SAFE DIAGRAM—A DESIGN AND RELIABILITY
TOOL FOR TURBINE BLADING

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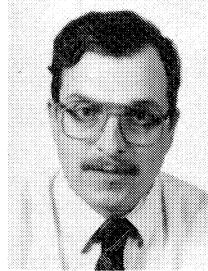
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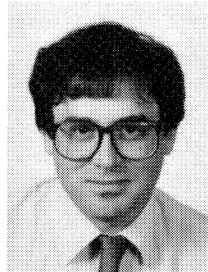
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ABSTRACT

The complex field of turbine blade vibration has long been in need of improved tools to help predict the reliability of blading.

The SAFE interference diagram is presented as such a tool. It presents much more information than the widely used Campbell diagram. In evaluating interferences, the SAFE diagram compares not only the frequencies of exciting harmonics with natural frequencies of blades, but also the shape of these harmonics with the normal mode shapes of a completely bladed disc including packeted blading. Examples are given of cases where the Campbell diagram predicts a dangerous resonance while the SAFE diagram shows that no resonances exist which

are supported by experience. Examples are also provided to show when the SAFE diagram can pinpoint what interference is likely to cause the largest blade vibration. Finally, it is shown how a simple change in packeting can be used to change the blade interference and to avoid dangerous operation.

INTRODUCTION

To meet the objective of designing reliable and trouble free blading in turbomachinery, the blade natural frequency analysis is of utmost importance. This is due to the fact that almost all blade failures can be attributed to metal fatigue, which is caused by variable aerodynamic loads acting on the blading. Resulting dynamic stresses depend on the natural frequency and the mode shape of the blade, the frequency and shape of the exciting force, and the energy dissipating mechanism present in the system included as damping.

In the early days of blade design, the natural frequency analysis was based on the assumption of a single beam cantilevered at the blade root. Prohl and Weaver [1] showed that in the case of a packeted assembly where a group of blades are connected by shrouding, many more natural frequencies and modes exist which could not be predicted by a single blade analysis. This is due to the fact that the group of blades behave as a system and are coupled through the shrouding. The magnitude of frequencies and the number of modes depends on the number of blades in the group and the stiffness of the shrouding. This type of analysis has explained many blade failures and helped designers to build more reliable blades.

The next important step in blade analysis was due to the realization that blades are mounted on a disk which can influence the dynamic behavior of blades. Calculations including the disk showed that frequencies can be affected and a new large number of modes exist which cannot be predicted by using a single packet analysis. In reality, all blades and the disk constitute one system which may or may not respond to an exciting force.

Singh and Schiffer [2] presented a finite element analysis for a packeted bladed disk assembly. They showed and discussed the features of dynamic behavior which is different than when the shrouding is 360 degrees.

Traditionally, a blade design is evaluated by using the Campbell Diagram. This paper discusses a method called the SAFE diagram used by the authors' company for design evaluation of packeted bladed disk assemblies. It has been used successfully not only in explaining failures and in developing reliable designs, which otherwise would not have been explained, based on the Campbell diagram.

MODE SHAPE OF A BLADED DISK AND FLUCTUATING FORCE

A brief description of the mode shapes of a bladed disk and the nature of fluctuating forces encountered in a turbine is provided to help understand the mechanism of resonance and also to depict graphically the necessary conditions of resonance.

MODE SHAPE

To define the dynamics of any mechanical structure, the eigenvalue (natural frequency) and the accompanying eigenvector (mode shape) are calculated. In the case of a bladed disk, the mode shapes have been described as nodal diameters and nodal circles. Singh and Schiffer [2] presented some calculated mode shapes for a packeted bladed disk in tangential vibration. The displacement, for example, of the tip of each blade when plotted with angular position, shows a sinusoidal characteristic which is designated as a diametral pattern. This is valid for either pure tangential or axial vibration of the blades. A bladed disk of 90 blades is shown in Figure 1 vibrating in what is called the first tangential mode. This wheel has 15 packets of blades with 6 blades in each packet. The important feature of the mode shape of Figure 1 is that not only are all six blades within each packet vibrating in phase but all 15 packets are also vibrating in phase. This is called the zero nodal diameter mode shape because there are no phase changes in the vibration of all 90 blades. The 1 nodal diameter mode shape of the same bladed disk is illustrated in Figure 2. There are two phase changes in the bladed disk which are noted by the radial lines forming the 1 nodal diameter mode shape. Notice that the six blades within each of the individual packets are still vibrating in phase with one another in the first tangential mode shape. The second bladed disk shown in Figure 2 has the same mode shape except the radial lines are shifted in location by 90 degrees.
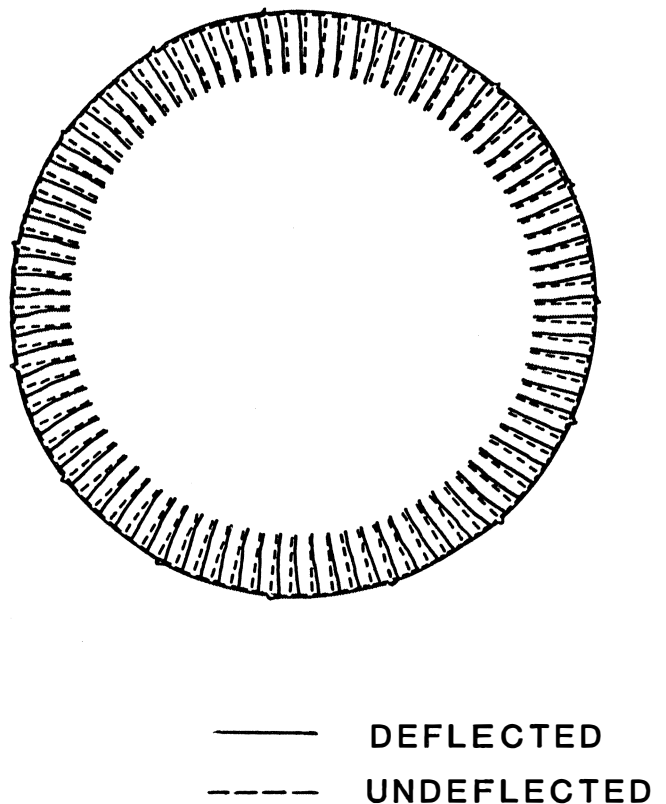

