ROTOR DESIGN AND VIBRATION RESPONSE

by

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ABSTRACT

A study is presented of a simple shaft configuration. This study shows the relative effect of variables of weight distribution and support stiffness on the response of the rotor. In particular, these variables are related to alternating load at the bearings due to an unbalance in the rotor. Comparisons are drawn between the study results and construction details in order that qualitative judgments may be made.

INTRODUCTION

The smoothness of operation of a rotor is of interest to all people concerned with the design, purchase or use of the rotor. The values of critical speeds, or the speed ranges in which critical speeds are prohibited are commonly specified in machinery purchase orders. It is the purpose of this paper to present performance data on a simple shaft which will lead to better fundamental understanding of the various factors affecting rotor performance and their relative value or influence.

MID SPAN WEIGHT

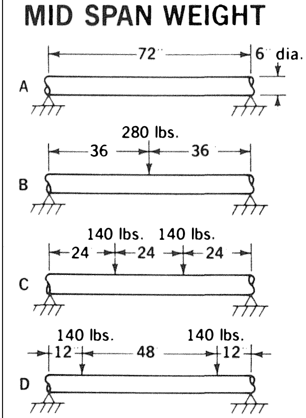
Figure 1 illustrates the effect of weight distribution in the mid span of the rotor. All critical speed values are for a 72" long by 6" diameter shaft on very stiff supports having a spring gradient of 10^9 pounds per inch. The results for the uniform shaft, Figure 1-a, gives a value of the ratio of second to first critical of 4.0. This is the true value of this ratio for any size uniform shaft on rigid supports. However, it can be radically different for various weight distributions within the rotor span.

A single wheel located at the shaft mid span, Figure 1-b, will have maximum effect in lowering the first critical speed, but will have minimal effect on the second critical speed since the wheel is located at the shaft node for the second critical. The ratio of second to first critical is 5.62, for the 280 pound weight which has been used in this example. The change in speed ratio from 4.0 to 5.62 is due to reduction in first critical alone since there was no change in second critical. This simple case assumed a dimensionless point weight for the wheel at mid span. It is apparent from this example that by changing the amount of weight at center span it is theoretically possible to make the ratios of second to first critical any desired value greater than 4.0.

Two equal weights placed at one-third span, Figure 1-c, have approximately the same effect on both first and second criticals so that the ratio of second to first critical remains approximately constant at 4.0 regardless of the amount of weight which is placed equally at these two locations.

The last example, Figure 1-d, illustrates the effect of locating equal weights at one quarter span. This is

MID SPAN WEIGHT



	Critical Speeds		
	1st RPM	2nd RPM	Ratio 2nd/1st
A	5,516	22,058	4.00
B	3,921	22,057	5.62
C	4,194	16,670	3.98
D	4,943	16,677	3.38

Figure 1. Effect of Mid Span Weight Location on Critical Speed.

the anti-node location for the shaft second critical and weight placed at this location has maximum effect in reducing the second critical speed relative to the reduction in first critical speed. Equal weights at this location give the smallest ratio of second to first critical.

In the practical design of most rotors, both the weight and location of the wheels on the shaft are determined by other considerations than critical speed placement. The examples which have been given are to demonstrate the types of changes in critical speed relationships that can be produced by variation in weight size and location between the two bearing supports.

OVERHUNG WEIGHT

Whereas the mid span weight may either raise or lower the ratio of second to first critical, the effect of the overhung weight is only to reduce this ratio. Figure 2 illustrates the effect of a 40 pound weight overhung various amounts on the same shaft and supports used in Figure 1. With increased overhang, there is significant reduction in first critical speed, such that with a 24″ overhang the ratio of second to first critical has dropped from 4.0 to a value of only 2.27. A large part of this effect is due not only to the 40 pound weight, but also to the very substantial overhung weight of the shaft extension itself. Inertial or gyroscopic effects of the wheel have not been included in this calculation. This can have a significant effect to raise the critical speeds if the inertia of the overhung weight is large compared to the shaft stiffness. It is apparent that overhung weights can be a major factor in depressing the second critical speed. In the design of high speed machinery the amount, location and possible unbalance of the overhung weight are important considerations in the design of the rotor.

