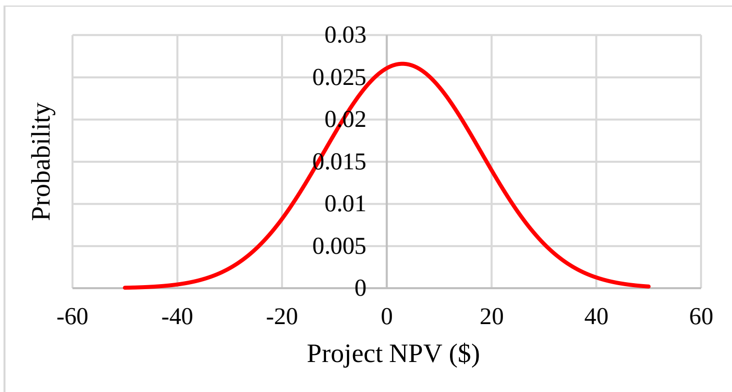

Fig. 16—"True" project value probability distribution for the simple-context effect-of-biases-on-VOPI-calculations experiment.

The expected value of the project was calculated by evaluating the following integral, where *f(x)* is the "true" project probability distribution (Fig. 16) and *x* is the project NPV:

$$EV = \int_{-\infty}^{\infty} x\, f(x)\, dx = \$3$$

The above equation calculates the EV of the upper decision tree in Fig. 15. The expected value with perfect information (EVWPI), the lower decision tree in Fig. 15, was calculated by evaluating the integral below:

$$EVWPI = \int_{-\infty}^{\infty} \max(0, x)\, f(x)\, dx$$

$$EVWPI = \int_{-\infty}^{0} 0\, f(x)\, dx + \int_{0}^{\infty} \mathrm{x}\, f(x)\, dx$$

$$EVWPI = \int_{0}^{\infty} \mathrm{x}\, f(x)\, dx = \$7.6$$

The above integral was evaluated numerically to calculate EVWPI of the project under consideration. The VOPI regarding the uncertain variable, project value, was calculated by subtracting EV from EVWPI.

$$VOPI = EVWPI - EV = \$7.6 - \$3 = \$4.6$$

Next, numerous combinations of overconfidence bias and directional bias (Fig. 14) were applied to the "true" project value distribution (Fig. 16) to obtain an array of truncated, biased uncertainty assessments regarding project value. Truncation parameters were determined based on the overconfidence bias and directional bias for each biased uncertainty assessment. EV and EVWPI based on the biased uncertainty assessments were calculated by numerically integrating the following integrals, where a and b are truncation parameters:

$$EV = \max\left(0, \int_{a}^{b} x\, f(x)\, dx\right)$$

$$EVWPI = \int_{a}^{b} \max(0, x)\, f(x)\, dx$$

The VOPI for project value was calculated under each combination of bias parameters based on the results of the above truncated integrals. The results indicate that overconfidence bias in a parameter's uncertainty assessment leads to underestimation of

VOPI for that parameter (**Fig. 17**), and the greater the overconfidence, the greater the underestimation of VOPI. This is because overconfidence yields narrower distributions (lower uncertainty), so there is less potential for uncertainty reduction through information acquisition.

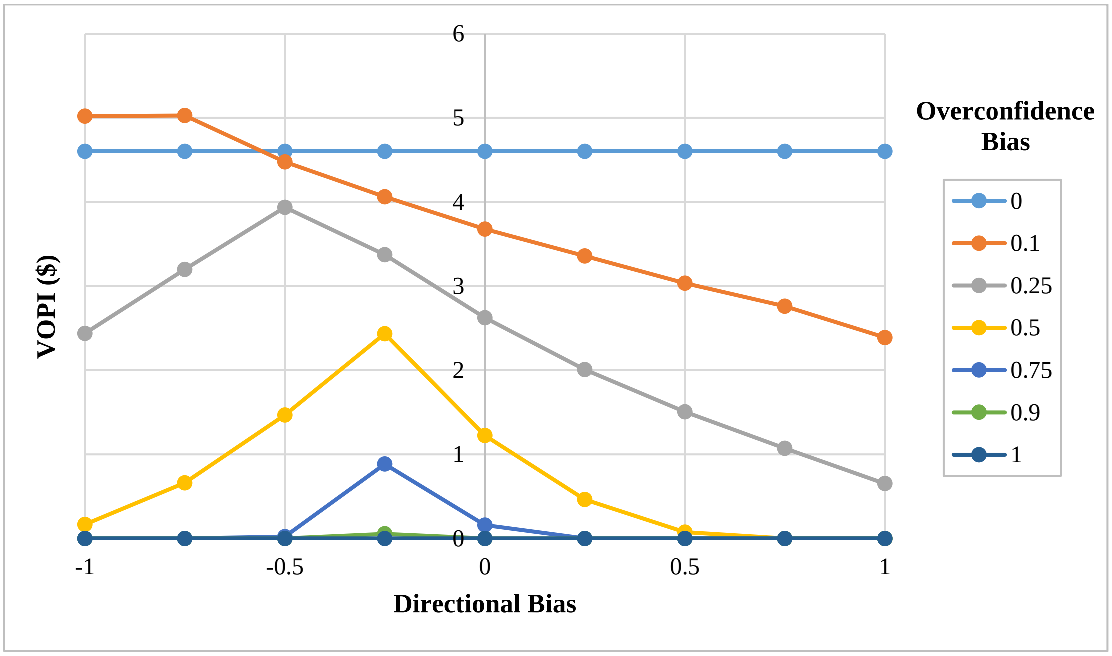

Fig. 17—Results from the simple-context go/no-go experiment investigating the effect of overconfidence and directional biases on VOPI.

These calculations also indicate that, in general, at low to moderate values of overconfidence, moving from pessimism (negative directional bias (DB)) towards optimism (positive DB) reduces VOPI in the decision context considered. This is because increasing DB moves the truncated estimated distribution in the positive direction, where there is less overlap of the zero-NPV value, which results in less chance of a decision change due to new information. At moderate to high values of overconfidence, there is a peak in VOPI and increasing pessimism (decreasing DB) from the peak reduces VOPI. At the peak, the estimated distribution straddles the zero-NPV value and, thus, there is the greatest opportunity for a decision change due to information. Decreasing DB from this point moves the truncated estimated distribution into negative values with less overlap of the zero-NPV value, thus reducing the opportunity for decision change. The results from this simple experiment are not necessarily representative of the impact that biases will have on VOPI for all scenarios. However, these results establish that, in general, overconfidence bias in uncertainty quantification (which is common in the industry) will reduce calculated VOPI values. This could have significant impacts on data acquisition practices in the industry.

6.2 Effect of Biases on VOPI Analysis for Well-Spacing Decision

To demonstrate the effects of biases in uncertainty quantification on VOPI calculations in a more meaningful context, the effect of overconfident and optimistic quantification of uncertainty on the VOPI calculation presented in Section 5 was investigated. This was accomplished by introducing bias in the quantification of uncertainty regarding the gas price, the variable previously calculated to have the highest VOPI. The expected NPV associated with 300, 400, and 500-ft well-spacing decisions was calculated for an array of discrete cases making up the probability space was calculated previously in the research presented in Section 5. In the assessment of uncertainty presented in Section 5, gas price uncertainty was considered discretely with a P10 value of 1.73 \$/Mcf, a P90 value of 4.12 \$/Mcf, and a P50 case value of 2.88 \$/Mcf (Table 18). A probability of 25% was assigned to the P10 case, a probability of 50% was assigned to the P50 case, and a probability of 25% was assigned to the P90 for calculation of VOPI (Table 19). This original quantification of uncertainty regarding gas price is summarized below in **Table 26**.

Table 26—Original assessment of uncertainty regarding the gas price.

Value	Probability
1.73 $/Mcf	25%
2.88 $/Mcf	50%
4.12 $/Mcf	25%

To assess the impact of bias in uncertainty assessment on multi-parameter VOPI calculations, the above gas price probability distribution was perturbed and the results were observed. Probability weightings of each considered discrete gas price were adjusted to reflect characteristics of overconfidence and optimism bias. In the biased uncertainty assessment, 1.73 $/Mcf gas price was assigned a probability of 10%, 2.88 $/Mcf gas price was assigned a probability of 65%, and 4.12 $/Mcf gas price was assigned a probability of 25% (**Table 27**). Assigning a higher probability to the 2.88 $/Mcf gas price mimics overconfidence bias by placing a considerably increased weight on this value. Skewing the probability assessment in the direction of the high case mimics optimism bias (**Fig. 18**).

Table 27—Biased assessment of uncertainty regarding the gas price.

