DYNAMIC CHARACTERISTICS OF FLUID-FILM BEARINGS

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ABSTRACT

The concept of cross-coupling in bearings is introduced and its influence on bearing dynamics is presented. Procedures for obtaining cross-coupled spring and damping coefficients are explained. The special case of pad pitch coupling in tilting-pad bearings is discussed and the method, assumptions and equations made in reducing the complete tilting pad matrix to the more conventional 4 × 4 is presented. The closed form solutions for point mass bearing whirl utilizing the cross-coupled coefficients is developed in the discussion of bearing stability. Discussions of various bearing types with recommended geometric considerations are presented. Data on steady-state performance; cross-coupled coefficients and whirl stability of various bearing types are included.

INTRODUCTION

Fluid-film bearings can significantly influence the dynamics of rotating shafts. They are a primary source of damping and thus can inhibit vibrations. Alternatively, they provide a mechanism for self-excited rotor whirl, and thus can be very destructive. Consequently, understanding of bearing dynamics characteristics is important. It is the purpose of this paper to provide some insight into bearing dynamics, present some background into analytical methods and representations, and discuss some particular bearings and factors that can influence dynamic characteristics. Steady-state and dynamic performance data is presented for several bearing types.

THE CONCEPT OF CROSS-COUPLING

A journal bearing derives load capacity from viscous pumping of the lubricant through a small clearance region. To generate pressure, the resistance to pumping must increase in the direction of fluid flow. This is accomplished by a movement of the journal such that the clearance distribution takes on the form of the familiar tapered wedge in the direction of rotation as shown on Figure 1.

The attitude angle γ on Figure 1 is the angle between the load direction and the line of centers. Thus, the displacement of the journal is not along a line that is coincident with the load vector, and a load in one direction causes not only displacements in that direction but orthogonal displacements as well.

Similarly, a displacement of the journal in the bearing will cause a load opposing the displacement, and a load orthogonal to it. Thus, there are strong cross-coupling influences introduced by the mechanism by which a bearing operates. The concept of cross-coupling is significant with regards to dynamic characteristics.

FUNDAMENTAL BEARING THEORY

The governing equation in bearing theory is the Reynolds Lubrication Equation:

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial Z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial Z}\right)=\frac{6U}{R}\frac{\partial h}{\partial\theta}+\frac{12\partial h}{\partial t} \quad (1)$$

The Reynolds equation is essentially a flow balance equation. The left hand side represents pressure induced flows in the θ and Z directions through a differential element. The first term on the right hand side represents shear flow of the fluid induced by the surface velocity of the journal, U. Note that this term contains the derivative of clearance with respect to distance. If this term is zero then there is zero pressure produced by hydrodynamic action; the term $\partial h/\partial\theta$ is the mathematical representation of the tapered wedge. The second term on the right hand side refers to a time rate of change in clearance, or a velocity of the mass center of the journal. It produces pressure by a fluid velocity normal to the bearing surfaces that attempts to squeeze fluid out of a restricted clearance space. This phenomenon is commonly called the "squeeze film" effect in bearing terminology. Since it is proportional to the velocity of the center of the journal, it is the phenomenon that produces viscous damping in a bearing.

Solution of this equation and ramifications for turbulence, external pressurization, etc., are accomplished by numerical methods using the digital computer. References [1] through [4] are some pertinent references describing numerical techniques for solving the Reynold's equation.

CROSS-COUPLED SPRING AND DAMPING COEFFICIENTS

For dynamic considerations, a convenient representation of bearing characteristics is by cross-coupled spring and damping coefficients. These are obtained as follows (refer to Figure 1).

1. The equilibrium position to support the given load is established by computer solution of Reynold's equation.
2. A small displacement to the journal is applied in the Y direction. A new solution of Reynold's equation is obtained and the resulting forces in the X and Y direction are produced. The spring coefficients are as follows:

$$K_{xy} = \frac{\Delta F_x}{\Delta y} \quad (2)$$

$$K_{yy} = \frac{\Delta F_y}{\Delta y} \quad (3)$$

where ΔF_x = difference in X forces between the displaced and equilibrium position

ΔF_y = difference in Y forces between the displaced and equilibrium position

Δy = displacement from equilibrium position in Y direction

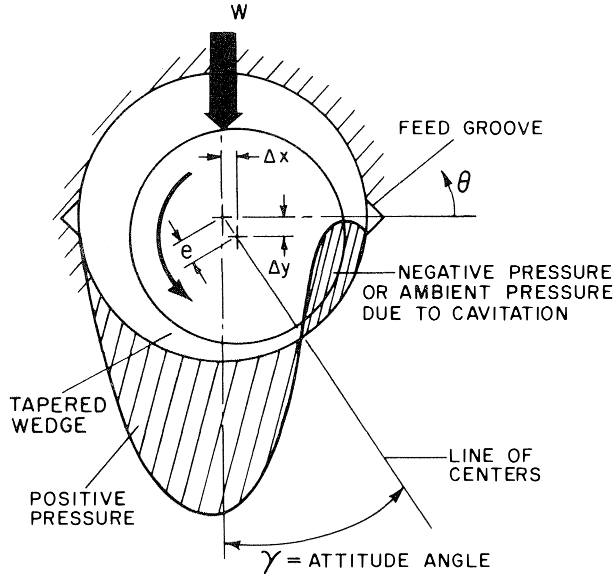

Figure 1. Two-Groove Cylindrical Bearing.

K_{xy} = stiffness in X direction due to a Y displacement

K_{yy} = stiffness in Y direction due to an X displacement

3. The journal is returned to its equilibrium position and an X displacement applied. Similar reasoning produces K_{xx} and K_{yx}.

The cross-coupled damping coefficients are produced in a like manner, except instead of displacements in the X and Y direction, velocities in these directions are consecutively applied with the journal in the equilibrium position. The mechanism for increasing load capacity is squeeze film in which the last term on the right hand side of equation (1) is actuated.

Thus for most fixed bearing configurations, there are a total of eight coefficients, four spring and four damping.

The total force on the journal is

$$F_i = K_{ij}X_j + D_{ij}\dot{X}_j \quad (4)$$

F_i = force in the i^{th} direction

Repeated subscripts implies summation:

$$K_{ij}X_j = K_{ix}X + K_{iy}Y \qquad \text{etc.}$$

It should be realized that the cross-coupled spring and damping coefficients represent a linearization of bearing characteristics. When they are used, the equilibrium position should be accurately determined as the coefficients are only valid about a small displacement region encompassing the equilibrium position of the journal. This is true because the spring and damping coefficients remain constant only about a small region of the equilibrium position.

Consider the two groove cylindrical bearing shown on Figure 1, with the geometrical and operating conditions indicated in Table 1.

TABLE 1. TWO-GROOVE, CYLINDRICAL BEARING, GEOMETRY AND OPERATING CONDITIONS

Journal Diameter, D	= 5 inches
Bearing Length, L	= 5 inches
Active Pad Angle, θ_P	= 160° (10° grooves on either side)
Radial Clearance, C	= 0.0025 inches
Operating Speed, N	= 5000 rpm
Lubricant Viscosity, μ	= 2×10^{-6} lb-sec/in^2
Eccentricity Ratio, ϵ	= 0.5
Load Direction, Vertical Down	

The computer solution would produce the following results:

Bearing load, W = 20,780 lbs

Power loss, HP = 15.51

Minimum film thickness, h_M = .00125 in

Side leakage, Q = 0.941 gpm

Spring and Damping Coefficients:

Spring Coefficients:

$$\begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix} = \begin{bmatrix} -12.14 \times 10^6 & -4.64 \times 10^6 \\ 28.3 \times 10^6 & -20.41 \times 10^6 \end{bmatrix}$$

Damping Coefficients:

$$\begin{bmatrix} D_{XX} & D_{XY} \\ D_{YX} & D_{YY} \end{bmatrix} = \begin{bmatrix} -2.85 \times 10^4 & 2.66 \times 10^4 \\ 2.69 \times 10^4 & -1.11 \times 10^5 \end{bmatrix}$$

The negative signs imply a positive stiffness because the restoring load is opposite to the applied load. Note for this bearing configuration there is very strong cross-coupling as evidenced by the magnitude of the off diagonal terms.

CROSS-COUPLED COEFFICIENTS FOR TILTING PAD BEARINGS

Tilting-pad bearings are used very extensively in high speed bearing applications because of their whirl-free characteristics. They present a special situation with respect to cross-coupled spring and damping coefficients. Consider the five-pad bearing shown on Figure 2. A tilt of pad 1 not only causes a righting moment on the pad but introduces forces on the shaft as well. Thus, there is cross-coupling between the pitch modes of the shoes and displacements of the journal. For a 5-shoe bearing there are 7 degrees of freedom and thus the spring and damping matrices are each 7 × 7. The 7 degrees of freedom are the X and Y motions of the journal and the pitch modes of the pads.

Reduction of the complete tilt-pad coefficient matrices to the reduced 4 × 4 matrix is often necessary because some rotor dynamics computer codes are only equipped to handle the 4 × 4 stiffness and damping matrices. The reduction method is described in Appendix A. It is important to realize that, in making the reduction, some approximations are made and accuracy is lost. An assumption must be made as to the vibrating frequency of the pads and also their inertia. In general, synchronous frequencies are selected. The reduced matrix cannot determine whether pad flutter instabilities exist and at times they give erroneous results with respect to stability of a tilting pad bearing system.

