FORCES ON CENTRIFUGAL PUMP IMPELLERS

by
Belgacem Jery
Graduate Student
Christopher E. Brennen
Professor
Thomas K. Caughey
Professor
and
Allan Acosta
Professor
Division of Engineering and Applied Science
California Institute of Technology
Pasadena, California

Belgacem Jery is a Graduate Student at the California Institute of Technology in Pasadena, California. Mr. Jery holds an Engineer Diploma from ENSAM in Paris, France. His M.S. in Mechanical Engineering degree is from Caltech, Pasadena, California. Mr. Jery has conducted graduate research on rotor instability in high performance turbomachines as part of his Ph.D. work. He has published several papers on this subject. He is a member of ASME and Societe des Ingenieurs ENSAM.

Christopher E. Brennen is a Professor of Mechanical Engineering in the Division of Engineering and Applied Science, at the California Institute of Technology in Pasadena, California. He holds B.A., M.A. and Ph.D. degrees in Engineering Science from Oxford University. Dr. Brennen was a Fulbright Scholar from 1969 to 1971. He has been honored by ASME, is a consultant and has many technical publications.

Thomas K. Caughey is a Professor of Applied Mechanics at the California Institute of Technology in Pasadena, California. He received B.Sc. and M.E. degrees from Glasgow University in 1948, an M.M.E. degree from Cornell University in 1952 and a Ph.D. degree in Engineering Science from California Institute of Technology in 1954. Dr. Caughey's fields of interest are Applied Mathematics, Dynamics, Vibrations, Acoustics, Stability, Non-linear Systems, Stochastic Processes, Chaos and Control Theory.

Allan J. Acosta, Professor of Mechanical Engineering at the California Institute of Technology since 1954, is currently involved in the study of internal flows in hydraulic machinery and fluid-structure interactions in these machines. He has also studied cavitation effects in hydraulic machinery and cavitation inception as it occurs in applications of naval hydrodynamics.

Prior to joining Caltech, he worked on model pumps for the Grand Coulee Irrigation Project. In 1954, he joined Caltech's Hydrodynamics Research Laboratory and worked on problems in centrifugal pump impellers, axial flow pumps and inducers and cavitating hydrofoils.

A Fellow of ASME, Dr. Acosta is a two-time recipient of the Knapp Award, and received the Moody Award, the 1975 ASME Melville Medal, and the Henry R. Worthington award in 1982. He has been chairman of the Ocean Engineering Division (1967) and the Fluids Engineering Division (1980).

A registered professional engineer in the State of California, he is also a member of the International Association of Hydraulic Research, the American Association for the Advancement of Science, the Society of Naval Architects and Marine Engineers and Sigma Xi.

ABSTRACT

Forces are exerted on a centrifugal pump impeller, due to the asymmetry of the flow caused by the volute or diffuser, and to the motion of the center of the impeller whenever the shaft whirls. Recent work in the measurement of these forces, as a function of the whirl speed to shaft speed ratio, and the influence of the volute, is reviewed. These forces may be decomposed into a steady force, a static stiffness matrix, a damping matrix and an inertia matrix. It is shown that for centrifugal pumps of the moderate specific speed typical of boiler feed stages, there is a region of potential shaft vibration excitation from the hydrodynamic forces if the operating speed is well above the first flexural critical speed.
INTRODUCTION

The need for more powerful, more efficient and more reliable turbomachines has grown substantially in the past decade. As a result, a great deal of effort is being devoted to solving increasingly complex design and manufacturing problems. One such problem is a self-excited rotor vibration phenomenon widely known now as rotor whirl. The research work described herein is an experimental investigation of rotor whirl related instabilities in centrifugal flow pumps. This investigation is part of an on-going program that started in 1977 at the California Institute of Technology, under the sponsorship of NASA, in conjunction with the development of the Space Shuttle Main Engine (SSME). A recent article by Ehrich and Childs [1] provides excellent background information on the subject of rotor whirl in general. As far as the present research is concerned, detailed descriptions of the test program, the experimental setup and the preliminary findings of the investigation exist elsewhere in the literature [2,3,4,5,6,7]. References [4] and [7] are especially recommended. Contributions from other sources are also noted in the literature [8,9,10,11,12,13,14]. More recent experimental test results are reported herein. A discussion is provided of the measured hydrodynamic stiffness, damping and inertia coefficients and their relation to the various impeller and volute or diffuser geometries and to the wide range of flow conditions. Finally, an example which illustrates the practical use of these rotodynamic coefficients is provided.

