

# PULSATIONS IN LIQUID PUMPS AND PIPING SYSTEMS

by

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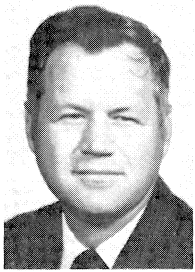
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## ABSTRACT

The existence of high intensity, low frequency pulsation problems in centrifugal compressor and pump piping systems has now been well documented throughout industry. Strong pulsations have been observed at frequencies ranging from two to several hundred Hz which typically are not harmonically related to (and do not vary with) pump rotor speed. Investigation has revealed that in most cases, the problem is not caused simply by pump or compressor characteristics, but instead is the result of dynamic interaction of the passive response of the piping, the head curve characteristics of the pump, dynamic flow damping, and location of the pump in the piping geometry.

Since pulsation problems are usually vibration fatigue problems of the piping or machine internals, problems can often be mitigated by effective vibration control. A more fundamental and often more economical approach, however, lies in controlling the pulsation levels and frequencies, either by controlling the generation sources or pulsation response of the piping. Unfortunately, these two phenomena cannot be effectively separated, and the problem must be solved with a more comprehensive system dynamic analysis. It has been shown, for example, that relative low level vortex formation at piping discontinuities can be amplified by the pump or compressor, but such amplification must be at one of the responsive acoustic length resonant modes of the piping system, and then only if the pump or compressor is situated at or near a velocity maximum in the standing wave field. Additionally, however, the compressor itself may influence the acoustic response (distributed impedance) of the piping as well as serve as an amplifying element. Flow or pressure drop is also an important factor in sustaining such pulsations as it affects modal damping.

Techniques have now been developed whereby many of the phenomena associated with low frequency pulsations in centrifugal pump and compressor systems can be effectively simulated either for design or problem solving. The paper will briefly describe the phenomena associated with low frequency pump pulsations and will illustrate use of the technology in several case histories.

## INTRODUCTION

It is well recognized in industry that a majority of piping vibration problems are induced by internal fluid pulsations. There is sometimes a tendency to attribute these pulsations solely to the action of pumps or compressors within the piping systems, however, and to direct remedial steps either toward modifying the pump internals or providing more effective pipe support. While each of these approaches are valid and useful, an alternate or complementary approach lies in modifying the transient flow (pulsation) response of the piping. When pulsation frequencies from a reciprocating pump, for example, coincide with a mechanical vibration resonance of the piping, the process of mismatching the driving force from the normal vibration response modes is quite effective. In addition, however, pulsation resonant frequencies within the piping can also amplify pulsation effects to the point that mechanical fatigue failures occur in the piping or pump components. Thus, successful treatment of resulting vibration problems can often be achieved merely by mismatching these pulsation resonances from major pump frequencies, and by providing acoustic filters to attenuate pulsation amplitudes over a wide range of frequencies. The acoustic filter approach is particularly applicable

to variable speed pump installations where operational flexibility dictates operation over a range of speeds and pulsation frequencies.

Similar pulsation problems exist with centrifugal pumps, but influence of the piping is often even more important in defining both pulsation frequencies and amplitudes. Pulsation problems with reciprocating pumps can now be adequately dealt with, and reliable techniques for predicting and solving such problems have been previously described in the technical literature (1,2). Therefore, this paper will deal chiefly with centrifugal pump pulsations and liquid piping systems.

To date, the various piping codes for plant design have not been able to deal effectively with piping pulsation and vibration problems largely because too little technology has been readily available for recognizing and avoiding such problems at the design stage. The only available approaches have been to include simplified dynamic criteria and to "beef-up" materials properties and static design parameters in the hope that dynamic problems would be minimized in the process. Very often, just the opposite effect has been achieved. Requirements for thicker pipe walls, stiffer branch connection fittings, and stringent flexibility requirements often intensify vibration problems.

By far the most difficult problem in analytically describing plant dynamics problems lies in predicting driving forces. In piping systems, for example, the problem has centered not around predicting the mechanical resonant frequencies and mode shapes of the complex, three-dimensional piping systems, but rather around predicting what the internal flow or pulsation forces are which excite the vibrations. These pulsation driving forces, as inferred earlier, are a function of both the pulsation response of the piping system and the source strength and frequencies of the pulsation generating mechanisms.