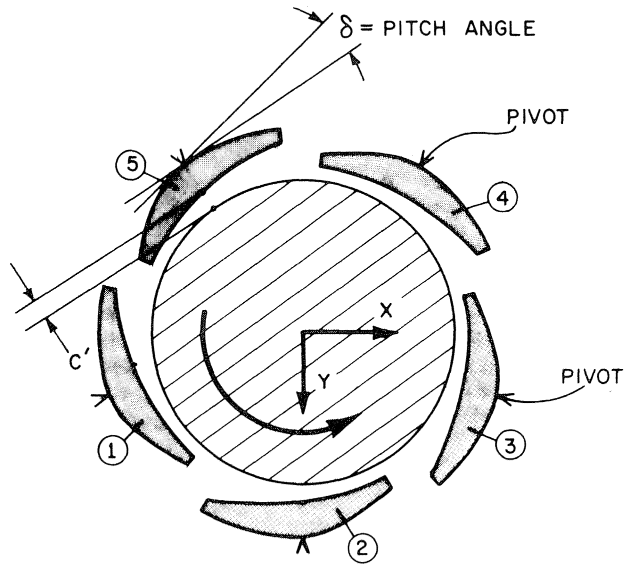

Figure 2. 5-Pad, Tilting-Pad Bearing.

An important geometric variable concerned with tilting-pad bearings is the preload ratio. The preload ratio is defined as follows:

$$\text{Preload Ratio} = \frac{C'}{C} = \frac{\text{Concentric Pivot Film Thickness}}{\text{Machined Clearance}}$$

The variable C′ is an installed clearance and is dependent upon the radial position of the pivot. On Figure 3 are shown two pads. Pad 1 has been installed such that the Preload Ratio is less than 1. For pad 2 the Preload Ratio is 1. The solid line represents the position of the journal in the concentric posi-

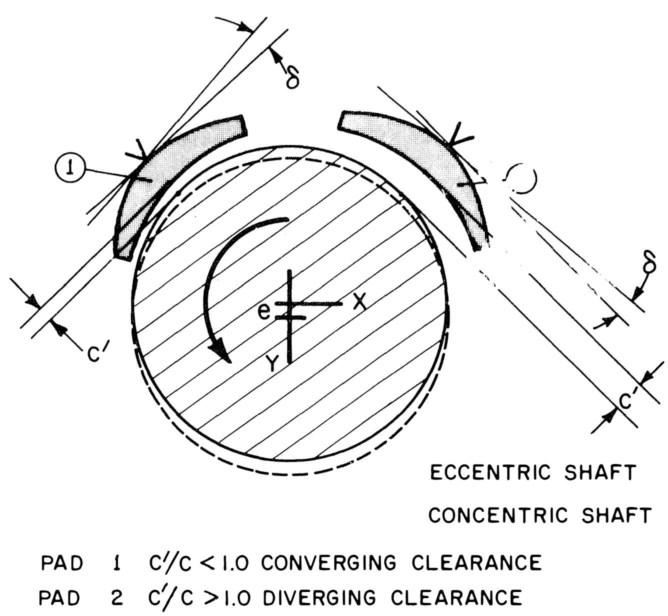

Figure 3. Tilting-Pad Bearing Preload.

tion. The dashed portion of the journal represents its position when load is applied to the bottom pads (not shown). Pad 1 is operating with a good converging wedge even though the journal is moving away from it. Pad 2 on the other hand is operating with a completely diverging film, which means that it is completely unloaded. Thus, bearings with installed pad Preload Ratios of 1 or greater will operate with some pads that are completely unloaded. Unloading not only reduces the overall stiffness of the bearing but it deteriorates stability because the upper pads do not aid in resisting cross-coupling influences. In the unloaded position they are also subject to flutter instability and to a phenomenon known as leading edge lockup, where the leading edge is forced against the shaft and is maintained in that position by the frictional interaction of the shaft and the pad. This is especially true of bearings that operate with low viscosity lubricants such as gas or water bearings. Thus, it is important to design bearings with preload, although for manufacturing reasons, it is common practice to produce bearings without preload.

Table 2 shows how the stiffness matrix for a five pad tilting-pad bearing can be interpreted. The columns represent displacements of the shaft in X and Y directions and the pitch motions of the pads. The rows represent forces on the shafts and moments on the pads. Thus, the moment stiffness on pad 2 due to a Y displacement of the shaft is $K_{\delta_2 Y}$, that is the intersection of the M_2 row with the ΔY column and has the units in-lbs/in.

The damping matrix is interpreted in a similar manner except instead of displacements, velocities are applied.

As an example of tilting-pad bearing dynamic characteristics the bearings described in Table 3 were computer analyzed.

Table 4 shows the spring and damping coefficient matrices for a preloaded bearing with a C′/C = 0.5. The complete matrices are filled and there are no unloaded pads. Also indicated are the reduced equivalent matrices produced by assuming a pitch inertia of zero and a synchronous vibratory frequency of 523.6 rad/sec (5000 rpm). For bearings in which the fluid-film stiffnesses are large, inertia characteristics of the pads are not meaningful and can be neglected. Note that for tilt-pad bearings the cross-coupling terms in the reduced matrix are very low.

TABLE 2. NOMENCLATURE FOR TILTING-PAD CROSS-COUPLED SPRING COEFFICIENTS INCLUDING PAD MOTIONS

	ΔX	ΔY	δ_1	δ_2	δ_3	δ_4	δ_5
F_x	K_{xx}	K_{xy}	$K_{x\delta_1}$	$K_{x\delta_2}$	$K_{x\delta_3}$	$K_{x\delta_4}$	$K_{x\delta_5}$
F_y	K_{yx}	K_{yy}	$K_{y\delta_1}$	$K_{y\delta_2}$	$K_{y\delta_3}$	$K_{y\delta_4}$	$K_{y\delta_5}$
M_1	$K_{\delta_1 x}$	$K_{\delta_1 y}$	$K_{\delta_1\delta_1}$				
M_2	$K_{\delta_2 x}$	$K_{\delta_2 y}$		$K_{\delta_2\delta_2}$			
M_3	$K_{\delta_3 x}$	$K_{\delta_3 y}$			$K_{\delta_3\delta_3}$		
M_4	$K_{\delta_4 x}$	$K_{\delta_4 y}$				$K_{\delta_4\delta_4}$	
M_5	$K_{\delta_5 x}$	$K_{\delta_5 y}$					$K_{\delta_5\delta_5}$

TABLE 3. TILTING-PAD BEARINGS, GEOMETRY AND OPERATING CONDITIONS

Journal Diameter, D = 5 inches
Bearing Length, L = 5 inches
Pad Angle, $\theta_p = 60°$
Pivot Location, Central
Operating Speed, N = 5000 rpm
Lubricant Viscosity, $\mu = 2 \times 10^{6}$ lb-sec/in²
Pad Preload, C′/C = 0.5, 1
Load Directly into Bottom Pad
Eccentricity Ratio, $\epsilon = 0.5$
Number of pads = 5
Clearance, C = .005 inches
Load, W = 2007 lb

Table 4

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES
TILTING PAD ASSEMBLY PROGRAM

TILT-PAD BEARING - L/D=0.5 , CP/C=0.5 , LOAD INTO PAD

DIMENSIONAL SPRING COEFFICIENTS FOR ECCENTRICITY RATIO OF .50000+00

-.2780+07	-.1551+07	.2472+07	-.1934+07	-.2691+07	-.3447+06	.4290+06
.3805+07	-.6985+07	.1175+07	.1142+08	.5030+06	-.5631+06	-.5019+06
-.1239+06	.3310+06	-.1192+07	.0000	.0000	.0000	.0000
-.1934+07	.1516+06	.0000	-.6526+07	.0000	.0000	.0000
-.9434+05	-.3406+06	.0000	.0000	-.1192+07	.0000	.0000
.5028+05	-.1944+05	.0000	.0000	.0000	-.1758+06	.0000
.3403+05	.4181+05	.0000	.0000	.0000	.0000	-.1758+06

DIMENSIONAL DAMPING COEFFICIENTSFOR ECCENTRICITY RATIO OF .50000+00

-.6801+04	.1115+04	.6155+03	-.1763+04	-.8422+03	.5751+01	.1270+03
.1103+04	-.1550+05	.5857+03	.4910+04	-.1121+03	-.1317+03	-.3522+02
.6204+03	.5873+03	-.1238+04	.0000	.0000	.0000	.0000
-.1763+04	.4938+04	.0000	-.5951+04	.0000	.0000	.0000
-.8471+03	-.1104+03	.0000	.0000	-.1238+04	.0000	.0000
.5291+01	-.1323+03	.0000	.0000	.0000	-.2769+03	.0000
.1275+03	-.3586+02	.0000	.0000	.0000	.0000	-.2769+03

PITCH INERTIA= .0000
FREQUENCY= .5236+03

EQUIVALENT BEARING STIFFNESS COEFFICIENTS

-.167133+07	-.821991+00
-.809083-02	-.549170+07

EQUIVALENT BEARING DAMPING COEFFICIENTS

-.333698+04	-.223769-02
-.197379-02	-.693475+04

Table 5 shows similar results for a non-preloaded bearing (C′/C = 1). Note that the last two pads are completely unloaded as evidenced by the zero columns for pads 4 and 5.

BEARING STABILITY

General Theory

It is the cross-coupling characteristics of a journal bearing that can promote self-excited instabilities in the form of bearing whirl. Motion in one direction produces orthogonal forces which in turn causes orthogonal motion. The process continues and an orbital motion of the journal results. This orbital motion is generally in the same direction as shaft rotation and subsynchronous in frequency. Half frequency whirl is the common term referred to in bearing terminology. Bearing whirl is a self-excited phenomenon and does not require external forces to promote it.

