MODELING AND ANALYSIS OF VSC-BASED DC LINES IN AC POWER FLOW PROBLEMS

An Undergraduate Research Scholars Thesis

by

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>2</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>3</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>4</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I  INTRODUCTION</td>
<td>5</td>
</tr>
<tr>
<td>Power System Topology</td>
<td>5</td>
</tr>
<tr>
<td>Voltage Problems in Power Systems</td>
<td>6</td>
</tr>
<tr>
<td>VSC Technology for Improved Power Delivery</td>
<td>9</td>
</tr>
<tr>
<td>II  THEORY AND MODELING OF VSC BASED DC LINES</td>
<td>11</td>
</tr>
<tr>
<td>Active and Reactive Power Dependencies</td>
<td>11</td>
</tr>
<tr>
<td>VSC Operation</td>
<td>12</td>
</tr>
<tr>
<td>VSC Steady State Representation</td>
<td>14</td>
</tr>
<tr>
<td>Simplified VSC Steady State Representation</td>
<td>19</td>
</tr>
<tr>
<td>Steady State Voltage Stability</td>
<td>21</td>
</tr>
<tr>
<td>III  MODEL SIMULATION AND VOLTAGE STABILITY ANALYSIS</td>
<td>23</td>
</tr>
<tr>
<td>VSC Steady State Model Simulation</td>
<td>23</td>
</tr>
<tr>
<td>Simplified VSC Steady State Model Simulation</td>
<td>25</td>
</tr>
<tr>
<td>PQV Capability Region</td>
<td>30</td>
</tr>
<tr>
<td>IV  CONCLUSION</td>
<td>33</td>
</tr>
<tr>
<td>Value of the Detailed VSC Steady State Model</td>
<td>33</td>
</tr>
<tr>
<td>Value of the Simplified VSC Steady State Model</td>
<td>33</td>
</tr>
<tr>
<td>VSC Flexibility</td>
<td>34</td>
</tr>
<tr>
<td>Future Work</td>
<td>34</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>35</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>36</td>
</tr>
</tbody>
</table>
ABSTRACT

Improving Microgrid Stability Through Voltage Source Converter Based DC Lines. (May 2015)

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Integration of renewable power generation has emerged as a critical challenge facing electric power systems. Increasing the penetration of such energy sources, while maintaining stability, will require creative solutions infused with new technology. Advancements in voltage source converters (VSC) make the device a viable option for high voltage DC (DC) transmission. Utilization of VSC based DC lines could improve stability performances impeding large wind farms connected to the grid by improving reactive power support. This thesis introduces a detailed method for incorporating VSC transmission lines into AC power flow models, and simplifies the proposed method into a combination of AC components. The simplified model will benefit grid planning efforts by reducing computation time through smaller Jacobian matrices, while the detailed method offers insights into the VSC’s operating condition and proximity to capability boundaries. In addition, the flexibility provided by VSC transmission is analyzed through the generation of PQV capability regions. Comparing the regions for a conventional AC connection versus a VSC-DC line shows the latter does not necessitate a change in voltage at a load bus when the power demand varies.
DEDICATION

Soli Deo gloria.
ACKNOWLEDGEMENTS

Firstly, I would like to thank my parents for their unwavering support and for instilling the value of education in me. I would also like to express my appreciation to Dr. Le Xie, who has introduced me to the world of research and guided me through every step of this process. Additionally, I thank Omar Urquidez for serving as my VSC expert and mentor throughout the past year. Without the help of these people, my research experience would have been much rougher or possibly nonexistent.
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>HVDC</td>
<td>High Voltage Direct Current</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage Source Converter</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated-Gate Bipolar Transistor</td>
</tr>
<tr>
<td>PQV</td>
<td>Active Power, Reactive Power, AC Bus Voltage</td>
</tr>
<tr>
<td>I</td>
<td>Current</td>
</tr>
<tr>
<td>R</td>
<td>Resistance</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

Power System Topology

Transmission System

The first phase in the power system topology is the transmission system, which is characterized by high voltage lines. After power is generated, the voltage is stepped up using transformers before being transferred to the load. These lines are used to carry power to load centers, because they minimize the amount of power lost in transfer. By having a high voltage level on the line, the current is dropped, reducing the $I^2R$ losses. This helps to maximize efficiency as energy is moved toward the load center.

Distribution System

When the power has neared the load center the voltage is stepped down to an appropriate level for consumer utilization. This portion of the power system is called the distribution system. Since the voltage demands vary customer to customer, the lines must be connected to various subsequent substations. The distribution lines typically carry power to consumers in a radial fashion.

Future of Power Transfer

The growth in technology, especially in power electronics such as voltage source converters (VSCs) and flexible AC transmission system (FACTS) devices, will improve power systems through their controllability [1]. For example, an MIT study concerning the future of electric
grids concluded that the “control flexibility of voltage source converters can improve system stability and facilitate the integration of remotely located renewable generation [1].” Naturally, a common technical challenge in transmission and distribution systems of the future is the incorporation of power electronics in traditional analysis that is largely based on AC-only models. Our specific contribution lies in how to incorporate a voltage source converter (VSC) based DC line into power flow analysis models for both transmission and distribution systems. This task involves accounting for the VSC’s influence on power by including its controllable parameters into a power flow model. Additionally, a model that functions on both a transmission and distribution systems should account for increased line losses when the voltage is dropped. Therefore, a useful model for VSC based DC lines will include VSC parameters which impact power flow control, DC line characteristics, and AC system characteristics.

**Voltage Problems in Power Systems**

*Voltage Problems in Transmission Systems*

Power system stability requires that a grid keep satisfactory steady state operation under normal operating conditions and be able to return to equilibrium after subjection to a disturbance. One of the most crucial aspects of power system stability is voltage stability. In order for the system to be classified as voltage stable, the bus voltages throughout the network must remain steady within an approved range [2]. Although this aspect of stability originally only posed problems for weak systems, as grids become increasingly congested the issue resurfaces [2]. This is particularly an issue for the US electric grid, as the consumption of our nation’s energy by electricity generation has grown from 14% in 1959 to 41% in 2009 [1]. Due to this large increase
in electricity demand, power engineers have been forced to consider voltage stability in their planning and analysis.

