Determination of Critical Head in Soil Piping

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Abstract: One of the main mechanisms of failure of levees is a phenomenon called "piping," which generally begins with the formation of a sand boil at the leeward side of the levee, and has been frequently observed to proceed upstream along the base of the levee through a slit formation. The issue of most important concern is to estimate the critical head that could promote the occurrence of piping. Considering the flow through porous media and coupling it with Bernoulli's equation and a critical tractive stress condition, a model is developed for the critical head. Using appropriate transformations, the proposed model takes on a form which supports Bligh's empirical findings. Another model based on critical velocity is also developed to estimate the critical head. The functional form of these two models is evaluated using the critical head versus porosity data from a number of laboratory studies conducted in the Netherlands. These models were found to perform better than did Terzaghi's model.

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Introduction

Levees have been built for flood control, irrigation works, recreation activities, prevention of saltwater intrusion, and navigation. Levee failures are common and are caused by a multitude of factors, such as poor construction, inadequate design, piping, and improper maintenance, to name but a few. The phenomenon of piping is commonly observed under levees, and involves subsurface erosion of soil particles in the land-facing zone of levees (see Fig. 1). Piping is a form of seepage erosion and refers to the development of subsurface channels in which soil particles are transported through porous media. Piping begins at the land-facing side of the structure where the flow lines converge. High seepage pressure may force a slit to develop; then the process of erosion develops backward under the levee, and if the process continues, the structure may be undermined and collapse.

Based on a large number of failures due to piping, Bligh (1910) proposed empirical rules for preventing piping. Bligh's work is considered to be pioneering, in that it was supported by a large number of field studies. Since the work of Bligh (1910), limited attempts have been made to develop alternative models of piping with a view to providing a theoretical basis for Bligh's empirical rules. Lane (1935) developed another set of empirical rules to safeguard structures against piping. Based on their experience with the U.S. Army Corps of Engineers, Turnbull and Mansur (1961a,b) summarized flood-induced seepage under levees. Peter (1974) examined the conditions leading to piping in the subsoil and near levees in the Mississippi region in the United States and in the Danube region in former Czechoslovakia, Hungary, and Yugoslavia. Khilar et al. (1985) developed criteria for piping in clayey soils. Meyer et al. (1994) and Budhu and Gobin (1995) also addressed the problem of seepage erosion and presented case studies. Meyer et al. (1994) stated that there was a distinct lack of models dealing with the piping phenomenon.

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Using analytical as well as numerical solutions of the underlying flow domain beneath a structure, Sellmeijer (1988) provided an expression for the critical head which should not be exceeded to avoid failure due to piping. Sellmeijer's work with relevant details is also available from Sellmeijer and Koenders (1991). Without emphasizing the need for another model, Weijers and Sellmeijer (1993) proposed a modified equation for the critical head. Based on the agreement between his model and that of Bligh (1910), Sellmeijer (1988) reasoned that his model was a suitable model. However, his model does not convert to Bligh's model and therefore it is difficult to interpret the Sellmeijer model as providing a theoretical basis to the Bligh model, because the two models have different forms. Thus, there is a need for the development of an alternate model for critical head computations which may faithfully mimic the role of different parameters with regard to the critical head.

The objective of this study was to undertake a comparative evaluation of two equations proposed by Sellmeijer and his associates, and to develop a critical head model that would provide a theoretical basis for Bligh's empirical model. For the purpose of developing the critical model, the Carman-Kozeny head loss model for flow through porous media (Rich 1961) as well as a capillary flow model (Thevanayagam and Nesarajah 1998) were employed.

Review of Literature

The analysis of the seepage zone below hydraulic structures has been extensively reported in the literature (Harr 1962). The equation describing the flow field is the Laplacian equation in the velocity potential ϕ, expressed as

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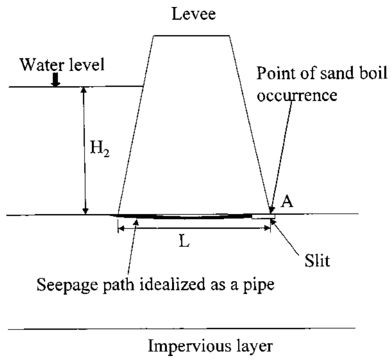

Fig. 1. Definition sketch

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \tag{1}$$

where x is measured left along the base from the point of sand boil occurrence (see Fig. 1), and y is measured downward from A.