SHAFT MODE FORMS

Figures 3 and 4 show the critical speeds and mode forms for the simple uniform shaft and for the uniform shaft with a 40 pound weight on a 24″ long overhang. This information is given not only for a very stiff support having a spring constant, K, equal to 10^9 pounds per inch, but also for a very soft support having K equal to 10^5 pounds per inch.

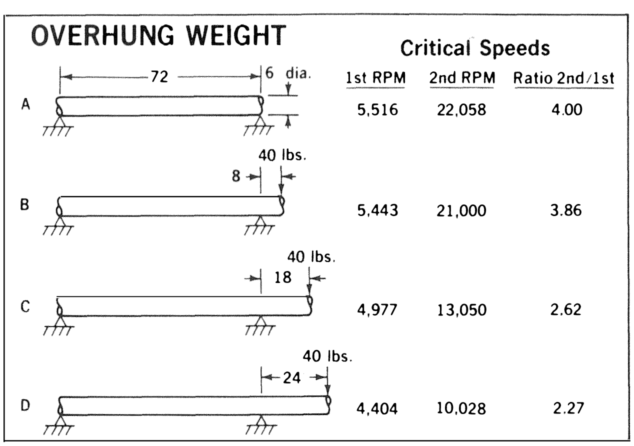


Figure 2. Effect of Overhung Weight on Critical Speed.

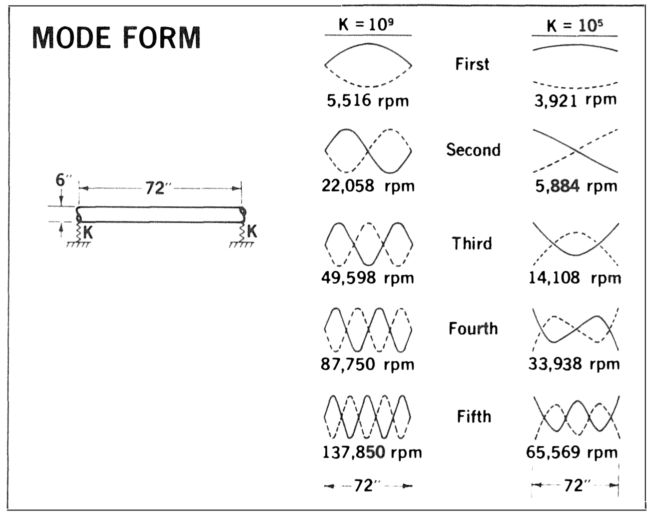


Figure 3. Mode Shapes for Simple Uniform Shaft for Both Stiff and Soft Supports.

In comparing Figure 3 and 4, it should be noted that the fifth critical of the overhung shaft, Figure 4, very closely resembles the fourth critical of the simple shaft, Figure 3, both in mode form and in value of critical speed. In fact, this same relation holds true for all the higher mode forms involving substantial bending in the shaft. At the higher mode forms, the extra length of the overhang begins to participate in the same general manner that it would if it had been included between the bearing supports.

This relationship becomes less obvious with decreasing critical speed where the shaft has less bending. The two lowest criticals, for both the simple shaft and for the shaft with the 24″ overhung weight, have minimum bending curvature in the shaft. In particular for softer supports, the shaft at the first two criticals tends to act as a rigid body with very little bending in the shaft; that is, the value of the critical speeds approach that of