## PIPING SYSTEM PULSATION RESPONSE

### *Piping Acoustics*

A piping system employing various discrete lengths, diameters, branches, constrictions, etc., constitutes a complex flow impedance network incorporating dynamic transfer characteristics which will amplify some pulsation frequencies and attenuate others. In this action, the piping characteristics are almost directly analogous to electrical circuitry or delay lines, wherein the inertial (inductance) and stiffness (capacitance) characteristics are quite linear, but the resistance is non-linear with flow. In flowing piping systems, the acoustic resistance is a direct function of the  $\rho V^2$  pipe frictional losses. These far overshadow conventional acoustic dissipation effects such as molecular relaxations for the range of flows normally encountered in industrial piping systems. This distributed impedance network, therefore, can amplify low level pulsation up to as much as a factor of 100 (i.e., acoustic Q's up to 100 or so), depending on frequency, pipe size, flow, termination impedance, etc.

Since the reactive flow impedances (distributed inertia and stiffness) of a piping system are linear, conventional acoustic theory can be used to define resonant frequencies, mode shapes and filtering characteristics even for very high flow velocities. The non-linear effects of damping only affect the amplitude or sharpness of the resonance peaks. Using elementary physics, resonances of individual piping segments can be very simply described from "organ pipe" resonance theory. The standing wave (pressure) mode shapes for various piping

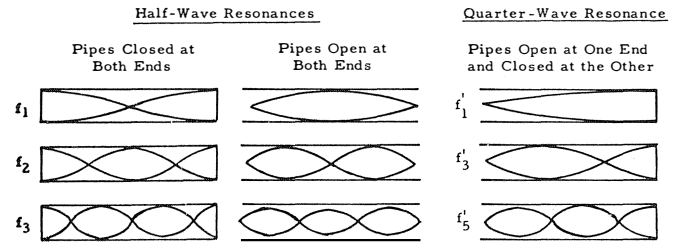


Figure 1. Organ Pipe Resonant Mode Shapes (standing pressure waves) for Various Pipe Configurations.

resonances are shown in Figure 1. It will be recalled that such resonance modes are characterized by:

- pressure maxima at closed ends
- velocity minima at closed ends
- pressure minima at open ends
- velocity maxima at open ends.

Since the basic equation defining wave length ( $\lambda$ ), pipe length ( $L$ ) and acoustic velocity ( $a$ ) is  $a = f\lambda$ , resonant frequencies for the various modes of the three configurations can be conveniently calculated from the following equations:

*Half-Wave Resonance* (Pipe closed at both ends or open at both ends)

$$f = N \frac{a}{2L} \quad \text{where } N = 1, 2, 3, \dots = \text{harmonic} \quad (1)$$

*Quarter-Wave Resonances* (Pipe open at one end and closed at the other)

$$f = (2N - 1) \frac{a}{4L} \quad \text{where } N = 1, 2, 3, \dots \quad (2)$$

When piping components are coupled together, more complex resonances can exist (i.e., of several interacting segments) in addition to the pure component resonances, and one component can be situated to detune a resonance within another (e.g., a quarter-wave stub or side branch Helmholtz resonator). These complex system interaction characteristics can best be analyzed by acoustic network analyses, as for example, delay line simulation techniques.

### *Velocity of Sound in Liquid Piping*

Another important consideration in liquid system analysis is the fact that acoustic velocity in the pipe (i.e., the velocity at which a pressure wave propagates) can be significantly affected by pipe wall stiffness. The velocity of sound through a fluid in an infinite medium or a rigid pipe is given by the expression:

$$c = 12 \sqrt{\frac{gE}{\rho}} \quad (3)$$

where

$c$  = velocity of sound of the medium, ft/sec

$E$  = bulk modulus of fluid, lb/in<sup>2</sup>

$\rho$  = density of the fluid, lb/ft<sup>3</sup>

$g$  = acceleration due to gravity, ft/sec<sup>2</sup>

Now consider that the fluid container or pipe is elastic. Considering both the elasticity of fluid and pipe, the above equation may be rewritten

$$a = 12 \sqrt{\frac{gK}{\rho}} \quad (4)$$

where

$a$  = velocity of sound in fluid in elastic container, ft/sec

$K$  = effective bulk modulus of liquid as modified by pipe expansion, lb/in<sup>2</sup>

It can be shown that the effective bulk modulus is

$$K = \frac{E}{\left(\frac{2RE}{E_1 t} + 1\right)} \quad (5)$$

where

- $E_1$  = elastic modulus of pipe wall, lb/in<sup>2</sup>
- $S$  = stress in the pipe wall induced by excess pressure  $\Delta p$ , psi
- $t$  = pipe wall thickness, in
- $R$  = initial inner radius of pipe, in

Substitution yields the velocity of sound relationship in pipe

$$a = 12 \sqrt{\frac{gE}{\rho}} \sqrt{\frac{1}{\left(1 + \frac{DE}{tE_1}\right)}} \quad (6)$$

where

$D$  = pipe diameter.