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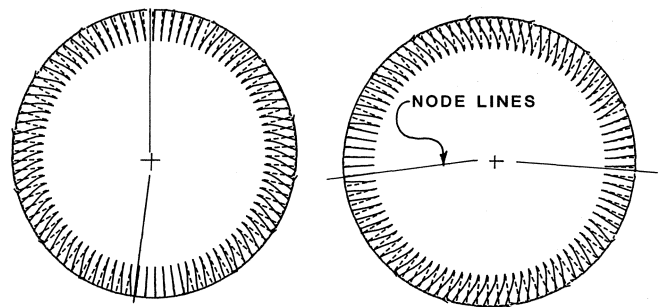Figure 1. Zero Nodal Diameter Mode.

Figure 2. One Nodal Diameter Mode.

Each nodal diameter mode shape occurs in a pair. The two nodal diameter mode shape of the same bladed disk is shown in Figure 3. Again, all the blades within each packet ars still vibrating in phase with each other in the first tangential mode shape. This trend will continue until each packet is out of phase with its neighbors. In this case where there is an odd number of packets (15) there will be a maximum of 14 phase changes between packets or seven nodal diameters.
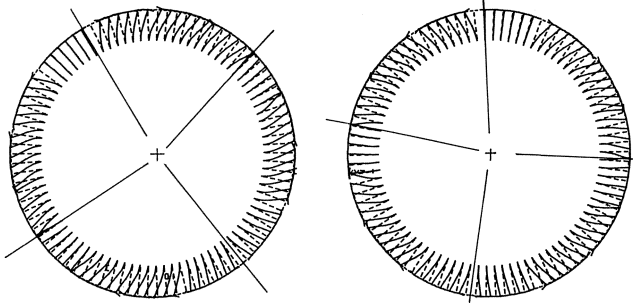

Figure 3. Two Nodal Diameter Mode.

The remainder of the first tangential family of mode shapes for a six blade packet is shown in Figure 4. These mode shapes are commonly called in phaseout of phase modes or fixed-supported modes. The significant characteristic of these mode shapes is that the number of phase changes within the packet varies from one to five. The packet with one phase change will form bladed disc nodal diameters eight through 15. The 15 nodal
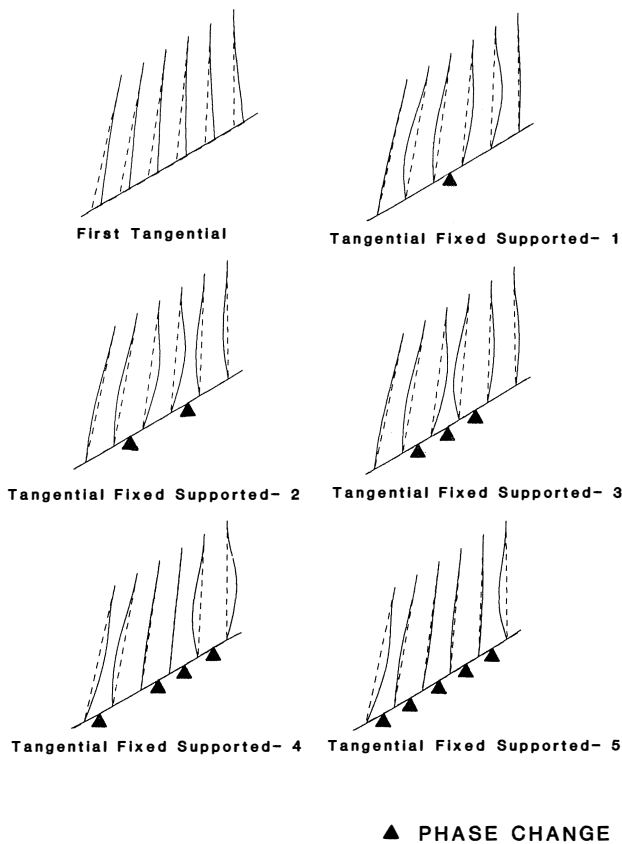

First Tangential   **Tangential Fixed Supported- 1**

Tangential Fixed Supported- 2   **Tangential Fixed Supported- 3**

Tangential Fixed Supported- 4   **Tangential Fixed Supported- 5**

▲ **PHASE CHANGE**

Figure 4. Tangential Mode Shapes For Six Blade Packet.

