A GAME-THEORETIC MODEL OF MUTUAL BENEFITS IN BILATERAL NUCLEAR SECURITY REGIMES

A Dissertation

by

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ABSTRACT

States collaborate to achieve common goals. In the interest of advancing nuclear security globally, states have previously formed bilateral partnerships that allow two states to cooperate in germane areas of the nuclear industry such as safeguarding nuclear material, securing nuclear weapons, and advancing peaceful uses of nuclear technologies. Specifically, some states collaborate in establishing state-level strategies on nuclear security measures in order to protect against possible non-state adversaries (e.g., the Cooperative Threat Reduction and Material Protection, Control, and Accounting Programs between the Russian Federation and the United States). In an attempt to quantify utilities, a methodology has been developed within this work that uses game-theoretic models to measure the value of cooperation. In certain bilateral regimes, the opportunity for influence arises due to asymmetry between the partners. The developed methodology has the potential to identify circumstances under which one state might influence another in securing the latter's nuclear assets against possible non-state actors by virtue of a potential collective benefit in a bilateral cooperative nuclear security regime.

The methodology employs three different, but related, game-theoretic models – two using non-cooperative approaches and one using a cooperative approach. Determining the existence and magnitude of utilities between uncorrelated and correlated strategies provides the opportunity to study various cooperative strategies between states. The bargaining solutions of the cooperative game that models agreements providing a net benefit to both parties were then used to evaluate utilities of each such viable cooperative strategy, and the results compared. This process was applied to four case studies exhibiting a temporal progression of cooperation between the Russian Federation (as successor to the Soviet Union) and the United States and a fifth case study assessing possible cooperation between modern-day Pakistan and the United States. A result of applying the methodology to the former bilateral regime illustrated the use of nuclear insecurity as a potentially profitable commodity (a stated concern of nuclear deterrence and nonproliferation scholars). Two notable conclusions include 1) the level of investment for independent action by the states can impact the nature of a collaborative regime and 2) the collective total (investment and consequential) costs of a bilateral regime can be reduced but will require additional investment by at least one state. We conclude that the methodology developed here has the potential to assist future decision makers and analysts in quantifying the value of state-level cooperation for nuclear security.

DEDICATION

I dedicate this work to my daughters, Gabi and Nico, and my beloved wife, Mariana.

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1. INTRODUCTION

During the 2008 U.S. presidential race, then-Senator Barack Obama promised to "lead a global effort to secure all nuclear weapons materials at vulnerable sites within four years" in an effort "to prevent terrorists from acquiring a nuclear bomb."[1] This promise continued under the President's term in the form of numerous Nuclear Security Summits where significant progress had been made with numerous nations around the world.[2] States with nuclear weapons and other special nuclear materials have expressed an understanding of the necessity for protecting against potential threats or misuse and, in their efforts, have defined their respective nuclear security postures to consist of robust material control and accounting systems as well as border defense systems.[3,4] States form regimes which can be defined as "specialized arrangements that pertain to well-defined activities, resources, or geographical areas and often involve only some subset of the members of international society."[5] Additionally, states have formed and are still forming bilateral and multilateral regimes that could help achieve their goals of enhancing their nuclear and national security (examples include the U.S.-Russia Material Protection, Control, and Accounting Program and the U.S.-Kazakhstan BN-350 Blend Down Program).[6] Understandably within these regimes, it is not always guaranteed that a state acts in the best interest of the regime or of the other state(s). This work focuses on assessing state-level strategies and actions using game-theoretic analysis with an eye specifically for influencing nuclear security implementation against potential non-state actors. Historically, game theory that is based on classical, discrete zero-sum, non-cooperative games has been used as an assessment tool for evaluating strategies regarding nuclear deterrence. The present work models relationships as cooperative, continuous non-zero sum games. Cooperative game strategies model the terms of the agreement that define and maintain a bilateral cooperative nuclear security regime to counter the external threat of non-state actors. If nuclear terrorists cannot be dissuaded by traditional direct deterrence, perhaps state-level suppliers can be deterred from facilitating the acquisition of a nuclear asset by a non-state adversary (by influencing the state's nuclear security).

1.1. Objective

The objective of this research is to develop, test, and analyze a two-player game-theoretic methodology for identifying both: (a) the circumstances under which one state might be influenced by another in securing its nuclear assets against non-state actors by virtue of a collective benefit from the formation of a bilateral cooperative nuclear security regime, and (b) the nature of such a regime as might result from bargaining between the two states. The regime mentioned in the objective refers to the bilateral arrangement between the aforementioned target and source states. Initially, we use non-cooperative game theory to determine the respective utility of each player's independent (or uncorrelated) strategies and the potential additional utility (otherwise referred to as a "surplus") they would receive if they were to coordinate their strategies. Starting with a set of uncorrelated strategies for two players, if a second strategy set can be agreed upon by both where at least one player benefits and the other is not made worse, then this is the definition of a Pareto improvement.[7] The magnitude of the Pareto improvement helps define the utility surplus for both players. Within this context, if the surplus utility is divided among the players as a way to compensate the player who gains lesser benefits, this is a Kaldor-Hicks improvement.[8] In the present work, we determine how a surplus can be distributed by quantitatively defining bargaining solutions of the surplus between the players.

Herein, both players are nation-states looking to define nuclear security with non-negative costs (a combination of one state's security measures and the other's interdiction investments). A hidden but vital component to this work is the non-descript, non-state actor that serves as the impetus of the bilateral arrangement. Though this work does not identify or specify non-state actors, it is essential to the understanding of the work to accept the threat the non-state actor poses to both the target and source state and, therefore, the reason why both states would enter into such an arrangement.

Calculating utilities in game theory requires a common metric for evaluating strategies. Therefore, in an attempt to assign a common measure of utility, this work uses estimated total incurred costs (in USD) to a nation-state in both securing and interdicting nuclear assets. This concept is further touched upon in Section 3.5.

A necessary (but not sufficient) condition for forming a partnership is that coordination of strategies between the two states will provide more total utility to the bilateral nuclear security regime than if each state were to seek maximum utility without such coordination. In addition to the bargaining solutions, this work discusses relevant data issues and results for prototypical problems as well as historically representative cases where coalitions have been utilized for the benefit of nuclear security within a bilateral regime. Overall, the developed approach uses non-cooperative game theory to determine the existence and the division of a surplus utility and co-opts certain aspects of cooperative game theory to define an appropriate bilateral nuclear security regime.

1.2. Research Motivation

The intent of this work is to develop and explore a quantitative method of evaluating the attainability of a bilateral nuclear security engagement between two advanced nuclear states. The resources expended on two primary manners of engagement (securing and interdicting) comprise the states' strategies and the methodology discussed within this document provides a method by which the net benefit of collaborative actions by nation-states can be assessed. For instance, even Cold War adversaries (the United States and the Soviet Union) entered into agreements that benefited the greater good.[9,10] In these cases, both states saw benefit from entering into bilateral accords with the other. Perceived as a symmetrical relationship in terms of nuclear capabilities and relative position in the global community, the U.S. and the Soviet Union both aspired to become the world's nuclear super-powers and were able to invest national resources to accomplish this. However, asymmetrical relationships are another paradigm that introduce a less clear list of benefits to both states (with a potential clearer benefit to one side over the other). Historically, many bilateral relations between two asymmetric states have occurred because the "lesser" state power has possessed something of value to the "grander" state power: economic commodities such as fossil fuels or political leverage. Determining a method to quantify the benefits and costs of bilateralism is the intent here and to answer the question: when will two states come to terms on a mutually beneficial bilateral nuclear security regime? Potentially in future work, the author has interest to expand this consideration by proceeding beyond influence and entering the area of coercion or punitive actions (e.g., economic sanctions) between the states.[11]

Bilateral, regional, and multilateral regimes have previously been used to address arms control, nuclear material safeguards, and nuclear security. For example, New START between the U.S. and Russia for arms control and the safeguards inspection regime between Argentina and Brazil provide references for studying the benefits of establishing bilateral regimes between adversarial states (which shared an initial lack of trust).[12,13] The eventual bilateral regimes that resulted have since created a pronounced level of trust in nuclear weapons control and material safeguards, respectively.[14]

Regional regimes, in the forms of safeguards inspectorates and geographical nuclear weapon free zones, have united regions with common goals such as applied nuclear material safeguards and prohibiting the presence of nuclear weapons within the vicinities. These accords have proven the capability of states to unite over nuclear activities. Examples include the regional ABACC (Brazilian-Argentine Agency for Accounting and Control of Nuclear Material) or EURATOM (European Atomic Energy Community) safeguards inspectorates (where France and the United Kingdom are subjected to EURATOM-administered safeguards).[13,15] However, as exhibited with the Agency for the Control of Armaments of the Western European Union, regional solidarity can break down.[16] Until 1987, the Agency of Western Europe was a regime where nations collected their conventional military forces declarations into a collective pool and then inspections would occur with all players. If this paradigm were applied to all declared nuclear-armed states (the United States, Russia, China, France, the United Kingdom, India, Pakistan, and the Democratic People's Republic of Korea), disparities would exist in the quantity of reported sites. As it occurred with the Agency of Western Europe, this disparity could create animosity between states whose sites would be randomly chosen for inspection more frequently than others with fewer sites. For example, where Russia could declare twenty different facilities, the DPRK would report two; and therefore, would get their sites selected ten times less frequently than Russian sites.

Multilateral regimes such as the Nuclear Nonproliferation Treaty Regime or International Atomic Energy Agency (IAEA) safeguards have also been established and are being maintained well beyond bilateral partnerships and regional consortia. Of particular interest are the Safeguards regime of the IAEA and the Convention on the Physical Protection of Nuclear Material (CPPNM).[17] The CPPNM focuses on nuclear security of civilian-use material, and both India and Pakistan adhere to it. The IAEA Safeguards regime is a system of numerous documents and over 140 states with agreements for declaring, inspecting, and monitoring special nuclear material.[18]

There is a long-documented history of states collaborating for meeting the comprehensive global nuclear weapon threat – consisting of weapons disarmament, establishing nuclear weapon free zones, safeguarding civilian nuclear material, countering nuclear threats, securing nuclear material around the world, controlling the international trade of nuclear weapons relevant technologies, and more. In addition to multilateral accords such as the Nonproliferation Treaty, the Comprehensive Test Ban Treaty, the

Additional Protocol, and regional accords for establishing nuclear weapon free zones (such as the Treaties of Tlatelolco, Raratonga, and Pelindaba), there have been many bilateral arrangements which have come into existence for the collective benefit of the partnership (a subset is included in Table 1.1). Notably, many such engagements involved (at least) one of the parties gaining other benefits unrelated to nuclear weapons and material.

Table 1.1. Bilateral and multilateral engagements addressing nuclear threats

Date	**Engagement/Treaty**	**Parties**
1972	Signing of Anti-Ballistic Missile (ABM) Treaty and SALT I	U.S.-U.S.S.R.
1988	Intermediate-Range Nuclear Forces (INF) Treaty in force	U.S.-U.S.S.R.
1991	ABACC is established (Argentina and Brazil)	Argentina-Brazil
1991	Cooperative Threat Reduction authorized by Congress	U.S.-Russia
1994	U.S.-China Statement on Missile Proliferation	U.S.-China
1994	Agreed Framework between U.S./DPRK	U.S.-DPRK
1994	Project Sapphire: 0.5T of HEU from Kazakhstan to U.S.	U.S.-Kazakhstan
1996	START II (reduction to 3000-3500)	U.S.-Russia
1998	Russian security upgrades under U.S. MPC&A Program	U.S.-Russia
1999	Lahore Declarations reduce tensions after nuclear tests	India-Pakistan
2003	Megatons to Megawatts eliminates 175T HEU	U.S.-Russia
2005	Russia and Canada deconstruct nuclear submarines	Russia-Canada
2009	New START talks begin between U.S. and RF	U.S.-Russia

States have shown that their actions can either be made independently of any other partner state or in conjunction with another. Bilateral alliances such as those shown in Table 1.1 have exhibited various similarities as well as differences – each has benefits and disadvantages that depend on several issues. The present work sets out to study the circumstances under which such benefits do or do not exist under bilateral actions.

In Verdier's treatise of the relationship between bilateralism and multilateral regimes, nuclear nonproliferation via multilateral instruments (such as the Nuclear Nonproliferation Treaty) is primarily achieved with "superpower bilateral (or dyadic) diplomacy."[19] He implies that without bilateralism, any attempt at an eventual multilateral arms and material control regime would be unsuccessful. Furthermore, bilateral regimes provide the opportunity to customize offers to reflect each state's specific circumstances as opposed to enforcing adherence to uniform terms across all participants. The present work can be seen as an attempt to quantitatively study the use of bilateralism in addressing nuclear security.

1.3. Background

Until the fall of the Soviet Union, in an essentially bipolar world, securing nuclear assets ready to be used at a moment's notice comprised applied nuclear security. However, post-Cold War, concerns were elevated by the potential of nuclear terrorism as economic strife affected Soviet nuclear security.[20] These concerns were magnified and brought to the forefront by the 9/11 terrorist attacks in 2001. The previously-held strategies for deterring nuclear attacks needed to be modified to meet the growing threat from terrorists: "traditional concepts of deterrence will not work against a terrorist enemy whose avowed tactics are wanton destruction and the targeting of innocents."[21]

1.3.1. Influencing Strategies

States[a] have endeavored to influence others' strategies in the nuclear realm for many years. As an example, the study of deterring a state from using nuclear weapons against another (nuclear deterrence) has benefited from the development of numerous analytical methods. Studies of the "waves" of deterrence theory study began with simple cost-benefit analyses by Brodie and Wolfers and more analytical approaches developed by Schelling, Kaplan, Dresher.[22,23,24,25,26] The latter wave weighed costs and gains in state-level strategies and determined the feasibility of influencing states by the *credible* threat of punishment or balancing influence via punishment (increasing costs) or denial (decreasing gains). Modern scholarly work (such as from Knopf) has grown as a response to the fall of the Soviet Union where non-state adversaries must be dissuaded from action as well.[27] Scholars have applied deterrence theory to nuclear terrorism – namely as a viable strategy against asymmetric threats, but disagreement lies in its implementation: 1) directly deter terrorist organizations themselves and 2) indirectly deter potential supplier states.[28] In the former case, some claim there exists a semblance of rationality in the leadership of terrorist organizations who care about the longevity of their organization, the long-term prospect of their message, and "high-valued targets including family members and supporters."[29]

In the latter implementation (indirect deterrence), researchers rely on rationality as the reason by which deterrence works. Castillo details numerous scenarios in which a non-

[a] "States" in this context refers to sovereign nations under a single system of government as opposed to federated states under partial sovereignty as a member of a federal union.

state actor acquires such a nuclear capability through a 'nuclearly' capable (and amoral) state and concludes that deterrence against rogue states is a feasible option.[30] With the concern of WMD proliferation by new states, Colby argues that outside a conventional defensive posture, deterring states from transferring weapons is the strongest strategy available to the nuclear powers.[31] Moreover, scholars such as Lieber and Press argue that deterrence is only as effective as the forensics capabilities of the regime: if material that is used against or interdicted by a deterring state can be attributed back to its source, the likelihood of a state *knowingly* supplying a nuclear weapon to a terrorist organization decreases.[32] Though discussed and written about, quantifying deterrence strategies (admittedly, indirectly) against non-state actors has not been studied as thoroughly as against states.[33] Thus, the discussion within this work is focused on using a game-theoretic model as a tool for assessing potential influence over state-level nuclear security strategies against an unspecified, non-state threat.

1.3.2. Nuclear Material Security

With lessons learned from post-Cold War joint U.S.-Russian activities, it was concluded that inhibiting terrorist activities included restricting their access to weapons grade material and nuclear assets in general.[34] Moreover, denying the acquisition of weapons and weapons-usable nuclear material is paramount to deterring terrorist actors. Particularly, nuclear terrorism was a large concern after the end of the Cold War while the newly-formed Russian Federation had thousands of sites with little to no protection, control, or accounting measures to ensure the security of such material.[35,36] This circumstance changed with the U.S.-Russian Cooperative Threat Reduction (CTR) program in which the U.S. collaborated with the Russian government to, among other activities, secure their nuclear material and assets.[37]

Securing weapons-usable material is vital to a responsible nuclear weapons control regime. Existing nuclear-weapon states (as defined by the Nuclear Nonproliferation Treaty or NPT) have adhered to numerous international and bilateral treaties and agreements to assure the security of their arsenals and material (with some deficiencies still in existence). A problem lies in the nuclear-armed states that are not NPT nuclear weapon states, where information is neither open for scrutiny nor even shared with a reliable partner. How secure are the assets in those states? How can a state ensure these assets do not find their ways into the hands of terrorists or groups wishing to do harm? By securing nuclear assets, a state can convey its commitment to the responsibility of having such items within their boundaries. The state's verifiable strategy of applied security against theft or non-authorized use is essential to the bilateral approach considered in the present work.

The concept of material control and accounting is well-known in international safeguards. The process of establishing state systems of accounting and control and the formulae for calculating material unaccounted for, material balance areas, material balance periods, and significant quantities (SQ) are well-established.[38] The concept of continuity of knowledge is essential in controlling high-valued material/assets such as nuclear weapons. Much can be gleaned and applied from the aforementioned concepts for securing nuclear assets.

Through international engagements and current events, the culture of nuclear security is growing stronger.[39,40] Current international and bilateral endeavors led by states/organizations such as the U.S., the Russian Federation, Japan, and the IAEA have contributed to the overall global nuclear security culture by raising awareness and understanding among the countries' current and future nuclear workforces. Programs such as the Partnership for Nuclear Security at the U.S. State Department, the Global Initiative to Combat Nuclear Terrorism (GICNT), the Integrated Support Center for Nuclear Security of the Japanese Atomic Energy Authority, and the IAEA's International Nuclear Security Education Network all facilitate interaction between nuclear professionals in various countries for the sake of enhancing nuclear security awareness, understanding, education, and application.[41,42,43,44]

1.3.3. Assessing State-Level Strategies

Many studies have been conducted to determine metrics for assessing state-level nuclear security strategies.[45,46,47] Tools for measuring effectiveness of collaborations have mostly been qualitative.[48,49] Some, however, attempt a quantitative approach. A 1992 U.S. Government Accountability Office report discusses the potential for evaluating the use of economic sanctions by describing the goals looking to be achieved and the difficulty in assessing incurred costs to a state with gross gains for that state.[50] In evaluating methods to assess border security metrics, Rosenblum and Hipsman define four ways to measure the effectiveness of border security and present nine various methods used to calculate the metrics.[51] To determine the number of unauthorized immigrants entering between ports of entry, the authors of that study explored a deterministic approach in utilizing a recidivistic model, a survey-based assessment, and observational data collecting. To evaluate a second metric (the number of unauthorized immigrants entering through official ports of entry), the authors referenced a program used by the U.S. Customs and Border Protection (but no reports on the result are available). They lastly use the number of immigrants overstaying their visas (gleaned from third-party and exit data) and the number of unauthorized immigrants living in the U.S. (calculated by using Census data between legal non-citizens from total non-citizens). Admittedly not precise, these methods comprise a comprehensive approach used for evaluating the effectiveness of border security and immigration control for the entire U.S. by today's government researchers.

Some have endeavored in ranking state-level security strategies thus introducing controversy such as the Nuclear Threat Initiative's (NTI) Nuclear Security Index (NSI).[52] In this index, states with nuclear materials are evaluated against each other in terms of adherence levels regarding nuclear security. Researchers at NTI have compiled multiple data points on relevant states to construct grades for states based on various nuclear security elements. Opening controversy with their analysis, NTI researchers attempt to quantify factors by assigning discrete numbers to how well a state's adherence is or how many accords the state has signed.[53] Though considered faulty by some in the international nuclear security industry, the NSI at least exhibits an initial attempt at evaluating state nuclear security efforts that can be further refined with time.[52,53]

1.4. Chapter Overviews

A summary of the remainder of this document follows. Chapter 2 provides a detailed discussion on game-theoretic analysis with a section each on how game theory is used, previous applications of game theory relevant to the nuclear industry, and a discussion consisting of combining some useful aspects of both non-cooperative and cooperative game theory analysis in Sections 2.4 and 2.5. Chapter 3 breaks the methodology into three sections: a top tier-level explanation of defining the model and players' strategies and how they are applied to the five case studies detailed in Chapters 4-8; a decomposition of the cost functions into terms and factors for which at least approximate state-level data might be available; and an explanation of how the functions are to be used in solving for the non-cooperative and cooperative strategies in each case study. Sections 3.4 and 3.5 focus on caveats of the present work including trust/commitment between nation-states, metrics of the game models, and the incommensurability of results. Chapters 4-8 detail five case studies meant to illustrate the use of the developed methodology: U.S.-U.S.S.R. relations during the height of the Cold War circa 1985, U.S.-Russia relations in three different historical contexts (1995, 2008, and 2015) and the U.S.-Pakistan relationship in 2008. For each case study, each chapter begins with a section that uses publicly available data to evaluate the necessary parameters for the threat cost functions. The second section consists of replicating observed characteristics of the relationship between the two states. Using a non-cooperative approach, we determine both uncorrelated and correlated solution concepts as a means to analyze and gain insight into the bilateral relationship for nuclear security. The third section uses the difference between both solution concepts to illustrate bargaining solutions between the states. The fourth section of each chapter provides a discussion of the results. After all case studies are presented, Chapter 9 includes a discussion on the methodology itself in three parts: summarizing results (particularly on the four Russia/USSR-US partnerships), defining performance indicators, and addressing the sensitivities of input parameters to the results defined in the first section of each case study. Chapter 10 closes with potential future work that falls outside of the current scope, as well as conclusions regarding the methodology.

2. GAME-THEORETIC ANALYSIS

Game theory is the process of modeling strategic interaction between two or more players. We introduce it in this chapter so as to ensure a common understanding of terminology, concepts, and expectations prior to showing its application in the methodology described in Chapter 3. Specifically, game theory models the interaction, preferences, strategies, and outcomes of self-interested players. A game of one player reduces the model to a decision problem – where the decision maker defines a set of strategies and their respective outcomes.[54] The latter is often defined by devising a function that quantifies outcomes of strategies. Mapping choices to real numbers for appropriate strategy building requires using an objective utility function, u: it might represent a gain (in which case it is to be maximized) or a cost (in which case it is to be minimized). The preceding point makes a common assumption of rationality: choosing strategies that optimize utilities (i.e., optimizing a cost-benefit difference).

A game-theoretic model exhibits some similarities with decision theory but the primary difference includes accounting for the effects of other players' decisions on each other's outcomes. For example, a player using expected utility theory to determine their payoff from a certain strategy must also account for the strategy of another and its effect on his own utility. Therefore, the player assumes the strategy that seems as the best response to the most likely strategy of his counterpart. Following this paradigm, a game theorist can then construct strategy sets between the players (also referred to as solution concepts). Various types of games have been theorized and used for analyzing numerous types of scenarios in several different fields of research: biology, computer science, arms control, psychology, and economics. Some scholars posit the benefit of using game theory to model for terrorism applications over decision theory because of the nature of defining strategies in light of an active counterpart.[55] Solution concepts for both decision theory and game theory can exhibit either discrete or continuous utilities. Most simple introductory game-theoretic models, as featured in the remainder of this chapter, are discrete. By contrast, the strategy spaces of the models used for the case studies of Chapters 4-8, are continuous.

The work discussed herein uses elements of both non-cooperative game theory (NCGT) and cooperative game theory (CGT). The former is concerned with the analysis of strategies where each player is making choices out of their own interest. The latter focuses on the analysis of cooperation between players where outcomes are bargained. To be clear, cooperation can, and often does, occur in NCGT models but only when a clear benefit is perceived by the players. Subsection 2.4 includes some considerations regarding how CGT is being applied in this developed methodology.

Firstly, we introduce basic elements of NCGT including solution concepts to define strategy spaces and previous applications as well as certain useful elements of CGT (e.g., solution concepts, bargaining theory, and utility transferability among players).

2.1. Elements of Game Theory

Modern game theory was born out of Von Neumann's 1928 seminal publication.[56] Since then, various game theoretic concepts have risen from numerous scholars and other published works. Historically (and still in use today), the most common method of representing basic games is exhibiting them in strategic (or normal) form. In this manner, each player's choices are shown in rows or columns with the outcome of their choices conveyed as payoffs or utilities respectively. Figure 2.1 below conveys the normal form of a simple, two-player game.

		Player 2	
Player 1		L	R
	T	(a,-a)	(b,-b)
	B	(c,-c)	(d,-d)

Figure 2.1. Simple 2-player game in normal form.

Players 1 and 2 have strategy sets with respective choices of T or B and L or R, respectively. The chosen strategies then yield four outcomes (a,-a), (b,-b), (c,-c), and (d,-d) where each variable represents the magnitude of the players' utilities. In this game, the utilities are of opposite magnitudes. This implies a zero-sum game of limited utility: what is won by one player is lost by the other. Non-zero sum games are when a player's payoff is not bounded by the other player's. In normal form, the temporal component is not included: the respective timing of players' decisions is not incorporated into the game. This means that the game is not specific for either simultaneous or sequential games. For this aspect, we change to a more detailed depiction of a game using the extensive form or a game tree as shown in Figure 2.2 below.

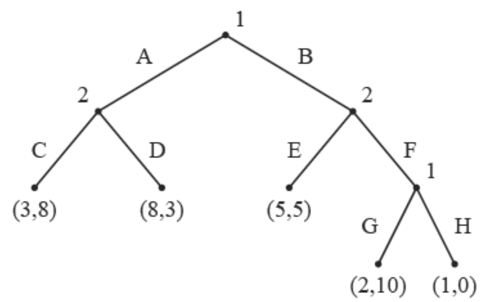

Figure 2.2. Simple 2-player game in extensive form

In Figure 2.2, the sequence of moves by the players is explicitly shown. Each node represents a player's choice between the letters over each branch and the final terminal nodes show the outcomes with respective payoffs for each player. This form of analysis allows for the modeling of the order of players' decisions as well as the information known to them at the time of their decision. Both forms can model sequential and simultaneous games but the displayed outcomes of the extensive form are limited to discrete payoffs whereas the normal form can model continuous payoffs by being able to incorporate a utility or objective function based on the players' strategic decisions. It is with this latter form, we devise the methodology in Chapter 3.

2.2. Solution Concepts

In game theory, a solution to a game is merely a prediction of the players' strategies and resultant payoffs. A player's array of rational choices forms the strategy set with his associated payoffs (which can be calculated using a utility function, *u*). A solution concept is the pairing of the most likely strategy of each player that often exhibits certain desirable properties. Some commonly-mentioned properties include the existence of a solution (such that the solution exists for all games which it is applied); the uniqueness of a solution (only one set of strategies forms the solution concept); an element of self-enforcement (where the players choose not to deviate from their strategies due to receiving higher payoffs); and the limitation of pareto optimality (in that collective improvement of utility is not achievable).[57] Though there are no universally-accepted methods in determining solutions for either simple matrix or repeated games, theorists can apply different strategy-defining principles so as to predict solution concepts within the game.[57] Assume the simple, 2-player game presented in Figure 2.3 where each player has two choices and they know their payoffs for each strategy set – e.g., $[T\ \ , L\ \ ]$ results in (5,5).

		Player 2	
Player 1		L	R
	T	(5,5)	(10,0)
	B	(0,10)	(1,1)

Figure 2.3. Normal form game with payoff structure

Player 1's strategy set is {T,B}. If he knows what his payoff is with either of his choices based Player 2's strategy set {L,R}, he can plan his strategy accordingly. The outcomes presented in Figure 2.3 are listed respectively for Player 1 and 2. Using these payoffs, Player 1 deduces that strategy T will always yield a greater payoff than B. Therefore, strategy T is the dominant strategy. Reciprocated from Player 2's perspective, strategy L is the dominant strategy because regardless of Player 1's strategy, Player 2 receives a payoff of 5 or 10. Hence, based on dominant strategies, the solution concept is $[T, L]$ with an outcome of (5,5).

Maximin and *minimax* strategies form the basis of the next two solution concepts. The *maximin* strategy of Player 1 consists of the choice whereby Player 1's worst-case payoff is maximized. Assuming a worst case scenario, the *maximin* strategy is a conservative choice that results in the best case minimal payoff. The *minimax* strategy of Player 1 is more of a defensive posture whereby Player 1 chooses the strategy which would limit Player 2's maximum payoff. Applying the *maximin* strategy to his strategy set in Figure 2.3, Player 1 would review his minimum payoffs and choose the strategy that would provide him the higher minimum: strategy T with a higher payoff of 5 over 0). Conversely, Player 2 would also review his payoffs and choose L with a higher payoff of 5 over 0. If Player 1 adopts the *minimax* strategy, he would choose the strategy that would limit Player 2's payoffs the most: strategy T. Vice versa, Player 2 would reciprocate and choose his own strategy L. Thus, the solution concept of $[T, L]$ occurs with each type of strategy: dominant, *maximin*, and *minimax*.

Additionally, two of the most fundamental solution concepts are the Pareto Optimum (PO) and the Nash Equilibrium (NE).[57,58] Both solution concepts are based on different principles than the aforementioned three. The PO solution concept is a strategy pair where there exists no other pair such that the payoff to at least one of the players is better and that to neither is worse. The only pair that exhibits this is $[T, L]$ again. Lastly, if each player reviews the outcomes and aims to choose the strategy that shall benefit them the most in light of a rational assumption for the other player's chosen strategy, the resulting solution concept is the NE (which is also referred to as the best response strategy set). Notable for the NE is the implication that this solution concept is self-enforcing in that each player prefers this strategy for their respective outcome based on the kind of counterpart they are playing against. For Figure 2.3, the NE is $[T, L]$ again.

The five previously discussed solution concepts are limited as pure strategies for explanation purposes only. The outcomes for each strategy pair are strictly discrete in Figure 2.3. However, in analysis of more realistic games, payoffs can reside on a continuous spectrum. For that reason, probabilities are incorporated into solution concepts as a way of deviating from strictly pure strategies to mixed strategies. By using mixed strategies, one can always determine a NE – this is referred to as the Mixed-Strategy Nash Equilibrium (MSNE).

<table>
<tr><td></td><td colspan="3">Player 2</td><td></td></tr>
<tr><td rowspan="3">Player 1</td><td></td><td>L</td><td>R</td><td></td></tr>
<tr><td>T</td><td>(8,2)</td><td>(1,6)</td><td>p</td></tr>
<tr><td>B</td><td>(3,5)</td><td>(4,1)</td><td>1-p</td></tr>
<tr><td></td><td></td><td>q</td><td>1-q</td><td></td></tr>
</table>

Figure 2.4. Solving for the MSNE

Consider the game specified, in normal form, by Figure 2.4 above. Assume the game is played once, the players make their choices simultaneously, and they both know the other's payoffs. When Player 2 chooses L, it behooves Player 1 to choose T. When Player 2 chooses R, it would behoove Player 1 to choose B. Conversely, when Player 1 chooses T, Player 2 should chose L. And when Player 1 chooses B, then Player 2 should choose L. Assuming only pure strategies, the NE is not a solution concept. However, by assuming the game is repeated with mixed strategies, every game has a Nash Equilibrium. Generally, probabilities are assigned as frequencies of a strategy being selected in an infinite number of trials. Specifically, in Figure 2.4, p and q represent how frequently Player 1 chooses T and Player 2 chooses L, respectively. Therefore, as Player 2 defines his best response strategy, he must first compute Player 1's expected payoffs from his pure strategies T and B in terms of Player 2's strategy choice based on q:

$$T = q \times 8 + (1 - q) \times 1 \tag{2.2.1}$$

$$B = q \times 3 + (1 - q) \times 4 \tag{2.2.2}$$

With his strategy set {T,B}, Player 1's highest expected payoff depends on which strategy Player 2 chooses. Plotting both Equations (2.2.1) and (2.2.2) in Figure 2.5 shows the relationship between Player 2's strategy frequency q and Player 1's payoff.

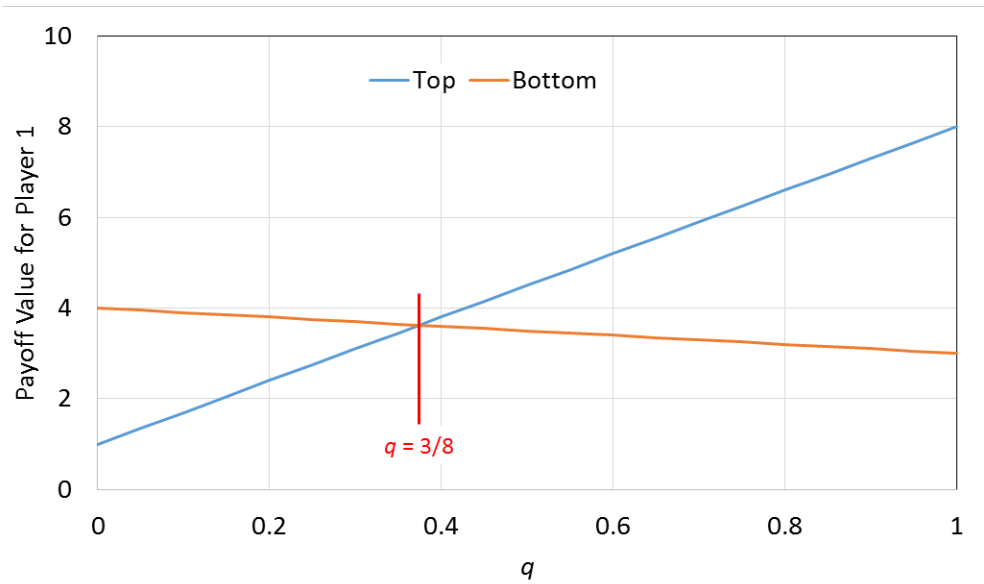

Figure 2.5. Expected payoffs for player 1

When Player 2 chooses strategy L more than 37.5% of the time, it behooves Player 1 to have chosen T. Conversely, it Player 2 chooses strategy L less than 37.5% of the time, Player 1 would receive a higher payoff if he chooses B. Therefore, in an effort to minimize Player 1's payoff, Player 2 chooses strategy L 37.5% of the time.

Conversely, to determine his best response mixed strategy, Player 1 would pursue Player 2's expected payoffs for his strategy set {L,R} per the expected utility Equations (2.2.3) and (2.2.4):

$$L = p \times 2 + (1-p) \times 5 \tag{2.2.3}$$

$$R = p \times 6 + (1-p) \times 1 \tag{2.2.4}$$

Solving for Player 1's best strategy yields $p = {}^{1}/_{2}$. Plotting Equations (2.2.3) and (2.2.4) as shown in Figure 2.6 below convey the relationship of Player 2's utilities with the probability of Player 1's strategy.

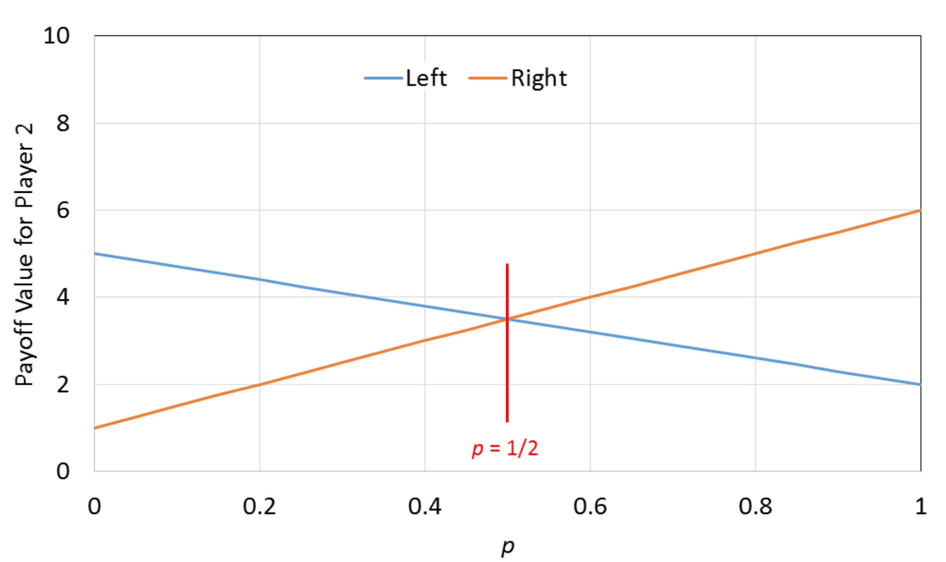

Figure 2.6. Expected payoffs for player 2

Similarly conveyed in Figure 2.5, Player 2 should choose L if Player 1 chooses T under 50% of the time or, vice versa, should choose R if Player 1 chooses T over 50% of the time. If this game will only be played once and Player 1 is interested minimizing Player

2's payoff, he should play T with a probability of 50%. Conclusively, the NE solution concept is $(^1/_2, ^3/_8)$.

2.3. Applied Game Theory on Nuclear Issues

Game theory has been used in the past as an analytical tool for analyzing hybrid technical-political problems by evaluating and assessing real-world situations such as gun control, war games, and safeguards inspections.[59,60,61] More specifically, previous game-theoretic studies have been conducted to help understand nuclear-related situations, such as state-level economic coercion, arms verification/nuclear safeguards inspections, and the Cuban Missile crisis.[62,63] These assessments employ non-cooperative games which are analyzed to identify the Nash Equilibrium and other solution concepts.[64] More so, on the Cuban Missile Crisis, Fraser and Hipel argue the benefits of advanced game-theoretic models for analyzing "games where information is incomplete or misleading."[65] In their 1996 work, Avenhaus and Canty apply game theoretic analysis for nuclear material safeguards inspections within the international safeguards regime.[61] They construct a game-theoretic model for determining the number and frequency of a state's safeguards inspections. They optimize the number of these inspections by finding the NE where the gains of both the inspector and the inspectee are maximized. In later work, they invoke Diamond in identifying minimax policies of players in a 2-player, zero-sum inspections game.[66]

Dresher provides an assessment of a structure for an arms inspection regime that gives maximum insurance that an agreement is not violated.[26] He frames the problem as “unlike sampling in manufacturing, sampling for arms control must take into account that the statistical population may be altered to conceal a violation” – this is where the “game of strategy (exists) between the inspector and the inspected.” He continues by defining the situation as a two-person (Inspector and Inspectee) zero-sum, repeated, non-cooperative game where v is the payoff to the inspector and, due to the zero-sum nature of the game, $-v$ is the payoff to the Inspectee (in Figure 2.7, the payoffs are from the perspective of the Inspector). As per Avenhaus, if n is the number of inspection opportunities and m is the number of inspections the Inspector chooses to conduct, then $m \leq n$. If a violation is made by the Inspectee and the Inspector had chosen to inspect, the payoff to the Inspector increases by 1. Contrarily, if a violation is made and the Inspector had chosen not to inspect, the payoff to the Inspector is -1. Conversely, the Inspectee who had violated and the inspector would have received -1 and +1, respectively. If the Inspectee had acted legally and not violated in either case, the change to the payoffs would have been that the number of inspection opportunities, n, would have decreased by one and, if the Inspector had inspected, the number of inspections, m, would have decreased by one as well (otherwise, with no inspection occurring, m does not change). The design of this game allows for the termination after a detected violation (where the Inspector would have received -1 payoff) or after the n periods (where the Inspector would have received a payoff of 0).

Inspector	Inspectee: Legal Action	Inspectee: Violation
Inspection	v(n-1,m-1)	+1
No inspection	v(n-1,m)	−1

Figure 2.7. Dresher's bimatrix game showing inspector's payoffs

Avenhaus then calculates a mixed-strategy equilibrium by employing probabilities p and q (similar to Section 2.2 where probabilities are more accurately defined as the frequency in an infinite amount of trials) to find the equilibrium value of the payoff, v, for the Inspector. Using the same approach as the previous section, the values for the Inspectee are based on p:

$$v_{legal} = p \cdot v(n-1, m-1) + (1-p) \cdot v(n-1, m) \tag{2.3.1}$$

$$v_{violation} = p \cdot (+1) + (1-p) \cdot (-1) \tag{2.3.2}$$

And the value for the Inspector are based on the probability q:

$$v_{inspect} = q \cdot v(n-1, m-1) + (1-q) \cdot (+1) \tag{2.3.3}$$

$$v_{not} = q \cdot v(n-1, m) + (1-q) \cdot (-1) \tag{2.3.4}$$

To find the equilibrium of their strategies, the values, v, for each must be equal. Hence, for the Inspectee, Equation (2.3.1) must equal Equation (2.3.2) to determine p:

$$p \cdot v(n-1,m-1) + (1-p) \cdot v(n-1,m) = p \cdot (+1) + (1-p) \cdot (-1)$$
$$p \cdot v(n-1,m-1) + v(n-1,m) - p \cdot v(n-1,m) = 2p-1$$
$$2p + p \cdot v(n-1,m) - p \cdot v(n-1,m-1) = v(n-1,m) + 1$$
$$p \cdot \left[2 + v(n-1,m) - v(n-1,m-1)\right] = v(n-1,m) + 1$$

$$p = \frac{v(n-1,m)+1}{2+v(n-1,m)-v(n-1,m-1)} \qquad (2.3.5)$$

And for the Inspector, Equation (2.3.3) must equal Equation (2.3.4) to determine *q*:

$$q \cdot v(n-1,m-1) + 1 - q = q \cdot v(n-1,m) + q - 1$$
$$q \cdot v(n-1,m-1) + q \cdot v(n-1,m) - 2q = -2$$
$$q\left[v(n-1,m-1) + v(n-1,m) - 2\right] = -2$$

$$q = \frac{2}{2-v(n-1,m-1)-v(n-1,m)} \qquad (2.3.6)$$

By combining Equations (2.3.5) and (2.3.6), Avenhaus and Dresher calculated the NE for the Inspector as:

$$v(n,m) = \frac{v(n-1,m)+v(n-1,m-1)}{2+v(n-1,m)-v(n-1,m-1)} \qquad (2.3.7)$$

In his more recent body of work (2011, 2012, and 2013), Avenhaus (with Krieger) analyzes unannounced interim inspections for safeguards, spent fuel storage facilities inspections in Europe, and pathways states may use to divert nuclear material for military purposes from declared civilian facilities.[67,68,69] In the latter publication, Avenhaus and

Krieger solve for the MSNE in a number of models consisting of an inspectorate and a state considering a path for material diversion. They form non-cooperative games with strategy sets of discrete payoffs for each player and incrementally complicate their simple form models to include real-world factors such as additional strategies, frequency of inspections, false alarms, and non-detection probabilities. Avenhaus and Krieger then conclude that determining how the equilibria of strategies affects the state player can be an effective model of deterrence of illegal activity under a comprehensive safeguards system.

In 1984, the U.S. Nuclear Regulatory Commission (NRC) sponsored a feasibility study for a strategic analysis of safeguards systems by A.J. Goldman.[70] This work investigates the use of game theory for nuclear material accountancy at a generic nuclear facility under the NRC. Goldman aims to set alarm thresholds for inventory differences in accounting by defining applied safeguards as a zero-sum, non-cooperative game between the facility and a generically-defined intelligent "diverter." The study continues to expand the potential for game theory in regulatory settings to become more complicated as more players and more options are assumed. Furthermore, Goldman expands on the possibility that these such games should not be considered zero-sum (what the diverter gains is what the defender loses) but more so should be studied further to understand the potential value of certain outcomes through analytic, though subjective, methods such as multi-attribute utility theory. Goldman also introduces the concept of ordinal payoffs as a way to remove subjectivity from determining a proper payoff function and states that they can be used as a "fallback position if the cardinal approach flounders." The problem of defining an appropriate utility function to determine payoffs is a common theme in game theory and is approached in the present work via financial costs to a state.

2.4. Non-Cooperative Game Theory: Cooperative Solution Concepts

Sections 2.1 and 2.2 discuss non-cooperative game theory (NCGT), its elements, and its solution concepts. As stated in the preamble to this chapter, NCGT has commonly been used for conducting state-level analyses assuming states act in their own best interest and they do not know their counterparts' strategies. However, herein, we begin to consider how things change when players are able to cooperate. Nash explicitly states that in order for cooperation to exist, players must be able to communicate their strategies and intent.[71] Hence, by incorporating communication between players, we can refine aforementioned non-cooperative games into cooperative ones. Under NCGT, solution concepts for games with cooperation can be attained in two ways: players reach an equilibrium through repeated play of the game or a third-party arbiter or 'umpire' prescribes an achievable equilibrium point to the players. The latter would include the possibility of the third-party serving as an enforcer and would necessitate communication (bargaining, negotiation) between the players.

Summarily, cooperative actions are not prohibited in NCGT as long as there exists an individual payoff to justify it as well as an agreement between the players. Solution concepts (as introduced in Subsection 2.2) such as the Nash Equilibrium, the Pareto Optimum, the *minimax* and *maximin* strategies can describe potential "solutions" of non-cooperative games. However, determining "solutions" (i.e., unique equilibrium points) of cooperative games has not proven to be as straightforward. Particularly, Shubik (pg. 3) alludes to the worthwhile yet eventually unsuccessful efforts by Harsanyi and Selten to "select a single equilibrium point that would be an appropriate solution for any game."[72] In sum, the segue into CGT from NCGT is not as seamless as one might hope.

To convey how one can arrive at a cooperative solution concept in a non-cooperative game, we present the Prisoner's Dilemma game as shown in Figure 2.8. The solution concept in this game is made up of each player choosing from his strategy set of {Silence, Confess} so as to provide an agreeable benefit to himself. The payoffs presented herein (i.e., the length of their jail sentences) refer to what each prisoner receives after they are apprehended for a crime and interrogated separately. In other words, if both players had previously coordinate their strategies and stay silent, they will each be sentenced only 2 years in prison. If one confesses in order to receive 1 year, then the other will be blamed for the crime and consequentially receive a full 10 years in prison. If both confess to the crime hoping to receive lighter sentences, they will equally be punished as accomplices to the other and both be sentenced to six years.

		Player 2	
		Silent	Confess
Player 1	Silent	2,2	10,1
	Confess	1,10	6,6

Figure 2.8. Prisoner's Dilemma game with payoffs for both players

Each solution concept discussed below encompasses some properties mentioned in Subsection 2.2. Table 2.1 summarizes these concepts and whether they exhibit various properties such as uniqueness, existence, optimality, and self-enforcement. It is beneficial to consider these while discussing each solution concept.

Table 2.1. Summary of concepts and properties

Concept \ Property	Existence	Uniqueness	Self-enforcing	Pareto Efficiency	K-H Criteria
Nash Equilibrium (NE)	Y	Y	Y	N	N
Pareto Optimality (PO)	Y	N	N	Y	Y
Maximin	Y	Y	Y	N	N
Minimax	Y	Y	Y	N	N
Kaldor-Hicks (K-H) Optimality	Y	N	N	Y	Y

The first non-cooperative solution concept we consider is the Nash Equilibrium (NE). If each player acts independently to maximize their own utility (i.e., minimizing their costs), each player chooses to Confess. Hence, the NE solution concept is (Confess, Confess). Particular to the NE solution concept is that neither player can improve their payoff by merely changing their own strategy – this adds the element of the strategies being self-enforced – where strategies are selected as the best response to the other player's perceived strategy. Hence, the strategies are self-enforced in that the players will not deviate for their own benefit. Due to the NE being the concept resulting from unilateral responses, it is neither pareto efficient nor meets Kaldor-Hicks criteria (meaning that one player receives enough benefit to bribe the other into this solution concept).

Let us assume the players have a pre-negotiated understanding where, if caught, they agree to remain silent – resulting to a solution concept of (Silent, Silent). Compared to the NE solution concept, both players receive less jail time. This is the Pareto Optimum (PO) where there exists no other strategy pair such that the payoff to at least one of the players is better and that to none is worse. This solution concept requires a great amount of trust, or some enforcement mechanism to ensure each player is compliant to the agreement because of assurance the other does not act selfishly and thereby condemn the compliant first player to a 10-year jail sentence. Per Table 2.1, the PO does exist but is not unique nor self-enforcing due to a PO solution concept lying on the efficiency frontier where any solution point can be agreed upon and that there is no guarantee that either player would not deviate from their stated strategy. However, the PO is pareto efficient (implying that whatever is gained by one player is lost by the other) and can meet Kaldor Hicks criteria of one player receiving enough of a payoff to provide to the other (explained in more detail later in this section).

In this particular game, both the *maximin* and *minimax* approaches yield the NE solution concept. When the players both choose to maximize their minimum utilities, the solution concept is (Confess, Confess) – each player considers the payoffs of their strategy set {Silent, Confess} and chooses the strategy that will result in the better worst-case scenario. Similarly, if the players choose to minimize the other's maximum, the solution concept is also (Confess, Confess) – each player considers the other player's potential maximum utility and chooses that which yields the lesser best-case scenario for the other player. It makes sense that in Table 2.1, both the maximin and minimax solution concepts exhibit the same properties as the NE.

The final solution concept, the Kaldor-Hicks (K-H) concept (where one player would receive enough utility to transfer to the other while still providing both a better result than the NE) is achieved by the (Silent, Confess) solution concept which yields a payoff of (10, 1) and where one player can transfer his entire utility to the other (if he wants). Hence, in that situation, with (Silent, Confess), one player would be able to ultimately bribe the other to an outcome of (0, 11) which would yield a more favorable payoff for player 2 than the NE (from Table 2.1). Furthermore, a K-H solution concept can include any amount of transferred utility ranging between (10, 1) and (0, 11) including the outcome of (5.5, 5.5) where one player transfers enough utility to the other so that both players receive an equal benefit. The K-H and PO solution concepts are not self-enforceable and are susceptible to a player's deviation from their initial strategy. Therefore, both solution concepts require a substantial amount of trust and a level of commitment between the players that may not reside in a typical prisoner's dilemma bimatrix game. This concept is paramount to a bilateral nuclear security regime between states however and is discussed more in Section 3.4.

The intent of presenting the PO in the aforementioned discussed Prisoner's Dilemma game is to introduce the precariously placed potential of players' cooperations which can serve as motivator for cooperative game theory (CGT). In CGT, players may collaborate by coordinating their strategies in order to gain a higher total payoff than if they were to play non-cooperatively. Scholarly research in CGT has been extensive in operations research, political science, computer science, maritime operations, and other such areas but not as much in nuclear policies and strategies.[73,74,75,76] There exist two important concepts necessary for a cooperative game: 1) the existence of a perceived additional benefit (i.e., surplus) by cooperating and 2) the players' pre-negotiations of that surplus[b]. Past scholars have studied the latter as an important issue in coalition formation and stability.[77,78,79,80] In a two-player cooperative game, the two players can form one coalition and negotiate between themselves to receive a higher payoff than if they were to act independently (e.g., when one player can transfer some or all of his utility to the other player in the form of a side payment). This concept is referred to as utility transferability and is expanded upon in Subsection 2.5.2.

2.5. Assessing Cooperation in Game Theory

Modern scholars in cooperative game theory (CGT) have focused substantially on alliance-building between players to form coalitions. As the number of players increases, so does the complexity of the potential solution concepts. However, by reducing a game

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[b] In other words, an important concept necessary for a cooperation game is that for all NE solution concepts, there exists a strategy pair that is a Kaldor-Hicks improvement on that NE, as well as a way to share the utility of that improvement.

to a simple bi-matrix game of two players, we can describe bargaining strategies for when players cooperate. Furthermore, in this document, we co-opt terminology commonly used in CGT to fit the needs of explaining our methodology (particularly in the following two subsections, in Section 3.2, and in each case study's third section of chapters 4-8). This is necessary due to the lack of a clear segue from NCGT to CGT. Shubik and Powers comment on this as recently as 2016 by stating there has yet to be a constructed theory that would "unify both types."[81] The methodology presented here does not fit this need *per se* but, rather, the intent of this work is to provide a way to gain insight from applying the logic described in Chapter 3.

2.5.1. Bargaining Strategies

In the aforementioned (preceding section preface), two-person, bi-matrix game with cooperation, we assume communication is made between the players so that both can achieve a mutually-agreed upon result that proves favorable for each. Within this section, utilities from correlated strategies are identified with a COR superscript and utilities from uncorrelated strategies are identified with a UNC superscript. Determining the solution concepts for both types of strategies allows the opportunity to identify the existence of any benefit from correlating strategies, by reviewing the difference in utility (as discussed in the beginning of this chapter) between the payoffs for the COR and the UNC strategies. The division of any additional benefit (otherwise referred to as the surplus) is negotiated between the two players is how one defines a player's bargaining strategy. The collection of bargaining strategies is then the game's (bargaining) solution concept.

