a ratio

$$\frac{M_p}{M_{np}} = \frac{\tau_p}{\tau_{np}} = \frac{0.5\tau_y(1 - \tau_{np} / \tau_y)SF_y}{ß_f SF_f \tau_y} \leq 0.16 \qquad (18)$$

yields. Hence, if the pulsating torque amplitude in a shaft section is not higher than about 16 percent of the rated nonpulsating torque, no danger of a fatigue failure exists, provided the shaft has been properly sized and designed with respect to its notch radii, and a shaft material withstanding any possible corrosion attack has been selected.

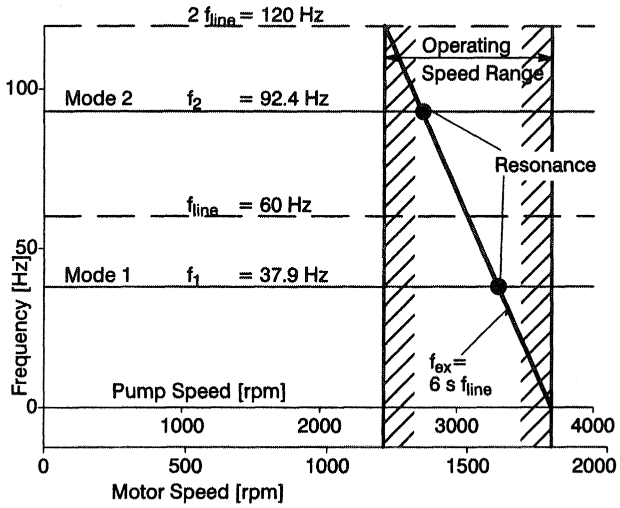

Figure 18. Calculation of Torque Response at Resonance.

Referring to the numbers shown in Table 3, the shaft train section between gear and pump can be considered safe with respect to high cycle fatigue, without a detailed assessment of each stress raising element, when impeller damping is taken into account; the shaft train section between motor and gear however has required a careful evaluation of all peak stresses to assure a safe running at the resonant speeds for extended time periods.

At startup, the shaft train is also subject to a transient dynamic excitation. When the motor is switched to the line, a transient torque which is described by the following Equation develops in the airgap of the motor:

$$M(t) = M_0(1\text{-}e^{-t/T1}e^{-t/T2}) + M_1e^{-t/T1}\sin(2\pi ft\text{-}\alpha) - M_1e^{-t/T2}\sin(2\pi ft+\alpha) \quad (19)$$

A graph of this function for the case investigated is shown in Figure 19. Excitation frequency is equal to line frequency which is 60 Hz. The transient dynamic torques developing between motor and gear and between gear and pump are shown in Figures 20 and 21. The maximum torque values have to be compared with the nominal (rated) torque transmitted which is 15124 Nm, between motor and gear and 7367 Nm between gear and pump. Thus,

- Between motor and gear M_{peak}/M_{rated} = 7572/15124 = 0.50
- Between gear and pump M_{peak}/M_{rated} = 2320/7367 = 0.31

A shaft train has to be designed related to nominal torque with a safety factor of three against gross yielding as the very minimum. Hence, the above peak torques can be taken by the shaft train without any danger of gross plastification or distortion. Another question is the number of starting transients allowable without any danger of low cycle fatigue failures at locations with high peak stresses.

For the transient excitation at startup, rotor damping in the case investigated has little influence on the dynamic torque along the train, because the excitation frequency is far from any torsional natural frequency. However, if resonance occurred, the amount of damping available would again be the determining parameter influencing the rotor response for a given excitation.

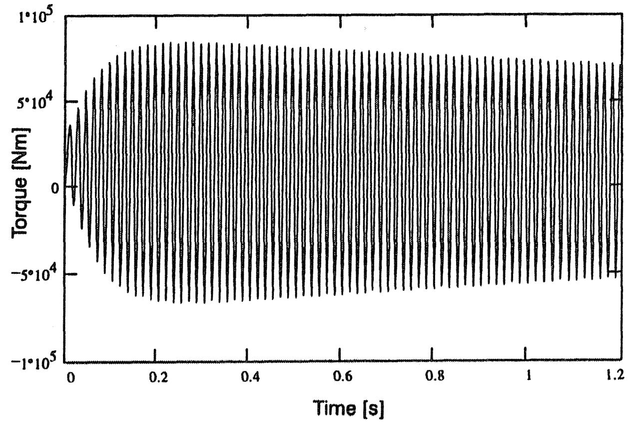

Figure 19. Airgap Torque during Motor Startup.