Although the solution of such an equation subject to appropriate boundary conditions can lead to evaluation of uplift pressures at any point on the base of the structure, several attempts have been made to assess the critical head, which is essentially the height of the stored water in the reservoir up to which the levee is safe against downstream erosion or the effect of piping. Among several studies on estimation of the critical head, the work by Bligh (1910) is considered seminal and is widely used in the Netherlands and other countries (Sellmeijer 1988). Analyzing a large number of failures from field studies, Bligh (1910) proposed that

$$\frac{L}{H_{\text{crit}}}=E \tag{2}$$

where L = length of the base of the levee perpendicular to the flow in the river; and H_{crit} = critical head. The value of the ratio E depends on the type of material and is given for four different types of materials in Table 1. Bligh (1910) assumed that the seepage path was mainly concentrated near the base of the structure and thus L in Eq. (2) refers to the base length of the levee.

Another useful work, which accounts for the vertical movement of flow lines and anisotropy of the porous medium, is by Lane (1935). Meyer et al. (1994) modified Lane's criteria by making use of horizontal and vertical permeabilities. The work by Bligh and Lane is noteworthy for stimulating interest in the piping phenomenon (Arulanadan et al. 1975; Peter 1982; Arulanadan and Perry 1983; Khilar et al. 1985). Considering the equilibrium of forces in the soil, Terzaghi (1929) (also refer to Peter 1982) proposed the following equation for the critical hydraulic gradient:

$$i_c=\frac{(\gamma_s-\gamma_w)}{\gamma_w}(1-n) \tag{3}$$

Table 1. Bligh's Thumb Rules for Obtaining $L/H_{\text{crit}}(=E)$ Sellmeijer 1988

Type of foundation material	E
Riverbeds of light sandy sand	18
Fine micaceous sand	15
Coarse-grained sand	12
Boulders or shingle and gravel and sand	5–9

where γ_s = specific weight of soil; γ_w = specific weight of the water; and n = clean bed porosity.

Using a capillary model, Khilar et al. (1985) suggested the following equation for evaluation of the critical gradient i_c $(=H_{\text{crit}}/L)$:

$$i_c=\frac{\tau_c}{2.828}\left(\frac{n}{K}\right)^{0.5} \tag{4}$$

where n = porosity; K = hydraulic conductivity; and τ_c = critical shear stress. In the case of clays, a maximum value of L/H_{crit} equal to 40 has been recommended by Khilar et al. (1985). This value of the gradient is also the maximum value in the pinhole test (Sherard et al. 1976), which is used to identify dispersive soils.

Sellmeijer (1988) considered the development of a slit (see Fig. 1) as an important parameter for computing the critical head. The slit should not grow to support the occurrence of piping. Considering the limiting equilibrium analysis of forces within the sand boil and slit, Sellmeijer (1988) proposed the following equations.

In the sand boil

$$\frac{\gamma_w}{\gamma_{\text{sub}}}+\frac{p}{q}\cot\Phi=1 \tag{5}$$

and

$$h=\frac{P}{q} \tag{6}$$

where γ_{sub} = submerged unit weight of soil $=(1-n)\gamma_p-n\gamma_w$; γ_w = unit weight of water; γ_p = unit weight of soil particles; n = porosity; $p=\partial\phi/\partial x$; $q=-\partial\phi/\partial y$; both derivatives are at $y=0$; h = height of sand boil; Φ = internal friction angle; and $P=\phi(x,0)$. The submerged unit weight of soil is directly obtained by recalling that the total volume of a soil V_t is made up of the volume of voids V_v and the volume of particles or solids V_s. Thus, $V_t=V_v+V_s$ and the porosity $n=V_v/V_t$. When the soil is submerged, one can write for $\gamma_{\text{sub,soil}}V_t=\gamma_w V_v+\gamma_p V_s$, which when divided by V_t gives γ_{sub}.

In the piping channel (Sellmeijer 1988)

$$Cq+p\left(\frac{3}{\pi\eta}\frac{a}{d}+1\right)\cot\Phi=\frac{\gamma_p}{\gamma_w} \tag{7}$$

$$pa^3=12\kappa Q \tag{8}$$

In Eqs. (7) and (8), C and η = coefficients; a = width of the slit; d = median particle size; γ_p = unit weight of submerged particles; and κ = intrinsic permeability. Q and q are related as $q=-dQ/dx$ or the discharge gradient specifying the flow variation in the x direction.

