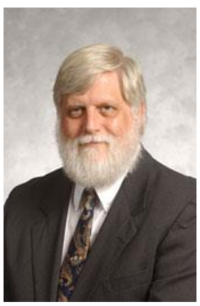
46TH TURBOMACHINERY & 33RD PUMP SYMPOSIA
HOUSTON, TEXAS | DECEMBER 11-14, 2017
GEORGE R. BROWN CONVENTION CENTER

PUMP AFFINITY LAWS MODIFIED TO INCLUDE VISCOSITY AND GAS EFFECTS

Gerald L. Morrison
Professor
Texas A&M University
College Station, Texas, USA

Abhay Patil
Research Engineer
Texas A&M University
College Station, TX, USA



Gerald L. Morrison is a Professor of Mechanical Engineering at Texas A&M University. He has been a member of the Turbomachinery Laboratory for over 35 years investigating aircraft jet noise, space shuttle turbopump and standard pump seals, and single and multiphase flow meters and pumps.

Abhay Patil is a Research Engineer working for Dr. Morrison at Turbomachinery Laboratory, TAMU. His research includes numerical and experimental investigations of performance and reliability of artificial lifting methods for multiphase flows. Dr. Abhay Patil holds a PhD degree in Mechanical Engineering from Texas A&M University, College Station.

ABSTRACT

The pump Affinity Laws have been modified to include the effects of viscosity. The results are simple 2D curves that represent the head-flow rate and efficiency-flow rate relationships for a specific pump. The new Modified Affinity Laws were validated using experimental data from several different types of pumps and CFD simulations of a mixed flow pump. An additional similitude relationship was developed to obtain a 2D curve showing how the addition of gas changes the head generated in a single stage. Since the gas volume changes with each stage due to pressure rise, this relationship can be used along with the Modified Affinity Laws to predict the performance of a single stage pump or the stage by stage performance of a multi stage pump operating with gas and varying viscosity.

INTRODUCTION

The performance of a pump changes significantly with pump speed, fluid density, fluid viscosity, and the presence of gas. Performance maps that specify how a pump will operate over a wide range of pump speed, fluid density, and fluid viscosity are not readily available. Add the effects of gas in the liquid and it is an educated guess as to how much head can be generated at a given flow rate and pump speed as well as the power required to operate the pump. Generally the information available is a water test that provides head generated and power required as a function of flow rate and pump speed. Figure 1 provides two examples. One shows the effect of impeller size and the other pump speed for a fixed size impeller. The graphs can be a bit complex and requires interpolation for the engineer or operator to determine how the pump will behave for each given operating condition.

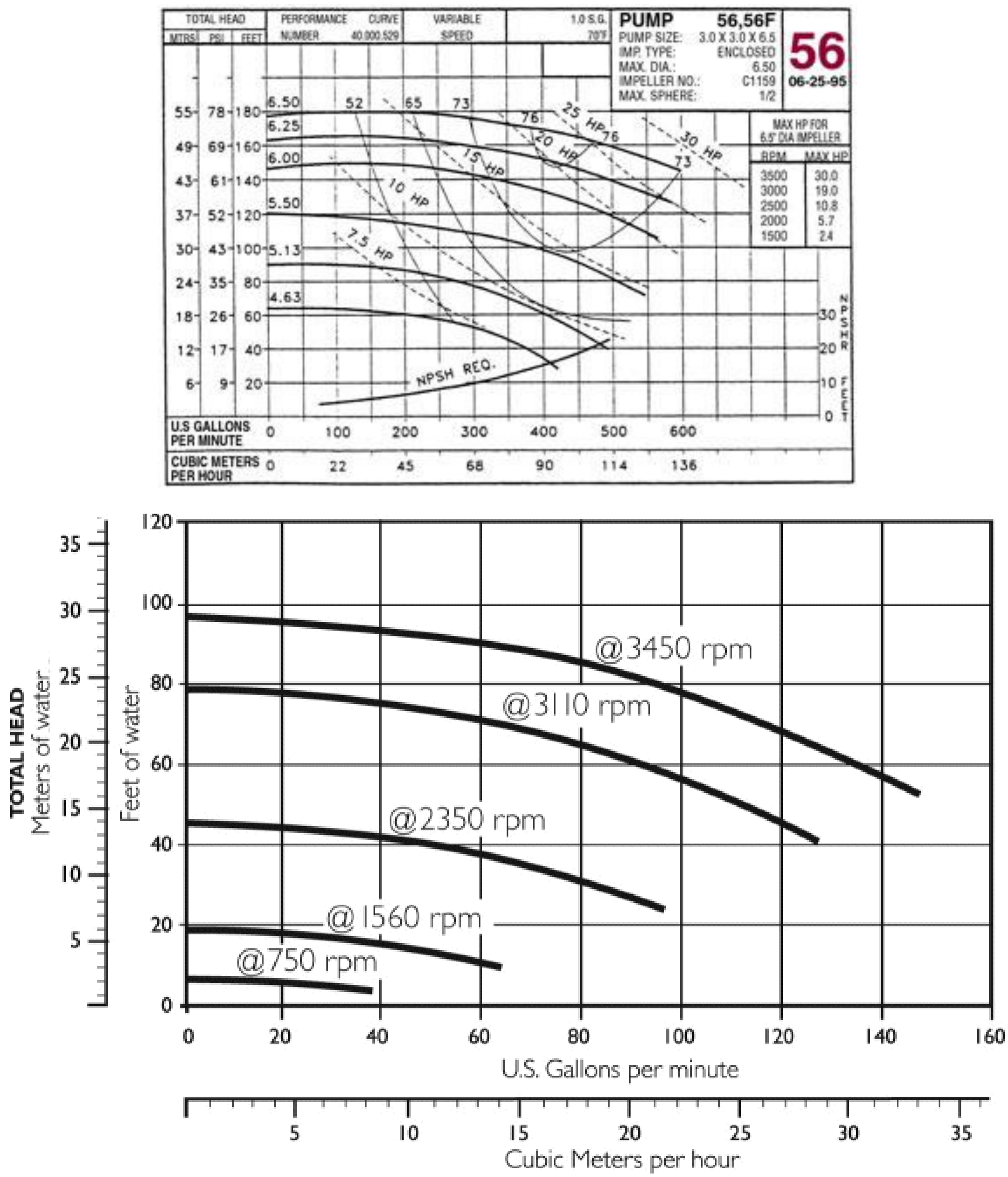


Figure 1. Typical Pump Performance Curves

Buckingham Pi groups [1] have been used for over 100 years to help solve these types of problems. Pipe flow is a perfect example where large amounts of data were analyzed and the Moody [2] friction factor charts were developed. These charts allow the design of piping systems and the prediction of the system's operation under varying conditions. The Buckingham Pi procedure consists of listing the important variables (density, viscosity, velocity, size, etc.) and producing nondimensional Pi groups such as Reynolds number, Mach number, and Froude number. This type of analysis was performed on pumps many years ago which included the effects of density, pump speed, impeller size, and flow rate upon the pressure generated and power required to operate the pump. From this analysis, the Affinity Laws were obtained that reduce the complex performance map into three distinct curves for head coefficient (ψ), power coefficient for the power supplied to the pump shaft (Π), and pump efficiency (η) as a function of flow coefficient (ϕ). Two curves (ψ vrs ϕ and η vrs ϕ) define the entire performance map of the pump for a single viscosity since the fourth nondimensional group can be calculated from the other three, i.e. $\Pi = f(\phi,\psi,\eta)$. If the viscosity of the pumped fluid remains close to the value of the fluid (usually water) used in the experimental test to obtain the data for the flow map, it is still a good representation of the pump performance. The Affinity Laws are given below where Q is the volumetric flow rate, ω is the pump speed, D_s is the pump impeller diameter, g is the acceleration due to gravity, H is the pump head ($\Delta P/\rho g$), and Γ is the pump shaft torque.