**Introduction to the Voltage Instability Phenomenon**

Voltage instability occurs when a bus voltage enters a progressive, uncontrollable decline after the system experiences a disturbance [2]. Typical disturbances include load fluctuations, line outages, loss of generators, and other contingencies. The upper limit of voltage stability, also known as the critical point, occurs at the load impedance which causes the receiving voltage to equal the voltage drop across the transmission line. This point also represents the maximum transmitted power. Moving beyond this limit signifies a transition into unstable operating conditions. System response in this region is characterized by an inability to raise the receiving end power by reducing the load impedance. This failure to control power through load adjustment can lead to successive drops in the voltage magnitude at the load bus, also known as voltage instability. While voltage instability is typically a local phenomenon, its effects can lead to significant system-wide failures. If voltage instability at one location impacts the ability of other buses to maintain acceptable voltages, the subsequent result can be low voltages throughout a sizable portion of the grid. This cumulative effect is known as voltage collapse [2].

Several factors can be identified as causes of voltage instability; however, most issues arise from insufficient reactive power support. This is due to the fact that increases in reactive power demand in a system result in voltage drops. In order to prevent voltage decay, appropriate quantities of reactive power must be injected into the system.

**Outline of Voltage Stability Analysis**
The two main qualities that need to be investigated when analyzing voltage stability include the nearness to voltage instability and the driving factors of voltage instability [3]. When determining the proximity to an unstable voltage operating condition values such as load, real power transfer, and available reactive power all need to be considered. Studying the mechanisms involved in a voltage instability situation involves determining the source of the instability and possible solutions for preventing the loss stability [3]. Classification of voltage instabilities can broken down into two groups- problems that are accompanied by frequency instabilities and problems in which there are insignificant changes in frequency [4]. The first class incorporates the connection between volatility in electromechanical values of a machine (angle and frequency) and its field and terminal voltages [4]. The other class of voltage issues occurs with relatively high independence from frequency. Studying these voltage problems draws upon static and dynamic analysis. Since voltage instability typically occurs in a mid/long term time frame (on the order of 30 seconds to tens of minutes), the validity of static methods remains for many attributes of the problem [2]. Such analysis can reveal information regarding key contributing factors to the source of the instability. For a more detailed analysis of the condition, dynamic simulations can be performed. Dynamics will shed insight into the system’s response to attempts aimed at restoring steady-state equilibrium [2]. Utilizing both of these analysis techniques in a complementary fashion can illustrate the nature of voltage instability.

*Voltage Problems in Distribution Systems*

The growth of distributed renewable resources and new technologies available to consumers, such as fuel cells and electric vehicles, are increasing interest in DC or mixed AC/DC distribution systems [5]. However, the issue of voltage stability discussed above presents several
challenges in systems with DC lines, due to the tie between voltage level and active power [5]. In a DC line, the concept of reactive power is nonexistent and drops in voltage are a result of line resistance. This means the voltage level cannot be manipulated by altering reactive power; rather, a change in voltage also implies a change in active power. Additionally at the distribution level, lower line voltages cause the issue of voltage drop to be further exasperated.

**VSC Technology for Improved Power Delivery**

Voltage source converter (VSC) based DC lines are emerging as a valuable option for system operators to improve grid stability. A VSC is a power electronics device capable of acting as both an inverter and rectifier. Spawning from improvements in semiconductor technology, its main component is the insulated-gate bipolar transistor (IGBT), which can be turned off and on multiple times per power frequency cycle [6]. This aspect offers significant advantages over previously used line commutated converters, since switching control is no longer dependent on the main current to switch off [6]. The operation of VSCs in a DC line can viewed as a black box converter between AC and DC as depicted to the following model [6]:

![Figure 1: Diagram of a DC transmission line using two VSCs](image)

**VSCs in the Transmission System**
The flexible characteristics of VSCs open the door for novel applications, which can positively influence grid stability [7]. VSCs offer both voltage and frequency control and can rapidly changed to meet active stresses on a system by manipulating both active and reactive power [7]. Since wind, solar, and other renewable energy sources threaten voltage and frequency stability due to their uncontrollable variability, VSC transmission provides a plausible solution for counteracting some of the fluctuations in generation [8]. Additionally, when power must be transferred over long distances, high voltage DC (HVDC) lines have proven there economic viability over AC lines. Traditionally, current source converters have been used for conversion between AC and DC in HVDC projects. Alternatively, VSCs provide a new option for implementing an HVDC scheme with greater controllability, which does not depend on naturally occurring current zeroes for commutation.

**VSCs in the Distribution System**

Power electronic conversion equipment, such as VSCs, also provide an interesting interplay between DC lines and an AC distribution system [5]. VSCs and similar devices which are capable of injecting reactive power could be used to manage AC voltages of connected buses [5]. In the distribution system, this characteristic of VSCs will allow for better interconnection of DC lines by reducing the impact of voltage drop.
CHAPTER II
THEORY AND MODELING OF VSC BASED DC LINES

Active and Reactive Power Dependencies
In AC systems, two types of power are necessary to sustain satisfactory operation: active power and reactive power. Controlling these values requires an understanding of the factors with the greatest influence on both forms of power. Such relationships can be derived from the equations used in steady state power flow analysis.

Derivation of Active Power and Voltage Angle Coupling
The formula for net active power injection is shown below in equation (1).

\[ P_l = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)) \]  \hspace{1cm} (1)

Since the reactance of a transmission line is much greater than the resistance, the \( B_{ik} \) terms will contribute significantly more than the \( G_{ik} \) terms. Additionally, the voltage angle difference is typically less than 0.2618 radians (15 degrees), which means the cosine term will be about 1 and the sine term will can be approximated as \( \theta_i - \theta_k \). Lastly, under normal operating conditions the voltage magnitude will be between 0.95 and 1.05 per unit. Applying these assumptions to equation (1) yields the simplified form of equation (2).

\[ P_l \approx \sum_{k=1}^{n} B_{ik}(\theta_i - \theta_k) \]  \hspace{1cm} (2)

It can be seen from this equation that the net power injection depends heavily on the voltage angle difference.

