A GENERALIZED AND SIMPLIFIED TRANSIENT TORQUE ANALYSIS FOR SYNCHRONOUS MOTOR DRIVE TRAINS

by
H. Ming Chen
Senior Analytical Engineer

Donald W. McLaughlin
Design Consultant

and
Stan B. Malanoski
Manager, Analysis and Design Engineering Branch
Mechanical Technology Incorporated
Latham, New York

H. Ming Chen is a graduate mechanical engineer from Rensselaer Polytechnic Institute, where he was a doctoral engineering candidate in System Engineering. Presently, he holds the position of Senior Mechanical Engineer and is a key technical contributor in the Research and Development Division of Mechanical Technology Incorporated. His fields of special competence, developed over the past twenty years in the analytical area, are rotor-bearing-foundation dynamics, heat transfer, finite element stress analysis, and computer-aided engineering analysis in general. Also, he has paralleled this analytical experience with field/experimental engineering experience in signature analysis, vibration trouble shooting and experimental stress analysis.

Mr. Chen is author or co-author of over a dozen technical papers and more than forty MTI Technical Reports. He is a member of ASME and a registered professional engineer in the State of New York.

Donald W. McLaughlin earned his B.S. in Mechanical Engineering from Michigan State University and his M.S. in Applied Mechanics from Rensselaer Polytechnic Institute. He has taken additional courses in statistics, reliability, and probabilistic methods in structural design. His career has been primarily in engineering analysis as a stress analyst, failure analyst, and manager, and he presently is a design consultant in the Research and Development Division of Mechanical Technology Incorporated.

Mr. McLaughlin has twenty-five years of experience in pressure vessel and rotating machinery stress analysis. He has specialized in metals engineering and in failure investigation and analysis as an adjunct to stress/deflection analysis.

The results of Mr. McLaughlin's work have been published in six technical papers and articles, and in over fifty internal technical reports. He also is a regular lecturer at MTI technical seminars on rotating machinery dynamics and investigation of machinery failures.

Stan B. Malanoski is a graduate mechanical engineer and Manager of Analysis and Design Engineering at Mechanical Technology Incorporated's Research and Development Division. He manages engineering personnel who provide designs and analyses in mechanical engineering with particular emphasis in the disciplines of machine design, rotor-bearing dynamics and vibration, stress, heat transfer, and piping analysis.

Mr. Malanoski's fields of special competence, developed over the past twenty-five years, are in management of engineering personnel and projects; dynamic analysis of practical rotor-bearing systems; squeeze-film damper design, analysis and application; development of efficient computer-based analyses and programs; and fluid-film bearings and bearing system designs, including process fluid (gas and liquid) lubrication. He is author or co-author of over forty technical publications and over two hundred Mechanical Technology Incorporated technical reports.

ABSTRACT

The start-up of synchronous motor drive trains is usually associated with torsionally excited vibrations and low-cycle fatigue problems. Traditional calculation methods used for analysis of such a system involve computerized integrations with very small time steps and many degrees of freedom.

A simple method is presented herein which uses the knowledge of system natural frequencies and mode shapes and a general dimensionless integration data plot. A sample problem is included to demonstrate the application of the method. A procedure is also included to relate the dynamic torque to the torsional low-cycle fatigue limit, thus establishing a safe number of starts.

INTRODUCTION

Large synchronous motors tend to have a decrease in acceleration just before they reach synchronous speed. At the same time, their vibratory exciting torque increases. This torque is at twice slip frequency, which coincides, for a moment, with the lowest torsional system natural frequency somewhere during the running up process. In other words, the
usually large inertia system is excited torsionally during every
start-up (Figure 1). The level of vibratory shear stress that is
reached depends on how fast the motor can pass the “critical
speed,” how much damping is available in the shafting, and,
of course, the size of the shafting at the weakest link, such as at
the bearing journal, shaft end, coupling, etc. It is not uncommon
to see shaft torsional low-cycle fatigue problems in these
machinery trains, especially those associated with motors of
“solid pole” structure [1].

![Figure 1. Example of Synchronous Motor Torsional Response.](image)

Traditionally, calculation of the transient torque is a
lengthy and costly task, because it involves computerized
integrations with very small time steps and many degrees of
freedom. Presented herein is a simple method which uses the
results of frequency and mode shape from the common Holzer
method for a torsional system and a specially generated dimen­
sionless integration data plot. The frequency and mode shape
determined by the Holzer method is preliminary to any tor­
sional system analysis, and it always precedes the transient
calculation.

