# A NEW METHOD FOR COMPUTING MINIMUM FLOW

by

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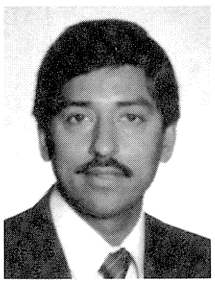

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## ABSTRACT

Appropriate specification of minimum flows is important to avert pump reliability problems. A method is presented for the calculation of minimum flow by allowing a satisfactory margin below the onset of flow recirculation in the impeller. A semi-analytical method is proposed for the calculation of recirculation onset. The margins depend upon size, speed, specific gravity of pumped fluid, NPSH available, intermittence of operation and mechanical design. Empirical charts are provided for estimating these margins.

## INTRODUCTION

Many pump reliability problems in the field are being attributed to the operation of the pump at flow rates well below its best efficiency point. Generally, the vendor and the user try to avert such problems by specifying a minimum flow, below which the pump should not be operated. Unfortunately, the criteria for specifying such a minimum flow are not clearly understood by all parties concerned. The user normally would expect the minimum flow to be the lower limit for reliable operation of the whole pump. This requirement includes reliability of bearings, seals, auxiliary systems, etc. On the other hand, the vendor's specification of minimum flow is typically not as broad-based, and reliability problems can, therefore, arise. The vendor normally represents minimum flow from a hydraulic, or cavitation view point. For example, historically, minimum flow was based upon the point at which the pump inefficiency raised the fluid temperature by a specified amount. Generally, this flow rate is a small fraction of best efficiency point flow (BEP flow) and extended operation near this value would lead to premature pump failure in most pumps.

## A STRATEGY FOR DETERMINING MINIMUM FLOW

Through the entire head *vs* flow operating range of a pump at constant speed, several important flow rates can be established. This is shown in Figure 1, with the positioning of the various flow points being somewhat arbitrary. These concepts are briefly described below. More detailed description and calculation will be given in the next section.

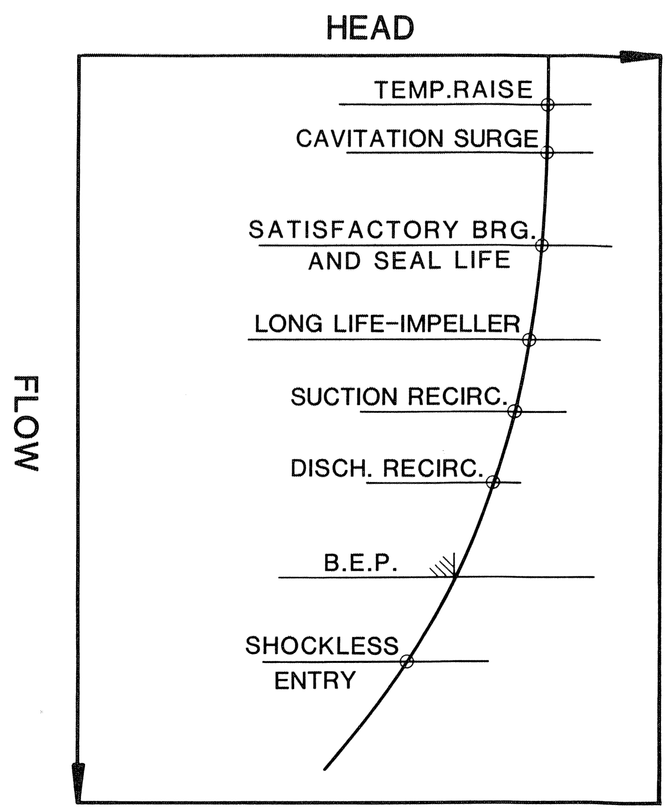


*Figure 1. Typical Head-Flow Characteristic of a Centrifugal Pump Indicating Several Important Flow Rates.*

The best efficiency point (BEP) is the flow rate at which the combined hydraulic losses of the pump unit are the lowest. The individual component losses in general are not all minimal at this point. The flow rate at which the relative flow to the impeller is perfectly aligned with the impeller vane is called the shockless entry point. For impellers designed to have high suction capacity, the shockless entry point is usually to the right of BEP.

At flows lower than BEP, recirculation flows can commence as shown in Figure 2. For a given NPSH available, an impeller starts undergoing cavitation damage at some flow rate lower than BEP (assuming that at BEP the available NPSH is adequate to prevent damage). The bearing and seal life begin to be impaired at some low flow rate because of the large dynamic forces generated by the hydraulic phenomena in impellers and volutes or diffusers. At very low flow rates, large scale axial and transverse motions occur because of two-phase flow regimes arising from severe cavitation. Also, as mentioned earlier, high pumping inefficiencies at low flows cause significant temperature rise of the pumped fluid.

