LESS COSTLY TURBOEQUIPMENT UPRATES THROUGH OPTIMIZED COUPLING SELECTION

by
Heinz P. Bloch
Senior Staff Engineer
Exxon Chemical Company
Florham Park, New Jersey

Heinz P. Bloch received his M.S. degree in Mechanical Engineering from Newark College of Engineering. As Senior Staff Engineer in the Technology Department he resolves complex machinery operating problems in Exxon Chemical Company's worldwide affiliates, selects machinery for major or unusual applications and develops maintenance programs to improve profitability.

Mr. Bloch has made contributions in the area of design consulting, trouble-shooting, field project engineering, start-ups and non-process quality control with assignments in the U. S., Canada, Europe, Latin America and the Far East. He is a registered professional engineer and holds patents in the field of high-speed machinery.

ABSTRACT

Compressor and driver shafts often prove to be overstressed in equipment uprate situations. This paper shows how a change from conventional gear-type couplings to the more recent diaphragm coupling design can lower the shaft stresses sufficient to avoid shaft replacement during power uprates of centrifugal compressors or compressor drivers.

Equipment uprate projects involving turbomachinery often *appear* to require time consuming and costly shaft replacements. However, closer examination of how the equipment vendor arrived at his maximum allowable stress levels may frequently show that such shaft replacements can be avoided without undue risk if the coupling selection is optimized. This conclusion is based on the fact that gear-type coupling have the potential of inducing in a shaft both torsional stresses *and* bending stresses, whereas diaphragm couplings tend to primarily induce torsional stresses and insignificant bending stresses at best.

The economic incentives of finding ways of salvaging major rotating equipment shafts illustrated on a steam turbine shaft originally rated to transmit 17,600 hp maximum at 6,400 rpm. If uprating the turbine to 19,600 hp were being considered, the required changeout of stationary steam path components would cost around $60,000, but a combined replacement cost of about $500,000 would probably be quoted for the main and spare rotor shafts. A rigorous calculation of shaft stresses shows the shaft factor of safety to be greater at 19,600 hp using a diaphragm coupling than at 17,600 hp using a conventional gear coupling.

MAXIMUM SHAFT STRESS CAN BE CALCULATED

Efforts to determine whether or not rotating equipment power uprates require shaft replacements should be preceded by shaft stress calculations. The coupling end of the steam turbine shaft used in our example would typically have the dimensions shown in Figure 1.

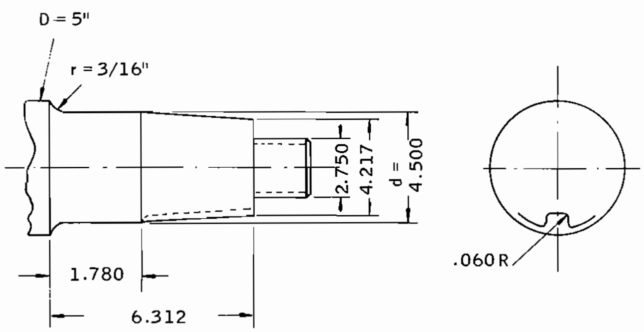

Figure 1. Shaft Dimension

Assuming the shaft material to be ASTM A-293 Class A steel, heat treatment and stabilization at 1,000°F would result in the following properties:

Ultimate strength in tension,	σ_{ut} =	105,000 psi
Ultimate strength in shear,	τ_{ut} =	60,600 psi
Endurance limit in shear,	τ_E =	30,300 psi
Minimum yield strength in tension,	σ_{yp} =	80,000 psi
Minimum yield strength in shear,	τ_{yp} =	40,000 psi
Endurance limit in tension,	σ_E =	52,500 psi

Most of these properties are used in a Soderberg diagram (Ref. 1) similar to Figure 2.

