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DERIVATION OF THE SCS-CN PARAMETER *S* FROM LINEARIZED FOKKER-PLANCK EQUATION

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Abstract

The potential maximum retention, S, of the Soil Conservation Service Curve Number (SCS-CN) method (SCS, 1956) was derived for a large set of published infiltration data ranging from Plainfield sand to Yololight clay using the relations between ψ (negative pressure) and θ (moisture content) and between K (hydraulic conductivity) and θ. The physical significance of S is explained using the diffusion term of the linearized Fokker-Planck equation for infiltration, which relates S to the storage and transmission properties of the soil. The derived S-values exhibit a strong looped relationship with the initial moisture content, analogous to that for curve numbers for three antecedent moisture conditions. The variation of S in vertical infiltration is also explained and discussed.

Key words: curve number, infiltration, Fokker-Planck equation, maximum soil retention, SCS-CN method.

INTRODUCTION

The event-based rainfall-runoff model of the Soil Conservation Service of the U.S. Department of Agriculture (SCS, 1956), known as SCS Curve Number (SCS-CN) method, has received much attention in recent years (Ponce and Hawkins, 1996; Bonta, 1997;

Yu, 1998; Mishra and Singh, 1999a; b; McCuen, 2002). The SCS-CN method combines the water balance equation and two fundamental hypotheses which, respectively, are:

$$P = I_a + F + Q \quad , \tag{1}$$

$$\frac{Q}{P - I_a} = \frac{F}{S} \quad , \tag{2}$$

$$I_a = \lambda S \quad , \tag{3}$$

where P is the total precipitation, I_a is the initial abstraction, F is the cumulative infiltration excluding I_a, Q is the direct runoff, S is the potential maximum retention, and λ is the initial abstraction coefficient, assumed to be equal to 0.2 for usual applications, but it can have a range of (0, ∞) (Mishra and Singh, 1999a). The value of $\lambda = 0.2$ is derived from a plot of I_a *versus* S that includes I_a (SCS, 1971). However, for $F = S$, in the water balance equation (1), S is separate from I_a, and thus S should exclude I_a. Thus, the second hypothesis (eq. 3) has sometimes been misinterpreted to consider I_a as a part of S. For S to exclude I_a, Chen (1981) derived $\lambda = 0.25$, while taking $\lambda = 0.2$ for S including I_a.

Coupling of eq. 2 with eq. 1 leads to the following popular form:

$$Q = \frac{(P - I_a)^2}{P - I_a + S} \quad . \tag{4}$$

Alternatively, eq. (4) can also be used for deriving S from P-Q data (Hawkins, 1993; Mishra and Singh, 1999a). In practice, S is determined from the scaling equation:

$$S = \frac{1000}{CN} - 10 \quad , \tag{5}$$

where S is in inches and CN is the curve number that varies from 0 to 100 and is derivable from the National Engineering Handbook (Section-4) NEH-4 tables for various catchment characteristics: soil type, land use, hydrologic condition, and antecedent moisture condition (AMC). AMC represents the amount of moisture available in the soil profile before the start of a storm and its value is defined by the precipitation amount of antecedent 5 days (SCS, 1956).

The SCN-CN hypothesis can be described using $C = S_r$ concept (Mishra and Singh, 2002; 2003), where C is the runoff coefficient and S_r is the degree of saturation of the soil profile. Parameter S is the maximum possible retention of water in the soil profile or the maximum cumulative amount of dynamic infiltration. Such a description is closely associated with the discussion of Mockus (1964) (in: Rallison, 1980) on the physical significance of S. However, the Mockus description does not distinguish dynamic or static portion of infiltration and takes into account only the volumetric properties of the soil excluding its transmission characteristics. To describe a dynamic system, it is necessary to account for both the storage and transmission properties of the medium (Ponce, 1989). For water transmission through soil systems, soil sorptivity, S_s, (Philip, 1957; 1969) and hydraulic conductivity, K, describe, respectively, the volumetric and transmission characteristics of the soil. Both S_s and K depend on (1) soil characteristics (type, texture, and structure) and initial moisture condition; (2) land use; (3) rainfall intensity; and (4) turbidity and temperature. A detailed description of these properties is available in the textbooks by Hillel (1976) and Marshall *et al.* (1996).

Succinctly, S_s and K can be described by the ψ–θ (negative pressure – moisture content) and K–θ relations, respectively, which characterise the phenomenon of moisture movement in unsaturated porous media (Collis-George, 1977; Feddes *et al.*, 1988; Mallants *et al.*, 1998). In such media, the adsorption and adhesion-cohesion forces resulting in concave air-water interfaces (also known as matric forces) lead to capillary action responsible for water retention. The matric suction is a function of soil type and moisture content, θ; the greater the moisture content, the lower will be the suction force and *vice versa* (Moore, 1939). This phenomenon can also be explained in terms of the potential defined as the work required to transfer a unit quantity of water from a standard reference state, viz. zero, to another state. Whether the water transfer actually takes place depends on the conductivity of the unsaturated soil or capillary conductivity that depends not only on the pore geometry but also on the water content. Thus, the objective of this paper is to derive S using both the volumetric and transmission properties and investigate the SCS-CN hypothesis (eq. 3) and the relation of AMC with the initial moisture content using a large set of infiltration data derived from laboratory tests.

ANALYTICAL DERIVATION OF *S*

For describing S in terms of storage and transmission properties of soils, the following linearized Fokker-Planck equation describing infiltration in unsaturated media (Philip, 1974) is utilized:

$$\frac{\partial \theta}{\partial t} + \frac{dK}{d\theta}\frac{\partial \theta}{\partial z} = D\frac{\partial^2 \theta}{\partial z^2} \tag{6}$$

where θ is the moisture content, K is the unsaturated hydraulic conductivity, D is the hydraulic diffusivity, z is the space coordinate, positive in the vertical downward direction, and t is the time. The non-linear form of eq. (6) (with its right hand side equal to $\partial[D(\theta)\partial\theta/\partial z]/\partial z$) results from the coupling of the following equations (Philip, 1969):

Darcy's law

$$U = -K(\theta)\frac{\partial \Phi}{\partial z} , \tag{7}$$

continuity equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial U}{\partial z} \quad , \tag{8}$$

and the energy equation or Buckingham equation

$$\Phi = \frac{p}{\rho g} + z = \psi + z \quad , \tag{9}$$

where U is the Darcy flux (or velocity), Φ is the total potential head, p is the capillary pressure, ρ is the fluid density, g is the gravitational acceleration, ψ is the pressure head $(= p/\rho g)$ that is a function of θ $(= \psi(\theta))$, z and t are, respectively, the space and time coordinates, the value of z is measured from the soil surface positive down below, and D is the moisture diffusivity defined as

$$D = K\mathrm{d}\psi/\mathrm{d}\theta \quad , \tag{10}$$

which can be derived from the relations (e.g., Feddes *et al.*, 1988): $C(\theta) = \mathrm{d}\theta/\mathrm{d}\psi$ and $D(\theta) = D = K(\theta)/C(\theta)$. The term $C(\theta)$ is called specific moisture capacity.