Value	Probability
1.73 $/Mcf	10%
2.88 $/Mcf	65%
4.12 $/Mcf	25%

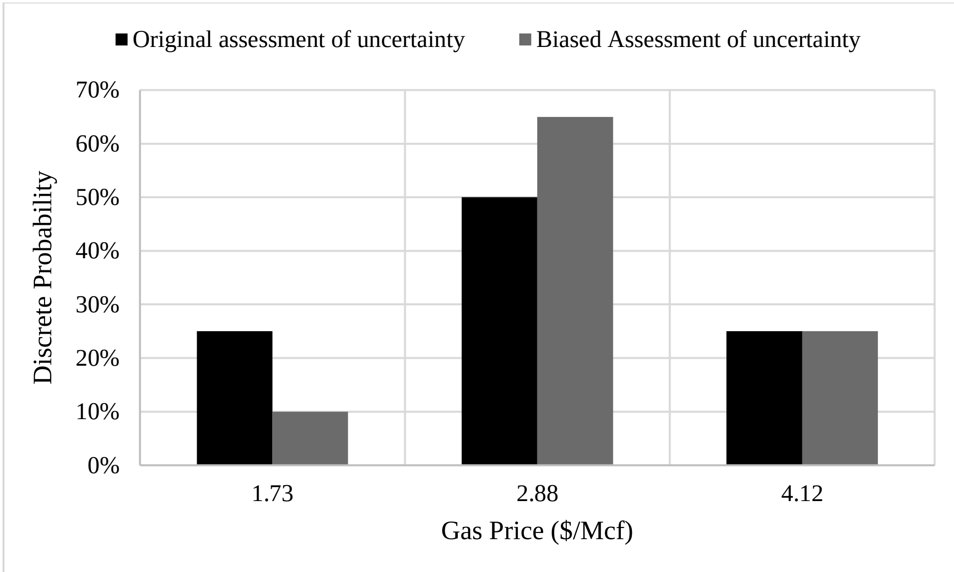

Fig. 18—Original assessment of uncertainty vs. biased assessment of uncertainty regarding gas price.

Using the probability weights displayed above (Table 27) for gas price and original probability weights for the other variables, the generalized multi-variable model calculated VOPI for each uncertain variable considered in the unconventional well-spacing decision context presented in Section 5. VOPI outputs are displayed below in **Table 28**.

Table 28—Multi-variable VOPI calculation outputs for unconventional well-spacing decision scenario under biased uncertainty assessment of gas price.

Decision at current uncertainty	400 ft spacing
EOL at current uncertainty	$2,750,215

VOPI Results		
Known Variable	**EOL**	**VOPI**
Matrix porosity	$2,043,689	$706,525
Gas Price	$2,195,409	$554,806
Created-fracture propagation	$2,263,679	$486,535
Natural-fracture density	$2,750,213	1
Matrix permeability	$2,750,214	1

Comparison of VOPI calculated under biased uncertainty assessment to the VOPI calculation under the original assessment of uncertainty (Table 23) reveals that the overconfident and optimistic assessment of uncertainty resulted in an undervaluation of the gas price VOPI by approximately 42%, or $400,000. The overall VOPI and the VOPI regarding created-fracture propagation were also significantly underestimated with biased assessment of gas price uncertainty, while VOPI for matrix porosity was significantly overvalued (**Table 29**).

Table 29—Effect of biased assessment of uncertainty regarding the gas price on each VOPI calculation.

Variable	Original VOPI	VOPI: biased uncertainty assessment	VOPI Over/under Estimation (%)	VOPI Over/under Estimation ($)
Overall VOPI	$3,457,590	$2,750,215	-20%	($707,375)
Gas Price	$963,425	$554,806	-42%	($408,619)
Created-fracture propagation	$546,202	$486,535	-11%	($59,667)
Matrix Porosity	529,594	$706,525	+33%	$176,931
Matrix Permeability	$0	$0	0%	$0
Natural-Fracture Density	$0	$0	0%	$0

The VOPI for gas price is underestimated because the assessment of uncertainty regarding gas price is overconfident. As shown in the previous section, if information that reduces uncertainty in an unknown variable has value, overconfident uncertainty assessment (lower uncertainty) necessarily leads to a decrease in calculated VOPI. The VOPI calculations were skewed for created-fracture propagation and matrix porosity as well. Information value for created-fracture propagation was underestimated, while information value for matrix porosity was overestimated. The biased uncertainty assessment also led to a distortion in the ranking of VOPI (Table 29). This is of particular interest, because one of the primary advantages of multi-variable VOPI analysis is that it indicates relative information values.

Though barely scratching the surface in characterizing all the negative effects that biases in the quantification uncertainty can have on VOPI analysis, the results presented in this section establish that introducing bias in the assessment of a key uncertainty can have a significant effect on VOPI analysis. It was shown that biased uncertainty assessment for a key variable can cause VOPI for some variables to be overestimated and VOPI for other variables to be underestimated. Furthermore, biased uncertainty assessment for a key variable can change the calculated VOPI ranking, which skews relative information values. This further enforces the importance of reliable uncertainty assessment for decision makers in unconventional reservoir development.

7. DISCUSSION

Reliable assessment of uncertainty is vital for optimization of the decision-making process in the context of unconventional reservoir development. However, to truly optimize the decision-making process it is important to go further than assessing uncertainty accurately. The decision maker must also assess which uncertainties are worth reduction through data acquisition. Hubbard presents the following process for application of VOI analysis to optimize data-acquisition decisions (Hubbard 2014):

1) Frame the decision context. The decision maker must understand what the key uncertainties are and what decision alternatives will be considered. A decision model must be built to connect possible values of uncertain variables to the EV of the project under each considered decision alternative.
2) Quantify the uncertainty related to key variables. Uncertainty of each variable that is treated as uncertain in the decision model must be quantified accurately. Otherwise, outputs of VOI calculations can be skewed.
3) Calculate overall VOPI and the VOPI for each uncertain variable. Using the probability distributions previously assigned to each uncertain variable as inputs to the decision model, calculate project EV under each considered decision alternative for all possible combinations of uncertain input variables. Then, calculate VOI for each uncertain variable.

4) Compare data-acquisition costs with calculated VOI for each uncertain variable. For variables with VOI higher than costs of acquiring related data, acquire the related data.
5) Repeat steps 2-4 until data-acquisition costs exceed VOI for all uncertain variables.
6) Proceed with the decision alternative with the lowest calculated EOL.

The work presented in this research established that Steps 1-3 of Hubbard's methodology for optimizing the data-acquisition decision process are translatable to an unconventional reservoir development context. Application of multi-variable VOPI methodology to an unconventional reservoir well-spacing decision showed that commodity price, created-fracture propagation, and matrix porosity may be the parameters for which further uncertainty reduction creates the most value. However, decision makers searching for a rational approach to data-acquisition decisions should apply multi-variable VOPI analysis to their specific decision contexts, rather than relying on the results of this research. VOPI calculations are dependent on the specific decision context faced by the decision maker and the specific information available to the decision maker. Therefore, the VOPI for key uncertain variables in unconventional reservoir development is specific to each decision scenario and operator (because uncertainty can vary by operator).

In the context of unconventional reservoir development, some uncertain parameters for which measurement reliability is low may be identified as having high VOPI. For example, microseismic data provides information that reduces the uncertainty related to created-fracture propagation to a certain extent. However, the uncertainty related to the propagation of the created fractures likely remains large after acquisition of microseismic data. The uncertainty related to created-fracture propagation has high VOPI in the decision context presented in this research, but it may have low VOI due to low measurement accuracy. If multi-variable VOPI analysis reveals high information value for an uncertain parameter, yet this uncertain parameter is not easily measured through data acquisition, then research efforts focused on advancing understanding of this uncertain parameter or developing more accurate measurement methods may have high value potential. Application of multi-variable VOPI analysis in an academic setting allows for researchers to identify high-value future research topics without incurring steep data-acquisition costs.

Results and conclusions from this research should be considered in the context of limiting assumptions that were made. First, the uncertain variables considered in the multi-variable VOPI analysis applied to the unconventional reservoir well-spacing decision were assumed to be independent. Second, uncertainties of the variables considered in the well-spacing VOPI analysis were quantified based on data available in the industry literature. Quantification of uncertainty based on data from an E&P company would provide more tangible results.

The application of the generalized multi-variable VOPI model presented in this thesis considers once decision variable. However, it could easily be used for VOPI calculations in decision contexts which consider multiple decision variables by decision variable combination. For example, a two-decision variable context consisting of three well-spacing alternatives and three well lateral length decision alternatives can be considered a single-decision variable context with nine variables.

The generalized multi-variable VOPI model could also be used for investigating the sensitivity of VOPI calculations to dependence between uncertain variables. If VOPI calculation outputs are highly sensitive to the correlation assumptions between particular uncertain variables, then effort to accurately quantify the correlation is necessary to ensure accurate VOPI calculations. However, if VOPI calculation outputs are not highly sensitive to the correlation assumptions between particular uncertain variables, then further effort to quantify the correlation may not be justified.

The major benefit of this research is that it provides a rational approach for determining the value of uncertainty reduction through data acquisition in unconventional reservoir development. Highlighting this approach should lead to increased industry and academic awareness of the power of multiple-variable VOPI analysis to justify data acquisition and focus research efforts. Increased awareness, if translated to increased application, should lead to improved decision making and financial performance in unconventional reservoir development.

8. CONCLUSIONS

- A generalized multi-variable VOPI model with the ability to accommodate a variable number of discrete uncertain variables and discrete decision alternatives was successfully developed.
- In the context of an unconventional well-spacing decision using Eagle Ford shale data obtained from the industry literature, gas price is the uncertain variable with the highest VOPI.
- In the context of an unconventional well-spacing decision using Eagle Ford shale data obtained from the industry literature, information related to created-fracture propagation and matrix porosity have significant VOPI.
- In the context of an unconventional well-spacing decision using Eagle Ford shale data obtained from the industry literature, information related to matrix permeability and natural fracture density do not have significant VOPI.
- Biased assessment of uncertainty for key uncertain variables can lead to significantly skewed VOPI calculations for all uncertain variables with information value and can change the VOPI rankings, leading to skewed estimation of relative information values.

