PRACTICAL CONSIDERATIONS FOR A RATED SPEED SHOP BALANCE

by

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ABSTRACT

The increasing availability of high speed balancing facilities requires the manufacturer and/or ultimate user to seriously consider a rates speed rotor balance. In addition, the development of the "influence coefficient" method of balancing as a complement to the modal technique has established increased balancing reliability.

The present paper develops the aspects of flexible rotor balancing which should be evaluated when considering the necessity and benefits of a rated speed balance. Three classes of industrial turbomachine rotors are investigated using a "rotor response" and an "influence coefficient" computer balance program. These results illustrate the concepts established in the analysis and present a practical example of how the influence coefficient and modal balancing methods complement one another, resulting in a complete approach to balancing.

INTRODUCTION

The increasing availability of high speed balancing facilities and continuing trend toward more flexible, high speed rotors has broadened the potential applications for balancing at rated speeds. In addition, the development of the "influence coefficient" method of balancing as a complement to the "modal" technique has established a methodical approach to balancing which will allow a multimass flexible rotor to be balanced with a reasonable effort. The present paper considers the aspects of flexible rotor balancing which should be considered when evaluating the necessity and benefits of a running speed shop balance.

A running speed balance can often serve as an alternative to a shop mechanical test, especially on a repair or replacement order where suitable cases and/or facilities may not be available. The benefits of having a rotor run at its operating speed prior to field installation or storage are substantial, and may well be the only considerations needed to justify the additional effort. In addition to demonstrating vibration levels, these tests can illuminate other potential mechanical problems such as loose fits, bent shafts and rotor instability.

The majority of turbomachine rotors operating successfully today have been balanced at low speeds (rigid rotor) or, at best at speeds substantially below their design speed. To expand on this observation, the current analysis considers the fundamental synchronous behavior of a flexible rotor and the requirements associated with a high speed, flexible rotor balance. These concepts are illustrated with case studies of three classes of turbomachine rotors for which a "paper" analysis was conducted using a "rotor response" and an "influence coefficient" computer balance program.

From the analysis and case studies, one can better understand how the "influence coefficient" and "modal" methods of balancing complement one another, resulting in a complete approach to high speed balancing.

ROTOR CRITICAL SPEEDS

A critical speed map for a typical multistage rotor is shown in Figure 1. This map presents the undamped critical speeds of the rotor as a function of bearing stiffness. Although the stiffness values on a critical speed map should represent the equivalent stiffness of the bearing and its supporting structure (pedestal), the present discussion considers the pedestals to be infinitely stiff for simplicity. In addition, the two bearings associated with the rotor are assumed to be identical and istropic (equal horizontal and vertical characteristics).

From an inspection of the rotor mode shapes, the critical speed map can be divided into three regions with respect to the shaft stiffness-bearing stiffness ratio. Note that the mode shapes actually represent the shape of the rotor as it performs synchronous whirl about the line of bearing centers.

> *Region I*: High shaft-bearing stiffness ratio. In this region, the first two critical speeds are characterized by rigid body motion as demonstrated by the mode shapes and the direct variation of the critical speed with the square root of bearing stiffness. The third mode, however, is a shaft bending (elastic) mode associated with essentially zero bearing stiffness, i.e. the free-free mode.

> *Region II:* Shaft and bearing stiffness of the same order. Here the mode shapes associated with the first and second critical speeds are a composite of rigid body and shaft bending modes. This can be visualized as elastic shaft deformations (shaft bending) superimposed on rigid shaft displacements.

Region III: Low shaft-bearing stiffness ratio. This region represents the area where the bearings appear to be rigid with respect to the shaft. The rotor will whirl with mode shapes composed almost entirely of elastic shaft deformations.

The stiffness values associated with fluid film bearings will normally be in the range of Region II, where some bearing motion occurs. Since the damping associated with the bearing oil film is normally the primary means of dissipating energy in the rotor-bearing system, Region III represents a potentially undesirable area of operation in that it corresponds to a rigid bearing system, resulting in undamped critical speeds. In addition, rotors in this area which operate above the first critical speed may be subject to nonsynchronous vibrations.

ROTOR BALANCE CRITERIA

Consider a symmetric, two bearing rotor under synchronous whirl. Divide the rotor into "n" segments along its axial length "L" to the degree necessary so that each segment may be considered as a concentrated mass M_i acting at a distance l_i from the shaft end. Associated with each segment "i", let m_i be an unbalance mass located at a distance e_i from the axis of rotation at zero speed; that is, the products m_ie_i represent point mass unbalances U_i resulting from the difference between the segment geometric and rotational axis, all general point unbalances, and any balance correction weights.