Based on analytical and numerical solutions of Eqs. (1) and (5)–(8), Sellmeijer (1988) concluded that the critical condition occurs when the slit length approaches one-half of the base length, and then the critical head H_{crit} is given by

$$H_{\text{crit}}=c^*\frac{\gamma_p}{\gamma_w}\tan(\theta)[1-0.65(c^*)^{0.42}]L \tag{9}$$

where

$$c^*=0.25\pi\eta\left(\frac{2d^3}{kL}\right)^{1/3} \tag{10}$$

and

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Table 2. Comparative Evaluation of Eqs. (15) and (16)

c	Eq. (15)	Eq. (16)
0.00001	9.84E−06	1.83E−05
0.0001	9.76E−05	0.00016
0.001	0.00095	0.0014
0.01	0.009	0.011
0.02	0.017	0.021
0.05	0.040	0.049
0.1	0.075	0.091
0.2	0.133	0.17
0.5	0.255	0.37
0.6	0.284	0.44
0.7	0.307	0.50
0.8	0.325	0.56
1	0.349	0.68

$$k = \frac{\upsilon}{g} K_h \tag{11}$$

In Eqs. (9)–(11), θ = bedding angle or angle of repose (degrees); η = drag coefficient; L = seepage length; υ = kinematic viscosity; and g = acceleration due to gravity.

In a subsequent modification, Weijers and Sellmeijer (1993) suggested the following expression for computation of the critical head:

$$H_{\text{crit}} = \alpha c^{**} \frac{\gamma_p}{\gamma_w} \tan(\theta)(0.68 - 0.10 \ln c^{**}) L \tag{12}$$

with

$$\alpha = \left(\frac{D}{L}\right)^{\beta}, \quad \beta = \frac{0.28}{[D/L]^{2.8} - 1.0} \tag{13}$$

and

$$c^{**} = \eta \left(\frac{d_{70}^3}{kL}\right)^{1/3} \tag{14}$$

where D = thickness of soil layer; and d_{70} = 70% finer (by weight) grain size diameter. Eqs. (13) and (14) were also tested using experimental data. However, the results were reported to be satisfactory in supporting only limited experimental observations.

Evaluation of Existing Relationships

Inspection of Eqs. (9) and (12) indicates that these two equations are similar with some differences in the functional representation of c^* or c^{**}. For comparative purposes, a situation with very large values of D for which α tends to unity is considered. The two functions, which are different in the proposed equations, are rewritten as

$$f_1 = c^*[1 - 0.65(c^*)^{0.42}] \tag{15}$$

and

$$f_2 = c^{**}(0.68 - 0.10 \ln c^{**}) \tag{16}$$

In Eqs. (15) and (16), based on comparative evaluation of Eqs. (10) and (14), c^* and c^{**} are found to be related as

$$c^* = 0.989 c^{**} \tag{17}$$

Table 2 indicates that, depending on the values of c^* or c^{**}, these two functions can differ appreciably from each other and thus the utility of these equations becomes questionable. This also signifies the need for the development of an alternative model for computation of the critical head. Eq. (16) always yields higher values of the critical head when compared with Eq. (15).

Development of Critical Head Model

Use of Head Loss Model through Porous Media

The proposed approach utilizes a head loss model through porous media and a critical tractive force model to describe the initiation of motion of media particles. Among the popular models of head loss for flow through porous media, use is made of the Carman-Kozeny head loss model (Rich 1961). To describe the head loss h_f through a porous medium, this model can be expressed as

$$h_f = f\left(\frac{L}{\phi d}\right)\left(\frac{1-n}{n^3}\right)\left(\frac{V_s^2}{g}\right) \tag{18}$$

where f = coefficient of friction; ϕ = shape factor = 1 for spherical particles; n = porosity of bed; g = acceleration due to gravity; and V_s = mean flow velocity. It should be noted that Eq. (18) is similar to the Darcy-Weisbach equation commonly used in pipe and channel flow. In Eq. (18), f is given by

$$f = 150 \frac{1-n}{N_{\text{Re}}} + 1.75 \tag{19}$$

In Eq. (19), R is the Reynolds number and is given by

$$\mathsf{R} = \frac{\phi d V_s}{\upsilon} \tag{20}$$

For R up to 1 (for laminar flow) and $n = 0.4$, the term containing R in Eq. (19) is as high as 90 and thus the term 1.75 can be neglected without much loss of accuracy.