Derivation of Reactive Power and Voltage Magnitude Coupling
The deductions made for the active power flow derivation are also valid for the net reactive power injection formula shown in equation (3).

$$Q_i = \sum_{k=1}^{n} |V_i|^2 |V_k| (G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k))$$  \hspace{1cm} (3)

Following the same logic from the previous derivation, equation (3) can be reduced to:

$$Q_i \approx -\sum_{k=1}^{n} |V_i|^2 |V_k| B_{ik}$$  \hspace{1cm} (4)

Expanding the $B_{ik}$ system characteristic yields additional insights into where the contributions are found in the AC network.

$$B_{ik} = -b_{ik}, \text{ for } i \neq k$$  \hspace{1cm} (5a)

$$B_{ik} = b_i + \sum_{k=1, k \neq i}^{n} b_{ik}, \text{ for } i = k$$  \hspace{1cm} (5b)

$$Q_i = -b_i + \sum_{k=1, k \neq i}^{n} b_{ik}(|V_i| - |V_k|)$$  \hspace{1cm} (6)

The first term of equation 6 relates to the reactive power contribution of the shunt susceptance at bus “$i$”, while the second term factors in the transfer of reactive power from connections.

Equation (6) also shows the strong coupling between voltage magnitude and reactive power.

**VSC Operation**

One of the most well practiced techniques for understanding an electric grid is steady-state power flow analysis. Such analysis provides insight about operating points, potential contingencies, and the impact of introducing new devices (such as VSC transmission lines) on system conditions. A challenge for the power systems industry associated with adding innovative technologies revolves around appropriate representation of such equipment. In the case of VSC based HVDC lines, it is important to understand key operation features before progressing into steady-state representation.
**Active Power Control**

VSC based DC lines, like other DC lines and traditional AC lines, is capable of transferring active power from a generation source to loads; however, it offers a significant advantage over its alternatives—controllability. VSCs are capable of varying the amount of active power they supply. The internal components of a VSC are capable of switching at extremely high frequencies relative to the system operating frequency. Therefore, it is capable of responding to load variations as they occur. Treating the converter as a black box as shown below in Figure 2, a high level concept of active power control can be easily examined [6].

![Figure 2: Active Power Flow from a VSC Connection to an AC Load](image)

Control of the active power supplied to an AC load is achieved by regulating the phase angle of the voltage generated on the inverter side of the VSC ($U_{\text{conv}}$). If the angle of $U_{\text{conv}}$, $\delta$, is leading the voltage angle of AC load, $\theta$, then active power ($P_{\text{conv}}$) will be transferred to the AC system. In contrast, if $\delta$ lags $\theta$, then $P_{\text{conv}}$ flows from the AC connection. This performance and control technique follows from the strong coupling between net active power injection and voltage angle difference derived in equation (2).

**Reactive Power Control**
VSCs are also able to supply reactive power to a connected AC load. Similar to active power, the amount of reactive power generated is controllable with relatively fast response times. Again, the fundamental idea behind reactive power control can be seen in Figure 3 through a black box representation of a VSC connected to an AC system [6].

![Figure 3: Reactive Power Flow from a VSC Connection to an AC Load](image)

Reactive power control is achieved by manipulating the magnitude of the voltage produced on the AC side of the VSC ($U_{\text{conv}}$). If $U_{\text{conv}}$ is greater than the AC system voltage, then reactive power will be passed to the AC connection. Conversely, a lower magnitude results in an extraction of reactive power from the AC system. This scheme is consistent with the logic found in equation (6), which outlines a heavy correlation between net reactive power injection and voltage magnitude. Since both active and reactive power vary with the manipulation of different characteristics, these values can be set and changed almost independently. However, it should be noted that the logic presented for power control with VSCs might be broken when converter angles are close to zero degrees or voltage magnitudes are numerically close to their AC counterparts. This is due to the fact that active power is slightly influenced by voltage magnitude and angle has a small impact on the reactive power.

**VSC Steady State Representation**
For transmission and distribution operators, a first step towards understanding the real-time situation is through conducting steady-state power flow analysis. Steady state analysis revolves around acquiring a valid solution to a systems power flow equations. It takes an instantaneous snapshot to reveal valuable information about the grids operating conditions under prescribed loadings. In order to incorporate VSC DC lines into a power system, its steady state representation must accurately describe its impact on power flow.

*Derivation of VSC-DC Power Flow Equations*

The first step in modeling a VSC based DC line for steady-state analysis involves mapping DC and AC variables to VSC power transfer equations. The following shows how to derive these equations. Figure 1 from the “VSC Technology for Improved Power Delivery” section of chapter one has been repeated below as a convenient reference for the nomenclature used for variable names.

![Figure 1: Diagram of a DC transmission line using two VSCs](image)

**DC Line Power Flow:**

Power transfer through the DC line can be described using equation (7) below.

\[ U_{d1} I_d - I_d^2 R_d = U_{d2} I_d \quad (7) \]
VSC/AC System Power Exchange:

As mentioned in the “VSC Operation” section of this chapter, power transfer between the AC system and the VSC depends heavily on two variables—$U_{\text{conv}}$ and $\delta$. The active power from the AC system to the VSC is given by equation (8).

$$ P = \frac{U_{\text{conv}} U_L \sin (\delta)}{X_{\text{conv}}} \quad (8) $$

Equation (9) shows the reactive power relationship between the VSC and the connected AC system.

$$ Q = \frac{U_L (U_L - U_{\text{conv}} \cos (\delta))}{X_{\text{conv}}} \quad (9) $$

The DC variables must be linked to the VSC variables to relate the DC power flow to the power exchanged between the VSC and the AC system. The following equation establishes this connection.

$$ U_{\text{conv}} = C k_\lambda U_d \quad (10) $$

These equations introduce two new parameters ($C$ and $k_\lambda$), which depend on the VSC’s operation. $C$ is a constant associated with VSC’s configuration and number of levels. For a two-level VSC in a three phase configuration, the value of $C$ equates to $\sqrt{6}/\pi$. The $k_\lambda$ term is a controllable parameter that represents a ratio of $U_{\text{conv}}$ to its maximum possible value, and it can take any value between 0 and 1. Substituting equation (10) into equation into equations (8) and (9) yields a set of power equations that depend on DC, AC, and VSC variables.

$$ P = \frac{C k_\lambda U_d U_L \sin (\delta)}{X_{\text{conv}}} \quad (11) $$

$$ Q = \frac{U_L (U_L - C k_\lambda U_d \cos (\delta))}{X_{\text{conv}}} \quad (12) $$
Equations (11) and (12) illustrate that the power flow through a VSC based DC transmission line varies with the DC line voltage \( U_d \), the AC system voltage \( U_L \), the voltage ratio \( k_\lambda \), and the phase angle difference \( \delta \) between the VSC output voltage and the AC system voltage.