$$\phi = \frac{Q}{\omega D_s^3}\ , \quad \psi = \frac{gH}{\omega^2 D_s^2}\ , \quad \Pi = \frac{\omega\Gamma}{\rho\omega^3 D_s^5}\ , \quad \eta = \frac{\rho g Q H}{\omega\Gamma} = \frac{\phi\psi}{\Pi}$$

Industrial applications of rotordynamic pumps cover a wide range of fluid viscosities and may also include the presence of gas and liquid mixtures. Currently, new performance maps which include the effects of viscosity must be generated for a given pump application. Figure 2 is an illustration of such a map for a mixed flow pump. Each map contains very large amounts of data covering the wide range of viscosities which is required to completely characterize the pump. Since the cost of providing the data (either through experiment or computer simulations) is high, it is rarely obtained. Even then if the data are obtained, it frequently remains proprietary so that each operator must obtain the data for their facilities. This work will first address how to modify the Affinity Laws to include the effects of viscosity so that only two curves are required to characterize the pump.

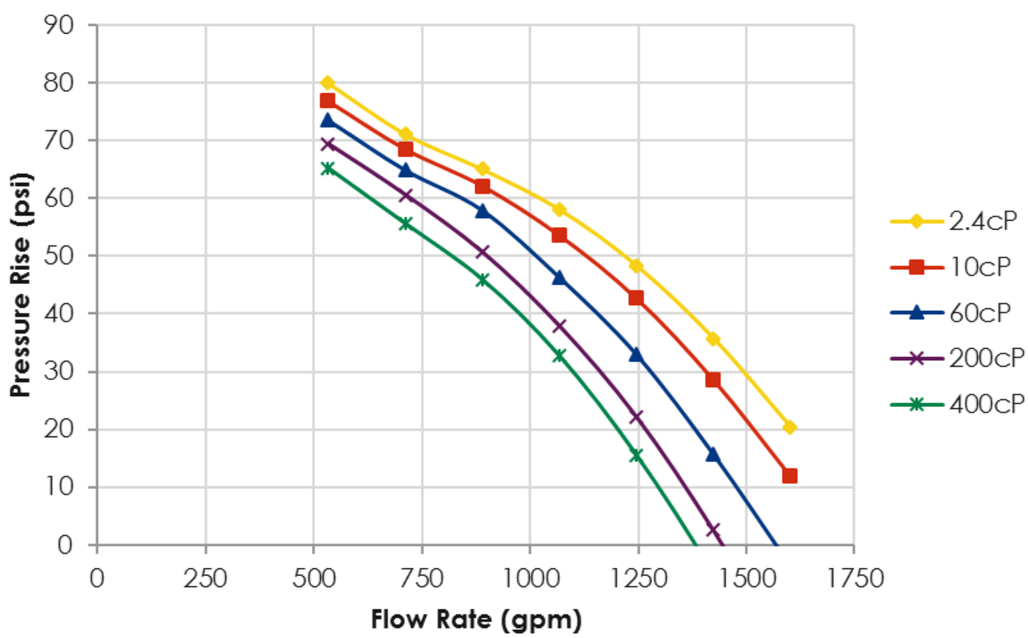


Figure 2. CFD simulation results showing viscosity effect upon the head-flow rate of a mixed flow pump.

Morrison et.al. [3] presented a study where both computer simulations and experimental data from different types of Electrical Submersible Pumps (ESP) were operated over a range of viscosities, pump speeds, and densities. The ESP is essential a multi stage rotordyanmic pump designed to be installed inside a well. The process presented here is applicable to all rotordyanmic pumps. Frank White [4] presents a Buckingham Pi analysis of rotordynamic pumps including the effects of viscosity. Since the viscosity changes the wall friction inside the pump, much like a pipe flow, he also included wall roughness (ε). The two additional nondimensional numbers are given as a Reynolds number, $\mathrm{Re}_\omega = \frac{\rho\omega D_s^2}{\mu}$, and the ratio of the wall roughness to the impeller diameter, $\frac{\varepsilon}{D_s}$.

The roughness factor has not been included in this analysis since it would require detailed analysis of the flow paths inside the pump with the head losses calculated as in a piping system. It was opted to treat this aspect of the pump as a lumped parameter, much like an elbow or tee, which is represented by an equivalent length which includes the effects of all the internal flow paths, including roughness. Thus, only the Reynolds number is included. This Reynolds number is based upon the pump speed, not the flow through the pump. Considering the velocity triangle of a pump, the flow speed through the pump depends upon both the rotational speed and the flow rate. Thus in this analysis, the fluid friction effects are considered a function of the flow through the pump which is represented by the flow coefficient and the rotational speed is represented by the Reynolds number. These two nondimensional groups are combined to represent the effects of viscosity.

The development of the Modified Affinity Laws utilized computer simulations of a mixed flow pump (Morrison et.al.[3]) since the simulations allowed specifying specific viscosities, densities, and rotational speeds. Only the flow through the main hydraulic paths was considered. After the Modified Affinity Laws are developed they will be applied to actual pump data to verify the technique. Figure 3 extends the data of Figure 2 by including the effects of pump speed. In this figure, the pressure rise is expressed in terms of the head coefficient and the flow rate as a flow coefficient.

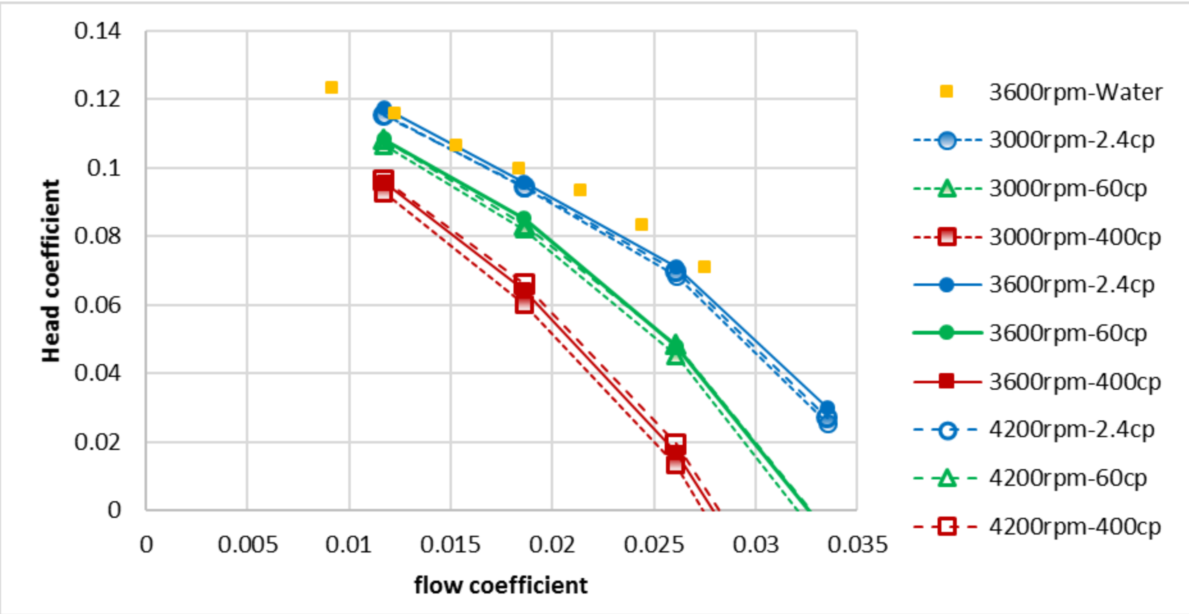


Figure 3. CFD simulation results for fluid viscosity and pump speed upon the head coefficient – flow coefficient relationship for a pump.