Assume that the mode shape of the deflected rotor may be described at each segment by the sum of a rigid body displacement Y_i, and an elastic displacement δ_i. For simplicity, let the unbalances lie in an axial plane. Thus, a summation of forces and moments for zero dynamic bearing forces requires that for a given angular speed "ω",

$$\sum F \equiv \sum_i M_i(Y_i+\delta_i)\omega^2 + \sum_i m_i(e_i+Y_i+\delta_i)\omega^2 = 0 \quad (1)$$

$$\sum M_{x=o} \equiv \sum_i M_i(Y_i+\delta_i)l_i\omega^2 + \sum_i m_i(e_i+Y_i+\delta_i)l_i\omega^2 = 0 \quad (2)$$

At low speeds, the rotor acts as a rigid body and the elastic displacements will be zero ($\delta_i \sim 0$). In addition, zero bearing forces require zero journal displacements. Since the shaft is acting as a rigid body, a zero shaft displacement at the journal segments requires that all $Y_i=0$. Thus, for a rigid rotor balance, Equations (1) and (2) are simply the equilibrium of forces and moments associated with the unbalances $m_ie_i \equiv U_i$,

$$\sum_i m_ie_i \equiv \sum_i U_i = 0 \quad (3)$$

$$\sum_i m_ie_il_i \equiv \sum_i U_il_i = 0 \quad (4)$$

Incorporation of Equations (3) and (4) (and hence $Y_i=0$) into Equations (1) and (2) yields the following additional relationships pertaining to a high speed balance.

$$\sum_i (M_i+m_i)\delta_i = 0 \quad (5)$$

$$\sum_i (M_i+m_i)\delta_il_i = 0 \quad (6)$$

Since the above equations must be satisfied for all mode shapes, even in the case of $m_i=0$, the most general solution requires that the elastic displacements vanish

$$\delta_i = 0 \quad (7)$$

It can be demonstrated both analytically and experimentally that the major contribution to the elastic deflection comes from the mode shape of the nearest critical speed [1]. For example, the first critical speed mode will normally describe the elastic deflections quite adequately in the speed range of 50% to 140% of the critical. Thus the deflection at segment "i" due to an unbalance at segment "i" may be written as

$$\delta_i = A_if(x_i)U_i \quad (8)$$

where f(x) describes the mode shape and A_i is a scale factor.

From the mode shape, the deflection at segment (j) due to an unbalance at segment (i) will be

$$\delta_j = A_if(x_j)U_i \quad (9)$$

which, from the Maxwell Reciprocal Theorem of Elasticity [2], may be transposed to

$$\delta_i = A_if(x_j)U_j \quad (10)$$

Therefore, the condition which satisfied Equation (7) is

$$\sum_j f(x_j)U_j = 0 \quad (11)$$

Note that the terms $A_if(x_j)$ in Equation (10) are the "influence coefficients" which will be discussed in the next section.

Extending Equations (3), (4) and (11) to the general case of a rotor with distributed mass M(x) and unbalance U(x) results in the following three governing equations.

$$\int_0^L U(x)dx = 0 \quad \text{rigid mode static balance} \quad (12)$$

$$\int_0^L x\,U(x)dx = 0 \quad \text{rigid mode dynamic balance} \quad (13)$$

total rotor balance

$$\int_0^L f(x)U(x)dx = 0 \quad \text{elastic mode balance (zero elastic displacements)} \quad (14)$$

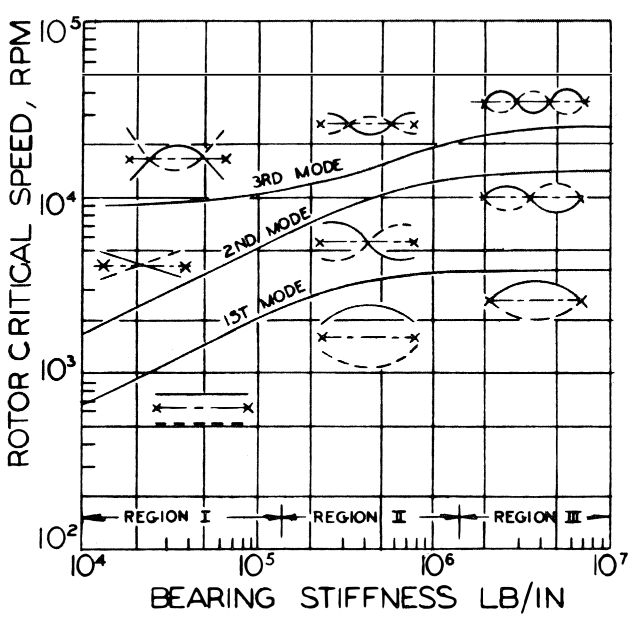


Figure 1. Critical Speed Map.

The above equations, which apply to each of two orthogonal axial planes chosen as coordinate planes on the rotor, represent the fundamental relationships for rotor balancing [3]. Note that the distributed unbalance U(x) includes balance correction weights and that Equation (14) must be satisfied for each flexible mode shape f(x) under consideration.

In Case Study I, it is demonstrated that the balancing criteria for the first flexural mode and the free-free mode are identical. With this result and the fact that all three governing relationships given by Equations 12-14 must be satisfied for a flexible rotor, it can be concluded that a flexible rotor, when balanced throughout its speed range on relatively stiff bearings, will remain in balance if operated with softer bearings. Referring to Figure 1, a flexible rotor balanced in Region III will thus retain its balance when operated in Region I or II.