This equation shows that the elasticity of pipe wall tends to reduce the velocity of sound from that of a fluid in rigid pipe. For compressible fluids, such as gases, the elasticity of pipe can be neglected. With liquids, the pipe elasticity must be considered, as shown in Figure 2.

### FLOW INSTABILITY IN PUMP SYSTEMS

The acoustic theory described above can be used in flow network analysis to provide piping system designs with controlled and predictable pulsation response or transient transfer characteristics. Many papers have been written describing the application of such techniques to piping system design for reciprocating compressor and pump installations (1,2) and for pipeline transient response (3). Examples of the effectiveness of acoustic filters in controlling and predicting piping pulsations from a triplex charging pump at a nuclear power plant are given in Figures 3, 4 and 5. Figure 3 compares predicted and measured pulsation amplitudes in the piping, while Figure 4 shows pulsation levels "before" and "after" installation of a liquid acoustic low pass filter. Figure 5 shows the piping systems involved.

Similar observations can be made regarding pulsations in centrifugal pumps, and the similar acoustic techniques can be employed. In such systems, the piping often plays an even more predominant role both in amplifying or attenuating pulsations, and in contributing to the basic generation mechanism

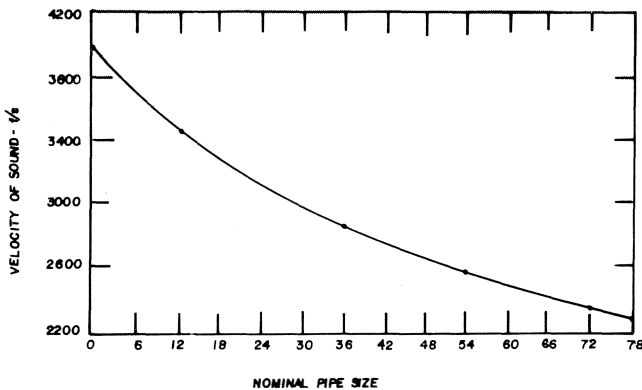


Figure 2. Velocity of Sound in Water (14.6 psia, 60°F) vs Nominal Pipe Size With 0.25 Wall Thickness.

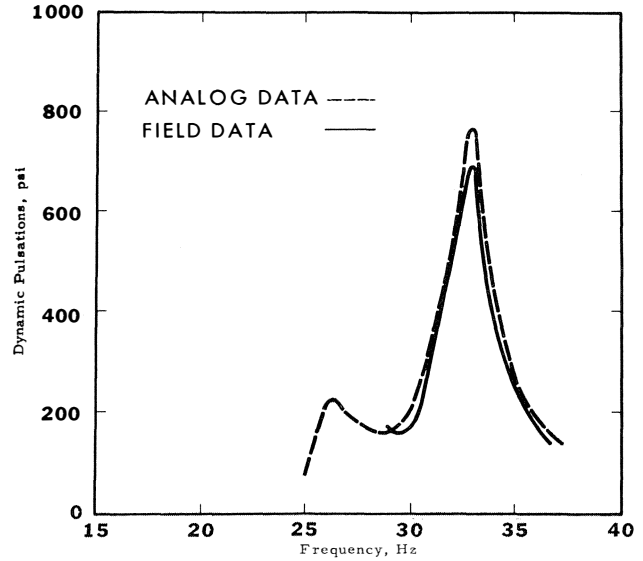


Figure 3. Comparison of Field and Analog Data on Pump Pulsations.

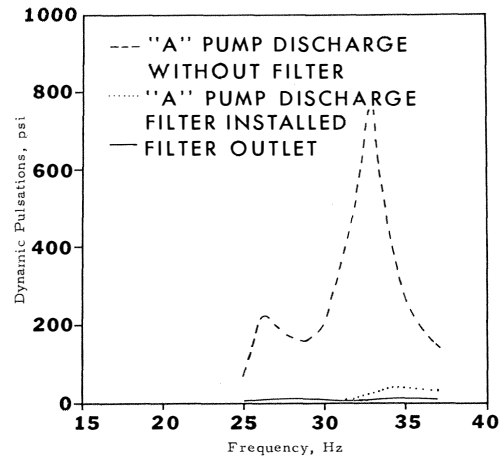


Figure 4. Effect of Acoustic Filter on Pulsations From a Reciprocating Triplex Pump.