By utilizing the Affinity Law coefficients, the effects of pump speed and density are included in the nondimensional groups resulting in a single curve for each viscosity for all the data. The effects of viscosity are clearly evident with the head produced decreasing with increasing viscosity. Using the Buckingham Pi analysis and assuming the viscosity effects can be normalized using a Reynolds number, Figure 4 was produced. In this figure, the log base 10 of the Reynolds number is used since in pipe flows the friction factor (effects of viscosity) can be calculated using a power function of the Reynolds number.

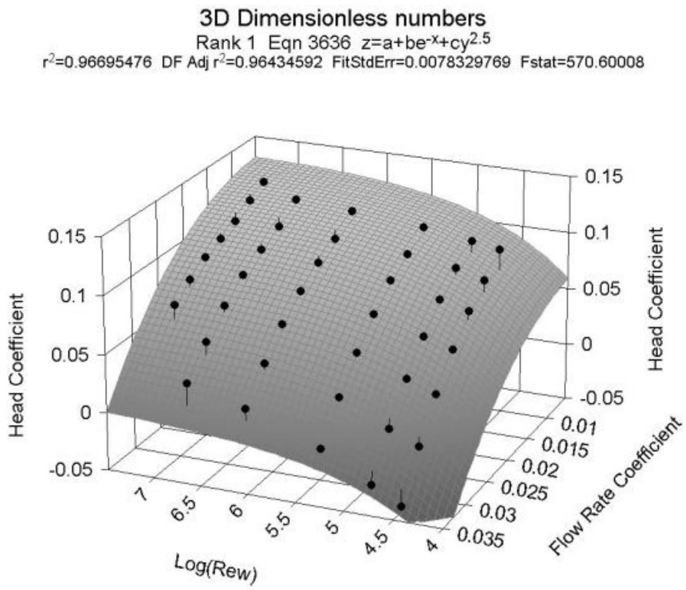


Figure 4: Head coefficient versus common logarithm of rotating Reynolds number and flow rate coefficient using data from Figure 3.

Figure 4 illustrates that there is a well behaved relationship of the head coefficient as a function of the flow coefficient and the Reynolds number. At this point, the method to represent the performance of a pump with varying flow rate, pump speed, fluid density, and fluid viscosity has been established. However, a large set of experimental or computational data is still required to establish the surface shown in Figure 4. This can be time consuming as well as expensive. Therefore, the present work continued with a second goal of obtaining a single simple 2D line which contains all the data shown in Figure 4.

Further analysis of the data shown in Figure 4 resulted in a collapse of the data to a single 2D curve. This was accomplished by multiplying the flow coefficient by the Reynolds number raised to a power. Figure 5 shows the results of this analysis. The CFD simulations collapse to a single 2D curve which includes the effects of pump speed, flow rate, fluid density, and fluid viscosity over a range of viscosities from 1 to 400 cp. This is valuable in that now only enough experimental data to establish the power to which the Reynolds number is raised must be obtained. Performing tests (experimental or CFD) for two viscosities, preferably at the extremes of the viscosity to be encountered, will be sufficient to generate the 2D performance curve. This will greatly reduce the cost of obtaining the performance map.

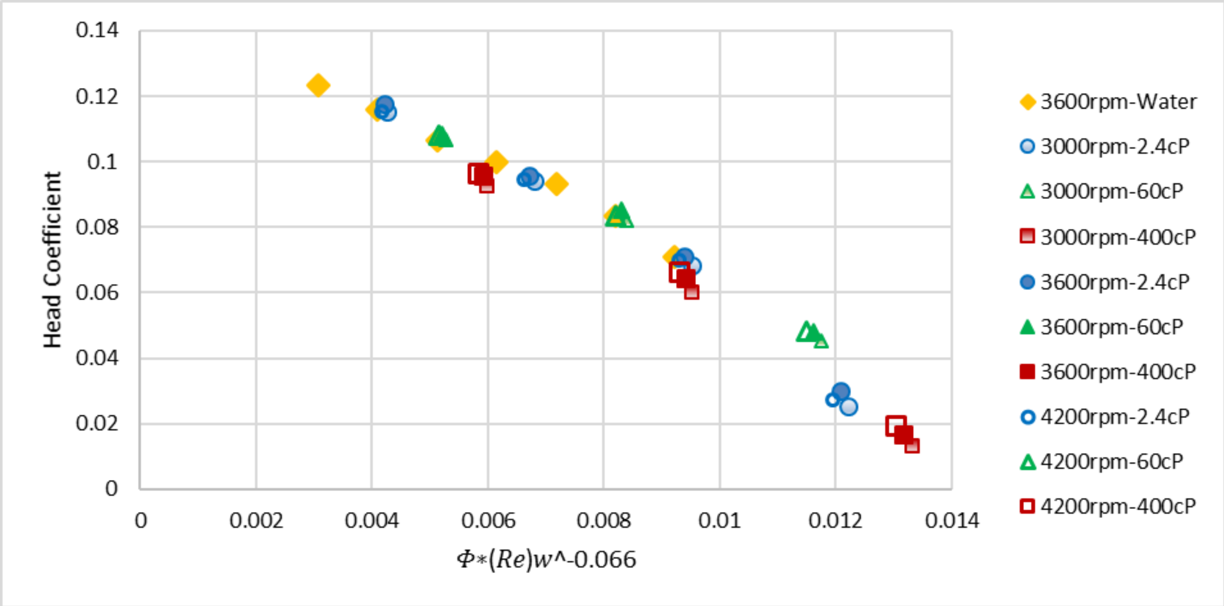
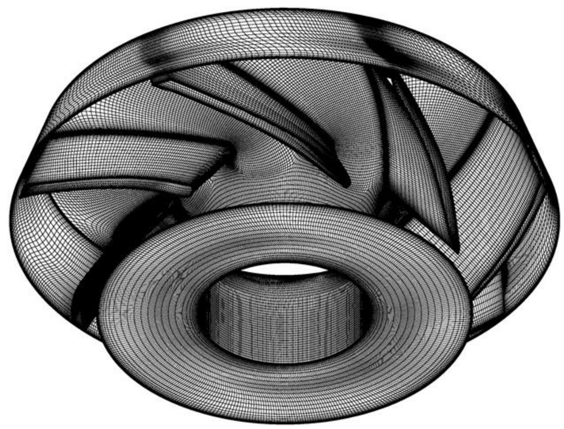


Figure 5. Modified Affinity Law relationship for a pump's head-flow rate data expressed in terms of head coefficient, flow coefficient, and rotational Reynolds number.

The data used in the analysis presented up to this point has been generated using CFD. The Modified Affinity Laws will now be used to analyze several different pumps for which varying viscosity data are available.

Validation using published test data of a mixed flow pump (Specific Speed: 3869):

To evaluate the applicability of the correlation to different pump designs, published data from a previous experimental research study was utilized. Figure 6 shows the impeller design of the semi-axial pump along with performance data obtained by Amaral, G. (acquired from Stel[5]) using fluids with different viscosities and densities. The figure also presents the data in dimensional (b) and Modified Affinity Laws format (c). The nondimensional data collapse to a common curve. The exponent of the Reynolds number is different from the valued obtained in the previous analysis. This is due to the different pump design which changes the frictional losses as well as the inclusion of the internal secondary flow leakage.