INFLUENCE COEFFICIENT VS MODAL BALANCING

The development of the influence coefficient method of balancing has stimulated discussion as to which method is preferred — influence coefficient or modal technique. Hence, some background on each technique is appropriate prior to considering when to high speed balance.

Both methods have the same objective: to place compensating balance weights opposite to the existing rotor unbalances. The rule of balancing is to make the corrections in the planes of unbalance. Since the unbalance distribution is usually not known and the use of a large number of balance planes impractical, any realistic balancing procedure must obtain a satisfactory balance with a minimum number of balance planes. As indicated in the following discussion, the two methods are generally used together, resulting in the most efficient approach to flexible rotor balancing.

Rigid Rotor

In the case of low speed, rigid rotor balancing, two balance planes are sufficient for satisfying Equations (12) and (13) and thus compensate for the inherent rotor static and dynamic unbalance. From the measured bearing reactions on the unbalanced rotor, the appropriate compensating weights for the two balance planes selected may be calculated from simple statics (Equations 12 and 13). This calculation is in fact the function that modern balancing machines perform electronically. This procedure is actually an elementary form of the influence coefficient method, where the "influence coefficients" relating the bearing reactions to an applied compensating weight are known a priori from statics.

Flexible Rotor

After rigid rotor balancing, Equation (14) states that the integral of the product of the flexible mode shape and the unbalance distribution must be zero for a high speed (flexible mode) balance. Thus, in the case of a sinusoidal family of mode shapes $f(x) = \text{Sin}\frac{K\pi x}{L}$, it is evident that the first flexural mode may be minimized with a single correction weight, and the Kth flexural mode with "K" correction weights. In order to maintain rigid rotor balance, two correction weights, added at mode nodal points, are needed to compensate for the flexible mode correction weights. Thus, (K+2) correction planes are normally used in order to balance a rotor in the vicinity of the Kth critical speed (see Figure 2).

At this point, the modal information has provided the number and approximate location of the balance planes. However, the correction weights are undefined since the original unbalance distribution is unknown. In continuing with the modal technique, one would now select the appropriate correction weight set for the mode of interest (see Figure 2) and make an estimate of the magnitudes of the flexible mode weights; the two rigid mode compensating weights are then determined. This trial set of weights is applied and the rotor operated at the subject speed(s). The procedure is repeated until an acceptable balance is obtained.

With the influence coefficient method, the above results concerning the number and approximate location of balancing planes are not essential. This concept is of great importance when the associated mode is complicated or in fact unknown. In addition, the rotor may be balanced at several speeds simultaneously. In many cases, however, the use of modal information minimizes the number of trial weights needed and maximizes their effect; i.e. the influence coefficient method is often used primarily to determine the correction weight magnitudes and circumferential position as opposed to plane number and location. Having selected the balance planes, a trial weight is placed in a plane and the rotor run at the subject speed(s). This procedure is repeated for each balance plane. From these results and the original run, a set of "influence coefficients" are determined which relate the reactions at the measuring points to unit weights in the planes.

With the influence coefficients, a set of equations can be written which expresses the total reactions at the measuring points as a function of all possible correction weight combinations. Technically, several measuring points along the rotor are needed in order for the influence coefficient method to effectively minimize the flexible mode displacements. This method is used successfully, however, with bearing measurements

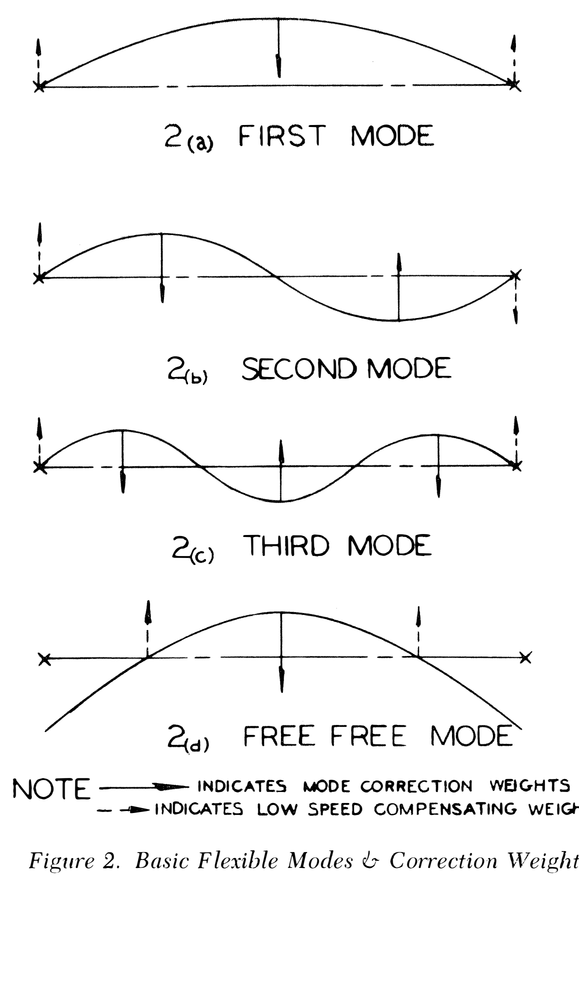


Figure 2. Basic Flexible Modes & Correction Weights.

alone, since these reactions will be a function of the elastic bending of the rotor unless located at a nodal point.