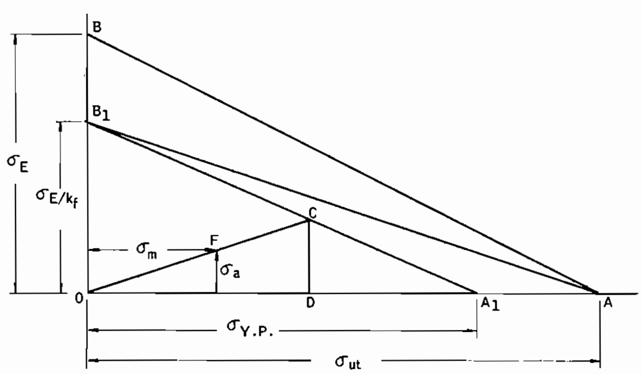

Figure 2. SODERBERG Diagram

In addition to some of the nomenclature given earlier for the shaft properties, the diagram uses σ_m (= steady tensile stress component), σ_a (= alternating tensile stress component), and k_f, the stress concentration factor for the particular keyway dimensions shown in Figure 1. The stress concentration factor for our sample keyway is 2.9 (Ref. 2).

The line B_1A_1 is considered to define the limiting stress condition for a specimen with stress concentrations. If the steady stress on the specimen is given by the absicca OD, then the limiting amplitude of the alternating stress is given by the ordinate DC, and point C will represent the limiting stress condition. The corresponding safe condition will be represented by point F with coordinates σ_m and σ_a (or τ_m and τ_a). These coordinates are obtained by dividing the coordinates of point C by the factor of safety n. From the similarity of triangles we have

$$OB_1A_1 \triangleq DCA_1, \therefore \frac{\overline{CD}}{\sigma_E/k_f} = \frac{\overline{DA}}{\sigma_{y.p.}} = \frac{\sigma_{y.p.} - \overline{OD}}{\sigma_{y.p.}},$$

$$\text{or} \quad \frac{\overline{CD}}{\sigma_E/k_f} + \frac{\overline{OD}}{\sigma_{y.p.}} = 1$$

Dividing this equation by n, we obtain for the safe stress condition (Point F)

$$\frac{\sigma_a}{\sigma_E/k_f} + \frac{\sigma_m}{\sigma_{y.p.}} = \frac{1}{n}$$

$$n = \frac{1}{k_f \frac{\sigma_a}{\sigma_E} + \frac{\sigma_m}{\sigma_{y.p.}}}$$

Eq. (1)

This is the expression for the factor of safety in the case of UNIAXIAL STRESSES.

In the case of combined stresses, the equivalent stresses for σ_a and σ_m should be substituted in the previous equation and Ref. 1 shows how the equation for TORSION reduces to equation (2). Using $k_{f'}$ instead of k_f

$$n = \frac{1}{\sqrt{3}\left(k_{f'} \frac{\tau_a}{\sigma_E} + \frac{\tau_m}{\sigma_{y.p.}}\right)}$$

Eq. (2)

For COMBINED BENDING AND TORSION, the factor of safety is obtained from

$$n = \frac{1}{\sqrt{\left(k_f \frac{\sigma_a}{\sigma_E} + \frac{\sigma_m}{\sigma_{y.p.}}\right)^2 + 3\left(k_{f'} \frac{\tau_a}{\sigma_E} + \frac{\tau_m}{\sigma_{y.p.}}\right)^2}}$$

Eq. (3)

If bending acts alone, τ_a and τ_m vanish and the expression for the factor of safety in the case of unilateral stresses results. When σ_a and σ_m vanish, we have the equation for n in torsion.

TORSIONAL STRESSES CAN BE READILY CALCULATED

For the desired uprate conditions given earlier, we calculate shaft torque output

$$T = \frac{(63{,}000)(19{,}600 \text{ HP})}{6{,}400 \text{ RPM}} = 193{,}000 \text{ lb-inches}$$

The steady torsional stress is, therefore,

$$\tau_m = \frac{16T}{\pi d^3} = \frac{(16)(193{,}000)}{\pi (4.5)^3} = 10{,}900 \text{ psi.}$$

Making the generally accepted assumption that the alternating torsional stress will not exceed 20% of the stead stress, we obtain

$$\tau_a = (0.2)(10{,}900) = 2{,}180 \text{ psi.}$$

BENDING MOMENTS ASSESSED FOR GEAR COUPLING

There are three relevant bending moments* caused by a gear coupling when transmitting torque with angular or parallel misalignment:

- Moment caused by shift of contact point. This moment acts in the plane of angular misalignment and tends to straighten the coupling. It can be expressed as

$$M_c = \frac{T}{D_p/2}\left(\frac{X}{2}\right)$$

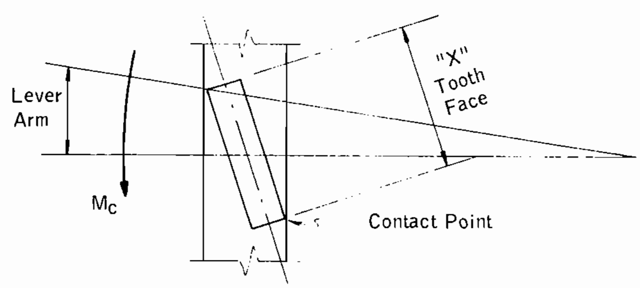

Figure 3. Shift in Contact Point

Where T is the shaft torque, D_p is the gear coupling pitch diameter and X is the length of tooth face (see Figure 3, above).

- Moment caused by coupling friction. This moment acts in a plane at right angle relative to the angular misalignment. It has the magnitude

$$M_f = T\mu$$

where μ is the coefficient of friction.

- Moment caused by turning torque through a misalignment angle α. It acts in the same direction as the friction moment M_F and can be expressed as

$$M_T = T \sin\alpha$$

*An additional moment, due to overhung weight of coupling, is usually negligibly small and is omitted from the bending moment analysis for both gear and diaphragm couplings.

The total moment is the vector sum of the individual moments;

$$M_{TOTAL} = \sqrt{M_c^2 + (M_F + M_T)^2}$$

A gear coupling suitable to transmit 19,600 HP at 6,400 RPM is assumed to have a pitch diameter $D_p = 9$ inches and a face width of 1.3 inches. Using a coefficient of friction $\mu = 0.3$ and a misalignment angle $\alpha = 0.0057°$ (approximately 0.001 in/in parallel offset), we calculate

$$M_c = \frac{193{,}000}{4.5} \times \frac{1.3}{2} = 27{,}878 \text{ \#-in}$$

$$M_F = (193{,}000)(0.3) = 57{,}900 \text{ \#-in}$$

$$M_T = (193{,}000)(\sin 0.057) = 193 \text{ \#-in}$$

$$M_{TOTAL} = \sqrt{(27{,}878)^2 + (57{,}900 + 193)^2} = 64{,}435 \text{ \#-in}$$

This moment is fixed relative to the actual angular misalignment α and is seen by the shaft as a rotating bending moment at a once-per-revolution frequency.

BENDING MOMENTS CALCULATED FOR CONTOURED DIAPHRAGM COUPLING

The contoured diaphragm coupling causes two bending moments:

- Moment caused by angular misalignment which results in bending the diaphragm,

  $$M_B = k_B \alpha$$

  In this expression, k_B equals the angular spring rate of the diaphragm (lb-in/degree) and α is the misalignment angle. This moment acts in the plane of angular misalignment, as did M_c in the gear coupling analysis.

- Moment caused by turning torque through a misalignment angle α. It can be expressed as

  $$M_T = T \sin \alpha$$

  The total moment is now

  $$M_{TOTAL} = \sqrt{M_B^2 + M_T^2}$$

A suitable contoured diaphragm coupling has a diaphragm diameter of 16.5 inches and an angular spring rate $k_B = 18{,}800$ lb-in/degree. Thus, for the misalignment angle used earlier,

$$M_B = (18{,}000)(0.057) = 1{,}080 \text{ \#-in}$$

$$M_T = (193{,}000)(\sin 0.057) = 193 \text{ \#-in}$$

$$M_{TOTAL} = \sqrt{(1{,}080)^2 + (193)^2} = 1{,}095 \text{ \#-in}$$

ALTERNATING STRESSES COMPARED

Comparing the bending moments caused by gear couplings with those resulting from contoured diaphragm couplings shows the former to be significant and the latter virtually negligible in comparison.

