TIPPING POINT INDICATORS IN DYNAMIC SYSTEMS

A Dissertation

by

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ABSTRACT

Many complex systems have a tipping point after which the system suddenly shifts to a qualitatively different state. This large, abrupt, and often irreversible change in the behavior makes their management challenging. Predicting when a system is near, at or beyond a tipping point is important in system design and management. The similarities in system behaviors raise the question whether there are generic indicators of tipping points. Although, "critical slowing down" is the most studied tipping point indicator suggested in the literature, the usefulness of this indicator in real systems is still under investigation. To transition from simple math models to the application of tipping point indicators in practice, a library of system dynamics models with tipping points was developed. Model complexity ranges from simple archetypes to more complex and realistic models. The library models were used to develop an improved definition of a tipping point and to study the underlying system structures that create a tipping point. Two tipping point types were identified: (1) change in the dominance of feedback loops, and (2) change in the direction of the dominant reinforcing loop. Testing the "slowing down" measures in the library models demonstrated and supported the importance of these indicators in identifying tipping points. Investigating the similarities in the system behaviors before the tipping point resulted in two new potential indicators: (1) distance of system conditions on a x_{t+1}-x_t graph, and (2) slope of a x_{t+1}-x_t graph. The proposed indicators are more practical and easier to use in the construction management field. The findings highlight some limitations: the clarity of the indicator signal depends on the complexity of the system, presence of noise in the system, and the time frame of the study.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

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All other work conducted for the dissertation was completed by the student independently.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Page

CHAPTER I

INTRODUCTION

Large construction projects are among the most complex dynamic systems. These projects consist of various interdependent components and feedback relationships. Due to their dynamic nature, construction projects may demonstrate unexpected behavior. Although these projects intend to add value to their stakeholders, many times they fail to meet their budget, schedule, or quality targets. An example is the National Ignition Facility (NIF), a laser-based research device at the Lawrence Livermore National Laboratory in Livermore, California. The NIF is the largest device of its kind and has the largest laser in the world. Although the NIF is an engineering success from a technical point of view, it is an example of a mega project failure when considering budget and schedule objectives. The construction of the NIF was completed six years later than initially scheduled and with more than $1 billion overrun (United States General Accounting Office 2000; Powell and Sawicki 1998).

Construction projects can have a tipping point after which the behavior of the system suddenly changes (Repenning et al. 2001; Taylor and Ford 2006; Morrison 2008; Taylor and Ford 2008) and project managers lose control of the project. This change in behavior is abrupt in comparison to the project evolution, making the management of the project more challenging. In addition, the size of the change in behavior is so large that crossing the tipping point can determine success or failure of the project. Crossing a tipping point can either degrade the system (e.g. "catastrophic shifts in rangelands" (Scheffer et al. 2009)) or improve it (e.g. developing a continuous proficiency with a new skill as described in the Morrison's learning curve model (Morrison 2008).

The failure of large construction projects can have enormous consequences. Good management practice tries to predict the future behavior of the system and identify conditions that enhance the system behavior to prevent failure. Having a reliable tool that predicts the future behavior of systems and gives warning signals whenever the system is approaching a potential failure can be valuable to project planners, managers and policy makers and help practitioners to better manage their projects. For example, on a project that is forecasted to finish after its deadline, overtime can decrease the labor deficit gap and accelerate the project progress. However, the same overtime can also have a delayed negative consequence of creating fatigue and reducing productivity, thereby making the project even more behind schedule (Reichelt 1999; Lyneis and Ford 2007). The hours of overtime have a tipping point after which the negative consequences overcome the benefits and adding overtime worsens the project performance in regard to schedule. How much overtime is possible without pushing the project into an increasing delay spiral? How can project managers recognize when a project is near that point? Having knowledge of this tipping point can prevent project failure and avoid additional and unnecessary overtime and delayed completion costs. In general, the purpose of this work is to (1) study system structures that can create tipping points, (2) test hypotheses based on the suggested tipping point indicators in the literature, and (3) identify new potential tipping point indicators. The results are then related to the design and management of construction projects.

Background

Tipping points[1] have been observed in many complex systems within diverse disciplines. In climate change, the change from icehouse earth to greenhouse earth 34 million years ago is an example of a tipping point (Dakos et al. 2008). Changes in global temperature (Scheffer et al. 2009) and regime shifts in the North Pacific climate (Beaulieu et al. 2012) are more recent examples of tipping points. The birth of the Sahara desert in South Africa (Scheffer 2009), sudden drops in fish populations (Scheffer et al. 2009; Beaulieu et al. 2012; Kuehn 2011), loss of vegetation in lakes (Scheffer et al. 2009; Kuehn 2011), and the dramatic shift in coral reefs (Scheffer 2009; Boettiger and Hastings 2012) are examples of tipping points in ecosystems. Tipping point dynamics have also been observed in medicine when studying asthma attacks, epileptic seizures (Scheffer et al. 2009), epidemics (Kermack and McKendrick 1927; Sterman 2000), and depression (van de Leemput et al. 2014). Sociologists are acquainted with tipping points too. Riots and revolutions (Granovetter 1978), smoking and fashion trends (Scheffer 2009; Gladwell 2006), and speeding (Connolly and Åberg 1993) have thresholds after which the behavior of the system suddenly changes. In business and development, learning curves (Morrison 2008) and diffusion of innovations (Sterman 2000) have been demonstrated to have tipping points. Finally, in finance, the collapses in exchange markets is believed by some to have been a tipping point (Scheffer et al. 2009;

[1] Several other terms have been used to refer to tipping points, e.g. critical transition (Scheffer 2009; Scheffer et al. 2009; Kuehn 2011; Hirota et al. 2011; Mrotzek 2011), phase change (Solé et al. 1996), and thresholds (Granovetter 1978; Sterman 2000; Kuehn 2011)

Beaulieu et al. 2012; Kuehn 2011).

Complex systems with a tipping point structure, regardless of their system details, have one thing in common: at some point, the system can cross a threshold and enters a state[2] that is far from its original state (Kuehn 2011). The similarities in these systems' behavior raise the question of whether diverse dynamic systems with embedded tipping point structures demonstrate similar behavior before the transition occurs. If so, are there generic indicators that can recognize this behavior and help to predict the crossing of a tipping point? A large body of literature in the fields of ecosystem and climate change has concentrated on addressing these questions.

Studies (both theoretical and laboratory experiments) have observed similar behaviors in systems before critical transitions, a phenomenon generally known as "slowing down" which is derived from the properties of a bifurcation in dynamical systems. At an equilibrium, the rate of change becomes zero and the system recovers from perturbations slower (Scheffer et al. 2009). The state of the system at any moment becomes more similar to its state at the previous moment (Scheffer et al. 2009; Dakos et al. 2010; Lenton et al. 2012). There are three statistical measures of critical slowing down that are easy to quantify and require monitoring the state variable[3] (Scheffer et al. 2009): (1) recovery rate (the amount of time that a system in equilibrium takes to recover from a small perturbation), (2) temporal autocorrelation (the correlation between a state

[2] State/basin of attraction or attractor is defined as a set of conditions or dynamic states toward which the system (with nonzero initial conditions) converges over time

[3] Dependent/state/target variable is defined as the variable whose value depends on the control variables and is being tested and measured

variable and its lagged value), and (3) variance (the squared deviation of the state variable from its mean).

Scheffer et al. (2009) introduce "spatial patterns"[4] as another indicator of tipping points. Some systems consist of connected units. The units tend to have a similar state to the units they are connected to. As an example, financial markets are known to influence one another. In such systems, the spatial coherence[5] increases near a tipping point. "Cross-correlation" can be used to measure this coherence (Scheffer et al. 2009).

Another approach to predicting a bifurcation tipping point which is based on the "critical slowing down" is the idea of tracking the natural frequency of a system response before an instability point. Virgin and Wiebe (2013) claim that as the system moves from an equilibrium towards an unstable point, the "undamped natural frequency of linear oscillations" tends to zero. However, this natural frequency cannot be observed in experiments. To overcome this problem, the authors apply the eigenvalue theory and show that as a "viscously damped nonlinear system" approaches instability (a tipping point), the damping ratio of the system increases as the critical point is getting closer, supporting the "critical slowing down" effect. They validate their theory by running an experiment and show that this phenomenon can be observed in real systems too.

"Flickering" (switching back and forth between alternate attractors) is another potential indicator which is not derived from the "critical slowing down" (Scheffer et al. 2012). When a system has more than one basin of attraction, the system might

[4] Defined as the perceptual structure, placement or arrangement of objects
[5] Describes the correlation between wave signals from one point to another

repeatedly flip between the two states due to large impacts. At some point, if "the underlying slow change in conditions persists," the system might permanently move to the alternative state. Wang et al. (2012) suggest that in the presence of flickering, the slowing down indicators are stronger. This phenomenon has been observed in models of "trophic cascades"[6] (Scheffer et al. 2009), as well as some climatic shifts and epileptic seizures (Scheffer et al. 2009). Flickering can statistically be observed in "the frequency distribution of states as increased variance and skewness as well as bimodality (reflecting the two alternative regimes)" (Scheffer et al. 2009).

Lamberson and Page (2012) offer a different indicator for a tipping point, they claim that before a tipping point the future state of a system is uncertain. However, when the system tips, there is only one possible outcome for the future state of the system, and the probability of the system being in a particular state converges to one. Lamberson and Page (2012) use Shannon entropy and define the entropy ratio (τ) (the ratio of the Shannon entropy to the expected uncertainty of the system in the next time step) to capture the "tippiness" of a system.

Problem Description and Research Questions

The large and abrupt change in the behavior of complex systems due to tipping points makes their management challenging. The success or failure of such projects depends on the ability to predict the future behavior of these systems. A reliable tool that

[6] An ecological phenomenon triggered by the addition or removal of top predators and involving reciprocal changes in the relative populations of predator and prey through a food chain, which often results in dramatic changes in ecosystem structure and nutrient cycling.

can predict the future behavior of the project in advance can help managers in making timely remedial policies and thereby saving projects. An example is the construction of the Limerick Unit 2 nuclear power plant. As described by Taylor and Ford (2008) unit 2 of the Limerick nuclear power plant is a 1,065 megawatt unit near Philadelphia. Construction began in June of 1974 and was scheduled to be completed in September of 1980. However, the project was only 36% complete in 1980. The construction was stopped in 1982 due to poor performance. In retrospect, if the project owners could have foreseen the future behavior of the project, they could have applied policies (e.g. overtime, using more qualified personnel, scope change, etc.) to prevent the project failure. The Limerick nuclear power plant suffered a tipping point failure. As the number of unfinished work packages (project backlog) increased, the reinforcing loops in the model (rework, schedule pressure and ripple effects) overpowered the balancing loop that made project progress. As a result, the backlog kept increasing and further worsening the situation.

Having practical indicators of tipping points can help avoid such failures. Broad classes of dynamic systems in different fields may demonstrate similarities in system behavior before a tipping point. This has encouraged researchers to find generic indicators of tipping points. “Critical slowing down” is one of the most studied indicators in the literature and is based on the properties of a bifurcation. Despite the theoretical background in bifurcation and nonlinear dynamics theories, most nonlinear systems have been found impossible to solve analytically (Strogatz 2014). Geometrical approaches such as phase space[7] (Strogatz 2014) and stability landscapes[8] (Scheffer 2009) help explain systems with a limited number of variables, however, they are also limited by the assumptions of homogeneity and constant environment (Scheffer 2009). The interactions among multiple components of complex systems, make their behavior different from the findings based on simple models (Scheffer 2009) and the theoretical research on the dynamic behavior of nonlinear and large systems (number of variables >>1) is "at the limits of current understanding" (Strogatz 2014). In summary, "though the mathematical generality of critical slowing down is promising, it does not guarantee that it is useful as an indicator in practice" (Van Nes and Scheffer 2007) and the robustness of the current indicators has not been fully studied (Scheffer et al. 2012; Lenton 2011).

To bridge the gap from simple math models to the application, this work applies a progressive approach starting with simple archetypes and moving on to previously validated models of real systems. Studying existing simple and relatively complex models will provide a better understanding of tipping point dynamics and usefulness of potential indicators. The ultimate goal is to apply this knowledge in practice when there is no formal model available.

This model-based approach is different from the current research in climate change and ecology that has mostly focused on data-based approaches. A data-based

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[7] a phase space is a space (a set with an imposed structure) the elements (phase points) of which (conventionally) represent the states of the system
[8] Graphic representation of the system based on a ball and bowl analogy

approach uses statistics to analyze the behavior of a system before a tipping point and test the effectiveness of potential indicators based on historical data. Hence, there is still a need to understand the underlying structural causes of a tipping point (Scheffer et al. 2012). System theory suggests that classes of systems share generic structures that generate similar behaviors (Sterman 2000). A model-based approach following system dynamics principles can explain the sudden change in the behavior of the system due to a tipping point by using the feedback structure of the system.

To develop transitional theory and bridge the gap between mathematical theories (such as bifurcation theory) and practice, a set of system dynamics archetypes and previously validated system dynamics models of realistic systems have been studied to develop knowledge about the system structures that can create tipping points. The findings are then used to test the existing critical slowing down indicators in realistic models and identify and test new indicators that can be beneficial to practitioners. This knowledge can be used in project management to apply preventive strategies before projects pass the tipping point threshold and get out of control, or in case the consequences of a tipping point are desirable, those strategies can be applied that expedite crossing the tipping point. The current research addresses these issues by investigating the following research question:

How can designers and managers of systems predict tipping point dynamics?

The research question is disaggregated into the following supporting questions. The applied approach to answering each question is described in the next section.

1. What are the necessary and sufficient conditions for a tipping point?

2. What system structure(s) create multiple attractors and a tipping point?
3. How do tipping point indicators behave in different types of tipping point structures?

Research Approach

To answer the research question, the following research steps are conducted:

1. Develop an improved definition of a tipping point.

   This step answers the first supporting research question. In Chapter II, various definitions from the literature have been used to find the necessary and sufficient conditions of tipping points. The results are used to improve and develop a more complete definition of a tipping point.

2. Build a classification of tipping points types.

   To answer the second supporting research question, a set of previously validated system dynamics models with embedded tipping point dynamics are collected in a model library. A taxonomy of tipping point types is developed based on the study of the feedback structures of the library models. Brief descriptions of the models in the library and the developed taxonomy can be found in Chapter III.

3. Develop research hypotheses, design the best testing approach and test the hypotheses

   This step answers the last supporting research question. Chapters IV, V, and IV are designated to research hypotheses. To facilitate the reading of the document each of these chapters is dedicated to a group of hypotheses that have similar backgrounds and testing procedures.

The first section of each chapter introduces the suggested indicator including a discussion about their benefits and shortcomings. Part of the hypotheses are derived from the existing tipping point indicators. Others are developed based on the similarities observed in the system behavior of library models when approaching a tipping point.

The detailed steps to test each hypothesis and the assumptions are described in the second section. The features of system dynamics approach are used to identify the best procedure to test each of the hypotheses.

To test the hypotheses, first, a causal loop diagram of the formally validated models is used to find the main reinforcing and balancing loops in the systems. Second, potential attractors and tipping point conditions of the systems are identified. Then, the procedure identified in the previous step is used to test each hypothesis for every model in the model library. The hypotheses are tested in two different settings: (1) with embedded randomness in the model and (2) without any randomness. The third section of Chapter V includes the results of the hypotheses testing, followed by a summary section that describes the results and discusses the outcome of each hypothesis testing.

4. Application in practice

   In Chapter VII, the two most practical indicators are used to predict a tipping point in a construction project model. Based on the prediction, preventive strategies are used to help the project and the outcome is compared with the case when the project passes the tipping point.

5. Draw conclusions and discuss the implications for research and project managers.

   The last chapter provides a summary of the work done and answers to the research supporting questions. It evaluates the current research describes the contributions to both research and practice. Future research opportunities are also identified in this chapter.

Scope of the Work

The current work studies the transitions in the system that are solely derived from the internal dynamics of the system. Noise-induced tips and system shifts due to external forces are excluded from the scope of the study. Although these types of transitions are equally important when studying the behavior of a system, they are difficult or even impossible to predict. In addition, it is possible for a system to have multiple tipping points and tipping variables[9], but for the purpose of this work, only one tipping point and tipping variable will be studied for each system.

The work focuses on three statistical metrics of "critical slowing down" (namely, recovery rate, temporal autocorrelation, and variance) and other indicators as discussed above are excluded from the study because:

a) Although spatial patterns might be a more powerful indicator of a tipping point because they provide more information than the warning signals in time series (Dakos et al. 2010), the changes in patterns and how they can be interpreted in

[9] High leverage control variables

different systems is not yet fully understood (Scheffer et al. 2012).

b) “Flickering” might be an indicator of having more than one attractor, but it does not guarantee that a transition happens; hence, it is not included as a tipping point indicator in this work.

c) The changes in the entropy ratio occur after the system has already crossed a tipping point, therefore, it cannot be used as a warning signal to predict a future tipping point.

Finally, although, there is a close relationship between tipping point dynamics and resilience[10], studying system resilience and its indicators are beyond the scope of this study. One definition of resilience is “the maximum disturbance a system can take without shifting to an alternative state” and has been suggested by Holling to measure the stability[11] of a system (Van Nes and Scheffer 2007)[12]. However, a major problem in measuring the resilience is the ambiguity about whether the disturbance should be applied to the control variables[13] or the state variable (Van Nes and Scheffer 2007). In addition, in complex models with multiple state variable, finding the maximum disturbance is not easy because the perturbation can be applied in multiple ways (Van Nes and Scheffer 2007).

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[10] Van Nes and Scheffer (2007) have shown that critical slowing down (particularly recovery rate) can be an indicator of instability in the system.

[11] defined as the system ability to absorb disturbances without being tipped to a different attractor (Van Nes and Scheffer 2007)

[12] The term “resilience” should not be confused with the term “engineering resilience” which addresses the recovery rate from a perturbation in the system. “Resilience” or “ecological resilience” is the width of the basin of attraction.

[13] Independent/control/ input variable is defined as the exogenous variables in the system that cause and influence the dependent variable

CHAPTER II

DEFINITION OF A TIPPING POINT

Gladwell (2006) defines one tipping point as “that one dramatic moment in an epidemic when everything can change all at once”. By making different examples like the shift in fashion trends and smoking, he shows that at the tipping point a small change in the system can cause dramatic changes and make the system to move away from its current stable state.

Per Bak et al. (1987; 1988) show that dynamical systems naturally move towards a “self-organized critical point”. This critical point is an equilibrium attractor in which the system tends to stay. They explain the idea by giving a simple example of a “pile of sand”. When the pile is being built, as new grains of sand are added, the pile grows and the slope increases. However, when the slope reaches a critical angle (“angle of repose”), if any more grain of sand is added it will slide off. Before the critical angle, the system is stable but as the critical angle is passed, the system moves away from its stable condition and collapses.

Sterman (2000) describes that in infectious diseases, there is a threshold (a tipping point) beyond which the diseases become an epidemic. Below a tipping point, negative feedback loops are dominant and the system is stable. If a new disease arrives, there will only be a few cases and people recover fast. However, when the system crosses the tipping point, the positive feedback loops dominate the negative feedback loops. In the case of a new disease, it spreads wildly and becomes an epidemic.

Repenning (2001) uses this idea and defines the tipping point in product development systems as “the balance between the workload and resources” in a project. When the project passes a threshold (a tipping point), the organization will have difficulty performing all the project tasks and cannot execute the project.

Similarly, Taylor and Ford (2006; 2008) define a tipping point as “a set of conditions that separate two very different, internally driven, behavior modes”. In their study, they show that when a development project crosses these conditions, it becomes unstable and will eventually fail.

Lamberson and Page (2012) define tipping points in social systems as “discontinuities between current and future states of a system”. At a tipping point, small changes in a variable will significantly impact the future state of the system. Wolfram in his book “A new kind of science” divided behaviors of a system into four types: stable points, cycles, randomness, and complexity (Lamberson and Page 2012). Accordingly, Lamberson and Page (2012) group tipping points into two categories: a system can either tip “within class” (e.g. from one equilibrium to another) or “between class” (e.g. from an equilibrium to chaos). The current work includes both classes.

Sole et al. (1996) give a full summary of different types of phase transitions. Near a critical point (an “instability point”), a system undergoes some qualitative changes. In a first-order phase transition, the system moves from one state to a different state, a phenomenon observed in many physical and biological systems. The best example of a first-order phase transition is the transition of water from liquid to gas. In a second-order phase transition, when the critical point is reached, the system has two potential new states. In other words, at the critical point, a bifurcation occurs and any perturbation will lead the system towards one of the new states.

Lenton (2011) describes a tipping point as a critical point at which a small change can provoke a nonlinear response in the system and change the future state of the system. He gives “Greenland ice sheet, Amazon rainforest and West African monsoon” as examples of systems that have been pushed passed their tipping point by human activities. He claims this transition from the current state of the system to another state resembles a “bifurcation” behavior, a theory in nonlinear dynamics.

In a separate work, Kuehn (2011), uses the terms “critical transition” and “tipping point” to describe the situation when a system is moved to an attractor far away from its original one. He also describes this change to be abrupt in comparison to regular system changes. He gives a summary of bifurcation theory as the mathematical framework which can be applied in critical transitions.

Many other researchers have used the term bifurcation and tipping point interchangeably as well. Ditlevsen and Johnson (2010) describe a tipping point as a bifurcation point beyond which a structural change happens in the system. Thompson and Sieber (2012) see the climate tipping point as an abrupt and often irreversible change in the system. They also relate this event to bifurcations where the state of the system becomes unstable and the system suddenly moves to a completely different state.

Scheffer et al. (2001; 2009; 2012) state that in systems with more than one stable equilibrium[14], when the system crosses a critical threshold (a bifurcation point), it is

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[14] Defined as a set of conditions or dynamic states toward which the system (with nonzero initial conditions) converges over time

either directed towards another (contrasting) state or is moved from a stable attractor to a cyclic or chaotic attractor. In other words, at bifurcation points, the system becomes very sensitive so that a small perturbation creates a large transition in the system. They refer to these transitions in ecosystem and climate as “critical transitions” and call the bifurcation points, tipping points. Furthermore, Scheffer et al. (2009) distinguish three different types of bifurcations: (1) “safe bifurcation” when the system is switched from one stable state to another stable state; (2) “explosive bifurcation” when the system moves far away from equilibrium but in the end returns to the initial stable state and (3) “dangerous bifurcation” where the dynamics of the system after tipping is different from what it has shown before. In system dynamics terms, a positive feedback drives the system towards a contrasting state.

In a slightly different definition, Van Nes and Scheffer (2007), Morrison (2008) and Ellison and Fudenberg (2003) describe a tipping point as an unstable equilibrium. Some complex systems have multiple equilibria and tipping points. When the system is close to an unstable equilibrium (a tipping point), it can easily be tipped to an “alternative basin of attraction” (Van Nes and Scheffer 2007). This new basin can either degrade the system [e.g. “catastrophic shifts in rangelands” (Scheffer et al. 2009)] or improve it [e.g. developing a continuous proficiency with a new skill as described in the learning curve model of Morrison (2008)].

Mrotzek (2011) gives an opposite definition of a tipping point. He claims when a system passes a critical point (tipping point), it moves away from its stable equilibrium to an unstable position, entering a catastrophic state.

These explanations of a tipping point have some features in common:

1. The system is believed to have more than one equilibrium state and the tipping point is at the unstable equilibrium.
2. There exists a threshold at which qualitative changes occur in the system (Kuehn 2011).
3. This change is relatively abrupt in comparison to regular system changes (Kuehn 2011).
4. The transition is either irreversible (Thompson and Sieber 2011; Beaulieu et al. 2012) or the energy required to move the system back to its original state is more than the required energy to tip the system. "The forward and backward switches occur at different critical conditions" (Scheffer et al. 2001).
5. There is a general agreement that this phenomenon is mathematically similar to the bifurcation theory in nonlinear dynamics.

However, reviewing the literature reveals a problem in defining a tipping point. Some authors have defined a tipping point as a point in time (Gladwell 2006; Lamberson and Page 2012). Some have explained the conditions and behavior of the system in proximity of a tipping point (Lenton 2011; Sieber et al. 2012). Other have more focused on the structure of the system (Scheffer et al. 2001; Scheffer et al. 2009; Scheffer et al. 2012; Ditlevsen and Johnsen 2010). For this study, an improved definition of a tipping point that includes different perspectives is used:

- Tipping point behavior:

  Tipping point behavior is the sudden, large and often irreversible shift of the system from its state to an alternative attractor. Near a tipping point, a system becomes sensitive to changes and any small perturbation can tip the system to the other attractor.

- Tipping point conditions:

  Tipping point conditions are the values of the system and tipping variable(s)[15] which make the system shift between the two attractors possible.

- Tipping point structure:

  Tipping point structure is the internal components and connections of the system that create qualitatively different basins of attraction and is based on the feedback structure of the system.

The improved definition of a tipping point has two important implications. First, tipping points occur when the system moves from one attractor to a qualitatively different attractor. If there is not a qualitative difference between the two attractors, the system is not considered to have a tipping point. An example is a system with a dominant goal seeking behavior. In this example, the system always has a stable attractor. Although the value of the dependent variable[16] at the attractor differs from one set of conditions to another, the two attractors are not qualitatively different from one another. Second, changes in the system behavior without any change in the system attractor are not considered as tipping points. In the previous example, if the system

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[15] System control variables that have the highest leverage and can cause the system to tip (excludes initial values of the dependent variable(s)). The value of tipping variable(s) is a proxy for the closeness to the tipping point.
[16] Dependent/state/target variable is defined as the variable whose value depends on the control variables and is being tested and measured

behavior changes from the normal goal seeking to a cyclic behavior around the stable attractor, this is not considered as a tipping point because there are not two distinct attractors in the system.

CHAPTER III

MODEL LIBRARY AND TIPPING POINT TAXONOMY

A Library of Tipping Point Models

The sudden change in the behavior of the system due to a tipping point can be explained by the feedback structure of the system[17]. To gather more knowledge about system structures that can create multiple attractors and thereby tipping points, a set of pre-validated system dynamics models were collected to develop a model library. The tipping point model library includes system archetypes and models of real systems that inherently have multiple attractors and have shown tipping point dynamics, including:

1. Models based on system archetypes (Senge 2006; Senge 2014)
2. Firefighting in product development (Repenning 2001; Black and Repenning 2001)
3. Forest fire management in Portugal (Collins et al. 2013)
4. Sustainable campus improvement program design using energy efficiency and conservation (Faghihi et al. 2014)
5. SIR epidemic: an epidemic model that considers a fixed population with three components (a) susceptible (b) infected, and (c) removed (Kermack and McKendrick 1927; Sterman 2000)
6. World 3, a model that studies the carrying capacity of the earth for human activities (Forrester 1971; Meadows et al. 1972)
7. Kaibab, a model that studies the overshoot of the deer in Kaibab Plateau in northern

[17] Tipping points that are caused by large external pulses are excluded from this study.

Arizona (Goodman 1997; Ford 2000)

8. Learning curve (Morrison 2008)
9. Fishbanks, a renewable resource management simulation (Meadows et al. 1993)
10. Arms Race (Senge 2006; Senge 2014)
11. Limerick construction project model (Taylor and Ford 2006; Taylor and Ford 2008)
12. Social impact bonds, a model that studies the benefits of using social impact bonds in rehabilitation programs of Peterborough prison to reduce prison population (White 2014)

The tipping point model library was used to develop and test the taxonomy of tipping point types as described here. However, for the purpose of hypothesis testing and because of the similarities in the system structures, eight of these models were selected for further study. These eight models fall into two groups based on their complexity. The first group of the selected models is four system dynamics archetypes, simple models that are the building blocks of complex models: (1) limits to growth, (2) fixes that fail, (3) reinforcing loop, and (4) escalation. The second group focuses on models of real systems that are more complex and have multiple stock variables: (1) fish banks, (2) Limerick project, (3) arms race and (4) social impact bonds.

Brief Descriptions of the Models in the Library

Limits to Growth Archetype

This system archetype (Senge 2006; Senge 2014) models the growth of a population limited by a fixed carrying capacity. The model consists of one reinforcing loop and one balancing loop. The reinforcing loop represents the effect of births on the population. The birth rate is the multiplication of a fractional change rate by the current population. As the population increases, the birth rate increases, further adding to the stock of the population. The balancing loop shows the impact of deaths on the population. At the beginning, the birth rate is a large number. But as the population increases and gets closer to the system's carrying capacity, the birth rate decreases over time which will reduce the stock of the population. A stock and flow diagram of "limits to growth" is shown in Figure 1 (based on Bourguet-Díaz and Pérez-Salazar (2003)). This archetype is particularly important because it is the base structure of many real systems other than population growth. Systems in which the growth of the stock is restricted by limiting conditions can be modeled using the limits to growth[18] formulation. Credit card balance, limits to the success of a product, the infectious population in the SIR epidemic model (Kermack and McKendrick 1927), learning curve (Morrison 2008) are some of the examples that use the limits to growth structure as their basis.

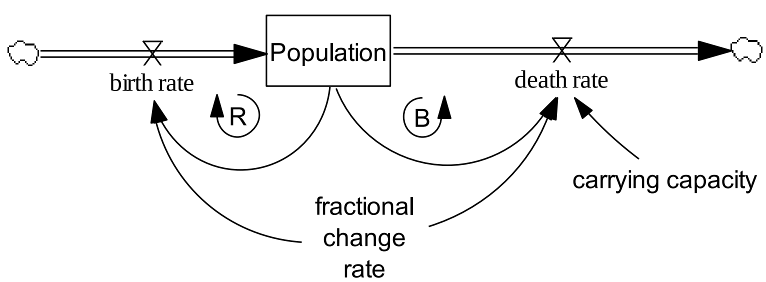


Figure 1 - Stock and flow diagram of limits to growth archetypes (based on Bourguet-Díaz and Pérez-Salazar (2003))

[18] Also known as logistic growth

Equations (modified from Bourguet-Díaz and Pérez-Salazar (2003)):

Population= INTEG (birth rate-death rate,5) *[dependent variable]*

*birth rate=(fractional change rate)*Population*

*death rate=(fractional change rate)*Population*Population/carrying capacity*

carrying capacity=10

fractional change rate=[-1,1, step=0.1] *[tipping variable]*

(a) (b)

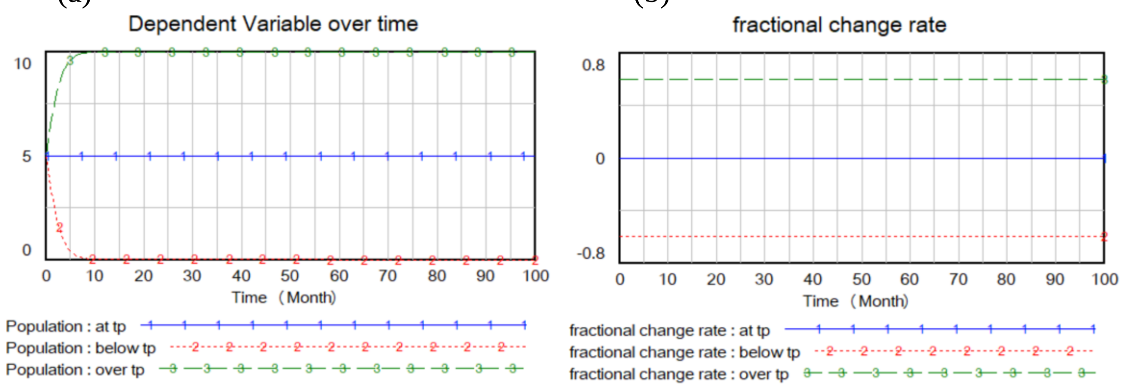


Figure 2 (a-b) - Limits to growth behavior graphs over time
2(a) behavior of population over time for three system conditions (at the tipping point, below the tipping point, and over the tipping point)
2(b) value of the fractional change rate over time for three system conditions

The fractional change rate is the tipping variable in the system which represents the capital growth rate. If the fractional change rate (f) is zero, the birth rate is equal to the death rate and the population neither grows nor decreases over time (graph 1 in Figure 2 (a-b)). However, this equilibrium is not stable and any change in the fractional change rate moves the system away from this unstable state (the tipping point). When the fractional change rate is negative (graph 2 in Figure 2b), death rate becomes larger than the birth rate and thus, the net birth rate is negative and the population goes extinct (graph 2 in Figure 2a). When the fractional change rate is positive (graph 3 in Figure 2b), the birth rate is larger than the death rate. In this case, the population increases over time until it reaches the carrying capacity (graph 3 in Figure 2a). The results are based on the assumption that the initial population is a positive number and much smaller than the carrying capacity of the system. Although in natural system, it is not possible to have a negative fractional change rate, in systems like the credit card balance or the limits to success of a new product, negative values of fractional change rates are conceivable, hence, this variable has been used as the tipping variable in the system.

Fixes That Fail Archetype[19]

This system archetype (Senge 2006; Senge 2014) models the quick fixes to a problem and the negative impacts of the delayed unintended consequences. The model is composed of a balancing loop and a reinforcing loop. The quick fix initially improves the problem systems but after some delay, the negative effects continue to accumulate over time and not only the problem symptoms return but also they may get worse than the original problem. Figure 3 shows a stock and flow diagram of this archetypes (based on Bourguet-Díaz and Pérez-Salazar (2003)).

[19] Shifting the burden archetype is an expansion of fixes that fail that includes both the short term and long term solutions to a problem as well as their unintended consequences. Because of the similarities in the model structure, shifting the burden has been excluded from the hypotheses testing.

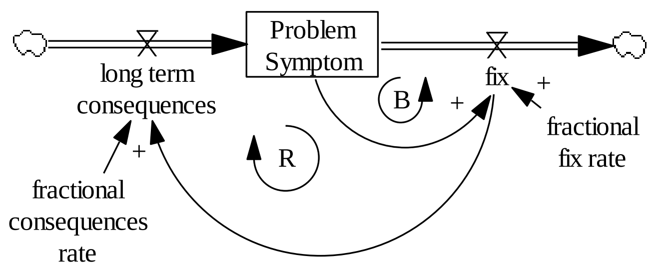


Figure 3 - Stock and flow diagram of fixes that fail archetype (based on Bourguet-Díaz and Pérez-Salazar (2003))

Equations (modified from Bourguet-Díaz and Pérez-Salazar (2003)):

Problem Symptom= INTEG (long term consequences -fix,50) *[dependent variable]*

*long term consequences=fractional consequences rate*DELAY1(fix, 1)*

fractional consequences rate=[0.1,1.9, step=0.1] *[tipping variable]*

*fix= fractional fix rate*Problem Symptom*

fractional fix rate=2

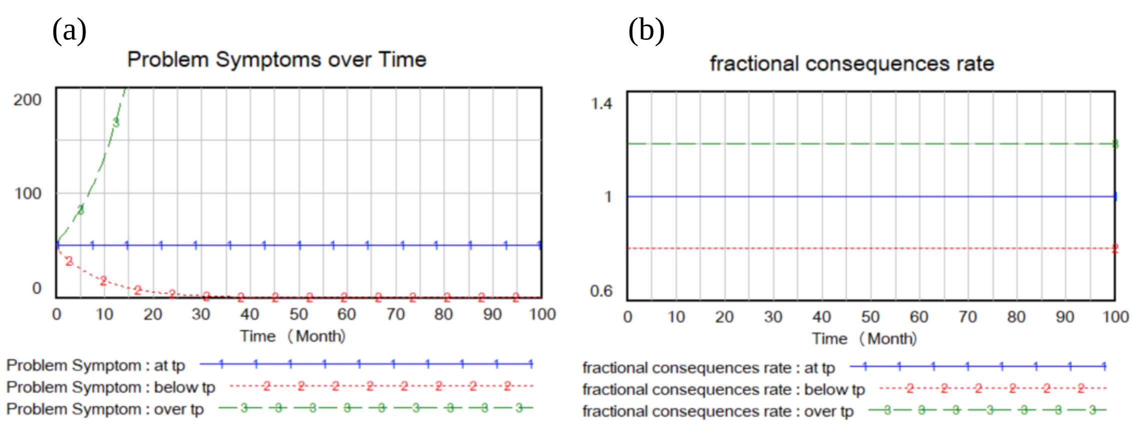


Figure 4 (a-b) - Fixes that fail behavior graphs
4(a) behavior of problem symptoms over time for three system conditions (at the tipping point, below the tipping point, and over the tipping point)
4(b) value of the fractional consequences rate over time for three system conditions

If the long-term consequences are equal to the fixes, the problem symptoms do not change over time and the system sits at its tipping point. If the long-term consequences are small enough then the problem symptoms disappear over time (graph 2 in Figure 4a). However, if the negative effects become larger than the fixed rate, the problem symptoms worsen over time (graph 3 in Figure 4a). The tipping variable in the system is the “fractional consequences rate” which is the rate at which a fix to a solution causes new issues.

Reinforcing Loop Archetype

This system archetype is a single reinforcing loop (Senge 2006; Senge 2014). There is not a real system that is solely made of a single reinforcing loop and a counter balancing loop eventually limits the reinforcing behavior of the system. However, the behavior of some systems can be dominated by a reinforcing loop for a period before the balancing mechanisms kick in. During this time, the dominant reinforcing loop can either turn in a virtuous cycle or a vicious cycle. A change in the direction of the dominant reinforcing loop creates a tipping point. Studying this archetype gives some insight into the mechanism of the change in the direction of a reinforcing loop by isolating the reinforcing behavior of the system from irrelevant balancing mechanisms. Figure 5 shows a stock and flow diagram of the reinforcing loop archetype (based on Bourguet-Díaz and Pérez-Salazar (2003)).

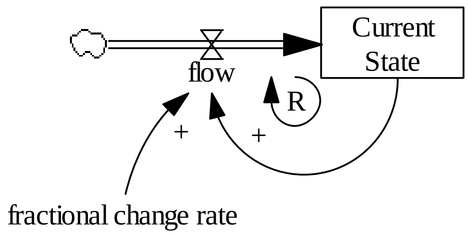


Figure 5 - Stock and flow diagram of the reinforcing loop archetype (based on Bourguet-Díaz and Pérez-Salazar (2003))

Equations (modified from Bourguet-Díaz and Pérez-Salazar (2003)):

Current State= INTEG (flow, 5) *[dependent variable]*

*flow=Current State * (fractional change rate)*

fractional change rate=[-1,1, step=0.1] *[dependent variable]*

The tipping variable in this archetype is the "fractional change rate" which can represent the growth rate in a real system such as an interest rate or a decay rate such as a loss rate. When the fractional change rate (f) is zero the current state of the system does not change over time and the system is standing at an unstable equilibrium (its tipping point). When the fractional change rate is a positive number, the current state grows exponentially over time (graph 2 in Figure 6a). When the fractional change rate is negative, the current state decays over time until it reaches an equilibrium at zero (graph 3 in Figure 6a).

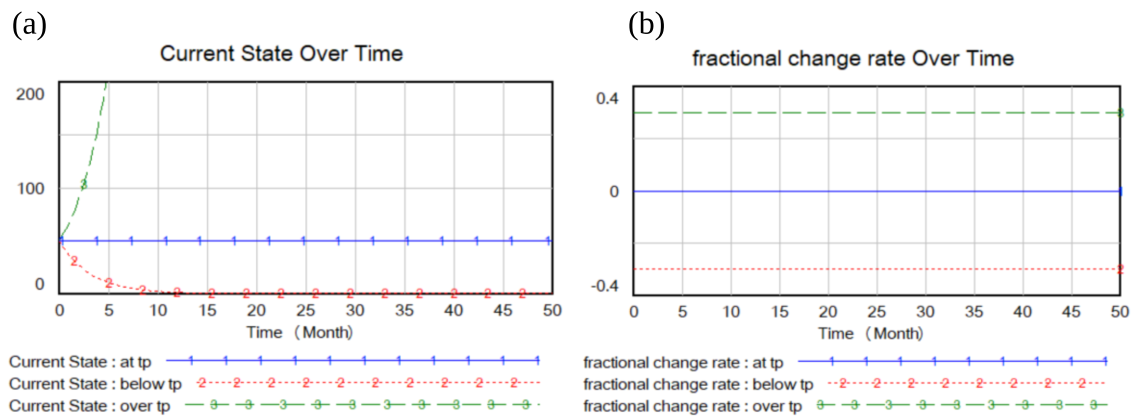
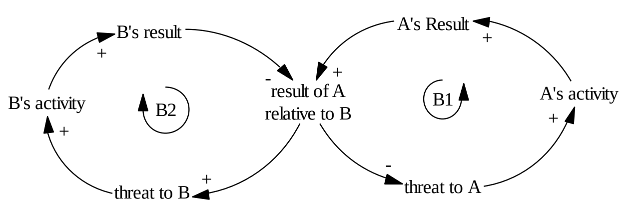


Figure 6 (a-b) - Reinforcing loop behavior graphs
6(a) behavior of the current state over time for three system conditions (at the tipping point, below the tipping point, and over the tipping point)
6(b) value of the fractional change rate over time for three system conditions

Escalation Archetype

Escalation archetype (Senge 2006; Senge 2014) models how the action and outcomes of an individual or a group affect the actions and outcomes of another individual/group. This archetype is made of two balancing loop which counter-interact each other in a way that creates a single reinforcing loop. Figure 7 shows how the interaction of two balancing loops can result in a reinforcing loop behavior. Imagine A and B are two competing companies. In the escalation dynamics, an increase in A's activities will cause B to increase their activity, resulting in a competition. If "A's results" are better than "B's results", the "results of A relative to B" goes up. B feels threatened by A and will increase their activity. As the results of B goes up, "the result of A relative to B" decreases. This cycle closes the B2 balancing loop in Figure 7a. But the "result of A relative to B" is a linking parameter between the two balancing loops. The latest decrease in the "result of A relative to B" will threaten A. A increases their activity to get more results and eventually the "results of A relative to B increases, making B feel threatened by A and we go through the B2 loop again. This archetype is important because several business problems can be modeled on its feedback structure. For example, the competition between two companies with the same product to gain market share can be modeled using the escalation archetype. If one of the companies reduces its product price, the second company will face a drop in its sale and be forced to lower its price, resulting in the first company lowering its price further. This competition goes on until both companies barely break even and are in danger of going out of business.

(a)



(b)

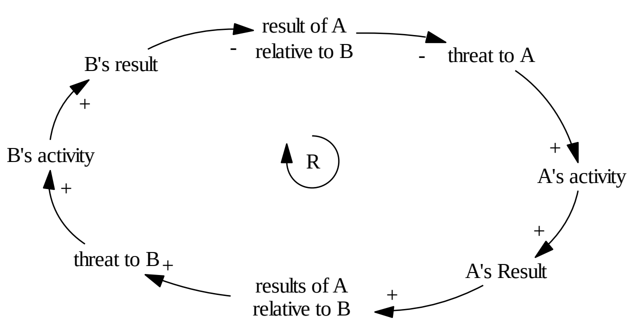


Figure 7 (a-b) - Escalation causal loop diagram
7(a) causal loop diagram of the escalation archetype showing the traditional two balancing loop
7(b) redrawn causal loop diagram of the escalation archetype showing one large reinforcing loop

Figure 8 shows the stock and flow diagram of the escalation archetype as used in this work (based on Bourguet-Díaz and Pérez-Salazar (2003)).

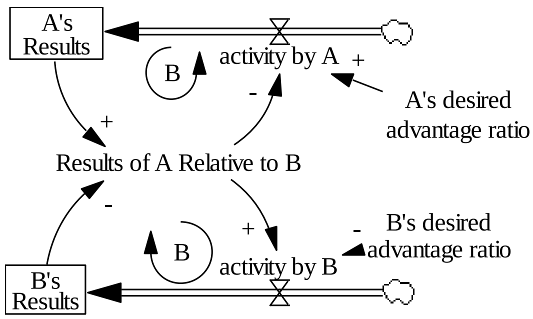


Figure 8 - Stock and flow diagram of escalation archetype (based on Bourguet-Díaz and Pérez-Salazar (2003))

Equations (modified from Bourguet-Díaz and Pérez-Salazar (2003)):

A's Results= INTEG (activity by A, 20) *[dependent variable]*

B's Results= INTEG (activity by B, 20)

activity by A= (A’s desired advantage ratio-Results of A Relative to B)

activity by B=- (B’s desired advantage ratio-Results of A Relative to B)

A’s desired advantage ratio=[0.1,1.9, step=0.1] *[tipping variable]*

B’s desired advantage ratio=1

Results of A Relative to B=A's Results/B's Results

(a) (b)

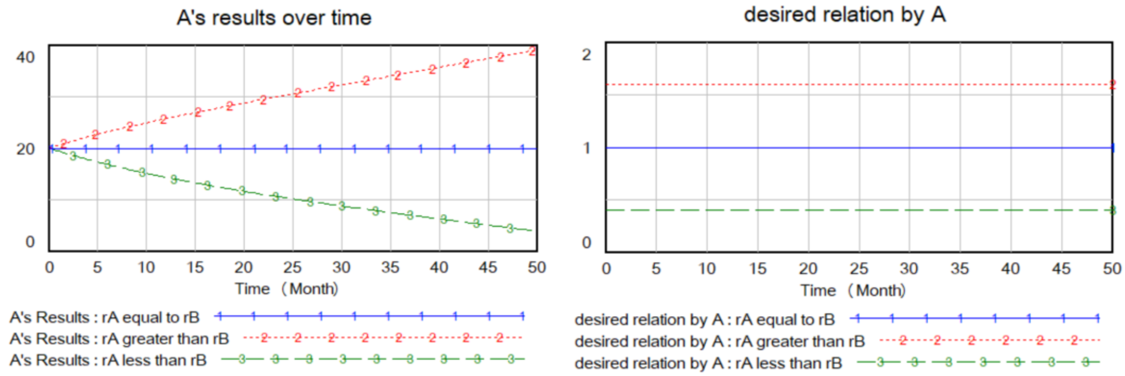


Figure 9 (a-b) - Escalation behavior graphs
9(a) behavior of the "A's Results" over time for three system conditions (at the tipping point, virtuous cycle, and vicious cycle)
9(b) value of the "A's desired advantage ratio" over time for three system conditions

"A's desired advantage ratio" is used as the tipping variable in this system. This variable represents the desired competitive goals of company A compared to the company B's success. When the "A's desired advantage" (r_A) is equal to the "B's desired advantage" (r_B), the system is in an unstable equilibrium. In this state, A's results and B's results do not change over time. If r_A is greater than r_B, A's results and B's results increase over time (Graph 2 in Figure 9a). If r_A is less than r_B, A's results and B's results decrease over time (graph 3 in Figure 9a).

Limerick Construction Project Model

This model is based on the Limerick Unit 2 nuclear power plant construction project as modeled by Taylor and Ford (2006; 2008). Unit 2 of the Limerick nuclear power plant is a 1,065 megawatt unit in Philadelphia. Construction began in June of 1974 and was scheduled to be completed in September of 1980. However, the project progress was only 36% in 1980 and construction was stopped in 1982 due to poor performance. The model (Figure 10) shows the flow of work packages between three main development activities: initial completion, quality assurance and rework. It consists of one balancing loop (project progress) and three reinforcing loops (rework cycle, ripple effects and “haste makes waste”). As work progresses, work packages are moved from initial completion backlog to quality assurance backlog. If there is no error found in the work, the work packages are approved and moved to the work releases. This balancing loop makes the project progress. However, in any activity, a proportion of work is erroneous due to various factors such as human errors, computation errors, etc. and needs to be redone. The first reinforcing loop represents an established project process known as rework cycle (Cooper 1980; Ford and Sterman 1998; Repenning and Sterman 2001; Ford and Sterman 2003a; Love et al. 1999; Lyneis and Ford 2007; Godlewski et al. 2012; Ye et al. 2014). The proportion of work that is erroneous requires rework and enter rework cycle. After the work is redone, it returns to quality assurance to be re-checked. The second reinforcing loop models the impact of schedule pressure on performance. “Haste makes waste” reinforcing loop (Lyneis and Ford 2007) degrade both time and quality performance and decrease the productivity. When a project is behind schedule the workforce works harder and faster in order to compensate for the anticipated delay. But working under pressure decreases the quality of work (Ford and Sterman 2003b) and productivity (Godlewski et al. 2012). As a result, more mistakes are generated and the project backlog[20] increases over time. The last reinforcing loop shows

[20] Defined as the number of work packages that need to be completed

the unintended side effects of changes in the project, aka “ripple effects” (Bower 2000; Love and Edwards 2004; Lyneis and Ford 2007; Ivanov et al. 2014). Changing a work package can affect other work packages, increasing the total amount of work that must be completed. This increase in the scope creates more side effects, which further increases project scope (R3). Figure 10 shows a stock and flow diagram of the Limerick model (Taylor and Ford 2006; Taylor and Ford 2008). See Taylor and Ford (2006; 2008) for model equations.