When measurements are taken at two (2) bearings and N speeds, there will be (2N) equations of the above type. If the number of balance planes equals (2N), then the correction weights for zero bearing reactions at every balance speed N may be calculated directly; this procedure is entitled the "exact point-speed" method. If the number of balance planes is less than (2N), there are more equations than unknown correction weights and an optimization technique is used whereby correction weights are determined which will yield the best results over all the speeds considered; this procedure is known as the "least squares" method of influence coefficients.

WHEN TO HIGH SPEED BALANCE

As stated in the Introduction, a shop balance at rated speed is often used primarily to verify mechanical integrity and operation of the rotor with the expectation that the measured unbalance amplitudes will confirm the adequacy of the low and/or medium speed production balance. This section will address the factors which will have a direct effect on whether a rated speed balance should be stipulated as part of the normal production procedure.

Based on the discussions contained in previous sections, a rated speed shop balance should be considered when:

1. The actual field rotor operates with characteristic mode shapes significantly different than those which occur during a standard production balance.

   This is a primary consideration since flexible rotor balancing must be performed with the rotor whirl configuration approximating the mode in question.

2. The operating speed(s) is in the vicinity of a major flexible mode resonance (damped critical speed).

   As these two speeds approach one another, a tighter balance tolerance will be required. Of special concern are those designs which have a low rotor-bearing stiffness ratio or bearings in the vicinity of mode nodal points.

3. The predicted rotor response of an anticipated unbalance distribution is significant.

   This type of analysis may indicate a sensitive rotor which should be balanced at rated speed. It will also indicate which components need to be carefully balanced prior to assembly.

4. The available balance planes are far removed from locations of expected unbalance and thus relatively ineffective at the operating speed.

   The rule of balancing is to compensate in the planes of unbalance when possible. A low speed balance utilizing inappropriate planes can have an adverse effect on the high speed operation of the rotor.

   In many cases, implementation of an incremental low speed balance as the rotor is assembled will provide an adequate balance, since compensations are being made in the planes of unbalance. This is particularly effective with designs incorporating solid rotor construction.

5. A very low production balance tolerance is needed in order to meet rigorous vibration specifications.

   Vibration levels below those associated with a standard production balanced rotor are often best obtained with a multiple plane balance at the operating speed(s).

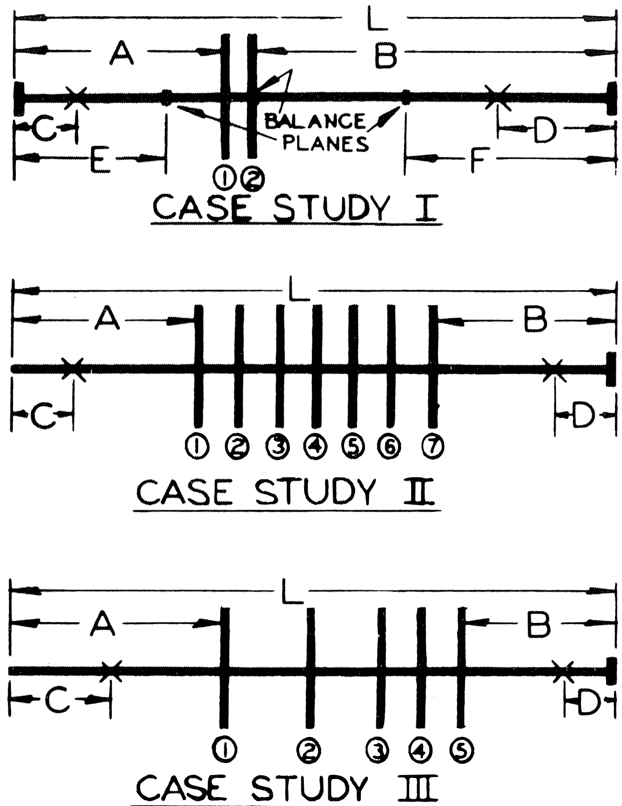
6. The rotor or other similar designs have experienced field vibration problems.

   Even a well designed and constructed rotor may experience excessive vibrations due to improper or ineffective balancing. This situation can often occur when the rotor has had multiple rebalances over a long service period and thus contains unknown balance distributions. A rotor originally balanced at high speed should not be rebalanced at low speed.

Three case studies on actual rotors are presented in the following section which will further illustrate these points.