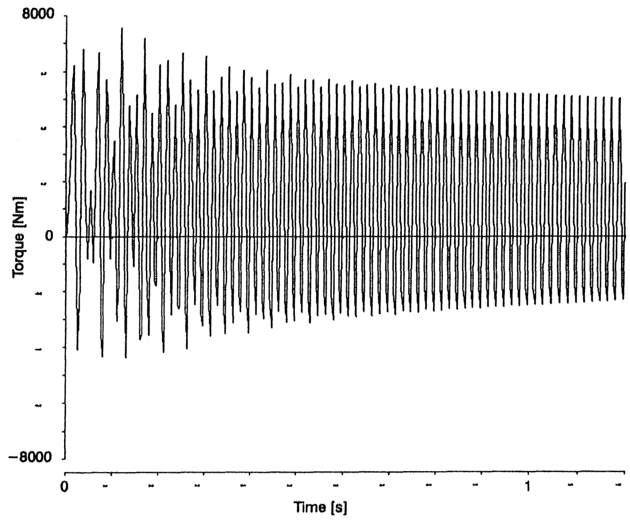

Figure 20. Torque between Motor and Gear.

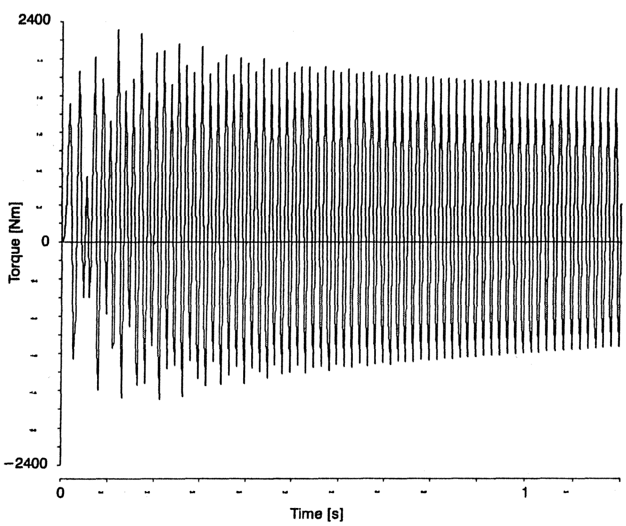

Figure 21. Torque between Gear and Pump.

Transient torque amplitudes in the airgap of the motor at startup or during electrical faults like phase-to-phase terminal short circuit are very high when compared with the nominal torque. The ratio peak torque to rated torque ranges from four to values beyond ten. Excitation frequencies are stator feed (line) frequency for startup and short circuit, and additionally twice stator feed frequency for short circuit, as shown in Table 1. Whenever possible, torsional natural frequencies should be separated from the above two excitation frequencies by at least 20 percent. If this is not possible, which is the case, e.g., for synchronous thyristor controlled variable speed motors, where the stator feed frequency is proportional to speed and, hence, variable [2], the behavior at resonance has to be carefully investigated.

Also, for constant speed asynchronous motors directly switched to the line, the behavior at startup should be investigated, even in a nonresonant case, if the mass moment of inertia of the driven components (gear, pump) reduced to the driver shaft, Thompson [8], is more than about 30 percent of that of the motor.

CONCLUSIONS

Based on the findings presented in this paper the following conclusions can be drawn:

• The interaction between impeller vanes and fluid pumped on a torsionally vibrating pump rotor results in an added mass moment of inertia and in torsional damping.

• Measurements carried out on a test rig have shown that added mass moment of inertia as well as damping depend strongly on the vibration frequency.

• A reasonably conservative approximation for the torsional damping term near best efficiency flow and for a frequency range from zero up to about four times running speed frequency is given by the quasi steady damping term as defined in section three of this paper.

• An approximation for the added mass term can be given as a percentage of the mass moment of inertia of the fluid contained in the channels between the impeller vanes.

• The damping from the impellers can considerably reduce the amplification factor at resonance such that a continuous run at a resonance between a torsional natural mode and an excitation by a variable speed electric motor is possible. However, every case has to be investigated and judged individually.

• During startup and at an electrical fault condition like a phase-to-phase short circuit, high transient torque pulsations develop in the airgap of electric motors. The frequencies of these transient torque pulsations are located at one and two times stator feed frequency. Resonance with these frequencies shall be avoided whenever possible.

• Further experimental and theoretical work needs to be done to clarify the dynamic torsional behavior of an impeller with respect to load and vibration frequency.

NOMENCLATURE

AF		Amplification factor
C		Constant
D	(Nms)	Damping coefficient
e		Basis for natural logarithm
f	(Hz)	Frequency
H		Impedance function
i		Imaginary unit
M	(Nm)	Torque
n	(rpm)	Shaft speed
n_{syn}	"	Synchronous shaft speed
P	(W)	Power
SF		Safety factor
t	(s)	Time
T	(s)	Time constant
α	(rad)	Phase angle
β		Fatigue notch factor
ε	(s^{-2})	Angular acceleration
φ	(rad)	Rotational angle
Θ	(kgm^2)	Mass moment of inertia
τ	(N/m^2)	Shear stress
ω	(s^{-1})	Shaft angular speed
Ω	(s^{-1})	Vibration angular frequency
ζ		Critical damping ratio

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