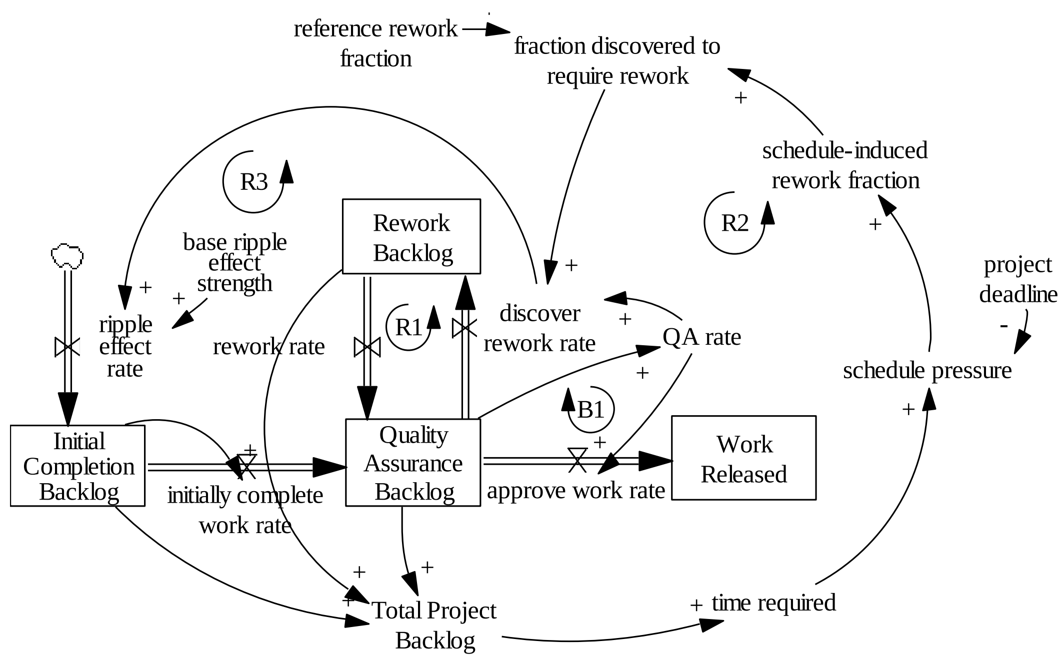


Figure 10- Limerick construction project model (adapted from Taylor and Ford (2008) with permission from ASCE)

Legend of Loops

R1-Rework cycle: Quality Assurance Backlog-Rework Backlog-Quality Assurance Backlog
R2- “Haste Makes Waste” loop: Quality Assurance Backlog-Total Project Backlog-time required-schedule pressure-schedule-induced rework fraction-fraction discovered to require rework-discover rework rate-rework backlog- Quality Assurance Backlog
R3-Ripple effects loop: Initial Completion Backlog- Total Project Backlog-time required-schedule pressure-schedule-induced rework fraction-fraction discovered to require rework-discover rework rate-ripple effect rate-Initial Completion Backlog
B1-Project progress loop: Quality Assurance Backlog-QA rate-approve work rate- Quality

Assurance Backlog

The tipping variable used in this work is the “base ripple effects strength”, which represents the complexity of the system and the level of interdependency between various tasks. To find the tipping variable, statistical screening (Ford and Flynn, 2005; Taylor et al., 2010) was used to find the correlation between the system exogenous variables and the dependent variable (“total project backlog”); the “base ripple effects strength” was found to have the highest correlation and, therefore, was identified as a high leverage variable and was used as the tipping variable in this study. To find the value of “base ripple effects strength” in a real project, a manager can compare the complexity of his/her project to previous projects and estimate the “base ripple effects strength”. When the base ripple effects strength is less than 0.67 (graph 3 in Figure 11b), work is being approved faster than adding extra work due to rework and ripple effects. As long as approve work rate is greater than the rework rate, the backlog of the project decreases over time and the project finally complete (graph 3 in Figure 11a). However, if the base ripple effects strength is greater than 0.67 (graph 2 in Figure 11b), the reinforcing loops become stronger than the balancing loop. The rework rate overpasses the approve work rate and, as a result, the backlog of the project increases constantly (graph 2 in Figure 11a) and the project eventually fails (Taylor and Ford 2006; Taylor and Ford 2008).

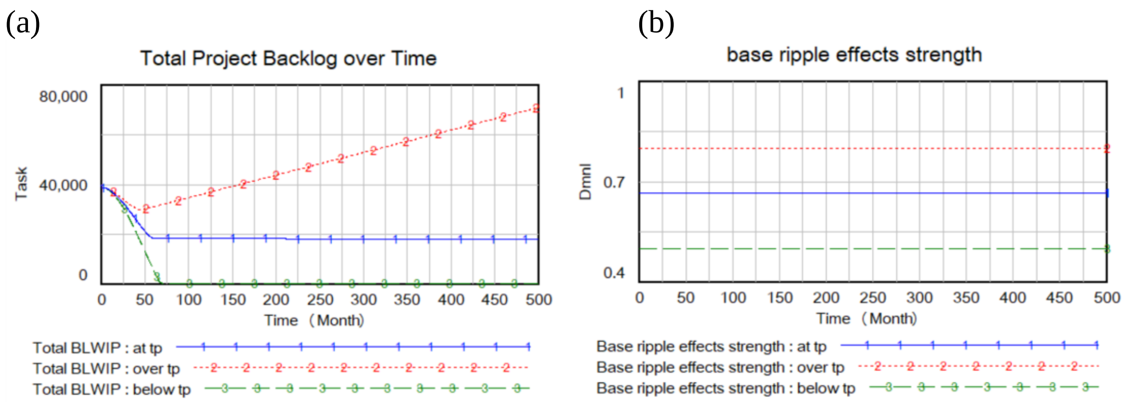


Figure 11-Limerick construction model behavior graphs
11(a) behavior of the total project backlog (BLWIP) over time for three system conditions (at the tipping point, below the tipping point, and over the tipping point)
11(b) value of the base ripple effects strength over time for three system conditions

Fish Banks[21]

Dennis Meadows developed the fish banks game to raise awareness of how natural resources can be used effectively (Whelan 2001). Whelan (2001) under the supervision of Jay Forrester compiled and published the game. The players of the game manage their own fishing company with the same operating cost and technology with the objective of maximizing their profits. Each fishing company starts with an equal amount of resources (money and ships). At the end of each simulation year, the players decide about purchasing new ships and whether they want to fish or not and where to fish. The

[21] Another model with a similar structure to fish banks is the Kaibab Plateau model. This model studies the overshoot of the deer in Kaibab Plateau in northern Arizona (Goodman 1997; Ford 2000). Because of the similarities in the model structures, only fish banks model has been included in the hypotheses testing.

model has three reinforcing loops (hatching and catching fish, and reinvestment) and two balancing loops (fish death and buying ship). Figure 12 shows the stock and flow diagram of the fish banks model (Whelan 2001).

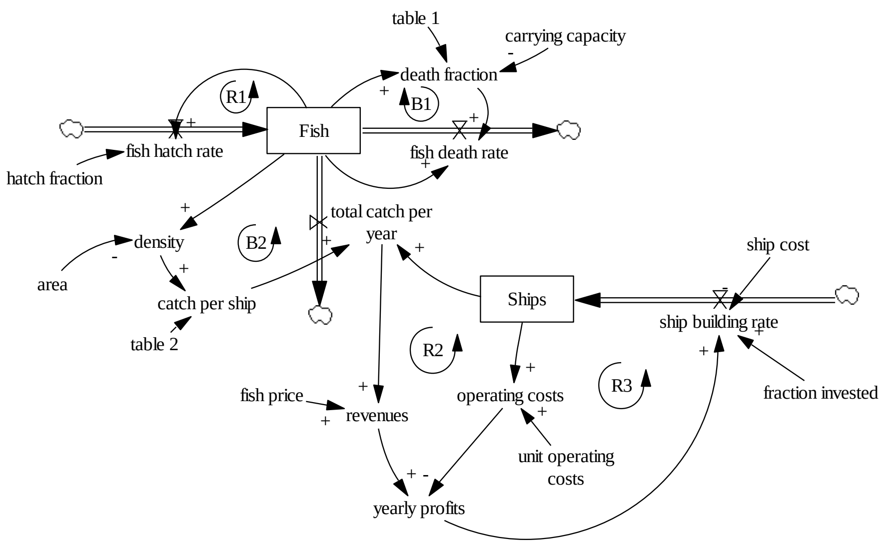


Figure 12-Fish banks game (adapted from Whelan (2001) with permission of MIT)

<u>Legend of Loops</u>
R1-Hatching: Fish-fish hatch rate-fish
R2- Reinvestment: Ships-total catch per year-revenues-yearly profits-ship building rate-Ships
R3-Buying ship: Ships-operating costs-yearly profits-ship building rate-Ships
B1-Fish Death: Fish-death fraction-fish death rate-Fish
B2-Catching Fish: Fish-density-catch per ship-total catch per year-Fish

Equations (Whelan 2001):

Fish= INTEG (fish hatch rate+pulse rate-fish death rate-total catch per year, 20)

[dependent variable]

*fish death rate=Fish*death fraction*

death fraction=table(Fish/carrying capacity)

table([(0,0)-(2,15)], (0,5.22), (0.2,5.23), (0.4,5.255), (0.6,5.345), (0.8,5.665), (1,6), (1.2,6.44), (1.4,7.13), (1.6,7.97), (1.8,9.32), (2,11)) *carrying capacity=1200*

*fish hatch rate=Fish*hatch fraction*

hatch fraction=[0.5:9.5] every 0.5 step *[tipping variable]*

*total catch per year= catch per ship*Ships*

catch per ship=table 2(density)

table 2([(0,0)-(10,40)], (0,0), (1,5), (2,10.4), (3,15.9), (4,20.2), (5,22.1), (6,23.2), (7,23.8), (8,24.2), (9,24.6), (10,25))

density=Fish/area

area=100

Ships= INTEG (ship building rate, 10)

*ship building rate=fraction invested*yearly profits/ship cost*

fraction invested=0.2

ship cost=300

yearly profits=revenues-operating costs

*revenues=fish price*total catch per year*

*operating costs=Ships*unit operating costs*

fish price=10

unit operating costs=250

When the system is in equilibrium, the fish hatching rate (the tipping variable) is equal to the sum of fish death rate and total catch per year; hence, the number of fish stays at the maximum sustainable number after an initial adjustment period. If the values of the variable change and the fish loss becomes greater than the fish hatch, the system gets out of equilibrium and fish population decays. As more fish is caught, more revenue and profit streams into the company. The company can spend the extra profit to buy more ships and catch more fish. But catching more fish decreases the fish population and after a certain point, the natural system is not able to recover from this sudden drop in the fish population and the fish population goes to zero (graph 2 in Figure 13a). If the fish hatching rate is sufficiently greater than the fish loss, the fish population shows an initial growth but this growth is eventually limited by the carrying capacity of the system (graph 3 in Figure 13a). If the fish hatch fraction increases further, the system enters a cyclic behavior and the fish population oscillates around a sustainable fish population.

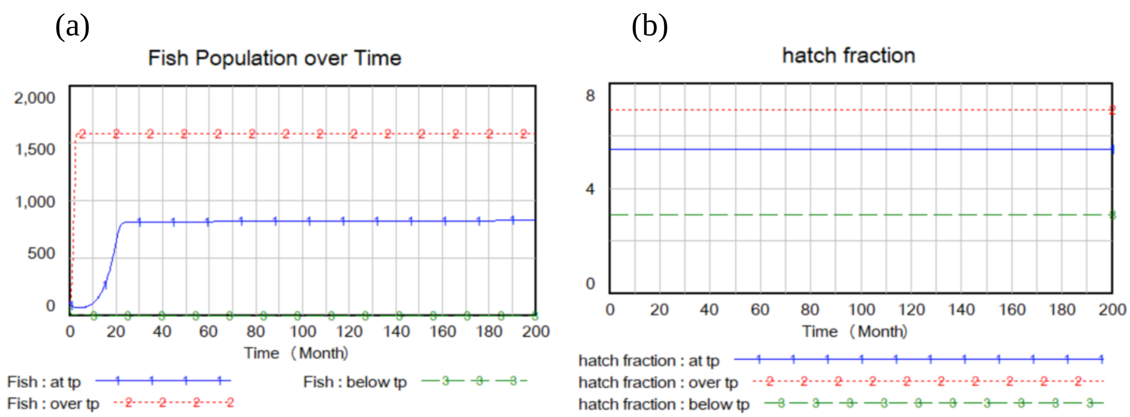


Figure 13- Fish banks behavior graphs
13(a) behavior of the fish population over time for three system conditions (at the tipping point, below the tipping point, and over the tipping point)
13(b) value of the hatch fraction over time for three system conditions

Arms Race

The arms race between the United States and the Soviet Union during the cold war can be modeled as an escalation archetype. Figure 14 shows a simplified stock and flow diagram of an arms race model (based on Ventana Systems (2012)). See Vensim user guide (Ventana Systems 2012) for the full stock and flow diagram and model equations. The model has multiple balancing and reinforcing loops. But the eventual behavior of the system is derived from the R1 reinforcing loop that encourages the arms race between the two countries.

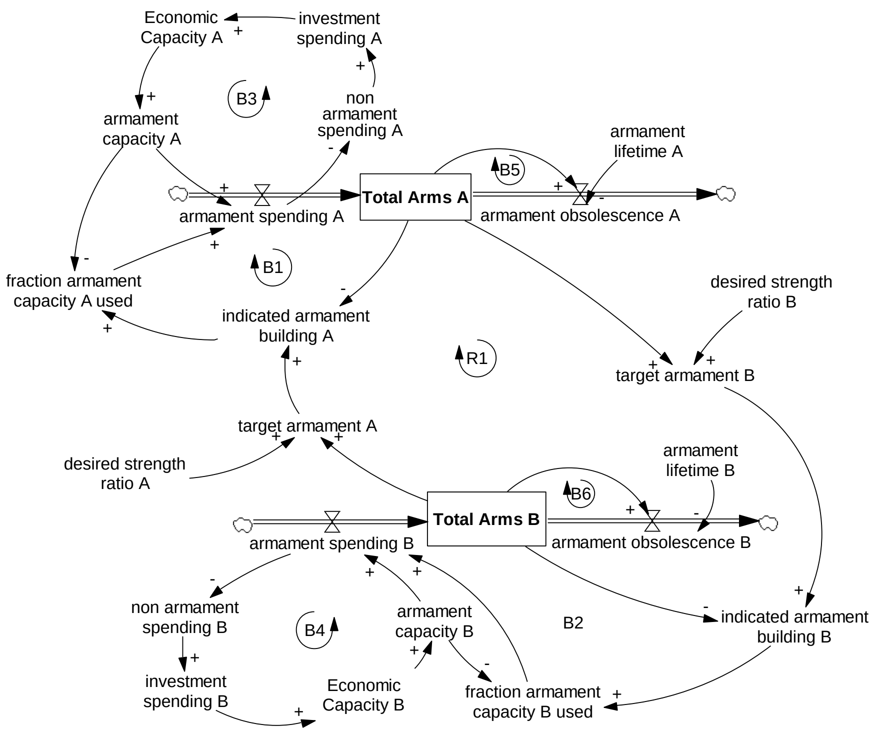


Figure 14-Arms race model (simplified from Vensim user guide (Ventana Systems 2012)

<u>Legend of Loops</u>

R1-Arms race: Total Arms A-target armament B-indicated armament building B-fraction armament capacity B used-armament spending B-Total Arms B-target armament A-indicated armament building A-fraction armament capacity A used-armament capacity spending A-Total Arms A

B1- Armament building A: Total Arms A-indicated armament building A-fraction armament capacity A used-armament spending A-Total Arms A

B2- Armament building B: Total Arms B-indicated armament building B-fraction armament capacity B used-armament spending B-Total Arms B

B3-Economy building A: armament spending A-non armament spending A-investment spending A-economic capacity A-armament capacity A-armament spending A

B4-Economy building B: armament spending B-non armament spending B-investment spending B-economic capacity B-armament capacity B-armament spending B

B5-Armament depletion A: Total Arms A-armament obsolescence A-Total Arms A

B5-Armament depletion B: Total Arms B-armament obsolescence B-Total Arms B

If both countries have the same military strength, the system is in equilibrium. However, if country A feels threatened by the military strength of country B, country A tries to increase its military strength. An increase in country A's military strength will alarm country B who will consequently increase its military strength, activating a vicious reinforcing cycle (graph 2 in Figure 15). The overall behavior of the system is reinforcing and it can either spin in the virtuous cycle of decreasing the arm force of both countries (graph 3 in Figure 15) or the vicious cycle of increasing the arm forces. The "desired strength ratio of A" is the tipping variable in the system. This ratio represents the armament goal of country A in proportion to their opponent country B armament.

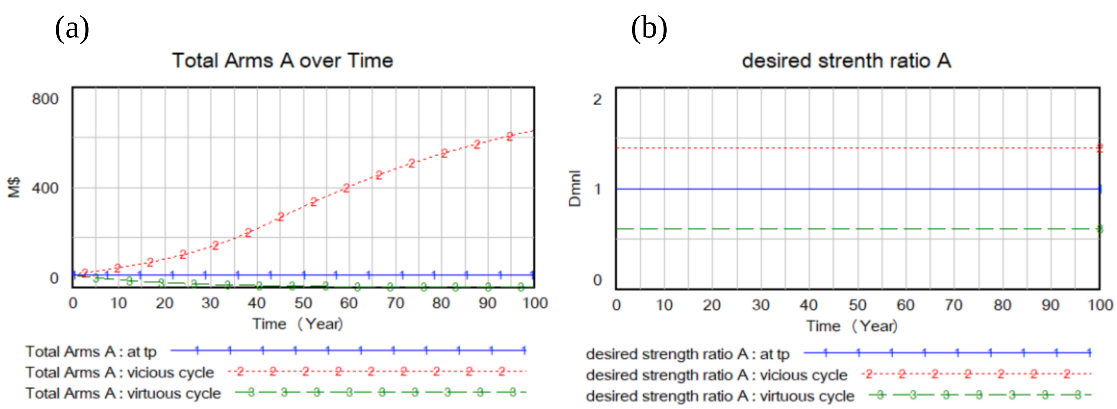


Figure 15-Arms race behavior graphs
15(a) behavior of the "total arms A" over time for three system conditions (at the tipping point, vicious cycle, and virtuous cycle)
15(b) value of the "desired strength ratio A" over time for three system conditions

Social Impact Bonds in Peterborough Prison

White (2014) models the impact of social bonds on prison rehabilitation programs in HM Prison Peterborough. This model shows the tradeoff between prisoner rehabilitation costs and recidivism. The model is based on the idea that rehabilitation programs can decrease the total prison population by decreasing the recidivism and reconviction. Dynamics of relations between social impact bonds and prison population is shown in Figure 16 (simplified from White (2014)). See White (2014) for full model equations.

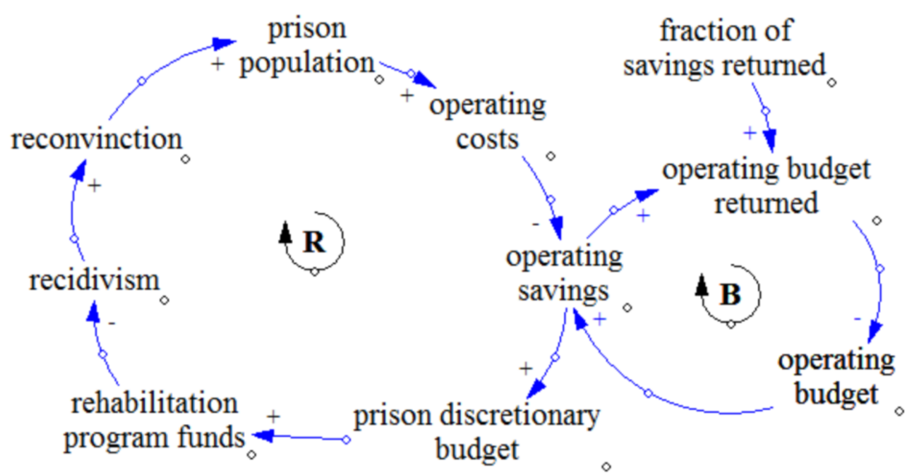


Figure 16-Social impact bonds model (simplified from White (2014))

If the money allocated to rehabilitation program is not enough, it initiates the vicious cycle where lack of rehabilitation program causes more recidivism and increases the prison population. As a result, the operating costs go up and even less money can be spent on rehabilitation programs (graph 2 in Figure 17a). However, if enough money is spent on the rehabilitation program through social impact bonds, the reinforcing loop works as a virtuous cycle. Rehabilitation programs decrease the number of reconviction and the prison population, resulting in more saving in operating costs and allowing more money to be spent on the rehabilitation programs (graph 3 in Figure 17a). The tipping variable used in this system, is the variable “fraction of savings returned” which is the surplus of the budget that is returned and reinvested in the rehabilitation program.

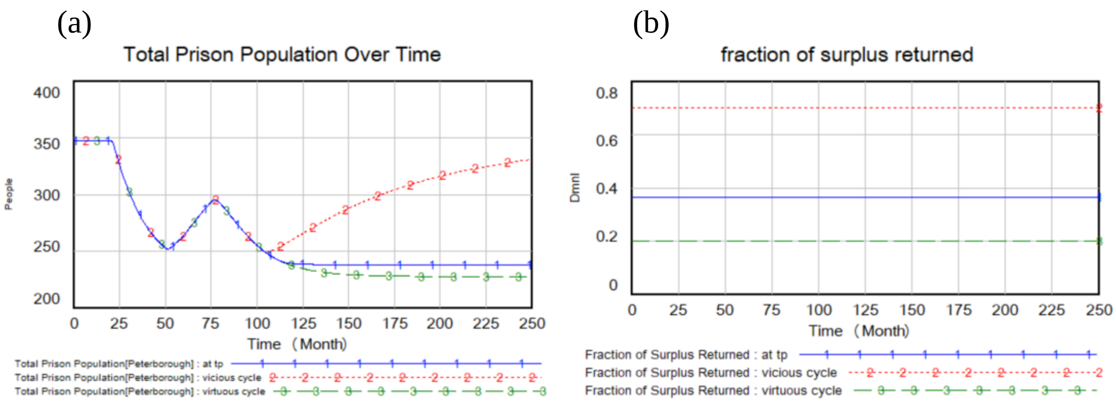


Figure 17-Social impact bonds behavior graphs
17(a) behavior of the total prison population over time for three system conditions (at the tipping point, vicious cycle, and virtuous cycle)
17(b) value of the fraction of surplus returned over time for three system conditions

A Taxonomy of Tipping Point Types

The sudden change in the behavior of the system due to a tipping point can be explained by the feedback structure of the system[22]. To study the tipping point structures, first, a causal loop diagram of the formally validated models was used to identify the main reinforcing and balancing loops in the systems. Second, the main stock variable was selected in each model. Then, the exogenous variables were changed and the systems were allowed to run naturally over a long period of time and under different conditions (different values of control variables) to identify the alternative attractors

[22] Tipping points that are caused by large external pulses are excluded from this study.

(stable or unstable). Finally, the causal loop diagram was revisited to identify any potential similarities in the feedback structures. After studying the feedback structure of the models, three potential tipping point types were identified.

Type I - Change in Loop Dominance

The first type of a tipping point is created by the change in the dominance in feedback loops. When there is a balance between the reinforcing loops and balancing loops, the system stands at an unstable equilibrium (the tipping point). If the balancing loop becomes dominant, the system moves towards a stable equilibrium. However, if the reinforcing loop becomes dominant, due to the nature of a reinforcing loop the impacts of the reinforcing loop are intensified over time and the reinforcing loop becomes stronger until it is limited by other balancing loops in the system. The SIR epidemics model (Kermack and McKendrick 1927; Sterman 2000) is an example of this type of tipping point. In case the balancing loops are dominant in the system, an arrival of a new disease will only cause a few infectious cases and the infected people will recover quickly. However, if the reinforcing loops become dominant in the system, the new disease will spread widely and becomes an epidemic. Figure 18 shows a typical behavior of type I tipping points.

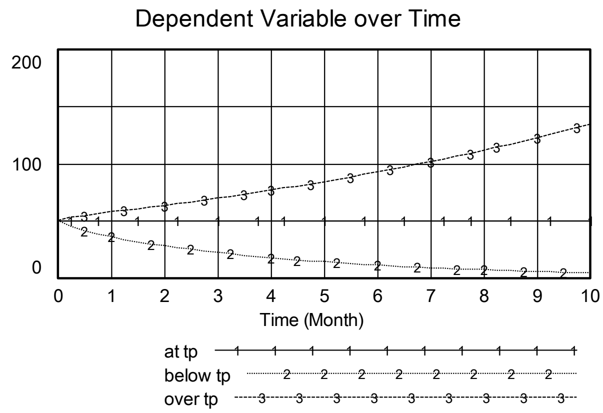


Figure 18-An example behavior of type I tipping points

Type II - Change in Loop Direction

The second tipping point type occurs when the dominant reinforcing loop starts spinning in the opposite direction. The dominant reinforcing loop has the potential to spin in two different directions: 1) vicious cycle and 2) virtuous cycle. At the tipping point, the direction of the reinforcing loop changes from vicious cycle to virtuous cycle or vice versa. In this case, there is no alternative stable equilibrium and the equilibrium state, if existing, is unstable. Escalation archetype is a good example of this type of tipping point. This archetype is made of two balancing loop which counter-interact each other in a way that creates a single reinforcing loop. There are two stock variables in the system. At the tipping point, both stock variables are standing at an equilibrium. However, this state is not stable. Any change in values of the parameters will move the system to either a vicious cycle or virtuous cycle, resulting in infinite increase or decrease of the state variables. Figure 19 shows a typical behavior of type II tipping points.

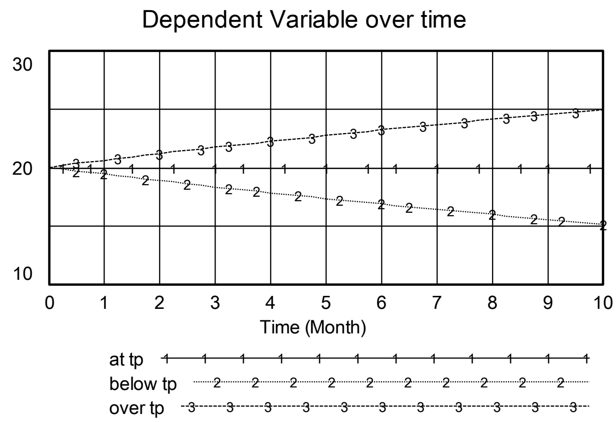


Figure 19-An example behavior of type II tipping points

Figure 20 describes type I and type II tipping points using an analogy of a ball and a bowl.

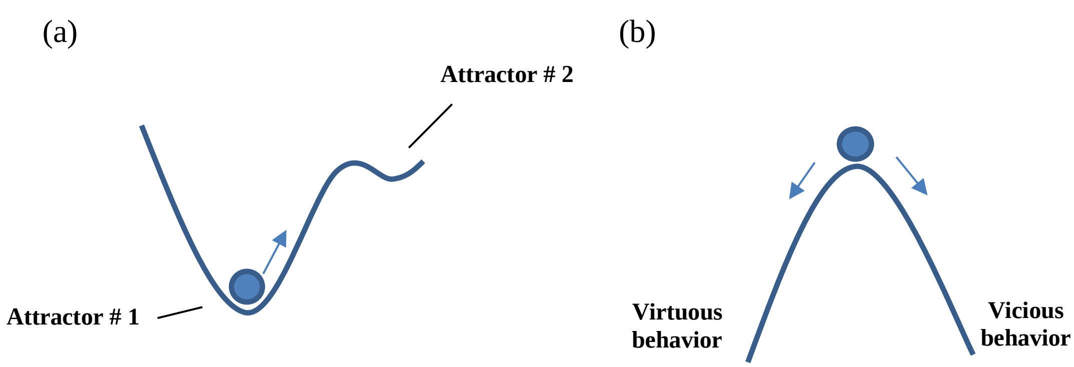


Figure 20 (a-b)-Ball and bowl analogy to describe tipping point types
Balls represent the state variable and arrows represent disturbances. Valleys are the attractors and the peaks are tipping points.
20(a) Tipping point type I: if the balancing loop is dominant the ball remains in attractor #1. If the reinforcing loop becomes dominant, the balls shifts to the attractor #2.
20(b) Tipping point type II: the system is standing on top of a hill and can potentially go down either side of the hill (vicious cycle vs. virtuous cycle).

Type III - Change in System Structure

In this type of a tipping point, there is a qualitative change in the system structure. Unlike the first type of tipping points, the reinforcing loops (if existing) and balancing loops are not working against each other. The balancing loop is dominant but because of the changes in the values of the parameters, the structure of the balancing loop will change and the system behavior will be like that of a reinforcing loop. A single balancing loop is an example of this type of tipping point. For hypothetical negative values of fractional change rate, the balancing loop is converted into a reinforcing loop. Figure 21 shows a typical behavior of type III tipping points. Because no example of this tipping point type was found in the models of real systems, it has been excluded from the hypotheses testing.

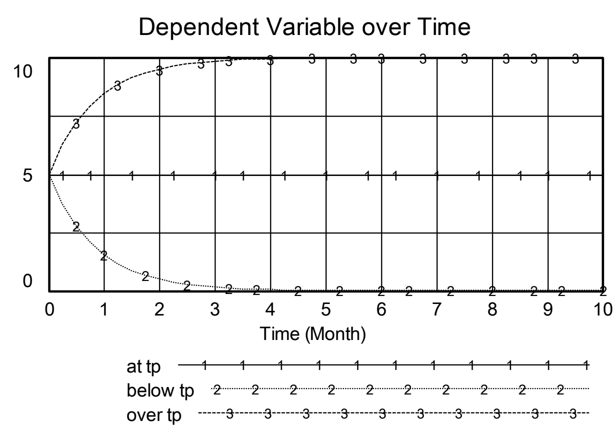


Figure 21-An example behavior of type III tipping points

After identifying the tipping point types, the models in the library were categorized in each type. See Table 1 for tipping point types and their examples from system dynamics archetypes and real systems.

Table 1-Taxonomy of tipping point types with examples

Examples of different TP Types in	TP Type I Change in Loop Dominance	TP Type II Change in Loop Direction	TP Type III Change in Structure
Archetypes	1. <u>Limits to growth</u> (Senge 2006; Senge 2014; Bourguet-Díaz and Pérez-Salazar 2003) 2. <u>Fixes that fail</u> (Senge 2006; Senge 2014; Bourguet-Díaz and Pérez-Salazar 2003)	1. <u>Reinforcing loop</u> (Senge 2006; Senge 2014; Bourguet-Díaz and Pérez-Salazar 2003) 2. <u>Escalation</u> (Senge 2006; Senge 2014; Bourguet-Díaz and Pérez-Salazar 2003)	1. Balancing Loop (Senge 2006; Senge 2014; Bourguet-Díaz and Pérez-Salazar 2003)
Models of real systems	1. <u>Limerick construction project model</u> (Taylor and Ford 2006; Taylor and Ford 2008) 2. <u>Fish banks</u> (Meadows et al. 1993; Whelan 2001)	1. <u>Arms race</u> (Ventana Systems 2012) 2. <u>Social Impact Bonds in Peterborough Prison</u> (White 2014)	

CHAPTER IV

HYPOTHESIS 1

Hypothesis 1 Introduction

Many researchers have used the term bifurcation and tipping point interchangeably (Kuehn 2011; Ditlevsen and Johnsen 2010; Sieber et al. 2012; Scheffer et al. 2001; Scheffer et al. 2009; Scheffer et al. 2012). According to the nonlinear dynamics theory, near a bifurcation, the solutions to the set of differential equations "no longer decay exponentially fast" (Strogatz 2014). "This lethargic decay is called *critical slowing down* in the physics literature" (Strogatz 2014). Although mathematical models of limited dependent variables show the "critical slowing down" near their critical points, the applicability of "critical slowing down" in realistic and more complex models with multiple variables has not been thoroughly studied.

The model library collected in this work includes a wide range of models from simple models with a limited number of state variables (system archetypes) to more complex models that represent real systems. The model library provides an opportunity to test the mathematical theory in more realistic settings without using an analytical approach. The first hypothesis will test if system dynamics models studied here show features of a bifurcation at their tipping points, i.e. when a system is closer to a tipping point, it will reach its stable equilibrium (the solution to the set of differential equations that model the system) slower than when it is far from the tipping point.

H1: As a system approaches a tipping point, the time required for the system to reach its stable equilibrium increases.

In a ball and bowl analogy (Figure 22 (a-c)), the ball represents the dependent variable and the valley is the stable equilibrium. The system is more stable when it is far from the tipping point, in the analogy, this can be shown by having a deeper valley (Figure 22b). When the system is close to the tipping point, it is less stable and can be shown as having a flatter valley (Figure 22c). The ball is standing at the top of the valley (initial conditions). At time=0, the ball starts rolling (Figure 22a) and it rests at the bottom of the valley (the attractor) at time=t (Figure 22b&c). Before the ball comes to a rest, it may oscillate around the attractor (e.g. fish banks model starts oscillating for some system conditions). The amount of time that the ball takes to move from its initial condition to the attractor is proposed as an indicator of a tipping point: as a system approaches a tipping point, the amount of time required to reach an attractor increases.

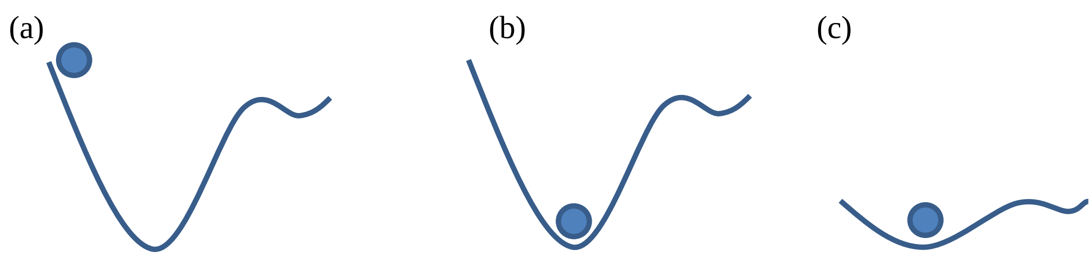


Figure 22 (a-c)- Ball and bowl analogy to describe hypothesis 1
22(a) system at its initial condition
22(b) system reaches its attractor faster; system is far away from the tipping point (the valley is deeper)
22(c) system reaches its attractor slower; system is closer to the tipping point (the valley is flatter)

Hypothesis 1 Testing Procedures

To test the first hypothesis, simulations are run for different values of the tipping variable(s) and the system is left to reach its attractor (without any external perturbation). The time required for the dependent variable to reach an attractor (the proposed tipping point indicator) is measured in two settings: (a) without any noise in the system and (b) with randomness in the system. The detailed steps are as follows:

1. Choose the dependent variable(s). See Chapter III for a stock and flow diagram of the models and equations.

   Example: the stock of "Problem Symptoms" in fixes that fail archetype.

2. Find system attractors. See Table 2 for details of attractors in each model.

   Example: fixes that fail archetypes has three attractors: zero (stable), initial problem symptoms (unstable), and infinity.

3. Find tipping variable(s). Use statistical screening (Ford and Flynn, 2005; Taylor et al., 2010) to find the variable(s) with the highest leverage on the dependent variable and identify them as tipping variable(s). The value of tipping variable(s) is a proxy for the closeness to the tipping point. Define tipping point conditions by finding the values of the control variables (including tipping variables) at the tipping point. See Table 2 for details of tipping point conditions in the models studied here.

   Example: the tipping variable in the fixes that fail model is the" fractional consequences rate" and its value is equal to one at tipping point conditions.

4. Set the tipping variable(s) at a value other than the tipping point conditions.

Example, in fixes that fail archetype, the testing can start from “fractional consequences rate” equal to 1.5.

5. Run simulations and measure how long it takes the dependent variable to move away from the original state and reach its stable equilibrium. In case the system did not have a stable state (e.g. escalation archetype), the amount of time that the dependent variable took to reach a defined large number was used instead.
6. Incrementally change the tipping variable(s) to move the system closer to or further from the tipping point and repeat step 5.

   Example, in fixes that fail archetype, the “fractional consequences rate” (tipping variable) was changed between 0.05 and 1.95 every 0.05 steps.
7. Design randomness into the system that would represent the noise that is present in reality and repeat steps 4-6. This is done by changing selected exogenous variables dynamically over time. The selected exogenous variables are changed at each time step based on a random distribution function within twenty percent of their original value. See Appendix A for details of the selected exogenous variable and the noise formulation.

   Example: in fixes that fail archetype, the variable “fractional fix rate” is changed randomly between 1.6 and 2.4 using a random uniform function.

   See Appendix C for the details of statistical analysis.

Table 2-Model-specific Tipping Point Conditions as Used in the Hypotheses Testing

Model	Dependent variable (dv)	Tipping variable[23] (tp var)	Attractors	Tipping point conditions[24]
Limits to growth archetype	Population	fractional change rate	tp var < 0: dv → 0 tp var = 0: dv = dv_0 tp var > 0: dv → +∞	Initial population<<Carrying capacity Initial population>0 fractional change rate = 0
Fixes that fail archetype	Problem Symptoms	fractional consequences rate	tp var < 1: dv → 0 tp var = 1: dv = dv_0 tp var > 1: dv → +∞	Initial problem symptom > 0 fractional fixes rate = 2 fractional consequences rate = 1
Reinforcing loop archetype	Current State	fractional change rate	tp var < 0: dv → 0 tp var = 0: dv = dv_0 tp var > 0: dv → +∞	Initial state > 0 fractional change rate = 0
Escalation archetype	A's Results	A's desired advantage ratio	tp var < 1: dv → -∞ tp var = 1: dv = dv_0 tp var > 1: dv → +∞	A's initial result=B's initial result > 0 A's desired advantage ratio = 1 B's desired advantage ratio = 1

[23] It should be noted that a system can have more than one tipping variable. In this study, however, only the control variable with the highest levarge on the dependent variable has been identified at the tipping variable.

[24] Although it is possible to have more than one tipping point in the system, in this study only one tipping point, as described in the table, has been studied in each model.

Table 2 Continued

Model	**Dependent variable (dv)**	**Tipping variable (tp var)**	**Attractors**	**Tipping point conditions**
Limerick construction project	Total Project Backlog	base ripple effects strength	tp var < 0.665: dv → 0 tp var = 0.665: dv = 17,882 tp var > 0.665: dv → +∞	base rework fraction = 0.3 base sensitivity to schedule pressure = 0.3 base ripple effects strength = 0.665 Project deadline=75 Initial scope=38,700
Fish banks	Fish Population	fish hatch	tp var < 5.25: dv → 0 tp var = 5.25: dv = 825 tp var > 5.25: dv → oscillation around 2,000	Ship cost=300 Fish price=10 Fraction invested=0.2 Carrying capacity=1200 Area=100 Initial fish=100 Initial ships=10
Arms Race	Total Arms A	desired strength A	tp var < 1: dv → 0 tp var = 1: dv = dv_0 tp var > 1: dv → +∞	Initial armament A=Initial armament B Desired strength B=1
Social impact bonds in Peterborough prison	Total Prison Population	fraction of surplus returned	tp var < 0.365: dv → 228 tp var = 0.365: dv = 238 tp var > 0.365: dv → 331	Initial first time offender=87 Initial reconvicted=260 Conviction cost=2.853 Program cost per member=1.5 Recidivism fraction=0.75 fraction of surplus returned=0.365

Hypothesis 1 Results

The results of hypothesis 1 testing are described in this section. The plot of the amount of time that the dependent variable takes to reach an attractor (without any external perturbation) versus the value of the tipping variable (used as a proxy for the closeness to the tipping point) has been used to test hypothesis 1.

Limits to Growth Archetype

Figure 23 shows the results of hypothesis 1 in limits to growth archetype. The x-axis is the tipping variable (fractional change rate) which shows the proximity to the tipping point. The y-axis represents the time that the system takes to reach its equilibrium state. For negative values of the tipping variable, the system goes to an equilibrium at zero and for positive values of the tipping variable, the system has a stable equilibrium at the carrying capacity of the system. As the system gets closer to the tipping point (tipping variable=0) from either side, the time to reach an attractor increases. For example, at tipping variable equal to 1 (Figure 23a), the system takes less than one hundred time steps to reach the equilibrium state, whereas, when the tipping variable is equal to 0.1 and the system is close to the tipping point, it takes almost 750 steps to reach the equilibrium. These results support the hypothesis. The time to reach an attractor increases as the system approaches the tipping point, even in the presence of dynamic exogenous variables (Figure 23b). The results of t-test show that for values of |fractional change rate|<0.7, the mean of time to reach an attractor increases significantly as the system moves towards the tipping point: for example, there is no statistical difference between the time to reach an attractor when the tipping variable changes from 1 to 0.9 but when the tipping variable changes from 0.6 to 0.5, the time to reach an attractor significantly increases.

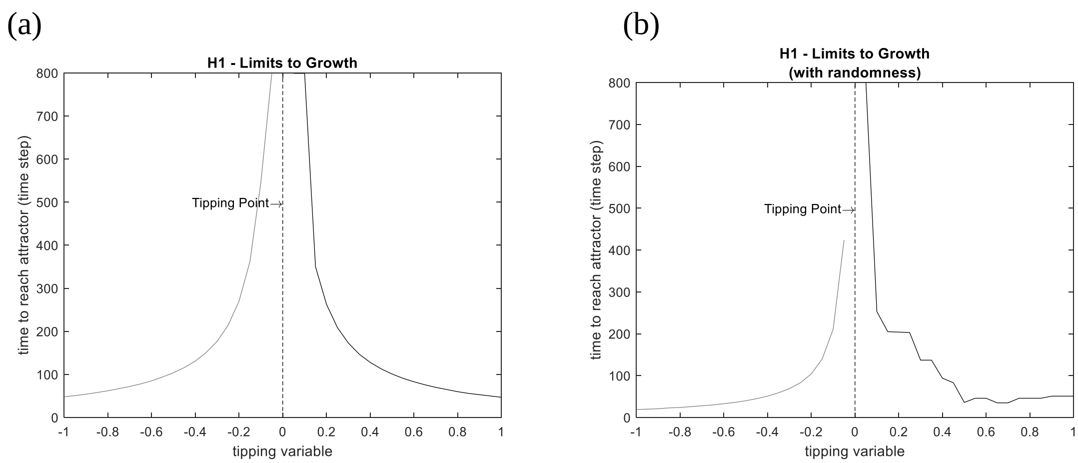


Figure 23 (a-b)- Results of H1 in limits to growth archetype
(Tipping variable: fractional change rate; dependent variable: population)
23(a) time to reach an attractor vs. tipping variable without any noise in the system
23(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Fixes That Fail Archetype

In the fixes that fail archetype, the fractional consequences rate is the tipping variable and the system has a tipping point when the fractional consequences rate is equal to one. For values of the tipping variable less than one, the system has a stable equilibrium at zero. For values of the tipping variable greater than one, the dependent variable increases over time. In the second case, the time it takes the dependent variable to reach a very large number was measured as the time to reach an attractor (assuming the system has an attractor at infinity). As shown in Figure 24a, the simulation results support the hypothesis. As the system gets closer to the tipping point (tipping variable=1 on the x-axis), the time to reach the stable equilibrium (y-axis) increases and the increase is non-linear. The difference between the mean time to reach an attractor is statistically significant for all values of the tipping variable. Although having randomness in the system (Figure 24b) changes the amount of time required to reach an attractor, it does not affect the increasing trend and the results still support the hypothesis.

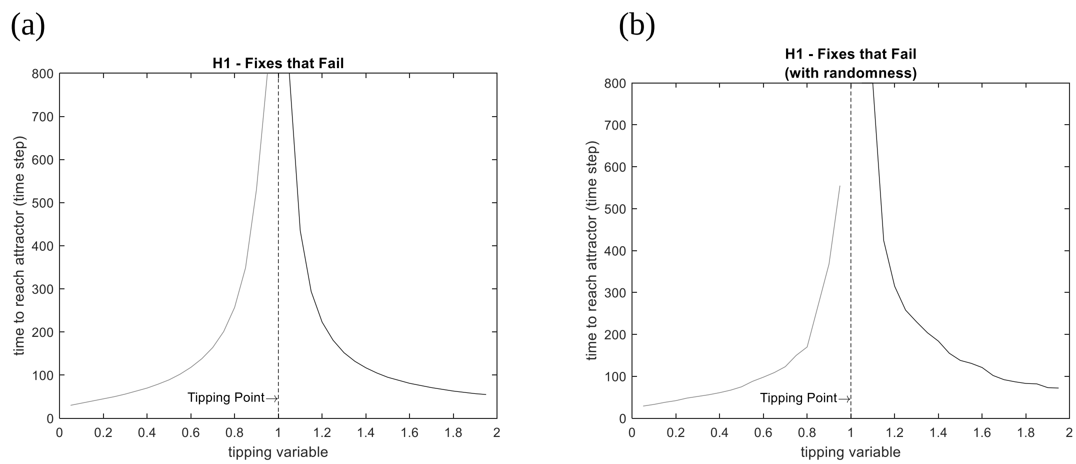


Figure 24 (a-b)- Results of H1 in fixes that fail archetype
(Tipping variable: fractional consequences rate; dependent variable: problem symptoms)
24(a) time to reach an attractor vs. tipping variable without any noise in the system
24(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Reinforcing Loop Archetype

The reinforcing loop archetype has a stable equilibrium at zero when the fractional change rate (tipping variable, x-axis) is negative. For positive values of the tipping variables, the stock of the system increases over time in which case the time to reach a very large number was measured as the time to reach an attractor. The simulation results of hypothesis 1 testing in the reinforcing loop archetype support the hypothesis (Figure 25). As the reinforcing loop archetype gets closer to the tipping point (tipping variable=0 as shown on the x-axis in Figure 25), the time to reach an attractor (y-axis) increases non-linearly. These results support the first hypothesis. Adding noise into the system (Figure 25b) does not affect the results. The changes in the time to reach an attractor are statistically significant when |tipping variable|<0.5.

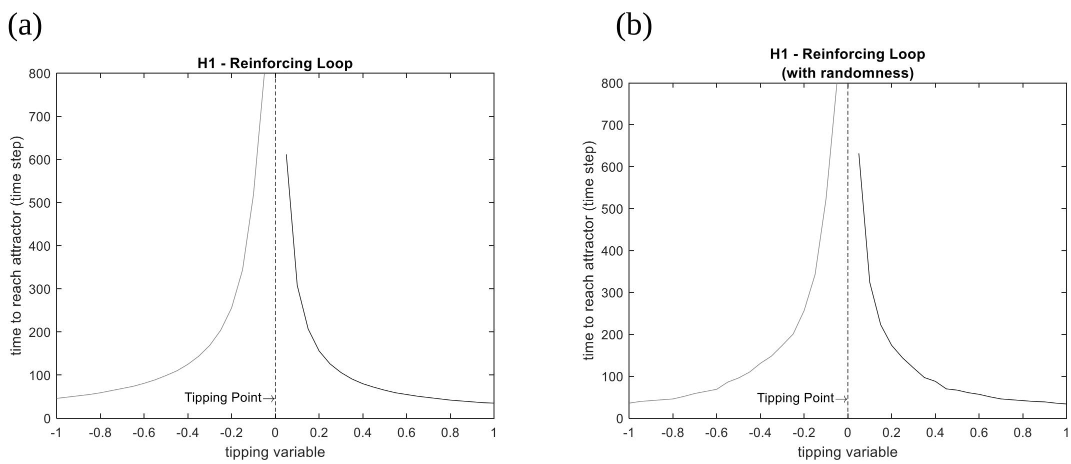


Figure 25 (a-b)- Results of H1 in reinforcing loop archetype
(Tipping variable: fractional change rate; dependent variable: current state)
25(a) time to reach an attractor vs. tipping variable without any noise in the system
25(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Escalation Archetype

The escalation archetype does not have a stable equilibrium. The dependent variable either increases or decreases infinitely depending on the value of the tipping variable. The time to reach a very large positive/negative number has been measured as the time to reach equilibrium, respectively. The results of testing H1 in the escalation archetype are demonstrated in Figure 26. As the system gets closer to the tipping point (Figure 26a, tipping variable =1 on the x-axis), the time to reach an attractor increases from less than 50 times steps to 400 time steps. The increase in the amount of time is non-linear. The insertion of randomness in the system does not have any impact on the results as shown in Figure 26b. The results of a t-test show a statistically significant change in the time to reach an attractor when the tipping variable is between 0.4 and 1.4.