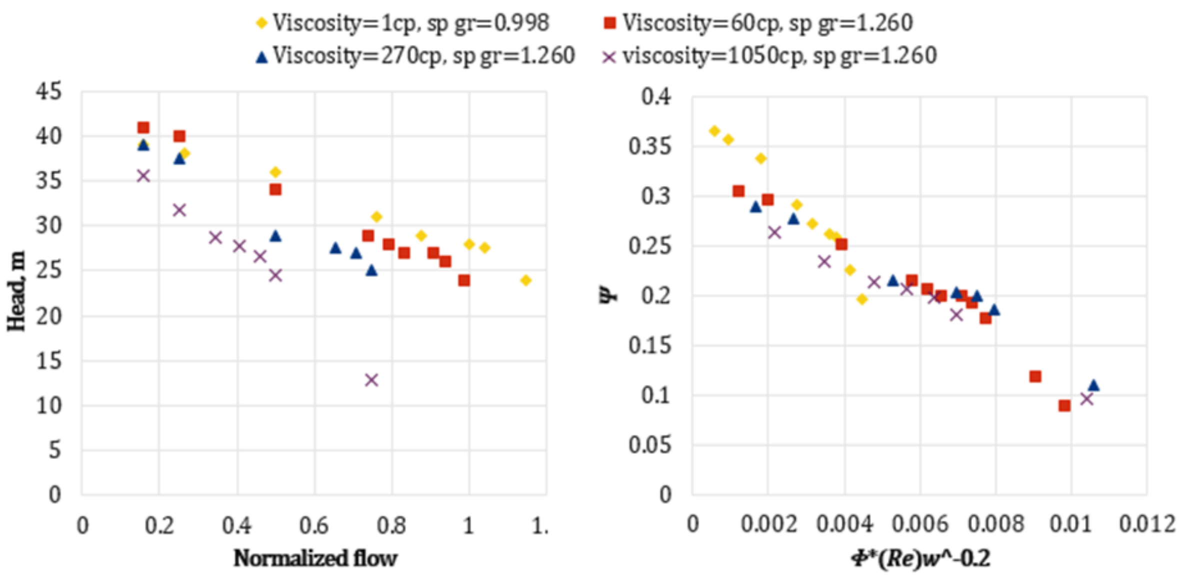


Figure 6: a) Pump head curve of semiaxial pump, 3500 RPM b) Empirical pump head curve using proposed affinity law for viscosity

Validation using viscosity test data of split vane impeller pump and helicoaxial pumps:

To further evaluate the effect of specific speed and pump design on the exponent of the Reynolds number, data from experimental tests of two different types of pumps, a split vane pump and a helicoaxial pump were utilized. Fluids under considerations were water (1cp, 998 kg/m3) and silicon oil (5cp, 918 kg/m3). The split vane pump is designed for better gas handing by inducing turbulence using a split vane impeller design as shown in the Figure 7 a). It has a specific speed of 3027. Figure 7 b) shows the helicoaxial pump in which the special design of the impeller produces flow primarily in the axial direction while generating lower wall shear stress compared to mixed flow pumps. It has a specific speed of 5281.

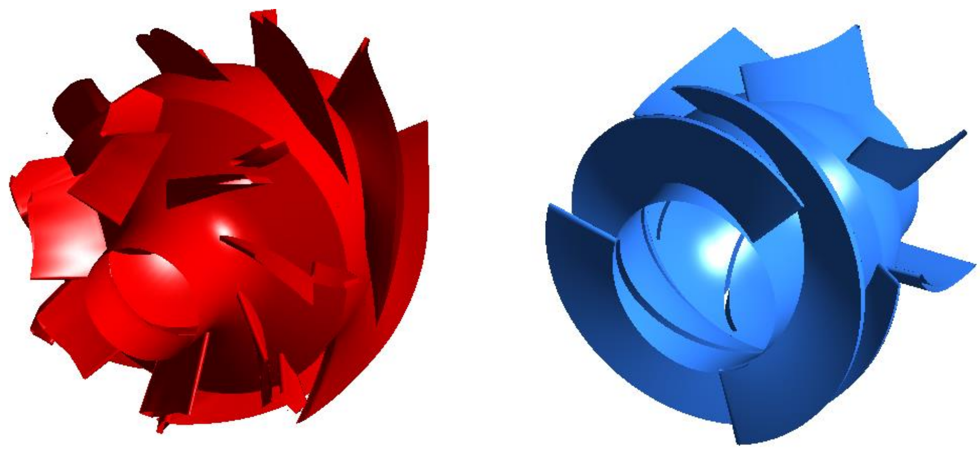


Figure 7: a) Split vane impeller pump b) Helicoaxial pump

Figure 8 a) shows the dimensional head generated by three stages of the split vane pump for 3000 rpm and 3600 rpm rotational speeds for water (1 cp) and silicone oil (5 cp) as a function of flow rate. Each operating conditions has a separate curve. Figure 8 b) utilizes the Modified Affinity laws which collapse all four curves to one common curve. These data have a slight increase in the Reynolds number exponent over the mixed flow pump of Figure 5. Even though the specific speed of the split vane pump is higher than previous pump, the slight increase in exponent represents higher viscous losses in split vane pump possibly due to higher levels of turbulence generated by the split vane impeller.

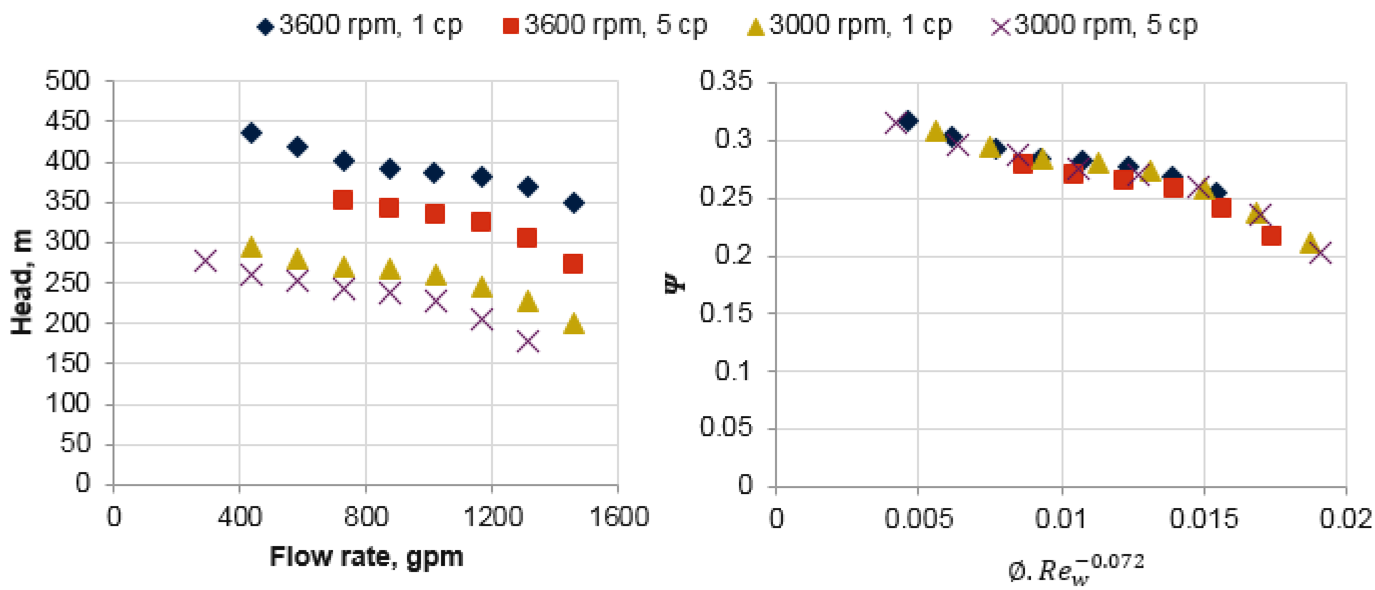


Figure 8: a) Pump performance curve of split vane pump b) Empirical pump head curve using proposed affinity law for viscosity

Figure 9 a) shows the dimensional pump performance of head generated by four stages of the helicoaxial pump for 3000 rpm and 3600rpm rotational speeds for both water and silicone oil. Figure 9 b) recasts the data into the Modified Affinity Laws format which collapses these data onto a single curve. These data show a slight decrease in the Reynolds number exponent. The special design of the impeller of helicoaxial pump causes lower shearing of the fluid inside the flow passage reducing the viscous losses which causes the exponent to decrease.