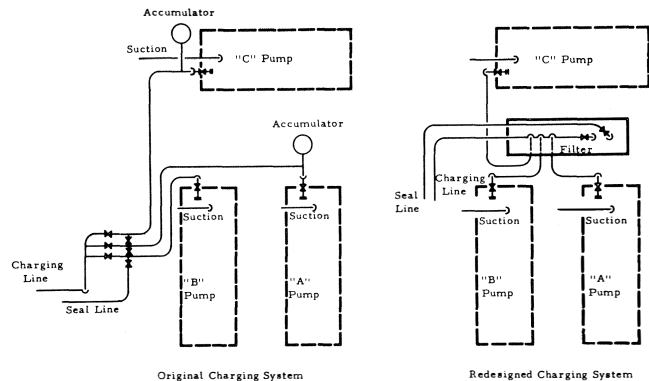


Figure 5. Original and Modified Piping Systems for Reactor Charging Pumps.

involved. In centrifugal pump systems, the mechanisms of pulsation generation are varied, and several situations will be discussed below to illustrate the important dynamic interaction between a pump and its attached piping.

#### *Centrifugal Response to External Pulsations*

One of the more interesting phenomena associated with centrifugal pumps and compressors is their ability to either amplify or attenuate pulsations introduced into the piping from an external source such as a nearby reciprocating machine. Such phenomena have been documented both in the laboratory and in industrial plants for gas compressors and strongly evidenced in a variety of liquid pump problems.

External pulsations, as from a reciprocating pump or compressor, are normally fixed in frequency at the reciprocating pump shaft speed and multiples thereof. Pulsation amplitudes, of course, are strongly influenced by the dynamic acoustic response of the attached piping system, and if a harmonic of pump speed corresponds to a strong normal mode acoustic resonance of the piping system, severe pulsations can result. Normally, when such pulsations are introduced into the suction or discharge of a centrifugal pump, they will be conducted through the impeller, and standing wave pulsation resonances can exist throughout the pump piping system. Further, under certain design conditions, the compressor can serve to amplify the pulsations to a higher level than would occur in a purely passive piping system. The mechanism of pulsation amplification by centrifugal machines is described more fully in the following section, but it should be re-emphasized that this action depends both on the piping system response and the pump characteristics.

#### *Flow Induced Pulsation and Centrifugal Pump Amplification*

Through use of piping acoustic network analysis techniques, solutions to many types of piping pulsation and vibration problems have now become routine, both in the plant design stage and after the plant becomes operational. One type of pulsation that still defies complete quantitative analysis or simulation, however, is that involving flow-induced pulsations. Such problems are typified by flow past an obstruction or piping discontinuity which produces vortex shedding and which, in turn, may be either amplified or attenuated (filtered) by the acoustic properties of the piping network and by interaction of a centrifugal pump with the piping system.

Action of centrifugal machines in amplifying these relatively low level pulsations up to significant and even destructive levels has been well documented in industrial plant systems, and experimental work in the laboratory has served to define many of the phenomena involved. Field data on a broad spectrum of centrifugal machines has shown strong pulsations at frequencies ranging from two to several hundred Hz which typically are not harmonically related to (and do not vary with) rotor speed. Investigation has revealed that in most cases, the problem is not caused simply by pump or compressor characteristics, but instead is the result of dynamic interaction of the passive response of the piping and pump characteristics. The action of the pump or compressor in causing amplification or attenuation of this vortex energy is quite complex, but basically is dependent upon:

1. The head curve slope and operating point.
2. System flow damping (in the piping).
3. The existence of strong reactive resonances in the piping, particularly if they coincide with vortex frequencies.
4. The location of the pump in the standing wave field (i.e., at a velocity maximum rather than a pressure maximum).

Such pulsations can build to extremely high levels and result in a variety of machine and piping problems such as destruction of machine internals, large torsional reactions, initiation of cavitation, large force reactions at valves, bends, etc. Amplitude, of course, is flow dependent and subject to coincidence of various resonances. Thus, pulsations may come and go as conditions change. Detailed testing of mode shape of the pulsation standing waves both in the field and laboratory show that the resonances observed are characteristic of the normal length resonances of the coupled piping components, and that the pulsation initiating source normally involves flow vortex formation at piping discontinuities (junctions, obstructions, etc.). When the frequency of this vortex formation excites one of the acoustic resonances of the piping system, and when the compressor is situated near a velocity maximum in this resonant system, then high amplitude, self-sustaining pulsations can result. Particular resonances can often be destroyed by moving the compressor to a velocity minimum, but often a new pulsation frequency will be generated such that the compressor is again situated near a velocity maximum for this higher mode oscillation. With proper care and detailed analysis of the relative strength of the various pulsation resonance modes of the piping, piping designs can be developed to avoid these strong resonances. Further, by controlling piping stub lengths (e.g., in by-pass piping, etc.) such that their quarter-wave stub resonances are far removed from the preferential von Karman vortex frequencies, the basic generation mechanisms can be substantially suppressed.