diameter mode shape is shown in Figure 5. The packet with two phase changes will form nodal diameter mode shapes 15 through 23. This trend will continue until the packet with five phase changes forms bladed disk mode shapes 38 through **45**. For a bladed disk with 90 blades, the maximum number of nodal diameters is 45, which occurs when a phase change occurs between each blade.
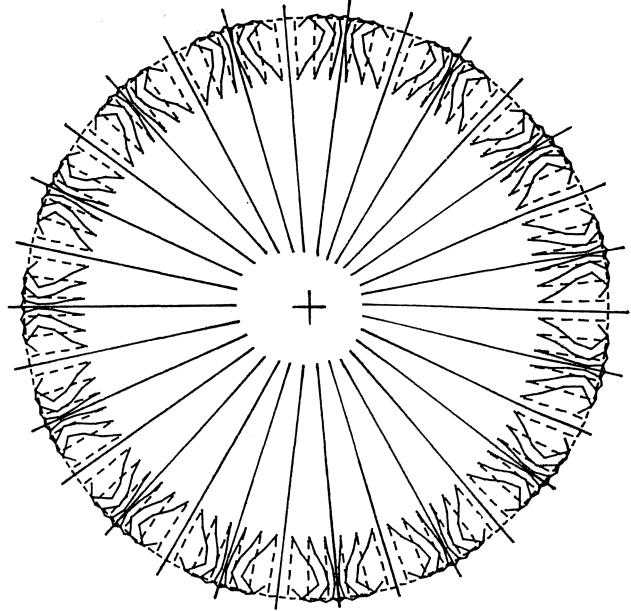

Figure 5. 15 Nodal Diameter Mode.

When this pattern shows a one sine (sine θ) pattern, it is designated as one nodal diameter. If it shows a two sine (sine 2θ) pattern, it is designated as two nodal diameter. The maximum number of nodal diameters in a bladed disk assembly is half of the number of blades for an even number of blades. For a disk having an odd number of blades, the maximum nodal diameter is half of the total number of blades minus one. Mathematically, the mth mode at one speed can be described as:

$$y_m (\theta, t) = -Y_m \cos (\omega_m t + m\theta) \qquad (1)$$

ω_m is the natural frequency and m is the number of nodal diameters. The frequency of the blading is dependent on the speed of the turbine, due to the centrifugal stiffening effect. Hence, the natural frequency, ω_m, nodal diameter (m), and the turbine speed are required to describe any one mode. Due to the three variables required, it can only be plotted as a surface. A typical plot for a packeted bladed disk is given in Figure 6.

FLUCTUATING FORCES

Forces imposed on the blade in one complete revolution emanate from the circumferential distortion in the flow. The pressure variation in the fluid flow causes the forces to vary circumferentially. These forces depend on the relative position of blading with respect to nozzles and any other interruptions like struts, etc. In each revolution, however, blades will experience the same force pattern making it a periodic force. The force can be broken into its harmonic contents using Fourier decomposition.

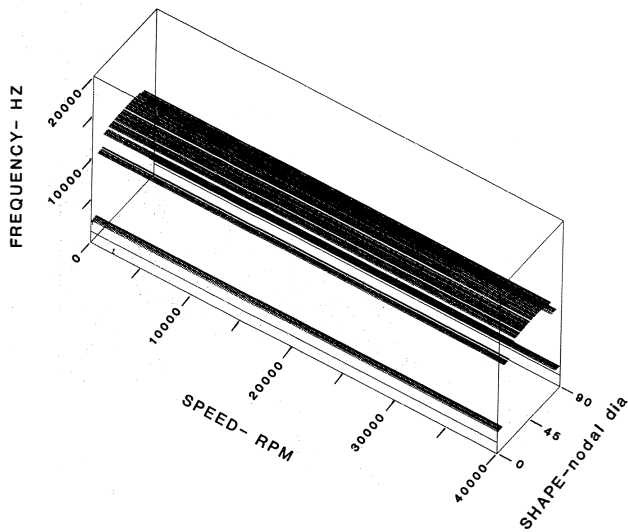$$F = F_0 + F_1 \sin (\omega_1 t + \theta_1) + \ldots + F_m \sin (\omega_m t + \theta_m) + \ldots \qquad (2)$$

Figure 6. Three Dimensional Representation for Modes of the Packeted Bladed Disc.

Figure 7. Three Dimensional Representation of Forcing in Steam Turbines.

The frequency of the harmonics depends on the speed of the turbine and the number of interruptions in the annulus, e.g., the number of nozzles, and is expressed as:

$$\omega = \frac{MN}{60} \qquad (3)$$

where

ω = frequency (HZ)
M = number of nozzles in a 360 arc
N = turbine speed (rpm)

Fluctuating forces are due to two primary sources in turbines. The first is known as nozzle passing frequency excitation and the second is running speed harmonic excitation. Nozzle passing frequency excitation is caused by nozzle vane trailing edge wakes as the flow leaves the vanes. A blade passes through nozzle vane wakes a number of times equal to the number of nozzle vanes in one revolution. Examination of equation (3) reveals that the harmonics of forcing can only be described as a surface with three variables, namely frequency, ω_m a shape M and speed of the turbine N. A typical plot is shown in Figure 7.

Running speed excitation is caused by circumferential variations in the fluid flow pressure usually due to extraction or exhaust ducts. Other low harmonic sources of excitation which are multiples of running speed, are support struts or diaphragm nonuniformities. For example, six symmetrically located support struts will give a six times running speed (6 × RS) or a sin 6θ excitation to the blades.

CONDITION OF RESONANCE IN A BLADED DISK

A mathematical discussion of the condition of resonance of the bladed disk is provided here and some important conclusions are drawn which will help give an understanding of the evolution of the SAFE diagram.