Assume v_i is the payoff for player, i, which is the result of evaluating a non-descript utility function, u, based on strategy μ_i. Strategy μ_i is either uncorrelated (player i acts regardless of the other players' strategies) or correlated (player i uses the cooperative strategy in conjunction with the other players' strategies). Equation (2.5.1) conveys the general formula for the total payoff to the regime of n players, v_r.

$$v_r = \sum_i^n v_i \tag{2.5.1}$$

Where $v_i = u(\mu_i)$ and u is the utility function used for defining payoff, v_i. A utility function is specific for each player i and reflect those players' interests. Hence, it is more accurate to use $v_i = u_i(\mu_i)$. For the case of only two players, the total payoff of the regime using the correlated strategies, μ_i^{COR}, is v_r as shown below:

$$v_r = v_1^{COR} + v_2^{COR} = u_1\left(\mu_1^{COR}, \mu_2^{COR}\right) + u_2\left(\mu_1^{COR}, \mu_2^{COR}\right)$$

For uncorrelated payoffs where players do not cooperate, the payoffs are computed via utility functions defined by independent strategies looking to merely maximize the player's utility:

$$v_i^{UNC} = \max u_i\left(\mu_i\right)$$

Each player's bargaining strategy is determined by 2 values: the total payoff for the regime when they correlate their strategies, v_r, and the respective uncorrelated payoffs for each player: v_i^{UNC}. For n players, the total payoff within the cooperative solution for each player is defined by Equation (2.5.2) with d_i denoting the percentage of the difference between the regime's payoff and the sum of the individual players' payoffs the player receives ($d_i = \{0,1\}$).

$$v_i = d_i \cdot \left[v_r - \left(\sum_i^n v_i^{UNC} \right) \right] \tag{2.5.2}$$

When two players are present, $d_{3-i} = 1 - d_i$. For a Nash Bargaining Solution or NBS, *n* players receive an even split of the surplus (i.e., $d_i = {}^1/_n$). Hence, for the two player game, Equation (2.5.2) is modified to show the NBS for both players:

$$v_1^{NBS} = 0.5 \cdot \left[v_r - \left(v_1^{UNC} + v_2^{UNC} \right) \right]$$
$$v_2^{NBS} = 0.5 \cdot \left[v_r - \left(v_1^{UNC} + v_2^{UNC} \right) \right]$$

The bracketed quantity in Equation (2.5.2) is referred to as the surplus payoff, v^{SUR} – the difference in payoffs for the regime between the payoff when players correlate their strategies, v^{COR}, and when they do not, v^{UNC}. Using the superscripts described above, the surplus is defined as Equation (2.5.3):

$$v^{SUR} = v^{COR} - v^{UNC} \tag{2.5.3}$$

Here $v^{COR} = v_r$ and $v^{UNC} = \sum_i^n v_i^{UNC}$ from Equation (2.5.2). Varying d_i provides different distributions of the surplus or bargaining strategy concepts – otherwise referred to as imputations of the game. The collection of all imputations across the range of values for d_i makes up the cooperative game's core (as the blue-shaded region in Figure 2.9).[c]

[c] Both imputations and core are used in traditional cooperative game theory to describe divisions among n-players and their various solutions but, herein this methodology, they are used to describe characteristics of the bargaining solution between two players.

The core represents the area of negotiations for both players where each player gains something and, hence, any solution concept residing therein is a viable outcome of the cooperative game.

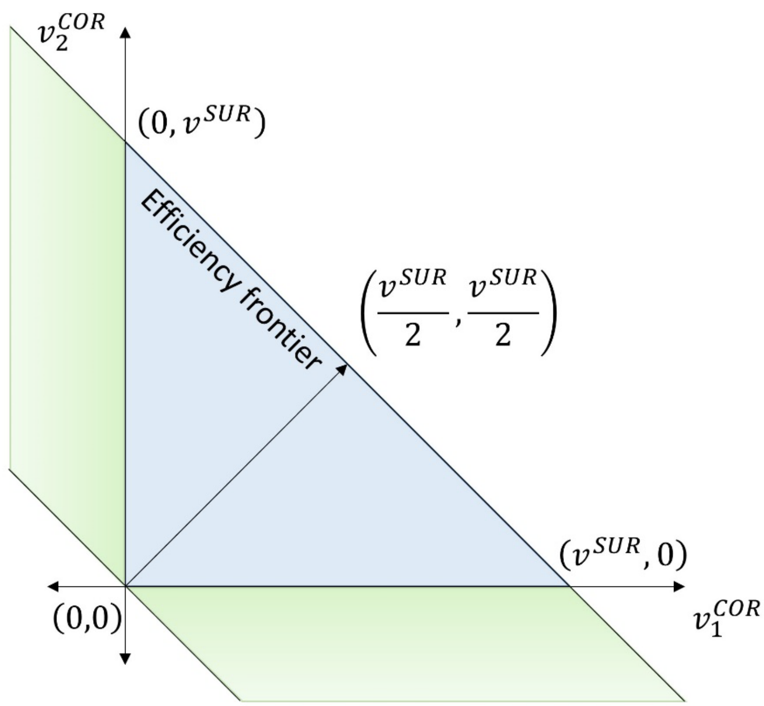

Figure 2.9. Solution space on surplus axes

Figure 2.9 exhibits various characteristics of bargaining solutions in cooperative games. Assume the normalization of all payoffs in regard to the uncorrelated payoffs – i.e., the uncorrelated payoffs (v_1^{UNC}, v_2^{UNC}) are set at the figure's origin (0, 0). Any positive amount of payoff to either player from the correlation of strategies leads to non-negative payoffs along the horizontal and vertical axes that which represent each player's correlated payoff v_1^{COR} and v_2^{COR} respectively. When either player receives the full surplus, points $(v^{SUR}, 0)$ and $(0, v^{SUR})$ are identified. The line segment between these two points can be referred to as the efficiency frontier of the game's solution space. Within this triangular area resides the game's core. The NBS is shown as the arrow with a +1 slope from the origin to exhibit the equal split of the surplus to each player. Any solution concept that resides within the core or falls on the efficiency frontier would be a Pareto improvement over the uncorrelated solution concept because both players would receive an increase in their respective payoff. The green-shaded regions below the x-axis and to the left of the y-axis signify solution concepts which would be an improvement for one player but not the other. When this happens, the potential exists for one player to supplement the other's loss – this is the definition of a Kaldor-Hicks improvement over the uncorrelated solution concept.

The applied methodology in each case study (Chapters 4-8) allows for other bargaining solution concepts that are within the game's core (beyond the NBS). In other scholarly work, the NBS bargaining solution is assessed from various solution concepts.[82] Conversely, in this work, we use a single uncorrelated solution concept for each case study in Chapters 4-8 and assess multiple bargaining solutions.

2.5.2. Visualizing Utility Transferability

In the two-player, bi-matrix game with cooperation, we begin to include the option of players making *side payments* to each other. This concept is essentially transferring one player's utility to another. Herein, we convey how the methodology assists in visualizing this concept. Assume an uncorrelated solution concept results in a solution placed at point (α,β), as shown in Figure 2.10 as point P at (2,3). Recalling Equation (2.5.3), the surplus, v^{SUR}, is the difference between a correlated payoff, v^{COR}, and the uncorrelated payoff, v^{UNC}. Players A and B then negotiate v^{SUR} into fractions (r for A and s for B where $r + s = 1$). The ratio *r:s* defines an allocation of the surplus (otherwise referred to as an imputation). Therefore, the total amount each player can receive is x and y, respectively, as shown below:

$$x-\alpha=r\left[v^{SUR}-\left(\alpha+\beta\right)\right] \tag{2.5.4}$$

$$y-\beta=s\left[v^{SUR}-\left(\alpha+\beta\right)\right] \tag{2.5.5}$$

Equations (2.5.4) and (2.5.5) can be reduced to Equation (2.5.6) shown in Figure 2.10:

$$y=\frac{s}{r}x+\left(\beta-\frac{s}{r}\alpha\right) \tag{2.5.6}$$

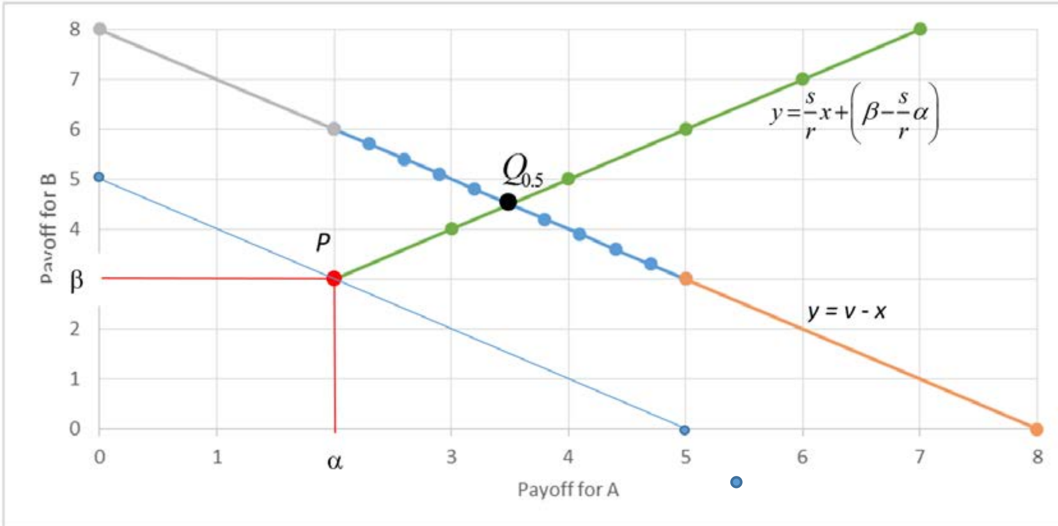

Figure 2.10. Nash Bargaining Solution at $Q_{0.5}$ where *r:s* = 1:1

To use the entire surplus, x and y must also satisfy $x + y = v$. The line (Q_i) – Equation (2.5.6) in point-slope form – is the utility transferability (UT) line. Moreover, this line exhibits the solution's Pareto Optimality for both players (meaning one player can only do better at the expense of the other).[78] Moreover, with a solution concept residing on line Q_i between α and β, the outcome therefore gives each player more or as much as they could have attained by acting alone – thus meeting a desirable characteristic of a solution concept referred to as individual rationality (page 6 in Leyton-Brown).[57] Specifically, Figure 2.10 conveys the Nash Bargaining Solution at $Q_{0.5}$. When the imputation is 0.5 (r:s ratio is 1:1 or when $r = s$), the collective payoff is 8 (A receives 3.5 and B receives 4.5) and the initial uncorrelated solution concept resides at point P: (2, 3).

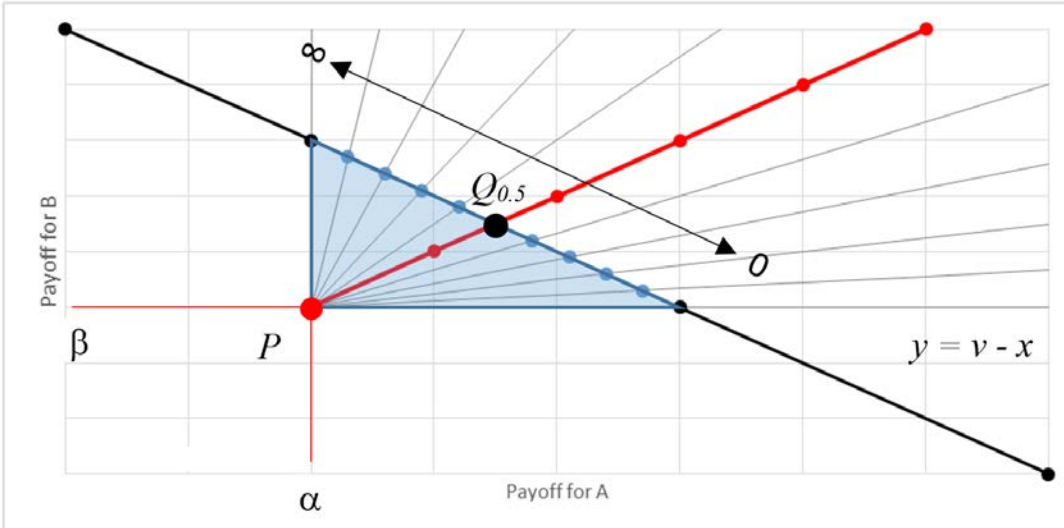

Figure 2.11. Nash Bargaining Solution with blue-shaded core

Figure 2.11 exhibits a general set of solution concepts to convey other imputations (allocations) which are shown as grey lines as they intersect the UT line as a ratio between r and s. As the fraction of s/r approaches zero, the distribution of the surplus becomes more favorable for player A (conversely as well). Any additional side payments that fall outside the blue-shaded region (i.e., $s/r < 0$) implies that a player will have to sacrifice a portion of his payoff from the initial uncorrelated point, P. When s/r is negative, the strategy set meets the criteria for a Kaldor-Hicks solution where side payments from one player to another is possible. As an imputation approaches zero (i.e., $s/r \rightarrow 0$), the distribution of the surplus becomes more favorable for player A (and vice versa). The coalition's total utility, v, is not exceeded. Any additional side payments that fall outside the core (i.e., Q_i<0) imply that a player will have to be made whole so as to return the cooperative strategy set back into the game's core. Making one player whole would require the other to provide a side payment to achieve a fair division of the surplus (Leyton-Brown, p. 14).[57] This *ex post* transfer of wealth leads to the possible questioning of credibility of this bargaining solution (or even the coalition's viability).

As will be shown, each case study begins solving correlated strategies as Kaldor-Hicks solutions. Then, as part of each case study, we vary the *r:s* ratio and generate variations of Equation (2.5.6) to devise different imputations exemplified graphically by the grey lines in Figure 2.11. When side payments between players are permitted, the strategic point P (α, β) can change to (α+d, β-d) where *d* is positive when player B makes a payment to player A (and vice versa if *d* is negative). By limiting *d* and assuming its value is seen equally by both players, the UT line exhibits a slope of -1.

3. GAME THEORETIC METHODOLOGY FOR ASSESSING BENEFITS

A foundation of game-theoretic analysis was included in Chapter 2. In Chapter 3, we use this foundation to discuss the developed methodology used to assess sets of strategies in various bilateral nuclear security regimes. These strategy sets are otherwise referred to as solution concepts and can be either cooperative or non-cooperative (as discussed in Section 2.4). as they pertain to the bilateral nuclear security regimes used in this work. Specific considerations of the described methodology include using characteristic or objective functions to compute players' utilities based on their strategies. The functions are defined in the Section 3.1 and, as mentioned in Section 2.2, the utilities they yield are continuous. Moreover, in this methodology, players' utilities are defined as costs, can be bargained, and are treated as transferable.

Both cooperative and non-cooperative game theories can be used to determine solution concepts, a distinction between both is the level of cooperation that is imposed on the bilateral regime. In conventional cooperative game theory (CGT), the cooperation is enforced by an unbiased third-party or by legal bindings. In assessing cooperation under non-cooperative game theory (NCGT), the players see the benefit in cooperating and, thus, set their strategies jointly through communication. It is this latter approach that is used herein.

The methodology will show how to determine the existence of a mutual benefit that would provide the incentive for both states to coordinate their strategies. If determined to exist, the surplus is then the bargaining chip between the states so that both are enticed to negotiate possible entrance into a bilateral regime. A cooperative game model is presented on how the surplus can be divided by the states. Furthermore, in the paradigm presented in this work, monetary investment or cost is the metric which inherently is a negative utility for the states (the terms “investments” and “strategic costs” are used interchangeably within this document). Therefore, the term “surplus”, in this game model actually implies how much a state can save in its investments.

The following four sections describe various details of the methodology: its framework and operational flow as used in the case studies in Chapters 4-8, input parameters used to help discern the various regimes, both non-cooperative and cooperative solution concepts, and considerations of the methodology as it stands in this composition such as players’ commitment issues and commensurability of results. Each chapter presents a snapshot in time of a bilateral relationship between states with the focus on nuclear security by using input parameters found for each case study’s year (the first four case studies center around US-Russia/Soviet relations): 1985, 1995, 2008, 2015, and a US-Pakistan partnership with 2008 input parameters. These snap shots in time were used due to the availability of the relevant data for those years in scholarly publications.

3.1. Methodology Framework and Operational Flow

For this methodology, the two players of the three game-theoretic models are a source state securing its nuclear assets (State A) and a target state interdicting the use of assets against itself (State B) . The utility for each state is defined by the characteristic function introduced here and developed in more detail in Subsection 3.1.2. It is based on annual investments for achieving their respective goals and the consequential costs from the inadequacy of achieving those goals.

The characteristic function is used to define each state's utility based on the state's strategy set. As stated in this chapter's preamble, the utilities of chosen strategies are in terms of cost due to the expense of nuclear security and any consequences that could arise. Therefore, the characteristic function used to calculate utilities for states' strategies can also be referred to as the objective cost function which computes continuous costs as part of determining solution concepts. States make annualized financial investments to achieve their strategic objective (i.e., strategic costs, C_i) and then bear the consequential costs (i.e., threat costs, T_i) of that investment. All costs are non-negative (though in future work, this assumption can be relaxed). For both states, as the strategic costs increase, the threat costs decrease (arguably approaching zero). This inverse correlation and their additive nature for determining a state's total costs (i.e., TC_i) provide the opportunity for optimizing strategic and threat costs in order to minimize each state's respective total cost. Moreover, the characteristic function exhibits reductions in threat cost per unit strategic cost as a negative slope as well as diminishing returns of investment. This implies that after the initial strategic investment, each subsequent additional investment produces less reduction in the threat cost.

The methodology's three game-theoretic models of each bilateral regime include determining two solution concepts by a non-cooperative game theoretic approach (where the states act unilaterally and then communicate their strategies) and one solution concept defined by a bargaining approach. First, the unilateral approach yields an initial solution concept where states' strategies are not communicated and, hence, are *uncorrelated*. This solution concept is an initial point where both states choose the independent, *uncorrelated* strategies in order to minimize their total costs based on assumptions of the other's strategy (otherwise, each state's best response strategy which in turn results in the Nash Equilibrium). Second, the next solution concept assumes communication occurs and both states *correlate* their strategies so as to gain a collective super-additive benefit (i.e., a surplus) if one exists. This resulting solution concept is a Pareto improvement over the initial Nash Equilibrium (where at least one state improves its payoff). This is mathematically represented in Equation (3.1.1) where *u* represents a generic cost function based on each *i*'th state's cost, x_i.

$$\max_{x_1,x_2}\left[u_1(x_1)+u_2(x_2)\right]\geq\max_{x_1}u_1(x_1)+\max_{x_2}u_2(x_2) \qquad (3.1.1)$$

Last, using *bargaining* strategies, the methodology provides the visualization of forming a coalition between the two states where the surplus can be divided between the two. Fair division of the surplus introduces the concept of utility transferability and can be accomplished *ex post* by way of side payments between the players.[83]

3.1.1. Defining the Game

The game consists of two states, A and B, serving in the following roles: State A is a potential source of a nuclear weapon being confiscated by a non-state adversary and State B is the potential target state against which said nuclear weapon would be used. As the source state, State A concentrates its annual strategic nuclear-security costs, C_A, on securing its nuclear weapons and State B, as the potential target state, invests its annual strategic costs, C_B, into a system for interdicting any potential nuclear threat at its borders. As a function of these investments, threat costs, T_A and T_B, represent the effect of said investment and the financial consequences of C_A and C_B. The total annual cost of the state's nuclear security strategy, TC_i, is the sum of the strategic, C_i, and threat costs, T_i. In looking for a minimal total cost, each state must optimize the balance between these two constituents. Herein this work, all costs are non-negative[d] and continuous for both states. Contrary to discrete games, continuous games allow for numerical strategies to consist of any real number (not just integers). The levels of commitment are defined by a set of all non-negative real numbers, $\mathcal{R}_+ = [0, \infty)$, signifying annualized costs interpreted as $C_i \in \mathcal{R}_+$.[e] Each state seeks to minimize its total cost, TC_i, based on Equations (3.1.2) and (3.1.3) below.

$$TC_A\left(C_A\right) = C_A + T_A\left(C_A\right) \tag{3.1.2}$$

$$TC_B\left(C_A, C_B\right) = C_B + T_B\left(C_A, C_B\right) \tag{3.1.3}$$

It is both useful and reasonable to assume the threat costs have certain properties – these are discussed in the following Section 3.2.

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[d] Strategic costs of either state can be negative. For example, if the source state chooses a strategy consisting of negative strategic costs, this would imply a decision to de-invest in its own security. If the target state chooses negative costs, it would be ensuring less interdiction capabilities for itself. It can be implied that the financial shortfalls resulting from the negative strategic costs can come from elsewhere outside the bilateral regime's structure but the model is not designed for this situation currently.

[e] Ostensibly, the cost to a state of a nuclear attack is substantial. A future modification to the methodology could annualize the costs of an attack (e.g., have the consequential costs pro-rated for a decade or one generation) but in the current incarnation of this methodology, we make a conservative assumption by front-loading the entire threat cost as a generic, worst possible scenario.

When considering State A's total cost, both strategic costs, C_A, and threat costs, T_A, are used. As shown in Figure 3.1, the profile of TC_A is based on Equation (3.1.2). The initial point, $TC_A(C_A = 0) = TC_A^{max} = T_A^{max}$, reflects a worst-case scenario of the highest potentially realizable threat cost (T_A^{max} an as-yet undefined maximum threat value when nothing is spent on securing nuclear material). Other characteristics of TC_A are $T_A(0, \infty) = \{T_{max}, 0\}$; $\frac{dT(C_A)}{dC_A} < 0$; and $\frac{d^2T(C_A)}{dC_A^2} > 0$.

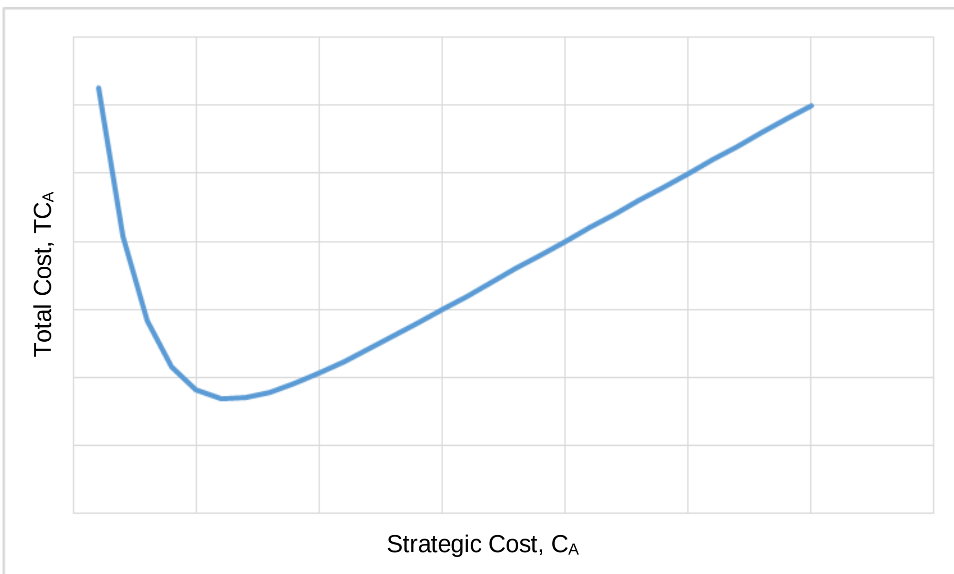

Figure 3.1. Total cost profile for State A

State B's threat cost is not only the cost of an inadequately funded interdiction system (financially defined as C_B) but it is also dependent on State A's level of investment in their security considering an asset from A can be used against B. It is for this reason that B's threat cost is not independent of A's strategy of securing its nuclear assets. Plotting an indicative trend for State B's threat cost based on both C_A and C_B would require the addition of a third dimension on the z-axis that would represent State A's investment (as shown in Figure 3.2 for illustrative purposes only).

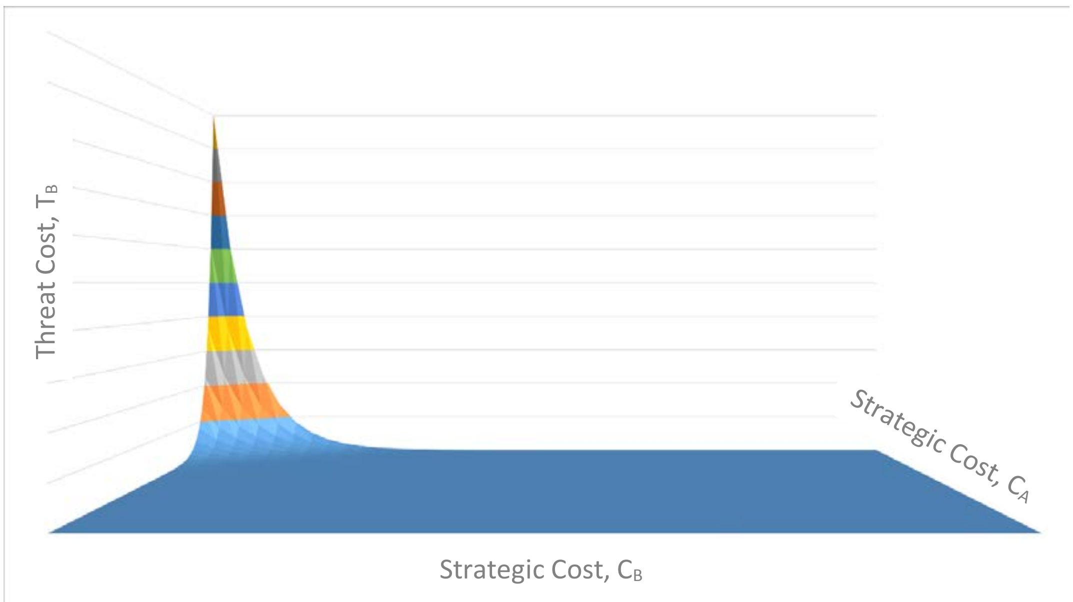

Figure 3.2. Trend of TC_B (z-axis) based on C_A and C_B (x- and y-axis)

Analogously for State B, Figure 3.3 shows the three-dimensional plot of B's total cost function, per Equation (3.1.3).

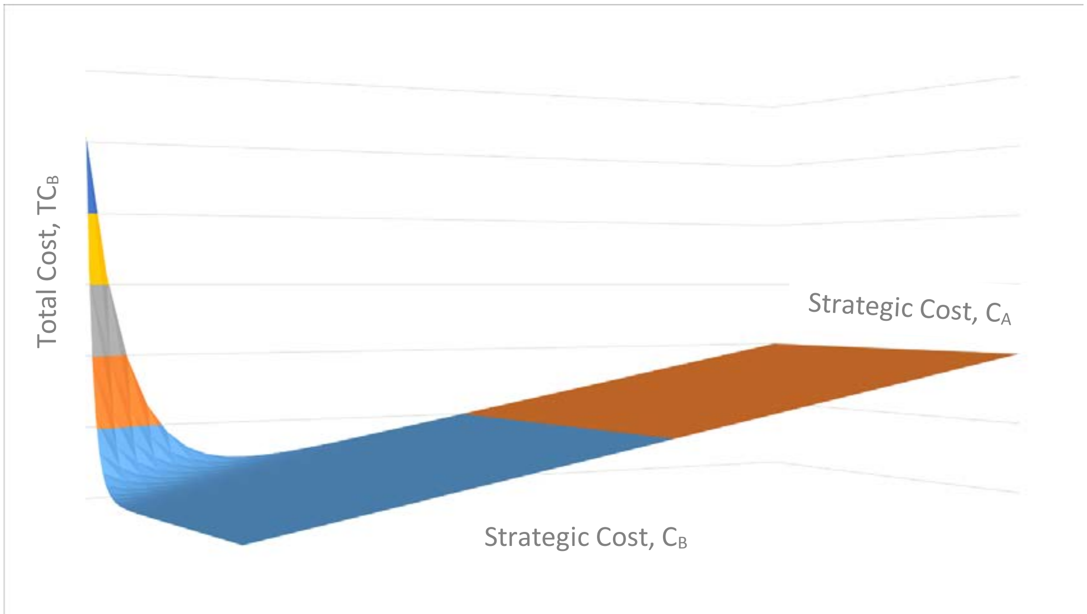

Figure 3.3. Total cost profile for State B

As stated in the preamble of this section, the first task is identifying the solution concept of the *uncorrelated* strategies between the states where each state determines the required strategic costs needed to minimize the total annualized costs, TC_A and TC_B, as shown in Equations (3.1.2) and (3.1.3). Below, Equations (3.1.4) and (3.1.5) show both summed to form the aggregate TC_{AB} – to be discussed in more detail in Subsection 3.2.1.

$$TC_{AB}\left(C_A,C_B\right)=TC_A\left(C_A\right)+TC_B\left(C_A,C_B\right) \tag{3.1.4}$$

$$TC_{AB}\left(C_A,C_B\right)=C_A+T_A\left(C_A\right)+C_B+T_B\left(C_A,C_B\right) \tag{3.1.5}$$

.

The *uncorrelated* solution concept is based on the assumption that each states seeks independently to minimize its individual total cost, without any regard[f] for the total cost of the other. This consists of defining each state's strategic cost (when acting independently) that would minimize their total cost. For State A, this is shown as Equation (3.1.6). Due to State B's threat cost being dependent on State A's strategy, State B should assume State A will set its strategy point at C_A' and define its own strategy, C_B', in response a la Equation (3.1.7).

$$C'_A = \underset{C_A \geq 0}{\operatorname{argmin}} \left[TC_A \left(C_A \right) \right] \tag{3.1.6}$$

$$C'_B = \underset{C_B \geq 0}{\operatorname{argmin}} \left[TC_B \left(C'_A, C_B \right) \right] \tag{3.1.7}$$

Strategy set (C'_A, C'_B) is a Nash Equilibrium because State B's response is the best response to State A's own best strategy. This is the initial focal point for study in the included case studies in Chapter 4-8. Inserting (C'_A, C'_B) into Equation (3.1.4) yields the solution concept $TC'_{AB}(C'_A, C'_B)$ as shown in Equation (3.1.8):

$$TC'_{AB}\left(C'_A, C'_B\right) = TC'_A\left(C'_A\right) + TC'_B\left(C'_A, C'_B\right) \tag{3.1.8}$$

An alternative solution concept is when State A decides to change its strategy causing State B to reconsider its own strategy. For example, given that State B is the U.S.

[f] "Without regard" refers to neither state having intent to help the other by decreasing its costs or harm it by increasing those costs.

and State A is a nondescript state with nuclear assets needing to be secured, the U.S. can adopt a strategy under the assumption that State A does not invest in its own security, $C_A = 0$. Conclusively, State A seeks to minimize the U.S.'s maximum utility and therefore, causes the U.S. to redefine its own strategic cost, C_B^m (where the superscript '*m*' refers to the minimax strategy) as such:

$$C_B^m = \underset{C_B \geq 0}{\operatorname{argmin}} \left[TC_B \left(\underset{C_A \geq 0}{\operatorname{argmax}} \left[TC_B \left(C_A, C_B \right) \right], C_B \right) \right] \tag{3.1.9}$$

Conversely, $C_A^m = 0$ so Equation (3.1.9) is reduced to Equation (3.1.10):

$$C_B^m = \underset{C_B \geq 0}{\operatorname{argmin}} \left[TC_B \left(0, C_B \right) \right] \tag{3.1.10}$$

Therefore, State B's strategy at the minimax focal point as shown in Equation (3.1.10) results from its potential partner, State A, investing nothing in nuclear security. The resulting strategy set $(0, C_B^m)$ is the minimax strategy.

Another solution concept results from States A and B *correlating* their strategies. This solution concept is determined by summing the two states' total cost objective functions and then determining the values of C_A and C_B that minimizes the regime's total cost. Equation (3.1.11) conveys this by solving for the cooperative solution concept, $TC_{AB}^*(C_A^*, C_B^*)$ where C_A^* and C_B^* are the *correlated* strategies.

$$TC_{AB}^* \left(C_A^*, C_B^* \right) = \min_{C_A \geq 0, C_B \geq 0} \left[TC_{AB} \left(C_A, C_B \right) \right] \tag{3.1.11}$$

Within this methodology, Equation (3.1.11) can be solved analytically as well as by using a number of readily-available computational tools: Microsoft Excel's solver or MatLab's suite of optimization functions (including *fminsearch*, *fminunc*, and *fmincon*). In the case studies of Chapters 4-8, the *fmincon* function in MatLab is used to determine the minimum TC_{AB} (in Chapters 4 and 5, the solution is first completed by hand). As shown in Figure 3.3, a minimum exists when the first derivative near C_A^* and C_B^* are negative and the second derivative of the characteristic function is positive. – this is discussed in more detail in Subsection 3.3.2. Figure 3.4 is an illustrative example of $TC_{AB}^*(C_A^*, C_B^*)$.

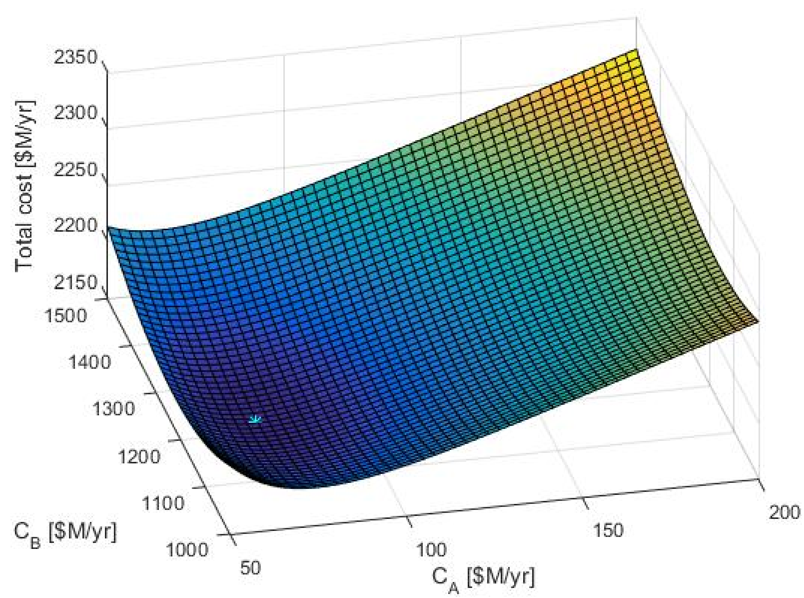

Figure 3.4. Global minimum of TC_{AB} using MATLAB's *fmincon* function

As discussed in Section 2.3, the difference between TC'_{AB} and TC^*_{AB} is referred to as the "surplus." But actually establishing a cooperative bilateral regime requires the division of said surplus via *bargaining solutions* between the players. The negotiation of the division of the surplus, v, is the metric by which either state will maintain or forego the bilateral regime.

$$v = TC'_{AB} - TC^*_{AB} \tag{3.1.12}$$

How this surplus is distributed through negotiations segues into the concept of utility transferability between the states (discussed in more detail in Subsection 3.2.2). Depending on how this is achieved, transferability can impact whether a bilateral arrangement is established as well as successfully maintained. Furthermore, transferring utility is a vital component in cooperative game theory (specifically in coalition games) but is reliant on trust and commitment between the players (discussed later in Section 3.3).

3.1.2. Parameters of the Threat Cost

Before delving into more detail of the solution concepts, let us first discuss how the characteristic functions are formed. The threat cost, T_i, used in Equation (3.1.2) is deconstructed to adequately define the game models. With the understanding that C_i is the cost of the state's security or interdiction, the realized threat cost, T_i, is based on that investment, C_i. The threat cost term of that equation is based on a conventional risk equation used in the nuclear safety industry comprised of the product of the frequency of a catastrophic safety event per unit time and the consequence of said event in terms of its magnitude.[84] Additionally, a conditional probability is included to signify the potentiality of an attack against a state. Overall (as previously alluded to in Section 3.1.1), the threat cost is characterized by the law of diminishing marginal utility: as a state invests in the security of its nuclear assets or the interdiction methods at its borders and after the initial benefit of a non-zero investment is realized, the impact of each additional unit of investment is reduced. State A's primary objective from the first game model is to secure the nuclear asset so as to prevent a potential attack on itself or B. State A's annual threat cost (deconstructed in Equation (3.1.13)) is based on a loss rate function, $L(C_A)$ which is dependent on the cost of securing State A's nuclear assets, C_A. In addition, the conditional probability of an attack on State A once an asset is out of its control (P_A), and the subsequent consequence to A (K_A) are also present. The parameters shown below are values obtained and determined from real-world data.

$$T_A(C_A) = L(C_A) \cdot P_A \cdot K_A \tag{3.1.13}$$

State B's threat cost, T_B, is displayed in Equation (3.1.14) below. Here, C_B is the annual amount State B invests in its interdiction capabilities. P_B is the non-interdiction probability for State B's functional nuclear material interdiction system (which is dependent on the investment of State B in its interdiction capabilities, C_B). K_B is the expected cost to State B of a nuclear attack against it by a terrorist organization (regardless from where the weapon is attained). $L(C_A)$ is the same aforementioned loss function (notice the dependence on State A's investment in security). The gamma parameter, Γ, conveys an attempt at considering other potential sources of a weapon for the non-descript, non-state adversary acquired outside of State A (otherwise, referred to later in this work as the asset source balance).

$$T_B\left(C_A,C_B\right)=\left[L\left(C_A\right)+\Gamma\right]\cdot P_B\left(C_B\right)\cdot K_B \tag{3.1.14}$$

Arguably, Equation (3.1.14) could incorporate the antithesis of P_A from Equation (3.1.13) as a way to signify that State B would face the threat of any probable nuclear attack not committed against State A, but this would be an unrealistic assumption in the real world and therefore is not considered in any model presented herein. This assumption however can be a modification encompassed an area of future work beyond the scope of this presented work.

It is necessary to discuss the parameters of the threat costs because they contribute to the total costs for each state and the regime per the first two game models respectively. The parameters are presented with various assumptions and example estimations in this section. Though numbers in the following subsections are shown with multiple significant digits, numbers in the actual case studies must be read as estimates valid to no more than two significant digits at best (Section 3.5 includes a more detailed discussion).

3.1.2.1. Loss Function

States with nuclear weapons invest substantial efforts in developing and maintaining such a capability. It is reasonable to assume that the state would therefore protect these high-valued assets (nuclear weapons or special nuclear material) against threats to some level of physical security above and beyond the baseline level that would be provided by prudent owners of any similarly valued objects and the general level of law and order prevailing in the state. This baseline would depend upon characteristics that fall outside our model, such as the state's economic situation, level of openness as a democracy, overall form of government, etc.

The loss rate, $L(C_A)$, as it appears in the threat cost formulae, $T_A(C_A)$ and $T_B(C_A, C_B)$ as shown in Equations (3.1.13) and (3.1.14), exhibits the following four properties that, in reality, also satisfy requirements of the threat cost formulae (included in Subsection 3.1.1).

1) $\lim_{C_A \to 0} L(C_A) = \alpha$ – as the annual investment into State A's nuclear security (resources devoted to A's strategic cost) decreases towards 0 (no additional investment outside the baseline cost), the annual loss approaches a certain limit of α (in terms of $\frac{NW}{yr}$ or $\frac{SQ}{yr}$, as is discussed in more detail in Subsection 3.2.1.1);
2) $\lim_{C_A \to \infty} L(C_A) = 0$ – as investment into State A's security increases to infinity, the "perfect" case of no loss is approached (hence, $T_A = 0$);
3) $\frac{dL}{dC_A} < 0$ – as State A invests in its strategic costs, the loss rate is always being reduced; and
4) $\frac{d^2L}{dC_A^2} > 0$ – as State A devotes more resources to security, the magnitude of the marginal reduction in threat costs is reduced (i.e., exhibiting the law of diminished returns).

Various functions can meet the assumptions outlined above. In the case of this work, the loss rate function is exponential and has units similar to α: $\frac{NW}{yr}$ or $\frac{SQ}{yr}$.

$$L(C_A) = \alpha \cdot e^{-\lambda \cdot C_A} \quad (3.1.15)$$

Specifically for this work, $L(C_A)$ is in terms of an annual loss rate of a nuclear asset from State A's control. As mentioned in Subsection 1.3.2, the term "asset" implies a nuclear weapon under State A's control as part of State A's nuclear weapons program or an amount of special nuclear material (plutonium or highly enriched uranium) where the fact of assembling a nuclear weapon cannot be discounted (i.e., a significant quantity or SQ).[38] The loss rate as seen by State A in a year when $C_A = 0$ is α – otherwise considered the maximum loss. As presented in Equation (3.1.15), lambda, λ (in terms of $\$M^{-1}$) is the reduction in loss rate per unit increase in strategic cost – otherwise, considered the relative marginal loss rate. A negative exponential trend conveys the concept that when funding for securing weapons suffers (i.e., decreases), the loss rate (and hence the total cost to the regime) increases.

In order to convey how α could be estimated, 1994 nuclear trafficking data are presented in Table 3.1, with Russia as State A and the United States as State B. This is the value of α used for a later case study modeling U.S.-Russian relations in 1995. Values of α for other case studies are specifically discussed in each respective case study in Chapters 4, 6, 7, and 8.

Table 3.1. Nuclear trafficking events in 1994

Month	**Location of Confiscation**	**Material**	**Mass [g]**	**SQ**
March	St. Petersberg, Russia	HEU (90%)	2972	0.107
May	Tengen-Wiechs, Germany	Pu	6.2	0.000775
June	Landshut, Germany	HEU (87.7%)	0.795	2.788e-5
July	Munich, Germany	Pu	0.24	0.00003
August	Munich Airport, Germany	Pu	363.4	0.045425
December	Prague, Czech Republic	HEU (87.7%)	2730	0.095768
TOTAL				**0.249**

The total SQ value shown in Table 3.1 was determined by tallying nuclear material trafficking data (for anything over 20% ^{235}U, 25kg of ^{235}U = 1 SQ; 8kg plutonium = 1 SQ) confirmed by the International Atomic Energy Agency (IAEA) and in terms of SQs. By modifying Equation (3.1.15) into Equation (3.1.16) below, we are able to estimate α for 1994 and use this value for the subsequent case study year of 1995 to be discussed later in Chapter 5:

$$\alpha = L(C_i) \cdot e^{\lambda \cdot C_i} \qquad (3.1.16)$$

We assume C_{94} = 0M USD due to published conclusions stating that American assistance programs for nuclear security in Russia had not yet produced tangible results and the claim that accounting for and storing nuclear materials was in need of substantial improvement after "a series of highly publicized incidents of theft and smuggling of

Russian nuclear materials."[34,85] The total recorded loss, α, attributed to Russia from that year then was 0.249 $\frac{\text{SQ}}{yr}$ per the modified Equation (3.1.16):

$$\alpha = L_{94} \cdot e^{\lambda \cdot C_{94}}$$
$$\alpha = 0.249 \cdot e^{\lambda \cdot (0)}$$
$$\alpha = \underline{0.249} \left[\frac{\text{SQ}}{yr} \right]$$

This value of α was estimated for 1994 as well as for Case Study 2 in Chapter 5.

Furthermore, when $C_A > 0$, the value for λ must also be considered. As previously mentioned, λ, as shown in Equation (3.1.15) represents the relative marginal loss rate of State A's additional investment in its security – i.e., the reduction in loss rate per unit increase of strategic investment. This is completed in Chapter 5 (the 1995 case study) and moreover, as computed in Subsection 5.1.1, both α and λ are computed with $C_A = 0$. Mathematically, λ is defined in Equation (3.1.17):

$$\frac{dL(C_A)}{dC_A} = -\lambda \cdot L(C_A)$$

$$\lambda = -\frac{1}{L(C_A)} \cdot \left[\frac{dL(C_A)}{dC_A} \right] \tag{3.1.17}$$

Conceptually, Equation (3.1.17) conveys the definition of λ as the absolute marginal loss rate per $L(C_A)$; otherwise, the relative marginal loss rate. Modifying

Equation (3.1.15) (shown as Equation (3.1.18) below) provides the manner in which specific values of λ were determined for each case study in Chapters 4-8.

$$\lambda = \frac{1}{-C_A^{ACT}} \cdot \ln\left[\frac{L\left(C_A^{ACT}\right)}{\alpha}\right] \tag{3.1.18}$$

For this, actual values of $L(C_A)$ and $C_A$were gleaned from publications and, with an estimated α value, λ was computed. This particular value of λ per case study was used because the assumption was made that this was the state's own best assumption for its relative marginal loss rate. By using the state's estimated λ, strategy sets and solution concepts were computed and evaluated per the methodology for each case study in Chapters 4-8.

In Section 9.2, a re-assessment is made for the various Russia-US case studies where $C_A > 0$. Both α and λ are estimated simultaneously by plotting data points for $L(C_A)$ and C_A and fitting a non-linear regression line to the data using the least-squares method. Results are discussed there and compared with the initial engineering judgement of the author as presented in this subsection.

3.1.2.2. Conditional Probability of a Successful Attack

Recalling Equation (3.1.13), P_A, appears as the conditional probability of a successful attack on State A by a non-state adversary assuming an asset was removed from State A's control. State A's competency in securing the asset itself is not factored into P_A – the security system in place is conceptually under the loss rate which includes the concept of an adversary gaining access into an asset's storage site and acquiring the asset itself (as presented in Subsection 3.2.1). The focus of P_A is what the non-state adversary is able to accomplish once the asset is under his/her control. From here, there are two considered scenarios. The first consists of the adversary attaining control of the asset but does not remove it (due to a physical security system component such as the detection system or the response force or to the adversary's preferred intent of making a statement by detonating an asset on site). Subsequently, they then detonate while on site. The second scenario consists of the adversary gaining control of the asset, successfully removing it from the site, transporting it successfully to the final target for detonation and detonating it. In both scenarios, each mentioned step has a probability of success for the adversary. In scenario 1 (*on-site* detonation), fewer steps are involved and hence, the value for P_A can be more than that of for scenario 2 (where more uncertainty occurs due to the number of steps that must take place). The consequence of such a detonation, however, would arguably be less than a transported weapon to a highly populated area due to the assumption that sites storing nuclear materials are more likely to be in sparsely populated areas with a nearby military presence.[86] In scenario 2 (*at-target* detonation), the asset must be removed successfully from the site (in light of the military presence and remote location of most nuclear asset storage sites) and transported to the final detonation point. The latter step would require an inadequacy of State A's recovery (and defensive) effort that associates another layer of uncertainty for the non-state adversary. If successful, however, the potential consequence of a detonation in a highly populated area can be substantially higher than that of the first scenario. The parameter value for P_A in both scenarios is highly dependent on State A’s competency of either addressing the threat on site or recovering and preventing its eventual use in an attack.

Other factors incorporated into the value of P_A depend on the non-state adversary and present more uncertainty in the computation of the total probability of an attack when combined in some form:

- The adversary’s intent in either making a statement or causing death and destruction
- The site’s security system detecting the adversary’s incursion
- The site’s security system correctly assessing the adversary’s incursion
- The adversary’s level of collusion with a potential insider
- The adversary gaining control of the asset
- The site’s security system responding to the adversary’s incursion in time
- The site’s security system neutralizing the adversary
- The adversary’s ability to detonate the asset (if a weapon) on site
- The adversary’s ability to remove the asset from the site
- The adversary’s ability to transport the asset away from the site
- The site’s and state’s abilities to recover the asset
- The adversary’s ability to weaponize the asset (if bulk fissile material)
- The adversary’s willingness in using its first attained asset
- The adversary’s ability to transport the asset to the target point
- The adversary’s ability to detonate the asset at the target point

First discussed in 2003, Bunn et. al. listed nine fundamental steps along the terrorist's pathway to a nuclear device where each one had its own associated probability of the terrorist completing that step, P_i.[87]

Table 3.2. Nine steps to nuclear terrorism

Steps	Action
S1	Form a highly capable group with extreme objectives
S2	Decide to escalate to the nuclear level of violence
S3	Steal nuclear weapons material
S4	Acquire nuclear weapons material
S5	Smuggle material to safe haven
S6	Construct nuclear explosive device
S7	Smuggle nuclear explosive device into target country
S8	Transport nuclear explosive device to target location
S9	Detonate nuclear explosive device

Using the nine steps in Table 3.2, Maerli devises a simple multiplicative method for combining each probability, P_i, into P.[88] In 2006, Bunn revisited the steps and incorporated, among others, concepts of collusion with an insider and black market acquisition into $P_{s(k)}$ or the probability that any given acquisition attempt k will be successful and ultimately lead to a terrorist nuclear attack.[89] Interestingly, he also devised two further probabilities that are used within this methodology: the success of converting "the acquired items to a nuclear capability" and "delivering and detonating the bomb once acquired."[89] His formula for $P_{s(k)}$ in Equation (3.1.19) includes the probabilities $P_{o(j)}$, $P_{i(j)}$,

$P_{b(j)}$, and $P_{n(j)}$ for a nuclear terrorist group *j* attempt a theft as an outsider from a facility, a theft with a facility insider, an acquisition of a nuclear asset on the black market, and the provision of the asset from a nation state. Each attempt *k* has some probability of being successful, *s:* $P_{os(j,k)}$, $P_{is(j,k)}$, $P_{bs(j,k)}$, and $P_{ns(j,k)}$. $P_{w(j,k)}$ is the probability of converting the asset into "a workable nuclear explosive that would in fact detonate" and $P_{d(j,k)}$ is the probability the adversary will "decide to, and be able to, deliver the bomb to its intended target and detonate it" (Bunn, p. 105).[89]

$$P_{s(k)} = \left(P_{o(j)} \times P_{os(j,k)} + P_{i(j)} \times P_{is(j,k)} + P_{b(j)} \times P_{bs(j,k)} + P_{n(j)} \times P_{ns(j,k)}\right) \cdot \left(P_{w(j,k)} \times P_{d(j,k)}\right) \quad (3.1.19)$$

Bunn's 2006 work provides the opportunity to draw differences between the *on-site* and *at-target* probabilities for the methodology described in this work.[89] Whereas Equation (3.1.19) explicitly deconstructs various acquisition attempts through an outsider attack, an insider attack, a black market acquisition, or a state provision, the methodology herein reduces this probability to merely gaining an asset from a facility/site. Only the outsider/insider difference can affect this probability. This, echoing some of Bunn's parameters and indices (a terrorist adversary *j* attempts an attack *k*), we simplify $P_{A(j,k)}$ as such:

$$P_{A(j,k)} = P_{a(j,k)} \times P_{s(j,k)} \times P_{r(j,k)} \times P_{w(j,k)} \times P_{d(j,k)} \quad (3.1.20)$$

Equation (3.1.20) uses $P_{a(j,k)}$, $P_{s(j,k)}$, $P_{r(j,k)}$, $P_{w(j,k)}$, and $P_{d(j,k)}$ for multiplicatively determining $P_{A(j,k)}$ where

- $P_{a(j,k)}$ is the probability of an attempt, *k*, to attack a facility with an asset by a terrorist *j*;

- $P_{s(j,k)}$ is the probability of a successful acquisition of a nuclear asset from attempt *k* by terrorist *j*;
- $P_{r(j,k)}$ is the probability of terrorist *j* removing an acquired nuclear asset from the facility that has been attacked in attempt *k*;
- $P_{w(j,k)}$ is the probability that terrorist *j* will weaponize the asset taken during attempt *k* into a nuclear capability able to detonate on command (echoing Bunn's 2006 description); and
- $P_{d(j,k)}$ is the probability that terrorist j will decide to, deliver to, and detonate the weaponized asset taken during attempt *k*.

Therefore, referring to the two aforementioned scenarios, $P_{A(j,k)}$, defined by Equation (3.1.20), can be modified to reflect either an *on-site* (where only $P_{a(j,k)}$ and $P_{s(j,k)}$ are of importance) or an *at-target* (where all five probabilities matter) nuclear terrorist attack. The consideration between probabilities for the *on-site* and *at-target* scenarios is discussed in more detail in Subsection 9.4. In sum, the P_A values used in Chapters 4-8 are estimates for the methodology to produce quantitative results that are intended to evoke more discussion.

