

EFFECTS OF FLUID-FILLED CLEARANCE SPACES ON CENTRIFUGAL PUMP AND SUBMERGED MOTOR VIBRATIONS

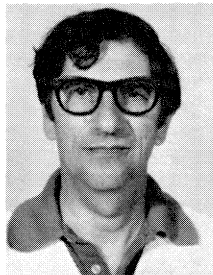
by

Henry F. Black

Professor of Mechanical Engineering

Heriot-Watt University

Edinburgh, U.K.



Henry F. Black graduated with honors in Mechanical Engineering from the University of London in 1949, and then worked with the Bristol Aeroplane Company on vibrations analysis. He joined the staff of the Mechanical Engineering Department at Heriot-Watt University in 1952, progressing to his present rank of professor of Mechanical Engineering, being elected to a Personal Chair in 1975.

Dr. Black has been active in research on rotor dynamics, ship motion reduction and surge instabilities of boiler feed networks, being the author of about 30 scientific and technical publications. He is the winner of the James Clayton Fund Prize of the Institution of Mechanical Engineers for published work on journal bearing performance in super-laminar regimes. He is an active consultant to a number of companies and agencies. A substantial part of his research and consultancy has been devoted to vibrations problems of centrifugal pumps.

ABSTRACT

Annular clearance spaces, such as impeller wearing rings, inter-stage bushes and balance drums in centrifugal pumps, or the fluid-filled clearance around the armature of a submerged motor, at a first glance, present the appearance of very weak journal bearings. The fluid has very low viscosity compared with lubricating oils, and clearances are much larger than in proper journal bearings. Two factors invalidate this deceptive view. Firstly, the Reynolds numbers of flows in these spaces are very high, rendering flows turbulent. This greatly increases effective viscosities. Secondly, fluid inertia effects not normally encountered in journal bearings become basic determining factors of the bearing forces raised by internal clearances of the type indicated.

In clearance spaces of centrifugal pumps, the high pressure differences across the clearance spaces bring about a sudden pressure drop owing to acceleration into the space. This converts the clearance space into a powerful hydrostatic bearing. As the journal rotates and vibrates, damping and cross-coupling forces are developed as well as radial stiffness. Here, fluid inertia is significant only in the entry loss effect which controls hydrostatic stiffness.

In the submerged motor, where the fine clearance surrounds a large length of shaft, fluid inertia effects in the clearance space itself become dominant.

Both types of clearance space alter the synchronous response and stability limits of the rotors (beyond recognition), as compared with the performance in air. In centrifugal pumps, generally, the internal clearances both raise and damp critical speeds and improve stability margins. There are, however, special circumstances where these clearances become de-

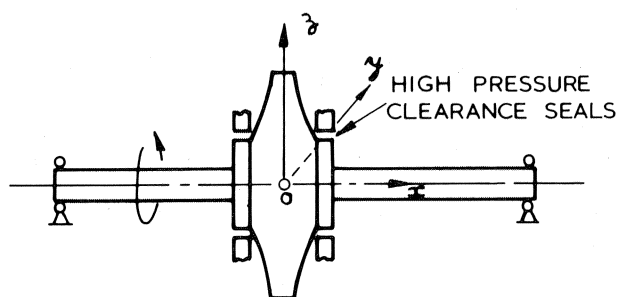
stabilizing elements. In submerged motors, critical speeds are substantially reduced and damped, and the type of destabilization normally associated with circular journal bearings may be encountered. The paper explains these phenomena and illustrates them with simple calculations.

INTRODUCTION

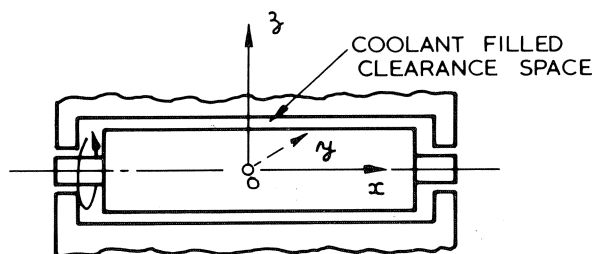
The rotors of all turbomachines are subject to lateral forces, often exhibiting cross-coupling arising from the dynamic properties of fluid films in fine clearance spaces like bearings and also from oscillatory phenomena in the working fluid. The latter include aerodynamic cross-coupling in compressors, forces arising in glands of compressors, gas turbines and steam turbines and forces arising in the clearance spaces around turbine blade rows.