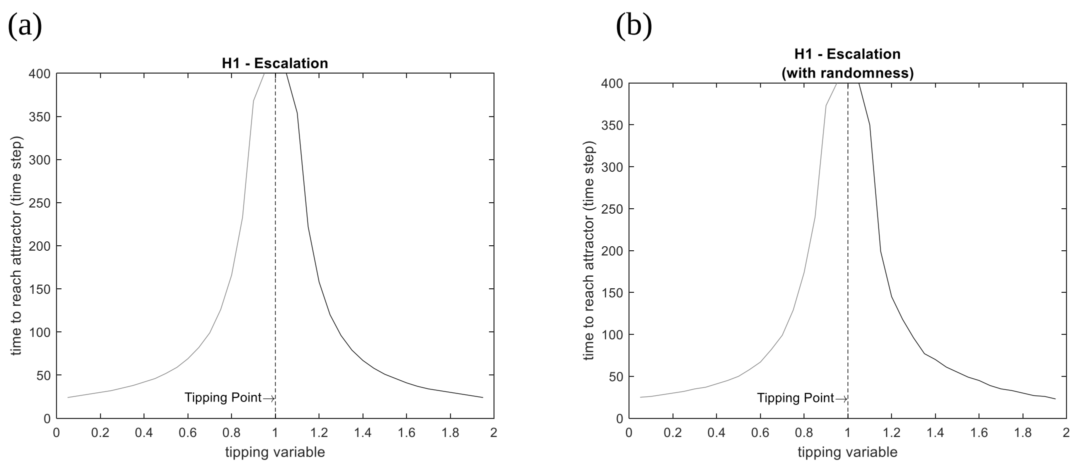


Figure 26 (a-b) - Results of H1 in escalation archetype
(Tipping variable: A's desired advantage ratio; dependent variable: A's results)
26(a) time to reach an attractor vs. tipping variable without any noise in the system
26(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Limerick Construction Project

Figure 27 shows the results of testing H1 in the first realistic model in the library (Limerick construction project). The tipping variable (base ripple effects strength) is shown on the x-axis. The system is at the tipping point when the tipping variable is equal to 0.665. The y-axis shows the time to reach an equilibrium state. As the system gets closer to the tipping point, the time to reach attractor increases (Figure 27a) and the results support the hypothesis. Based on the results of the t-test, for values of base ripple effect strength between 0.35 and 0.75, an increase in the time to reach an attractor can be an indicator of approaching a tipping point. The introduction of randomness to the system changes the time to reach attractor but does not impact the general increasing trend ((Figure 27b).

(a) (b)

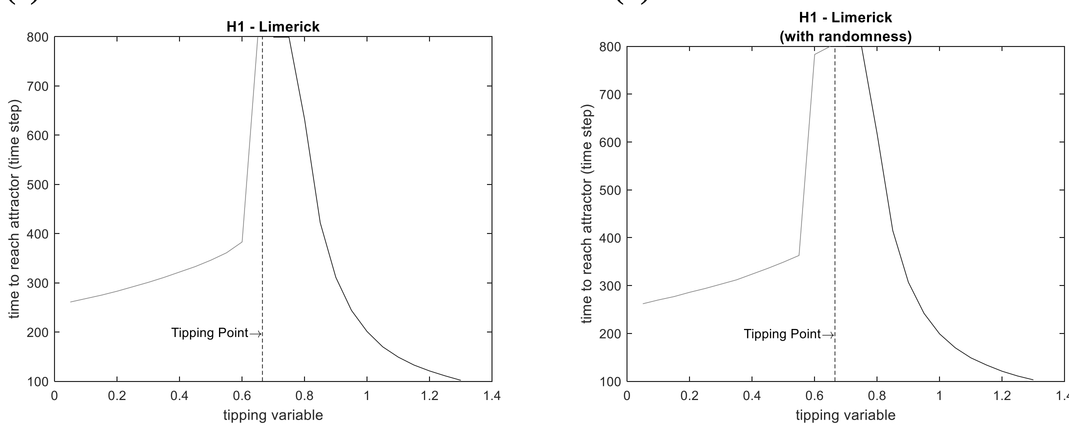


Figure 27 (a-b)- Results of H1 in Limerick construction project model
(Tipping variable: base ripple effects strength; dependent variable: total project backlog[25])
27(a) time to reach an attractor vs. tipping variable without any noise in the system
27(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Fish Banks

Figure 28 shows the results of testing H1 in the fish banks model. The system has two equilibrium states: one when the fish population is equal to zero and the other when the fish population is equal to the carrying capacity of the system. The hatch fraction is

[25] Defined as the number of work packages that need to be completed

the tipping variable and the fish population stands at a tipping point when the tipping variable is equal to 5.25 (as shown on the x-axis in Figure 28). As the tipping variable gets closer to 5.25, the time it takes the fish population to reach an equilibrium state increases (Figure 28a) and the simulation results support the hypothesis. Based on the results of the t-test, the difference in the time to reach attractor is statistically significant when the tipping variable is between 3.5 and 7.5. When randomness is introduced to the system, the amount time to reach the attractor increases (Figure 28b). This is due to the cyclic behavior of the system and the fact that an intrinsic randomness will cause the cycle to dampen slower, hence it takes the system longer to reach an attractor. In presence of randomness, the increase in the time to reach an attractor is gradual when the tipping variable is less than 5.25 (system is approaching an equilibrium at fish=0) but on the other side of the tipping point when the system is going towards the carrying capacity, the increase in time to reach an attractor is abrupt (sudden jump from 5 to 400 time steps when the tipping variable changes from 8.5 to 8 in Figure 28b).

(a) (b)

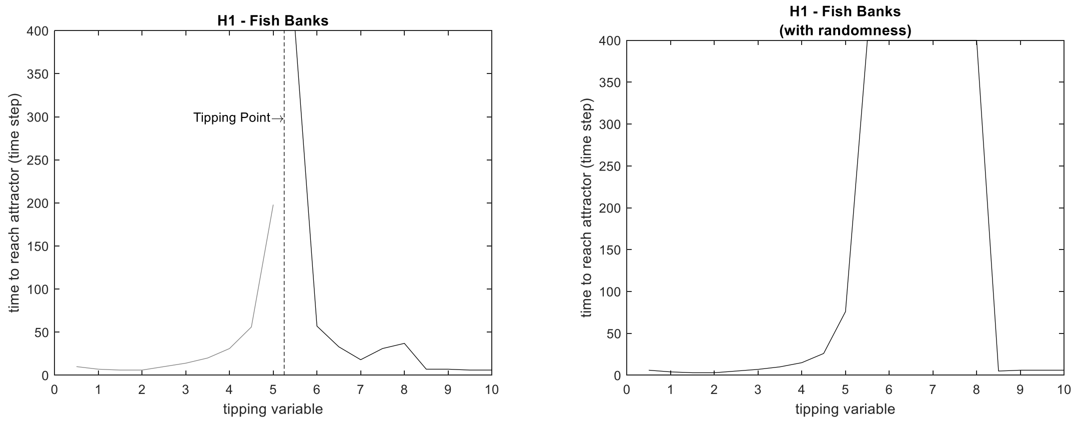


Figure 28 (a-b) - Results of H1 in fish banks model
(Tipping variable: hatch fraction; dependent variable: fish population)
28(a) time to reach an attractor vs. tipping variable without any noise in the system
28(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Arms Race

In the arms race model, the system is standing at a tipping point when the tipping variable is equal to one (x-axis in Figure 29). At the tipping point, the total arms of country A and B are equal. If the tipping variable is less than one, the total arms of both countries decrease over time until there are no arms left. When the tipping variable is larger than one, the total arms increases. In this case, the amount of time that the system takes to reach a very large number has been used as the time to reach an attractor. When the system gets closer to the tipping point (the tipping variable approaches 1), the time to reach an attractor increases (Figure 29a) and the results support the hypothesis. When the system is farther away from the tipping point (tipping variable less than 0.55 or greater than 1.35) there is no significant change in the time to reach an attractor. The presence of dynamic variables in the system (Figure 29b) does not influence the general behavior of the system.

(a) (b)

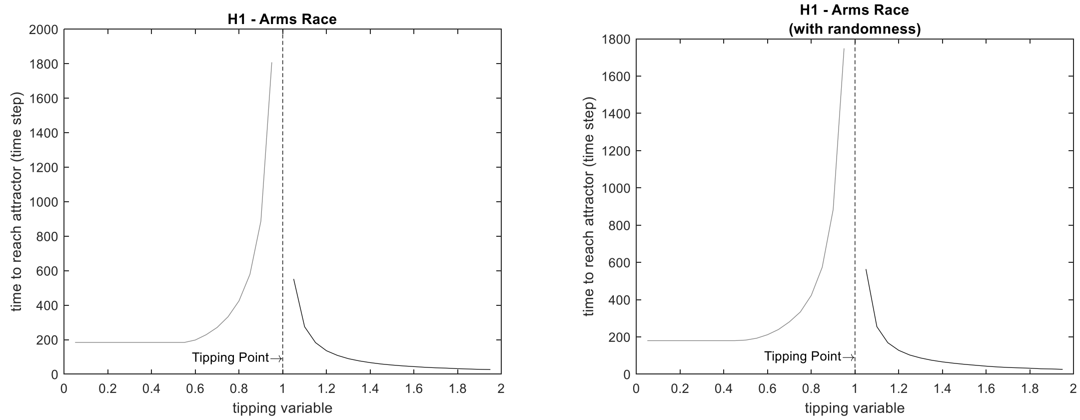


Figure 29 (a-b)- Results of H1 in arms race model
(Tipping variable: desired strength ratio A; dependent variable: total arms A)
29(a) time to reach an attractor vs. tipping variable without any noise in the system
29(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Social Impact Bonds

The social impact bonds model has a tipping point when the fraction of surplus returned (the tipping variable) is equal to 0.365. Depending on the value of the tipping variable, the total prison population moves towards either of its two equilibrium states. Figure 30 shows the H1 testing results in the social impact bonds model. The x-axis is the tipping variable (the tipping point indicator) and the y-axis is the time to reach an attractor. The results of testing H1 in social impact bonds model partially support the hypothesis: If the tipping variable is less than 0.365, there is no change in the time required to reach the attractor until the system is almost at the tipping point (Figure 30a). This can be due to the constraints applied to the system (based on the case study) which prevent the system from showing a pure bifurcation behavior. On the other hand, for values of tipping variable greater than 0.365, the time to reach the equilibrium increases gradually when the system approaches the tipping point (Figure 30a). The increase in the time to reach an attractor is statistically significant when the tipping variable is between 0.365 and 0.6. Simulation results of the social impact bonds model after adding a noise in the system show a similar trend and the hypothesis is partially supported (Figure 30b): the results support the hypothesis when the system approaches the tipping point from one side but there is no change in the time required to reach an attractor when the system approaches the tipping point from the other side.

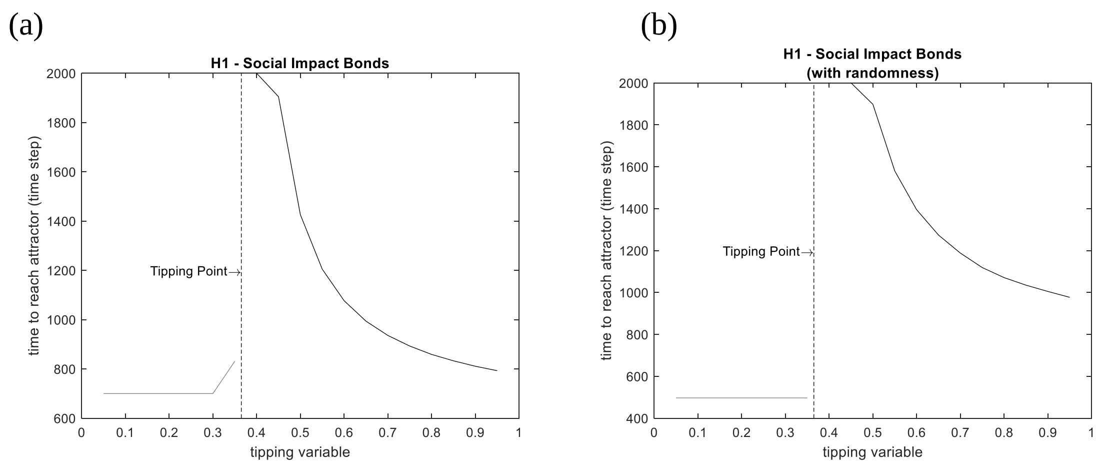


Figure 30 (a-b)- Results of H1 in social impact bonds model
(Tipping variable: fraction of surplus returned; dependent variable: total prison population)
30(a) time to reach an attractor vs. tipping variable without any noise in the system
30(b) time to reach an attractor vs. tipping variable with dynamic exogenous variables

Hypothesis 1 Summary

The first hypothesis tests whether the properties of a bifurcation identified in mathematical models are also observed in system dynamics archetypes and realistic models studied here. The time to reach an attractor is used as an indicator of approaching a tipping point. To test the hypothesis the value of the tipping variable has been used as a proxy of the closeness of the system to the tipping point. All models in the library (both type I and type II) either fully or partially support the hypothesis: the system reaches the equilibrium state slower when the system conditions are close to the tipping point conditions. This indicates that the models in the library have the properties of a bifurcation at their tipping point and that an increase in the time to reach an equilibrium can be an indicator of approaching a tipping point. The relationship between the time required to reach an attractor and the value of the tipping variable is non-linear. When the system is far away from the tipping point, the change in the amount of time required to reach the attractor is not noticeable. But when the system gets closer to the tipping point, the time to reach an attractor significantly increases. This result supports the concerns stated in the literature that predicting a tipping point by using critical slowing down might happen too late (Dakos et al. 2010). The existence of intrinsic randomness in the system impacts the amount of time required to reach the attractor but the general behavior trend remains unchanged. The simulations were replicated multiple times and a standard t-test and ANOVA was used to compare the means. See Appendix C for detailed statistical testing results.

CHAPTER V

HYPOTHESES 2-4

"Critical slowing down", a phenomenon which is derived from the properties of a bifurcation in dynamical systems, is one of the most studied tipping point indicators in the literature. Three ways have been suggested in the literature to measure "critical slowing down" (Scheffer et al. 2009): (1) recovery time of the dependent variable from a small perturbation, (2) lag-1 autocorrelation[26] of the dependent variable, and (3) variance of the dependent variable. The climate change and ecology literature have used historical data to trace the changes in these measures before a known tipping point. In the systems studied, an increase in recovery time, lag-1 autocorrelation and variance were observed before a critical transition. However, the robustness of these indicators has not been fully studied (Scheffer et al. 2012; Lenton 2011). Boettiger and Hastings (2012) have identified some difficulties of using slowing down measures as a tipping point indicator. The increase in variance and autocorrelation should be measured across replicates. But in practice, replicates are rarely available and therefore, a moving window in time in a single replicate is used. The overlap with the consecutive window and the size of the window are arbitrary choices that can influence the results. Another problem that arises from a lack of data, is interpolating from existing data to create "evenly spaced points" for analysis purposes. Such interpolations create artificial autocorrelation. Using computer models instead of historic data can address some of these issues. The realistic models in the library represent

[26] degree of similarity between a given time series and a lagged version of itself over successive time intervals

the complexity of real systems but at the same time provide the benefits of using computer simulations in a controlled setting to test the effectiveness of critical slowing down measures as indicators of a tipping point.

Hypothesis 2 Introduction

The literature has identified three statistical measures of critical slowing down that are easy to quantify and require monitoring the state variable (Scheffer et al. 2009). The first measure is the recovery time. At an equilibrium, the rate of change becomes zero and the system recovers from perturbations slower (Scheffer et al. 2009). A tipping point is an unstable equilibrium, therefore, an increase in recovery time after a small (experimental) perturbation can be a sign of critical slowing down and of approaching a tipping point. The second research hypothesis is based on this measure:

H2: As a system approaches a tipping point, the recovery time after a small perturbation increases.

A ball and bowl analogy[27] (Figure 31) can be used to explain this hypothesis:

[27] Although a ball and bowl analogy can help explaining these concepts, it should not be considered as a perfect representation of systems. Its sole purpose is to give a simplified graphical representation of the concepts.

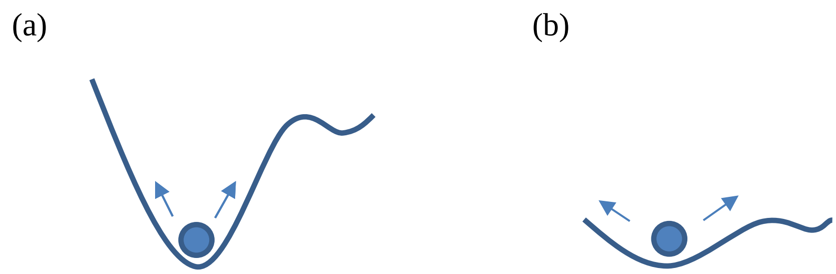


Figure 31- Ball and bowl analogy to describe hypothesis 2
22(a) low recovery time: system is far away from the tipping point (the valley is deeper)
22(b) high recovery time: system is closer to the tipping point (the valley is flatter)

The ball represents the dependent variable and the valleys are the attractors. The system is more stable when it is far from the tipping point, in the analogy, this can be shown by having a deeper valley (Figure 31a). When the system is close to the tipping point, it is less stable and can be shown as having a flatter valley (Figure 31b). The ball is standing at the bottom of a valley before it is perturbed either to the right or left (arrows show the direction of perturbance). The ball starts oscillating and eventually rests at the bottom of the valley again. In hypothesis 2, the time it takes the ball to return to the stable equilibrium is measured and used as an indicator of approaching a tipping point. When the system is far away from the tipping point (Figure 31a), any perturbation decays faster than when the system is close to the tipping point (Figure 31b). Hence, an increase in the recovery time of the dependent variable from a small perturbation is an indicator of approaching a tipping point: as a system approaches a tipping point, the recovery time from a small perturbation increases.

Hypothesis 3 Introduction

The second measure of critical slowing down is the temporal autocorrelation of the state variable. As the system becomes more sluggish before a tipping point, its current state becomes more similar to its previous state. The critical slowing down is revealed by an increase in the lag-1 autocorrelation[28] of the state variable (Scheffer et al. 2009).

H3: As a system approaches a tipping point, the average lag-1 autocorrelation of the dependent variable increases.

In the ball and bowl analogy (Figure 31), imagine that the vertical position of the ball represents the value of the dependent variable. After the ball is disturbed, it started oscillating until it finally rests at the bottom of the valley again. When the system is close to the tipping point (Figure 31b), the valley is flatter, and a perturbation will cause little changes in the position of the ball on the vertical axis. At any time, the vertical position of the ball is very similar to its previous position: the vertical position of the ball is highly correlated to its previous position. Hence, the lag-1 autocorrelation of the dependent variable (the ball) can be used as an indicator of approaching a tipping point: as a system approaches a tipping point, the lag-1 autocorrelation of the dependent variable increases.

---

[28] The autocorrelation measures the similarity of a variable to a lagged version of itself

Hypothesis 4 Introduction

The last measure of the critical slowing down studied here is the variance of the state variable. As the system gets closer to the tipping point, any change in the system will decay slower. This "accumulating effect increases the variance of the state variable" (Scheffer et al. 2009).

H4: As a system approaches a tipping point, the variance of the state variable increases.

Hypothesis 4 looks at the variance of the dependent variable which can be illustrated as the displacement of the ball in a ball and bowl analogy (Figure 31). When the system is far from the tipping point, the valley is deeper (Figure 31a) and it is harder to move the ball. But when the system is close to the tipping point ((Figure 31b), the valley is flatter and a small perturbation will move the ball more easily. The ball moves more in a flatter valley (close to the tipping point) than in a deep valley (far from the tipping point): the ball has a larger variance when close to the tipping point. Hence, an increase in the variance of the dependent variable can be an indicator of approaching a tipping point: as a system approaches a tipping point, the variance of the dependent variable increases.

Hypothesis 2-4 Testing Procedures

To test the second hypothesis 2-4, the system is first set to its stable state. A small perturbation that has no risk of pushing the system over the tipping point is implemented in the system. The time for the dependent variable to return to the equilibrium state is then measured to test hypothesis 2. To test hypotheses 3 and 4, the lag-1 autocorrelation and the variance of the dependent variable are measured. The experiment is repeated for different conditions of the system and pulse sizes. Changing the value of the tipping variable changes the width of the basin of attraction and makes the system more or less stable (farther from or closer to the tipping point); i.e. the value of the tipping variables is used as a proxy of proximity to the tipping point. The detailed procedure is as follows:

1. Choose the dependent variable(s). See Chapter III for a stock and flow diagram of the models and equations.

   Example: the stock of “Problem Symptoms” in fixes that fail archetype.

2. Find system attractors. See Table 2 for details of attractors in each model.

   Example: fixes that fail archetypes has three attractors: zero (stable), initial problem symptoms (unstable), and infinity.

3. Find tipping variable(s). Use statistical screening (Ford and Flynn, 2005; Taylor et al., 2010) to find the variable(s) with the highest leverage on the dependent variable and identify them as tipping variable(s). The value of tipping variable(s) is a proxy for the closeness to the tipping point. Define tipping point conditions by finding the values of the control variables (including tipping variables) at the tipping point. See Table 2 for details of tipping point conditions in the models studied here.

   Example: the tipping variable in the fixes that fail model is the” fractional consequences rate” and its value is equal to one at tipping point conditions.

4. Set the tipping variable(s) at a value other than the tipping point conditions and

let it reach its stable state. See Table 2 for details of tipping conditions and system attractors in each model.

Example: in the fixes that fail model the stable equilibrium is at zero. For any value of the "fractional consequence rate" less than one, the system moves towards this attractor.

5. Apply a small perturbation in the dependent variable.

   Example: in the fixes that fail archetype, the problem symptoms (dependent variable) was pulsed.

6. H2: Measure the amount of time that it takes the dependent variable to return to the equilibrium state (recovery time).

7. H3: Measure the lag-1 autocorrelation of the state variable from the time that the perturbation was applied.

8. H4: Measure the variance of the state variable from the time that the perturbation was applied.

9. Incrementally change the tipping variable(s) to move the system closer to or further from the tipping point and repeat steps 5-8.

   Example, in fixes that fail archetype, the "fractional consequences rate" (tipping variable) was changed between 0.05 and 0.95 every 0.05 steps.

10. Repeat steps 4-9 for different pulse sizes.

    Example, in fixes that fail archetype, the pulse size is between -10 and 10 with increments of 2.

11. Design randomness into the system that would represent the noise that is present

in reality and repeat steps 4-10. This is done by changing selected exogenous variables dynamically over time. The selected exogenous variables are changed at each time step based on a random distribution function within twenty percent of their original value. See Appendix A for details of the selected exogenous variable and the randomness formulation.

Example: in fixes that fail archetype, the variable “fractional fix rate” is changed randomly between 1.6 and 2.4 using a random uniform function.

See Appendix C for the details of statistical analysis.

Hypothesis 2-4 Results

The results of hypothesis 2-4 testing are discussed in this section. After the system reaches a stable equilibrium, a perturbation is put in the dependent variable of the system. The recovery time from the perturbation, lag-1 autocorrelation of the dependent variable and the variance of the dependent variable are used to test H2 through H4 respectively. The value of the tipping variable is used as a proxy for the closeness to the tipping point.

Limits to Growth Archetype

Figure 32 shows the results of hypothesis 2 in limits to growth archetype. The x-axis is the tipping variable (fractional change rate) which shows the proximity to the tipping point. The y-axis represents the recovery time from a perturbation. For negative values of the tipping variable, the system goes to an equilibrium at zero and for positive values of the tipping variable, the system has a stable equilibrium at the carrying capacity of the system. As the system gets closer to the tipping point (tipping variable=0) from either side, the recovery time from a perturbation increases. For example, the recovery time from a perturbation when the tipping variable is equal to 1 is less than fifty time steps (Figure 32a). When the system gets closer to the tipping point and the tipping variable is equal to 0.1, the recovery time increases to four hundred time steps (Figure 32a). Similar behavior is observed when there are dynamic exogenous variables in the system: the recovery time increases as the system approaches the tipping point (Figure 32b). The test results support the hypothesis. The results of t-test[29] show that for values of $-0.8<$fractional change rate<0.7, the mean recovery time increases significantly as the system moves towards the tipping point: for example, there is no statistical difference between the time to reach an attractor when the tipping variable changes from 1 to 0.9 but when the tipping variable changes from 0.6 to 0.5, the time to reach an attractor significantly increases ($\alpha=0.05$).

[29] The t-test was performed for pulse size equal to ±3.

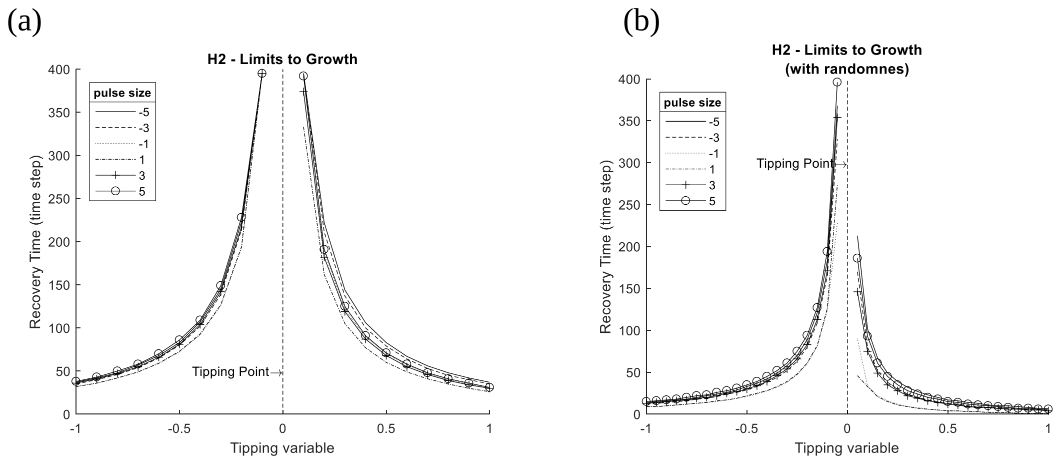


Figure 32 (a-b)- Results of H2 in limits to growth archetype
(Tipping variable: fractional change rate; dependent variable: population)
32(a) recovery time vs. tipping variable without any noise in the system
32(b) recovery time vs. tipping variable with dynamic exogenous variables

Figure 33 (a-b) shows the test results of hypothesis 3 in the limits to growth archetype. The x-axis shows the tipping variables (a proxy for the tipping point) and the y-axis represent the lag-1 autocorrelation of the dependent variable. the results show that, as system conditions get closer to tipping point conditions (tipping variable=0), the temporal autocorrelation increases linearly. The results are robust in the presence of randomness in the system. And the difference in the mean recovery time is statistically significant for all values of the tipping variable (i.e. [-1,1] as studied here). See Appendix C for the details of the results of statistical analysis.

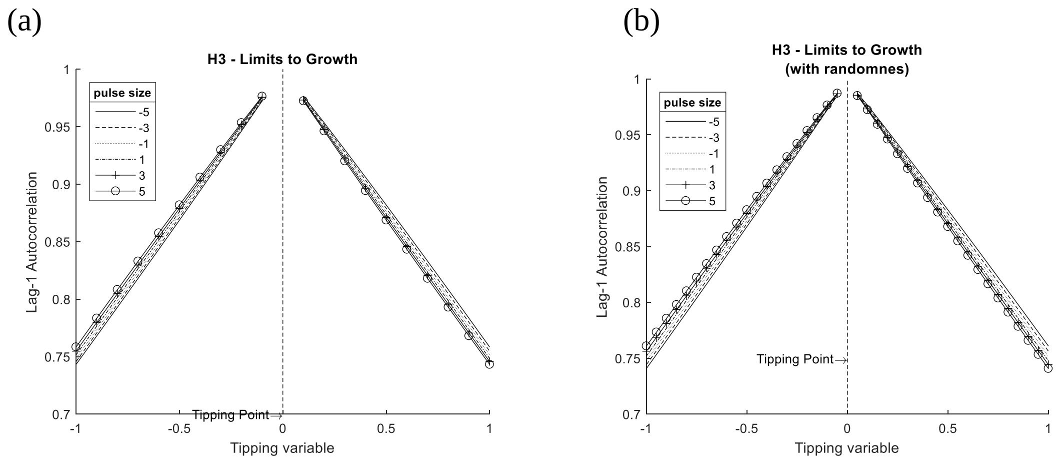


Figure 33 (a-b)- Results of H3 in limits to growth archetype
(Tipping variable: fractional change rate; dependent variable: population)
33(a) lag-1 autocorrelation vs. tipping variable without any noise in the system
33(b) lag-1 autocorrelation vs. tipping variable with dynamic exogenous variables

Hypothesis 4 test results in limits to growth archetype are shown in Figure 34 (a-b). The x-axis shows the tipping variable and the y-axis represents the variance in the dependent variable. In general, the results support the hypothesis. As system conditions get closer to tipping point conditions (tipping variable=0), for larger pulse sizes, the average variance increases (Figure 34a). However, the amount of increase depends on the pulse size and for very small perturbations there is no change in the variance. Standard t-test and ANOVA was performed for the mean variance when the pulse size is equal to ±3. The results show a significant increase in the mean variance as the system moves towards the tipping point when -0.6<tipping variable<0.7.

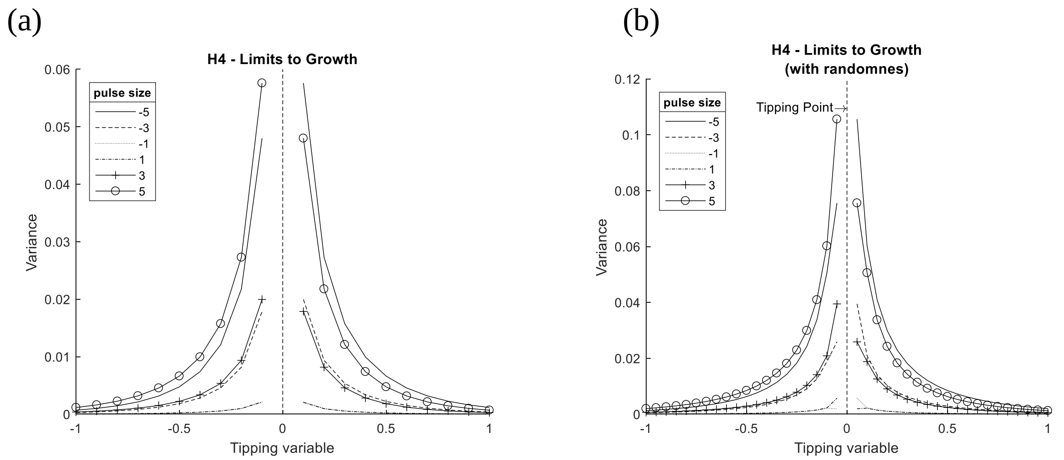


Figure 34 (a-b)- Results of H4 in limits to growth archetype
(Tipping variable: fractional change rate; dependent variable: population)
34(a) variance of the dependent variable vs. tipping variable without any noise in the system
34(b) variance of the dependent variable vs. tipping variable with dynamic exogenous variables

Fixes That Fail Archetype

In the fixes that fail archetype, the fractional consequences rate is the tipping variable and the system has a tipping point when the fractional consequences rate is equal to one. For values of the tipping variable less than one, the system has a stable equilibrium at zero. For values of the tipping variable greater than one, the dependent variable increases over time. In order to test H2-H4, the system needs to have a stable equilibrium, therefore, these hypotheses have been tested in the first case when the tipping variable is smaller than one. As shown in Figure 35 (a-b), the simulation results support the hypothesis. As the system gets closer to the tipping point (tipping variable=1 on the x-axis), the recovery time from a perturbation (y-axis) increases and the increase is non-linear (Figure 35a). The results are robust over different pulse sizes. In addition, the difference between the mean recovery is statistically significant for all values of the tipping variable[30]. Although having randomness in the system (Figure 35b) creates some fluctuations in the graphs, it does not affect the general increasing trend and the results still support the hypothesis.

(a) (b)

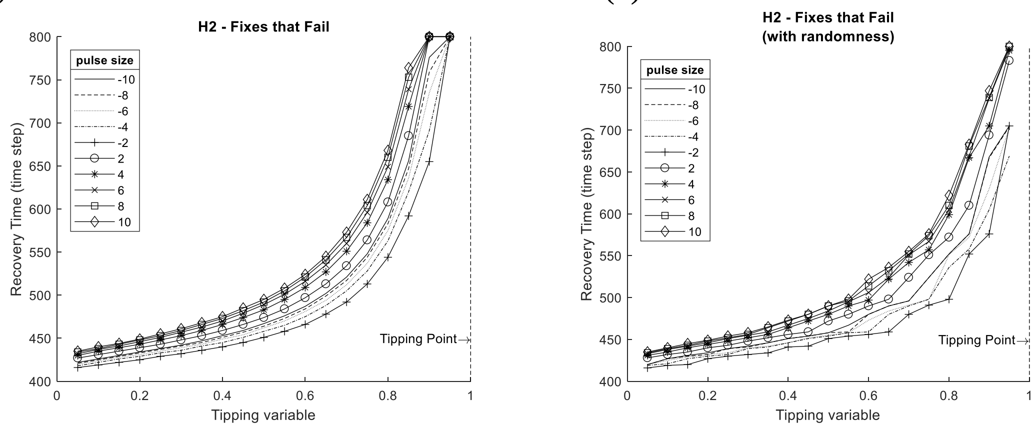


Figure 35 (a-b)- Results of H2 in fixes that fail archetype
(Tipping variable: fractional consequences rate; dependent variable: problem symptoms)
35(a) recovery time vs. tipping variable without any noise in the system
35(b) recovery time vs. tipping variable with dynamic exogenous variables

The results of testing H3 in fixes that fail archetype support the hypothesis (see Figure 36 (a-b)). As the system approaches the tipping point (at tipping variable equal to 1), temporal autocorrelation increases. The results are robust for different pulse sizes and the relationship between the lag-1 autocorrelation and the tipping variable is non-linear. The results are supported by the standard t-test and ANOVA.

---

[30] Standard t-test and ANOVA was performed for pulse size equal to ±4. See Appendix C for details.

(a) (b)

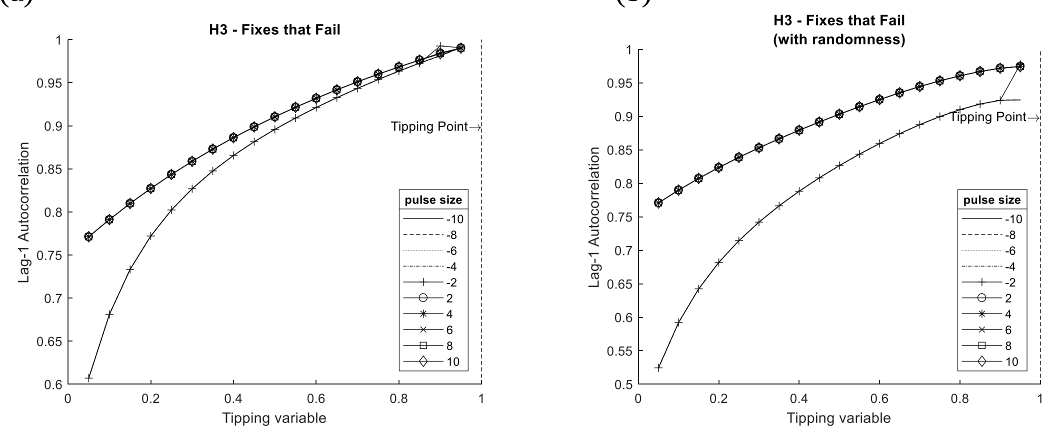


Figure 36 (a-b)- Results of H3 in fixes that fail archetype
(Tipping variable: fractional consequences rate; dependent variable: problem symptoms)
36(a) lag-1 autocorrelation vs. tipping variable without any noise in the system
36(b) lag-1 autocorrelation vs. tipping variable with dynamic exogenous variables

Figure 37 (a-b) show the results of testing H4 in fixes that fail archetype. When the system is far away from the tipping point there is no change in the variance of the dependent variable but as the system approaches the tipping point, the average variance increases (Figure 37a). The results of standard t-test and ANOVA show significant differences in the mean variance. Inserting randomness in the system affects the values of the variance but the general increasing trend is unchanged (Figure 37b).

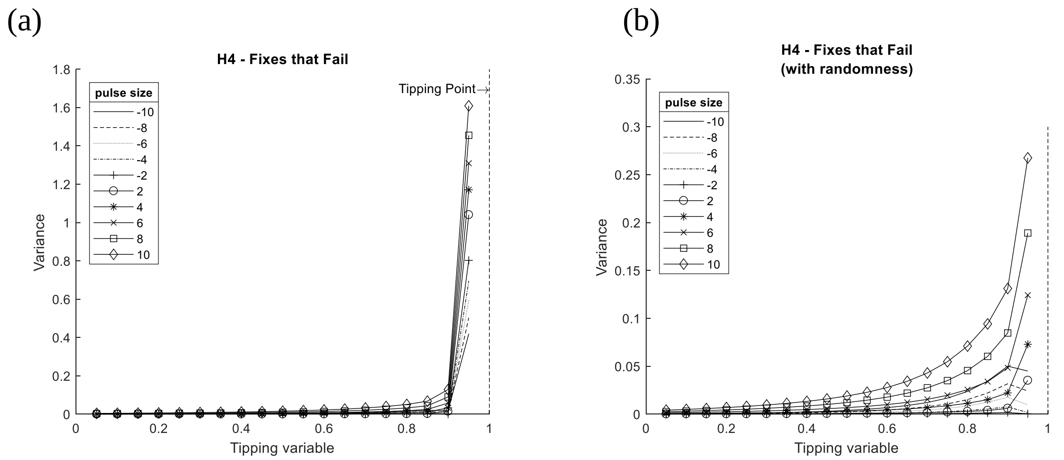


Figure 37 (a-b)- Results of H4 in fixes that fail archetype
(Tipping variable: fractional consequences rate; dependent variable: problem symptoms)
35(a) variance of the dependent variable vs. tipping variable without any noise in the system
35(b) variance of the dependent variable vs. tipping variable with dynamic exogenous variables

Reinforcing Loop Archetype

The reinforcing loop archetype has a stable equilibrium at zero when the fractional change rate (tipping variable, x-axis) is negative. The simulation results of hypothesis 2 testing in the reinforcing loop archetype support the hypothesis (Figure 38 (a-b)). As the reinforcing loop archetype gets closer to the tipping point (tipping variable=0 as shown on the x-axis in Figure 38a), the recovery time (y-axis) increases non-linearly. Adding noise into the system (Figure 38b) does not affect the results. The changes in the recovery time are statistically significant when -0.9<tipping variable. See Appendix C for the detailed results of t-test and ANOVA[31].

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[31] The statistical analysis was performed for the cases where pulse size is equal to ±2

(a) (b)

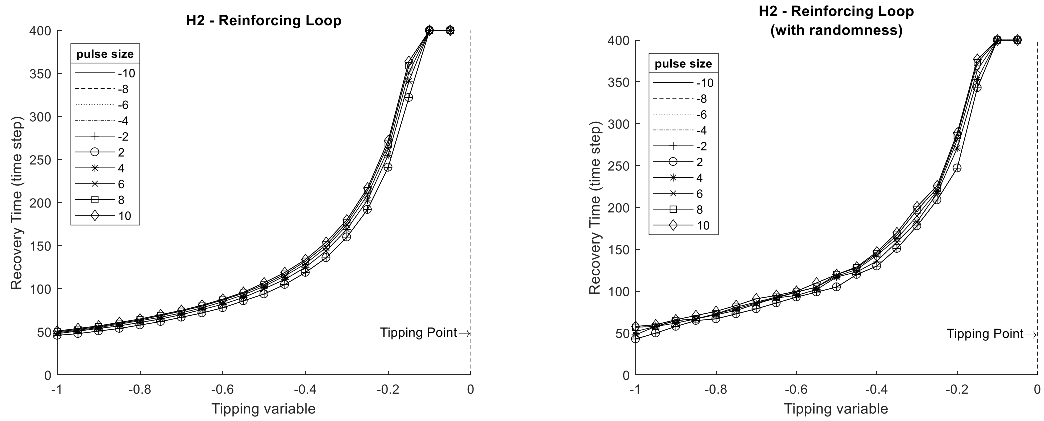

Figure 38 (a-b)- Results of H2 in reinforcing loop archetype
(Tipping variable: fractional change rate; dependent variable: current state)
38(a) recovery time vs. tipping variable without any noise in the system
38(b) recovery time vs. tipping variable with dynamic exogenous variables

Figure 39 (a-b) shows the results of testing H3 in the reinforcing loop archetype. The hypothesis has been tested for values of tipping variable less than 0 when the system has a stable equilibrium. The results show that as the system gets closer to the tipping point (at tipping variable equal to 0), the lag-1 autocorrelation increases (Figure 39a). Despite the small changes in the lag-1 autocorrelation, statistical analysis shows that the increase in the lag-1 autocorrelation as the system approaches the tipping point is significant. The results are robust for different pulse sizes and the insertion of randomness in the system does not affect the increasing trend of the temporal autocorrelation but it does change the shape from a linear relationship to a non-linear relationship.

(a) (b)

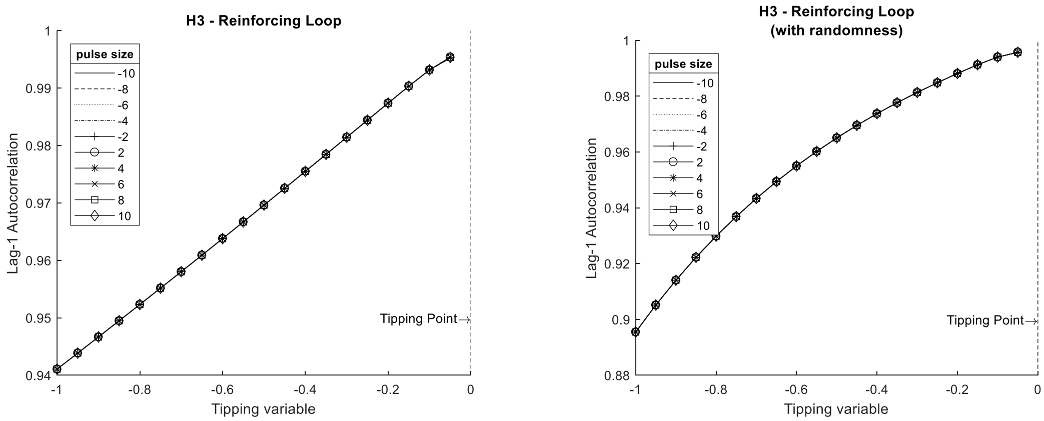

Figure 39 (a-b)- Results of H3 in reinforcing loop archetype
(Tipping variable: fractional change rate; dependent variable: current state)
39 (a) lag-1 autocorrelation vs. tipping variable without any noise in the system
39(b) lag-1 autocorrelation vs. tipping variable with dynamic exogenous variables

Figure 40 (a-b) shows the results of H4 testing in the reinforcing loop archetype. The average variance of the state variable increases as the system gets closer to the tipping point (tipping variable equal to 0 as shown in the x-axis in Figure 40a). However, the results depend on the pulse size: for small perturbations, there is no change in the variance. Standard t-test and ANOVA was performed to compare the mean of variance for different values of the tipping variable when the pulse is equal to ±2. The results show that despite the small change in the variance (for these pulse sizes), the difference is still statistically significant and the results support the hypothesis.

(a) (b)

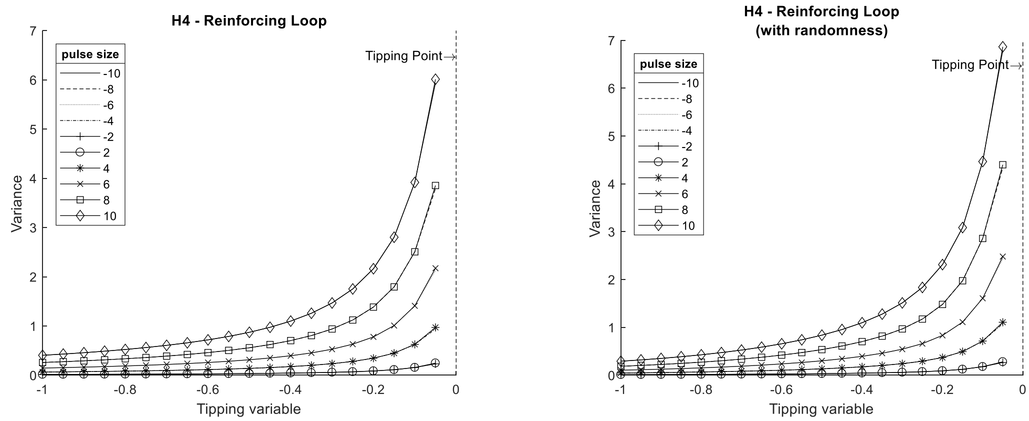

Figure 40 (a-b)- Results of H4 in reinforcing loop archetype
(Tipping variable: fractional change rate; dependent variable: current state)
25(a) variance of the dependent variable vs. tipping variable without any noise in the system
25(b) variance of the dependent variable vs. tipping variable with dynamic exogenous variables

Escalation Archetype

Since the escalation archetype does not have any stable equilibriums, hypothesis 2,3, and 4 could not be tested.

Limerick Construction Project

Figure 41 (a-b) shows the results of testing H2 in the Limerick construction project model. The tipping variable (base ripple effects strength) is shown on the x-axis. The system is at the tipping point when the tipping variable is equal to 0.665. The y-axis shows the recovery time from a perturbation. the system has a stable equilibrium for the values of base ripple effects strength less than 0.665. As the system gets closer to the tipping point, the time to reach attractor increases (Figure 41a) and the results support the hypothesis. The results do not depend on the pulse size. Based on the results of the t-test, for values of base ripple effect strength is greater than 0.35, an increase in the recovery time can be an indicator of approaching a tipping point[32]. In the presence of randomness in the system, the recovery time increases only when the system conditions are very close to the tipping point (Figure 41b).

(a) (b)

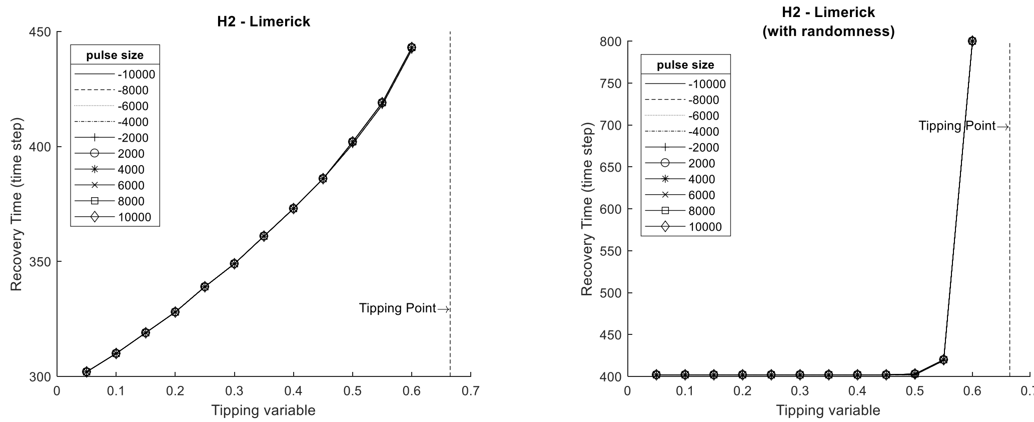

Figure 41 (a-b)- Results of H2 in Limerick construction project model
(Tipping variable: base ripple effects strength; dependent variable: total project backlog[33])
41(a) recovery time vs. tipping variable without any noise in the system
41(b) recovery time vs. tipping variable with dynamic exogenous variables

The results of testing H3 in Limerick model are illustrated in Figure 42 (a-b) As the value of the tipping variable approaches the tipping conditions (0.665 on the x-axis in Figure 42a), the temporal autocorrelation of the state variable in Limerick construction project model increases (y-axis in Figure 42a). Although the increase in the temporal autocorrelation is small, the results support the hypothesis. Since in system dynamics models the state variables are defined by their value in the previous step the

[32] The statistical analysis was performed for the value of pulse size equal to ±2000
[33] Defined as the number of work packages that need to be completed

autocorrelation tends towards 1 very quickly, therefore, even slight increases observed in lag-1 autocorrelation can be considered significant. Similar to the results of hypothesis 2, the increase in the lag-1 autocorrelation of the dependent variable is significant when base ripple effect strength (tipping variable) is greater than 0.35. When there are dynamic exogenous variables in the system (Figure 42b), the increase in the lag-1 autocorrelation is significant only when the system is very close to the tipping point. The existence of noise in the system makes the signal of tipping point indicator less clear as shown in Figure 42b.