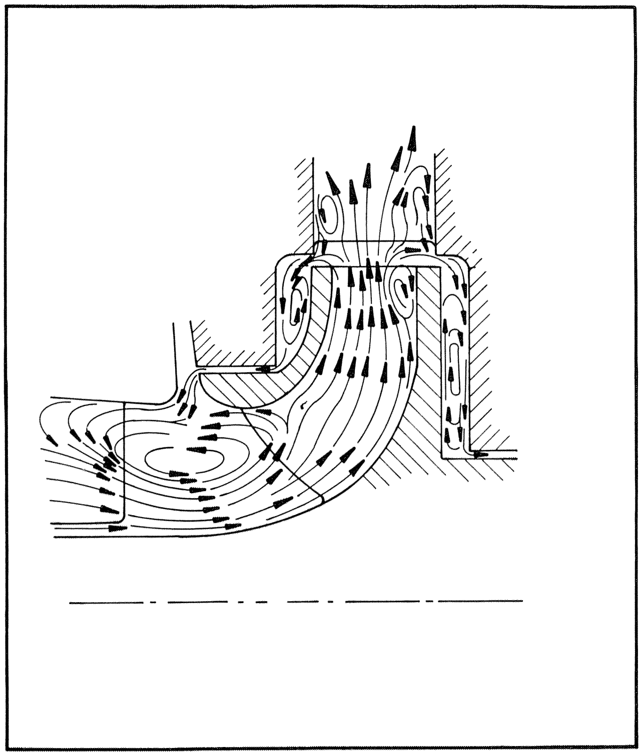

*Figure 2. Recirculating Flow Pattern in the Meridional Plane of a Centrifugal Impeller.*

Specification of minimum flow must recognize the purpose for which it is done. For example, for aircraft fuel pumps, vaporization of the inlet flow is the main concern rather than seal or bearing life. Consequently, temperature rise is the criterion for setting up minimum flows. For low energy pumps operating with marginal NPSH, the criterion could be impeller life from a cavitation damage point of view. For high energy pumps, even modest recirculation may be intolerable because of severe vibrations, in which case recirculation onset points will establish minimum flow.

Apart from these special cases, for a broad range of centrifugal pumps, overall pump reliability is perhaps the best criterion for specifying minimum flow rates at which hydraulic interactions cause forces which compromise seal and bearing reliability. The basic assumption in this strategy is that bearing and seal life is affected by flow rates. The calculation outlined below demonstrates the validity of this assumption.

The radial load exerted by an impeller varies with flow. The static radial thrust coefficient K defined as

$$K = F_r/\rho\ H\ D_2\ b_2$$

is about 0.025 for double volute pumps at the best efficiency point. The value of K increases as the flow rate is reduced. For single volute pumps, the ratio of K at shutoff to that at BEP can be as high as 20 [1]. For double volutes, this ratio can be much smaller (as low as 1.5 to 2.0). However, the dynamic forces increase rapidly with reduction in mass flow. The work of Kanki [2] shows that near shutoff, strong subsynchronous and vane passing force components exist. K values of about found for diffuser and double-volute pumps. Based on these data, the variation of K with flow for a 4.0 in discharge process pump with an overhanging impeller has been hypothesized as shown in Figure 3. From these impeller forces, bearing loads can be calculated and the average bearing life expectancy can be estimated. Since bearing life varies inversely as the cube of load, life decreases dramatically with flow as shown in Figure 3. At BEP, the expected life is indefinitely large, but as the flow goes down, it decreases rapidly. For example, at 40 percent flow, expected life is only about six months.

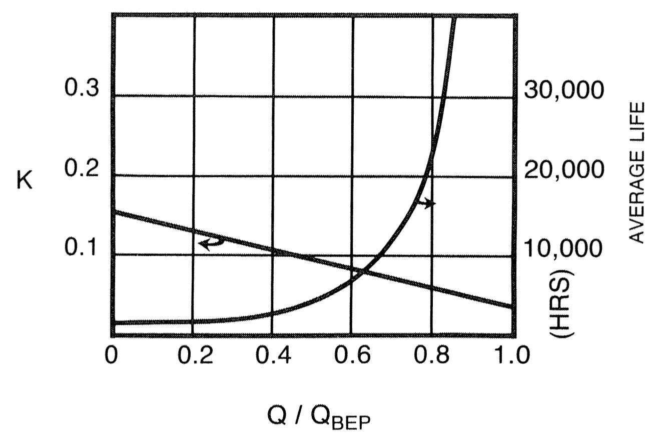


*Figure 3. Reduction in Anticipated Average Bearing Life Due to Increasing Radial Loads at Low Flows.*

Seal life depends strongly upon shaft deflections. Even though no empirical rules are available to relate seal life to shaft deflection, it is evident that increasing forces tend to decrease seal life.