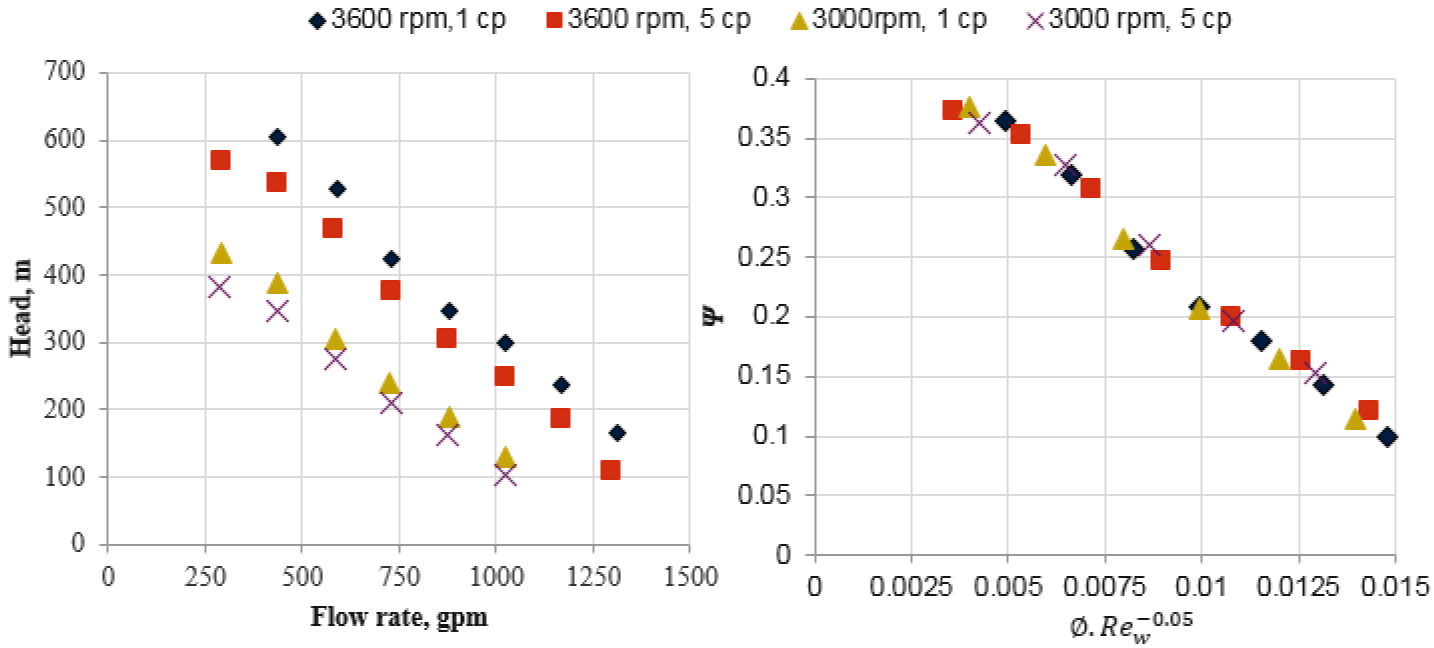


Figure 9: a) Pump performance curve of helicoaxial Pump b) Empirical pump head curve using proposed affinity law for viscosity

The cases presented so far cover a viscosity range from 1 to 400 cp. However, as viscosity further increases, there is a probability that the flow inside the pump will become laminar. If this occurs, there should be a change in the Reynolds number dependence just as there is in pipe flow. A study by Le Fur[8] was presented at the Pump Symposium in 2015. This study included pump performance data for water (1 cp) and then oil with viscosities ranging from 486 to 1639 cp. Figure 10 presents these data.

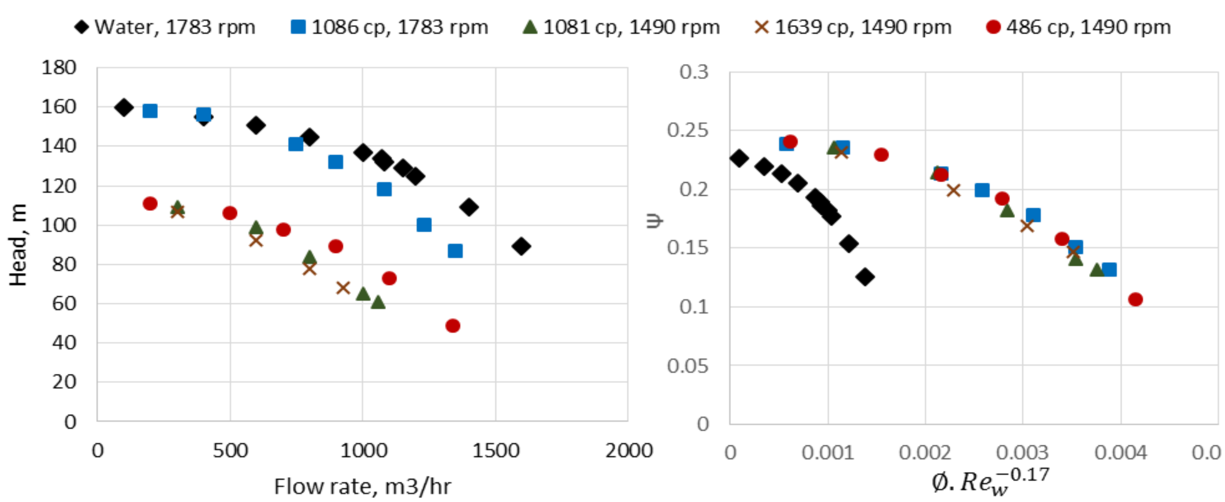


Figure 10. Pump data from Le Fur[8] showing how a large change in viscosity changes the flow from turbulent to laminar.

These data show that there is a step change in the head-flow rate relationship in both the dimensional and nondimensional representations. Using the Modified Affinity Laws, the high viscosity cases collapse to a common curve and the water curve is separate. The transition from laminar to turbulent flow inside a pump was observed in the CFD simulations of Morrison et.al.[3].

Based on the above results, it is seen that the Modified Affinity Laws for head prediction produced a single common curve for all operating conditions and viscosities for all the pumps under consideration when the flow inside the pump is either laminar or turbulent. Table 4 shows the value of exponent for different pump designs which possessed turbulent flow. No relationship could be formed between the exponent and specific speed at this time. However, further analysis is in progress to correlate the dependence of geometric parameters on the value of the exponent to reliably predict the effect of viscosity on specific pump designs. For the laminar flow Fur[8] data, the exponent increased to 0.17 which is higher than all the other cases.