CASE STUDY I

This study considers a two stage turbine rotor supported on tilting pad bearings with both turbine discs located near the mid-span. The relative dimensions are given in Figure 3. Based on the rotor-bearing stiffness ratio, the field rotor will



	CASE STUDY I	CASE STUDY II	CASE STUDY III
A/L	0.35	0.3	0.35
B/L	0.6	0.3	0.25
C/L	0.1	0.1	0.17
D/L	0.2	0.1	0.08
E/L	0.25	—	—
F/L	0.35	—	—

Figure 3. Rotor Configurations.

operate in the vicinity of the Region I-Region II boundary shown in Figure 1. Thus, the first and third critical speeds of the rotor will involve elastic deformation, with the third critical being the free-free bending mode. The rotor at operating speed will reflect this third mode shape.

Historically, the bare rotor without blades or couplings has been balanced at approximately 15% and 70% of the rated speed. At the 15% speed, the bare rotor mode shape is characteristic of a first flexural mode superimposed on rigid body motion, whereas the 70% speed is associated with a second type rigid body mode (Figure 4). At rated speed, the bare rotor would basically retain the rigid mode shape. Three balance planes are used and are located at one of the stages and at each of the seal areas, approximately midway between the stages and the bearings (Figure 3). After balancing, the blades and coupling halves are assembled with intermediate low speed balance corrections at the location of each assembled component.

Consider the case where the intermediate balance corrections to the assembled components is exact; then only the original bare rotor unbalance and corrections need be considered. Since the rotor is of a simple geometry with integral discs, the primary source of bare rotor unbalance will be that associated

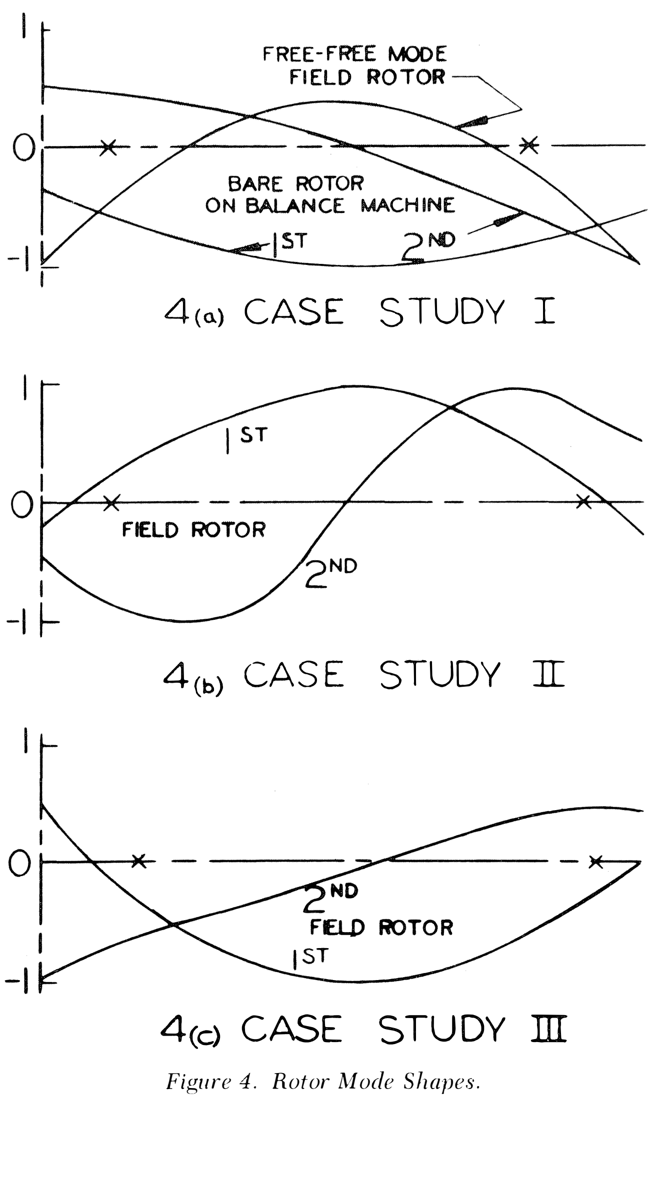


Figure 4. Rotor Mode Shapes.

with the grinding of the journals. This operation is held to a concentricity of 0.0005 inches TIR and therefore each journal could be offset by 0.00025 inches with respect to the rotor geometric center. This suggests three basic unbalance distributions, namely a uniform offset of the entire rotor, a triangular offset, and a dynamic offset (journal offsets out of phase).