In each revolution of the bladed disk, blades pass through a field of pressure fluctuation due to nozzle or any other interruptions in the flow field. This fluctuation in pressure imposes a time varying force on the blades. In general such forces can be broken into harmonic contents using Fourier analysis. The frequency of any harmonic is an integer multiple of rotational speed and the number of interruptions The nth harmonic of the force can be expressed as:
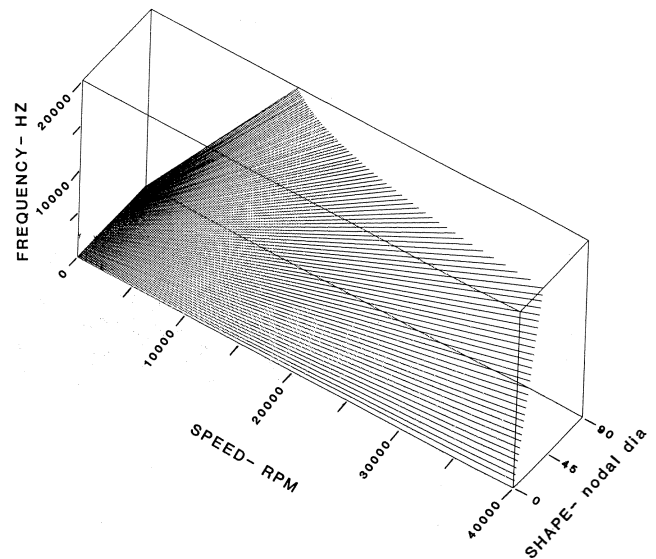
$$f_n(\theta,t) = F_n \sin n(\omega t + \theta) \qquad (4)$$

where

F_n = amplitude of the nth harmonic
θ = angular position on the disk
t = time

The condition of resonance is defined as when the forces imposed on the blade do positive work. When a force is applied on a moving mass the work is defined as:

$$W = \int_0^1 F dx \qquad (5)$$

where

W = is the work done by the force, F, to move the body to a distance 1.

The work done by the nth harmonic of the force on the mth nodal diameter mode of the bladed disk in one period can be expressed by using equations (1), (4), and (5) as

$$W = \int_0^{2\pi} \int_0^T f_n(\theta,t) \; \frac{\partial}{\partial t} \; y_m(\theta,t) \; \frac{N}{2\pi} \; dtd\theta \qquad (6)$$

Finally,

$$W = \begin{cases} \pi NF_nY_m \text{ for } n = m \text{ and } \omega_m = n\omega \\ 0 \text{ for } n \neq m \text{ and } \omega_m \neq n\omega \end{cases} \qquad (7)$$

The first of the above results defines the condition of resonance for the bladed disk. It can be stated as follows:

• The natural frequency ω_m of the bladed disk must be equal to the frequency of the exciting force

$$\omega_m = m\omega \qquad (8)$$

The number of nodal diameters must coincide with the harmonic of the force n.

$$m = n \qquad (9)$$

The second result defines that the work done will be zero when either of the above conditions is not satisfied.

Summarizing the results for a true resonance in the case of a bladed disk, each of the conditions is necessary, but not sufficient. Both of these must exist together for a true resonance to occur.

TRUE RESONANCE IN THE PACKETED BLADED DISK CAMPBELL DIAGRAM

Determination of probable resonances has been traditionally made by using a Campbell diagram. A Campbell diagram depicts turbine speed on the horizontal axis and frequency on the vertical axis. The natural frequencies and the frequencies of exciting forces are plotted. The coincidence of natural frequencies with the exciting frequencies have been used as the definition of resonance. A typical Campbell diagram and the depicted points where natural frequencies are equal to the exciting frequencies are shown in Figure 8. Experience based on turbines in the field and calculations of response have demonstrated that only a few points among the numerous points in figure 8 show high response levels hence providing concern for the reliability of the blading.

Figure 9. Three Dimensional Interference Representation of Resonance Conditions.
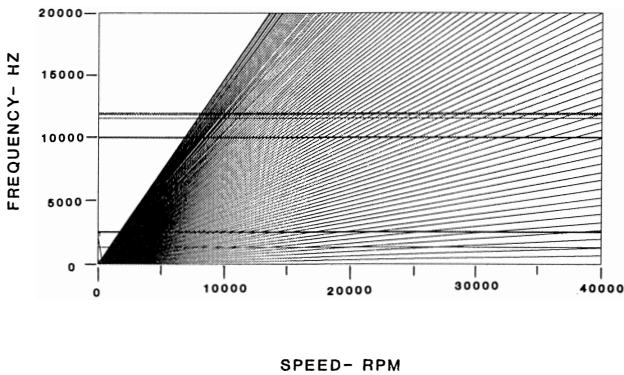

Figure 8. Campbell Diagram.

There is a need to eliminate the points which are of no concern in the evaluation of reliability and also to explain why these points are of no concern to designers. This aspect is considered in the following sections.

GRAPHICAL REPRESENTATION OF TRUE RESONANCE

The true resonance in a bladed disk of a turbine is defined in Equation (7) by combining the definition of modes given in Equation (1) and force defined in Equation (4). The three dimensional plots of Equation (1) and Equation (3) have been given in Figure 6 and Figure 7. Graphically, the condition of resonance can be denoted by the intersection of two surfaces shown in Figure 6 and Figure 7. Such a plot is provided in Figure 9. The interference points shown as circles are the true resonance points. Even though a three dimensional plot such as that in Figure 9 helps define conceptually the resonances, two dimensional views are desired to make design decisions. Three planar views of resonance points are given in Figures 10, 11, and 12.