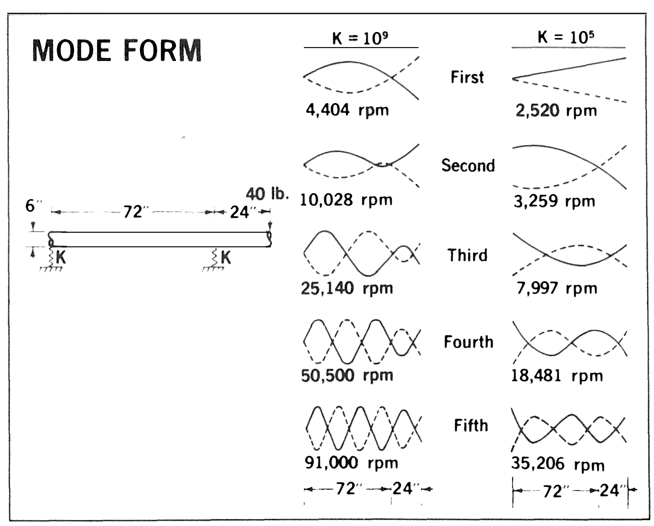


Figure 4. Mode Shapes for Shaft with Overhung Weight for Both Stiff and Soft Supports.

a simple mass on supporting springs. It could, therefore, be expected that the value of these two lower criticals would be especially susceptible to support stiffness. Likewise, it can be expected that changing shaft stiffness will have minimum effect on the first two critical speeds if the support structure is very soft.

SUPPORT STIFFNESS—SPRING GRADIENT

Figure 5 shows the relation of critical speeds to support stiffness for the simple uniform shaft. This curve illustrates the fact deduced from the mode shapes; namely, that the first two criticals are much more strongly influenced by support stiffness than are the higher criticals which involve greater bending curvature in the shaft. The support stiffness is expressed as an equivalent spring with a spring gradient of K pounds/inch, at each bearing location. In the case of an actual rotor, the equivalent support stiffness, K, is influenced by both bearing oil film stiffness and the stiffness of the supporting structure.

Typical values of spring gradient of the oil film of a pivoted shoe journal bearing are given in Figure 6. The oil film spring gradient is a function of details of bearing design, speed, oil viscosity and bearing loading. For other variables held constant, which is usually the case in machinery design, the spring gradient of the oil film increases markedly as the loading on the bearing is increased from 50 psi to 250 psi on bearing projected area. Approximately a 5 to 1 increase in bearing oil film stiffness is obtained for a given bearing design simply by increasing the bearing loading from 50 psi to 150 psi. A further increase in loading to 250 psi would gain about 2 to 1 more in oil film stiffness. Most high speed machinery design is faced with the problem of trying to increase, rather than decrease, rotor critical speeds. The curve in Figure 6 indicates that maximum values of oil film stiffness and, therefore, of rotor critical speeds are obtainable at the higher bearing loadings.

The stiffness of supporting structures is usually obtainable only by complex analysis or by direct measurement. However, in order to give an order of magnitude feel for the meaning of the spring gradients which may

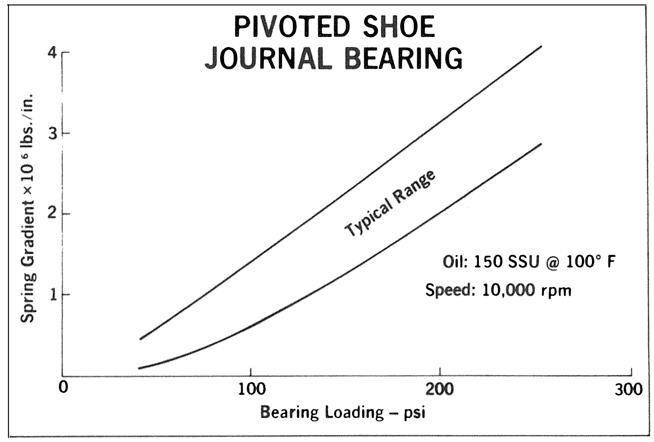


Figure 6. Typical Spring Gradients for a Pivoted Shoe Journal Bearing at 10,000 RPM.

be used in rotor response calculations, simple examples are given in Figures 7 and 8. Figure 7 shows spring gradients for 4 different diameter solid steel cylinders resting on infinitely stiff supports. It is apparent from the 30" high solid steel cylinders in Figure 7 that a spring gradient as high as $K = 10^9$ lb./in. or as low as $K = 10^5$ lb./in. are not likely to be found in practical construction. Figure 8 indicates that a 36" x 12" x 10 foot long wide flange beam weighing 194 pounds per foot and resting on an infinitely stiff foundation would have a spring gradient of only approximately 10^7 lb./in. An 18" x 8¾" wide flange would have a spring gradient of 10^6 lb./in. and a 10" x 5¾" wide flange would have a gradient of only 10^5 lb./in. These numbers are indicative of the problem of designing steel supporting structures to give good operating machinery. They also indicate one reason why the machinery manufacturer always expresses a preference for a reinforced concrete foundation. These examples, of course, greatly oversimplify the real life problem of structures in order to illustrate the general significance of spring gradient numbers.