Since the onset of recirculation signals the generation of strong forces, the strategy of determining minimum flow involves providing an allowable margin below the onset point.

The techniques for calculating these factors are described below.

$$Q_{min} = Q_R\ \ k_1\ k_2\ k_3\ k_4\ k_5 \tag{1}$$

where

$Q_R$ = recirculation onset point (suction or discharge, whichever is larger)
$k_1$ = factor to account for power density
$k_2$ = factor to account for liquid specific gravity
$k_3$ = factor to account for margin of available NPSH over required NPSH
$k_4$ = factor to account for intermittency of operation
$k_5$ = factor to account for specific mechanical design margins

The techniques for calculating these factors are described later.

## COMPUTATION OF MINIMUM FLOW

*Recirculation Onset*

The phenomenon of recirculation has been extensively researched and many publications dealing with the theoretical aspects as well as control may be found in the literature. Recirculation flow patterns are complex. The view of the meridional plane is shown in Figure 2. In the blade-to-blade plane, there is a strong convection of the fluid particles toward the pressure side, as the suction side is covered by separation cavities. When the convected bubbles reach the pressure surface, they implode because of the higher ambient pressure and cause damage on the backside of the vane. Thus, recirculation damage often is not seen easily on the impeller unless care is taken to look for damage on the backside using mirrors. In extreme cases, the backside damage is large enough that a hole is poked through and then it is visible on the suction side.

In general, the onset of recirculation is gradual, and the flow rates at which it begins are different for suction and discharge. Of all the publications dealing with the recirculation phenomena that may be found in the literature, the work of Fraser [3] provides the simplest method for calculation. The discharge recirculation onset point $Q_{DR}$ is given in as: [3]

$$Q_{DR} = \pi\, D_2^{\,2}\ \ b_2\, \tfrac{\omega}{2}\, \overline{C}_{DR} \qquad (2)$$

The quantity $\overline{C}_{DR}$ was empirically found in Reference 3 to be a function of the exit vane angle $\beta_2$.
The suction recirculation onset is given in [3] as:

$$Q_{SR} = \tfrac{\pi}{8}\, \omega\, D_1^{\,3}\, (1 - \gamma^2)\, \overline{C}_{SR} \qquad (3)$$

with $\overline{C}_{SR}$ given as a function of the vane inlet angle $\beta_1$.

For $$\frac{D_1}{D_2} > 0.5,\ Q_{SR} \geq Q_{DR}$$

Using these relationships, suction and discharge recirculation points can be calculated.

The suction recirculation onset predicted by Equation 3 is in good agreement with test data for normal vane angles. For $\beta$ values less than about 15°, this equation appears to predict rather low values for the onset. To improve this situation, the present study has utilized the following equation:

$$\overline{C}_{SR} = \tan\beta\, \{1 - 0.2091\, (\beta - 9.5)^{0.4}\} \qquad (4)$$

Even though the above equations are simple, their use is limited because they require a knowledge of vane inlet and discharge angles. An approximation to the suction recirculation onset point can be made in the following way.

It is generally understood that suction specific speed and suction recirculation onset are linked together. The higher the $S_s$, the higher the onset point is expected to be. The classical expression for NPSH at shockless entry point is:

$$2g\ \mathrm{NPSH} = a_1\, c_{m1}^{\,2} + a_2\, W_1^{\,2} \qquad (5)$$

where $a_1$ and $a_2$ are constants.

In the absence of prewhirl $W_1^{\,2} = U_1^{\,2} + c_{m1}^{\,2}$

Also at shockless entry point $c_{m1} = U_1 \tan\beta_1$.

Substituting these in (5), we get

$$2g\,\mathrm{NPSH} = (a_1 + a_2)\, U_1^{\,2} \left[\tan^2\beta_1 + \frac{a_2}{a_1 + a_2}\right] \qquad (6)$$

The flow rate at shockless entry point is:

$$Q_{SE} = \tfrac{\pi}{8}\, \omega\, D_1^{\,3}\, (1 - \gamma^2)\, \tan\beta_1 \qquad (7)$$

$$S_s = \frac{N\,\sqrt{Q_{SE}}}{(\mathrm{NPSH})^{0.75}} = 358.6\sqrt{1-\gamma^2}\,\frac{\sqrt{\tan\beta_1}}{\frac{a_1 + a_2}{2g}\,(\tan^2\beta_1 + \frac{a_2}{a_1 + a_2})^{0.75}} \qquad (8)$$

Dividing Equation (3) by Equation (7), after using (4), we get

$$\frac{Q_{SR}}{Q_{SE}} = 1 - 0.2091\,(\beta_1 - 9.5)^{0.4}$$

Since Equations (8) and (9) are functions only of $\beta_1$, it is clear that the ratio $Q_{SR/QSE}$ is uniquely related to $S_s$.