In flexible rotors, the changes mentioned so far mostly have a minor effect on critical speeds. At relatively low speeds, including the first critical, they contribute to small damping. At high speeds, they imply the risk of unstable sub-synchronous whirling.

In centrifugal high speed water pumps and motors fully submerged in water, the fluid in the fine clearance spaces surrounding the rotor (Figure 1) exerts very significant journal bearing-like forces, which radically alter the synchronous



SINGLE - STAGE DOUBLE - ENTRY PUMP



ROTOR OF SUBMERGED MOTOR

Figure 1. Rotor of Submerged Motor.

vibrations response and may also induce subsynchronous whirl. The effects of the fine clearances in the two types of machine show marked qualitative differences.

In the case of the high pressure clearance seals in centrifugal pumps, the action is primarily hydrostatic together with some forward driving cross-coupling and damping. All of these are proportional to the pressure drops across the seals which in turn increase, almost in proportion, to the square of running speed. The dynamic coefficients and their effects on rotors have been studied theoretically and experimentally by Lomakin, Black, Jessen, Murray, et al. [1, 2, 3, 4, 5, 6].

In submerged motors, the coolant filled clearance space around the armature acts like a simple non-cavitating journal bearing, but with vastly exaggerated fluid inertia effects. These arise partly because the clearance is much greater than normal in journal bearings and partly because the "bearing" envelops most of the rotor. The dynamic coefficients and their effects on a model rotor have been studied by Fritz [7].

EFFECTS OF HIGH PRESSURE SEALS IN CENTRIFUGAL PUMPS

The lateral forces exerted by the seals indicated in Figure 1 are partly hydrostatic and partly hydrodynamic. Both of these components are greatly influenced by the fluid flow regime in the clearance space, which is very unlike that in normal journal bearings; the flow is dominantly axial, and thoroughly turbulent. The hydrostatic action is caused by the wholly inertial pressure drop at the inlet, as indicated in Figure 2. Hydrodynamic action arises due to shaft rotation and vibration. The derivations of appropriate dynamic coefficients and some experimental results supporting the theory are treated in [1], [2] and [3]. Summarizing, the linearized lateral forces are of form

$$\begin{bmatrix} F_y \\ F_z \end{bmatrix} = - \begin{bmatrix} k\omega^2 & k_c\omega^2 \\ -k_c\omega^2 & k\omega^2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} - \begin{bmatrix} 2k_c\omega & 0 \\ 0 & 2k_c\omega \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} \quad (1)$$

omitting some insignificantly small terms related to fluid accelerations in the clearance space. The dynamic coefficients are correlated to speed because the pump head, and hence, the pressure drop across seals is nearly proportional to the square of speed.

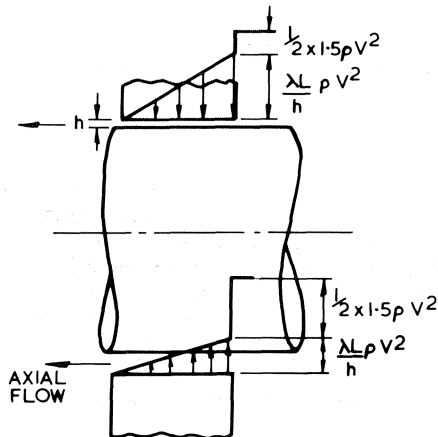


Figure 2. Hydrostatic Bearing Action of Eccentric High Pressure Seals.

The constants k and k_c , which have the physical dimensions of mass, are functions of P_o , ω_o , R , L , c , ρ and λ .