EXPERIMENTAL OBJECTIVE

When introducing a new turbomachine design, engineers generally agree that model testing is the only sure way to predict the dynamic behavior, especially for operation off-design. If done for every new product, this can prove to be very costly. However, the lack of theoretical data leaves very little choice to the designer. Empiricism, intuition, and past practical experience are usually brought into play. Another alternative is to conduct a limited, but very carefully selected, set of experiments, in which key parameters are varied and tested in a systematic fashion. Results of such tests can be extended to a much larger class of machines. Contributing from other sources by the use of simple scaling (similarity) laws. This was the approach chosen in the current experimental work. The objective herein is to provide designers of centrifugal flow pumps with a deeper understanding of the role of hydrodynamic impeller-volute/diffuser interactions in the excitation or damping of rotor whirl. From a rotordynamics point of view, this means providing impeller-volute/diffuser hydrodynamic stiffness, damping and inertia coefficients to be used in determining the critical speeds and the Onset Speed of Instability (OSI).

EXPERIMENTAL PROCEDURE

To measure these rotodynamic coefficients, it was decided to perform what amounts to a set of forced vibration tests. The rotor of a centrifugal pump is forced into a whirling motion in which the impeller center is made to follow a circular orbit, at a constant angular rate Ω, while the impeller axis remains parallel to the pump centerline. This takes place in addition to and independently of the normal impeller rotation at the constant angular rate ω. The whirl orbit center coincides with the volute or diffuser geometric center 0 (Figure 1). The orbit radius ε is constant and its value of 1.3 mm was carefully chosen: too large an ε would void the assumption of small perturbations; too small an ε would result in forces not large enough to be measured accurately. The word “forces” here refers to the components of the generalized hydrodynamic force vector (F) acting on the impeller as it undergoes simultaneous whirl and rotation.

All six components of (F) are sensed by a rotating dynamometer, strategically placed between the impeller and the drive shaft to avoid any drive system noise and unwanted forces. By design, both the shaft and the dynamometer could be considered infinitely rigid for the purpose of these measurements. Since whirl is usually associated with the lateral deflection of the rotor, only two of vector (F) components will be discussed, namely, the two lateral hydrodynamic forces Fx and Fy, acting at the impeller center, in a plane perpendicular to the machine axis of rotation. Referring to Figure 1, these forces can be written in the form:

\[
\begin{bmatrix}
F_x \\
F_y \\
F_{aox} \\
F_{aoy}
\end{bmatrix} = \begin{bmatrix}
F_{ox} \\
0
\end{bmatrix} + [A] \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

(1)

where x and y represent the coordinates of the impeller center measured from the center of the volute 0. In both this equation and all the equations and results which follow, dimensionless quantities are used. The linear form of equation (1) assumes small offsets x and y as allowed by the choice of ε. At the present time little is known of the possible non-linear effects.

![Figure 1. Schematic Diagram Representing the Lateral Forces in the Rotating Dynamometer Frame (Fx, Fy), in the Stationary Volute Frame (Fx, Fy) and in the Local Polar Coordinate Frame (F0x, F0y).](image)

When x and y are fixed in time, Fx and Fy represent the steady lateral forces, and can be viewed as the sum of two forces: a fixed force (Fx and Fy) would be only forces present, should the impeller center coincide with the volute center, i.e., x = y = 0 and a quantity proportional to the displacement, which represents a static fluid “stiffness” effect. These two quantities have been the subject of previous reports. [3,4,5,6]. Herein, the analysis and experiments for the case of non-zero whirl velocity, in which x and y are harmonic functions of time, (x = x(t) = ε cos Ωt, y = y(t) = ε sin Ωt), will be discussed. Damping and inertia effects are associated with the harmonic velocities and accelerations ẋ, ẏ, ẍ, ÿ of the impeller center. In this case, equation (1) can be generalized to the form:
\[
\begin{pmatrix}
F_x(t) \\
F_y(t)
\end{pmatrix} = \begin{pmatrix}
F_{x_0} \\
F_{y_0}
\end{pmatrix} + [A(\Omega/\omega)] \begin{pmatrix}
\epsilon_T^2 \cos \Omega t \\
\epsilon_T^2 \cos \Omega t
\end{pmatrix}
\]

