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Evaluation of Models of Border Irrigation Recession

R. S. RAM*; V. P. SINGH†

Recession characteristics play an important role in the design of border irrigation systems. This is particularly true of those borders which have closed downstream ends where surface storage is large even after the cessation of inflow. This paper examines some of the existing models of border irrigation recession. Using experimental data these models are evaluated and compared with each other in regard to their ability to predict recession flow.

1. Introduction

Although recession flow during irrigation has long been a subject of theoretical and experimental research, relatively little is known about it. It is usually assumed that when the inflow at the upstream of a border is cut off, water recedes first vertically and then horizontally due to the combined effect of infiltration and surface flow,[1] as shown in *Fig. 1*. Should there be a bund at the downstream end, water gets impounded and recedes predominantly by infiltration after cessation of horizontal recession[2] as shown in *Fig. 2*. The approaches to modelling recession flow are principally of 3 kinds: (1) hydraulic, (2) volume balance and (3) empirical.

The equations of continuity and momentum form the basis of a hydraulic approach. Solution of these equations is complicated by a lack of a prior knowledge of the boundaries of the solution domain. These boundaries must be determined along with the solution, and are responsible for

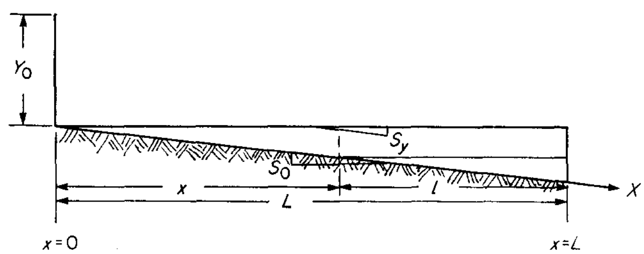

Fig. 1. Schematic surface profiles during vertical and horizontal recession phases in a freely draining border

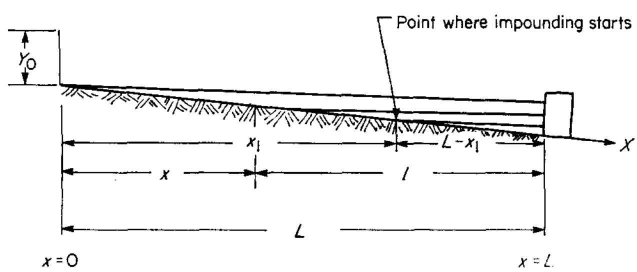

Fig. 2. Schematic surface profiles during vertical and horizontal recession phases in a border with closed end

* Department of Civil Engineering, Mississippi State University, Mississippi 39762, U.S.A.

† Department of Civil Engineering, Louisiana State University, Baton Rouge, Louisiana 70803, U.S.A.

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NOTATION

A_1	$= -\frac{1}{2}\alpha_1 (m_0S_0)^{1/2} L^{-1/2}$
A_2	$= -\frac{1}{2}\alpha_2 (m_0S_0)^{2/3} L^{-1/3}$
a	exponent in the Kostyakov cumulative infiltration equation (7)
B	$= -f/2m_0S_0L$
b	exponent in horizontal recession equation (22)
C	Chezy's roughness coefficient, $m^{1/2}/s$
f	average infiltration rate, m/s
f_c	minimum infiltration rate, m/s
f_s	average infiltration rate, m/s
f_1	f defined by Eqn (6)
f_2	f defined by Eqn (6)
f_3	f defined by Eqn (9)
f_4	f defined by Eqn (9)
f_5	f defined by Eqn (17)
f_6	f defined by Eqn (17)
F	quantity defined by Eqn (12)
I	cumulative infiltration (m) in Eqns (7) and (23)
I_1	cumulative infiltration (m) at end of T_1
I_2	cumulative infiltration (m) at end of T_2
I_3	cumulative infiltration (m) at end of T_3
I_{r_1}	instantaneous rate of infiltration (m/s) at end of T_1
I_{r_2}	instantaneous rate of infiltration (m/s) at end of T_2
I_{r_3}	instantaneous rate of infiltration (m/s) at end of T_3
I_L	cumulative infiltration (m) at time $t_t - t_L$
I_{L-l}	cumulative infiltration (m) at time $t_t - t_{L-l}$
I_R	cumulative infiltration (m) at time $T_1 - t_L$
I_l	cumulative infiltration (m) at time $T_1 - t_{L-l}$
K_h	constant in Eqn (22)
K_i	infiltration constant, m/s^a
K_r	recession constant, s
L	total length of border, m
l	length of horizontal recession (m) measured from downstream end
m_0	slope constant
n	Manning's roughness coefficient
q_0	inflow rate, $m^3\, m^{-1}\, s^{-1}$
q_1	$= q_0 - f_sL$
r	constant in Eqn (22)
R_l	quantity defined by Eqn (15)
R_L	quantity defined by Eqn (13)
S	surface storage, m^3
S_0	border slope, m/m
S_y	$= Y_n/L$
t	time of horizontal recession measured after completion of vertical recession for horizontal recession length (x) from upstream end, min
t_x	time of advance to point x measured from upstream end, min
t_r	time of horizontal recession measured after completion of vertical recession for length l measured from downstream end, min