3.1.2.3. Cost of Detonation

The parameter K_i appears in both Equations (3.1.13) and (3.1.14) and occurs in either the *on-site* or the *at-target* scenario introduced in the previous subsection. The former is discussed after the latter in this subsection. Overall, K_i is the expected consequences (in terms of cost) of a successful attack on a state. Specifically, K_A is the consequence of a successful (*on-site* or *at-target*) attack on State A using an asset obtained from State A and K_B is the consequence of a successful attack on State B using an asset obtained from State A. Generally, calculating K_i necessitates parsing the parameter into smaller, more manageable constituents as shown in Equation (3.1.21) – specifically for a successful *at-target* attack.[90]

$$K_i = K_i^{hu} + K_i^{ec} + K_i^{in} \tag{3.1.21}$$

The individual costs from the loss of lives (K^{hu}), the economic impact (K^{ec}) of such losses, and the loss of national infrastructure (K^{in}) contribute additively to the overall consequences. For example, assuming State B is the U.S., an *at-target* attack on the current largest impact target (read: highest population density) would be on Manhattan, New York City (with a maximum population density of 25,846 people per square kilometer)[91] This is used as an example for calculating K^{hu}. With an estimated yield of 10 kilotons (kT) for an improvised nuclear detonation comprised of highly enriched uranium, a surface detonation could create an air blast of 5 psi of overpressure at a radius of 0.77 km.[92] This is enough for widespread fatalities due to collapsing buildings especially in a highly populated area such as New York City (0.77-km blast radius is shown in yellow in Figure 3.5 on Manhattan Island – used herein as part of Google Earth's fair use copyright policy).[93]

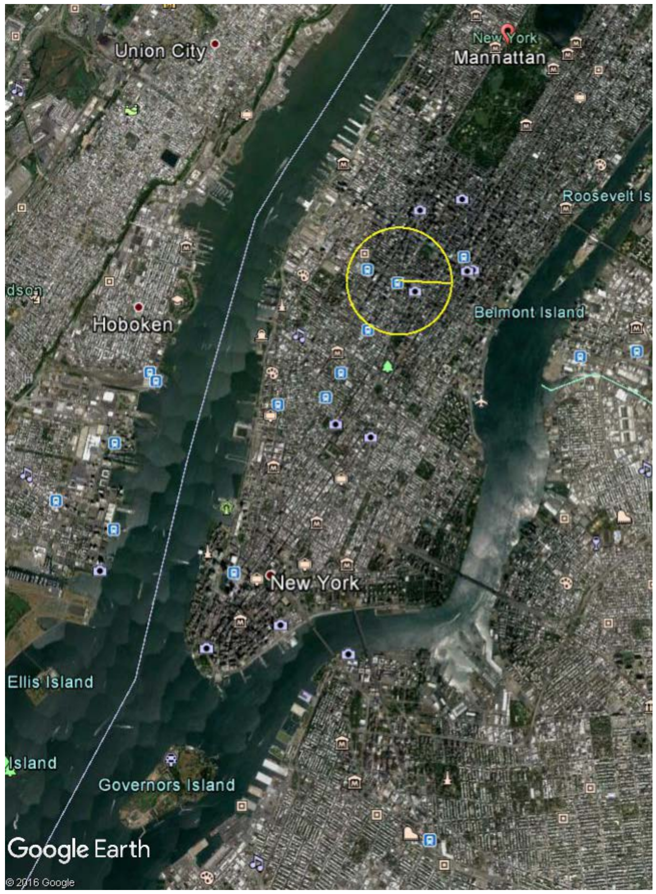

Figure 3.5. 5-psi blast radius of 10-kT device on Manhattan

According to a medical study on nuclear war by Daugherty et al., the number of fatalities from the 15kT Hiroshima detonation was 50% at approximately 7.5 psi (as shown in Figure 3.6).[94]

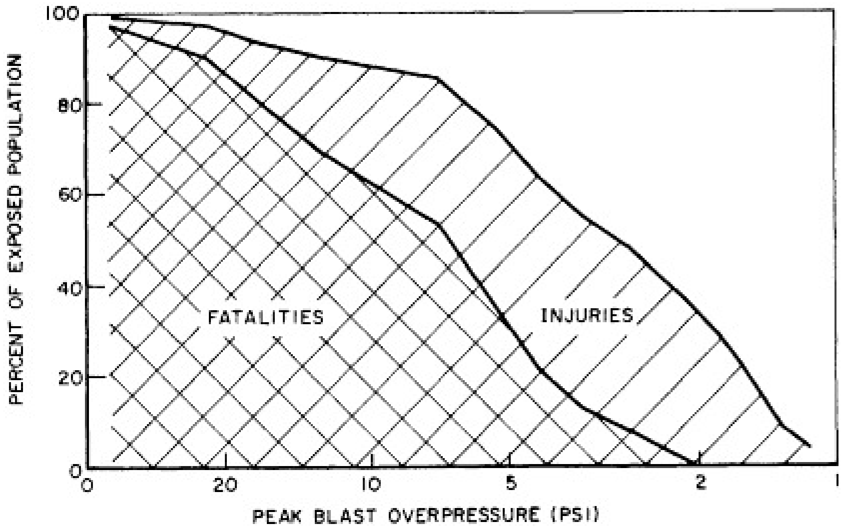

Figure 3.6. Hiroshima casualty rates

$$K^{hu} = \left[\frac{\text{ppl}}{\text{km}^2}\right] \cdot \left[\pi \cdot r_b^2\right] \cdot \left[50\%\ \text{fatality}\right] \cdot \left[\text{VSL}\right] \quad (3.1.22)$$

Using 7.5 psi as the peak blast pressure limit with a 10kT detonation produces a radius with a blast range, r_b, of 0.77km.[95] Using this value for r_b and the 2016 value of human life set at $9.6 million, the consequence of the immediate loss of life in 2016 assuming NYC's maximum population density, K^{hu}, is computed at 96.84B USD per Equation (3.1.22).[96]

$$K^{hu} = \left[25{,}846 \frac{\text{ppl}}{\text{km}^2}\right] \cdot \left[\pi \cdot (0.77\text{km})^2\right] \cdot [0.5] \cdot \left[\frac{\$9.6\text{M}}{\text{person}}\right] = \$231{,}081\text{M}$$

For the same year, the associated economic loss (K^{ec}) from losing that number of people, in $M, is computed by taking the population density and multiplying by the area at that blast radius, a 50% fatality rate, and the gross domestic product per capita. Inputting similar values from the example above into Equation (3.1.23) and with 2015 GDP per capita in the U.S. as $56,115 yields $K^{ec} = 1{,}351\text{M USD}$.[97]

$$K^{ec} = \left[\frac{\text{ppl}}{\text{km}^2}\right] \cdot \left[\pi \cdot r_b^2\right] \cdot \left[\text{50\% fatality}\right] \cdot \left[\text{GDP}_{\text{per capita}}\right] \qquad (3.1.23)$$

Effects on a state's infrastructure (K^{in}) depend on the location of an attack and can include replacing telecommunications, resurrecting homes, reestablish electricity distribution, rebuilding transportation means, mitigating societal fears, and, in general, recovering to normalcy. Quantified values of such critical infrastructure reestablishment were found by studying losses from and reconstruction costs after (both natural and man-made) catastrophic events: the 2015 Nepal earthquake, the 2010 Haiti earthquake, the 2008 Sichuan earthquake, the 2005 Hurricane Katrina, the 2008 Hurricane Ike, the 2012 Hurricane Sandy, the 1986 Chernobyl nuclear accident, and the 2011 Fukushima-Daichi incident. Numbers for recovery and reconstruction range drastically depending on infrastructure capabilities prior to any event, number of people displaced or otherwise impacted, the severity of the event, and more. For example, as of March 2016, the reconstruction and recovery costs associated with the Fukushima-Daichi incident in Japan has been estimated to exceed $250 billion.[98] However, for the reconstruction and recovery of the island nation of Haiti after its 2010 earthquake, the estimated amount was stated as $14 billion.[99]

Case studies 1-5 presented in Chapters 4-8 use computed values for K_i that are based on published literature. An example estimation for an *at-target* K_i uses a 10kT nuclear terrorist's device (either a stolen 10kT device or a manufactured one using stolen fissile material) to attack 2016 NYC and consists of $K^{hu} = 231{,}081$M USD, $K^{ec} = 1{,}351$M USD, and $K^{in} = 67{,}600$M USD (based on a recovery estimate from the 2012 Hurricane Sandy natural disaster in NYC). Therefore, $K_{16} = 231{,}081 + 1{,}351 + 67{,}600 = 300$B USD.

For estimating K_i for a successful *on-site* attack, the measure of the adversary's success is lower than that of the former discussion. Particularly, a non-state adversary infiltrating a site with nuclear assets, gaining control over at least one asset, and detonating it requires fewer steps than physically removing the asset and transporting it to a target. For this reason, the *on-site* consequence is estimated at 10% of the consequence for the *at-target* attack. Various reasons for this assumption are that if an adversary were to detonate an asset located at a site (either storage or military), the potential impact would be arguably less because usually sites with nuclear assets are located away from heavily populated areas (assumed at about 1%). Moreover, the investment in the proximate humans to a storage or military facility would increase that value by at least one order of magnitude due to the increased investment in handlers, operators, and/or soldiers located at the site. Hence, the supposed impact on human life, economic loss, and infrastructure can be relaxed to roughly 10% for computation purposes in Chapters 4-8.

For each case study, the *at-target* consequence is used to evaluate T_A. Subsection 9.4 provides a comparison between the consequence on State A of an *on-site* detonation and an *at-target* detonation.

3.1.2.4. Non-interdiction Probability and μ Value

The parameter for non-interdiction probability, $P_B(C_B)$ (cf. Eq. (3.2.1), is based on two factors: the invested cost on interdiction, C_B, and the marginal reduction rate from each investment, μ. The form of the relationship is shown in Equation (3.1.24).

$$P_B\left(C_B\right) = e^{-\mu \cdot C_B} \tag{3.1.24}$$

In an assumption taken from a similar field (i.e., international safeguards), the non-interdiction probability, P_B, is set at 5%.[100] This threshold is chosen following many instances in similar industries where a 95% probability is accepted and stated as a reasonable goal: the U.S. Nuclear Regulatory Authority guide on nuclear material physical inventories and the Australian safeguards and Non-Proliferation Office's publication on how the IAEA safeguards measures support detecting undeclared nuclear activities.[101,102] The range of P_B is 0 to 1 and relates to the C_B range of infinity to 0, respectively. With 5% set as a plausible non-interdiction probability and an actual annual investment cost gleaned from published literature, the value of μ for year y is determined via Equation (3.1.25) below.

$$\mu_y = \frac{-1}{C_B^{ACT}} \cdot \ln\left[P_B\left(C_B^{ACT}\right)\right] \tag{3.1.25}$$

For each year of a case study, μ will be estimated using published values of C_B and a plausible assumption for P_B explained in each chapter. Additionally in Section 9.2, a reassessment of the final strategies will be discussed with adjusted values of μ as determined by a non-linear regression line similar to what was mentioned in Subsection 3.2.1.

3.1.2.5. Expanding the Potential Source of the Asset

When concerned with the defined threat cost for State B, the parameter Γ represents the potential for a source of a nuclear asset to originate from beyond State A (i.e., to expand the source from where the asset comes). Though the bilateral arrangements discussed throughout this document express the primary source is State A, in reality, State B should support an interdiction system independent of the asset's source. If only State A should be considered the origin of a threatening asset to State B, $\Gamma = 0$. If it is to be assumed that an asset threatening State B can originate equally from State A as *not* State A, then $\Gamma = \mathrm{L}(C_A)$ – this implies the likelihood from State A still outweighs the likelihood from all other nuclearly capable nation states. If the origin of the asset is more likely to come from outside State A, then $\Gamma > \mathrm{L}(C_A)$. The discussion for each case study includes more details for parameter Γ per year.

3.1.3. Results in Terms of Percent Utility Gained

The methodology developed in this dissertation lends itself to the creation of a basic method for evaluating utility gained or lost through correlating strategies. Assume the Nash Equilibrium solution concept (exhibiting *uncorrelated* strategies) as a baseline from which to compute players' utilities and the received utilities of any subsequent solution concept is compared to that. In an effort to quantify the change of utilities, results for each case study are presented near the end of each chapter's case study as a percent additional utility received by each state and the regime. The *uncorrelated* total cost from the NE is used as a baseline, TC_{UNC}. Any subsequent strategy, TC_x, is then evaluated against TC_{UNC} as shown in Equation (3.1.26):

$$\Delta U = TC_{UNC} - TC_x$$
$$\%U = \frac{\Delta U}{TC_{UNC}}$$
$$\%U = \frac{TC_{UNC} - TC_x}{TC_{UNC}}$$
$$\%U = 1 - \frac{TC_x}{TC_{UNC}} \tag{3.1.26}$$

As will be shown in the final sections of Chapters 4-8, when the utility dynamics calculated via Equation (3.1.26) above are positive for both states, then both states could receive a positive benefit in the regime. A negative number represents when no benefit results and, furthermore, a state may then experience an increase in its total costs.

3.2. Determining the Solution Concepts

As was discussed in Chapter 2, the bulk of traditional game theory consists of non-cooperative game theory (NCGT) – analyzing players' decisions and utilities in response to other players' actions. Assuredly, NCGT does not imply that players never cooperate in a game – only that they will cooperate if there is something to gain. Cooperative solution concepts arise when players decide to coordinate their strategies to achieve a mutually beneficial Pareto Optimum that is not a Nash Equilibrium (recalling the Prisoner's Dilemma game of subsection 2.4 where either player is tempted to change their strategy to increase their payoff). The present work begins by determining two solution concepts in a two-state game (consisting of *uncorrelated* and *correlated* strategies) and then determining a bargaining solution concept that evaluates the benefits of cooperation for both players.

In section 3.1 we describe the frame upon which the methodology is based (the concept of strategic and threats costs), the objectives guiding the use of the methodology, the construction of the characteristic functions for both states, and a method used in each case study on how one could convey results. Within this section, we discuss the determination of solution concepts. Below are six assumptions for State A's characteristic function for computing its utility:

1. The threat cost of State A, T_A, is a function of State A's strategic cost, C_A – discussed in Section 3.2.
2. T_A is in terms of positive cost – with these models, cost is positive
3. The derivative of T_A is negative – this signifies that for each incremental investment made, there is a reduction in threat
4. The second derivative of T_A is positive – this signifies the nature by which with each subsequent increment of strategic cost, the observed reduction in threat decreases (otherwise, subjected to diminishing returns)
5. If there is no strategic investment, the threat cost, T_A, will exhibit a maximum cost
6. As the level of strategic investment approaches infinity, the threat cost will asymptotically approach 0.

Conversely, State B's utility characteristic function assumes much the same as above except for two elements:

1. The threat cost of State B, T_B, is a function of State B's strategic investment, C_B, and of State A's strategic investment, C_A.
2. The addition of an asset source expansion parameter, Γ, signifies the potential for a source of the nuclear threat against State B and leads to a possible positive slope for T_B (as $C_A \rightarrow \infty$ but not as $C_B \rightarrow \infty$).

With the above assumptions in mind, the *uncorrelated* and *correlated* strategies for States A and B are described in Subsection 3.2.1. Subsection 3.2.2 includes the bargaining solution that utilizes a cooperative game theoretic approach.

3.2.1. *Uncorrelated* and *Correlated* Solution Concepts

As discussed in Subsection 3.1.1, within non-cooperative game theory, two players in a game will evaluate and decide strategies to maximize their own payoffs while assuming the other player will do the same. If a player's individual payoff can be increased by coordinating a strategy with the other player, then cooperation may result. The former solution concept resulting from both players choosing their respective best-response strategies (to the other player's assumed strategy) yields the Nash Equilibrium. In the methodology described herein, this is referred to as the *uncorrelated* solution concept. The latter results from the presence of an additional benefit that can be had by the players coordinating their strategies – i.e., the *correlated* solution concept.

Specifically in this work, the first defined game model consists of state-level players, States A and B, coming to a Nash Equilibrium by considering their options and adopting a respective *uncorrelated* strategy that would provide the highest expected utility. The second game model is defined as both states *correlating* their strategies to attain a potential additional payoff (a decrease of total costs). When the surplus is present, the states' *correlate* their strategies within the bilateral regime and then proceed to the bargaining solutions discussed later in Subsection 3.2.2. Below, we step through the process of solving for the *uncorrelated* and *correlated* strategies for both State A (the source state) and State B (the *target* state) focuses on interdicting any asset used for an attack at its borders.

Computing the *uncorrelated* solution concept requires recalling Equation (3.1.6) and solving for the minimized strategic cost for State A, C_A' by solving when $\frac{dTC_A}{dC_A} = 0$. Based on State A's threat cost (Equation (3.1.13) in the previous Subsection 3.1.2), the following derivation begins with Equation (3.2.1):

$$TC_A\left(C_A\right) = C_A + P_A \cdot K_A \cdot \left[\alpha \cdot e^{-\lambda \cdot C_A}\right] \tag{3.2.1}$$

$$\left.\frac{dTC_A}{dC_A}\right|_{C_A = C'_A} = 1 - \alpha \cdot \lambda \cdot P_A \cdot K_A \cdot e^{-\lambda \cdot C'_A} = 0$$

$$1 = \alpha \cdot \lambda \cdot P_A \cdot K_A \cdot e^{-\lambda \cdot C'_A}$$

$$\frac{1}{\alpha \cdot \lambda \cdot P_A \cdot K_A} = e^{-\lambda \cdot C'_A}$$

$$\ln\left(\frac{1}{\alpha \cdot \lambda \cdot P_A \cdot K_A}\right) = -\lambda \cdot C'_A$$

$$C'_A = \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right) \tag{3.2.2}$$

Equation (3.2.2) is the specific solution to Equation (3.1.6) and is only valid when $C_A' > 0$. As the four-factor term $(\alpha \cdot \lambda \cdot P_A \cdot K_A)$ decreases to 1, $C_A' \rightarrow 0$. Therefore, when

$(\alpha \cdot \lambda \cdot P_A \cdot K_A) < 1$, C'_A should equal zero. Conceptually, this implies that within the bilateral structure, State A is the only investor in its uncorrelated strategy of securing its assets and when it invests nothing, the four-variable term to be less than 1. Hence, $C'_A = 0$. Conclusively, the two values for C'_A can be written as

$$C'_A = \max\left\{0, \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right\} \tag{3.2.3}$$

Solving for C'_B requires using Equation (3.1.7) and determining at which point is $TC_B(C'_A, C_B)$ at a minimum over $C_B > 0$ (unless $C'_B = 0$):

$$TC_B\left(C_A, C_B\right) = C_B + e^{-\mu \cdot C_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right) \tag{3.2.4}$$

$$\left.\frac{dTC_B}{dC_B}\right|_{C_A = C'_A, C_B = C'_B} = 1 - \mu \cdot e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right) = 0$$

$$1 = \mu \cdot e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)$$

$$e^{-\mu \cdot C'_B} = \frac{1}{\left\{\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right\}}$$

$$-\mu \cdot C'_B = \ln\left[\frac{1}{\left\{\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right\}}\right]$$

$$C'_B = \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right] \tag{3.2.5}$$

Similarly to Equation (3.2.3), $C_B = 0$ if $\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right)\right] < 1$ due to the fact that this methodology currently assumes only non-negative values of costs (as discussed in Subsection 3.1.1). Therefore:

$$C'_B = \max\left\{0, \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right)\right]\right\} \tag{3.2.6}$$

Inserting the corresponding minimum values for C'_A and C'_B into Equations (3.3.1) and (3.3.4) yields the total costs for State A and State B, TC'_A and TC'_B, respectively (with the two possible values of C'_A and C'_B).

$$TC'_A\left(C'_A = 0\right) = \alpha \cdot P_A \cdot K_A$$

$$TC'_A\left(C'_A = \frac{1}{\lambda} \cdot \ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right) = \frac{1}{\lambda} + \frac{1}{\lambda} \cdot \ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)$$

$$TC'_B(0,0) = K_B \cdot \left(\alpha + \Gamma\right)$$

$$TC'_B\left(0, \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]\right) = \frac{1}{\mu} + \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha + \Gamma\right)\right]$$

$$TC'_B\left(\frac{1}{\lambda} \cdot \ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right), 0\right) = \frac{K_B}{\lambda \cdot P_A \cdot K_A} + K_B \cdot \Gamma$$

$$TC'_B\left(\frac{1}{\lambda} \cdot \ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right), \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]\right) = \frac{1}{\mu} + \frac{1}{\mu}\ln\left(\frac{\mu \cdot K_B}{\lambda \cdot P_A \cdot K_A} + \Gamma \cdot \mu \cdot K_B\right)$$

The sum of TC'_A and TC'_B is the total cost, TC'_{AB} (shown in Equation (3.2.7)), of the regime's uncorrelated strategies: State A securing its assets while State B invests in its interdiction methods. Values of parameters λ, P_A, K_A, K_B, Γ, and μ (based on various literature findings for each case study described in Chapters 4-8) are needed to adequately evaluate TC'_A , TC'_B, and TC'_{AB}.

$$TC'_{AB} = TC'_A + TC'_B \tag{3.2.7}$$

To determine the states' *correlated* strategies, the individual objective utility (cost) functions, Equations (3.2.1) and (3.2.4), are combined into one function (Equation (3.2.8) based on Equation (3.1.5)) and values of C_A and C_B are solved for when the aggregate

TC_{AB} (first referenced in Subsection 3.1.1 and shown in Equation (3.2.8)) is at a minimum – otherwise, when the total derivative, $dTC_{AB} = 0$ and when $\frac{d^2TC_{AB}}{dTC_A^2} > 0$ and $\frac{d^2TC_{AB}}{dTC_B^2} > 0$ as shown in Equation (3.2.9) below.

$$TC_{AB}\left(C_A, C_B\right) = C_A + P_A K_A\left[\alpha e^{-\lambda C_A}\right] + C_B + e^{-\mu C_B} K_B\left[\alpha e^{-\lambda C_A} + \Gamma\right] \quad (3.2.8)$$

And the function to solve for C_A and C_B as the minima:

$$TC^*_{AB}\left(C^*_A, C^*_B\right) = \min_{C_A \geq 0, C_B \geq 0}\left(C_A + P_A K_A\left[\alpha e^{-\lambda C_A}\right] + C_B + e^{-\mu C_B} K_B\left[\alpha e^{-\lambda C_A} + \Gamma\right]\right) \quad (3.2.9)$$

If $\partial TC_A(C_A^*) = 0$ and $\partial TC_B(C_A^*, C_B^*) = 0$, then strategy set $TC_{AB}^*(C_A^*, C_B^*)$ can be a global minimum when the second partial derivatives are also positive. If neither partial derivative equals zero, then the minimum probably occurs at some location where either C_A or C_B are negative. If this occurs, then the correlated strategy set is defined at the x- or y-axis for the strategic cost that is 0 and the non-negative value of the other state's strategic value. The minima of these models, TC_{AB}^*, were assumed as the global minima due to the simplicity of the objective functions. Within the methodology for each case study, the difference between TC_{AB}^* and TC'_{AB} is the surplus, *v*:

$$v = TC'_{AB} - TC^*_{AB} \quad (3.2.10)$$

Dividing the existing surplus between the two players provides the opportunity to visualize and consider various bargaining solutions between the players – the subject of the subsequent subsection.

3.2.2. Bargaining Solutions and Utility Transferability

The existence of a surplus (as computed by Equation (3.2.10) above) provides an opportunity for the players to negotiate their expected utilities and plotting the optimal divisions of the surplus creates a linear utility transferability line in this case with a slope of -1 (i.e., a Pareto optimal efficiency frontier).[103] With the cooperative game theoretic approach, States A and B act as two players who may form a solution concept consisting of their correlated strategies. This solution concept may yield a surplus that must be divided under some pre-negotiated terms to attain a bilateral nuclear security regime. The simplest example of a pre-negotiated bargaining solution is to simply share the surplus evenly to ensure fairness – otherwise, referred to as the Nash Bargaining Solution (NBS). In order to achieve the correlated solution concept though, one player may have to invest more strategic costs than required with their initial uncorrelated strategy. If a player cannot afford the investment, an idea is for the other player to subsidize or partially contribute to their cost via prepayment for a larger percentage of the eventual surplus (invoking the concept of utility transferability or side payments). This concept (not unlike Myerson's analysis of the "threat game" discussed in Section 2.4) is discussed more within each case study but in this section, is described generally.

Under the *uncorrelated* solution concept (i.e., the Nash Equilibrium of both states acting independently), State A contributes a certain financial cost for securing its nuclear assets, C_A, and State B concerns itself with investing in interdiction of assets, C_B. If A and B *correlate* their strategies, the need may arise for State B to contribute to State A's cost of nuclear security. This leads to the strategic cost of securing State A's assets, referred to now as C_S, to be separated into that contributed by State A, C_A, and that contributed by State B, C_{AB}.

$$C_S = C_A + C_{AB} \tag{3.2.11}$$

This also leads to the investment that is made for State B's strategy of interdiction, now referred to as C_I, to be the remainder of State B's total investment after its contribution to State A's security, C_{AB}.

$$C_I = C_B - C_{AB} \tag{3.2.12}$$

Within the discussion in subsection 3.1.1 where C_A and C_B are introduced, a further distinction should be made to define costs *of* a strategy for a state (C_S and C_I) and costs *to* a state for a strategy (C_A, C_B, and C_{AB}). Reformatting Equations (3.2.1) and (3.2.4) with C_S and C_I to define the total costs of these strategies leads to Equations (3.2.13) and (3.2.14) below.

$$TC_S\left(C_S\right) = C_S + P_A K_A\left[\alpha e^{-\lambda C_S}\right] \tag{3.2.13}$$

$$TC_I\left(C_S, C_I\right) = C_I + e^{-\mu C_I} K_B\left[\alpha e^{-\lambda C_S} + \Gamma\right] \tag{3.2.14}$$

Using both equations here and recalling Equation (3.1.5) provides the opportunity to define the new total amalgamated cost of the securing and interdicting state strategies, TC_{SI}:

$$TC_{SI}\left(C_S, C_I\right) = C_S + P_A K_A\left[\alpha e^{-\lambda C_S}\right] + C_I + e^{-\mu C_I} K_B\left[\alpha e^{-\lambda C_S} + \Gamma\right] \tag{3.2.15}$$

Substituting C_S and C_I with Equations (3.2.11) and (3.2.12) yields

$$\begin{aligned} &TC_{SI}\left(C_A, C_B, C_{AB}\right) = \\ &\left(C_A + C_{AB}\right) + P_A K_A\left[\alpha e^{-\lambda\left(C_A + C_{AB}\right)}\right] + \left(C_B - C_{AB}\right) + e^{-\mu\left(C_B - C_{AB}\right)} K_B\left[\alpha e^{-\lambda\left(C_A + C_{AB}\right)} + \Gamma\right] \end{aligned}$$

Which is simplified to yield Equation (3.2.16):

$$TC_{SI}\left(C_A,C_B,C_{AB}\right)=C_A+P_AK_A\left[\alpha e^{-\lambda(C_A+C_{AB})}\right]+C_B+e^{-\mu(C_B-C_{AB})}K_B\left[\alpha e^{-\lambda(C_A+C_{AB})}+\Gamma\right]$$

(3.2.16)

Equation (3.2.16) results in a third variable not present in TC_{AB}: C_{AB} or the contribution to State A's security borne by State B. This variable is important for determining the *correlated* solution concept because through C_{AB} can State B hope to persuade State A in adopting certain strengthened applied security strategies. Moreover, in the cooperative game model, C_{AB} can come from the calculated surplus and therefore, defines a bargaining solution. Specifically, when $C_{AB} = 0$, then $C_S = C_A$ and $C_I = C_B$ meaning each state bears the cost for security and interdiction independently. If $C_{AB} = C_A^* - C_A'$, then State B bears the additional amount of State A's security that is needed to reach the correlated strategy set (C_A^*, C_B^*). State A then only contributes at the level of its uncorrelated strategy (C_A') yet receives the full benefit of the correlated threat cost (T_A') while State B bears all strategic costs no borne by State A. Furthermore, the portion of State B's contribution to State A can be either taken from State B's portion of the surplus or, if the amount is greater than State B's portion, from side payments outside the surplus. Case studies included in Chapters 4-8 show a range of bargaining solutions (in terms of utility transferability) beyond what is introduced in this subsection.

In sum, within each case study, parameters (e.g., α, K_A, K_B, etc.) for both States A and B will be determined using available data form literature. Furthermore, actual investments from contemporary years in security (C_A^{ACT}) and interdiction (C_B^{ACT}) for A and

B respectively will be used to calculate values such as λ_y and μ_y which will then be used as estimates for computing appropriate *uncorrelated* and *correlated* strategies for both States A and B. Once the *uncorrelated* and *correlated* solutions are determined, the ways by which the surplus can be shared between State A and B help visualize various options for cooperation. As shown in the case studies in chapters 4-8, the defined boundaries of the surplus divisions provide the opportunity to study State B's potential to influence State A's security strategy.

3.2.3. Computational Tools for Solving for Solution Concepts

To calculate the strategy sets, costs are calculated using a MATLAB script and the bargaining solutions were determined via MSExcel.[g] Input parameters for each case study are specified in Subsection 3.1.2 and are set within the code. The parameters were collected from various government and scholarly publications and used to define specific circumstances of the various case studies. Once input into each case study, the methodology computed the Nash Equilibria for each state's *uncorrelated* strategy per Equations (3.1.6) and (3.1.7). The MATLAB minimization routine *fmincon* was used to solve for the *correlated* strategies where the process requires determining the minimized total regime cost (TC^*_{AB}) based on two independent variables: C_A and C_B. The MATLAB results were then inserted into the pre-programmed MSExcel spreadsheet that visualized the various *correlated* solution concepts and computed the associated bargaining solution

[g] The MATLAB script and MSExcel spreadsheets are included in Appendix A.

strategy sets which divided the surplus between the states and the percent changes in utilities for the three entities (State A, State B, and the regime between States A and B).

3.3. Trust and Commitment Issues

Game theory provides a way to analyze strategies made by rational players. With no guarantees of players' actions, overall trust between the players can be a potential issue when trying to determine equilibria and solution concepts. Furthermore, in cooperative games, actions can include the formation of coalitions or partnerships. Therefore, with both non-cooperative and cooperative type games, trust is a vital element for calculating utilities and is especially significant in simultaneous, single-play games when players lack knowledge of others' intentions, utilities, or strategies. Furthermore, communication is key to countering this lack of knowledge. Either overt (or indirect) communication between players is vital to game theory. Lacking overt communication methods, single-play games present a challenge of uncertainty between the players. Therefore, overcoming uncertainty to establish cooperation requires pre-emptive communications or "a leap of faith" between the players. After establishing a series of repeated games however, players are able to observe actions and use them as a means of continued communication which provides a better understanding of the type of counterpart they are playing against. Given time, the ability of predicting strategies can segue into a semblance of confidence and potentially trust between players.[104]

In the Prisoner's Dilemma game presented in Figure 2.8, the Pareto Optimum is not the Nash Equilibrium. This conveys a trust issue with the Pareto Optimum: a unilateral change, without response from the other player in a single-play game, can result in a more favorable payoff for the changing player but a worse payoff for the opposing player (which results in a worse collective payoff for both). This implication shows there lies a moral conundrum: despite the incentive for an originating player to change their strategy, will they do so if it negatively impacts the collective payoff?

A game theorist can mitigate the impact of any lack of trust by introducing a "nature" component that provides a probability for the "type" of player playing the game.[105] This adds a passive a third "player" that can impact game results. However, actions by (and imposed on) players themselves can also mitigate the impact of any such moral conundrums. For example, instead of utilizing a passive third "player" to estimate variability in the type of players, there exists an option of utilizing an active third player that would serve as an enforcer of any game play. If trust and commitment are issues among players, there exists the option of (creating and) using a third party actor to monitor the game play. Historically, this has been accomplished through the establishment of the International Atomic Energy Agency (IAEA) in 1967 or the Joint Comprehensive Plan of Action with Iran which defines the IAEA as the third party enforcer.

Another action indirectly alluded to by Schelling as he details the necessity of communicating commitment in order to make a credible threat: "if a threat is anything more than an assertion that is intended to appeal to the other player by power of suggestion… it must involve some notion of commitment – real or fake – if it is to be anything."[25] He alludes to establishing a red line in order to compel a player to act. Examples of this include the U.S. government establishing the quarantine line for the

Soviets during the Cuban Missile Crisis in 1962, the large U.S. military presence on the Korean Peninsula poised to stand against North Korean aggression, or President Obama setting a red line on chemical weapons in Syria in 2012.[106,107,108] Another method alluded to by Snyder consists of communicating a threat of simple denial to an aggressor of his/her accomplishment – it "is more likely than reprisal action to promise a rational means of defense in case deterrence fails."[109] These various actions taken by players in game theory rely on the credibility of their communicated intentions.

Presented within this work, the game models consist of two players where there is no guarantee of trust. Hence, various trust and commitment issues do arise in the assessments in Chapters 4-8. Firstly, we address that the game players are states seeking strategies to address nuclear security concerns. Second, the states' utility metrics are in the form of financial investment/savings. When the states decide on strategies, they do so with the expectation that the other state acts rationally. This forces the states to compute utilities so as to select their preferred decision – otherwise known as expected utility theory (EUT). Based on the von Neumann-Morgenstern utility theorem, EUT is the general method by which a decision is made in order to maximize the expected utility from the potential outcomes of various options.[80]

Referring to the second step in the preceding paragraph, computing utilities can be accomplished through devising a utility function by which the value of decisions are made. In the models used in this work, the states' expected utilities were solely measured with costs because of the dearth of publications in quantifiable measurements of security and interdiction.[110] Though this introduced a potential lapse in connecting monetary investment to effectiveness of a state's nuclear security posture, it was the most logical assumption to make in terms of establishing a metric. Related to the defined metric, in each case study, an assumption was made that states had interest in reducing costs and therefore, were amenable to cooperation. Furthermore, connecting investments with achieving state objectives also implies the lack of government corruption. According to the 2015 global corruptions perceptions index, some bilateral regimes discussed herein include two states that fall at opposite sides of the spectrum of perceived government corruption.[111]

Regarding cooperation enforcement, no case study in Chapters 4-8 have a third-party arbiter with punitive powers to enforce agreements. Ostensibly, this lack of enforcer opens the game to be abused by one or both the states and, thus, does not imbue either player with a sense of commitment through fear of reprisal. Primarily, this work assumes the existence of potential mutually beneficial results is what drives the states to cooperate and correlate their strategies. However, a third-party entity could be utilized to convince the states of said benefit or to provide a contingency surplus to be shared by both.

Specific to the game models presented in each case study, a state's lackluster commitment to strategic objectives could raise red flags. Specifically, if a state's total *uncorrelated* costs do not surpass a certain threshold, there exists a potential for the overall decrease in costs (achieved through correlating strategies) to exceed those *uncorrelated* costs and cause the state's strategy to consist of a negative total cost. This leads the state to potentially receive a positive utility from cooperating with the other state – a position described in 2005 by Corr that explains how a state can hold the lack of security as a commodity and by which it receives a profit.[33] He continues by identifying a growing concern first applied to Russia but later to North Korea: the "threat of insecure storage [of nuclear assets] as a resource or commodity to sell." Illustrating this theory quantitatively could serve the community and future decision makers when analyzing the benefits and costs to entering bilateral arrangements such as the Joint Comprehensive Plan of Action between the P5+1 states and Iran or the Agreed Framework between the U.S. and North Korea.[112,113] As is conveyed in Section 9.6, this potential outcome can result from a state's inadequate investment which conversely conveys a lack of commitment to nuclear security (i.e., if the state is not committed to the idea of nuclear security, it may rely on the partner to pay for it).

Overall, within some of the case studies discussed in Chapters 4-8, the mere existence of a benefit of cooperation is not the sole basis upon which the players should correlate their strategies. Obviously, by considering the aforementioned limitations to the methodology as well as other outside political and economic factors, cooperation between players can easily be inhibited. Specific cases are discussed in more detail in Section 9.1.

3.4. Incommensurability and Plausibility of Data

Before each case study is discussed individually, it should be noted that the obtained utilities presented in each should be considered evocative, not definitive. One of the primary challenges of applying the above methodology was computing utilities based on acquired and calculated parameters that were an attempt to adequately represent the real-world situation of each case study. Ostensibly, the parameters inserted into each state's total cost function should not be considered absolute but should still be considered plausible enough to gain insight into the bilateral nuclear security regimes. This concept of relying on logic rather than values for assisting decision makers understand and assess complex situations is explained by Snyder in that "the factors involved [are] highly intangible, unpredictable, unmeasurable, and incommensurable except in an intuitive way."[105] He expounds on the concept of "the essential logic of deterrence" instead of the potentially intangible values by which decisions can be set. He continues on the need for developing and understanding the underlying logic "to predict, to measure, and in some sense, to make incommensurable factors commensurate if [decision makers] are to reach wise decisions." He closes with the proposition: "logic is just as applicable to imprecise quantities as to precise ones; to express it in mathematical terms can provide a useful check on intuitive judgment and may bring to light factors and relationships which judgment would miss."[105]

Therefore, this methodology strives to provide insight into bilateral nuclear security regimes (specifically in various bilateral regimes formed in the latter half of the 20th century) regardless of the precision of the various parameters. Ascertained through openly-available literature, state-determined investment values in Chapter 4-8 were used to define a focal point that represented the actual investments made by both states. This focal point was used to compute the α and λ parameters which were then used to determine the next focal point which represented the *uncorrelated* solution concept. This solution concept is the Nash Equilibrium per Equations (3.1.6) and (3.1.7). From this latter focal point is where the eventual *correlated* solution concept (i.e., the Pareto Optimal solution) would be calculated. The location of the three aforementioned focal points relied heavily on the data we attained through openly-available publications. Some data proved difficult to acquire due to the level of sensitivities associated with nuclear security measures and costs. Ostensibly, the computed focal points could differ with other sources of data and, therefore, some conclusions could change. However, an honest effort was made to replicate observed strategies between two states during particular times in recent history per each case study. Assuredly, with more (accurate) data beyond what was currently openly available, focal points could be defined differently and credibility in the results could increase.

3.5. Introducing the Case Studies

Particular parameters of five case studies are modeled in Chapters 4-8. Initially, only nuclear weapons were considered. However, loss rate data and other relevant information for weapons-usable material are more readily available in open-source searches and literature reviews. Therefore, in the interest of connecting the methodology to citable data, the case studies included after this chapter consist of aggregate data of relevant security information for both nuclear weapons and special fissionable material (uranium, plutonium, and thorium) in units of significant quantities.[h] Significant quantities are used herein as a common metric for identifying assets for nuclear-armed states that

[h] 1 SQ = 8kg Plutonium = 8 kg Uranium-233 = 25kg Uranium-235 (HEU) = 75kg Uranium-235 (LEU) = 10t Uranium-235 (natural) = 20t Uranium-235 (depleted) = 20t Thorium

also include weapons-usable materials per the definitions used by the International Atomic Energy Agency Safeguards Glossary.[38]

1) Case Study 1: United States – Union of Soviet Socialist Republics (1985) – This case study is modeled to replicate the American-Soviet state-level relationship during the height of the Cold War in the mid-1980s when both states had no collaborations in nuclear security. Furthermore, at this point, the perceived threat from the Soviet Union did not include any non-state adversary or terrorist obtaining a nuclear weapon from the U.S.S.R. and using it against the U.S. or its allies. The main threat in 1985 consisted of the state-level threat from the U.S.S.R. in launching the barrage of nuclear weapons to the U.S. For that, the U.S. and U.S.S.R. entered into many arms control and disarmament agreements but none addressed applied security of the other nations' assets. This case study was chosen to confirm assumptions in the model and replicate a quintessential historical bilateral relationship (which essentially was non-existent regarding securing nuclear weapons or materials). Data from this "snap shot" in time was acquired through U.S. Congressional testimonies, Soviet Duma publications regarding federal budgets, news articles, and published historical analyses by both Russian and American scholars. The conclusions from this case study convey the potential for a large benefit to both sides if cooperation had occurred. However, due to the obvious external circumstances between the rivals, state-level collaborations (especially coordination of nuclear security strategies) were minimal regardless of the potential benefit. Furthermore, Soviet society was so heavily restricted and policed that not many scholars nor government officials seriously considered the thought of

nuclear assets escaping from Soviet control. This concept is discussed in greater detail in Chapter 4.

2) Case Study 2: United States – Russian Federation (1995) – This case study exemplifies the infancy stages of the Comprehensive Threat Reduction Program between the U.S. and Russia that began with the Nunn-Lugar Act in assisting the newly-emerging state of Russia to regain control of its nuclear arsenal. After the fall of the Soviet Union, a perceived threat emerged among U.S. policy makers recognizing the lack of adequate security measures protecting Russian special nuclear material and nuclear weapons. In addition to addressing the security of special nuclear material through collaborations led by the U.S. Department of Energy's Material Protection, Control, and Accounting (MPC&A) Program Office, the U.S. Department of Defense worked closely with the Russian Ministry of Defense in regaining control of and re-securing its arsenal of nuclear weapons.[114] This case study incorporates a strong sense of collaboration between the partner states as well as the understanding that Russia was potentially the main source of any nuclear weapon that would have been used against the United States. Moreover, at that time, Russia's economic struggles lent credibility to the perception that a terrorist threat would most likely attain a nuclear weapon from Russia rather than from another nuclear-weapons state. Data for this case study was acquired through numerous publications released by both American and Russian legislatures, scholars, and other reports. The year of 1995 was selected as the high point of collaboration between the former Cold War rivals – especially for nuclear security. After various reports of loose nuclear materials were interdicted in 1994, the push for

collaboration from both states was heightened. As will be shown in Chapter 5, the potential for correlating nuclear security strategies for the benefit of both states was high.

3) Case Study 3: United States – Russian Federation (2008) – This case study exemplifies the relationship between the U.S. and Russia in the midst of the MPC&A and CTR where the U.S. was heavily invested in nuclear-related collaborations with Russia. This case study recognizes the existing bilateral arrangements between both for such things as cooperating against the common threat of nuclear terrorism (e.g., via the Global Initiative to Combat Nuclear Terrorism) but that the Russian Federation continued to receive financial assistance from the U.S. to support nuclear security endeavors.[115] Notable in this case study is the continued level of cooperation between the two states, the growing Russian economy, and the advent of other nuclear threats in the world. Beyond a representative year of complex collaboration between the states, this year was chosen for the easily accessible data gleaned from scholarly publications on government expenditures for nuclear materials smuggling and interdiction for both Russia and the U.S. from academia, government, national laboratories, and non-governmental organizations such as think tanks and other research entities. As conveyed in Chapter 6, a waning interest in collaboration began to take hold between the two state governments for various reasons: a strengthening Russian economy, the existence of other nuclear terrorist threats, and appropriations fatigue expressed by some U.S. legislators. These points are described further in Chapter 6.

4) Case Study 4: United States – Russian Federation (2015) – This case study incorporates more recent developments between the U.S. and Russia – particularly, the dissolution of the joint MPC&A Program between the states in 2013 as well as the annexation of Crimea by the Russian military which led to the eventual suspension of all nuclear security activities between the countries. With this in mind, it is assumed in the U.S. that Russia has assumed many nuclear security responsibilities and understand the necessity of appropriate nuclear security and therefore, is continuing to properly secure all potential targeted materials and assets. However, the U.S. is not familiar with exact measures in Russia because of the lack of collaboration between the states. This factor is included in this case study – as is the fact that the number of potential sources of a nuclear weapon being commandeered by a non-state actor and used against the U.S. has increased. Though previously Russia was the only security concern, in modern times, other states such as the Democratic Peoples' Republic of Korea or Pakistan have become nuclear concerns as well. This year was chosen as the most recent year (at the time of this writing) of available data through scholarly publications, official American and Russian government records, and various other news articles. As is conveyed in Chapter 7, the potential for benefit from collaboration still existed but due to other domestic and external issues, the interest in cooperative strategies waned on both sides.
5) Case Study 5: United States – Pakistan (2008) – This case study incorporates a very collaborative nature between the two states due to the continued relationship both states have in the particular interest of jointly meeting the terrorist threat. Of interest

with Case Study 5, the U.S. provides financial contributions specifically for enhancing the security of the Pakistani nuclear arsenal.[116] Furthermore, in spite of regional influences and interests, the U.S. and Pakistan maintain a strong collaborative relationship beyond mere nuclear security. Importantly, it is considered that in today's world, Pakistan is not the only potential source for a nuclear weapon to be appropriated and used against the U.S. The data from the U.S. on nuclear security (i.e., interdiction) stems from the same 2008 data acquired for Case Study 3. The data for Pakistan securing its nuclear assets in 2008 was retrieved from various news articles and a small number of scholarly publications. Despite this limitation, the conclusions show that cooperation proves at least somewhat beneficial to both sides of this nuclear security regime.

4. CASE STUDY 1: U.S.S.R. – U.S. (1985)

This case study consists of the relationship between the United States (U.S.) and the Union of Soviet Socialist Republics (U.S.S.R.) during the final years of the Cold war, circa 1985. Under the party leadership of Mikhail Gorbechev and his policies of *Perestroika* and *Glasnost*, the U.S.S.R. was inching closer towards an open society by allowing more international trade and influence from the rest of the world.[117] During this time, the bilateral relationship, though adversarial, consisted of actions and strategies in the limited scopes of space exploration, nuclear disarmament, weapons testing, and others. The largest inventories of nuclear weapons were in the U.S. and the U.S.S.R. and, specifically for securing weapons, there was no interaction between the two. The lack of cooperation embodied the nature of the relationship for this case study. However, this case study also provides whether any collective benefit could have been achieved if the U.S.S.R. and/or the U.S. had they correlated their nuclear security strategies. We detail the parameter evaluations of the model in Section 4.1 and, in Sections 4.2 and 4.3, delve into the calculations of the various strategies and hypothetical bargaining solutions between the states. Lastly, Section 4.4 includes a presentation of the results.

4.1. Evaluating the Game Model Parameters

This case study models two parties in a nuclear security arrangement with no cooperation and also with cooperation. Specifically, the source state (State A or the U.S.S.R.) is interested in securing its own assets so that it is not used against it and the target state (State B or the U.S.) is interested in an asset not being released from under State A's control and subsequently used against State B. Though the objectives of both states are different (hence, their interests), the potential consequences of an event can impact both states substantially. Therefore, it is assumed that the potential non-state adversary does not serve the interest of either player. A terrorist organization potentially could use an asset to attack the target state, but there is the probability the asset could be used against the source state also. The latter provides incentive for the source state to secure its weapons. As modelled in the uncorrelated-strategy game, the target state makes no attempt to influence the source state to enhance its security further. Rather, the target state adopts interdiction as its countermeasure to this threat. As was the case during the Cold War, each state commits to interdiction and security independently and cooperation between the two was non-existent. In reference to the total cost model found in Equations (3.2.1) and (3.2.4), many parameters are identified for State A (α, λ, P_A, and K_A) and State B (μ, K_B, and Γ) in the remainder of this Section 4.1. Sections 4.2 and 4.3 detail how to calculate the appropriate uncorrelated, correlated, and bargaining strategies corresponding to the defined parametric values in the pursuit of minimizing the total costs to each state.

4.1.1. State A Parameters

In order to solve the non-cooperative games for the states' strategic costs, parameter values tailored to the 1985 era were determined under various assumptions. When dealing with the Soviet Union (as well as Russia in Chapters 5-7), it is "exceedingly difficult if not impossible to comprehend the total historical and current costs of the

Soviet/Russian nuclear weapons program" – let alone the nuclear security program.[118] With this in mind, inferences and other best estimates from a myriad of historical publications were used to estimate spending effectiveness and other parameters for this Soviet case. Though not an explicitly-defined parameter of the methodology, the overall level of security within the Soviet Union during this time (where society was under heavy government control) is roughly estimated by defining α subsequently. Because the population was closely watched and their actions were so scrutinized and policed, it is a safe assumption that the likelihood of a nuclear-related crime within the Soviet Union in 1985 was low.

Estimating the first parameter, α_{85}, necessitates determining the probability of a single unlikely event during the 36-year time span between the Soviet Union having a nuclear device (1949) and the year of this case study (1985). Deemed as a rare event in their work, Quigley and Revie determine the probability of such an event corresponds to the reciprocal of 2.5 times the number of years.[119] Therefore, with a sample size of 36, the rate of loss for this rare event (an asset being removed from the Soviet Union's control in the midst of the Cold War) is

$$\alpha_{85} = \frac{1}{2.5 \cdot (36\text{yr})} = 0.011 \left[\frac{\text{SQ}}{\text{yr}} \right] \tag{4.1.1}$$

With $\alpha_{85} = 0.011 \left[\frac{\text{SQ}}{\text{yr}}\right]$, the marginal reduction in loss, λ, for the Soviet government in 1985 is computed after more assumptions and other information are extracted from published reports and statements. First, due to a lack of actual lost assets in 1985 (and before), a rough estimation of loss rate is made using a relative value from the 1955

AFSWP report for acceptable losses of a 10-100kT weapon (of the like prevalent under Soviet control around 1985).[120,121] The number of "accepted accidental events" stated in the report was 3e-4 events/year when only 2422 weapons were in existence in the U.S. Adjusting this value per weapon for the U.S. and re-estimating for Soviet numbers in 1985 (39,197) yields an estimated value for the loss rate as 0.00486 events.[122] Also, based on a stated estimation of funds (2M USD) spent on "fissionable material accountability" from 1957 and converting from 1957 to 1985 yields 7.66M USD via the Bureau of Labor Statistics of the U.S. Department of Labor.[123,124] Hence, using α_{85}, an estimated loss rate and an estimated cost for security, with Equation (3.1.18) yields the following:

$$\lambda_{85} = \frac{1}{-C_{85}} \cdot \ln\left[\frac{L(C_{85})}{\alpha_{85}}\right]$$

$$\lambda_{85} = \frac{1}{-7.66} \cdot \ln\left[\frac{0.00486}{0.011}\right]$$

$$\lambda_{85} = 0.108\left[\$M^{-1}\right]$$

For this case study, P_A, for lack of a better value is estimated as the number of years when a fatal terrorist attack occurred during the entire time of existence of the Soviet Union (particularly because for the ten-year span of this case study, no fatal terrorist attacks occurred in Moscow). Therefore, using only 1973 and 1977 as the two years when a fatal attack occurred in Moscow from the entire existence of the Soviet Union (1922 – 1991) leads the frequency of such an attack 2 years in 69 – or 0.029.[125,126] Hence, P_A = 0.03.[i]

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[i] As stated in Subsection 3.2.2, this value (along with many others) is meant to be evocative for the sake of proving the logic presented herein – not to provide the definitiveness of the results.

Recalling Subsection 3.2.3 in defining the consequences, K_A, of an attack suffered by State A requires several calculations and estimations. For calculating the *at-target* K_A value, the following data are used: 8,580,000 as the population of Moscow that year[127]; 878.7 square kilometers as the land area of the city of Moscow[128]; $1,780 as the GDP per capita of Russia[129]; 0.77 kilometers as the radius range of a 10-kT nuclear detonation[92] and a 50% survivability; 32,000USD as the value for statistical life for an average Russian from Moscow[130]; and the rough estimate of rebuilding Moscow after a large-scale attack, K^{ni}, is given by using the cost used to rebuild Mexico City in 1985 after a devastating 7.6 magnitude earthquake as 5 billion USD.

$$K_A^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right](50\%\ \text{fatality})\left[\text{VSL}\right] = \left(\frac{8.58\text{M}}{878.7}\right)\cdot\left[\pi \cdot 0.77^2\right]\cdot(0.5)\cdot\left[\$32{,}000\right] = \$291\text{M}$$

$$K_A^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right](50\%\ \text{fatality})\left[\text{GDP}_{\text{per capita}}^{1985}\right] = \left(\frac{8.58\text{M}}{878.7}\right)\cdot\left[\pi \cdot 0.77^2\right]\cdot(0.5)\cdot\left[\$1{,}780\right] = \$16.2\text{M}$$

$$K_A^{ni} = \$5{,}000\text{M}$$

$$K_A = K_A^{hu} + K_A^{ec} + K_A^{ni} = \$291\text{M} + \$16\text{M} + \$5{,}000\text{M} = \underline{\$5{,}307\text{M}}$$

Therefore, in this model, *at-target* K_A = 5.3B USD.

4.1.2. State B Parameters

The parameter μ circa 1985 was estimated by examining the beginning of the American radiological/nuclear interdiction system as well as border porosity threshold U.S. lawmakers strived to achieve. In early 1984, an estimated 600 tons of radioactive steel (alloyed with cobalt-60 taken from a medical radiotherapy device) was shipped to 23

U.S. states and Canada from Mexico. As some material was detected near Los Alamos National Laboratory, a national effort was put in place later that year by the U.S. Customs Service and the Nuclear Regulatory Commission to install radiation monitors at 22 border crossings.[131] Though never implemented, the estimated total cost of the first year of that program was stated as 6.1M USD. With this in mind and interdictions only having occurred by chance, the estimated non-detection probability, P_B, for this case study is set at 0.99 (a reasonable value due to an ineffective national interdiction system). Hence, the estimated value for μ is 0.00164 according to Equation (3.1.24) – when compared to other case studies, this value of μ is lower and therefore, implies that investment to State A's security was more beneficial to State B than investing in their own interdiction as discussed in Section 4.3:

$$P_B(C_B) = e^{-\mu \cdot C_B}$$
$$\mu = \frac{-\ln\left(P_B(C_B)\right)}{C_B} = \frac{-\ln(0.99)}{6.1} = \underline{0.00164}$$

For State B, Equations (3.1.21), (3.1.22), and (3.1.23) are used to estimate K_B and its constituents (K^{hu}, K^{ec}, and K^{ni}):

$$K_B = K_B^{hu} + K_B^{ec} + K_B^{ni}$$

Where, in using values from Glasstone and Viscusi[95,132]:

$$K_B^{hu} = \left(\frac{\text{ppl}}{\text{km}^2}\right)\left[\pi \cdot r_{th}^2\right](50\%\ \text{fatality})\left[\frac{\$1.33\text{M}}{\text{person}}\right]$$
$$K_B^{hu} = \left(\frac{1.464\text{M}}{59.1\text{km}^2}\right)\cdot\left[\pi \cdot (0.77\text{km})^2\right]\cdot(0.5)\cdot\left[\frac{\$1.33\text{M}}{\text{person}}\right]$$
$$K_B^{hu} = \$30{,}683\text{M}$$

And K^{ec} for 1985 uses the estimated population and land area of Manhattan, New York City (1.464M people and 59.1 km^2) and the gross domestic product per capita ($18,264.4) to produce the following values[133,134]:

$$K_B^{ec} = \left(\frac{\text{ppl}}{\text{km}^2}\right)\left[\pi \cdot r_{th}^2\right]\left(50\%\ \text{fatality}\right)\left[\text{GDP}_{\text{per capita}}^{1985}\right]$$
$$K_B^{ec} = \left(\frac{1.464\text{M}}{59.1\text{km}^2}\right)\left[\pi \cdot \left(0.77\text{km}\right)^2\right]\left(0.5\right)\left[\frac{18{,}269.4}{1{,}000{,}000}\right]$$
$$K_B^{ec} = \$421.5\text{M}$$

K^{ni} is estimated such as for State A by comparing the similar event of the Mexico City earthquake of 1985. Again, the financial estimated impact to the city's infrastructure exceeded 5 billion USD. Therefore, without a better alternative, K^{ni} = $5,000 M, and the summation of K_B is shown below:

$$K_B = K_B^{hu} + K_B^{ec} + K_B^{ni} = \$30{,}683\text{M} + \$422\text{M} + \$5{,}000\text{M} = \underline{\$36{,}105\text{M}}$$

As is conveyed in the previous case study, the value of Γ is an attempt to convey the equal likelihood of an attack against State B using an asset from another source that is not State A. Hence, in this case study, we do not expand the potential source of an asset and therefore, $\Gamma=0$.