The tendencies for instability described above are more prominent at low flows, as one would expect from surge technology. The basic reasons for this is that acoustic damping in a flowing system is predominantly afforded by flow friction effects. Normally, for flow velocities higher than 2 or 3 fps, frictional effects far overshadow conventional acoustic resistance effects, as described previously. Thus, at very low flows, the dynamic system is very lightly damped and very susceptible to resonant buildup from relatively weak oscillation sources. Nevertheless, the pulsations can appear at flows substantially above the conventional surge point of the machine.

To illustrate the concepts described above, consider a simple transfer pump system shown in Figure 6. The piping shown will have various resonance modes, all of which have pressure minima at the reservoir junctions (typical open-end organ type acoustic resonances). All even harmonics of this resonance will also have pressure minima at the center of the span where the pump is situated. From resonance theory, we also know that each pressure minimum point will also exhibit a velocity maximum, and thus, the pump position shown will subject the pump impeller to rather high flow oscillations and

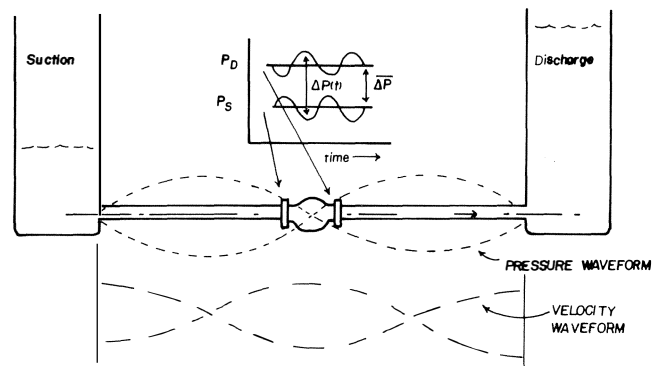


Figure 6. Representation of a Second Mode Piping Resonance and Its Effective Pump Head.

dynamic forces. If, now, external pulsations (either a pump or flow turbulence) at the piping inlet coincide with the fundamental frequency depicted (or an even harmonic), the piping will tend to substantially amplify this pulsation component merely by piping resonance. Further, the standing pressure wave which results from this resonance will cause a fluctuating head imposed across the pump. Pressure waveforms are depicted at various points along the pipe. Note that suction and discharge pulsations are out-of-phase, since the pump is at a pressure node. If, now, this fluctuating head is imposed on the pump, the pump will respond with a further modulation of flow due to its head curve characteristics, as shown in Figure 7. If this flow modulation is in-phase with the conventional dynamic flow component of the pulsations, amplification of the pulsation will result, and pressure (and velocity) amplitudes will build up to the point allowed by system damping.

The above description of effects is an over simplification of the processes involved in pulsation amplification and attenuation, as very decided changes in head curve (pump performance) and flow stability can result from the introduction of external pulsations. Nevertheless, it illustrates the basic phenomena involved.

**CENTRIFUGAL ANALOG**

In pursuit of more definitive and predictable techniques for solving pulsations in centrifugal pumps and compressor systems, a centrifugal analog simulator has recently been evolved. This device, shown in Figure 8, is an electronic model of a centrifugal pump which exhibits both the flow and pulsation response characteristics of the machine. The analog is tuned on the basis of the head curve of the machine to be simulated, and is used in conjunction with delay line sections which simulate the flow response of pipe segments. This unit has now been used successfully in simulating and solving a rather broad spectrum of pulsation problems for both pumps and compressors. The case histories that follow illustrate the use and value of this approach, and illustrate more graphically some of the pulsation phenomena described above.

**CASE HISTORIES**

*Case 1: Coupling Failures Caused by Low Frequency Pulsations*

A centrifugal pump used as a waterflood unit was driven by a reciprocating engine through a 12:1 speed increaser. The speed range of the engine was 360-460 rpm, whereas the pump speed varied from 4340 to 5548 rpm. Numerous coupling failures were experienced on one unit while a similar unit had very little problems. A field investigation of torsional and lateral vibrations, and of system pulsations revealed that a pulsation resonance at 7 Hz was present in the piping of the unit which had been experiencing the coupling failures.