**Incorporation of VSC-DC Lines in Power Flow Models**

Including a VSC transmission line into a power flow model requires several initial assumptions. Equations (11) and (12) contain four variables that determine the power flow; therefore, two of the variables must be “set” (i.e. assume a prescribed value) to reach a solution. Setting these values depends on the VSC control scheme implemented. For example, the VSC could be used to control the AC bus voltage and DC line voltage, which means that \( U_L \) and \( U_d \) would be known. Additionally, the real and reactive power must be assumed or solved for prior to finding the unknown variables. Typically, the desired real power is known. Determining the reactive power depends on the selected control scheme. For AC voltage control, the reactive power should be allowed to change, meaning that it must be found in a similar manner to solving for the reactive power produced by a generator. In other words, the AC system power flow equations should be solved first to determine the reactive power the VSC must supply (or absorb).

Alternatively, if the AC bus voltage is not controlled, then the desired reactive power for the VSC must be known. A series of equations is provided below as a general guide for solving a mixed AC/VSC-DC system using Newton’s method.

\[
P_i = P_{gi} - P_{li} = P_{ac} + P_{vsc} \quad (13)
\]

Where \( P_{ac} \) is given by equation (1) and \( P_{vsc} \) is given by equation (11).

\[
Q_i = Q_{gi} - Q_{li} = Q_{ac} + Q_{vsc} \quad (14)
\]

Where \( Q_{ac} \) is given by equation (3) and \( Q_{vsc} \) is given by equation (12).
where \( x_{ac} \) consists of \( V_i/θ_i \), and \( x_{vsc} \) is a pair from \( V_i/V_d/k_i/δ \).

The system in (15) can be broken up into AC and VSC equations, which can be solved separately. The next four equations can be used to find the AC solution.

\[
P_{ac} = P_i - P_{vsc} \quad (16)
\]

\[
Q_{ac} = Q_i - Q_{vsc} \quad (17)
\]

Where \( P_{vsc} \) and \( Q_{vsc} \) are known, or there is AC voltage control and the bus is treated as a PV bus.

\[
f_{ac}(x_{ac}) = \begin{bmatrix}
P_{2ac}(x_{ac}) - P_{2ac} \\
\vdots \\
P_{Nac}(x_{ac}) - P_{Nac} \\
Q_{2ac}(x_{ac}) - Q_{2ac} \\
\vdots \\
Q_{Nac}(x_{ac}) - Q_{Nac}
\end{bmatrix} = 0 \quad (18)
\]

The Newton-Raphson iteration for the AC system is presented below in equation (19). It should be repeated until satisfactory convergence is reached.

\[
[f_{ac}][Δx_{ac}] = -[f_{ac}(x_{ac})] \quad (19)
\]

After finding the AC system’s solution, \( Q_{vsc} \) can be calculated (if it was not already known) using equation (17), and the VSC equations can be solved.

\[
f_{vsc}(x_{vsc}) = \begin{bmatrix}
P_{2vsc}(x_{vsc}) - P_{2vsc} \\
\vdots \\
P_{Nvsc}(x_{vsc}) - P_{Nvsc} \\
Q_{2vsc}(x_{vsc}) - Q_{2vsc} \\
\vdots \\
Q_{Nvsc}(x_{vsc}) - Q_{Nvsc}
\end{bmatrix} = 0 \quad (20)
\]

Again, Newton’s method can be used to find the solution to the system of equations.

\[
[f_{vsc}][Δx_{vsc}] = -[f_{vsc}(x_{vsc})] \quad (21)
\]
After reaching convergence in equation (21), all system variables ($x_{ac}$ and $x_{vsc}$) will be known and the power flow analysis complete.

**Simplified VSC Steady State Representation**

The complexity of the steady state model for VSC-DC can be reduced if the values of for the VSC parameters do not need to be known. The next two sections introduce concepts for simplifying the VSC transmission line down to AC components that mimic the VSC’s impact on the AC system.

**VSC as Injection/Extraction**

If the VSC is operating without controlling the AC bus voltage, then the VSC can be treated as an active power extraction/injection at its connection points. At the sending side of the link, the VSC extracts active power from the bus like a load. On the receiving side, active power is injected into the bus. Thus, it acts as a constant power source supplying the amount of active power that was extracted from the sending side, minus a small line loss. On both ends of the VSC transmission line, reactive power can be removed or added independent of the reactive power operation at the other end and any active power settings. This means that the VSC-DC line can be represented as only a constant PQ “load” on the sending side and a constant PQ “generator” on the receiving side. Since DC lines have low resistance, the amount of power lost during transmission is typically negligible relative to the total amount of power sent. However, if neglecting this loss is not permissible, it can be easily calculated using the DC line voltages and added to the “load”. The Matlab Simulink model below (constructed using the Power Systems Analysis Toolbox) depicts a VSC transmission line between buses 2 and 3.
VSC as PV Bus

VSCs are capable of supplying reactive power to an AC system, and the amount can be varied quickly. An appropriate controller design could take advantage of this attribute. Since voltage magnitude is strongly coupled to reactive power, a VSC could be used to supply or drain the necessary reactive power needed to maintain the desired bus voltage. By operating in this fashion and providing constant active power support, both buses connected to the VSC transmission line would experience voltage regulation. This means that the buses can be classified as PV buses in a power flow model, as long as the reactive power requirement is within the VSC’s operating limits. An example of a three bus system with reactive power controlled VSCs is implemented in Figure 5.
Steady State Voltage Stability

Measuring the level of voltage stability in the steady state time frame is accomplished through the use of PV and QV curves. These curves relate the voltage magnitude at a bus to various active and reactive power contributions. They can be used to check a particular buses proximity to voltage instability.