The constant $a_1$ represents the non-uniformity of the velocity profile at the eye. A good discussion of its effects may be found in Spring's article [4]. For approximate calculation, an assumption of $a_1 = 1.2$ is generally adequate.

The constant $a_2$ represents the dynamic depression of the static pressure and is a function of the details of the vane

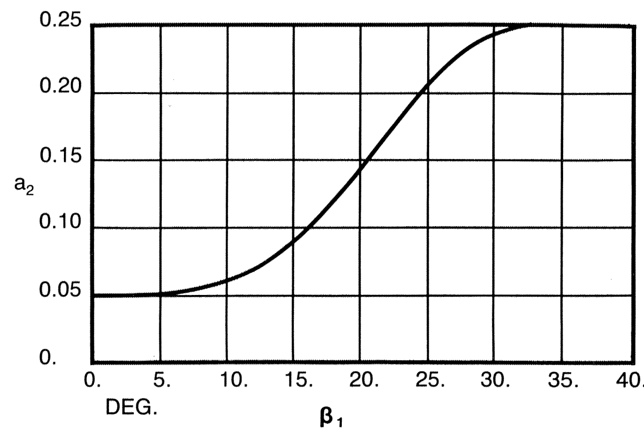


*Figure 4. Variation of NPSH Coefficient $a_2$ with Vane Inlet Angle.*

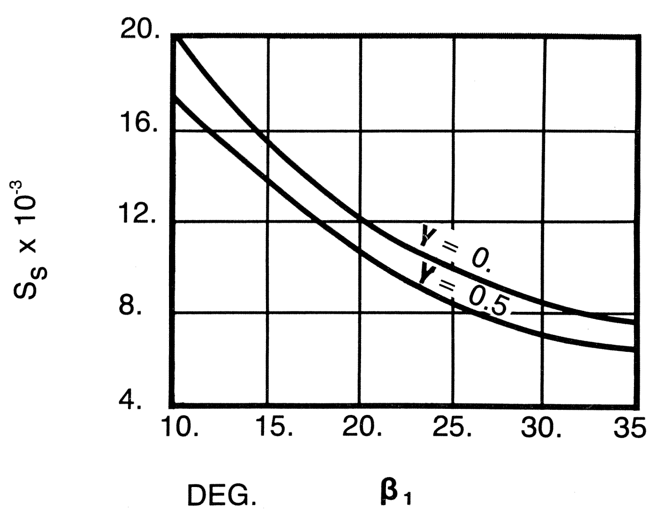


*Figure 5. Variation of Suction Specific Speed with Vane Inlet Angle.*

geometry. Spring states that a varies from about 0.05 to 0.15 [4]. In the present author's experience, a depends strongly on $\beta_1$. A typical variation is shown in Figure 4. Using $a_1 = 1.2$ and $a_2$ given by Figure 4, suction specific speed can be plotted as a function of $\beta_1$ using Equation (8), as shown in Figure 5. Equation (9) is plotted in Figure 6. Using Figures 5 and 6, the relationship between suction recirculation onset and suction specific speed can be graphically plotted as shown in Figure 7. This figure demonstrates the expected tendency that high suction specific speed machines tend to recirculate closer to the shockless entry point.

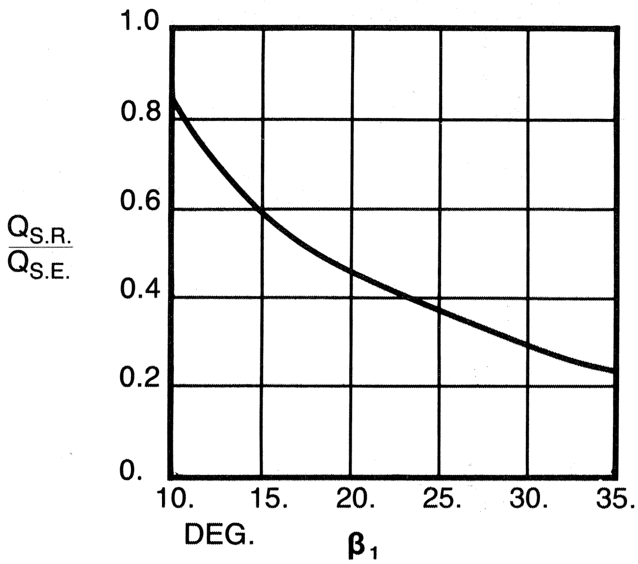


*Figure 6. Variation of the Ratio of Suction Recirculation Onset Flow to Shockless Entry Flow with Vane Inlet Angle.*