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APPENDIX A

FIGURES AND TABLES NOT EMBEDDED IN TEXT

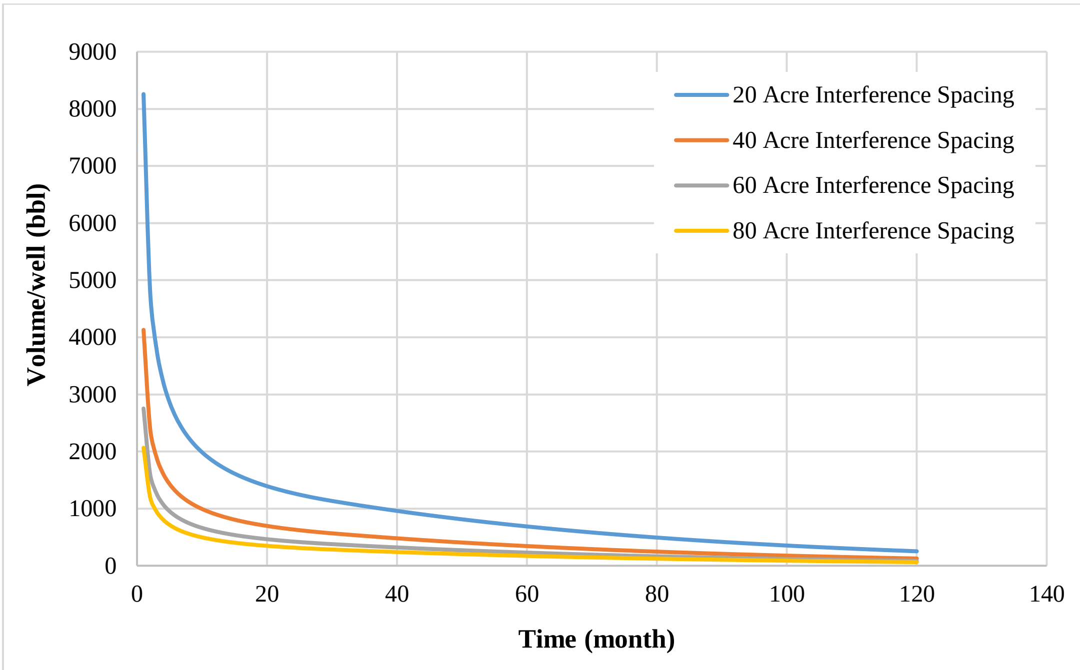
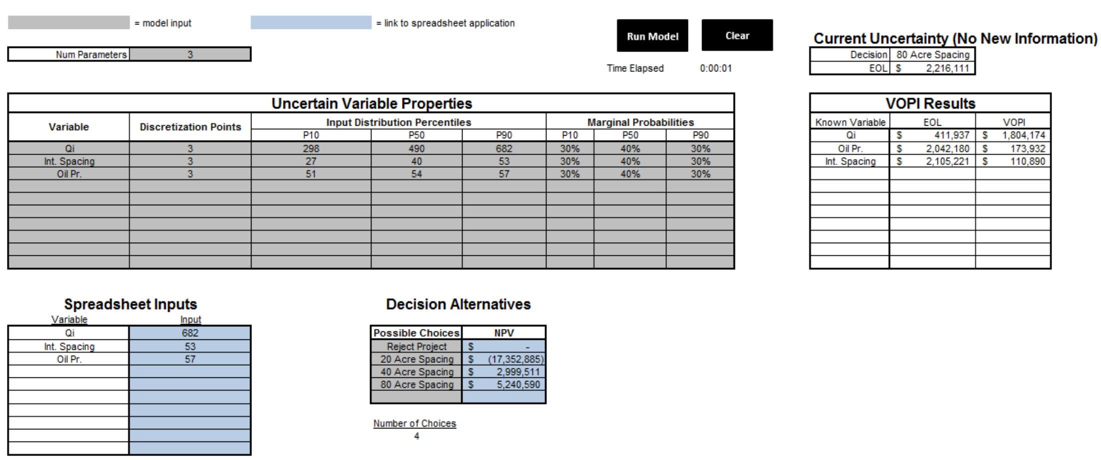


Fig. A-1—Graphical visualization of effect of "Interference Spacing" from the synthetic model (Well spacing = 20).



= model input

= link to spreadsheet application

Run Model

Clear

Num Parameters	3

Time Elapsed 0:00:01

Current Uncertainty (No New Information)

Decision	80 Acre Spacing
EOL	$ 2,216,111

Uncertain Variable Properties

Variable	Discretization Points	Input Distribution Percentiles			Marginal Probabilities		
		P10	P50	P90	P10	P50	P90
Qi	3	298	490	682	30%	40%	30%
Int. Spacing	3	27	40	53	30%	40%	30%
Oil Pr.	3	51	54	57	30%	40%	30%

VOPI Results

Known Variable	EOL	VOPI
Qi	$ 411,937	$ 1,804,174
Oil Pr.	$ 2,042,180	$ 173,932
Int. Spacing	$ 2,105,221	$ 110,890

Spreadsheet Inputs

Variable	Input
Qi	682
Int. Spacing	53
Oil Pr.	57

Decision Alternatives

Possible Choices	NPV
Reject Project	$ -
20 Acre Spacing	$ (17,352,885)
40 Acre Spacing	$ 2,999,511
80 Acre Spacing	$ 5,240,590

Number of Choices
4

Fig. A-2—Generalized multi-variable VOPI model input and calculations for the three-variable synthetic model scenario.

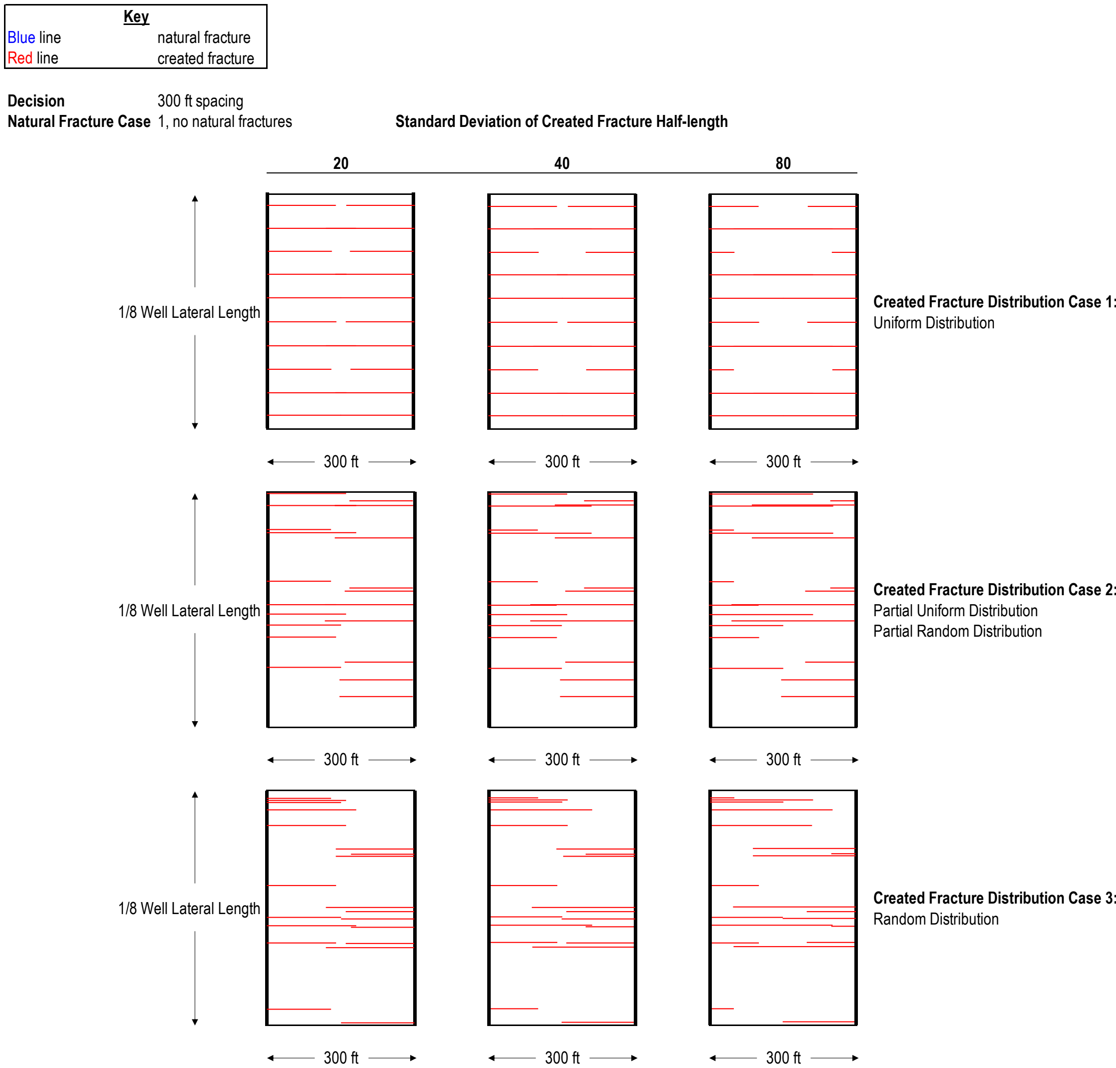


Fig. A-3—Discrete fracture network cases considered under the no natural fractures case when the well-spacing decision is 300 ft: Non-mirror image wells.