Soil water flow is highly nonlinear and is described by the nonlinear Fokker-Planck equation. The nonlinearity is caused by the dependence of both the hydraulic conductivity and soil water pressure on the soil moisture content. This nonlinear equation has not yet been solved under general conditions and has only been solved for simplified flow conditions under a number of restrictive assumptions (Feddes *et al.*, 1988). Therefore, numerical modelling is frequently resorted to, for example, CREAMS (Knisel, 1980), LEACHM (Wagenet and Hutson, 1987), MACRO (Jarvis *et al.*, 1991), and WAVE (Vanclooster *et al.*, 1994) are a few popular numerical models among many others. Owing to the fact that the SCS-CN method is a simple, one-parameter, event-based, lumped model that works with the mean values (Ponce and Hawkins, 1996), its parameter S is derived analytically from linear Fokker-Planck equation (eq. 6) using the linear perturbation theory. Here, D is assumed constant in accordance with the SCS-CN assumption of the constant parameter S (CN) for a rainstorm. It is acknowledged that for soils D is seldom constant and as a result linearization of the Fokker-Planck equation is only an assumption which has long been employed in hydrology. A brief discussion of other limitations of such an analysis follows.

Harmonic analysis of a wave phenomenon forms an initial step to gain insight to the mechanism of a system, whether linear or non-linear, in several branches of mechanics and physics (Crawford, 1965; Pedlosky, 1979) and the resulting linear solutions often prove adequate for practical purposes (Kundzewicz and Dooge, 1989). In open channel hydraulics, linear analyses have been carried out assuming either complex frequency and real wave number (Ponce and Simons, 1977) or real frequency and complex wave number

(Vreugdenhill, 1972; de Souza and Price, 1980) and both yield an incomplete description of wave characteristics (Kundzewicz and Dooge, 1989). A feasible solution for complex frequency and wave number is not possible because of the unknown relation between these complex parameters. In the following analytical derivation, we utilise the approach of Ponce and Simons (1977) for the reason that the derived wave criteria (Ponce *et al.*, 1978) perform satisfactorily in field applications (Mishra and Seth, 1996; Mishra *et al.*, 1997; Mishra and Singh, 1999c; 2001a; and others).

Assume that K–θ and ψ–θ relations follow a power relation (Campbell, 1974):

$$K = a\theta^{m} , \tag{11}$$

$$\psi = a_1 \theta^{m_1} , \tag{12}$$

where a and a_1 are coefficients, and m and m_1 are exponents. Equation (12) was proposed by Visser (1969). For other forms of ψ–θ relations, the work of Mallants *et al.* (1998) can be referred. Campbell (1974) tested these relations on five soils, Botany, Guelph, Cecil Ap, Cecil B2, and Cecil B3, in θ ranges: 0.06–0.36, 0.25–0.53, 0.05–0.40, 0.28–0.47, and 0.23–0.40 (volume/volume), respectively. Gardener *et al.* (1970) also found eq. (12) to be sufficiently accurate over a limited range. Following Ponce and Simons (1977), the characteristic length χ (Todd, 1980) can be defined as

$$\chi = \frac{\psi_0}{\partial\psi/\partial z}, \tag{13}$$

where ψ_0 is the negative pressure at the initial moisture content, θ_0. In eq. (13), $\partial\psi/\partial z$ can be replaced by $\partial\theta/\partial z$ using ψ–θ relation (12). From eqs. (11) and (12) the derivatives for K and ψ can be derived, respectively, as

$$\frac{dK}{d\theta} = ma\theta^{m-1} = m\frac{K}{\theta} \tag{14}$$

and

$$\frac{d\psi}{d\theta} = m_1 a_1 \theta^{m_1 - 1} = m_1\left(\frac{\psi}{\theta}\right) . \tag{15}$$

Substitution of eq. (15) into eq. (10) leads to

$$D = m_1\left(\frac{K}{\theta}\right)\psi . \tag{16}$$

Equation (16) implies that for a given θ, D is directly proportional to the energy expended in unit time (or power) per unit weight of the absorbate, and the proportionality constant m_1 is a ψ–θ characteristic as mentioned above.

Using eqs. (14) and (16), eq. (6) can be linearized as

$$\frac{\partial\theta}{\partial t} + \frac{mK_0}{\theta_0}\frac{\partial\theta}{\partial z} = \frac{m_1 K_0 \psi_0}{\theta_0}\frac{\partial^2\theta}{\partial z^2} , \tag{17}$$

where K_0 and ψ_0 correspond to θ_0. It is assumed that a linear perturbation analysis holds. Then,

$$\theta' = \theta - \theta_0 , \tag{18}$$

where θ' is the perturbation about θ_0, and eq. (17) can be rewritten as:

$$\frac{\partial\theta'}{\partial t} + \frac{mK_0}{\theta_0}\frac{\partial\theta'}{\partial z} = \frac{m_1 K_0 \psi_0}{\theta_0}\frac{\partial^2\theta'}{\partial z^2} . \tag{19}$$

For the foregoing wave analysis, (θ'/θ) is assumed to follow a sinusoidal form as

$$\frac{\theta'}{\theta_0} = \Omega \exp\left[\mathrm{i}\left(\hat{\sigma}\hat{z} - \hat{\beta}\hat{t}\right)\right] , \tag{20}$$

where

$$\hat{\sigma} = \frac{2\pi}{L}\chi , \quad \hat{\beta} = \frac{2\pi}{T}\left(\frac{\chi}{K_0}\right), \quad \hat{z} = \frac{z}{\chi} , \quad \hat{t} = \frac{t K_0}{\chi} , \tag{21}$$

Ω is the non-dimensional wave amplitude, L is the wavelength, T is the time period, $\hat{\sigma}$ is the dimensionless wave number, $\hat{\beta}$ is the dimensionless frequency factor, and i is the imaginary unit.

