

RELIABILITY CRITERIA FOR TURBINE SHAFT BASED ON COUPLING TYPE—GEARED TO HAVE FLEXIBILITY

by

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ABSTRACT

A turbine shaft end at the coupling is exposed not only to constant torque load but to various variable loads as well. These externally imposed variable loads come from sources like compressor surge, misalignment between turbine shaft, and the driven machine, to name a few. These loads can be bending or torsional in nature. To assure reliability, the design method should include the influence of such loads on the final sizing of the shaft

end. The design method should also include the effects of shrink fits, keyways, fillets, chrome plating and fretting on the fatigue properties of the shaft material.

Due to variable nature of loads superimposed on the steady torque load, the design method must be based on fatigue behavior of materials. Many fatigue theories have been used in the design of components of turbomachinery. These depend on the nature of the load, type of material and component being designed. A designer should realize not only the nature of loads imposed on the shaft and their source but also have an estimate of the magnitude of those loads.

A design method is described which includes many of the factors governing the useful life of the shaft at the coupling end. This method is based on the von Mises reliability criteria, including the influence of the driven machine and the choice of coupling type.

This design method has resulted in many successful designs throughout the years and has helped in selecting a less costly but reliable option in many rerate situations.

INTRODUCTION

The mechanical power generated by a turbine is transferred to the driven machine through the shaft at the coupling end. Size of shaft end and its load carrying capacity depend on the method of connection between the turbine and driven shafts. Other variables influencing the size of shafts are torque load, externally-imposed variable loads, and material properties of the shafting.

The selection of material is based on its monotonic as well as on its fatigue properties. Sizing, based on constant torque and monotonic properties of material, is considered to be straight forward, but experience shows that distress in the shaft end is mostly due to fatigue.

This presentation will first include a method which is based on von Mises stress theory which has been used extensively in shaft design. Secondly, discussions and suggestions will be provided for many of the concerns which do arise in design of the shaft end. Maintenance practices and procedures necessary for reliable operation of steam turbine are not addressed.

FACTORS TO BE CONSIDERED IN DESIGN

Many factors must be understood because of their importance in the evolution of a comprehensive design philosophy. Claims are not made herein either for the list to be complete or for dis-

cussions to be exhaustive. It is hoped, however, that these will set a basis by which factors can be included or excluded when the application dictates. Also, the suggested numerical value of some factors can be modified when more knowledge is obtained.

Configuration of Shaft End

Two methods of attaching the coupling halves to shaft ends are shrunk-on hub and integral hub.

Shrunk-on hub construction has an interference between shaft and hub bore. The shaft end can be either tapered or cylindrical in shape. Keys are also used in such a construction; the numbers of keys can be one or more. The amount of interference is specified such that shrink-fit through friction will carry all or part of the torque load. The size of the key(s) should be such that the full load can be carried by the key(s).

In such a design, two extremes can occur. First, the full load is carried by the shrink stress and key(s) are used as a back up. Secondly, there is no shrink and the full load is carried by the key(s). The first option provides a redundancy, but is associated with higher cost for precisely controlling the interference. Due to microscopic relative motion between hub bore and shaft surface, fretting fatigue damage can occur, which is detrimental to fatigue life. The second option is associated with lower cost, but has high stresses near the keyway. This option does not provide any redundancy if the key fails.

Integral hub design provides a hub at the shaft-end which bolts onto the coupling half. This type of construction provides minimal overhang weight which helps the rotordynamics. It reduces potential problems in mounting and unmounting of the coupling half as in the shrunk-on case, e.g., loose fit, scoring of shaft surface, and the bore of the hub. The stress imposed on the shaft end is reduced by elimination of the key(s) and shrink-fit. However, the integral hub can be damaged in handling. There can be physical constraint where this type of design might not be feasible, such as when an unsplit seal is used.

Influence of Coupling Type

Discussion of the available coupling types and their use in a turbomachinery train is beyond the scope of this presentation, but the influence of coupling on shaft end size is important and will be discussed. The design method to be presented later will account for cyclic load which can result due to misalignment of two shafts. Cyclic load which results due to misalignment depends on coupling stiffness. Couplings with larger stiffness will impose larger cyclic load on the shaft end for a given misalignment, compared to more flexible coupling. Bloch [1] presented a method to calculate the cyclic load due to misalignment and this will be discussed later.