Table 5

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES
TILTING PAD ASSEMBLY PROGRAM

TILT-PAD BEARING - L/D=0.5 , CP/C=1.0 , LOAD INTO PAD

DIMENSIONAL SPRING COEFFICIENTS FOR ECCENTRICITY RATIO OF .50000+00

-.1819+06	-.2207+06	.3755+06	-.1976+06	-.3886+06	.0000	.0000
.5356+06	-.6566+06	.1444+06	.1712+07	.1039+06	.0000	.0000
-.1667+05	.1701+05	-.7197+05	.0000	.0000	.0000	.0000
-.1976+06	-.1612+05	.0000	-.6670+06	.0000	.0000	.0000
.3490+04	-.2356+05	.0000	.0000	-.7197+05	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000

DIMENSIONAL DAMPING COEFFICIENTSFOR ECCENTRICITY RATIO OF .50000+00

-.8950+03	.1035+03	.2502+02	-.2245+03	-.5529+02	.0000	.0000
.1030+03	-.2089+04	.5963+02	.4147+03	-.3353+02	.0000	.0000
.2436+02	.5941+02	-.1653+03	.0000	.0000	.0000	.0000
-.2245+03	.4176+03	.0000	-.7576+03	.0000	.0000	.0000
-.5463+02	-.3375+02	.0000	.0000	-.1653+03	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000

PITCH INERTIA= .0000
FREQUENCY= .5236+03

EQUIVALENT BEARING STIFFNESS COEFFICIENTS

-.657487+05	-.870456-01
-.299103-01	-.478170+06

EQUIVALENT BEARING DAMPING COEFFICIENTS

-.547140+03	.644531-05
-.182555-04	-.115504+04

The cross-coupled spring and damping coefficients provide a convenient way of representing a bearing in a stability analysis. Consider a journal of mass M operating in a bearing. The journal can be considered to have two degrees of freedom X and Y (refer to Figure 1). The governing equations are:

$$M\ddot{X} + D_{XX}\dot{X} + D_{XY}\dot{Y} + K_{XX}X + K_{XY}Y = 0 \tag{5}$$

$$M\ddot{Y} + D_{YY}\dot{Y} + D_{YX}\dot{X} + K_{YY}Y + K_{YX}X = 0 \tag{6}$$

Assume a sinusoidal response of the form:

$$X = X_0 e^{\beta t} \tag{7}$$

$$X = Y_0 e^{\beta t} \tag{8}$$

where β is a complex variable.

$$\beta = \alpha + i\omega \tag{9}$$

By Euler expansion of $e^{\beta t}$, another way to write equations (7) and (8) would be:

$$X = X_0 e^{\alpha t} (\cos \omega t + i \sin \omega t) \tag{10}$$

$$Y = Y_0 e^{\alpha t} (\cos \omega t + i \sin \omega t) \tag{11}$$

An interpretation of the variables α and ω are shown on Figure 4. The real part of $\beta = \alpha$ is called the growth or attenuation factor. The imaginary part is the frequency of vibration. A positive real part means that response to a disturbance grows in time. The growth factor is similar to the logarithmic mean decrement which is common in vibration theory.

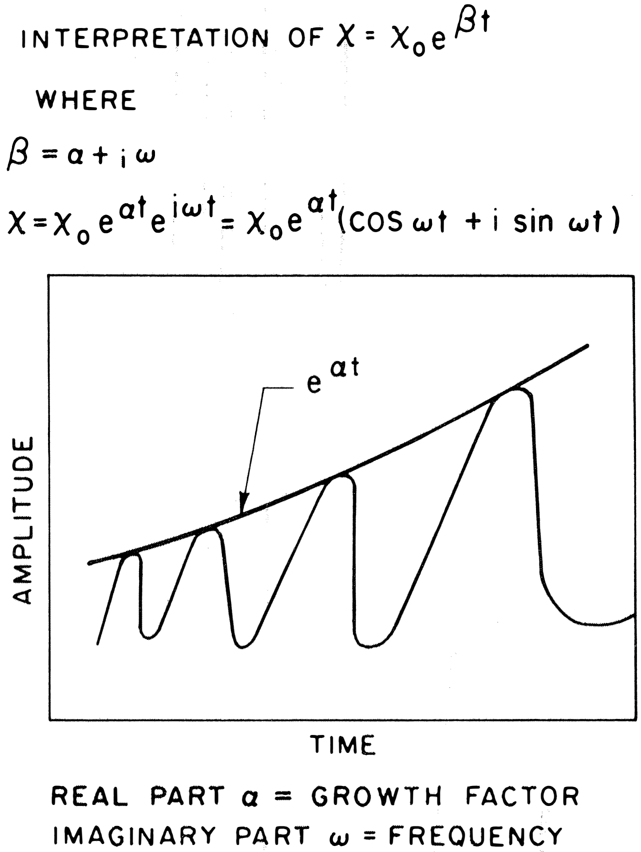

Figure 4. Interpretation of Growth Factor and Orbital Frequency.

$$\alpha\tau = \ln \frac{X_{n+1}}{X_n} \tag{12}$$

where τ = period of vibration
X_{n+1} = amplitude at time n+1
X_n = amplitude at time n

Thus, the growth factor α is a measure of the growth or decay of the journal to a small disturbance. A positive growth factor implies an instability.

The solution of equations (5) and (6) are obtained by substituting equations (7) and (8), which produces the following:

$$\begin{bmatrix} (M\beta^2 + D_{XX}\beta + K_{XX}) & (D_{XY}\beta + K_{XY}) \\ (\beta D_{YX} + K_{YX}) & (M\beta^2 + \beta D_{YY} + K_{YY}) \end{bmatrix} \begin{Bmatrix} X_0 \\ Y_0 \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix} \tag{13}$$

To obtain a solution, the determinant of the coefficient matrix must vanish. Expansion produces a polynomial in β that can be solved for the roots of β which in turn provide the growth factors and frequencies of vibration.

It is possible to obtain a closed form solution of equations (5) and (6) for the critical mass and resulting orbital frequency. The critical mass M_C is defined as that mass above which an instability will occur. At the threshold of instability the real part of $\beta = \alpha$ goes to zero and

$$\beta = i\omega$$

Substituting into equation (13), expanding the determinant and separating real and imaginary components produces the following equations:

$$M_C^2\omega_C^4 - M_C\underbrace{(K_{yy}+K_{xx})}_{A}\omega_C^2 + \underbrace{(D_{yx}D_{xy}-D_{xx}D_{yy})}_{B}\omega_C^2 + \underbrace{(K_{xx}K_{yy}-K_{xy}K_{yx})}_{C} = 0 \qquad (14)$$

$$\underbrace{M_C\,[D_{yy}+D_{xx}]\,\omega_C^2}_{D} + \underbrace{[D_{xy}K_{yx}+D_{yx}K_{xy}-D_{xx}K_{yy}-D_{yy}K_{xx}]}_{E} = 0 \qquad (15)$$

The two equations can be solved for M_C and ω_C which are the critical mass and orbital frequency, respectively. Results (in terms of the above defined constants) are as follows:

$$M_C = \frac{BED}{E^2 - AED + CD^2} \qquad (16)$$

$$\omega = \sqrt{\frac{-(AED + E^2 + CD^2)}{BD^2}} \qquad (17)$$

Thus, if the cross-coupled coefficients are known, it is possible to determine the minimum mass that will cause an instability and the orbital frequency. If the mass acting on the bearing exceeds or equals the minimum value, then an instability will occur.

OPERATING CONDITIONS THAT AFFECT BEARING STABILITY

Cavitation

Figure 1 shows a pressure distribution in a journal bearing. Positive pressure is generated in the converging wedge because the journal is pumping fluid through a restriction. On the downstream side of the minimum film thickness the journal is pumping fluid out of a diverging region. In this region the pressure reduces. The pressure becomes either a negative which is defined as pressure below ambient pressure, or the film cavitates and reduces to atmospheric pressure as the lubricant releases entrapped air.

With respect to stability, cavitation is a more desirable condition than the development of negative pressure. From examination of Figure 1, it is seen that the negative pressure pulls the journal in an orthogonal direction and increases the cross-coupling. The more eccentric the bearing, the larger the negative pressure or cavitated region.

In lightly loaded bearings that are pressure fed, negative pressures can occur because in the divergent region pressures have not approached atmospheric pressure. Cavitation does not occur and the bearing is prone to instability. Thus, the feed pressure and load on a bearing are two additional parameters that affect stability.

Lobe bearings are another type that are often used because of their excellent anti-whirl characteristics. Figure 5 shows two types of lobe bearings; the symmetric lobe bearing and the canted lobe bearing. The symmetric lobe bearing is designed so that in the concentric position the minimum film thickness occurs at the center of each lobe. Note that this permits a region of converging film followed by a region of diverging film. Thus, depending upon the ambient pressure, it is possible to have negative pressures in a symmetric lobe bearing. Under very high ambient conditions a symmetric lobe bearing can go unstable.

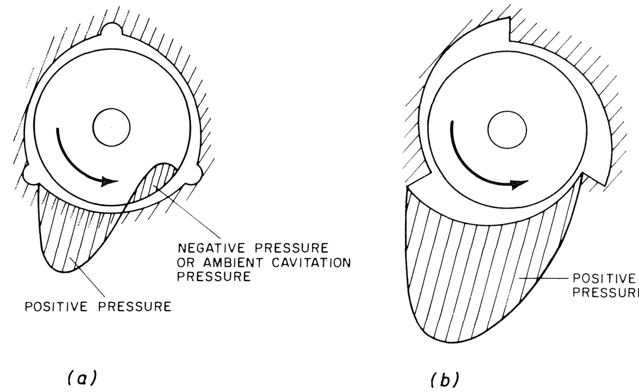

Figure 5. Lobe Bearing Configurations.

The canted lobe bearing is designed to have a completely converging wedge and positive pressure throughout its arc length. It has superior stability characteristics than the symmetric lobe bearing. Its steady-state characteristics are also superior.