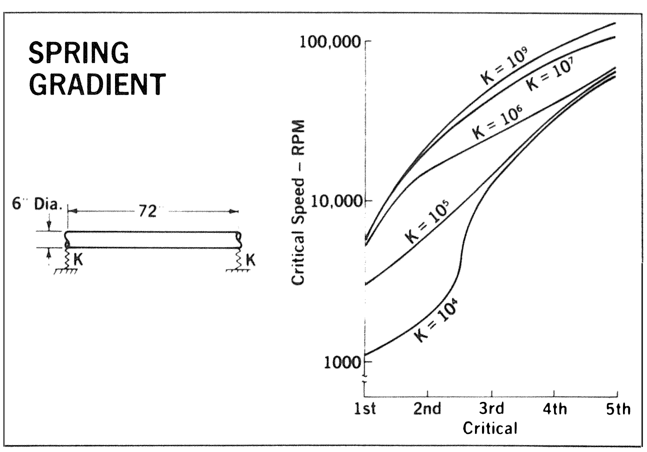


Figure 5. Effect of Support Spring Gradient on Critical Speeds.

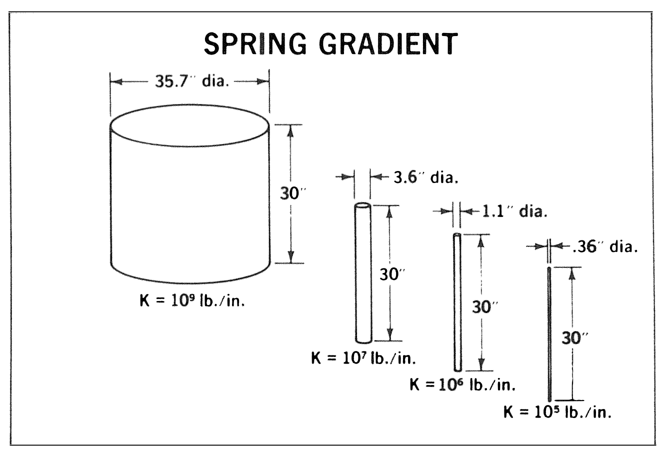


Figure 7. Spring Gradients for 30" Tall Solid Cylinders of Steel on an Infinitely Stiff Support.

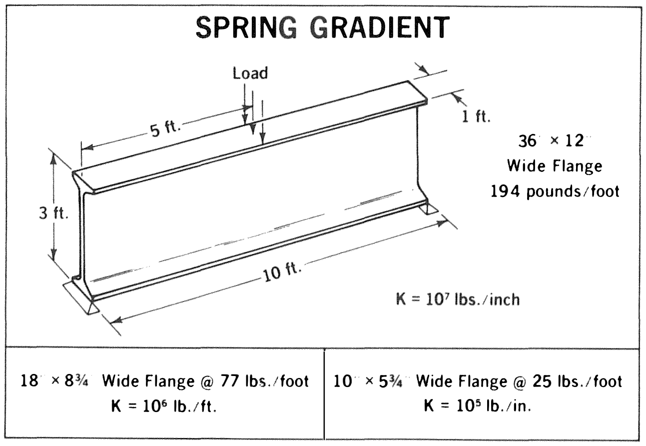


Figure 8. Spring Gradients for 10 Foot Long Wide Flange Beams on an Infinitely Stiff Support.