Loads Imposed on Shaft End

The distress in the shaft end is usually of fatigue in nature. Estimating the required steady-state torque is straightforward, and is easy to include in the design process. Variable loads imposed on shaft-ends come from many sources, e.g., compressor surge or any other operation different from normal that can provide variable torque. Misalignment of shafts also produces an alternating bending moment. These loads, in combination with steady-torque load, may be responsible for distress in shaft end and may be responsible for eventual failure. Knowledge of the nature of these loads, the sources, the magnitude, and the frequency of these occurrences is essential for a reliable design.

Many times, complete information about these forces is not known; but in some ways their effects should be included. Also, a design method should be simple. Keeping the preceding factors in mind, use of a "Service Factor" was evolved. It is used in shaft sizing to account for variable loads which will depend on a particular application.

Thoma [2] has given a very good account of the "Service Factor" and has discussed in considerable detail its evolution and provided values for many applications. "Service Factor" is defined as the ratio of peak torque to maximum continuous torque. Shaft size is calculated for a load equal to the service factor multiplied by contracted torque. It is implied that the resulting design will withstand cyclic load regardless of its magnitude and frequency of occurrence. Essentially, the fatigue problem is converted into a static one, at least to account for specific types of driven machinery. The choice of one value of service factor for a given type of driven machinery cannot possibly include all different types of process and manufacture.

Distress or appearance of a crack in the shaft end involves a process of nucleation, initiation, and crack growth under cyclic loading. It is hardly debatable a) that there always will exist variable loads in turbomachinery operation, b) that these variable loads influence the life of shaft end, and c) that the design method should be based on principles of fatigue. What is debatable, however, is its nature, its magnitude, and its frequency of occurrence.

3D STRESS SYSTEM ON SHAFT END SURFACE

After all the loads (nature and magnitude) are identified, the next step is to identify and predict stresses. The stress system is three-dimensional. A shrunk on and integral hub design is shown in Figure 1. The stress at point A on the shaft surface under the coupling hub can be shown as in Figure 2. Explanation of each stress component is provided in Table 1.

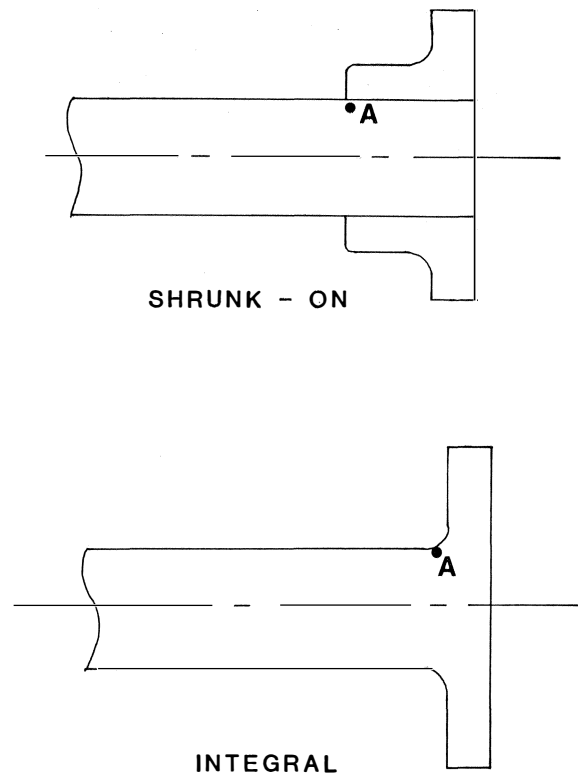


Figure 1. Shrunk-on and Integral Hub Construction.

A simple formula is adequate to calculate these stresses. For a more accurate estimate, a finite element analysis should be performed.