The second quantity on the right hand side of Equation (2) represents the unsteady forces resulting from the combined motions of the impeller. Notice that what was a pure force matrix \([A]\) in equation (1) is now a “generalized” force matrix \([A(\Omega/\omega)]\). Indeed, the primary purpose of the unsteady measurements was to establish the functional form of this matrix when \(\omega/\Omega\) is varied. However, the matrices required for input into most rotodynamic analyses are the stiffness \([K]\), damping \([C]\), and inertial \([M]\), matrices defined by the dimensionless matrix equation:

\[
\begin{pmatrix}
F_x \\
F_y
\end{pmatrix} = \begin{pmatrix}
M_{xx} & M_{xy} \\
M_{yx} & M_{yy}
\end{pmatrix} \begin{pmatrix}
\dot{x} \\
\dot{y}
\end{pmatrix} - \begin{pmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{pmatrix} \begin{pmatrix}
\dot{x} \\
\dot{y}
\end{pmatrix} - [K] \begin{pmatrix}
x \\
y
\end{pmatrix}
\]

It follows, by comparing the definitions (2) and (3), that

\[
\begin{align*}
A_{xx} &= M_{xx} \frac{\Omega^2}{\omega^2} - C_{xx} \frac{\Omega}{\omega} - K_{xx} \\
A_{xy} &= M_{xy} \frac{\Omega^2}{\omega^2} + C_{xy} \frac{\Omega}{\omega} - K_{xy} \\
A_{yx} &= M_{yx} \frac{\Omega^2}{\omega^2} - C_{yx} \frac{\Omega}{\omega} - K_{yx} \\
A_{yy} &= M_{yy} \frac{\Omega^2}{\omega^2} + C_{yy} \frac{\Omega}{\omega} - K_{yy}
\end{align*}
\]

The identities (4) suggested the following data acquisition and reduction procedure: for each impeller-volute/diffuser combination, pump speed \(\omega\) and flow coefficient \(\phi\), a set of force measurements was taken varying only the whirl velocity \(\Omega\). These measurements were processed, resulting in a set of generalized stiffness matrices, one for each value of \(\omega/\Omega\).

At this point it should be observed that Equation (3) makes the a priori assumption that the elements \(A_{xx}\) of the matrix \([A(\Omega/\omega)]\) vary quadratically with \(\Omega/\omega\). In the absence of any fundamental reason supporting this assumption, it was decided to report the measured values of these elements in the form of curves of \(A_{xx}\) versus \(\Omega/\omega\). The elements of \([M], [C],\) and \([K]\) are estimated from these curves, using least squares quadratic fits.

**TEST MATRIX**

**TREATMENT OF TARE FORCES**

Three primary sources of tare forces were identified:
(a) gravitational and pure mass inertial loads on the rotor
(b) buoyancy force on the submerged impeller and dynamometer
c) parasitic hydrodynamic forces on the external surfaces of the submerged dynamometer.

Forces (a) were removed by subtracting the forces measured in a “dry” run, where the impeller was operated in air, from the forces measured in a “wet” run, where the same impeller was operated in water at the same speeds \(\Omega\) and \(\omega\). Thus, each data point was determined from the results of two separate tests.

Forces (b) were also made in the software to subtract out force (b) which is simply equal to the weight of the dry rotor, minus that of the submerged rotor (as measured by the dynamometer when the rotor was still). A recent modification of the hardware (enlargement of the gap surrounding the dynamometer shield) made it possible to neglect forces (c). Indeed, these forces now represent less than three percent of the impeller forces. Before this modification, parasitic forces were as high as 10 percent of the impeller forces. It was then necessary to subtract these parasitic forces. This required performing two additional runs for each eventual data point: a wet run without impeller and a dry run without impeller.

Subsequent to this and other minor hardware modifications, a complete recalibration of the dynamometer took place, and all the previous measurements were repeated. The present data include results from these reruns for the sake of completeness.

**TEST MATRIX**

To date, over 75 sets of measurements have been made. In each of these sets \(\Omega\) was varied while \(\omega\) and \(\phi\) were held constant. This brings the total of individual test runs to over 900. Altogether three impellers (one five-bladed (Impeller X), one six-bladed (Impeller Y) and a solid dummy (Impeller S)) and seven volute/diffuser volutes (Volute A and D, Diffuser Volutes E, F, G and H; and N, referring to the absence of any volute) were tested under various combinations of pump speeds (500, 1000, 1500, and 2000 rpm) and flow coefficients (\(\phi = 0\) (shut-off), 0.03, 0.06, 0.092 (design \(\phi\) for Impeller X), 0.110, and \(\phi = \phi_{\text{max}}\) (throttle full open)).

Appendix A gives a summary of the characteristics of the various volutes/diffusers and the different impellers. All tests were conducted using the same impeller face seal clearance of 0.013 mm.