The cyclic bending stress imposed on a gear coupling-equipped shaft can be computed from

$$\sigma_a = \frac{M_{TOTAL} \times C}{I},$$

where C and I are the shaft radius and shaft area moments of inertia, respectively. Thus

$$\sigma_a = \frac{(64{,}435)(2.25)}{\pi d^4/64}$$

$$= \frac{(64{,}435)(2.5)(64)}{\pi(4.5)^4} = 7{,}206 \text{ psi.}$$

In addition, there is a mean tensile stress acting on the shaft cross-sectional area. This means stress equates to

$$\sigma_m = \frac{T\mu}{(D_p/2)(\pi C^2)\cos\theta}$$

$$= \frac{(193{,}000)(0.3)}{(4.5)\,\pi\,(2.25)^2\,(0.94)} = 860 \text{ psi.}$$

where $\theta = 20°$, the pressure angle assumed for the gear teeth.

The cyclic bending stress seen by the diaphragm coupling — equipped shaft can be obtained by a rapid ratio calculation:

$$\frac{\sigma_a \text{ (Diaphragm Coupling)}}{\sigma_a \text{ (Gear Coupling)}} = \frac{M_{TOTAL} \text{ (Diaphragm Coupling)}}{M_{TOTAL} \text{ (Gear Coupling)}}$$

$$\sigma_a \text{ (Diaphragm Coupling)} = \frac{(1{,}095)(7{,}026)}{64{,}435} = 122 \text{ psi.}$$

The mean tensile stress acting on the cross-sectional area of a diaphragm coupling-equipped shaft depends on how far the diaphragm is displaced axially from its neutral rest position and the axial spring rate of the diaphragm. Assuming the diaphragm of this sample case were displaced by its maximum permissible distance of 0.100 inches, it would exert a force of 1,950 lbs on the shaft cross-sectional area (Ref. 4). This would cause a mean stress

$$\sigma_m = \frac{1{,}950\#}{\pi C^2} = \frac{1{,}950}{\pi(2.25)^2} = 125 \text{ psi.}$$

SHAFT FACTOR OF SAFETY CAN NOW BE EVALUATED

Before actually calculating the shaft factors of safety for torque transmission with either coupling type, one must determine the stress concentration factors k_f and $k_{f'}$. Using values of r/D , D/d and R/d from Figure 1, Reference 3 gives stress concentration factors $k_f = 1.95$ and Reference 2 gives $k_{f'} = 2.9$. The stress concentration factor $k_{f'}$ results from the keyway and must be used in torsional stress calculations. Factor k_f takes into account the shaft step going from 4.5" to 5.0" diameters. It must be used in the bending stress calculation.

Since the gear coupling-equipped shaft is loaded in torsion *and* (potentially) bending, we employ the expression for combined torsion and bending and obtain the factor of safety (from equation 3):

$$n = \frac{1}{\sqrt{\left(k_f \frac{\sigma_a}{\sigma_E} + \frac{\sigma_m}{\sigma_{y.p.}}\right)^2 + 3\left(k_{f'} \frac{\tau_a}{\sigma_E} + \frac{\tau_m}{\sigma_{y.p.}}\right)^2}}$$

$$= \frac{1}{\sqrt{\left(1.95 \frac{7{,}206}{52{,}500} + \frac{860}{80{,}000}\right)^2 + 3\left(2.9 \frac{2{,}180}{52{,}500} + \frac{10{,}900}{80{,}000}\right)^2}}$$

$$= 1.90$$

Using the same mathematical expression to calculate the factor of safety for the shaft equipped with the contoured diaphragm coupling, we obtain

$$n = \frac{1}{\sqrt{\left(1.95 \frac{122}{52{,}500} + \frac{125}{80{,}000}\right)^2 + 3\left(2.9 \frac{2{,}180}{52{,}500} + \frac{10{,}900}{80{,}000}\right)^2}}$$

$$= 2.25$$

The value $\left(1.95 \frac{122}{52{,}500} + \frac{125}{80{,}000}\right)^2$ in the above expression is so small that application of the equation for pure torsion (equation 2) would have given the same factor of safety, n = 2.25.