PV Curve Analysis

A PV curve maps specific active power loads to voltages. It can be constructed using a series of iterations which increment in small steps until a power flow solution no longer exists. Each increment should only increase the active power (i.e. the reactive power must be kept the same). A sample PV curve is presented below in Figure 6.

![PV Curve](image)

*Figure 6: Sample PV Curve*

The upper portion of the PV curve represents stable operating conditions, which is characterized by a decline in voltage magnitude as the power demand increases. The critical point occurs at the right side of the curve, where the upper and lower portions meet. If the system moves into the
lower part of the graph, then the voltage magnitude becomes unstable. Operation in the unstable area means that the power systems attempts to raise the voltage will fail, resulting in a possible collapse. It is also worth noting that the load flow Jacobian becomes singular for the lower part of the curve. This implies that eigenvalue analysis can also be used to investigate voltage stability.

*PV Curves for VSC*

Construction of PV curves for buses connected to VSCs dictates use of either the full detailed method without AC voltage control or the extraction/injection method. This is due to the constant reactive power supply and demand requirement for PV curves. When the VSC is operating without any reactive power control, this demand is not a problem. Steady state voltage stability for implementations with reactive power control, however, should not be neglected. Therefore, to account for the range of reactive power a controlled VSC can supply, at least two PV curves should be created using stable operating points. One of the curves should represent the lower reactive power limit for the VSC, while the other should correspond to the upper bound. The result will be a PV “region” that outlines all of the possible active power and voltage magnitude combinations. For a more accuracy regarding the critical side of the region, additional curves can be included using intermediate reactive power values.
CHAPTER III
MODEL SIMULATION AND VOLTAGE STABILITY ANALYSIS

VSC Steady State Model Simulation

The test system presented in Figure 7 represents a simple two bus system, for the purpose of testing the VSC steady state model derived in the previous chapter. The variable values the proposed model converges to should satisfy each of the VSC power flow equations and follow the logic presented in the “VSC Operation” section.

![Figure 7: 2 Bus Test System with VSC-DC Line](image)

Table 1 outlines all of the system parameters. The VSC control scheme used for the simulation is AC voltage control; therefore, the voltage at bus 2 is known. Additionally, it is assumed that the DC line voltage is known.

<table>
<thead>
<tr>
<th>R</th>
<th>X_c</th>
<th>P_vsc2</th>
<th>U_d2</th>
<th>V_2</th>
<th>V_1</th>
<th>P_2</th>
<th>Q_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.2</td>
<td>-0.8</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>-0.8</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

*Note: All values are in per unit.*
Following the procedure in the “VSC Steady State Representation” section of Chapter II, the first step is to solve the AC system. Since there is no AC transmission line found in Figure 7, the active and reactive AC power flows are both zero. The next step is to calculate the unknown VSC-DC variables using the Newton-Raphson method. A Matlab function and script were written to perform this action. (The code can be found in the Appendix for reference.) The results yielded by the code are shown in Table 2.

Table 2. Converged VSC-DC Variables

<table>
<thead>
<tr>
<th>U_d1</th>
<th>δ_1</th>
<th>k_1</th>
<th>U_d2</th>
<th>δ_2</th>
<th>k_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5027</td>
<td>9.1043°</td>
<td>0.8644</td>
<td>1.5</td>
<td>-8.4282°</td>
<td>0.9335</td>
</tr>
</tbody>
</table>

*Note: DC line voltages are in per unit.*

The validity of the converged values in Table 2 can be confirmed by plugging them into equations (8) through (10), and verifying the power demands are met without violating the VSC operation logic presented in Chapter II.

Table 3. Two Bus, VSC-DC System Power Flow Results

<table>
<thead>
<tr>
<th>U_conv1</th>
<th>P_vsc1</th>
<th>Q_vsc1</th>
<th>U_conv2</th>
<th>P_vsc2</th>
<th>Q_vsc2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0128</td>
<td>0.8014</td>
<td>≈ 0</td>
<td>1.0918</td>
<td>-0.8</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

*Note: All values are in per unit.*

The active and reactive power transfer between bus 2 and its VSC connection (P_vsc2 and Q_vsc2) show that load (P_2 and Q_2) is satisfied. It is also important to check if the converter output
voltage ($U_{\text{conv}}$) and angle ($\delta$) correspond with understandings of VSC operation. The value of $U_{\text{conv}2}$ is greater than $V_2$, implying that reactive power is being transferred to bus 2. Additionally, $\delta_2$ is negative, indicating that it is leading the bus 2 voltage angle. Therefore, active power is being transmitted to bus 2. Since bus 2 is a load bus, active and reactive power should indeed be moving from the VSC to the AC system. A similar analysis can be performed on bus 1 and the VSC connected to it. In this case $\delta_1$ is positive, which means that the VSC is receiving active power. $P_{\text{vsc}2}$ is positive, confirming this assertion. The reactive power demand by the VSC connected to bus 1 was set to zero to show the special case, when the operation logic fails. According to the idealized relationship derived in the previous chapter, for zero reactive power transfer $U_{\text{conv}1}$ should be set equal to $V_1$; however, the actual value for $U_{\text{conv}1}$ found in the simulation is slightly greater than $V_1$. This is necessary to offset the effect of $\delta_1$ on reactive power transmission between bus 1 and the connected VSC. From inspection it is clear that the variables in Table 2 converged to values that satisfy the power flow equations.

**Simplified VSC Steady State Model Simulation**

*Testing VSC as PV Bus Simplified Model*

A two bus mixed AC/VSC-DC system will be used to compare the accuracy of the simplified “VSC as PV bus” model presented in Chapter II with the detailed model used in the simulation above. Figure 8 below depicts the test system, with all known parameters listed in Table 4.
Table 4. Two Bus, Mixed AC/VSC-DC System Parameters (With AC Voltage Control)

<table>
<thead>
<tr>
<th>System AC Parameters</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>X_{ac}</td>
<td>V_1</td>
<td>Θ_1</td>
<td>V_2</td>
<td>P_{L2}</td>
</tr>
<tr>
<td>0.005</td>
<td>0.34</td>
<td>1</td>
<td>0°</td>
<td>1</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System VSC-DC Parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>X_c</td>
<td>P_{vsc2}</td>
<td>U_{d2}</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>-0.2</td>
<td>1.5</td>
<td></td>
</tr>
</tbody>
</table>

*Note: All values are in per unit (except angle).*

A Matlab function was written to handle a two bus AC system. This code can be used in conjunction with the detailed, VSC model code used for the simulation in the previous section. (The code for the two bus AC function can be found in the appendix.) The power flow solution found using the detailed model is found in the table below.
The simplified PV bus model introduced in the previous chapter represents the VSC transmission line as an AC load and PV generator. For the model to be useful, it should reach the same power flow results as the detailed model. The Power Systems Analysis Toolbox was used to construct and simulate the simplified PV bus model.

