EXPERIMENTAL INVESTIGATION OF BALANCING FLEXIBLE ROTORS IN LARGE MOTORS AND GENERATORS

by

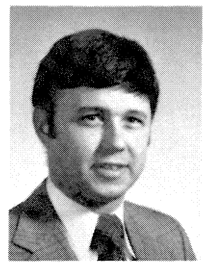
W. H. Miller

Manager — Advanced Quality Control Engineering

Large Motor and Generator Department

General Electric Company

Schenectady, New York



William H. Miller graduated from Cornell University in 1975 and joined the Elliott Company in Jeannette, Pennsylvania, as a development engineer. His work concentrated on the analysis of bearings, seals, and rotor vibrations. This work gained him his first patent award for the invention of a high-pressure shaft seal. In 1975, he joined the Turbo-Products Division of Ingersoll-Rand Company, Phillipsburg, N.J. While there, he collaborated with Dr. Kirk on their award-winning paper.

Mr. Miller later joined Mechanical Technology, Inc., Latham, New York, where his research and development in the area of compliant foil bearings has produced several new designs to improve the foil bearings performance. These designs have been filed with the U.S. patent office and to date three patents have been awarded.

In 1978, he accepted a position with the General Electric Company's Materials and Processes Laboratory as a rotor dynamist and bearings analyst. Recently he was named manager of Advanced Quality Control Engineering for GE's Large Motor and Generator Department in Schenectady, New York. In May 1980, he received ASLE's Walter D. Hodson award for the best published paper written by a member under thirty-five years of age.

In addition to his ASLE membership, Mr. Miller is a member of the ASME and is a licensed professional engineer in the State of New York.

ABSTRACT

The balancing of large motor and generator rotors is accomplished by a combination of balance machine corrections plus the calculated corrections determined by an Influence Coefficient Balancing computer program. The method has been applied to an 8000 HP two-pole induction machine rotor and the results are presented. The vibration is measured at several locations and speeds and the results are used to (a) ensure that the vibration levels are not excessive as the rotor speed is increased and (b) calculate the balance correction weights using a least squares Influence Coefficient Method. Measured and calculated results are presented which provide a check on the method. The results show that a multi-plane balance correction was necessary to minimize the rotor unbalance through the second critical speed.

INTRODUCTION

The manufacture of a large industrial motor or generator employs many processes, one of which is rotor balancing. The process of rotor balancing has been examined by many distinguished investigators in an attempt to replace the art of balancing with a scientific approach. Their effort have resulted in the development of two analytical techniques designed to effect rotor balancing. The first method to be implemented is known as the Influence Coefficient Method. It was first proposed by Thearle [1] as a systematic means to minimize the rotor amplitudes due to the unbalanced mass distribution in the rotor. Many researchers have extended and improved upon the method [2, 3, 4], and today it has nearly universal acceptance.

The Modal Balancing method is much newer and is a direct outgrowth of the eigenvalue formulation used to calculate "critical speeds" and mode shapes [5]. The classical modal method considered the undamped system which yields planar mode shapes. The work of Lund [6] included the effects of damping for which the analysis yields non-planar helical mode shapes. This development, while physically more realistic, complicates the solution which results in a reduction in the ease of understanding. A controversy in the literature, germain in nature to the present paper, has ensued. The question arises whether or not a low speed balance, implemented using a balance machine, is necessary or even contributes anything to the refinement of the rotor balance [7, 8]. There are many "theories" of rotor balancing, and this paper sets forth an approach which has been used successfully for many years.

ROTOR DESIGN ANALYSIS

The undamped eigenvalue analysis [9] of the rotor-bearing system enables one to calculate the system natural frequencies and planar mode shapes. When the rotor is relatively rigid, as in the case of a motor or generator, the two rigid-body modes correspond to a stiff shaft on flexible supports (i.e., the translational or static unbalance mode and conical or dynamical unbalance mode). The lowest "flexible rotor" mode corresponds to the third mode, which is the natural frequency of the rotor itself. These natural frequencies and mode shapes corresponding to the rigid and flexible rotor modes are represented schematically by the left hand portion of the "Critical Speed Map" (Figure 1).