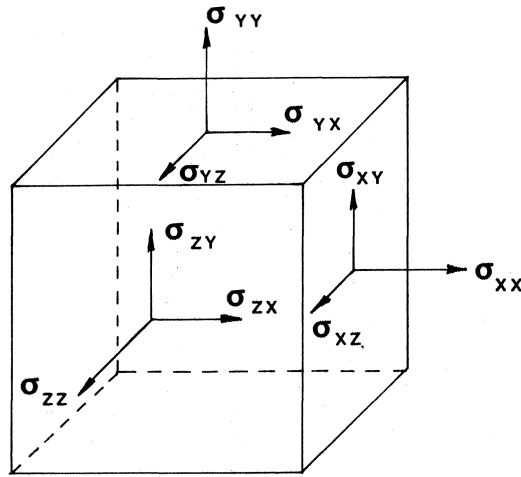
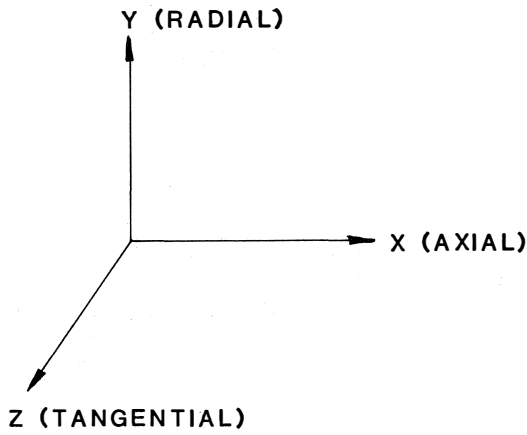


Figure 2. 3D Stress System at the Surface of Shaft End.

Table 1. 3D Stress System for Shaft End.

Stress	Nature	Cause	Comment
σ_{xx} (Axial)	Normal	Due to Axial Load and Bending Moment	Thrust, Misalignment of Coupling, etc.
σ_{yy} (Radial)	Normal	Shrink Stress Centrifugal	Friction of Interference and Speed
σ_{zz} Tangential	Normal	Shrink Stress Centrifugal	Interference and Speed
$\sigma_{yx}=\sigma_{xy}$	Shear Stress	Sliding in Axial Direction	Friction
$\sigma_{xz}=\sigma_{zx}$	Shear Stress	Due to Torque	
$\sigma_{yz}=\sigma_{zy}$	Shear Stress	Sliding around Shaft	Friction

DESIGN METHODOLOGY FOR SHAFTING

A reliable design, hence reliable operation of the shaft end, depends on many factors. Some of those are:

- Type of load.
- Type of coupling.
- Type of applications.
- Resulting stress systems.
- Mode of failure e.g., fatigue.

ASME CODE FOR TRANSMISSION SHAFTING

This method, though empirical in nature, tries to capture many factors. It accounts for fatigue by factors like the service factor. Hence, it suffers in perception that it is not based on theory of material fatigue.

In an effort to make the procedure simple, the ASME code defined allowable shear stress to be the lesser value of 0.3σ and $0.18 \sigma_u$.

A formula was provided for the calculation of shear stress to be:

$$\tau_d = \frac{16}{\pi d^3} \left[(C_m M)^2 + (C_t T)^2 \right]^{1/2} \tag{1}$$

and supplied the values for C_m and C_t which depended on application. This was based on maximum shear stress theory and tried to account for variable loads through factors C_m and C_t . The attempt was not to ignore fatigue but to make it simple for a wide range of applications.

Method Based on Fatigue

A method is described here which is based on von Mises criteria. It has been used in many shaft end designs. This is a general formula which contains alternating stress in axial direction and alternating torsional stress.

$$\frac{1}{n} = \left[\left\{ \frac{\sigma_{xx} + \sigma_{yy}}{\sigma_{yp}} + \frac{K_f (\sigma_{xxa} + \sigma_{yya})}{\sigma_e} \right\}^2 + 3(\mu_1^2 + \mu_2^2) \left\{ \frac{\sigma_{yy}}{\sigma_{yp}} + \frac{\sigma_{yya}}{\sigma_e} \right\}^2 + 3 \left\{ \frac{\sigma_{zx}}{\sigma_{yp}} + \frac{K_{sf} \sigma_{zxa}}{\sigma_e} \right\}^2 \right]^{1/2} \tag{2}$$