From eqs. (19), (20), and (21) the following characteristic equation can be derived:

$$\hat{\beta} = \frac{m}{\theta_0}\hat{\sigma} - \mathrm{i}\left(\frac{m_1\psi_0}{\theta_0\chi}\right)\hat{\sigma}^2 , \tag{22}$$

from which real and imaginary components can, respectively, be extracted as

$$\hat{\beta}_R = \frac{m}{\theta_0}\hat{\sigma} \quad \text{and} \quad \hat{\beta}_I = -\frac{m_1\psi_0}{\theta_0\chi}\hat{\sigma}^2 . \tag{23}$$

By definition, the non-dimensional wave celerity ($\hat{c}$) and logarithmic decrement (δ) of the wave can, respectively, be derived as follows:

$$\hat{c} = \frac{\hat{\beta}_R}{\hat{\sigma}} = \frac{m}{\theta_0} \tag{24}$$

and

$$\delta = 2\pi \frac{\hat{\beta}_I}{\hat{\beta}_R} = -2\pi \left(\frac{m_1}{m}\right)\left(\frac{\psi_0}{\chi}\right)\hat{\sigma} \ , \tag{25}$$

where θ_0 is the moisture content at $t = 0$; $\hat{c} = c / K_0$, c is the dimensional wave celerity; the "–" sign indicates that the wave propagates in the z-direction and attenuates; and the "+" sign indicates that the wave travels opposite to the z-direction and amplifies. From the above, c and δ can also be expressed, respectively, as

$$c = \frac{mK_0}{\theta_0} \tag{26}$$

and

$$\delta = -2\pi \left(\frac{D}{c\chi}\right)\hat{\sigma} \ . \tag{27}$$

The derivation of eq. (26) is analogous to deriving the Seddon speed from the stage-discharge relation in open channel flow (Mishra and Singh, 2001b). Here K_0/θ_0 represents the velocity of the moisture movement (Young, 1957). Equation (27) can also be written, alternatively, as

$$\delta = -4\pi^2 \frac{m_1 \psi_0 \theta_0}{m^2 K_0 T} \ . \tag{28}$$

For a physical description, δ can also be defined as (Ponce *et al.*, 1978):

$$\delta = \ln(\theta_T) - \ln(\theta_i) \tag{29}$$

or

$$\theta_T = \theta_i e^{\delta} \ , \tag{30a}$$

where θ_i and θ_T are the initial moisture content and the moisture content at time, $t = T$, respectively.

Equation (30a) can be written alternatively in terms of the water column equivalent to F as

$$F_T = F_i e^{\delta} \ , \tag{30b}$$

where F_T and F_i represent the cumulative infiltration at times T and i (initial), respectively. Similarly, for the maximum cumulative infiltration values or S, eq. (30b) can be recast as

$$S_T = S_i e^{\delta} \ , \tag{30c}$$

where S_T is the potential maximum retention after the time period, T, and S_i is the initial value of S.

Since eq. (6) approximates the diffusion process, δ will be a negative quantity and, therefore, $S_T < S_i$. Equations (30b) and (30c) help describe the cyclic effects of the absorption–desorption process of cumulative infiltration. In an absorption process, F starts from 0 and ends at S and the following desorption process takes F to 0. In the subsequent absorption cycle, F again reaches S and so on. Here, S_i corresponds to S of the first cycle and S_T stands for S of the second cycle. It is for this reason that the maximum cumulative infiltration or S is of prime interest in this paper, analogous to the flood peak description in flood routing. Thus, the same soil will yield different S-values in absorption–desorption cyclic process owing to the hysteretic effect (Chang and Sonu, 1993). The difference in S, however, depends on the magnitude of δ, which, in turn, depends on K–θ, ψ–θ, initial moisture content, and the time period.

Parallel to the hysteresis-based studies dealing with open channel flows (Mishra and Seth, 1996; Mishra and Singh, 1999c; 2001a; and others), the time period, T, can be described approximately equal to twice the time base of infiltration, T_b, or $T_b = T/2$. Thus, δ can be rewritten in terms of T_b as

$$\delta = -2\pi^2 \frac{m_1 \psi_0 \theta_0}{m^2 K_0 T_b} , \tag{31}$$

which can be derived from soil characteristics: ψ–θ and K–θ relations, initial moisture content, and infiltration data.

Description of S in terms of S_s and K

Collis-George (1977) related the maximum dynamic infiltration amount with the soil sorptivity, S_s, and hydraulic conductivity, K. This relation can be described in terms of S as

$$S = \frac{\pi}{4} \frac{S_s^2}{K} , \tag{32}$$

which can be derived from eq. (6). According to Collis-George (1977), eq. (32) is difficult to use over the whole time range of infiltration unless the experimentally derived maximum cumulative infiltration (or S) is appropriate for the S_s and K values. At small values of time, eq. (32) gives a good description of the maximum cumulative infiltration or S. Therefore, he generalised this equation in terms of a parameter N (Philip, 1974) that relates the integrated effects of capillary and gravitational phenomenon as

$$N = \frac{S}{Kt_c} = \frac{S_s^2}{SK} = \frac{S_s}{K(t_c)^{1/2}} , \tag{33}$$

where t_c is a characteristic time, and S represents the integrated effects of capillary phenomenon and the product of K and t_c of the gravitational phenomenon. Thus, using eq. (33), eq. (32) can be generalized as

$$S = \frac{1}{N}\frac{S_s^2}{K} \ . \tag{34}$$

For $N = 4/\pi$, eq. (34) reduces to eq. (32).

Description of S in terms of K–θ and ψ–θ relations

For defining the SCS-CN parameter S in terms of K–θ and ψ–θ relations, it is necessary to investigate S_s, in what follows. Philip (1957) defined the moisture diffusivity, D, as

$$\int_{\theta_0}^{\theta_1} D\,\mathrm{d}\theta = \frac{S_s^2}{2} \ . \tag{35}$$

Coupling of eq. (35) with eq. (10) yields

$$\int_{\psi_0}^{\psi_1} K\,\mathrm{d}\psi = \frac{S_s^2}{2} \ . \tag{36}$$

Equation (36) leads to the definition of S_s as

$$S_s = \frac{2\alpha}{\beta+1}\left[\psi_1^{\beta+1} - \psi_0^{\beta+1}\right] , \tag{37}$$

where

$$\alpha = \frac{a}{(a_1)^{m/m_1}} \qquad \text{and} \qquad \beta = \frac{m}{m_1} \ . \tag{38}$$

The right hand side of eq. (37) represents twice the area below the K–ψ relation. The hysteresis in ψ–θ relation (Mein and Larson, 1971) leads to reduction in the sorptivity of the soil or in the above-described S in the absorption–desorption cyclic process.