The presence of a low frequency, high amplitude pulsation (35 psi p-p) at 7 Hz indicated the possibility that the acoustical resonance was amplifying torsional vibrations at the first torsional natural frequency near 7 Hz. Torsional vibration measurements made (Figure 9) showed good correlation with the measured pulsation levels in the piping (Figure 10).

The two pumps and their piping systems were simulated using the SGA centrifugal analog module in conjunction with the electroacoustic piping analog to determine if the piping had an acoustic response near the first torsional natural frequency. Results of this analysis are given in Figure 11, which shows a definite resonance near 7 Hz. The other unit had similar characteristics; however, the acoustical response frequency

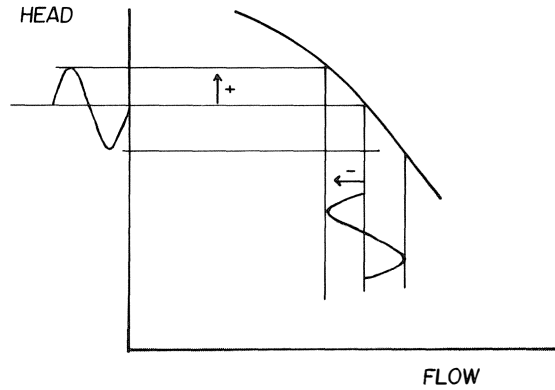


Figure 7. Pump Head Curve Showing the Effect of Head Modulation on Flow.

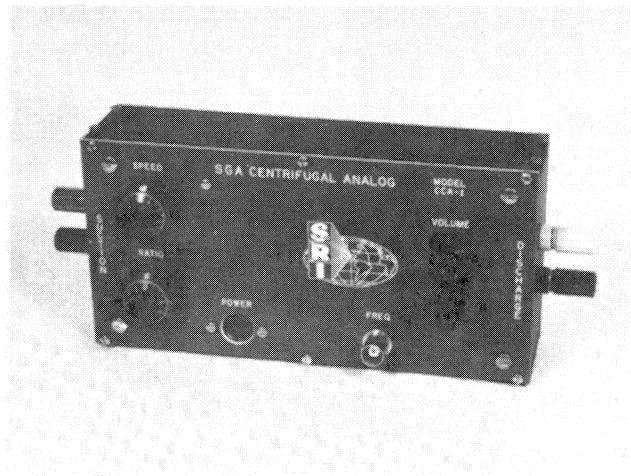


Figure 8. Electrical Analog for Simulating the Transient Flow Properties of a Centrifugal Pump or Compressor.

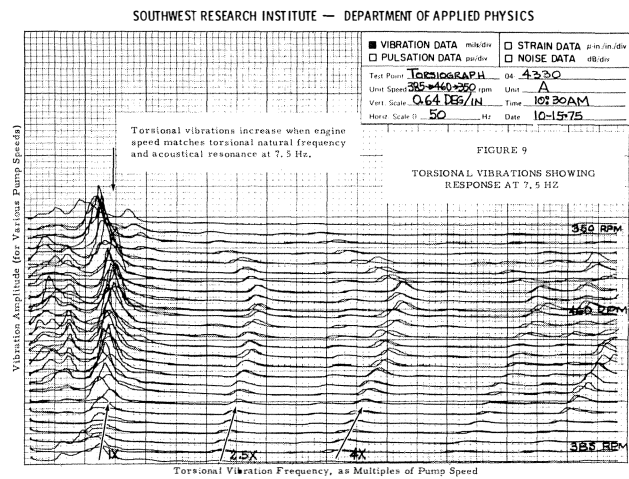


Figure 9. Torsional Vibrations Showing Response at 7.5 Hz.

near 7 Hz was at a slightly lower frequency. Note the location of the torsional natural frequency relative to the acoustic resonance. An acoustical filter to reduce these pulsations was designed and is in process of being installed at the time of writing of this paper.

This problem illustrates that low frequency pulsations can be amplified by piping responses and cause troublesome failures, particularly if a torsional response is near.