In practice the two fundamental modes, as depicted by the center region of Figure 1, are somewhat flexible and may require more than two balance planes to achieve acceptable vibration levels. Hence, we will define a "flexible rotor" in this paper to be any rotor which requires more than two balance planes to achieve acceptable vibration levels.

Since most conventional electric machines are locked into 60 Hz power grids, their speeds are necessarily 3600 RPM and below. A few electric machines have been designed to operate above their rigid bearing critical speed. (The obvious exceptions are the large utility generators and AC adjustable speed

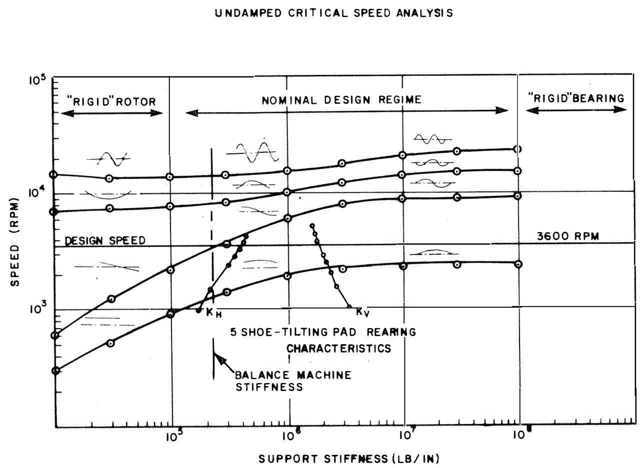


Figure 1. 8000 HP — Two-Pole Induction Motor Undamped Critical Speed Map with Speed Dependent Tilting Pad Bearing Characteristics.

drives which incorporate a power converter both of which are beyond the scope of this paper.)

An analysis of the 8000 HP two-pole induction motor design [10] was made using a calculation procedure similar to that of Lund [5] and Ruhl [11]. The response to unbalance was computed as a function of speed for a 30 oz-in unbalance placed at the rotor midspan. The magnitude of the unbalance was chosen to simulate a complete rotor prior to the factory balance. Figure 2 is a plot of the response analysis which shows three significant points: (a) the rotor system has one "critical" speed in the design speed regime — well removed from running speed, (b) the rotor exhibits significant bending as compared to the bearing deflection as the rotor passes through this "critical" speed, and (c) the vibration levels in the pre-balanced simulation are acceptable at the design speed.

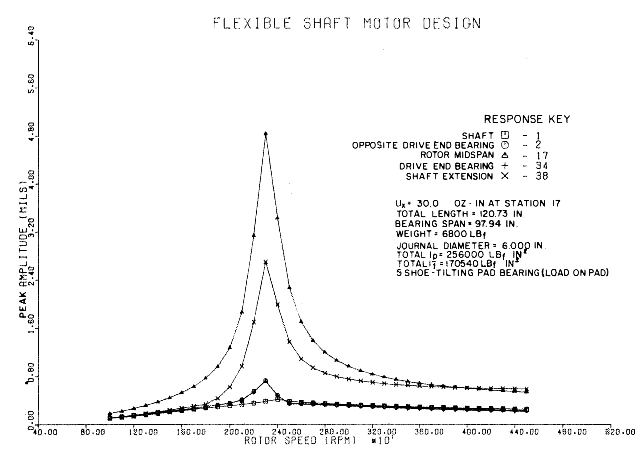


Figure 2. Calculated Response to Unbalance as a Function of Speed (As Designed — Pre-Balance Simulation).

ROTOR BALANCING CONSIDERATIONS

In preparing to balance the "flexible" rotor it was recognized that the balance machine response would differ from the motor design response. An analysis of the rotor supported on sleeve bearings and flexible pedestals was made to determine the balance machine system response. The results of the rotor response analysis are given in Figure 3. The plot shows several significant results: (a) the rotor-bearing system has two critical speeds in the operating speed regime (first critical — 1500 RPM; second critical — 3600 RPM) and the rotor will have to be balanced through two critical speeds in order to achieve the design speed of 3600 RPM in the balance machine, (b) the first "critical" speed is a translational mode (midspan — maximum vibration), and (c) the second "critical" speed is a conical mode (bearing journal amplitudes approximately ten times the rotor midspan amplitude).