Another aspect of the development of a critical head model is the use of a relationship of critical tractive stress. When a fluid flows through a porous medium, it exerts a shear stress on the porous media particles. As this shear stress increases, there is a limit beyond which a further increase in the shear stress will lead to transport of particles. This limiting shear stress is also known as critical shear stress. For erosion of soil particles to take place, it is evident that the shear stress acting on these particles must exceed the critical shear stress. For clay particles, the critical shear stress τ_c used by Khilar et al. (1985) is

$$\tau_c = 10 d_{50} \tag{21}$$

In Eq. (21), τ_c is in g/m^2 and d_{50} is in mm. Eq. (21) was originally proposed by Lane (Khilar et al. 1985) for sand particles. The variation of critical shear stress is generally nonlinear with d (Swamee and Ojha 1994). However, within smaller intervals, the shear stress can still be described using a linear model as

$$\tau_c = c d_{50} \tag{22}$$

where c is a coefficient which may vary with d. In the case of clay particles, electrical attractive forces and electrical repulsive forces influence the intergranular stress and thus these forces may also influence the magnitude of c in Eq. (22) or the functional dependency of shear stress on the particle diameter. However, for granular soils, silts and clays of low plasticity, the magnitude of electrical attractive forces and electrical repulsive forces are small and for all practical purposes, these can be neglected (Das 1983). Thus, for the present work in which a macroscopic view is adopted to describe the initiation of motion of particles, Eq. (22)

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is considered for subsequent use because of simplicity. Nevertheless, it is acknowledged that the representation of the critical shear stress acting on particles in a porous medium is not the same as it is in open channel flow. In fact, it is quite likely that pressure forces may be more dominant when compared with the velocity forces. It is well known that the shear strength of the soil is dependent on integranular effective stress, cohesion, and the angle of internal friction. Thus, the representation of shear stress versus particle diameter may not be as simple as given by Eq. (22).

Under the assumption that the flow through porous media (below the levee) can also be idealized as an assemblage of flows through a network of parallel pipes of diameter d (equal to the mean grain size), it is possible to compute certain characteristics of such flows, including the shear stress τ acting at the wall of the pipe, which can be expressed as (Albertson et al. 1964)

$$\tau=\gamma_w\frac{d}{4}\frac{\partial h}{\partial x} \tag{23}$$

Using Eqs. (22) and (23), an estimate of the maximum permissible gradient of h with x or h_f/L, or h_f, can be obtained as

$$h_f=\frac{4c}{\gamma_w}L \tag{24}$$

The coefficient c is introduced to accommodate various types of materials, such as sand, silt, and clay.

The head loss model involves the velocity V_s. Applying Bernoulli's theorem between the upstream water surface and to the point of the sand boil occurrence, an expression for velocity V_s can be written as

$$V_s=\sqrt{2g(H_2-h_f)} \tag{25}$$

where H_2 = depth of water standing at the levee; and h_f = head loss in the pipe (see Fig. 1).

Since the additive term 1.75 in Eq. (19) is not significant in influencing f at the lower R values, it can be dropped to simplify the expression for f. Furthermore, substitution of R from Eq. (20) with a shape factor of unity simplifies the expression for h_f in Eq. (18) as

$$h_f=\frac{150\upsilon}{gd}\left(\frac{L}{d}\right)\left(\frac{(1-n)^2}{n^3}\right)V_s \tag{26}$$

Substituting for V_s from Eq. (25) into Eq. (26) leads to a quadratic expression in h_f

$$h_f^2+2g\alpha_1^2L^2h_f-2g\alpha_1^2L^2H_2=0 \tag{27}$$

where

$$\alpha_1=\frac{150\upsilon}{gd^2}\left(\frac{(1-n)^2}{n^3}\right) \tag{28}$$

Eq. (27) is a quadratic equation whose solution yields to the following real root of h_f:

$$h_f=-g\alpha_1^2L^2+\sqrt{g^2\alpha_1^4L^4+2g\alpha_1^2L^2H_2} \tag{29}$$

Substituting for h_f from Eq. (24) into Eq. (29) and solving for H_2 leads to the following expression for the critical head:

$$H_2=H_{\text{crit}}=\frac{16c^2}{2g\gamma_w^2\alpha_1^2}+\frac{4cL}{\gamma_w} \tag{30}$$

or

$$H_{\text{crit}}=\frac{16c^2gd^4}{45{,}000\upsilon^2\gamma_w^2}\frac{n^6}{(1-n)^4}+\frac{4cL}{\gamma_w} \tag{31}$$

Eq. (31) can be rearranged as

$$\frac{L}{H_{\text{crit}}}=\frac{\gamma_w}{4c}\left[1+\frac{4cgd^4}{45{,}000\upsilon^2\gamma_w}\frac{n^6}{(1-n)^4}\frac{1}{L}\right]^{-1} \tag{32}$$