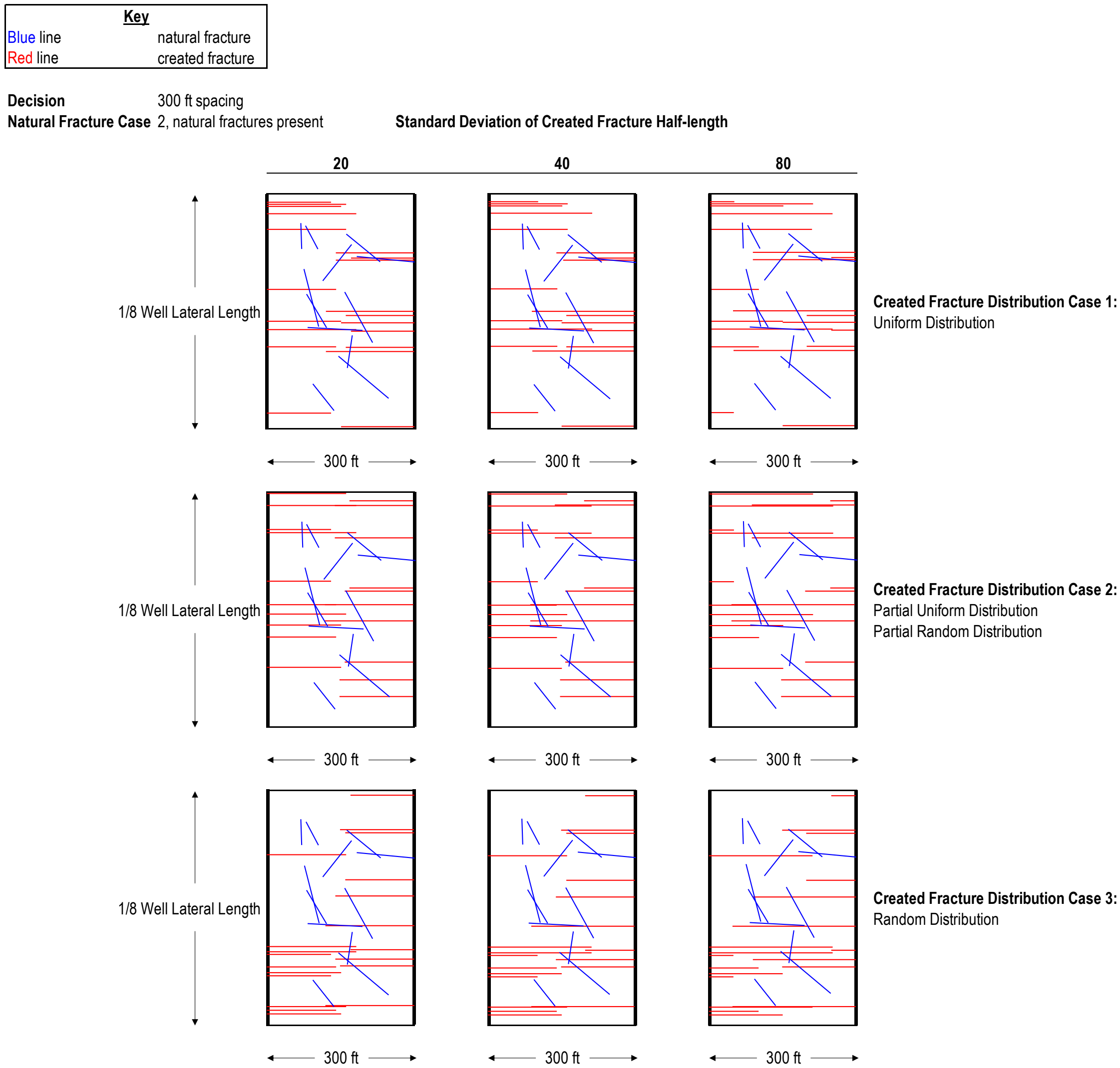


Fig. A-4—Discrete fracture network cases considered under the natural fractures present case when the well-spacing decision is 300 ft: Non-mirror image wells.

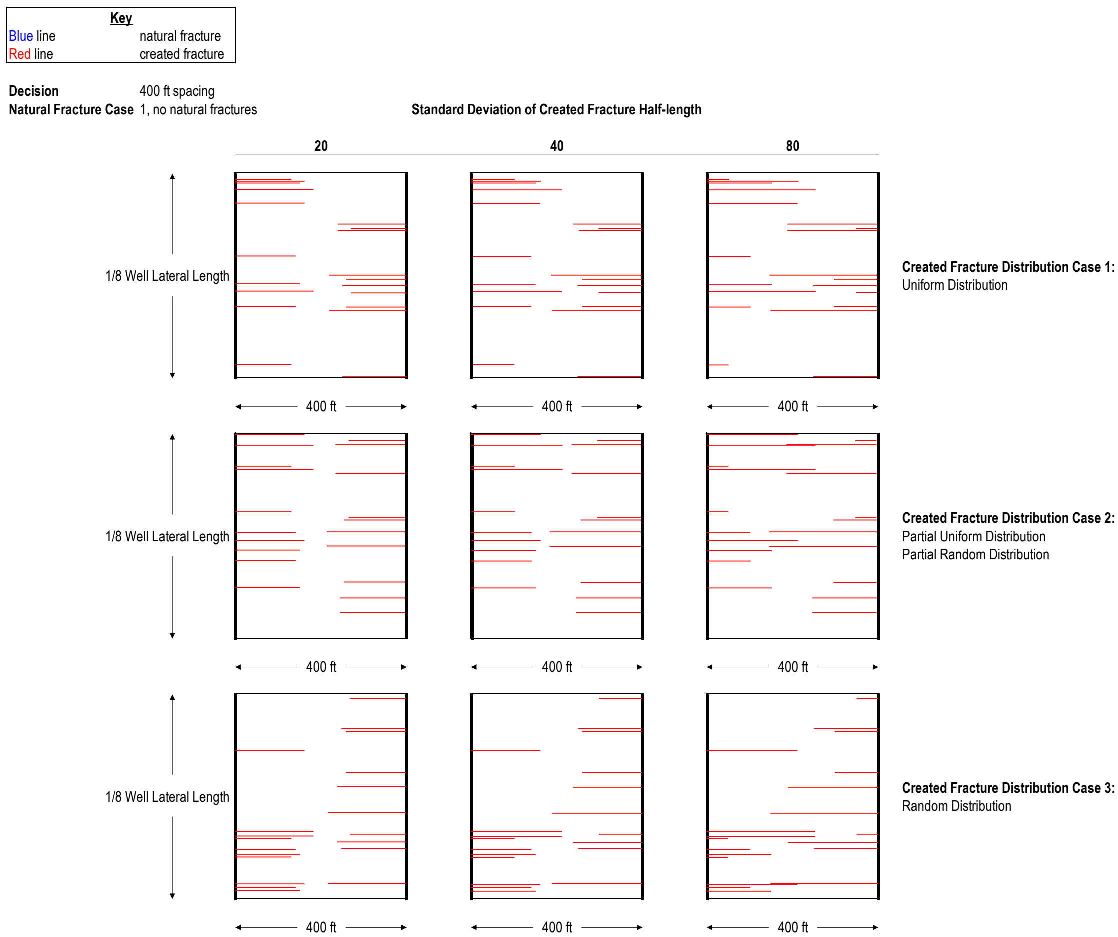


Fig. A-5—Discrete fracture network cases considered under the no natural fractures case when the well-spacing decision is 400 ft: Non-mirror image wells.