As a precautionary measure to limit the inertial loads and mechanical vibrations on the rotor assembly, the maximum whirl speed was kept at 1200 rpm. The following ranges for the value of \(\Omega/\omega\) resulted:

\(\begin{align*}
\text{(a)} & \quad -0.6 \leq \Omega/\omega \leq 0.6 \quad \text{for 2000 rpm pump speed} \\
\text{(b)} & \quad -0.7 \leq \Omega/\omega \leq 0.8 \quad \text{for 1500 rpm pump speed} \\
\text{(c)} & \quad -0.9 \leq \Omega/\omega \leq 1.1 \quad \text{for 1000 rpm pump speed} \\
\text{(d)} & \quad -2.0 \leq \Omega/\omega \leq 2.0 \quad \text{for 500 rpm pump speed}
\end{align*}\)

The negative sign in the inequalities refers to the negative whirl directions. Thus, both subsynchronous and supersynchronous whirl motions were explored in both directions.

**TEST RESULTS**

Data such as the average lateral forces \((F_{x_0}, F_{y_0})\), the steady and unsteady thrust, torque and bending moments and the hydraulic performance curves \((\phi(\psi))\) will be presented in later reports. The measured hydrodynamic force matrices \([A(\Omega/\omega)]\) results are presented herein. Data on these matrices can be presented in any one of three formats:

(a) graphs of the individual elements of \([A(\Omega/\omega)]\) as a function of \(\Omega/\omega\).
(b) graphs of the average normal and tangential forces \((F_N\) and \(F_T)\) given by:

\[
F_N = \frac{1}{2}(A_{xx} + A_{yy}) \\
F_T = \frac{1}{2}(-A_{xy} + A_{yx})
\]

(c) tables of stiffness, damping and inertia coefficients obtained from second order least squares fits as defined in Equations (3) and (4).

The first format has the advantage of reporting the data in their raw and complete form, but is not always convenient to work with. Typical results from a sample case are shown in Figure 2, using this format. The measurements were made using the combination Volute A/Impeller X, which was tested more extensively than any other combination. Volute A was designed to match Impeller X at its shock-free entry condition with a flow coefficient \(\phi = 0.092\). The elements of \([A(\Omega/\omega)]\) obtained when
this combination was tested at the design flow coefficient, with the pump running at 1000 rpm and the impeller face seal clearance set at 0.13 mm, are shown in Figure 2, plotted against $I/J$ (or $\Omega/\omega$, equivalently, see NOMENCLATURE).

![Figure 2. The Elements of the Hydrodynamic Force Matrix, $A$, as a Function of $\Omega/\omega$, for the Volute A/Impeller X Combination at 1000 RPM and the Impeller X Design Flow Coefficient $\phi = 0.092$.](image1)

![Figure 3. The Dimensionless Average Normal and Tangential Forces, $F_N$, $F_T$, Corresponding to Figure 2.](image2)

**Effect of Pump Speed**

Conventional scaling of forces in pumps imply that data obtained when varying only the pump speed should be identical when plotted in the appropriate dimensionless form. This is indeed the case for both $F_N$ and $F_T$ (for both design and off design flow conditions), when the present non-dimensionalization is used, as is demonstrated in Figures 4 and 5.

**Effect of Flow Coefficient**

The dependence of the hydrodynamic forces on the flow coefficient is illustrated in Figure 5 for the 1000 rpm pump speed. The data shown were obtained with a diffuser volute, diffuser Volute E and Impeller X. Both $F_N$ and $F_T$ show a clear dependence on the value of $\phi$ throughout the whirl speed range. Of particular interest, however, is the shift in the value $\Omega/\omega$ at which $F_T$ changes sign. At shutoff ($\phi = 0$), this value was negative ($\Omega/\omega = -0.2$), indicating a whirl stabilizing tangential force for all speed ratios. As $\phi$ is increased to 0.060, the sign change occurred in the positive whirl region and at a much higher speed ratio ($\Omega/\omega = +0.5$). Accordingly, $F_T$ had a destabilizing effect in the positive subsynchronous whirl region between $\Omega/\omega = 0$ and $\Omega/\omega = 0.5$. However, as $\phi$ was increased even further, an interesting reduction in the critical value of $\Omega/\omega$ was evident—at and above design ($\phi = 0.092$ and $\phi = \phi_{\text{max}}$) the destabilizing effect was confined to the region $0 < \Omega/\omega < 0.3$.

Further study of these curves revealed interesting information concerning the curvature of the graph of the average tangential force. At shutoff, this curvature undergoes several changes (from positive, to negative, to positive again). As the flow coefficient is increased, these changes become less pro-
nounced, ending in a uniform positive curvature at maximum flow.