4.2. Non-Cooperative Game Theory

The 1985-era values, estimated as described in the preceding section, of the parameters for States A and B for Case Study 1 are summarized in Table 4.1. All regime solutions and strategy sets are determined using these values but other future analysts are encouraged to modify the values of the game parameters per their own investigation/knowledge.

Table 4.1. Estimated parametric values for CS1

α	0.011	[SQ]
λ_{85}	0.108	[$\$M^{-1}$]
P_A	0.03	[ ]
K_A	5,307	[$M]
μ	0.00164	[$\$M^{-1}$]
K_B	36,105	[$M]
Γ	0	[SQ]

Of note, λ is almost two orders of magnitude greater than μ – the implication of such is State B receives a greater reduction in its threat cost from investing in State A's security rather than State B's interdiction measures. A comparison of the parametric values for each case study is included in Section 9.1.

4.2.1. Non-Cooperative Solution: Uncorrelated Strategies

The *uncorrelated* strategies for Case Study 1 are determined by calculating the Nash Equilibrium: the best response strategy for each state trying to minimize their respective total costs as shown in Equations (3.1.6) and (3.1.7). By using values from

Table 4.1, the strategic costs for State A and B can be evaluated for each state's uncorrelated strategy, C'_A and C'_B, by using Equations (3.2.2) and (3.2.5), respectively. C'_A and C'_B are then used in Equations (3.1.13) and (3.1.14) to calculate T_A and T_B and lastly into Equation (3.1.2) for each state. Below is the series of calculations for calculating C'_A and C'_B (with all units in millions USD).

$$C'_A = \max\left\{0, \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right\}$$
$$C'_A = \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)$$
$$C'_A = \frac{1}{0.108}\ln\left(0.011 \cdot 0.108 \cdot 0.03 \cdot 5307\right) = \frac{1}{0.108}\ln\left(0.189\right) = -15.4$$
$$C'_A = \max\left\{0, -15.4\right\} = \underline{0}$$

$$T_A\left(C'_A\right) = P_A \cdot K_A \cdot \left[\alpha \cdot e^{-\lambda \cdot C'_A}\right] = 0.03 \cdot 5307 \cdot \left[0.011 \cdot e^{-0.108(0)}\right] = \underline{1.7}$$
$$TC'_A\left(C'_A\right) = C'_A + T_A\left(C'_A\right) = 0 + 1.7 = \boxed{1.7}$$

Using the values from Table 4.1 produces the result $C'_A = 0$, meaning that there is no requirement for the Soviet Union to have made an additional strategic investment in its own nuclear security even for an *at-target* attack on a center of population like Moscow. For this reason, the consequence of an *on-site* attack is not used to determine another value for C'_A – if the *uncorrelated* strategy yields no need for investing against a more impactful attack, then there is less benefit to investing against a lesser attack at the facility which houses the nuclear asset originally.

Similarly, determining the value of C'_B:

$$C'_B = \max\left\{0, \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]\right\}$$
$$C'_B = \frac{1}{0.00164}\ln\left[0.00164 \cdot 36105 \cdot \left(0.011 \cdot e^{-0.108(0)} + 0\right)\right]$$

$$C'_B = \frac{1}{0.00164}\ln\left[0.651\right] = -261.4$$
$$C'_B = \max\left\{0, -261.4\right\} = \underline{0}$$

$$T_B\left(C'_A, C'_B\right) = e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)$$
$$T_B\left(C'_A, C'_B\right) = e^{-0.00164(0)} \cdot 36105 \cdot \left(0.011 \cdot e^{-0.108(0)} + 0\right) = \underline{397.2}$$
$$TC'_B\left(C'_A, C'_B\right) = C'_B + T_B\left(C'_A, C'_B\right) = 0 + 397.2 = \boxed{397.2}$$

The total cost for both states in this non-cooperative bilateral regime amounts to 398.9M USD (where both states take unilateral action).The grand majority of State A's total cost is borne by State B, in the form of threat costs, but, in the next subsection, it will be conveyed how it would have been more cost effective to invest directly into State A's strategy.

4.2.2. Non-Cooperative Solution: Correlated Strategies

Seeking to understand the observed lack of cooperation between Cold War foes in 1985 regarding nuclear security, we replicate the parameters of the states' relationship and use them in TC_{AB} to determine whether cooperation would have provided any benefit to the two states.

$$TC_{AB}\left(C_A,C_B\right)=C_A+P_A\cdot K_A\cdot\left[\alpha\cdot e^{-\lambda\cdot C_A}\right]+C_B+e^{-\mu\cdot C_B}\cdot K_B\cdot\left[\alpha\cdot e^{-\lambda\cdot C_A}+\Gamma\right]$$

(4.2.1)

Continuing by solving for the correlated strategies, the values calculated in this section were based on finding the global minimum in the multivariate total regime cost equation as shown in Equation (4.2.1). To ensure the values were located at a minimum, the partial derivatives were set to zero at the solution pair.

$$\frac{\partial TC_{AB}}{\partial C_A}=1-\lambda\cdot P_A\cdot K_A\cdot\alpha\cdot e^{-\lambda\cdot C_A^*}-\lambda\cdot e^{-\mu\cdot C_B^*}\cdot K_B\cdot\alpha\cdot e^{-\lambda\cdot C_A^*}=0$$

$$\frac{\partial TC_{AB}}{\partial C_B}=1-\mu\cdot e^{-\mu\cdot C_B^*}\cdot K_B\cdot\left[\alpha\cdot e^{-\lambda\cdot C_A}\right]=0$$

$$\lambda\cdot P_A\cdot K_A\cdot\alpha\cdot e^{-\lambda\cdot C_A^*}+\lambda\cdot e^{-\mu\cdot C_B^*}\cdot K_B\cdot\alpha\cdot e^{-\lambda\cdot C_A^*}=\mu\cdot e^{-\mu\cdot C_B^*}\cdot K_B\cdot\alpha\cdot e^{-\lambda\cdot C_A^*}$$

$$\lambda\cdot P_A\cdot K_A+\lambda\cdot e^{-\mu\cdot C_B^*}\cdot K_B=\mu\cdot e^{-\mu\cdot C_B^*}\cdot K_B$$

$$\lambda\cdot P_A\cdot K_A\cdot e^{\mu\cdot C_B^*}=\mu\cdot K_B-\lambda\cdot K_B$$

$$e^{\mu\cdot C_B^*}=\frac{K_B}{\lambda\cdot P_A\cdot K_A}\left(\mu-\lambda\right)$$

$$C_B^*=\frac{1}{\mu}\ln\left[\frac{K_B}{\lambda\cdot P_A\cdot K_A}\left(\mu-\lambda\right)\right]$$

$$C_B^*=\frac{1}{0.00164}\ln\left[36105\left(0.00164-0.108\right)\right]$$

$$C_B^*=\frac{1}{0.00164}\ln\left(-3840.12\right)=undefined\rightarrow C_B^*=\underline{0}$$

$$\frac{\partial TC_{AB}}{\partial C_A} = 1 - \lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} - \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} = 0$$

$$1 = \lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} + \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*}$$

$$e^{\lambda \cdot C_A^*} = \lambda \cdot P_A \cdot K_A \cdot \alpha + \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha$$

$$e^{\lambda \cdot C_A^*} = \alpha \cdot \lambda \left(P_A \cdot K_A + e^{-\mu \cdot C_B^*} \cdot K_B \right)$$

$$C_A^* = \frac{1}{\lambda} \ln \left[\alpha \cdot \lambda \left(P_A \cdot K_A + e^{-\mu \cdot C_B^*} \cdot K_B \right) \right]$$

$$C_A^* = \frac{1}{0.108} \ln \left[0.011 \cdot 0.108 \left(0.03 \cdot 5307 + e^{-0.00164(0)} \cdot 36105 \right) \right] = \underline{35.2}$$

Calculating the second partial derivatives for determining the concavity utilizes the following:

$$\frac{\partial^2 TC_{AB}}{\partial C_A^2} = \alpha \cdot \lambda^2 \cdot P_A \cdot K_A \cdot e^{-\lambda \cdot C_A} + \alpha \cdot \lambda^2 \cdot e^{-\mu \cdot C_B} \cdot K_B \cdot e^{-\lambda \cdot C_A}$$

$$\frac{\partial^2 TC_{AB}}{\partial C_B^2} = \mu^2 \cdot e^{-\mu \cdot C_B} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A}$$

$$\frac{\partial^2 TC_{AB}}{\partial C_A \partial C_B} = \lambda \cdot \mu \cdot e^{-\mu \cdot C_B} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A}$$

And:

$$\frac{\partial^2 TC_{AB}(x,y)}{\partial C_A^2} = \alpha \lambda^2 P_A K_A e^{-\lambda x} + \alpha \lambda^2 e^{-\mu y} K_B e^{-\lambda x}$$

$$\frac{\partial^2 TC_{AB}(x,y)}{\partial C_A^2} = 9.7e-3 > 0$$

$$\frac{\partial^2 TC_{AB}(x,y)}{\partial C_B^2} = \mu^2 e^{-\mu y} K_B \alpha e^{-\lambda x}$$

$$\frac{\partial^2 TC_{AB}(x,y)}{\partial C_B^2} = 2.28e-3 > 0$$

Therefore, as stated in Subsection 2.2.1, both requirements are satisfied for point (C_A^*, C_B^*) to be considered a minimum. Hence, when $C_A^* = 35.2$ and $C_B^* = 0$ for this case study, $TC_{AB}(C_A^*, C_B^*)$ is at its minimum. This implies the correlated strategic costs, as exhibited by C_i^*, are $C_A^* = 35.2$ and $C_B^* = 0$. The remaining threat and total costs are calculated based on the above values for C_A^* and C_B^*:

$$T_A\left(C_A^*\right) = P_A \cdot L\left(C_A^*\right) \cdot K_A$$
$$T_A\left(C_A^*\right) = P_A \cdot \alpha \cdot K_A \cdot e^{-\lambda \cdot C_A^*}$$
$$T_A\left(C_A^*\right) = 0.03 \cdot 0.011 \cdot 5307 \cdot e^{-0.108(35.2)} = \underline{0.04}$$

$$T_B\left(C_A^*, C_B^*\right) = P_B\left(C_B^*\right) \cdot K_B \cdot \left[L\left(C_A^*\right) + \Gamma\right]$$
$$T_B\left(C_A^*, C_B^*\right) = e^{-\mu \cdot C_B^*} \cdot K_B \cdot \left[\alpha \cdot e^{-\lambda \cdot C_A^*} + \Gamma\right]$$
$$T_B\left(C_A^*, C_B^*\right) = e^{-0.00164(0)} \cdot 36105 \cdot \left[0.011 \cdot e^{-0.108(35.2)} + 0\right] = \underline{9.3}$$

$$TC_i\left(C_i\right) = C_i + T_i\left(C_i\right)$$
$$TC_A\left(C_A^*\right) = C_A^* + T_A\left(C_A^*\right)$$
$$TC_A\left(C_A^*\right) = 35.2 + 0.04 = \underline{35.2}$$

$$TC_B\left(C_B^*\right) = C_B^* + T_B\left(C_B^*\right)$$
$$TC_B\left(C_B^*\right) = 0 + 9.3 = \underline{9.3}$$

$$TC_{AB}\left(C_A, C_B\right) = TC_A\left(C_A\right) + TC_B\left(C_A, C_B\right)$$
$$TC_{AB}\left(C_A, C_B\right) = 35.2 + 9.3 = \underline{44.5}$$

Table 4.2 includes all calculated values thus far.

Table 4.2. Annualized costs [in $M] from the non-cooperative game

	Uncorrelated [$M]	Correlated [$M]
C_A	0	35.2
$T_A(C_A)$	1.7	0.04
$TC_A(C_A)$	1.7	35.2
C_B	0	0
$T_B(C_A, C_B)$	397.2	9.3
$TC_B(C_A, C_B)$	397.2	9.3
TC_{AB}	398.9	44.5

Figure 4.1 shows the three dimensional profile of the strategic and total costs to the states. The red star represents the *uncorrelated* (unilateral) Nash Equilibrium between the states at point (0, 0, 398.9). The cyan star represents the correlated, cooperative strategy point at (35.2, 9.3, 44.5) if both states collaborate in strategies. The total cost, TC_{AB} is reduced substantially but requires an investment increase by State A (this is discussed in more detail in Section 9.1). This and all other 3-D plots were created using the MATLAB script included in Appendix A.

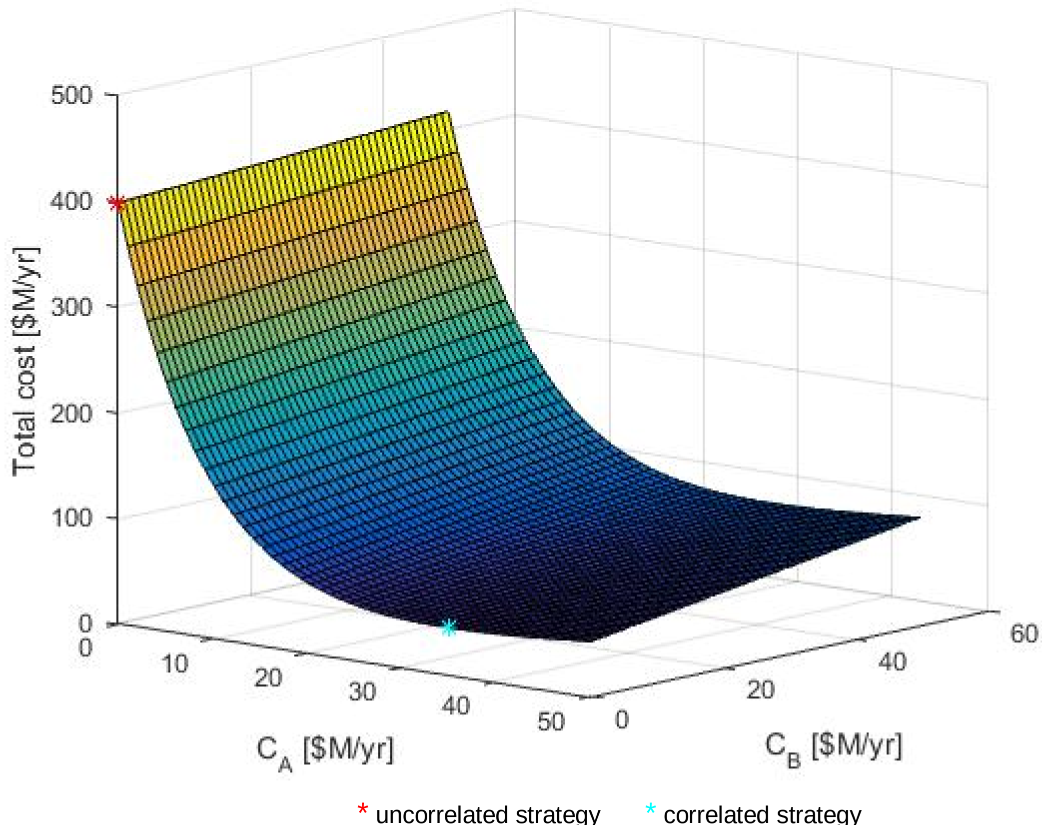

Figure 4.1. Total costs vs strategic costs for states A and B

4.3. Bargaining Solution Concepts and Utility Transferability

A two-dimensional representation of the various strategy points is included in Figure 4.2.

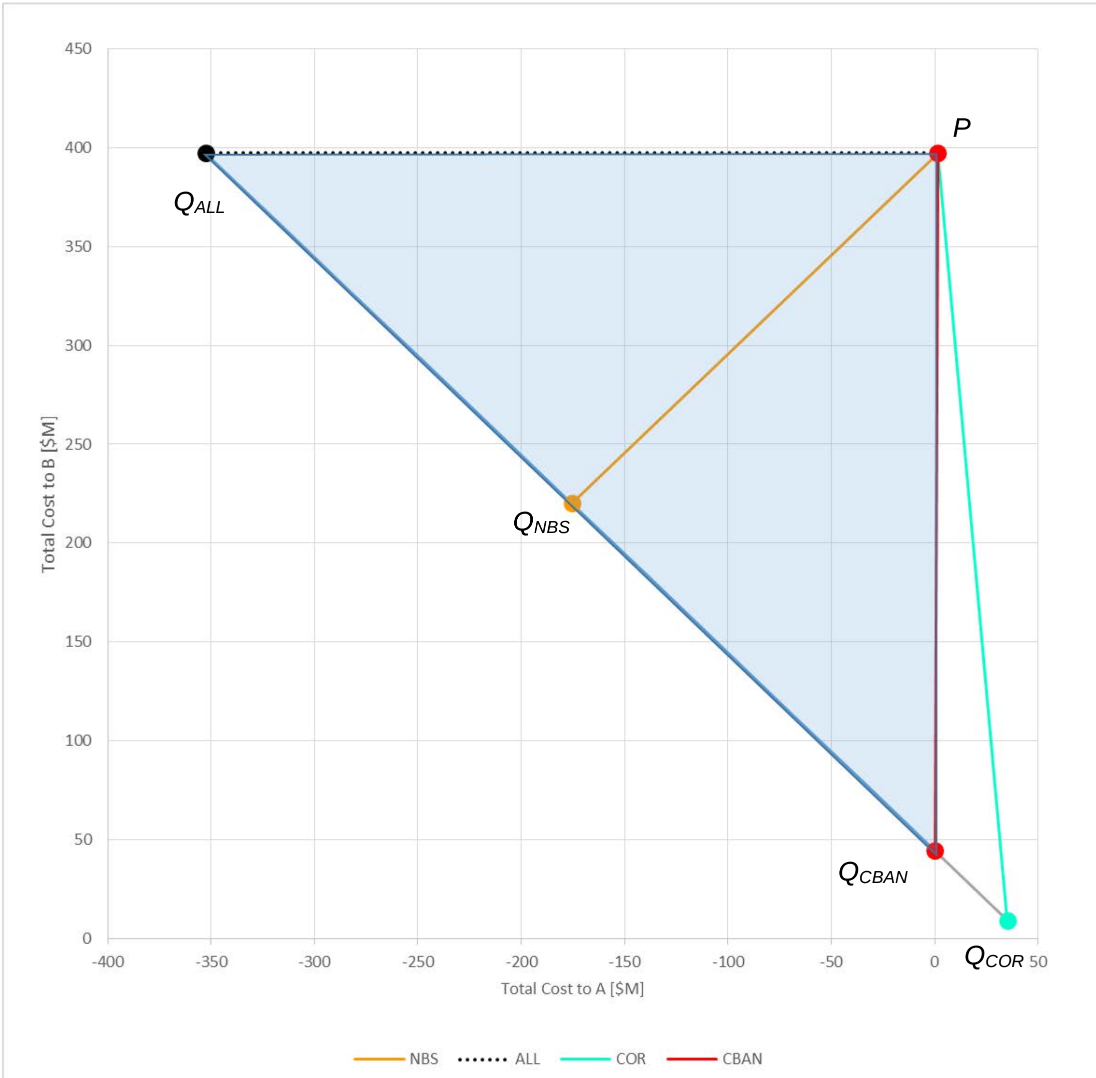

Figure 4.2. Imputations of the U.S.-U.S.S.R. game (1985)

As introduced in Subsection 2.5.1, the blue-shaded triangle represents the core of the cooperative game while any line segment that intersects the hypotenuse of the triangle defines an imputation (otherwise referred to as a bargaining solution or surplus split between the states). The cyan dot represents the *correlated* strategy set. This correlated strategy point lies on the line of utility transferability (UT) but does not exist in the game's core. Thus, as per the discussion in Subsection 2.5.2, the solution's viability is not certain. The surplus, visualized in Figure 4.1 as the distance between the points along the z-direction (i.e., the difference in total costs) and computed as $398.9 - 44.5 = 354.4$M, is now the bargaining medium between the two states.

Table 4.3. Annualized costs [in $M] from the non-cooperative game

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutral [$M]
C_A	0	35.2	0
$T_A(C_A)$	1.7	0.04	0.04
$TC_A(C_A)$	1.7	35.2	0.04
C_B	0	0	35.2
$T_B(C_A, C_B)$	397.2	9.3	9.3
$TC_B(C_A, C_B)$	397.2	9.3	44.5
TC_{AB}	398.9	44.5	44.5

Table 4.3 summarizes the various costs for each strategy. The third column conveys a renegotiated strategic option between the states. The C_{BA}-Neutral solution concept (shown as Q_{CBAN} in Figure 4.2) represents State B's contribution (C_{BA}) of all of State A's additional burden over its original strategic cost, C_A. This bargaining solution is

shown in Figure 4.2 as the intersection point of the red line and the dark blue UT line at (0, 44.5).

The orange line segment which bifurcates the hypotenuse is the NBS imputation (i.e., a 50-50 split of the savings between the states) and leads to the strategic point of (-175.4, 220) where each state receives half of the 354.4M USD savings surplus. Because the Soviet Union's correlated position is substantially better located than the U.S.'s position, they stand to receive a substantial negative cost (i.e., a profit). Lastly, the black dot located at (-352.6, 397.2) signifies a cooperative solution defined by the Soviet Union receiving the benefit of the entire surplus. This implies that the U.S. would not have received any benefit from the cooperation with the Soviet Union (in fact, this would have facilitated the opportunity for them to make a profit off their nuclear insecurity as insinuated in Corr's theory from Section 3.4 where if $TC_A < 0$, there would be a manner by which the Soviet Union would have profited).[33]

4.4. Analysis and Discussion

Recall that the states in this case study were the Soviet Union (U.S.S.R) as State A and the United States (U.S.) as State B, Table 4.4 shows the results for the four aforementioned bargaining strategies discussed in Section 4.3: the correlated strategy point (Q_{COR}), the NBS point (Q_{NBS}), the C_{BA} neutralization equivalent point (Q_{CBAN}), and the all-surplus-to-A point (Q_{ALL}). Additionally, the strategic point signifying the uncorrelated, unilateral strategic point, P, between the U.S.S.R. and the U.S. is also shown. Lastly, as a point of reference, the actual amounts for total cost per the definitions in

Chapter 3 are included as Q_{ACT} for reference. The aforementioned solutions were calculated in Sections 4.2 and 4.3. Moreover, the total costs to each state and the regime are shown. The metric used to assess which solution proves more favorable to the U.S. or the U.S.S.R. is the column with the s/r values. This quantity conveys the ratio of B's dividend of the surplus savings over A's from correlating their strategies. A negative value implies the strategic point falls out of the core (the blue-shaded triangle region) in Figure 4.2. A value of 1 implies the split is 50% to the Soviet Union and 50% to the U.S. – otherwise referred to as the Nash Bargaining Solution. When $s/r > 1$, more of the surplus goes to the U.S. Conversely, when $0 \leq s/r < 1$ (such as in Q_{CBAN} and Q_{ALL}), the Soviet Union receives more of the surplus.

Table 4.4. Imputations of the cooperative game for case study 1

	Savings Split for A [$M]	Savings Split for B [$M]	$\frac{s}{r}$	TC_A [$M]	TC_B [$M]	TC_{AB} [$M]
P	--	--	--	1.7	397.2	398.9
Q_{COR}	-33.54	387.9	-11.57	35.24	9.3	44.54
Q_{NBS}	177.18	177.18	1.00	-175.48	220.02	44.54
Q_{CBAN}	1.66	352.7	212.47	0.04	44.5	44.54
Q_{ALL}	354.36	0	0.00	-352.66	397.2	44.54
Q_{ACT}	--	--	--	9.06	831.2	840.26

The reader can compare these specific results to Figure 4.2 and assess which imputations/solutions could serve which state. If the Soviet Union and the U.S. had been able to overcome any political adversity and correlate strategies, the resulting strategic point, Q_{COR}, would have required the Soviet Union to invest slightly more than what it had already deemed necessary and even supplement the U.S.'s split of the surplus by an additional 33.5M USD. Q_{NBS}, Q_{CBAN}, and Q_{ALL} are located within the game's core and therefore seem as viable options (see Subsection 2.5.1) for bargaining a hypothetical bilateral nuclear security regime between the Soviet Union and the U.S. circa 1985 (discussed in Subsection 2.5.2). Importantly, if inclined to have desired a bilateral arrangement with the Soviet Union at this time for securing nuclear assets, the American government could have used this methodology to analyze any bargaining process with the Soviet Union using financial incentives outlines in Table 4.4.

Presenting the final results as utility percent advantages (as discussed in Section 3.1.2), Table 4.5 displays values to aid in determining how much each state would benefit in terms of gained or lost utility compared to the *uncorrelated* solution concept. Those with positive values exhibit those strategic points beneficial to that state – negative numbers imply a lost utility.

Table 4.5. Tabulated change in utility per imputation

	U_A	U_B	U_{AB}
Q_{COR}	-19.73	0.98	0.89
Q_{NBS}	104.22	0.45	0.89
Q_{CBAN}	0.98	0.89	0.89
Q_{ALL}	208.45	0.00	0.89

As exhibited by the results in Table 4.5, all but one strategy solution results in an advantage over the Nash equilibrium (unilateral) strategy point, P, for both states. The correlated strategy point, Q_{COR}, is a bargained solution where one state receives a negative benefit while the other does not. The Q_{ALL} strategy point shows a substantial benefit for the Soviet Union yet nothing for the U.S. – not a seemingly favorable regime for the U.S. to enter. In this case, an argument could be made about the U.S. instead focusing on its own expenditures for a substantive defensive posture. Though, conceptually with so much to gain for the Soviet Union, a great opportunity hypothetically exists within the core where the U.S. could influence the U.S.S.R. to play the game by selecting a strategy point which greatly benefits the latter (such as Q_{NBS} or Q_{ALL}). However, in light of the antagonistic relationship between the two states during this time in their history, any interaction would have had to supersede substantial impedances including domestic politics of collaborating with a cold war enemy. Lastly, despite the potential for the Soviet Union to have gained a negative cost (and thus a profit from the relationship), the utility benefit for the regime (U_{AB}) exemplifies a collective gain over the original *uncorrelated* strategy set at point P of 89%.

5. CASE STUDY 2: RUSSIA – U.S. (1995)

This second case study is a model of the bilateral regime[5] between the U.S. and the Russia Federation after the fall of the Soviet Union in late December 1991. That same year, American President H.W. Bush signed the Soviet Nuclear Threat Reduction Act of 1991 into law, commonly referred to as the "Nunn-Lugar" legislation that paved the way for establishing the Cooperative Threat Reduction (CTR) Program which helped establish the Russian-U.S. relationship focused on securing nuclear assets.[135] The relationship developed into a regime per Young's definition of a "specialized [set of] arrangements that pertain to well-defined activities and resources."[136] Resulting from insight gathered by Senators Sam Nunn and Richard Lugar, the CTR Program articulated two primary objectives: 1) to facilitate the transportation, storage, safeguarding, and destruction of nuclear weapons in the Soviet Union, its republics, and any successor states; and 2) to assist in the prevention of weapons proliferation. After a 1991 visit by Nunn to Moscow, there was an understood need for the U.S. and the four newly-independent states (Russia, Belarus, Kazakhstan, and Ukraine) to collaborate in securing former Soviet nuclear weapons. More so, upon his return from Moscow, Nunn called for "confidence-building measures and military exchanges as part of an effort to put in place quickly some measures that could shore up stability in the Soviet military and convey Washington's goodwill and support of a safe transition to a post-Soviet world."[137] This would aim at building trust between the states in an effort to open the collaboration for and show a commitment to enhancing nuclear security in the region (as discussed in Section 3.3, both trust and commitment would be integral to an effective nuclear security regime). In December 1994, the Budapest Memorandum was the agreement where Belarus, Ukraine, and Kazakhstan would surrender the former Soviet nuclear weapons within their borders to Russia by late 1996 in exchange for Russia accepting the states' sovereignties as well as accepting responsibility for the Soviet nuclear arsenal and the disarmament activities.[138] With that, the U.S. and Russia would collaboratively secure nuclear weapons and weapons-usable material under the CTR Program for over 23 years (until it was cancelled on January 1, 2015).[139]

In 1991, Russian President Boris Yeltsin established the Federal Inspectorate for Nuclear and Radiation Safety (known as Gosatomnadzor). It was originally formed to create a national system of radioactive and nuclear material control and accounting and regulate personnel handling nuclear materials.[140] Though it was a civilian regulatory body, it had authorization to inspect both civilian and military nuclear facilities that handled special nuclear material (eliciting the ire of both the Ministry of Defense's 12th Main Directorate and the Ministry of Atomic Energy).[141] Until 1995 when the authority over accounting and safety for nuclear weapons was transferred to the 12th Main Directorate, Gosatomnadzor was the sole entity responsible for nuclear material accounting and domestic inspections in the Russian Federation.[142,143] While it was still the main inspecting entity, in 1994, Gosatomnadzor stated the government would invest 1B USD to secure Russian nuclear materials for the subsequent 5-7 years.[144]

During these first few years of Russian existence, many instances of nuclear trafficking that originated from Russian nuclear facilities were beginning to occur. Though signed into law in late 1991, the Nunn-Lugar act was not able to secure funds to enhance nuclear security in Russia until fiscal year 1993 (beginning October 1992). At this point, most of the American funding was concentrated in securing nuclear weapons in Russia under the CTR program by the Department of Defense (DOD) while a small amount of funding was provided to Russia for securing weapons-usable nuclear material by the Department of Energy (DOE) under the newly formed Material Protection, Control, and Accounting (MPC&A) Program. Together, the American and Russian governments collaborated in securing nuclear weapons and weapons-usable materials in Russia through collaborative engagements between the DOD, DOE, and Gosatomnadzor for two subsequent decades. Bukharin provides more details on these and other of the various components (e.g., disposition of former weapons materials jointly declared excess to needs of the time) of the bilateral nuclear security regime that came into place circa 1995, along with other elements focused on securing personnel (e.g. former Soviet weapons designers), rather than materials *per se*.[145] Some of these latter elements include the Lab-to-Lab program focusing on promoting civil research collaborations between staff of the respective nuclear weapons laboratories and the similarly motivated International Science and Technology Center that attempted to pair former Soviet nuclear weapons designers with technical experts from across the EC and Japan, as well as from private industry in the U.S.

Case Study 2 models the relationship between the U.S. and Russia in 1995 as a regime after the first tangible benefits of both the CTR and the MPC&A Programs were beginning to be observed. Similar to Case Study 1 (1985), the *source* state (State A) is taken here as the Russian Federation and the *target* state (State B) as the United States. This chapter focuses on ascertaining the adequacy of the game-theoretic model described in Chapter 3 as a way to replicate the cooperative strategies under the CTR program and quantitatively determine various bargaining solutions between the states.

5.1. Evaluating the Game Model Parameters

As stated, Case Study 2 models a bilateral regime between the U.S. and the Russian Federation in 1995 when the collaboration under the CTR was underway. Here, both states invest in their respective strategic actions but State A's (RF) investment in securing its nuclear assets is not as strong as it once was during the Cold War when the Soviet Union relied on the "regimented, security-obsessed regime [to] hold down the risks of insider theft."[146] This was due in part to the economic down-turn that the RF suffered in the early 1990s as well as the general disorganization which caused "a deterioration of the fissile material management infrastructure."[147] When security decreased, the U.S. supplemented the gaps in funding via monetarily supporting the modernization of the Russian nuclear security complex. The assumptions made here reflect an attempt to make a plausible estimate with realistic data. The model can be run with other assumed input values for future analyses. Just as in Case Study 1 (1985), the set of parameters that must be evaluated for States A and B consist of (α, λ, P_A, K_A) and (μ, K_B, Γ), respectively.

5.1.1. State A Parameters

From Subsection 3.2.1.1 for 1994, α_{94} = 0.249 as the maximum loss rate when nothing is spent on securing nuclear assets by State A. A database by Zaitseva accounts for loss events of highly-enriched uranium and plutonium-239 originating from the former Soviet Union in 1995.[148] Table 5.1 below shows the total number in terms of significant quantities per year as $L(C_{95})$ = 0.0149 SQ/yr.

Table 5.1. Nuclear trafficking events in 1995 for computing $L(C_{95})$

Date	**Location of Confiscation**	**Material**	**Mass [g]**	**SQ**
June	Moscow, Russia	HEU (21%)	1700	0.0143
June	Prague, Czech Republic	HEU (87.7%)	0.415	>0.001
June	Ceske Budejovice, Czech Republic	HEU (87.7%)	16.9	>0.001
TOTAL				**0.0149**

Gosatomnadzor's statement in 1994 dictated that the Russian government would dedicate 1B USD total, over the subsequent 5-7 years, to enhance Russian nuclear security.[138] Using this estimation at best, the most would be $200M annually ($1B divided by 5 years). This also corresponds to Bunn's cited quote from the leadership (Commander Valynkin) of the 12th Main Directorate (responsible for securing Russia's nuclear weapons) that their budget was "only half as large as the entire U.S. assistance for Russian warhead security" (the reallocated budget for the Cooperative Threat Reduction program under the U.S. Department of Defense was $401M in 1995).[141,149] Therefore, the estimate of C_A = 200M USD is plausible. With these values, Equation (3.1.18) yields the following estimate for λ_{95}.

$$\lambda_{95} = \frac{-1}{C_{95}} \cdot \ln\left[\frac{L(C_{95})}{\alpha}\right]$$
$$\lambda_{95} = \frac{-1}{200} \cdot \ln\left[\frac{0.0149}{0.249}\right]$$
$$\lambda_{95} = \underline{0.0141}$$

Provided in Subsection 3.2.2, the definition of P_A (the perceived probability of an attack against State A using A's asset) is estimated from dividing the number of years immediately preceding and following the year of interest in which a terrorist attack occurred in Moscow with the number of years an attack did not occur. For the year of this case study (1995), over a 10-year span from January 1, 1991 until December 31, 2000, there was at least one fatal terrorist attack in Moscow 6 out of the 10 years.[150] Hence, for Case Study 2, P_A = 0.6.

The consequence of a detonation in K_A is calculated similarly to Case Study 1 with the values computed by utilizing Equations (3.1.21), (3.1.22), and (3.1.23) for an *at-target* attack. Equation (3.1.21) is used to estimate K_A for this case study and the assumed target is the most populous city in 1995 Russia: Moscow (approximately 9.2M inhabitants in just under 880 square kilometers per the 1995 entry for Moscow in the Encyclopedia Britannica).[151] Using a median value of life taken from a 2015 study by the Russian Center for Strategic Researches (4.5 million 2015 rubles) and adjusting to 1995 U.S. dollars yields

46,885USD.[124] Using calculated data from the Glasstone Nuclear Bomb Effects computer[j] for a 10-kT yield like in Chapter 4 and utilizing a 34B USD annual estimate of the reconstruction of Chengdu (a city of similar size of 1995 Moscow) following the 2008 Szichuan earthquake produces the following components for K_A.[95] Lastly, the gross domestic product per capita per Russian citizen in Moscow is assumed as 2,665.7USD taken from the World Bank data website.[97]

$$K_A^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right]\left(50\% \text{ fatality}\right)\left[\text{VSL}\right]$$
$$K_A^{hu} = 10{,}471 \cdot \left[\pi \cdot \left(0.77\text{km}\right)^2\right] \cdot \left(0.5\right) \cdot \left[\$46{,}885\right]$$
$$K_A^{hu} = \$457\text{M}$$

$$K_A^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right]\left(50\% \text{ fatality}\right)\left[\text{GDP}_{\text{per capita}}^{1994}\right]$$
$$K_A^{ec} = 10{,}471 \cdot \left[\pi \cdot \left(0.77\text{km}\right)^2\right] \cdot \left(0.5\right) \cdot \left[\$2{,}663.4\right]$$
$$K_A^{ec} = \$26\text{M}$$

$$K_A^{ni} = \$34{,}000\text{M}$$

$$K_A = K_A^{hu} + K_A^{ec} + K_A^{ni} = \$457\text{M} + \$26\text{M} + \$34{,}000\text{M} = \underline{\$34{,}483\text{M}}$$

Hence, K_A = 34.48 billion USD for an *at-target* attack in Case Study 2.

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[j] Using 7.5 psi for overpressure as stated in Subsection 3.2.3 and using r_b for the blast radius and r_{th} for the thermal radius.

5.1.2. State B Parameters

Based on a memo to the U.S. Senate from Harvard University's Belfer Center for Science and International Affairs, the U.S. was "poorly prepared to detect weapons being imported into the U.S." in 1997.[152] Therefore, the non-interdiction probability of the U.S. $P_B(C_B)$ is estimated at 75%. Furthermore, the authors of the memo accounted the U.S. Congress' budget for nuclear material interdiction at U.S. borders as \$61.1M to meet the threat of smuggling weapons of mass destruction. With this information (as close as the dollars were in comparison with 1995 currency) and that of the U.S. border system being inadequately equipped from 1994-1998, Equation (3.1.24) is reformulated to calculate μ (in terms of the annual number of events per \$M spent):

$$\mu = \frac{-\ln\left[P_B\left(C_B\right)\right]}{C_B} = \frac{-\ln\left[0.75\right]}{61.1} = \underline{0.00471}\left[\$M^{-1}\right]$$

Equation (3.1.21) is used to estimate K_B as the cost of an attack for the time of this case study (circa 1995) on Manhattan Island (59.2 km^2) as the most populated American city. Using estimated values of statistical life as defined in Viscusi and adjusting to 1995 dollars, the value of statistical life for the average American that year was approximately \$3.89M.[126] Using this, the same 1995 cost estimate of 40B USD for rebuilding portions of Los Angeles after the 1994 Northridge Earthquake (as in Subsection 4.1.1), and using the same fatality percentage from a similar 10-kT blast radius results in the following elements for K_B:

$$K_B^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right]\left(50\%\ \text{fatality}\right)\left[\text{VSL}\right]$$

$$K_B^{hu} = \left(\frac{1.514\text{M}}{59.2\text{km}^2}\right)\cdot\left[\pi\cdot\left(0.77\text{km}\right)^2\right]\cdot\left(0.5\right)\cdot\left[\$3.89\text{M}\right]$$

$$K_B^{hu} = \$92,652\text{M}$$

$$K_B^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right]\left(50\%\ \text{fatality}\right)\left[\text{GDP}_{\text{per capita}}^{1994}\right]$$

$$K_B^{ec} = \left(\frac{1.514\text{M}}{59.2\text{km}^2}\right)\cdot\left[\pi\cdot\left(0.77\text{km}\right)^2\right]\cdot\left(0.5\right)\cdot\left[27,776.6\right]$$

$$K_B^{ec} = \$661.6\text{M}$$

$$K_B^{ni} = \$40,000\text{M}$$

$$K_B = K_B^{hu} + K_B^{ec} + K_B^{ni} = \$92,652\text{M} + \$662\text{M} + \$40,000\text{M} = \underline{\$133,314\text{M}}$$

Lastly, in this case study, the gamma value, Γ, is considered to be equal to zero because, until the late 1990s, the primary nuclear weapon threat would continue to be perceived as originating from the Russian arsenal. Hence, nuclear security for thwarting a non-state adversary's acquisition *from another source* is not considered for this case study.

5.2. Non-Cooperative Game Theory

Table 5.2 presents a tabulation of the parameters for Case Study 2 ascertained in Section 5.1. The values here form the basis for calculating the uncorrelated and correlated strategies for the non-cooperative solutions for States A and B as described in Subsection 3.3.1. In Section 5.3, the calculated correlated strategies provide the opportunity to explore cooperative solution concepts. Lastly, Section 5.4 is a section that provides an analysis of the various results presented in Sections 5.2 and 5.3.

Table 5.2. Estimated parametric values for CS2

α	0.249	[SQ]
λ_{95}	0.0141	[M^{-1}]
P_A	0.6	[ ]
K_A	34,483	[$M]
μ	0.00471	[M^{-1}]
K_B	133,314	[$M]
Γ	0	[SQ]

Again, a comparison of the parametric values for each case study is included in Section 9.1.

5.2.1. Non-Cooperative Solution: Uncorrelated Strategies

As discussed in Section 3.3.1, the uncorrelated strategies are determined by calculating the best response strategies (also referred to as the Nash Equilibrium strategies as discussed in Section 2.2) using Equations (3.2.2) and (3.2.5). By using values from Table 5.2, the strategic costs for State A and B are computed for each state's optimal uncorrelated strategy, C'_A and C'_B, respectively (with all units in millions USD).

$$C'_A = \max\left\{0, \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right\}$$

$$C'_A = \frac{1}{0.0141}\ln\left(0.249 \cdot 0.0141 \cdot 0.6 \cdot 34483\right) = 304.3$$

$$C'_A = \max\left\{0, 304.3\right\} = \underline{304.3}$$

$$C'_B = \max\left\{0, \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]\right\}$$

$$C'_B = \frac{1}{0.00471}\ln\left[0.00471 \cdot 133314 \cdot \left(0.249 \cdot e^{-0.0141(304.3)} + 0\right)\right] = 163.0$$

$$C'_B = \max\{0, 163\} = \underline{163}$$

The uncorrelated strategic costs are then used to calculate the threat costs for both states using Equations (3.1.13) and (3.1.14).

$$T_A(C'_A) = P_A \cdot K_A \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A}\right)$$

$$T_A(C'_A) = 0.6 \cdot 34483 \cdot \left(0.249 \cdot e^{-0.0141(304.3)}\right) = 71.0$$

$$T_B(C'_A, C'_B) = e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)$$

$$T_B(C'_A, C'_B) = e^{-(0.00471)\cdot(163)} \cdot 133314 \cdot \left(0.249 \cdot e^{-0.0141(304.3)} + 0\right) = 212.4$$

Lastly, the total costs TC'_A and TC'_B are the sum of the respective state's strategic (C'_i) and threat (T_i) costs as shown in Equation (3.1.2).

$$TC'_A(C'_A) = C'_A + T_A(C'_A) = 304.3 + 71.0 = \underline{375.3}$$

$$TC'_B(C'_A, C'_B) = C'_B + T_B(C'_B) = 163 + 212.4 = \underline{375.4}$$

Therefore, the combined total cost for both states in this non-cooperative bilateral regime amounts to 750.7M USD with State B investing a similar amount as State A.

$$TC'_{AB} = TC'_A + TC'_B = 375.3 + 375.4 = \boxed{750.7}$$

5.2.2. Non-Cooperative Solution: Correlated Strategies

Utilizing the same process as in Subsection 4.2.2 for Case Study 1, values for the correlated strategies (C_A^* and C_B^*) are ascertained via the partial derivatives discussed in Subsection 3.2.1:

$$\frac{\partial TC_{AB}}{\partial C_A} = 1 - \lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A} - \lambda \cdot e^{-\mu \cdot C_B} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A}$$

$$\frac{\partial TC_{AB}}{\partial C_B} = 1 - \mu \cdot e^{-\mu \cdot C_B} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A}$$

Setting each partial derivative to zero allows us to solve for C_B^* first and then for C_A^*.

$$\frac{\partial TC_{AB}}{\partial C_A} = 1 - \lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} - \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} = 0$$

$$\frac{\partial TC_{AB}}{\partial C_B} = 1 - \mu \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \left[\alpha \cdot e^{-\lambda \cdot C_A} \right] = 0$$

$$\lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} + \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} = \mu \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*}$$

$$\lambda \cdot P_A \cdot K_A + \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B = \mu \cdot e^{-\mu \cdot C_B^*} \cdot K_B$$

$$\lambda \cdot P_A \cdot K_A \cdot e^{\mu \cdot C_B^*} = \mu \cdot K_B - \lambda \cdot K_B$$

$$e^{\mu \cdot C_B^*} = \frac{K_B}{\lambda \cdot P_A \cdot K_A} (\mu - \lambda)$$

$$C_B^* = \frac{1}{\mu} \ln \left[\frac{K_B}{\lambda \cdot P_A \cdot K_A} (\mu - \lambda) \right]$$

$$C_B^* = \frac{1}{0.00471} \ln \left[\frac{133314}{0.0141 \cdot 0.6 \cdot 34483} (0.00471 - 0.0141) \right]$$

$$C_B^* = \frac{1}{0.00471} \ln(-4.29) = \textit{undefined} \rightarrow C_B^* = \underline{0}$$

$$\frac{\partial TC_{AB}}{\partial C_A} = 1 - \lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} - \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} = 0$$

$$1 = \lambda \cdot P_A \cdot K_A \cdot \alpha \cdot e^{-\lambda \cdot C_A^*} + \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha \cdot e^{-\lambda \cdot C_A^*}$$

$$e^{\lambda \cdot C_A^*} = \lambda \cdot P_A \cdot K_A \cdot \alpha + \lambda \cdot e^{-\mu \cdot C_B^*} \cdot K_B \cdot \alpha$$

$$e^{\lambda \cdot C_A^*} = \alpha \cdot \lambda \left(P_A \cdot K_A + e^{-\mu \cdot C_B^*} \cdot K_B \right)$$

$$C_A^* = \frac{1}{\lambda} \ln \left[\alpha \cdot \lambda \left(P_A \cdot K_A + e^{-\mu \cdot C_B^*} \cdot K_B \right) \right]$$

$$C_A^* = \frac{1}{0.0141} \ln \left[0.249 \cdot 0.0141 \left(0.6 \cdot 34483 + e^{-0.00471(0)} \cdot 133314 \right) \right] = \underline{446.8}$$

C_B^* is determined as undefined. Hence, for practical application in this case, $C_B^* = 0$. Equations (3.1.13), (3.1.14), (3.1.2), and (3.1.4) are used to calculate the threat costs, $T_A(C_A^*)$and $T_B(C_A^*, C_B^*)$, and the total state and regime costs, $TC_i(C_A^*, C_B^*)$ and $TC_{AB}(C_A^*, C_B^*)$, in terms of millions of USD.

$$T_A\left(C_A^*\right) = P_A \cdot L\left(C_A^*\right) \cdot K_A$$
$$T_A\left(C_A^*\right) = P_A \cdot \alpha \cdot K_A \cdot e^{-\lambda \cdot C_A^*}$$
$$T_A\left(C_A^*\right) = 0.6 \cdot 0.249 \cdot 34483 \cdot e^{-0.0141(446.8)} = \underline{9.5}$$

$$T_B\left(C_A^*, C_B^*\right) = P_B\left(C_B^*\right) \cdot K_B \cdot \left[L\left(C_A^*\right) + \Gamma\right]$$
$$T_B\left(C_A^*, C_B^*\right) = e^{-\mu \cdot C_B^*} \cdot K_B \cdot \left[\alpha \cdot e^{-\lambda \cdot C_A^*} + \Gamma\right]$$
$$T_B\left(C_A^*, C_B^*\right) = e^{-0.0047(0)} \cdot 133314 \cdot \left[0.249 \cdot e^{-0.0141(446.8)} + 0\right] = \underline{61.5}$$

$$TC_i\left(C_i\right) = C_i + T_i\left(C_i\right)$$
$$TC_A\left(C_A^*\right) = C_A^* + T_A\left(C_A^*\right)$$
$$TC_A\left(C_A^*\right) = 446.8 + 9.5 = \underline{456.3}$$

$$TC_B\left(C_B^*\right) = C_B^* + T_B\left(C_B^*\right)$$
$$TC_B\left(C_B^*\right) = 0 + 61.5 = \underline{61.5}$$

$$TC_{AB}\left(C_A, C_B\right) = TC_A\left(C_A\right) + TC_B\left(C_A, C_B\right)$$
$$TC_{AB}\left(C_A, C_B\right) = 456.3 + 61.5 = \underline{517.8}$$

The results included in Table 5.3 as the uncorrelated and correlated results utilize respectively.

Table 5.3. Annualized costs [in $M] from the non-cooperative game

	Uncorrelated [$M]	Correlated [$M]
C_A	304.3	446.8
$T_A(C_A)$	71.0	9.5
$TC_A(C_A)$	375.3	456.3
C_B	163.0	0
$T_B(C_A, C_B)$	212.4	61.5
$TC_B(C_A, C_B)$	375.4	61.5
TC_{AB}	750.7	517.8

Figure 5.1 shows a 3D representation of the solution space (pairs of strategic costs) between State A (the RF) and State B (the U.S.) in terms of total regime cost versus total costs to both states.

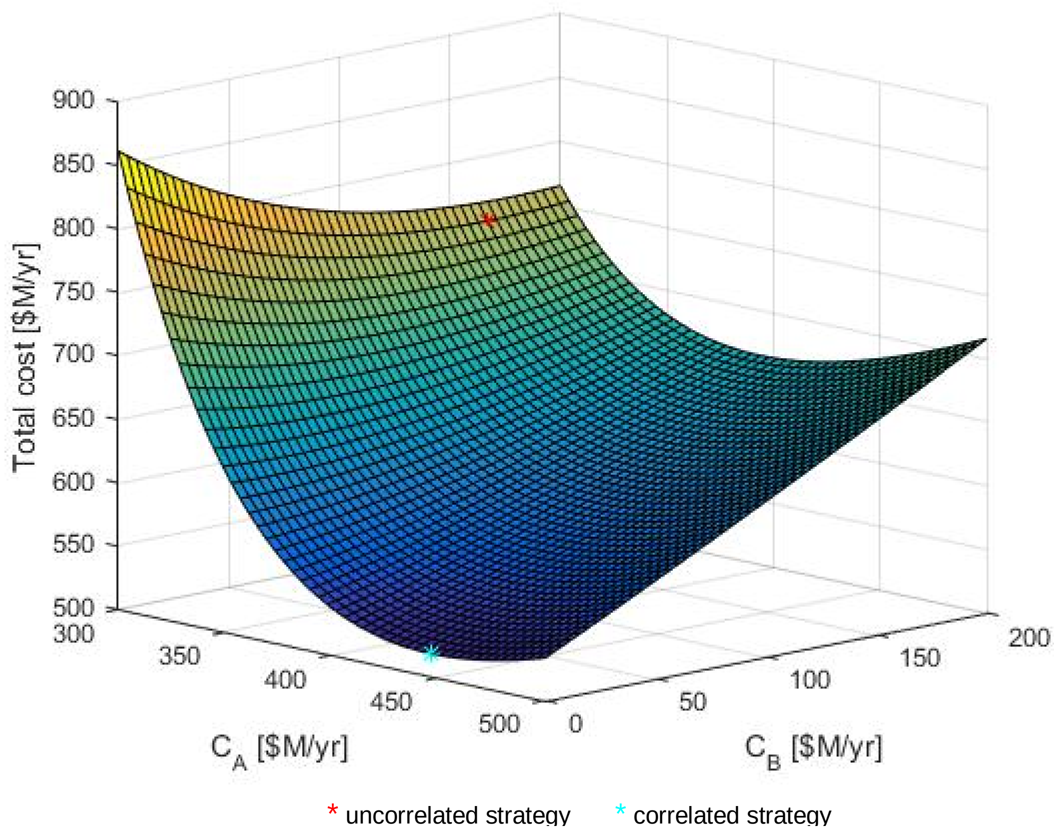

Figure 5.1. 3D representation of total annual regime cost for case study 1

In this plot, the cyan dot represents the correlated strategies for States A and B and the red dot displays the uncorrelated strategies. The vertical displacement between the two strategies is the surplus, *v* (below calculated using Equation (3.1.12)).

$$v = TC'_{AB} - TC^{*}_{AB}$$
$$v = 750.7 - 517.8 = \underline{232.9}$$

In this case study, the surplus (i.e., the amount in savings for the collective partnership) amounts to 232.9M USD which results from the two states correlating their strategies. How best to distribute the surplus in savings is discussed in the Section 5.3 (page 134) as transferring the utility within the relationship.

5.3. Bargaining Solution Concepts and Utility Transferability

Figure 5.2 shows a simple two dimensional plot of the cooperative game structure (total cost to each of the two states) of Case Study 2. The horizontal axis represents the total cost to the RF (TC_A) and the vertical axis represents the total cost to the U.S. (TC_B). Of note, the red point (304.3, 163, 750.7) signifies the *uncorrelated* strategies by both states (i.e., the NE strategies). The cyan point (446.8, 0, 517.8) graphically represents the calculated combined strategies of the states achieved by coordinating strategies for minimizing the cost to the regime (i.e., correlated strategies as in Figure 5.1). The triangular blue-shaded region represents the core of the cooperative game where various imputations (or ratios of surplus divisions) between the players are included. The collection of imputations forms the basis for the cooperative game: the *correlated* strategies depend on the negotiated bargaining solutions. The hypotenuse of the core is the efficiency frontier of the strategies – otherwise referred to as the line of utility transferability (UT) line where, through side payments, a bargaining solution, can be hypothetically achieved benefiting both states.