SAFE DIAGRAM

A closer look at Figure 10 shows that this is a Campbell diagram and the circles denote true resonances. One can argue here that one must be able to show only the true resonance
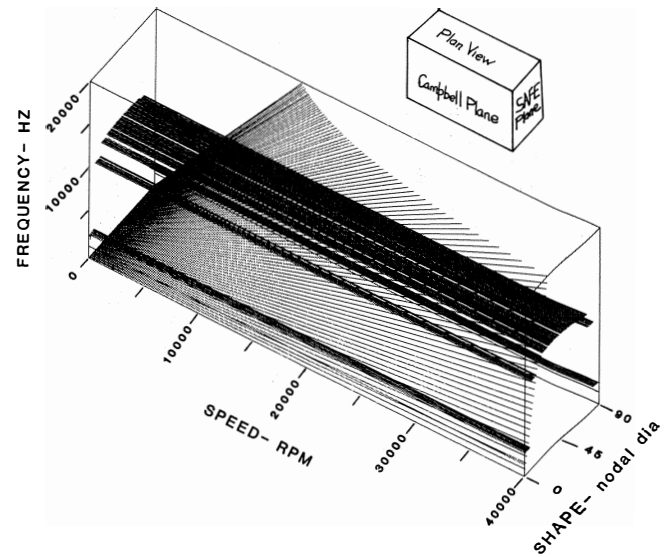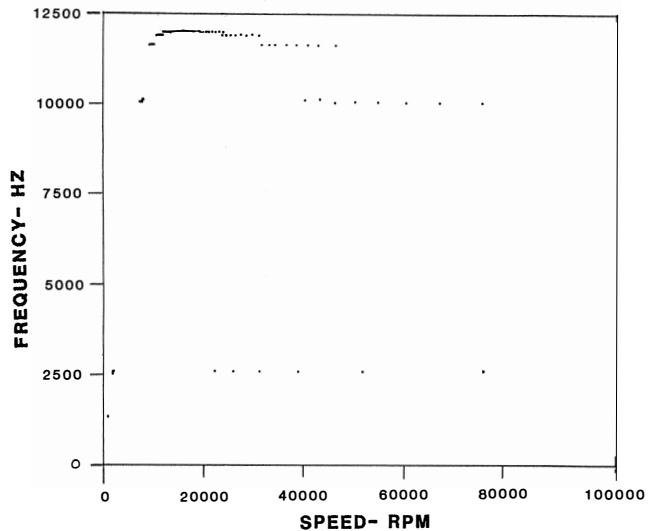

Figure 10. Plot of Interference Points on Campbell Plane.

points as in Figure 10 to avoid a lot of unnecessary concern and anxiety over superfluous frequency coincidence. As it turns out, if one must use a two dimensional depiction of a three dimensional surface, the plot in Figure 11 provides a clearer alternative. Diagrams like this have been in use by the authors' company for many years under the name of the SAFE diagram and have helped in explaining failures, and also successes unexplained based on the traditional Campbell diagram.

The SAFE (Singh's Advanced Frequency Evaluation) diagram differs from the Campbell diagram in that it plots frequency vs nodal diameters (mode shape) rather than frequency vs speed. The concept of mode shapes in turbine blade vibration can best be explained by looking at the blade vibration of an entire bladed disk rather than a single blade or a packet of blades. This can be done for blade vibration in either the axial direction or the tangential direction as explained earlier The first family of tangential modes is plotted on the SAFE diagram of Figure 11. The Campbell diagram of Figure 10 and the SAFE diagram
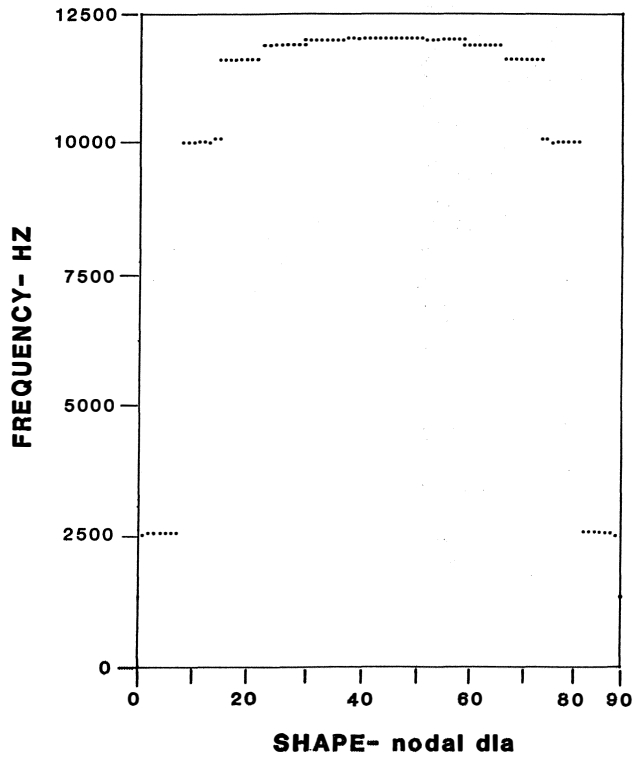
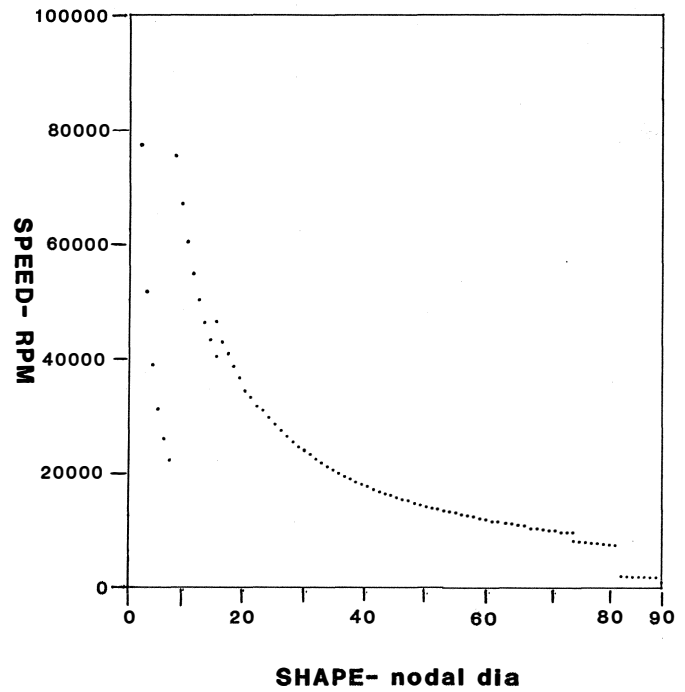Figure 11. Plot of Interference Points on SAFE Plane.