ROTOR RESPONSE

The latest design tool for rotors is the analytical ability to predict the response of a rotating rotor to the influence of an unbalance. Figure 9 shows, for a simple uniform rotor, a plot against speed of alternating force at the bearing nearest the unbalance. In this example, the unbalance is placed at the quarter point between the supports in order to effectively excite the first and second criticals and somewhat less effectively excite higher criticals. The alternating bearing loading is shown for three different values of support stiffness, K. All cases are for the same damping, C = 170 lb.-sec./in. This damping value is considerably lower than will be found in normal design, and was chosen so that the unbalance effect at the resonant speeds would be amplified more than normal for this study.

This plot, showing alternating load on the bearing, is indicative not only of the duty imposed on the bearing but it is also an indirect measure of the forces which may be transmitted to other parts of the machine and its supporting structure. The plot also contains the information for vibration amplitude, which is a more common form of shafting performance evaluation. Bearing loading is equal to the vectorial sum of damping force, plus spring force. For a first approximation it is sufficient to consider only the spring force. In which case, bearing loading is approximately equal to the product of vibration amplitude and spring gradient. It is, therefore, possible to have an evaluation of vibration amplitude from the curves which give bearing loading. For example, at the first critical speed of Figure 9, the shaft vibration amplitude with the softest spring support is almost 5 times as much as with the stiffest spring support even though the alternating bearing loadings are about the same for either of the spring gradients.

A dotted line is shown on Figure 9 to represent the centrifugal force due to the unbalance. The degree of sensitivity to unbalance of a given design can be evaluated not only in terms of the bearing loading but also in terms of amplification of the loading. For example, with the 0.7 x 10^6 spring gradient the amplification at the second critical is only 1.26, which is very low value. With the 2.0 x 10^6 spring gradient the amplification is 6.8, indicating a somewhat sensitive rotor when operating at the resonant speed. The amplification at first critical is about 10 which indicates an even greater response to unbalance. These relatively high amplifications are to be expected, however, because of the low value of damping used for these calculations.

A similar type example is given in Figure 10 for the simple shaft with the 40 pound weight on the 24″ long overhang. In this case the same unbalance, as used in the previous case, is assumed to be at the end of the shaft overhang. The bearing loadings and particularly the amplification is worse at the second critical speeds than in the previous case. This demonstrates in a different manner the importance of the overhung weight, not only in its effect on the location of the second and higher criticals, but also it indicates the additional vibration amplitude that may be generated by alternating forces at this location, not only by unbalance but also by other alternating forces such as those that might be due to coupling misalignment.

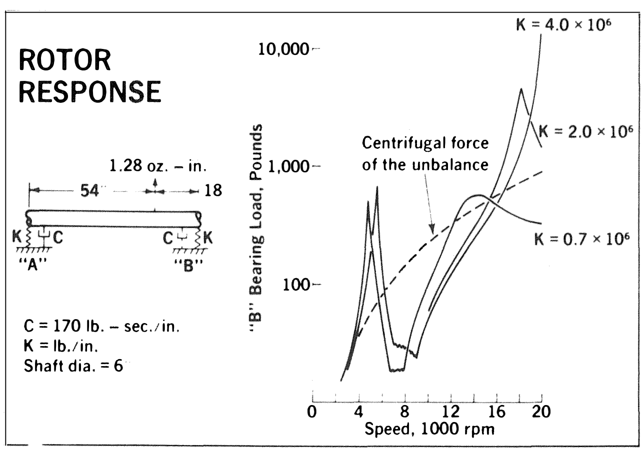


Figure 9. Unbalance Response of a Uniform Simple Shaft Excited at Quarter Point.

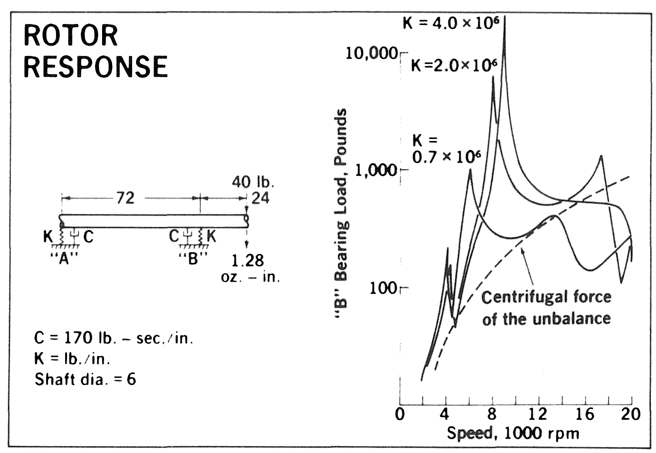


Figure 10. Unbalance Response of a Shaft with Overhung Weight Excited at the Overhang.