t_v	time of vertical recession after inflow is cut-off, min
t^*	total time of horizontal recession to the point where impounding starts, min
t_t	total time measured from the start of inflow until recession is complete, s
t_{L-l}	time of advance to point $L-l$, s
t_i	time of opportunity for infiltration, in Eqns (7) and (23), s
t_c	time of cut-off of inflow, s
T	total time to the point where impounding starts, min
T_1	time measured from the start of inflow to completion of vertical recession, s
T_2	time measured from the start of inflow to completion of recession at the downstream end, s
T_3	$= T_1 - T_L$
T_L	time of advance to the end of the border, s
T_{0i}	time of opportunity in region of impounding, min, defined in Eqn (25)
U_z	quantity defined by Eqn (14)
U_{z1}	quantity defined by Eqn (16)
V_0	$= q_0 t_c$
W	width of border, m
x	length of recession measured from the upstream end, m
x_1	length of border to the point where impounding starts in a closed end border, as measured from upstream end, m
x_r	$= l/L$
Y_0	normal depth of flow at upstream end, m
Y_c	critical depth of flow, m
Y_n	normal depth of flow corresponding to recession flow rate q_1, m
α	kinematic friction parameter
α_1	α defined by Eqn (4)
α_2	α defined by Eqn (5)
β	exponent indicating quality of flow

the arising of free boundary problems in irrigation hydraulics, as illustrated by Sherman and Singh.[3] Several investigators have employed these equations to describe recession flow. By utilizing continuity and momentum equations, Su[4] developed a simplified expression for vertical recession. An expression for horizontal recession was, however, developed from the continuity equation alone. In a similar vein, several other investigators[5-8] developed mathematical models to describe recession flow, subject to infiltration, under various conditions.

The equations of hydraulics are difficult to solve, even numerically; their computer solutions are often expensive. Further, the model results may not always agree well with field measurements of recession because of inaccuracies associated with measurements in estimation of infiltration, roughness and geometric characteristics. These difficulties have led to simplified hydraulic models: (1) kinematic wave models[3,9] and (2) zero inertia models.[10,11]

On the other hand, the volume balance approach is based on a spatially lumped form of the continuity equation and a storage discharge relationship, linear or non-linear, and has been employed by several investigators.[12-14] The parameters in the storage equation are evaluated empirically. In an empirical approach algebraic equations are postulated for recession flow where the constants are correlated with border hydraulic and geometric characteristics.[15]

In this paper, an attempt is made to examine critically some of the simplified recession models. These models are evaluated using limited experimental data and compared with each other in the light of their validity for applications in irrigation design.

2. Models of vertical recession

The vertical recession time, as shown in *Figs 1* and *2*, is the time from the cessation of inflow to the time when the depth of flow at the upstream end becomes zero. Vertical recession models, currently in use, are considered below. Only pertinent remarks are made about the models; for their details see the references cited.

2.1. *SWP model*

The model developed by Shockley, Woodward and Phelan,[16] henceforth designated as the SWP model, expresses the time of vertical recession as

$$t_v = \frac{Y_0^2}{120\, S_0 q_0}, \qquad \ldots(1)$$

where t_v is the time of vertical recession (min), Y_0 is the normal depth of flow at the upstream end (in m), as shown in *Fig. 1*, S_0 is the border slope (m/m) and q_0 is the inflow rate ($m^3\, m^{-1}\, s^{-1}$). From now on, a symbol will be defined when it appears for the first time. For easy referencing, all the symbols are given in the Notation.

Utilizing the equation of continuity Chen[1] developed a similar model. The SWP model is based on 2 assumptions: (1) there exists a free water surface with a depth Y_0 at the upstream end after inflow is cut off and (2) the water recedes with a rate q_0.

2.2. *SCS model*

The Soil Conservation Service of the U.S. Department of Agriculture,[17] SCS in short, used a modified form of Eqn (1),

$$t_v = \frac{q_0^{0\cdot 2}\, n^{1\cdot 2}}{120\, S_0^{1\cdot 6}}, \qquad \ldots(2)$$

where n is Manning's roughness coefficient.

2.3. *Strelkoff model*

Strelkoff[13] used the following relationship for the time of vertical recession:

$$t_v = \frac{Y_0 L}{120\, q_0}, \qquad \ldots(3)$$

where L is length of the border (m).

The principal assumptions of this model are: (1) the surface profiles at the time when inflow is cut-off and the vertical recession is complete are straight lines and (2) the triangular volume of length L and upstream height Y_0 recedes by infiltration and surface outflow at the combined rate q_0. If $L = Y_0/S_0$, then Eqn (3) reduces to Eqn (1). The assumptions leading to Eqns (1)–(3) may not generally hold in flat borders.

3. Models of horizontal recession

3.1. *Kinematic approach*

3.1.1. *Sherman–Singh model*

Utilizing kinematic wave theory Sherman and Singh[3] derived explicit expressions for the time of horizontal recession for a free draining border as

$$t = \frac{1}{60}\left[\frac{x}{\alpha f^{\beta-1}}\right]^{1/\beta},$$

where β is 5/3 for Manning's relation and 3/2 for Chezy's relation. Accordingly, a will change and is designated as α_1 for Chezy's relation and α_2 for Manning's relation. For α_1, α_2 and β the resulting equations are

$$t = \frac{1}{60}\left[\frac{x}{\alpha_1 f^{1/2}}\right]^{2/3}, \qquad \alpha_1 = CS_0^{1/2} \qquad \ldots(4)$$

and

$$t = \frac{1}{60}\left[\frac{x}{\alpha_2 f^{2/3}}\right]^{3/5}, \qquad \alpha_2 = \frac{1}{n} S_0^{1/2}, \qquad \ldots(5)$$

where t is the time of horizontal recession (min) for length x (m) measured from the upstream end, f is the average infiltration rate (m/s), n is Manning's roughness coefficient (s/m$^{1/3}$) and C is Chezy's roughness coefficient (m$^{1/2}$/s).