(a) (b)

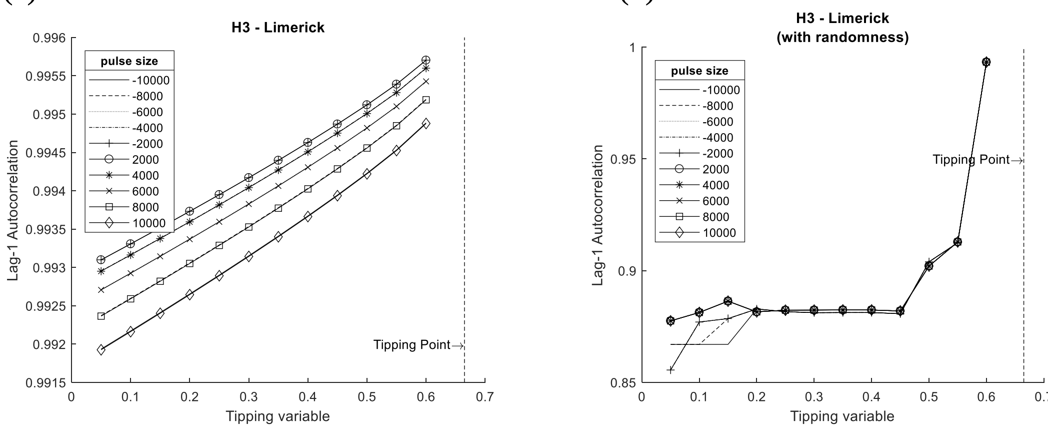

Figure 42 (a-b)- Results of H3 in Limerick construction project model
(Tipping variable: base ripple effects strength; dependent variable: total project backlog[34])
42(a) lag-1 autocorrelation vs. tipping variable without any noise in the system
42(b) lag-1 autocorrelation vs. tipping variable with dynamic exogenous variables

In Limerick construction project model, the results of testing H4 show an increase in the variance of the state variable as the system gets closer to the tipping point

[34] Defined as the number of work packages that need to be completed

and support the hypothesis (Figure 43a). The results are robust for different pulse sizes and the increase in the mean variance is statistically significant when the tipping variable is greater than 0.35. The noise in the system (Figure 43b) increases the size of the variance and results are shown on a logarithmic scale.

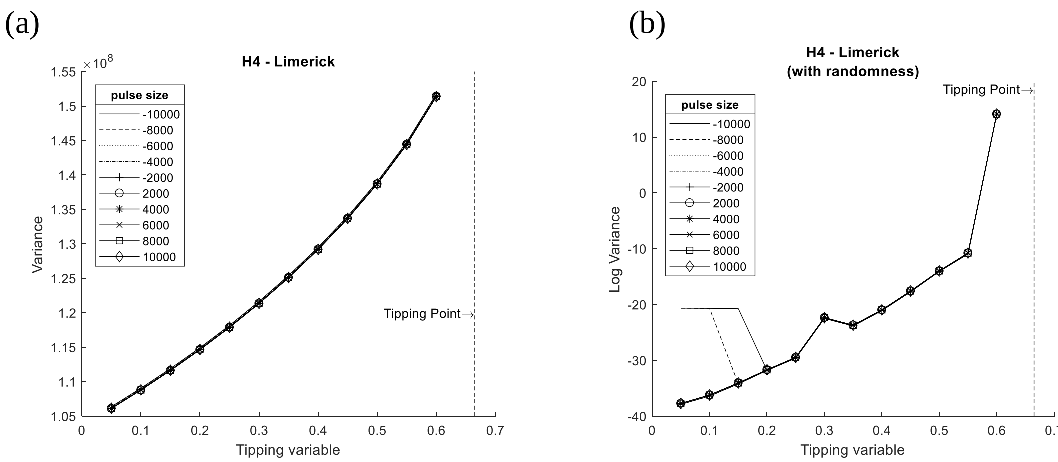

Figure 43 (a-b)- Results of H4 in Limerick construction project model
(Tipping variable: base ripple effects strength; dependent variable: total project backlog[35])
43(a) variance of the dependent variable vs. tipping variable without any noise in the system
43(b) variance of the dependent variable vs. tipping variable with dynamic exogenous variables

Fish Banks

Figure 44 (a-b) shows the results of testing H2 in the fish banks model. The system has two equilibrium states: one when the fish population is equal to zero and the other when the fish population is equal to the carrying capacity of the system. The hatch fraction is the tipping variable and the fish population stands at a tipping point when the

[35] Defined as the number of work packages that need to be completed

tipping variable is equal to 5.25 (as shown on the x-axis in Figure 44a). for hatch fraction less than 5.25, the system goes towards the stable equilibrium at zero. For values of the tipping variable between 5.25 and 7, the system starts oscillating but the oscillation dampens and eventually, the system reaches the carrying capacity. For values of the hatch fraction larger than 7, the system oscillates around the carrying capacity with a relatively large amplitude. As the tipping variable gets closer to 5.25, the recovery time from a perturbation in the dependent variable increases (Figure 44a) and the simulation results support the hypothesis. Based on the results of the t-test, the difference in the time to reach attractor is statistically significant when the tipping variable is between 3.5 and 7.5. The recovery time increases from tipping variable values 7 to 8 (Figure 44a) because at those values the fish population behavior becomes oscillatory with large amplitudes. This causes the calculations of the recovery time to sometimes reflect the oscillations. This suggests that caution may be needed when systems create oscillations. In presence of randomness (Figure 44b), the increase in the recovery time is gradual when the tipping variable is less than 5.25 but on the other side of the tipping point when the system enters a cyclic behavior, the noise in the system accentuates the existing oscillation and make the calculation of the recovery time inaccurate.

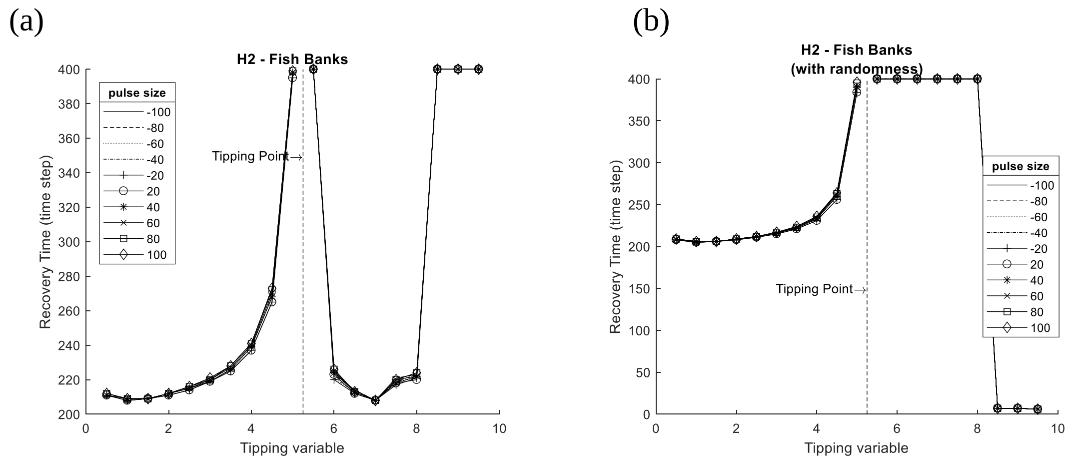

Figure 44 (a-b) - Results of H2 in fish banks model
(Tipping variable: hatch fraction; dependent variable: fish population)
44(a) recovery time vs. tipping variable without any noise in the system
44(b) recovery time vs. tipping variable with dynamic exogenous variables

The results of hypothesis H3 testing are shown in Figure 45 (a-b). In fish banks model, as the system gets closer to the tipping point (tipping variable equal to 5.5 as shown in the x-axis in Figure 45a), the lag-1 autocorrelation increases. The results support the hypothesis and are robust for different pulse sizes. Since the lag-1 autocorrelation of the dependent variable is measured from the time a perturbation was put in the system until the time that the system returned to its equilibrium state, the value of the recovery time affects the results of hypothesis 3 testing. As discussed earlier, when the tipping variable is larger than 5.25 and the system starts oscillating, the presence of randomness in the system interferes with the calculations and make the recovery time for these conditions inaccurate. Hence, the shape of the graph is different for these system conditions when there is oscillation in the system and the behavior of the tipping point indicator is less clear (Figure 45b).

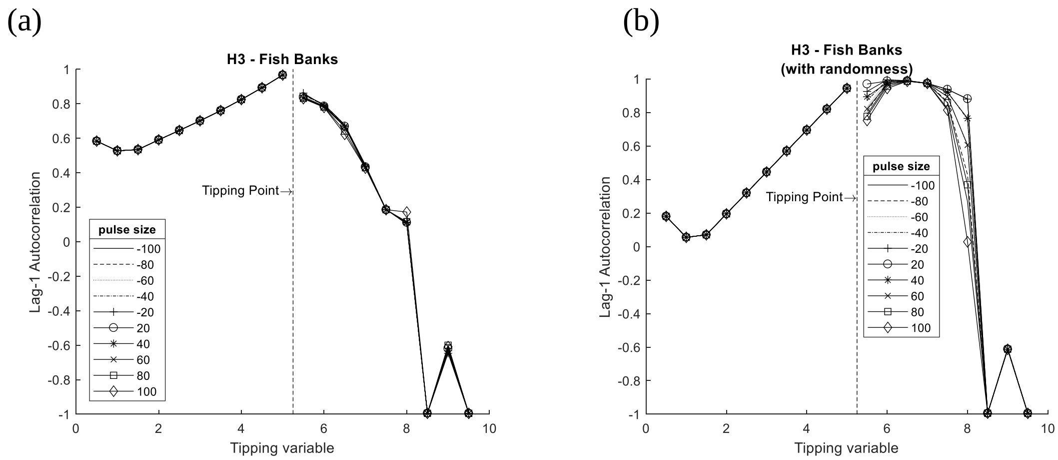

Figure 45 (a-b) - Results of H3 in fish banks model
(Tipping variable: hatch fraction; dependent variable: fish population)
45(a) lag-1 autocorrelation vs. tipping variable without any noise in the system
45(b) lag-1 autocorrelation vs. tipping variable with dynamic exogenous variables

The results of H4 testing in the fish banks model generally support the hypothesis. For the values of the tipping variable less than 5.25 (the tipping point condition), the variance of the dependent variable increases when the system is very close to the tipping point. When the tipping variable is greater than 5.5, there is a drop in the variance followed by an increase as the system is approaching the tipping point which is due to the cyclic behavior of the system for these tipping variable values as explained above. The increase in the variance in both cases is detectable when the system is very close to the tipping point.

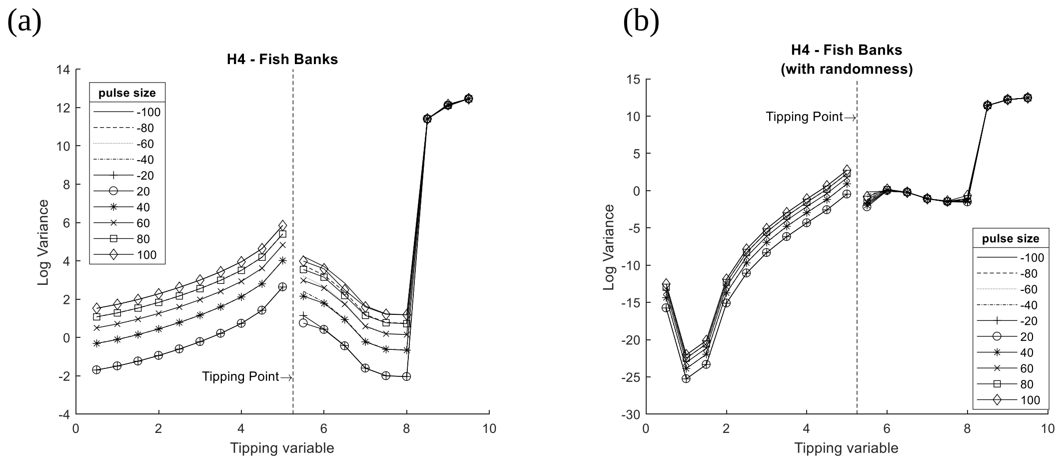

Figure 46 (a-b) - Results of H4 in fish banks model
(Tipping variable: hatch fraction; dependent variable: fish population)
46(a) variance of the dependent variable vs. tipping variable without any noise in the system
46(b) variance of the dependent variable vs. tipping variable with dynamic exogenous variables

Arms Race

Similar to the escalation archetype, due to lack of a stable equilibrium, it is not possible to test H2, H3, and H4 in the arms race model.

Social Impact Bonds

The social impact bonds model has a tipping point when the fraction of surplus returned (the tipping variable) is equal to 0.365. Depending on the value of the tipping variable, the total prison population moves towards either of its two equilibrium states. Figure 47 (a-b) shows the H2 testing results in the social impact bonds model. The x-axis is the tipping variable and the y-axis is recovery time. The results of testing H1 in social impact bonds model partially support the hypothesis: If the tipping variable is less than 0.365, there is no change in the time required to reach the attractor but the recovery time depends on the pulse size (Figure 47a). When the tipping variable is greater than

0.365 the recovery time does not change until the system is very close to its tipping point (Figure 47a). These results might be because the system has multiple stock variables and the effect of the perturbation is smoothed before reaching the main dependent variable. The results of t-test show that the change in the recovery time is significant only when the tipping variable is less than 0.5[36]. Having random exogenous variables in the system does not affect the results (Figure 47b).

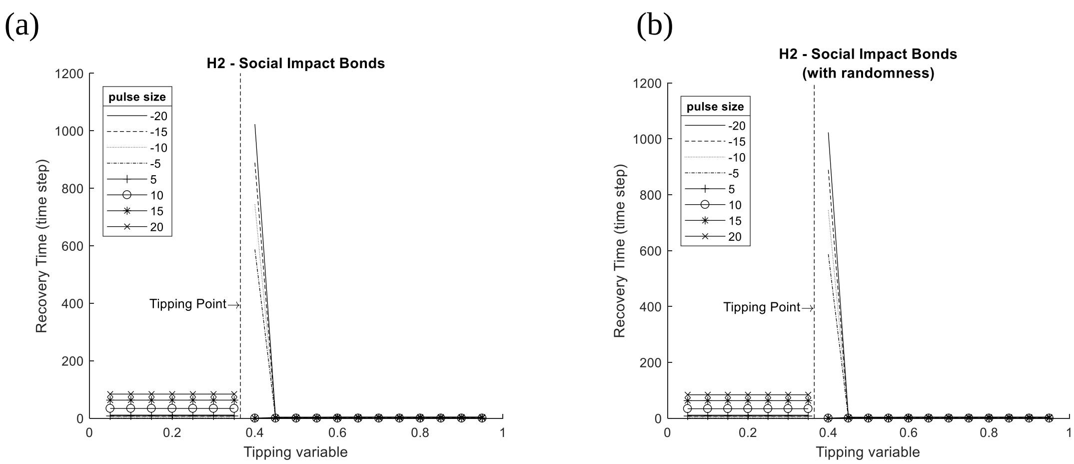

Figure 47 (a-b)- Results of H2 in social impact bonds model
(Tipping variable: fraction of surplus returned; dependent variable: total prison population)
47(a) recovery time vs. tipping variable without any noise in the system
47(b) recovery time vs. tipping variable with dynamic exogenous variables

Figure 48 (a-b) shows the results of testing H3 in social impact bonds model. There is no change in the lag-1 autocorrelation of the dependent variable when the tipping variable (fraction of surplus returned) is less than 0.365 (Figure 48a). For values of the tipping variable greater than 0.365, there is an increase in the recovery time when

[36] Standard t-test and ANOVA were performed for pulse size equal to ±10.

the system is very close to the tipping point conditions (tipping variable <0.6). See Figure 48a for the results of H3 and Appendix C for the results of the statistical testing.

(a) (b)

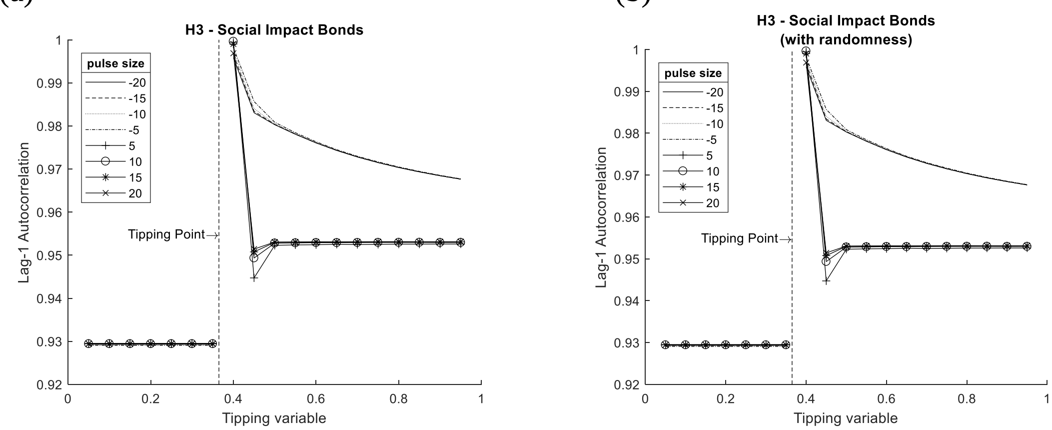

Figure 48 (a-b)- Results of H2 in social impact bonds model
(Tipping variable: fraction of surplus returned; dependent variable: total prison population)
48(a) lag-1 autocorrelation vs. tipping variable without any noise in the system
48(b) lag-1 autocorrelation vs. tipping variable with dynamic exogenous variables

The results of hypothesis 4 testing in the social impact bonds model are similar to the previous hypotheses as shown in Figure 49 (a-b). When the tipping variable is less than the value at the tipping conditions, there is no change in the variance of the total prison population (the dependent variable); when the tipping variable is greater than the value at the tipping condition there is a sudden increase in the variance as the system gets close to the tipping point (Figure 49a). The increase in the variance is statistically significant only when the tipping variable is less than 0.5.

(a) (b)

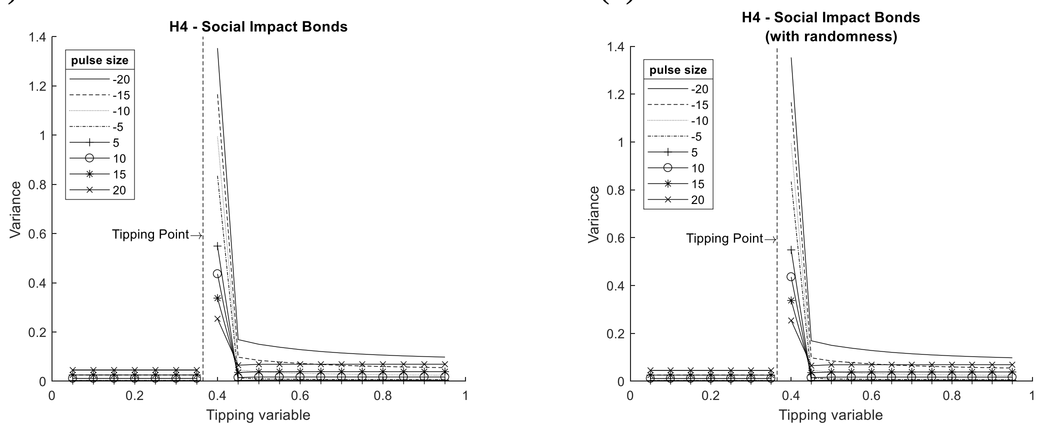

Figure 49 (a-b)- Results of H4 in social impact bonds model
(Tipping variable: fraction of surplus returned; dependent variable: total prison population)
49(a) variance of the dependent variable vs. tipping variable without any noise in the system
49(b) variance of the dependent variable vs. tipping variable with dynamic exogenous variables

Hypothesis 2-4 Summary

Regarding hypothesis 2, in all the models except for one (social impact bonds partially supports the hypothesis), the non-linear relationship between the recovery time and the tipping variable support the hypothesis and current literature. The systems recover from a perturbation slower when getting close to a tipping point. This indicates that the indicator that is useful in the literature models is also useful for the system dynamics archetypes and models of realistic systems studied here. However, the caveat is that in the models of the real systems (e.g. in Limerick construction project model), the recovery time from a perturbation can be an indicator of a tipping point only when the system is very close to the tipping conditions. When the system is farther away from the tipping conditions, there is either not a noticeable change in the recovery time from a perturbation or this indicator gives false alarms (e.g. fixes that fail model). The results of all type I tipping point models generally support the hypothesis. Not all type II tipping points could be tested for this hypothesis and out of two models that were tested, the results of the social impact bonds only partially support the hypothesis (for one of the stable equilibriums). The results also depend on the size of the perturbation. For very small pulses, there is no change in the recovery time.

Hypothesis 3 focuses on an increase in lag-1 autocorrelation as a potential indicator of a tipping point. Similar to the results of the previous hypothesis, all type I models support the hypothesis. Conclusions cannot be made for type II models: While the results of reinforcing loop archetype support the hypothesis, two of the type II models could not be tested because they do not have a stable equilibrium, and the results of social impact bonds model only partially support the hypothesis. However, it should be noted that due to the nature of a system dynamics model, the value of the stock variables at each step is dependent on its value in the previous time step. Therefore, lag-1 autocorrelation might not be the best indicator to look at in a system dynamics model.

There is some contradiction regarding using the increase in variance as an indicator of a tipping point (Lenton et al. 2012; Dakos et al. 2012) as defined in hypothesis 4. Dakos et al. (2012) numerically show that “variance may be amplified or dampened as the ecosystem approaches the bifurcation”. They suggest that the trend in variance depends on the existing noise in the system and the sensitivity of the state variable. The results of the models (all type I models and one of type II models) studied here generally support the hypothesis for larger perturbations. The results show that the increase in variance is very sensitive to the size of the perturbation. For small perturbation, there is no change in the variance. This supports the work by Dakos et al.

(2012) regarding the impact of the sensitivity of the state variable to the control variables that were used to disturb the system. In addition, the increase in variance only happens when the system is very close to the tipping point conditions. When the system is far away from the tipping point, there is not a noticeable change in the variance of the state variable.

In summary, the findings show that the clarity of the indicator signal depends on the closeness to the tipping point. Also, when the system is oscillating, the tipping point indicator might give false alarms. Another limitation in using the "critical slowing down" measures is that intrinsic noise (present in real systems) decreases the clarity of the tipping point indicator signal. Due to the features of system dynamics approach, the recovery time is the most useful indicator among the three indicators studied here when dealing with system dynamics models. The simulations were replicated multiple times and a standard t-test and ANOVA was used to compare the means. See Appendix C for detailed statistical testing results.

CHAPTER VI

HYPOTHESIS 5-6

Hypothesis 5-6 Introduction

Although critical slowing down can be an indicator of a tipping point, it might not be easy to test or observe by practitioners and project managers. To find more practical indicators of a tipping point that are easy to use in practice, the behaviors of the state variable in the models were studied further. Different plots (e.g. dependent variable over time, change in the first and second derivative of the dependent variable, changes in the net flow, changes in input rate:output rate ratio, etc) were investigated to find similarities in behavior before a tipping point. The x_{t+1}-x_t graphs showed some potential for identifying new tipping point indicators. The x_{t+1}-x_t graphs have been previously used in both system dynamics literature (Black and Repenning 2001; Richardson 1995; Taylor and Ford 2006) to study the changes in the dominance of feedback loops and in nonlinear dynamics (Strogatz 2014) to study the properties of bifurcations. When exploring the x_{t+1}-x_t graphs of the models in the library, some similarities before a tipping point were observed. The last two hypotheses are based on these observations:

H5: As a system approaches a tipping point, the points on the x_{t+1}-x_t graph get closer to each other.

H6: As a system approaches a tipping point, the slope of the x_{t+1}-x_t graph begins less than one and is decreasing until the system is near the tipping point. After the system crosses the tipping point, there is a sudden shift in the slope from a value less than one to greater than one.

Hypothesis 5-6 Testing Procedures

To test these two hypotheses, the value of the tipping variable is changed over a period of time so the system gets closer to the tipping point over time and passes the tipping point. The graph of x_{t+1}-x_t is drawn and the distance between the points and the slope of the graph are found to test hypotheses 5 and 6 respectively. The details are as follows:

1. Choose the dependent variable(s). See Chapter III for a stock and flow diagram of the models and equations.

   Example: the stock of “Problem Symptoms” in fixes that fail archetype.

2. Find system attractors. See Table 2 for details of attractors in each model.

   Example: fixes that fail archetypes has three attractors: zero (stable), initial problem symptoms (unstable), and infinity.

3. Find tipping variable(s). Use statistical screening (Ford and Flynn, 2005; Taylor et al., 2010) to find the variable(s) with the highest leverage on the dependent variable and identify them as tipping variable(s). The value of tipping variable(s) is a proxy for the closeness to the tipping point. Define tipping point conditions by finding the values of the control variables (including tipping variables) at the tipping point. See Table 2 for details of tipping point conditions in the models studied here.

   Example: the tipping variable in the fixes that fail model is the” fractional consequences rate” and its value is equal to one at tipping point conditions.

4. Set the tipping variable to decrease over time using a ramp function.

Example: in fixes that fail archetype, the “fractional consequences rate” (the tipping variable) is changed from 1.1 to 0.9 over a period of twenty months.

5. Run simulations and plot x_{t+1} vs x_t.
6. H5: Find the distance between the points on the graph.
7. H6: Find the slope of x_{t+1} vs x_t graph.
8. Set the tipping variable to increase over time using a ramp function.

   Example: in fixes that fail archetype, the “fractional consequences rate” (the tipping variable) is changed from 0.9 to 1.1 over a period of twenty months.
9. Repeat steps 5-7.
10. Design randomness into the system that would represent the noise that is present in reality and repeat steps 4-9. This was done by changing selected exogenous variables dynamically over time. The selected exogenous variables are changed at each time step based on a random distribution function within twenty percent of their original value. See Appendix A for details of the selected exogenous variable and the noise formulation.

    Example: in fixes that fail archetype, the variable “fractional fix rate” is changed randomly between 1.6 and 2.4 using a random uniform function.

Hypothesis 5-6 Results

The results of testing hypothesis 5 are shown in this section. There are two different types of graphs for each model. In each set, the top two graphs show the results when there is no randomness in the system. The bottom two graphs show the results when having randomness in the system[37].

The first set of graphs demonstrates the relationship between x_{t+1} and x_t. The x-axis shows x_t and the y-axis represent x_{t+1}. The numbers on the graph represent the time and the time at which the system reaches the tipping point is stated in the title of the graph. The second set of graphs shows the distance between the points on the x_{t+1}-x_t plot. The x-axis is time while the y-axis is the distance between the two consecutive points on the x_{t+1}-x_t plot. The third set of graphs shows the slope of the x_{t+1}-x_t plot. The x-axis is time while the y-axis is slop between the two consecutive points on the x_{t+1}-x_t plot.

The graphs on the left show the results when the tipping variable is decreased over time until the system crosses the tipping point. Whereas, in the graphs on the right the tipping variable increases over time until the system passes the tipping point.

Limits to Growth Archetype

The limits to growth archetype has two stable equilibriums. For negative values of the tipping variable, the system goes to an equilibrium at zero and for positive values of the tipping variable, the system has a stable equilibrium at the carrying capacity of the system. To test hypothesis 5 and hypothesis 6, the tipping variable was changed between -0.1 and 0.1 over a period of 20 time steps. The system starts from one side of the tipping point and gradually approaches the tipping point. At time ten, the system is at the tipping point conditions and eventually shifts to the other side of the tipping point.

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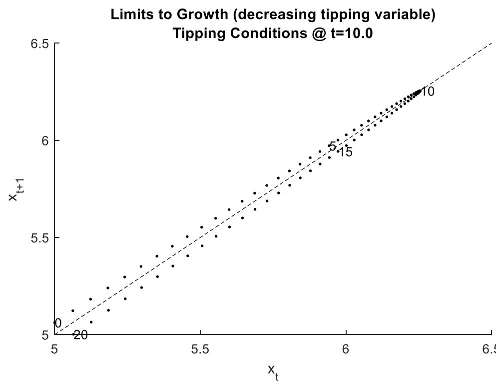
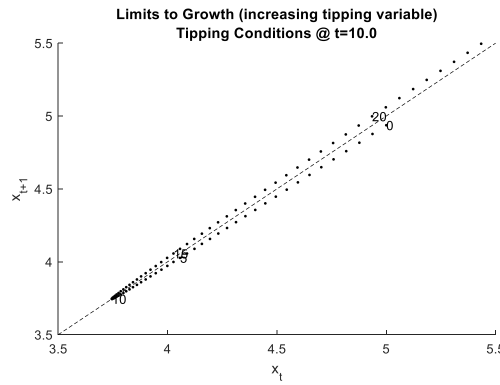
[37] The selected exogenous variables are changed at each time step based on a random distribution function within twenty percent of their original value. See Appendix A for more details.

Figure 50 (a-d) shows the x_{t+1}-x_t graph of limits to growth archetype. The graph shows the value of population stock at time t+1 versus time t. As it is illustrated in the graphs, the density of points increases near time 10. Also, the slope of the graph shifts from one side of the y=x to the other side: when the system crosses the tipping point, the x_{t+1}-x_t graph crosses the y=x line.

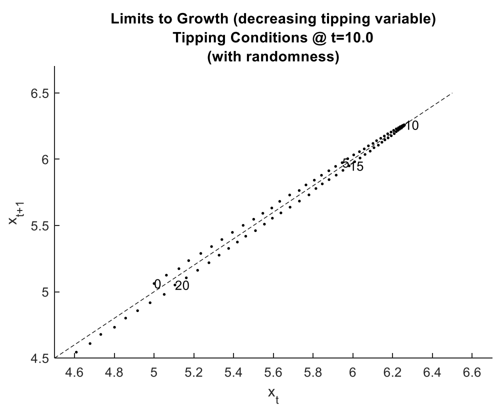
(a)



(b)



(c)



(d)

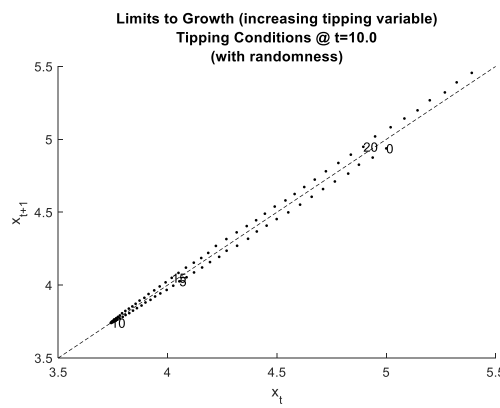


Figure 50 (a-d)- x_{t+1}-x_t graph in limits to growth archetype
(Tipping variable: fractional change rate; dependent variable (x): population)
50(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
50(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
50(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
50(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

Figure 51 (a-d) shows the results of H5 testing in the limits to growth archetype. The results support the hypothesis: as the system gets closer to the tipping point at time=10, the distance between the points decreases (Figure 51 a&b) and the system stays longer near the tipping point, i.e., the system slows down. Having noise in the system causes some fluctuations in the graph but the general trend is not affected by the randomness (Figure 51 c&d).

(a) (b)

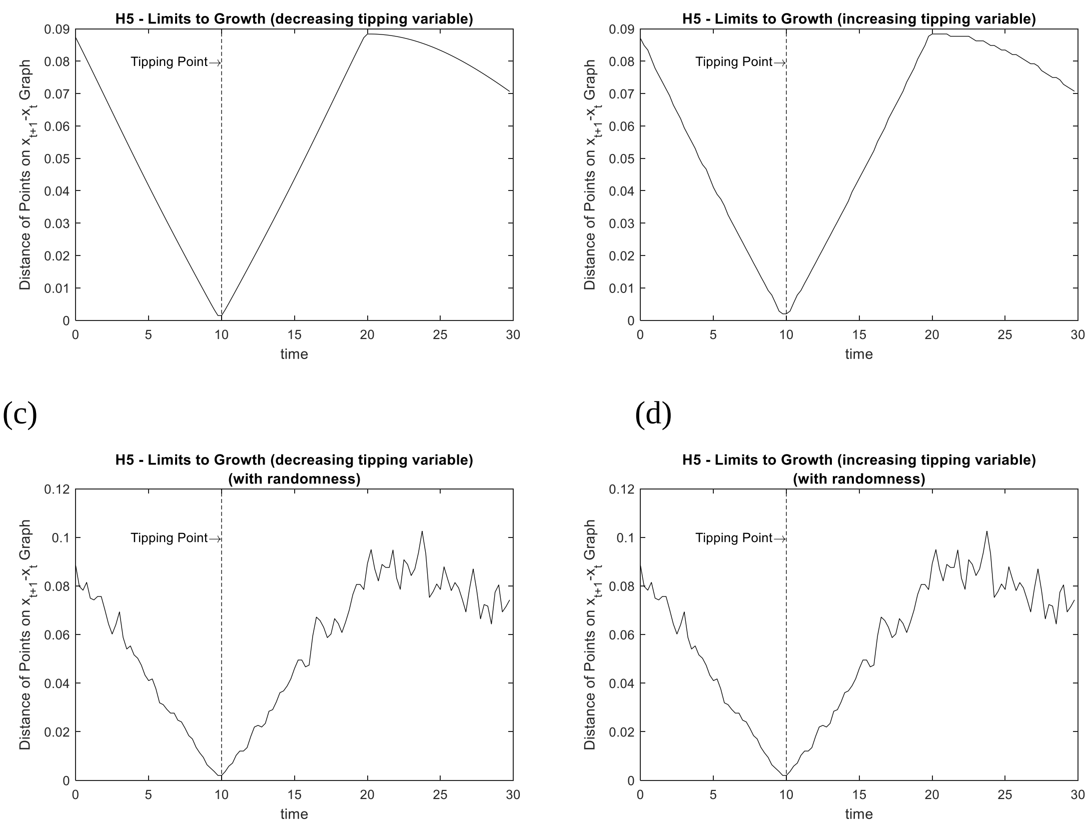


Figure 51 (a-d)- Results of H5 in limits to growth archetype
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: fractional change rate; dependent variable (x): population)
51(a) results when the tipping variable decreases over time without any noise in the system
51(b) results when the tipping variable increases over time without any noise in the system
51(c) results when the tipping variable decreases over time with dynamic exogenous variables
51(d) results when the tipping variable increases over time with dynamic exogenous variables

The results of H6 testing in the limits to growth archetype support the hypothesis as shown in Figure 52 (a-d). As the system gets closer to the tipping point at time=10, the slope of the x_{t+1}-x_t graph decreases from one to zero before the tipping point and abruptly rises to over one after the tipping point (Figure 52 a&b). Intrinsic noise in the system causes fluctuations in the graph but the general shape of the graph does not change: the slope decreases before the tipping point and abruptly jumps to more than one when the system crosses the tipping point (Figure 52 c&d). However, because of the fluctuations, it might be difficult to foresee an upcoming tipping point, although the sudden change in the slope after the system has passed the tipping point is easy to observe. This implies that hypothesis 6 might have some shortcomings in predicting a tipping point and be better used as an indicator that the system has already crossed a tipping point.

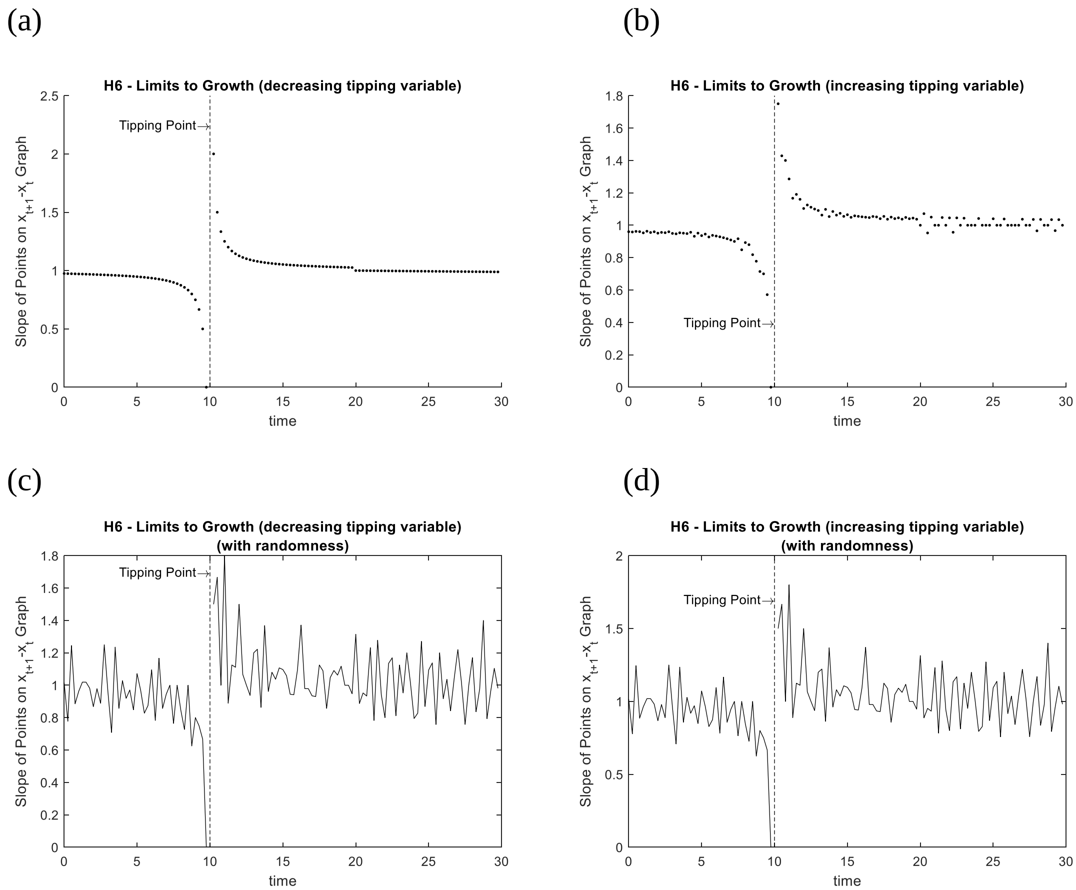


Figure 52 (a-d)- Results of H6 in limits to growth archetype
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: fractional change rate; dependent variable (x): population)
52(a) results when the tipping variable decreases over time without any noise in the system
52(b) results when the tipping variable increases over time without any noise in the system
52(c) results when the tipping variable decreases over time with dynamic exogenous variables
52(d) results when the tipping variable increases over time with dynamic exogenous variables

Fixes That Fail Archetype

In fixes that fail archetype, the fractional consequences rate is the tipping variable and the system has a tipping point when the fractional consequences rate is equal to one. To test hypothesis 5 and 6, the fractional consequences rate was gradually changed between 0.9 and 1.1 over time. The system starts from one side of the tipping point and moves towards the tipping point conditions. It reaches the tipping point conditions at time equal to 4.5. Then, the system passes the tipping point and continues its course to the alternate attractor. Figure 53 (a-d) shows the relationship between the dependent variable (problem symptoms) in two consecutive time steps in the fixes that fail archetype. At the tipping point (t=4.5), there is an increase in the density of the points on the graph and the slope of the graph changes at this point too.

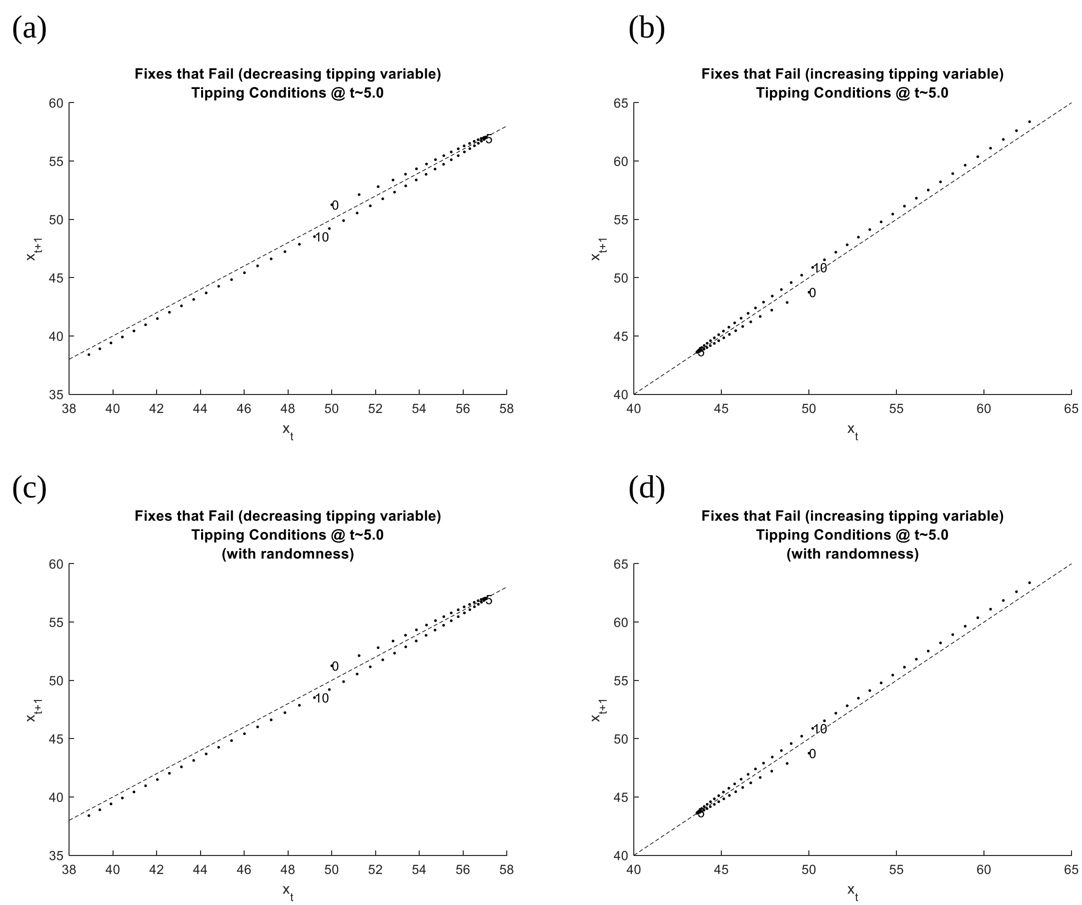
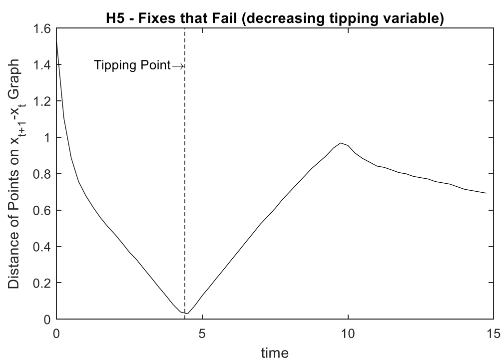


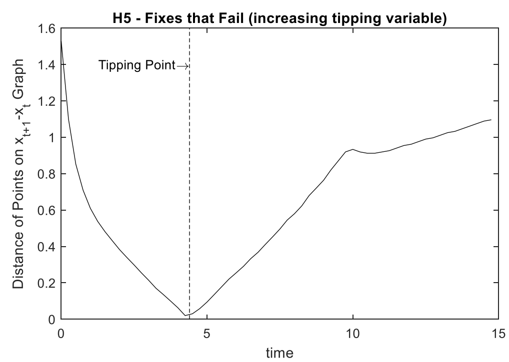
Figure 53 (a-d)- x_{t+1}-x_t graph in fixes that fail archetype
(Tipping variable: fractional consequences rate; dependent variable (x): problem symptoms)
53(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
53(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
53(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
53(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

Figure 54 (a-d) shows the results of testing H5 in fixes that fail archetype. As seen in the graph, the results support the hypothesis and as the system gets closer to the tipping point (at time=5) the distance between the points on the x_{t+1}-x_t graph decreases (Figure 54 a&b). Having randomness in the system does not affect the results and the hypothesis is still supported (Figure 54 c&d).

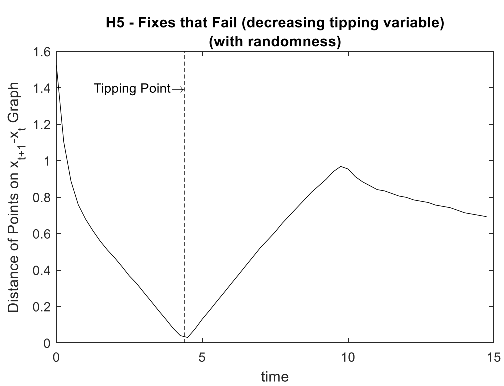
(a)



(b)



(c)



(d)

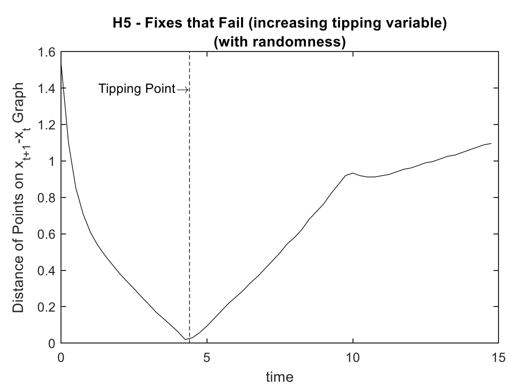


Figure 54 (a-d)-Results of H5 in fixes that fail archetype
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: fractional consequences rate; dependent variable (x): problem symptoms)
54(a) results when the tipping variable decreases over time without any noise in the system
54(b) results when the tipping variable increases over time without any noise in the system
54(c) results when the tipping variable decreases over time with dynamic exogenous variables
54(d) results when the tipping variable increases over time with dynamic exogenous variables

Figure 55 (a-d) shows the results of testing H6 in fixes that fail archetype. As seen in the graph, the results support the hypothesis and as the system gets closer to the tipping point (at time=4.55), there is first a decrease in the slope of the x_{t+1}-x_t graph before the tipping point followed by an abrupt change in the slope after the system passes the tipping point. The slope remains less than one on one side of the tipping point and as the system crosses the tipping point, the slope changes to a value greater than one (Figure 55 a&b). The results are robust even in the presence of dynamic exogenous variables in the system (Figure 55 c&d).

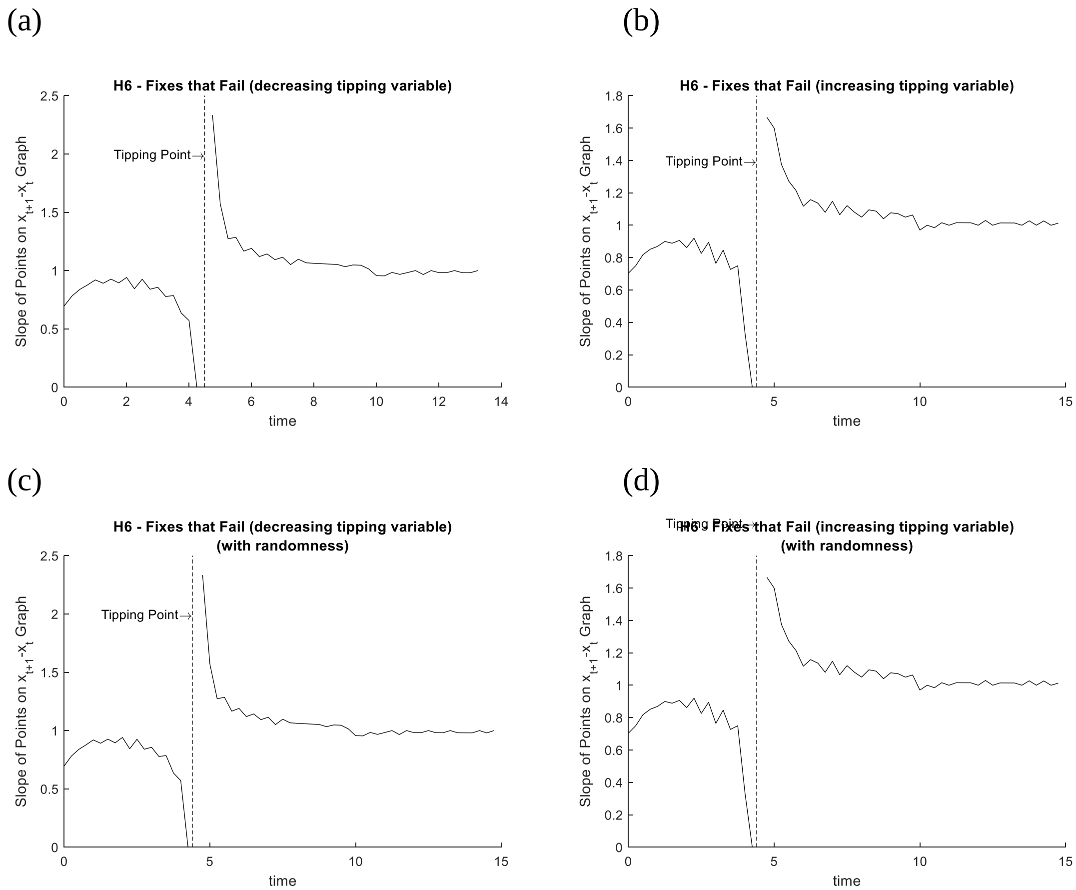


Figure 55 (a-d)-Results of H6 in fixes that fail archetype
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: fractional consequences rate; dependent variable (x): problem symptoms)
55(a) results when the tipping variable decreases over time without any noise in the system
55(b) results when the tipping variable increases over time without any noise in the system
55(c) results when the tipping variable decreases over time with dynamic exogenous variables
55(d) results when the tipping variable increases over time with dynamic exogenous variables

Reinforcing Loop Archetype

The reinforcing loop has a tipping point when the tipping variable is equal to zero. The tipping variable was gradually changed from -0.1 to 0.1 to make the system approach and pass the tipping point conditions. The system reaches the tipping point at time 10. Figure 56 (a-d) shows x_{t+1}-x_t graph of the dependent variable in the reinforcing loop archetype. At the tipping point (time=10) the points on the graph get closer to each other, showing the slowing down phenomenon.