Hybrid Bearings

At times, externally pressurized bearings have been resorted to for stability improvement. The philosophy is that externally pressurized bearings are not subject to high attitude angles as can be the case with hydrodynamic journal bearings. Although this is generally true, hybrid bearings can still be subject to considerable cross-coupling. Figure 6 shows a schematic arrangement of a hybrid bearing. Oil is fed through restrictors from an external source into pocket recesses. From there it exits into the clearance region between recesses. Lubricant is also pumped into and out of recesses by the rotating shaft by viscous drag in the same manner as in a purely hydrodynamic bearing.

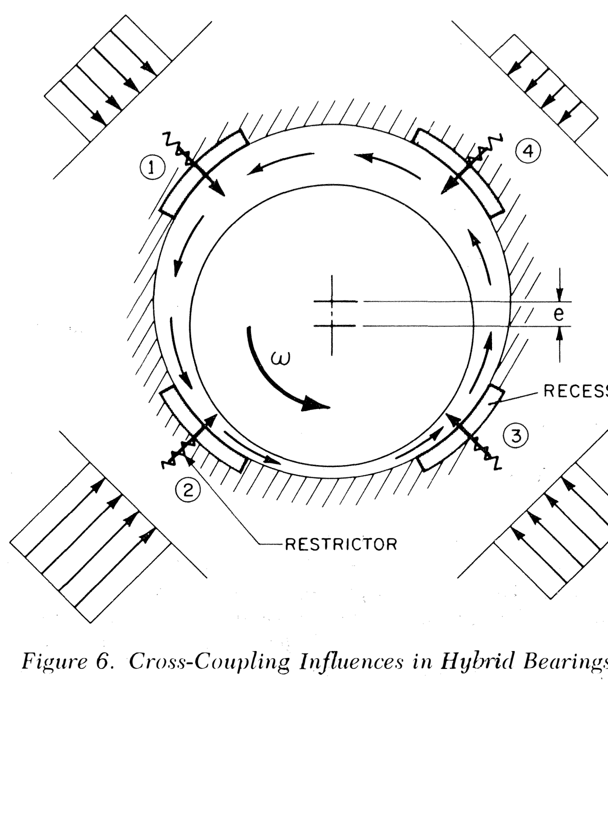

Figure 6. Cross-Coupling Influences in Hybrid Bearings.

Consider recess 2 on Figure 6. Oil is pumped from the shaft via a converging wedge and it augments the pressure in the recess provided by the external system. The net result is a higher pressure level in the bearing domain covered by recess 2 than would occur without rotation. Now consider recess 3. Here the journal is pumping fluid out of the recess into a diverging film, so that the hydrodynamic action tends to reduce the pressure in this recess domain. By similar reasoning, it can be shown that recess 4 operates at a lower pressure level than recess 1. The net result of this variation in pressure levels due to rotation is that cross-coupling forces are introduced and the hybrid bearing may not prevent instability.

STEADY STATE AND DYNAMIC PERFORMANCE OF COMMON BEARING CONFIGURATIONS

The bearing configuration plays an important role with respect to whirl stability as well as other performance parameters. Described below are some of the more common journal bearings used in industry with qualitative and quantitative assessments concerning their steady-state and dynamic performance characteristics.

1. 360° Cylindrical Journal Bearing (Plain)

This type bearing may be a complete circular shell or consist of two halves joined at a horizontal diametral parting line (see Figure 1). It is commonly used because of its simple construction. The bearing has high load capacity. It is, however, whirl prone especially at low load conditions. Generally, the bearing will produce more heat than other types, and can be contamination sensitive, since trapped particles have few escape routes in the clearance region. Because of its simplicity, the bearing should be used if possible, but it should be carefully checked for whirl stability and potential thermal problems. It is generally used for medium speed and medium to heavy load applications.

Computer performance of a two-groove cylindrical bearing, as shown on Figure 1, was completed as a function of eccentricity ratio. The active pad length was 160°, and two length to diameter ratios (L/D) were examined, 0.5 and 1.0. The computer printout produces both dimensional and nondimensional information. The nondimensional parameters have complete generality with respect to viscosity, clearance and speed and thus can be used for a variety of bearing applications. The dimensional information refers to the bearing and operating conditions described in Table 1, except a range of eccentricities were evaluated.

On Figure 7 is shown the load parameter $\overline{W}$ as a function of eccentricity ratio ϵ for L/D ratios of 0.5 and 1.0.

The L/D = 0.5 curve may have higher load parameter, but not higher load capacity, than the L/D = 1 bearing. When dimensionalizing, the parameter is multiplied by L^3. This particular set of curves were plotted on semi-log paper because of the steep variation of load with eccentricity ratio.

Figure 8 shows variations in dimensionless minimum film thickness H_M, power loss $\overline{P}$ and side leakage $\overline{Q}$ as a function of eccentricity ratio. The side leakage is the minimum flow required to supply the bearing without causing supply starvation and operation at reduced clearances and higher temperatures. Although dimensionless power loss is lower for L/D = 1.0, the dimensional quantity would be higher because of the length squared factor that is multiplied by $\overline{P}$ in the conversion to the dimensional quantity.

Figure 9 shows attitude angle γ (see Figure 1) as a function of eccentricity ratio. These angles were computed on the basis

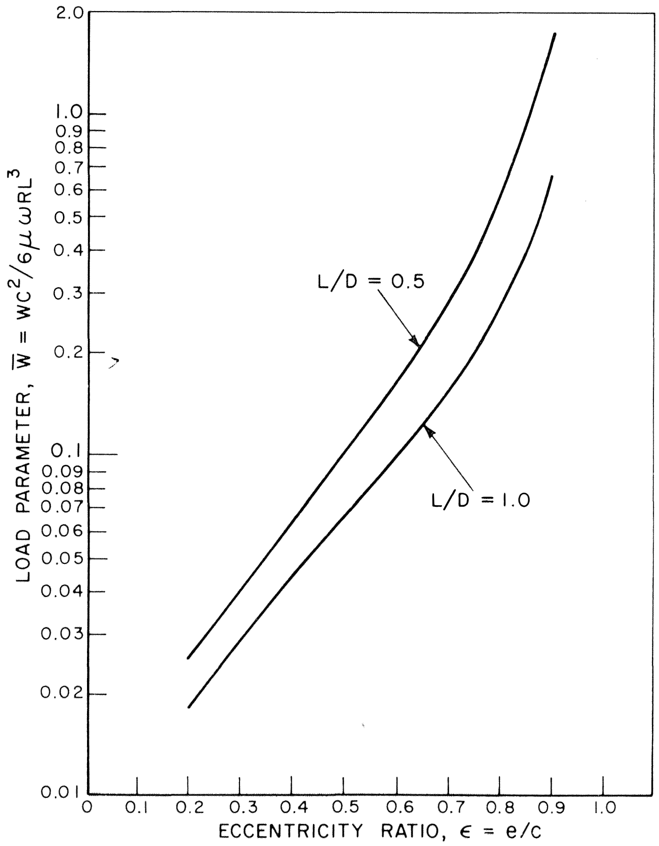

Figure 7. Dimensionless Load Parameter vs. Eccentricity, Two-Groove Cylindrical Bearing, Vertical Load Direction.

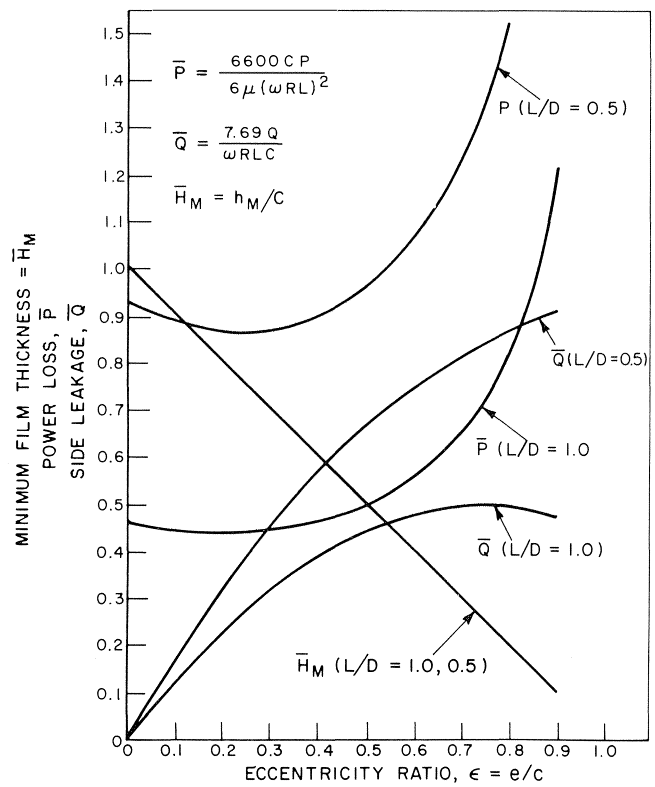

Figure 8. Steady-State Dimensionless Parameters, Two-Groove Cylindrical Bearing, Vertical Load Direction.