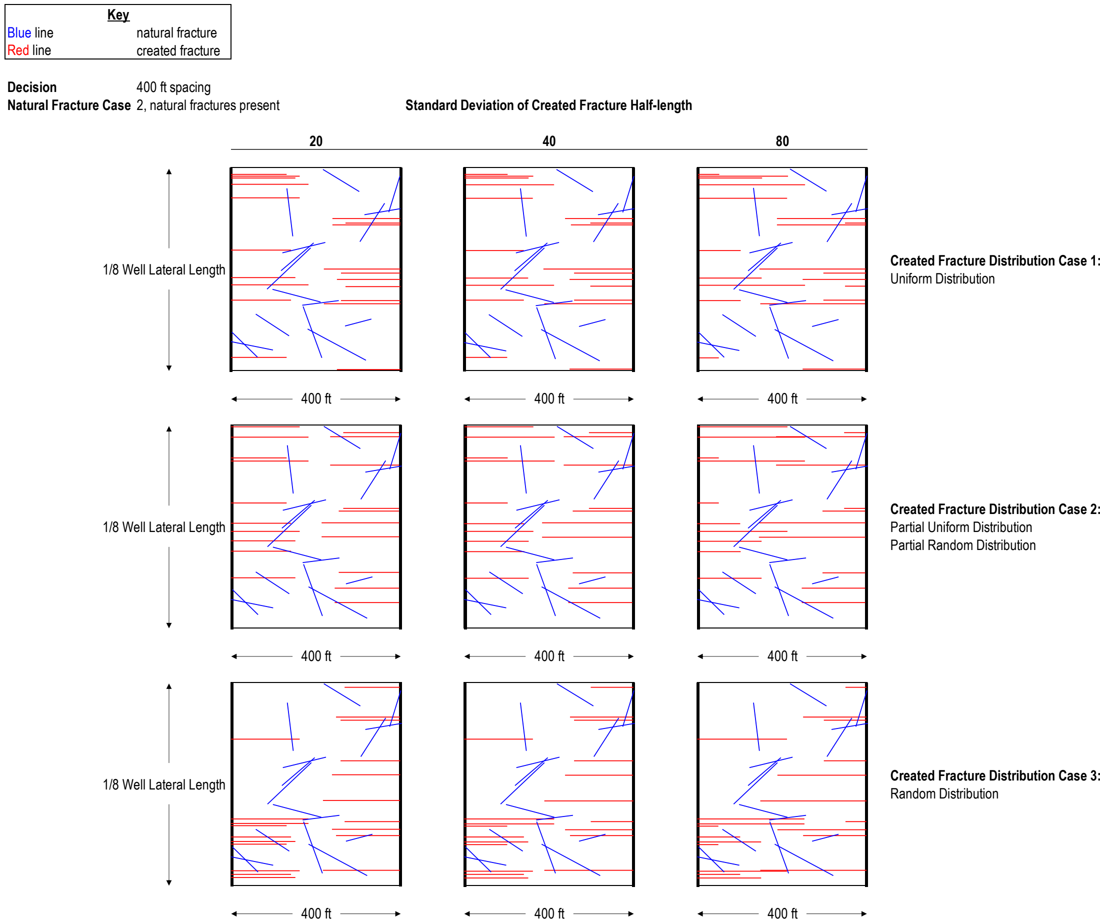


Fig. A-6—Discrete fracture network cases considered under the natural fractures present case when the well-spacing decision is 400 ft: Non-mirror image wells.

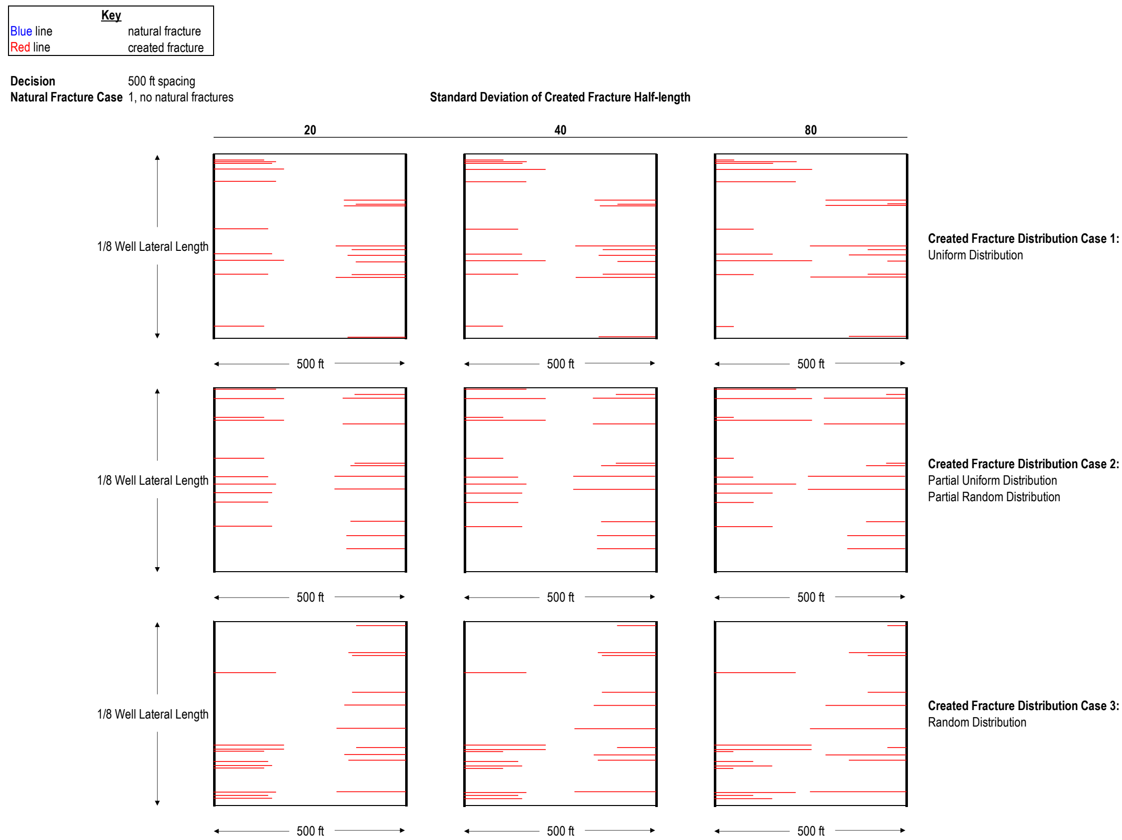


Fig. A-7—Discrete fracture network cases considered under the no natural fractures case when the well-spacing decision is 500 ft: Non-mirror image wells.

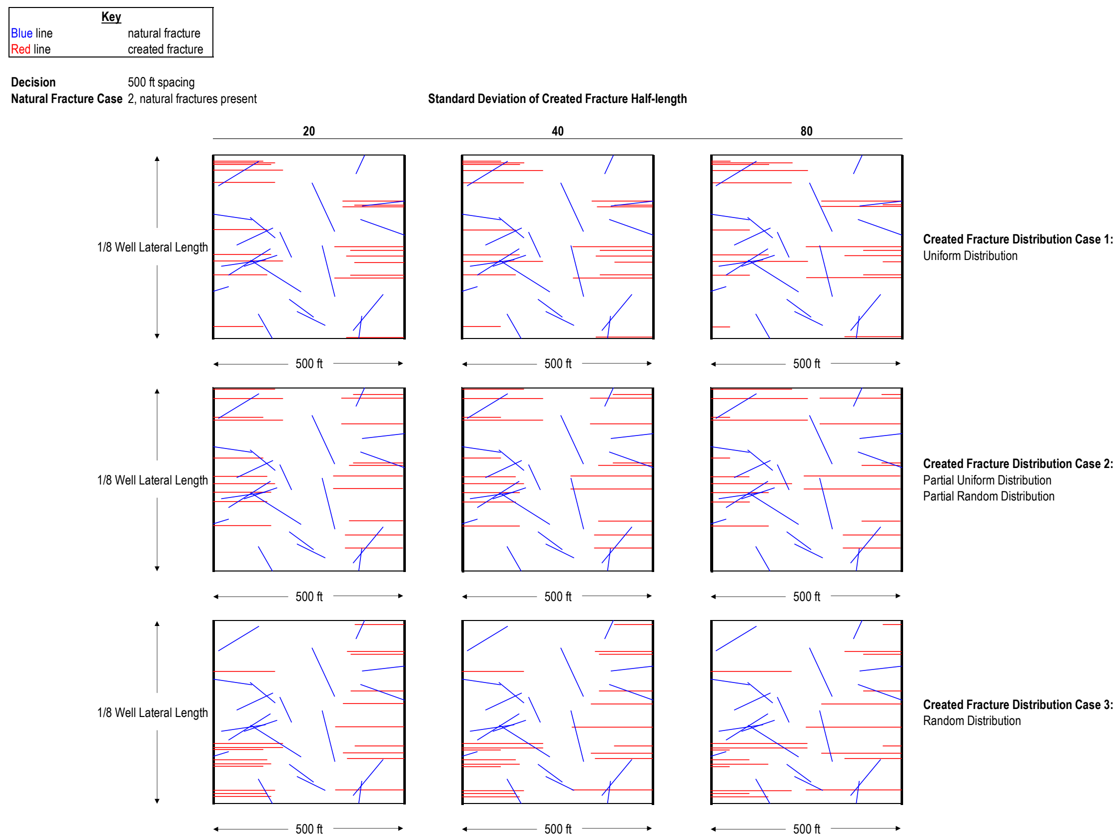


Fig. A-8—Discrete fracture network cases considered under the natural fractures present case when the well-spacing decision is 500 ft: Non-mirror image wells.

Table A-1—Rank and aggregate score of each "uncertainty" response in the Crisman/Berg-Hughes industry member survey.

Uncertainty	Rank	Score
Direction/nature of hydraulic fracture propagation	1	44
Oil and gas prices	2	42
Individual well productivities	3	33
Natural-fracture system: orientation	4	30
Permeability distribution	5	27
Mechanical Stratigraphy	6	26
Natural-fracture system: fracture fill and conductivity	7	24
Lateral variations in lithology	8	21
Reservoir Pressure	9	15
Stress state	10	13
Pressure-volume-temperature behavior in nano-pores	11	13
Effective fracture height	12	9
Effective fracture length	12	9
Natural-fracture system: density & distribution	14	7
Minerology	15	5
Dynamic models of perm and porosity associated with organic matter	15	5
Total organic content distribution	17	5
Borehole measurements responses	18	4
Correlation between commodity price & rig schedule	18	4
Proppant transport	18	4
Drilling hazards	21	3
Hydraulic fracture system: change over time	21	3
Cementing (Well Integrity) in horizontal Wells	21	3
Cluster Efficiency	21	3
Permeability through time & geomechanical effects in the stimulated reservoir volume	25	3
Natural-fracture interaction with hydraulic fractures	25	3
Drilling time	27	2
Volume calibrated fracture measurement	27	2
Reservoir quality & characterization	29	2
Fluid properties	29	2
Migration in non-source rock plays	29	2
Petrophysical log calibration & modeling	29	2
Organic geochemistry/organofacies of source rocks	29	2
Mobile water saturation	34	2
Critical gas saturation	34	2
Residual condensate/oil saturation	34	2
Seismicity	37	1

Table A-2—Rank and aggregate score of each "decision" response in the Crisman/Berg-Hughes industry member survey.