To better illustrate the evolution of the curvature with increasing flow coefficients, two additional sets of measurements were taken at selected intermediate values of $\phi$ (0.030 and 0.110). The corresponding $F_T$ curves are presented separately in Figure 7 (for clarity). Particularly noteworthy is the fact that, at $\phi = 0.030$, $F_T$ changed sign twice in the positive whirl speed region: first, sloping upward at around $\Omega/\omega = 0.05$, then downward at around $\Omega/\omega = 0.2$.

Effects of Volute Geometry

Effects of the volute geometry are presented in Figures 8 and 9. In Figure 8, comparison is made between the results obtained when Impeller X was operated inside Volute A, and when the same impeller was operated inside the pump casing (without any volute—crudely approximating the case where the impeller is whirling in an infinite media). One clearly observes a much stronger interaction due to the presence of a volute, resulting in a much wider destabilizing region ($0 \leq \Omega/\omega \leq 0.6$.
Figure 7. The Average Tangential Force for Volute E/Impeller X at 1000 RPM Pump Speed and Two Flow Coefficients: $\phi = 0.030$ and $\phi = 0.110$.

Figure 8. The Average Normal and Tangential Forces for Impeller X with and without Volute A. The pump speed is 1000 rpm and the flow coefficient is $\phi = 0.060$.

Figure 9. The Average Normal and Tangential Force for Impeller X and Four Different Diffuser Volutes (see APPENDIX A). The pump speed is 1000 rpm and the flow coefficient is $\phi = 0.060$.

compared to $0\leq \Omega/\omega \leq 0.4$ with no volute). On the other hand, the presence of diffuser guide vanes (or their number or orientation) appeared to have little effect on $F_N$ and $F_T$ (Figure 9). Data in this figure were obtained with the same diffuser volute. Diffuser Volute D was first tested with no vanes (D-0), then with two different sets of six vanes at two different angles (D-F6 and D-G6), and finally with a set of 12 vanes (D-H12). Detailed information on the impellers and volutes utilized in this testing is presented in APPENDIX A.

Effect of Impeller Geometry

Although Impeller X and Impeller Y have a different number of vanes (five for X and six for Y), their hydraulic characteristics are quite similar. It is therefore not a great surprise that the forces measured on these two impellers had very much the same magnitude and phase. This is demonstrated in Figure 10, where the pump speed was 1000 rpm and the flow coefficient was kept at the Impeller X design value ($0.092$).

Presently, skin friction and leakage flow through the impeller face seals are being investigated. Preliminary measurement results are presented in Figure 11. The comparison here is between the forces measured on Impeller X and those measured on a consolidated dummy Impeller S, duplicating the exterior geometry of Impeller X). Both measurements were made at shutoff ($\phi = 0$), with the pump running at 1000 rpm. A noticeable difference can be observed in both $F_N$ and $F_T$. 
FORCES ON CENTRIFUGAL PUMP IMPELLERS

Figure 10. The Average Normal and Tangential Forces for Diffuser Volute E and Two Different Impellers (Impeller X and Impeller Y). The pump speed is 1000 rpm and the flow coefficient is $\phi = 0.092 = \text{Impeller X design flow coefficient.}$

Figure 11. The Average Normal and Tangential Forces for Volute A and Two Different Impellers (Impeller X and Impeller S). The pump was run at shutoff at 1000 rpm.

Table 1. Impeller Rotodynamic Coefficients

<table>
<thead>
<tr>
<th>VOLUTE IMPEL</th>
<th>1000</th>
<th>1500</th>
<th>1000</th>
<th>1500</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPEED PHI</td>
<td>0.892</td>
<td>0.892</td>
<td>0.892</td>
<td>0.892</td>
</tr>
<tr>
<td>X</td>
<td>2.096</td>
<td>1.884</td>
<td>3.642</td>
<td>2.447</td>
</tr>
<tr>
<td>Y</td>
<td>3.499</td>
<td>3.139</td>
<td>5.228</td>
<td>4.881</td>
</tr>
<tr>
<td>Cx</td>
<td>2.096</td>
<td>1.884</td>
<td>3.642</td>
<td>2.447</td>
</tr>
<tr>
<td>Cy</td>
<td>3.499</td>
<td>3.139</td>
<td>5.228</td>
<td>4.881</td>
</tr>
<tr>
<td>Mx</td>
<td>6.986</td>
<td>6.697</td>
<td>13.174</td>
<td>12.994</td>
</tr>
</tbody>
</table>