Since the bare rotor is balanced in the vicinity of the first flexural and second rigid modes, one would expect the fully assembled rotor to have residual unbalance at the operating speed. This may not be the case, however, since the first flexural mode and the free-free mode have similar shapes. From Figure 4, the elastic (flexural) portion of the first mode may be approximated by $f_1(x)=0.5\ \mathrm{Sin}\frac{\pi x}{L}$ and the free-free mode by $f_3(x)=1.3\ \mathrm{Sin}\frac{\pi x}{L}-1$. Substituting into Equation (14) for flexible balancing gives

$$\int_0^L U(x)f_1(x)dx=.5\int_0^L U(x)\mathrm{Sin}\frac{\pi x}{L}\,dx=0 \qquad (15)$$

$$\int_0^L U(x)f_3(x)dx=1.3\int_0^L U(x)\mathrm{Sin}\frac{\pi x}{L}\,dx-\int_0^L U(x)dx=0 \qquad (16)$$

The last term in Equation (16) is zero from the first rigid rotor criteria (Equation 12) and the flexible balancing requirements for the first flexural and the free-free modes are therefore identical. It should be noted that Equations (15) and (16) are the basis upon which a soft bearing balance machine functions; the rotor is balanced in the free-free mode for field operation through the first flexural mode.

Since three balance planes are used, the rigid rotor and the first flexural mode can be balanced using speeds up to 15% rated speed. Incorporating speeds up to the first critical would enhance the quality of balance. Balancing at 70% of rated speed provides a good rigid rotor dynamic balance because of its rigid body mode. An improved balance can only be obtained by a rated speed balance with the fully assembled rotor. A bare rotor balance at rated speed may not be sufficient since it would basically retain a rigid body mode shape rather than a free-free configuration.

Table I presents the results of response calculations on this rotor-bearing system. The assumed inherent unbalance corresponds to a uniform .00025 inch rotor eccentricity. The results are tabulated in terms of inch-ounce unbalance by converting the sum of the absolute bearing reactions into unbalance at each speed. The balancing tolerance for this rotor is 0.4 in-oz per plane. The correction weights for the balanced rotors were determined using influence coefficients obtained from response calculations with trial weights. Realistic bearing stiffnesses and damping were incorporated.

The response data presented in Table I may be summarized as follows:

1. If the assembled rotor were built-up without a bare rotor balance, the effects of the assumed unbalance distribution would amount to approximately three balance tolerances at the design speed (Run 2). In fact, the high speed response is better than that at low speed on the basis of bearing reactions. This is due in part to a

bearing location being near the free-free mode nodal point (Figure 4).

2. An assembled rotor with a good bare rotor balance will operate below one (1) balance tolerance at rated speed (Run 4).
3. A three plane balance of the assembled rotor at rated speed will provide a better operating balance at the expense of the low speed balance (Runs 4 and 5).
4. A rotor balanced at rated speed should not be rebalanced at low speeds as inferred from Item (3).
5. The first flexural and the free-free mode cannot be fully balanced simultaneously with the present balance plane locations since improvement at one mode causes a deterioration at the other (Runs 4 and 5). Changing the end plane locations would allow a simultaneous balance (Run 6).

In summary, these rotors should operate satisfactorily with a 70% rated speed balance. However, further improvement in balance could be accomplished with a rated speed balance. Because of the rated speed mode shape, the response at the rated speed will have a strong dependence on the assembled coupling unbalance.

Run No.	Description	15% Rated Speed	70% Rated Speed	Rated Speed	Over-Speed
1	unbalanced bare rotor	3.2	2.1	-	-
2	unbalanced assembled rotor	3.4	1.9	1.1	0.6
3	balanced bare rotor @15% and 70% rated speed	0.03	0.06	0.06	0.24
4	assembled rotor with bare rotor balance of Run 3	0.03	0.08	0.15	0.24
5	assembled rotor re-balanced @ 15%, 70%, rated and overspeed	0.32	0.06	0.06	0.05
6	assembled rotor with bare rotor balance 15% and 70% rated speed - end balance planes moved outward	0.03	0.03	0.08	0.11

TABLE I. CASE STUDY I: CALCULATED TOTAL BEARING REACTIONS (in-oz)

CASE STUDY II

This case considers a seven stage centrifugal compressor supported on tilting pad bearings as illustrated in Figure 3. The rotor-bearing stiffness ratio places the unit in Region II of the critical speed map, Figure 1. The rotor operates below the second critical speed but has a second flexible mode shape at rated speed with a nodal occurring between stages four and five.

The standard method of production involves the assembly of balanced wheels and shaft with interference fits. Shaft run-out inspections and low speed balance corrections are made after each wheel assembly. After assembly of the last wheel, the rotor is balanced at low speeds (10% of rated speed) on the first and last stages to within one tolerance per plane. This balancing procedure did not provide consistent rotor performance and has been successfully revised to include multiplane balancing at 65% of rated speed, where the rotor exhibits some second mode flexural characteristics.

From the concepts presented in previous sections, it can be concluded that this rotor cannot be consistently balanced for field operation by means of a two plane, low speed balance. To illustrate this point, response calculations were made assuming each stage to have a one tolerance unbalance (0.425 in-oz per plane) with stages one thru four out of phase with stages five, six and seven.

These results are presented in Table II where the sum of the absolute bearing reactions are again expressed in terms of unbalance (in-oz). The following conclusions are obtained for the unbalance configuration under study.