The principal limitation of Eqn (4) or (5) lies in the assumption of constant infiltration rate f which is not explicitly defined. Since infiltration rate is not constant during irrigation, we consider the following definitions for f, designated as f_1 and f_2:

$$f_1 = \frac{1}{2}\left(\frac{I_1}{T_1}+\frac{I_2}{T_2}\right), \qquad f_2 = \tfrac{1}{2}(I_{r_1}+I_{r_2}). \qquad \ldots(6)$$

Cumulative infiltration I_1 and I_2 (both in m) and instantaneous rates I_{r_1} and I_{r_2} (both in m/s) at the end of vertical recession time T_1 (s) and completion of horizontal recession time T_2 (s), respectively, can be calculated by using the Kostyakov infiltration equation,[18]

$$I = K_i\, t_i^{a}, \qquad \ldots(7)$$

where I is the cumulative infiltration (m) for time t_i (s) and K_i (m/s^a) and a are infiltration constants.

3.2. *Volume balance approach*

3.2.1. *Wu model*

Wu[12] expressed the time of horizontal recession as

$$t_r = \frac{5}{180} K_r \ln\left(\frac{f_c K_r^{2\cdot5}\,\alpha_2^{1\cdot5}+L^{1\cdot5}}{f_c K_r^{2\cdot5}\,\alpha_2^{1\cdot5}+ l^{1\cdot5}}\right), \qquad \ldots(8)$$

where t_r is the time of horizontal recession (min) for length l (m) measured from the downstream end. In Eqn (8) f_c (m/s) is assumed as minimum infiltration rate. This assumption is reasonable if irrigation time is sufficiently long. The accuracy of Eqn (8) may depend upon a correct determination of the recession constant K_r(s) which may require series of experimental runs measuring recession in the border and obtaining it by data analysis. Infiltration rate will be minimum at the end of recession for a given irrigation. Therefore, f_c is the instantaneous infiltration rate at the end of T_2. However, the average rate at the end of T_2 may aso be taken as minimum rate. Since minimum rate is not clearly defined by Wu,[12] we consider the following additional definitions designated as f_3, f_4 and f_5:

$$f_3 = \frac{I_3}{T_3}, \; T_3 = T_1 - T_L, \; f_4 = \frac{I_2}{T_2}, \; f_5 = I_{r1}, \; f_c = I_{r2}, \qquad \ldots(9)$$

where I_3 is the cumulative infiltration (m) at the end of T_3 (s) and T_L is the time of advance to the end of border (s).

3.2.2. *Strelkoff model*

Strelkoff[13] derived an algebraic relationship for the time of horizontal recession:

$$t_r = \frac{S_y}{f_s F^{3/2}} (R_L - R_l), \qquad \text{...(10)}$$

where the terms in Eqn (10) can be defined as

$$S_y = \frac{Y_n}{L}, \qquad \text{...(11)}$$

$$F = \frac{1}{n}\left(\frac{S_0^{1/2} S_y^{5/3}}{f_s}\right), \qquad \text{...(12)}$$

$$R_L = \int_0^{U_z} \frac{dz}{1+z^{2/3}}, \qquad \text{...(13)}$$

$$U_z = F^{3/2} L, \qquad \text{...(14)}$$

$$R_l = \int_0^{U_{z_1}} \frac{dz_1}{1+z_1^{2/3}}, \qquad \text{...(15)}$$

$$U_{z_1} = F^{3/2}, \qquad \text{...(16)}$$

where Y_n is the normal depth of flow (m) corresponding to recession flow rate q_1 given in the Notation and f_s is the average of infiltration rates at the upstream and downstream ends (m/s).

Eqn (10) requires evaluation of integrals as given by Eqns (13)–(16) as part of its solution. Although Strelkoff[13] has given a graphical solution of these integrals, the solution of Eqn (10) is not amenable to a single calculation. The assumption of average rate of infiltration f_s (m/s) as the average of instantaneous rates at the upstream and downstream ends is reasonable. We also considered the average of average infiltration rates at the upstream and downstream ends for comparison. These rates are defined as

$$f_s = \tfrac{1}{2}(I_{r_3} + I_{r_1}), \qquad f_6 = \frac{1}{2}\left(\frac{I_3}{T_3} + \frac{I_1}{T_1}\right), \qquad \text{...(17)}$$

where I_{r3} is the instantaneous infiltration rate (m/s) at the end of T_3.

3.2.3. *Singh–McCann model*

Singh and McCann[14] defined mathematical models for horizontal recession utilizing a spatially lumped form of continuity equation

$$Wfl = a(m_0 S_0)^{\beta} l^{\beta} W = \frac{dS}{dt}, \qquad l(0) = L, \qquad \text{...(18)}$$

and a storage discharge relationship of the form

$$S = m_0 S_0\, W l^2, \qquad \text{...(19)}$$

where W is border width (m), m_o the slope constant, f the constant infiltration rate and S the surface storage (m^3). The recession models were developed by taking β as 3/2 and 5/3, respectively as

$$A_1(x_r^{0\cdot5} - 1) + B\, ln\, \frac{A_1 + B}{A_1 x_r^{0\cdot5} + B} = 30\, A_1^2\, t_r \qquad \text{...(20)}$$

and

$$(x_r^{1/3}-1)+\sqrt{\frac{B}{A_2}}\left(\tan^{-1}\sqrt{\frac{A_2}{B}}-\tan^{-1}\sqrt{\frac{A_2}{B}}x_r^{1/3}\right) = 20A_2\,t_r, \quad \text{...(21)}$$

where A_1, B and A_2 are constants as given in the Notation and x_r is the ratio l/L.