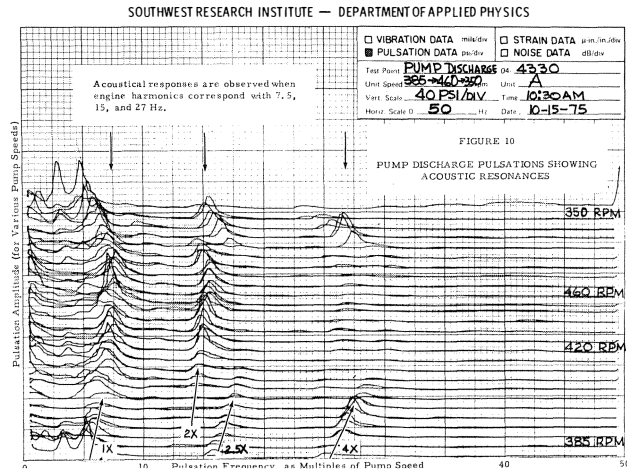


Figure 10. Pump Discharge Pulsations Showing Acoustic Resonances.

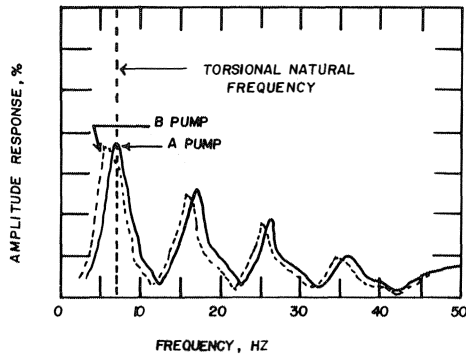


Figure 11. Acoustical Responses of Units A & B Measured on Electroacoustical Analog.

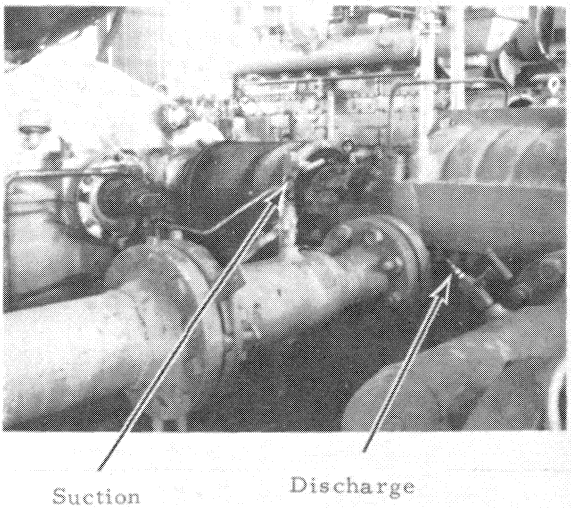


Figure 12. Test Points for Measuring Pump Pulsations.

Case 2: Crankshaft Failures Caused by Liquid Pump Pulsation

A 10-stage centrifugal pump (Figure 12) was driven by an eight cylinder engine at pump speed through a gear box with a gear ratio of 5.625. The engine speed range was 690-800 rpm with the corresponding speeds of 3880-4500 rpm on the pump.

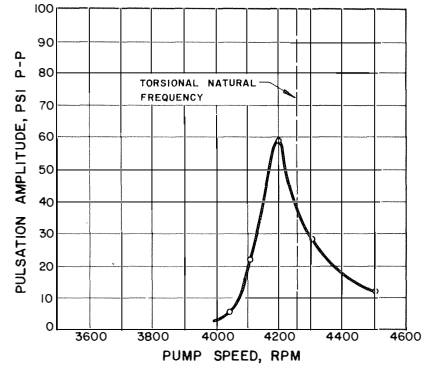


Figure 13. Suction Pulsations at Pump Speed.

There were repeated torsional crankshaft failures in one unit; however, other units consisting of the same engine, gear box, and pump, but with slightly different piping, had no crankshaft failures. Torsional vibrations, lateral vibrations, and pulsations were measured over the speed range of the unit to determine torsional natural frequencies and amplitudes and the effect of the liquid pulsations upon the dynamic stresses. Torsional natural frequencies of 1020 and 4260 cpm were measured which agreed closely with the calculated values of 970 and 4300 cpm. In addition, there was a strong acoustic resonance in the piping at 70 Hz which caused pulsation amplitudes of 60 psi p-p when the pump speed was at 4200 rpm (Figure 13). Amplification of pulsations could have been caused by the torsional natural frequency since increased torsional vibrations could cause increased pulsation amplitudes. In liquid pumps, however, the system damping is usually high enough to attenuate this effect. Measurements of pulsations on the units with no failures revealed that the acoustic pulsation resonance was near 80 Hz, therefore, not coincidental with the torsional resonance.

The mode shape of the second torsional natural frequency indicates that the maximum stress should occur between the seventh and eighth cylinders where the failures had occurred. Torsional vibrations measured on the end of the crankshaft are given in Figure 14. At the maximum vibration conditions (760/4275 rpm), the calculated stresses were in excess of the applicable endurance limit.