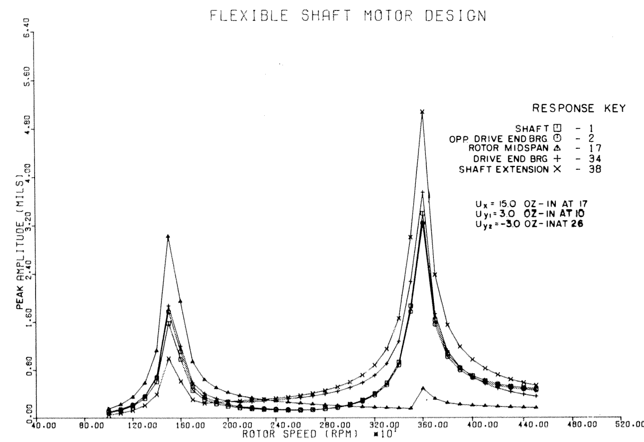


Figure 3. Pre-Balance Simulation of Response to Unbalance as a Function of Speed (Rotor-sleeve Bearings — Flexible Supports).

VIBRATION LIMITS

As a safety precaution for both equipment and personnel, some manufacturers have for many years observed specified maximum allowable levels of vibration during the balancing process. The vibration limits similar to those of Figure 4 are specified as a function of speed, and experience has shown these limits to be reasonable for product lines.

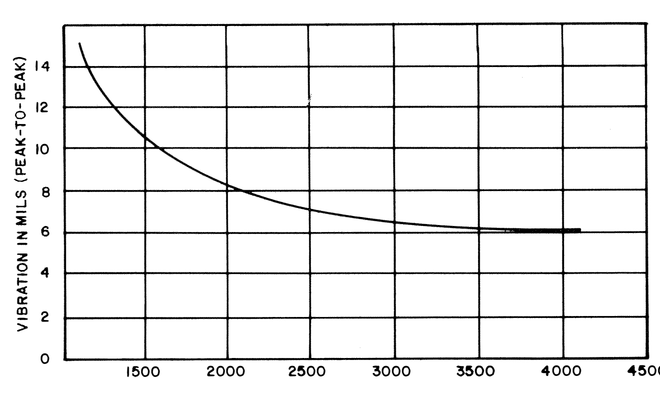


Figure 4. Speed Dependent Maximum Vibration Limits During Balancing Process.

The procedure for modal balancing is to run the rotor up to the neighborhood of a critical speed and measure the resulting deflections. These rotor amplitudes are primarily due to that particular critical speed with very little contribution from the higher order modes. Kellenberger's paper on modal balancing [8] concluded that the two rigid body modes also needed to be balanced separately. Bishop and Parkinson [7] disagreed with Kellenberger, stating that (based on their eigenvalue formulation) the rigid body modes were not necessary to balance.

This author's experience with industrial size machines strongly supports Kellenberger's N + 2 method since many of the large motors and generators operate through or near a fundamental mode, i.e., the translational and/or conical modes. These modes, while well below the shaft natural frequency, can experience significant bending as the rotor speed approaches a resonant frequency. Several "rigid rotor" balancing corrections must be made in order to pass through these modes. This influence is best illustrated by an example:

Speed	3600 RPM
Rotor weight	6800 lb
Allowable residual Unbalance/plane	$U = \frac{4W}{N}$ (oz − in)

$$= \frac{4 \times 6800}{3600} = 7.55 \text{ oz in/plane} .$$

Now assuming that the rotor center of gravity (c.g.) shifts:

$$\text{c.g.} = \frac{7.55 \text{ oz} - \text{in}}{68000 \text{ lb} \times 16 \text{ oz/lb}} = 6.93 \times 10^{-5} \text{ in} .$$

One can see from this example that a rotor does not have to bend very much during operation for a balance correction to be required. Since the allowable residual unbalance is inversely proportional to speed and the maximum allowable vibration is a function of speed, it is easy to understand why several multi-plane, multi-speed corrections are required.