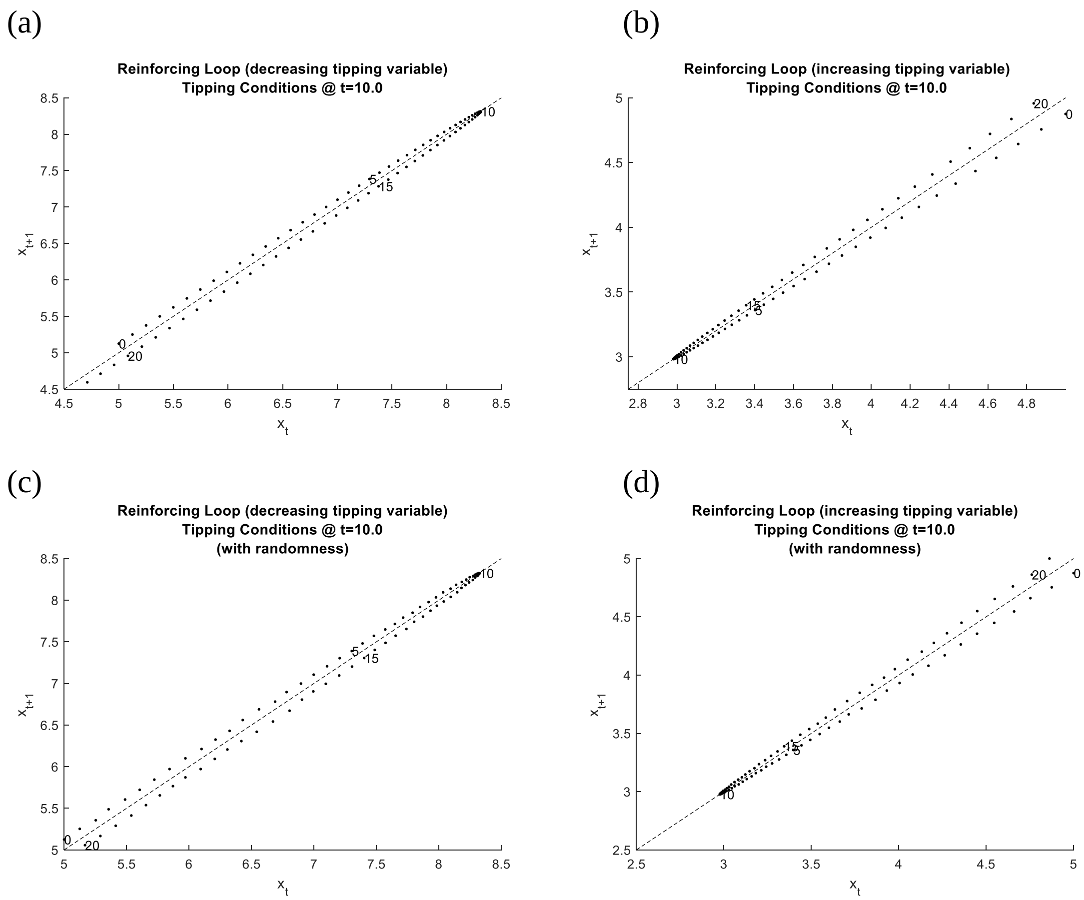


Figure 56 (a-d)- x_{t+1}-x_t graph in reinforcing loop archetype
(Tipping variable: fractional change rate; dependent variable (x): current state)
56(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
56(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
56(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
56(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

The results of testing H5 in reinforcing loop archetype support the hypothesis (see Figure 57 (a-d)). As the system approaches the tipping point, the value of the dependent variable at each step gets closer to its value at the previous time step and x_{t+1}

and x_t get closer to each other (Figure 57 a&b). Having random exogenous variables in the system causes fluctuations in the graph but the over trend stills supports the hypothesis: as the system approaches the tipping point (time=10), the distance between x_{t+1} and x_t decreases (Figure 57 c&d).

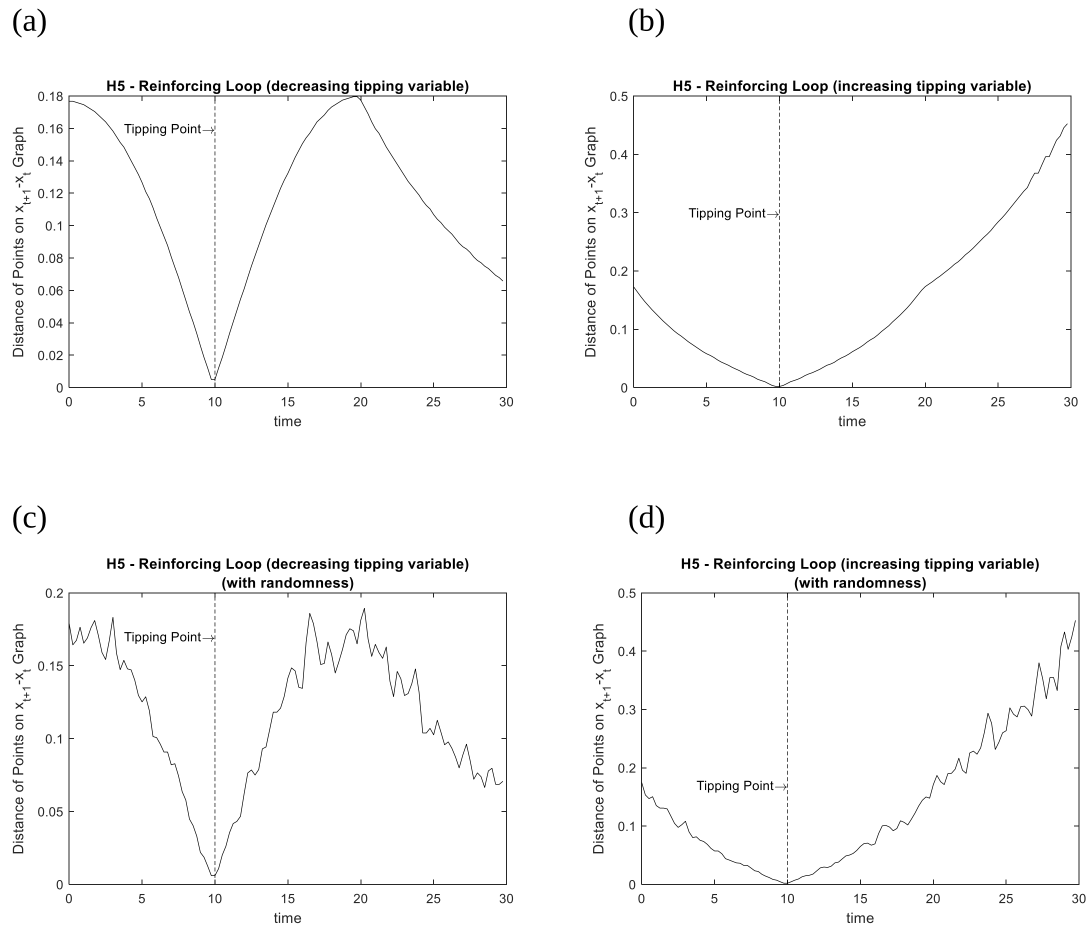


Figure 57-Results of H5 in reinforcing loop archetype
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: fractional change rate; dependent variable (x): current state)
57(a) results when the tipping variable decreases over time without any noise in the system
57(b) results when the tipping variable increases over time without any noise in the system
57(c) results when the tipping variable decreases over time with dynamic exogenous variables
57(d) results when the tipping variable increases over time with dynamic exogenous variables

The results of testing H6 in reinforcing loop archetype support the hypothesis (see Figure 58 (a-d)): the slope of the x_{t+1}-x_t graph decreases as the system gets closer to the tipping point at time=10 and abruptly changes from less than one to greater than one at the tipping point. Having randomness in the system (Figure 58 c&d) causes some fluctuations in the graph that can make predicting the tipping point difficult but the sudden change in the slope is still observable, indicating that hypothesis 6 can be more useful for confirming that a system has crosses its tipping point.

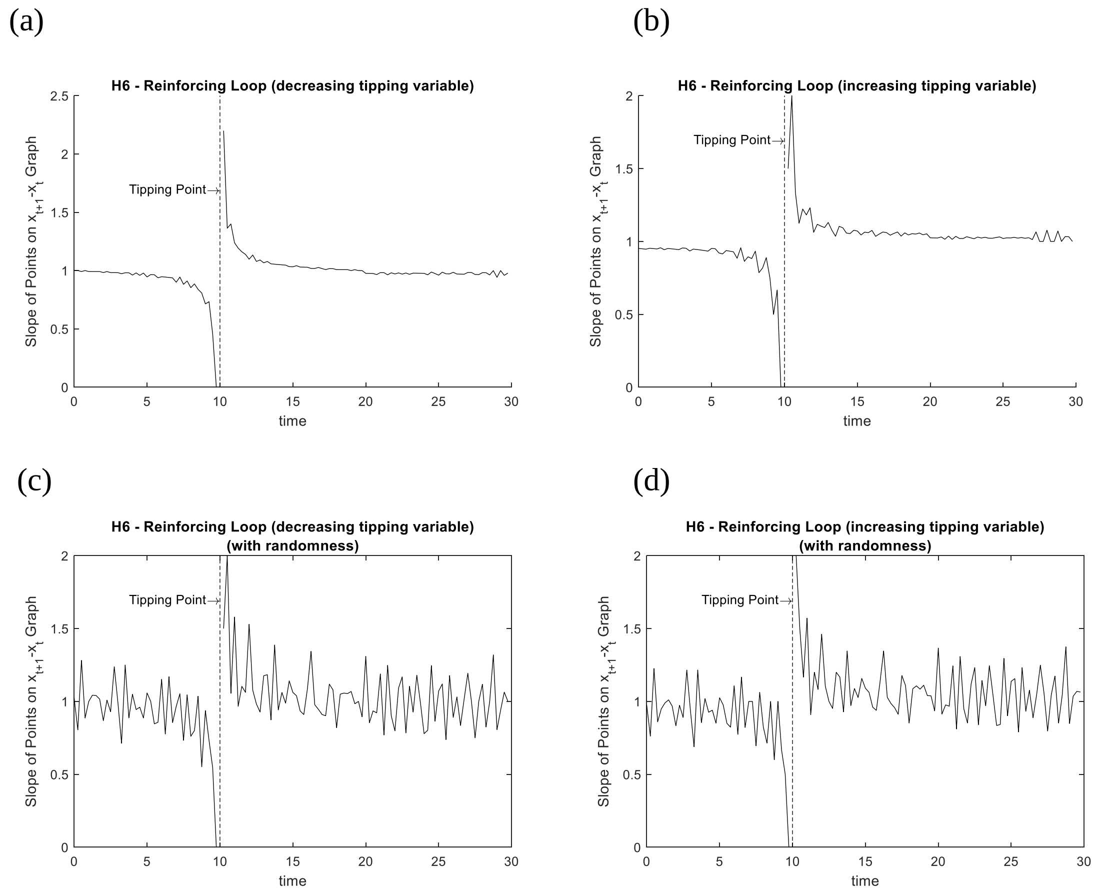


Figure 58-Results of H6 in reinforcing loop archetype
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: fractional change rate; dependent variable (x): current state)
58(a) results when the tipping variable decreases over time without any noise in the system
58(b) results when the tipping variable increases over time without any noise in the system
58(c) results when the tipping variable decreases over time with dynamic exogenous variables
58(d) results when the tipping variable increases over time with dynamic exogenous variables

Escalation Archetype

The escalation archetype has a tipping point when the tipping variable (A's desired advantage ratio) is equal to one. To test the last two hypotheses, the tipping variable was change between 0.9 and 1.1 overtime imitating conditions when the system will start from one side of a tipping point, approaches the tipping point conditions, and passes the tipping point. As the system approaches the tipping point, the value of the dependent variable (A's results) at each time becomes very similar to its value at the previous time step. This will increase the density of point on the x_{t+1}-x_t graph as shown in Figure 59 (a-d).

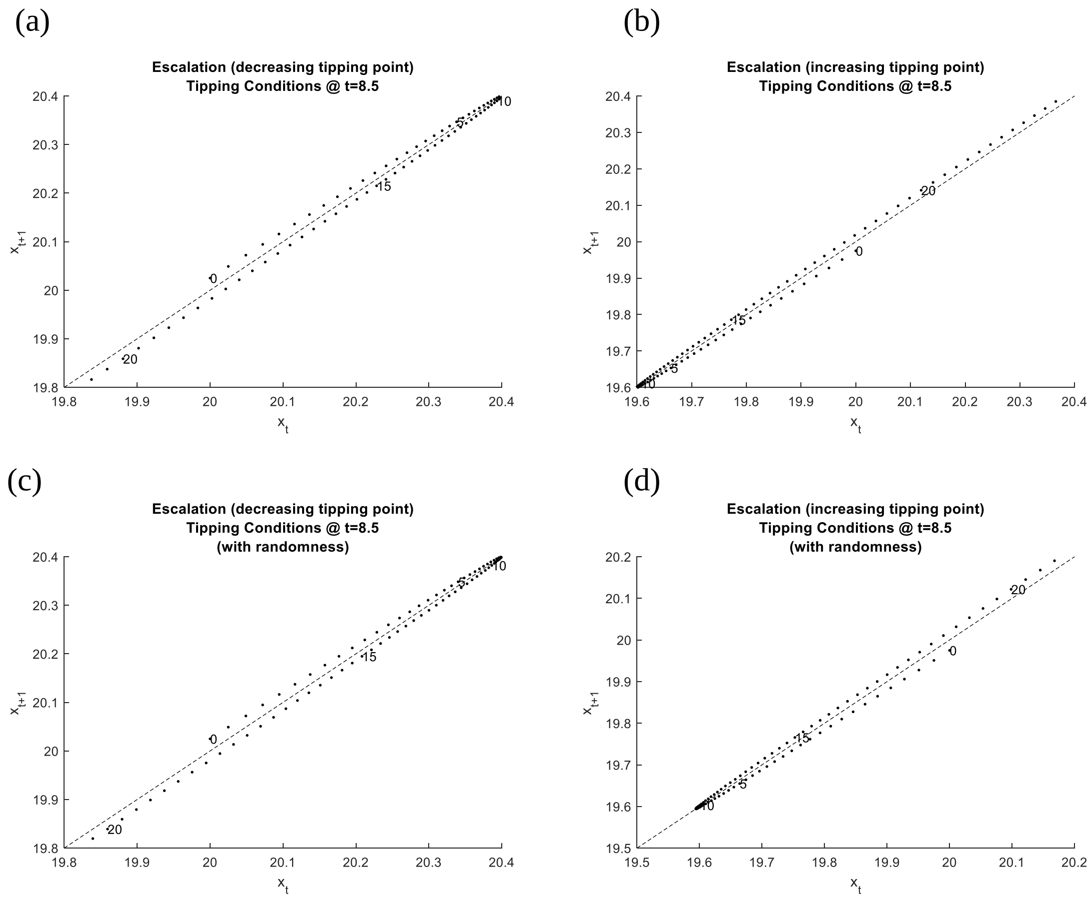
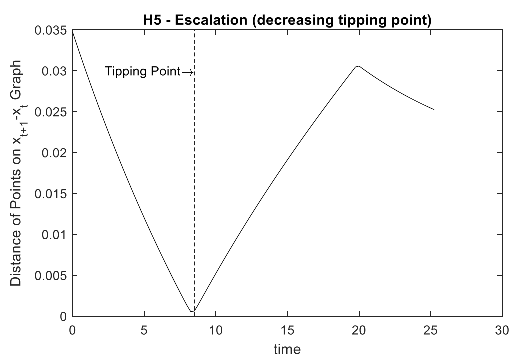


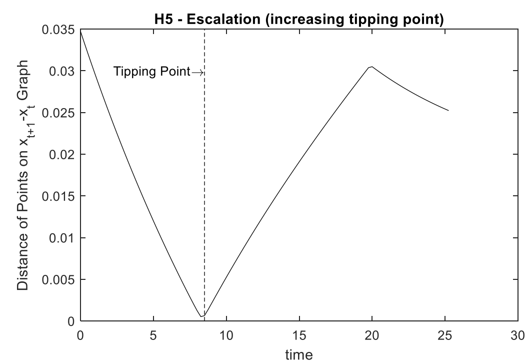
Figure 59- x_{t+1}-x_t graph in escalation archetype
(Tipping variable: A's desired advantage ratio; dependent variable (x): A's results)
59(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
59(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
59(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
59(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

Figure 60 (a-d) shows the results of testing H5 in the escalation archetype. The results support the hypothesis and as the system approaches the tipping point, the distance between the points on x_{t+1}-x_t graph decreases (Figure 60 a&b). Having noise in the system (Figure 60 c&d) causes minor fluctuations but the results still support the hypothesis.

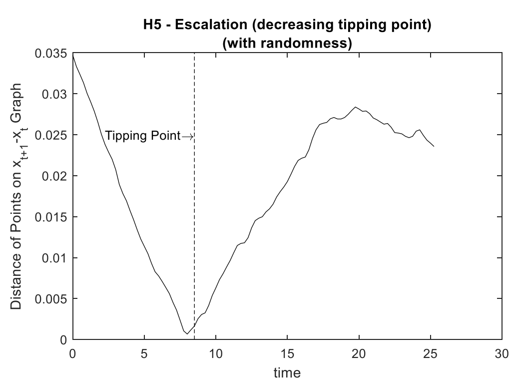
(a)



(b)



(c)



(d)

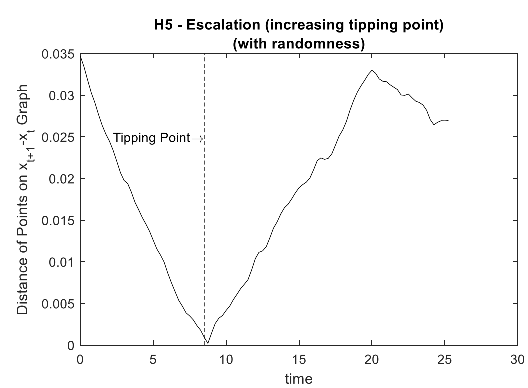


Figure 60-Results of H5 in escalation archetype
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: A’s desired advantage ratio; dependent variable (x): A’s results)
60(a) results when the tipping variable decreases over time without any noise in the system
60(b) results when the tipping variable increases over time without any noise in the system
60(c) results when the tipping variable decreases over time with dynamic exogenous variables
60(d) results when the tipping variable increases over time with dynamic exogenous variables

Figure 61 (a-d) shows the changes of the slope of x_{t+1}-x_t graph over time. Before the tipping point, the slope starts near one and decreases as the system approaches the tipping point. After the system has passed the tipping point, the slope suddenly changes to a number greater than one (Figure 61 a&b). Similar results are observed in the presence of randomness in the system (Figure 61 c&d) with some fluctuations.

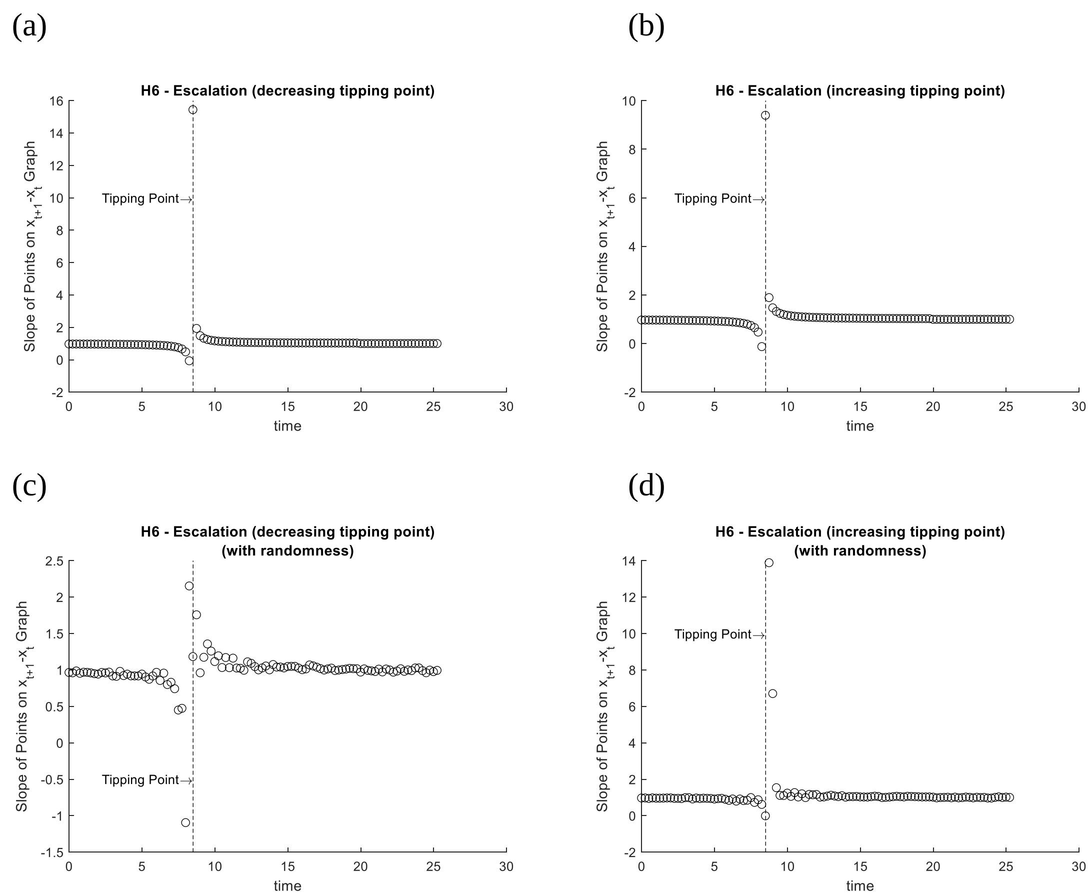


Figure 61-Results of H6 in escalation archetype
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: A's desired advantage ratio; dependent variable (x): A's results)
61(a) results when the tipping variable decreases over time without any noise in the system
61(b) results when the tipping variable increases over time without any noise in the system
61(c) results when the tipping variable decreases over time with dynamic exogenous variables
61(d) results when the tipping variable increases over time with dynamic exogenous variables

Limerick Construction Project

Figure 27 shows the results of testing H1 in the first realistic model in the library (Limerick construction project). The base ripple effects strength is the tipping variable and at the tipping point, its value is 0.665. The tipping variable was changed over time in the [0.5, 0.8] range to test hypothesis 5 and 6. Using this method, the system starts from one side of the tipping point and gradually approaches the tipping point (near time 50), then the system crosses the tipping point and continues its course. Figure 62 (a-d) shows x_{t+1}-x_t graph of Limerick construction project model. When the tipping variable decreases over time (Figure 62 a&c), the system starts from the "bad" side of the tipping point, the project is behind schedule and the backlog[38] of the project will increase over time if no action is taken. Decreasing the tipping variable over time moves the system towards the tipping point and shifts the system to the "good" side of the tipping point. After crossing the tipping point, the project starts recovering and eventually will get completed when the backlog goes to zero. Figure 62 b&d depict the opposite scenario when the system starts from the "good" side of the tipping point but because of the increase in the tipping variable is pushed to the other side and as a result, the system moves to its alternate attractor where the project backlog increases over time. In both cases the density of point on the x_{t+1}-x_t graph increase when the system is near the tipping point (time~50). In Figure 62 a&b the slope of the graph starts from one side of y=x line and at the tipping point, it crosses the y=x line and continues on the other side.

[38] Defined as the number of work packages that need to be completed

In all cases, the slope is very close to one.

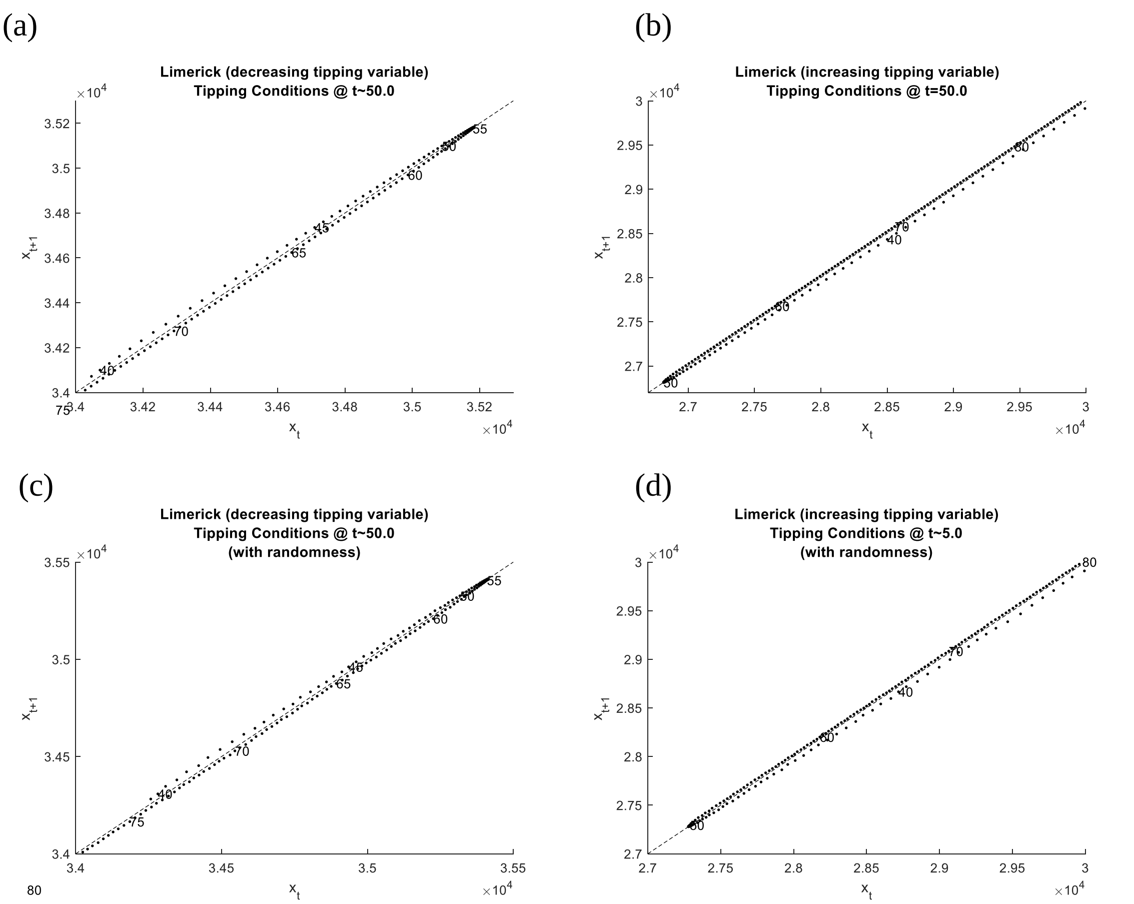


Figure 62- x_{t+1}-x_t graph in Limerick construction project model
(Tipping variable: base ripple effects strength; dependent variable (x): total project backlog)
62(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
62(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
62(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
62(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

Figure 63 (a-d) shows the results of testing H5. As expected from observing the x_{t+1}-x_t graphs, the results support the hypothesis and as the system approaches the tipping point the value of the dependent variable at each time step becomes very similar to its value at the previous time step resulting in a decrease in the distance of points on the x_{t+1}-x_t graph (Figure 63 a&b). When exogenous dynamic variables are designed in the system, some fluctuations are observed in the trend of the distance of points over time but the overall trend of the graph supports the hypotheses (Figure 63 c&d).

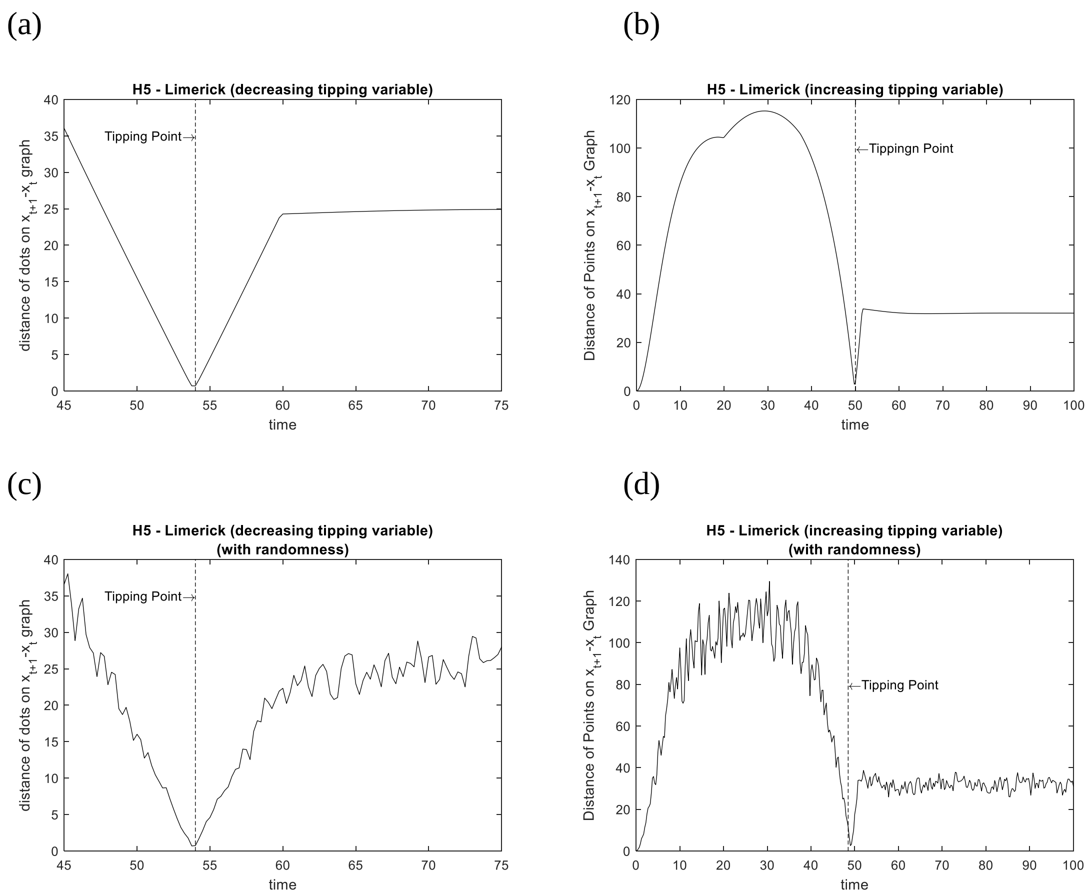


Figure 63- Results of H5 in Limerick construction project model (distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: base ripple effects strength; dependent variable (x): total project backlog)
63(a) results when the tipping variable decreases over time without any noise in the system
63(b) results when the tipping variable increases over time without any noise in the system
63(c) results when the tipping variable decreases over time with dynamic exogenous variables
63(d) results when the tipping variable increases over time with dynamic exogenous variables

The results of hypothesis 6 testing in the Limerick construction project are shown in Figure 64 (a-d). When there is no randomness in the system (Figure 64 a&b) the slope of the x_{t+1}-x_t graph decreases as the system approaches the tipping point. After the system crosses the tipping point, there is a sudden increase in the slope. These results support hypothesis 6 although the slope remains very close to 1 during the simulation. When there is randomness in the system (Figure 64 c&d), there are a lot of fluctuations in the graph but the sudden shift in the slope after the system crosses the tipping point is still recognizable.

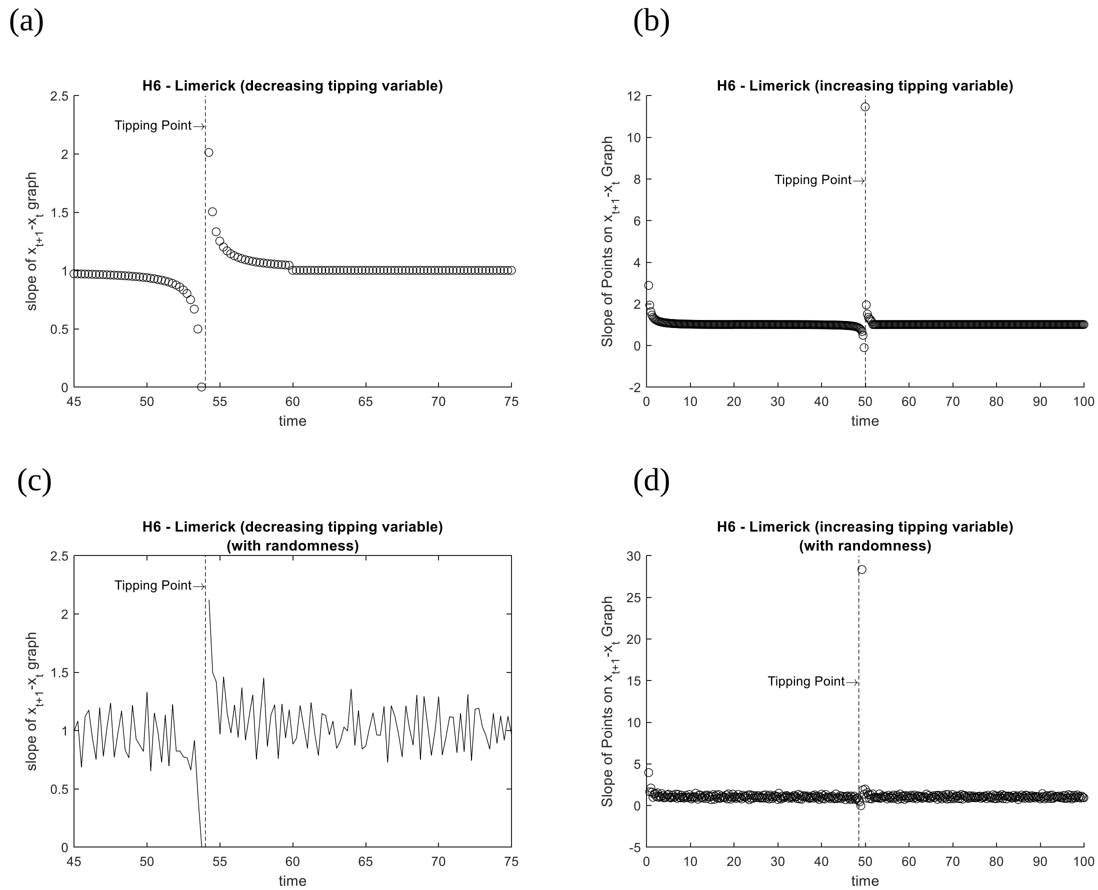


Figure 64- Results of H6 in Limerick construction project model
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: base ripple effects strength; dependent variable (x): total project backlog)
64(a) results when the tipping variable decreases over time without any noise in the system
64(b) results when the tipping variable increases over time without any noise in the system
64(c) results when the tipping variable decreases over time with dynamic exogenous variables
64(d) results when the tipping variable increases over time with dynamic exogenous variables

Fish Banks

The fish banks model has a tipping point when the tipping variable (hatch fraction) is equal to 5.25. To test hypothesis 5 and 6, hatch fraction was gradually changed between 4.5 and 6 over a period of time. The system starts from one side of the tipping point. If the tipping variable is not changed, the system will eventually move towards its attractor. However, because of the changes in the tipping variable, system conditions get close to the tipping point and the system is tipped over (at time = 3). After this, the system path is altered and it will move towards its second attractor. Figure 65 (a-d) shows x_{t+1}-x_t graphs of the fish banks model. As the system approaches the tipping point at time equal to 3, the density of dots on the graph increases and the points get closer to each other. In addition, there is a sudden change in the slope of the graph when the system passes the tipping point conditions: the graph moves from one side of y=x to the other side.

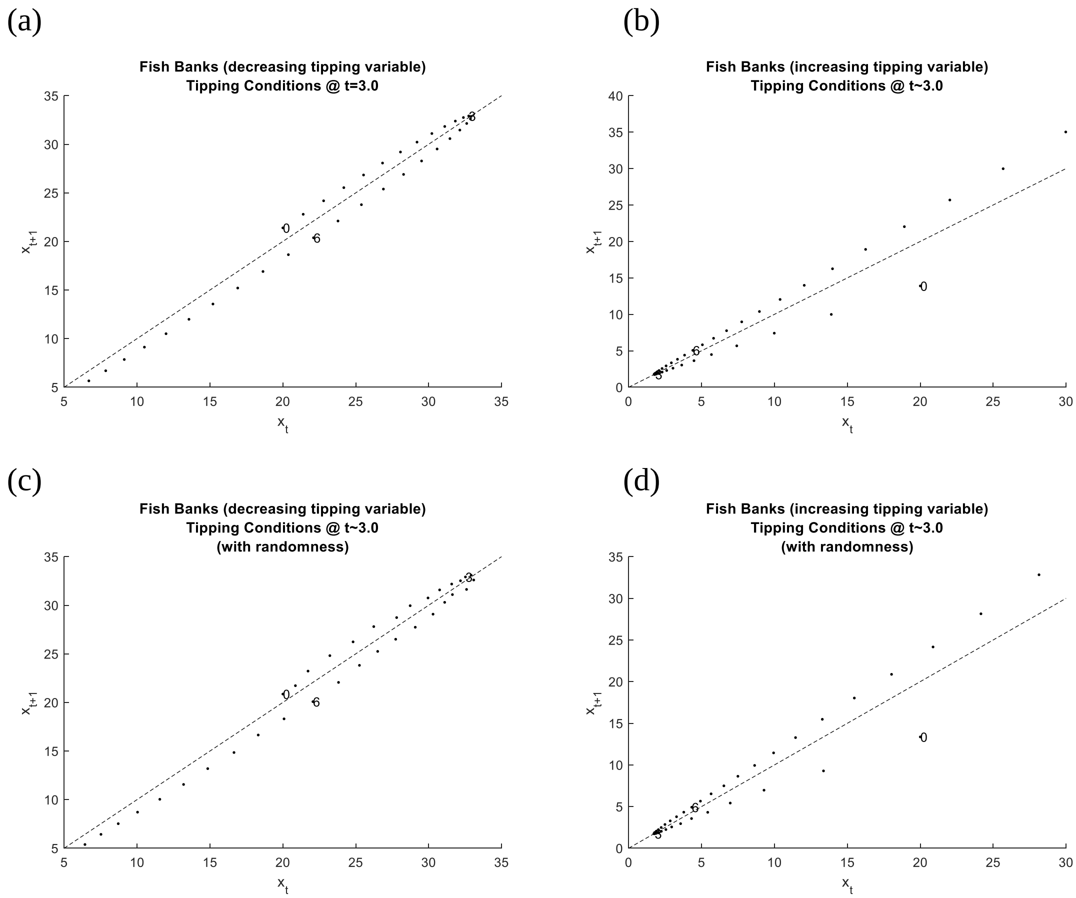


Figure 65- x_{t+1}-x_t graph in fish banks model
(Tipping variable: hatch fraction; dependent variable (x): fish population)
65(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
65(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
65(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
65(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

Figure 66 (a-d) shows the results of testing H5 in the fish banks model. As the system approaches the tipping point, the dots on the x_{t+1}-x_t graph get closer to each other (the distance between the points decreases). These results support the hypothesis. Having noise in the system causes fluctuation in the result but does not change the decreasing trend of the distance between the points (Figure 66 c&d).

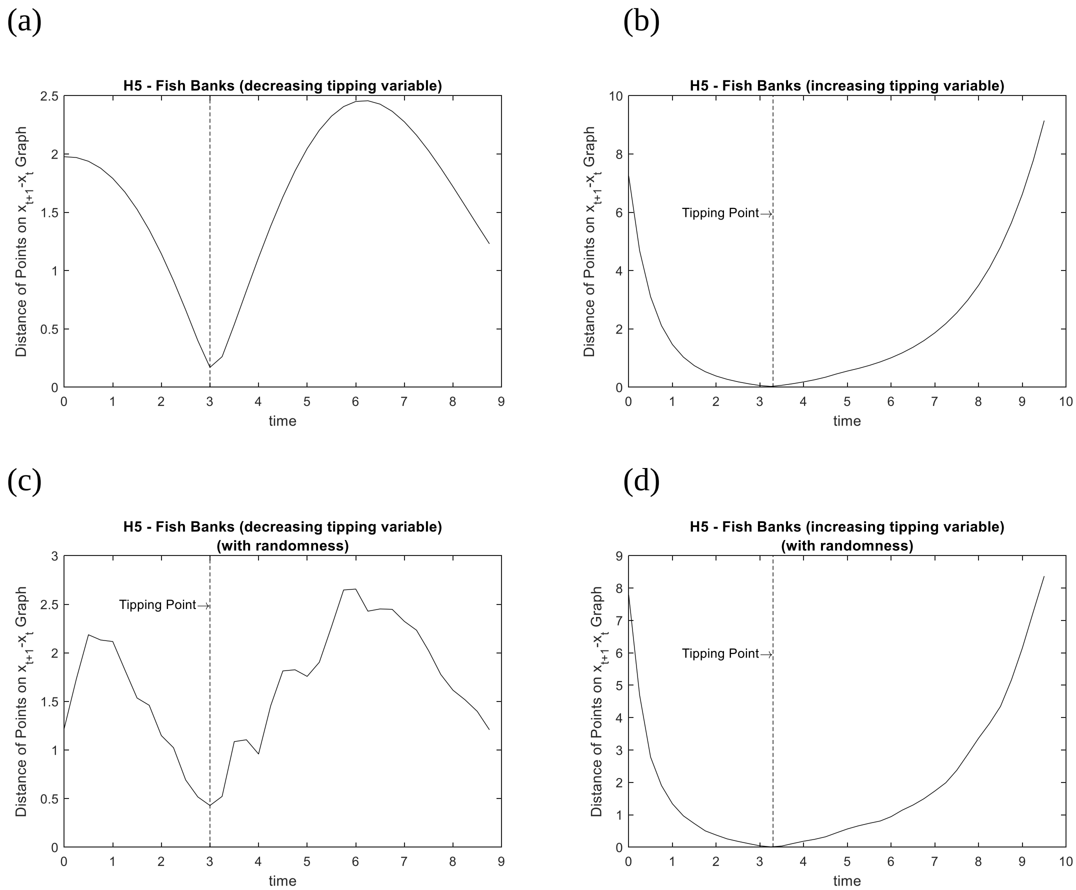


Figure 66- Results of H5 in fish banks model
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: hatch fraction; dependent variable (x): fish population)
66(a) results when the tipping variable decreases over time without any noise in the system
66(b) results when the tipping variable increases over time without any noise in the system
66(c) results when the tipping variable decreases over time with dynamic exogenous variables
66(d) results when the tipping variable increases over time with dynamic exogenous variables

Figure 67 (a-d) shows the slope of x_{t+1}-x_t graph over time. As the system approaches the tipping point, the slope decreases before a tipping point. When the system crosses the tipping point, there is a sudden shift to a value greater than one (Figure 67 a&b). In the presence of intrinsic randomness in the system (Figure 67 c), the jump in the slope happens a little after the system has passed the tipping point. In general, the results support hypothesis 6.

(a) (b)

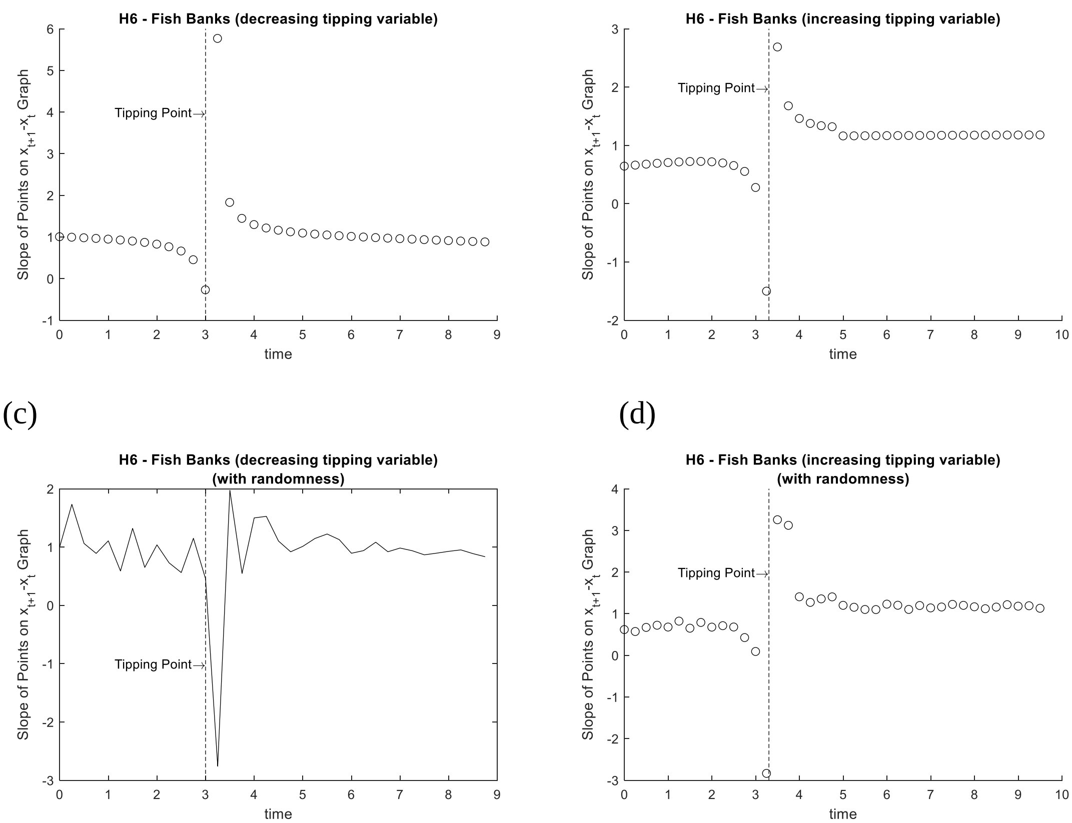


Figure 67 (a-d)- Results of H6 in fish banks model
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: hatch fraction; dependent variable (x): fish population)
67(a) results when the tipping variable decreases over time without any noise in the system
67(b) results when the tipping variable increases over time without any noise in the system
67(c) results when the tipping variable decreases over time with dynamic exogenous variables
67(d) results when the tipping variable increases over time with dynamic exogenous variables

Arms Race

In the arms race model, the system is standing at a tipping point when the tipping variable (desired strength ratio of A) is equal to one. On one side of the tipping point the total arms increase over time while on the other side the total arms decrease over time creating two different attractors in the system depending on the value of the tipping variable. At the tipping point, the total arms of country A and B are equal. To test hypothesis 5 and 6, the tipping variable was changed between 0.9 to 1.1, gradually moving the system towards the tipping point. After the system crosses the tipping point (around time 50), it changes its current path and starts moving towards its alternate attractor. Figure 68 (a-d) show the x_{t+1}-x_t graph in the arms race model. As the system gets closer to the tipping point (time~50) the density of dots on the plot increases, that is the value of the dependent variable (total arms A) becomes very similar to its value at the previous time step, this can be represented in a decrease in the distance between the points. In addition, when the system crosses the tipping point, the slope of the graph becomes greater than one.