![Diagram of Simplified Model (VSC as PV bus)](image)

*Figure 9: Simplified Model (VSC as PV bus)*
Table 6. Mixed AC/VSC-DC Power Flow Results (Simplified PV Bus)

<table>
<thead>
<tr>
<th>AC Power Flow Solution</th>
<th>VSC-DC Power Flow Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{1ac}$</td>
<td>$P_{1vsc}$</td>
</tr>
<tr>
<td>0.6018</td>
<td>0.2001</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: All values are in per unit (except angles).

Comparing Table 5 and Table 6 indicates that for VSC transmission using an AC voltage control scheme, the simplified PV bus model yields the same power flow solutions. The disadvantage of using the simplified method lies in the lack of information about the VSC parameters. In the simulation example above, the values for $k_\lambda$ and $\delta$ remain unknown.

*Testing VSC as PQ Bus Simplified Model*

If the VSC is not being used to control the AC voltage at its connected bus, then the control scheme will maintain a constant active and reactive power transfer. In this case, the “VSC as a PQ bus” simplified model introduced in Chapter II can be used as an alternative to the detailed model. This option can be verified as a reliable option for solving the steady state power flow problem by cross-referencing the results with the solution provided by the detailed model. The test system set-up will be the same as the one shown in Figure 8. The system parameters can be found in Table 7.
Table 7. Two Bus, Mixed AC/VSC-DC System Parameters (No AC Voltage Control)

<table>
<thead>
<tr>
<th>System AC Parameters</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>X_{ac}</td>
<td>V_1</td>
<td>\Theta_1</td>
<td>P_{L2}</td>
<td>Q_{L2}</td>
</tr>
<tr>
<td>0.005</td>
<td>0.34</td>
<td>1</td>
<td>0°</td>
<td>0.8</td>
<td>0.4</td>
</tr>
</tbody>
</table>

| System VSC-DC Parameters |  |  |  |  |
|---------------------------|--------|--------|-------|
| X_c                       | P_{vsc2} | Q_{vsc2} | U_{d2} |
| 0.2                       | -0.2   | -0.1   | 1.5   |

*Note: All values are in per unit (except angle).*

The detailed and simplified models can be solved with the same procedures used to verify the PV bus simplified model. The solutions for both methods can be found in the tables below, as well as the Power System Analysis Toolbox diagram.

Table 8. Mixed AC/VSC-DC Power Flow Results (Detailed, No AC Voltage Control)

<table>
<thead>
<tr>
<th>AC Power Flow Solution</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P_{1ac}</td>
<td>Q_{1ac}</td>
<td>V_1</td>
<td>\Theta_1</td>
<td>P_{2ac}</td>
<td>Q_{2ac}</td>
</tr>
<tr>
<td>0.6031</td>
<td>0.5133</td>
<td>1</td>
<td>0°</td>
<td>-0.6</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

| VSC-DC Power Flow Solution |  |  |  |  |  |  |
|-----------------------------|--------|--------|-------|-------|-------|
| P_{1vsc}                    | Q_{1vsc} | U_{d1} | \delta_1 | k_1 | P_{2vsc} | Q_{2vsc} | U_{d2} | \delta_2 | k_2 |
| 0.2001                      | 0       | 1.5007 | 2.2918° | 0.8553 | -0.2   | -0.1   | 1.5    | -3.1054° | 0.7455 |

*Note: All values are in per unit (except angles).*
Table 9. Mixed AC/VSC-DC Power Flow Results (Simplified, No AC Voltage Control)

<table>
<thead>
<tr>
<th>AC Power Flow Solution</th>
<th>VSC-DC Power Flow Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{1ac} )</td>
<td>( Q_{1ac} )</td>
</tr>
<tr>
<td>0.6031</td>
<td>0.5133</td>
</tr>
<tr>
<td>( P_{1vsc} )</td>
<td>( Q_{1vsc} )</td>
</tr>
<tr>
<td>0.2001</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: All values are in per unit (except angles).

Evaluating the results found in tables 8 and 9, reveals that the same power flow solution is found using both methods. This shows that the simplified model provides an accurate alternative to the detailed model, when the VSC parameters (\( U_d \), \( k_2 \), and \( \delta \)) are not required.

**PQV Capability Region**
For a conventional AC transmission line, there is a one-to-one relationship between a bus’s loading and its voltage magnitude. In other words, plotting stable AC bus voltages as a function of active and reactive power yields a surface that outlines all of the possible loading and voltage (PQV) combinations. The figure below shows the shape of the PQV capability region at the receiving end of two bus AC system.

One advantage of using VSC transmission is its ability to maintain steady AC bus voltages for multiple different loadings. Therefore a change in loading does not necessitate a change in voltage. Mapping loadings to achievable AC bus voltages produces a shape with volume. An example PQV capability region for the receiving end of a two bus system with a VSC-DC line is provided Figure 12.
Figure 11 and Figure 12 show only a portion of the complete capability regions for their respective power delivery schemes; however, both provide enough information to visualize the relationship between achievable loading and voltage combinations. (The Matlab code used to generate these figures can be found in the Appendix.)
CHAPTER IV
CONCLUSION

Value of the Detailed VSC Steady State Model

The detailed model introduced in Chapter II provides information about controllable parameters for the VSC, as well as a viable solution to the steady state power flow problem. Calculating the parameter values offers valuable insight into VSC’s operating condition. Particularly, a detailed model can be used to generate a capability region for a particular VSC line. Knowledge of a solution’s location within this region will let system operators discern proximity to unattainable loading conditions. The detailed model also allows the impact of the AC voltage ($U_L$), DC line voltage ($U_d$), VSC output voltage ratio ($k_\lambda$), and VSC output angle ($\delta$) to be investigated. This can be helpful for improving intuition about the abilities of a VSC based DC line and developing better control schemes.