Two bargaining solutions lie on the UT line: the initial correlated strategy (in cyan) and the Nash Bargaining Solution (NBS) in orange. As aforementioned, the red dot (375.3, 375.4) is the *uncorrelated* strategic pairing for both states and, when they correlate their strategies, the cyan point (456.3, 61.5) on the UT line shows the resulting strategy set under the assumption that Russia bore the entire strategic cost of securing its materials and the U.S. only covered the strategic costs and interdiction efforts. This is the first step in determining the minimum total cost to both States A and B collectively. Of note, this optimal correlated strategy point lies outside the game's core – meaning that if State A were required to bear the entire strategic cost of securing its own materials, then its total costs would be larger than if will have to invest more than the amount they would if they were to adopt an optimal uncorrelated strategy. The NBS constitutes a 50-50 split of the surplus between the players (intersecting at the orange point in Figure 5.2). In this solution, the two states evenly split the surplus/savings gained from correlating their strategies (232.9M USD) so each gains a savings, in total cost, of 116.45M USD – hence, the NBS solution occurs at (258.8, 258.9).

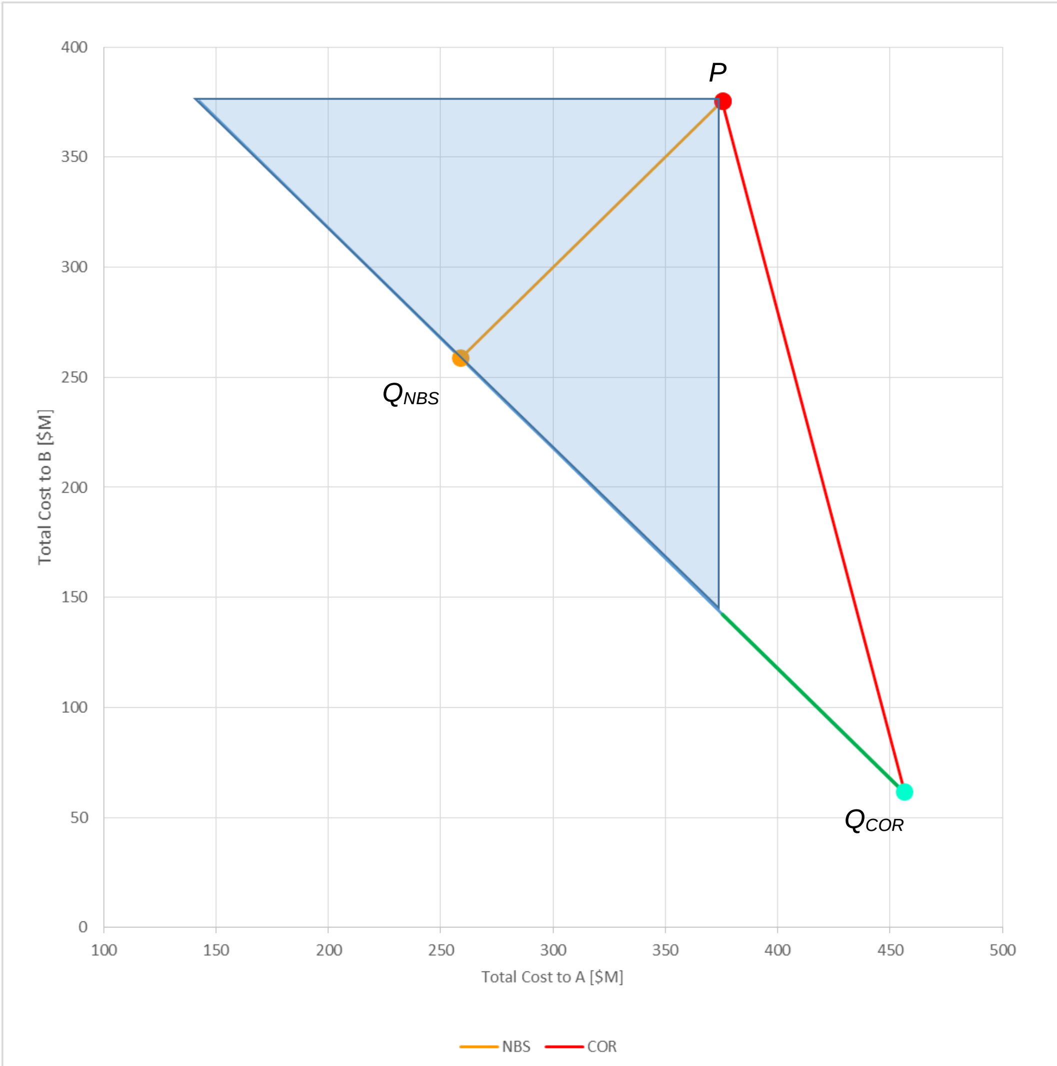

Figure 5.2. Imputations of the U.S.-RF game (1995)

Figure 5.2 presents two simple bargaining solutions for States A and B in this situation. A deduction from this data is that, when correlation occurred, the majority of the observed reduction in the regime's total cost occurs by reducing State B's total cost through investing in State A's security. As discussed in Subsection 5.1.2, the national interdiction program in the U.S. was not a robust system. Therefore, the only alternative available under to the U.S. was to assist in securing Russia's nuclear assets. In addition to other potential solutions, Table 5.3 introduces two other solution concepts in the appended third and fourth columns termed as the C_{BA}*-Neutral* (C_{BA}-N) and the C_{BA}*-Subsidize* (C_{BA}-S) strategies.

Table 5.4. Uncorrelated, correlated, C_A-Neutral, and C_A-Subsidize strategies

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutral [$M]	C_{BA}-Subsidize [$M]
C_A	304.3	446.8	304.2	0
$T_A(C_A)$	71.0	9.5	9.5	9.5
$TC_A(C_A)$	375.3	456.3	313.7	9.5
C_B	163.0	0	142.6	446.8
$T_B(C_A, C_B)$	212.4	61.5	61.5	61.5
$TC_B(C_A, C_B)$	375.4	61.5	204.1	508.3
TC_{AB}	750.7	517.8	517.8	517.8

In the C_{BA}*-N* strategy, which is an abbreviation for the Strategic Cost Neutralization (C_{BA}-N) strategy point, the states agree to correlate their strategies so as to arrive to the UT line but State B bears the maximum burden of paying for all additional strategic cost, say C_{BA}, required to secure A's nuclear assets beyond the level attained by following the uncorrelated strategy. For this case study, C_{BA} = 142.6M USD in order to achieve the collective benefit. This leads to $TC_A = 313.7$M and $TC_B = 204.1$M (with T_B still at 61.5M). This solution allows both states to have a lesser total cost than if both were not to correlate their strategies but the split in surplus savings would favor State B.

In the C_{BA}-S strategy, State A's entire strategic cost (including that above and beyond the additional investment to reach the correlated strategy point) is borne by State B: a total of 446.8M USD (304.2M USD for A's original strategic cost and 142.6M USD of additional strategic cost necessary from somewhere in order to achieve the correlated strategic cost). This leaves State A with only the burden of the calculated threat cost of 9.5M USD – seeing that no other state can absorb the burden of one state's threat cost. State B's total costs can range between the C_{BA}-S and C_{BA}-N solution concepts. These two strategic points are identified on the gray utility transferability (UT) line in Figure 5.3 as the purple (9.5, 508.3) and the red (313.7, 204.1) points respectively.

The C_{BA}-N strategy point appears at the endpoint of the red line at (313.8, 204). The purple point signifying the C_{BA}-S strategy point shows the intercept where State B bears the entire strategic investment for securing State A's materials: 365.8M. This ex-core strategy set presents a substantial benefit to State A: Russia. Similarly, though the correlated (456.3, 61.5) strategy point is on the UT line, it is also an ex-core strategy point which is not ideal to the regime. Another potential strategic point shown in Figure 5.3 (the intersect of the UT line by the black dotted line) occurs if State B were to bear all of State A's additional security costs (142.6M) as well as allow State A to benefit from all the savings (232.9M) after the establishment of the bilateral regime. Therefore, State B's investment would not shift from its original uncorrelated investment strategy. This point, signified as the intersection of the black dotted line and the UT line, occurs at (142.4, 375.4).

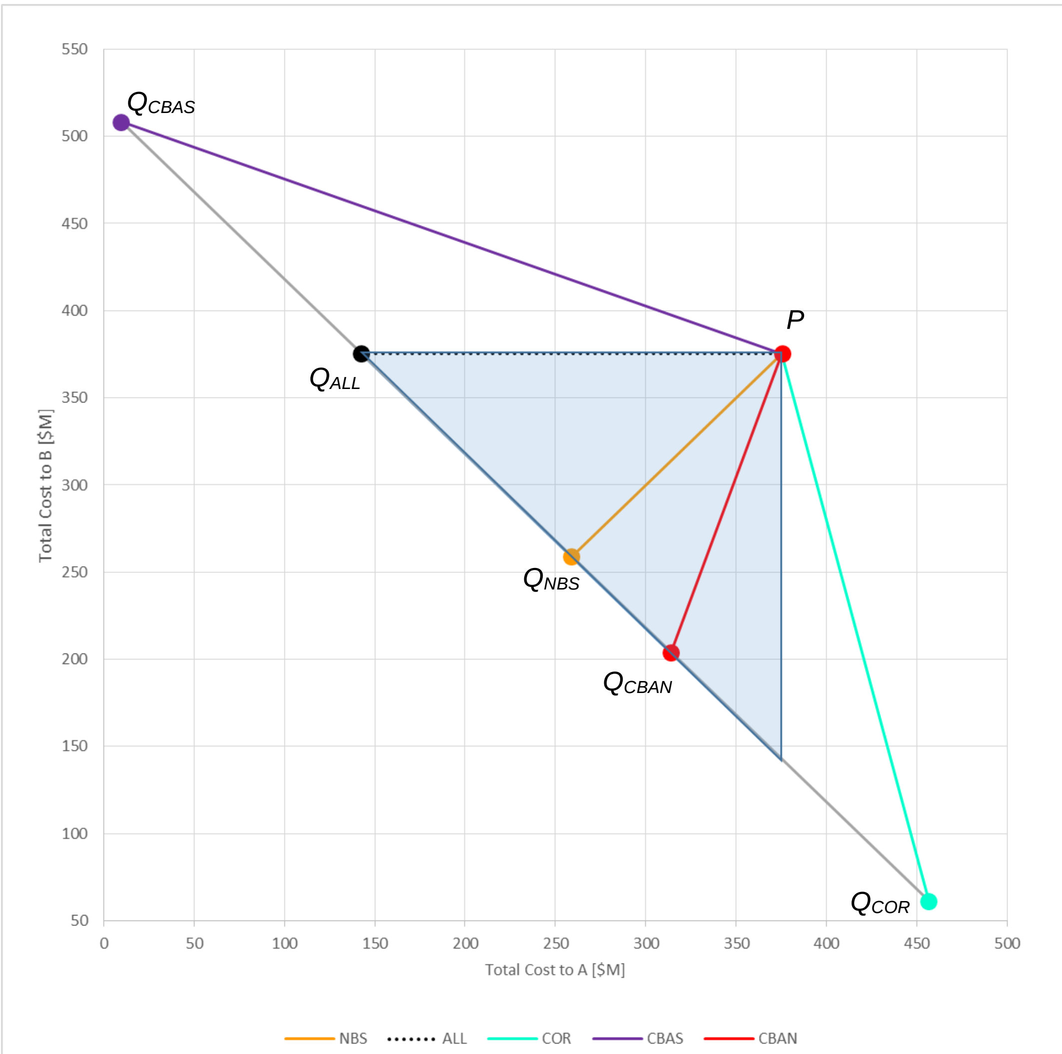

Figure 5.3. Imputations of the U.S.-RF (1995) + C_{BAN}

These potential bilateral regime strategies segue into the next discussion (Section 5.4) of parsing strategic costs of securing State A's materials into separate contributions by the game's players – not unlike the relationship exhibited by former-Cold War enemies the U.S. and Russia in the mid-1990s (such as in this case study). The resulting total cost of the regime (and the corresponding individual state costs) shows a direct benefit to both states in this situation. With State A and B's threat costs being reduced and State B's strategic cost decreasing to zero (when considering the only perceived threat of a nuclear security event against State B is from State A), the resulting strategy of both can be inferred as to invest heavily into State A's nuclear security.

5.4. Analysis and Discussion

The strategic points presented in Figures 5.2 and 5.3 are detailed in Table 5.5.

Table 5.5. Imputations of the cooperative game

	Savings Split for A [$M]	Savings Split for B [$M]	$\frac{s}{r}$	TC_A [$M]	TC_B [$M]	TC_{AB} [$M]
P	--	--	--	375.3	375.4	750.7
Q_{COR}	-81	313.9	-3.88	456.3	61.5	517.8
Q_{NBS}	116.45	116.45	1.00	258.85	258.95	517.8
Q_{CBAN}	61.5	171.4	2.79	313.8	204	517.8
Q_{CBAS}	365.8	-132.9	-0.36	9.5	508.3	517.8
Q_{ALL}	232.9	0	0.00	142.4	375.4	517.8
Q_{ACT}	--	--	--	519	170780	171299

Each imputation is a strategy Q_i with *i* referring to the strategy name. P is the initial state in Figures 5.2 and 5.3 at 375.3, 375.4) which represent the Nash equilibrium (TC'_A, TC'_B) of the *uncorrelated* game. The total regime cost is also included in the final column. The first two columns in Table 5.5 correspond to the surplus (or savings) split in actual quantities of $M for each state and the third column conveys the resulting ratio of the savings split as discussed in Subsection 2.3.2. The blank cells imply there is no surplus to divide between the players.

The $s/_r$ value denotes the ratio of savings that B receives over that of which A receives (when $s/_r > 1$, B receives more of the surplus but when $0 < s/_r < 1$, A receives more). When $s/_r < 0$, the savings division, and therefore, the imputation, resides outside the core of the game. The Q_{COR} imputation conveys the initial assessment of a correlated solution concept where the total regime cost is minimized. The Q_{NBS} imputation is the Nash Bargaining Solution. The Q_{CBAN} imputation simulates State B receiving 2.79 times more of the surplus than A but also can imply that A's additional strategic cost (beyond C'_A) can be neutralized by State B as well. Alternatively and though outside the core, the Q_{CBAS} imputation (with $s/_r = -0.36$), shows State A receiving a favorable quantity of the surplus to subsidize its entire strategic cost (C'_A). Additional annual investments of this ilk could prove problematic for State B to maintain over the long term.

Between the Q_{ALL} and Q_{CBAS} imputations shows an interesting result of the methodology. Previous scholars have introduced concepts of certain states' abilities of utilizing nuclear insecurity as a commodity.[33] In this case study, there is an area of cooperation between the states that, though the imputation does reside in the cooperative game's core (hence, it is a viable strategic solution), one state can assuredly claim a clear benefit over the other by receiving a negative total cost in the regime. Obviously, the gained benefit could provide incentive for State A to continue with the cooperative regime. Though State B would receive no additional benefit in the form of saving from its investment, TC_B for the Q_{ALL} imputation is equal to its utility for B's uncorrelated strategy. Actual investments determined through published data are included at the bottom of Table 5.4 as Q_{ACT}. If Q_{ACT} is assumed as an accurate representation of the regime's actual strategies in 1995, two conclusions can be made: 1) the states could have better managed their respective strategies for the benefit of all (e.g., by investing more in Russian nuclear asset security) and 2) the actual strategies taken during 1995 were not a Nash Equilibrium between the states.

Table 5.6 is constructed using values from Equation (3.1.26). All U_i values are based of the change in utility from the NE solution concept (i.e., the *uncorrelated* strategy set).

Table 5.6. Tabulated change in utility per imputation

	U_A	U_B	U_{AB}
Q_{COR}	-0.22	0.84	0.31
Q_{NBS}	0.31	0.31	0.31
Q_{CBAN}	0.16	0.46	0.31
Q_{CBAS}	0.97	-0.35	0.31
Q_{ALL}	0.62	0.00	0.31

Both states would see some change in utilities based on which solution concept is achieve. Some solution concepts result in positive utility change for only one state (Q_{COR}, Q_{CBAS}, and Q_{ALL}) and others convey a positive change in utility for both states (Q_{NBS} and Q_{CBAN}). The overall lesson with this case study is that assuming State A and State B would come to a consensus on the terms of a bilateral agreement for nuclear security, both states could benefit according to the U_{AB} values in the third column and any s/r ratio between 0 and 1 could help influence the Russian strategy. Lastly, the collective benefit to the regime, U_{AB} = 31%, shows a positive change in utility from the *uncorrelated* strategy set.

6. CASE STUDY 3: RUSSIA – U.S. (2008)

Case Study 3 consists of the bilateral security regime between the Russian Federation and the U.S. as States A and B, respectively. The difference here is the year of the game model: 2008. This year is used herein due to detailed information being available through various publications by Bunn and Schwartz as well as the 2008 fiscal year being in the midst of a successfully, long-running cooperative relationship between the states. As opposed to 1985 (when a relationship was non-existent), 1995 (when the first year had yet to yield tangible results from the CTR and MPC&A), and 2015 (after the relationship had deteriorated between the states' governments), 2008 represents a year identified by substantial cooperation for the collective benefit of securing special nuclear material against any perceived attack in Russia.

Near the end of 2008, U.S.-Russian relations were at a low due to the strained relationship in the wake of that year's Russian-Georgian war. The change in the American administration proved an opportunity to reset the relationship and plow ahead with other nuclear-related accords such as the NewSTART treaty.[153] However, other activities regarding nuclear security were also in full swing: the Global Threat Reduction Initiative (announced in 2004 for minimizing nuclear material around the world – large component consisted of removing HEU from Russian and Russian-provided reactors); the Second Line of Defense (established in 1998 to equip susceptible border crossings around Russia to minimize nuclear traffic); and the Global Initiative to Combat Nuclear Terrorism (launched in 2006 under Russian and American leadership /to thwart the advent nuclear terrorism) among others.[154,155,156] Though these programs were primarily multilateral, we simplify the concept into a bilateral model in the remaining sections of this chapter.

6.1. Evaluating the Game Model Parameters

Data for the 2008 case study were attained through publications from various research organizations. These published studies (identified in the subsequent sections) discuss investments in nuclear security by the U.S. and Russia in light of a newly emerging nuclear terrorist threat. Leadership in both states pushed for increased collaboration in securing nuclear assets and, especially after the 2008 election of Barack Obama, a strong push was made to facilitate programs for securing all (special) nuclear materials. Hence, because the security of nuclear materials was on the forefront of many minds, many studies were published on state-level investments in nuclear security.

6.1.1. State A Parameters

In this case study, like the previous two, State A is the Russian Federation (RF). Financial contribution data were attained from 2007 documents and estimated for 2008 for this case study. Specifically for calculating λ_{08} and α_{08}, estimates were needed for plausible Russian nuclear security expenditures and a 2008 Russian material loss rate. According to Bunn, the Russian Finance ministry committed approximately 30M USD in funding for sustaining "security measures at nuclear weapon sites" in 2007.[157] Bunn expands this by stating that though individual sites were financially responsible for providing adequate security and accounting measures at their respective sites, the Ministry of Internal Affairs (MVD), the Federal Security Service (FSB), and the 12th Main Directorate of the Ministry of Defense all contributed essential security components as well. In addition to the published 30M USD for nuclear weapons security, the U.S. also provided an additional 87.1M USD under the CTR in 2007 for enhancing the "weapons storage security" in Russia (allocated from a total budget of 372M USD under the CTR).[158] Thus, in 2007, a total of just under 120M USD was spent on nuclear weapons security in the RF. Furthermore, in 2007, the total Russian military budget amounted to 1100B RUB (equivalent to 44B USD) according to Cooper.[159] When only considering the Russian contribution of 30M USD as a percentage of the total federal budget in 2007 (44B USD), the percentage of the federal budget allocated to nuclear weapons security is 0.06%. With the American 87.1M USD contribution in 2007, the total amount spent on nuclear weapons security that year in Russia consisted of roughly 0.266% of the total Russian military budget. Therefore, assuming 0.266% of the entire Russian military budget is allocated to weapons security and using 1394B RUB as the 2008 Russian military budget (55M USD) with the USD-RUB exchange rate that year, the total 2008 expenditure on nuclear weapons security is estimated at 146.38M USD.

Regarding (non-weapon) nuclear materials, Rosatom proposed a seven-year budget for a new Federal Targeted Program (FTP) to target nuclear safety and security in 2007.[160] The amount listed in this report was 132B RUB (5.28B USD per year for 7 years) starting in 2008 and included provisions for (1) nuclear material protection, control, and accounting; (2) material consolidation and reduction; (3) facility clean-up; (4) spent nuclear fuel storage; and (5) site restoration. Only constituting a fifth of the total budget and only 1 out of the seven years, the amount for 2008 is approximately 150.9B USD. Therefore, for computing λ_{08}, C_{08} = 146.4M + 150.9M = 297.3M.

For the 2008 case study, α_{08} is estimated as slightly lower than in 1995 at $0.2\,\frac{\mathrm{SQ}}{\mathrm{yr}}$. Though an arbitrary reduction, it can be justified by the fact that, by 2008, Russia and the U.S. had had over ten years of the Cooperative Threat Reduction Program and the Material Protection, Control, and Accounting programs to secure nuclear assets throughout Russia. And, with such longstanding arrangements, it can be argued that the overall rate of asset loss if nothing had been spent on security in 2008 would have been lower. Furthermore, with the Russian economy being substantially in a better financial situation than in 1995, the expenditures on securing nuclear materials assisted in reducing the potential threat of nuclear theft. Only one major security event occurred involving special nuclear material from Russia: a theft of three natural uranium fuel rods from the Chepetsky Mechanical Plant in Glazov consisted of a total of just over 30 kilograms of natural uranium (as shown in Table 6.1).[152] The definition of significant quantities for natural uranium is 10,000 kg U_{nat} = 1 SQ.

Table 6.1. Nuclear trafficking events in 2008 for computing L_{08}

Date	**Location of Confiscation**	**Material**	**Mass [g]**	**SQ**
December	Glazov, Russia	Natural U	30700	3.07e-3
TOTAL				**3.07-3**

Hence, using α_{08} and the appropriate loss rate, $L(C_{08})$, shown in Table 6.1, λ_{08} is computed below.

$$\lambda_{08} = \frac{1}{-C_{08}} \cdot \ln\left[\frac{L(C_{08})}{\alpha}\right]$$
$$\lambda_{08} = \frac{1}{-297.3} \cdot \ln\left[\frac{0.00307}{0.2}\right]$$
$$\lambda_{08} = 0.014\left[\$M^{-1}\right]$$

Applying the same method as in the previous 1995 case study for determining the probability (related to the frequency) of large-scale fatal terrorist events in the near ten years (January 1, 2004 through December 31, 2013) yields a value of 0.3 for terrorist acts that occurred in 2003, 2004, and 2010. We use $P_A = 0.3$ as a rough estimate for an *at-target* detonation with the understanding that if we consider an *on-site* detonation, P_A could be higher (as discussed in Subsection 3.2).

Lastly, the *at-target* consequence parameter, K_A, is as in both Chapters 4 and 5. The various constituents listed below are used in Equation (3.1.21).

- 11,294,000 for the interpolated 2008 population of Moscow[146];
- 2,511 km^2 as the land area[153];
- 11,635 USD is estimated as the 2008 GDP per capita[97];
- 64,950 USD as the adjusted value of statistical life in 2008[124];
- the range of 0.77 km to simulate a single 10kT yield explosion producing a minimum 7.5 psi overpressure effect needed create a 50% fatality rate (which is

based on the most common weapon in current operation[k] within the Russian nuclear arsenal)[154]; and

- an estimate of 63,390M USD for rebuilding a vast portion of Moscow based on a modern city-wide rebuild such as post-2012 Hurricane Sandy on New York City[l].

$$K_A^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right]\left(50\%\ \text{fatality}\right)\left[\text{VSL}\right]$$
$$K_A^{hu} = \left(\frac{12.166\text{M}}{2{,}511}\right)\cdot\left[\pi\cdot\left(0.77\text{km}\right)^2\right]\cdot\left(0.5\right)\cdot\left[\$64{,}950\right]$$
$$K_A^{hu} = \$293.1\text{M}$$
$$K_A^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right]\left(50\%\ \text{fatality}\right)\left[\text{GDP}_{\text{per capita}}^{1985}\right]$$
$$K_A^{ec} = \left(\frac{12.166\text{M}}{2{,}511}\right)\cdot\left[\pi\cdot\left(0.77\text{km}\right)^2\right]\cdot\left(0.5\right)\cdot\left[\$11{,}635\right]$$
$$K_A^{ec} = \$52.5\text{M}$$
$$K_A^{ni} = \$63{,}390\text{M}$$

$$K_A = K_A^{hu} + K_A^{ec} + K_A^{ni} = \$293.1\text{M} + \$52.5\text{M} + \$63{,}390\text{M} = \underline{\$63735.6\text{M}}$$

Therefore, in this model, *at-target* K_A = 63.736B USD.

6.1.2. State B Parameters

The values for the U.S. parameters (μ, P_B, K_B, and Γ) are for 2008. Recalling Equation (3.1.24) and using Schwartz' estimations, 2008 expenditures by federal agencies

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[k] As of January 2016, 460 warheads are on 46 R-36M2 (SS-18 Voyevoda) with an estimated yield of 750 - 1000kT, 150 warheads are on 150 Topol class missiles with estimated yields of 800kT each, and 292 warheads on 73 RS-24 missiles with at least 1MT yields each according to http://russianforces.org/missiles.

[l] A Category 3 hurricane, Sandy struck New York City in October 2012 and was estimated to have caused $67.6B in damage according to NOAA: http://www.ncdc.noaa.gov/billions/events.

on nuclear security are categorized into five groups. Crossing multiple departments and program offices, Schwartz defines the following five groups: nuclear forces and operational support; deferred environmental and health costs; missile defense; nuclear threat reduction; and nuclear incident management. The nuclear threat reduction category is further parsed into three more subcategories: prevention/securing; elimination; and nonproliferation. Care was taken to further deconstruct the subcategorization of expenditures for prevention and securing into two separate groups for that year: $2,673.3M and $437.1M, respectively.[112]. Using the former value for investment by the U.S. and a common non-detection probability, P_B, of 5% (as described in Subsection 2.2.2), the value for μ is calculated below:

$$\mu = \frac{-\ln\left[P_B\left(C_B\right)\right]}{C_B} = \frac{-\ln\left[0.05\right]}{2673.3} = \underline{0.00112}$$

Invoking Equation (3.1.21) again from Subsection 2.2.2, K_B is the consequence of a nuclear detonation on U.S. soil with a Russian 10kT device. The individual costs from the loss of lives (K^{hu}) requires using previously defined cost of life estimations.[126] The economic impact of losing those lives is based on GDP per capita for 2008 (K^{ec}) and the similar loss from Subsection 6.1.1 (K^{ni}) such as New York City (adjusted for inflation).

$$K_B^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right]\left(50\%\text{ fatality}\right)\left[\text{VSL}\right]$$

$$K_B^{hu} = \left(\frac{1.632\text{M}}{59.2\text{km}^2}\right)\cdot\left[\pi\cdot\left(0.77\text{km}\right)^2\right]\cdot\left(0.5\right)\cdot\left[\$8.5\text{M}\right]$$

$$K_B^{hu} = \$218{,}232\text{M}$$

$$K_B^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right]\left(50\%\ \text{fatality}\right)\left[\text{GDP}_{\text{per capita}}^{2008}\right]$$

$$K_B^{ec} = \left(\frac{1.632\text{M}}{59.2\text{km}^2}\right) \cdot \left[\pi \cdot \left(0.77\text{km}\right)^2\right] \cdot \left(0.5\right) \cdot \left[\$48{,}401\right]$$

$$K_B^{ec} = \$1{,}243\text{M}$$

$$K_B^{ni} = \$63{,}400\text{M}$$

$$K_B = K_B^{hu} + K_B^{ec} + K_B^{ni} = \$218{,}232\text{M} + \$1{,}243\text{M} + \$63{,}400\text{M} = \underline{\$282{,}875\text{M}}$$

The last value for the case study, Γ, signifies the potential for a nuclear threat to arise from elsewhere beyond Russia. In 2008, the number of states with nuclear weapons had increased by one – the addition of North Korea increased the number of nuclear-armed states (P5 + India, Pakistan, and North Korea). Moreover, security in the latter two countries (Pakistan and North Korea) had been a continual concern against a non-state (and sub-state) threat.[161] To accommodate this additional threat, Case Study 3 uses $\Gamma > 0$. The magnitude of Γ calls into question how much of State B's nuclear terrorist threat comes from State A. Assuming Γ is the same as the loss rate for State A, then State B's threat would come equally from within State A and anywhere that is not State A. However, if Γ is greater than the attained loss rate, then the assumption is State B bears at least the same level of threat (if not more) from beyond State A than State A itself. Due to the long term collaborations on nuclear material and weapon security between the U.S. and the RF until 2008, $\Gamma = 0.00307$ SQ/yr.

6.2. Non-Cooperative Game Theory

The estimated values from Section 6.1 are collected in Table 6.2. From these, the non-cooperative solutions will be shown in the subsequent subsections.

Table 6.2. Estimated parametric values for case study 3

α	0.2	[SQ]
λ_{08}	0.014	[$\$M^{-1}$]
P_A	0.3	[ ]
K_A	63,736	[$M]
μ	0.00112	[$\$M^{-1}$]
K_B	282,875	[$M]
Γ	0.00307	[SQ]

Notably, though it seems that m has not increased much from the 1985 case study (conveying a porous border), the 2008 value relies on the increased rate of interdiction but also additional expense of such a national system.

6.2.1. Non-Cooperative Solution: Uncorrelated Strategies

Recalling Equations (3.2.3) and (3.2.6) for determining C'_A and C'_B leads to the following results:

$$C'_A = \max\left\{0, \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right\}$$

$$C'_A = \frac{1}{0.01479}\ln\left(0.2 \cdot 0.01479 \cdot 0.3 \cdot 63736\right) = 283.6$$

$$C'_A = \max\left\{0, 283.6\right\} = \underline{283.6}$$

$$T_A(C'_A) = P_A \cdot K_A \cdot \left[\alpha \cdot e^{-\lambda \cdot C'_A}\right]$$
$$T_A(C'_A) = 0.3 \cdot 63736 \cdot \left[0.2 \cdot e^{-0.014(283.6)}\right] = \underline{71.2}$$

$$TC'_A(C'_A) = C'_A + T_A(C'_A) = 283.6 + 71.2 = \boxed{354.8}$$

In this case, State A's strategic annual investment is 283.6M USD which yields the total cost of 354.8M USD. Per the methodology in Section 2.4, solving for the American NE requires solving for C'_B:

$$C'_B = \max\left\{0, \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]\right\}$$
$$C'_B = \frac{1}{0.00112}\ln\left[0.00112 \cdot 282875 \cdot \left(0.2 \cdot e^{-0.014(283.6)} + 0.00307\right)\right] = 684.3$$
$$C'_B = \max\{0, 684.3\} = \underline{684.3}$$

$$T_B(C'_A, C'_B) = e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)$$
$$T_B(C'_A, C'_B) = e^{-0.00112(684.3)} \cdot 282875 \cdot \left(0.2 \cdot e^{-0.014(283.6)} + 0.00307\right) = \underline{892.3}$$

$$TC'_B(C'_A, C'_B) = C'_B + T_B(C'_A, C'_B) = 684.3 + 892.3 = \boxed{1576.6}$$

$$TC'_{AB} = TC'_A + TC'_B = 354.8 + 1576.6 = \boxed{1931.4}$$

As shown above, the total cost to State B (the U.S.) is much greater than that for State A (the Russian Federation) and the total cost to the regime of the two states with no cooperation is 1,931M USD.

6.2.2. Non-Cooperative Solution: Correlated Strategies

The correlated, C_{BA}-neutralization and C_{BA}-subsidization solutions are computed using the aforementioned MATLAB script and presented in Table 6.3 as an attempt to quantify the bilateral relationship between Russia and the U.S. in 2008. Notable here is the fact that the costs to State B are all greater than those costs to State B in the 1995 case study. The reason for this stems from $\Gamma \neq 0$ seeing that the driving factor for State B's threat cost is external beyond State A.

Table 6.3. Annualized costs [in $M] from case study 3 (2008)

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutralized [$M]	C_{BA}-Subsidized [$M]
C_A	283.6	476.7	283.6	0
$T_A(C_A)$	71.2	4.7	4.7	4.7
$TC_A(C_A)$	354.8	481.4	288.3	4.7
C_B	684.3	44.8	237.9	521.5
$T_B(C_A, C_B)$	892.3	892.3	892.3	892.3
$TC_B(C_A, C_B)$	1576.6	937.1	1130.2	1413.8
TC_{AB}	1931.4	1418.5	1418.5	1418.5

The red and cyan points represent the uncorrelated and correlated results in Figure 6.1.

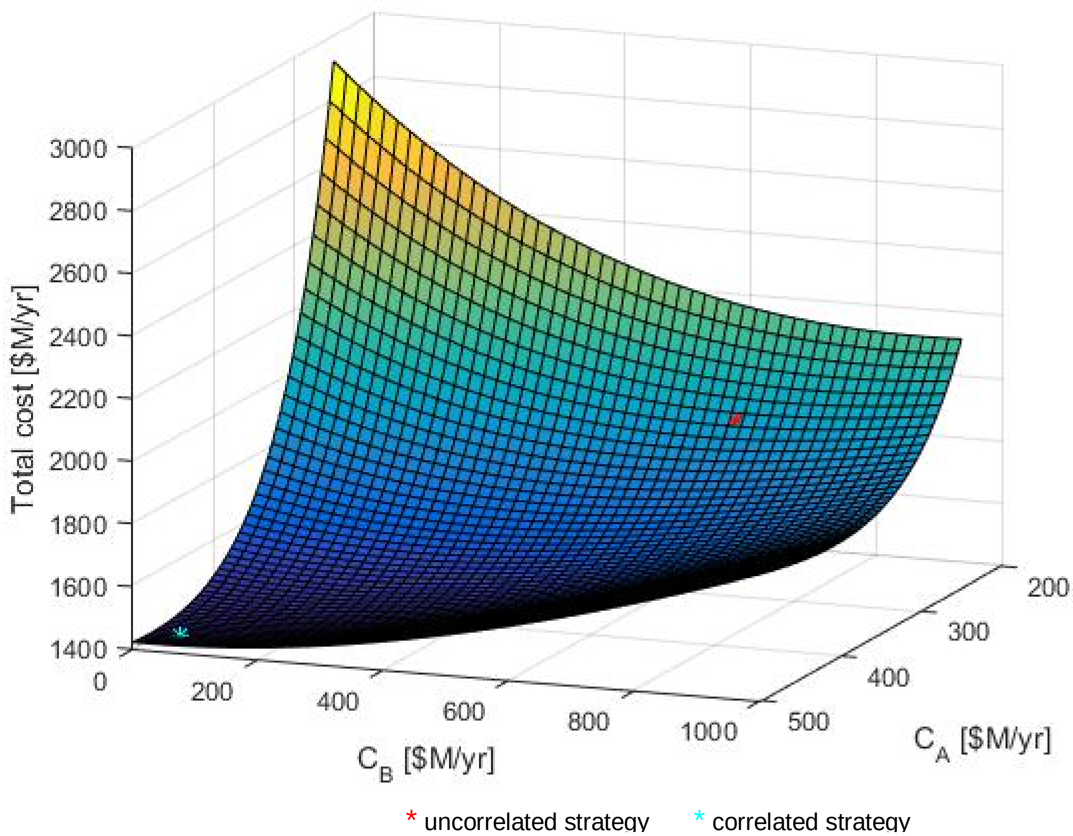

Figure 6.1. Case study 3: Russia-US (2008)

As is shown, the vertical displacement between the uncorrelated and correlated strategy points amounts to 512.9M USD ($1931.4 - 1418.5 = 512.9$). Thus, the 512.9M USD surplus in savings defines the range for the bargaining solutions discussed in Section 6.3.

6.3. Bargaining Solution Concepts and Utility Transferability

In Figure 6.2, five different potential bargaining solutions are shown (along with the *uncorrelated* strategy set in red). The line containing the hypotenuse of the blue shaded region (the cooperative game's core) is the utility transferability (UT) line with a slope of -1. This signifies that the surplus savings is distributed wholly between the Russian Federation (A) and the U.S. (B). The uncorrelated strategy point determined in Subsection 6.2.1. is the red point at (354.8, 1576.6) and the correlated strategy from Subsection 6.2.2 is the cyan point at (481.4, 937.1) – replicating the same color scheme from Figure 6.1. The slope between these two solution points is -5.05 (substantially less than the UT line's slope of -1). The greater difference of these lines' slopes conveys the strength of the collaborative solution benefitting both states. The orange line segment splitting the hypotenuse evenly conveys the Nash bargaining solution which is a commonly discussed result for games with cooperation signifying the equal distribution of the surplus between the game's players. This point at (98.4, 1320.1) lies within the game's core conveying this solution's plausibility. Importantly, the second red point at (288.3, 1130.2) signifies the when B neutralizes A's additional investment in its own security to reach the collaborative strategic solution. A previously-discussed solution (introduced in previous chapters) is the purple dot at (4.7, 1413.8) where B subsidizes all of A's strategic costs including C'_A (a.k.a. the C_{BA}-subsidized bargained solution). This implies that A will receive 350.1M USD of the 512.9M USD surplus. Meanwhile, the black-dotted line segment ends at the black point at (-158.1, 1576.6). This bargaining solution strategy point implies A receives a negative total cost.

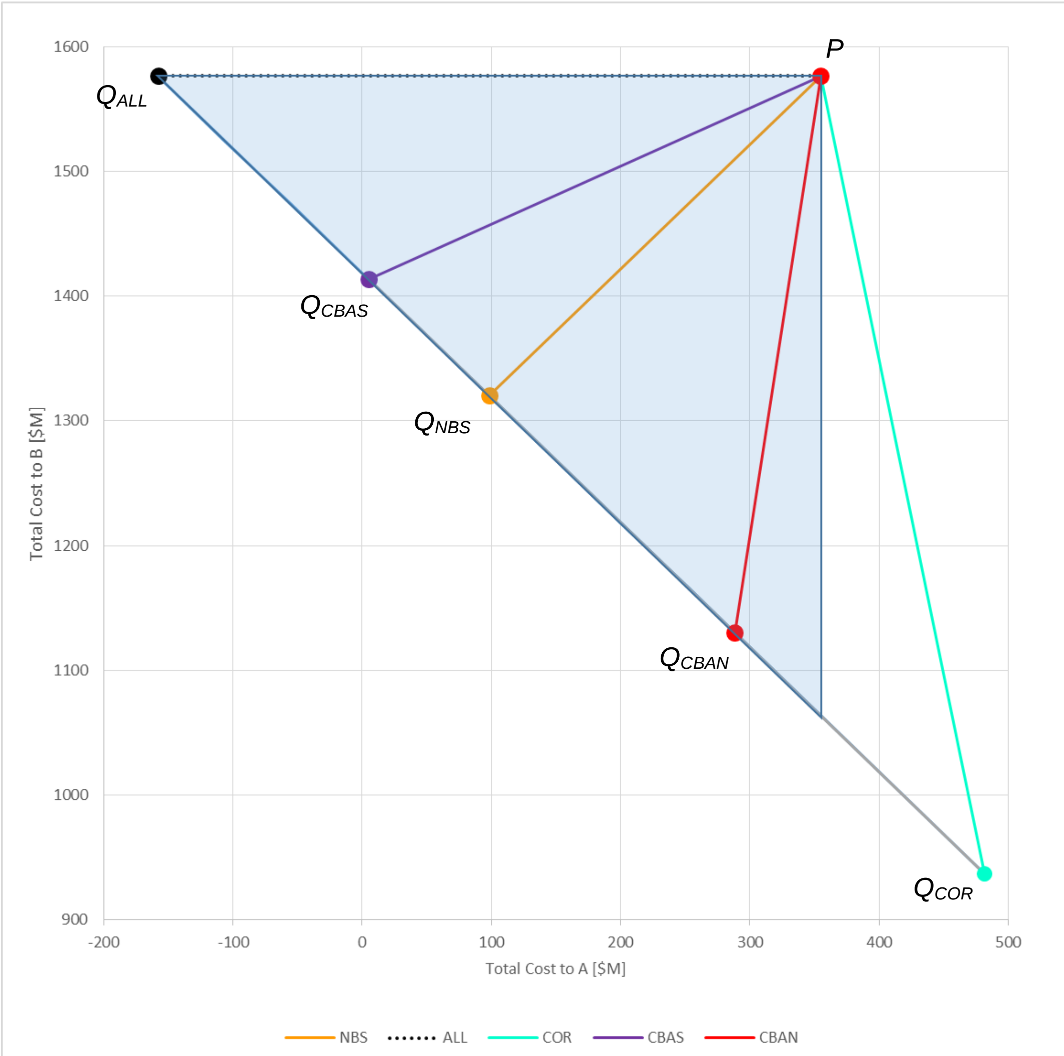

Figure 6.2. Russia-US (2008) – cooperative game core

State A's solution space between (0, 1418.5) and Q_{ALL} at (-158.1, 1576.6) on the UT line resides in the game's core which shows a feasibility of a strategic bargaining solution between the states where State A receives a negative cost benefit. Alternatively stated, the RF would receive a profit in agreeing to enter into a cooperative agreement with the U.S. if the pre-negotiated bargaining point resided in this area of the game's core. As shown in the next subsection, if the states were to agree to this strategic point, Russia could use its nuclear insecurity as a means to make a profit. Therefore, this figure conveys Corr's underlying theory of nuclear insecurity as a commodity as mentioned in Section 3.4 but discussed in more detail in Subsection 9.4.[33]

6.4. Analysis and Discussion

The uncorrelated, best-response solution, P, and bargaining solutions described in the previous section, Q_i, are displayed in Table 6.4 with more detailed data to help describe each strategy set. Considering the *uncorrelated* strategy set for Russia and the U.S. provides no collective benefit, no data are included as a savings split (the first three columns). Q_{COR} conveys the utility (in terms of total cost) for each state if they are to collectively pursue a strategy that minimizes the bilateral regime's total (the combined strategic and threat) cost. For this 2008 Russia-U.S. case study, the regime's minimized total cost amounts to 1418.5M USD. Therefore, any solution on the UT line (Q_i in Table 6.4) will maintain the total regime's minimized cost. Q_{NBS} represents the Nash bargaining solution which exhibits a 50-50 split in the surplus saving of 512.9M USD and therefore holds a s/r value of 1. Next, Q_{CBAN} is the C_{BA}-neutralized solution set for when B neutralizes A's additional strategic cost so that both can benefit from the collaboration. In this imputation, the U.S. invests the additional 193.1M USD $(Q_{CBAN}(B) - Q_{COR}(B) = 1130.2 - 937.1 = 193.1)$ for Russia to achieve the equivalent expenditure of the correlated strategy. This implies that only 66.5M USD goes to Russia for its additional investment (over its uncorrelated strategy responsibility of 354.8M USD). The s/r ratio leans heavily for the U.S. at 6.71.

Table 6.4. Imputations of the cooperative game

	Savings Split for A [$M]	Savings Split for B [$M]	$\frac{s}{r}$	TC_A [$M]	TC_B [$M]	TC_{AB} [$M]
P	--	--	--	354.8	1576.6	1931.4
Q_{COR}	-126.6	639.5	-5.05	481.4	937.1	1418.5
Q_{NBS}	256.45	256.45	1.00	98.35	1320.15	1418.5
Q_{CBAN}	66.5	446.4	6.71	288.3	1130.2	1418.5
Q_{CBAS}	350.1	162.8	0.47	4.7	1413.8	1418.5
Q_{ALL}	512.9	0	0.00	-158.1	1576.6	1418.5
Q_{ACT}	--	--	--	362.5	3236.9	3599.4

Q_{CBAS} is the bargaining solution where the U.S. pays for all of Russia's investment (signifying a potential attempt at influencing it to maintain the nuclear security bilateral regime between the two states). In this imputation, the U.S. covers all costs for Russia (save for Russia's threat cost) and the solution strategy point is at (4.7, 1413.8). This also signifies that the surplus split is slightly more favorable to Russia: $s/r = 0.47$. Q_{ALL} is the solution point for when both states agree to provide all surplus savings (512.9M USD) to Russia ($s/r = 0$). This point is displayed at (-158.1, 1576.6). Notably, Q_{ALL} yields negative values for Russia (though still residing within the game's core). With this solution, it can be conveyed that Russia is receiving a benefit from the collaboration in the form of a negative cost (i.e., receives a profit as part of these solutions sets). This illustrates

Corr's theory of nuclear insecurity potentially being used as a commodity from which a state can receive a profit. As unfair as these solutions are, they are not wholly unrepresentative of historical nuclear security arrangements. Specifically in 2008, the Russian-U.S. relationship for applying nuclear security was beginning to receive some concern domestically in the U.S. for providing too much to Russia.[162] The results in Table 6.4 seem to illustrate this point especially when one considers how historically, the relationship between the U.S. and Russia must have resided to the right of Q_{CBAS} (where the U.S. subsidizes all of Russia's strategic costs – similar to the situation in 1995). A more precise assessment of the regime's bargained solution concept could be surmised with more readily available data on how Russia was investing in their nuclear security.

Table 6.5. Tabulated benefits in utility results per imputation

	U_A	U_B	U_{AB}
Q_{COR}	-0.36	0.41	0.27
Q_{NBS}	0.72	0.16	0.27
Q_{CBAN}	0.19	0.28	0.27
Q_{CBAS}	0.99	0.10	0.27
Q_{ALL}	1.45	0.00	0.27

Table 6.5 presents the utility benefits (as discussed in Subsection 3.1.2.) for each state and the regime. As in previous case studies, positive values exhibit strategies that are beneficial to the state or regime whereas negative values imply lost utility. For the regime in 2008, the last column, U_{AB}, shows a collective benefit of 0.27 – thus conveying the

existence of at least some benefit in utility to both states (over the utility of the uncorrelated strategies). Q_{COR} is the only imputation where one state receives a negative utility (relating to the only solution point falling outside the game's core in Figure 6.2). Q_{NBS} and Q_{CBAS} yield similar utility benefits for both states. Interestingly, as can be seen with Q_{ALL}, when $U_A > 1$, State A receives a negative cost at the detriment to State B despite the regime's benefit from any collaboration.

7. CASE STUDY 4: RUSSIA – U.S. (2015)

In the modern era of deterring non-state adversaries from attaining a nuclear attack capability (previously discussed in Subsection 1.3 as the fourth wave by Knopf), the danger has been elevated to the forefront of many minds.[163] The numerous terrorist attacks since the mid-1990s have shined a light where scholars and decision makers look to propose ideas in diminishing the potential nuclear hazard presented by those who would do harm. One of the largest attempts by any coalition of states (achieved by Russia and U.S.), the Cooperative Threat Reduction program, as first mentioned in Case Study 1, was aimed at curbing the non-state nuclear threat and was reaching its planned expiration in 2013.[164] As of the program's expiration, from the U.S. perspective there was "no legal basis for continued cooperation on security for nuclear weapons."[165] In light of numerous previous successful joint activities (e.g., securing weapons of mass destruction, eliminating nuclear and chemical weapons, and facilitating the removal of nuclear weapons out of former Soviet republics), the Russian government elected to sign a new bilateral framework with the U.S. under the umbrella of the Multilateral Nuclear Environmental Program in the Russian Federation (MNEPR).[166] The new program constituted a narrower scope consisting of collaborations "in several areas of nonproliferation collaboration, including protecting, controlling, and accounting for nuclear materials."[166] Though not ideal, only non-weapons usable material would continue to be secured under a similar program meant to continue components of the old material protection, control, and accounting program. However, relations between the U.S. and the

Russian Federation soured after a series of events in 2014.[m] Due to this and other mounting domestic political pressures in Russia, as well as in the U.S., all collaborations between the two states ceased.[n] Furthermore, Russia did not participate in the final 2016 Nuclear Security Summit. As unfortunate as this was for the international nuclear security regime, Russia maintained a stated commitment to continue unilaterally its efforts in advancing the security enhancements at various facilities. In contrast to the first two case studies, the threat of a non-state adversary acquiring a nuclear weapon from a source other than Russia is non- zero. Hence, $\Gamma > 0$. In sum, the year 2015 was chosen (as explained in Section 3.5) as the closest year with full retrievable data at the time of this composition.

7.1. Evaluating the Game Model Parameters

An attempt at representing plausible modern-day estimates for nuclear security was made. Since the end of the CTR program on January 1, 2015, published documents with information on Russia's nuclear security efforts have been scarce. With the lack of collaboration between the U.S. and Russia, joint activities between the U.S. Department of Defense (DOD) and the Russian 12th Main Directorate, the U.S. Department of Energy (DOE) and Russia's Rosatom, and the U.S. Nuclear Regulatory Commission and the Russian Gosatomnadzor have lacked any publications with reportable data. Rosatom was established in 2007 after Rostekhnadzor (the result of splitting the responsibilities of Gosatomnadzor in 2004) to maintain material protection, control and accountancy on non-

[m] The annexation of the Crimean Peninsula from Ukraine was cause for heightened tensions between the states and thusly, many future joint plans were scrapped in response.
[n] The only continued collaboration was under space exploration between the respective space agencies.

military nuclear material. As mentioned, the 12^{th} Main Directorate collaborated with the U.S. DOD under the CTR on securing nuclear weapons, but Rosatom has worked extensively with the U.S. DOE's National Nuclear Security Administration to meet the needs of securing material against non-state use. Collaborations continued until political pressure strained the relationship. On both sides, tensions rose: among other things, the U.S. was critical of Russia's regional incursions into Georgia and the Ukraine and the Russian government had reserved issues with American insistence on weapons/material inspections and verification activities under the CTR. [167,168,169] In all, the relationship deteriorated at both ends. With this, the game is defined by populating it with values for the following parameters: State A (α, λ, P_A, K_A) and State B (μ, K_B, Γ).

7.1.1. State A Parameters

For current statuses of funding the security of a nuclear weapons program, it is difficult to find accurate representative numbers. For this reason, the various input data values estimated here are to serve as a basis for making sensible estimates inferred from publications and by subject matter experts in nuclear security, arms control, and Russian economic studies. The first estimated parameter is α as 0.15 assets lost per year when no funding is applied to nuclear security. This further reduction from the 2008 case study is attributed to the strengthening Russian economy (which could have contributed to reducing the financial incentive an adversary may have had to divert material) and the increase in power and influence by the Russian government on the world stage as a leader against nuclear terrorism through numerous international endeavors: GINCT, UNSCR1540, etc.[170] With α, we compute λ_{15} (an estimated reduction in loss rate) as discussed in Subsection 3.1.2.1.For the loss rate and corresponding expense for security by Russia in 2015, inferring data from published articles and other scholarly research lead to plausible parameter inputs used in this case study. For example, the loss rate of 2015 was calculated using the same method in the previous two case studies. Table 7.1 includes the tabulated recorded cases of nuclear trafficking of weapons-usable nuclear material from various published sources in terms of significant quantities from Russia.[143,171,172]

Table 7.1. Nuclear trafficking incidents for calculating loss rate in 2015

Date	**Location of Confiscation**	**Material**	**Mass [g]**	**SQ**
August	Chisinau, Moldova	DU	1800	2.7e-7
June	Chisinau, Moldova	HEU	7	5.6e-4
January	Blagoveshchensk, Russia	Th	2200	1.1e-4
June	Moscow, Russia	Th	56000	2.8e-3
TOTAL				**3.47e-3**

Similar to the methods presented in the 2008 case study in Chapter 6, the investment for nuclear security in Russia was estimated using various documents with data from 2007 and a Russian National financial report on nuclear and radiation safety from 2015. The combination of loss rate data from 2015 in Table 7.1 and Russian expenditure estimates were used to make plausible assumptions of 2015 data for this case study (Russian expenditures for 2015 were difficult to attain at the time of this writing – therefore, 2007 data was used and several adjustments were made). The Russian Duma federal budget allocated a total of 3,973B RUB for military expenditures (using the 2015 exchange rate of 0.02 USD to 1 RUB, this amounts to 79.46B USD).[173,174] The Russian military budget allocation for nuclear weapon security can be estimated at around 190.7M USD (0.24% of the Russian military budget for securing nuclear weapons if assumed at the 2007 level of both Russian and American contributions). Regarding (non-weapon) nuclear materials, the publically-available Fourth National Report of the Russian Federation states that, in 2007, Rosatom proposed an eight year budget for a Federal Targeted Program (FTP) in Nuclear and Radiation Safety Assurance which, among other tasks, consisted of "ensuring and monitoring" nuclear and radiological material and activities during normal operations at Rosatom facilities – nuclear material protection, control, and accounting.[175] The amount listed in this report was 20B RUB for all eight years. Using the exchange rate of roughly 1 USD to 25 RUB from 2007 and estimating the 2015 amount by calculating 1/8 of the original amount yields approximately 100M USD.[176] Therefore, the total amount for C_{15} used to calculate λ_{15} per Equation (3.1.18) is 290.7M USD – as shown below:

$$\lambda_{15} = \frac{1}{-C_{15}} \cdot \ln\left[\frac{L(C_{15})}{\alpha}\right]$$
$$\lambda_{15} = \frac{1}{-290.7} \cdot \ln\left[\frac{0.00347}{0.15}\right]$$
$$\lambda_{15} = 0.0129\left[\$M^{-1}\right]$$

Unlike previous case studies, an assumption for P_A was made using any fatal terrorist attack in Russian for the preceding decade to 2015. Using the 2010 bombings of the Moscow metro station and the 2011 bombing at Moscow's international Domodedovo

Airport, P_A is set at 0.2 assuming an at-target detonation (contrary to an on-site detonation, as discussed in Subsection 3.2.2, where the conditional probability of a detonation on-site given material is confiscated by a terrorist group is certain).[177,178] P_A = 0.2 can be considered a conservative estimate due to actions taken by all nuclear weapon states (including Russia in 2015) that have invested heavily in nuclear forensics – which by assumption contributes substantially to the nuclear security effort by increasing the probability of correct and expedited attribution. Hence, if nuclear forensics is enhanced, it can be assumed that the probability of an attack on Russia using its own material or weapon is lower than 20%. This probability can be changed by any methodology user in future campaigns.