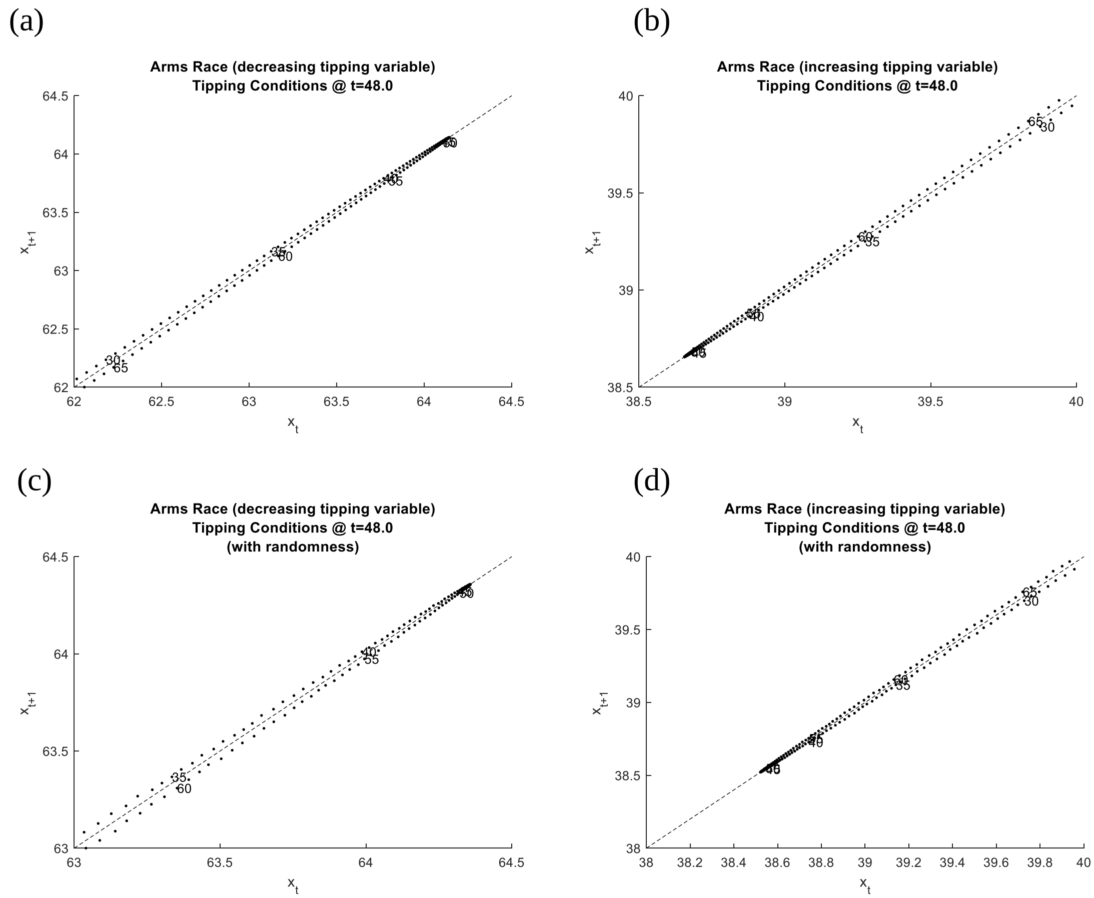


Figure 68- x_{t+1}-x_t graph in arms race model
(Tipping variable: desired strength ratio A; dependent variable (x): total arms A)
68(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
68(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
68(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
68(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

The simulations of arms race model support hypothesis 5. As the system approaches the tipping point (around time 50), the distance between the dots on the x_{t+1}-x_t graph decreases (Figure 69 a&d). When there is randomness in the system (Figure 69 c&d), minor fluctuations are present but the general decreasing trend is intact.

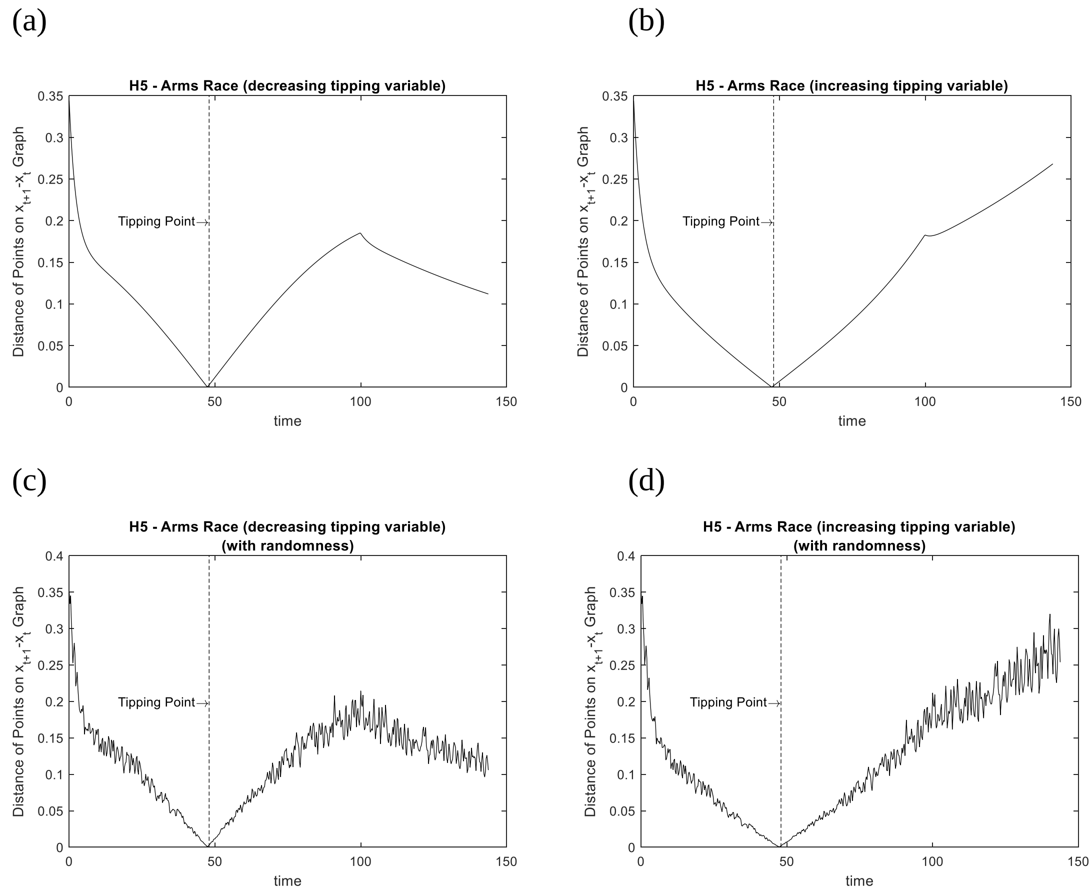


Figure 69- Results of H5 in arms race model
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: desired strength ratio A; dependent variable (x): total arms A)
69(a) results when the tipping variable decreases over time without any noise in the system
69(b) results when the tipping variable increases over time without any noise in the system
69(c) results when the tipping variable decreases over time with dynamic exogenous variables
69(d) results when the tipping variable increases over time with dynamic exogenous variables

The results of testing hypothesis 6 are demonstrated in Figure 70 (a-d). There is an abrupt change in the slope of the x_{t+1}-x_t graph when the system passes the tipping point (near time 50). But the decreasing trend when approaching the tipping point (as observed in the other models) cannot be easily detected in the arms race model. When there is no noise in the system, the x_{t+1}-x_t graph remains very close to the y=x line (the slope of the x_{t+1}-x_t graph remains close to one), therefore, the decrease in the slope before the tipping point is not noticeable (Figure 70 a&b). When there is randomness in the system (Figure 70 c&d) the fluctuations in the slope interfere with any existing trend in the graph. This indicates that the changes in the slope over time is more useful to identify past tipping points instead of predicting a future one.

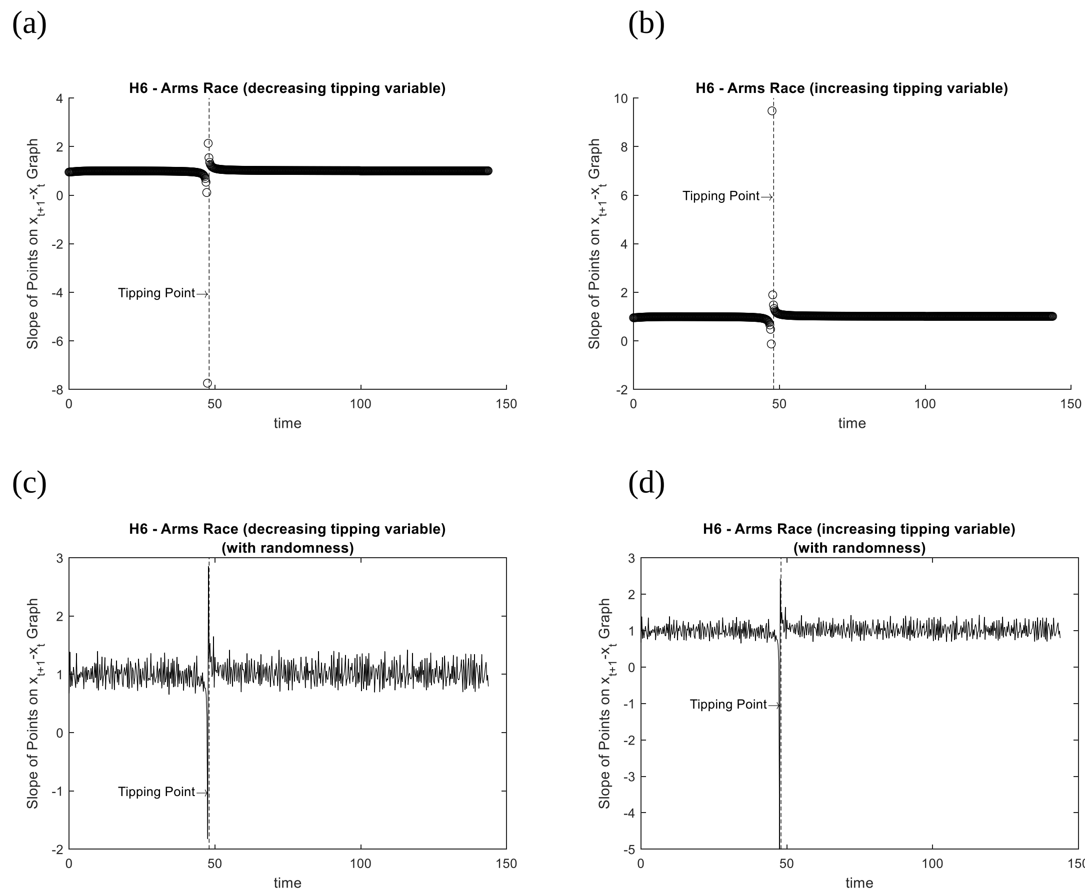


Figure 70- Results of H6 in arms race model
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: desired strength ratio A; dependent variable (x): total arms A)
70(a) results when the tipping variable decreases over time without any noise in the system
70(b) results when the tipping variable increases over time without any noise in the system
70(c) results when the tipping variable decreases over time with dynamic exogenous variables
70(d) results when the tipping variable increases over time with dynamic exogenous variables

Social Impact Bonds

The social impact bonds model has a tipping point when the fraction of surplus returned (the tipping variable) is equal to 0.365. Depending on the value of the tipping variable, the total prison population moves towards either of its two equilibrium states. To test hypothesis 5 and 6, the tipping variable was changed in the [0.3,0.5] range, forcing the system to move towards the tipping point. As a result of the change in the tipping variable, the system shifts from one stable equilibrium to another. Figure 71 (a-d) shows the x_{t+1}-x_t graphs in the social impact bonds model. When there is no noise in the system (Figure 71 a&b), the graphs show a similar trend as the other models studied here. There is an increase in the density of the dots near the tipping point and the slope of the graph shifts from less than one to greater than one. However, no particular trend can be identified in the graphs when there is randomness in the system (Figure 71 c&d)

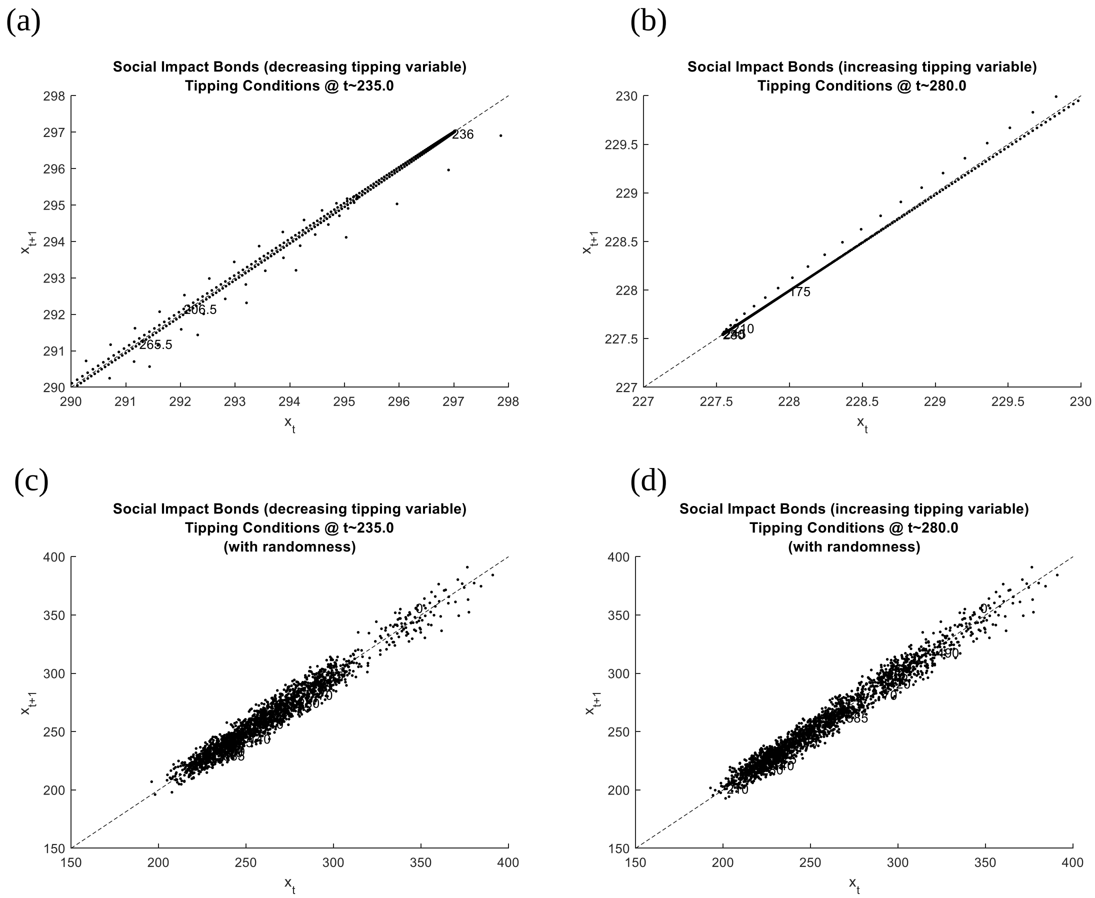


Figure 71- x_{t+1}-x_t graph in social impact bonds model
(Tipping variable: fraction of surplus returned; dependent variable (x): total prison population)
71(a) x_{t+1} vs. x_t when the tipping variable decreases over time without any noise in the system
71(b) x_{t+1} vs. x_t when the tipping variable increases over time without any noise in the system
71(c) x_{t+1} vs. x_t when the tipping variable decreases over time with dynamic exogenous variables
71(d) x_{t+1} vs. x_t when the tipping variable increases over time with dynamic exogenous variables

Figure 72 (a-d) shows the changes in the slope of x_{t+1}-x_t graph over time. When there is no noise in the system and the tipping variable decreases over time (Figure 72a), the results support the hypothesis. As the system approaches the tipping point, the distance between the dots on the graph decrease. However, similar trend is not observed when the tipping variable is increasing over time (Figure 72b). This might be due to the fact that by the time any change is applied to the tipping variable, the system has already reached its equilibrium state and therefore at each time the value of the dependent variable is exactly the same as its value in the previous time step ($x_{t+1}=x_t$) and the distance is zero. When there is randomness in the system (Figure 72 c&d), no pattern can be identified because of the fluctuations in the system, hence, the results are inconclusive.

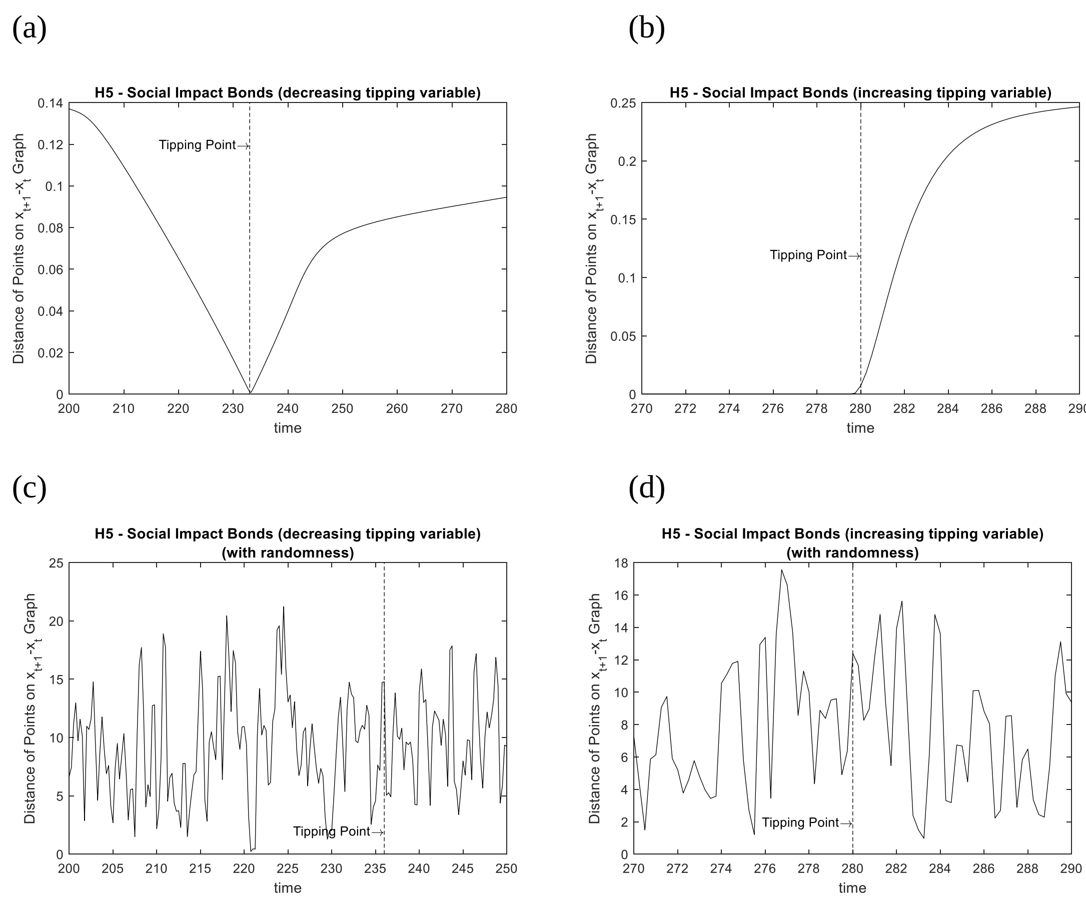


Figure 72- Results of H5 in social impact bonds model
(distance of points on x_{t+1}-x_t graph vs. time)
(Tipping variable: fraction of surplus returned; dependent variable (x): total prison population)
72(a) results when the tipping variable decreases over time without any noise in the system
72(b) results when the tipping variable increases over time without any noise in the system
72(c) results when the tipping variable decreases over time with dynamic exogenous variables
72(d) results when the tipping variable increases over time with dynamic exogenous variables

The results of hypothesis 6 testing are shown in Figure 73 (a-d). In the case that there is no randomness in the system and the tipping variable is decreasing over time (Figure 73 a), the slope of x_{t+1}-x_t graph decreases as the system gets closer to the tipping point and suddenly shifts to a value greater than one after the system crosses the tipping point which supports the hypothesis. However, a similar trend is hardly recognized in the case where the tipping variable is increasing (Figure 73 b). No distinguishable pattern is identified in the graphs when there is noise in the system (Figure 73 c&d). In conclusion, the results of hypothesis 6 testing in the social impact bonds model are inconclusive.

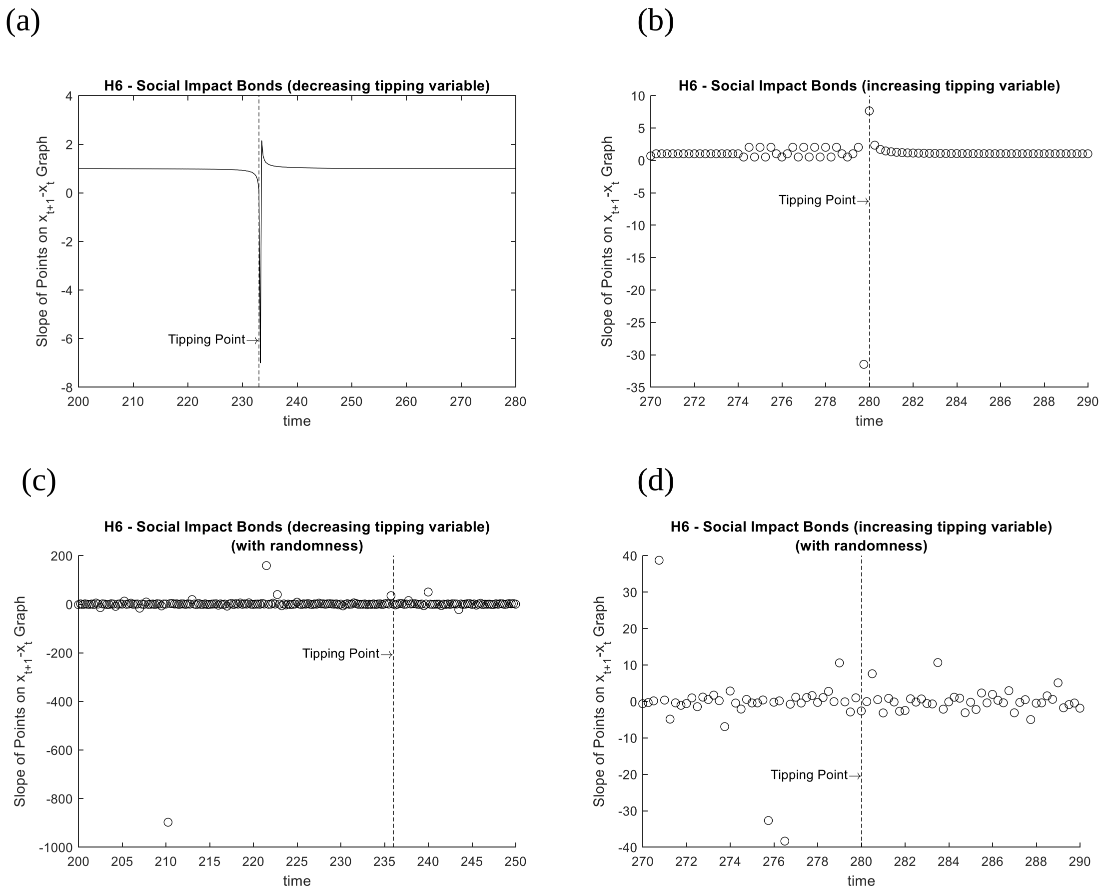


Figure 73- Results of H6 in social impact bonds model
(slope of x_{t+1}-x_t graph vs. time)
(Tipping variable: fraction of surplus returned; dependent variable (x): total prison population)
73(a) results when the tipping variable decreases over time without any noise in the system
73(b) results when the tipping variable increases over time without any noise in the system
73(c) results when the tipping variable decreases over time with dynamic exogenous variables
73(d) results when the tipping variable increases over time with dynamic exogenous variables

Hypothesis 5-6 Summary

An increase in the density of point on a x_{t+1}-x_t graph can be an indicator of a tipping point and a new measure of critical slowing down. As a system approaches a tipping point the state variable becomes more similar to its previous time step value resulting in more number of points on the graph near a tipping point. This density can be measured by finding the distance between two consecutive points on the graph. The results of testing this hypothesis in the models in the library show that the hypothesis is supported in simple systems (with few stock variables). In the complex models of the library, in basic cases where the exogenous variables are static and there is no noise in the system, the results support the hypothesis. However, in the presence of randomness in the system, due to the fluctuation caused by the noise, any existing decrease in the distance between the points cannot be identified and the indicator signal is not clear.

Observing x_{t+1}-x_t graphs also reveals a trend in the changes in the slope of the graph over time. As the system approaches the tipping point, the slope remains less than one and decreases until the system reaches the tipping point. At this point and as the system crosses the tipping point, the slope abruptly shifts to a value greater than one. In other words, when drawing x_{t+1}-x_t graphs, if the graph crosses the y=x line, it is an indicator of crossing a tipping point. This behavior is similar to the phenomenon called "avoided crossing" in physics that occurs near a critical point in systems with a limited number of state variables. The simulation results of simple models (less than 3 stock variables) support the hypothesis. Simulations results of the complex models are inconclusive. If there is no randomness in the system, the aforementioned behavior is observed in the models but in some cases, the slope of the x_{t+1}-x_t graph tends to stay very close to one which makes identifying the existing decrease in the slope before the tipping point very difficult. But in all cases, the sudden change in the slope after the system has passed the tipping point is easily recognizable, indicating that the slope of x_{t+1}-x_t graph might be more useful to identify past tipping points instead of predicting a future one. In the presence of the randomness in the system, no trend can be recognized because of the noise-induced fluctuations.

CHAPTER VII

TESTING THE APPLICATION OF TIPPING POINT INDICATORS IN PRACTICE

In this chapter, the two new indicators (distance of points on the x_{t+1}-x_t graph and slope of x_{t+1}-x_t graph) are used to predict and manage a tipping point in the Limerick construction project model. Based on the prediction, a preventive strategy is used to avoid crossing the tipping point and save the project. This is used to show how project managers can use these tipping point indicators to avoid having projects fail due to tipping point dynamics.

This model is based on the Limerick Unit 2 nuclear power plant construction project as modeled by Taylor and Ford (2006; 2008). See Chapter III for a description of the model and Appendix A for model equations. The model has two attractors. On the "good" (desirable) side of the tipping point, the total project backlog[39] decreases over time and the project finally completes successfully, on the "bad" (undesirable) side of the tipping point, the total project backlog increases over time and the project eventually fails in respect to its schedule performance. See Chapter III for a full description of the attractors and the tipping point conditions. For the purpose of this chapter, the system conditions are defined in a way to simulate the project such that it starts on the "good" side and tips towards the "bad" side of the tipping point. The behavior of the total project backlog (the dependent variable) over time is shown in Figure 74. At the beginning, the project backlog decreases over time and it looks like the project is making progress.

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[39] Defined as the number of work packages that need to be completed

However, the extra work caused by rework, schedule pressure and ripple effects strength builds over time and eventually tips the system at around 60 months[40]. After this time, the project gets out of control, the total project backlog increases constantly and the project will miss its schedule performance target.

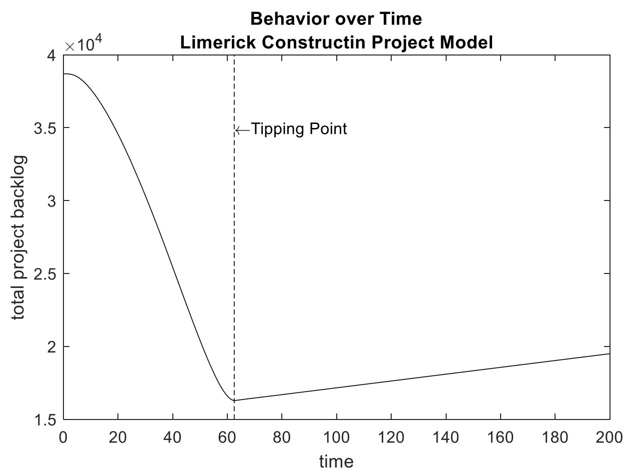

Figure 74-Behavior graph of total project backlog in Limerick construction project model

H5 and H6 indicators (distance of points on the x_{t+1}-x_t graph and slope of x_{t+1}-x_t graph) are used to predict the tipping point at around time 60 in the Limerick construction project model. These two indicators are the most practical ones studied in this work because they are easy to use and do not require extensive statistical knowledge. The data required to use these indicators is the status of the total project backlog at each time interval (weekly or monthly) which is already collected during

[40] The tipping point time is dependent on the tipping conditions including the value of the tipping variable.

periodic project reports. The x_{t+1}-x_t graph can easily be drawn using widely available software such as MS Excel and monitored based on the data. From the x_{t+1}-x_t graph, it is easy to find the distance between the dots and the slope of the graph using basic geometric equations.

Figure 75 shows the x_{t+1}-x_t graph built based on the value of the total project backlog between month 50 and 90 as shown in Figure 74. The y-axis shows the total project backlog at time t+1, and the x-axis is the total project backlog at time t. The arrows show the direction of the graph as the project progresses. The graph starts on the right side of the y=x line (slope is less than one). As the project continues, the dots on the graph get closer to each other, and at time 62.5 (circled on the graph), the trend crosses the y=x line and moves towards the other direction with a slope slightly greater than one. Time 62.5 is the tipping point of the system.

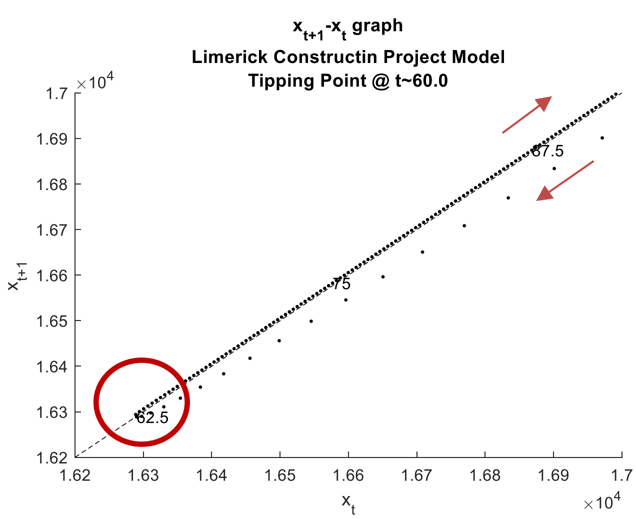

Figure 75- x_{t+1}-x_t graph of Limerick construction project (x: total project backlog)

From Figure 75, the distance between the dots (H5 indicator) is calculated and the changes of the distance over time are shown in Figure 76. At the beginning of the project, the distance increases over time. Between time 39 and 43, there is no change in the distance between the dots. After this point, the distances between the dots start decreasing until it reaches its minimum value at 62.5 months (the tipping point). At time 43, the distance is 180. In three months, there is 1% decrease in the distance between the dots. The percentage decrease goes up to 5% at time 49 and 10% at time 52.

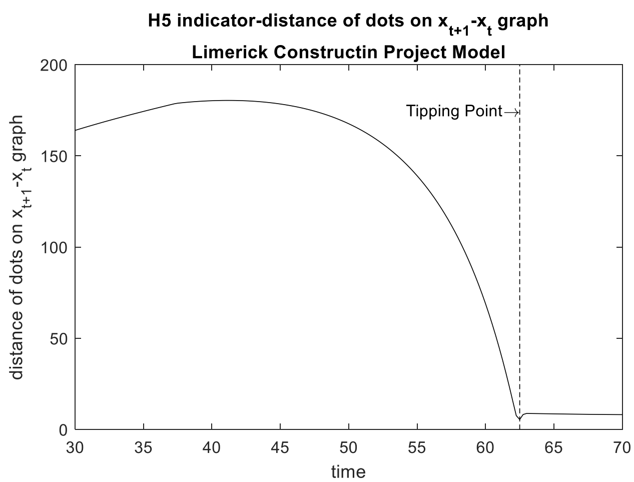

Figure 76-Distance of points on the x_{t+1}-x_t graph in Limerick construction project model

A project manager who is monitoring this indicator can observe a decrease in the distance of dots after time 43. At the beginning, the decrease is small and it might be considered as temporary. At time 49 (6 months after the decreasing trend starts), the project manager will observe a 5% decrease and be aware of an upcoming tipping point in the project. This can provide time to take action before the tipping point is crossed.

Adding more staff to the project is one preventive strategy to help the project. To apply this strategy in the Limerick model, the total project staff was increased by 45% at time 49 for the remainder of the project[41]. Figure 77 (a-b) shows the total project backlog over time when this strategy is used and the total project performance without any managerial response. When the extra staff is added from time 49. the project never passes the tipping point and instead, the backlog decreases over time until the project completes at time 95 (Figure 77a). If the project manager waits till the 10% decrease threshold (month 52), he/she will need to increase the project staff by 60% to get similar results.

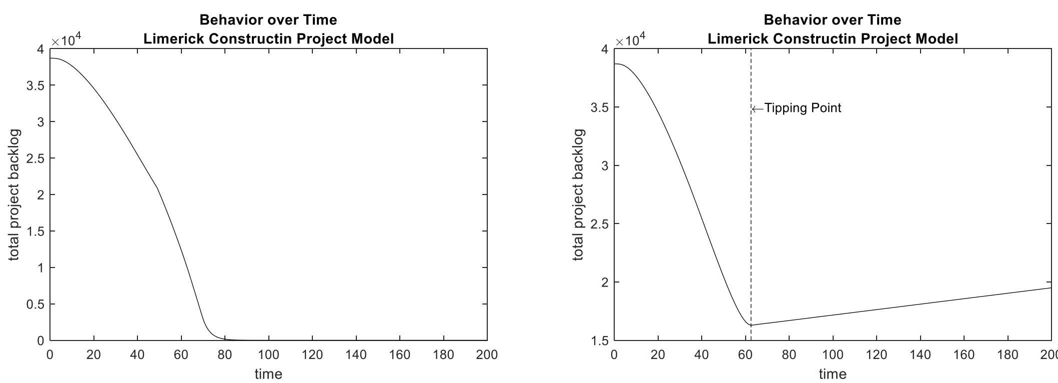

Figure 77 (a-b)- Behavior graph of total project backlog in Limerick construction project model with and without managerial response
77(a) project performance with managerial response (adding staff): after increasing the project staff by 45% at time 49, the total backlog will continue decreasing and the project will complete at time 95
77 (b) project performance without managerial response: the project crosses the tipping point at around 60 and it will not finish

[41] The initial project staff is 2350 and an extra 1058 were added after time 49

If the project manager was unaware of the tipping point, and does not add extra staff to the project on time, the project would cross the tipping point at time 62.5, and the backlog would increase over time without any chance for the project to complete (Figure 77b). After this point, adding staff alone does not help the project, and it should be combined with other strategies (e.g. reducing project scope) to have any effect on the project schedule. See Taylor and Ford (2008) for a case study of the tipping point in the Limerick construction project. This supports the idea that crossing a tipping point is either irreversible (Thompson and Sieber 2011; Beaulieu et al. 2012) or the energy required to move the system back to its original state is more than the required energy to tip the system (Scheffer et al. 2001).

Next, the slope of the x_{t+1}-x_t graph (H6 indicator) is used to predict the tipping point. The slope starts from near one and remains very close to one (Figure 78). It starts decreasing near the tipping point (t=60) and suddenly changes to a value greater than one after the tipping point.

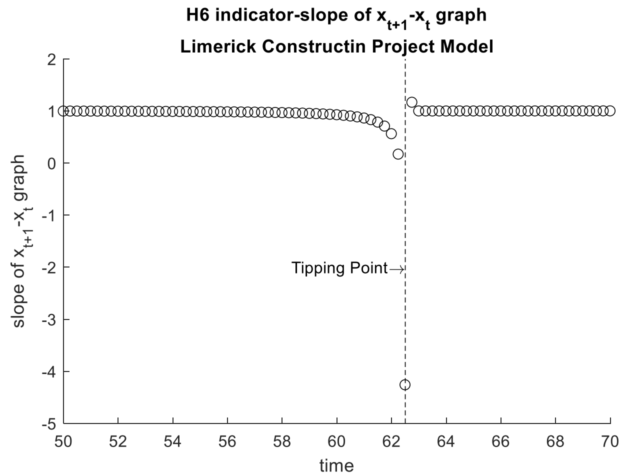

Figure 78- Slope of the x_{t+1}-x_t graph in Limerick construction project model

The slope starts decreasing at 60 months. In one month (t=61), the decrease is over 5%, and in five months (t=61.25) the decrease is 10%. This indicator predicts the tipping point much later than the previous indicator (compared to 49 months for 5% decrease in the distance between the points and 52 months for 10% decrease). If the project manager starts the strategy of adding more staff at time 61, he/she will need to use four times the initial staff in order to finish the project in 100 months. See Figure 79 for the changes of total project backlog over time using this strategy. This example demonstrates that the H6 indicator can trace a tipping point when the system is very close to the tipping point. This might not give enough time to the practitioners to apply effective strategies to avoid the tipping point. Therefore, the H6 indicator is more useful in identifying past tipping points.

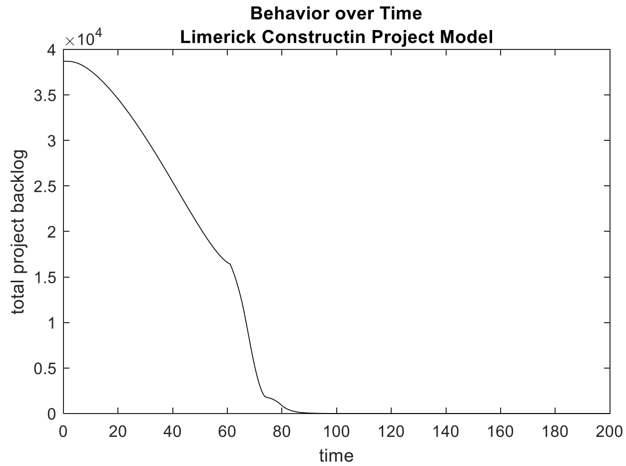

Figure 79- Behavior graph of total project backlog in Limerick construction project model after using six times the initial staff starting at time 61

The other tipping point indicators studied in hypotheses 1-4 are not straightforward to use. The H1 indicator (time to reach an equilibrium state without any perturbation) was mainly tested to find if the models studied show properties of bifurcation near their tipping point. Although it is possible to use this indicator in practice mainly to compare the state of two different projects, it might not be very practical to use this indicator for predicting a tipping point in a single project. By the time enough data has been gathered to test the indicator, the project will be almost near completion.

Using the existing "critical slowing down" measures (i.e. H2-H4 indicators, namely, recovery time from a perturbation, lag-1 autocorrelation, and variance of the state variable) has some limitations when the system model is not available. Testing these indicators requires perturbing the system which might be ethically or practically impossible. However, in construction projects, change orders frequently provide natural disturbances in the system. Adding a change order can be considered as a perturbation to the project schedule. The time it takes the project to get back on schedule can be compared to results of previous change order incidents. If it takes the project significantly longer amounts of time to get back on track (for the same type and volume of work), it might be an indicator that the project is deteriorating and getting close to a tipping point. Although the temporal autocorrelation and variance of the project backlog can be used in the same way, by the time any increase is observed in these two measures, the system might be so close to the tipping point that preventive policies are ineffective. Therefore, the recovery time is a better indicator of a tipping point in a construction project although it is still more difficult to measure than the indicators based on x_{t+1}-x_t graphs as described earlier in this chapter.

CHAPTER VIII

CONCLUSIONS AND DISCUSSION

Summary

Tipping point dynamics have been observed in various systems in different disciplines. Because of the large negative consequences of tipping point-induced failures, an indicator that can predict tipping points can be very valuable to designers and managers of complex systems. "Critical slowing down" is one of the most studied tipping point indicators in the literature. However, not all tipping points show slowing down. Scheffer et al. (2012) suggest that slowing down should be seen as an indicator of a "potential" change in the system, and the robustness of the current indicators still needs to be fully studied (Scheffer et al. 2012; Lenton 2011). In addition, the research so far has focused on statistical analysis of historical data; hence, the system structures that result in a tipping point have not yet been thoroughly investigated (Scheffer et al. 2012). The "slowing down" indicators are derived from the properties of bifurcation in nonlinear dynamics. These findings are based on simple models with few dependent variables, and the theoretical research on the dynamic behavior of large and complex systems is still limited (Strogatz 2014). As engineers, our interest is to be able to use the suggested indicators in application with the focus on construction projects. To bridge the gap between simple math models and application, this work has applied a progressive approach in which first system dynamics archetypes that are models with less than three stock variables are studied. The archetypes represent similar behavior patterns in systems and are the foundation of the models of complex systems. Next, previously validated system dynamics models of realistic systems are studied. These models are relatively complex and have more variables. The purpose of this approach is to gain more understanding of system structures that create tipping points and the usefulness of tipping point indicators by studying existing simple and relatively complex models in a controlled environment. The ultimate goal is to apply this knowledge and extend the findings to practice when no formal model of the projects is available.

This work started by studying the existing definitions of a tipping point in the literature. Further investigation revealed a problem in the current definitions of a tipping point: each researcher has focused on a different perspective of a tipping point. Some see the tipping point as a point in time (e.g. Gladwell 2006) while others describe the system behavior or system conditions near or at a tipping point (e.g. Lenton 2011; Sieber et al. 2012). *This research focused on improving the definition of a tipping point by defining three different terms: tipping point conditions, tipping point behavior and tipping point structure.* Each of the terms focuses on one aspect of a tipping point, and together they provide a more thorough definition than the current definitions in the literature. This improved definition, as described in Chapter II, answers the first research question: "What are the necessary and sufficient conditions for a tipping point?"

Next, *a library of models with tipping point behaviors was developed*. The model library consists of system dynamics archetypes as well as models of real systems that have been previously developed and validated in the system dynamics literature. *The feedback loop structure of library models was studied to develop a taxonomy of tipping point structures.* Type I tipping points dynamics are caused because of the change in the dominance of feedback loops whereas type II tipping points happen due to a change in the direction of the dominant reinforcing loop. The model library and the identified tipping point types, as described in Chapter III, answer the second research question: "What system structure(s) create multiple attractors and a tipping point?"

To answer the final research question ("How do tipping point indicators behave in different types of tipping point structures?"), the behavior of four tipping point indicators (derived from "critical slowing down" literature) and two potential new indicators (inspired by the behavior of state variables in the library models) were formulated in the form of six hypotheses. The best procedure to test each hypothesis was developed based on the features of system dynamics models. The hypotheses were then tested in the selected models of the library.

In both type I and type II, an increase in the time to reach an equilibrium without any perturbation is an indicator of a tipping point. The only exception is the social impact bonds model (an example of a type II tipping point) in which the results support the hypothesis when approaching the tipping point from one side but not from the other side. These results show that the models in the library have the properties of a bifurcation at their tipping point. However, in realistic models, the increase in time required to reach an equilibrium is only noticeable when the system is very close to the tipping point, which may not provide enough time to apply preventive actions. These results emphasize the concerns of Dakos et al. (2010) that the slowing down behavior might happen too late.

An increase in the recovery time from a perturbation (one of the measures of "critical slowing down") can be an indicator of tipping points in type I models. However, in realistic models, the results depend on the time frame studied, and these indicators are only useful when the system is very close to the tipping point conditions. The systems recover from a perturbation slower when getting close to a tipping point, but when the system is further away from its tipping point, recovery time might either remain constant or give false alarms as in the fixes that fail archetype (due to the oscillatory behavior). The value of recovery time varies depending on the size of perturbation, but the general behavior does not change. The results of type II models are not conclusive. Only two of the type II models could be tested using the suggested testing procedures. Of the two models that could be tested, the results of one (the reinforcing loop archetype) support the hypothesis whereas the results of the social impact bonds model only partially support the hypothesis (there is no change in the recovery time when approaching the tipping point from one side).

An increase in the lag-1 autocorrelation of the dependent variable (the second "critical slowing down" measure) can be used as an indicator of a tipping point in type I models, but definite conclusions cannot be made for type II models. It should be noted that, by definition, in a system dynamics model, the value of the stock variables at each time step are calculated based on their value in the previous time step. Therefore, lag-1 autocorrelation might not be the best indicator to look at in a system dynamics model.

An increase in the variance of the dependent variable (the third measure of "critical slowing down") should be used with caution as a tipping point indicator. The results of all type I models and one of the type II models support the hypothesis. However, the results are sensitive to the size of the perturbation. Some doubts about using the increase in variance as an indicator of a tipping point have been stated in the literature (Lenton et al. 2012; Dakos et al. 2012). Dakos et al. (2012) suggest that the direction of changes in the state variable variance (decrease or increase) depends on the sensitivity of the state variable to the existing noise in the system. Although the results here do not show any major decrease in the variance, they are very sensitive to the size of the perturbation which supports the critics on using variance as an indicator. Dakos et al. (2012) suggest that the sensitivity of the state variable to the control variables that were used to disturb the system can impact the results. The closeness of the system to the tipping point also affects the results. In other words, the increase in the variance only starts when the system is very close to the tipping point.

Two new tipping point indicators (i.e. decrease in the distance between two consecutive points on x_{t+1}-x_t graphs and sudden change in the slope of the x_{t+1}-x_t graph) are suggested based on the similarities in the behavior of the dependent variable as the system approaches a tipping point. The distance between two consecutive points on x_{t+1}-x_t graphs (where x is the dependent variable) is a promising practical indicator of a tipping point. In all models studied (both type I and type II), the distance between the points decreases as the system approaches the tipping point. This indicator can be considered a new measure of critical slowing down. As the system approaches a tipping point, the state of the system becomes very similar to its state at the previous time step. Hence, the density of the points on a x_{t+1}-x_t graph increases, and the points get closer to each other as the system approaches the tipping point. This indicator is easy to use in practice. Keeping the x_{t+1}-x_t graphs up to date is easy and does not require additional information other than what is normally gathered when managing projects. A manager can keep track of the project backlog and use a decrease in the distance between the points on the x_{t+1}-x_t graphs as an indicator of approaching a tipping point. See Chapter VII for a sample application of this indicator in a construction project model.

Observing x_{t+1}-x_t graphs reveals a trend in the changes in the slope over time. As the system approaches the tipping point, the slope remains less than one and decreases over time until the system reaches the tipping point at which the slope abruptly changes to a value over one (the x_{t+1}-x_t graph crosses the y=x line at the tipping point). The simulation results of all models (both type I and type II) show this trend when there is no noise in the system. However, when there is randomness in the system, the trend is not easily recognized due to the fluctuations caused by the noise in the system, and the results are inconclusive for those conditions. Also, the decreasing trend in the slope starts when the system is very close to the tipping point. Hence, it might not provide enough time to apply preventive strategies in the system. The abrupt change in the slope at the tipping point is observed in both conditions (with and without randomness). Therefore, this indicator is more useful in identifying a past tipping point than predicting a future one. An application of this indicator in a construction project model is described in Chapter VII. Table 3 summarizes the results of testing the hypotheses in the library models studied here. The detailed descriptions of hypothesis 1, hypothesis 2-4 and hypothesis 5-6 are described in Chapters IV, V and VI respectively.

Table 3-Summary of results of testing hypotheses in the tipping point library models

TP Type	Model	Hypothesis 1	Hypothesis 2	Hypothesis 3	Hypothesis 4	Hypothesis 5	Hypothesis 6
Type I	Limits to growth archetype	Supported	Supported	Supported	Partially supported	Supported	Supported
	Fixes that fail archetype	Supported	Supported	Supported	Partially supported	Supported	Supported
	Limerick construction project model	Supported	Supported	Supported	Partially supported	supported	supported
	Fish banks model	Supported	Supported with limitations	Supported with limitations	Partially supported	Supported	Supported
Type II	Reinforcing loop archetype	Supported	Supported	Supported	Partially supported	Supported	Supported
	Escalation archetype	Supported	Could not be tested	Could not be tested	Could not be tested	Supported	Supported
	Arms race model	Supported	Could not be tested	Could not be tested	Could not be tested	Supported	Supported
	Social Impact Bonds in Peterborough Prison	Partially supported	Partially supported	Partially supported	Partially supported	supported	supported

Research Contributions

This work has taken steps to bridge the gap between the bifurcation theory in nonlinear dynamics and practice in the system dynamics field. Investigating and analyzing the current definitions of a tipping point lead to identifying potential points of confusion in the accepted definitions. Each researcher has focused on a different perspective of a tipping point. A more thorough definition of a tipping point that includes all aspects and can be used across different disciplines seems necessary. The ***improved definition of a tipping point,*** as suggested here, adds rigor to the meaning of tipping points. Three aspect of tipping points (namely behavior, conditions and structures) are brought together in the improved definition, which encourages researches to have a holistic approach when studying tipping points instead of concentrating on a point in time (as some of the current definitions suggest).

Unlike most of the tipping point literature that uses a statistical data-based approach in identifying tipping point indicators, a model-based approach was used here. This approach utilizes the system dynamics principles to study the feedback structures of the systems with tipping points. By investigating a set of models, the underlying system structures that can create multiple attractors and thereby give rise to a tipping point were identified. The knowledge was used to develop ***a taxonomy of tipping point types***. This taxonomy can help distinguish different tipping point dynamics in complex models.

In addition, the ***library of system dynamics tipping point models*** gathered here brings existing case studies in the system dynamics literature into one place. Although system dynamicists have studied and researched tipping points individually, their efforts have not been gathered in one place before. Having a library of models is beneficial for future studies of tipping points. Researchers can expand the model library and use the models to further investigate tipping point dynamics. This can also help standardize tipping point discussion by having a set of models that everyone can refer back to.