**SIMPLIFIED METHOD**

During the start-up, the motor will pass a speed band
width around the critical speed where the system will be
excited. Since the system is usually lightly damped, the speed
band is relatively small, compared to the total speed range.
The rate of speed increase and the motor exciting torque in the
band width can be practically assumed as constants. The
vibratory motion of the system at this critical speed is repre­
sented by the modal equation:

\[
\frac{d^2 q_1}{dt^2} + 2\xi \omega_1 \frac{dq_1}{dt} + \omega_1^2 q_1 = \frac{V T}{I_1} \sin \left(2\pi \frac{2n N_s (1 - \frac{N_1}{N_s})}{N_1} t \right)
\]

where
- \( q_1 \) = modal (angular) displacement of first mode
- \( \xi \) = system damping ratio
- \( V \) = first mode shape displacement at motor
- \( V_i \) = first mode shape displacement at \( i \)th station of rotor
  model
- \( T \) = motor vibratory torque amplitude at \( 2 \times \) slip
  frequency
- \( I_1 \) = first modal inertia* = \( \sum I_i \omega_i^2 \) (lb-in-sec^2)
- \( I_i \) = inertia at \( i \)th station of rotor model
- \( \omega_1 \) = first torsional natural frequency (rad/sec)
- \( N_s \) = synchronous speed (cps)
- \( n \) = number of poles = 3 for \( N_s = 20 \) cps
  = 2 for \( N_s = 30 \) cps
- \( N \) = motor speed (cps)
- \( t \) = time (sec)

One can solve for \( q_1 \) as a function of time, and then the
vibratory torque at any shaft location (say between station \( i \) and
station \( i + 1 \)) with stiffness \( K_i \) is calculated as

\[
T_i = q_1 (V_{i+1} - V_i) K_i
\]

To solve Equation 1 by integration in time, one first specifies

\[
N = N_0 + \frac{1}{2\pi} \frac{T_0}{I_1} t
\]

where
- \( N_0 \) = initial speed (cps)
- \( T_0 \) = steady state driving torque minus the load (lb-in)
- \( I_1 \) = total system inertia (lb-in-sec^2)

Then substitute Equation 3 into Equation 1 and let

\[
T = \omega_1 t
\]

\[
N_1 = \omega_1 / 2\pi
\]

\[
B = 2n (N_s - N_0) / N_1
\]

Equation 1 becomes

\[
\frac{d^2 q_1}{dt^2} + 2\xi \omega_1 \frac{dq_1}{dt} + q_1 = A \sin \left( B - \alpha \tau \right)
\]

with

\[
A = VT/I_1 \omega_1^2
\]

and

\[
\alpha = 2n T_0 / I_1 \omega_1^2
\]

When the instantaneous forcing \( B \) frequency is close to 1, the
system, represented by Equation 4, will be excited. When
solving for \( q_1 \), it is sufficient to integrate the equation in the
range \( 0.8 \leq (B - \alpha \tau) \leq 2.0 \). One can use the Runge-Kutta
method or the simple Euler’s scheme for the integration. As
shown in the Appendix, a recursive formula derived from the
convolution integral method provides another alternative.

The maximum amplitude in Figure 2 is generated by
using \( \alpha = 1 \). Note that, in Figure 2, \( q_1 \) approaches \( \frac{1}{2\xi} \) as \( \alpha \)
approaches zero. This is the steady state torsional resonance
amplitude. By using Equations 2, 5, and 6 and Figure 2, one
can easily calculate the maximum torque in the system with
known first mode frequency and mode shape.

*If the torsional frequency analysis applies a consistent mass approach, such as
the finite element method, the modal inertias have to be calculated by modal
transformation. Frequently they are normalized to the value of 1.0 with respect
to the mode shapes.
PRACTICAL EXAMPLE

Figure 3 shows a synchronous motor driving an axial compressor through a single, step-up gearbox. The motor is rated at 25,000 hp, with synchronous speed at 1200 rpm. The torsional system can be mathematically represented by three inertias: the motor, the gear and the compressor. The first mode frequency and mode shape are calculated by the Holzer method and are shown in Figure 4. The system torques versus speed characteristics during start-up are presented in Figure 5. Figure 6 is the torsional Campbell diagram showing the location of the critical speed where the motor’s 2 × slip frequency coincides with the first torsional natural frequency at 972 rpm.