The value of m_0 in Eqn (18) is not specified and should be determined by trial and error. In the present study its value was assumed to be 1·0. The constant infiltration rate f was not defined. Therefore, Eqn (6) was used.

3.3. *Empirical approach*

3.3.1. *Ram–Lal model*

Ram and Lal[15] proposed the following form of empirical relationship of horizontal recession time:

$$t = \left[\frac{xY_0^r}{K_h Y_c^r}\right]^{1/b}, \quad \text{...(22)}$$

where values of K_h, r and b were reported as 8·17, 0·775 and 1·16, respectively, and Y_c is the critical depth of flow. Eqn (22) is simple but does not explicitly account for infiltration; it is location specific. Its constants are strongly correlated with those in the Kostyakov infiltration equation. This equation was originally developed for the soil group with infiltration characteristics expressed by

$$I = 0{\cdot}0003642\,t_i^{0{\cdot}608}. \quad \text{...(23)}$$

Eqn (22) may be used with appropriate values of K_h, r and b for specified soil characteristics.

4. Recession of impounded water

In borders with closed downstream ends, water becomes impounded against the bund, as shown in *Fig. 2*. Models to describe the recession of impounded water are considered below.

4.1. *Ram–Lal model*

Ram and Lal[19] proposed the following relationship for calculation of recession time of impounded water:

$$t_r = T_{0_i}-T+t_x, \quad \text{...(24)}$$

where T_{0_i} is the time of opportunity in the region of impounding and T is the total time to the point where impounding starts (both in min). These quantities can be expressed as

$$T_{0_i} = \left[\frac{(x-x_1)S_0}{60^a K_i}+(T-t_x)^a\right]^{1/a}, \quad \text{...(25)}$$

$$T = \frac{t_c}{60}+t^*+t_v, \quad \text{...(26)}$$

where t_c is the time at which inflow is cut-off (s), x_1 is the length of border (m) to the point where impounding starts at the time t^* (min) and t_x is the time of advance (min) for length x in the region of impounding.

Eqn (24) is based on the assumption that after completion of horizontal recession water recedes only by infiltration, and there is no surface flow. This may not always be true, especially on steep slopes.

4.2. *Strelkoff model*

Assuming a linear water surface profile at the end of vertical recession, Strelkoff[13] expressed the approximate form of the volume balance as

$$\frac{S_0 L^2}{2}+\frac{I_1+I_3}{2}L = V_0, \qquad \text{...(27)}$$

where V_0 is the volume of inflow (m^3). Since I_1 is the cumulative infiltration at the end of T_1, Eqn (27) can be used to determine T_1 by trial and error. After completion of the vertical recession at time T_1, the recession length x was expressed as

$$x = \frac{I_L+I_{L-l}-I_R-I_l}{2S_0}, \qquad \text{...(28)}$$

where V_0, I_L, L_{L-l}, I_R and I_l are as given in the Notation.

If the cut-off time of inflow is sufficiently large for a trapezoidal water volume to form on the surface, T_1 would be computed from Eqn (27). Assumptions of linear water surface profile for large times of inflow may be valid on relatively flat borders. However, these equations need field verification.

Separate relationships for computation of impounded water for small inflow times were developed by Strelkoff which need verification by field data.

5. Model testing and discussion

The mathematical models for vertical and horizontal recession were tested using irrigation data on open end borders by Roth.[20] Four sets of data, designated as I, II, III and IV, on q_0, S_0, K_r, n, C, K_l and a were used as given in Table I. The depth of flow, Y_0, was used in recession models as calculated by Manning's equation. The models for recession of impounded water could not be tested due to lack of data.

It may be remarked that the data by Roth[20] were selected as they contained all pertinent information required for comparison of these recession models. Unfortunately, such data are not easily available. It is realized that the testing of models performed in this study is based on this limited set of data, but is hoped that this study might encourage others to test and validate the various recession models using a wide variety of data.

5.1. *Models of vertical recession*

Table I shows observed times and the times calculated by the SWP, SCS and Strelkoff models [respectively, Eqns (1), (2) and (3)] for vertical recession. Table II gives the absolute and per cent deviations between observed and computed t_r for each model. Results of each model are described below.

5.1.1. *SWP model*

Calculated and observed vertical recession times, as given in Table I, are in good agreement for the data sets I and IV. However, this is not the case with data sets II and III. The percentage differences, given in Table II, between calculated and observed times are as high as 302 for the data set II and 45 for the data set III. Thus, the model is not consistent in its performance. The borders for which calculations were made are on very flat slopes. Consequently, the assumption to remove the triangular volume of water at rate q_0 may not be valid. It is plausible that the model may give better results on steep slopes.