Figure 12. Plan View of Interference Points.

CASE 1   BKTS IN GROUP 6

BKT.HGT. 1.299   NO. OF NOZZLES 78

Figure 13. Campbell Diagram Showing Interference with 1 × NPF for Case 1.

of Figure 11 can be directly compared, since they are plotting identical blade frequencies.

CASE STUDIES USING THE SAFE DIAGRAM

Three cases are chosen to demonstrate the use of the SAFE diagram These cases help to show various advantages in making reliability decisions.

Case 1—No Design Change Based on the SAFE Diagram

This case illustrates that by using the bladed disk concept and the SAFE diagram a costly design change was averted.

In the head end stages of steam turbines, the blade heights are relatively short (less than two inches in height) and when their natural frequencies are plotted on a traditional Campbell diagram a situation similar to that depicted in Figure 13 frequently occurs. What is shown in the Campbell diagram is an interference between the first harmonic of nozzle passing frequency (1 × NPF) and the three lowest natural frequency modes of a packeted bladed wheel within the operating speed range. These modes are the first tangential, first axial and axial-rocking modes. Since nozzle passing frequency is one of the prime sources of excitation in turbines and the lowest natural blade frequencies are the most easily excited, a large response would be expected. Frequently design changes have been made in order to avoid these interferences. The design changes made are either in the number of nozzle vanes to change the exciting frequency or in the blade to change the natural frequency. Sometimes, both changes are required to accomplish the goal of avoiding the interferences. These design changes often result in compromising the stage efficiency.

When the same frequencies in the Campbell diagram of Figure 13 are plotted in the SAFE diagram of Figure 14, a different result occurs. The SAFE diagram shows that there is no reso-

nance, since only the frequencies match but not the mode shape (nodal diameters) and, therefore, the response will be small. The shape of the 1 × NPF exiting frequency with 78 nozzles occurs at the 72 nodal diameter location of Figure 14 for this case (150 rotating blades), since it is reflected off the 75 nodal diameter
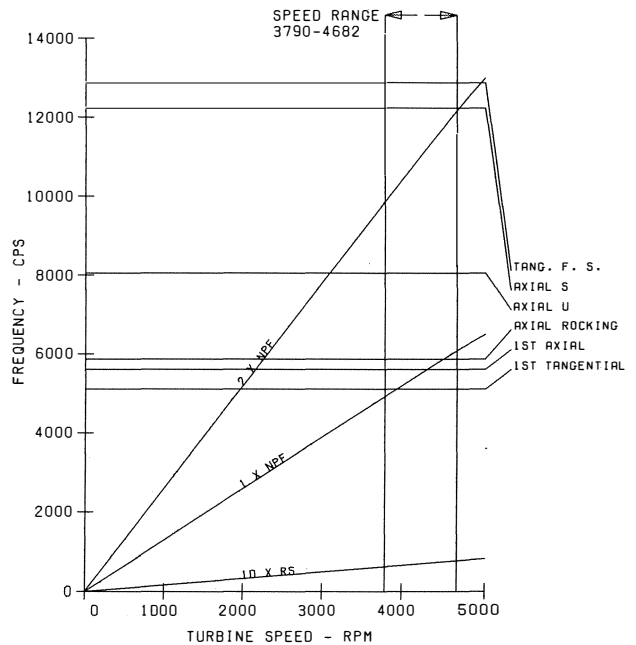
line. Detailed description of the construction of the SAFE diagram and the validity of reflecting speed lines and NPP excitation is shown in the literature [3]. Since the number of nozzle vanes is generally about half the number of rotating blades, the shape of the 1 × NPF exciting frequency and the mode shapes of the first three packeted blade frequencies will never match, and a true resonance will not occur. The only occasion when 1 × NPF excitation would be resonant with the lowest three blade frequencies is when the number of nozzle vanes and the number of rotating blades are almost equal. Then both the frequency and mode shape would match.
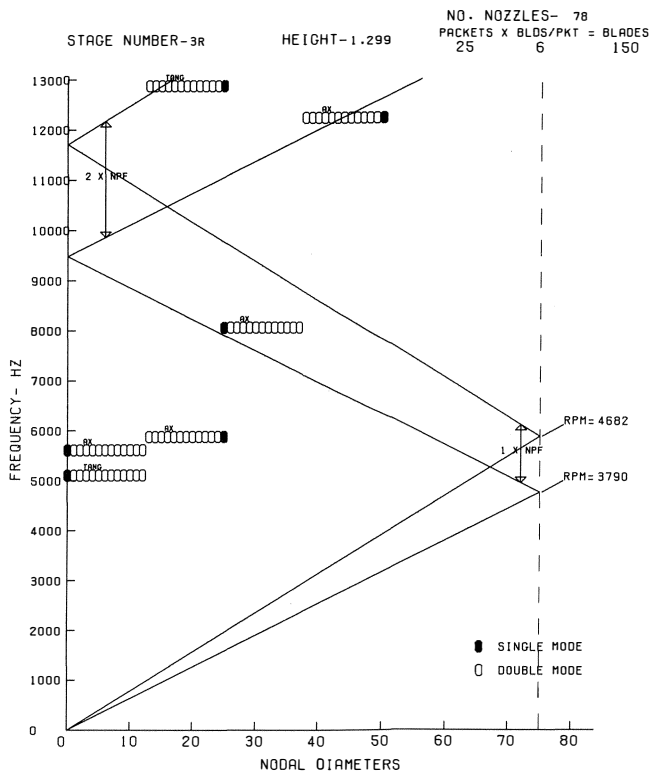

Figure 14. SAFE Diagram for Case 1.