Flexible rotor designs can have a significant separation between the horizontal and vertical fundamental modes such that as many as four "flexible" modes must be balanced in the operating regime of an industrial machine. This phenomena comes about due to the asymmetry in the fluid-film bearing dynamic characteristics, with the vertical direction much stiffer than the horizontal direction for a typical sleeve bearing.

BALANCING PROCEDURE

The rotor was installed in a soft mount balancing machine on fluid-film sleeve bearings. A sketch of the rotor is presented in Figure 5 which shows that balance correction weights can be applied at five locations along the rotor, i.e., each fan, and three balance correction locations in the rotor body. The center balance provision was used as a single plane location since weights in this area were distributed axially among the three planes.

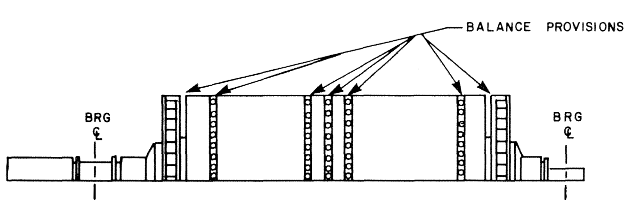


Figure 5. Sketch of 8000 HP — Two-Pole Induction Rotor.

Figure 6 shows the locations of the noncontacting probe surfaces which were machined into the rotor assembly. The mechanical runout of each probe surface was measured and limited to 0.2 mils. The noncontact probes were mounted in the horizontal direction on free-standing supports fixed to ground. The probe signals were input to a vibration analyzer which provided an analog readout of amplitude and triggered a strobe light. A stationary coordinate system was affixed to the opposite drive end pedestal so that the phase angle could be measured using the strobe light.

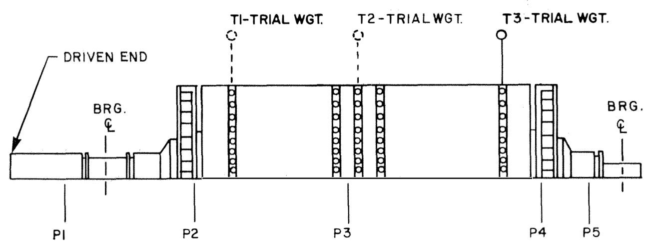


Figure 6. Sketch of Rotor Indicating Trial Weight and Non-Contact Probe Locations.

The balance machine was used to pre-balance the rotor, correcting for unbalance up to about 1800 RPM in the soft mode. At this point the hydraulic lift was shut off and the floating pedestals were locked. The balance machine was again used to further refine the balance up to about 1800 RPM. The influence of the 1500 RPM critical speed was noted since the correction weights called for by the balance machine did not reduce the vibration levels as might otherwise be expected. Moreover, as repeated corrections were made, the balance machine continued to call for additional weight but at various angles. It was obvious that the computer analysis would need to be employed for the remainder of the rotor speed range.

The rotor speed was increased while a careful watch was made of the vibration amplitudes. The speed was increased until the midspan amplitude reached approximately 4.2 mils at 2800 RPM. A trial weight of 20.8 grams was applied at 0° at planes T_1, T_2, and T_3 in turn, and the vibration measured at 2400 and 2800 RPM. This data was input to the least squares influence coefficient computer program and a balance correction shot was calculated. The balance correction was computed to be:

	W (Grams)	θ (Degrees)
T_1	57.1	209
T_2	79.1	138
T_3	10.6	227

The resulting vibration at the two speeds is given in Table 1 which shows that a minor improvement in the balance condition had been achieved. The decision to attempt higher speeds was made for two reasons:

(a) the vibration levels were below the limits specified by Figure 4.

(b) the bearing housing vibration had been reduced at 2800 RPM.

The rotor speed was increased to 3000 RPM, then to 3200

TABLE 1. MEASURED VIBRATION DATA (VIBRATION-MILS, ANGLES-DEGREES, TRIAL WEIGHT-GRAMS).