Absolute differences between observed and calculated vertical recession times are only 0·18, 2·42, 1·36 and 0·49 min compared with irrigation time of 181·4, 179·7, 179·0 and 179·3 min for data sets I, II, III and IV, respectively. In such cases, the model may be used without adversely

TABLE I

Time of vertical recession by different recession models: W = 5·89 m, L = 91·46 m

Data set	*Time of irrigation, min*	*Observed vertical recession time, min*	q_0, $m^3\ m^{-1}s^{-1}$	S_0, *m/m*	K_r, *s*	n	C, $m^{\frac{1}{2}}/s$	K_i m/s^a	a	*SWP*,[16] *min*	*Strelkoff*,[13] *min*	*USDA*,[7] *min*
I	181·4	1·6	0·0017	0·001	1044	0·021	29·16	0·0023	0·394	1·42	7·50	1·42
II	179·7	0·8	0·0024	0·001	750	0·039	14·36	0·0032	0·358	3·22	9·64	3·22
III	179·0	3·0	0·0032	0·001	768	0·024	22·10	0·0155	0·116	1·64	6·13	1·64
IV	179·3	1·7	0·0024	0·001	768	0·028	18·63	0·0055	0·266	2·19	7·94	2·19

TABLE II

Absolute and percentage deviations between calculated and observed times of vertical recession

Data set	*Observed vertical recession time, min*	*Calculated recession time, min*								
		SWP[16]			*Strelkoff*[13]			*SCS–USDA*[7]		
		Calculated t_v, *min*	*Absolute deviation*	*Percentage deviation*	*Calculated* t_v, *min*	*Absolute deviation*	*Percentage deviation*	*Calculated* t_v, *min*	*Absolute deviation*	*Percentage deviation*
I	1·60	1·42	0·18	112·5	7·50	5·90	368·8	1·42	0·18	112·5
II	0·80	3·22	2·42	302·5	9·64	8·84	1105·0	3·22	2·42	302·5
III	3·00	1·64	1·36	45·3	6·13	3·13	104·3	1·64	1·36	45·3
IV	1·70	2·19	0·49	28·8	7·94	6·24	367·0	2·19	0·49	28·8

TABLE III

Time of horizontal recession by different recession models

Data Set I: f_1 = 2·86 cm/h, f_2 = 1·13 cm/h

Distance from upstream end, m	*Observed recession time, min*	*Recession time (min) calculated from Eqn*							
		(4)	*(5)*	*(20)*	*(21)*	*(4)*	*(5)*	*(20)*	*(21)*
9·15	5·00	3·85	5·36	1·09	0·77	5·26	7·78	1·11	0·86
45·73	14·00	11·27	14·07	6·19	4·60	15·36	20·43	6·32	5·15
91·46	17·00	17·88	21·33	19·62	22·28	24·39	30·96	20·75	24·97
Average percentage deviation	—	12·65	9·28	42·73	53·47	18·16	54·52	42·42	51·15
Data Set II: f_1 = 2·89 cm/h, f_2 = 1·03 cm/h									
9·15	8·50	6·16	7·80	2·15	1·28	8·68	11·76	2·23	1·53
45·73	21·50	18·02	20·49	12·17	7·62	25·38	30·89	12·96	9·13
91·46	33·50	28·61	31·05	36·89	36·95	40·29	46·83	40·79	44·26
Average percentage deviation	—	16·36	5·68	36·70	52·88	14·07	37·62	25·57	48·96
Data Set III: f_1 = 1·30 cm/h, f_2 = 0·14 cm/h									
9·15	6·00	5·83	7·78	1·32	0·88	12·17	18·79	1·34	1·00
45·73	26·00	17·05	20·41	7·54	5·27	35·58	49·35	7·66	5·97
91·46	41·00	27·06	30·93	24·60	25·58	56·48	74·80	25·93	28·92
Average percentage deviation	—	26·36	17·96	59·95	66·93	43·81	98·43	59·24	63·80
Data Set IV: f_1 = 2·03 cm/h, f_2 = 0·54 cm/h									
9·15	8·00	5·82	7·40	1·70	1·05	9·05	12·57	1·74	1·22
45·73	24·00	17·01	19·44	9·66	6·30	26·45	33·01	9·92	7·28
91·46	35·00	27·00	29·46	30·44	30·56	41·99	50·04	32·84	35·29
Average percentage deviation	—	24·65	14·69	48·70	59·84	12·11	38·56	47·04	55·18

affecting the design of an irrigation system. However, care should be exercised where vertical recession times are large.

5.1.2. *SCS model*

This model is the same as the SWP model except that the depth of flow, Y_0, is expressed by Manning's equation. The results for this model are, therefore, the same as for the SWP model as is evident from Tables I and II.

5.1.3. *Strelkoff model*

The absolute differences between calculated and observed recession times for the data sets I, II, III and IV are 5·9, 8·84, 3·13 and 6.24 min, respectively, as shown in Table II. These differences are very high. Thus, the model cannot be used at all for prediction purposes. If length L is substituted by Y_0/S_0 in the model, the model will give the same results as the SWP model.