Rotodynamic Matrices

As mentioned earlier, rotodynamicists much prefer the $[K]-[C]-[M]$ format. Dimensionless values of the elements of the $[K]$, $[C]$ and $[M]$ matrices obtained from second order least squares fits to the elements of $[A](\phi/\omega)$ are presented in Table 1. It is clear from Figures 6 and 7 that the curvature of the graph of the average tangential force, $F_T$, is small and somewhat uncertain below the design flowrate. This result is in appreciable departure from the pure quadratic behavior and, hence, discrepancies in the off-diagonal terms of some of the inertia matrices, $[M]$. On the other hand, the elements of the stiffness matrices, $[K]$, are in good agreement with the measurements of Chamier, et al. [3,4,5]. Selected values from Table 1 are used in the rotodynamic analysis of the SSME High Pressure Oxygen Turbopump (HPOTP). This analysis was carried out by Childs and Moyer and is appended herein.

SUMMARY AND CONCLUSIONS

Measurements of the complete hydrodynamic force matrices were presented for various impeller-volute/diffuser geometries and various pump operating conditions. The common assumption of skew-symmetry for these matrices is justified. Conventional scaling of forces with pump speed was perfectly demonstrated. The results obtained at and above design flow conditions closely support the commonly used quadratic variation with whirl frequency. As expected, the rotodynamic coefficients are very sensitive to the value of the flow coefficient, especially below design conditions. On the other hand, the presence, number, or orientation of diffuser guides have surprisingly little effect on the results.
As for the effect of the impeller geometry, a definite conclusion should not be made until impellers with more dissimilar characteristics are tested. To this end, the pump inlet section will be modified to accommodate the HPOTP inducer/impeller which will be tested in the near future. Also, leakage flow and preliminary cavitation tests are underway.

**NOMENCLATURE**

- \( [A] \) dimensionless hydrodynamic force matrix
- \( [C] \) hydrodynamic damping matrix as defined by Equation (3)
- \( b_2 \) impeller discharge width
- \( \{F\} \) 6-component generalized force vector
- \( F_1, F_2 \) components of instantaneous lateral force on impeller in the rotating balance reference frame
- \( F_x, F_y \) components of instantaneous lateral force on impeller in fixed laboratory reference frame \((X, Y)\) non-dimensioned by \( \rho r_2^2 \omega^2 b_2^2 \)
- \( F_{o1} F_{o2} \) values of \( F_x \) and \( F_y \) on impeller axis remains coincident with the origin of the \((X, Y)\) coordinate system
- \( F_{N1}, F_{N2} \) components of lateral force on impeller normal to and tangential to the whirl orbit, non-dimensioned by \( \rho r_2^2 \omega^2 b_2^2 \), averaged over one orbit
- \( F_{N1}, F_{N2} \) values of \( F_{N1}, F_{N2} \) averaged over one orbit
- \( \Omega \) integers such that \( \Omega = \omega t \)
- \( \Omega \) hydrodynamic stiffness matrix as defined by Equation (3)
- \( [M] \) hydrodynamic inertia matrix as defined by Equation (3)
- \( N \) pump rpm = \( 60 \omega / 2 \pi \)
- \( r_2 \) impeller discharge radius
- \( t \) time
- \((X, Y)\) fixed laboratory reference frame
- \( x, y \) instantaneous coordinates of impeller center in fixed laboratory reference frame \((X, Y)\), non-dimensioned by \( r_2 \)
- \( \tilde{x}, \tilde{y}, \tilde{x}, \tilde{y} \) first and second time derivatives of impeller position, non-dimensioned using the distance \( r_2 \) and the time \( \omega^{-1} \)
- \( \epsilon \) radius of circular whirl orbit
- \( \rho \) density of water
- \( \phi \) pump flow coefficient based on impeller discharge area and tip speed
- \( \psi \) pump total head coefficient, total head rise divided by \( \rho r_2^2 \omega^2 \)
- \( \omega \) radian frequency of pump shaft rotation
- \( \Omega \) radian frequency of whirl motion = \( \omega t \)

**APPENDIX A**

<table>
<thead>
<tr>
<th>IMPELLER NAME</th>
<th>DIAMETER</th>
<th>WIDTH</th>
<th>ANGLE</th>
<th>BLADES</th>
<th>SPEED</th>
<th>COEFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>141.9</td>
<td>15.8</td>
<td>23</td>
<td>5</td>
<td>0.57</td>
<td>0.892</td>
</tr>
<tr>
<td>Y</td>
<td>142.1</td>
<td>16.5</td>
<td>38</td>
<td>6</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>142.1</td>
<td>16.5</td>
<td>38</td>
<td>6</td>
<td>0.95</td>
<td></td>
</tr>
</tbody>
</table>

**REFERENCES**

FORCES ON CENTRIFUGAL PUMP IMPELLERS


ACKNOWLEDGMENTS

The authors are indebted to the NASA George Marshall Space Flight Center, Huntsville, Alabama, for their continued sponsorship of this research under contract NAS8-33108. We are also most grateful for the help given by graduate students D. Adkins and R. Franz, undergraduate W. Goda and staff member D. Brennen. Belgacom Jery would also like to thank Byron-Jackson Pump Division, Borg-Warner Industrial Products, Inc., for financial support.