Decision	Rank	Score
Well spacing	1	73
How to complete well	2	47
Should we be in play/basin?	3	34
Lateral landing zone	4	32
Stimulation number of stages	5	30
How much do we pay for leases?	6	17
Where do we buy leases?	7	17
Stimulation fluid volume	8	15
Production - how to flow back well	9	14
Appraisal program - number and locations of test wells	10	11
Type and amount of proppant	11	11
Stimulation fluid type	12	10
Well lateral length	13	9
Coring program - number of wells, cored interval, types of analyses	14	7
Logging program - types of logs	15	6
Cluster spacing	16	5
Acquire scanning electron microscopy data-polished	17	4
Enhanced oil recovery for unconventional reservoirs	18	3
Optimize fracture size to avoid over-stimulation when well spacing too close	19	3
Water disposal/management	20	2
Stimulated reservoir volume optimization for stacked play (through space & time)	20	2
Well azimuth	22	2
Well spacing - Horizontally	22	2
Acquire microseismic data?	24	2
Facilities design & optimization in early stages	25	1
Artificial Lift Type, Timing, Efficiency	27	1

Table A-3—Probability distributions for q_i, IntSpacing, and Oil Pr.

Parameter 1: q_i (bbl/day)		
Percentile	Value	Probability
P10	298	30%
P50	490	40%
P90	682	30%

Parameter 2: IntSpacing (acres)		
Percentile	Value	Probability
P10	27	30%
P50	40	40%
P90	53	30%

Parameter 3: Oil Pr. ($/bbl)		
Percentile	Value	Probability
P10	51	30%
P50	54	40%
P90	57	30%

Table A-4—Joint-probability matrix for the three-variable synthetic model.

		q_i (bbl/day)				
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**	**IntSpacing (marginal)**	**Oil Pr. (marginal)**
51	**27**	2.70%	3.60%	2.70%	**30%**	**30%**
	40	3.60%	4.80%	3.60%	**40%**	
	53	2.70%	3.60%	2.70%	**30%**	
54	**27**	3.60%	4.80%	3.60%	**30%**	**40%**
	40	4.80%	6.40%	4.80%	**40%**	
	53	3.60%	4.80%	3.60%	**30%**	
57	**27**	2.70%	3.60%	2.70%	**30%**	**30%**
	40	3.60%	4.80%	3.60%	**40%**	
	53	2.70%	3.60%	2.70%	**30%**	
	q_i (marginal)	**30%**	**40%**	**30%**		

Table A-5—NPV for each possible realization of q_i, IntSpacing, and Oil Pr. under each decision alternative.

		Decision =		20 Acre Spacing
		q_i (bbl/day)		
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**
51	**27**	($23,543,132)	($12,463,900)	($1,384,668)
	40	($29,041,520)	($21,511,940)	($13,982,359)
	53	($31,871,576)	($26,169,024)	($20,466,472)
54	**27**	($22,223,005)	($10,291,524)	$1,639,957
	40	($28,144,346)	($20,035,567)	($11,926,788)
	53	($31,192,098)	($25,050,888)	($18,909,678)
57	**27**	($20,902,877)	($8,119,147)	$4,664,582
	40	($27,247,171)	($18,559,193)	($9,871,216)
	53	($30,512,620)	($23,932,752)	($17,352,884)

		Decision =		40 Acre Spacing
		q_i (bbl/day)		
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**
51	**27**	($8,689,124)	($1,159,544)	$6,370,037
	40	($8,689,124)	($1,159,544)	$6,370,037
	53	($11,519,180)	($5,816,628)	($114,076)
54	**27**	($7,791,950)	$316,829	$8,425,608
	40	($7,791,950)	$316,829	$8,425,608
	53	($10,839,702)	($4,698,492)	$1,442,718
57	**27**	($6,894,775)	$1,793,203	$10,481,180
	40	($6,894,775)	$1,793,203	$10,481,180
	53	($10,160,224)	($3,580,356)	$2,999,512

Table A-5—Continued

Oil Pr. ($/bbl)	IntSpacing (acres)	Decision = 80 Acre Spacing q_i (bbl/day) 298	490	682
51	**27**	($4,344,562)	($579,772)	$3,185,018
	40	($4,344,562)	($579,772)	$3,185,018
	53	($4,344,562)	($579,772)	$3,185,018
54	**27**	($3,895,975)	$158,415	$4,212,804
	40	($3,895,975)	$158,415	$4,212,804
	53	($3,895,975)	$158,415	$4,212,804
57	**27**	($3,447,387)	$896,601	$5,240,590
	40	($3,447,387)	$896,601	$5,240,590
	53	($3,447,387)	$896,601	$5,240,590

Oil Pr. ($/bbl)	IntSpacing (acres)	Decision = Reject Project q_i (bbl/day) 298	490	682
51	**27**	$0	$0	$0
	40	$0	$0	$0
	53	$0	$0	$0
54	**27**	$0	$0	$0
	40	$0	$0	$0
	53	$0	$0	$0
57	**27**	$0	$0	$0
	40	$0	$0	$0
	53	$0	$0	$0

Table A-6—Optimal decision matrix for three-variable synthetic model.

		q_i (bbl/day)		
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**
51	**27**	Reject Project	Reject Project	40 Acre Spacing
	40	Reject Project	Reject Project	40 Acre Spacing
	53	Reject Project	Reject Project	80 Acre Spacing
54	**27**	Reject Project	40 Acre Spacing	40 Acre Spacing
	40	Reject Project	40 Acre Spacing	40 Acre Spacing
	53	Reject Project	80 Acre Spacing	80 Acre Spacing
57	**27**	Reject Project	40 Acre Spacing	40 Acre Spacing
	40	Reject Project	40 Acre Spacing	40 Acre Spacing
	53	Reject Project	80 Acre Spacing	80 Acre Spacing

Table A-7—Project value matrix with perfect information for three-variable synthetic model.

		q_i (bbl/day)		
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**
51	**27**	$ 0	$ 0	$ 6,370,037
	40	$ 0	$ 0	$ 6,370,037
	53	$ 0	$ 0	$ 3,185,018
54	**27**	$ 0	$ 316,829	$ 8,425,608
	40	$ 0	$ 316,829	$ 8,425,608
	53	$ 0	$ 158,415	$ 4,212,804
57	**27**	$ 0	$ 1,793,203	$ 10,481,180
	40	$ 0	$ 1,793,203	$ 10,481,180
	53	$ 0	$ 896,601	$ 5,240,590

Table A-8—Opportunity loss for each possible realization of q_i, IntSpacing, and Oil Pr. under each decision alternative.

		Decision = 20 Acre Spacing		
		q_i (bbl/day)		
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**
51	**27**	$23,543,132	$12,463,900	$7,754,704
	40	$29,041,520	$21,511,940	$20,352,396
	53	$31,871,576	$26,169,024	$23,651,490
54	**27**	$22,223,005	$10,608,353	$6,785,651
	40	$28,144,346	$20,352,396	$20,352,396
	53	$31,192,098	$25,209,303	$23,122,482
57	**27**	$20,902,877	$9,912,350	$5,816,598
	40	$27,247,171	$20,352,396	$20,352,396
	53	$30,512,620	$24,829,353	$22,593,474

		Decision = 40 Acre Spacing		
		q_i (bbl/day)		
Oil Pr. ($/bbl)	**IntSpacing (acres)**	**298**	**490**	**682**
51	**27**	$8,689,124	$1,159,544	$0
	40	$8,689,124	$1,159,544	$0
	53	$11,519,180	$5,816,628	$3,299,094
54	**27**	$7,791,950	$0	$0
	40	$7,791,950	$0	$0
	53	$10,839,702	$4,856,907	$2,770,086
57	**27**	$6,894,775	$0	$0
	40	$6,894,775	$0	$0
	53	$10,160,224	$4,476,957	$2,241,078

Table A-8—Continued

Oil Pr. ($/bbl)	IntSpacing (acres)	Decision = 80 Acre Spacing, q_i (bbl/day): 298	490	682
51	27	$4,344,562	$579,772	$3,185,018
	40	$4,344,562	$579,772	$3,185,018
	53	$4,344,562	$579,772	$0
54	27	$3,895,975	$158,415	$4,212,804
	40	$3,895,975	$158,415	$4,212,804
	53	$3,895,975	$0	$0
57	27	$3,447,387	$896,601	$5,240,590
	40	$3,447,387	$896,601	$5,240,590
	53	$3,447,387	$0	$0

Oil Pr. ($/bbl)	IntSpacing (acres)	Decision = Reject Project, q_i (bbl/day): 298	490	682
51	27	$0	$0	$6,370,037
	40	$0	$0	$6,370,037
	53	$0	$0	$3,185,018
54	27	$0	$316,829	$8,425,608
	40	$0	$316,829	$8,425,608
	53	$0	$158,415	$4,212,804
57	27	$0	$1,793,203	$10,481,180
	40	$0	$1,793,203	$10,481,180
	53	$0	$896,601	$5,240,590

Table A-9—EOL for each decision alternative at current level of uncertainty.

Decision	20 Acre Spacing	40 Acre Spacing	80 Acre Spacing	Reject Project
EOL	$ 20,991,476	$ 3,562,293	$ 2,216,111	$ 2,374,526

Table A-10—Opportunity loss for each possible realization of q_i, IntSpacing, and Oil Pr. under each decision alternative if q_i is known perfectly. EOL values associated with each potential value of q_i and each decision alternative.