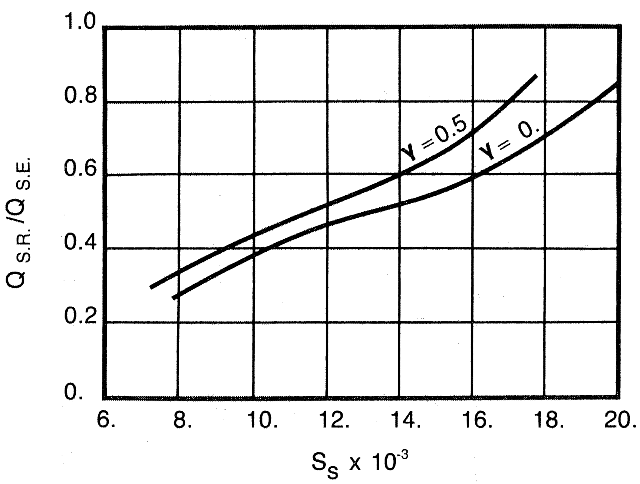


*Figure 7. Variation of the Ratio of Suctional Recirculation Onset Flow to Shockless Entry Flow with Suction Specific Speed.*

To simplify the calculations still further, if we assume that $Q_{SE} = 1.2\ Q_{BEP}$, the recirculation point can be plotted as a ratio to BEP flow against $S_s$. This variation is presented in Figure 8.

When vane angle information is available, the onset point can be calculated in two ways: one by using Equations (2) and (3) and the other by using Figure 6 and Equation (7). In general, these results may not agree because of the assumptions involved. In

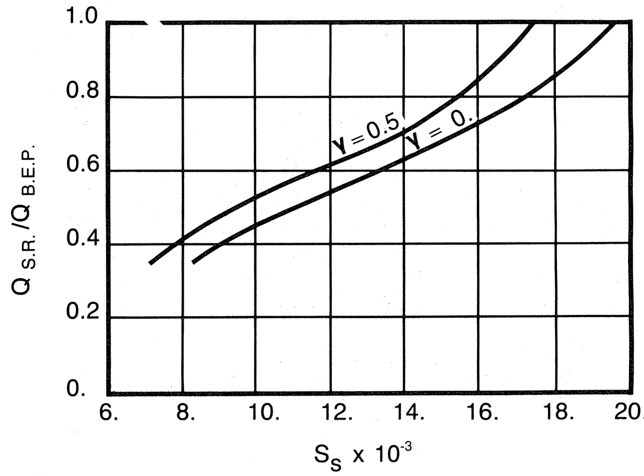


*Figure 8. Variation of the Ratio of Suction Recirculation Onset Flow to BEP Flow with Suction Specific Speed.*

such cases, an average between the two values may be desirable. If β information is not available, Figure 8 can be used as a reasonable estimate.

*Example*

A single stage end suction process pump of 1750 specific speed was tested for recirculation onset. The static pressure in the inlet pipe was measured just ahead of the impeller for various flows. The absolute flow angle was also measured near this location as close to the pipe wall as practical. These variations are shown in Figure 9. The sudden increase in pressure is indicative of the beginning of recirculation as explained by Fraser [3]. The presence of prerotation also shows that recirculation has commenced. It must be pointed out that there is considerable uncertainty in the measurement of flow angles near the wall, particularly as the flow is quite unsteady. The recirculation onset shown in Figure 9 appears to be about 1000 gpm, when one considers the static pressure increase. It appears to be about 1200 gpm, if one observes the beginning of inlet prewhirl. The discrepancy is due to both the uncertainty in angle measurement (as much as 15 de-