1. The response of the unbalanced rotor (Run 1) and of the rotor balanced at 10% rated speed to one tolerance per plane, dynamic, (Run 2) indicates that the unbalance at rated speed is approximately five (5) times the low speed value.
2. A good low speed balance is required to obtain operating bearing forces on the order of one balancing tolerance (Run 3).
3. If the rotor is balanced on the first and last stage at both 10% and 65% of rated speed, the resulting two plane balance yields no improvement at running speed balance compared to item 2 above (Runs 4 vs 3).
4. A four plane influence coefficient balance utilizing the two out pairs of stages results in improved characteristics. The same weight corrections were obtained for balance speed sets of (1) 10% and 65% rated speed and (2) 10%, 65% and 100% rated speed (Run 5).
5. From item 4 above, multiplane balancing at speeds above 65% rated speed is not necessary. However,

Run No.	Description	10% Rated Speed	65% Rated Speed	Rated Speed	Over-Speed
1	unbalanced rotor	0.91	1.4	4.1	6.5
2	rotor balanced at low speed on stages 1 and 7 to within one tolerance per plane (assumed out of phase)	0.43	0.72	2.2	3.5
3	rotor balanced at low speed on stages 1 and 7	0.01	0.19	0.39	0.77
4	rotor balanced at 10% and 65% speed on stages 1 and 7	1.1	0.05	0.41	0.84
5	rotor balanced on stages 1,2,6 and 7 at 10% and 65% speed <u>or</u> at 10%, 65% and rated speed	0.03	0.03	0.15	0.25
6	rotor balanced at low speed on stages 2 and 7	0.007	0.12	0.12	0.16

TABLE II. CASE STUDY II:CALCULATED TOTAL BEARING REACTIONS (in-oz) balancing at higher speeds would involve larger reactions and a more pronounced mode shape, thus providing increased balancing accuracy.

6. Since the location of the low speed balance planes influences the high speed balance (Run 6 vs 3), the actual unbalance distribution will in turn affect the results of a low speed balance utilizing specific planes. Therefore, consistent rated speed performance will not be obtained among those rotors balanced only at low speeds.

The above conclusions indicate that a multiplane balance at or above 65% of rated speed will be needed in order to provide consistently balanced rotors. The variation of high speed response with respect to the selection of the two planes used during low speed balance (item 6 above) raises the question as to whether this type of balancing procedure could ever lead to successful flexible mode operation. In other words, can a two plane rigid rotor balance be equivalent to a (K+2) plane balance at the Kth mode. An affirmative answer is concluded from the following paragraph and provides additional insight into the successful operation of some flexible rotors with only a rigid rotor balance.

Consider the first and second flexural modes of a classical symmetric rotor supported by rigid bearings. Associate a symmetric and antisymmetric balance distribution with the first and second mode, respectively, as shown in Figure 5. The unbalance per unit length is constant, but does not necessarily have to extend along the entire rotor length. The balancing requirements given by Equations (12), (13), and (14) can be investigated with respect to determining the location of two symmetric balance planes which would satisfy both the rigid and flexible mode criteria. The results are given in Figure 5 and demonstrate that such planes do exist, and hence a two plane low speed balance can be equivalent to a conventional (K+2) plane flexible balance. Two additional concepts are evident from Figure 5. First, the balance planes will be located within the extent of the unbalance, i.e. balancing external to the unbalance distribution will not suffice. Secondly, the correction planes for the second mode do not coincide with the antinode (quarter span) as is often assumed when selecting second mode balance planes.

CASE STUDY III

This case study involves a rotor, running in the vicinity of its second critical speed, which operates successfully with only a two plane low speed balance. The rotor is a five stage centrifugal compressor of approximately four times the weight of the unit in Case Study II (Figure 3). The rotor-bearing stiffness ratio is such that the unit operates at the lower extremity of Region II on the critical speed map, Figure 1. The first mode is primarily a flexural mode, whereas the second mode is a rigid-elastic composite as shown in Figure 4.

The assembly of this rotor is similar to that discussed in the previous case study. An antisymmetric balance distribution is again assumed, with the first two stage unbalances being out of phase with the remaining three stages. The unbalance per stage is taken to be 1.8 in-oz, equivalent to one balancing tolerance. The final low speed balance is carried out on the first and last stage.

Table III summarizes the rotor response calculations in a manner identical to that of the previous studies. These results demonstrate that the rotor will operate quite satisfactorily at rated speed if a good two plane low speed balance is obtained. In fact, the original unbalance at low speed is only magnified by a factor of two at the operating speed, thus indicating moderate sensitivity to this flexural mode.