TABLE IV

Time of horizontal recession by different recession models

Data Set I

Distance from upstream end, min	*Observed recession time, min*	*Recession time (min) calculated from Eqn*						
		(8)	*(8)*	*(8)*	*(8)*	*(10)*	*(10)*	*(22)*
		Infiltration rate, cm/h						
		f_3 (3·47)	f_4 (2·79)	f_5 (1·16)	f_c (1·10)	f_6 (3·20)	f_s (1·26)	— —
9·15	5·00	2·56	2·81	3·63	3·67	1·05	1·37	2·05
45·73	14·00	13·57	15·20	21·29	21·61	5·72	7·88	8·25
91·46	17·00	25·03	28·92	47·32	48·59	13·60	22·75	14·99
Average percentage deviation	—	26·02	32·50	74·84	77·60	47·50	35·79	32·22

Data Set II

		f_3 (3·59)	f_4 (2·73)	f_5 (1·09)	f_c (0·98)	f_6 (3·32)	f_s (1·19)	— —
9·15	8·50	2·98	2·98	3·16	3·17	1·52	1·85	2·61
45·73	21·50	17·73	18·53	20·28	20·42	8·48	10·74	10·47
91·46	33·50	45·33	50·49	68·47	70·67	21·46	33·22	19·05
Average percentage deviation	—	26·54	28·97	38·81	39·94	53·55	48·85	47·82

Data Set III

		f_3 (1·65)	f_4 (1·19)	f_5 (0·16)	f_c (0·13)	f_6 (1·54)	f_s (0·17)	— —
9·15	6·00	2·93	3·04	3·32	3·33	1·18	1·83	2·07
45·73	26·00	17·86	18·89	21·68	21·76	6·89	10·21	8·33
91·46	41·00	44·76	51·11	92·58	96·40	21·56	35·33	15·14
Average percentage deviation	—	24·05	24·71	38·36	39·49	62·72	48·51	60·16

Data Set IV

		f_3 (2·60)	f_4 (1·91)	f_5 (0·58)	f_c (0·51)	f_6 (2·38)	f_s (0·63)	— —
9·15	8·00	2·88	2·99	3·25	3·26	1·37	1·63	2·24
45·73	24·00	17·41	18·46	20·89	21·04	7·79	9·50	9·21
91·46	35·00	42·41	48·24	72·11	74·79	21·16	32·19	16·75
Average percentage deviation	—	26·82	28·35	39·32	40·60	58·02	48·28	55·32

TABLE V

Maximum and minimum absolute deviation of observed time from calculated time of horizontal recession along length of border by different irrigation recession models

Type of average infiltration rate	*Absolute deviation (%) calculated from Eqn*													
	(4)	*(5)*	*(20)*	*(21)*	*(8)*	*(10)*	*(22)*	*(4)*	*(5)*	*(20)*	*(21)*	*(8)*	*(10)*	*(22)*
	Minimum							*Maximum*						
f_1	2·83	0·53	10·12	10·32	—	—	—	34·43	29·50	78·74	86·82	—	—	—
f_2	2·12	36·41	6·17	0·83	—	—	—	102·81	213·13	78·23	84·77	—	—	—
f_3	—	—	—	—	2·26	—	—	—	—	—	—	64·04	—	—
f_4	—	—	—	—	0·40	—	—	—	—	—	—	71·17	—	—
f_5	—	—	—	—	1·03	—	—	—	—	—	—	178·38	—	—
f_c	—	—	—	—	1·22	—	—	—	—	—	—	185·80	—	—
f_6	—	—	—	—	—	19·99	—	—	—	—	—	—	82·82	—
f_s	—	—	—	—	—	0·83	—	—	—	—	—	—	79·66	—
Independent of infiltration	—	—	—	—	—	—	11·77	—	—	—	—	—	—	71·28

5.2. *Models of horizontal recession*

The times of horizontal recession were calculated for distances of 9·15, 18·29, 27·44, 36·59, 45·73, 54·88, 64·02, 73·17, 82·32 and 91·46 m measured from the upstream end. However, to conserve space, calculated and observed times are given only for distances 9·15, 45·73 and 91·46 m, as shown in Tables III and IV. The agreement between observed and computed times was measured in terms of maximum, minimum and average percentage deviations, as shown in Table V.

Using infiltration rates of 0·1, 0·5, 1·0, 1·5, 2·0, 3·0, 4·0 and 5·0 cm/h, recession times were calculated for data set I by different recession models, and are given along with observed times in Table VI. Average percentage absolute deviations for all sets of data are given in Table VII. The results for different models are discussed below.

5.2.1. *Sherman–Singh model*

The Sherman–Singh model is given by Eqn (4) when Chezy's equation is used to express kinematic friction parameter α. The model is given by Eqn (5) when α is expressed by Manning's equation. Calculated times by Eqn (4) are less than observed ones in almost all cases for infiltration rate f_1 (Table III). This is also true for Eqn (5) (Table III). The maximum and minimum percentage absolute deviations along the length of the border were found to be 2·83 and 34·43 from Eqn (4) and 0·53 and 29·50 from Eqn (5), as shown in Table V. The average absolute percentage deviation for different sets of data varied between 12·65 and 26·36 for Eqn (4) and 5·68 and 17·96 for Eqn (5) (Table V). However, the recession time was overestimated if f_2 was used. The results in Table V also show that the infiltration rate near f_1 is the best for recession models given by Eqns (4) and (5).

Both equations were found to be very sensitive to changes in infiltration rate. For example, the absolute percentage deviation for Eqn (4) varied between 9·96 for infiltration rate 2·0 cm/h and 52·17 for 0·5 cm/h for data set I, as shown in Table VII. The increase in infiltration rate beyond 2·0 cm/h led to increased error. Similar trends were observed for other sets of data. Eqn (5) also showed a similar trend. However, Eqn (5) gives better results than Eqn (4).