It is first instructive to examine synchronous whirling of a simple single-mass rotor as indicated in Figure 1. The bearings are supposed to have a simple radially symmetric stiffness and the seals at the double-entry impeller are the only source of system damping. If response to mass unbalance is synchronous, circular whirling may be assumed. Then, with

$$r = y + jz, \\ m\ddot{r} + 2k_c\omega\dot{r} + (k_s + k\omega^2 - jk_c\omega^2)r = m\Delta\omega^2 e^{j\omega t} \quad (2)$$

and the synchronous whirl radius becomes

$$|r| = \frac{m\Delta\omega^2}{\sqrt{(k_s - \bar{m}\omega^2)^2 + (k_c\omega^2)^2}}, \quad (3)$$

where $\bar{m} = m - k$.

Note that, provided $k < m$, the peak amplitude occurs at the critical speed

$$\omega_c = \sqrt{\frac{k_s}{m - k}} \quad (4)$$

This is always greater than the critical speed in air,

$$\omega_n = \sqrt{\frac{k_s}{m}} \quad (5)$$

The peak amplitude is

$$|r|_{\max} = \frac{m}{k_c} \cdot \Delta \quad (6)$$

If $k > m$, a resonance is never reached; i.e., the first critical speed of the shaft is entirely suppressed.

Figure 3 shows synchronous responses for a rotor with the following data:

$$\begin{aligned} m &= 386 \text{ lb} \\ k_s &= 22500 \text{ lb/in} \\ P_o &= 300 \text{ lb/in}^2 \\ \omega_o &= 300 \text{ rad/sec} \\ R &= 2.5 \text{ in} \\ L &= 1 \text{ in} \\ \lambda &= 0.01 \\ \rho &= 0.036 \text{ lb/in}^3 \end{aligned}$$

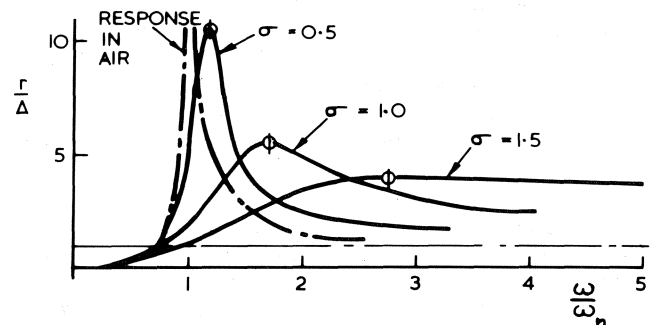


Figure 3. Synchronous Response for a Rotor.

The seal strength may be considered to be varied by clearance; i.e., $\sigma = 0.5, 1.0, 1.5$ are equivalent to clearance gaps of 0.02, 0.01, 0.0067 inches, respectively. It will be seen that these practically sized gaps produce very substantial increases in critical speed and attendant flattening of the peak response. It is also apparent that wear of the seal bushes (neck rings) can substantially change the synchronous response.

Equation (1) shows that seal forces embody forward driving cross-coupling. The possibility of unstable running zones must therefore be suspected. To look at this somewhat more realistically, it is assumed that some viscous damping from the bearings is present in addition to the seal forces. Equation (2) is then modified to,

$$m\ddot{r} + (c + 2k_c\omega)\dot{r} + (k_s + k\omega^2 - jk_c\omega^2)r = 0 \quad (7)$$

or

$$\ddot{r} + (2\xi\omega_n + \frac{2k_c}{m}\omega)\dot{r} + (\omega_n^2 + \frac{k}{m}\omega^2 - j\frac{k_c}{m}\omega^2)r = 0 \quad (8)$$

At a stability boundary, circular whirling, $r = Re^{j\omega t}$, may be supposed. Then, substituting in equation (8) and separating real and imaginary parts,

$$\Omega^2 = \omega_n^2 + \frac{k}{m}\omega^2 \quad (9a)$$

and

$$\Omega = \frac{\frac{k_c}{m}\omega^2}{2\xi\omega_n + 2\frac{k_c}{m}\omega} \quad (9b)$$

define the onset speed of unstable whirling.

It is first noted that when $\xi = 0$,

$$\omega = \frac{2\omega_n}{\sqrt{1 - 4\frac{k}{m}}} \quad (10)$$

i.e., the onset speed is always greater than twice the critical speed in air. Further, if $k > \frac{1}{4}m$, the onset speed moves up to infinity or the rotor is then inherently stable.