For

$$\frac{4cgd^4}{45{,}000\upsilon^2\gamma_w}\frac{n^6}{(1-n)^4}\frac{1}{L}\ll 1 \tag{33}$$

and expanding Eq. (32) using the binomial theorem, neglecting the second and higher order terms in the binomial expansion, one obtains

$$\frac{L}{H_{\text{crit}}}=\frac{\gamma_w}{4c}\left[1-\frac{4cgd^4}{45{,}000\upsilon^2\gamma_w}\frac{n^6}{(1-n)^4}\frac{1}{L}\right] \tag{34}$$

Eq. (34) is similar in appearance to Bligh's empirical relation. For finer particles the bracketed term will be very nearly 1. In the case of coarse particles, the bracketed term of Eq. (34) will also be less than unity (as d is increased). This result may lead to L/H_{crit} being smaller in magnitude for coarse particles than it is for finer particles if the value of c is assumed to be not very sensitive to d. If one compares this behavior with Table 1, which gives the values of the empirical rules of Bligh, a similarity can be found. With $c=10\ \text{kg/m}^3$ [Eq. (21)] and $\gamma_w=1{,}000\ \text{kg/m}^3$, for very fine particles, L/H_{crit} approaches 25. It can be seen that this limit is not significantly far from what was suggested by Bligh (1910). With the increase in d, this ratio decreases, as also manifested by Table 1.

Use of Critical Velocity Concept

The concept of critical velocity has been popular in the literature on sediment transport. If the flow velocity exceeds the critical flow velocity V_c one should expect initiation of the motion of particles. V_c can also be expressed as (Garde and Ranga Raju 1985).

$$V_c=\sqrt{\frac{\tau_c}{\rho}}=\sqrt{\frac{cd}{\rho}} \tag{35}$$

Using Eq. (35) in Eqs. (25) and (26), the following expression for the critical head, which corresponds to $V_s=V_c$, can be obtained:

$$H_{\text{crit}}=\frac{cd}{2\gamma_w}+\frac{150\upsilon}{gd}\left(\frac{L}{d}\right)\frac{(1-n)^2}{n^3}\sqrt{\frac{cd}{\rho}} \tag{36}$$

Evaluation of Performance

To evaluate the performance of the models developed, use is made of the critical head versus porosity observations reported by Weijers and Sellmeijer (1993) for a variety of sands. Although the details pertaining to the size of the different sand types and lengths of the dike are not given, it is believed that the authors might have used the same set of other variables, i.e., length of the levee L, diameter d, and a fluid with the same kinematic viscosity and mass density, while presenting the influence of porosity on the critical head. Going through the literature, it is clear that no information is available on the quality of data reported by Weijers and Sellmeijers (1993) and used by a number of other investigators. It is assumed that the data are of acceptable quality. Eqs. (31)

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Table 3. Porosity and Corresponding Observed Critical Head for Different Sand Types (Weijers and Sellmeijer 1993)

Type of sand	Porosity n	Critical head (m)
Dune sand	0.352	0.370
	0.355	0.340
	0.363	0.270
	0.370	0.245
	0.390	0.220
	0.410	0.165
River sand 1A	0.35	0.40
	0.36	0.36
	0.37	0.30
	0.39	0.20
Coarse sand	0.32	0.78
	0.34	0.66
	0.37	0.44

and (36) are considered for their ability to account for porosity variations. To proceed further, Eq. (31) can be written in a general form as

$$H_{\mathrm{crit}}=a\,\frac{n^6}{(1-n)^4}+b \tag{37}$$

with

$$a=\frac{16c^2gd^4}{45{,}000\upsilon^2\gamma_w^2}\quad\text{and}\quad b=\frac{4cL}{\gamma_w} \tag{38}$$

Similarly, Eq. (36) can be written in a general form as

$$H_{\mathrm{crit}}=a\,\frac{(1-n)^2}{n^3}+b \tag{39}$$

with

$$a=\frac{150\upsilon}{gd}\left(\frac{L}{d}\right)\sqrt{\frac{cd}{\rho}}\quad\text{and}\quad b=\frac{cd}{2\gamma_w} \tag{40}$$

Assuming that the parameters a and b are the same in each of the H_{crit} versus n figures of Weijers and Sellmeijer (1993), corresponding to the observed values of H_{crit}, the objective was set to minimize the following error function R^2:

$$R^2=1-\frac{\Sigma(H_{\mathrm{crit,observed}}-H_{\mathrm{crit,estimated}})^2}{\Sigma(H_{\mathrm{crit,observed}}-H_{\mathrm{crit,average}})^2} \tag{41}$$