ADDENDUM

SAMPLE ROTORDYNAMIC CALCULATIONS USING CAL-TECH ROTORDYNAMIC COEFFICIENTS

by

Dara W. Childs
Director, Turbomachinery Laboratories
Department of Mechanical Engineering
Texas A&M University
College Station, Texas

and

David S. Moyer
Space Shuttle Systems Engineer
McDonnell Douglas Technical Services Company
Houston, Texas

INTRODUCTION

Engineers may well be curious as to the influence of the impeller-diffuser forces on the rotordynamic characteristics of an actual piece of turbomachinery. As a partial answer to this point, some calculated results are presented for the HPOTP (High Pressure Oxygen Turbopump) of the SSME (Space Shuttle Main Engine). The rotating assembly for this unit is illustrated in Figure A-1. The high operating speed range of 20,900-30,380 rpm and the relatively high specific gravity of liquid oxygen (1.137) are obviously not typical of commercial equipment; however, the results do show the very significant influence of the impeller rotordynamic coefficient.

IMPELLER ROTORDYNAMIC MODEL

The Cal-Tech data presented in Table 1 defines the following constant-coefficient face-displacement model

\[
\begin{bmatrix}
F_X \\
F_Y \\
M_X \\
M_Y
\end{bmatrix}
= \begin{bmatrix}
K_{XX} & K_{XY} & 0 & 0 \\
K_{YX} & K_{YY} & 0 & 0 \\
C_{XX} & C_{XY} & 0 & 0 \\
C_{YX} & C_{YY} & 0 & 0
\end{bmatrix}\begin{bmatrix}
X \\
Y \\
X \\
Y
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
M_{XX} & M_{XY} & 0 & 0 \\
M_{YX} & M_{YY} & 0 & 0
\end{bmatrix}\begin{bmatrix}
X \\
Y
\end{bmatrix}
\]

(A.1)
However, an inspection of the data in this table shows that the matrices are approximately symmetric; hence, for the results presented here, the following simplified model is used

\[
\begin{pmatrix}
F_X \\
F_Y
\end{pmatrix}
= 
\begin{bmatrix}
K & K \\
K & C
\end{bmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
+ 
\begin{bmatrix}
C & C \\
-\omega & M
\end{bmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
+ 
\begin{bmatrix}
M \\
-\omega & M
\end{bmatrix}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
\]  

(A.2)

Figure A.1 HPOTP Rotating Assembly.

The dimensional entries of (A.2) are related to the nondimensional entries of Table 1 as follows:

\[
\begin{align*}
K &= (K_X + K_Y) C_{\omega^2} / 2 \\
k &= (K_X - K_Y) C_{\omega^2} / 2 \\
C &= (C_X + C_Y) C_{\omega^2} / 2 \\
c &= (C_X - C_Y) C_{\omega^2} / 2 \\
M &= (M_X + M_Y) C_{\omega^2} / 2 \\
m &= (M_X - M_Y) C_{\omega^2} / 2
\end{align*}
\]

where * denotes the nondimensional entries of Table 1. Further,

\[
\begin{align*}
C_l &= \rho \pi b_2 \mu_2 \\
\rho &= \text{fluid density} \\
b_2 &= \text{impeller discharge radius} \\
b_2 &= \text{impeller discharge width.}
\end{align*}
\]

For the main impeller of the HPOTP, \(b_2 = 85\) mm, and \(b_2 = 25.4\) mm, while for the boost impeller, \(b_2 = 66\) mm, and \(b_2 = 6.9\) mm. The fluid density for both impellers is approximately 1100 kg/m³.

Most computer codes in use today for pump rotordynamics analysis do not account for the added-mass coefficients of Equation (A.2). The authors of this addendum extended the procedure of [A.1] to account for added mass terms in carrying out the calculations presented here.