$\sigma_{yya} = 0$ no variable load in radial stress

$\mu_1 = \mu_2 = \mu$ coefficient of friction same in axial sliding and sliding around shaft.

$$\frac{1}{n} = \left[\left\{ \frac{\sigma_{xx} + \sigma_{yy}}{\sigma_{yp}} + \frac{K_f \sigma_{xxa}}{\sigma_e} \right\}^2 + 6 \left\{ \mu \frac{\sigma_{yy}}{\sigma_{yp}} \right\}^2 + 3 \left\{ \frac{\sigma_{zx}}{\sigma_{yp}} + \frac{K_{sf} \sigma_{zxa}}{\sigma_e} \right\}^2 \right]^{1/2} \tag{3}$$

Explanation of each stress term is provided in Table 1.

Fatigue Strength

Normally, the fatigue strength of a material is determined in a test by fatiguing a polished specimen. The stress system is uniaxial in general. Real parts made out of the material are of different size and shape, and the real stress system is far from being uniaxial. Hence, the properties obtained by such tests should not be used directly in the design.

A procedure is followed to modify the measured fatigue strength specific to the application at hand. This is usually accomplished by defining multipliers to account for the factors like surface finish, size, and survival rates. This modified value of fatigue strength is used in design, such as:

$$\sigma'_f = K_1 \cdot K_2 \cdot K_3 \cdot \sigma_f \quad (4)$$

where

- σ'_f = Modified fatigue strength, psi
- K_1 = Surface factor, depends on finish
- K_2 = Size factor
- K_3 = Reliability factor
- σ_f = Fatigue strength of polished specimen

Typical curves and values of these factors can be found in books by Lipson and Sheth [3].

FRETTING FATIGUE

The subject of fretting fatigue in the context of crack initiation and its propagation is very complex indeed. Waterhouse [4, 5] gave a very good account of the subject.

A brief review is provided for understanding and to show that its influence can be built into a design process.

Fretting is a term used to define the damage that results when two surfaces in contact (under a normal load or clamping load) experience a relative motion. In the absence of environmental media that may contribute to the process (e.g., in vacuo), the process may be purely physical in nature, but in the presence of a corrosive environment, a chemical surface reaction also may be involved in the process.

The damage of the surface is generally plastic flow of material, a debris of oxide, and a cavity is formed on the surface, initiating minute cracks. There is no agreement among researchers on the mechanism of fretting damage, but two variables listed below are thought to be essential for this phenomenon to take place.

- Normal pressure on the contacting surfaces
- Relative motion between the two surfaces

The influence of the following parameters has been noted in technical publications.

- Normal pressure, magnitude, and distribution
- Amount of slip
- Coefficient of friction
- Smoothness or roughness of surfaces
- Hardness of surfaces
- Frequency of relative motion
- Surface treatments
- Lubricants
- Temperature
- Environment
- State of stress in nearby zone, etc.

Test data on simplified laboratory specimens under simplified loading condition indicates:

- There is a reduction in fatigue strength of the material.
- There may not be an endurance limit of the material as assumed in classical fatigue analysis.

This is shown in Figure 3.

A predictive analytical tool is almost nonexistent, and material properties needed for the problem at hand does not exist. However, based on knowledge and insight gained from literature, the following model of fretting fatigue damage and ultimate failure of shaft coupling key interface is visualized.

Any fracture can be thought of proceeding in the following stages:

- Nucleation of crack
- Initiation of crack
- Propagation of crack

Nucleation of Crack

Nucleation of a crack can be thought of on a molecular level or at least on a grain boundary level. The grain boundary, being

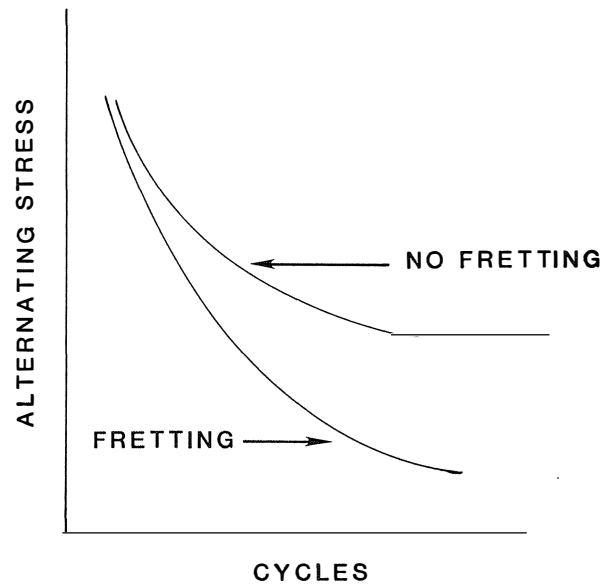


Figure 3. Fatigue Behavior of Material With and Without Fretting.