To estimate a value for the *at-target* K_A, we solve for Equation (3.1.21) using its various constituents:

- 12,166,000 for the 2015 population of Moscow[146];
- 2,511 km^2 as the current land area[153];
- 9,092 USD is estimated as the 2015 GDP per capita[97];
- 71,500 USD as the value of statistical life in 2015[124];
- the range of 0.77 km to simulate a single 10kT yield explosion producing a minimum 7.5 psi overpressure effect needed create a 50% fatality rate (which is

based on the most common weapon in current operation[o] within the Russian nuclear arsenal)[154]; and

- an estimate of 67,600M USD for rebuilding a vast portion of Moscow based on a modern city-wide rebuild such as post-2012 Hurricane Sandy on New York City[p].

$$K_A^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right](50\%\ \text{fatality})\left[\text{VSL}\right]$$
$$K_A^{hu} = \left(\frac{12.166\text{M}}{2{,}511}\right)\cdot\left[\pi\cdot(0.77\text{km})^2\right]\cdot(0.5)\cdot[\$71{,}500]$$
$$K_A^{hu} = \$323\text{M}$$

$$K_A^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right](50\%\ \text{fatality})\left[\text{GDP}_{\text{per capita}}^{1985}\right]$$
$$K_A^{ec} = \left(\frac{12.166\text{M}}{2{,}511}\right)\cdot\left[\pi\cdot(0.77\text{km})^2\right]\cdot(0.5)\cdot[\$9{,}092]$$
$$K_A^{ec} = \$41\text{M}$$

$$K_A^{ni} = \$67{,}600\text{M}$$

$$K_A = K_A^{hu} + K_A^{ec} + K_A^{ni} = \$323\text{M} + \$41\text{M} + \$67{,}600\text{M} = \underline{\$67{,}964\text{M}}$$

Therefore, in this model, the *at-target* K_A = 64.964B USD.

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[o] As of January 2016, 460 warheads are on 46 R-36M2 (SS-18 Voyevoda) with an estimated yield of 750 - 1000kT, 150 warheads are on 150 Topol class missiles with estimated yields of 800kT each, and 292 warheads on 73 RS-24 missiles with at least 1MT yields each according to http://russianforces.org/missiles.

[p] A Category 3 hurricane, Sandy struck New York City in October 2012 and was estimated to have caused $67.6B in damage: http://www.ncdc.noaa.gov/billions/events.

7.1.2. State B Parameters

In this case study, State B's interdiction efforts (i.e., the U.S.) are characterized by the advent of the domestic portion of the Global Nuclear Detection Architecture (GNDA) as of 2006 under the U.S. Department of Homeland Security's Domestic Nuclear Detection Office (DNDO). In general, the GNDA is the global "framework for detecting (through technical and non-technical means), analyzing, and reporting on nuclear and other radioactive materials that are out of regulatory control."[4] Supported by various American departments and other entities (e.g., the Departments of Defense, Energy, Justice, State, etc.), the DNDO focuses on the integrated role of interdiction through domestic detection, identification, and neutralization measures. Per a statement by DNDO Director Huban Gowadia, the annual operating budget of the DNDO domestic protection efforts amounts to approximately 180M USD.[179] Additionally, the DOE, DOD, and DOS also contribute to State B spending to help secure State A's interdiction mission as well. According to the Nuclear Threat Initiative *Securing the Bomb Budget Tool*, a total of approximately 1.455B USD was spent on interdicting nuclear smuggling by the three aforementioned departments (excluding the DNDO) in 2015.[180] Therefore, the total amount from all four organizations is estimated at 1.635B USD. In calculating μ, this value is used as the investment amount in order to achieve a non-detection probability of 5% per the modification of Equation (3.1.24) shown below.

$$\mu = \frac{-\ln\left[P_B\left(C_B\right)\right]}{C_B} = \frac{-\ln\left[0.05\right]}{1635\text{M}} = \underline{0.00183}\ \left[\$\text{M}^{-1}\right]$$

Of note, the increase in value of μ between 2008 and 2015 signifies a decrease in investment of State B's interdiction. This can be the result of any number of reasons which are touched upon further in Section 9.1.

Below is the process used for determining K_B – the financial consequence to the U.S. of a direct nuclear attack on the most populous major city using the same method as used in the preceding subsection for 2015 Moscow with a 2015 Manhattan (population of 1,644,518[127] and a 59.2-km^2 land area[181]). Beyond the estimate used for simulating an attack on Moscow, for Americans of New York, the estimates include a 9.4M USD value of statistical life and a 55,837 USD GDP per capita for 2015.[126,97] All else being equal yields the following:

$$K_B^{hu} = \left[\frac{\text{ppl}}{\text{km}^2}\right]\cdot\left[\pi \cdot r_{th}^2\right]\cdot\left[50\%\ \text{fatality}\right]\cdot\left[\frac{\$9.4\text{M}}{\text{person}}\right]$$

$$K_B^{hu} = \left(\frac{1.645\text{M}}{59.2\text{km}^2}\right)\cdot\left[\pi \cdot (0.77\text{km})^2\right]\cdot(0.5)\cdot\left(\frac{\$9.4\text{M}}{\text{person}}\right)$$

$$K_B^{hu} = \$243{,}190\text{M}$$

$$K_B^{ec} = \left[\frac{\text{ppl}}{\text{km}^2}\right]\cdot\left[\pi \cdot r_{th}^2\right]\cdot\left[50\%\ \text{fatality}\right]\cdot\left[\text{GDP}_{\text{per capita}}^{2015}\right]$$

$$K_B^{ec} = \left(\frac{1.645\text{M}}{59.2\text{km}^2}\right)\cdot\left[\pi \cdot (0.77\text{km})^2\right]\cdot(0.5)\cdot\left(\frac{55{,}837}{1{,}000{,}000}\right)$$

$$K_B^{ec} = \$1{,}445\text{M}$$

$$K_B^{ni} = \$67{,}600\text{M}$$

$$K_B = K_B^{hu} + K_B^{ec} + K_B^{ni} = \$243{,}190\text{M} + \$1{,}445\text{M} + \$67{,}600\text{M} = \underline{\$312{,}235\text{M}}$$

Finally, contrary to the first two case studies, and like Case Study 3, Case Study 4 incorporates the non-zero value of Γ as a means to signify that, in the modern world, the Russian Federation is not the only source of a nuclear weapon for a non-state actor. The estimate for Γ (the calculated loss rate for this scenario) is increased one order of magnitude to signify the perceived non-state threat would come from elsewhere much more likely than from Russia. Relatively, the number of nuclear devices and weapons-grade nuclear material in other countries with a lower level of applied nuclear security outweighs that of within the borders of 2015 Russia. Hence, for Equation (3.2.4), $\Gamma = 10 \times L(C_{15}) = 0.0347$ SQ/yr.

7.2. Non-Cooperative Game Theory

Table 7.2 below compiles all the previously discussed input variables for determining the strategies in this section.

Table 7.2. Estimated parametric values for CS4

α	0.15	[SQ]
λ_{15}	0.0129	[$M^{-1}]
P_A	0.2	[ ]
K_A	67,963	[$M]
μ	0.00183	[$M^{-1}]
K_B	312,235	[$M]
Γ	0.0347	[SQ]

In this case study, the value of μ is greater than the 2008 case study value of 0.00112 $\$M^{-1}$ due to a decreased investment in State B's system of interdiction while the non-interdiction probability was estimated as the same: 5%. This could be the result of the impact of investment was greater in 2015 than 2008 in the United States or that as much investment did not have to occur due to the raised nuclear security awareness around the world thanks for multilateral endeavors such as GICNT, the Nuclear Security Summits, and the IAEA's general global outreach for enhancing nuclear security.[2,42] The following two subsections, 7.2.1 and 7.2.2 show the calculation of the various costs for the players: the Russian Federation as State A and the United States as State B.

7.2.1. Non-Cooperative Solution: Uncorrelated Strategies

Determining the appropriate state-level uncorrelated strategies requires solving for the strategic costs in Equations (3.2.3) and (3.2.6). This is shown below using the values in Table 7.2.

$$C'_A = \max\left\{0, \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right\}$$

$$C'_A = \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)$$

$$C'_A = \frac{1}{0.0147}\ln\left(0.15 \cdot 0.013 \cdot 0.2 \cdot 67963\right) = 252.7$$

$$C'_A = \max\left\{0, 252.7\right\} = \underline{252.7}$$

$$T_A(C'_A) = P_A \cdot K_A \cdot \left[\alpha \cdot e^{-\lambda \cdot C'_A}\right]$$
$$T_A(C'_A) = 0.2 \cdot 67963 \cdot \left[0.15 \cdot e^{-0.013(252.7)}\right] = \underline{77.2}$$

$$TC'_A(C'_A) = C'_A + T_A(C'_A) = 252.7 + 77.2 = \boxed{329.9}$$

The strategic annual investment therefore is 252.7M USD which yields the total cost of 329.9M USD. This is estimated as the best case scenario for the Russian Federation (with the aforementioned parameters). Solving for the best-response strategies for the U.S. requires solving for C'_B:

$$C'_B = \max\left\{0, \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]\right\}$$
$$C'_B = \frac{1}{0.0018}\ln\left[0.0018 \cdot 312235 \cdot \left(0.15 \cdot e^{-0.013(252.7)} + 0.0347\right)\right] = 1713.6$$
$$C'_B = \max\{0, 1713.6\} = \underline{1713.6}$$

$$T_B(C'_A, C'_B) = e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)$$
$$T_B(C'_A, C'_B) = e^{-0.0018(1713.6)} \cdot 312235 \cdot \left(0.15 \cdot e^{-0.013(252.7)} + 0.0347\right) = \underline{545.8}$$

$$TC'_B(C'_A, C'_B) = C'_B + T_B(C'_A, C'_B) = 1713.6 + 545.8 = \boxed{2259.4}$$

$$TC'_{AB} = TC'_A + TC'_B = 329.9 + 2259.4 = \boxed{2589.3}$$

As shown above, the total cost to the U.S. is much greater than that for the RF. The total cost to the regime of the two states with no cooperation therefore, is 2589.3M USD according to the model. Subsection 7.2.2 shows how to calculate the cost of the correlated strategies where then we compare the uncorrelated and the correlated costs to the regime.

7.2.2. Non-Cooperative Solution: Correlated Strategies

The correlated strategy is where the total regime, as defined in Equation (3.2.8), exhibits a global minimum. Following confidence in the results calculated in Chapters 4-6 for Case Studies 1-3, the correlated results for Case Study 4 are shown in Table 7.3. Compared to the total amounts invested and the potential consequential costs, T_i, there is not much benefit to the regime (with subscript 'AB') in correlating strategies. Especially exhibited when comparing TC_{AB} values, the surplus only amounts to 24.9M USD according to this model. Though potentially viable as a means for initiating and maintaining a bilateral arrangement for some states, this amount would not invigorate either the U.S. or the Russian Federation to overcome a multitude of (domestic and international) political challenges for collaborating on nuclear security.

Table 7.3. Annualized costs [in $M] from the non-cooperative game

	Uncorrelated [$M]	Correlated [$M]
C_A	252.7	308.9
$T_A(C_A)$	77.2	37.2
$TC_A(C_A)$	329.9	346.1
C_B	1713.6	1672.4
$T_B(C_A, C_B)$	545.8	545.8
$TC_B(C_A, C_B)$	2259.4	2218.2
TC_{AB}	2589.3	2564.4[q]

As of summer 2016, there is no collaboration between the U.S. and Russia in nuclear security. With the fallout of the CTR/MPC&A and eventually the MNEPR, joint activities and collaborative efforts between Russia and the U.S. seem to be diminishing by both sides for various reasons. Therefore, in the hope of analyzing the benefit of collaboration, this case study will also consider the possibility of a collaborative game in order to understand the mutual gains for each player and how to maintain it annually.

Figure 7.1 presents the strategies on a 3D plot. With $\Gamma > 0$, one can see the increasing trend for TC_{AB} as strategic costs increase from the correlated strategy point as represented by the cyan star. Again, the vertical difference between the red and cyan points on Figure 7.1 amounts to 24.9M USD in total cost to the regime. With this amount, Section

[q] Discrepancies are due to rounding in calculations.

7.3 includes a discussion of various bargaining solutions where this surplus in savings can be potentially split between the U.S. and the Russian Federation.

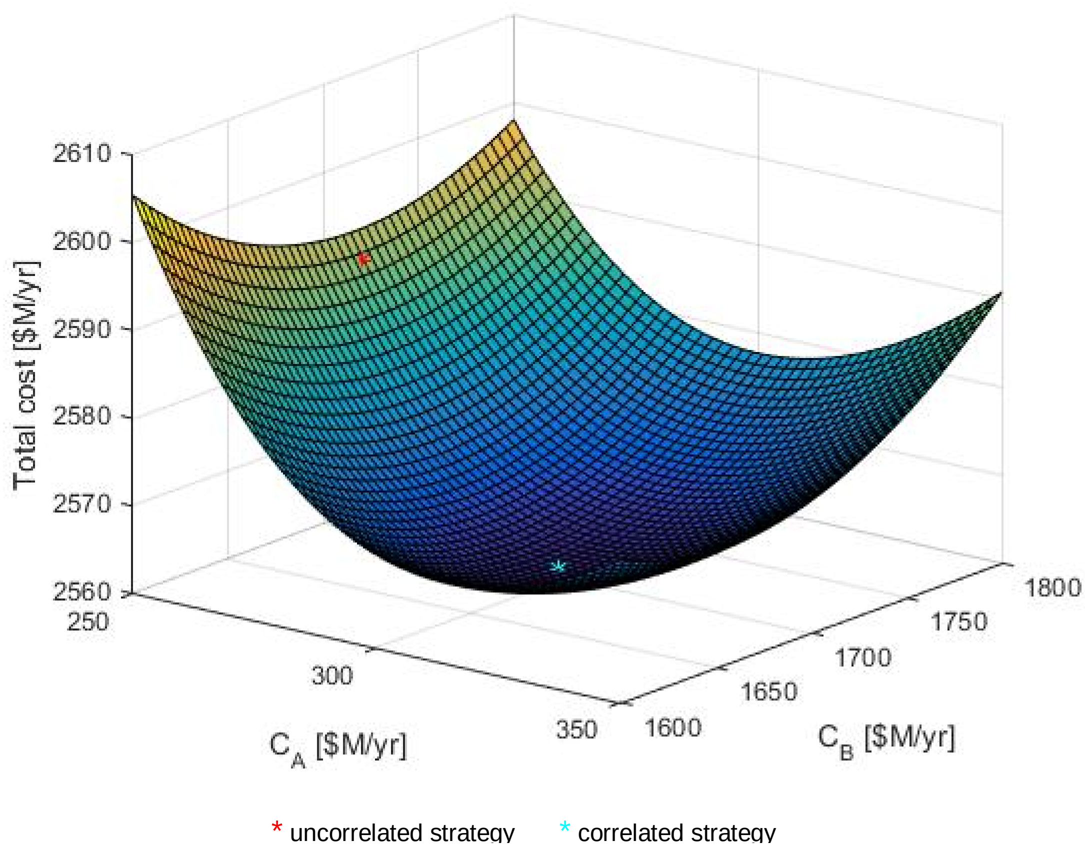

Figure 7.1. Total costs vs strategic costs for states A and B

7.3. Bargaining Solution Concepts and Utility Transferability

Table 7.4 repeats the data found in Table 7.3 but includes a third solution concept under the C_{BA}-Neutralization (C_{BA}-N) strategy. The *uncorrelated* and *correlated* solution concepts show an interesting scenario in that the additional strategic security investment $(C_A^* - C_A')$ to Russia bears no effect on the threat cost to the U.S. $(T'_B = T_B^*)$ because Russia does not present the only threat to the U.S. (i.e., $\Gamma \neq 0$). Hence, Russia's reduced threat correlated cost, $T_A(C_A^*)$, is the only observed benefit of the additional investment to the security of material shown in Table 7.4.

Table 7.4. Annualized costs [in $M] from the non-cooperative game

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutralized [$M]
C_A	252.7	308.9	252.7
$T_A(C_A)$	77.2	37.2	37.2
$TC_A(C_A)$	329.9	346.1	289.9
C_B	1713.6	1672.4	1728.7
$T_B(C_A, C_B)$	545.8	545.8	545.8
$TC_B(C_A, C_B)$	2259.4	2218.2	2274.5
TC_{AB}	2589.3	2564.4	2564.4

The C_{BA}-N strategy in the third column infers the U.S. is contributing to the additional cost of securing nuclear assets in Russia to the amount of 56.2M USD per $[C_A^* - C'_A]$. This amount is contributed by the U.S. as C_{BA}. Because $\Gamma > 0$, the American strategy must still defend against other threats originating from beyond Russia. This effect also occurred in Case Study 3 although it was not as drastic – though $\Gamma > 0$, State B could invest all the surplus to State A's security and the final strategic and total cost to State B was still less than the *uncorrelated* costs as shown in Table 6.3. Hence, no significant diminished investment occurs between C_B^{UNC} and C_B^{COR} and no decrease occurs from $T_B(C'_A, C'_B)$ to $T_B(C_A^*, C_B^*)$. Therefore, for the C_{BA}-neutralized strategic solution, the American investment cost is calculated as the following:

$$C_B = C_B^* + C_{BA} = C_B^* + \left(C_A^* - C'_A\right) = 1672.4 + \left(308.9 - 252.7\right) = 1728.6$$

The new C_B amount yields no difference in T_B and therefore, yields TC_B = 2,274.4M (which provides minimal benefit over its uncorrelated expense of 2,259.4M USD). Furthermore, as seen in Figure 7.2, the *correlated* solution (albeit on the UT line) falls outside the game's core heavily favoring State A's utility in that it would require the RF in this case to receive all the savings surplus in addition to a side payment from the U.S. This is shown as the red point at (289.9, 2274.4).

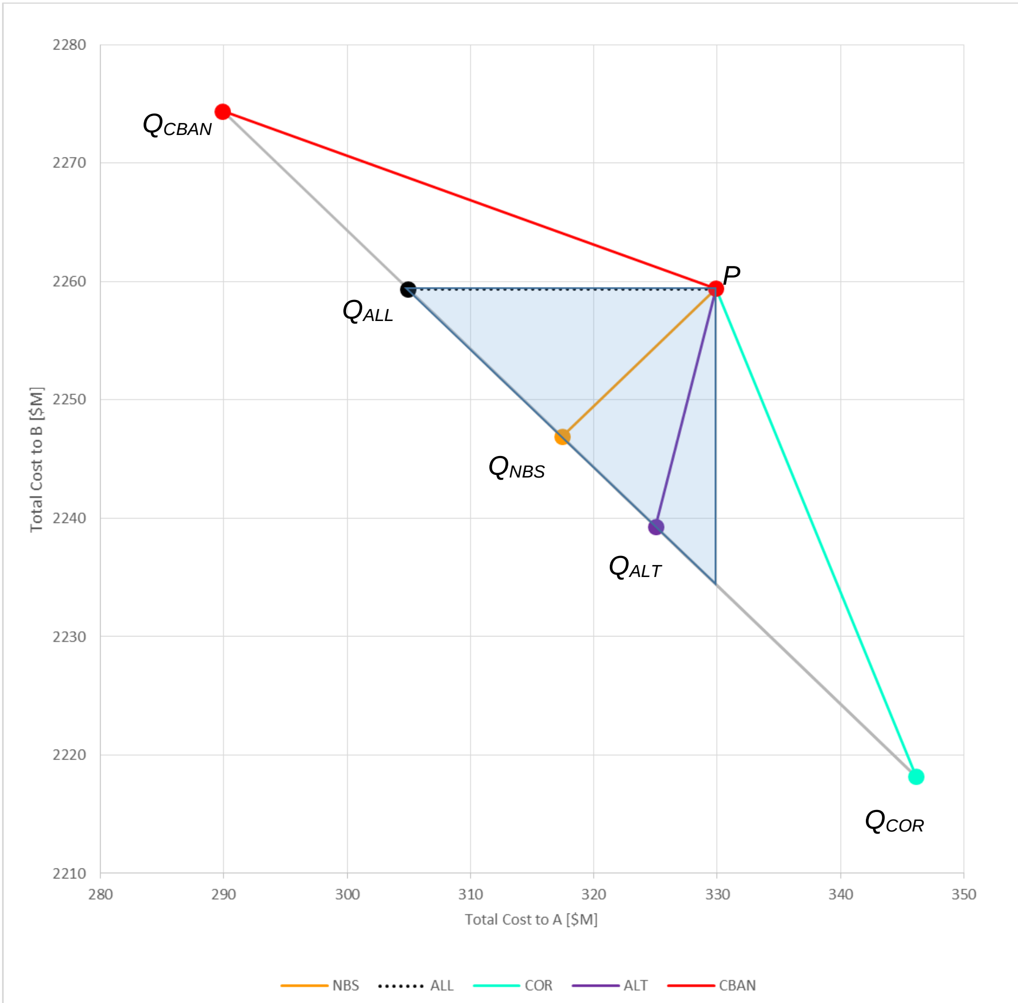

Figure 7.2. Imputations of CS3: the RF-U.S. game (2015)

The orange point in Figure 7.3 occurs at the Nash Bargaining Solution (317.4, 2246.9) where each state receives an equal share of the cost savings of 12.5M USD. The black strategic point at (304.9, 2259.4) signifies all the savings surplus is received by Russia and therefore conveys that the total cost of the U.S. does not change while the Russian Federation's total cost diminishes substantially. As opposed to Case Study 2 (data in Table 4.2), the U.S. receives no reduction in *threat* cost if it contributes to Russia's security due, in part, to $\Gamma > 0$ (i.e., the threat for the U.S. still exists so reducing the conceivable threat cost regarding Russia provides no benefit). Hence, the minimum total cost achievable by the U.S. in this case is only 2,218.2M USD. Lastly, an alternative bargaining solution is included as the purple ALT segment to convey another, more favorable bargaining solution for the U.S. that falls within the game's core: (325, 2239.3).

7.4. Analysis and Discussion

Table 7.5 shows the details of all the imputations between the 2015 Russian Federation and the United States: the *uncorrelated*, Nash Equilibrium strategy; the *correlated* strategy; the Nash bargaining solution; the C_{BA}-Neutralization strategy; the all-surplus transfer solution; the previously-discussed alternative solution; and the best attempt at the actual strategy point used in Section 7.1 to determine the α, λ and μ values. The last imputation, Q_{ACT}, can assist one in understanding the benefit for the U.S. to cooperate with Russia in 2015. Conversely, not much would be gained by Russia in this case because the total cost of 337.9M did not provide a significant benefit to Russia.

Table 7.5. Imputations of the cooperative game for CS 4

	Savings Split for A [$M]	Savings Split for B [$M]	$\frac{s}{r}$	TC_A [$M]	TC_B [$M]	TC_{AB} [$M]
P	--	--	--	329.9	2259.4	2589.3
Q_{COR}	-16.2	41.2	-2.54	346.1	2218.2	2564.3
Q_{NBS}	12.5	12.5	1.00	317.4	2246.9	2564.3
Q_{CBAN}	40	-15	-0.38	289.9	2274.4	2564.3
Q_{ALT}	4.9	20.1	4.10	325	2239.3	2564.3
Q_{ALL}	25	0	0.00	304.9	2259.4	2564.3
Q_{ACT}	--	--	--	337.9	2230.9	2568.8

To convey the final utility percent advantage (as defined in Section 3.1.2), Table 7.6 shows the utility benefits per imputation for Russia (U_A), the U.S. (U_B), and the collective regime (U_{AB}). Those with positive values exhibit those strategy points lying within the game's core.

Table 7.6. Tabulated benefits in utility results per imputation

	U_A	U_B	U_{AB}
Q_{COR}	-0.05	0.02	0.01
Q_{NBS}	0.04	0.01	0.01
Q_{CBAN}	0.12	-0.01	0.01
Q_{ALT}	0.01	0.01	0.01
Q_{ALL}	0.08	0.00	0.01

These results convey how various strategy sets benefit the two sides in this case study. Impressively, half of the imputations exhibit some individual utility benefit over unilateral strategies – albeit none being substantial. Notably, U_{AB} for all solutions never exhibits a substantial collective benefit for the regime (0.01). These minimal values lead one to consider the benefit of cooperation between the states in Case Study 4: Russia-U.S. (2015). Imputation Q_{ALL} yields a zero benefit for the U.S. and therefore has low credibility within the regime. However, depending on the reliance of other U.S. interests with Russia or within the region, these imputations cannot be discounted without fully assessing the international situation. In all, these solution concepts illustrate how adopting various strategies could have assisted in defining the nuclear security regime between Russia and the U.S. in 2015.

8. CASE STUDY 5: PAKISTAN – U.S. (2008)

Case Study 5 is an attempt to demonstrate the utilization of this methodology with a different pair of states. Here, we focus on the precarious relationship between the U.S. and Pakistan. This case study serves as a trial of the methodology on a completely different potential bilateral partnership than the previous four case studies. As regards potential targets for terrorist acquisition of nuclear materials or weapons, Pakistan has a small-scale civil nuclear program consisting of four reactors at the Chashma Nuclear Power Plant in the Punjab province that provide less than 5% of its power.[111,182] However, the perhaps larger concern is the state's nuclear weapons program which reportedly stockpiles 110-130 weapons and, by one unofficial account, has produced 3000 kilograms of highly enriched uranium and 200 kilograms of plutonium since its beginning.[111] The weapons and material are under the stewardship of the Pakistani Strategic Plans Division (SPD) in the military. In July 2004, retired Major General Mahmud Ali Durrani published a document for Sandia National Laboratories on Pakistan's Strategic Thinking and the Role of Nuclear Weapons.[183] In it, Durrani identifies four Pakistani nuclear policy objectives: 1) "deter all forms of external aggression that endanger [Pakistani] national security," 2) do so by developing and maintaining "an effective combination of conventional and strategic forces," 3) deter all adversaries by "securing strategic assets and threatening nuclear retaliation," and 4) "stabilize strategic deterrence in the South Asia region."

Echoing the aforementioned objectives in his December 2011 report, Banuri states "India's massive conventional military buildup, the India-U.S. nuclear deal, and [India's pursuit of missile defense systems] forced Pakistan to make qualitative and quantitative adjustments" to their arsenal.[184]

With the close proximity to areas of terrorist activities and the concern over government/leadership stability, Pakistan has received numerous offers of international collaborative activities, including with the U.S. Particularly, the U.S. and Pakistan have long collaborated against Taliban activities in the region but with Pakistan's leadership wary of American actions. For example, as the U.S. was preparing to attack the Afghan Taliban after the September 11, 2001 attacks in the U.S., then-President Musharraf reportedly "ordered the Pakistani nuclear arsenal redeployed to new, secret locations."[185] According to a televised speech by Musharraf, the move was a result of Musharraf's insecurity with American intentions and whether "the U.S. would decide to conduct military strikes against Pakistan's nuclear assets if the government did not assist against the Taliban."[186]

In 2007 and 2008, expressions of doubt regarding the adequacy of security surrounding the Pakistani nuclear arsenal were raised in public – both internally to Pakistan and outside. In late 2007, Former Prime Minister Benazir Bhutto publically questioned the stability of Musharraf's control over the nuclear weapons arsenal in an interview.[187] Furthermore, numerous U.S. officials began to question publicly the level of security surrounding nuclear weapons in Pakistan (especially in times of political turmoil – which were beginning to occur more frequently). Given the original intent of the arsenal as a state-level deterrent, Krepon openly questioned the movements of nuclear assets across Pakistan to counter Indian aggression as a vulnerability to nuclear security because of less-then-adequate transportation security (compared to heavily-guarded storage sites).[188]

In light of these perceptions, various governments (particularly the U.S.) began encouraging and promoting collaborations with Pakistan. In her January 2005 congressional confirmation hearing, then U.S. State Secretary Rice responded to an inquiry about the status of nuclear weapons in Pakistan during and after a radical Islamic coup with an affirmation that the situation had been considered and a contingency plan had been discussed.[189] Later in 2007, former U.S. Deputy Secretary of State Armitage confirmed American assistance in securing Pakistani nuclear weapons and deemed them "fairly secure."[190] However, former DOE Director Mowatt-Larssen of the Office of Intelligence and Counterintelligence did admit later in 2009 that with the lack of transparency, it was difficult to ensure how assistance in the amount of $100M was being spent but that, upon a subsequent visit, the "money was well spent."[191] Mowatt-Larssen's qualitative statement harkens back to Section 3.3's *Trust and Commitment* discussion regarding a lack of self-enforcement between the game's players – the U.S. did not have a method by which to measure how Pakistan was investing in its nuclear security program.

One defining aspect of the Pakistani nuclear program is the shadow cast by the black market network for nuclear technologies operated by Abdul Qadeer Khan. Since the revelation of his large-scale network, many officials in the U.S. government agreed that Pakistan had "increased its efforts to prevent [further] nuclear proliferation."[111]

8.1. Evaluating the Game Model Parameters

For this case study, the players of the game are assigned as State A (Pakistan) and State B (the U.S.). Due to the difficulty of finding published numbers for the numerous parameters, this case study consists of defining plausible values for 2008 and determining the corresponding strategies and their calculated costs. Despite a level of distrust between the partners[r], there is a proven record of cooperation between the two. Furthermore, in light of modern day relationships, Pakistan can definitely not be considered the sole threat of a nuclear attack on the U.S. Therefore, $\Gamma > 0$. Other parameters are discussed in the following two subsections.

8.1.1. State A Parameters

Pakistan presents many issues not observed in case studies between the R.F. and the U.S. The father of the Pakistani nuclear bomb, A.Q. Khan, embodied the pinnacle of nuclear proliferation (specifically nuclear technologies). Though distributing weapons and material were not the *modus operandi* of Khan's network, his detrimental effect on the nonproliferation regime was (and continues to be) felt globally. Furthermore, the prominent existence of non-state and even some sub-state adversaries within Pakistan constitute a large nuclear security concern.[192] Therefore, the lack of evidence of nuclear trafficking cannot conservatively be considered as a lack of activity by terrorists to acquire nuclear materials and/or weapons.

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[r] Surmised as the US assisting Pakistan in securing its material because of distrust and Pakistan mistrusting American intentions in weapons security due to lax security.

For this case study, α and an estimated loss rate, $L(C_{PAK})$, are used in Equation (3.1.18) to estimate λ:

$$\lambda_t = \frac{1}{-C_{PAK}} \cdot \ln\left[\frac{L(C_{PAK})}{\alpha}\right]$$

As mentioned earlier, in the introductory preface to this chapter, Mowatt-Larssen states there was at one time around 2008 an investment from the U.S. to Pakistan for securing its weapons (C_{PAK} = 100M USD). In the particular case of Pakistan circa 2008, there are no recorded loss events of nuclear materials or weapons.[155] Therefore, as a plausible estimate, $L(C_{PAK})$is assigned a small value by basing it off the 1955 AFSWP report for acceptable losses of a 10-100KT weapon (similar to the process presented in Chapter 4).[163] With the close connection between Pakistan and the U.S., it is conceivable that the published numbers could have been the starting point of a discussion for defining loss rates between the states (but no record supports this assertion – this is merely an assumption by the author). The acceptable loss value in 1955 for 10-100kT yield weapons is defined as 2e-3 events per year. Just as in Case Study 1 from Chapter 4, α is estimated using Quigley's assumption of the likelihood of an event occurring is the inverse of 2.5 times the number of years since Pakistan has had a nuclear arsenal (18 years as of this document).[114] Hence, recalling Equation (4.1.1) in Chapter 4, α=0.022 [SQ/yr]. Therefore, assuming an averaged loss rate, $L(C_{PAK}) = 0.002$ events in 2008, C_{PAK} = 100M USD and α=0.022, λ_{PAK} is computed below:

$$\lambda_{PAK} = \frac{1}{-100} \cdot \ln\left[\frac{0.002}{0.022}\right]$$
$$\lambda_{PAK} = 0.024\left[M^{-1}\right]$$

Terrorists have been highly active in Pakistan since at least 2000: 2008 alone saw 2,148 terrorist attacks with 2,267 fatalities and 4,558 injuries.[193,194] Using the same estimation process as the previous three case studies, each five-year span surrounding 2008 (except 2003) saw a large-scale, high-casualty terrorist attack in Karachi.[193] Therefore, P_A = 0.9 for an *at-target* detonation in Case Study 5.

Estimating the K_A value requires using the following assumptions for 2008-era Karachi (as the most populous city in Pakistan and assumed as the primary target for terrorist activity) in Equation (3.1.21) for determining K_A^{hu}, K_A^{ec}, and K_A^{ni}:

- 2008 Karachi is estimated to have 18,000,000 inhabitants and a total land area of 3530 square kilometers[195];
- the estimated Pakistani GDP per capita according to the World Bank is 1,042.80 USD[97];
- an averaged 548,000 USD value of statistical life[196];
- the range of 0.77 km to simulate a rough yield of at most 10kT with a 50% fatality rate[197]; and
- a 2.4B USD estimate taken from reconstruction costs for the similarly-sized Houston, Texas after Hurricane Ike passed over that city in 2008.[198]

$$K_A^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right]\left(50\%\text{ fatality}\right)\left[\text{VSL}\right] = \left(\frac{22.5\text{M}}{3{,}527}\right)\cdot\left[\pi \cdot 0.77^2\right]\cdot\left(0.5\right)\cdot\left[\$548{,}000\right] = \$3{,}256\text{M}$$

$$K_A^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right]\left(50\%\text{ fatality}\right)\left[\text{GDP}_{\text{per capita}}^{2008}\right] = \left(\frac{22.5\text{M}}{3{,}527}\right)\cdot\left[\pi \cdot 0.77^2\right]\cdot\left(0.5\right)\cdot\left[\$1{,}042.80\right] = \$6.2\text{M}$$

$$K_A^{ni} = \$2{,}400\text{M}$$

$$K_A = K_A^{hu} + K_A^{ec} + K_A^{ni} = \$3{,}256\text{M} + \$6\text{M} + \$2{,}400\text{M} = \underline{\$5{,}662\text{M}}$$

Therefore, in this model, the consequence of an *at-target* detonation, K_A, is 5.662B USD.

8.1.2. State B Parameters

The values for the U.S. parameters (μ, P_B, K_B, and Γ) are estimated circa 2008. Recalling Equation (3.1.24) and invoking values published by Schwartz of the deconstructed U.S. government spending on nuclear security for 2008.[199] Using his estimates (as in Case Study 3), the expenditures for interdiction that year was $2,673.3M which, with a common non-detection probability, P_B, of 5% (as described in Subsection 2.2.2), the value for μ is computed as:

$$\mu = \frac{-\ln\left[P_B\left(C_B\right)\right]}{C_B} = \frac{-\ln\left[0.05\right]}{2673.3} = \underline{0.00112}$$

Using Equation (3.1.21) again, from Subsection 2.2.2, K_B is the consequence of a nuclear detonation on U.S. soil with a 10kT device originating from Pakistan. The economic impact of losing individual lives is based on GDP per capita for 2008 (K^{ec}) and the loss of infrastructure of an analogous event in a highly populated area (K^{ni}) such as New York City (adjusted for inflation).[126]

$$K_B^{hu} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_b^2\right]\left(50\%\text{ fatality}\right)\left[\text{VSL}\right] = 10{,}358 \cdot \left[\pi \cdot 0.77^2\right] \cdot \left(0.5\right) \cdot \left[\$8.5\text{M}\right] = \$81{,}996\text{M}$$

$$K_B^{ec} = \frac{\text{ppl}}{\text{km}^2}\left[\pi \cdot r_{th}^2\right]\left(50\%\text{ fatality}\right)\left[\text{GDP}_{\text{per capita}}^{2008}\right] = 10{,}358 \cdot \left[\pi \cdot 0.77^2\right] \cdot \left(0.5\right) \cdot \left[\$48{,}401\right] = \$467\text{M}$$

$$K_B^{ni} = \$63{,}400\text{M}$$

$$K_A = K_A^{hu} + K_A^{ec} + K_A^{ni} = \$81{,}996\text{M} + \$467\text{M} + \$63{,}400\text{M} = \underline{\$145{,}863\text{M}}$$

The last parameter estimate required to populate the input parameters for the model corresponding to this case study is Γ which, if positive, implies that the threat is not isolated to only coming from Pakistan. In 2008, there existed seven other nuclear weapon states – all of which could be a source for materials to be used in an attack on the U.S. For simplicity purposes, we set the value for Γ as equal to $L(C_{PAK})$ for assuming the likelihood of material originating in Pakistan is the same as if it were to originate from any of the other nuclear weapon states. In Section 8.3, this assumption will be relaxed by increasing Γ over $L(C_{PAK})$ to signify the greater possibility of material being used from elsewhere beyond Pakistan.

8.2. Non-Cooperative Game Theory

The estimated parameter values determined in the preceding section are presented together in Table 8.1. State B's parametric values (all but Γ) are repeated from Case Study

3 in Chapter 6 due to the efforts put forth by the U.S. in 2008 were the same regardless of the bilateral regime being assessed. Contrarily to Case Study 3 however, Γ conveys how the origination of threats did not solely come from Pakistan in this case – in 2008, Pakistan was far from the only state with nuclear assets that could have been used against the U.S. For this reason, the U.S. would have benefitted from Pakistan securing its own assets while the U.S. would have better invested in its interdiction efforts to thwart threats from beyond Pakistan. Furthermore, the value of λ is computed at 0.024 $\$M^{-1}$ based on Quigley explained in Subsection 8.1.1 which shows how effective investments in nuclear security measures were in Pakistan. Lastly, with Pakistan's history with nuclear weapons being much less than Russia's that same year, α is much less as well.

Table 8.1. Estimated parametric values for CS5

α	0.022	[SQ]
λ_{PAK}	0.024	[$M^{-1}]
L(C_{PAK})	0.002	[events]
P_A	0.9	[ ]
K_A	5,662	[$M]
μ	0.00112	[$M^{-1}]
K_B	145,863	[$M]
Γ	0.002	[SQ]

8.2.1. Non-Cooperative Solution: Uncorrelated Strategies

Solving for the total costs of the uncorrelated strategies requires using Equations (3.2.3), (3.2.6), and (3.2.7) as shown below for States A (Pakistan) and B (the U.S.). First, the costs for Pakistan:

$$C'_A = \max\left\{0, \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right)\right\}$$

$$C'_A = \frac{1}{\lambda}\ln\left(\alpha \cdot \lambda \cdot P_A \cdot K_A\right) = \frac{1}{0.024}\ln\left(0.022 \cdot 0.024 \cdot 0.9 \cdot 5{,}662\right) = 41.2$$

$$C'_A = \max\left\{0, 41.2\right\} = \underline{41.2}$$

$$T'_A\left(C'_A\right) = P_A \cdot K_A \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A}\right)$$

$$T'_A = 0.9 \cdot 5{,}662 \cdot 0.022 \cdot e^{-0.024 \cdot (41.2)}$$

$$T'_A = \underline{41.7}$$

$$TC'_A\left(C'_A\right) = C'_A + T'_A\left(C'_A\right) = 41.2 + 41.7 = \boxed{82.9}$$

For the U.S.:

$$C'_B = \frac{1}{\mu}\ln\left[\mu \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)\right]$$

$$C'_B = \frac{1}{0.00112}\ln\left[0.00112 \cdot 145{,}863 \cdot \left(0.022 \cdot e^{-0.024(41.2)} + 0.002\right)\right] = 454.7$$

$$C'_B = \max\{0, 454.7\} = \underline{454.7}$$

$$T'_B(C'_A, C'_B) = e^{-\mu \cdot C'_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C'_A} + \Gamma\right)$$

$$T'_B(C'_A, C'_B) = e^{-0.00112(454.6)} \cdot 145{,}863 \cdot \left(0.022 \cdot e^{-0.024(41.2)} + 0.002\right)$$

$$T'_B(C'_A, C'_B) = \underline{892.3}$$

$$TC'_B\left(C'_A, C'_B\right) = C'_B + T'_B\left(C'_B\right) = 454.6 + 892.3 = \boxed{1346.9}$$

For the regime:

$$TC'_{AB} = TC'_A + TC'_B = 82.9 + 1346.9 = \boxed{1429.8}$$

8.2.2. Non-Cooperative Solution: Correlated Strategies

Using values presented in Table 8.1 with Equation (3.2.8), strategic and threat costs are included in Table 8.2. Of note, these results convey the consequences of an *at-target* detonation (which will be replaced with an *on-site* detonation assumption later in this subsection).

Table 8.2. Annualized costs for CS5

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutralized [$M]
C_A	41.2	182.6	41.2
$T_A(C_A)$	41.7	1.4	1.4
$TC_A(C_A)$	82.9	184	41.6
C_B	454.7	0	141.3
$T_B(C_A, C_B)$	892.4	332	332
$TC_B(C_A, C_B)$	1347.1	332	473.3
TC_{AB}	1430	516	516

For the perceived threat against the U.S. from Pakistan, the *uncorrelated* strategy for the U.S. is 454.7M USD and for Pakistan, it is 41.2M USD. Figure 8.1 represents the two solutions in a 3D plot with the red star signifying the uncorrelated strategy point (41.2, 454.7, 1430) and the cyan star signifying the correlated strategy point (182.6, 0, 516). The best *correlated* strategy for the U.S. in this situation is to invest in Pakistan's nuclear security. This strategy set is displayed in Table 8.2 in the middle column where the total regime cost is equal to that of the correlated strategies and the surplus is 914M USD. The last column, $C_{BA\text{-}N}$, displays results for when the U.S. pays the additional amount it would take in strategic costs to secure assets in Pakistan – where the latter state's threat cost decreases but the U.S. receives no such benefit from its further investment.

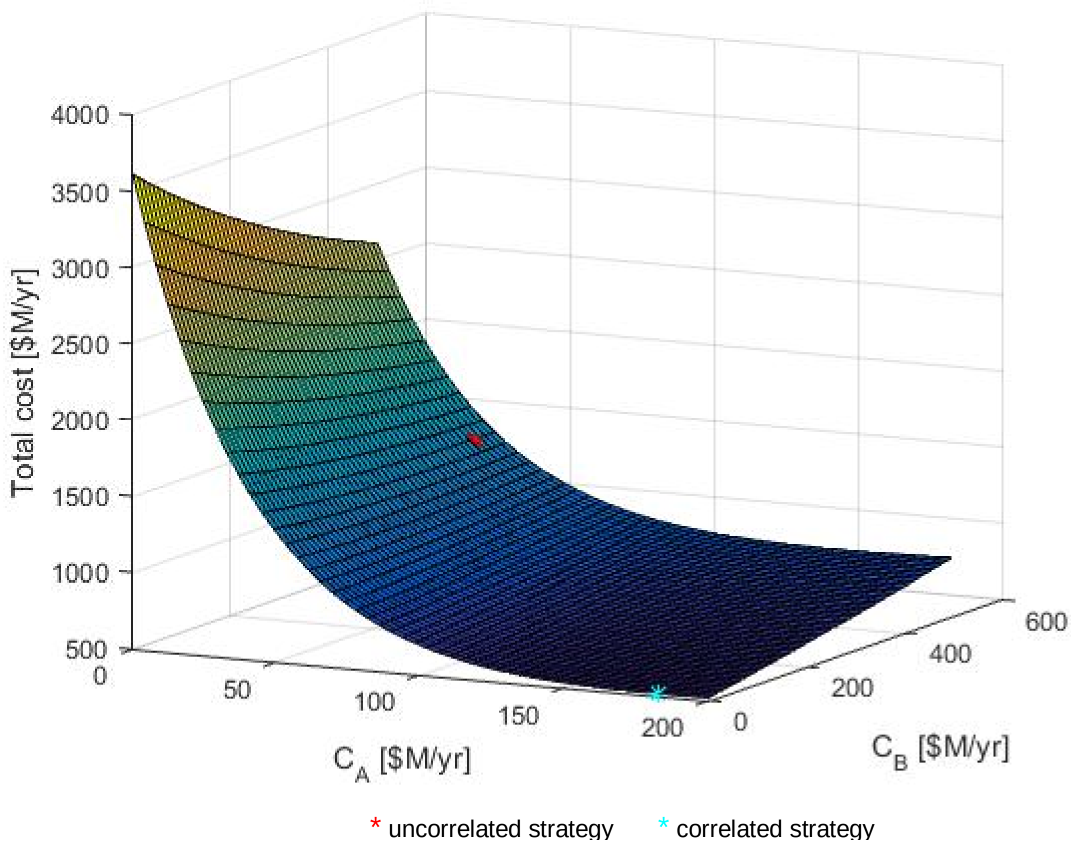

Figure 8.1. Uncorrelated and correlated strategy points between states A and B

Changing Γ can reflect another scenario: where the threat from Pakistan is not nearly the most pressing. To demonstrate this, results for $\Gamma = 10 \times L(C_{PAK})$, are shown in Table 8.3 below.

Table 8.3. Annualized costs for CS5 – $\Gamma > L(C_{PAK})$

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutralized [$M]
C_A	41.2	134.6	41.2
$T_A(C_A)$	41.7	4.4	4.4
$TC_A(C_A)$	82.9	139	45.6
C_B	1363.1	1095.1	1188.5
$T_B(C_A, C_B)$	892.4	892.4	892.4
$TC_B(C_A, C_B)$	2255.5	1987.5	2080.9
TC_{AB}	2338.4	2126.5	2126.5

In Table 8.3, one can infer many important distinctions: there is a reduction in surplus between the *uncorrelated* and *correlated* strategies when Γ is greater; and due to the increased Γ, the threat cost for the U.S. (T_B) between the *uncorrelated* and *correlated* strategies does not change because the partner country does not constitute the primary threat of the U.S.

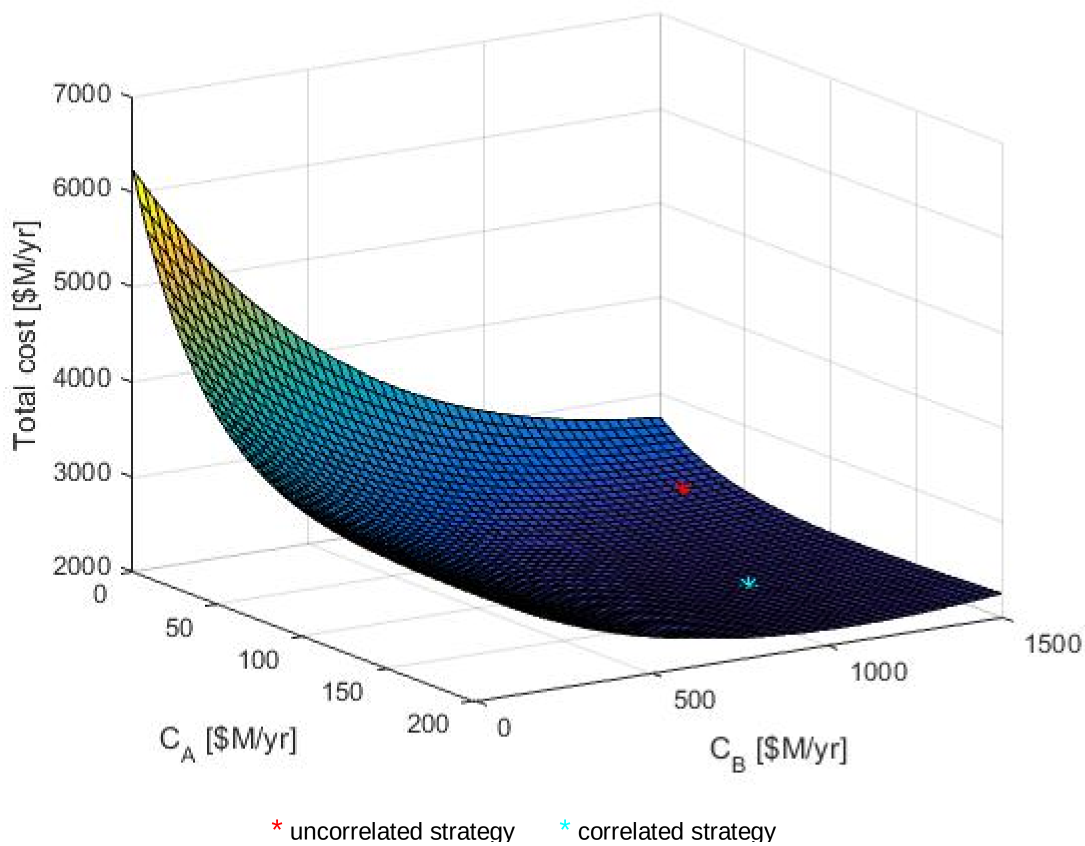

Figure 8.2. Uncorrelated and correlated strategy points between states A and B

We repeat Tables 8.2 and 8.3 (as well as Figures 8.1 and 8.2) with a modified success of attack by a non-state adversary seeking to confiscate a nuclear weapon in State A. This includes an *on-site* detonation at the facility where the asset is held initially. This leads to $P_A = 1$ and a reduced K_A due to the likelihood of the asset being on a military site far from a center of population and any vital national infrastructure elements. The effects of these adjustments are shown below in Table 8.4 and Figure 8.3 with an *on-site* -detonation with $\Gamma = L(C_{PAK})$ and Table 8.5 and Figure 8.4 with an *on-site* detonation with $\Gamma > L(C_{PAK})$.

Table 8.4. Annualized costs for CS5 – on-site detonation

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutralized [$M]
C_A	45.6	98.8	45.6
$T_A(C_A)$	41.7	11.7	11.7
$TC_A(C_A)$	87.3	110.5	57.3
C_B	0	0	53.2
$T_B(C_A, C_B)$	136.6	59.2	59.2
$TC_B(C_A, C_B)$	136.6	59.2	112.4
TC_{AB}	223.9	169.7	169.7

If P_A = 1 (meaning a non-state adversary makes an unsuccessful attempt at removing a nuclear weapon at a military facility and then detonates on site), then both major solution concepts (*uncorrelated* and *correlated* strategies) imply the lack of benefit for State B to invest in its own interdiction. The reduction in B's threat cost from *uncorrelated* to *correlated* can be ascribed to State A's increase of nuclear security.

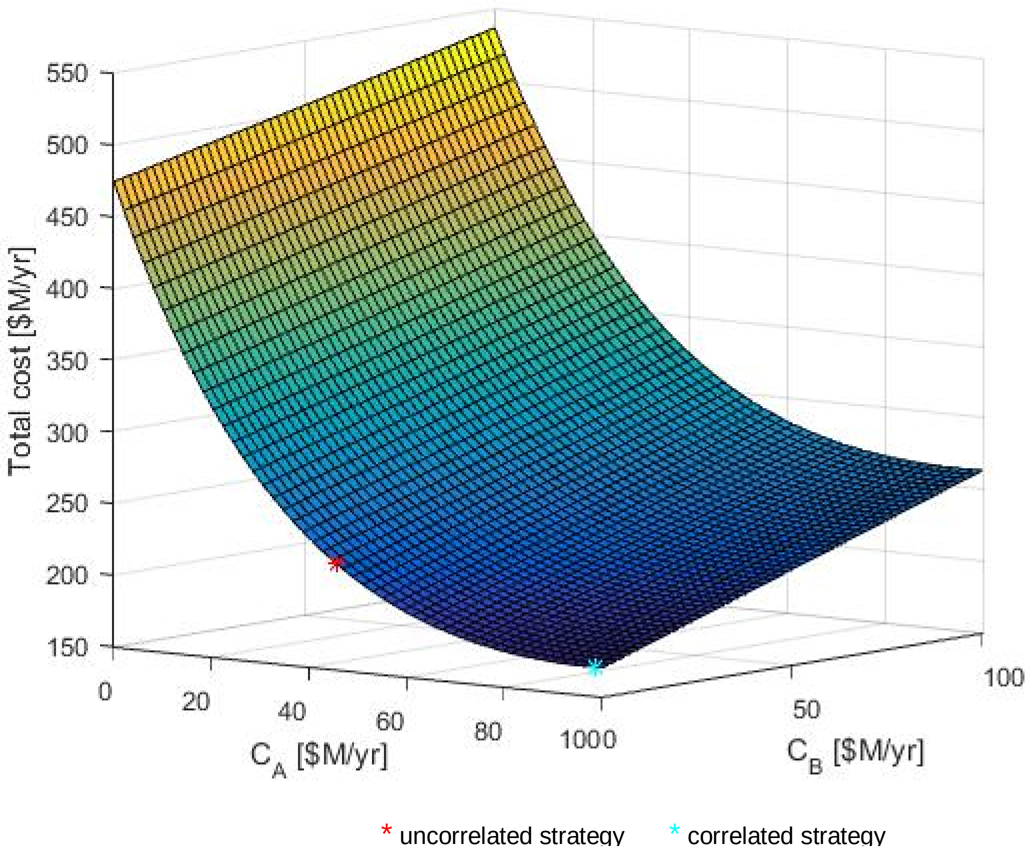

Figure 8.3. Uncorrelated and correlated strategy points between states A and B

Table 8.5 includes the costs for an *on-site* detonation with $\Gamma = 10 \times L(C_{PAK})$.