		Decision =		20 Acre Spacing			
		Known Variable: q_i (bbl/day)					
Unknown Variable		298		490		682	
Oil Price ($/bbl)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
50	27	$23,543,132	9%	$12,463,900	9%	$7,754,704	9%
	40	$29,041,520	12%	$21,511,940	12%	$20,352,396	12%
	53	$31,871,576	9%	$26,169,024	9%	$23,651,490	9%
54	27	$22,223,005	12%	$10,608,353	12%	$6,785,651	12%
	40	$28,144,346	16%	$20,352,396	16%	$20,352,396	16%
	53	$31,192,098	12%	$25,209,303	12%	$23,122,482	12%
57	27	$20,902,877	9%	$9,912,350	9%	$5,816,598	9%
	40	$27,247,171	12%	$20,352,396	12%	$20,352,396	12%
	53	$30,512,620	9%	$24,829,353	9%	$22,593,474	9%
EOL		$27,282,269		$19,181,939		$17,113,398	

		Decision =		40 Acre Spacing			
		Known Variable: q_i (bbl/day)					
Unknown Variable		298		490		682	
Oil Price ($/bbl)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
50	27	$8,689,124	9%	$1,159,544	9%	$0	9%
	40	$8,689,124	12%	$1,159,544	12%	$0	12%
	53	$11,519,180	9%	$5,816,628	9%	$3,299,094	9%
54	27	$7,791,950	12%	$0	12%	$0	12%
	40	$7,791,950	16%	$0	16%	$0	16%
	53	$10,839,702	12%	$4,856,907	12%	$2,770,086	12%
57	27	$6,894,775	9%	$0	9%	$0	9%
	40	$6,894,775	12%	$0	12%	$0	12%
	53	$10,160,224	9%	$4,476,957	9%	$2,241,078	9%
EOL		$8,706,275		$1,752,756		$831,026	

Table A-10—Continued

		Decision =		80 Acre Spacing			
		Known Variable: q_i (bbl/day)					
Unknown Variable		298		490		682	
Oil Price ($/bbl)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
50	27	$4,344,562	9%	$579,772	9%	$3,185,018	9%
	40	$4,344,562	12%	$579,772	12%	$3,185,018	12%
	53	$4,344,562	9%	$579,772	9%	$0	9%
54	27	$3,895,975	12%	$158,415	12%	$4,212,804	12%
	40	$3,895,975	16%	$158,415	16%	$4,212,804	16%
	53	$3,895,975	12%	$0	12%	$0	12%
57	27	$3,447,387	9%	$896,601	9%	$5,240,590	9%
	40	$3,447,387	12%	$896,601	12%	$5,240,590	12%
	53	$3,447,387	9%	$0	9%	$0	9%
EOL		$3,895,975		$406,574		$2,948,963	

		Decision =		Reject Project			
		Known Variable: q_i (bbl/day)					
Unknown Variable		298		490		682	
Oil Price ($/bbl)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
50	27	$0	9%	$0	9%	$6,370,037	9%
	40	$0	12%	$0	12%	$6,370,037	12%
	53	$0	9%	$0	9%	$3,185,018	9%
54	27	$0	12%	$316,829	12%	$8,425,608	12%
	40	$0	16%	$316,829	16%	$8,425,608	16%
	53	$0	12%	$158,415	12%	$4,212,804	12%
57	27	$0	9%	$1,793,203	9%	$10,481,180	9%
	40	$0	12%	$1,793,203	12%	$10,481,180	12%
	53	$0	9%	$896,601	9%	$5,240,590	9%
EOL		$0		$564,989		$7,161,767	

Table A-11—Min EOL and associated decision for each potential q_i information value for three-variable model.

q_i (bbl/day)	298	490	682
Min EOL	$ 0	$ 406,574	$ 831,026
Optimal Decision	Reject Project	80 Acre Spacing	40 Acre Spacing

Table A-12—Opportunity loss for each possible realization of q_i, IntSpacing, and Oil Pr. under each decision alternative if IntSpacing is known perfectly. EOL values associated with each potential value of IntSpacing and each decision alternative.

		Decision =			**20 Acre Spacing**		
		Known Variable: IntSpacing (acres)					
Unknown Variable		**27**		**40**		**53**	
Oil Price ($/bbl)	**q_i (bbl/day)**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**	**OL**	**Conditional Probability**
51	**298**	$ 23,543,132	9%	$ 29,041,520	9%	$ 31,871,576	9%
	490	$ 12,463,900	12%	$ 21,511,940	12%	$ 26,169,024	12%
	682	$ 7,754,704	9%	$ 20,352,396	9%	$ 23,651,490	9%
54	**298**	$ 22,223,005	12%	$ 28,144,346	12%	$ 31,192,098	12%
	490	$ 10,608,353	16%	$ 20,352,396	16%	$ 25,209,303	16%
	682	$ 6,785,651	12%	$ 20,352,396	12%	$ 23,122,482	12%
57	**298**	$ 20,902,877	9%	$ 27,247,171	9%	$ 30,512,620	9%
	490	$ 9,912,350	12%	$ 20,352,396	12%	$ 24,829,353	12%
	682	$ 5,816,598	9%	$ 20,352,396	9%	$ 22,593,474	9%
EOL		**$ 13,085,083**		**$ 22,829,126**		**$ 26,447,668**	

Table A-12—Continued

Unknown Variable		Decision = 40 Acre Spacing					
		Known Variable: IntSpacing (acres)					
		27		40		53	
Oil Price ($/bbl)	q_i (bbl/day)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
51	298	$ 8,689,124	9%	$ 8,689,124	9%	$ 11,519,180	9%
	490	$ 1,159,544	12%	$ 1,159,544	12%	$ 5,816,628	12%
	682	$ 0	9%	$ 0	9%	$ 3,299,094	9%
54	298	$ 7,791,950	12%	$ 7,791,950	12%	$ 10,839,702	12%
	490	$ 0	16%	$ 0	16%	$ 4,856,907	16%
	682	$ 0	12%	$ 0	12%	$ 2,770,086	12%
57	298	$ 6,894,775	9%	$ 6,894,775	9%	$ 10,160,224	9%
	490	$ 0	12%	$ 0	12%	$ 4,476,957	12%
	682	$ 0	9%	$ 0	9%	$ 2,241,078	9%
EOL		$ 2,476,730		$ 2,476,730		$ 6,095,272	

Unknown Variable		Decision = 80 Acre Spacing					
		Known Variable: IntSpacing (acres)					
		27		40		53	
Oil Price ($/bbl)	q_i (bbl/day)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
51	298	$ 4,344,562	9%	$ 4,344,562	9%	$ 4,344,562	9%
	490	$ 579,772	12%	$ 579,772	12%	$ 579,772	12%
	682	$ 3,185,018	9%	$ 3,185,018	9%	$ 0	9%
54	298	$ 3,895,975	12%	$ 3,895,975	12%	$ 3,895,975	12%
	490	$ 158,415	16%	$ 158,415	16%	$ 0	16%
	682	$ 4,212,804	12%	$ 4,212,804	12%	$ 0	12%
57	298	$ 3,447,387	9%	$ 3,447,387	9%	$ 3,447,387	9%
	490	$ 896,601	12%	$ 896,601	12%	$ 0	12%
	682	$ 5,240,590	9%	$ 5,240,590	9%	$ 0	9%
EOL		$ 2,635,145		$ 2,635,145		$ 1,238,365	

Table A-12—Continued

		Decision =	Reject Project				
		Known Variable: IntSpacing (acres)					
Unknown Variable		27		40		53	
Oil Price ($/bbl)	q_i (bbl/day)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
51	298	$ 0	9%	$ 0	9%	$ 0	9%
	490	$ 0	12%	$ 0	12%	$ 0	12%
	682	$ 6,370,037	9%	$ 6,370,037	9%	$ 3,185,018	9%
54	298	$ 0	12%	$ 0	12%	$ 0	12%
	490	$ 316,829	16%	$ 316,829	16%	$ 158,415	16%
	682	$ 8,425,608	12%	$ 8,425,608	12%	$ 4,212,804	12%
57	298	$ 0	9%	$ 0	9%	$ 0	9%
	490	$ 1,793,203	12%	$ 1,793,203	12%	$ 896,601	12%
	682	$ 10,481,180	9%	$ 10,481,180	9%	$ 5,240,590	9%
EOL		$ 2,793,560		$ 2,793,560		$ 1,396,780	

Table A-13—Min EOL and associated decision for each potential IntSpacing information value for three-variable model.

IntSpacing (acres)	27	40	53
Min EOL	$ 2,476,730	$ 2,476,730	$ 1,238,365
Optimal Decision	40 Acre Spacing	40 Acre Spacing	80 Acre Spacing

Table A-14—Opportunity loss for each possible realization of q_i, IntSpacing, and Oil Pr. under each decision alternative if Oil Pr. is known perfectly. EOL values associated with each potential value of Oil Pr. and each decision alternative.