Table 1: Value of exponent for different pump designs

Pump Type	Specific Speed	Re Exponent
Mixed flow	2758	0.066
Split vanes	3027	0.072
Semi-axial	3817	0.2
Helico-axial	5284	0.05

Computer generated data and experimental data have been used to verify the Modified Affinity Laws ability to efficiently include the effects of viscosity by collapsing all of the head-flow rate data for each pump onto single curve. The experimental data included a mixed flow pump, a split vane impeller mixed flow pump, and a helicoaxial pump. This covers a wide range of pump types and specific speeds. By representing the performance of the pump with one 2D curve, the Modified Affinity Laws significantly reduces the amount of data required to obtain a pump performance curve which includes the effects of viscosity. This results in a large reduction in the amount of data required to perform the analysis resulting in a large saving in time and money.

Pump Efficiency

The efficiency of a pump is obtained by using the head and flow rate produced by the pump to calculate the hydraulic power added to the fluid and dividing that by the power required to operate the pump. As before, the CFD data will be utilized to develop this relationship. FLUENT provides the torque required to rotate the impeller which is used to calculate the power supplied to the impeller. The efficiency as a function of the flow coefficient for all the pump speeds and viscosities considered is presented in Figure 11. As was observed for the head coefficient, the pump efficiency is segregated into three curves each representing a different viscosity for all three pump speeds and flow rates. The best efficiency point degrades significantly (72% to 45%) and shifts to a lower flow coefficient (0.02 to 0.01) as the viscosity increases. Figure 10 indicate that viscosity significantly changes a pump's performance but for a given viscosity the standard Affinity Laws are valid. The task is now how to collapse the three curves to one common curve by modifying the affinity laws.

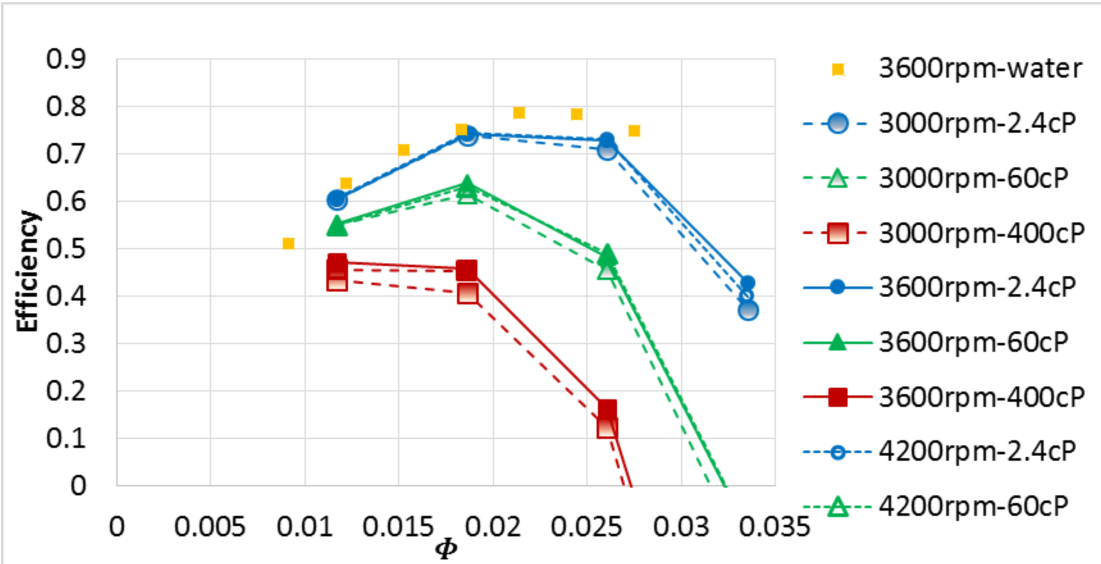


Figure 11: Pump efficiency versus flow rate coefficient with trend lines

Development of the Modified Affinity Law for viscosity-Efficiency Prediction

Further analysis was conducted to develop the Affinity Law modification for the efficiency of a pump by considering the flow inside the hydraulic path of the API 610 mixed flow pump. The goal was to obtain detailed hydraulic path information to help identify new scaling laws and hence losses inside the front and back seals and thrust surfaces were neglected. The efficiency was calculated using the CFD simulated torque required by the pump. Figure 12 a) presents the efficiency as a function of flow rate. The BEP shifts to lower flow rates and lower efficiency as viscosity is increased. The graph presents a daunting task to collapse to common curve. However, by using flow the coefficient Φ and Reynolds number ($\boldsymbol{Re_w}$) it can be accomplished as shown in the Figure 12 b). Utilizing the form of $\phi^b.Re_w{}^a$ for the independent variable and efficiency as the dependent variable, the widely scattered data in Figure 11a) is collapsed to a very compact single curve. This modified flow coefficient brings all the curves together so that there is only one value of the modified flow coefficient for all shaft speeds and viscosities for the Best Efficiency Point. Note the power on the Reynolds number has changed to -0.033 and exponent of 0.575 is added to flow coefficient on the X-axis to compensate for friction loss on either side of BEP with varying viscosity. On the Y-axis, an additional term of $\Phi * \boldsymbol{Re_w}$ with common *a* exponent is added to the efficiency. The higher exponent of this term represents higher viscous loss reducing the magnitude of the pump efficiency. Further study is in progress to validate the proposed terms using experimental data**.**

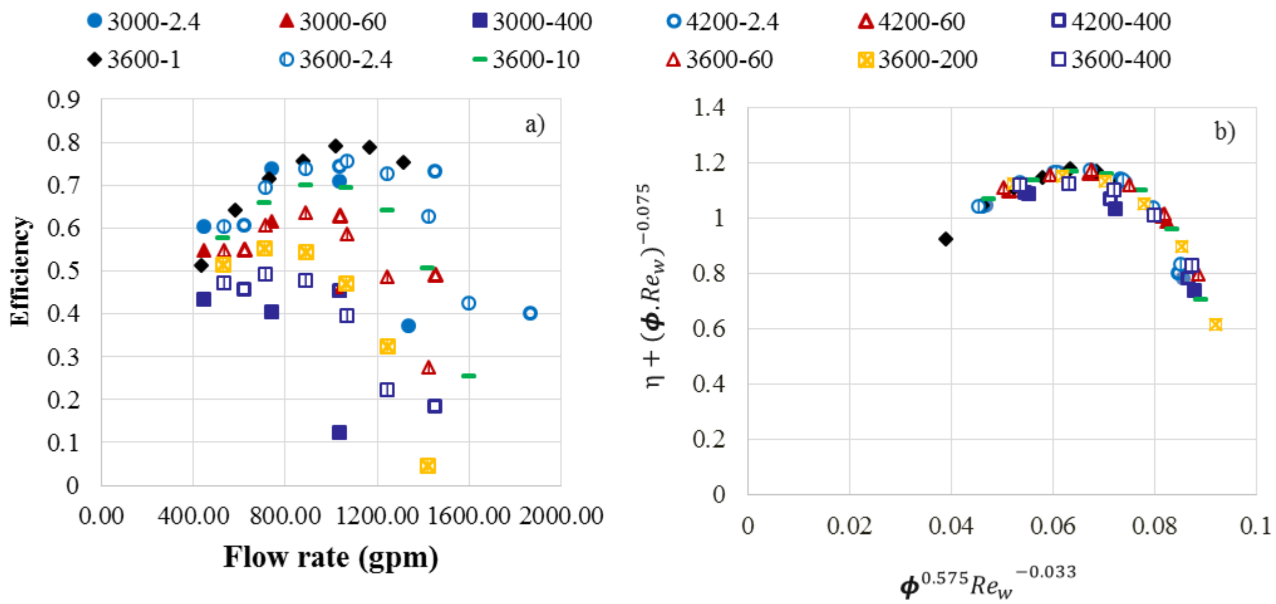


Figure 12 a) Pump efficiency curve of mixed flow pump b) Efficiency prediction curve using proposed affinity law for viscosity

Le Fur[8] presented efficiency data which is presented in Figure 13 as a function of flow rate and using the Modified Affinity Law relationships. The water flow has a much higher efficiency while the laminar flow cases attain a much lower efficiency, 40% compared to 70%. However, for the case of efficiency, a single curve for both the laminar and turbulent flows can be obtained.