Other approximate expressions for S_s and, in turn, S, can be derived taking D as a constant quantity similar to the existing SCS-CN method that considers the parameter S to be constant for a rainstorm on a watershed. Since S varies with θ_0, D will also vary with it. Thus, an integration of eq. (35) yields

$$D(\theta_1 - \theta_0) = \frac{S_s^2}{2} \ , \tag{39}$$

where $\theta_1 > \theta_0$. In the context of a rainstorm, θ_0 and θ_1 can be taken as the moisture contents at the beginning and end of the storm, respectively. Substitution of eq. (39) into eq. (16) leads to

$$S_s^2 = 2m_1\left[\underbrace{\bar{\psi}(K_1 - K_0)}_{\text{I}} + \underbrace{\bar{K}(\psi_1 - \psi_0)}_{\text{II}}\right] , \tag{40}$$

where (K_0, ψ_0) and (K_1, ψ_1) correspond to θ_0 and θ_1, respectively, and $\overline{\psi}$ and $\overline{K}$ correspond to the average ψ and K, respectively, computed for the θ-range of (θ_0, θ_1). In eq. (40), the I and II parts implicitly represent energy expenditure in unit time for unit weight of the absorbate, respectively, at constant pressure and at constant volume (which can be derived from Darcy's law, eq. (7), following Rankine, 1870), which, in turn, are analogous to specific heats at constant pressure, c_p, and constant volume, c_v, respectively (Mishra and Singh, 2001b). Thus, S_s is analogous to the hysteresis defined by Mishra and Seth (1996), Mishra and Singh (1999c), and others as the energy expenditure in wave propagation during one time period.

In eq. (40), the first part dominates when $\theta_1 \rightarrow \theta_{sat}$ (= θ at saturation), as shown by the ψ–θ relation (Philip, 1969), whereas K varies rapidly with θ. However, the second part dominates when $\theta_1 \rightarrow \theta_0$, where the variation in K with θ is insignificant and ψ varies rapidly. Thus, eq. (40) covers the whole range of the θ-variation.

The above feature of the first part leads to

$$S_s^2 = 2m_1 \overline{\psi}(K_1 - K_0) \ , \qquad (41)$$

which is analogous to the following relation (Young, 1964) for $K_1(=K) \gg K_0$:

$$S_s^2 = 2(n-\theta)K\psi_e \ . \qquad (42)$$

Here, ψ_e is the Green-Ampt effective wetting front suction. A comparison of eq. (41) with eq. (42) yields $m_1\overline{\psi} = (n-\theta_0)\psi_e$ or $\psi_e = m_1\overline{\psi}/(n-\theta_0)$. It is noted here that ψ_e and $\overline{\psi}$ represent the Green-Ampt effective wetting front suction and average suction, respectively; and m_1 and $(n-\theta_0)$ represent the pore size distribution index (Singh, 1997) and the effective soil porosity, respectively.

Then, an expression for the SCS-CN parameter S can be derived from eq. (34) for $K_1 (=K) \gg K_0$ as

$$S = \frac{1}{N}\left(\frac{S_s^2}{K_1 - K_0}\right) . \qquad (43)$$

Substitution of eq. (41) into eq. (43) yields

$$S = \frac{2}{N} m_1 \overline{\psi} \ , \qquad (44)$$

where m_1 represents the ψ–θ characteristic. From eq. (44) it can be inferred that for a given soil, S is directly proportional to the average head and, therefore, this equation describes the variation of S with time conforming to the infiltration decay pattern experienced in field or laboratory observations. As time progresses, infiltration rate decreases or, in other words, θ increases or ψ decreases, and thus, $\overline{\psi}$ decreases. Reduction in $\overline{\psi}$ decreases S or increases CN (eq. 5), consistent with the hypothesis of Mishra *et al.* (1999) that describes the temporal variation of S using the storage concept of the general infiltration model (Singh and Yu, 1990). Consequently, it is possible to quantify the S (or CN) variation within the storm duration. Furthermore, based on the volumetric concept, S can be defined as

$$S = (n - \theta_0) D_s \ , \tag{45}$$

where D_s is the depth above the impeding layer of the soil profile. A comparison of eq. (45) with eq. (44) yields $D_s = (2/N) m_1 \overline{\psi} / (n - \theta_0)$. It follows that D_s depends on pore size distribution, average suction, and effective porosity, and thus, it is a physically determinable, but θ-dependent, quantity.

The general form of eq. (44) can also be derived using eq. (40) as

$$S = \frac{2}{N} m_1 \left[\overline{\psi} + \frac{\overline{K}(\psi_1 - \psi_0)}{(K_1 - K_0)} \right] . \tag{46}$$

From eq. (46) it is evident that S is also a function of ψ_0 and K_0 which themselves depend on the initial moisture content, θ_0. It is consistent with the concept of AMC that is identical to θ_0. Equation (46) actually quantifies all the four runoff producing watershed characteristics, viz., soil type, land use, hydrologic condition of land surface, and AMC, used for derivation of CN from the NEH-4 tables. These watershed properties eventually affect K–θ and ψ–θ relations, and thus, affect the S or CN computation.

For practical purposes, an average value of S can be derived for $\psi_1 \ll \psi_0$ and $K_0 \ll K_1$ (here, ψ_1 and K_1 correspond to θ_{sat}) and, therefore, eq. (46) converges to

$$S = \frac{2}{N} m_1 \left[\overline{\psi} - \frac{\overline{K}}{K_1} \psi_0 \right] . \tag{47}$$

It is noted that since ψ is influenced by the magnitude of the depth of storm precipitation (Philip, 1969), S also depends on the precipitation magnitude.

Description of S in terms of intrinsic sorptivity

S can also be expressed in terms of intrinsic sorptivity, ς, defined by Philip (1969) as

$$\varsigma = \left(\frac{\mu}{\sigma} \right)^{1/2} S_s \ , \tag{48}$$

where μ is the dynamic viscosity and σ is the surface tension (different from $\hat{\sigma}$ which is the wave number) of the absorbate. Here ς is analogous to the intrinsic permeability, a characteristic of the internal geometry of the medium and is independent of the properties of the absorbate (Philip, 1969). Equation (48) can be expressed for S_s as

$$S_s^2 = \frac{\varsigma^2 \sigma}{\mu} . \tag{49}$$

Therefore, S can also be derived from eqs. (49) and (43) as

$$S = \frac{1}{N} \frac{\varsigma^2 \sigma}{\mu (K_1 - K_0)} . \tag{50}$$

Alternatively, the intrinsic sorptivity can be expressed using eqs. (40) and (48) as

$$\varsigma = \sqrt{\frac{2m_1 \mu}{\sigma} \left[\overline{\psi} (K_1 - K_0) + \overline{K} (\psi_1 - \psi_0) \right]} \tag{51}$$

and can be explained in terms of c_p and c_v, as above. It is important to note that S is inversely proportional to μ implying that CN increases with μ and *vice versa*. Lee (1983) found that the freezing of the soil of high moisture content decreased infiltration to zero whereas freezing at low moisture content increased infiltration to twice its normal rate. Such behaviour is, however, attributed to the variation of μ with temperature and moisture content.