		Decision =		20 Acre Spacing			
		Known Variable: Oil Price ($/bbl)					
Unknown Variable		51		54		57	
q_i (bbl/day)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	27	$ 23,543,132	9%	$ 22,223,005	9%	$ 20,902,877	9%
	40	$ 29,041,520	12%	$ 28,144,346	12%	$ 27,247,171	12%
	53	$ 31,871,576	9%	$ 31,192,098	9%	$ 30,512,620	9%
490	27	$ 12,463,900	12%	$ 10,608,353	12%	$ 9,912,350	12%
	40	$ 21,511,940	16%	$ 20,352,396	16%	$ 20,352,396	16%
	53	$ 26,169,024	12%	$ 25,209,303	12%	$ 24,829,353	12%
682	27	$ 7,754,704	9%	$ 6,785,651	9%	$ 5,816,598	9%
	40	$ 20,352,396	12%	$ 20,352,396	12%	$ 20,352,396	12%
	53	$ 23,651,490	9%	$ 23,122,482	9%	$ 22,593,474	9%
EOL		$ 21,819,012		$ 20,873,202		$ 20,321,637	

		Decision =		40 Acre Spacing			
		Known Variable: Oil Price ($/bbl)					
Unknown Variable		51		54		57	
q_i (bbl/day)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	27	$ 8,689,124	9%	$ 7,791,950	9%	$ 6,894,775	9%
	40	$ 8,689,124	12%	$ 7,791,950	12%	$ 6,894,775	12%
	53	$ 11,519,180	9%	$ 10,839,702	9%	$ 10,160,224	9%
490	27	$ 1,159,544	12%	$ 0	12%	$ 0	12%
	40	$ 1,159,544	16%	$ 0	16%	$ 0	16%
	53	$ 5,816,628	12%	$ 4,856,907	12%	$ 4,476,957	12%
682	27	$ 0	9%	$ 0	9%	$ 0	9%
	40	$ 0	12%	$ 0	12%	$ 0	12%
	53	$ 3,299,094	9%	$ 2,770,086	9%	$ 2,241,078	9%
EOL		$ 4,181,028		$ 3,444,019		$ 3,101,255	

Table A-14—Continued

		Decision =			80 Acre Spacing		
		Known Variable: Oil Price ($/bbl)					
Unknown Variable		51		54		57	
q_i (bbl/day)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	27	$ 4,344,562	9%	$ 3,895,975	9%	$ 3,447,387	9%
	40	$ 4,344,562	12%	$ 3,895,975	12%	$ 3,447,387	12%
	53	$ 4,344,562	9%	$ 3,895,975	9%	$ 3,447,387	9%
490	27	$ 579,772	12%	$ 158,415	12%	$ 896,601	12%
	40	$ 579,772	16%	$ 158,415	16%	$ 896,601	16%
	53	$ 579,772	12%	$ 0	12%	$ 0	12%
682	27	$ 3,185,018	9%	$ 4,212,804	9%	$ 5,240,590	9%
	40	$ 3,185,018	12%	$ 4,212,804	12%	$ 5,240,590	12%
	53	$ 0	9%	$ 0	9%	$ 0	9%
EOL		$ 2,204,131		$ 2,097,837		$ 2,385,789	

		Decision =			Reject Project		
		Known Variable: Oil Price ($/bbl)					
Unknown Variable		51		54		57	
q_i (bbl/day)	Int Spacing (acres)	OL	Conditional Probability	OL	Conditional Probability	OL	Conditional Probability
298	27	$ 0	9%	$ 0	9%	$ 0	9%
	40	$ 0	12%	$ 0	12%	$ 0	12%
	53	$ 0	9%	$ 0	9%	$ 0	9%
490	27	$ 0	12%	$ 316,829	12%	$ 1,793,203	12%
	40	$ 0	16%	$ 316,829	16%	$ 1,793,203	16%
	53	$ 0	12%	$ 158,415	12%	$ 896,601	12%
682	27	$ 6,370,037	9%	$ 8,425,608	9%	$ 10,481,180	9%
	40	$ 6,370,037	12%	$ 8,425,608	12%	$ 10,481,180	12%
	53	$ 3,185,018	9%	$ 4,212,804	9%	$ 5,240,590	9%
EOL		$ 1,624,359		$ 2,256,252		$ 3,282,390	

Table A-15—Min EOL and associated decision for each potential Oil Pr. information value for three-variable model.

Oil Pr. ($/bbl)	51	54	57
Min EOL	$ 1,624,359	$ 2,097,837	$ 2,385,789
Optimal Decision	Reject Project	80 Acre Spacing	80 Acre Spacing

Table A-16—Overall EOL if each unknown variable is perfectly known and associated VOPI for three-variable model.

Known Variable	EOL	VOPI
q_i	$ 411,937	$ 1,804,174
Oil Pr.	$ 2,042,179	$ 173,932
IntSpacing	$ 2,105,221	$ 110,890

Table A-17—NPV array from generalized model solution to the synthetic three-variable scenario. Results match the manual solution in Table A-5.

Potential Outcome Num.	Reject Project	20 Acre Spacing	40 Acre Spacing	80 Acre Spacing
1	$ 0	$ (23,543,132)	$ (8,689,124)	$ (4,344,562)
2	$ 0	$ (12,463,900)	$ (1,159,544)	$ (579,772)
3	$ 0	$ (1,384,668)	$ 6,370,036	$ 3,185,018
4	$ 0	$ (29,041,520)	$ (8,689,124)	$ (4,344,562)
5	$ 0	$ (21,511,940)	$ (1,159,544)	$ (579,772)
6	$ 0	$ (13,982,360)	$ 6,370,036	$ 3,185,018
7	$ 0	$ (31,871,576)	$ (11,519,180)	$ (4,344,562)
8	$ 0	$ (26,169,024)	$ (5,816,628)	$ (579,772)
9	$ 0	$ (20,466,473)	$ (114,077)	$ 3,185,018
10	$ 0	$ (22,223,004)	$ (7,791,949)	$ (3,895,975)
11	$ 0	$ (10,291,524)	$ 316,829	$ 158,415
12	$ 0	$ 1,639,957	$ 8,425,608	$ 4,212,804
13	$ 0	$ (28,144,345)	$ (7,791,949)	$ (3,895,975)
14	$ 0	$ (20,035,567)	$ 316,829	$ 158,415
15	$ 0	$ (11,926,788)	$ 8,425,608	$ 4,212,804
16	$ 0	$ (31,192,098)	$ (10,839,702)	$ (3,895,975)
17	$ 0	$ (25,050,888)	$ (4,698,492)	$ 158,415
18	$ 0	$ (18,909,679)	$ 1,442,717	$ 4,212,804
19	$ 0	$ (20,902,877)	$ (6,894,775)	$ (3,447,387)
20	$ 0	$ (8,119,148)	$ 1,793,203	$ 896,601
21	$ 0	$ 4,664,581	$ 10,481,180	$ 5,240,590
22	$ 0	$ (27,247,171)	$ (6,894,775)	$ (3,447,387)
23	$ 0	$ (18,559,193)	$ 1,793,203	$ 896,601
24	$ 0	$ (9,871,216)	$ 10,481,180	$ 5,240,590
25	$ 0	$ (30,512,620)	$ (10,160,224)	$ (3,447,387)
26	$ 0	$ (23,932,752)	$ (3,580,357)	$ 896,601
27	$ 0	$ (17,352,885)	$ 2,999,511	$ 5,240,590

Table A-18—OL array from generalized model solution to the synthetic three-variable scenario. Results match the manual solution in Table A-8.

Potential Outcome Num.	Reject Project	20 Acre Spacing	40 Acre Spacing	80 Acre Spacing
1	$ 0	$ 23,543,132	$ 8,689,124	$ 4,344,562
2	$ 0	$ 12,463,900	$ 1,159,544	$ 579,772
3	$ 6,370,037	$ 7,754,705	$ 0	$ 3,185,018
4	$ 0	$ 29,041,520	$ 8,689,124	$ 4,344,562
5	$ 0	$ 21,511,940	$ 1,159,544	$ 579,772
6	$ 6,370,037	$ 20,352,396	$ 0	$ 3,185,018
7	$ 0	$ 31,871,576	$ 11,519,180	$ 4,344,562
8	$ 0	$ 26,169,024	$ 5,816,629	$ 579,772
9	$ 3,185,018	$ 23,651,492	$ 3,299,095	$ 0
10	$ 0	$ 22,223,004	$ 7,791,950	$ 3,895,975
11	$ 316,829	$ 10,608,353	$ 0	$ 158,415
12	$ 8,425,608	$ 6,785,652	$ 0	$ 4,212,804
13	$ 0	$ 28,144,346	$ 7,791,950	$ 3,895,975
14	$ 316,829	$ 20,352,396	$ 0	$ 158,415
15	$ 8,425,608	$ 20,352,396	$ 0	$ 4,212,804
16	$ 0	$ 31,192,098	$ 10,839,702	$ 3,895,975
17	$ 158,415	$ 25,209,304	$ 4,856,907	$ 0
18	$ 4,212,804	$ 23,122,484	$ 2,770,087	$ 0
19	$ 0	$ 20,902,876	$ 6,894,775	$ 3,447,387
20	$ 1,793,203	$ 9,912,350	$ 0	$ 896,601
21	$ 10,481,180	$ 5,816,598	$ 0	$ 5,240,590
22	$ 0	$ 27,247,170	$ 6,894,775	$ 3,447,387
23	$ 1,793,203	$ 20,352,396	$ 0	$ 896,601
24	$ 10,481,180	$ 20,352,396	$ 0	$ 5,240,590
25	$ 0	$ 30,512,620	$ 10,160,224	$ 3,447,387
26	$ 896,601	$ 24,829,354	$ 4,476,958	$ 0
27	$ 5,240,590	$ 22,593,476	$ 2,241,079	$ 0