If ξ is not zero, equations (9) define a quartic polynomial for the onset speed. This has no analytical solution, but is easily solved numerically.

In Figure 4, mappings of stability boundaries are shown for the rotor previously specified. Again, seal strength is varied by assuming various clearances. Both plain and heavily serrated neck-rings are considered. In the latter case, the semi-empirical treatment described in [5] is applied. The "nominal clearance," shown in Figure 4, is 0.017 inches.

With the plain seals, at nominal clearance, the rotor is inherently stable at all conceivable speeds. The seals would have to wear to more than twice nominal clearance before there was any likelihood of instability at high speed.

With the much weaker bearing forces of the serrated seals, the margin of inherent stability disappears, although very high speeds would be needed to drive the rotor unstable with nominal clearances. A much smaller percentage of wear is needed to put the rotor into a region of much higher risk.

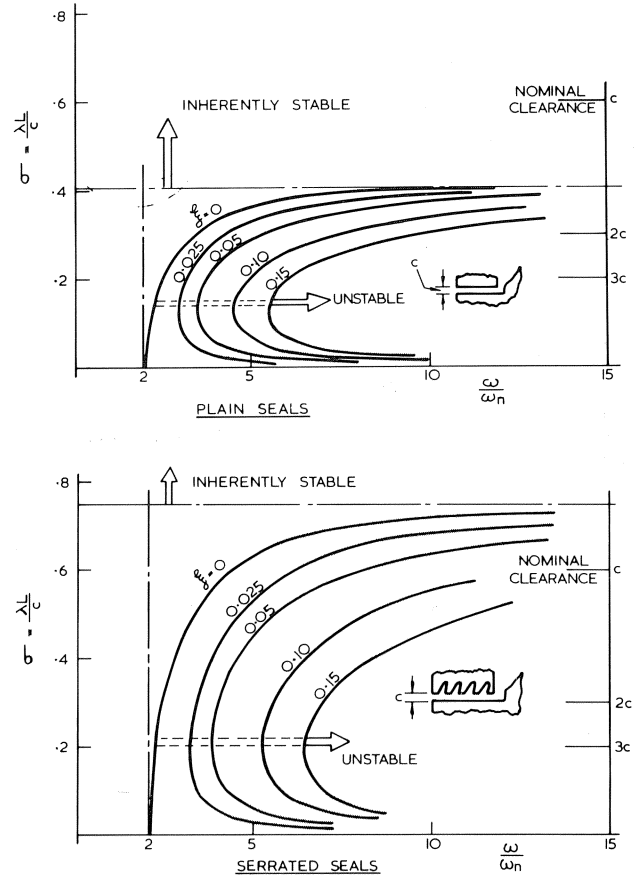


Figure 4. Mappings of Stability Boundaries for a Rotor.

EFFECTS OF COOLANT FLUID IN SUBMERGED MOTORS

The fluid film surrounding the rotor of a typical submerged motor (Figure 1) generally has a relatively large gap and a high kinematic viscosity when compared with the circular journal bearing it resembles superficially. Unlike the centrifugal pump seal, the axial flow velocities are likely to be negligibly small compared with the drag flow. As Reynolds numbers are very high, the fluid flow is superlaminar and fluid inertia forces play a very significant part in the journal dynamics. Figure 5 indicates the nature of the fluid inertia forces acting when the film is not cavitating and journal motions about the centered position are small. When these are put together with the usual drag induced forces, dynamic coefficients are given by,

$$\begin{bmatrix} F_y \\ F_z \end{bmatrix} = \begin{bmatrix} \frac{1}{4}m_a\omega^2 & -\bar{k}\omega \\ \bar{k}\omega & \frac{1}{4}m_a\omega^2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} + \begin{bmatrix} -2\bar{k} & -m_a\omega \\ m_a\omega & -2\bar{k} \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -m_a & 0 \\ 0 & -m_a \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{z} \end{bmatrix} \quad (11)$$

where

$$\bar{k} = 6\pi\mu_e L_a \left(\frac{r}{c}\right)^3 \quad (12)$$

and added mass

$$m_a = \rho\pi r^2 L_a \frac{r}{c} \quad (13)$$

Here, μ_e is an effective viscosity, given by

$$\mu_e = 0.0053 R_r^{0.75} \mu \quad (14)$$

as derived by Black [8].