Value of the Simplified VSC Steady State Model

The simplified models presented in the second chapter offer a less computationally expensive alternative to the detailed model. The detailed model requires Newton-Raphson formulations for both the AC and VSC-DC lines, doubling the number of Jacobian matrices that need to be inverted. Since the simplified models represents the VSC transmission line using pre-existing AC system concepts (PV generators and PQ loads), the power flow solution can be found utilizing conventional methods. In the case of the “VSC as a PV bus” model, the VSC-DC line actually reduces the size of the Jacobian matrix if it is connected to a load bus. This reduces the matrix inversion time less than both the detailed model and in a purely AC line system. These
models are valuable as a time saving alternative to detailed model, provided the VSC system parameters do not need to be known. This characteristic is valuable when large numbers of power flow simulations are being run, for example during long term system planning.

**VSC Flexibility**

The flexibility of VSC-DC is one of its most important advantages over alternative power delivery schemes. The PQV capability region for a bus with a VSC line connection illustrates this concept. The VSC allows the power demand to vary without necessitating an adverse affect on the bus voltage. This property is a result of the fast response time that characterizes the power electronics used in a VSC, as well as the strong decoupling between active and reactive power. VSC transmission could prove a viable option for connecting renewable resources to the grid. Since renewable generation, such as wind or solar, offers limited reactive power support and has high variability, VSC-DC with its flexibility to quickly change its control parameters offers a possible answer to the challenges associated with incorporating these energy sources in the existing grid.

**Future Work**

Future work for the steady state model includes writing Matlab code that can be integrated with the existing MATPOWER program. This will introduce the model to be utilized by others interested in power system operation and VSC based DC lines. Additionally, detailed models for dynamic time frames also need to be investigated in order to fully understand the effects and possible benefits of incorporating VSCs into transmission and distribution systems.
REFERENCES


Matlab Code for 2-Bus VSC Power Flow:

```matlab
function [Pvsc_i, Qvsc_i, Vd_i, d_i, k_i, Pvsc_r, Qvsc_r, Vd_r, d_r, k_r] = VSC_PF( P2, Q2, V2, V1, Vd_i, Xc, R)
%The purpose of this function is to perform a NR iteration on a 2 bus system and return the variables for a VSC-DC line
% This function assumes that the VSC is operating to keep the AC voltage constant. Vd, k, and d represent the DC variables. It is assumed that the DC voltage on the inverter side (Vd_i) is known as well as the AC voltage at the connected bus. Additionally, this function operates on a committed real power delivery to the load. It solves for the unknown DC variables on the inverter side (k_i and d_i), then solves for the DC variables on the rectifier side (Vd_r, k_r, and d_r).

%Solve for unknown DC variables on the inverter side
dd_i = 1;
d_i = 0;
dk_i = 1;
k_i = .9;

%Do NR until satisfactory convergence of unknown variables
while abs(dd_i) > 1e-6 || abs(dk_i) > 1e-6
    Pvsc_i = (sqrt(6)/pi)*(k_i*Vd_i*V2*sin(d_i)/Xc);
    L11 = (sqrt(6)/pi)*(k_i*Vd_i*V2*cos(d_i)/Xc);
    L12 = (sqrt(6)/pi)*(Vd_i*V2*sin(d_i)/Xc);

    Qvsc_i = (V2*(V2-(sqrt(6)/pi)*k_i*Vd_i*cos(d_i)))/Xc;
    L21 = (sqrt(6)/pi)*(k_i*Vd_i*sin(d_i)/Xc);
    L22 = (-sqrt(6)/pi)*(Vd_i*cos(d_i)/Xc);

    N = [L11 L12; L21 L22];
    N_inv = inv(N);

    delta_i = N_inv*[P2-Pvsc_i; Q2-Qvsc_i];
    dd_i = delta_i(1,1);
    dk_i = delta_i(2,1);
    d_i = d_i+dd_i;
    k_i = k_i+dk_i;
end

%Solve for unknown DC variables on the rectifier side
dd_r = 1;
d_r = 0;
dk_r = 1;
k_r = .9;
Id = Pvsc_i/Vd_i;
P1 = Id^2*R - Pvsc_i;
Vd_r = P1/(-Id);
Q1 = 0;
```

APPENDIX
%Do NR until satisfactory convergence of unknown variables
while dd_r > 1e-6 || dk_r > 1e-6
    Pvsc_r = (sqrt(6)/pi)*(k_r*Vd_r*V1*sin(d_r)/Xc);
    L11 = (sqrt(6)/pi)*(k_r*Vd_r*V1*cos(d_r)/Xc);
    L12 = (sqrt(6)/pi)*(Vd_r*V1*sin(d_r)/Xc);
    Qvsc_r = (V1*(V1-(sqrt(6)/pi)*k_r*Vd_r*cos(d_r)))/Xc;
    L21 = (sqrt(6)/pi)*(k_r*Vd_r*sin(d_r)/Xc);
    L22 = -(sqrt(6)/pi)*(Vd_r*cos(d_r)/Xc);

    M = [L11  L12; L21  L22];
    M_inv = inv(M);

    delta_r = M_inv*[P1-Pvsc_r; Q1-Qvsc_r];
    dd_r = delta_r(1,1);
    dk_r = delta_r(2,1);
    d_r = d_r+dd_r;
    k_r = k_r+dk_r;
end
end

Matlab Code for 2-Bus AC Power Flow

function [ P2, Q2, V2, Theta2, P1, Q1, V1, Theta1, dTheta2 ] = ACPF_2bus( P2, V2, V1, Theta1, G12, B12, G11, B11, G22, B22 )
%This function calculates the AC power flow for a 2 bus system with PV bus.
% The function returns all of the AC parameters in a 2 bus system

dTheta2 = 1;
Theta2 = 0;