Description of S for vertical infiltration

The vertical infiltration phenomenon is described by the equation derived from eq. (6) eliminating the product of two derivatives on the left hand side of this equation (Philip, 1969). Here gravity forces are insignificant compared to matric forces. Similar to the above wave solutions it can be shown that the wave celerity in vertical infiltration is equal to zero implying that m is equal to zero in eq. (26) since neither K nor will θ be zero. It infers that for such a soil exhibiting only vertical infiltration, K should be independent of θ. It is possible when $K = K_s$ (K at saturation) or $\theta = \theta_{sat}$. In this case, the moisture profile moves with a velocity of K/θ with $\delta = \infty$, implying that the whole moisture is absorbed up to the distance the profile travels.

The physical significance of the SCS-CN parameter S is described for this case following Philip (1957; 1969), who described S_s using the relation

$$S_s = x t^{-1/2} , \tag{52}$$

where x is the vertical depth of flow from the ground surface. Substitution of eq. (52) into eq. (32) leads to

$$S = \frac{1}{N} \frac{x^2 t^{-1}}{K} \tag{53}$$

or, alternatively, from eq. (33), K can be defined as

$$K = \frac{x}{\sqrt{t\,t_c}} \quad . \tag{54}$$

Equation (54) describes variation of K with x and t in vertical infiltration. At a given location x, K initially reduces more sharply than it does at longer times, for t_c represents a characteristic time. An alternative form of eq. (53) describing S is as below:

$$S = \frac{1}{N}\left(\frac{x t^{-1}}{K}\right) x \quad . \tag{55}$$

Under free fall situations:

$$x = \frac{1}{2} g t^2 \quad . \tag{56}$$

Substituting t from eq. (56) into eq. (55) leads to

$$S = \frac{1}{2N}\left(\frac{\sqrt{2 g x}}{K}\right) x \quad . \tag{57}$$

The ratio in the bracketed part of eq. (57) is the ratio of two speeds: That in the numerator describes the velocity of flow under free fall and that in the denominator is the average velocity of flow under the unit hydraulic gradient (Darcy's law, eq. 4). Here, the inverse of the ratio is defined by M as follows:

$$M = \frac{K}{\sqrt{2 g x}} \quad , \tag{58}$$

which appears to be a hydraulic number characterizing the flow transmission in unsaturated porous media, analogous to the Froude number defining the hydraulic properties of flow in open channels. Since K can assume any value ($\leq$ or $>$ $\sqrt{2 g x}$), M can also assume any value ($\leq$ or $>$ 1). As the minimum value that K can assume is equal to zero, M can have a range of $(0, \infty)$ for $x > 0$. Therefore, eq. (58) can be written as

$$S = \frac{1}{2N}\frac{x}{M} \quad . \tag{59}$$

Equation (59) implies that in a given flow regime (or constant M), S increases linearly or CN decreases with the depth of flow. Near the ground surface ($x \rightarrow 0$) $S \rightarrow 0$ and CN$\rightarrow$100 implying that during a storm, surface runoff can commence even before the soil storage is fully exhausted.

Description of S for kinematic wave situation

Another hypothetical case of kinematic wave derivable from eq. (6) that represents the linear form of a diffusion wave is considered. The kinematic wave form can be derived by ignoring the second-order term containing D at the right hand side of eq. (6). From a linear analysis it can be shown that the kinematic wave celerity, c_k, is the same as that of eq. (6) as

$$c_k = \frac{mK_0}{\theta_0} \quad . \tag{60}$$

It implies that such a wave propagates in the z-direction, and $\delta = 0$ implies that these waves do not attenuate. Since $D = 0$ or $S_s = 0$, $S = 0$. Following Ponce *et al.* (1978), a criterion for kinematic wave application for 95% accuracy can be derived as follows:

$$\frac{T_b K_0 m^2}{m_1 \psi_0 \theta_0} \geq 385 \quad . \tag{61}$$

APPLICATION

The applicability of the above derivations was investigated using the infiltration data observed from laboratory tests on soils given in Table 1 (Mein and Larson, 1971). The parameters of the power relations fitted to the available K–θ, ψ–θ, and K–ψ relations for these soils are described in Table 2. The infiltration tests were conducted for varying

Table 1

Soil characteristics for Minnesota region, USA

Soil type	Number of tests	K_s (cm/s)	Porosity	Reference
Plainfield sand (PFS) (disturbed sample)	12	3.44×10^{-3}	0.477	Black *et al.* (1969)
Columbia sandy loam (CSL) (disturbed sample)	36	1.39×10^{-3}	0.518	Laliberte *et al.* (1966)
Guelph loam (GL) (air dried, sieved)	12	3.67×10^{-4}	0.523	Elrick and Bowman (1964)
Ida silt loam (ISL) (undisturbed sample)	42	2.92×10^{-5}	0.530	Green (1962)
Yololight clay (YLC) (disturbed sample)	12	1.23×10^{-5}	0.499	Moore (1939)

Table 2

Fitted parameters of the K–θ, ψ–θ, K-θ, and K–ψ relations for the soils in Table 1

Soil type	K–θ			ψ–θ			K–ψ	
	a	m	r^2	a_1	m_1	r^2	α	β
Plainfield sand	0.8399	6.7160	0.9842	-2.0654	-1.7034	0.9530	4.81E-02	-3.94
Columbia sandy loam	0.9107	9.0279	0.9339	-6.4616	-1.4820	0.9213	1.05E-05	-6.09
Guelph loam	1.3824	12.359	0.9602	-1.5532	-4.0433	0.9896	3.60E-01	-3.06
Ida silt loam	0.3280	14.845	0.9982	-6.6796	-3.3730	0.9998	7.69E-05	-4.40
Yololight clay	0.0041	8.6778	0.9934	-0.5435	-4.8768	0.9988	1.21E-02	-1.78

uniform rainfall intensity and initial moisture contents. For the given magnitudes of rainfall intensity and initial moisture content, the available information on the time to ponding, time base, and maximum cumulative infiltration amount from infiltration tests were averaged to characterize the infiltration process of a soil in a given situation. The derived information is listed in Table 3 along with the hydraulic conductivity and magnitudes of suction for initial (ψ_0) and saturated (ψ_{sat}) conditions. It is noted that the time base of an infiltration curve was taken as the last value of the published data.