SHAFT STILL ADEQUATE UNDER UPRATE HORSEPOWER CONDITIONS

At this point, we may want to recall that the above analysis was made to determine if the steam turbine shaft would require replacement should the output horsepower be increased beyond the manufacturer's maximum permissible rating of 17,600 HP. Rather than engaging in a debate as to safe maximum absolute values of stress in turbine shafts, we merely investigate what factor of safety the shaft design embodied while equipped with a suitable gear coupling transmitting 17,600 HP at 6,400 RPM:

$$\text{Shaft torque } T = \frac{(63{,}000)(17{,}600)}{6{,}400} = 173{,}250 \text{ \#-in}$$

$$\text{Steady torsional stress } \tau_m = \frac{16T}{\pi d^3} = \frac{(16)(173{,}250)}{\pi(4.5)^3}$$

$$= 9{,}680 \text{ psi.}$$

$$\text{Alternating tors. stress } \tau_a = (0.2)(9{,}680) = 1{,}940 \text{ psi.}$$

$$M_C = \frac{173{,}250}{4.5}\left(\frac{1.3}{2}\right) = 25{,}025 \text{ \#-in}$$

$$M_F = (173{,}250)(0.3) = 52{,}000 \text{ \#-in}$$

$$M_T = (173{,}250)(\sin 0.057) = 173 \text{ \#-in}$$

$$M_{TOTAL} = \sqrt{(25{,}025)^2 + (52{,}000 + 173)^2}$$

$$= 57{,}864 \text{ \#-in}$$

$$\sigma_a = \frac{(57{,}864)(2.25)\,64}{(\pi)(4.5)^4} = 6{,}472 \text{ psi.}$$

$$\sigma_m = \frac{(173{,}250)(0.3)}{(4.5)\,\pi\,(2.25)^2\,0.94} = 770 \text{ psi}$$

$$n = \frac{1}{\sqrt{\left(1.95 \frac{6{,}472}{52{,}500} + \frac{770}{80{,}000}\right)^2 + 3\left(2.9 \frac{1{,}940}{52{,}500} + \frac{9{,}680}{80{,}000}\right)^2}}$$

$$= 2.14$$

The shaft factor of safety, while certainly adequate for operation at 17,600 HP with gear-type couplings, is thus shown to be only 2.14 in the original design case with conventional gear couplings. Equipping the same shaft with contoured diaphragm couplings makes operation at 19,600 HP not only possible, but will actually increase the factor of safety to 2.25.

WHICH CONTOURED DIAPHRAGM COUPLING?

Several manufacturers have both the capability and experience to provide diaphragm couplings for critical petrochemical machinery. In reviewing this experience, machinery engineers may want to compare the combined stresses acting in diaphragms of operating couplings with the stresses that would act in the diaphragms of couplings being considered, since this, in essence, establishes prior experience at a given stress level. Proper installation reviews and an understanding of axial shaft growth of coupled machines should serve as verification that the manufacturer's maximum permissible axial diaphragm deflection will not be exceeded. Non-contacting proximity probes can be adapted for continuous monitoring of diaphragm displacement and have been installed on process compressor trains where the owner wanted to be sure beyond any doubt (Ref. 5).

REFERENCES

1. Timoshenko, F. P., "Strength of Materials — Advanced Theory and Problems," Van Nostrandt-Reinhold Publishing, N.Y. 3rd Edition, 1956.

2. Peterson, R. E., "Stress Concentration Design Factors," John Wiley and Sons, Inc., New York, 1953.

3. Wilson, C. E., Jr. "Mechanisms — Design Oriented Kinematics." American Technical Society, Chicago, Ill., 1969.

4. Bendix Fluid Power Corporation, Utica, N.Y. Technical Bulletin, "Contoured Diaphragm Couplings," 1973.

5. Bently-Nevada Company, Minden, Nevada. Technical Bulletin on Proximitor Alignment Systems, 1974.