Table 8.5. Annualized costs for CS5 – on-site detonation – $\Gamma > L(C_{PAK})$

	Uncorrelated [$M]	Correlated [$M]	C_{BA}-Neutralized [$M]
C_A	45.6	98.8	45.6
$T_A(C_A)$	41.7	11.7	11.7
$TC_A(C_A)$	87.3	110.5	57.3
C_B	0	0	53.2
$T_B(C_A, C_B)$	399.2	321.8	321.8
$TC_B(C_A, C_B)$	399.2	321.8	375
TC_{AB}	486.5	432.3	432.3

Despite $\Gamma = 10 \times L(C_{PAK})$, there is no impact on State B's *correlated* strategic cost if it collaborates with Pakistan. However, as Γ increases, State B's potential threat cost, T_B, is increased (compared to T_B in Table 8.4). This can imply the fact that when only considering an *on-site* detonation in State A, State B should still invest in its interdiction capabilities for all threats beyond State A but will still experience a reduction in its threat cost when State A's security is enhanced through an increase in C_A: $C'_A \rightarrow C^*_A$.

Table 8.6. Variants of CS5

Figure	Table	Location	Γ [NW]	C'_A [$M]	C^*_A [$M]	C'_B [$M]	C^*_B [$M]	TC'_{AB} [$M]	TC^*_{AB} [$M]
8.1	8.2	A.T.	0.002	41.2	182.6	454.7	0	1430	516
8.2	8.3	A.T.	0.02	41.2	134.6	1363.1	1095.1	2338.4	2126.5
8.3	8.4	O.S.	0.002	45.6	98.8	0	0	223.9	169.7
8.4	8.5	O.S.	0.02	45.6	98.8	0	0	486.5	432.3

Table 8.6 recaps the four previous figures and tables with adjusted values for Γ and the two forms locations of detonations: *on-site* (O.S.) and *at-target* (A.T.). Herein, the uncorrelated, C'_i, and correlated costs, C_i^*, reflect how modifying these parameters affects the relationship between Pakistan and the U.S. For example, modifying Γ can have a substantial effect to both C_i^{UNC} or C_i^{COR}. For the regime, modifying Γ and estimating the success of the non-state adversary in stealing a nuclear asset both have substantial impacts on both the uncorrelated and correlated total costs. These effects are expected due to how the strategies are defined in Equations (3.2.3), (3.2.6), (3.2.7), and (3.2.9).

8.3. Bargaining Solution Concepts and Utility Transferability

The initial assumption for this discussion will consist of using parameters from Figure 8.2 and Table 8.3 where a non-state adversary successfully gained control of a nuclear asset from Pakistan but yet, does not pose the largest threat to the U.S. (i.e., $\Gamma = 10 \times L(C_{PAK})$). The data are shown in Figure 8.4. Here, the red point located at (82.9, 2255.5) represents the uncorrelated strategy point between Pakistan and the U.S. whereas the cyan point located at (139, 1987.5) represents the correlated strategy point. In order to achieve the benefit, Pakistan must invest an additional, compared to its optimal *uncorrelated* strategy annual cost, 56.1M USD per the results in Table 8.3, $139 - 82.9 = 56.1$M. With the savings surplus as 211.9M USD, if split evenly and provided to each state (each receives the benefit of 105.95M in savings), the result is the NBS strategy shown as the orange segment's intersection with the UT line at (-23, 2150.5). The black dotted line infers all the 211.9M USD surplus in savings is provided to Pakistan meaning only they receive that benefit and not the U.S. The solid red line signifies the U.S. paying for the entire amount of Pakistan's security measures (including over the *uncorrelated* strategic costs), which is shown as Q_{CBAN} in Figure 8.5. This bargaining strategy point, shown at (45.6, 2080.9), conveys how the U.S. can offer to pay for Pakistan's additional security measures and both states still gain more benefit over acting unilaterally without correlating their strategies.

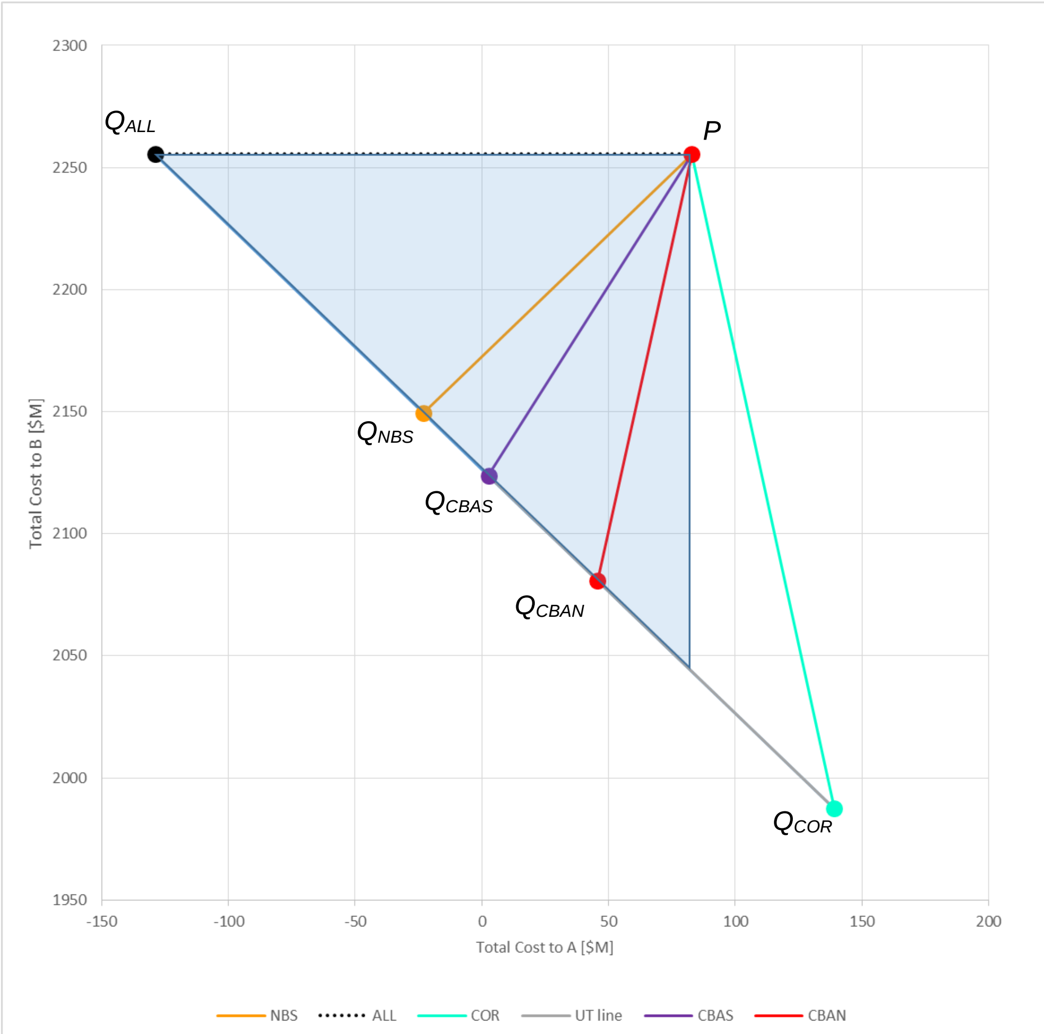

Figure 8.4. Imputations of modified CS5

Lastly, the purple line signifies the U.S. offering to pay Pakistan's entire strategic cost for securing their nuclear weapons with only Pakistan bearing the threat cost, T_A; also Pakistan bearing only that cost. This is the Q_{CBAS} (cost of A's security being completely subsidized by B) bargaining strategy point. Hence, that strategy is (C_A,C_B)= (2.7, 2123.8), in millions of USD annually. Interestingly, this point signifies an improvement over the

NBS strategy point for State B (but not State A). More notably, more than half the game's core is in the negative strategy space for State A, meaning State A can receive a negative total cost when it collaborates with State A. This is discussed more in Chapter 9.

8.4. Analysis and Discussion

Figure 8.4 graphically conveys various bargaining strategies available between Pakistan and the U.S. Detailed results and data are included in Table 8.7.

Table 8.7. Imputations of game for CS5

	Surplus Split for A [$M]	Surplus Split for B [$M]	$\frac{s}{r}$	TC_A [$M]	TC_B [$M]	TC_{AB} [$M]
P	--	--	--	82.9	2255.5	2338.4
Q_{COR}	-56.1	268	-4.78	139	1987.5	2126.5
Q_{NBS}	105.95	105.95	1.00	-23.05	2149.55	2126.5
Q_{CBAN}	37.3	174.6	4.68	45.6	2080.9	2126.5
Q_{CBAS}	80.2	131.7	1.64	2.7	2123.8	2126.5
Q_{ALL}	211.9	0	0.00	-129	2255.5	2126.5
Q_{ACT}	--	--	--	110.2	2702.5	2812.7

Table 8.7 shows the various imputations in table format for if Pakistan invests in its own nuclear security. The six strategy points exhibit a wide array of s/r ratios. Though Q_{NBS} is a 1:1 ratio, it can be construed as unfair by State B due to the mere fact that it leads to a negative total cost (i.e., a profit) for State A. Q_{ALL} and Q_{CBAN} also occur within the

game's core but at different ratios signifying which state receives the savings benefit (i.e., the reduction in total cost). Q_{ALL} represents an agreed cooperative solution where all 211.9M USD of the savings surplus goes to Pakistan: $s/r = 0$. Signifying the contribution of B to A, Q_{CBAN}, the ratio s/r is 4.68. Only Q_{COR} exhibits a negative s/r value of -4.78 signifying the points fall outside the game's core and hence are non-viable.

Table 8.8 displays the change in utility as compared to the *uncorrelated* strategy for each state and the regime as discussed in Subsection 3.1.2. All but one imputation (Q_{COR}) exhibit a positive utility. However, there is not sufficient proof of the benefit in savings to the U.S. (State B) with all being 12% or less. Lastly, the change in utility for the regime is only 9% over uncorrelated strategies.

Table 8.8. Tabulated benefits in utility results per imputation

	U_A	U_B	U_{AB}
Q_{COR}	-0.68	0.12	0.09
Q_{NBS}	1.28	0.05	0.09
Q_{CBAN}	0.45	0.08	0.09
Q_{CBAS}	0.97	0.06	0.09
Q_{ALL}	2.56	0.00	0.09

If more accurate data can be acquired (i.e., beyond what currently is publically available), Case Study 5 should be repeated. It is the author's intent to show how the methodology in Chapter 3 potentially provides insight in a stand-alone case study such as 2008 Pakistan-U.S. in addition to being useful for comparative analysis as exhibited in Case Studies 1-4 (Chapters 4-7).

9. DISCUSSION OF THE METHODOLOGY

The methodology described in Chapter 3 was used to estimate the strategic costs of nuclear security regimes resulting from two states either correlating their strategies or choosing to act independently. When choosing the former solution, the methodology approximates the result of negotiations leading to agreement on some of the details – but not enforcement – of a cooperative arrangement. The methodology was applied to five case studies in Chapters 4-8: the Soviet Union and the U.S. in 1985; Russia and the U.S. in 1995; Russia and the U.S. in 2008; Russia and the U.S. in 2015; and Pakistan and the U.S. in 2008. Plausible values of the model input parameters were assembled from various literature sources and used to construct the *uncorrelated* and *correlated* strategies as well as to construct the cooperative game cores – the set of possible plausible strategies corresponding to use of the correlated-strategy solution as an agreed starting point for negotiations on how to split the surplus deriving from correlating strategies. The objective of this was to develop and utilize a method in which using certain input parameters can help assess the utility of state-level strategies between two states in a bilateral nuclear security regime. Three game models were devised via non-cooperative (an *uncorrelated* strategy set and a *correlated* strategy set) and cooperative (a bargained solution strategy) approaches. Specific results are included in the fourth section of each case study's chapter but an overall discussion of the methodology and results are presented in Section 9.1 (except for Case Study 5: Pakistan-U.S. due to a lack of comparative data for drawing conclusions).

In each case study, two states were designated as players A and B: the *source* state was the former while the *target* state was the latter. The *target* state, State B, in all case studies was the United States. The *source* state, State A, varied between the Soviet Union and the Russian Federation during different time periods and Pakistan in the fifth case study (Chapter 8). That State B always represented the U.S. is not definitive – the intent of the methodology is to be applied to any two states where one would like to evaluate the potential for developing a bilateral nuclear security regime. Unique to each state was the determination (in sequence) of α and λ. An alternative method of determining α and λ is discussed in Section 9.2.

The methodology developed herein relies on the numerous input parameters to yield plausible results to aid in discussion. Section 9.3 addresses how results can be impacted by the changes in various parameters.

Echoing Subsection 3.1.2.2., Section 9.4. delves deeper into the distinction between different P_A parametric values – specifically between a non-state adversary gaining access and detonating a device on the facilities' site or gaining access and detonating a device at a pre-determined target (which includes transporting the asset, successful extraction from the facility with the full asset, and other considerations). The discussion is based heavily on Bunn's 2006 work on risk management for security.[89]

Section 9.5 identifies some characteristic indicators that can assist in analyzing results from the methodology. Computed using results from the case studies discussed in Chapters 4 through 7, the indicators are presented as an attempt to provide a qualitative validation of the model's computed results with observed characteristics from the bilateral regimes. Particular focus is given on the State A's actions and results due to the asymmetric nature of the regimes previously presented: State A faces the decision of whether to implement a nuclear security program prior to cooperation with State B or not. Also, an indicator is included to represent State B's implementation of an interdiction program at their border. Though present results are fragmentary, future work can continue in this area to solidify and more accurately draw conclusions.

Lastly, section 9.6 presents a concept that was exhibited in case studies 2 and 3 (Chapters 5 and 6, respectively): the potential for profit from correlating strategies. Discussed by Corr, one state faces the possibility of "exporting" its nuclear insecurity by using it as a negotiating tactic for receiving a more favorable utility.[33] By the placement of the correlation game plot along the total cost axes (as exhibited in Figures 4.2 and 6.2), part of the core (and therefore some imputations of the bargaining strategies between the states) occurs as a negative total cost for State A which infers how a lack of security of nuclear material and/or weapons could potentially be 'sold' as a commodity. As discussed therein, Corr's point will be illustrated.

9.1. Comparison of Model Results with Observed Actions

The case studies presented in Chapters 4-7 provide an opportunity to evaluate the methodology as it was applied to specific cases where historical strategies and actions can be inferred. Though we do not know actual state expenditures on discrete objectives such as nuclear security or interdiction with utmost certainty, much can still be deduced from observing actions during each snapshot in time which the case studies represent. As a reference for the following discussion, results from case studies 1-4 are rounded to the nearest $1M and displayed in Table 9.1. Results are categorized by year and separated into three separate solution concepts: the *uncorrelated* strategies, the *correlated* strategies, and the C_{BA}-neutralized (or *bargained*) strategies (where State B contributes the additional strategic cost for State A's security so that State A does not have to pay over its *uncorrelated* strategic cost). As shown in each case study, to achieve correlation of strategies, the total cost of at least one state must be increased from the uncorrelated solution concept.

Table 9.1. Costs of state strategies [$M]

		C_A	$T_A(C_A)$	$TC_A(C_A)$	C_B	$T_B(C_A, C_B)$	$TC_B(C_A, C_B)$	TC_{AB}
1985	UNC	0	2	2	0	397	397	399
	COR	35	0	35	0	9	9	45
	C_{BA}-N	0	0	0	35	9	45	45
1995	UNC	304	71	375	163	212	375	750
	COR	447	10	457	0	61	61	518
	C_{BA}-N	304	10	314	143	61	204	518
2008	UNC	284	71	355	684	892	1577	1931
	COR	477	5	482	45	892	937	1419
	C_{BA}-N	284	5	289	238	892	1130	1419
2015	UNC	253	78	330	1714	546	2260	2590
	COR	309	37	346	1672	546	2218	2564
	C_{BA}-N	253	37	290	1729	546	2275	2564

In Case Study 1 (Soviet Union-United States), both states' *uncorrelated* strategies were to not invest in their respective strategic costs. The resulting disparity of each state's threat cost is substantial which implies the benefit to State B if State A were to secure its own nuclear materials. As shown in the *correlated* costs, State A could have increased its strategic cost to 35M USD so as to decrease the threat costs to both states. Under a bargaining solution concept, State B could have hypothetically covered the increased strategic cost of 35M USD for State A and both would have been better off in terms of total costs to each (0,45) but this can set up a moral conundrum and opened the game up to substantial abuse by one state. In 1995, the situation had been changed with the fall of the Soviet Union. As conveyed in Table 9.1, strategic costs for State A securing its nuclear assets were more than those in the previous case study (1985) due to the deterioration of the Soviet police state. State B's increased *uncorrelated* strategic and threat costs also reflect this. As the primary source of a nuclear threat, the security of State A's assets has the most impact in reducing the total regime cost. Hence, additional investment in its own nuclear security (signified by the *correlated* strategic cost of 447M USD) is paramount to reducing the regime's (primarily from State B's perspective) total cost: 518M USD. This connection leads to bargained solution concept where State B bears the additional strategic cost of State A's nuclear security (the difference between the *correlated* and *uncorrelated* strategic costs), both states would reduce in total costs. For Case Study 3 (2008) between Russia and the U.S., the methodology produced results that reflect 1) the improved Russian economy and 2) the heightened nuclear threat from beyond Russia ($\Gamma > 0$). The latter leads to State B's increased strategic cost for interdicting a nuclear asset from any source state as well as the resultant threat cost that is unchanged between the *uncorrelated*, *correlated*, and bargained solution concepts due to threats not being realized from just Russia. Furthermore, the drop between the *uncorrelated* and *correlated* total regime costs (TC_{AB}) convey that there existed some benefit for both states if they cooperated: $1931M - 1419M = \underline{512M}$. That difference between the *uncorrelated* and *correlated* total costs for the regime is greatly reduced in the 2015 case study: $2590M - 2564M = \underline{26M}$. This suggests there was not much incentive for the states to correlate their strategies in the hopes of receiving any substantial benefit in reduced total costs. Furthermore, as is shown in Table 9.1, State B's strategic costs for 2015 do not convey a favorable bargaining strategy: the *uncorrelated* total cost (2260M) is less than the total cost it receives with a *correlated* or bargained strategy (2275M). Hence, more so than in 2008, there is little to no incentive for State B to enter into the bilateral regime with State A.

Table 9.2. Comparison of observed and indicated[s] results

	State A – Security		State B – Interdiction		Regime – Cooperation		
	Indicated? [Y/N]	Observed? [Y/N]	Indicated? [Y/N]	Observed? [Y/N]	Indicated? [Y/N]	Observed? [Y/N]	Surplus? $(TC'_{AB} - TC^{*}_{AB})$
1985	N	N	N	N	Y	N	356
1995	Y	Y	Y	Y	Y	Y	232
2008	Y	Y	Y	Y	Y	Y	512
2015	Y	Y	Y	Y	Y	N	26

[s] Though used throughout this section, "indicated" behavior implies "with bargaining."

Table 9.2 presents a summary of observations from the numerical data in Table 9.1. Using values from the latter, various behaviors were indicated to have been possible to occur by the methodology. For example, C_A' and C_B' provide insight on whether nuclear security and interdiction were indicated to have been implemented independently by States A and B, respectively. The observed columns for each are deduced from historical observations between the partners (discussed previously in each case study's chapter). The three columns under the Regime – Cooperation describe whether cooperation was indicated from the existence of any surplus present, whether cooperation was observed, and the quantity of said surplus ($TC_{AB}' - TC_{AB}^*$), if it existed. For both States A and B, the indicated and the observed results are consistent for each. The Regime – Cooperation indicated and observed results differ. Whereas the methodology's results indicate cooperation would have yielded a positive surplus in 1985 and 2015, cooperation did not occur due to extenuating circumstances landing beyond the scope of this study. For instance, as discussed in Chapter 4, in 1985, there existed many impediments to nuclear security collaboration between the U.S. and the Soviet Union stemming from being on opposite sides of the long-standing Cold War. Regardless of the perceived benefit for both sides in collaborating in the particular area of nuclear security, domestic and international political pressures inhibited such discussions and engagement. As discussed in Chapter 7, 2015 signified the beginning of a new era between the U.S. and the Russian Federation: all technical collaboration had since ceased despite perceived mutual benefits due to political and economic pressures. For these reasons in both case studies, though the methodology indicates a benefit from cooperation, cooperation did not occur. In contrast, data for 1995 and 2008 convey that there was a surplus to be had ($232M and $512M, respectively, to be divided between the states) and there was cooperation between States A and B also.

Comparing results with historically observed actions exhibited by the states in each case study discussed above, we can illustrate how the methodology can assist in analyzing bilateral nuclear security regimes. For example in 1985, the Soviet Union and United States had very limited engagement within the nuclear industry save for disarmament activities through test bans and arsenal drawdowns. Securing nuclear assets did not occur as a collaborative effort at any level between governments. Though results in Tables 9.1 and 9.2 show the regime would have benefited from any collaboration (by virtue of TC_{AB}^{COR}), none occurred that year. In 1995, the Russian Federation was struggling financially and therefore, did welcome engagement with the U.S. through government engagements at the ministerial–departmental levels on upgrading security measures and accounting methods at various nuclear facilities, training and educational methods of nuclear asset security, and an overall enhancement of the nuclear security culture throughout the Russian military and civilian nuclear complexes. In 2008, nuclear security collaborations continued between both states but with the strengthening Russian economy and the growing threat of nuclear terrorism from other corners of the world (e.g., Pakistan, Iran, North Korea), the recognized need for the U.S. to assist Russia secure its nuclear assets waned in the form of deteriorating congressional support for American funds to Russia (as introduced in Section 6.1). This diminished support continued until 2013 when the Russian government ceased engagements with the U.S. in securing its nuclear assets.

Particularly in 2015, there was no collaboration between the U.S. and Russia in securing the latter's nuclear assets. Conversely to 1985 when no cooperation existed despite an advantage for the regime to do so, cooperation did not exist in 2015 as there was no substantial benefit for the regime. This can be attributed to the expanded threat and the waning interest on both sides to continue. Respectively represented within the game models as $\Gamma \neq 0$ and the decrease in α, P_A, and P_B for the states (shown in Tables 9.2 and 9.3).

Table 9.3. Estimated parametric values for case studies 1-4

	α [SQ]	λ [$M^{-1}]	P_A [ ]	K_A [$M]	μ [$M^{-1}]	P_B [ ]	K_B [$M]	Γ [SQ]
1985	0.011	0.108	0.03	5,307	0.00164	0.99	36,105	0
1995	0.249	0.0141	0.6	34,483	0.00471	0.75	133,314	0
2008	0.2	0.014	0.3	63,736	0.00112	0.05	282,875	0.00307
2015	0.15	0.0129	0.2	67,963	0.00183	0.05	312,235	0.0347

Reflecting on the latter observed actions between the U.S. and the Soviet Union/Russia provides an opportunity to evaluate the validity of the results from the methodology in Table 9.1. For example, the methodology indicated that despite a reduction in total correlated costs between the U.S. and Russia indubitably due to the CTR and MPC&A, cooperation did not exist in 1985.[37,110] Reasons for this can be explained by four of the estimated parameters shown to be substantially different in 1985 than other years in Table 9.3: α, λ, P_A, and K_A. Compared to the parameters from 1985, the 1995 estimated parametric values of α, P_A, and K_A in Table 9.3 provide insight about the methodology's results that is supported by historical observations. With the porous Russian borders in 1995, the leakage rate of nuclear assets was high – providing to a relatively higher value of α (refer to Section 5.1). Furthermore, the central government's strangle on the population as a police state during the tumultuous post-Cold War years had allowed crime to rise as well as a growing anti-Russian sentiment.[200] Therefore, the probability of a violent terrorist attack on a major Russian metropolitan area was considered – P_A was derived from the number of years with a fatal attack in Moscow (6) in the 10 years surrounding the year in question: 0.6. Lastly, the value for K_A was an increase from the previous case study due primarily to the increase in the median value of statistical life in 1995 Russia from 1985 Soviet Union (refer to Section 5.1).

From 1995 to 2008, K_A increased (as value of life increased and the economy grew), α decreased (estimated for an increased awareness of nuclear security after 16 years of collaboration with the U.S. to secure nuclear assets), and P_A decreased (due to the enhanced security measures implemented in nuclear facilities). By 2008, several substantial shifts had emerged between Russia and the U.S. For example, the domestic political regime within the U.S. was beginning to raise questions such as if the Russian government was using American funds to secure its nuclear assets or was it assisting other, more nefarious actions like supporting militaristic activities. Furthermore, the September 11, 2001 terrorist attacks altered the terrorism landscape in that the largest terrorist act the world had ever seen had originated from somewhere beyond Russia. Between 1995 and 2008, the number of nuclear-armed states had increased to include two states with precarious security situations: Pakistan and North Korea. For this reason, the terrorist threat definition had changed substantially: it was not limited to nuclear terrorists acquiring assets from Russia only. For this reason, Γ (the source of nuclear threat to State B that is beyond State A) was non-zero – opposed to $\Gamma = 0$ in 1985 and 1995. With $\Gamma > 0$, the results in Table 9.1 for 2008 represent a regime where State B's threat cost would not be impacted by increasing State A's strategic cost but the regime's total cost would still decrease through collaboration. This echoes the continued relationship between the U.S. and Russia in 2008 during which both the CTR and the MPC&A programs were being implemented.

Lastly, by 2015, collaborative security efforts between Russia and the U.S. had ended. The nuclear security threat for the U.S. had expanded well beyond the threat posed by terrorists acquiring a weapon from Russia – states like Pakistan and North Korea had grown their nuclear arsenals since 2008. Overall, the estimated parameters in Table 9.3 for Case Study 4 do not show a substantial change from the previous case study (2008) other than Γ for the U.S. With an increase of ten times greater, Γ shows how the nuclear threat shifted greatly away from Russia and thus, limited the amount of cost savings the U.S. was able to achieve through collaborating with Russia that year. From Table 9.1, the *uncorrelated* strategic cost (1714M USD) to the *correlated* strategic cost (1672M USD), the only amount of reduced strategic cost for the U.S. amounted to 42M USD – not a substantial amount of received savings in cost to warrant a new nuclear security collaboration. Furthermore, if the U.S. would have agreed to an arrangement where they would have contributed to Russia's strategic costs for securing nuclear assets, the maximum strategic cost under a bargaining solution concept could have potentially amounted to 1729M USD – more than the *uncorrelated* amount of 1714M USD. Hence, there was no incentive to collaborate from the U.S.'s perspective. This reflects what actually did transpire in 2015 between the U.S. and Russia: no collaboration.

Returning to the value of μ in each case study, one can see the trajectory of the value across the four case studies. In 1985, the value represented a lack of investment in the interdiction capabilities of the U.S. (as per Equation (3.1.24) in Subsection 3.1.2). The nondetection probability of an asset crossing a border was high and there was not much spent on the control system. In 1995, the need was recognized to invest greater resources into a national interdiction system (suggested by α and μ from Table 9.3) and, hence, both are increased from 1985. However, the reduction in T_B for the U.S. was actually achieved through investing in securing Russian nuclear assets. In 2008, μ is the lowest value compared to all case studies considering that substantial investment was being made into American interdiction – which could have stemmed from a growing concern of continuing any investment in nuclear security in Russia. The μ value increased in 2015 reflecting a decreased investment in interdiction but still achieving a similar non-detection probability as in 2008 (5%). This could potentially be due to any number of global and/or American endeavors of the time: the GICNT, nuclear security culture awareness efforts by the U.S. and the IAEA, Nuclear Security Summits, and others.[2,42]

With the estimated parameters present in Table 9.3, real-world, historical observations could be replicated (as shown in Table 9.1). Despite limitations of the results and lacking consideration of some externalities (such as domestic opinions, budgetary restrictions, shifting state objectives, etc.), the results can roughly illustrate estimations of state actions within a bilateral nuclear security regime.

9.2. Re-estimating Parametric Values

Subsection 3.2.1 included a description of estimating the parametric values α and λ for each case study. Specifically, α was estimated for 1985 using Quigley and for 1995 with data from Zaitseva.[114,143] And λ was estimated using retrieved data with admittedly questionable assumptions (especially regarding assumed data for the 1985 USSR-U.S. case study where the value of $L(C_A)$ was estimated from old data).[119] This section focuses on an alternate approach to estimating these parameters to re-compute results shown in Table 9.1 for Case Studies 2-4 from Chapters 5-7 (Case Study 1 was omitted because of a lack of confidence in the parameters from 1985). Determination of a non-linear regression line by the least-squares method was used to estimate constant values for α and λ using the compiled values of $L(C_A)$ and C_A^{ACT} from Sections 5.1, 6.1, and 7.1. (shown below in Table 9.4).

Table 9.4. $L(C_A)$ and C_A^{ACT} from case studies 2 (1995), 3 (2008), and 4 (2015)

	C_A^{ACT} [$M]	$L(C_A)\left[\frac{SQ}{yr}\right]$
1995	200	0.0149
2008	297	0.00307
2015	291	0.00347

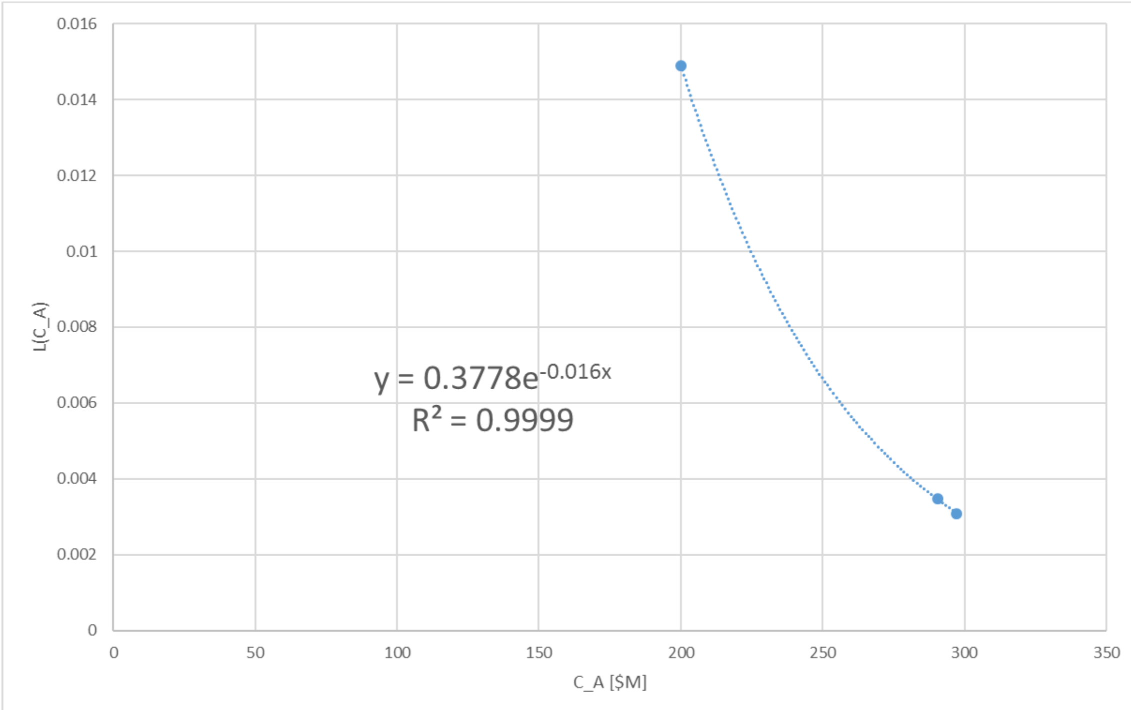

Figure 9.1. Non-linear regression line with least-squares fit

Plotted in Figure 9.1, we derive an exponential regression line of $y = 0.3778e^{-0.016x}$ where $\alpha = 0.3778\left[\frac{SQ}{yr}\right]$ and $\lambda = 0.016[\$M^{-1}]$. These values of α and λ are then used to recreate results from Table 9.1 which are incorporated as *italicized* results in Table 9.5 (Table 9.1 results are not italicized). As can be seen, results do not change much but with more data points for $L(C_A)$ and C_A^{ACT} from more years, a more accurate analysis could be made. Moreover, trends continue to reflect historically observed activities. An example of this is that despite there existing a slight reduction in total costs for the regime in 2015 by correlating strategies, such a reduction was not sufficient to convince the U.S. to increase its strategic costs as shown as C_{BA}-N (in the last row). For future applications of the methodology, it would serve the analyst better to determine the α and λ parameters, from more data, prior to determining the strategies resulting from the game-theoretic model presented here.

Table 9.5. Costs of state strategies [$M]

		C_A	$T_A(C_A)$	$TC_A(C_A)$	C_B	$T_B(C_A,C_B)$	$TC_B(C_A,C_B)$	TC_{AB}
1995	UNC	304	71	375	163	212	375	750
		247	*63*	*310*	*320*	*212*	*532*	*842*
	COR	447	10	457	0	61	61	518
		422	*4*	*426*	*0*	*59*	*59*	*485*
	C_{BA}-N	304	10	314	143	61	204	518
		247	*4*	*251*	*175*	*59*	*234*	*485*
2008	UNC	284	71	355	684	892	1577	1931
		297	*63*	*360*	*623*	*892*	*1515*	*1875*
	COR	477	5	482	45	892	937	1419
		467	*4*	*471*	*36*	*892*	*928*	*1399*
	C_{BA}-N	284	5	289	238	892	1130	1419
		297	*4*	*301*	*206*	*892*	*1098*	*1399*
2015	UNC	253	78	330	1714	546	2260	2590
		275	*63*	*338*	*1699*	*546*	*2245*	*2583*
	COR	309	37	346	1672	546	2218	2564
		321	*30*	*351*	*1665*	*546*	*2211*	*2562*
	C_{BA}-N	253	37	290	1729	546	2275	2564
		275	*30*	*305*	*1711*	*546*	*2257*	*2562*

Furthermore, as introduced in Subsection 3.2.4, the value for μ would also benefit from being revisited. The parameters C_B^{ACT} and $P_B(C_B^{ACT})$ for Case Studies 1-4 are shown in Table 9.6.

Table 9.6. C_B^{ACT} and $P_B(C_B^{ACT})$ for case studies 1-4

	C_B^{ACT} [$M]	$P_B(C_B^{ACT})$ [ ]
1985	6.1	0.99
1995	61.1	0.75
2008	2673	0.05
2015	1635	0.05

This allows us to formulate a non-linear regression line to determine values for μ and an additional multiplicative scalar, η, in Figure 9.2.

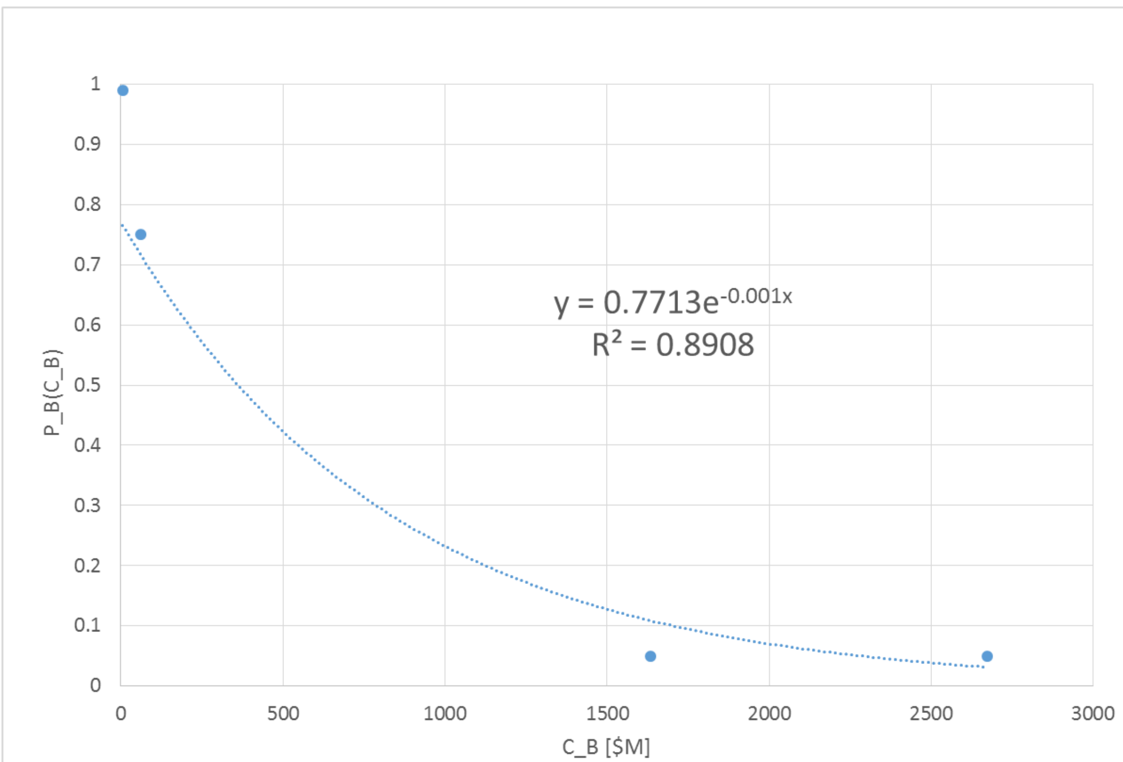

Figure 9.2. Non-linear regression line for probability of non-detection for state B

The resulting equation – based on a modified exponential model of Equation (3.1.24) – $P_B(C_B) = \eta \cdot e^{-\mu C_B}$ – is

$$P_B(C_B) = 0.7713e^{-0.001 \cdot C_B} \tag{9.2.1}$$

Here, the values μ = 0.001 [$M^{-1}] and η = 0.7713 were both obtained by applying least-squares regression to the plot of the data points (η can be interpreted as the non-interdiction probability when there is no further effort put into interdiction; i.e., $C_B = 0$).

Replacing Equation (3.1.24) with Equation (9.2.1) yields a modified Equation (3.2.4) for recalculating $TC_B(C_A, C_B)$:

$$TC_B(C_A, C_B) = C_B + \eta \cdot e^{-\mu \cdot C_B} \cdot K_B \cdot \left(\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right) \tag{9.2.2}$$

Equation (9.2.2) yields new equations for $TC_{AB}(C_A, C_B)$, $T'_B(C_A, C_B)$, and $T_B^*(C_A, C_B)$ that are used to reproduce Table 9.5 in Table 9.7 with cost strategies for Case Studies 1-4 using new values for α, λ, μ, and η – new results are *italicized*.

Table 9.7. Costs of state strategies [$M]

		C_A	$T_A(C_A)$	$TC_A(C_A)$	C_B	$T_B(C_A, C_B)$	$TC_B(C_A, C_B)$	TC_{AB}
1995	UNC	304	71	375	163	212	375	750
		247	*63*	*310*	*0*	*739*	*739*	*1049*
	COR	447	10	457	0	61	61	518
		406	*5*	*411*	*0*	*58*	*58*	*469*
	C_{BA}-N	304	10	314	143	61	204	518
		247	*5*	*252*	*159*	*58*	*217*	*469*
2008	UNC	284	71	355	684	892	1577	1931
		297	*63*	*360*	*324*	*1000*	*1324*	*1684*
	COR	477	5	482	45	892	937	1419
		454	*5*	*459*	*0*	*727*	*727*	*1186*
	C_{BA}-N	284	5	289	238	892	1130	1419
		297	*5*	*302*	*157*	*727*	*884*	*1186*
2015	UNC	253	78	330	1714	546	2260	2590
		275	*63*	*338*	*2247*	*1000*	*3247*	*3584*
	COR	309	37	346	1672	546	2218	2564
		345	*20*	*365*	*2166*	*1000*	*3166*	*3531*
	C_{BA}-N	253	37	290	1729	546	2275	2564
		275	*20*	*296*	*2235*	*1000*	*3235*	*3531*

As can be gleaned from Table 9.7, modifying all four parameters does impact the specific values but not the trends: a *correlated* strategy for State B will always yield less cost than an *uncorrelated* strategy and a bargained strategy can provide a manner by which one state can contribute all additional security to another state.

These results convey the potential for re-estimating parameters that describe the "effectiveness" of applied security and interdiction. Initially in Chapters 4-7, α, λ, and μ (η was not used previous to this section) were estimated and derived from published works. This section describes how, by using a least squares fit, parameters can be estimated for various snapshots in time but, more importantly, with other ways to estimate parameters even if individual results change, the trends may not. In the following section, we explore how sensitive are the results to the estimated parametric values.

9.3. Sensitivities of Model Results to Uncertainties in Parameters

The game-theoretic methodology underling this dissertation relies on real-world estimates of seven input parameters taken from publically-available literature and based on plausible assumptions. They are exhibited in Table 9.3 in Section 9.1. Precise values were not achievable due to the limitations of the available data. Therefore, this section outlines the process and conclusions on how sensitive the model's results were relative to the estimated input parameters. Each parameter was decreased and increased by an order of magnitude and the impact on the *uncorrelated* and *correlated* total costs for each state and the regime was recorded.

The results presented in Table 9.8 are based on Case Study 3 (Russia-U.S.) from Chapter 6. Each of the seven parameters was multiplied by 10% and then 1000%. Then the fractional changes in the results compared to the original values are displayed in terms of orders of magnitude under each total cost for State A, State B, and the combination of State A and State B. For example, if α is decreased by an order of magnitude, the maximum impact to the total *uncorrelated* cost to State A is 20 percent. Another example is if K_B is reduced by a factor of 10, the total uncorrelated strategy for State B will exhibit a 90% change.

Table 9.8. Sensitivity attributed to parameters in CS3

Parameters		Fractional Change of TC_A		Fractional Change of TC_B		Fractional Change of TC_{AB}	
		UNC	COR	UNC	COR	UNC	COR
α	X0.1	0.2	0.2	0.3	0.1	0.2	0.1
	X10	0.0	0.1	0.1	0.0	0.1	0.0
λ	X0.1	4.4	4.4	1.0	1.0	1.6	2.1
	X10	0.9	0.9	0.4	0.1	0.5	0.3
P_A	X0.1	0.5	0.0	1.0	0.0	0.7	0.0
	X10	0.2	0.0	0.3	0.0	0.2	0.0
K_A	X0.1	0.5	0.0	1.0	0.0	0.7	0.0
	X10	0.5	0.1	0.4	0.0	0.2	0.0
μ	X0.1	0.0	0.0	0.2	0.0	0.2	0.0
	X10	0.0	0.2	0.8	0.6	0.6	0.5
K_B	X0.1	0.0	0.2	0.9	0.9	0.7	0.6
	X10	0.0	0.0	1.3	2.2	1.1	1.4
Γ	X0.1	0.0	0.0	0.3	0.8	0.2	0.6
	X10	0.0	0.2	0.9	2.2	0.8	1.4

The objective of Table 9.8 is to convey which parameter has the most impact on results. Of note, λ, K_B, and Γ have the largest impact on the total calculated costs for the states and the regime. It can therefore be reasoned that the use of this methodology would require some confidence in these three values for computing, with any kind of certainty, TC_A, TC_B, and TC_{AB}. As is shown above, the methodology discussed within this document allows for uncertainties in <u>some</u> estimated parametric values without impacting results. We suspect that for more sensitive parameters, the use of data beyond what is publically available would allow the methodology to yield more accurate results.

9.4. On-Site versus At-Target Events

As was discussed within Subsection 3.1.2.2 while defining P_A and K_A, this study included the consideration of two types of nuclear terrorist events (also discussed in the case studies). This section includes a discussion on both P_A and K_A and how, if evaluated differently, they can impact the overall results of the methodology. Assuming a completely successful attack ($P_A \cong 1$) would include the non-state actor succeeding in almost every action they attempts – that is most likely not the case in the real world. As was introduced in Subsection 3.1.2.2, P_A can be deconstructed into numerous other probabilities of events regarding the non-state adversary. This line of thought is expanded from Bunn et. al.'s treatise on "blocking the terrorist pathway to the bomb."[87] Table 9.8 presents a compilation of Bunn's nine steps (and another six steps) to block a terrorist attempting a successful nuclear attack.

Table 9.9. Constituent probabilities for P_A

#	Elements	Steps for a successful nuclear attack
1	intent	The adversary's intent in either making a statement or causing death and destruction
2	non-detection	The site's security system not detecting the adversary's incursion
3	non-assessment	The site's security system correctly assessing the adversary's incursion
4	insider threat	The adversary's level of collusion with a potential insider
5	acquisition	The adversary gaining control of the asset
6	non-response	The site's security system responding to the adversary's incursion in time
7	non-neutralization	The site's security system neutralizing the adversary
8	detonation (on-site)	The adversary's ability to detonate the asset (if a weapon) on site
9	removal	The adversary's ability to remove the asset from the site
10	successful transport to base	The adversary's ability to transport the asset away from the site
11	non-recovery	The site's and state's abilities to recover the asset
12	weaponization	The adversary's ability to weaponize the asset (if bulk fissile material)
13	use at first	The adversary's willingness in using its first attained asset
14	successful transport to target	The adversary's ability to transport the asset to the target point
15	detonation (at-target)	The adversary's ability to detonate the asset at the target point

Assuming all fifteen steps are required to successfully detonate one weapon at a pre-determined target, one can change the probability of an element occurring to decrease the overall risk of a nuclear terrorist event. For example, in an attack on a facility to steal an asset, an intruding adversary enters and reaches an asset undetected. Once the asset is in their control, an alarm is triggered, assessed and a response force is dispatched to neutralize the intruding threat. Based on the facility's physical security system capability, the intruder may have time to alter their objective and choose to detonate the asset while still in the facility. If this is the case, only steps 1-8 from Table 9.9 are in play. If the adversary is able to egress the facility successfully with the asset (whether detected or undetected), other steps are needed for the non-state adversary to achieve the goal of detonating the asset at a pre-selected target. This would mean steps 9-15 are also needed for the adversary to achieve success. For this reason, P_A varies when an attempt is made either on-site or at-target and, in echoing Bunn's treatise on mathematically modelling nuclear terrorism risk, Equation (9.4.1) shows the multiplicative nature of calculating the annual P_A value:[195]

$$P_A = P_a \times P_s \times P_r \times P_w \times P_d \tag{9.4.1}$$

Recalling Subsection 3.1.2.2, P_A is categorized into five different event probabilities:

- P_a is the probability of a facility with an asset being attacked (this depends on what intent the adversary may have – to attain a weapon or merely the fissile material needed for a weapon);
- P_s is the probability of successfully gaining control of a nuclear asset via the incursion attack, given that an attack occurs;
- P_r is the probability of the non-state adversary removing the nuclear asset, given that an attack occurs and the attacker gains control of a nuclear asset (for example

the attacker might fail to remove the asset because of action taken by the site's response force);

- P_w is the probability of weaponizing or converting that asset into a nuclear capability, given that an attack occurs, the attacker gains control of a nuclear asset and successfully removes the nuclear asset ; and
- P_d is the probability of successfully delivering and detonating the asset at the target, given that an attack occurs, the attacker gains control of a nuclear asset, successfully removes the nuclear asset and weaponizes it or otherwise converts it into a nuclear asset.

Deductively, we see which event probabilities are vital for an on-site (P_a and P_s) and an at-target (all five matter) type of eventual nuclear attack. For an on-site attack, one could assume the initial theft plan was inhibited by the facility's physical security system or the adversary meant to merely sabotage the facility. Either way, the assumed events associated with the probabilities P_r, P_w and P_d are not vital to occurrence of a successful on-site attack, so can each be discounted by equating those respective probabilities to unity. This leaves the distinction between *on-site* P_A ($P_A^{o.s.}$) and *at-target* P_A ($P_A^{a.t.}$) as:

$$P_A^{o.s.} = P_a \times P_s \qquad (9.4.2)$$

$$P_A^{a.t.} = P_a \times P_s \times P_r \times P_w \times P_d \qquad (9.4.3)$$

The distinctions between $P_A^{o.s.}$ and $P_A^{a.t.}$ per Equations (9.4.2) and (9.4.3) implies there are different expected consequences, K_i, depending on the type of attack. The detonation location ranges between a storage location typically staffed by military personnel and military-grade equipment to a highly-populated civilian location like a large metropolis with associated infrastructure and potential for economic loss. Similarly to how K_i is computed in Subsection 3.1.2.3, $K_A^{o.s.}$ and $K_A^{a.t.}$ signify the expected consequence of a detonation for an on-site and an at-target nuclear attack on State A which correspond to $P_A^{o.s.}$ and $P_A^{a.t.}$, respectively. Therefore, there is an inverse relationship between P_i and K_i depending on the detonation location.

The last parameter modification for determining an on-site detonation is λ. Per Equation (3.1.18) in Subsection 3.2.1.2 and particularly applied to the 2008 Russia-U.S. case study in Section 6.1.1, λ is recalculated with the adjusted value of $C_A^{ACT} = 146.6$M USD solely based on the expenditures for nuclear weapon security (relevant here because an *on-site* detonation would require incursion into a site with nuclear weapons). Reflecting this change also implies a modification for $L(C_A^{ACT})$ as applied to a nuclear weapon site which arguably, can be assumed as very low in 2008. This would cause the value in the parenthetical argument for the natural logarithm of Equation (3.1.18) to decrease asymptotically towards negative infinity. Specifically for this discussion (maintaining α at 0.2 $^{SQ}/_{yr}$), $C_A^{ACT} = 146.4$M USD in Equation (3.1.18), therefore, $\lambda_{08} = 0.0285$M USD^{-1}.

This distinction between the detonation locations leads the discussion to consider the potential effects the adversary's success has on the results for each case study. Each case study can begin with the consideration of an outside adversary gaining access to a military site in State A with intent to steal a nuclear weapon for eventual use (against either State A or B). Even with the intruding adversary detected and interdicted by the site's response force, there is the possibility that the adversary is not able to escape the site with the asset yet nor is the adversary neutralized. In this situation, the probability of the adversary attempting to detonate the device (either in the absence of successfully removing the device or if this was the original intent) increases. Table 9.9 takes the results of case study 3 ($P_A = 0.3$, $K_A = 63{,}756$M USD, and $\lambda_{08} = 0.0285$M USD^{-1}) and recalculates the states' strategies with an *on-site* probability and detonation (keeping all other parameters equal to those in the original case study in Chapter 6). For illustrative purposes, assuming the worst case scenario as $P_A = 1.0$, $K_A = 6{,}373$M USD as a rough estimate for a detonation successfully committed at a site where nuclear weapons are held is shown in Table 9.10 as on-site detonation 1 (limited civilian casualties and infrastructure destruction that would otherwise impact State A on a large scale).

Table 9.10. Reformulated CS3 strategies using on-site detonation 1 and 2

	At-target Detonation (original results)		On-site Detonation 1		On-site Detonation 2	
	UNC	COR	UNC	COR	UNC	COR
C_A	283.6	476.7	126	259.3	0	258.5
$T_A(C_A)$	71.2	4.7	35.1	0.8	35	0
$TC_A(C_A)$	354.8	481.4	161.1	260.1	35	258.5
C_B	684.3	44.8	891.8	10.7	3716.4	11.5
$T_B(C_A, C_B)$	892.3	892.3	892.3	892.3	892.3	892.3
$TC_B(C_A, C_B)$	1576.6	937.1	1784.1	903	4608.7	903.8
TC_{AB}	1931.4	1418.5	1945.2	1163.1	4643.7	1162.3

The modified values for P_A and K_A lower State A's uncorrelated and correlated strategic costs in on-site detonation 1 and 2. If State A's threat is only an on-site detonation, then the decision for when State A will invest in its own security is shown as UNC C_A under Detonation 1. Continuing with this logic, K_A can effectively be lowered to when State A does not perceive enough of a threat from an on-site detonation (i.e., when $C_A' = 0$) to expend any of its own resources to provide adequate security to its own nuclear assets. This case, as presented as on-site detonation 2, occurs when $K_A = 175$M USD (as opposed to the previously-defined value of 63,756M USD). Hence, if the consequence of a detonation from an adversary decreases by virtue of the location of the attack (say at a site with nuclear assets) to a low enough value, there may not be enough incentive for State A to invest enough to secure its own assets against an attack upon itself.[t] This will present a chain reaction imposed upon State B's uncorrelated strategies as can be seen in Table 9.7 for the *uncorrelated* strategies for State B: C_B' increases as $T_B(C_A, C_B)$ does not change values. This exercise can be applied to all case studies and respective values of P_A and K_A.