PROBE LOCATION			P_1	SEISMIC	P_2	P_3	P_4	P_5	SEISMIC
TRIAL WEIGHT & LOCATION	SHOT NO.	SPEED	SHAFT	BEARING HOUSING	T_1	T_2	T_3	SHAFT	BEARING HOUSING
Grams/ Degree Lag	RES	2400	1.5 / 310	.4 / 10	.34 / 15	3.9 / 258	2.3 / 310	.8 / 305	.55 / 320
	RES	2800	2.0 / 320	1.3 / 340	.25 / 285	4.2 / 260	2.8 / 308	1.0 / 285	.85 / 290
20.8/0°	T_1	2400	1.35 / 310	.4 / 350	.22 / 5	4.0 / 255	2.2 / 313	1.05 / 290	.75 / 300
	T_1	2800	1.9 / 320	1.3 / 340	.4 / 275	4.5 / 260	2.7 / 315	1.2 / 280	1.0 / 282
20.8/0°	T_2	2400	1.75 / 305	.55 / 320	.29 / 40	4.2 / 258	2.2 / 307	90 / 290	.88 / 290
	T_2	2800	2.5 / 310	1.6 / 330	.2 / 50	4.6 / 258	2.8 / 303	1.05 / 280	1.0 / 305
20.8/0°	T_3	2400	1.7 / 310	.45 / 340	.4 / 45	4.4 / 252	2.2 / 310	.84 / 310	.32 / 210
	T_3	2800	1.75 / 330	1.7 / 310	.35 / 80	4.8 / 258	2.9 / 303	.78 / 290	1.5 / 0
3-Plane Computer Balance Shot	RES	2400	1.2 / 325	.35 / 330	.90 / 0	5.2 / 260	2.8 / 320	.55 / 85	.75 / 75
	RES	2800	2.2 / 315	.95 / 320	.9 / 80	5.5 / 255	2.8 / 315	.64 / 90	.78 / 110

RPM where the outboard shaft extension vibration began to approach the process limit. Therefore, a second balance attempt was made.

A two-plane balance was first attempted since the rotor had achieved speeds well above the first critical speed. This test would also give some insight as to whether the multi-plane approach was necessary. A trial weight of 17.9 grams was applied to T_1 at 110° and the vibration measured at the five planes. Next, a trial weight of 24.5 grams was placed at T_3 at 110°. Table 2 gives the vibration amplitudes measured with the trial weight applied and the residual vibration with the computer calculated correction in place. The calculated correction was:

	W (Grams)	θ (Degrees)
T_1	33.5	281
T_3	43.9	104

The computed correction is, as would be expected, a couple, since the rotor is running well above the first critical and approaching the second critical speed. A review of the vibration data at 3000 RPM shows that the rotor and bearing housing vibration have both increased. Hence, the two-plane correction was detrimental to the rotor performance.

The two-plane correction was removed and a trial weight of 24.5 grams was applied at T_2 at 110°. The vibration was measured at all five planes at 3000 RPM and 3200 RPM. This data, coupled with the two-plane trial weight data, was used to compute a three-plane balance correction which is given below:

	W (Grams)	θ (Degrees)
T_1	36.8	315
T_2	33.1	188
T_3	22.6	69

The resulting vibration amplitudes are given in Table 3. Comparing the 3000 RPM vibration measurements from Tables 2 and 3, one can see that the three-plane correction was very effective. Both the rotor and bearing housing vibration had been reduced.

Next, the rotor was carefully accelerated in 100 RPM

TABLE 2. TWO-PLANE BALANCE CORRECTION, MEASUREMENT, AND RESULTS.