Stages with Campbell diagrams similar to Figure 13 have occurred on numerous occasions in the authors' experience. However, the failure rate of these short blade stages has been negligible. Most of the turbines are mechanical drive units that operate over a large speed range, and therefore, the interferences between 1 × NPF and the lowest blade frequencies are inevitable at some time in operation. The only explanation for the excellent reliability of these blades is the SAFE diagram.

CASE 2—THE SAFE DIAGRAM HELPS PINPOINT AN AREA OF CONCERN QUICKLY

This case study is presented to illustrate how use of the SAFE diagram is used to quickly determine if a bladed disk interference exists. Bladed disk natural frequencies and mode shapes are calculated. By plotting these on a SAFE diagram along with rotor speed and exciting frequency data, a determination of interference is made. Finally, blade response will be calculated at the interference frequency.

The turbine disk consists of 18 packets of six blades each. This disk is a uniform 1.0 in thick with a bore diameter of 6.0 in. Additional information is given below:

Number of Blades — 108
Blade Height — 4.0 in
Disk Diameter — 19.0 in
Number of Nozzles — 20
Rotor Speed — 8000 rpm

To begin this case study, a traditional Campbell diagram shown in Figure 15 is constructed from data from blade packet natural frequency calculations. For simplicity, only axial modes are considered here. The Campbell diagram is drawn here for comparison with the SAFE diagram which is introduced next. There is an interference in the operating speed range with 1 × NPF and the axial "U" mode. The axial "U" mode is characterized by a "U" shaped bending of the shroud in the axial direction. Nozzle passing frequency is 20 times running speed. That is, there are 20 nozzles and for every revolution a blade "feels" 20 impulses. Maximum deflection is at the shroud midspan and there are two node points along the length of the shroud.
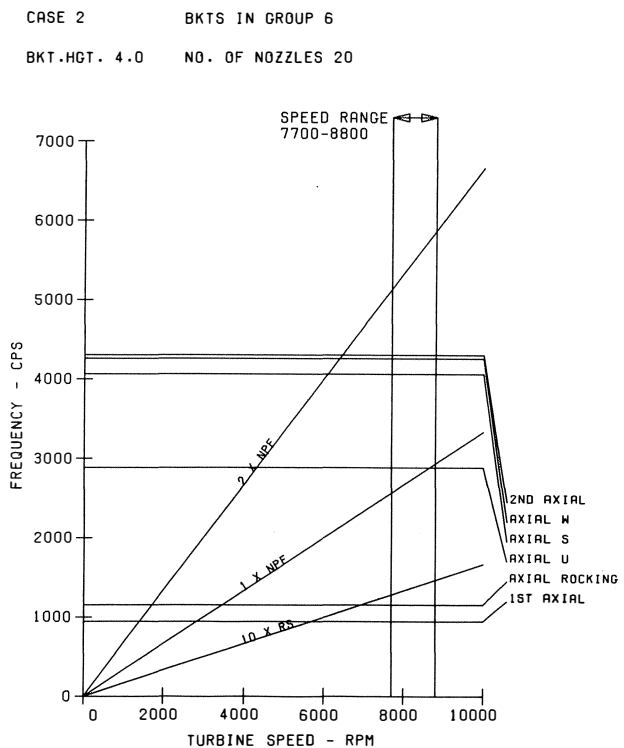

Figure 15. Campbell Diagram for Case 2.

Construction of the SAFE diagram begins with calculation of bladed disk natural frequencies and mode shapes. This is an important difference; the source of data for construction of the Campbell diagram is a packet calculation while the SAFE diagram is constructed from data from an entire bladed disk calculation. The computer program TBDYNE [4] has been developed expressly for the purpose of solving bladed disk problems. TBDYNE is a finite element program which solves for the natural frequency and mode shapes of bladed disks. It is also capable of performing a forced response analysis.

The SAFE diagram for this case is shown in Figure 16. Again, for clarity, only axial modes are included. The model used in this case consists of three elements for the disk from the bore to the outer diameter. Therefore, disk flexibility is included in this analysis, which is another advantage of this type of analysis. This feature is evident because the frequencies of lower nodal diame-

ter modes are significantly reduced by inclusion of disk flexibility as shown in Figure 16. Also, axial natural frequencies at higher nodal diameters are lower on the SAFE diagram than on the Campbell diagram, because the flexibility of the disk is included in the analytical model.
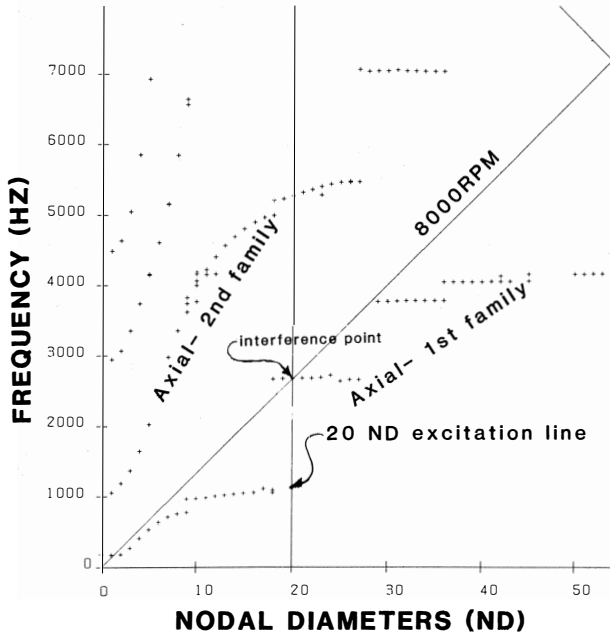

Figure 16. SAFE Diagram for Case 2.