Let the rotor indicated in Figure 1 be rigid and supported on bearings having simple radial stiffness $k_b/2$. Considering only cylindrical whirling, the forces defined in equation (11) lead to

$$\begin{aligned} (m + m_a) \ddot{r} + (2\bar{k} - jm_a\omega)\dot{r} + (k_b - \frac{1}{4}m_a\omega^2 - j\bar{k}\omega) \\ = m\Delta\omega^2 e^{j\omega t} \end{aligned} \quad (15)$$

Considering first synchronous whirling, the amplitude is given by,

$$R = \frac{\frac{m}{(m + \frac{1}{4}m_a)} \Delta\omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi\omega_n\omega)^2}} \quad (16)$$

where

$$\omega_n = \sqrt{\frac{k_b}{(m + \frac{1}{4}m_a)}} \quad (17)$$

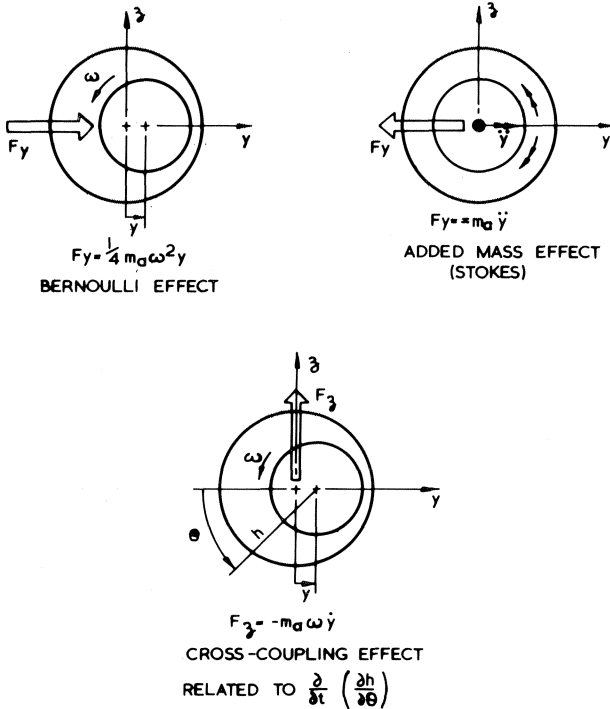


Figure 5. Inertia Effects in Long Journal Bearing Configuration.

and

$$\xi = \frac{\bar{k}}{2\sqrt{k_b(m + \frac{1}{4}m_a)}} \quad (18)$$

These results show that:

- (i) the natural frequency is reduced by the added mass effects of fluid inertia;
- (ii) as expected, damping is derived from the drag effects in the fluid, but the damping ratio is reduced by the added mass.

The onset of instability can be examined by omitting the right-hand side of equation (15) and looking for a quasi-stable whirl, as given by

$$r = Re^{j\Omega t}$$

Then

$$(k_b - \frac{1}{4}m_a\omega^2) - (m + m_a)\Omega^2 + m_a\Omega\omega = 0 \quad (19a)$$

and

$$2\Omega - \omega = 0 \quad (19b)$$

Equations (19) show that unstable whirling is half-speed at onset, and the onset speed is

$$\omega = 2 \sqrt{\frac{k_b}{m}} \quad (20)$$

The whirl onset speed is therefore completely unaffected by the fluid inertia effects.

The following numerical results demonstrate that the effects described are very large in normal circumstances.

Suppose $r = 4$ in, $L_a = 16$ in, $c = 0.04$ in

$$m = 228 \text{ lb}$$

$$\mu = 10^{-7} \frac{\text{lb sec}}{\text{ft}^2}$$

$$\rho = 60 \text{ lb/ft}^3$$

$$k_b = 147,260 \text{ lb/in.}$$

Then, the added mass, $m_a = 2753$ lb.

Note that this is about twelve times the rotor mass, and $m + \frac{1}{4}m_a = 926$ lb.

Then ω_n (in air) = 500 rad/sec

ω_n (in fluid) = 247.5 rad/sec;

i.e., the natural frequency drops by a factor of just over two.