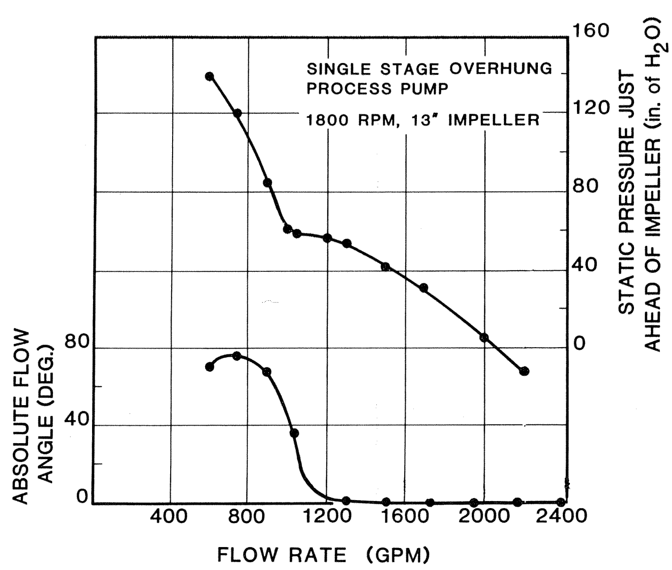


*Figure 9. Experimentally Measured Suction Recirculation Onset—Variation of Absolute Flow Angle and Static Pressure at Impeller Inlet with Flow Rate.*

grees near the wall) and also to the fact that the static pressure tap was located slightly upstream of the wedge probe.

For this impeller $\beta_1$ = 21 degrees giving $\dot{C}_{SR}$ = 0.171 from Equation (4). Substituting this in Equation (3), with $D_1$ = 6.875 in, $\omega$ = 188.5 1/sec, and $\gamma$ = 0.418 gives $Q_{SR}$ = 880 gpm. Using the second method requires knowledge of $S_S$, which from test data, is 11150. From Figure 7, $Q_{SR}/Q_{SE}$ = 0.46. From Equation (7), $Q_{SE}$ = 1980 gpm. Therefore, $Q_{SR}$ = 910 gpm which is close to the 880 gpm calculated by the first method. Thus, the prediction of about 900 gpm agrees reasonably well with the measured value between 1000 and 1200 gpm.

To check the validity of the calculation for higher specific speed, comparison is made for a vertical pump of $N_s$ = 7000. The variation of absolute angle with radius at the inlet is shown in Figure 10 for two flow rates. With BEP at 9500 gpm, it can be seen that minor recirculation is evident at 6000 gpm, while at 4000 gpm, recirculation dominates nearly the entire inlet area.

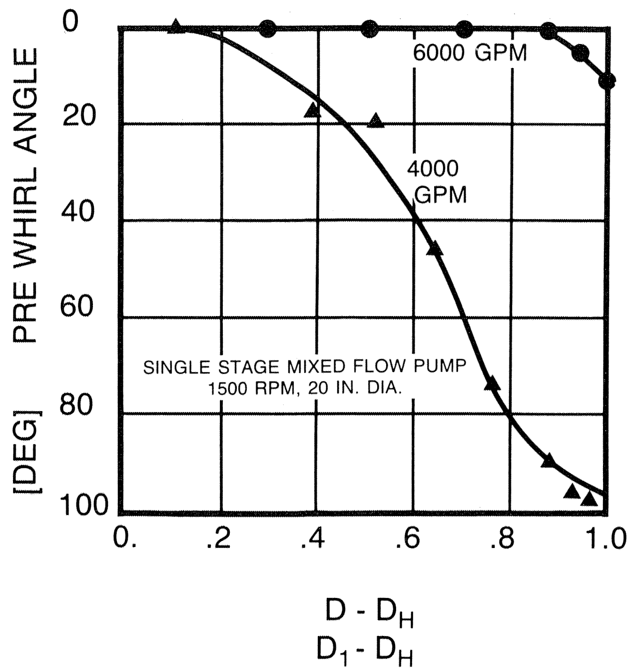


*Figure 10. Variation of Absolute Flow Angle at the Inlet for a 7000 Specific Speed Mixed Flow Pump.*

For this impeller, $\beta_1 \cong 22.5°$ giving $\overline{C}_{SR}$ = 0.173. With $D_1$ = 12.125 in, $\gamma$ = 0.289 and $\omega$ = 158.1 1/sec, we get $Q_{SR}$ = 5000 gpm. Using the second method with the known $S_s$ = 12650, the result is $Q_{SR}/Q_{SE}$ = 0.51. $Q_{SE}$ is calculated to be 11970 gpm from Equation (7). Thus, recirculation onset flow is 11970 × 0.51 = 6100 gpm. Between the two methods, a value fairly close to the observed onset is achieved.

In the above examples, $S_S$ has been calculated at BEP. Strictly, the computation method is intended to be used with $S_s$, calculated at shockless entry point. Fortunately, in most cases, the error caused by this discrepancy is small. For example, in the second case, $S_s$ calculated at 11,970 gpm is 11,000. Then $Q_{SR}/Q_{SE}$ = 0.46, and onset flow is 5500 gpm. In the first case, $S_s$ at shockless entry is 10,900 giving $Q_{SR}$ = 890 gpm.