TABLE VI

Effect of infiltration rate on time of horizontal recession

Infiltration rate 0·1 cm/h

Distance from upstream end, m	*Observed recession time, min*	*Recession time (min) calculated from Eqn*						
		(4)	*(5)*	*(20)*	*(21)*	*(8)*	*(10)*	*(22)*
9·15	5·00	11·78	20·50	1·12	0·97	4·48	2·13	2·06
45·73	14·00	34·47	53·86	6·39	5·82	29·08	11·98	8·25
91·46	17·00	54·72	81·63	21·71	28·23	112·67	41·93	14·99
Average percentage deviation	—	153·76	292·52	42·37	48·32	177·70	41·56	32·22

Infiltration rate 0·5 cm/h

9·15	5·00	6·89	10·77	1·12	0·92	4·11	1·52	2·06
45·73	14·00	20·16	28·29	6·37	5·47	25·51	8·89	8·24
91·46	17·00	32·00	42·88	21·28	26·51	68·29	29·82	14·99
Average percentage deviation	—	52·17	110·50	42·36	49·81	117·35	37·39	32·22

Infiltration rate 1·0 cm/h

9·15	5·00	5·47	8·16	1·11	0·87	3·73	1·41	2·05
45·73	14·00	16·00	21·44	6·32	5·20	22·16	8·19	8·26
91·46	17·00	25·40	32·49	8·11	13·52	50·81	24·69	14·99
Average percentage deviation	—	22·65	61·74	42·40	50·91	82·38	35·72	32·22

Infiltration rate 1·5 cm/h

9·15	5·00	4·78	4·39	1·10	0·84	3·42	1·33	2·06
45·73	14·00	15·78	20·34	6·29	5·00	19·63	7·61	8·24
91·46	17·00	22·19	27·63	20·48	24·27	41·47	21·26	14·99
Average percentage deviation	—	42·48	51·75	10·16	38·88	61·52	35·96	32·22

Infiltration rate 2·0 cm/h

9·15	5·00	4·34	6·19	1·09	0·81	3·15	1·25	2·06
45·73	14·00	12·70	16·24	6·25	4·84	17·62	7·05	8·24
91·46	17·00	20·16	24·63	20·16	23·46	35·35	18·58	14·99
Average percentage deviation	—	9·97	24·77	42·56	52·45	46·74	37·32	32·22

Infiltration rate 3·0 cm/h

9·15	5·00	3·79	5·26	1·09	0·76	2·72	1·08	2·06
45·73	14·00	11·09	13·82	6·18	4·56	14·66	5·95	8·24
91·46	17·00	17·61	20·94	19·55	22·11	27·59	14·36	14·99
Average percentage deviation	—	13·29	7·75	42·76	53·62	30·10	45·54	32·22

Infiltration rate 4·0 cm/h

9·15	5·00	3·45	4·69	1·08	0·72	2·04	0·91	2·06
45·73	14·00	10·08	12·32	6·11	56·32	12·56	4·84	8·24
91·46	17·00	16·01	18·66	19·02	21·07	22·76	10·94	14·99
Average percentage deviation	—	19·37	7·20	42·98	54·58	23·21	54·80	32·22

Infiltration rate 5·0 cm/h

9·15	5·00	3·20	4·29	1·07	0·69	2·15	0·69	2·06
45·73	14·00	9·36	11·26	6·05	4·13	10·99	3·65	8·24
91·46	17·00	14·85	17·07	18·54	20·04	19·42	7·84	14·99
Average percentage deviation	—	24·49	10·83	43·22	55·41	21·46	64·09	32·22

TABLE VII

Effect of infiltration rate on percentage deviation between observed and calculated times of horizontal recession

Data set	Infiltration rate, cm/h	Average deviation (%) from Eqn						
		(4)	(5)	(20)	(21)	(8)	(10)	(22)
I	0·1	153·76	292·52	42·37	48·32	177·70	41·56	32·22
	0·5	52·17	110·51	42·36	49·81	117·38	37·39	32·22
	1·0	22·66	61·74	42·41	50·92	82·38	35·72	32·22
	1·5	10·16	38·88	42·48	51·75	61·52	35·96	32·22
	2·0	9·96	24·77	42·56	52·45	46·73	37·31	32·22
	3·0	13·29	7·75	42·76	53·61	30·10	45·53	32·22
	4·0	19·36	7·20	42·98	54·58	23·21	54·80	32·22
	5·0	24·49	10·83	43·22	55·41	21·45	64·09	32·22
II	0·1	137·81	236·31	35·21	44·76	58·22	29·01	47·82
	0·5	42·85	80·98	35·32	47·11	46·24	38·20	47·82
	1·0	15·25	39·96	35·55	48·85	39·70	40·14	47·82
	1·5	4·08	19·86	35·83	50·17	35·37	43·03	47·82
	2·0	6·64	7·82	36·13	51·27	32·11	46·20	47·82
	3·0	17·29	6·96	36·77	53·05	28·12	51·85	47·82
	4·0	24·03	16·08	37·43	54·51	25·71	57·01	47·82
	5·0	28·82	22·47	38·08	56·76	25·01	61·93	47·82
III	0·1	61·08	127·92	59·21	63·53	41·09	38·81	60·16
	0·5	9·29	24·04	59·48	65·18	30·30	57·82	60·16
	1·0	21·48	13·47	59·78	66·34	25·41	60·85	60·16
	1·5	29·28	20·92	60·06	67·27	23·92	62·59	60·16
	2·0	34·91	26·55	60·33	68·02	24·86	64·02	60·16
	3·0	41·99	34·77	60·83	69·28	32·79	66·50	60·16
	4·0	46·46	40·87	61·31	70·33	38·81	68·72	60·16
	5·0	49·65	45·14	61·76	71·24	43·57	70·80	60·16
IV	0·1	90·08	163·65	46·45	53·72	53·79	34·50	55·32
	0·5	14·93	42·81	46·98	55·08	40·73	46·87	55·32
	1·0	6·89	10·43	47·58	56·88	34·19	50·99	55·32
	1·5	17·51	5·57	48·13	58·44	30·49	53·83	55·32
	2·0	24·22	14·10	48·66	59·75	27·93	56·28	55·32
	3·0	32·66	25·60	49·64	61·89	26·38	60·64	55·32
	4·0	37·98	32·70	50·55	63·65	28·11	64·61	55·32
	5·0	41·77	37·67	51·40	65·17	33·16	68·39	55·32