With $\nu = \frac{\omega}{247.5}$, and, bearing in mind the variation of effective viscosity with Reynolds number (equation (14)), the synchronous response equation (16) becomes,

$$\frac{R}{\Delta} = \frac{0.246 \nu^2}{\sqrt{(1 - \nu^2)^2 + 0.519 \nu^{3.5}}} \quad (21)$$

compared with

$$\frac{R}{\Delta} = \frac{0.245 \nu^2}{|1 - 0.245 \nu^2|} \quad (22)$$

in air.

Figure 6 indicates these responses, showing also the response if the gap c were widened to 0.08 inches.

The onset speed of unstable whirling (equation (20)) is

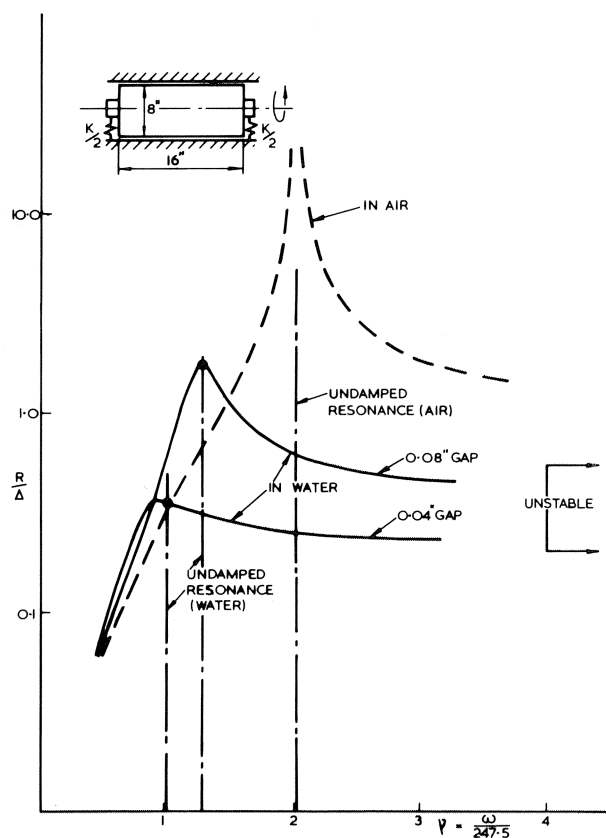


Figure 6. Response to Mass Unbalance.

simply twice the natural frequency in air; i.e., 1000 rad/sec. Figure 7 shows an eigenvalue (damped natural frequency) history. It is seen that two damped natural frequencies are present. The one with very high damping is a backward whirl. The one which goes unstable at 1000 rad/sec is a forward whirl, remaining at half-speed whirl when the stability limit is crossed.

CONCLUSION

In both centrifugal pumps and submerged motors, fluid filled clearance spaces dominate lateral vibration response.

In pumps, high pressure clearance seals act as hybrid bearings whose stiffness increases sharply with running speed; there are also cross-coupling and damping forces present. As a result, critical speeds can be greatly increased or sometimes entirely suppressed. High speed subsynchronous whirl can be present if the bearing forces from the seals are weak. Alternatively, subsynchronous whirling may appear when these forces have been weakened by substantial wear.

In submerged motors, the coolant fluid around the armature forms a substantial hydrodynamic bearing. Because this bearing envelops the entire rotor, and also because of the relatively large clearance space leading to high Reynolds numbers, very substantial fluid inertia effects are present. These increase the effective mass of the rotor in vibrations very substantially, leading to great reductions of critical speeds. The cross-coupling and damping forces normally present in low Reynolds number journal bearings are also present. Subsynchronous whirl onset occurs at twice the critical speed in air.

The rotor dynamics of these machines cannot be represented even approximately without consideration of the effects described.