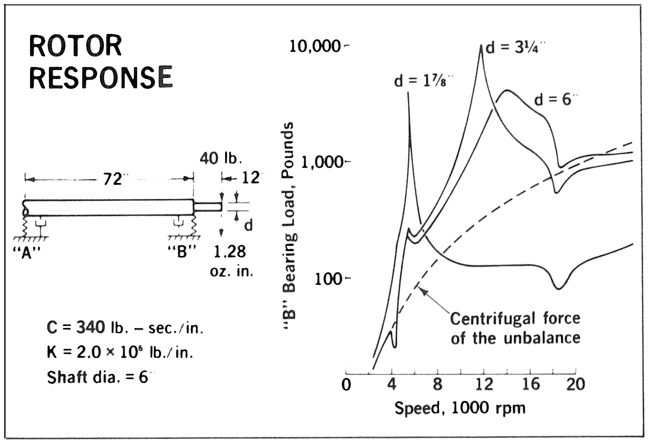


Figure 11. Unbalance Response of a Shaft for Various Diameter Overhangs.

The previous examples have shown a general increase in rotor response with speed. This is not surprising considering that the centrifugal force due to the unbalance increases as the square of the speed. The examination of actual rotor performance does not always indicate a greater rotor response at increased speed. The curves shown on Figure 11 have been drawn to illustrate one reason why this is so. The overhung shaft for this example has been reduced in length to 12", and design variation is accomplished by changing shaft overhang diameter. The factors of spring constant and damping coefficient are held constant. The same unbalance is used as in previous cases and it is at the end of the overhang. The smallest sized shaft, 1-7/8" diameter, was selected to tune the natural frequency of the cantilevered overhang to the first natural frequency of the shaft. It will be seen that this design shows a major peak at the first critical with below normal response at higher speeds. The intermediate size shaft, 3¼" diameter, was selected to tune the overhang to the second critical of the system. This design shows an increased sensitivity and response at the second critical and only normal response at the first critical. The larger 6" diameter shaft is the same as has been used in previous examples and is included as a base for comparison purposes. Since there is little damping in the rotor material itself, and all damping is assumed to be in the bearing oil film, the system responses become sharper and more pronounced as the hypothetical natural frequency of the cantilevered overhang becomes more closely tuned to the over-all system natural frequencies. This is one reason why relatively small changes in overhang distance or weight in some cases cause major changes in rotor response.

DAMPING

Damping in oil lubricated bearings is an important factor in the consideration of rotor behavior. Increased damping tends to increase the critical speed of the rotor. This is usually a small factor. Increased damping also tends to decrease the amplitude of vibration, and, therefore, the alternating bearing loading. In systems where the amplification is large, the response of the system at resonance will vary approximately inversely with damping. For example, the peak bearing loading given in Figure 9 for K = 2.0 x 10^6 pounds/inch would reduce from 4900 pounds to 2400 pounds if the bearing damping were increased from 170 pound-second/inch to 340 pound-second/inch. Under some conditions the damping will tend to eliminate the vibration peak which is normally associated with a critical speed. This is what happens to the rotor response for K = 0.7 x 10^6 in Figure 9 when the bearing damping is increased from 170 pound-second/inch to 340 pound-second/inch. The peak response at the second critical is eliminated with this increase in damping.

Values of bearing damping may be expected to be from about 300 pound-second/inch to 800 pound-second/inch with the usual bearing designs and oil viscosities common to high speed rotating equipment. The values of damping used in this study have purposefully been made lower than would be expected in actual practice in order to emphasize the effect of other variables in the design.