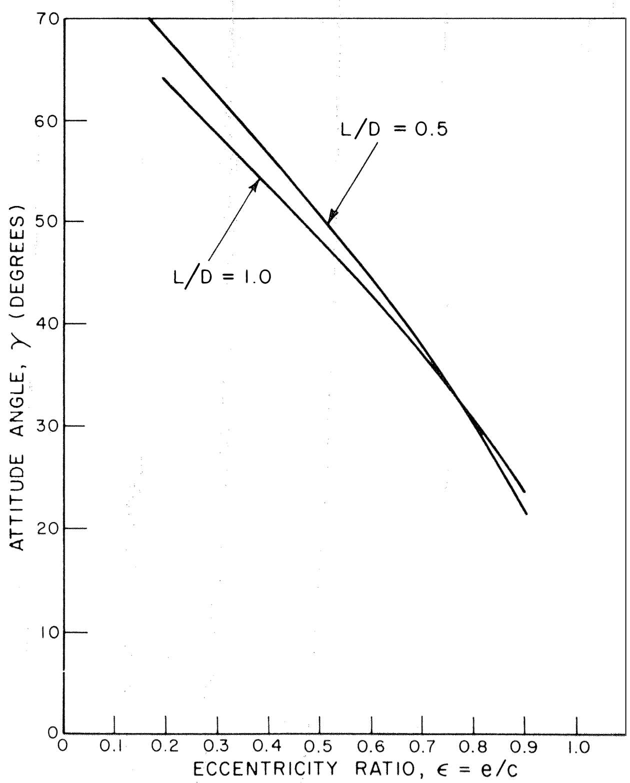

Figure 9. Attitude Angle vs. Eccentricity Ratio, Two-Groove Cylindrical Bearing, Vertical Load Direction.

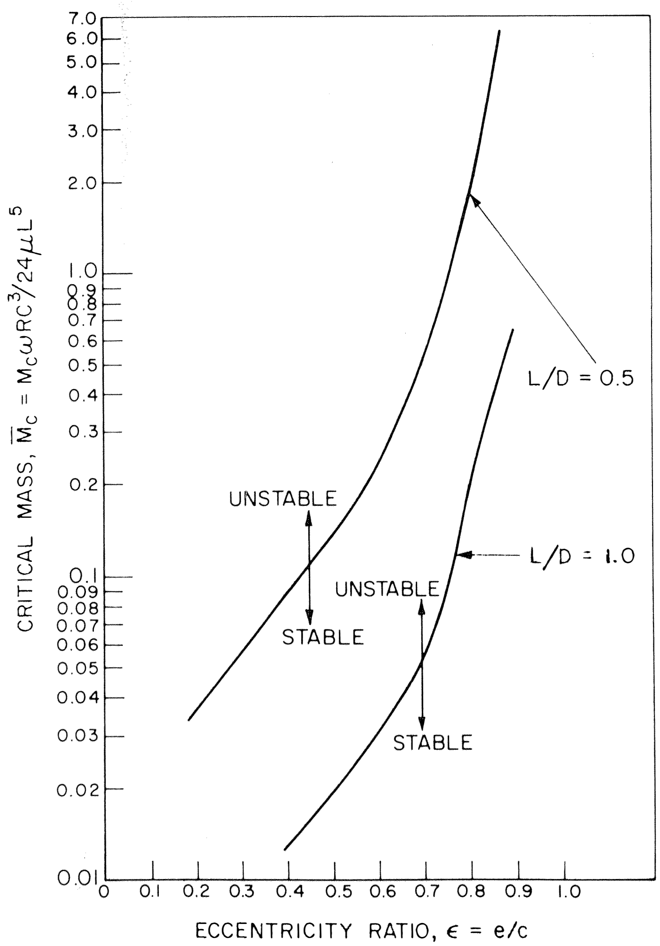

Figure 10. Critical Mass vs. Eccentricity, Two-Groove Cylindrical Bearing, Vertical Load Direction.

of a cavitating bearing. Note the attitude angle (and cross-coupling) reduces with increased eccentricity. Loading a bearing increases stability.

Figure 10 shows the critical mass parameter $\overline{M}_c$ as a function of eccentricity ratio. If the mass being supported by the journal exceeds the critical mass, then the bearing will be unstable. At low eccentricity ratios the unstable region is much larger than at high eccentricity ratios. Cavitation tends to reduce the cross-coupling influences. Tables 6 and 7 show tabulated performance of both the bearings. The dimensional information refers to the operating conditions indicated in Table 1.

2. Cylindrical Bearing with Axial Slots

The bearing is a plain cylindrical bearing that is separated into three or four sectors by axial grooves. The axial groove bearing is a little less simple and has less load capability than a full cylindrical bearing. The axial slots usually serve as feeding grooves and interrupt the downstream pressure distribution to reduce the cross-coupling influences. It, therefore, has better whirl stability than the cylindrical bearing. Oil feed through the axial slots also helps keep the bearing cooler than the plain cylindrical bearing. The slots also offer an outlet for contaminants so that it is less contamination sensitive than a plain cylindrical bearing. Thus, this bearing should be used as an alternative to a plain cylindrical bearing if it can correct a reason for elimination of a plain bearing such as whirl and overheating.

3. Elliptical and Lobe Bearings (see Figure 5)

Elliptical and lobe bearings are often used because they provide better resistance to whirl than cylindrical configurations. They do so because they have multiple load producing pads that assist in preventing large attitude angles and cross-coupling.

Elliptical or two-lobe bearings generally have poor horizontal stiffness because of the large clearances along the major diameter of the ellipse. The split elliptical configuration, however, is easier to manufacture than the other types, because it is simply a cylindrical bearing in which material has been removed from the horizontal parting line. The other types of lobe bearings are complicated to manufacture. Lobe bearings are usually clearance and tolerance sensitive.

The canted lobe configuration, as shown on Figure 5, has better performance characteristics than the symmetric bearing. The reason is that a positive pressure is usually developed throughout the lobe, while for the converging lobe, negative pressures or cavitation develops in the diverging region of the clearance. A 2:1 ratio between concentric machined clearance and installed minimum clearance is generally a reasonable compromise, although for a particular application this ratio should be varied to determine an optimum value. Generally, lobe bearings are used for high speed, low load applications where whirl is a problem.

Tables 8 and 9 indicate computer generated performance for symmetric three lobe bearings. The bearings are manufactured so that the concentric minimum film thickness is half of the machined clearance. The pads are each 110° in angular

Table 6

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES INCOMPRESSIBLE HYDRODYNAMIC JOURNAL BEARING ANALYSIS
TWO-GROOVE BEARING L/D=0.5

ECC	ANG-ECC DEGREES	ANG-RXN DEGREES	BEARING FORCE LBS	BEARING FORCE NON DIM	POWER LOSS HP	POWER LOSS NON DIM	MINIMUM CLEARANCE INCH	MINIMUM CLEARANCE NON DIM
.000	.00	.00	.0000	.0000	.7250+01	.9308+00	.2500-02	.1000+01
.200	338.43	89.55	.9830+03	.2503-01	.6769+01	.8691+00	.2000-02	.8001+00
.400	326.33	90.56	.2571+04	.6547-01	.7028+01	.9023+00	.1502-02	.6008+00
.500	320.65	90.97	.3988+04	.1015+00	.7492+01	.9620+00	.1250-02	.5000+00
.600	314.50	90.96	.6405+04	.1631+00	.8340+01	.1071+01	.1005-02	.4019+00
.700	307.99	90.53	.1109+05	.2823+00	.9554+01	.1227+01	.7511-03	.3004+00
.800	300.97	90.27	.2241+05	.5706+00	.1178+02	.1513+01	.5003-03	.2001+00
.900	292.28	89.52	.6628+05	.1688+01	.1713+02	.2200+01	.2518-03	.1007+00

ECC	SIDE LEAKAGE GPM	SIDE LEAKAGE NON-DIM	TOTAL INLET FLOW GPM	TOTAL INLET FLOW NON-DIM	CRITICAL MASS LB	CRITICAL MASS NON-DIM	THRESHOLD FREQ. CPM	THRESHOLD FREQ. NON-DIM
.000	.0000	.0000	-.2127+01	-.2000+01	.9832+01	.1110-03	.2497+04	.9987+00
.200	.3382+00	.3180+00	-.2065+01	-.1961+01	.3091+04	.3490-01	.2541+04	.1017+01
.400	.6028+00	.5668+00	-.2022+01	-.1901+01	.7752+04	.8754-01	.2525+04	.1010+01
.500	.7089+00	.6665+00	-.1984+01	-.1865+01	.1158+05	.1308+00	.2610+04	.1044+01
.600	.7950+00	.7475+00	-.1941+01	-.1825+01	.2063+05	.2329+00	.2445+04	.9779+00
.700	.8695+00	.8175+00	-.1894+01	-.1780+01	.1493+06	.1685+01	.1155+04	.4621+00
.800	.9276+00	.8721+00	-.1844+01	-.1734+01	STABLE			
.900	.9722+00	.9141+00	-.1796+01	-.1688+01	STABLE			

NON DIM SPRING COEFFICIENTS / NON DIM DAMPING COEFFICIENTS

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.10745-03	-.53167-01	.11483+00	-.31655-02	-.53233-01	-.98569-08	-.30515-02	-.11498+00
.200	-.48203-01	-.64472-01	.14634+00	-.41487-01	-.68032-01	.25833-01	.25640-01	-.14489+00
.400	-.11261+00	-.62053-01	.25021+00	-.15383+00	-.85373-01	.58804-01	.59476-01	-.23653+00
.500	-.19005+00	-.80422-01	.38166+00	-.26993+00	-.13011+00	.11113+00	.11189+00	-.35467+00
.600	-.28884+00	-.54901-01	.60825+00	-.57458+00	-.15528+00	.16452+00	.16190+00	-.55299+00
.700	-.43321+00	.50840-01	.10605+01	-.14090+01	-.16544+00	.21787+00	.22086+00	-.93877+00
.800	-.85188+00	.33885+00	.23633+01	-.42643+01	-.24263+00	.41085+00	.39904+00	-.20527+01
.900	-.21114+01	.20889+01	.84199+01	-.24428+02	-.38622+00	.90149+00	.95767+00	-.72901+01

DIM SPRING COEFFICIENTS (LBS/IN) / DIM DAMPING COEFFICIENTS (LBS-SEC/IN)