The excitation line at 8000 rpm passes through the 20 nodal diameter mode. This identifies the location as a real resonance and design changes should be made to avoid the situation. Either the structure must be changed or the forcing mechanism must be removed. Listed below are changes that could be made to avoid this interference.

- Change the number of blades in a packet
- Change the number of nozzles
- Modify blade flexibility
- Modify wheel flexibility
- Change shroud flexibility
- Move the operating speed range

The first two choices are the most practical and the ones most often used. The next case examined will continue Case 2, and observe the effect of making the necessary changes to avoid resonances.

CASE 3—THE SAFE DIAGRAM WILL HELP MAKE A SIMPLE CHANGE TO AVOID RESONANCE

Now that the cause of the resonance has been identified, this case will show how changes can be made to avoid resonance. To verify the SAFE diagram and to show improvements in the bladed disk, a forced response analysis is performed. In this analysis the basic equation of motion

$$m\ddot{x} + c\dot{x} + Kx = F(t) \qquad (10)$$

is solved for displacement. Blade stress can then be calculated from the displacement solution. For this analysis, a unit force of

1.0 lb was applied at the blade tip to the first blade in the packet. Forces were applied to the other blades in the packet using the appropriate phase shift based on the circumferential locations.

A calculation was performed at the 20 nodal diameter interference point of Figure 16 of Case 2. The frequency at this point is 2674.7 Hz. Results are shown in Table 1 for stresses at the blade base.

Table 1. Response of Six Blade Packet Due to 20 ND Excitation at 2674 Hz.

Blade Sequence in packet	Axial stress (psi)
1	2580
2	294
3	2160
4	2160
5	295
6	2570

Since the mode in interference is the axial "U" described earlier, the above pattern of stress in not unexpected. Blade stress is relatively low for blades two and five, because these blades are near node locations, and there is relatively little motion in these blades.

Changing the number of blades in a packet will eliminate the interference of Case 2. The SAFE diagram for the same bladed disk of Case 2 is shown in Figure 17, but the number of blades in a packet has been reduced to three. This change has two results, it increases the (frequency of the) axial mode of concern and it "rearranges" the nodal diameter pattern of the axial modes. If a response calculation is performed at the same frequency, stress levels should be greatly reduced. The results of the calculation are shown in Table 2.
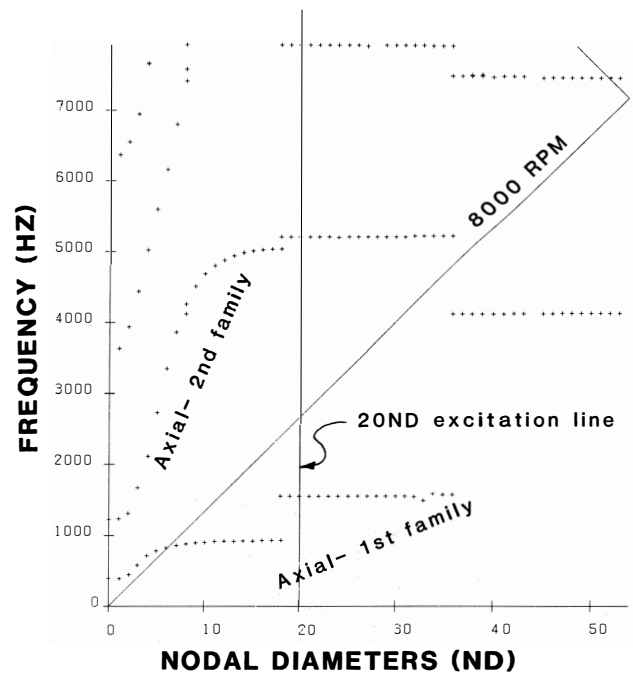

Figure 17. SAFE Diagram for Case 3.

Table 2. Response of Three Blade Packet Due to 20 ND Excitation at 2674 Hz.

Blade sequence in packet	Axial stress (psi)
1	180
2	20
3	137

These numbers represent greatly reduced stress levels from those of the six blade packet. Changing the packeted disk natural frequency has eliminated the interference and thus increased the reliability of the wheel design.

CONCLUSION

The tools to calculate natural frequencies of blading have become faster, more economical and more accurate in recent years. Still, the unplanned and untimely outage of turbomachinery due to blade failures are numerous. The study presented herein and the proposed use of the SAFE diagram have helped to identify possible problem areas and have also helped in less costly, faster design changes. The concept of the SAFE diagram is based on the matching of natural frequency and its associated mode shape with the exciting frequency and its shape. This can be used on any rotating, symmetrical system where the frequency of excitation is a function of rotational speed. Such cases are encountered in axial compressors, gas turbines, pump, and fans.

The SAFE diagram provides more information in the blade reliability decision process than the traditional Campbell diagram. The use of the SAFE diagram is proposed in blade reliability evaluation.

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