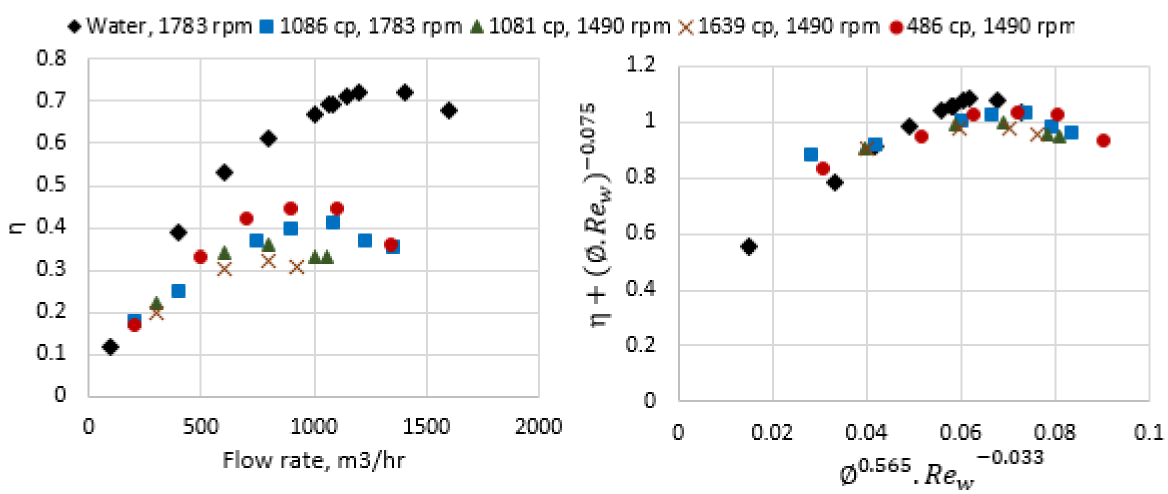


Figure 13. Pump efficiency curve for Fur[8] data.

EFFECTS OF ADDED GAS

Pumps used by energy companies to extract fluids from wells frequently must accommodate not only liquids but gas as well. The addition of gas greatly increases the data required to provide operators an accurate performance map covering the wide ranges of density, viscosity, and gas content that may be present during the life of a well.

Experimental data for the split vane impeller pump (Figure 7 a)) operating with water and air obtained by Kirkland[6] and is presented in Figure 13.

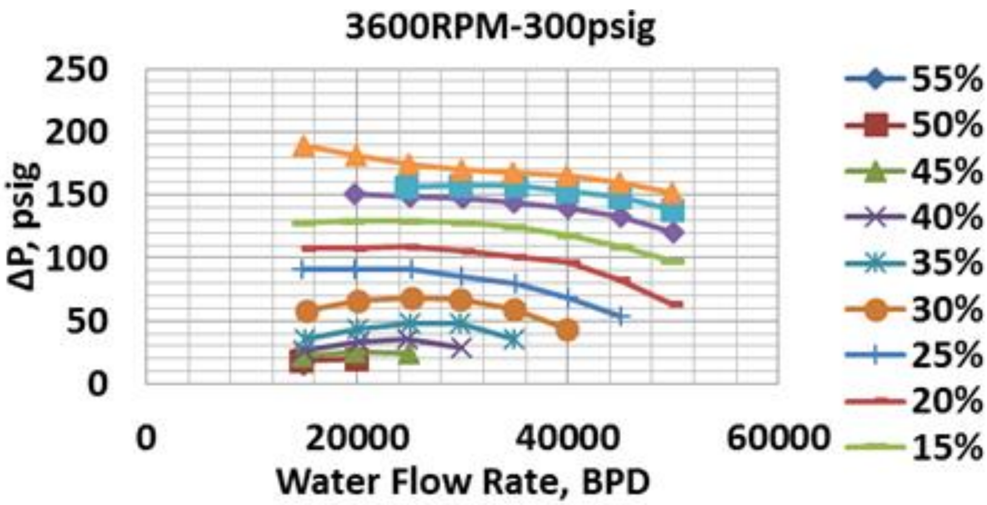


Figure 14: Three Stage performance data rotating speed is 3600 RPM and pump inlet pressure is 300psig

Figure 14 shows a wide range of pump performance variation with the gas content with head produced decreasing with increasing gas content. It was also observed that the range of flow rates possible was reduced. In order to collapse these data, a head ratio is defined [Sahand et al OTC]. It is used to compare the stage by stage performance of similar or different multiphase pumps. It is defined as

$$Head\ ratio = \frac{h}{h_{homo}}$$

where h is the head obtained from experimental data with gas present and $h_{homo}$is the homogenous head obtained from liquid only flow. The head ratio varies from 0 to 1, where 1 is the liquid only condition and zero is when no head is developed. Using this technique on the data of Kirkland[], Figure 15 is obtained. This figure shows that this similitude relationship is valid for all liquid flow rates over a range of Gas Volume Fraction (GVF) at the inlet up to the point when the head developed has been decreased by 80%. This relationship is also independent of the fluid pressure. Therefore, as the flow progresses through a multi stage pump, the GVF at the inlet of each stage can be calculated and the head ratio used in conjunction with the Modified Affinity Laws to predict the head rise in each stage. This permits the user to know how a pump will perform with varying fluid viscosity, fluid density, and gas content. It will also allow the user and manufacturer to determine the number of stages a pump will require to produce the head required over the range of fluid viscosities and gas content.