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.16879+04	-.83515+06	.18037+07	-.49724+05	-.31940+04	-.59142-03	-.18309+03	-.68985+04
.200	-.75716+06	-.10127+07	.22988+07	-.65167+06	-.40819+04	.15500+04	.15384+04	-.86934+04
.400	-.17688+07	-.97473+06	.39303+07	-.24163+07	-.51224+04	.35282+04	.35686+04	-.14192+05
.500	-.29852+07	-.12633+07	.59951+07	-.42400+07	-.78064+04	.66675+04	.67137+04	-.21260+05
.600	-.45370+07	-.86238+06	.95544+07	-.90255+07	-.93169+04	.98711+04	.97141+04	-.33180+05
.700	-.68048+07	.79860+06	.16659+08	-.22132+08	-.99266+04	.13072+05	.13252+05	-.56326+05
.800	-.13381+08	.53227+07	.37122+08	-.66983+08	-.14558+05	.24651+05	.23943+05	-.12316+06
.900	-.33166+08	.32812+08	.13226+09	-.38371+09	-.23173+05	.54089+05	.57460+05	-.43741+06

Table 7

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES INCOMPRESSIBLE HYDRODYNAMIC JOURNAL BEARING ANALYSIS

TWO-GROOVE BEARING L/D=1.0

ECC	ANG-ECC DEGREES	ANG-RXN DEGREES	BEARING FORCE LBS	BEARING FORCE NON DIM	POWER LOSS HP	POWER LOSS NON DIM	MINIMUM CLEARANCE INCH	MINIMUM CLEARANCE NON DIM
.000	.00	.00	.0000	.0000	.1450+02	.4654+00	.2500-02	.1000+01
.200	334.29	89.45	.5713+04	.1819-01	.1377+02	.4419+00	.2001-02	.8006+00
.400	323.42	90.91	.1417+05	.4511-01	.1450+02	.4654+00	.1502-02	.6007+00
.500	317.61	90.33	.2078+05	.6613-01	.1551+02	.4979+00	.1251-02	.5004+00
.600	313.20	90.77	.3114+05	.9912-01	.1745+02	.5601+00	.1002-02	.4009+00
.700	307.96	90.81	.4928+05	.1569+00	.2030+02	.6516+00	.7511-03	.3004+00
.800	301.05	90.08	.8775+05	.2793+00	.2538+02	.8146+00	.5003-03	.2001+00
.900	293.76	89.51	.2081+06	.6625+00	.3759+02	.1207+01	.2548-03	.1019+00

ECC	SIDE LEAKAGE GPM	SIDE LEAKAGE NON-DIM	TOTAL INLET FLOW GPM	TOTAL INLET FLOW NON-DIM	CRITICAL MASS LB	CRITICAL MASS NON-DIM	THRESHOLD FREQ. CPM	THRESHOLD FREQ. NON-DIM
.000	.0000	.0000	-.4254+01	-.2000+01	.4539+02	.1602-04	.2497+04	.1998+01
.200	.5002+00	.2351+00	-.4050+01	-.1904+01	.1418+05	.5004-02	.2521+04	.2017+01
.400	.8403+00	.3950+00	-.3767+01	-.1771+01	.3534+05	.1247-01	.2561+04	.2049+01
.500	.9408+00	.4423+00	-.3605+01	-.1695+01	.5416+05	.1911-01	.2464+04	.1971+01
.600	.1023+01	.4810+00	-.3434+01	-.1615+01	.8460+05	.2985-01	.2437+04	.1950+01
.700	.1060+01	.4982+00	-.3260+01	-.1532+01	.1487+06	.5246-01	.2334+04	.1867+01
.800	.1046+01	.4917+00	-.3098+01	-.1456+01	.5480+06	.1934+00	.1638+04	.1310+01
.900	.1026+01	.4824+00	-.2966+01	-.1394+01	.1851+07	.6530+00	.1412+04	.1129+01

NON DIM SPRING COEFFICIENTS / NON DIM DAMPING COEFFICIENTS

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.63968-04	-.26796-01	.85818-01	-.47136-02	-.13414-01	-.36153-08	-.23277-02	-.42963-01
.200	-.28100-01	-.31715-01	.10667+00	-.29336-01	-.16725-01	.75678-02	.75062-02	-.54136-01
.400	-.74314-01	-.45945-01	.17123+00	-.85002-01	-.29454-01	.22982-01	.22977-01	-.87355-01
.500	-.96614-01	-.36953-01	.22538+00	-.16242+00	-.29739-01	.27664-01	.28034-01	-.11523+00
.600	-.14978+00	-.37344-01	.33296+00	-.29153+00	-.41766-01	.46105-01	.46464-01	-.17029+00
.700	-.24148+00	-.21602-01	.55044+00	-.60228+00	-.58453-01	.77514-01	.77162-01	-.28129+00
.800	-.42191+00	.63992-01	.10875+01	-.16100+01	-.78020-01	.13004+00	.13193+00	-.55792+00
.900	-.10560+01	.34460+00	.34077+01	-.70779+01	-.18376+00	.41698+00	.40509+00	-.18515+01

DIM SPRING COEFFICIENTS (LBS/IN) / DIM DAMPING COEFFICIENTS (LBS-SEC/IN)

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.80385+04	-.33673+07	.10784+08	-.59233+06	-.12877+05	-.34707-02	-.22346+04	-.41245+05
.200	-.35312+07	-.39854+07	.13404+08	-.36865+07	-.16056+05	.72651+04	.72060+04	-.51970+05
.400	-.93386+07	-.57736+07	.21518+08	-.10682+08	-.28276+05	.22063+05	.22058+05	-.83861+05
.500	-.12141+08	-.46436+07	.28322+08	-.20411+08	-.28549+05	.26557+05	.26913+05	-.11062+06
.600	-.18821+08	-.46928+07	.41841+08	-.36635+08	-.40096+05	.44261+05	.44605+05	-.16348+06
.700	-.30345+08	-.27146+07	.69170+08	-.75684+08	-.56114+05	.74413+05	.74076+05	-.27004+06
.800	-.53019+08	.80415+07	.13666+09	-.20232+09	-.74900+05	.12484+06	.12665+06	-.53560+06
.900	-.13270+09	.43304+08	.42822+09	-.88944+09	-.17641+06	.40031+06	.38888+06	-.17774+07

Table 8

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES INCOMPRESSIBLE HYDRODYNAMIC JOURNAL BEARING ANALYSIS

THREE-LOBE BEARING L/D=0.5 LOAD INTO PAD

ECC	ANG-ECC DEGREES	ANG-RXN DEGREES	BEARING FORCE LBS	BEARING FORCE NON DIM	POWER LOSS HP	POWER LOSS NON DIM	MINIMUM CLEARANCE INCH	MINIMUM CLEARANCE NON DIM
.000	.00	.00	.8916-04	.9082-08	.6345+01	.1629+01	.2500-02	.5000+00
.100	325.93	89.77	.5294+03	.5393-01	.6407+01	.1645+01	.2189-02	.4379+00
.200	325.93	89.18	.1250+04	.1273+00	.6670+01	.1713+01	.1835-02	.3669+00
.300	324.41	89.80	.2518+04	.2565+00	.7184+01	.1845+01	.1414-02	.2829+00
.400	320.42	90.09	.5880+04	.5989+00	.8238+01	.2116+01	.9324-03	.1865+00
.450	317.67	90.89	.1043+05	.1062+01	.9158+01	.2352+01	.6654-03	.1331+00

ECC	SIDE LEAKAGE GPM	SIDE LEAKAGE NON-DIM	TOTAL INLET FLOW GPM	TOTAL INLET FLOW NON-DIM	CRITICAL MASS LB	CRITICAL MASS NON-DIM	THRESHOLD FREQ. CPM	THRESHOLD FREQ. NON-DIM
.000	.6541+00	.3075+00	-.3986+01	-.1874+01	.3704+04	.3346+00	.2398+04	.9590+00
.100	.6779+00	.3187+00	-.3979+01	-.1871+01	.4455+04	.4025+00	.2375+04	.9500+00
.200	.7371+00	.3465+00	-.3960+01	-.1862+01	.8004+04	.7231+00	.2170+04	.8681+00
.300	.8148+00	.3831+00	-.3929+01	-.1847+01	.1514+05	.1367+01	.2061+04	.8243+00
.400	.9347+00	.4394+00	-.3890+01	-.1829+01	.4459+06	.4028+02	.5295+03	.2118+00
.450	.1034+01	.4862+00	-.3869+01	-.1819+01	STABLE			

NON DIM SPRING COEFFICIENTS / NON DIM DAMPING COEFFICIENTS

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.30904+00	-.44358+00	.44548+00	-.30788+00	-.46220+00	-.23799-03	-.17754-02	-.46486+00
.100	-.29975+00	-.48604+00	.44766+00	-.44179+00	-.43125+00	-.14699-01	-.11912-01	-.54667+00
.200	-.35166+00	-.57011+00	.52808+00	-.81566+00	-.48431+00	-.20590-01	-.17853-01	-.68233+00
.300	-.62569+00	-.70632+00	.10622+01	-.16270+01	-.62476+00	.12764+00	.13102+00	-.12953+01
.400	-.12264+01	-.71615+00	.27467+01	-.50145+01	-.87839+00	.52684+00	.51642+00	-.30006+01
.450	-.21400+01	-.61606-01	.58764+01	-.11447+02	-.93130+00	.12837+01	.12664+01	-.60255+01

DIM SPRING COEFFICIENTS (LBS/IN) / DIM DAMPING COEFFICIENTS (LBS-SEC/IN)