The existing "critical slowing down" measures were tested in the library models and this study ***demonstrated and supported them as important potential tipping point indicators*** that are applicable to a broad range of systems including system dynamics models. Although "critical slowing down" measures have been widely studied in the climate change and ecosystem literatures, there has not been any attempt to use them in the construction management or system dynamics before.

The findings of this study also highlighted some limitations of "critical slowing down measures". The results ***demonstrated that the clarity of the tipping point indicators' signal depends on the complexity of the system***. While the indicators clearly predict tipping points in simple models with a limited number of state variables, interpretation of the signals in more complex models are not always easy and there might be some missed or false alarms when using the indicators. It was also found that having noise in the system highly affects the performance of tipping point indicators. The fluctuations caused ***by intrinsic noise in the system interfere with the tipping point indicators*** and it is not always possible to separate the noise-induced fluctuations from the indicator signals.

The study of the library models' behavior before a tipping point enhanced the tipping point body of knowledge by ***identifying two potential new tipping point***

indicators. The distance and slope of x_{t+1}-x_t graph are proposed as being more useful to practitioners than the previous indicators. These indicators are easy to use and the data required to use them as indicators is not difficult to gather. The ease of application of these two indicators were demonstrated in a construction project model in Chapter VII.

Research Implications for Practitioners

This work has some important implications for practice. The improved definition of a tipping point will help the practitioners to better recognize a tipping point in the system and avoid the confusion caused by the incomprehensive accepted definitions. While the assumption might be to see the tipping point as a point in time, the improved definition highlights that a combination of system behaviors, system conditions and system structures is responsible for tipping point dynamics. These three factors should be considered together when studying tipping point dynamics rather than focusing on a single point in time.

The practitioners can compare their system behavior to the behavior of the system archetypes and other models gathered in the library and recognize their system as one of the library. This will result in a better understanding of the underlying feedback structures in their systems and how different system components interact with each other. The taxonomy of the tipping point types in combination with the model library will enable the practitioners to classify the tipping point they are facing which will facilitate collecting data for further investigation of the system.

The results of hypotheses testing support and strengthen the importance of "critical slowing down indicators". Although these indicators are not easy to use in project management as discussed in Chapter VII, they provide useful tools to compare the stability of different systems. These indicators can be used to rank systems and the knowledge will help the system designers to identify the strength and weaknesses of their systems and enhance their future designs.

The two new indicators suggested in this work (the distance of slope of x_{t+1}-x_t graphs) are potential useful tools to recognize tipping points. In a x_{t+1}-x_t graph each point represents the condition of the system at a point in time. These two indicators are easy to use by practitioners because the data required to build x_{t+1}-x_t graphs is already gathered during periodical project reports. Monitoring x_{t+1}-x_t graphs is straightforward and do not require extensive knowledge of statistics. These indicators were applied in the Limerick construction project model as an example to show their ease of use. The indicators were used to predict the tipping point in the system and a common preventive strategy (add resources) was used to keep the project crossing the tipping point conditions and thereby helping the project meet its schedule target. See Chapter VII for details of the application. The exponential nature of the decrease in the distance and slope of x_{t+1}-x_t graphs will also allow the managers to estimate the closeness of the system to the tipping point.

The findings of this work showed that the clarity of the indicators' signal depends on the complexity of the system and size of noise in the system. This helps set expectations on how clear these indicators can predict tipping points. Practitioners need to be aware of the limitations of the indicators and not have unrealistic expectations of their effectiveness.

Future Work

The results of this effort have identified future research opportunities in tipping point studies. There is still a need for further theoretical work to quantify and formulate the strength of feedback loops in a practical and accessible way. This quantification will be beneficial for both tipping point types developed here. Tracking the strength of the dominant feedback loop compared to the competing loops can predict a future change in the dominance (tipping point type I). Also, comparing the feedback strength when approaching different existing attractors in type II tipping points can be used to develop new tipping point indicators.

There is ambiguity in the literature whether the state variable or the tipping variable(s) should be perturbed to test the hypotheses (van Nes and Scheffer 2007). This study disturbed the state variable to measure "critical slowing down". Future work can disturb other variables and compare the results to this work. In addition, the choice of state and control variables might affect the results and should be studied further. These investigations will move the research closer to operationalizing tipping point indicators.

This work has applied a model-based approach to test tipping point indicators. To further this work, these indicators should be tested with real projects data. This test could determine if the effects of noise are as observed in this dissertation. It can also determine if in a real project, the indicator can be recognized early enough to turn the project around.

Finally, the relationship between the indicators studied here and resilience of systems and its measures is another area that needs further investigation. These investigations can be used to rank the levels of the fragility of the systems as suggested by Scheffer et al. (2012). Understanding tipping point dynamics and its relationship to stability and resilience of system will be beneficial for system designers. This knowledge can be utilized to design robust systems that are resilient to unexpected changes.

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APPENDIX A

MODEL EQUATIONS

Limits to Growth Archetype

Model Equations

(1) birth rate=(noise+fractional change rate)*Population
(2) carrying capacity=10
(3) death rate=(noise+fractional change rate)*Population*Population/carrying capacity
(4) fractional change rate=[-1,1, step=0.1]
(5) noise=0
(6) Population= INTEG (birth rate+pulse rate-death rate,1)

Dynamic Variables

noise=0.2*fractional change rate*RANDOM UNIFORM(-1, 1, 0)

Perturbation Parameters

pulse rate=pulse switch*pulse size*PULSE(pulse time, pulse duration)

pulse time=100

pulse duration=TIME STEP*4

pulse size=[-5,5, step=2]

Simulation Control Parameters

FINAL TIME = 200

INITIAL TIME = 0

SAVEPER = TIME STEP

TIME STEP = 0.25

Fixes that Fail Archetype

Model Equations

(1) fix=fractional fix rate*Problem Symptom
(2) fractional consequences rate= [0.1,1.9, step=0.1]
(3) fractional fix rate=2
(4) long term consequences=fractional consequences rate*DELAY1(fix, 1)
(5) Problem Symptom= INTEG (long term consequences+pulse rate-fix,50)

Dynamic Variables

fractional fix rate=2+0.5*2*RANDOM UNIFORM(-1, 1, 1)

Perturbation Parameters

pulse rate=pulse switch*pulse size*PULSE(pulse time, pulse duration)
pulse time=100
pulse duration=TIME STEP*4
pulse size=[-10,10, step=2]

Simulation Control Parameters

FINAL TIME = 200
INITIAL TIME = 0
SAVEPER = TIME STEP
TIME STEP = 0.25

Reinforcing Loop Archetype

Model Equations

(1) Current State= INTEG (flow+pulse rate, 5)
(2) flow=Current State * (fractional change rate+noise)
(3) fractional change rate=[-1,1, step=0.1]

Dynamic Variables

Noise= 0.2*fractional change rate*RANDOM UNIFORM(-1,1,0)

Perturbation Parameters

pulse rate=pulse switch*pulse size*PULSE(pulse time, pulse duration)
pulse time=100
pulse duration=TIME STEP*4
pulse size=[-10,10, step=2]

Simulation Control Parameters

FINAL TIME = 200
INITIAL TIME = 0
SAVEPER = TIME STEP
TIME STEP = 0.25

Escalation Archetype

Model Equations

(1) A's desired advantage ratio=[0.1,1.9, step=0.1]
(2) activity by A= (desired relation by A-Results of A Relative to B)
(3) activity by B= (desired relation by B-Results of A Relative to B)
(4) A's Results= INTEG (activity by A, 20)
(5) B's desired advantage ratio=1
(6) B's Results= INTEG (activity by B, 20)
(7) Results of A Relative to B=A's Results/B's Results

Dynamic Variables

desired relation by B=1+0.2*1*RANDOM UNIFORM(-1,1,0)

Simulation Control Parameters

FINAL TIME = 50
INITIAL TIME = 0
SAVEPER = TIME STEP
TIME STEP = 0.25

Limerick Construction Project Model

Model Equations

For full model documentation see Taylor and Ford ((2006; 2008)). Some of the equations have been modified to simplify the model.

(1) Actual IC fraction= INTEG (Change IC fraction,Prop fraction of IC resource demand)
(2) Actual release productivity=ZIDZ(Work Released, Time)
(3) Actual RW fraction= INTEG (Change RW fraction, Prop fraction of RW resource demand)
(4) Approve work rate=QA rate-Discover rework rate
(5) Base rework fraction=0.3
(6) Base ripple effects strength=[0.05,1.25, step=0.05]
(7) Base sensitivity to schedule pressure=0.3
(8) Change IC fraction=(Prop fraction of IC resource demand-Actual IC fraction)/Staff Adjustment Time
(9) Change QA fraction=(Prop fraction QA resource demand-Actual QA fraction)/Staff Adjustment Time
(10) Change RW fraction=(Prop fraction of RW resource demand-Actual RW fraction)/Staff Adjustment Time
(11) Discover rework rate=Fraction discovered to require rework*QA rate
(12) Fraction discovered to require rework=MIN(1,Base rework fraction+Rework fraction due to schedule pressure)
(13) IC labor required=Total Work avail for IC/IC staff productivity
(14) IC process rate=Total Work avail for IC/Minimum IC duration
(15) IC resource rate=IC staff productivity*IC Staff
(16) IC Staff=Actual IC fraction*Total project staff
(17) IC staff productivity=1
(18) Initial Completion Backlog= INTEG (ripple effects rate-Initially complete work rate),Scope initial)
(19) Initially complete work rate=MAX(MIN(IC process rate,IC resource rate),0)
(20) Max Effective Schedule Pressure Ratio=2
(21) Minimum IC duration=1
(22) Minimum RW duration=1
(23) Minimum QA duration=1
(24) Percent complete=Work Released/(Total project work)*100
(25) Planned project duration fraction used to adjust Release Productivity=0.5
(26) Planned Release Productivity=XIDZ(Scope initial,Project deadline,1)
(27) Project deadline=75
(28) Prop fraction of IC resource demand=ZIDZ(IC labor required, Total labor required to complete available work)

(29) Prop fraction QA resource demand=ZIDZ(QA labor required, Total labor required to complete available work)
(30) Prop fraction of RW resource demand=ZIDZ(RW labor required, Total labor required to complete available work)
(31) Quality Assurance Backlog= INTEG (Initially complete work rate-Discover rework rate+Rework rate-Approve work rate,0)
(32) QA labor required=Quality Assurance Backlog/QA staff productivity
(33) QA process rate=Quality Assurance Backlog/Minimum QA duration
(34) QA resource rate=QA staff productivity*QA Staff
(35) QA rate=MIN(QA resource rate,QA process rate)
(36) QA Staff=Actual QA fraction*Total project staff
(37) QA staff productivity=1
(38) Actual QA fraction= INTEG (Change QA fraction,Prop fraction QA resource demand)
(39) Release productivity for forecasting= (((MAX(0,Time to transition to Actual Release Productivity-Time))/Time to transition to Actual Release Productivity) *Planned Release Productivity) + (MIN(1,Time/Time to transition to Actual Release Productivity))*Actual release productivity
(40) Rework Backlog= INTEG (Discover rework rate-Rework rate,0)
(41) Rework fraction due to schedule pressure=Schedule pressure switch*Base Sensitivity to schedule pressure*(Schedule Pressure Ratio-1)
(42) Rework rate=MIN(RW process rate, RW resource rate)
(43) ripple effects rate=Ripple effects switch*(Discover rework rate)*Base Ripple effects strength
(44) RW labor required=Rework Backlog/RW staff productivity
(45) RW process rate=Rework Backlog/Minimum RW duration
(46) RW resource rate=RW Staff*RW staff productivity
(47) RW Staff=Actual RW fraction*Total project staff
(48) RW staff productivity=1
(49) Scope initial=38700
(50) Schedule Pressure Ratio=MAX(1, MIN(Max Effective Schedule Pressure Ratio, XIDZ(Time required, Time available,Time required/TIME STEP)))
(51) Staff Adjustment Time=4
(52) Time available=MAX(1,Project deadline-Time)
(53) Time required=ZIDZ(Total BLWIP,Release productivity for forecasting)Time to transition to Actual Release Productivity=Project deadline*Planned project duration fraction used to adjust Release Productivity
(54) Total BLWIP=Initial Completion Backlog+Quality Assurance Backlog+Rework Backlog+pulse rate)
(55) Total labor required to complete available work=IC labor required+QA labor required+RW labor required
(56) Total project staff=2350
(57) Total project work=Total BLWIP+Work Released
(58) Total projeect work not in IC BLWIP=Total project work-Initial Completion

Backlog
(59) Total Work avail for IC=Total project work-Total projeect work not in IC BLWIP
(60) Work Released= INTEG (Approve work rate,0)

Dynamic Variables

IC staff productivity= RANDOM UNIFORM(-1,1,3)*1*0.2+1
QA staff productivity= RANDOM UNIFORM(-1,1,4)*1*0.2+1
RW staff productivity= RANDOM UNIFORM(-1,1,5)*1*0.2+1
Minimum IC duration= RANDOM UNIFORM(-1, 1,2)*1*0.2+1
Minimum QA duration= RANDOM UNIFORM(-1, 1,0)*1*0.2+1
Minimum RW duration= RANDOM UNIFORM(-1, 1,1)*1*0.2+1

Perturbation Parameters

pulse rate=pulse size*pulse switch*PULSE(pulse time, pulse duration)
pulse time=100
pulse duration=TIME STEP*4
pulse size=[-10000,10000, step=2000]

Simulation Control Parameters

FINAL TIME = 500
INITIAL TIME = 0
SAVEPER = TIME STEP
TIME STEP = 0.25

Fish Banks Model

Model Equations

(1) area=100
(2) carrying capacity=1200
(3) catch per ship=table 2(density)

(4) death fraction=table(Fish/carrying capacity)
(5) density=Fish/area
(6) Fish= INTEG (fish hatch rate+pulse rate-fish death rate-total catch per year, 20)
(7) fish death rate=Fish*death fraction
(8) fish hatch rate=Fish*hatch fraction
(9) fish price=10
(10) fraction invested=0.2
(11) hatch fraction=[0.5,9.5, step=0.5]
(12) operating costs=Ships*unit operating costs
(13) revenues=fish price*total catch per year
(14) Ships= INTEG (ship building rate, 10)
(15) ship building rate=fraction invested*yearly profits/ship cost
(16) ship cost=300
(17) table([(0,0)-(2,15)], (0,5.22), (0.2,5.23), (0.4,5.255), (0.6,5.345), (0.8,5.665), (1,6), (1.2,6.44), (1.4,7.13), (1.6,7.97), (1.8,9.32), (2,11))
(18) table 2([(0,0)-(10,40)], (0,0), (1,5), (2,10.4), (3,15.9), (4,20.2), (5,22.1), (6,23.2), (7,23.8), (8,24.2), (9,24.6), (10,25))
(19) total catch per year= catch per ship*Ships
(20) unit operating costs=250
(21) yearly profits=revenues-operating costs

Dynamic Variables

area=100+0.2*100*RANDOM UNIFORM(-1, 1, 3)

fish price=10+0.2*10*RANDOM UNIFORM(-1, 1, 0)

ship cost=300+0.2*300*RANDOM UNIFORM(-1, 1,1)

unit operating costs=250+0.2*250*RANDOM UNIFORM(-1, 1, 0)

Perturbation Parameters

pulse rate=pulse size*pulse switch*PULSE(pulse time, pulse duration)

pulse time=50

pulse duration=TIME STEP

pulse size=[-100,100, step=20]

Simulation Control Parameters

FINAL TIME = 100

INITIAL TIME = 0

SAVEPER = TIME STEP*4

TIME STEP = 0.25

Arms Race Model

Model Equations

(1) Total Arms A= INTEG (armament spending A + pulse rate-armament obsolescence A, initial armament A) /dependent variable/
(2) initial armament A=50
(3) armament capacity A=Economic Capacity A * max capacity to armament A
(4) armament capacity B=Economic Capacity B * max capacity to armament B
(5) armament lifetime A=20
(6) armament lifetime B=20
(7) armament obsolescence A=Total Arms A/armament lifetime A
(8) armament obsolescence B=Total Arms B/armament lifetime B
(9) armament spending A=armament capacity A * fraction armament capacity used A
(10) armament spending B=armament capacity B * fraction armament capacity used B
(11) capacity degradation A=Economic Capacity A/capacity lifetime A
(12) capacity degradation B=Economic Capacity B/capacity lifetime B
(13) capacity lifetime A=30
(14) capacity lifetime B=30
(15) desired strength ratio A=[0.1,1.9, step=0.1]
(16) desired strength ratio B=1
(17) Economic Capacity A= INTEG (growth in capacity A - capacity degradation A, initial economic capacity A)
(18) Economic Capacity B= INTEG (growth in capacity B - capacity degradation B, initial economic capacity B)
(19) fraction armament capacity used A= WITH LOOKUP (ZIDZ(indicated armament building A,armament capacity A), ([(0,0)-(10,1), (0,0), (1,1), (10,1)], (0,0), (0.4,0.4), (2,0.8), (3,0.9), (5,1), (10,1)))
(20) fraction armament capacity used B= WITH LOOKUP (ZIDZ(indicated armament building B,armament capacity B), ([(0,0)-(10,1), (0,0), (1,1), (10,1)], (0,0), (0.4,0.4), (2,0.8), (3,0.9), (5,1), (10,1)))
(21) fraction spending to investment A=0.3
(22) fraction spending to investment B=0.3
(23) growth in capacity A=investment spending A * investment effectiveness A
(24) growth in capacity B=investment spending B * investment effectiveness B
(25) indicated armament building A=MAX(0,armament obsolescence A + (target armament A - Total Arms A)/time to correct armament A)
(26) indicated armament building B=MAX(0,armament obsolescence B + (target armament B Total Arms B)/time to correct armament B)

(27) initial armament B=50
(28) initial economic capacity A=100
(29) initial economic capacity B=100
(30) investment effectiveness A=0.15
(31) investment effectiveness B=0.15
(32) investment spending A= non armament spending A * fraction spending to investment A
(33) investment spending B=non armament spending B * fraction spending to investment B
(34) max capacity to armament A=0.4
(35) max capacity to armament B=0.4
(36) non armament spending A=Economic Capacity A - armament spending A
(37) non armament spending B=Economic Capacity B - armament spending B
(38) target armament A=Total Arms B * desired strength ratio A
(39) target armament B=Total Arms A * desired strength ratio B
(40) time to correct armament A=5
(41) time to correct armament B=5
(42) Total Arms B= INTEG (armament spending B-armament obsolescence B, initial armament B)

Dynamic Variables

armament lifetime A= 20+0.2*20*RANDOM UNIFORM(-1,1,12)

armament lifetime B= 20+0.2*20*RANDOM UNIFORM(-1, 1, 11)

capacity lifetime A=0.2*30*RANDOM UNIFORM(-1, 1,0)+30

capacity lifetime B=0.2*30*RANDOM UNIFORM(-1, 1,10)+30

fraction spending to investment A=0.3+0.2*0.3*RANDOM UNIFORM(-1, 1,3)

fraction spending to investment B=0.3+0.2*0.3*RANDOM UNIFORM(-1, 1,8)

initial economic capacity A=100+0.2*100*RANDOM UNIFORM(-1, 1,1)

initial economic capacity B= 100+0.2*100*RANDOM UNIFORM(-1, 1,6)

investment effectiveness A=0.15+0.2*0.15*RANDOM UNIFORM(-1, 1, 2)

investment effectiveness B=0.15+0.2*0.15*RANDOM UNIFORM(-1, 1,7)

time to correct armament A=5+0.2*5*RANDOM UNIFORM(-1, 1, 5)

time to correct armament B=5+0.2*5*RANDOM UNIFORM(-1,1,13)

time to correct armament A=5+0.2*5*RANDOM UNIFORM(-1, 1, 5)

time to correct armament B=5+0.2*5*RANDOM UNIFORM(-1,1,13)

Simulation Control Parameters

FINAL TIME = 700
INITIAL TIME = 0
SAVEPER = TIME STEP
TIME STEP = 0.25

Social Impact Bonds Model

Model Equations

(1) "Cohort Active?"[Cohort 1]=IF THEN ELSE(Actual Cohort Start Times[Cohort 1]<Time,IF THEN ELSE(Actual Cohort Closing Times[Cohort 1]>=Time,1,0),0)
(2) "Cohort Active?"[Cohort 2]=IF THEN ELSE(MIN(Actual Cohort Closing Times[Cohort 1],Actual Cohort Start Times[Cohort 2])<Time,IF THEN ELSE(Actual Cohort Closing Times[Cohort 2]>=Time,1,0),0)
(3) "Cohort Active?"[Cohort 3]= IF THEN ELSE(MIN(Actual Cohort Closing Times[Cohort 2],Actual Cohort Start Times[Cohort 3])<Time,IF THEN ELSE(Actual Cohort Closing Times[Cohort 3]>=Time,1,0),0)
(4) "Cohort Active?"[Cohort 4]=IF THEN ELSE(Actual Cohort Start Times[Cohort 4]<Time,IF THEN ELSE(Actual Cohort Closing Times[Cohort 4]>=Time,1,0),0)
(5) "Cohort Active?"[Overall]=1
(6) "Investors Cohorts Paid? Amount"[Cohorts]=Owner Success Payment*"Payment Consideration?"[Cohorts]
(7) "Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,Release Program]=Program Entry Rate[Cohorts,Prison Systems,Release Program]*Program Recidivism Fraction[Release Program]
(8) "Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohorts,Prison Systems,Release Program]=Repeat Offender Program Entry Rate[Cohorts,Prison Systems,Release Program]*Program Recidivism Fraction[Release Program]
(9) "Payment Consideration?"[Cohort 1]=IF THEN ELSE(Cohort Payment Time[Cohort 1]<=Time,IF THEN ELSE(Cohort Payment Time[Cohort 1]+Time Program has to Pay>=Time,1,0),0)
(10) "Payment Consideration?"[Cohort 2]=IF THEN ELSE(Cohort Payment Time[Cohort 2]<=Time,IF THEN ELSE(Cohort Payment Time[Cohort 2]+Time Program has to Pay>=Time,1,0),0)
(11) "Payment Consideration?"[Cohort 3]=IF THEN ELSE(Cohort Payment Time[Cohort 3]<=Time,IF THEN ELSE(Cohort Payment Time[Cohort 3]+Time Program has to Pay>=Time,1,0),0)
(12) "Payment Consideration?"[Cohort 4]=IF THEN ELSE(Cohort Payment Time[Cohort 1]<=Time,IF THEN ELSE(Cohort Payment Time[Cohort 4]+Time Program has to Pay>=Time,1,0),0)
(13) "Payment Consideration?"[Overall]=1

(14) "Post-SIB Program Payments"=Owner Program Payments
(15) "Prison Holding-First Offence"[Prison Systems]= INTEG (New Offender Incarceration Rate[Prison Systems]-SUM("Rehabilitated Prisoner Release Rate-First Offence"[Overall,Prison Systems,Release Program!])-SUM("Non-Rehabilitated Prisoner Release Rate- First Offence"[Overall ,Prison Systems,Release Program!])+SUM(pulse rate[Overall,Prison Systems,Release Program!]),Initial First time Offender Prisoner Holding*Offender Multiplier between Prison Systems[Prison Systems])
(16) "Prison Holding-Repeat Offence"[Cohort 1,Prison Systems]= INTEG (SUM(Repeat Offender Arrest Rate[Cohort 1,Prison Systems,Release Program!]-"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 1,Prison Systems,Release Program!]-"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohort 1,Prison Systems,
(17) "Prison Holding-Repeat Offence"[Cohort 2,Prison Systems]= INTEG (SUM(Repeat Offender Arrest Rate[Cohort 2,Prison Systems,Release Program!]-"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 2,Prison Systems,Release Program!]-"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohort 2,Prison Systems,
(18) "Prison Holding-Repeat Offence"[Cohort 3,Prison Systems]= INTEG (SUM(Repeat Offender Arrest Rate[Cohort 3,Prison Systems,Release Program!]-"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 3,Prison Systems,Release Program!]-"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohort 3,Prison Systems,
(19) "Prison Holding-Repeat Offence"[Cohort 4,Prison Systems]= INTEG (SUM(Repeat Offender Arrest Rate[Cohort 4,Prison Systems,Release Program!]-"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohort 4,Prison Systems,Release Program!]-"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 4,Prison Systems
(20) "Prison Holding-Repeat Offence"[Overall,Prison Systems]= INTEG (SUM(Repeat Offender Arrest Rate[Overall,Prison Systems,Release Program!]-"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Overall,Prison Systems,Release Program!]-"Rehabilitated Prisoner Release Rate-Repeat Offence"[Overall,Prison Systems,Release Program!])+SUM(pulse rate 1[Overall,Prison Systems,Release Program!]),Initial Reconvicted Prisoner Holding*Offender Multiplier between Prison Systems[Prison Systems])
(21) "Program Continues?"=max("Cohort Active?"[Cohort 4], "Successful Program?"[Cohort 4])
(22) "Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,Release Program]=Program Entry Rate[Cohorts,Prison Systems,Release Program]*(1-Program Recidivism Fraction[Release Program])
(23) "Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohorts,Prison Systems,Release Program]=Repeat Offender Program Entry Rate[Cohorts,Prison Systems,Release Program]*(1-Program Recidivism Fraction[Release Program])
(24) "Successful Program?"[Cohort 1]=IF THEN ELSE(Reconviction Change[Cohort

1]>=Minimum Change in Reconvictions[Cohort 1],1,0

(25) "Successful Program?"[Cohort 2]=IF THEN ELSE(Reconviction Change[Cohort 2]>=Minimum Change in Reconvictions[Cohort 2],1,0)

(26) "Successful Program?"[Cohort 3]=IF THEN ELSE(Reconviction Change[Cohort 3]>=Minimum Change in Reconvictions[Cohort 3],1,0)

(27) "Successful Program?"[Cohort 4]=IF THEN ELSE(Reconviction Change[Cohort 4]>=Minimum Change in Reconvictions[Cohort 4],1,0)

(28) "Successful Program?"[Overall]=IF THEN ELSE(Reconviction Change[Overall]>=Minimum Change in Reconvictions[Overall],1,0)

(29) "Surplus Returned?"=IF THEN ELSE(Investment Finalization Time[Cohort 4]<Time,1,0)

(30) ,Release Program!]),0)

(31) [Cohort 2,Prison Systems,Normal]-Repeat Offender Arrest Rate[Cohort 2,Prison Systems,Normal],0)

(32) [Cohorts,Prison Systems,Release Program!]))*"Cohort Active?"[Cohorts]

(33) [Overall,Prison Systems,Normal]-Repeat Offender Arrest Rate[Overall,Prison Systems,Normal],Initial Undiscovered Reoffenders*Offender Multiplier between Prison Systems[Prison Systems])

(34) Accumulated Reconvictions Prevented Percent[Cohorts]=ZIDZ((Number of Reconviction[Cohorts,Comparison]/Offender Multiplier between Prison Systems[Comparison]-Number of Reconviction[Cohorts,Peterborough]/Offender Multiplier between Prison Systems[Peterborough]),(Number of Reconviction[Cohorts,Comparison]/Offender Multiplier between Prison Systems[Comparison]))*100

(35) Actual Cohort Closing Times[Cohorts]= INTEG (Cohort Time[Cohorts],0)

(36) Actual Cohort Start Times[Cohorts]= INTEG (Cohort Time Delay Rate[Cohorts],0)

(37) Amount Investors are willing to Invest= INTEG (-Investing,Expected Program Investment amount)

(38) Available Capacity=XIDZ(Funds Available for the Program,Program Cost per Member,99999)

(39) Average Prisoner Incarceration Rate for Peterborough=57.8333

(40) Average Time between Release and Reconviction[Prison Systems]=Normal Recidivism Discovery Time

(41) Budget Excess Recovered="Surplus Returned?"* (Budget Excess/MOJ Surplus return check*Fraction of Surplus Returned-MOJ SIB Savings Rate)

(42) Budget Excess= INTEG (Budget Surplus-Budget Excess Recovered-MOJ SIB Savings Rate-"Post-SIB Program Payments",0)

(43) Budget Peterborough=(Total Prison Population[Comparison]*monthly cost+Conviction Cost*SUM(Repeat Offender Arrest Rate[Overall,Comparison,Release Program!]))/Offender Multiplier between Prison Systems[Comparison]

(44) Budget Surplus=Budget Peterborough-Peterborough Operations Costs

(45) cashflow=Investor Return Rate-Investment Rate

(46) Change in Max Program Costs=max(0,Program Costs-Max Program Costs)/Time to Adjust Maximum Program Cost

(47) Cohort Addition of New Offenders[Cohorts,Prison Systems]=SUM(("Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,Release Program!]+"Non-Rehabilitated Prisoner Release Rate- First Offence"

(48) Cohort Collection of New Offenders[Cohorts,Prison Systems]= INTEG (Cohort Addition of New Offenders[Cohorts,Prison Systems],0)

(49) Cohort Expense Rate=Program Costs

(50) Cohort initiate switch[Cohort 1]=IF THEN ELSE(Cohort Collection of New Offenders[Cohort 1,Peterborough]<= Max People in Cohort[Cohort 1] , IF THEN ELSE((Actual Cohort Start Times[Cohort 1]+Max Cohort Length[Cohort 1]) >Time, 1 , 0), 0)

(51) Cohort initiate switch[Cohort 2]=IF THEN ELSE(Cohort Collection of New Offenders[Cohort 2,Peterborough]<= Max People in Cohort[Cohort 2] , IF THEN ELSE((Actual Cohort Start Times[Cohort 2]+Max Cohort Length[Cohort 2]) >Time, 1 , 0), 0)

(52) Cohort initiate switch[Cohort 3]=IF THEN ELSE(Cohort Collection of New Offenders[Cohort 3,Peterborough]<= Max People in Cohort[Cohort 3] , IF THEN ELSE((Actual Cohort Start Times[Cohort 3]+Max Cohort Length[Cohort 3]) >Time, 1 , 0), 0)

(53) Cohort initiate switch[Cohort 4]=IF THEN ELSE(Cohort Collection of New Offenders[Cohort 3,Peterborough]<=Max People in Cohort [Cohort 3],IF THEN ELSE(Actual Cohort Start Times[Cohort 4]+3*Max Cohort Length[Cohort 4]>Time,1,0),0)

(54) Cohort initiate switch[Overall]=1

(55) Cohort Investment switch[Cohorts]=IF THEN ELSE(Actual Cohort Start Times[Cohorts]<Time, IF THEN ELSE(Investment Finalization Time[Cohorts]>=Time, 1,0),0)

(56) Cohort Payment Time[Cohort 1]=Actual Cohort Closing Times[Cohort 1]+Time after Cohort for Reconviction Collection+Time for System Update

(57) Cohort Payment Time[Cohort 2]=Actual Cohort Closing Times[Cohort 2]+Time after Cohort for Reconviction Collection+Time for System Update

(58) Cohort Payment Time[Cohort 3]=Actual Cohort Closing Times[Cohort 3]+Time after Cohort for Reconviction Collection+Time for System Update

(59) Cohort Payment Time[Cohort 4]=Actual Cohort Closing Times[Cohort 4]+Time after Cohort for Reconviction Collection+Time for System Update+ Time Program has to Pay

(60) Cohort Payment Time[Overall]=0

(61) Cohort Time Delay Rate[Cohort 1]=IF THEN ELSE(Max People in Cohort[Cohort 1]>Cohort Collection of New Offenders[Cohort 1,Peterborough],IF THEN ELSE(Latest Cohort Start Times[Cohort 1]>Time,1,0),0)*Cohort initiate switch[Cohort 1]

(62) Cohort Time Delay Rate[Cohort 2]=IF THEN ELSE(Max People in

Cohort[Cohort 2]>Cohort Collection of New Offenders[Cohort 2,Peterborough],IF THEN ELSE(Latest Cohort Start Times[Cohort 2]>Time,1,0),0)*Cohort initiate switch[Cohort 1]

(63) Cohort Time Delay Rate[Cohort 3]=IF THEN ELSE(Max People in Cohort[Cohort 3]>Cohort Collection of New Offenders[Cohort 3,Peterborough],IF THEN ELSE(Latest Cohort Start Times[Cohort 3]>Time,1,0),0)*Cohort initiate switch[Cohort 2]

(64) Cohort Time Delay Rate[Cohort 4]=IF THEN ELSE(Max People in Cohort[Cohort 4]>Cohort Collection of New Offenders[Cohort 4,Peterborough],IF THEN ELSE(Latest Cohort Start Times[Cohort 4]>Time,1,0),0)

(65) Cohort Time Delay Rate[Overall]=0

(66) Cohort Time[Cohorts]=Cohort initiate switch[Cohorts]

(67) Cohorts:Overall, Cohort 1, Cohort 2, Cohort 3, Cohort 4

(68) Conviction Cost=2.853

(69) Cost per Participant per month=Program Cost per Member/Defined Year

(70) Cummulative Investor Returns= INTEG (Owner Success Payment, No Initial Worth)

(71) Cummulative Program Cost= INTEG (Program Costs,No Initial Worth)

(72) Defined Year=12

(73) Desired Payment Value[Cohorts]=Unpaid Investor Earned Returns[Cohorts]*"Payment Consideration?"[Cohorts]*"Successful Program?"[Cohorts]

(74) Earned Investor Return payments[Cohorts]= "Investors Cohorts Paid? Amount"[Cohorts]

(75) Expected Program Investment amount=5000

(76) Fraction for Investor share=0.3

(77) Fraction of Surplus Returned= [0.05,0.95, step=0.05]

(78) Funds Available for the Program= INTEG (MOJ Payments-Cohort Expense Rate,Expected Program Investment amount)

(79) Funds Earned by SIB= INTEG (MOJ SIB Savings Rate-MOJ SIB Payment Rate,0)

(80) Initial First time Offender Prisoner Holding=86.75

(81) Initial Reconvicted Prisoner Holding=260.25

(82) Initial Undiscovered Reoffenders=1041

(83) Interval time between Cohort completion and Payment Time[Cohort 1]=Time after Cohort for Reconviction Collection+Time for System Update+Time Program has to Pay|

(84) Interval time between Cohort completion and Payment Time[Cohort 2]=Time after Cohort for Reconviction Collection+Time for System Update+Time Program has to Pay

(85) Interval time between Cohort completion and Payment Time[Cohort 3]=Time after Cohort for Reconviction Collection+Time for System Update+Time Program has to Pay

(86) Interval time between Cohort completion and Payment Time[Cohort 4]=Time

Program has to Pay

(87) Interval time between Cohort completion and Payment Time[Overall]=0

(88) INV IRR=IF THEN ELSE(Investment Starts=0,0,IF THEN ELSE(Investment Return starts=0,-1,IRR))

(89) Invested Amount= INTEG (Investment Rate,0)

(90) Investing=PULSE(Actual Cohort Start Times[Cohort 1]+1,1)*Investment Payment per Cohort+PULSE(Actual Cohort Start Times[Cohort 2]+1,1)*Investment Payment per Cohort+PULSE(Actual Cohort Start Times[Cohort 3]+1,1)*Investment Payment per Cohort

(91) Investment Finalization Time[Cohort 1]=Actual Cohort Closing Times[Cohort 1]

(92) Investment Finalization Time[Cohort 2]=Actual Cohort Closing Times[Cohort 2]

(93) Investment Finalization Time[Cohort 3]=Actual Cohort Closing Times[Cohort 3]

(94) Investment Finalization Time[Cohort 4]=Cohort Payment Time[Cohort 4]+Interval time between Cohort completion and Payment Time[Cohort 4]

(95) Investment Finalization Time[Overall]=Actual Cohort Closing Times[Overall]

(96) Investment Payment per Cohort=Expected Program Investment amount/number of payments

(97) Investment Rate=(Investing)

(98) Investment Return starts=IF THEN ELSE(Cummulative Investor Returns>0,1,0)

(99) Investment Starts=IF THEN ELSE(Invested Amount>0,1,0)

(100) Investor Account= INTEG (Investor Return Rate-Investment Rate, 0)

(101) Investor Balance=Cummulative Investor Returns+Amount Investors are willing to Invest

(102) Investor Earned Return Accumulation[Cohorts]=Investor Earned[Cohorts]*Cohort Investment switch[Cohorts]

(103) Investor Earned[Cohorts]=(Savings from Reduced Reconvictions[Cohorts])*Fraction for Investor share

(104) Investor Return Rate=Owner Success Payment

(105) IRR=INTERNAL RATE OF RETURN(cashflow,Defined Year,0,0)

(106) Latest Cohort Start Times[Cohorts]=-0.5,20,44,68,20

(107) Max Cohort Length[Cohort 1]=Standard Cohort Length

(108) Max Cohort Length[Cohort 2]=Standard Cohort Length

(109) Max Cohort Length[Cohort 3]=Standard Cohort Length

(110) Max Cohort Length[Cohort 4]=Max Cohort Length[Cohort 1]+Max Cohort Length[Cohort 2]+Max Cohort Length[Cohort 3]

(111) Max Cohort Length[Overall]=0

(112) Max People in Cohort[Cohort 1]=Standard People in Cohort

(113) Max People in Cohort[Cohort 2]=Standard People in Cohort

(114) Max People in Cohort[Cohort 3]=Standard People in Cohort

(115) Max People in Cohort[Cohort 4]=Max People in Cohort[Cohort 1]+Max People in Cohort[Cohort 2]+Max People in Cohort[Cohort 3]

(116) Max People in Cohort[Overall]=1e+015

(117) Max Program Costs= INTEG (Change in Max Program Costs,0)

(118) Maximum Payment Accumulation[Cohorts]=IF THEN ELSE(Cohort Payment

Time[Cohorts]<Time,IF THEN ELSE(Desired Payment Value[Cohorts]>Maximum Successful Program Payment Value[Cohorts],Desired Payment Value[Cohorts]-Maximum Successful Program Payment Value[Cohorts],0)/TIME STEP,0)

(119) Maximum Successful Program Payment Value[Cohorts]= INTEG (Maximum Payment Accumulation[Cohorts],0)

(120) Minimum Change in Reconvictions[Cohorts]=1,0.1,0.1,0.1,0.075

(121) MOJ Account= INTEG (Budget Peterborough-Budget Surplus-Peterborough Operations Costs,0)

(122) MOJ Payments=Owner Program Payments

(123) MOJ SIB Payment Rate=Owner Success Payment

(124) MOJ SIB Savings Rate=Investor Earned Return Accumulation[Cohort 4]

(125) MOJ Starts Paying=73

(126) MOJ Surplus return check=1

(127) monthly cost=Yearly Cost of Incarceration/Defined Year

(128) New Offender Incarceration Rate[Comparison]=Offender Multiplier between Prison Systems[Comparison]*New Offender Incarceration Rate[Peterborough]

(129) New Offender Incarceration Rate[Peterborough]=Average Prisoner Incarceration Rate for Peterborough*Offender Multiplier between Prison Systems[Peterborough]

(130) No Initial Worth=0

(131) Normal Recidivism Discovery Time=6

(132) Normal Recidivism Fraction=0.75

(133) number of payments=3

(134) Number of Reconviction[Cohorts,Prison Systems]= INTEG (Rate of Reconviction[Cohorts,Prison Systems],0)

(135) Offender Multiplier between Prison Systems[Prison Systems]=1,10

(136) Offender Release Rate[Cohorts,Prison Systems,Release Program]= "Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,Release Program]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohorts,Prison Systems,Release Program]+"Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,Release Program]+"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohorts,Prison Systems,Release Program]

(137) Offenders entering Program[Cohorts]="Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Peterborough,SIB]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohorts,Peterborough,SIB]+"Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Peterborough,SIB]+"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohorts,Peterborough,SIB]

(138) Offenders' Willing to enter program Fraction=0.7

(139) Offense:First, Repeat

(140) One Service= INTEG (Program Payments from SIP-Program Costs,No Initial Worth)

(141) Owner Program Payments=max(IF THEN ELSE(MOJ Starts Paying<Time, (Program Payment Rate+0*Program Payments from SIP)*("Successful

Program?"[Cohort 4]),0),0)
(142) Owner Success Payment=(Payment for Cohort 1+Payment for Cohort 2+Payment for Cohort 3+Payment for Cohort 4)/Time Program has to Pay
(143) paying early=IF THEN ELSE(Actual Cohort Closing Times[Cohort 4]<Time,0,MOJ Payments)
(144) Payment for Cohort 1=Maximum Successful Program Payment Value[Cohort 1]*"Payment Consideration?"[Cohort 1]
(145) Payment for Cohort 2=Maximum Successful Program Payment Value[Cohort 2]*"Payment Consideration?"[Cohort 2]
(146) Payment for Cohort 3=Maximum Successful Program Payment Value[Cohort 3]*"Payment Consideration?"[Cohort 3]
(147) Payment for Cohort 4=Maximum Successful Program Payment Value[Cohort 4]*"Payment Consideration?"[Cohort 4]
(148) Peterborough Operations Costs=Total Prison Population[Peterborough]*monthly cost+SUM(Repeat Offender Arrest Rate[Overall,Peterborough,Release Program!])*Conviction Cost
(149) Population in Program[Cohorts]=Rehabilitated Offenders[Cohorts,Peterborough,SIB]+Undiscoverd Reoffenders[Cohorts,Peterborough,SIB]
(150) Prison Population Percent Reduction=(1-(Total Prison Population[Peterborough]/Offender Multiplier between Prison Systems[Peterborough])/(Total Prison Population[Comparison]/Offender Multiplier between Prison Systems[Comparison]))*100
(151) Prison Systems:Peterborough, Comparison
(152) Probationary Period Length=12
(153) Program Cost per Member=1.5
(154) Program Costs=Population in Program[Overall]*Cost per Participant per month
(155) Program Entry Rate[Cohorts,Prison Systems,Normal]="Prison Holding-First Offence"[Prison Systems]/Time spent in prison for a Conviction*"Cohort Active?"[Cohorts]-Program Entry Rate[Cohorts,Prison Systems,SIB]
(156) Program Entry Rate[Cohorts,Prison Systems,SIB]=MIN("Prison Holding-First Offence"[Prison Systems]*Offenders' Willing to enter program Fraction,max(Space Available in Program,0))/Time spent in prison for a Conviction*"Cohort Active?"[Cohorts]*Program Switch[Prison Systems]
(157) Program Payment Rate=MIN(Budget Excess/Time MOJ Backlog Payments,Max Program Costs)
(158) Program Payments from SIP=Program Costs
(159) Program Recidivism Fraction[Normal]=Normal Recidivism Fraction
(160) Program Recidivism Fraction[SIB]=Normal Recidivism Fraction-Recidivism Reduction Fraction*Normal Recidivism Fraction*"Program Continues?"
(161) Program Savings Collected by Government= INTEG (Budget Excess Recovered,0)
(162) Program Switch[Peterborough]=IF THEN ELSE(Time>=20,1,0)
(163) Rate of Offenders Coming off of Probation[Cohorts,Prison Systems,Release

Program]=

(164) Rate of Reconviction[Cohorts,Prison Systems]=SUM(Repeat Offender Arrest Rate[Cohorts,Prison Systems,Release Program!])*Reconviction Termination[Cohorts]

(165) Recidivism Reduction Fraction=0.25

(166) Reconviction Change[Cohorts]=1-XIDZ(Reconviction Fraction[Cohorts,Peterborough] ,Reconviction Fraction[Cohorts,Comparison],1)

(167) Reconviction Fraction[Cohorts,Prison Systems]=ZIDZ(Number of Reconviction[Cohorts,Prison Systems],Cohort Collection of New Offenders[Cohorts,Prison Systems])

(168) Reconviction Termination[Cohorts]= IF THEN ELSE(Actual Cohort Closing Times[Cohorts]+Time after Cohort for Reconviction Collection<Time,0,1)

(169) Reconvictions Prevented[Cohorts]=Rate of Reconviction[Cohorts,Comparison] /Offender Multiplier between Prison Systems[Comparison]-Rate of Reconviction[Cohorts,Peterborough]/Offender Multiplier between Prison Systems[Peterborough]

a. Rehabilitated Offenders[Cohorts,Prison Systems,Release Program]/Probationary Period Length

(170) Rehabilitated Offenders[Cohorts,Prison Systems,Release Program]= INTEG ("Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,Release Program]+"Rehabilitated Prisoner Release Rate-Repeat Offence"[Cohorts,Prison Systems,Release Program]-Rate of Offenders Coming off of Probation[Cohorts,Prison Systems,Release Program],0)

(171) Release Program!]),0)

(172) Release Program!]),0)

(173) Release Program!]),0)

(174) Release Program:Normal, SIB

(175) Repeat Offender Arrest Rate[Cohorts,Prison Systems,Release Program]=(Undiscoverd Reoffenders[Cohorts,Prison Systems,Release Program])/Average Time between Release and Reconviction[Prison Systems]

(176) Repeat Offender Program Entry Rate[Cohort 1,Prison Systems,SIB]=MIN("Prison Holding-Repeat Offence"[Cohort 1,Prison Systems]*Offenders' Willing to enter program Fraction/Time spent in prison for a Conviction,max(Space Available in Program,0)/Time spent in prison for a Conviction-Program Entry Rate[Cohort 1,Prison Systems,SIB])*Program Switch[Prison Systems]

(177) Repeat Offender Program Entry Rate[Cohort 2,Prison Systems,SIB]=MIN("Prison Holding-Repeat Offence"[Cohort 2,Prison Systems]*Offenders' Willing to enter program Fraction/Time spent in prison for a Conviction,max(Space Available in Program,0)/Time spent in prison for a Conviction-Program Entry Rate[Cohort 2,Prison Systems,SIB]-Repeat Offender Program Entry Rate[Cohort 1,Prison Systems,SIB])*Program Switch[Prison Systems]

(178) Repeat Offender Program Entry Rate[Cohort 3,Prison Systems,SIB]=MIN("Prison Holding-Repeat Offence"[Cohort 3,Prison Systems]*Offenders' Willing to enter program Fraction/Time spent in prison for a Conviction,max(Space Available in

Program,0)/Time spent in prison for a Conviction-Program Entry Rate[Cohort 3,Prison Systems,SIB]-Repeat Offender Program Entry Rate[Cohort 1,Prison Systems,SIB]-Repeat Offender Program Entry Rate[Cohort 2,Prison Systems,SIB])*Program Switch[Prison Systems]

(179) Repeat Offender Program Entry Rate[Cohort 4,Prison Systems,SIB]=(Repeat Offender Program Entry Rate[Cohort 1,Prison Systems,SIB]+Repeat Offender Program Entry Rate[Cohort 2,Prison Systems,SIB]+Repeat Offender Program Entry Rate[Cohort 3,Prison Systems,SIB])*Program Switch[Prison Systems]

(180) Repeat Offender Program Entry Rate[Cohorts,Prison Systems,Normal]="Prison Holding-Repeat Offence"[Cohorts,Prison Systems]/Time spent in prison for a Conviction-Repeat Offender Program Entry Rate[Cohorts,Prison Systems,SIB]

(181) Repeat Offender Program Entry Rate[Overall,Prison Systems,SIB]=MIN("Prison Holding-Repeat Offence"[Overall,Prison Systems]*Offenders' Willing to enter program Fraction/Time spent in prison for a Conviction, max(Space Available in Program,0)/Time spent in prison for a Conviction-Program Entry Rate[Overall,Prison Systems,SIB] -Repeat Offender Program Entry Rate [Cohort 4,Prison Systems,SIB])*Program Switch[Prison Systems]

(182) Savings from Reduced Reconvictions[Cohorts]=Total Cost of Incarceration *(Reconvictions Prevented[Cohorts])

(183) Social Impact Partnership= INTEG (nvesting+Owner Program Payments-Program Payments from SIP,No Initial Worth)

(184) Space Available in Program=Available Capacity-Population in Program[Overall]

(185) Standard Cohort Length=24

(186) Standard People in Cohort=1000

(187) Time after Cohort for Reconviction Collection=12

(188) Time for System Update=6

(189) Time MOJ Backlog Payments=12

(190) Time Program has to Pay=3

(191) Time spent in prison for a Conviction=1.5

(192) Time to Adjust Maximum Program Cost=1

(193) Total Cost of Incarceration=monthly cost*Time spent in prison for a Conviction+Conviction Cost

(194) Total Prison Population[Prison Systems]="Prison Holding-First Offence"[Prison Systems]+"Prison Holding-Repeat Offence"[Overall,Prison Systems]

(195) Total Program Participants[Cohorts]= INTEG (Offenders entering Program[Cohorts],0)

(196) Undiscoverd Reoffenders[Cohort 1,Prison Systems,Normal]= INTEG ("Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohort 1,Prison Systems,Normal]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 1,Prison Systems,Normal]-Repeat Offender Arrest Rate[Cohort 1,Prison Systems,Normal],0)

(197) Undiscoverd Reoffenders[Cohort 2,Prison Systems,Normal]= INTEG ("Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohort 2,Prison Systems,Normal]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"