%Find Theta2
while abs(dTheta2) > 1e-6
    P2x = V2*V1*(G12*cos(Theta2-Theta1)+B12*sin(Theta2-Theta1))+(V2^2)*G22;
    J = V2*V1*(-G12*sin(Theta2-Theta1)+B12*cos(Theta2-Theta1));
    dP2 = P2-P2x;
    dTheta2 = inv(J)*dP2;
    Theta2 = Theta2+dTheta2;
end

%Find Q2
Q2 = V2*V1*(G12*sin(Theta2-Theta1)-B12*cos(Theta2-Theta1))+(V2^2)*(-B22);

%Find P1
P1 = (V1^2)*(G11)+V1*V2*(G12*cos(Theta1-Theta2)+B12*sin(Theta1-Theta2));

%Find Q1
Q1 = (V1^2)*(-B11)+V1*V2*(G12*sin(Theta1-Theta2)-B12*cos(Theta1-Theta2));
end
Matlab Code for Generating 2-Bus AC PQV Region

Sub-function to Calculate Active Power

```matlab
function [ Pr ] = ActivePwr( Vs, Zline, Zln_angle, Zload, PF_angle )
%This function calculates the amount of active power received for a 2 bus system
% The following system parameters must be known: voltage source magnitude, polar transmission line parameters, polar load parameters

F = 1+((Zload./Zline).^2)+2*(Zload./Zline).*cosd(Zln_angle-PF_angle);
Pr = (Zload./F).*((Vs/Zline)^2).*cosd(PF_angle);
end
```

Sub-function to Calculate Reactive Power

```matlab
function [ Qr ] = ReactivePwr( Vs, Zline, Zln_angle, Zload, PF_angle )
%This function calculates the amount of reactive power received for a 2 bus system
% The following system parameters must be known: voltage source magnitude, polar transmission line parameters, polar load parameters

F = 1+((Zload./Zline).^2)+2*(Zload./Zline).*cosd(Zln_angle-PF_angle);
Qr = (Zload./F).*((Vs/Zline)^2).*sind(PF_angle);
end
```

Sub-function to Calculate Receiving End Voltage Magnitude

```matlab
function [ Vr ] = RecVoltage( Vs, Zline, Zln_angle, Zload, PF_angle )
%This function calculates the receiving end voltage for a 2 bus system
% The following system parameters must be known: voltage source magnitude, polar transmission line parameters, polar load parameters

F = 1+((Zload./Zline).^2)+2*(Zload./Zline).*cosd(Zln_angle-PF_angle);
Vr = ((1./F).^(.5)).*(Zload./Zline)*Vs;
end
```

Script to Generate AC PQV Region

```matlab
%Define system variable ranges and constants
a = log10(.34);
b = log10(34);
Zld = logspace(a,b,100);
Phi = -50:1:89;
Zln = .34;
Alpha = 90;
Vs = 1;

%Create active power received matrix. Rows vary by PF angle. Columns
```
% vary by load impedance magnitude.
Pr = zeros(length(Phi), length(Zld));
i = 1;
for i = 1:140
    PF_ang = Phi(i);
    Pr_i = ActivePwr(Vs, Zln, Alpha, Zld, PF_ang);
    Pr(i,:) = Pr_i;
end

% Create reactive power received matrix. Rows vary by PF angle. Columns vary by load impedance magnitude.
Qr = zeros(length(Phi), length(Zld));
i = 1;
for i = 1:140
    PF_ang = Phi(i);
    Qr_i = ReactivePwr(Vs, Zln, Alpha, Zld, PF_ang);
    Qr(i,:) = Qr_i;
end

% Create receiving end voltage matrix. Rows vary by PF angle. Columns vary by load impedance magnitude.
Vr = zeros(length(Phi), length(Zld));
i = 1;
for i = 1:140
    PF_ang = Phi(i);
    Vr_i = RecVoltage(Vs, Zln, Alpha, Zld, PF_ang);
    Vr(i,:) = Vr_i;
end

% Create 3-D surface plot to show the PQV-Region
surf(Pr, Qr, Vr,
    'EdgeColor', 'none',
    'LineStyle', 'none',
    'FaceLighting', 'phong')

Matlab Code to Generate 2-Bus VSC PQV Region

Sub-function to Calculate VSC Active Power Transfer

function [ Pvsc ] = VSCActive( Vi, Vd, k, d, X )
% This function calculates the active power transferred by a VSC

    Pvsc = (sqrt(6)/pi)*k*Vd*Vi*sind(d)/X;
End

Sub-function to Calculate VSC Reactive Power Transfer

function [ Qvsc ] = VSCReactive( Vi, Vd, k, d, X )
% This function calculates the reactive power transferred by a VSC

    Qvsc = (Vi*(Vi-(sqrt(6)/pi)*k*Vd*cosd(d)))/X;
end
Script to Generate VSC PQV Region

%Define system variable ranges and constants
Vd = 1.5;
Vi = linspace(0.95, 1.05, 100);
d = -90:1:0;
k = linspace(0, 0.99, 100);
Xc = .2;

%Create blank VSC power transfer matrices
Pvsc = zeros(length(Vi), length(d)*length(k));
Qvsc = zeros(length(Vi), length(d)*length(k));

%Fill the VSC power transfer matrices element by element
for x = 1:length(Vi)
    for y = 1:length(d)
        for z = 1:length(k)
            %temporary variables
            Vi_x = Vi(x);
d_y = d(y);
k_z = k(z);
            %column value for the Pvsc and Qvsc matrices
            c = length(k)*(y-1)+z;
            %Add values to Pvsc 1 element at a time
            Pvsc_xyz = VSCActive(Vi_x, Vd, k_z, d_y, Xc);
Pvsc(x,c) = Pvsc_xyz;
            %Add values to Qvsc 1 element at a time
            Qvsc_xyz = VSCReactive(Vi_x, Vd, k_z, d_y, Xc);
Qvsc(x,c) = Qvsc_xyz;
        end
    end
end

%Create AC bus voltage matrix to plot Vi as a function of Pvsc and Qvsc
Vi_plot = zeros(length(Vi), length(d)*length(k));
for i = 1:length(Vi)
    Vi_plot(i,:) = Vi(i);
end

%Plot VSC PQV Region
surf(Pvsc, Qvsc, Vi_plot,'EdgeColor','none','LineStyle','none','FaceLighting','phong')