**ROTORDYNAMIC CALCULATIONS**

Over the past several years, the HPOTP has experienced excessive subsynchronous motion at a frequency associated with a second critical speed mode shape. The zero-running-speed undamped modes are illustrated for the first and second rotor bending modes in Figure A-2. Observe that the first critical speed mode shape involved primarily the overhung turbine mode with relatively small bearing deflections, while the second mode involved large deflections at the main impeller and large bearing deflections. Hence, the first mode response is expected to be insensitive to impeller forces, while the second mode would be quite sensitive.

The addendum authors recently published the result of a rotodynamic analysis concerning subsynchronous vibration problems of the HPOTP [A-2] when operating at high speeds. The rotordynamics model used in [A-2] included a structural dynamics model for the rotor and the housing. For linear analysis, the bearings were modeled as linear springs. Liquid seals at the boost impeller were modeled according to Equation (A-2) using the analysis procedure of [A-3]. Gas seals in the turbine area were accounted for by a model similar to Equation (A-2), except that the mass matrix was dropped. Seal coefficients were calculated based on Nelson's analysis [A-4]. The turbine clearance excitation forces used the model of Thomas [A-5] and Allford [A-6].

The obvious point of interest here is the influence that the impeller coefficients had on the rotordynamics characteristics of the HPOTP. This point is addressed by calculating the OSI (Onset Speed of Instability) and synchronous bearing reactions for the following impeller models:

- **Stiffness Coefficients** only. This was the first "static-only" data published by Chamieh, et al. [3] and was used in [A-2].
- **Full model**, including stiffness, damping, and added-mass coefficients.
- **Reduced model**, obtained by dropping the added-mass terms.
- **Reduced model**, obtained by dropping the added-mass terms and the cross-coupled damping terms.

The nondimensional data for Volute A, \(\phi = 0.092\) of row 4 in Table 1 are used.

The synchronous reactions for bearings 1 and 4 due to rotating imbalance based on only the stiffness-matrix model for the impellers are illustrated in Figure A-3. The bearing-reaction response was dominated by first and second critical speeds at approximately 13,500 rpm and 31,170 rpm, respectively. Some smaller peaks are evident, due to housing resonances. The
calculated OSI for this configuration was approximately 30,500 rpm. The cross-coupled stiffness coefficient from the main impeller was the source of the instability, and the OSI was arbitrarily set to the calculated value by adding direct damping at the main impeller. The OSI of 30,500 rpm was chosen to yield agreement with field data.

The results presented in Figure A-4 illustrate the predicted synchronous-response characteristics including the stiffness, damping, and added mass terms. The second critical speed was calculated to lie at 33,060 rpm, while the predicted OSI was greater than 55,000 rpm.

The results presented in Figure A-5 illustrate the calculated bearing reactions if the stiffness and damping matrices are included, but the mass matrix is dropped. The second critical speed is seen to be heavily damped and was elevated to approximately 39,000 rpm. For this impeller model, the OSI was also greater than 55,000 rpm. By comparison with Figure A-4, the obvious conclusion is that the mass-matrix contribution cannot be neglected, and rotordynamics-analysis procedures for pumps, which are not able to account for the added-mass matrix at impellers, should be avoided.

The cross-coupled-damping coefficients in the models of equations (A-1) and (A-2) act as "gyroscopic-stiffening" elements to raise the rotor's critical speed, and the Cal-Tech cross-coupled damping coefficients are relatively high. The direct mass coefficients tend to compensate for the cross-coupled damping coefficients in depressing the rotor critical speeds, and this explains the sharp elevation of the second critical speed from Figure A-4 to Figure A-5 after the mass matrix has been dropped. From this reasoning, one could expect that dropping both the mass matrix and the cross-coupled damping matrix might yield a simplified, but still reasonable, model. The synchronous response prediction for this type of impeller model is illustrated in Figure A-6. The second critical speed is predicted to be at 32,340 rpm, which is in reasonable agreement with the results of Figure A-6 for the complete impeller model; however, the second critical speed is much more heavily damped and the predicted OSI is now in excess of 75,000 rpm. Clearly, the proposed simplified model is inadequate.

CONCLUSIONS
The sample calculations presented herein support the following conclusions:
(a) Impeller-diffuser forces have a very significant impact on the calculated rotordynamic characteristics of pumps, particularly with respect to damping and stability
(b) The complete model for the impeller, specifically including the mass terms, must be included in rotordynamic calculations to achieve reasonable results.

REFERENCES
A.1 Childs, D. W., "The Space Shuttle Main Engine High-Pressure Fuel Turbopump Rotordynamic Instability Prob-


ACKNOWLEDGMENT

This work was supported in part by NASA Contract NAS8-34505, from the George C. Marshall Space Flight Center.