Observations of recirculation in practice are not precise. The position of the pressure tap, or depth of immersion of velocity probe below the pipe wall, strongly affect what is recorded as recirculation onset. Since tiny areas of recirculation may exist immediately ahead of the impeller at nearly every flow condition, care must be exercised in identifying recirculation. The present calculations can be expected to agree with measured data within about 10 to 20 percent.

*Allowable Margin below Recirculation Onset*

The minimum flow to be specified will be less than or equal to the recirculation onset point. The allowable margin will be determined by the factors shown in Equation (1). These factors and their determination are enumerated below.

*Power Density*: The forces can be expected to scale as $N^2$ $D^4$ with N being speed scaling factor and D size scale. Since $N^2 D^4 \cong N \times ND^3 \cong N \times Q$, the factor $k_1$ can be expected to be a function of flow and speed. Based on this, an empirical representation of $k_1$ is shown in Figure 11.

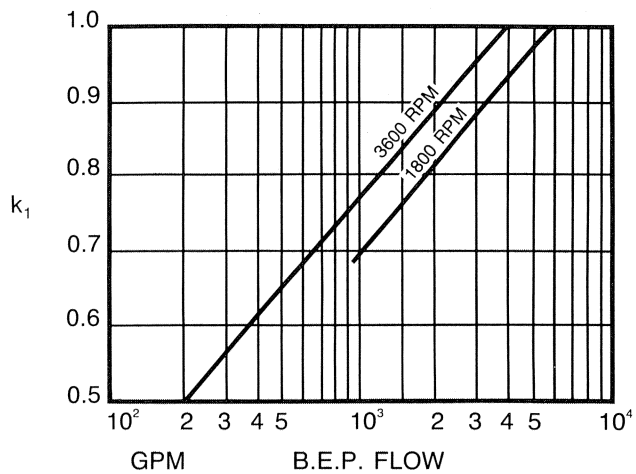


*Figure 11. Empirical Factor to Account for Size and Speed Effects.*

*Specific Gravity*: Since forces can be expected to increase directly with specific gravity of the pumped liquid, $k_2$ is represented as equal to specific gravity.

*NPSH*: Occurrence of cavitation increases the magnitude of unsteady forces. Even when available NPSH is significantly greater than NPSH based on three percent head drop, cavitation bubbles are present. Therefore, the factor $k_3$ can be expected to be a function of the ratio R = NPSH available/NPSH for three percent head drop. If R is much greater than 1.0, reduction in minimum flow due to this effect may be allowed. Further, the cavitation effect is more pronounced for end-suction pumps as the region in front of the impeller and close to the centerline becomes filled with vapor cavities which may grow and collapse, causing significant unsteady forces. The variation of $k_3$ with R allowing for smaller margin below recirculation onset for end suction pumps is shown in Figure 12.

*Intermittence of Operation*: As the anticipated life is affected by whether the pump operates continuously or intermittently at minimum flow, the factor $k_4$ is used.

$k_4$ = 1.0 for continuous operation
= 0.7 for intermittent operation (i.e., less than about 25% of total operating time).

*Mechanical Design*: The quality of mechanical design has a strong influence on minimum flow. Shaft deflections, bearing load factors, choice of mechanical seal and the suitability of seal environment, etc. affect minimum flow. Unfortunately, these are hard to quantify. Generally, for a medium size pump, if the ratio $L^3/D^4$ (where L = overhang length and D is shaft diameter at the seal) is below 100, $k_5$ can be taken to be less than 1.0. Simi-

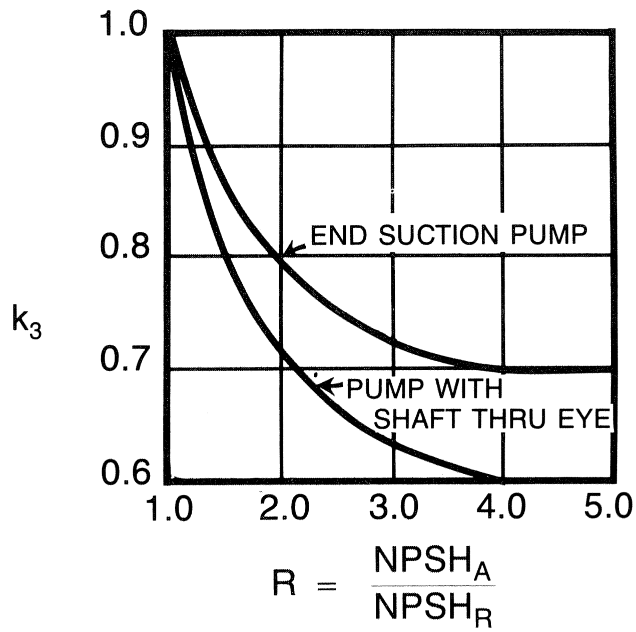


*Figure 12. Empirical Factor to Account for Margin of NPSH Available Over NPSH Required.*

larly, bearing service factors in excess of 1.5 and shaft deflections at the seal less than about two mils may lead to $k_5$ being taken as less than 1.0.