Computation of CN

As apparent from eq. (44), the determination of parameter S requires the determination of N a priori for field applications. From several infiltration data sets derived from laboratory and field tests, Collis-George (1977) determined the value of N ranging from 0.54 to 5.8 for various soils. Table 3, however, indicates the variation of N from 0.005 to 0.094 for the above soils. Here, S values are derived following Collis-George (1977) as

$$S = F_{max} - K_s T_b \tag{62}$$

and S_s-values are derived from eq. (37) that allows variation of D with θ. Here, F_{max} is the maximum cumulative infiltration. The computed S-values are shown in Table 3. Apparently, S varied from 2.87 to 4.21 cm for Plainfield sand, from 3.12 to 6.09 cm for Columbia sandy loam, from 2.45 to 3.46 cm for Guelph loam, from 0.52 to 1.55 cm for Ida silt loam, and from 0.97 to 2.18 cm for Yololight clay. On the whole, S-values tended to decrease with the soil type from Plainfield sand to Yololight clay in the given order. Table 3 also indicates the variation of the depth above the impeding layer, D_s, (cm)

Table 3

Computation of parameters

Soil	i_0 [cm/s]	θ_0 [vol/vol]	θ_{sat} [vol/vol]	t_p [s]	T_b [s]	K_0 [cm/s]	h_0 [cm]	h_{sat} [cm]	F_{max} [cm]	I_a [cm]	δ	S_s^2 [cm²/s]	N	t_c [s]	$S=F_{max}-K_s*T_b$ [cm]	CN	D_s [cm]	KW parameter
Plainfield sand	1.38E-02	0.13	0.472	104	1145	9.41E-07	-66.7	-7.4	8.154	1.44	-0.66	2.73E-02	0.051	650	4.21	85.8	12.15	3.29E-03
	1.38E-02	0.23	0.472	77	789	4.34E-05	-25.2	-7.4	5.583	1.06	-0.96	2.66E-02	0.072	309	2.87	89.9	11.61	1.56E-01
	2.75E-02	0.13	0.472	25	1070	9.41E-07	-66.7	-7.4	7.954	0.68	-0.71	2.73E-02	0.051	668	4.27	85.6	12.31	3.07E-03
	2.75E-02	0.23	0.472	60	796	4.34E-05	-25.2	-7.4	5.738	1.64	-0.95	2.66E-02	0.069	338	3.00	89.4	12.14	1.58E-01
Columbia sandy loam	2.78E-03	0.25	0.518	2636	3034	3.34E-06	-50.4	-17.1	8.312	7.33	-0.76	1.61E-02	0.045	1043	4.10	86.1	15.28	4.42E-02
	5.56E-03	0.125	0.518	599	1540	6.40E-09	-140.8	-17.1	7.183	3.33	-1.49	1.62E-02	0.037	1574	5.04	83.4	12.83	3.08E-05
	5.56E-03	0.15	0.518	553	2090	3.32E-08	-107.5	-17.1	8.377	3.08	-1.10	1.62E-02	0.034	1854	5.47	82.3	14.87	2.37E-04
	5.56E-03	0.2	0.518	490	1676	4.46E-07	-70.2	-17.1	7.110	2.72	-1.37	1.61E-02	0.039	1416	4.78	84.2	15.03	2.93E-03
	5.56E-03	0.25	0.518	420	1523	3.34E-06	-50.4	-17.1	6.270	2.34	-1.50	1.61E-02	0.045	1072	4.15	85.9	15.50	2.22E-02
	5.56E-03	0.318	0.518	324	1219	2.93E-05	-35.3	-17.1	4.817	1.80	-1.88	1.57E-02	0.057	619	3.12	89.1	15.61	1.75E-01
	8.34E-03	0.125	0.518	244	970	6.40E-09	-140.8	-17.1	5.699	2.04	-2.36	1.62E-02	0.043	1172	4.35	85.4	11.07	1.94E-05
	8.34E-03	0.25	0.518	170	1495	3.34E-06	-50.4	-17.1	6.490	1.42	-1.53	1.61E-02	0.042	1210	4.41	85.2	16.46	2.18E-02
	1.11E-02	0.125	0.518	136	914	6.40E-09	-140.8	-17.1	5.678	1.51	-2.51	1.62E-02	0.043	1203	4.41	85.2	11.22	1.83E-05
	1.11E-02	0.15	0.518	127	1971	3.32E-08	-107.5	-17.1	8.827	1.40	-1.16	1.62E-02	0.031	2294	6.09	80.7	16.54	2.23E-04
	1.11E-02	0.2	0.518	117	1737	4.46E-07	-70.2	-17.1	7.731	1.30	-1.32	1.61E-02	0.035	1752	5.32	82.7	16.72	3.03E-03
	1.11E-02	0.25	0.518	90	1457	3.34E-06	-50.4	-17.1	6.479	1.00	-1.57	1.61E-02	0.042	1233	4.45	85.1	16.62	2.12E-02
	1.11E-02	0.318	0.518	75	1039	2.93E-05	-35.3	-17.1	4.606	0.83	-2.21	1.57E-02	0.056	635	3.16	88.9	15.81	1.49E-01

Guelph loam	1.47E-03	0.3	0.523	1702	3668	4.77E-07	-202.0	-21.4	4.592	2.50	-4.33	9.43E-03	0.075	1117	3.25	88.7	14.55	1.09E-03
	1.47E-03	0.35	0.523	1246	2855	3.20E-06	-108.3	-21.4	3.496	1.83	-5.57	9.19E-03	0.094	652	2.45	91.2	14.15	9.12E-03
	2.94E-03	0.3	0.523	376	3187	4.77E-07	-202.0	-21.4	4.628	1.11	-4.99	9.43E-03	0.070	1268	3.46	88.0	15.51	9.47E-04
	2.94E-03	0.35	0.523	305	2513	3.20E-06	-108.3	-21.4	3.540	0.90	-6.33	9.19E-03	0.088	746	2.62	90.7	15.13	8.02E-03
Ida silt loam	5.84E-05	0.25	0.53	32495	39597	3.79E-10	-717.0	-56.9	2.255	1.90	-7.87	8.85E-04	0.024	1364	1.10	95.9	3.92	5.47E-06
	5.84E-05	0.4	0.53	14232	39715	4.06E-07	-146.9	-56.9	1.843	0.83	-7.85	8.50E-04	0.036	549	0.68	97.4	5.26	1.79E-02
	1.17E-04	0.25	0.53	7449	37689	3.79E-10	-717.0	-56.9	2.651	0.87	-8.27	8.85E-04	0.017	2717	1.55	94.2	5.54	5.20E-06
	1.17E-04	0.3	0.53	5907	38962	5.67E-09	387.6	-56.9	2.500	0.69	-8.00	8.84E-04	0.020	2100	1.36	94.9	5.92	1.24E-04
	1.17E-04	0.35	0.53	5123	18976	5.59E-08	-230.5	-56.9	1.473	0.60	-16.43	8.77E-04	0.029	962	0.92	96.5	5.11	8.59E-04
	1.17E-04	0.4	0.53	2963	39734	4.06E-07	-146.9	-56.9	1.967	0.35	-7.85	8.50E-04	0.031	766	0.81	96.9	6.21	1.79E-02
	1.17E-04	0.43	0.53	2515	19304	1.19E-06	-115.1	-56.9	1.082	0.29	-16.15	8.05E-04	0.043	334	0.52	98.0	5.18	3.03E-02
	1.75E-04	0.25	0.53	3745	18501	3.79E-10	-717.0	-56.9	1.841	0.66	-16.85	8.85E-04	0.021	1912	1.30	95.1	4.65	2.55E-06
	1.75E-04	0.4	0.53	1853	19281	4.06E-07	-146.9	-56.9	1.272	0.32	-16.17	8.50E-04	0.035	591	0.71	97.3	5.45	8.70E-03
	2.33E-04	0.25	0.53	2384	17530	3.79E-10	-717.0	-56.9	1.909	0.56	-17.79	8.85E-04	0.019	2206	1.40	94.8	4.99	2.42E-06
	2.33E-04	0.3	0.53	1674	19035	5.67E-09	-387.6	-56.9	1.778	0.39	-16.38	8.84E-04	0.022	1690	1.22	95.4	5.31	6.07E-05
	2.33E-04	0.35	0.53	1248	19260	5.59E-08	-230.5	-56.9	1.553	0.29	-16.19	8.77E-04	0.027	1118	0.99	96.2	5.50	8.72E-04
	2.33E-04	0.4	0.53	763	19926	4.06E-07	-146.9	-56.9	1.315	0.18	-15.65	8.50E-04	0.034	632	0.73	97.2	5.64	8.99E-03
	2.33E-04	0.43	0.53	671	19447	1.19E-06	-115.1	-56.9	1.097	0.16	-16.03	8.05E-04	0.042	348	0.53	98.0	5.29	3.05E-02
Yololight clay	4.92E-05	0.25	0.499	38783	48342	2.44E-08	-469.2	-16.1	2.313	1.91	-17.30	3.78E-04	0.007	7818	1.72	93.7	6.90	1.56E-04
	4.92E-05	0.35	0.499	20782	29127	4.53E-07	-90.9	-16.1	1.326	1.02	-28.71	3.01E-04	0.008	3107	0.97	96.3	6.49	6.41E-03
	9.84E-05	0.25	0.499	9874	48017	2.44E-08	-469.2	-16.1	2.766	0.97	-17.42	3.78E-04	0.005	12529	2.18	92.1	8.74	1.55E-04
	9.84E-05	0.35	0.499	5048	28549	4.53E-07	-90.9	-16.1	1.525	0.50	-29.29	3.01E-04	0.006	4572	1.17	95.6	7.88	6.28E-03