5.2.2. *Wu model*

The Wu model [Eqn (8)] underestimated the horizontal recession time in the beginning of the border and overestimated towards the end for all sets of data, as shown in Table IV. The minimum infiltration rate f_c (Wu[12]), as given by Eqn (9), resulted in inferior results compared with the one when f_3 [given by Eqn (9)], which is the average infiltration rate at the downstream end of the border, was used as infiltration capacity. This indicates that the proper selection of an infiltration rate may give better results for the same recession model. Changes in infiltration rate improved the results significantly (Table V). For infiltration rate f_c (1·1 cm/h) the average absolute deviation was 77·6% which dropped to 21·46% when an average rate of 5·0 cm/h was used for data set I (Table VII). This is comparable with the results with f_3 (deviation 26·02%). It is, therefore, advisable to use f_3 as the infiltration rate.

5.2.3. *Strelkoff model*

The Strelkoff model [Eqn (10)] underestimated the recession time throughout the border length for almost all sets of data (Table IV). It gives better results with infiltration rate f_5, expressed by Eqn (17), compared with the results with f_6. However, Tables VI and VII show that the model was not very sensitive to changes in infiltration rates.

5.2.4. *Singh–McCann model*

The Singh–McCann model is given by Eqn (20) when α and β are expressed by Chezy's equation. Similarly, this model is given by Eqn (21) when α and β are expressed by Manning's equation. Both equations underestimated the time of horizontal recession for almost all sets of data and all points along the length of the border (Table III). Calculated times in the beginning of the border were too small compared with observed times. However, these times improved considerably towards the end for both Eqns (20) and (21). The maximum, minimum and average percentage absolute deviations (Table VII) indicate that there is a wide range of variations between calculated and observed recession times. Even the changes in infiltration rate did not influence the results significantly (Tables VI and VII).

5.2.5. *Ram–Lal model*

The Ram–Lal model [Eqn (22)], which is purely an empirical equation, does not account for infiltration. The average absolute percentage deviations for different sets of data were between 32·2 and 60·16 (Table IV), while the minimum and maximum deviations were 11·77 and 71·28% among all 4 sets of data (Table V). The results may improve if recession constants developed for local conditions are used in the model.

6. Comparison of models

Among all models for determination of the time of horizontal recession, the Sherman–Singh model (α expressed by Manning's equation) predicted recession time most closely, followed by the same model for α expressed by Chezy's equation with f_1 as the infiltration rate. In order of accuracy of predictions on the basis of average absolute percentage deviation, the Sherman–Singh model was followed by the Strelkoff model with f_5, the Singh–McCann model with α_1 and f_1, the Ram–Lal model, the Wu model with f_c and the Singh–McCann model with α_2 and f_2. However, when f_3 was used in the Wu model, it gave better results than the Singh–McCann, Strelkoff and Ram–Lal models. Similarly, in order of sensitivity to infiltration, these models can be ranked as the Sherman–Singh, Strelkoff, Wu, Singh–McCann and Ram–Lal models.

7. Conclusion

The Sherman–Singh model predicted horizontal recession time reasonably well, within 29·5% of deviation from the observed time, provided the average of average infiltration rates at the end of completion of vertical and horizontal recession times is used and the kinematic friction parameter is expressed by Manning's equation. The Wu model also gave predictions with 45·3% of accuracy, if minimum infiltration rate is taken as average infiltration rate at the lower end of the border after completion of vertical recession. The Singh–McCann model and the Strelkoff model are simple and can be used within an accuracy of 85% deviation from observed data.

The Sherman–Singh model is sensitive to infiltration and may result in poor results if infiltration parameters are not estimated accurately while the Singh–McCann model, the Strelkoff model and the Wu model are not as sensitive. The Ram–Lal model gave results within 60% of observed recession times. The results may be improved by accurate estimation of empirical constants in the equation.

Vertical recession was not predicted accurately by any of the models. However, the SWP model may be used in irrigation system design without undue restriction because the vertical recession time is usually very small compared with the time of irrigation.

It should be emphasized that the above conclusions are based on a set of limited experimental data presented by Roth.[20] Therefore, these recession models need to be tested further and compared using a variety of data before more definitive conclusions can be reached. However, it may be appropriate to note that the conclusions reached in this study are in support of those reached in individual model developments.

Models for recession of impounded water in closed downstream borders need field evaluation for short and long time of inflow cut-off.

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