under alternating load, might weaken and can create a site for crack initiation. In many circumstances, the grain boundary may be stronger than the strength of the material and adjacent molecules may break the bond at a local spot, creating a crack initiation site.

Initiation of Crack

Depending on the energy being imparted to the stressed material, more molecular bonds may be broken, giving an initiation of a crack either transgranular or intergranular. This process may occur simultaneously at several places in the volume of material.

Propagation of Crack

If more energy is available, the stressed material will try to relieve the energy being imparted by creating a free surface; hence, crack growth. This part of the fracture process is best understood and can be handled using fracture mechanics concepts. This is beyond the scope of this study.

FRETTING MODEL

When the material is only in surface distress, as in fretting, conceptual explanations can be provided (Waterhouse [4], Figure 4).

When surface asperities come in contact under a normal load, the asperities may be deformed. In addition, fatigue load is applied and, if the breaking force is overcome (depending on normal load, slip amplitude, and fatigue loads), then damage and debris are created after the first contact cycle. The early damage, however, will not necessarily result in reduced fatigue life if the fretting is discontinued.

After sufficient cycles, the damage produces cracks emanating from the damaged areas. These cracks always initiate in the slippage region. It is not clear, however, that contact over an entire region of pressured contact will eliminate the deleterious effects of fretting on fatigue. In the last portion of the figure, it is suggested that surface fatigue can be a factor. This is likely for only low normal pressure on rough surfaces. The development of fatigue as shown has been proposed as a model to explain the fretting fatigue mechanism. This proposed model is mainly mechanical in nature. It does not suggest the effect of environmental contamination and/or the aspects of chemical effects.

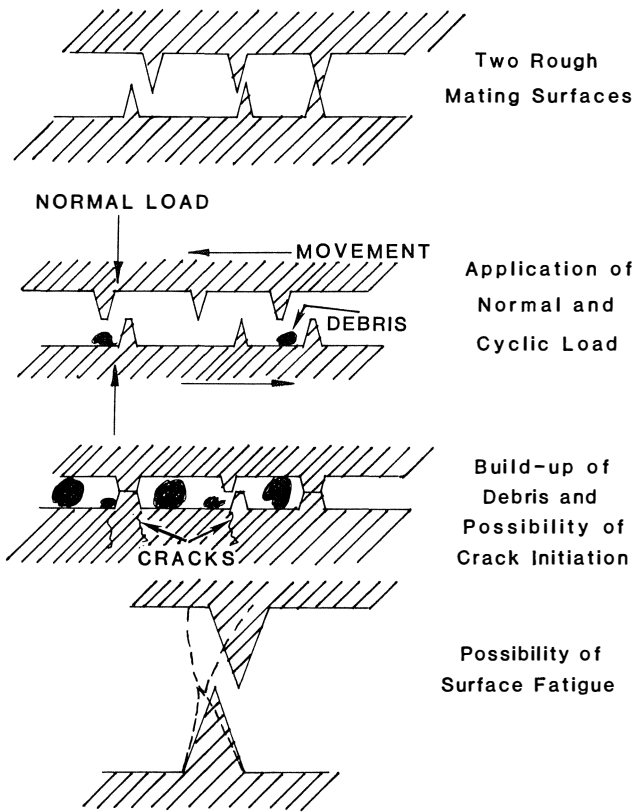


Figure 4. Conceptual Surface Distress in Fretting Fatigue.