(derived using eq. 45) with the soil type. To distinguish the S-computations utilizing linear and non-linear assumptions the S-values determined from eq. (44) using $N=4/\pi$ were plotted against each other and it was found that the assumption of D not varying with θ allowed several fold over-estimation of S compared to that computed using D varying with θ. Furthermore, $N=4/\pi$ exhibited a contrasting trend of S-variation with the soil type; in general, S should decrease with the soil changing from Plainfield sand to Yololight clay.

The above-derived N-values were examined for their correlation with the available physical parameters and these were found to show a good correlation with the initial moisture content, θ_0, rainfall intensity, i_0, and time base, T_b, as expressed below:

$$y = 1782.5\, x^{-1.8887}, \quad R^2 = 0.9580 , \tag{63}$$

where $y = N \times i_0/\theta_0$ (cm/s), x is the time base (in s), and R^2 is the coefficient of determination. The derived S-values (eq. 63) for both the observed and predicted ones plotted in terms of respective values of CN are shown in Fig. 1. Although there exists a large scatter in the data points, the figure indicates that CN-values can be adequately derived from physical characteristics of the soil.

The above computations, however, require information on T_b *a priori*. Therefore, it is useful to derive an empirical relation for determination of T_b. To this end, the product of i_0 and θ_0 was found to exhibit a correlation with the time base, as shown in Fig. 2 and expressed mathematically as

$$y = 12.153\, x^{-0.7539}, \quad R^2 = 0.9536 , \tag{64}$$

where $y = T_b$ (in s) and $x = i_0 \times \theta_0$ (in cm/s).

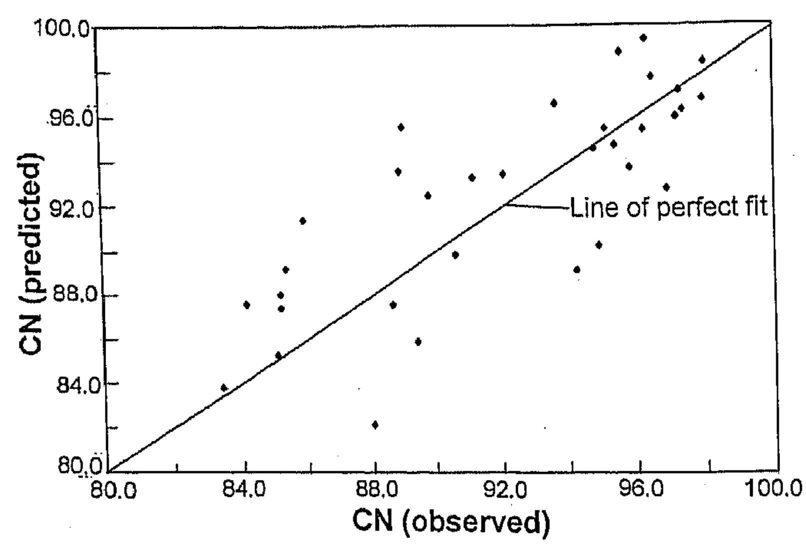

Fig. 1. Observed CN *versus* predicted CN.

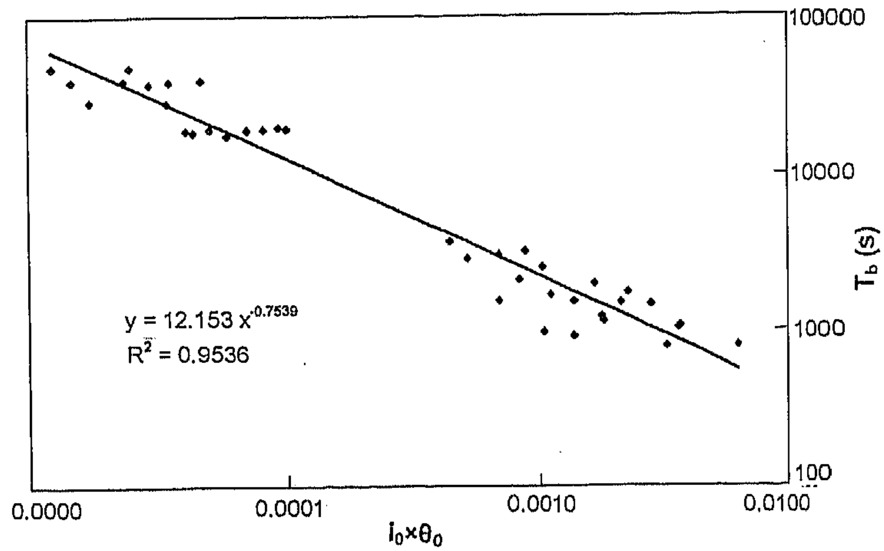

Fig. 2. Relation between rainfall intensity, i_0 (in cm/s), initial moisture content, θ_0 (in vol/vol), and time base of an infiltration decay curve, T_b (in s).