(198) Undiscoverd Reoffenders[Cohort 3,Prison Systems,Normal]= INTEG ("Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohort 3,Prison Systems, Normal] +"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 3,Prison Systems,Normal]-Repeat Offender Arrest Rate[Cohort 3,Prison Systems,Normal],0)

(199) Undiscoverd Reoffenders[Cohort 4,Prison Systems,Normal]= INTEG ("Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohort 4,Prison Systems, Normal]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohort 4,Prison Systems,Normal]-Repeat Offender Arrest Rate[Cohort 4,Prison Systems,Normal], 0)

(200) Undiscoverd Reoffenders[Cohorts,Prison Systems,SIB]= INTEG ("Non-Rehabilitated Prisoner Release Rate- First Offence"[Cohorts,Prison Systems,SIB]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"[Cohorts,Prison Systems,SIB]-Repeat Offender Arrest Rate[Cohorts,Prison Systems,SIB],0)

(201) Undiscoverd Reoffenders[Overall,Prison Systems,Normal]= INTEG ("Non-Rehabilitated Prisoner Release Rate- First Offence"[Overall,Prison Systems,Normal]+"Non-Rehabilitated Prisoner Release Rate- Repeat Offence"

(202) Unpaid Investor Earned Returns[Cohorts]= INTEG (Investor Earned Return Accumulation[Cohorts]-Earned Investor Return payments[Cohorts],No Initial Worth)

(203) Value Paid Early= INTEG (paying early,0)

(204) Yearly Cost of Incarceration=39

Dynamic Variables

Conviction Cost=2.853+0.2*2.853*RANDOM UNIFORM(-1, 1, 2)

Fraction for Investor share=0.5+0.2*0.5*RANDOM UNIFORM(-1,1,9)

Program Cost per Member=1.5+0.2*1.5*RANDOM UNIFORM(-1, 1, 0)

Yearly Cost of Incarceration=39+0.2*39*RANDOM UNIFORM(-1,1,8)

Perturbation Parameters

pulse rate=pulse switch*pulse size*PULSE(pulse time, pulse duration)

pulse time=500

pulse duration=TIME STEP*4

pulse size=[-20,20, step=5]

Simulation Control Parameters

FINAL TIME = 2000
INITIAL TIME = 0
SAVEPER = TIME STEP
TIME STEP = 0.25

APPENDIX B

STATISTICAL SCREENING

Limits to Growth Archetype

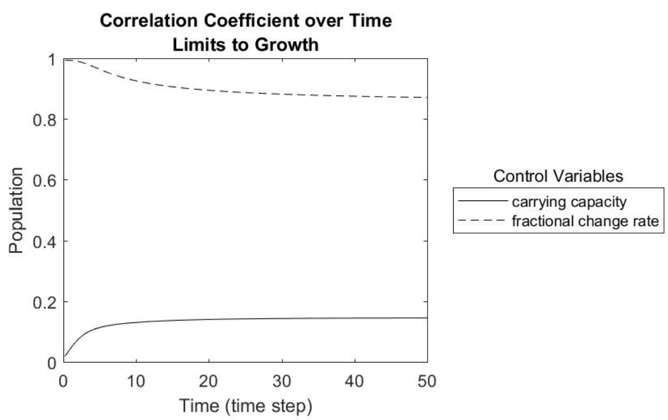

Figure 80-Statistical screening result of the limits to growth archetype

Fixes that Fail archetype

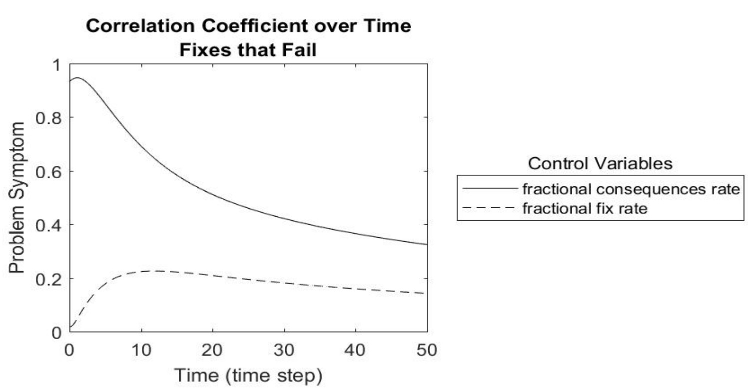

Figure 81-Statistical screening result of the fixes that fail archetype

Escalation Archetype

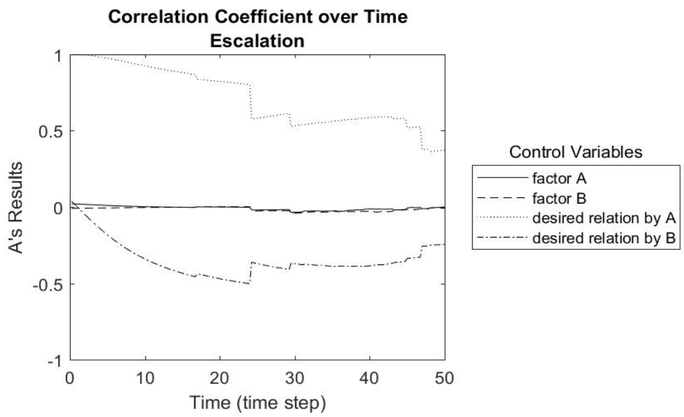

Figure 82- Statistical screening result of the escalation archetype

Limerick Construction Project Model

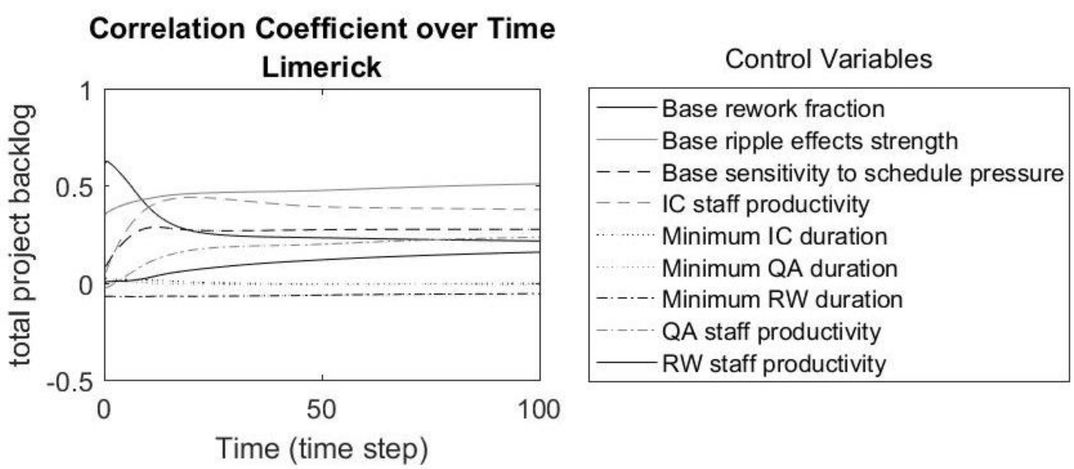

Figure 83- Statistical screening result of the Limerick construction project model

Fish Banks Model

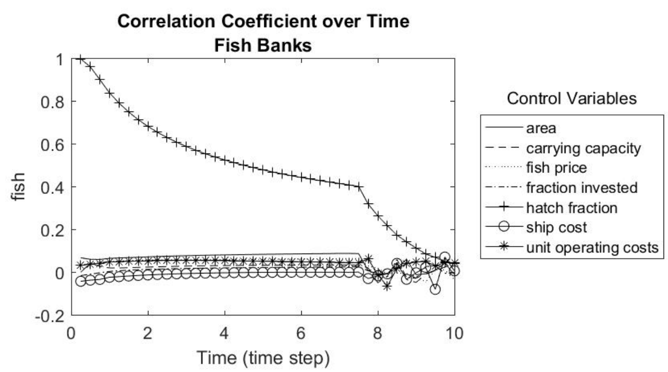

Figure 84- Statistical screening result of the fish banks model

Arms Race Model

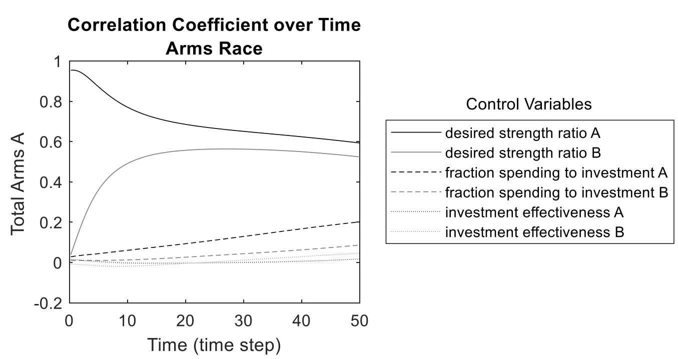

Figure 85- Statistical screening result of the arms race model

Social Impact Bonds Model

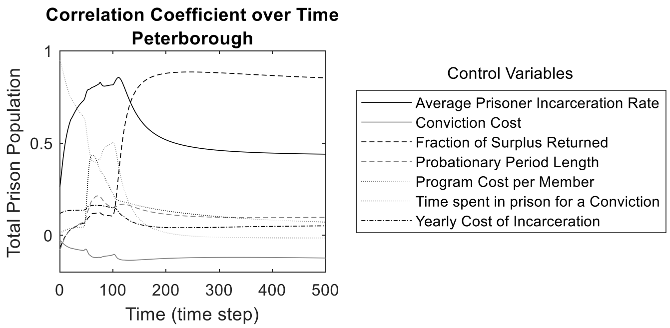

Figure 86- Statistical screening result of the social impact bonds model

APPENDIX C

STATISTICAL TESTING RESULTS

Limits to Growth Archetype

H1-Limits to Growth

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1980
Error Mean Square	441.3212
Critical Value of Studentized Range	5.01859
Minimum Significant Difference	10.543

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	459.260	100	0.1
	A	457.330	100	-0.1
	B	225.980	100	-0.2
	B	222.630	100	0.2
	C	149.270	100	-0.3
	C	146.420	100	0.3
	D	116.270	100	0.4
	D	110.620	100	-0.4
	E	87.720	100	-0.5
	E	86.560	100	0.5
	F	72.220	100	-0.6
G	F	70.880	100	0.6
G	H	61.370	100	-0.7
	H	60.180	100	0.7
I	H	52.980	100	-0.8
I	H	52.040	100	0.8

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
I	J	46.530	100	-0.9
I	J	45.800	100	0.9
	J	41.440	100	-1
	J	40.600	100	1

H2-Limits to Growth (pulse size=+3)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1980
Error Mean Square	175.7969
Critical Value of Studentized Range	5.01859
Minimum Significant Difference	6.6541

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	393.090	100	-0.1
	B	365.770	100	0.1
	C	219.060	100	-0.2
	D	184.240	100	0.2
	E	144.620	100	-0.3
	F	122.400	100	0.3
	G	105.100	100	-0.4
	H	86.390	100	0.4
	H	83.170	100	-0.5
	I	68.840	100	0.5
	I	67.270	100	-0.6
	J	57.790	100	-0.7
	J	56.800	100	0.6
	K	48.620	100	-0.8

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
L	K	46.090	100	0.7
L	M	41.100	100	-0.9
N	M	38.670	100	0.8
N	M	36.710	100	-1
N	O	34.080	100	0.9
	O	29.700	100	1

H2-Limits to Growth pulse size=-3

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1980
Error Mean Square	150.0936
Critical Value of Studentized Range	5.01859
Minimum Significant Difference	6.1484

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	395.000	100	0.1
	A	392.010	100	-0.1
	B	215.910	100	-0.2
	B	211.820	100	0.2
	C	142.510	100	-0.3
	C	141.130	100	0.3
	D	103.510	100	-0.4
	D	99.600	100	0.4
	E	81.810	100	-0.5
	E	79.540	100	0.5
	F	66.170	100	-0.6
	F	65.590	100	0.6

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	G	56.770	100	-0.7
H	G	53.350	100	0.7
H	I	47.740	100	-0.8
J	I	44.950	100	0.8
J	K	40.360	100	-0.9
J	K	39.630	100	0.9
	K	36.190	100	-1
	K	34.690	100	1

H3-limits to growth (pulse size=+3)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1980
Error Mean Square	0.000328
Critical Value of Studentized Range	5.01859
Minimum Significant Difference	0.0091

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.975826	100	-0.1
A	0.973080	100	0.1
B	0.951733	100	-0.2
B	0.947390	100	0.2
C	0.928371	100	-0.3
C	0.923042	100	0.3
D	0.903381	100	-0.4
D	0.894558	100	0.4
E	0.879907	100	-0.5
E	0.871053	100	0.5

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
F	0.854642	100	-0.6
F	0.847822	100	0.6
G	0.833564	100	-0.7
H	0.818433	100	0.7
I	0.806434	100	-0.8
J	0.790863	100	0.8
K	0.777133	100	-0.9
K	0.769168	100	0.9
L	0.756216	100	-1
M	0.743425	100	1

H3-limits to growth (pulse size=-3)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1980
Error Mean Square	0.000328
Critical Value of Studentized Range	5.01859
Minimum Significant Difference	0.0091

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.975480	100	0.1
A	0.973459	100	-0.1
B	0.951672	100	0.2
B	0.947452	100	-0.2
C	0.928763	100	0.3
C	0.922624	100	-0.3
D	0.901595	100	0.4
D	0.896417	100	-0.4

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
E	0.878899	100	0.5
E	0.872093	100	-0.5
F	0.856252	100	0.6
G	0.846180	100	-0.6
H	0.827319	100	0.7
H	0.824750	100	-0.7
I	0.799948	100	0.8
I	0.797392	100	-0.8
J	0.778264	100	0.9
K	0.768062	100	-0.9
L	0.752371	100	1
L	0.747249	100	-1

H4-Limits to Growth (pulse size=+3)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1980
Error Mean Square	7.767E-7
Critical Value of Studentized Range	5.01859
Minimum Significant Difference	0.0004

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	0.0203649	100	-0.1
	B	0.0179253	100	0.1
	C	0.0094607	100	-0.2
	D	0.0082663	100	0.2
	E	0.0055215	100	-0.3

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	F	0.0047860	100	0.3
	G	0.0034347	100	-0.4
	H	0.0027909	100	0.4
	I	0.0023262	100	-0.5
	J	0.0018784	100	0.5
K	J	0.0015802	100	-0.6
K		0.0013045	100	0.6
K	L	0.0011664	100	-0.7
M	L	0.0008439	100	0.7
M	L	0.0008074	100	-0.8
M	N	0.0005663	100	0.8
M	N	0.0005451	100	-0.9
M	N	0.0004143	100	-1
M	N	0.0004140	100	0.9
	N	0.0002910	100	1

Fixes that Fail Archetype

H1-Fixes that Fail

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1782
Error Mean Square	0
Critical Value of Studentized Range	4.94110
Minimum Significant Difference	0

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	647.0	100	0.9

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
B	435.0	100	1.1
C	317.0	100	0.8
D	223.0	100	1.2
E	203.0	100	0.7
F	152.0	100	1.3
G	145.0	100	0.6
H	117.0	100	1.4
I	110.0	100	0.5
J	95.0	100	1.5
K	87.0	100	0.4
L	81.0	100	1.6
M	71.0	100	1.7
N	69.0	100	0.3
O	63.0	100	1.8
P	57.0	100	1.9
Q	56.0	100	0.2
R	44.0	100	0.1

H2-Fixes that Fail (pulse size=+4)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	891
Error Mean Square	0
Critical Value of Studentized Range	4.39738
Minimum Significant Difference	0

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	800.0	100	0.9

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
B	634.0	100	0.8
C	551.0	100	0.7
D	509.0	100	0.6
E	483.0	100	0.5
F	466.0	100	0.4
G	453.0	100	0.3
H	443.0	100	0.2
I	435.0	100	0.1

H2-Fixes that Fail (pulse size=-4)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	891
Error Mean Square	45.4779
Critical Value of Studentized Range	4.39738
Minimum Significant Difference	2.9655

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	691.8300	100	0.9
B	564.5700	100	0.8
C	505.7500	100	0.7
D	475.8900	100	0.6
E	457.8300	100	0.5
F	445.3500	100	0.4
G	436.6800	100	0.3
H	429.2100	100	0.2
I	422.6600	100	0.1

H3-fixes that fail (pulse size=+4)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	891
Error Mean Square	0
Critical Value of Studentized Range	4.39738
Minimum Significant Difference	0

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.9841	100	0.9
B	0.9683	100	0.8
C	0.9511	100	0.7
D	0.9319	100	0.6
E	0.9104	100	0.5
F	0.8862	100	0.4
G	0.8587	100	0.3
H	0.8272	100	0.2
I	0.7911	100	0.1

H3-fixes that fail (pulse size=-4)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	891
Error Mean Square	8.703E-6
Critical Value of Studentized Range	4.39738
Minimum Significant Difference	0.0013

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.9809475	100	0.9
B	0.9634733	100	0.8
C	0.9434920	100	0.7
D	0.9212697	100	0.6
E	0.8960124	100	0.5
F	0.8661841	100	0.4
G	0.8281124	100	0.3
H	0.7751937	100	0.2
I	0.6884219	100	0.1

H4-Fixes that Fail (pulse size=+4)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	891
Error Mean Square	0
Critical Value of Studentized Range	4.39738
Minimum Significant Difference	0

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.03424	100	0.9
B	0.00875	100	0.8
C	0.00577	100	0.7
D	0.00411	100	0.6
E	0.00307	100	0.5
F	0.00237	100	0.4
G	0.00187	100	0.3
H	0.00150	100	0.2
I	0.00122	100	0.1

H4-Fixes that Fail (pulse size=-4)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var
Note: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	991
Error Mean Square	0
Critical Value of Studentized Range	4.39628

Comparisons significant at the 0.05 level are indicated by *.**				
tpvar Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
0.9 - 0.8	0.004253	0.004253	0.004253	***
0.8 - 0.7	0.000000	0.000000	0.000000	***
0.7 - 0.6	0.000000	0.000000	0.000000	***
0.6 - 0.5	0.000000	0.000000	0.000000	***
0.5 - 0.4	0.000000	0.000000	0.000000	***
0.4 - 0.3	0.000000	0.000000	0.000000	***
0.3 - 0.2	0.000000	0.000000	0.000000	***
0.2 - 0.1	0.000000	0.000000	0.000000	***

Reinforcing Loop Archetype

H1-Reinforcing Loop

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t
Note: This test controls the Type I experimentwise error rate.

Alpha	0.05
Error Degrees of Freedom	4079
Error Mean Square	14785.17
Critical Value of Studentized Range	5.05080

Comparisons significant at the 0.05 level are indicated by ***.				
tpvar Comparison	**Difference Between Means**	**Simultaneous 95% Confidence Limits**		
-0.1 - -0.2	110.11	66.13	154.09	***
0.1 - 0.2	157.26	113.28	201.24	***
-0.2 - -0.3	175.89	131.91	219.87	***
0.2 - 0.3	120.52	76.54	164.50	***
-0.3 - -0.4	60.08	16.10	104.06	***
-0.4 - -0.5	36.46	-7.52	80.44	
0.3 - 0.4	50.04	6.06	94.02	***
-0.5 - -0.6	19.54	-24.38	63.47	
0.4 - 0.5	17.01	-26.97	60.99	
-0.6 - -0.7	15.77	-28.10	59.63	
-0.7 - -0.8	8.72	-35.14	52.59	
0.5 - 0.6	10.77	-33.21	54.75	
-0.8 - -0.9	6.62	-37.25	50.49	
0.6 - 0.7	8.91	-35.07	52.89	
-0.9 - -1	7.09	-36.78	50.96	
0.7 - 0.8	6.46	-37.52	50.44	
0.8 - 0.9	3.34	-40.64	47.32	
0.9 - 1	3.93	-40.05	47.91	

H2-Reinforcing Loop (pulse size=+2)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	990
Error Mean Square	1.637545
Critical Value of Studentized Range	4.48436
Minimum Significant Difference	0.5738

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	58.0300	100	-0.1
B	30.2000	100	-0.2
C	20.9900	100	-0.3
D	16.2400	100	-0.4
E	13.4400	100	-0.5
F	11.6500	100	-0.6
G	10.4400	100	-0.7
H	9.4000	100	-0.8
I	8.6000	100	-0.9
I	8.1000	100	-1

H2-Reinforcing Loop (pulse size=-2)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	990
Error Mean Square	7.612505
Critical Value of Studentized Range	4.48436
Minimum Significant Difference	1.2373

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	121.8100	100	-0.1
B	61.8200	100	-0.2
C	41.8100	100	-0.3
D	31.5900	100	-0.4
E	25.4100	100	-0.5
F	21.6000	100	-0.6
G	18.8600	100	-0.7

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
H	16.5700	100	-0.8
I	14.9700	100	-0.9
J	13.7000	100	-1

H3-reinforcing loop (pulse size=2)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	990
Error Mean Square	4.393E-6
Critical Value of Studentized Range	4.48436
Minimum Significant Difference	0.0009

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.9931633	100	-0.1
B	0.9874158	100	-0.2
C	0.9815247	100	-0.3
D	0.9755243	100	-0.4
E	0.9693650	100	-0.5
F	0.9637817	100	-0.6
G	0.9582821	100	-0.7
H	0.9522688	100	-0.8
I	0.9466901	100	-0.9
J	0.9411936	100	-1

H3-reinforcing loop (pulse size=-2)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	990
Error Mean Square	4.393E-6
Critical Value of Studentized Range	4.48436
Minimum Significant Difference	0.0009

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.9931622	100	-0.1
B	0.9874158	100	-0.2
C	0.9815247	100	-0.3
D	0.9755243	100	-0.4
E	0.9693650	100	-0.5
F	0.9637817	100	-0.6
G	0.9582821	100	-0.7
H	0.9522688	100	-0.8
I	0.9466901	100	-0.9
J	0.9411936	100	-1

H4-Reinforcing Loop (pulse size=2)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	990
Error Mean Square	0.00001
Critical Value of Studentized Range	4.48436
Minimum Significant Difference	0.0014

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.1568232	100	-0.1
B	0.0868097	100	-0.2
C	0.0590583	100	-0.3
D	0.0440696	100	-0.4
E	0.0346115	100	-0.5
F	0.0287653	100	-0.6
G	0.0244979	100	-0.7
H	0.0209294	100	-0.8
I	0.0183357	100	-0.9
J	0.0162681	100	-1

H4-Reinforcing Loop (pulse size=-2)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	990
Error Mean Square	0.00001
Critical Value of Studentized Range	4.48436
Minimum Significant Difference	0.0014

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.1567496	100	-0.1
B	0.0868097	100	-0.2
C	0.0590583	100	-0.3
D	0.0440696	100	-0.4
E	0.0346115	100	-0.5
F	0.0287653	100	-0.6
G	0.0244979	100	-0.7

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
H	0.0209294	100	-0.8
I	0.0183357	100	-0.9
J	0.0162681	100	-1

Escalation Archetype

H1-Escalation

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1782
Error Mean Square	51.83015
Critical Value of Studentized Range	4.94110
Minimum Significant Difference	3.5573

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	200.000	100	0.9
	A	200.000	100	1.1
	A	200.000	100	1.2
	A	200.000	100	0.8
B	A	199.060	100	0.7
B		196.080	100	1.3
	C	171.500	100	0.6
	D	155.650	100	1.4
	E	132.290	100	0.5
	F	119.570	100	1.5
	G	103.170	100	0.4
	H	94.540	100	1.6

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	I	83.500	100	0.3
	J	77.650	100	1.7
	K	69.980	100	0.2
	L	66.370	100	1.8
	M	60.180	100	0.1
	M	57.750	100	1.9

Limerick Construction Project Model

H1-Limerick

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1188
Error Mean Square	131490.6
Critical Value of Studentized Range	4.63095
Minimum Significant Difference	167.93

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	1561.07	100	0.7
	B	1312.54	100	0.6
	C	983.75	100	0.8
	D	730.01	100	0.5
E	D	603.42	100	0.9
E	F	454.38	100	0.4
E	F	440.52	100	1
G	F	383.72	100	0.3
G	F	344.63	100	0.2
G	F	320.58	100	0.1

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
G	F	319.31	100	1.1
G		257.35	100	1.2

H2-Limerick (pulse size=+2000)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	693
Error Mean Square	17429.54
Critical Value of Studentized Range	4.18186
Minimum Significant Difference	55.209

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	653.50	100	0.65
A	607.81	100	0.55
B	518.08	100	0.45
C	446.58	100	0.35
C	422.56	100	0.25
C	403.99	100	0.15
C	402.00	100	0.05

H2-Limerick (pulse size=-2000)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	693

Error Mean Square	17459.36
Critical Value of Studentized Range	4.18186
Minimum Significant Difference	55.257

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	653.18	100	0.65
A			
A	607.61	100	0.55
B	517.99	100	0.45
C	446.51	100	0.35
C			
C	422.52	100	0.25
C			
C	403.97	100	0.15
C			
C	402.00	100	0.05

H3-Limerick (pulse size=2000)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	693
Error Mean Square	0.122729
Critical Value of Studentized Range	4.18186
Minimum Significant Difference	0.1465

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.54500	100	0.65
A	0.49414	100	0.55
B	0.29553	100	0.45
C	0.13360	100	0.35
C	0.09299	100	0.25
C	0.00106	100	0.15
C	-0.00249	100	0.05

H3-Limerick (pulse size=-2000)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	693
Error Mean Square	0.121143
Critical Value of Studentized Range	4.18186
Minimum Significant Difference	0.1456

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.53941	100	0.65
A	0.49405	100	0.55
B	0.29545	100	0.45
C	0.13009	100	0.35
C	0.09077	100	0.25
C	-0.00249	100	0.05
C	-0.00408	100	0.15

H4-Limerick (pulse size=+2000)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	693
Error Mean Square	1.403E13
Critical Value of Studentized Range	4.18186
Minimum Significant Difference	1.57E6

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	4149070	100	0.45
B	A	2619763	100	0.55
B	C	1920279	100	0.35
D	C	416036	100	0.25
D		76288	100	0.65
D		9979	100	0.15
D		9926	100	0.05

H4-Limerick (pulse size=-2000)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	693
Error Mean Square	1.378E13
Critical Value of Studentized Range	4.18186
Minimum Significant Difference	1.55E6

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	4110271	100	0.45
B	A	2581012	100	0.55
B	C	1900881	100	0.35
D	C	405361	100	0.25

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
D		69686	100	0.65
D		9926	100	0.05
D		9913	100	0.15

Fish Banks Model

H1-Fish Banks

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1881
Error Mean Square	2462.681
Critical Value of Studentized Range	4.98101
Minimum Significant Difference	24.718

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	400.000	100	6
	A	400.000	100	6.5
	B	223.530	100	7
	C	198.550	100	5
	D	55.800	100	4.5
E	D	31.230	100	4
E	F	25.000	100	7.5
E	F	20.540	100	3.5
E	F	14.230	100	3
E	F	10.400	100	0.5
E	F	10.130	100	9
E	F	9.990	100	2.5

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
E	F	9.830	100	8
E	F	7.320	100	10
E	F	7.000	100	1
E	F	6.800	100	8.5
	F	6.290	100	9.5
	F	6.190	100	2
	F	6.000	100	1.5

Arms Race Model

H1-Arms Race

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1782
Error Mean Square	233548
Critical Value of Studentized Range	4.94110
Minimum Significant Difference	238.79

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	3036.38	100	0.9
	B	1714.81	100	0.8
	B	1635.47	100	1.1
	C	1043.36	100	0.7
	D	789.46	100	0.6
E	D	719.95	100	0.5
E	D	703.01	100	0.4
E	D	700.35	100	0.2

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
E	D	694.41	100	0.3
E	D	675.91	100	0.1
E		528.83	100	1.2
	F	231.58	100	1.3
	F	175.92	100	1.4
	F	150.82	100	1.5
	F	102.11	100	1.6
	F	88.39	100	1.7
	F	81.02	100	1.8
	F	72.27	100	1.9

Social Impact Bonds Model

H1-Social Impact Bonds

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	1881
Error Mean Square	149498.4
Critical Value of Studentized Range	4.98101
Minimum Significant Difference	192.59

Means with the same letter are not significantly different.							
Tukey Grouping					**Mean**	**N**	**tpvar**
			A		1972.48	100	0.45
			A		1963.50	100	0.4
			A		1790.99	100	0.5
			B		1530.53	100	0.55
			C		1329.98	100	0.6

Means with the same letter are not significantly different.							
Tukey Grouping					Mean	N	tpvar
	D		C		1264.90	100	0.35
	D		C	E	1221.93	100	0.65
	D	F	C	E	1145.24	100	0.7
	D	F	G	E	1081.27	100	0.75
	H	F	G	E	1039.93	100	0.8
	H	F	G	I	1000.72	100	0.85
J	H	F	G	I	969.52	100	0.9
J	H		G	I	942.27	100	0.95
J	H		K	I	881.54	100	0.1
J			K	I	818.85	100	0.15
J			K	I	808.26	100	0.2
J			K		792.73	100	0.25
J			K		790.83	100	0.3
			K		745.55	100	0.05

H2-Social Impact Bonds (pulse size=+10)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	594
Error Mean Square	34246.56
Critical Value of Studentized Range	4.04330
Minimum Significant Difference	74.825

Means with the same letter are not significantly different.			
Tukey Grouping	Mean	N	tpvar
A	109.89	100	0.4
B	0.00	100	0.5
B	0.00	100	0.6

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
B	0.00	100	0.7
B	0.00	100	0.8
B	0.00	100	0.9

H2-Social Impact Bonds pulse size=-10

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for t

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	594
Error Mean Square	60635.74
Critical Value of Studentized Range	4.04330
Minimum Significant Difference	99.564

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	255.47	100	0.4
B	0.00	100	0.5
B	0.00	100	0.6
B	0.00	100	0.7
B	0.00	100	0.8
B	0.00	100	0.9

H3-Social Impact Bonds (pulse size=10)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05

Error Degrees of Freedom	594
Error Mean Square	0.000243
Critical Value of Studentized Range	4.04330
Minimum Significant Difference	0.0063

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	0.968017	100	0.4
B	0.961066	100	0.9
B	0.961050	100	0.8
B	0.961026	100	0.7
B	0.961010	100	0.6
B	0.960514	100	0.5

H3-Social Impact Bonds (pulse size=-10)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for acf

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	594
Error Mean Square	0.000038
Critical Value of Studentized Range	4.04330
Minimum Significant Difference	0.0025

Means with the same letter are not significantly different.				
Tukey Grouping		**Mean**	**N**	**tpvar**
	A	0.9911886	100	0.4
	B	0.9850347	100	0.5
	C	0.9810182	100	0.6
D	C	0.9785677	100	0.7
D	E	0.9764046	100	0.8
	E	0.9748993	100	0.9

H4-Social Impact Bonds (pulse size=10)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	594
Error Mean Square	4.763304
Critical Value of Studentized Range	4.04330
Minimum Significant Difference	0.8825

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	2.5114	100	0.4
B	0.0156	100	0.9
B	0.0156	100	0.8
B	0.0156	100	0.7
B	0.0156	100	0.6
B	0.0153	100	0.5

H4-Social Impact Bonds (pulse size=10)

The ANOVA Procedure

Tukey's Studentized Range (HSD) Test for var

Note: This test controls the Type I experimentwise error rate, but it generally has a higher Type II error rate than REGWQ.

Alpha	0.05
Error Degrees of Freedom	594
Error Mean Square	5.502579
Critical Value of Studentized Range	4.04330
Minimum Significant Difference	0.9485

Means with the same letter are not significantly different.			
Tukey Grouping	**Mean**	**N**	**tpvar**
A	2.9561	100	0.4
B	0.0373	100	0.5
B	0.0305	100	0.6
B	0.0274	100	0.7
B	0.0251	100	0.8
B	0.0238	100	0.9

APPENDIX D

COMPUTER CODES

```
% calculates and plots the correlation coefficient between the
% dependent variable and control variables over time in Matlab

model='XXX';
m='H1-XXX'; %name used to save the figures
dvar='XXX'; %dependent variable
cvar={'AAA','BBB'};  %control variables
cvarNO=2; %umber of control variables
start=cvarNO+2; %the column where the time series start
timestep=0.25;

[row,col]=size(data);
dv=data;

%adds time values to the first row of the matrix
dv=[zeros(1,col);dv];
for j=start+1:col
  dv(1,j)=timestep*(j-start);
end

%calculates the correlation coefficients
cormat=zeros(cvarNO+1,col-cvarNO-1); %correlation coefficient matrix
cormat(1,:)=dv(1,start:col);
for k=2:cvarNO+1
        for j=start:col
     cormat(k,j-start+1)=corr2(data(:,k),data(:,j));
   end
end

%saves the correlation coefficient matrix in an excel file
filename=sprintf('Correlation Matrix-%s',m);
xlswrite(filename,cormat);

% plots and saves correlation coefficient over time for all control variables
strg1=sprintf('Correlation Coefficient over Time');
strg2=sprintf('%s', m);

plotname=sprintf('H0-Sensitivity Analysis-%s',m);
f=figure;
set(0,'defaultaxescolororder',[0 0 0]); %black and white
```

```
set(0,'defaultaxeslinestyleorder',{'-';'--';':';'-.';'-+';'-o';'-*';'-x';...
    '-s';'-d';'-^';'-v';'->';'-<';'-p';'-h'}); %change line types
plot(cormat(1,:),cormat(2:end,:));
xlabel('Time (time step)');
ylabel(dvar);
%ylim([-.5,.5]);         %change if necessary
%xlim([0,50]);     %change if necessary
pbaspect([1.2 1 1]);
title({strg1,strg2});
leg = legend(cvar,'location','eastoutside');
hlt = text('Parent', leg.DecorationContainer,'String', 'Control Variables', ...
'HorizontalAlignment', 'center','VerticalAlignment', 'bottom', 'Position',...
[0.5, 1.05, 0], 'Units', 'normalized');
saveas(f,plotname,'jpeg');
savefig(f,plotname);

%undo grayscale
set(0, 'DefaultAxesColorOrder', 'remove');
set(0, 'DefaultAxeslinestyleOrder', 'remove');
```

```
% calculates 'H1-time to reach an attractor' in Matlab
% draws the graphs of time to reach an attractor vs tipping variable

%ENTER THE FOLLOWING VALUES MANUALLY
model='XXX';
m='H1-XXX'; %name used to save the figures
cvarNO=000; %number of control variables
tpvar=000;  %value of tipping variable at tp conditions
eq1=000;    %stable equilibrium 1
eq2=000;    %stable equilibrium 2
largenumber=100000;
smallnumber=0.00001;
start=cvarNO+2;
tpvarstep=0000;
timestep=0.25;

H1=data;
sim=H1(:,1);
tp=H1(:,2);
[row, col]=size(H1);
t=[sim tp zeros(row,1)];

for i=1:row
   for j=col:-1:start
      if abs(H1(i,j)-eqv1)<smallnumber || abs(H1(i,j)-eqv2)<smallnumber ...
         || abs(H1(i,j))>largenumber …
         ||abs(max(H1(i,j:col))-min(H1(i,j:col)))<smallnumber
         %choose one or more arguments depending on the system attractors
        t(i,end)=(j-start);
      end
      if abs(t(i,end))<0.00001 && abs(H1(i,2))>0.00001
         %assigns the last simulation time if the system does not reach
         %an equilbrium during the simulation
         t(i,end)=(col-start);
      end
   end
end

%saves the results in two different tables: below and above tipping point
loc=t(:,2)-tpvar>0.00001;
tp=t(loc,:);
loc=t(:,2)<tpvar;
tn=t(loc,:);
```

```
strg1=sprintf('H1 - %s', model);
strg1a=sprintf('(with randomness)' %use if there is noise in the system
strg2=sprintf('tipping variable');
strg3=sprintf('time to reach attractor (time step)');
plotname=sprintf('%s',m);

f=figure;
set(0,'defaultaxescolororder',[0 0 0;0.5 0.5 0.5]); %greyscale
set(0,'defaultaxeslinestyleorder',{'-';'--';':';'-.';'-+';'-o';'-*';'-x';...
    '-s';'-d';'-^';'-v';'->';'-<';'-p';'-h'}); %change line types
pbaspect([1.2 1 1]);
plot(tp(:,2),tp(:,end),tn(:,2),tn(:,end));
hold on;
line([tpvar tpvar],get(gca,'YLim'),'LineStyle','--')
txt1 = 'Tipping Point\rightarrow';
text(tpvar,500,txt1,'HorizontalAlignment','right');
title(strg1); %use title(strg1,strg1a) if there is noise in the system
xlabel(strg2);
ylabel(strg3);

saveas(f,plotname,'jpeg');
savefig(f,plotname);
```

```
% calculates H2-recovery time, H3-variance and H4-autocorrelation in Matlab
% draws the graphs for different values of pulse sizes

%ENTER THE FOLLOWING VALUES MANUALLY
model='XXX';
m='XXX'; %name used to save the figures
cvarNO=000; %number of control variables
tpvar=000;  %value of tp variable at tp conditions
eq1=000;    %stable equilbrium 1
eq2=000;    %stable equilbrium 2
largenumber=100000;
smallnumber=0.00001;
start=cvarNO+2;
tpvarstep=0000;
pulsestep=000;
timestep=0.25;
pulsetime=000/timestep+start;   %enter pulse time to calculate column no

%deletes zero pulses
loc=abs(data(:,3))<0.00001;
H2=data(~loc,:);

sim=H2(:,1);
pulse=H2(:,3);  %check the column number
tp=H2(:,2);   %check the column number
[row, col]=size(H2);
t=[sim pulse tp zeros(row,1)];

for i=1:row
   for j=col:-1:pulsetime+000  %check when the pulse starts affecting
      if abs(H1(i,j)-eqv1)<smallnumber || abs(H1(i,j)-eqv2)<smallnumber ...
         || abs(H1(i,j))>largenumber …
         ||abs(max(H1(i,j:col))-min(H1(i,j:col)))<smallnumber
         %choose one or more arguments depending on the system attractors
         t(i,end)=(j-pulsetime);
      end
      if abs(t(i,end))<0.00001 && abs(H1(i,2))>0.00001
         %assigns the last simulation time if the system does not reach
         %an equilbrium during the simulation
         t(i,end)=(col-pulsetime);
      end
   end
end
```

```
H3=H2(:,pulsetime:end);
[row1, col1]=size(H3);
variance=[sim pulse tp zeros(row,1)];
acf=[sim pulse tp zeros(row,1)];
r=zeros(1,col1);
for i=1:row
    H3arraytemp=H3(i,1:end);
    variance(i,end)=var(H3arraytemp);
    m=mean(H3arraytemp);
    [row1, col1]=size(H3arraytemp);
    for j=1:col1-1
        r(1,j)=r(1,j)+(H3arraytemp(1,j)-m)*(H3arraytemp(1,j+1)-m);
    end
    acf(i,end)=r(1,:)/variance(i,end)/(col1-1);
end

pulse=unique(pulse);
pulsesize=size(pulse,1);
minpulse=min(pulse);
maxpulse=max(pulse);

tp=unique(tp);
tpsize=size(tp,1);
mintp=min(tp);
maxtp=max(tp);

%plots graphs for positive pulse
pulsesize=size(pulse,1);
plotname=sprintf('H2-%s',m);

strg12=sprintf('H2 - %s',model);
strg22=sprintf('(with randomness)');
strg32=sprintf('Tipping variable');%=%s', tpvar);
strgy42=sprintf('Recovery Time (time step)');

f=figure;
set(0,'defaultaxescolororder',[0 0 0;]); %blacka and white
set(0,'defaultaxeslinestyleorder',{'-';'-';'--';'--';':';':';'-.';'-.';...
    '-+';'-+';'-o';'-o';'-*';'-*';'-x';'-x';'-s';'-s';'-d';'-d';'-^';'-^';...
    '-v';'-v';'->';'->';'-<';'-<';'-p';'-p';'-h';'-h'}); %change line types;
hold on;
pbaspect([1.2 1 1]);
for i=minpulse:pulsestep:maxpulse
    loc1=t(:,3)>tpvar & abs(t(:,2)-i)<0.00001;
```

```
    ttemp1=t(loc1,:);
    loc2=t(:,3)<tpvar & abs(t(:,2)-i)<0.00001;
    ttemp2=t(loc2,:);
    p1=plot(ttemp1(:,3),ttemp1(:,end),'HandleVisibility', 'off');
    p2=plot(ttemp2(:,3),ttemp2(:,end),'displayname',num2str(i));
    leg=legend('show','location','northwest'); %change the location
    title(leg,'pulse size');
end
hold on;
line([tpvar tpvar],get(gca,'YLim'),'LineStyle','--','HandleVisibility','off')
txt1 = 'Tipping Point\rightarrow';
text(tpvar,50,txt1,'HorizontalAlignment','right'); %change the location
title(strg12); %use title(strg12,strg22) if there is noise in the system
xlabel(strg32);
ylabel(strgy42);
saveas(f,plotname,'jpeg');
savefig(f,plotname);

strg13=sprintf('H3 - %s',model);
strg33=sprintf('Lag-1 Autocorrelation');
plotname=sprintf('H3-%s',m);

f=figure;
set(0,'defaultaxescolororder',[0 0 0;]); %blacka and white
set(0,'defaultaxeslinestyleorder',{'-';'-';'--';'--';':';':';'-.';'-.';...
    '-+';'-+';'-o';'-o';'-*';'-*';'-x';'-x';'-s';'-s';'-d';'-d';'-^';'-^';...
    '-v';'-v';'->';'->';'-<';'-<';'-p';'-p';'-h';'-h'}); %change line types;
hold on;
pbaspect([1.2 1 1]);
for i=minpulse:pulsestep:maxpulse
    loc1=acf(:,3)>tpvar & abs(acf(:,2)-i)<0.00001;
    acftemp1=acf(loc1,:);
    loc2=acf(:,3)<tpvar & abs(acf(:,2)-i)<0.00001;
    acftemp2=acf(loc2,:);
    p1=plot(acftemp1(:,3),acftemp1(:,end),'HandleVisibility','off');
    p2=plot(acftemp2(:,3),acftemp2(:,end),'displayname', num2str(i));
    leg=legend('show','location','northwest'); %change the location
    title(leg,'pulse size');
end
hold on;
line([tpvar tpvar],get(gca,'YLim'),'LineStyle','--','HandleVisibility','off')
txt1 = 'Tipping Point\rightarrow';
text(tpvar,25,txt1,'HorizontalAlignment','right'); %change the location
title(strg13); %use title(strg13,strg22) if there is noise in the system
```

```
xlabel(strg32);
ylabel(strg33);
saveas(f,plotname,'jpeg');
savefig(f,plotname);

strg14=sprintf('H4 - %s',model);
strg34=sprintf('Variance');
plotname=sprintf('H4-%s',m);

f=figure;
set(0,'defaultaxescolororder',[0 0 0;]); %blacka and white
set(0,'defaultaxeslinestyleorder',{'-';'-';'--';'--';':';':';'-.';'-.';...
    '-+';'-+';'-o';'-o';'-*';'-*';'-x';'-x';'-s';'-s';'-d';'-d';'-^';'-^';...
    '-v';'-v';'->';'->';'-<';'-<';'-p';'-p';'-h';'-h'}); %change line types;
hold on;
pbaspect([1.2 1 1]);
for i=minpulse:pulsestep:maxpulse
    loc1=variance(:,3)>tpvar & abs(variance(:,2)-i)<0.00001;
    variancetemp1=variance(loc1,:);
    loc2=variance(:,3)<tpvar & abs(variance(:,2)-i)<0.00001;
    variancetemp2=variance(loc2,:);
    p1=plot(variancetemp1(:,3),variancetemp1(:,end),'HandleVisibility','off');
    p2=plot(variancetemp2(:,3),variancetemp2(:,end),'displayname', num2str(i));
    leg=legend('show','location','northwest'); %change the location
    title(leg,'pulse size');
end
hold on;
line([tpvar tpvar],get(gca,'YLim'),'LineStyle','--','HandleVisibility','off')
txt1 = 'Tipping Point\rightarrow';
text(tpvar,0.03,txt1,'HorizontalAlignment','right'); %change the location
title(strg14); %use title(strg14,strg22) if there is noise in the system
xlabel(strg32);
ylabel(strg34);
saveas(f,plotname,'jpeg');
savefig(f,plotname);

%undo grayscale
set(0, 'DefaultAxesColorOrder', 'remove');
set(0, 'DefaultAxeslinestyleOrder', 'remove');
```

```
% calculates H5-distance and H6-slope of xt+1-xt graph in Matlab

%ENTER THE FOLLOWING VALUES MANUALLY
model='XXX';
m='H1-XXX'; %name used to save the figures
tptime=000; %time when the tipping variables reaches the values of the tipping condition
timestep=0.25;
dv0='000';

H5=data;
[row,col]=size(H5);

dvslope=zeros(1,col);
dvdistance=zeros(1,col);

time=H5(1,:);
dv=H5(2,:); %check the column number

for j=1:col-2;
   dvslope(1,j)=(dv(1,j+2)-dv(1,j+1))/(dv(1,j+1)-dv(1,j));
   dvdistance(1,j)=sqrt((dv(1,j+2)-dv(1,j+1))^2+(dv(1,j+1)-dv(1,j))^2);
end;

strg15=sprintf('H5 - %s', model);
strg16=sprintf('H6 - %s', model);
strg2=sprintf('%s',model);
strg3=sprintf('Tipping Conditions @ t=%.1f', tptime);
strg4=sprintf('x_{t}');
strg5=sprintf('x_{t+1}');
strg6=sprintif('(with randomness)');

%change the values to zoom in the graph
dx=0;
dy=0;
zoom=3*tptime;
labelstep=20;
labelend=zoom/timestep;

% x t+1-x t
plotname=sprintf('x2-x1-%s',m);
f=figure;
set(0,'defaultaxescolororder',[0 0 0]); %black and white
scatter(dv(1,1:zoom/timestep-1),dv(1,2:zoom/timestep),'.');
```

```
ref=refline(1,0)
ref.Color='black';
ref.LineStyle='--';
%xlim([1 1.6]); %change if necessary
%ylim([1 1.6]); %chnage if necessary
axis([3.5 5.5 3.5 5.5]);
xlabel(strg4);
ylabel(strg5);
pbaspect([1.5 1 1]);
title(strg2,strg3); %use title(strg2,strg3,strg6) if there is noise in the system
for ii=1:labelstep:labelend-10;
    num2str(time(ii));
    text(dv(ii)+dx,dv(ii+1)+dy,num2str(time(ii)));
end
saveas(f,plotname,'jpeg');
savefig(f,plotname);

% distance x t+1-x t
plotname=sprintf('H5-%s',m);
f=figure;
set(0,'defaultaxescolororder',[0 0 0]); %black and white
plot(time(1:zoom/timestep),dvdistance(1:zoom/timestep));
hold on;
line([tptime tptime],get(gca,'YLim'),'LineStyle','--')
txt1 = 'Tipping Point\rightarrow';
text(tptime,0.08,txt1,'HorizontalAlignment','right'); %change location
xlabel('time');
ylabel('Distance of Points on x_{t+1}-x_{t} Graph');
pbaspect([1.5 1 1]);
title({strg15});  %use title(strg15,strg6) if there is noise in the system
saveas(f,plotname,'jpeg');
savefig(f,plotname);

% slope x t+1-x t
plotname=sprintf('H6-%s',m);
f=figure;
set(0,'defaultaxescolororder',[0 0 0]); %black and white
line(time(1:zoom/timestep),dvslope(1:zoom/timestep));%,'.');
hold on;
line([tptime tptime],get(gca,'YLim'),'LineStyle','--')
txt1 = 'Tipping Point\rightarrow';
text(tptime,2,txt1,'HorizontalAlignment','right'); %change location
xlabel('time');
ylabel('Slope of x_{t+1}-x_{t} Graph');
```

```
pbaspect([1.5 1 1]);
title({strg16});  %use title(strg16,strg6) if there is noise in the system
saveas(f,plotname,'jpeg');
savefig(f,plotname);

%undo grayscale
set(0, 'DefaultAxesColorOrder', 'remove');
set(0, 'DefaultAxeslinestyleOrder', 'remove');
```

```
*Performs ANOVA and t-test in SAS

ods graphics on;

title 'Hx-XXX'; /*hypothesis number and model name*/
data results;
set results;
Label tpvar="Tipping Variable" t="XXX"; /*indicator being tested*/
run;

data results;
set results;
proc means data=results mean std;
class tpvar;
var t;
run;

ods graphics on;
proc glm data=results;
class tpvar;
model t = tpvar;
means tpvar / hovtest ;
run;

ods graphics off;
data results;
set results;
proc anova;
    class tpvar;
    model t=tpvar;
    means tpvar;
    means tpvar / tukey;
  run;
  ods graphics off;
quit;
```