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.60681+06	-.87097+06	.87470+06	-.60453+06	-.34665+04	-.17850+01	-.13315+02	-.34865+04
.100	-.58856+06	-.95433+06	.87898+06	-.86745+06	-.32344+04	-.11024+03	-.89339+02	-.41000+04
.200	-.69048+06	-.11194+07	.10369+07	-.16016+07	-.36324+04	-.15443+03	-.13390+03	-.51175+04
.300	-.12285+07	-.13869+07	.20857+07	-.31946+07	-.46857+04	.95731+03	.98265+03	-.97147+04
.400	-.24080+07	-.14062+07	.53930+07	-.98460+07	-.65879+04	.39513+04	.38732+04	-.22505+05
.450	-.42020+07	-.12096+06	.11538+08	-.22475+08	-.69847+04	.96277+04	.94979+04	-.45191+05

Table 9

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES INCOMPRESSIBLE HYDRODYNAMIC JOURNAL BEARING ANALYSIS
THREE-LOBE BEARING L/D=1.0 LOAD INTO PAD

ECC	ANG-ECC DEGREES	ANG-RXN DEGREES	BEARING FORCE LBS	BEARING FORCE NON DIM	POWER LOSS HP	POWER LOSS NON DIM	MINIMUM CLEARANCE INCH	MINIMUM CLEARANCE NON DIM
.000	.00	.00	.3645-03	.4641-08	.1276+02	.8191+00	.2500-02	.5000+00
.100	328.13	89.43	.2029+04	.2583-01	.1293+02	.8299+00	.2204-02	.4407+00
.200	328.13	89.75	.4733+04	.6026-01	.1356+02	.8706+00	.1859-02	.3717+00
.300	325.96	90.17	.9298+04	.1184+00	.1471+02	.9442+00	.1439-02	.2878+00
.400	321.92	90.72	.1979+05	.2520+00	.1707+02	.1096+01	.9608-03	.1922+00
.450	317.52	90.59	.3425+05	.4361+00	.1919+02	.1232+01	.6626-03	.1325+00

ECC	SIDE LEAKAGE GPM	SIDE LEAKAGE NON-DIM	TOTAL INLET FLOW GPM	TOTAL INLET FLOW NON-DIM	CRITICAL MASS LB	CRITICAL MASS NON-DIM	THRESHOLD FREQ. CPM	THRESHOLD FREQ. NON-DIM
.000	.6912+00	.1625+00	-.7450+01	-.1751+01	.1334+05	.3766-01	.2358+04	.1887+01
.100	.7220+00	.1697+00	-.7419+01	-.1744+01	.1587+05	.4480-01	.2324+04	.1859+01
.200	.7883+00	.1853+00	-.7331+01	-.1723+01	.2377+05	.6710-01	.2269+04	.1815+01
.300	.8823+00	.2074+00	-.7195+01	-.1691+01	.5239+05	.1479+00	.1959+04	.1568+01
.400	.1011+01	.2375+00	-.7023+01	-.1651+01	STABLE			
.450	.1142+01	.2683+00	-.6932+01	-.1629+01	STABLE			

NON DIM SPRING COEFFICIENTS / NON DIM DAMPING COEFFICIENTS

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.13405+00	-.19698+00	.19675+00	-.13450+00	-.10435+00	.12795-03	-.12761-03	-.10435+00
.100	-.12010+00	-.21328+00	.20819+00	-.19874+00	-.97446-01	-.55953-02	-.47660-02	-.12762+00
.200	-.16139+00	-.24830+00	.29663+00	-.32841+00	-.10798+00	.56873-02	.55583-02	-.18938+00
.300	-.24895+00	-.25473+00	.45176+00	-.67716+00	-.11745+00	.24901-01	.25111-01	-.27963+00
.400	-.46234+00	-.22392+00	.10029+01	-.18596+01	-.16162+00	.88394-01	.86341-01	-.56168+00
.450	-.75292+00	.19220-01	.21029+01	-.41641+01	-.15858+00	.21793+00	.21450+00	-.11342+01

DIM SPRING COEFFICIENTS (LBS/IN) / DIM DAMPING COEFFICIENTS (LBS-SEC/IN)

ECC	KXX	KXY	KYX	KYY	DXX	DXY	DYX	DYY
.000	-.21057+07	-.30941+07	.30906+07	-.21128+07	-.12522+05	.15354+02	-.15313+02	-.12522+05
.100	-.18866+07	-.33501+07	.32703+07	-.31218+07	-.11693+05	-.67143+03	-.57191+03	-.15315+05
.200	-.25351+07	-.39003+07	.46595+07	-.51587+07	-.12958+05	.68247+03	.66699+03	-.22725+05
.300	-.39106+07	-.40012+07	.70962+07	-.10637+08	-.14095+05	.29881+04	.30133+04	-.33555+05
.400	-.72625+07	-.35174+07	.15754+08	-.29211+08	-.19394+05	.10607+05	.10361+05	-.67401+05
.450	-.11827+08	.30190+06	.33033+08	-.65409+08	-.19029+05	.26152+05	.25740+05	-.13610+06

extent. The dimensional values are for the following operating conditions:

Bearing Diameter, D = 5 inches

Bearing Length, L = 5 inches

Pad Angle, $\theta_p = 110°$

Machined Clearance, C = 0.005 inches

Journal Speed, N = 5000 rpm

Lubricant Viscosity, $\mu = 2.5 \times 10^{-6}$ lb-sec/in^2

Tilting Pad Bearing

The tilting-pad bearing is the most whirl free bearing, and is the reason they are so extensively used. Performance is not as clearance sensitive as in most bearings. As a rule of thumb the machined radial clearance should be .002 times the radius. The pivot clearance C′ should equal half the value of C so that C′/C = 0.5. The tilting-pad bearing does not provide as much film damping as other types, and they are generally more expensive to manufacture. For high-speed applications, where pad vibrations are high, a common problem is fretting corrosion of the pivots.

Tables 10 and 11 are dimensional performance parameters for tilting pad bearings for the operating conditions given on Table 2. The preload ratio C′/C = 0.5. Table 10 is for a L/D ratio of .5 and Table 11 is for a L/D ratio of 1.

Externally Pressurized or Hybrid Bearings

Because of external pressure, these bearings generally have high load capacity and stiffness. The external flow through the clearance region has a cooling effect, and clearances and tolerances are generally more liberal than hydrodynamic bearings. A primary disadvantage is the necessity of an external source of pressure supply.

They are applied when there is insufficient hydrodynamic generating speed, or where very high load capacity and stiffness are required. They are sometimes applied to prevent whirl, but as discussed previously, rotational speed can offset recess pressures and thus introduce cross-coupling and promote whirl.

Table 10

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES
TILTING PAD ASSEMBLY PROGRAM

TILT-PAD BEARING - L/D=0.5 , CP/C=0.5 , LOAD INTO PAD

STEADY STATE PERFORMANCE

ECCENTRICITY RATIO	LOAD CAPACITY LBS	SIDE LEAKAGE GPM	POWER LOSS HP	MINIMUM FILM IN
.000000	.782535-04	.134778+01	.681701+01	.216298-02
.200000+00	.908581+03	.133508+01	.697021+01	.170496-02
.300000+00	.155316+04	.131907+01	.718044+01	.147622-02
.400000+00	.233385+04	.129603+01	.747448+01	.124765-02
.500000+00	.362154+04	.126528+01	.794062+01	.101857-02
.600000+00	.587998+04	.122902+01	.867650+01	.789650-03
.700000+00	.104537+05	.117854+01	.989553+01	.568930-03

ECCENTRICITY RATIO	INLET FLOW GPM	ANGLE TO ECCENTRICITY	ANGLE TO REACTION
.000000	-.625750+01	.270000+03	.000000
.200000+00	-.624981+01	.270000+03	.900000+02
.300000+00	-.624011+01	.270000+03	.900000+02
.400000+00	-.622621+01	.270000+03	.900000+02
.500000+00	-.620761+01	.270000+03	.900000+02
.600000+00	-.618487+01	.270000+03	.900000+02
.700000+00	-.615445+01	.270000+03	.900000+02

DIMENSIONAL SPRING COEFFICIENTS,LBS/IN

ECC	KXX	KXY	KYX	KYY
.000000	-.117807+07	-.247032+00	.258399+00	-.117807+07
.200000+00	-.125865+07	.255924-01	.179873+00	-.152416+07
.300000+00	-.135755+07	.600032-01	.160494+00	-.206018+07
.400000+00	-.149358+07	-.319897+00	.111931+00	-.320668+07
.500000+00	-.167133+07	-.821991+00	-.809083-02	-.549170+07
.600000+00	-.189552+07	-.111687+01	-.282250+00	-.107203+08
.700000+00	-.212527+07	-.233290+01	.487781-01	-.245667+08

DIMENSIONAL DAMPING COEFFICIENTS,LB-SEC/IN

ECC	DXX	DXY	DYX	DYY
.000000	-.274320+04	.161420-02	-.159249-02	-.274320+04
.200000+00	-.284382+04	.715348-03	-.150699-02	-.315557+04
.300000+00	-.296423+04	.718185-03	-.174423-02	-.376544+04
.400000+00	-.312840+04	-.810821-03	-.195946-02	-.490817+04
.500000+00	-.333698+04	-.223769-02	-.197379-02	-.693475+04
.600000+00	-.359429+04	.365984-03	-.175858-02	-.109527+05
.700000+00	-.391708+04	-.117049-01	-.114547-02	-.218268+05

Table 11

THE FRANKLIN INSTITUTE RESEARCH LABORATORIES
TILTING PAD ASSEMBLY PROGRAM

TILT-PAD BEARING - L/D=1.0 , CP/C=0.5 , LOAD INTO PAD

STEADY STATE PERFORMANCE

ECCENTRICITY RATIO	LOAD CAPACITY LBS	SIDE LEAKAGE GPM	POWER LOSS HP	MINIMUM FILM IN
.000000	.161710-06	.547046-02	.679676+01	.499756-02
.200000+00	.486335+03	.413890+00	.731929+01	.367058-02
.300000+00	.789525+03	.518635+00	.766601+01	.316130-02
.400000+00	.124599+04	.590512+00	.810330+01	.267855-02
.500000+00	.200715+04	.632215+00	.868206+01	.220998-02
.600000+00	.343341+04	.644679+00	.950546+01	.174406-02
.700000+00	.656442+04	.642943+00	.108234+02	.127939-02