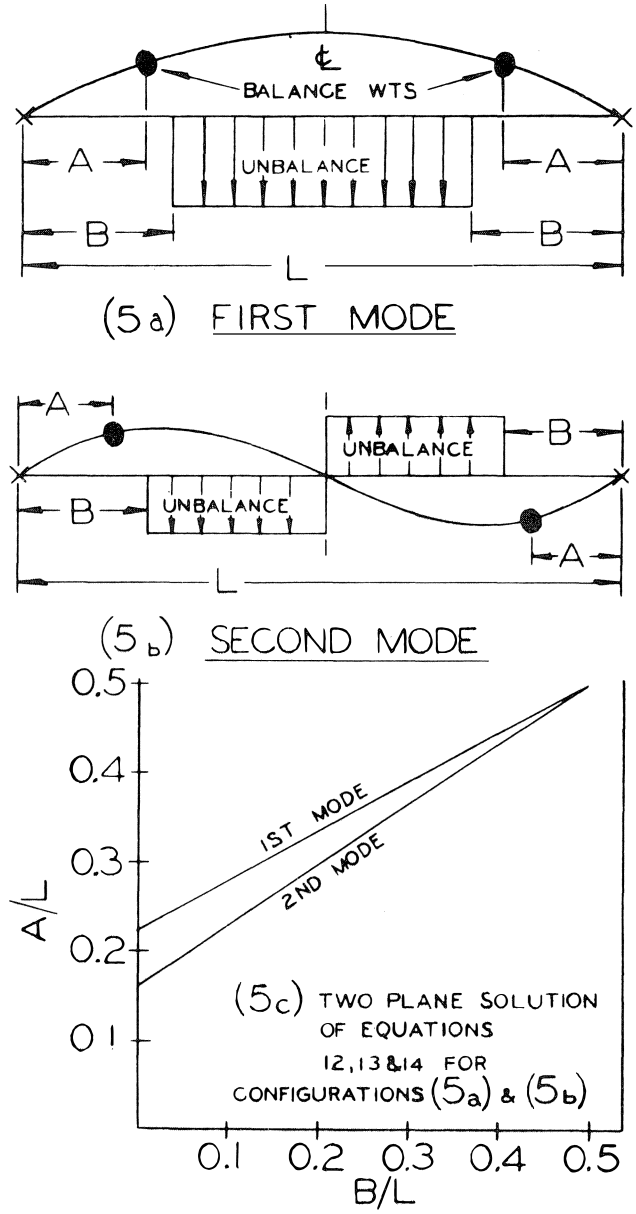


Figure 5. Two Plane Flexible Rotor Balance.

Run No.	Description	10% Speed	Rated Speed	Over-Speed
1	unbalanced rotor	2.8	5.5	6.3
2	rotor balanced at low speed stages 1 and 5 to within one tolerance per plane (assumed out of phase)	1.9	3.8	4.4
3	rotor balanced at low speed on stages 1 and 5	0.003	0.19	0.26

TABLE III. CASE STUDY III: CALCULATED TOTAL BEARING REACTIONS (in-oz)

The reason for this rotor behavior is evident from an inspection of the second mode shape in Figure 4. The elastic deformation occurs primarily outside of the area in which unbalances are located. Therefore, the flexible balance criteria of Equation (14) will be satisfied, and only the rigid body balance requirements of Equations (12) and (13) need be fulfilled.

SUMMARY & CONCLUSIONS

The most important benefit of conducting a rated speed shop balance on any high speed turbomachine rotor is that it provides an opportunity to verify the mechanical integrity, stability and dynamic behavior of the rotor. This can be of major importance on a repair or replacement order when a shop mechanical test may not be practical. In addition, a rated speed balance provides a means for correcting problem rotors through the application of distributed balance corrections based on dynamic measurements along the length of the rotor.

The circumstances which would indicate that a rated speed balance may be needed are:

1. When a flexible rotor operates in a mode significantly different than that which occurs during standard production balance.
2. When the operating speed(s) is in the vicinity of a major flexible mode resonance.
3. When a response analysis with anticipated unbalances indicates a sensitive rotor.
4. When the available balance planes are far removed from the locations of expected unbalance.
5. When a very low production balance tolerance is required in order to meet rigorous vibration specifications.
6. When the particular rotor or similar rotor designs have encountered reoccurring field vibration problems.
7. When the rotor contains unknown balance distributions because of multiple rebalances during long periods of service.
8. Whenever the rotor was originally balanced as a flexible rotor (high speed balance).

Modal analysis complements the influence coefficient method of balancing. With a sufficient number of balancing planes and measuring stations, any rotor may be balanced at speed(s) with the influence coefficient technique; however, modal information provides the knowledge necessary to perform this operation in the most expeditious manner. With available rotor response and balancing computer programs, sound balancing decisions can be made prior to the installation of the first trial weight.

REFERENCES

1. Gunter, E. J., Barret, L. E., and Allaire, P. E., "Balancing of Multimass Flexible Rotors" Proceeding of the Fifth Turbomachinery Symposium, Texas A&M University, 1976
2. Den Hartog, J. P., *Advanced Strength of Materials*, McGraw Hill, 1952, p. 226.
3. Kellenberger, W. "Balancing Flexible Rotors on Two Generally Flexible Bearings" Brown Boveri Review, 1967.