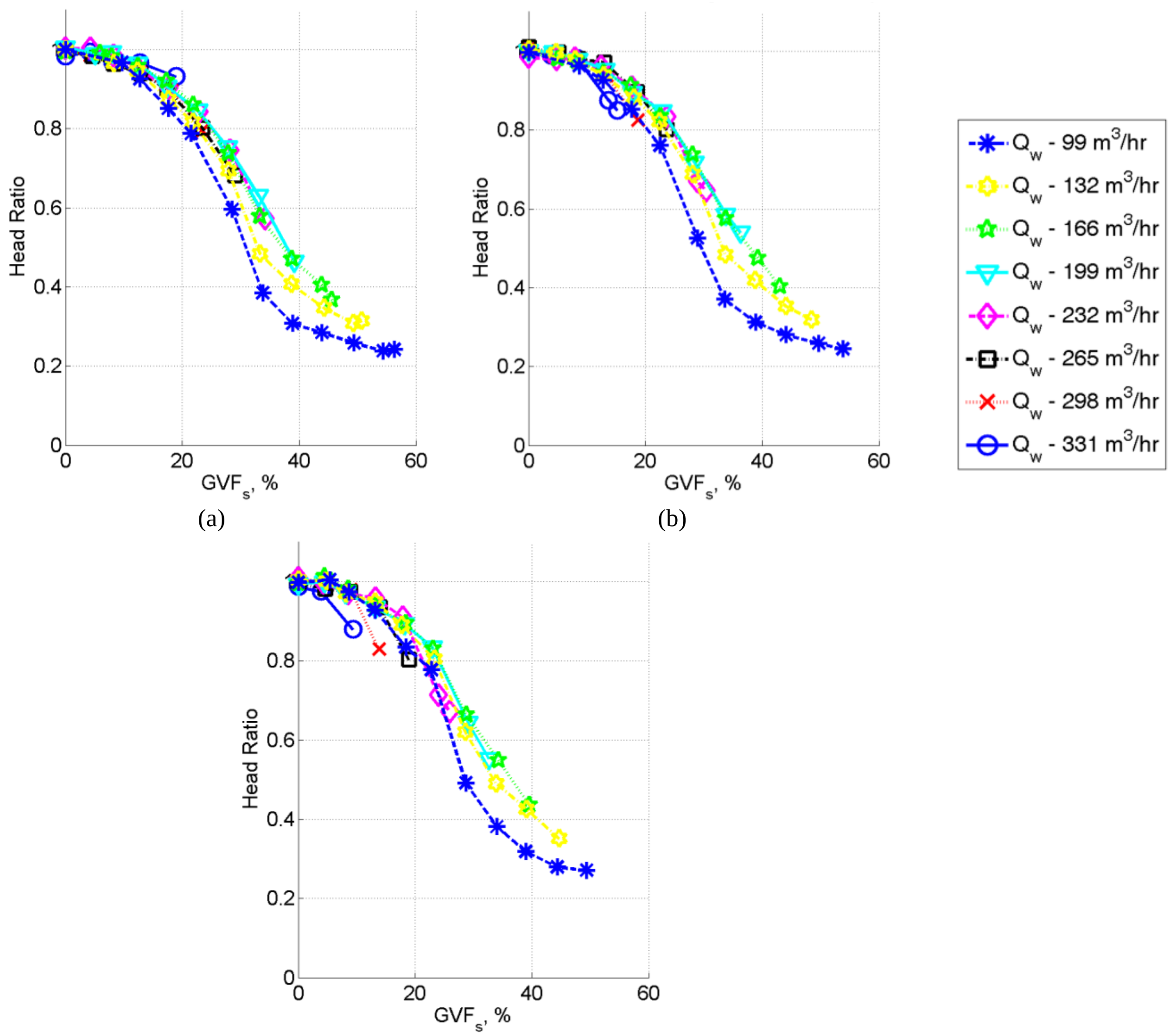


Fig.15: Rotating speed effects on 3rd stage head ratio (a) 3600 *RPM* (b) 3300 *RPM* and (c) 3000 *RPM* for 2.07 *MPa* pump inlet pressure for the split vane impeller pump, Q_w is the water flow rate.

The split vane impeller pump produced very homogenous two phase flow. This is the best case scenario for this type of analysis. This analysis may introduce uncertainties in predicting the head degradation if different pump type is chosen. Uncertainty is also higher for first stage due to inlets losses and separated flow at the inlet. Work is underway to obtain more data for various types of pumps and fluids with a wide range of viscosity to determine the applicability of this technique to other flow conditions.

CONCLUSIONS

The pump Affinity Laws have been modified to include the effects of viscosity. The results are simple 2D curves that represent the head-flow rate and efficiency-flow rate relationships for a specific pump. The new Modified Affinity Laws were validated using experimental data from several different types of pumps and CFD simulations of a mixed flow pump. A rotational Reynolds number was added to the nondimensional groupings normally used. By multiplying the flow coefficient by this rotational

Reynolds number raised to a power, the effects of viscosity were taken into account. The value of this exponent takes into account the internal viscous affects and varies for different specific pump designs. Data by Le Fur[8] included laminar and turbulent flows inside a pump. The rotational Reynolds number dependency changed between the laminar and turbulent cases. This necessitates that each pump series must undergo individual testing (experimental or CFD) to determine this exponent value and if the pump transitions between laminar and turbulent flow. The pump efficiency was also dependent upon the rotational Reynolds number. This relationship was validated using the CFD simulations and Le Fur's[8] data. The Turbomachinery Laboratory is modifying a closed loop test facility to operate over a wide viscosity range (at least 1 to 1000 cp) with pumps up to 250 HP to obtain the data necessary to provide more detailed experimental verification.

An additional similitude relationship was developed to obtain a 2D curve showing how the addition of gas changes the head generated in a single stage. This relationship expresses the head ratio of the liquid and gas mixture to that of only liquid. This relationship is based upon experimental data obtained in the Turbomachinery Laboratory for a three stage split vane impeller pump. The single curve is valid for all pump flow rates, pump speeds, stage inlet pressure, and GVF up to when 80% of a single pump stage head has been lost due to the presence of gas. Since the gas volume changes with each stage due to the pressure rise in each stage, this relationship can be used along with the Modified Affinity Laws to predict the performance of a single stage pump or the stage by stage performance of a multi stage pump operating with gas and varying viscosity.

The development of these scaling laws for rotordynamic pumps provides a relatively easy technique to predict the performance of a single or multi stage pump for varying shaft speed, flow rate, fluid viscosity, fluid density, and gas content. The Modified Affinity Laws and the head ratio/GVF relationship not only make the use of the simple resulting equations easy, but significantly reduces the amount of data required to obtain the performance curves for a specific pump. The Modified Affinity Laws only requires a pump be tested at two different viscosities to determine the value of the rotational Reynolds number exponent. It is best to have these two cases span the range of expected viscosities that will be present.

NOMENCLATURE

ESP	Electrical Submersible Pump
GVF	Gas Volume Fraction
D_s	Pump Impeller Diameter
G	Acceleration Due to Gravity
H	Head
P	Pressure
ΔP	Pressure Rise
Q	Volumetric Flow Rate
Re_ω	Rotational Reynolds Number = $\rho\ \omega\ D_s^2/\mu$
Γ	Pump Shaft Torque
ω	Pump Shaft Speed, rpm
ϕ	Flow Coefficient
ψ	Head Coefficient
Π	Power Coefficient
η	Efficiency
ρ	Fluid Density
μ	Fluid Viscosity, Absolute
ε	Surface Roughness

REFERENCES

[1] Buckingham, E., "On Physically Similar Systems: Illustrations of the Use of Dimensional Equations,"Physical Review, 4, 4, 1914, pp.345-376.

[2] Moody, L.F., Friction Factors for Pipe Flow," Transactions of ASME, 66, 8, November 1944, pp. 671-684.

[3] Morrison Gerald., Wenjie Yin, Rahul Agarwal, and Abhay Patil, "Evaluation of Effect of Viscosity on an Electrical Submersible

Pump,' FEDSM2017-69157, ASME 2017 Fluid Engineering Division Summer Meeting, July 31 – August 3, 2017, Waikoloa, Hawaii, USA

[4] White, Frank M., "Fluid Mechanics," McGraw Hill, 4th Edition, 1999, ISBN0-07-069716-7.

[5] Stel, H., Sirino T., P.R.Pamella., "CFD Investigation of the Effect of Viscosity on a Three-Stage Electric Submersible Pump." In ASME 2014 4th Joint US-European Fluids Engineering Division Summer Meeting collocated with the ASME 2014 12th International Conference on Nanochannels, Microchannels, and Minichannels. 2014. American Society of Mechanical Engineers.

[6] Kirkland, Klayton, "Kirkland, K. (2012). "Design and fabrication of a vertical pump multiphase flow." MS, Texas A&M University.

[7] Pirouzpanah, S., Gudigopuram, S. R., Morrison, G. L., "Two-phase Flow Characterization in a Split Vane Impeller Electrical Submersible Pump", Journal of Petroleum Science and Engineering, Volume 148, January 2017, Pages 82-93

[8] Le Fur, Brigitte, Moe, C.K., and Cerru, F., High Viscosity Test of a Crude Oil Pump," 44th Turbomachinery and 31st Pump Symposium, September 14-17, 2015, Houston Texas, USA.