From Figure 5, one can read the following at 81% of the synchronous speed (0.81 × 1200 = 972 rpm):

\[ T = 0.71 \text{ P.U.} \]
\[ T_m = 1.025 \text{ P.U.} \]
\[ T = 0.175 \text{ P.U.} \]
\[ T_o = 0.85 \text{ P.U.} \]

where

\[ \text{P.U.} = 63,025 \times \text{hp/rpm} = 1.313 \times 10^6 \text{ in-lb} \]

Using Equation 5,

\[ A = 1.296 \times 10^{-3} \text{ rad} \]

Using Equation 6 with \( n = 3 \), the rotor acceleration parameter is

\[ \alpha = 4.23 \times 10^{-3} \]

From Figure 2, the amplification factor is 9.15, assuming
\( \xi = .04 \). Therefore, the maximum modal displacement is 
\((q_1)_{\text{max}} = 9.15 A = 1.186 \times 10^{-3} \text{ rad} \).

The maximum vibratory torque between the motor and the gear is, by Equation 2,
\[
T_{1-2} = (q_1)_{\text{max}}(V_1 - V_2) K_{1-2} = 4.27 \text{ P.U.}
\]

Since the steady state torque is 1.025 P.U. at 81% speed, the maximum total torque is
\[
T = T_{1-2} + T_{m} = 5.29 \text{ P.U.}
\]

The weakest section between the motor and the gear is at the motor bearing journal, with \( d = 9.5 \text{ in.} \) The shear stress due to the maximum total torque is
\[
\tau_s = 16 T/\pi d^3 = 41.3 \times 10^3 \text{ psi}
\]

COMPARISON WITH TRADITIONAL METHOD

The above example was treated by the traditional method [3], with three degrees of freedom. The result of the motor shaft torque is presented as the bottom trace of Figure 7. The corresponding simplified one-mode approach resulted in the top trace of Figure 7. The similarity between these two traces at the torsional resonance is obvious, and the accuracy of the simplified method is within 5%. Note that the top trace is plotted in the time scale \((t)\), not the normalized time \((\tau)\). Also, it is vertically shifted by the amount of steady state driving torque \((T_m)\).

LOW CYCLE FATIGUE ANALYSIS

In the lifetime of a synchronous motor drive machine, the number of starts is usually estimated to be 1,000 or more. During every start-up, the weak links in the rotor train will experience several high peak stress cycles. Here, in the example, the motor bearing journal, being a weak link, will have several instantaneous peak stresses higher than its yield when the stress concentration factor is considered. One must be sure that the “accumulated” damage at the torsional resonance will not break the journal in the designed life. Material fatigue data are available in the form shown in Figure 8, where
\[
\beta = 1 + \eta (K - 1)
\]
\[
K = \text{stress concentration factor}
\]
\[
\eta = \text{notch sensitivity factor}
\]
\[
\tau_w = \text{shear fatigue limit (psi)}
\]
\[
\tau_s = \text{calculated peak shear stress (psi)}
\]

For every peak at resonance, there corresponds a peak shear stress. The occurrence of each peak consumes a certain amount of the designed life. Table 1 shows how the peak stresses are related to the life in the conventional way. It indicates that the sum of all the consumed life fractions is 94%. Although it is less than 100%, and the 1,000 starts may be achievable, it does not leave much safety margin. Therefore, to increase the journal diameter from 9.5 in. to 10 in. is necessary. In order to avoid the detailed calculations of Table 1, which also need the time integration of the resonance peaks, it is proposed herein that the sum of the consumed life fraction be \( \Sigma f_n = 5 \times \text{maximum peak life fraction} \), and also that it be less than 1.0. Experience suggests that this simple but conservative rule provides enough safety margin.

<table>
<thead>
<tr>
<th>Peak No.</th>
<th>Calculated Shear Stress (1000 psi)</th>
<th>Stress Ratio</th>
<th>Cycles to Failure</th>
<th>1000 Starts Life Fraction ( f_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.9</td>
<td>0.557</td>
<td>645142</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
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<td>0.654</td>
<td>306690</td>
<td>0.003</td>
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<tr>
<td>3</td>
<td>23.4</td>
<td>0.727</td>
<td>175583</td>
<td>0.006</td>
</tr>
<tr>
<td>4</td>
<td>28.9</td>
<td>0.896</td>
<td>47787</td>
<td>0.021</td>
</tr>
<tr>
<td>5</td>
<td>33.1</td>
<td>1.029</td>
<td>17189</td>
<td>0.058</td>
</tr>
<tr>
<td>6</td>
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<td>1.211</td>
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</tr>
<tr>
<td>7</td>
<td>41.3</td>
<td>1.284</td>
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<td>0.410</td>
</tr>
<tr>
<td>8</td>
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<td>1.187</td>
<td>5134</td>
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</tr>
<tr>
<td>9</td>
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<tr>
<td>10</td>
<td>23.0</td>
<td>0.715</td>
<td>192658</td>
<td>0.005</td>
</tr>
</tbody>
</table>

\[ \Sigma f_n = \text{0.941} \]

NOTES:

\( \beta = 1.5, \tau_w = 32,000 \text{ psi} \)
\[
\Sigma f_n = 5 \times .410 = .205 \text{ (by simple rule)}
\]
Refer to Figure 7 for peak number.
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DISCUSSION

A number of important points related to the forcing frequency, the damping ratio, gear backlash, critical torque, and extension of the present procedure are discussed in the following:

"Instantaneous" Forcing Frequency

In Equation 4, the forcing frequency may appear to be \((B - \alpha \tau)\). But, one will find out in integration that at peak resonance, the value of \((B - \alpha \tau)\) is far from 1.0. Also, it is different for a different value of \(B\) assigned, while the resonance peak amplitudes stay the same. The truth is that we should be dealing with the “instantaneous” forcing frequency, i.e.,

\[
\frac{d}{d\tau} \left[ (B - \alpha \tau) \tau \right] = B - 2\alpha \tau
\]

and that this frequency will not change with the value of \(B\). It is, however, a function of the rotor acceleration parameter \(\alpha\).

Damping Ratio

It is evident from Figure 2 that the damping ratio, \(\xi\), of the first system torsional mode is one of the two dominant parameters for evaluating the peak resonance torque. Damping ratios of 0.03 to 0.05 are the common values used for a torsional system without a large damping element, such as a Holset coupling. In practice, there are different physical interpretations of the damping ratio. For example, some engineers specify different \(Q\) (which equals \(\frac{\xi^2}{\omega}\)) factors for different sections of the shafting. Strictly speaking, when one assigns a damping ratio to a lightly damped torsional mode, it means that every stiffness element in the model is in parallel with an equivalent, viscous damping of the value

\[
C = 2 \xi \omega
\]

where \(\omega\) = the modal frequency (rad/sec). The damping is proportional to the stiffness [4] only.

For the first torsional mode, only one section of the shafting is twisting the most. Therefore, it is reasonable to take the system damping ratio as the same as that section.

Gear Backlash

The authors’ experiences suggest that the amount of the backlash in the synchronous motor gear system do not have significant effects on the first torsional mode.

Critical Torque

Also, the most serious transient torque problem is not at the instance of switch-on, nor in cases of short-circuits, but at the first mode resonance speed.

Procedure Extension

The one-mode approach presented herein may be extended to systems with large damping and non-linear stiffness elements, as long as the first system model damping is not larger than 0.20 [4]. Further study is needed in the areas of equivalent damping ratio and linearized stiffness.

CONCLUSION

The simplified torsional transient method presented herein provides a fast alternative for evaluating the low-cycle fatigue problem frequently encountered in synchronous motor machinery. It is ideal for decision-making at the early design stage, and for field trouble-shooting.

APPENDIX

The impulse response of the system represented by Equation 4 is

\[
h(\tau) = e^{-\xi \tau} \sin \tau
\]

for small \(\xi\) and \(A = 1\).

The forced response can be calculated by the convolution integral:

\[
q(\tau) = \int_0^\tau \sin [(B - \alpha \lambda) \lambda] e^{-\xi (\tau - \lambda)} \sin (\tau - \lambda) d\lambda
\]

or

\[
q(\tau) = e^{-\xi \tau} [F_1(\tau) \sin \tau + F_2(\tau) \cos \tau] \quad (A1)
\]

where

\[
F_1(\tau) = \int_0^\tau \sin [(B - \alpha \lambda) \lambda] e^{\xi \lambda} \cos \lambda d\lambda \quad (A2)
\]

\[
F_2(\tau) = -\int_0^\tau \sin [(B - \alpha \lambda) \lambda] e^{\xi \lambda} \sin \lambda d\lambda \quad (A3)
\]

Let \(\Delta \tau\) be a finite increment of \(\tau\). Using equation A1, the following recursive equation is derived:

\[
q(\tau + \Delta \tau) = e^{\xi \Delta \tau} \cos \Delta \tau q(\tau)
\]

\[
+ e^{-\xi (\tau + \Delta \tau)} \left[ (F_1 \cos (\tau + \Delta \tau) - F_2 \sin (\tau + \Delta \tau)) \right.
\]

\[
+ \left. (\Delta F_1 \sin (\tau + \Delta \tau) + \Delta F_2 \cos (\tau + \Delta \tau)) \right]\] \quad (A4)

where

\[
\Delta F_1 = F_1(\tau + \Delta \tau) - F_1(\tau) \quad (A5)
\]

\[
\Delta F_2 = F_2(\tau + \Delta \tau) - F_2(\tau) \quad (A6)
\]

Equations A1 to A6 can be implemented into a simple computer routine for calculating the transient modal response.

REFERENCES