*Example*

The minimum flow computation is now made for the process pump example used in the recirculation onset calculation for an assumed set of operating conditions as below:

$$Q = 1800 \text{ gpm yields } k_1 = 0.8 \text{ from Figure 11.}$$

Pumped fluid is a hydrocarbon of specific gravity 0.8 giving $k_2 = 0.8$. Available NPSH is 1.5 times the required NPSH. Then $k_3 = 0.87$ from Figure 12.

Continuous operation at minimum flow implies $k_4 = 1.0$. For conservatism $k_5$ is taken as 1.0. Then

$$Q_{min} = 880 \times 0.8 \times 0.8 \times 0.87 = 490 \text{ gpm.}$$

Heald and Palgrave [5] provide an empirical method for calculating minimum flow. Using the known suction specific speed of 11,100, and curve B of Figure 12 [5], results in getting the minimum flow to BEP ratio of 0.36. Further, with $R = 1.5$ for hydrocarbon, $k_m$ from Figure 13 [5] is found to be 0.74. Thus, Q min = 1800 × 0.74 × 0.36 = 480 gpm. This result is in close agreement with calculation using present method.

For the example given in by Heald and Palgrave [5], the present method gives close results. For $S_s = 14{,}000$, $Q_{SR}/Q_{BEP} = 0.68$ for $\gamma = 0.4$ from Figure 8. $Q_{SR} = 1600 \times 0.68 = 1088$ gpm. $k_1 = 0.85$ for 1600 gpm pump at 3600 rpm from Figure 11. $k_2 = 0.85$ and $k_3 = 0.87$ from Figure 12. Thus, $Q_{min} = 1088 \times 0.85 \times 0.85 \times 0.87 = 680$ gpm which is very close to the value previously calculated by Heald and Palgrave [5].

## MINIMUM FLOW FOR IMPELLER LIFE

Cavitation damage may restrict impeller life in marginal NPSH cases. The method of specifying minimum flow for such applications involves determination of the required NPSH for long life. Analytical methods for such calculations do not exist. However, for approximate specifications, the method of Gopalakrishnan [6] may be applicable. Here, NPSH required to avoid damage is calculated as a function of flow. The flow rate at which the required NPSH exceeds the available NPSH can then be specified as the minimum flow at which long impeller life can be expected.

## CONCLUSIONS

In the majority of industrial pump applications, a strategy for specifying the minimum flow can be based upon obtaining satisfactory bearing and seal life. The method presented herein involves calculating the recirculation onset point first and then applying a margin below this point.

A method is provided for calculating the recirculation onset points. For suction recirculation, the method is a slight variation of Fraser's report [3]. Since this technique requires some detailed information about the hydraulic design, a simpler method requiring knowledge of suction specific speed only is also given.

The allowable margins depend upon size, speed, specific gravity of pumped liquid, available NPSH, intermittence of operation and mechanical design. Empirical charts are provided to compute these factors.

It must be pointed out that in spite of the directness of the proposed method, allowances must be made in practice to account for specific pump designs and applications.

## NOMENCLATURE

| | |
|---|---|
| $a_1, a_2$ | coefficients used in NPSH calculations |
| b | impeller width |
| $\overline{C}$ | non-dimentional recirculation onset velocity |
| $c_m$ | meridional velocity |
| $D_1$ | eye diameter |
| $D_H$ | hub diameter |
| $F_r$ | radial force |
| g | acceleration due to gravity |
| H | pump head |
| $k_1-k_5$ | coefficients for calculating minimum flow |
| K | radial force coefficient |
| N | rpm |
| Q | flow |
| R | NPSH available/NPSH required for 3% head drop |
| $S_s$ | suction specific speed |
| U | peripheral velocity at impeller eye |
| W | velocity relative to impeller |
| ß | vane angle |
| $\gamma$ | ratio $D_H/D_1$ |
| $\rho$ | density |
| $\omega$ | angular velocity |

*subscripts*

| | |
|---|---|
| 1 | impeller inlet |
| 2 | impeller exit |
| BEP | best efficiency point |
| DR | discharge recirculation |
| min | minimum |
| SE | shockless entry |
| SR | suction recirculation |
| R | recirculation (either suction or discharge whichever is larger) |

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## ACKNOWLEDGEMENT

The author is indebted to the management of BW/IP International, Inc., for permission to present this paper.