TESTING

Testing is required to correlate analysis with experience. Although correlation of the performance of actual rotors with predictions is the most meaningful type of information, the testing of models has a useful function. The arrangement in Figure 12 shows a steam turbine driving a high speed model test rotor through a step up gear. Instrumentation is placed at five locations along the length of the shaft and simultaneous recordings can be made of the shaft performance as speed or other system variables are changed.

The speed-vibration traces shown in Figure 13 are one type of useful information which can be obtained from this type of testing. Phase angles along the shaft may also be obtained and the shaft mode forms may be determined to compare with prediction. This type of testing is most useful in comparing differences of performance for differences in design detail of the rotor system, such as different bearing designs.

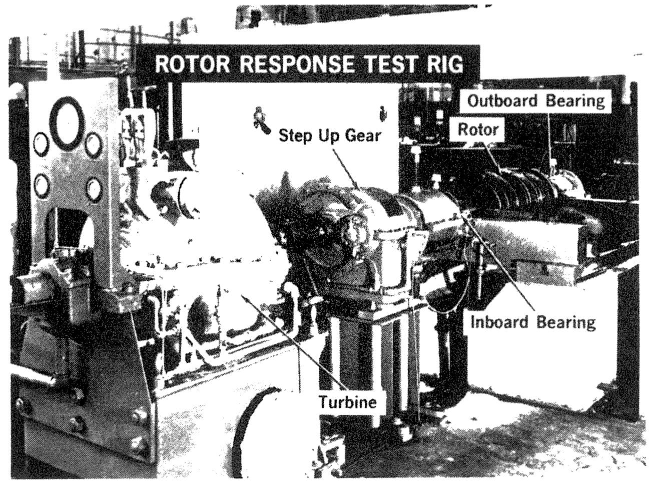


Figure 12. Model Rotor Testing for Dynamic Response to Unbalance.

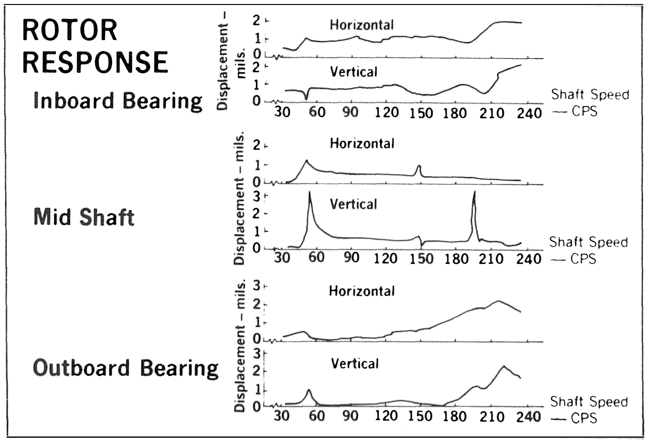


Figure 13. Rotor Vibration Response Due to Unbalance.

It is expected that continued work in the field of rotor dynamics, both analytically and experimentally, will bring increased accuracy to the prediction of rotor vibration response. The entire field of support structure response, including the effect of both structure mass and stiffness, is largely in the formative stage. Most information available on this subject is a matter of experience rather loosely correlated through over-simplified models. There is a need for continued work in this field.

SUMMARY

The vibration response of a rotor is dependent not only on the design variables of the rotor itself but also on the equivalent stiffness and damping of the rotor support structure. The shaft extension beyond the bearings and the overhang weight on this extension is an important design factor in the determination of the vibration response of high speed rotors.

Bearing alternating load is a useful measure of rotor response. It gives a measure not only of the forces transmitted to other parts of the machinery and foundation, but also indicates the degree of sensitivity of the rotor to unbalance.

It is expected that continued work in the field of rotor dynamics, both analytically and experimentally, will bring increased accuracy to the prediction of rotor vibration response. The entire field of support structure response, including the effect of both structure mass and stiffness, is largely in the formative stage. Most information available on this subject is a matter of experience rather loosely correlated through oversimplified models. There is a need for continued work in this field.

ACKNOWLEDGEMENT

The author gratefully acknowledges the help of Mr. R. P. Kolb and Mr. N. Mizne of De Laval Research and Advanced Product Development Department, in the construction of this paper.