The N-values derived using the approximate relation (eq. 46) were compared with the above derived from the non-linear analysis. The former were found to be much higher than the latter. This implies that the approximate average relations do not have the ability to capture the characteristics of the K–θ and ψ–θ relations. They are, however, useful for understanding the physical process. It is noted that in the above computations, S has included I_a whereas in the water balance equation (eq. 1), F is separate from I_a. Alternatively, for $F=S$ (eq. 2), S excludes I_a in eq. (1). Therefore, an investigation of S for I_a is in order.

Investigation of S for I_a

The initial abstraction amount, I_a, was taken equal to the product of the rainfall intensity and time to ponding, t_p. To investigate S for I_a these were plotted against each other for four cases: (1) S included I_a as well as the static portion of infiltration, F_c; (2) S included I_a and excluded F_c; (3) S excluded I_a and included F_c; and (4) S excluded both I_a and F_c. Here, F_c is taken equal to the product of K_s and T_b (eq. 62). The fitted relations for these cases are as follows:

Case 1:

$$y = 0.2749x, \quad R^2 = 0.3561 \,, \tag{65a}$$

Case 2:

$$y = 0.4240x, \quad R^2 = 0.4103 \,, \tag{65b}$$

where $y = I_a$ (cm) and x-axis represents the S values for the respective cases, as described above. It is worth emphasizing that Case 1 and Case 2 show spurious relations for both the x- and y-axes include I_a. Furthermore, the above relations (eqs. 65a, b) exhibit a poor fit, for the resulting R^2-values are very low in all the cases. For Cases 3 and 4, the value of R^2 was zero, indicating that there exists no relation. The ratios of y to x in these equations represent the magnitudes of initial abstraction coefficient, λ. Evidently, $\lambda = 0.275$ (approx.) for Case 1 and for other three cases, these are 0.42, 0.33, and 0.57 (approx.), respectively. For Case 1, the derived λ-value is close to 0.25 derived by Chen (1981).

Relation between S and AMC

In the available NEH-4 tables, S includes both I_a and F_c whereas S excludes both. Therefore, two cases including and excluding both I_a and F_c were considered for evaluating the impact of the antecedent moisture content (AMC) on S. Since AMC represents the amount of antecedent moisture before the start of a storm, the initial moisture content, θ_0, is a variant of AMC. The plot between S and θ_0 for S excluding both I_a and F_c is shown in Fig. 3. Apparently, the figure exhibits a decreasing trend of S with increasing θ_0, which is consistent with the general notion that S or CN decreases or increases with the increasing θ_0. Furthermore, it also depicts a loop in the S–θ_0 relation. The apparent lower and upper enveloping curves are analogous to AMC III and AMC I, respectively. Similar inferences were derived from the plot (not shown) depicting the relation between θ_0 and S including both I_a and F_c.

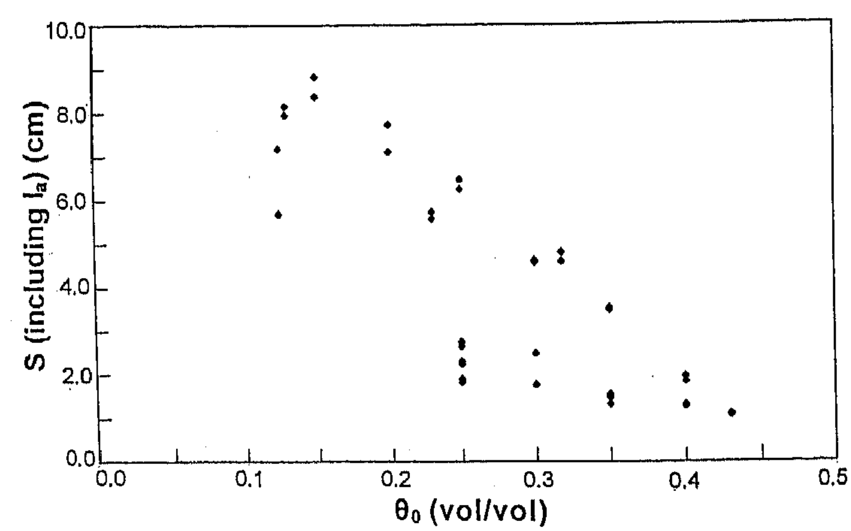

Fig. 3. Relation between potential maximum retention, S (including initial abstraction, I_a) and initial moisture content, θ_0.

Applicability of kinematic wave approximation

The kinematic wave parameter that is equivalent to the left hand side of eq. (61) is computed for the infiltration data available for all soils and given in Table 3. Its values range between 2.42×10^{-6} and 0.175, which are far less than 385. It implies that the infiltration process is not kinematic in nature, rather it is either a linear (eq. 6) or non-linear (Philip, 1969) diffusion or higher order dynamic wave phenomenon.

SUMMARY AND CONCLUSIONS

Parameter S of the SCS-CN method was derived for a wide range of soils, viz., Plainfield sand, Columbia sandy loam, Guelph loam, Ida silt loam, and Yololight clay. Its physical significance was described using the K–θ and ψ–θ characteristics of the soil. Expressions were derived for both linear and non-linear solutions. Using a linear solution of the Fokker-Planck equation for infiltration, the variation of S in vertical infiltration and the applicability of kinematic wave hypothesis formulation were described. The linear solution enabled derivation of a general expression relating S with the K–θ and ψ–θ characteristics of soils. The following inferences are drawn from the study:

– The expression for S derived from a volumetric analysis is a specific form of the general expression derived from a linear solution.

– S is an index of the energy expenditure in unit time per unit weight of absorbate and is a function of two specific heats, c_p and c_v, and is analogous to the attenuation driving parameters (for example, phase difference, logarithmic decrement, hysteresis) in open channel flow. Therefore, S is primarily responsible for infiltration decay.

– S depends on the K–θ and ψ–θ relations describing, respectively, the transmissivity and storage properties of soil and the magnitude of storm precipitation. S increases with the increase in soil sorptivity, with ψ and the magnitude of precipitation, and decreases with K. Higher initial moisture tends to increase S for a given sorptivity, which depends on the initial values of ψ, K, and θ. S or CN decreases or increases with progress of time within a storm duration.

– In vertical infiltration, S is directly proportional to the characteristic depth of flow in a given flow regime. S or CN increases or decreases with the depth of flow.

– Kinematic wave formulation is not suitable for simulating infiltration process for the soils studied.

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