In Eq. (41), the symbol Σ represents the summation for all the values of a particular model output. Table 3 contains the digitized data from the graphs of H_{crit} versus n presented by Weijers and Sellmeijer (1993). Using these data, Figs. 2 and 3 were constructed. The best linear fit equations in respect of the present model along with the coefficient of determination (R^2) are given in Table 4. Higher values of R^2 associated with the present models reflect the fact that their functional dependence on the porosity is acceptable. For comparative purposes, the utility of functional representation of other models can also be examined. As the Khilar et al. (1985) model is for clay, this model was not considered. In view of Eq. (3), a model relating the critical head to the term $(1-n)$ was also calibrated for three types of sand data. Needless to say, the fit was not good as it was characterized by very low values of R^2. This comparison indicates that Eqs. (37) and (39) are superior to Eq. (3) in terms of the functional dependence on porosity for a piping situation.

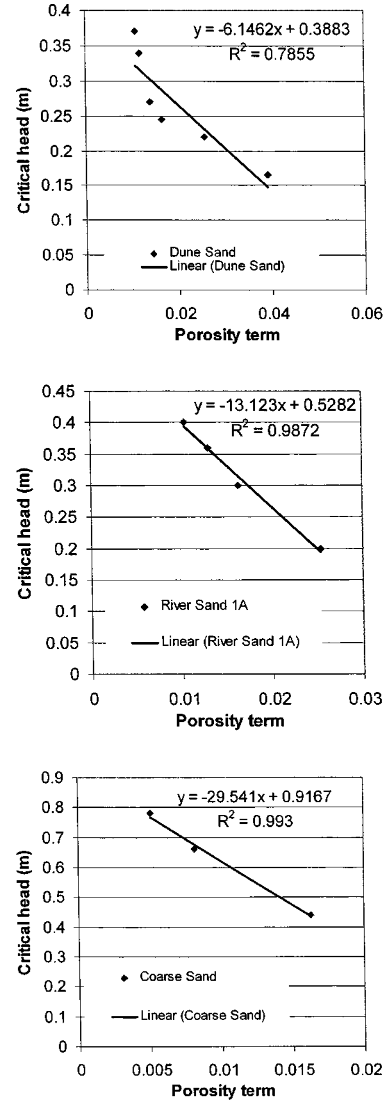

Fig. 2. Calibration of present model [Eq. (37)] for dune sand, river sand IA, and coarse sand data. Variation of critical head with porosity term in (top) Eq. (37); (center) Eq. (31); and (bottom) Eq. (31).

Results and Discussion

Inspection of Figs. 2 and 3 and Table 4 indicates that the proposed model, based on Eqs. (37) and (39), is capable of accommodating the variation in porosity. On the contrary, a capillary-flow-based model lacks any sensitivity to porosity variations (see the Appendix) and its use is thus not justified.

Soils normally consist of particles of different sizes. In a flood event, it is likely that, depending on the available head, finer particles get washed away. This increases the effective diameter and the porosity, which lowers the permissible L/H_{crit} value. Thus, in the next event, if a flood of the same stage occurs, the probability of failure will certainly be on the higher side. The present model reflects this behavior. In view of Figs. 2 and 3, if one has an idea about the change in porosity, the critical head can be computed, corresponding to a new value of porosity. Thus, the present model can be utilized to plan safety measures. In such cases, Bligh's rules cannot be used, for they are independent of porosity for any such planning.

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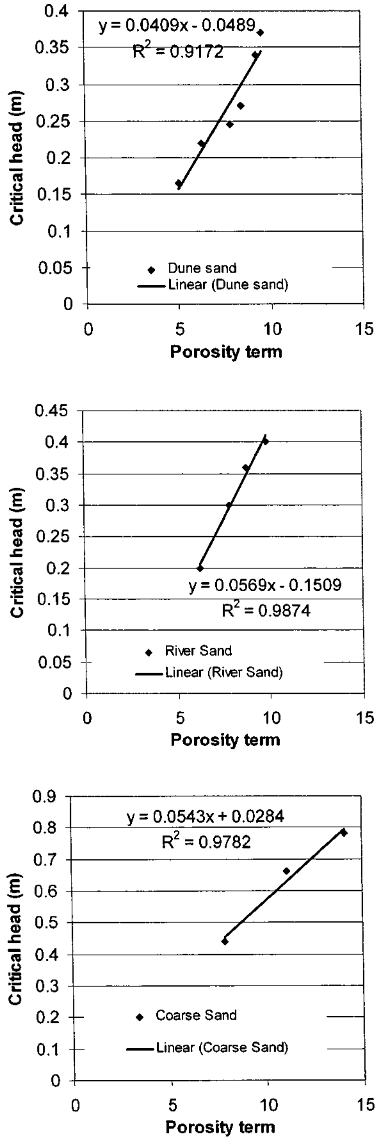

Fig. 3. Calibration of present model [Eq. (39)] for dune sand, river sand 1A, and coarse sand data. Variation of critical head with porosity term in (top) Eq. (39); (center) Eq. (37); and (bottom) Eq. (37).