Since the acoustic response of the piping was amplifying the torsional vibrations, an acoustic filter was designed to eliminate the resonance. A filter bottle designed on the SGA electroacoustical analog was installed, and pulsation levels were reduced to less than 10 psi p-p. This modification greatly reduced the torsional vibrations caused by pump speed pulsations and eliminated crankshaft failure.

Case 3: Failure of Pump Internals

A four stage centrifugal pump suffered repeated failures of the splitter between the pump stages. A detailed field study revealed that the cause of the problems was an acoustic resonance of the long crossover which connected the second stage discharge with the third stage suction (Figure 15). The resonant frequency was a half-wave acoustical resonance.

$$f = \frac{A}{2L} \tag{7}$$

where

A = speed of sound, ft/sec

L = length, ft.

The speed of sound in water is a function of the temperature and at 310°F, the speed of sound was 4770 ft/sec. The length of

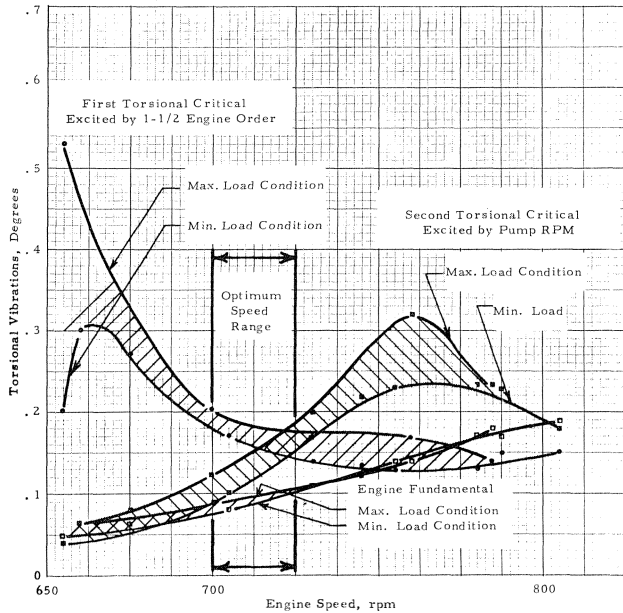


Figure 14. Torsional Vibrations as a Function of Pump Speed at Minimum and Maximum Load Conditions.

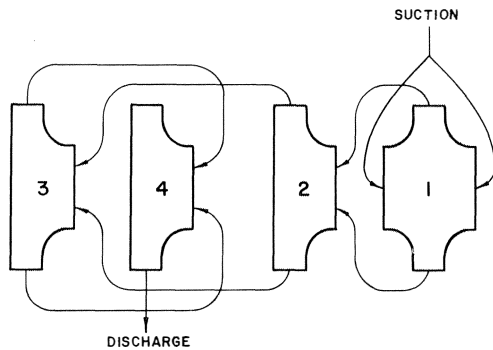


Figure 15. Flow Schematic of 4 Stage Pump.

the crossover was 5.75 ft. The acoustical natural frequency is

$$f = \frac{4770}{2(5.75)} = 415 \text{ Hz} \quad (8)$$

The acoustical resonant frequency was excited by the blade passage frequency (7 times running speed). Coincidence occurs at  $(415)(60) / 7 = 3560$  rpm.

Pulsations measured in the crossover showed pulsation amplitudes of 100 psi p-p. These pulsations were attenuated to

the point that at the suction and discharge flanges, the amplitude was less than 10 psi p-p.

Since the severe vibrations occurred when the speed was 3560 rpm for a water temperature of 310°F, several possible changes could be made to eliminate the possibility of coincidence of resonances during normal pump operation. One possible change was to reduce the diameter of the impellers and operate the pump at a higher speed. Another possible change would be to change to 6 or 8 blades to change the blade passage frequency. The impeller diameter change was the quickest and was carried out in the field, and the failures were eliminated.

CONCLUSION

The ability of centrifugal machines to amplify or attenuate piping pulsations has been documented in both the field and the laboratory. Co-existing acoustic resonances of the piping system can further amplify pulsations at frequencies corresponding to conventional organ pipe resonances of pipe components or by interacting system acoustic resonances of multiple components. The net result is that relatively low level excitation sources such as vortex formation at junctions in the piping systems can be amplified to levels which cause failure of piping and compressor or pump internals. Recent research into the mechanisms of these self-sustaining resonant buildups provides prediction techniques whereby such pulsation problems can be minimized, and case histories have proven the applicability of acoustically modifying pump and piping internal design to solve system vibration problems.

ACKNOWLEDGMENT

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