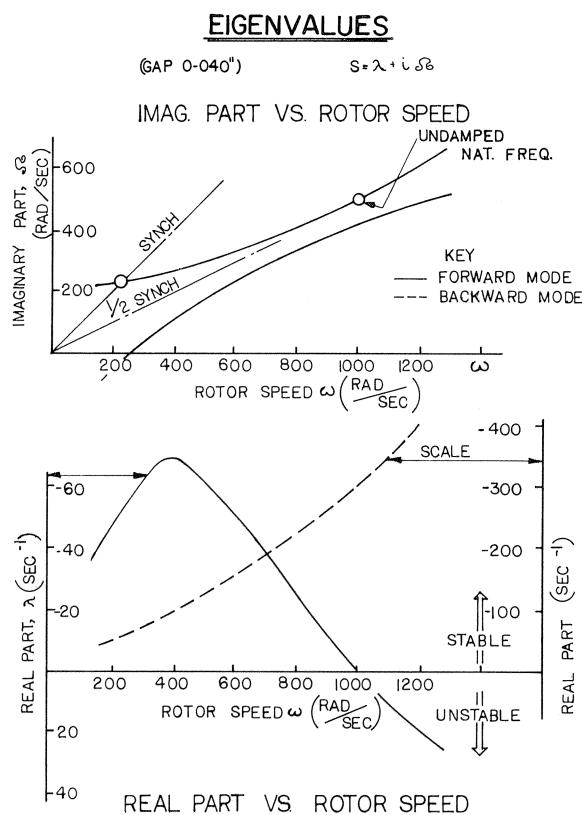


Figure 7. Damped Natural Frequency History (Real and Imaginary Eigenvalues).

REFERENCES

1. Bedcer, F. S. and Lomakin, A. A., "Determination of Critical Speed of a Pump Rotor with Reference to Forces Arising in the Seal Gaps," *Steam and Gas Turbine Engineering*, 1957, n.5, (in Russian).
2. Black, H. F., "Effects of Hydraulic Forces in Annular Pressure Seals on the Vibrations of Centrifugal Pump Rotors," *Journal of Mech. Eng. Sci.*, 1969, v.11, n.2, pp. 206-213.
3. Black, H. F. and Murray, J. L., "The Hydrostatic and Hybrid Bearing Properties of Annular Pressure Seals in Centrifugal Pumps," *BHRA*, 1969, RR 1026.
4. Black, H. F. and Jenssen, D. N., "Dynamic Hybrid Bearing Characteristics of Annular Controlled Leakage Seals," "Advanced Class Boiler Feed Pumps," *I. Mech. E.*, 1970, pp. 92-100.
5. Black, H. F. and Cochrane, E. A., "Leakage and Hybrid Bearing Properties of Serrated Seals in Centrifugal Pumps," *6th International Conference on Fluid Sealing*, 1973, 65, pp. 61-69.
6. Black, H. F., Brown, R. D., France, D., and Jenssen, D. N., "Theoretical and Experimental Investigations Relating to Centrifugal Pump Rotor Vibrations," *Plant Engineering* (London), 1974, v. 18, n. 7, pp. 16-21.
7. Fritz, R. J., "The Effects of an Annular Fluid on the Vibrations of a Long Rotor," *ASME, Series D*, 1970, v. 92, pp. 923-929.
8. Black, H. F., "Empirical Treatment of Hydrodynamic Journal Bearing Performance in the Superlaminar Regime," *Journal of Mech. Eng. Sci.*, 1970, v. 12, n. 2, pp. 116-122.

NOMENCLATURE

c	clearance in seal, journal bearing, etc.	R	seal radius, whirl amplitude
F_y, F_z	lateral bearing forces	R_r	rotational Reynolds number, $\rho r \omega c / \mu$
k	seal stiffness coefficient	V	axial fluid velocity in seal gap
k_c	seal damping and cross-coupling coefficient	y, z	lateral motion coordinates
k_s	shaft stiffness	Δ	mass eccentricity
\bar{k}	journal bearing cross-coupling and damping coefficient	λ	axial flow friction factor in seal gap
k_b	bearing stiffness	ξ	damping ratio
L	seal length	μ	fluid viscosity
L_a	armature length	μ_e	effective turbulent viscosity
m	rotor mass	ρ	density of fluid
m_a	added mass	σ	$\lambda L / c$
P, P_o	pressure drop, pressure drop at rated speed	ω_o	rated pump speed
r	armature radius, whirl radius co-ordinate	ω	shaft speed
		ω_n	critical speed