PROBE LOCATION			P_1	SEISMIC	P_2	P_3	P_4	P_5	SEISMIC
TRIAL WEIGHT & LOCATION	SHOT NO.	SPEED RPM	SHAFT	BEARING HOUSING	T_1	T_2	T_3	SHAFT	BEARING HOUSING
Grams/ Degree Lag	RES	3000	3.2 / 310	1.8 / 305	1.0 / 20	5.3 / 255	3.3 / 318	1.5 / 100	1.2 / 95
	RES	3200	5.0 / 290	4.0 / 275	2.8 / 0	5.5 / 265	4.7 / 310	4.0 / 65	3.4 / 60
17.9 / 110°	T_1	3000	2.5 / 305	1.8 / 310	1.35 / 15	5.5 / 260	3.6 / 312	1.7 / 90	1.3 / 80
	T_1	3200	5.2 / 270	3.8 / 260	2.8 / 0	5.6 / 270	4.6 / 310	3.0 / 60	3.0 / 50
24.5 / 110°	T_4	3000	4.0 / 320	2.5 / 325	1.2 / 48	5.4 / 255	3.9 / 318	2.2 / 120	1.9 / 125
	T_4	3200	6.8 / 265	5.8 / 260	3.7 / 8	5.7 / 270	5.4 / 308	5.2 / 55	4.8 / 45
2-Plane Computer Balance Shot	RES	3000	6.0 / 310	4.0 / 300	1.7 / 68	5.4 / 254	4.9 / 318	3.4 / 110	3.0 / 100
	RES	3200	/	/	4.2 / 2	6.4 / 270	6.4 / 303	/	/

increments to the full design speed of 3600 RPM. The rotor vibration amplitudes peaked at 3400 RPM and then decreased at the two higher speed points.

The decision was made to attempt a correction at 3400 RPM using three balance planes. Table 4 gives the vibration measurements and each trial weight magnitude and location. The computer calculated balance correction was:

	W (Grams)	θ (Degrees)
T_1	68.8	344
T_2	17.1	74
T_3	10.8	343

Figure 7 is a plot of the mode shape before and after the balance correction. It can be seen that the computer calculated correction effectively removed the couple component and a small residual static vibration remains. This is typical for a well balanced rotor.

The rotor was run at several lower speeds to ensure that the high speed corrections had not diminished lower speed refinements. At both the 3000 RPM and 2800 RPM speed points the rotor vibration was acceptable and had not changed significantly from the previous vibration measurements. The engineering vibration requirements were 2.0 mils or less (filtered) on the bearing housing and, having met this criteria, the flexible rotor was balanced.

ERROR ANALYSIS

A comparison of the measured and predicted rotor response at the five probe locations, at 3400 RPM, is given in Table 5. This comparison shows a good agreement between the measured and computed phase angle and a difference of approximately two times in some of the amplitude measurements. Several test arrangement factors affect the accuracy of the least squares influence coefficient method prediction of the balance correction vector and the rotor response. The method assumes a linear system which is approximately true since the speed dependent fluid-film bearing characteristics are nonlinear.

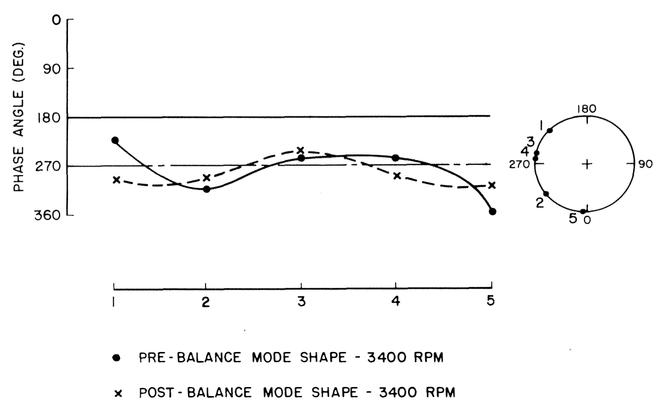


Figure 7. Mode Shape Before and After Three-Plane Balancing at 3400 RPM.

TABLE 3. THREE-PLANE BALANCE, MEASUREMENTS, AND RESULTS.