Considering the similarity of the behavior of the proposed model with the empirical rules of Bligh (1910), and its ability to calibrate reasonably well the observed critical heads for different sizes of particles in the experimental data from the Netherlands, it is expected that the present work will offer a better understanding of the piping phenomenon. Also, there remain many unresolved issues including the consideration of the depth of the underlying formation, role of the slit, sensitivity with *c*, etc., in the analysis and further calibration as well as development of the proposed model beyond laminar flow range merit attention in the future. The relationship between model parameters and calibration coefficient could not be established in the present investigation, largely because full information about the soil types and sufficient quantity of data are lacking. For all three types of sand, the value of *a* was not always positive, which must be as is apparent from Eqs. (38) and (40). The value of *a* is positive only in the case of coarse sand and that too with the use of a critical head model based on the critical velocity concept. The nature of *a* is found to be highly sensitive to the porosity term. For example, *a* becomes positive if the porosity term is taken as the inverse of the one reported in Eq. (37), for the porosity terms in Eqs. (37) and (39) are the reverse of each other. This fact is also supported by existing head loss equations (Rich 1961) which differ considerably in terms of their representation of the porosity term. Thus, the influence of the use of different head loss expressions will constitute the subject of subsequent investigations. One of the key issues in this work has been to study the variation of the critical head with porosity, and it is interesting to see that Eqs. (37) and (39) perform better than Eq. (3). Between Eqs. (37) and (39), Eq. (39) appears to be a better choice as the model parameters associated with the porosity term are always positive. For three types of sand considered here, the variability between these model parameters is also less.

The model parameter *b* also happens to be positive for the coarse sand data with the use of Eq. (39). However, the errors inherent in the data may also lead to a change in the sign of the *b* parameter. It appears that with the availability of additional data in future, the relative merit in using Eq. (37) or Eq. (39) can be better judged. It is noted, however, that the piping phenomenon is also dependent on the intergranular soil stresses, which have not been considered in the present analysis. It is possible that, because of this simplification in the present analysis, the positivity of the model coefficients has not always been preserved.

In the proposed model, the length of the seepage path has been considered as the base length of the structure. However, piping need not necessarily follow along the base of the structure. In fact, the word "piping" may sometimes refer to the removal of soil along discontinuities in an earthen structure or its foundation

Table 4. Calibrated Model Parameters and Error Statistics

Model type	Type of data	Calibrated equation	R^2
Present [Eq. (37)]	Dune sand	$H_{crit} = -6.1462$ [porosity term of Eq. (37)] $+0.3883$	0.7855
	River sand 1A	$H_{crit} = -13.123$ [porosity term of Eq. (37)] $+0.5282$	0.9872
	Coarse sand	$H_{crit} = -29.541$ [porosity term of Eq. (37)] $+37+0.915867$	0.9479
Present [Eq. (39)]	Dune sand	$H_{crit} = 0.0409$ [porosity term of Eq. (39)] -0.0489	0.9172
	River sand 1A	$H_{crit} = 0.0569$ [porosity term of Eq. (39)] -0.1509	0.9874
	Coarse sand	$H_{crit} = 0.0543$ [porosity term of Eq. (39)] $+0.0284$	0.9782
Terzaghi	Dune sand	$H_{crit} = 0.43\,(1-n)$	0.2212
	River sand 1A	$H_{crit} = 0.50(1-n)$	0.1062
	Coarse sand	$H_{crit} = 0.96(1-n)$	0.2595

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(Van Zyl and Harr 1981). An analysis of discontinuities within the structure or foundation is beyond the scope of this paper. However, whenever flows occur through such discontinuities, these can always be idealized as pipe flows, and the length of such paths can be used in place of the base length of the structure.

One of the assumptions of the study has been the use of the atmospheric pressure at point A in Fig. 1. This is a reasonable assumption when there is no depth of water available. However, if a certain flow depth is available in the land-facing side of the levee, the pressure term, equal to the specific weight of water times the flow depth, must be included in the energy equation.