9.5. Characteristic Indicators

The work described herein presents a challenge in verifying the model (i.e. demonstrating that the methodology solves the problem correctly) or validating the results (demonstrating the problem solved in some way faithfully replicates the real-world

[t] As a reminder, $T_A(C_A)$ is the perceived consequential threat cost of an *at-target* nuclear detonation on State A. Hence, T_A does not change regardless whether a device is detonated *on-site* or *at-target.* State A's preparation is the same.

situation). Some degree of confidence in the analysis can be provided by the results in each case study (Chapters 4-8); while it would not be accurate to claim the model solutions reproduce real-world results to a high degree of fidelity, the latter tend to have a degree of qualitative similarity to the former, especially if account is taken of known externalities in the model. The sensitivity results in Section 9.3 suggest solutions of the various game-theoretic models can be sensitive to the undoubtedly large uncertainties in the values of the various input parameters. The intent of this section is to develop and illustrate an assessment (otherwise, a qualitative validation) of the methodology's results. Qualitative validation is based on determining boundaries of the input parameters where the nature of the results abruptly change; solutions of game-theoretic models commonly display such abrupt changes in their qualitative nature. The upshot of the explanation offered here is that a rather low degree of accuracy in values of input parameters suffices, if one is willing not to ask too much of the model results; i.e., suffice for qualitative reproduction of real-world results.

State A's characteristic indicator is the marginal reduction of its threat cost (Equation (9.5.1)): the magnitude of reduction in threat cost should be greater than the magnitude of increasing the strategic cost (as shown in Equation (9.5.2)). For example, from State A's *uncorrelated* solution strategy, C_A^{UNC} (as discussed in Subsection 3.2.1), any unit increase of C_A beyond C_A^{UNC} should exhibit at least a unit decrease in T_A. When this occurs, it signifies the existence of a perceived benefit in more investment into State A's nuclear security. As investments continue, the subsequent reductions in threat cost can decrease per unit of strategic cost so long as there remains a reduction in threat costs (as noted in Equation (9.5.3)).

$$T_A(C_A) = \alpha \cdot e^{-\lambda \cdot C_A} \cdot P_A \cdot K_A \tag{9.5.1}$$

$$-\left.\frac{\partial T_A(C_A)}{\partial C_A}\right|_{C_A=0} > 1 \tag{9.5.2}$$

$$\frac{\partial^2 T_A(C_A)}{\partial C_A^2} > 0 \tag{9.5.3}$$

Solving for Equation (9.5.2) yields the *alpaca* indicator (based on α, λ, P_A, K_A) as shown in Equation (9.5.4) for when $C_A = 0$:[u]

$$-\left.\frac{\partial T_A(C_A)}{\partial C_A}\right|_{C_A=0} > 1$$

$$-\frac{\partial}{\partial C_A}\left.\left(\alpha \cdot e^{-\lambda \cdot C_A} \cdot P_A \cdot K_A\right)\right|_{C_A=0} > 1$$

$$\left.\alpha \cdot \lambda \cdot e^{-\lambda \cdot 0} \cdot P_A \cdot K_A\right|_{C_A=0} > 1$$

$$\boxed{\alpha \cdot \lambda \cdot P_A \cdot K_A > 1} \tag{9.5.4}$$

If *alpaca* is equal to 1, then the amount of investment equals the amount of return. If it is below 1, then the amount gained from reducing the threat cost does not match the

[u] This signifies that State A either (1) sees no benefit from investing into its own nuclear security measures and therefore, $C_A = 0$ or (2) State A only considers its nuclear assets secure by way of the inherent security of the state (as was the case in 1985 Soviet Union).

amount invested. Inserting the parameters into Equation (9.5.3) yields the following signifying diminished returns as C_A increases:

$$\frac{\partial^2}{\partial C_A^2}\left(\alpha \cdot e^{-\lambda \cdot C_A} \cdot P_A \cdot K_A\right) > 0$$

$$\frac{\partial^2}{\partial C_A^2}\left(\alpha \cdot e^{-\lambda \cdot C_A} \cdot P_A \cdot K_A\right) = \alpha \cdot \lambda^2 \cdot e^{-\lambda \cdot C_A} \cdot P_A \cdot K_A > 0$$

$$\alpha \cdot \lambda^2 \cdot P_A \cdot K_A > 0$$

Applying this assessment to Case Studies 1-4 (the Soviet Union/Russia-U.S. bilateral nuclear security regimes) as discussed in Chapters 4-7, Table 9.11 shows (for each case) the parametric values (α, λ, P_A, and K_A), the *uncorrelated* strategic cost (C_A^{UNC}) at which point *alpaca* is computed and the values of *alpaca*. As stated in its title, the values in Table 9.11 reflect an *at-target* attack within State A (per the discussion in Section 9.4).

Table 9.11. *Alpaca* values for case studies 1-4 (at-target parameters)

Year	α $\left[{}^{SQ}/_{yr}\right]$	λ [$\$M^{-1}$]	P_A [ ]	K_A [\$M]	C_A^{UNC} [\$M]	*alpaca* $\left[{}^{SQ}/_{yr}\right]$
1985	0.01	0.1066	0.03	5,307	0	0.187
1995	0.25	0.0141	0.6	34,483	304.2	72.93
2008	0.20	0.0140	0.3	63,736	283.6	53.72
2015	0.15	0.0129	0.2	67,963	252.7	26.42

The value of the *alpaca* indicator in each case study provides additional insight about the bilateral regime. In using the values in Table 9.11 (with at-target parameters), the models yield *alpaca* values that can be used for a qualitative comparison between the indicated and observed behavior of State A: whether State A independently invested in a nuclear security program (i.e., if there was a perceived marginal reduction in threat cost per investment in strategic cost). Table 9.12 includes the *alpaca* values for each case study as well as a summary of if State A should have proceeded with securing its nuclear assets and whether they did or not.

Table 9.12. Summary of the *alpaca* characteristic indicator (at-target)

Columns	1	2	3	4
Year	Computed *alpaca* $\left[{}^{SQ}/_{yr}\right]$	Is nuclear security indicated? [Y/N]	Was nuclear security observed? [Y/N]	If observed, was nuclear security (I)ndependent or (C)ooperative?
1985	0.187	N	N	--
1995	72.93	Y	Y	C
2008	53.72	Y	Y	C
2015	26.42	Y	Y	I

Using Table 9.12, a rough assessment can be made by evaluating whether the methodology (applied to each model using specific parameters representing the nuclear security regime) indicates State A's nuclear security investment for that year. In column 1, if *alpaca* ≤ 1, then there was no indicated nuclear security program to be pursued because the marginal reduction in threat cost would not have been realized with such a program (shown in column 2). However, if *alpaca* > 1, then a nuclear security program was indicated. By comparing the methodology's indications (column 2) with whether a nuclear security was historically observed or not (column 3), then we can state the methodology's results are consistent with history. Conversely, if columns 2 and 3 do not match, then the methodology did not yield accurate results. Upon review, the data presented in Table 9.12 convey that the methodology was able to accurately indicate the observed behavior of the Soviet Union/Russian Federation in all four case studies. As discussed in Chapter 4, the Soviet Union did not rely much beyond its closed and heavily policed society for securing its nuclear assets against unlawful access or diversion. This observation was indicated by the methodology. The remaining *alpaca* values in Table 9.12 present a consistency between a indicated and observed nuclear security program by Russia (as discussed in Subsections 5.1.1, 6.1.1, and 7.1.1). Further, column 4 in Table 9.12 distinguishes the manner by which nuclear security was observed to be applied in State A. As aforementioned, if *alpaca* > 1, nuclear security was indicated (and in these cases, observed). However, some case studies consisted of State A independently implementing a nuclear security program (I) while others convey State A cooperated (possibly after initiating their own independent security program) with their game counterpart to apply nuclear security (C). This added insight touches upon the possible future need of categorizing the *alpaca* value so as to determine if we can deduce how collaboration occurs between states. For example, a correlation could be made between *alpaca* and the ratio of State A's strategic cost (C_A) over the cost to secure (C_S) as a way to determine how security is applied in State A: $alpaca \propto \frac{C_A}{C_S}$.

Specifically, in the 1985 case study, the *alpaca* value suggests that State A is not effectively convinced of the danger of an *at-target* detonation. Therefore, it does not invest anything additional to secure its own nuclear assets. Conceptually, this lack of concern can be attributed to the heightened level of overall security in the Soviet Union circa 1985. The implication is such that there was not much concern of an outside terrorist threat entering a nuclear weapons site, attaining a weapon, escaping with said weapon, and successfully detonating it at a targeted site later. Conversely, the *alpaca* values for the other case studies are substantially greater than one. The implication with these values is that Russia (in 1995, 2008, and 2015) either did receive or would have received a reduction in their threat costs with the associated displayed strategic costs (even assuming an *at-target* attack despite having a lower probability of occurring). Hence, when comparing *alpaca* between the four years, it can be stated that there was a benefit for State A to invest in its own nuclear security for the latter three cases. Some other inferences that can be gleaned from this data include the facts that: 1) there was a lack of nuclear security (explaining the jump in *alpaca* between 1985 and 1995), 2) there existed a gradual acceptance of a perceived threat (explaining the reduction in *alpaca* between 1995 and 2015), and/or 3) a sense of nuclear security culture was growing through and permeating through the industry (explaining the gradual reduction in *alpaca* over the three years).

Conversely, if the analysis is to consider instead *on-site* detonation threat, the *alpaca* indicators change leading to an analysis of State A unilaterally investing in its own security. To consider the *on-site* threat, values for λ, P_A, and K_A are modified per the discussion in Section 9.4. P_A represents the worst-case scenario ($P_A = 1$) that a non-state adversary enters a site, acquires control over a weapon, and detonates it on-site. K_A is decreased to 10% of the original K_A value to signify the consequence of losing personnel, facilities, and all else on a remote military site away from civilians and state infrastructure. And for Case Studies 1, 2, and 4, λ is reassessed using only nuclear weapons security (assuming an event would only occur on a military site with nuclear weapons) – similarly to the discussion in Section 9.4 for Case Study 3. The input parameters for determining λ are discussed in Chapters 4-7.

Table 9.13. *Alpaca* values for case studies 1-4 (on-site parameters)

Year	α $\left[{SQ}/{yr}\right]$	λ [$M^{-1}]	P_A [ ]	K_A [$M]	C_A^{UNC} [$M]	*alpaca* $\left[{SQ}/{yr}\right]$
1985	0.01	0.1066	1	530.7	0	0.63
1995	0.25	0.0282	1	3,448.3	113.2	24.31
2008	0.20	0.0285	1	6,373.6	125.9	36.36
2015	0.15	0.0198	1	6,796.3	152.0	20.13

The C_A^{UNC} values of Table 9.13 reflect the impact of changing the detonation location in all four cases: if the considered threat in each case study is only the on-site detonation, State A's *uncorrelated* cost strategy is understandably less than the at-target detonation values. The *alpaca* indicator value for each case study are the same as those values in Table 9.11 signifying the lack of reduction in threat cost per incremental strategic cost in 1985 (opposed to the values for 1995, 2008, and 2015 where *alpaca* is greater than 1). Lastly, as mentioned previously, the accuracy of the input parameters need not be so precise because the qualitative behavior of State A (even with on-site parameters) is still understood – as it is in Table 9.11.

Similar analyses can be made for State B and the regime of States A and B. For example, another analysis can be completed in evaluating the point at which the amount of State B's investment in its strategic cost is less than the amount of reduced threat cost to State B. Analogously to *alpaca,* State B's characteristic indicator is based on Equation (3.1.14), expanded below in Equation (9.5.5), and can be used to determine State B's security strategy of an interdiction program given State A will proceed with its optimal uncorrelated strategy.

$$T_B\left(C_A, C_B\right) = \left[\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right] \cdot e^{-\mu \cdot C_B} \cdot K_B \tag{9.5.5}$$

Based on its threat cost, $T_B(C_A, C_B)$ as notated in Equation (9.5.5), one can infer that State B's strategy of investing in interdiction relies on more than just its own strategic investment (but State A's as well). A logical assumption is that if the marginal reduction in threat cost, when $T_B = 0$, is greater than the initial unit cost of interdiction and assuming State A adopts its *uncorrelated* strategy, C_A^{UNC} (recall $C_i' = C_i^{UNC}$), then State B should adopt a strategy of investing in its own interdiction measures. The inequality (9.5.6) mathematically exhibits State B's marginal reduction in threat cost per (any positive) investment of strategic cost.

$$-\left.\frac{\partial T_B\left(C_A, C_B\right)}{\partial C_B}\right|_{C_A = C_A', C_B \geq 0} > 1 \tag{9.5.6}$$

Also, it can be argued that State B continues to invest as long as there is a continued marginal reduction in threat cost that exceeds the increase in strategic cost as strategic costs increase (exhibited by Equation (9.5.7)).

$$\frac{\partial^2 T_B\left(C_A, C_B\right)}{\partial C_B^2} > 0 \tag{9.5.7}$$

Inserting Equation (9.5.5) into Equation (9.5.6) defines the characteristic indicator *algum* which is the characteristic indicator for State B's perceived marginal reduction in T_B. Similar to the above discussion regarding *alpaca*, we use $C_B^{UNC} = 0$ to derive the simple formula for *algum* shown in Equation (9.5.8):

$$-\frac{\partial T_B\left(C_A, C_B\right)}{\partial C_B}\bigg|_{C_A = C_A^{'}, C_B \geq 0} > 1$$

$$-\frac{\partial}{\partial C_B}\left(\left[\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right] \cdot e^{-\mu \cdot C_B} \cdot K_B\right)\bigg|_{C_A = C_A^{'}, C_B \geq 0} > 1$$

$$-\left(\left[\alpha \cdot e^{-\lambda \cdot C_A^{'}} + \Gamma\right] \cdot -\mu \cdot e^{-\mu \cdot 0} \cdot K_B\right)\bigg|_{C_A = C_A^{'}, C_B = C_B^{'} = 0} > 1$$

$$\boxed{\left(\alpha \cdot e^{-\lambda \cdot C_A^{'}} + \Gamma\right) \cdot \mu \cdot K_B > 1} \tag{9.5.8}$$

Continuing with Equation (9.5.7):

$$\frac{\partial^2}{\partial C_B^2}\left[\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right] \cdot e^{-\mu \cdot C_B} \cdot K_B > 0$$

$$\frac{\partial^2}{\partial C_B^2}\left[\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right] \cdot e^{-\mu \cdot C_B} \cdot K_B = \frac{\partial^2}{\partial C_B^2}\left[\alpha \cdot e^{-\lambda \cdot C_A} + \Gamma\right] \cdot \mu^2 \cdot e^{-\cdot C_B} \cdot K_B > 0$$

Taking a similar path as *alpaca*, Table 9.14 displays the various *algum* values as indicators for State B to unilaterally invest in its own interdiction using input parameters discussed in each case study (for at-target detonations). Of note, computing *algum* requires an assessment on State A's strategy. Hence, C_A^{UNC} is included in the table.

Table 9.14. *Algum* values for case studies 1-4 (at-target parameters)

Year	α $\left[{}^{SQ}/_{yr}\right]$	λ [$M^{-1}]	C_A^{UNC} [$M]	μ [$M^{-1}]	K_B [$M]	Γ $\left[{}^{SQ}/_{yr}\right]$	C_B^{UNC} [$M]	*algum* $\left[{}^{SQ}/_{yr}\right]$
1985	0.01	0.1066	0	0.0016	36,105	0	0	0.65
1995	0.25	0.0141	304.2	0.0047	133,314	0	162.7	2.15
2008	0.20	0.0140	283.6	0.0011	282,875	0.0031	688.2	2.16
2015	0.15	0.0129	252.7	0.0018	312,234	0.0347	1,734.5	22.69

The data presented in Table 9.14exhibit values for State B's uncorrelated strategic costs with the assumption that State A adopts unilateral strategies to thwart any at-target detonation in State A using State A's asset. In contrast to the *alpaca* indicator, values for the *algum* indicator (for State B) do not exhibit large ratios from unity. Analogously, the implication of this is that State B's results are more susceptible to the obtained input parameters and thus, more care should be taken regarding their accuracy. Furthermore, when *algum* < 1, it can be perceived that there is not enough of a perceived threat to State B to justify expenses in an interdiction system – as was the case in 1985. In that year, as explained in Subsection 4.1.2, there was almost no concerted effort in the U.S. to invest in nuclear material interdiction at national borders or ports of entry.[v] In the other case studies (especially with the values close to the transitional quantity of 1 in 1995 and 2008), it is suggestive that the *algum* indicator values imply a potential sensitivity of the results

[v] Recalling the 1984 case of 600 tons of radioactive steel shipped from Mexico into the U.S. and only detected once a portion of the shipment reached Los Alamos National Laboratory.

to uncertainties of the input parameters. Notably in the 2015 case study, algum is much greater than other years possibly due to the increase in Γ as mentioned in Subsection 7.1.2.

Table 9.15. *Algum* values for case studies 1-4 (on-site parameters)

Year	α $\left[{}^{SQ}/_{yr}\right]$	λ [$M⁻¹]	C_A^{UNC} [$M]	μ [$M⁻¹]	K_B [$M]	Γ $\left[{}^{SQ}/_{yr}\right]$	C_B^{UNC} [$M]	*algum* $\left[{}^{SQ}/_{yr}\right]$
1985	0.01	0.1066	0	0.0016	36,105	0	0	0.65
1995	0.25	0.0282	113.2	0.0047	133,314	0	395.9	6.46
2008	0.20	0.0285	125.9	0.0011	282,875	0.0031	894.9	2.72
2015	0.15	0.0198	152.0	0.0018	312,234	0.0347	1,758.3	23.69

Echoing Table 9.14, if State A only considers the threat of an on-site detonation, the resulting *uncorrelated* strategic cost for State A impacts State B's *uncorrelated* strategic cost and thus, impacts the resulting *algum* indicator values as shown in the far right column of Table 9.15. It can be rationalized that if State A invests less in securing its assets because the potential threat cost of an on-site detonation is less than the potential threat cost of an at-target detonation, then State B will have to invest more in its own strategic costs for interdiction.

Table 9.16. Summary of the *algum* characteristic indicator (at-target)

Columns	1	2	3
Year	Computed *algum* $\left[{}^{SQ}/_{yr}\right]$	Is interdiction indicated? [Y/N]	Was interdiction observed? [Y/N]
1985	0.65	N	N
1995	2.15	Y	Y
2008	2.16	Y	Y
2015	22.69	Y	Y

Table 9.16 presents a summary of the *algum* indicator for each case study assuming the former at-target scenarios. As with the *alpaca* indicator, the *algum* indicator provides an assessment of how well the methodology was able to provide results that accurately reflected observed interdiction by State B for each year. If it was observed that State B did invest in an interdiction program (column 3), the methodology should have indicated that (in column 2) by yielding *algum* > 1 (in column 1). Conversely, if no observed interdiction program existed in State B, the methodology would have accurately indicated that if *algum* ≤ 1. If any alternative results are produced, the agreement between the observed and indicated behaviors would represent an error in parameter estimation or an externality that would explain the lack of agreement in results. As can be seen in Table 9.16, the *algum* indicator does confirm that the methodology's results agree with historical behavior (as discussed in Subsections 4.1.2, 5.1.2, 6.1.2, and 7.1.2).

Similar analyses can be completed for the joint benefit of both States A and B (as part of the bilateral regime) if they are to cooperate – a qualitative assessment of the benefit of cooperation pending on the marginal reduction of respective threat costs. However, this would require the consideration of various bargaining solutions described under each case study such as how much would State B contribute to State A's security: State B's subsidization of State A's additional strategic cost, State B's neutralization of State A's complete strategic cost, or any another imputation between the two. This analysis is left for future work.

9.6. Potential for Regime Abuse

Within the bilateral regime, there exists some situations where there exists a potential for abusing the collaborative relationship. For example, State A can enter into a cooperative arrangement and, if a bargaining solution concept is achieved between both, not use State B's contributions for securing its nuclear assets. For another example, as introduced in the preamble to Chapter 9, the concept of a state profiting from its nuclear insecurity was illustrated in two of the five case studies: Russia-U.S. in 1985 and 2008 respectively (discussed in Chapters 4 and 6). First described by Corr, the concept he discusses is that one state recognizes its insecurity as an asset that can be traded or "exported" to another country in exchange for monetary gain.[33] In both case studies, the placement of the *uncorrelated* solution concept had an impact on whether this phenomenon arose. In general, whenever a solution concept consists of State B contributing any amount to State A's nuclear security, C_{AB}, (as is commonly done by the American nonproliferation community), there is the potential for abuse. Though the amount of abuse is subjective with any value of C_{AB}, certainly, in the specific case of $TC_i < 0$ (for any player i), the situation could easily be regarded as potentially abusive for the non-i player. In sum, $TC_A < 0$ is only one interpretation of the abuse potential by State A and the clearest demonstration of Corr's concern.

We return to the discussion from Subsection 3.3.1 where the strategic cost for securing nuclear assets in State A, C_S, is the combination of A's investment, C_A, and B's contribution to A's security, C_{AB}. Conversely, in Equation (3.2.12), the cost to State B for interdiction is its strategic investment, C_B, minus B's contribution to State A's security, C_{AB}. The value of C_{AB} is what defines B's diverted investment to A's efforts and has a direct effect on the resulting solution concept of (TC_A, TC_B). Specifically, when the surplus, v, is greater than State A's *uncorrelated* total cost, TC'_A, the amount of State B's contribution, C_{AB}, to the cost of State A's security, C_S, can yield a negative total cost for State A.[w] As C_{AB} increases, so does State A's temptation to deviate from its *correlated* strategy. This may occur because as State B's contribution grows, the value of State A's investment decreases (discussed in Subsection 3.1.2.1 as diminished returns). As State A does not receive the same marginal reduction rate in threat costs, it can divert investments elsewhere. Corr's concept is graphically displayed by the red tinted triangle in Figure 9.3 below (repeated from Chapter 6).

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[w] Conversely, if $C'_A > v$, State A's *uncorrelated* strategic cost is high enough that no division of the surplus, v, would cause A to receive a negative cost. This implies that State A accepts that there is enough of a nuclear security threat that it adopts a nuclear security strategy consisting of C'_A.

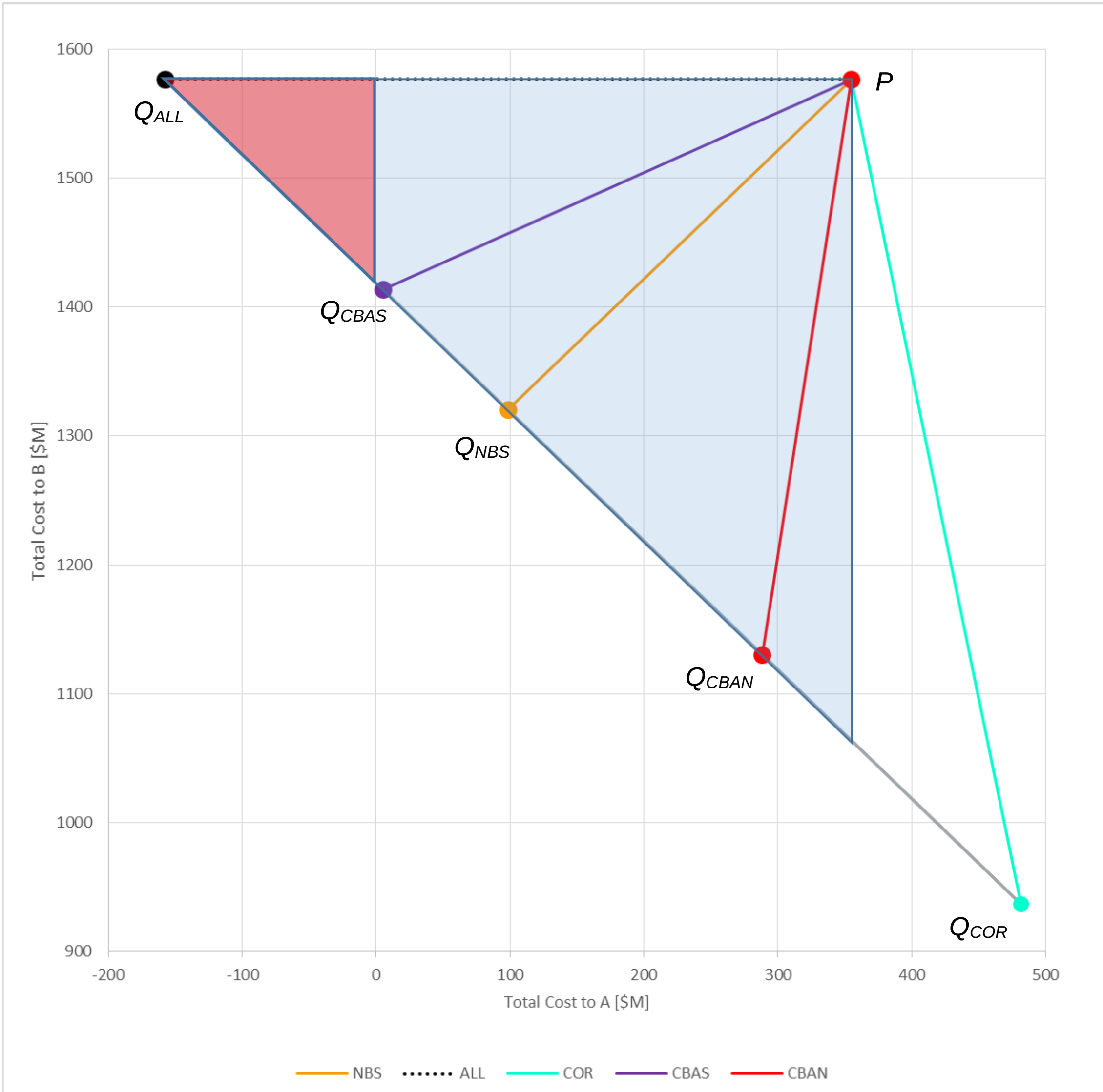

Figure 9.3. State A's profit from nuclear insecurity (shown in red)

In Figure 9.3, solution concepts are plotted using the total costs to both States A and B: TC_A and TC_B, respectively. Point P represents the uncorrelated solution concept (TC_A', TC_B') which can be used as the baseline by which other solution concepts (Q_i)

present in the figure are evaluated against. As shown, all solution concepts identified by Q_i exhibit a reduction in total cost to at least either, if not both, states. Therefore, one can conceptualize how negotiations between the states would occur in order to receive a reduction in their total costs – the magnitude of which would depend on the final bargained solution concept.

Utilizing Equation (3.2.9) results in Q_{COR} or (TC_A^*, TC_B^*) – the initial potential bargained solution concept between the states. In the example presented in Figure 9.3, Q_{COR} exhibits the unconstrained case where TC_A is increased but TC_B decreases. The increase in TC_A assumes State A does not receive as much reduction in its threat cost, T_A, as it increases its investment in its strategic cost, C_A. Contrarily, TC_B is reduced because it sees the direct benefit in its T_B reduction (which is dependent on the increase of C_A). Should State B contribution, C_{AB}, to State A's uncorrelated strategic cost, C_A', TC_B would rise and TC_A would fall. As alluded to previously, the magnitude of C_{AB} allows for any solution concept represented in Figure 9.3, Q_i (including Q_{ALL} – the extreme case of State B allowing State A to keep all of the surplus, *v*, thus allowing $TC_A < 0$).

Corr's illustrated concept here begs the discussion of whether it is in State B's interest to contribute to State A's strategic cost of nuclear security (and if so, to what level). It is a precarious situation if, as a baseline, it is determined that $TC_A' < v$. In this case, the bargained solution concept must be carefully negotiated from State B's perspective so as not to result in $TC_A < 0$ (though State B would have to assess whether a reduced T_B value is worth having TC_A be negative). Conversely, it would behoove State A to have C_{AB} be as high as possible so as to receive the negative TC_A and possibly abuse the cooperation between both states. On the other hand, if State B considers the hazard of cooperative action with State A is too high, it may forgo a collaborative approach and, instead, choose to solely invest its efforts into interdiction (irrespective of the source state) – a strictly defensive (and ostensibly isolationist) posture by State B.

State B's latter strategy infers the question of whether a cooperative approach to nuclear security benefits the overall goal of nuclear nonproliferation – especially in the face of multiple potential source states. This idea represents an obvious struggle of the global nonproliferation regime from the perspective of a single target state with resources. As the definition for State B includes the threat coming from just one other state, consider State B's strategy of creating and maintaining a number of bilateral regimes with different partner states to address all nuclear security threats. This endeavor can be argued to be far more cumbersome than focusing on interdiction. There is also the concern that State A must be assumed to understand the nuclear security threat as well. When $T_A = 0$, State A has no incentive to invest in any security because there is no threat to it. This leads to a weak bargaining position for State B. From State A's perspective, an appreciation of and a stated awareness to State B of the threat is necessary to enter into a potentially fruitful bilateral regime. In sum, State A could entice State B into entering into a bilateral regime but State B is left with defining whether it receives more benefit than adopting a unilateral defensive posture of interdiction.

10. CONCLUSIONS

A new methodology was developed for assessing strategies within bilateral nuclear security regimes based on game-theoretic analysis. To demonstrate the potential for providing insights into state-level security strategies on a bilateral basis, the methodology was applied to five case studies which consisted of three country partnerships: the Soviet Union and the U.S., the Russian Federation and the U.S., and Pakistan and the U.S. Though the U.S. was used as a partner in each case study, other states could easily be used in future applications if plausible data could be estimated to define input parameters.

Two notable conclusions were derived from this work: 1) the values of the *uncorrelated* total costs for each state (which depends greatly on the *uncorrelated* strategic costs) can impact the nature of the collaborative bilateral regime, and 2) correlation of states' strategies in a bilateral regime will assist in reducing the total costs but will almost always require additional investment by at least one state (as seen in figures 4.2. 5.2, 6.2, 7.2, and 8.5). The first conclusion, as was alluded to in Section 9.6, centers on the reduction of a state's temptation to profit from participating in a nuclear security bilateral regime. This is achieved by assuring the *uncorrelated* total costs (or Nash Equilibrium solution concept) for each state are greater than the surplus, v. In each specific case study, the only way this was to be accomplished was for having State A increase its total cost, TC_A, by raising its strategic cost, C_A, (which would also decrease State A's threat cost, T_A, a function of C_A) – recalling Equation (3.1.2). The implication of this is that State A understands its own nuclear threats and once convinced of State A's sincerity, State B could then decide whether to enter into the bilateral regime with State A.

The second conclusion infers that constructing and maintaining a cooperative regime (even for a mutually beneficial topic like securing nuclear assets) is not a simple task. The models presented herein do not contain external incentives that may exist which would convince states to enter or not into such a regime – though possible to explore in future work, this implies threat costs would be defined differently.. When comparing the *uncorrelated* and *correlated* total costs in each case study within Chapters 4-8 (as shown in Tables 4.2, 5.3, 6.3, 7.3, and 8.3), cooperation occurs among the asymmetric partnerships by the *target* state investing in the *source* state's security. Elevating a *source* state's nuclear security culture can raise awareness of nuclear security threats so that the state will unilaterally care about securing its assets. If the *source* state does not internalize the nuclear threat, it will be unlikely to care enough to invest. This was the situation in all case studies between the U.S. and the Soviet Union/Russia/Pakistan. Therefore, this methodology conveys that, though cooperation is possible, the *target* state will have to invest in the *source* state's security to reduce their threat (as mentioned in Section 9.1) and it would behoove the *target* state to set an initial threshold the *source* state would have to surpass so as to exhibit their commitment to nuclear security before the *target* state begins to invest (as discussed in Section 9.6).

Each case study discusses what can be done after determination that a cooperative regime can be established: bargaining the collective strategies so as to ensure a utility benefit for both partner states. In the third section of each chapter, one can see various ways to devise potentially beneficial cooperative regimes and, more so, how to visualize them. Representing the bargaining solutions allows for an analyst to evaluate and provide comparisons of various strategies within the regime.

10.1. Future Directions

Applied game theory is being pursued internationally by scholars in several disciplines: network security, energy markets, global feeding, etc.[201,202,203] Scholars and researchers throughout the U.S. and Europe have progressed the state of the art of game theory to particular areas of interest but much more can be done. Specifically with regards to the work presented herein, the following subsections outline ideas for readers to consider pursuing based on the developed methodology.

10.1.1. A Characteristic Indicator for the Cooperative Regime

The characteristic indicators discussed in Section 9.5 focus on States A and B individually. However, a cooperative performance indicator can be useful in delineating when States A and B, acting bilaterally to further secure State A's assets, are able to reduce the collective regime's total costs compared to the combined total costs resulting from their respective *uncorrelated* strategic costs. The mutual benefit received by both states, in the form of the surplus, can then be divided between the two states when their strategies are *correlated*. This third characteristic indicator can convey the benefit of cooperation when the aggregate marginal reduction in threat costs for both A and B, T_{AB} (shown below as Equation (10.1.1) in generic form with C_S and C_I as discussed in Subsection 3.2.2 –

where $C_S = C_A + C_{AB}$ and $C_I = C_B - C_{AB}$) is greater than unity. The total expanded threat cost of the regime, T_{AB}, is represented as Equation (10.1.2) and the regime's benefit of cooperation shown as Equation (10.1.3) denotes when there is a favorable reduction in regime threat cost per strategic investment (by both states) for the regime:

$$T_{AB}\left(C_S, C_I\right) = T_A\left(C_S\right) + T_B\left(C_S, C_I\right) \tag{10.1.1}$$

$$T_{AB}\left(C_S, C_I\right) = \alpha e^{-\lambda C_S} P_A K_A + e^{-\mu C_I} K_B \left[\alpha e^{-\lambda C_S} + \Gamma\right] \tag{10.1.2}$$

$$BC_{AB} = -\frac{\partial T_{AB}}{\partial C_S} > 1 \tag{10.1.3}$$

The aforementioned equations serve as a springboard for future work in this area. Characteristic indicators for the regime could prove beneficial for qualitative assessments of the methodology in future work – especially when other sources of securing State A's assets are considered. Moreover, we envision a benefit from developing more formal models of bargaining between states when establishing and maintaining cooperative bilateral nuclear security regimes.

10.1.2. Coercion

A strategy briefly introduced in Section 1.2 included the concept of coercion as a way to influence nuclear security. Otherwise described as providing negative incentives, coercion has been considered as a third viable security option beyond the two solutions presented in the methodology: *uncorrelated* strategies and *correlated*. One reason for defining the loss rate as it was presented in Equation (3.1.15) was to accommodate the impact of strategic costs on State A's loss rate. The concept was to allow the possibility of State A to incur so much cost that it would impede advancements in securing the nuclear material of interest. Other scholars have expressed similar concepts qualitatively. For example, in his book, Drezner describes two differing cases of economic coercion: the United Nations Security Council vote on imposing multilateral sanctions to Iraq in 1990 for the invasion of Kuwait and the United States' imposition of unilateral economic sanctions on Israel in 1991 to thwart the latter's construction of new housing in the occupied territories of the West Bank and Gaza.[11] The former case was unsuccessful because the multilateral economic sanctions did not help and so military action was taken to coerce Iraq's concession. The latter case unilateral economic sanctions on Israel was considered a success because construction was halted. These examples show that results from coercion in international relations are not always consistent. In sum, a quantitative assessment could prove useful.

Conceptually, coercion could be incorporated into the game theory model by permitting negative strategic costs which would impact the loss rate (as described in Equation (3.1.15)) as shown in Figure 10.1.

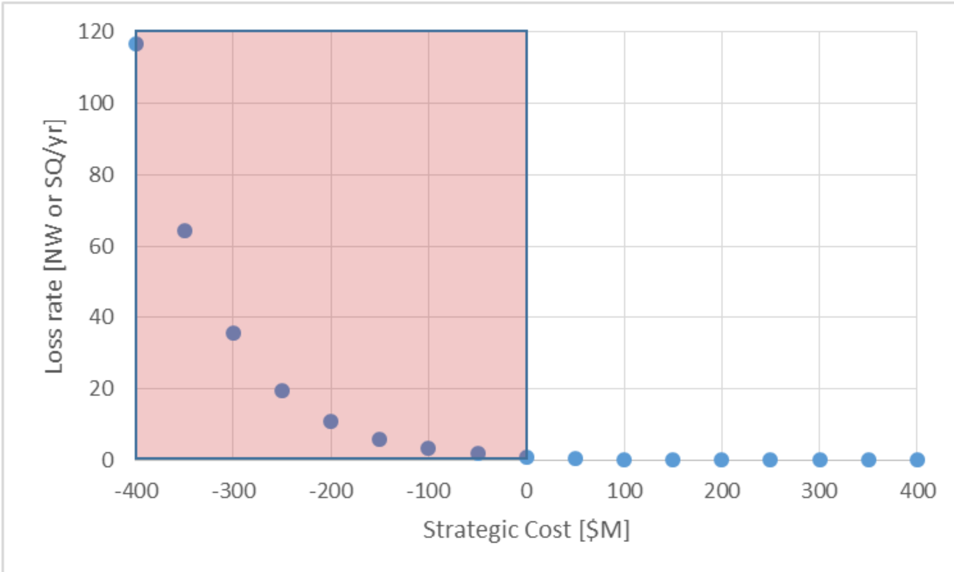

Figure 10.1. Impact of negative investment on loss rate

The red-shaded region visualizes the state having such a level of economic sanctions imposed on it that the funds it would otherwise use for securing assets would be used in areas of a higher priority than nuclear security. Therefore, this could have an opposite effect on influencing said state to increase the security of its nuclear assets.

10.1.3. Retaliation Factor

Subsection 3.1.2 describes the various input parameters that are used to model the bilateral nuclear security regime. In an effort to better explore the threat costs to State A, a retaliation factor parameter was initially considered to convey State B’s potential response to State A if a nuclear asset were lost and used against State B. Though difficult to estimate the cost to State A in retaliation, bounded assumptions could be made in relation to the K_B value. Equation (10.1.4) displays the modified cost function to State A with the incorporated retaliation factor, $R(C_A)$.

$$TC_A(C_A) = C_A + P_A \cdot K_A \cdot \left[\alpha \cdot e^{-\lambda \cdot C_A}\right] + R(C_A) \tag{10.1.4}$$

If considering a strictly military attack that is proportional to the initial attack on State B (where B suffers the cost of K_B), the following relation can be defined as in Equation (10.1.5):

$$R(C_B)\left[\frac{\$M}{event}\right] = \eta \cdot K_B\left[\frac{\$M}{event}\right] \tag{10.1.5}$$

Where $\eta = 1$ if the retaliation is reciprocated in kind (the impacts on States A and B are similar). When the retaliation from B impacts A greater than the initial attack on State B (the punishment is harsher), $\eta > 1$. This line of thought leads one to consider the price of an attack. Quantifying this cost incurred by either State A or B assumes the same three primary variables similar to the definition of K_B in Subsection 3.1.2.3: the loss of life (K^{hu}), the economic impact (K^{ec}) of such losses, and the loss of national infrastructure (K^{ni}). While other variables such as military loss, emotional distress, and loss of domestic political clout are also greatly affected by a nuclear attack, the most impactful seem to be these three.[90]

Furthermore, the value of η is important in the retaliation factor because this would signify how the initial attack on State B by a non-state actor using material from State A relates to the retaliatory attack onto State A by State B for not properly securing the special nuclear material (or nuclear weapon). When $\eta < 1$, the impact of the initial attack on State

B is greater than the retaliatory attack on State A. This could occur when State B is attacked in a highly populated area affecting B's economy greatly and B does not retaliate in kind so as not to adversely affect State A's command and control center. When $\eta > 1$, the impact of the retaliation on State A's infrastructure is stronger than that on B with the initial attack. This would occur if a weapon taken from State A is detonated as a display of force against B by a non-state actor and B punishes State A with a military invasion for allowing such a transgression to occur. Simply put, the parameter η would define the magnitude of a retaliatory action on State A by State B. Therefore, this would constitute a new calculation of the total cost to State A, TC_A, as shown in Equation (10.1.4) which would impact State A's objective function, resulting strategy, as well as State B's interdiction strategy (investment to interdiction may change if money is invested in State B's retaliatory efforts).

10.1.4. Multilateral Coalitions

Another ripe area of game theory that was introduced initially and can be applied to the state-level nuclear security regime is the use of this methodology with multilateral regimes. Not without precedence and continuously pursued (such as the International Partnership for Nuclear Disarmament Verification[x]), multilateral regimes exist for nuclear nonproliferation and security activities. This methodology can be easily adapted to allow

[x] As mentioned during a Nonproliferation and Arms Control technical division meeting during the 2016 Annual meeting of the Institute of Nuclear Materials Management, the group of 25 states (as of July 2016) is a public-private partnership between the U.S. State Department and Nuclear Threat Initiative for furthering to understand the complex technical challenges involved in nuclear verification.

for multiple states to join the regime and the players would evolve from being individual states to coalitions of states where multiple work together to attain an additional benefit in utility beyond what they would individually receive.

10.2. Summation

In sum, a quantitative assessment can be completed by evaluating uncorrelated and correlated strategies between two states in a bilateral nuclear security regime. The game theoretic methodology presented here for assessing bilateral regime strategies provides an opportunity to gauge surplus divisions and whether or not states gain more utility by correlating their strategies. This methodology has led to two notable conclusions:

1. States attempting to address nuclear security concerns can benefit from a bilateral collaborative regime where states can correlate their strategies. Though the regime's total (i.e., collective) cost can be reduced, it will almost always necessitate additional investment by at least one state. This asymmetrical partnership can lead to potential imbalances between the states in such a regime.
2. In some case studies where a strategy set showed potential as a fair surplus division between the states, the resulting strategy point would fall as a negative total cost for State A. This would mean that many imputations of the game's core between States A and B would provide State A an unfair advantage in receiving a negative cost (otherwise, a positive utility). In 'playing' with State B, State A would receive an additional benefit beyond the subsidization of its costs and possibly reap a profit from entering the bilateral regime at a position of insufficient nuclear security. This

could in turn lead State A to use its level of insecurity as a commodity to be sold off to State B.

The primary conclusions above can contribute positively to the future discussion of applying game theoretic analysis as a way to assess utilities in state-level nuclear security strategies within bilateral regimes. The work presented herein will form the basis for future publications in game theory, risk assessment, and other relevant scholarly publications. Hopefully, others, through this work, will appreciate the feasibility of using game theory to contribute to the overall discussion of assessing state-level nuclear security strategies.

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APPENDIX A

MATLAB Code for Uncorrelated and Correlated Strategies

```
clc;
clear;
res=50;  % resolution of discretization
format shortG
cs=input('Which case study? 1-1985; 2-1995; 3-2008; 4-2015; 5-2008 Pakistan
');
if cs==1            %1985 case
    C_A_ACT=7.66;
    C_B_ACT=6.1;
    LR=0.00486;
    alpha=.249;
    lambda=-(log(LR/alpha))/C_A_ACT;
    mu=.00164;  % P_B=99%
    P_A=0.03;           %user-defined probability of stolen weapon used against
B
    K_A=9447;            %negligible cost to A of attack on B with A's weapon
    K_B=171488;          %cost of attack on most populous target in B: 1985 NYC
    gamma=0;
elseif cs==2                %1995 case
    C_A_ACT=200;
    C_B_ACT=61.1;
    LR=0.0149;
    alpha=.249;
    lambda=-(log(LR/alpha))/C_A_ACT;
    mu=-(log(0.75))/C_B_ACT;         %from 1997 expenditures 0.75 porosity
    P_A=0.6;            %user-defined probability of stolen weapon used against
B
    P_B=0.75;
    K_A=40996;           %from Odling-Smee's 2006 IMF paper
    K_B=550618;         %cost of attack on most populous target in B: 1994 NYC
    gamma=0;
elseif cs==3    % 2008 case
    C_A_ACT=297.3;
    C_B_ACT=2673.3;
    LR=0.00307;
    alpha=.249;
    lambda=-(log(LR/alpha))/C_A_ACT;
    mu=-(log(0.05))/C_B_ACT;
    P_A=0.3;            %user-defined probability of stolen weapon used against
A
    K_A=70818;          %cost to A of attack on B with A's weapon - Russia is
part of IC
    K_B=1836023;
    gamma=LR;       %2008 Russia is not the only source of NW: Pakistan, China,
new DPRK
elseif cs==4            %2015 case
    C_A_ACT=290.7;
    C_B_ACT=1635;
    LR=0.00347;
    alpha=0.249;
    lambda=-(log(LR/alpha))/C_A_ACT;
    mu=-(log(0.05))/C_B_ACT;
```

```
    P_A=0.2;            %user-defined probability of stolen weapon used against
B
    K_A=75417;          %cost to A of attack on B with A's weapon - Russia is
part of IC
    K_B=2126830;         %cost of attack on most populous target in B: 2015 NYC
    gamma=10*LR;    %2015 Russia is not the only source of NW: Pakistan (by
far), DPRK (more now), India?
elseif cs==5    % PAK case
    C_A_ACT=100;
    C_B_ACT=2673.3;
    LR=0.002;   % or 8.26e-5?
    alpha=1;    %18 years and mayb eonly 1 NW lost? 0.055
    lambda=-(log(LR/alpha))/C_A_ACT;
    P_A=0.9;
    K_A=9931;
    mu=0.00112;
    K_B=253793;
    gamma=10*LR;    %2008 Pakistan is not the only source of NW: Russia, DPRK,
India?
end;
z=input('Zoomed in? 0-no; 1-yes  ');
if z==0
    if cs==1        %1985 case
        c_A=linspace(0,100,res); % strategic costs for A
        c_B=linspace(0,500,res); % strategic costs for B
    elseif cs==2    %1995 case
        c_A=linspace(0,1000,res); % strategic costs for A
        c_B=linspace(0,1000,res); % strategic costs for B
    elseif cs==3    %2008 case
        c_A=linspace(0,5000,res); % strategic costs for A
        c_B=linspace(0,5000,res); % strategic costs for B
    elseif cs==4    %2015 case
        c_A=linspace(0,10000,res); % strategic costs for A
        c_B=linspace(0,10000,res); % strategic costs for B
    elseif cs==5    % PAK case
        c_A=linspace(0,5000,res);     % to see negative values, remove lines
101 and 162
        c_B=linspace(0,5000,res);     % to see negative values, remove lines
114 and 165
    end;
else
    if cs==1        %1985 case
        c_A=linspace(0,20,res); % strategic costs for A
        c_B=linspace(0,500,res); % strategic costs for B
    elseif cs==2    %1995 case
        c_A=linspace(300,600,res); % strategic costs for A
        c_B=linspace(0,500,res); % strategic costs for B
    elseif cs==3    %2008 case
        c_A=linspace(250,500,res); % strategic costs for A
        c_B=linspace(1500,2500,res); % strategic costs for B
    elseif cs==4    %2015 case
        c_A=linspace(250,350,res); % strategic costs for A
        c_B=linspace(2700,2800,res); % strategic costs for B
    elseif cs==5    %Pak case
        c_A=linspace(100,150,res);
        c_B=linspace(1550,1650,res);
    end;
end;
act=input('Include actual expenses in green? 0-no; 1-yes  ');
[C_A, C_B]=meshgrid(c_A,c_B);
```

```
C_A_UNC=(1/lambda)*(log(alpha*lambda*P_A*K_A));  % minimum strategic cost for A
-- equation 2.3.3
if C_A_UNC<0
    C_A_UNC=0;
end
C_B_UNC=(1/mu)*log((mu*K_B)*((alpha*exp(-lambda*C_A_UNC))+gamma));

if C_B_UNC<0
    C_B_UNC=0;
end


T_A_UNC=alpha*P_A*K_A*exp(-lambda*C_A_UNC);   % minimum threat cost for A
T_B_UNC=K_B*exp(-mu*C_B_UNC)*((alpha*exp(-lambda*C_A_UNC))+gamma);
TC_UNC=C_A_UNC+T_A_UNC+C_B_UNC+T_B_UNC;   %  minimum total cost of A and B
separate

% ----------- Tabulating C_A vs C_B vs TC -----------------
for i=1:res
    for j=1:res
        T_A(i,j)=P_A*K_A*alpha*exp(-lambda*C_A(i,j)); % threat cost for A
        TC_A(i,j)=C_A(i,j)+T_A(i,j);  % strategic cost + threat cost for A
        T_B(i,j)=K_B*(exp(-mu*C_B(i,j)))*((alpha*exp(-lambda*C_A(i,j)))+gamma);
        TC_B(i,j)=C_B(i,j)+T_B(i,j); % strategic cost + threat cost for B
        TC_AB(i,j)=TC_A(i,j)+TC_B(i,j); % total costs for A + B
    end
end
surf(C_A,C_B,TC_AB);
hold on
view(135,30);
xlabel('C_A [$M/yr]')
ylabel('C_B [$M/yr]')
zlabel('Total cost [$M/yr]')

% ----------- Calculating correlated strategies -----------------
fun=@(x)x(1)+x(2)+P_A*K_A*(alpha*exp(-lambda*x(1)))+(exp(-
mu*x(2))*K_B*((alpha*exp(-lambda*x(1)))+gamma));
A = [];
b = [];
Aeq = [];
beq = [];
lb = [c_A(1),c_B(1)];
ub = [c_A(res),c_B(res)];
s_A = c_A(1)+((c_A(res)-c_A(1))/2);
s_B = c_B(1)+((c_B(res)-c_B(1))/2);
[x,fval]=fmincon(fun,[s_A,s_B],A,b,Aeq,beq,lb,ub);
if x(1)<1e-3
    x(1)=0;
end
if x(2)<1e-3
    x(2)=0;
end

% ---------------- PRIME signifies UNCORRELATED strategies -------------
C_A_UNC;
TC_A_UNC=C_A_UNC+T_A_UNC;
C_B_UNC;
TC_B_UNC=C_B_UNC+T_B_UNC;
```

```
TC_UNC;

% ---------------- STAR signifies CORRELATED strategies -----------------
C_A_COR=x(1);
T_A_COR=alpha*P_A*K_A*exp(-lambda*C_A_COR);
if T_A_COR <= 1e-3
    T_A_COR=0;
end
TC_A_COR=C_A_COR+T_A_COR;
C_B_COR=x(2);
T_B_COR=K_B*exp(-mu*C_B_COR)*((alpha*exp(-lambda*C_A_COR))+gamma);
if T_B_COR <= 1e-3
    T_B_COR=0;
end
TC_B_COR=C_B_COR+T_B_COR;
TC_COR=TC_A_COR+TC_B_COR;

C_A_ADJ=C_A_UNC;
C_B_ADJ=C_B_COR+(C_A_COR-C_A_UNC);
T_A_ADJ=alpha*P_A*K_A*exp(-lambda*C_A_COR);    % minimum threat cost for A
if T_A_ADJ <= 1e-3
    T_A_ADJ=0;
end
T_B_ADJ=T_B_COR;
if T_B_ADJ <= 1e-3
    T_B_ADJ=0;
end
TC_A_ADJ=C_A_ADJ+T_A_ADJ;   %  minimum total cost of A and B separate
TC_B_ADJ=C_B_ADJ+T_B_ADJ;   %  minimum total cost of A and B separate
TC_ADJ=TC_A_ADJ+TC_B_ADJ;   %  minimum total cost of A and B separate

%------- strategy points -------
plot3(C_A_UNC,C_B_UNC,TC_UNC,'*r');
plot3(x(1),x(2),fval,'*c');
if act==1
    plot3(C_A_ADJ,C_B_ADJ,TC_ADJ,'*g');
end
T_A_ACT=P_A*K_A*alpha*exp(-lambda*C_A_ACT);
TC_A_ACT=C_A_ACT+T_A_ACT;
T_B_ACT=K_B*exp(-mu*C_B_ACT)*((alpha*exp(-lambda*C_A_ACT))+gamma);
TC_B_ACT=C_B_ACT+T_B_ACT;
TC_AC_ACT=TC_A_ACT+TC_B_ACT;
TC_ACT=TC_A_ACT+TC_B_ACT;

U_A=1-(TC_A_ADJ/TC_A_UNC);
U_B=1-(TC_B_ADJ/TC_B_UNC);
U_AB=1-(TC_ADJ/TC_UNC);

ACT=[C_A_ACT;T_A_ACT;TC_A_ACT;C_B_ACT;T_B_ACT;TC_B_ACT;TC_ACT];
UNC=[C_A_UNC;T_A_UNC;TC_A_UNC;C_B_UNC;T_B_UNC;TC_B_UNC;TC_UNC];
COR=[C_A_COR;T_A_COR;TC_A_COR;C_B_COR;T_B_COR;TC_B_COR;TC_COR];
ADJ=[C_A_ADJ;T_A_ADJ;TC_A_ADJ;C_B_ADJ;T_B_ADJ;TC_B_ADJ;TC_ADJ];
f=figure('Position',[200 200 500 200]);
d=[UNC,COR,ADJ, ACT];
cnames={'Uncorrelated','Correlated','Renegotiated','Actual'};
rnames={'C_A';'T_A';'TC_A';'C_B';'T_B';'TC_B';'TC'};
t=uitable('Parent',f,'Data',d,'ColumnName',cnames,'RowName',rnames);
t.Position(3) = t.Extent(3);
```

```
t.Position(4) = t.Extent(4);
alpaca=alpha*lambda*P_A*K_A;

surplus = TC_UNC-TC_COR
alpaca=alpha*lambda*P_A*K_A
algum=mu*K_B*(alpha*exp(-lambda*C_A_UNC)+gamma)
```