PROBE LOCATION			P_1	SEISMIC	P_2	P_3	P_4	P_5	SEISMIC
TRIAL WEIGHT & LOCATION	SHOT NO.	SPEED RPM	SHAFT	BEARING HOUSING	T_1	T_2	T_3	SHAFT	BEARING HOUSING
24.5 / 110°	T_2	3000	4.2 / 295	2.0 / 310	1.4 / 42	5.4 / 256	3.4 / 310	1.6 / 110	1.6 / 110
	T_2	3200	6.4 / 255	4.6 / 250	3.9 / 0	6.0 / 268	4.4 / 308	4.0 / 40	4.0 / 35
	RES	3000	2.1 / 305	1.0 / 270	1.6 / 0	5.6 / 260	3.0 / 312	1.2 / 50	1.1 / 40
	RES	3200	3.1 / 260	2.2 / 250	2.8 / 350	6.2 / 262	3.9 / 310	2.5 / 25	2.3 / 10
	RES	3300	3.8 / 250	2.9 / 235	3.1 / 327	7.1 / 260	4.3 / 292	3.0 / 15	2.7 / 5
	RES	3400	4.5 / 220	4.0 / 210	3.2 / 322	8.3 / 250	4.3 / 265	3.9 / 0	3.5 / 350
	RES	3500	4.4 / 180	4.6 / 160	2.2 / 303	7.5 / 238	2.3 / 242	4.0 / 330	3.8 / 325
	RES	3600	3.2 / 165	4.0 / 145	1.8 / 315	6.8 / 235	1.6 / 243	3.2 / 320	3.5 / 320

Experimental factors which contribute to the inaccuracy of the predictions were:

(a) The non-contract probes P_2 and P_4 did not measure the rotor response in the balance correction planes. The probes measured the response several inches outboard of the balance correction plane on the outside diameter of the fan structure. This was done since it was easier from a manufacturing viewpoint to control the runout of the solid fan structure as compared to the rotor lamination stack. This offset between measurement and correction planes introduces an error in the measured response data input to the computer program. An evaluation of the magnitude of this influence was not made during this study.

(b) The phase angle was measured using a coordinate system fixed to the balance machine pedestal. The resolution of the phase angle was approximately ± 5°. This accuracy was a contributing factor in the error in the computed balance correction vector and the predicted response since it represented a limiting accuracy in the data input to the computer program.

(c) The third experimental condition which affected the agreement between the predicted and measured results was the placement of the balance correction weights. The balance provision in the rotor consisted of a multiplicity of drilled and tapped holes around the outside diameter of the rotor body. These discrete locations precluded applying the balance correction exactly as calculated by the computer program.

While these limitations in the experimental arrangement existed, the method was able to be applied successfully and acceptable rotor response levels were obtained.

The rotor was installed in a stator and vibration coast down plots were made to study the rotor response. The results are presented and discussed in detail in reference [10]. In general, these tests showed good agreement with the placement of the first critical speed as compared to Figure 2 of the present paper.

CONCLUSIONS

The balancing of a large induction rotor was effected by a combination of balance machine pre-balance and a multi-plane, multi-speed least squares influence coefficient balancing technique. The rotor was balanced through the first two critical speeds (1500 RPM — translational and 3400 RPM — conical modes). It was necessary to balance both modes in order to achieve the 3600 RPM design speed. The actual motor design has only a first "critical" speed at 2500 RPM in the design speed range since the motor frame and endshield assembly are significantly stiffer than the balance machine supports. This direct comparison of an industrial rotor operated to design speed of 3600 RPM on two different flexible supports provides a good check on the design simulation models and the balance calculation procedures. Good agreement was observed between the predicted and measured critical speeds and mode shapes.

The multi-plane, multi-speed least squares influence coef-

TABLE 4. THREE-PLANE BALANCE, MEASUREMENTS, AND RESULTS.