The present study refers to only one type of seepage erosion. The literature also refers to other types of seepage erosion, such as heave and internal erosion. Heave is the upward movement of soil when subjected to a high seepage gradient in the vicinity of the exit flow. Heave is generally analyzed by considering a balance of upward seepage force and the submerged weight of the soil (Van Zyl and Harr 1981). Similarly, in the process of internal erosion, finer particles may migrate locally to a coarser layer leading to the formation of a cavity. To avoid seepage erosion, all three types of seepage erosion should be investigated under a given hydraulic gradient. In this study, an attempt was made to address various issues related to the piping phenomenon. It must be emphasized at this point that by solving the Laplacian equation alone one cannot predict the occurrence or other relevant details of different types of seepage erosion.

Conclusions

A physically based model for computation of the critical head is developed, which also provides a theoretical basis for Bligh's (1910) empirical rules. The critical head is found to depend on the length of the structure, and soil and fluid properties. Highly porous soils have lower values of length to the critical head ratios in comparison to the less porous soils. The case similar is with larger particles, which show a higher permissible critical head when compared with finer particles. The model developed mimics the characteristics of Bligh's rules, which are based on a large number of field studies. The functional form of the model seems consistent, as seen from its calibration against the available critical head versus porosity data.

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Appendix: Expressions for Critical Head Based on Capillary Model

The use of the capillary model is also common in the literature on flow through porous media. Recently, Thevanayagam and Nesarajah (1998) used this model to study the flow characteristics through a soil matrix. Considering a capillary channel of internal radius R or the hydraulic radius R_H, the fluid flow velocity V_c through this channel under a small hydraulic gradient i along the tube is given by the Poiseuille equation

$$V_c=\left(\frac{\gamma_w}{2\mu}\right)R_H^2 i \tag{42}$$

With the following substitutions:

$$R_H=\frac{d}{4} \tag{43}$$

$$i=\frac{h_f}{L} \tag{44}$$

$$V_c=\sqrt{2g(H_2-h_f)} \tag{45}$$

and using Eqs. (24) and (29), one obtains the following expression for the critical head ($H_{\text{crit}}=H_2$):

$$H_{\text{crit}}=\frac{\gamma_w^2 d^4}{128\mu^2 g}\frac{c^2}{\gamma_w^2}+\frac{4c}{\gamma_w}L \tag{46}$$

Similarly to Eq. (32), Eq. (46) can also be rearranged as

$$\frac{H_{\text{crit}}}{L}=\frac{4c}{\gamma_w}\left(1+\frac{\gamma_w^2 d^4}{512\mu^2 gL}\frac{c}{\gamma_w}\right) \tag{47}$$

or

$$\frac{L}{H_{\text{crit}}}=\frac{\gamma_w}{4c}\left(1+\frac{\gamma_w^2 d^4}{512\mu^2 gL}\frac{c}{\gamma_w}\right)^{-1} \tag{48}$$

For

$$\frac{\gamma_w^2 d^4}{512\mu^2 gL}\frac{c}{\gamma_w}\ll 1 \tag{49}$$

Eq. (49) can be expanded using the binomial theorem and be simplified to

$$\frac{L}{H_{\text{crit}}}=\frac{\gamma_w}{4c}\left(1-\frac{\gamma_w^2 d^4}{512\mu^2 gL}\frac{c}{\gamma_w}\right) \tag{50}$$

It can be seen that Eq. (50) is also in a form similar to Eq. (34); however, it does not include the porosity term.

Notation

The following symbols are used in this paper:

a,b = coefficients;
c^*,c^{**} = model constants;
c_1,c_2 = dimensionless groups;
D = thickness of soil layer;
d = median size of particle;
E = constant;
f_1,f_2 = functions;
g = acceleration due to gravity;
H_{crit} = critical head;
H_1 = height of water on landward side of levee;
H_2 = height of water on side of levee exposed to water body;
h = height of sand boil;
h_f = head loss due to friction;
k = hydraulic conductivity;
L = base width of dyke/levee;
l = length of slit;
N = number of observations;
n^- = porosity;
p = partial derivative of velocity potential with x;
Q = flow in slit;

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R = Reynolds number;
α = function;
β = function;
γ_p = unit weight of particles;
γ_s = unit submerged weight of soil;
γ_w = unit weight of water;
η = constant, also known as drag coefficient;
θ = bedding angle;
κ = intrinsic permeability;
μ = dynamic viscosity;
ρ = mass density of liquid (water);
τ_c = critical stress;
υ = kinematic viscosity;
Φ = internal friction angle; and
ϕ = velocity potential.

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