ECCENTRICITY RATIO	INLET FLOW GPM	ANGLE TO ECCENTRICITY	ANGLE TO REACTION
.000000	-.212742+02	.270000+03	.000000
.200000+00	-.116512+02	.270000+03	.900000+02
.300000+00	-.110369+02	.270000+03	.900000+02
.400000+00	-.104031+02	.270000+03	.900000+02
.500000+00	-.975117+01	.270000+03	.900000+02
.600000+00	-.908137+01	.270000+03	.900000+02
.700000+00	-.840165+01	.270000+03	.900000+02

DIMENSIONAL SPRING COEFFICIENTS,LBS/IN

ECC	KXX	KXY	KYX	KYY
.000000	.490050+04	.446122-01	-.373899-01	.490050+04
.200000+00	-.481362+05	.735402-02	-.368298-01	-.146557+06
.300000+00	-.821041+05	.594458-01	-.404917-01	-.317103+06
.400000+00	-.124298+06	.466357-01	-.489055-01	-.646882+06
.500000+00	-.173371+06	-.242629+00	-.991995-02	-.134300+07
.600000+00	-.228236+06	.539387+00	.980044-02	-.300598+07
.700000+00	-.293360+06	-.128152-01	.181217+00	-.779922+07

DIMENSIONAL DAMPING COEFFICIENTS,LB-SEC/IN

ECC	DXX	DXY	DYX	DYY
.000000	-.757105+03	.544446-06	.221916-05	-.757105+03
.200000+00	-.122266+04	-.464575-04	-.492440-05	-.120505+04
.300000+00	-.144691+04	-.386528-04	.242970-04	-.163980+04
.400000+00	-.162711+04	-.207253-03	.271425-04	-.235474+04
.500000+00	-.177461+04	-.470742-03	.546406-04	-.363135+04
.600000+00	-.189643+04	.142827-02	.296823-04	-.617205+04
.700000+00	-.204289+04	-.507335-02	.130138-03	-.120677+05

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APPENDIX A — METHOD OF REDUCING TILTING-PAD STIFFNESS AND DAMPING MATRICES

The equations of motion accounting for all degrees of freedom are:

- pitch of i-th pad

$$I_P\ \ddot{\delta}_i = K_{\delta\delta}\ \delta_i + D_{\delta\delta}\ \dot{\delta}_i + K_{\delta X}\ X + D_{\delta X}\ X + K_{\delta Y}\ Y + D_{\delta Y}\ \dot{Y} \qquad \text{(A-1)}$$

- shaft translation

$$M\ \ddot{X} = K_{XX}\ X + D_{XX}\ \dot{X} + K_{XY}\ Y + D_{XY}\ \dot{Y} + \sum_{i=1}^{\text{no. pads}} (K_{X\delta}\ \delta + D_{X\delta}\ \dot{\delta}) \qquad \text{(A-2)}$$

$$M\ \ddot{Y} = K_{YX}\ X + D_{YX}\ \dot{X} + K_{YY}\ Y + D_{YY}\ \dot{Y} + \sum_{i=1}^{\text{no. pads}} (K_{Y\delta}\ \delta + D_{Y\delta}\ \dot{\delta}) \qquad \text{(A-3)}$$

Shaft orbit motion components are described in terms of equivalent coefficients as:

$$M\ \ddot{X} = K'_{XX}\ X + D'_{XX}\ \dot{X} + K'_{XY}\ Y + D'_{XY}\ \dot{Y} \qquad \text{(A-4)}$$

$$M\ \ddot{Y} = K'_{YX}\ X + D'_{YX}\ \dot{X} + K'_{YY}\ Y + D'_{YY}\ \dot{Y} \qquad \text{(A-5)}$$

where the primed coefficients are the equivalent coefficients.

As a consequence of assuming that pad and shaft vibrate at the same frequency, ν rad/sec, periodic motions can be described by:

$$X = X_0 e^{i\nu t};\ Y = Y_0 e^{i\nu t};\ \delta = \delta_0 e^{i\nu t} \qquad \text{(A-6)}$$

Substitution of Equation (3-6) into Equations (3-1 to 3-5) yields:

$$\delta_0 = \frac{(K_{\delta X} + i\nu D_{\delta X})X_0 + (K_{\delta Y} + i\nu D_{\delta Y})Y_0}{-(\nu^2 I_P + i\nu D_{\delta\delta} + K_{\delta\delta})} \qquad \text{(A-7)}$$

$$-\nu^2 M\ X_0 = (K_{XX} + i\nu D_{XX})X_0 + (K_{XY} + i\nu D_{XY})Y_0 + \Sigma\ (K_{X\delta} + i\nu D_{X\delta})\delta_0 \qquad \text{(A-8)}$$

$$-\nu^2 M\ Y_0 = (K_{YX} + i\nu D_{YX})X_0 + (K_{YY} + i\nu D_{YY})Y_0 + \Sigma\ (K_{Y\delta} + i\nu D_{Y\delta})\delta_0 \qquad \text{(A-9)}$$

$$-\nu^2 M\ X_0 = (K'_{XX} + i\nu D'_{XX})X_0 + (K'_{XY} + i\nu D'_{XY})Y_0 \qquad \text{(A-10)}$$

$$-\nu^2 M\ Y_0 = (K'_{YX} + i\nu D'_{YX})X_0 + (K'_{YY} + i\nu D'_{YY})Y_0 \qquad \text{(A-11)}$$

After substituting Equation (A-7) into Equations (A-8) and (A-9) and equating Equation (A-8) to Equation (A-10) and Equation (A-9) to Equation (A-11), expressions for inertia dependent equivalent coefficients can be extracted. Stiffness values are obtained by collecting like real terms; damping results from collecting like imaginary terms. Then,

$$K'_{ij} = K_{ij} - \Sigma\Big\{[K_{j\delta}\ K_{\delta j}\ K_{\delta\delta} + (K_{i\delta}\ K_{\delta j} - D_{i\delta}\ D_{\delta j}\nu^2)\ I_P\nu^2 + (D_{\delta\delta}\ D_{i\delta}\ K_{\delta j} + D_{\delta\delta}\ D_{\delta j}\ K_{i\delta} - D_{i\delta}\ D_{\delta j}\ K_{\delta\delta})] / [(I_P\nu^2 + K_{\delta\delta})^2 + D^2_{\delta\delta}\ \nu^2]\Big\} \qquad \text{(A-12)}$$

$$D'_{ij} = K_{ij} - \Sigma\Big\{[-K_{i\delta}\ K_{\delta j}\ D_{\delta\delta} + (D_{i\delta}\ K_{\delta j} + K_{i\delta}\ D_{\delta j})I_P\nu^2 + (D_{\delta\delta}\ D_{i\delta}\ D_{\delta j}\ \nu^2 + K_{\delta\delta}\ D_{\delta j}\ K_{i\delta} + D_{j\delta}\ K_{\delta j}\ K_{\delta\delta})] / [(I_P\nu^2 + K_{\delta\delta})^2 + D^2_{\delta\delta}\ \nu^2]\Big\} \qquad \text{(A-13)}$$

The subscripts i and j assume the necessary X and Y values to yield expressions for the 8 dynamic coefficients.

NOMENCLATURE

ANG-ECC = Angle from X axis to line of centers
ANG-RXN = Angle from X axis to load reaction
C = Machined bearing clearance, in.
C′ = Pivot film thickness with journal in concentric position, in.
D = Journal diameter, in.
D_{ij} = Damping coefficient in i direction due to a j displacement, lbs-sec/in
e = Journal displacement along line of centers, in.
F_i = Force on shaft in i direction, lbs
ΔF_i = Fluid-film force differential in i direction, lbs
h = Film thickness, in.
h_M = Minimum film thickness, in.
$\overline{H}_M$ = Dimensionless minimum film thickness = h_M/C
I_P = Pitch inertia of pad, lb-sec-in
K_{ij} = Stiffness in i direction due to a j displacement
L = Bearing length, in.
M = Journal mass, lb-sec²/in
M_C = Critical mass, lb-sec²/in
$\overline{M}_c$ = Dimensionless mass parameter = $M_c\omega RC^3/24\mu L^5$
M_i = Moment on i-th pad, in-lbs
N = Journal speed, rpm
p = Film pressure, psi
P = Viscous power loss, HP
$\overline{P}$ = Dimensionless power parameter = $6600CP/6\mu(\omega RL)^2$
θ_p = Pad angle, degrees
Q = Bearing side leakage, gpm
$\overline{Q}$ = Dimensionless bearing side leakage = $7.69Q/RLC$
R = Journal radius, in.
t = Time, sec
Threshold Freq (Non-Dim) = Orbiting frequency/rotational speed
U = Journal surface velocity, in/sec
W = Applied load, lbs.
$\overline{W}$ = Dimensionless load parameter = $WC^2/6\mu\omega RL^3$
W_C = Orbital frequency of critical mass, rad/sec
ΔX = Displacement in X direction, in.
ΔY = Displacement in Y direction, in.

Z = Distance in axial direction, in.

α = Growth factor, 1/sec

β = Complex exponent = $\alpha + i\omega$

γ = Attitude angle (angle between load direction and line of centers), degrees

δ = Pad pitch angle, rad

ϵ = Eccentricity ratio = e/c

θ = Arc length, rad

μ = Absolute viscosity, lb-sec/in^2

ν = Complex exponent for computing reduced matrices for tilting-pad bearings

τ = Period of vibration, sec

ω = Orbital frequency, rad/sec