PROBE LOCATION			P_1	SEISMIC	P_2	P_3	P_4	P_5	SEISMIC
TRIAL WEIGHT & LOCATION	SHOT NO.	SPEED RPM	SHAFT	BEARING HOUSING	T_1	T_2	T_3	SHAFT	BEARING HOUSING
20.4 / 20°	T_1	3400	3.6 / 240	3.2 / 210	2.7 / 330	7.1 / 255	3.7 / 290	3.2 / 10	2.9 / 0
19.7 / 20°	T_2	3400	4.8 / 225	4.5 / 210	2.6 / 308	7.6 / 248	3.5 / 256	4.1 / 345	3.6 / 340
20.4 / 20°	T_3	3400	5.1 / 205	5.4 / 175	3.4 / 312	7.8 / 244	3.6 / 245	4.8 / 340	4.4 / 330
	RES	3400	1.6 / 300	0.9 / 235	1.25 / 305	6.9 / 250	2.3 / 290	1.0 / 330	0.9 / 310
3-Plane Computer Balance Shot	RES	3600	1.15 / 210	2.0 / 140	0.6 / 290	6.2 / 230	1.0 / 250	2.0 / 310	1.6 / 300
	RES	3000			1.3 / 340	5.6 / 250	2.4 / 315		
	RES	2800			0.8 / 340	5.1 / 253	2.0 / 322		

TABLE 5. COMPARISON OF MEASURED AND PREDICTED VIBRATION AT 3400 RPM

Probe	1	2	3	4	5
Meas. A/θ	1.6/300	1.25/305	6.9/250	2.3/290	1.0/330
Calc. A/θ	.76/306	.48/314	3.3/234	2.9/44	1.3/307

ficient technique was shown to be effective in minimizing rotor vibrations. The vibrations arise from the complicated mass unbalance inherent in the rotor operating through a "helical" shaped, second critical speed. This technique coupled with a good low speed balance machine pre-balance has been used to balance several flexible shaft rotors.

REFERENCES

1. Thearle, E. L., "Dynamic Balancing of Rotating Machinery in the Field," Trans. ASME, Vol. 56 (1934), pp. 745-753.
2. Goodman, T. P., "A Least-Squares Method for Computing Balance Corrections," *Journal of Engineering for Industry*, Trans. ASME, Vol. 86, No. 3 (Aug. 1964), pp. 273-279.
3. Tessarzik, J. M., and Badgley, R. H., "Experimental Evaluation of the Exact Point-Speed and Least-Squares Procedures for Flexible Rotor Balancing by the Influence Coefficient Method," *Journal of Engineering for Industry*, Trans. ASME, Vol. 96, No. 2 (May 1974).
4. Gunter, E. J., et al, "Balancing of Multimass Flexible Rotors," *Proceedings of the Fifth Turbomachinery Symposium*, Texas A&M University, College Station, Texas, 1976.
5. Lund, J. W., "Stability and Damped Critical Speeds of a Flexible Rotor in Fluid-Film Bearings," *Journal of Engineeringfor Industry*, Trans. ASME, Series B, Vol. 96, No. 2 (May 1974).
6. Lund, J. W., "Modal Response of a Flexible Rotor in Fluid-Film Bearings," *Journal of Engineering for Industry*, Trans. ASME, Series B, Vol. 96, No. 2 (May 1974), pp. 525-533.
7. Bishop, R. E. D., and Parkinson, A. G., "On the Use of Balancing Machines for Flexible Rotors," *Journal of Engineeringfor Industry*, Trans. ASME, Series B, Vol. 94, No. 2 (May 1972), pp. 561-576.
8. Kellenberger, W., "Should a Flexible Rotor be Balanced in N or (N + 2) Planes?", *Journal of Engineering for Industry*, Trans. ASME, Series B, Vol. 94, No. 2 (May 1972), pp. 548-560.
9. Prohl, M. A., "A General Method for Calculating Critical Speeds of Flexible Rotors," *Journal of Applied Mechanics*, Vol. 12, Trans. ASME, Vol. 67 (1945), pp. A142-A148.
10. Dawson, R. N. and Eis, R. O., "Specifying and Designing Flexible Shaft Electric Machines," General Electric Company, Large Motor and Generator Department (to be published).
11. Ruhl, R. L., "Dynamics of Distributed Parameter Rotor Systems: Transfer Matrix and Finite Element Techniques," Ph.D. Dissertation, Cornell University, Ithaca, N.Y., 1970.
12. Lund, J. W. and Tonnesen, J., "Analysis and Experiments on Multi-Plane Balancing of a Flexible Rotor," *Journal of Engineering for Industry*, (February 1972), pp. 233-242.