Even though full quantitative understanding is not available, a fatigue strength reduction factor can be extracted from laboratory test data, and can be used in an equation to reduce the fatigue strength. The modified fatigue strength of Equation (4) can be further modified by a factor K_d to account for fretting fatigue.

$$\sigma_f'' = K_d \cdot \sigma_f' \quad (5)$$

APPLICATION OF DESIGN PROCEDURE

Neither all of the factors influencing the reliability of the shaft end is understood nor can all of these be quantified. An attempt must be made, however, to include most of these factors in some quantitative way in the design process. The following steps are required to size a shaft-end.

- Specify constant power, speed, and type of driven machine.
- Choose type of coupling to be used.
- Calculate all loads and resulting stresses.
- Choose the material and determine reduction factors.
- Use equation (3) to calculate factor of safety.
- Alternatively, if a minimum value of a factor of safety is defined, a minimum value of diameter can be calculated.

A computer program has been developed based on Equation (3). Capabilities have been built to include the effect of coupling types, and effect of fretting fatigue. This program can calculate shaft end size based on either shrunk-on, keyed, nonkeyed, or integral hub construction. This program also invokes API 671 to include a multiplier (1.75) if required.

Equation (3) is simplified where the alternating load imposed on shaft end is bending σ_{xxa} and torsional σ_{zxa} . The steady-state

continuous torque is applied in conjunction with these loads. Equation (3) reduces to the following when service factor is specified:

$$\frac{1}{n} = \frac{32}{\pi d^3} \left[\left(\frac{K_{t1} \cdot M_f}{2 \cdot K_d \cdot \sigma_f'} \right)^2 + \frac{3}{4} \left(\frac{1}{2 \sigma_{yp}} + \left(\frac{SF-1}{SF+1} \right) \frac{K_{t2}}{K_d \cdot \sigma_f'} \right)^2 \right]^{1/2} \cdot (SF+1) \cdot T_o \quad (6)$$

where

- K_{t1} and K_{t2} are geometric stress concentration factors
- M_f is moment factor, depends on coupling type, equations have been given by Bloch [1] and provided in Appendix.
- K_d fatigue reduction factor due to fretting
- S.F. service factor and is defined as a ratio of peak torque divided by maximum continuous torque. Peak torque is equal to the sum of maximum continuous torque and variable torque.

When a variable torque is specified, equation (6) can be rewritten as

$$\frac{1}{n} = \frac{32}{\pi d^3} \left[\left(\frac{K_{t1} \cdot M_f}{2 \cdot K_d \cdot \sigma_f'} \right)^2 + \frac{3}{4} \left(\frac{1}{2 \sigma_{yp}} + \frac{K_{t2}}{(2+K_v) \cdot K_d \cdot \sigma_f'} \right)^2 \right]^{1/2} \cdot (2+K_v) \cdot T_o \quad (7)$$

where K_v is the ratio of variable torque by continuous torque.

Equation (6) and equation (7) will provide the same results because

$$SF = 1 + K_v$$

(b) When API 671 applies, the continuous torque is simply multiplied by 1.75 to take care of variable torque.

$$\frac{1}{n} = \frac{32}{\pi d^3} \left[\left(\frac{K_{t1} \cdot M_f}{K_d \cdot \sigma_f'} \right)^2 + \frac{3}{4} \left(\frac{1}{\sigma_y} \right)^2 \right]^{1/2} \cdot (1.75 T_o) \quad (8)$$

Equations (6), (7), and (8) provide a means to calculate the factor of safety for a given shaft end, and/or to calculate shafts size when a factor of safety is specified.

Example

An example is provided to demonstrate the applicability of equations (Equations (6), (7), and (8)). After that, results of a sensitivity study will be presented so that some generalized comments can be made.

Design Example

A turbine shaft is to be designed for an output of 7650 hp at 4300 rpm. The desired minimum factor of safety is 2. (Assume API 671 applies). Use service factor of 1.3 for this application.

Gear Type Coupling

Coupling Information

- Gear
- Face width = 0.875

- Pitch Dia = 6.0
- Maximum Misalignment Angle = 0.5 degrees
- Coefficient of friction at gear teeth = 0.15
- Moment factor = 0.2157 (using equation from APPENDIX)

Calculate

$$\begin{aligned} \text{Torque} &= \frac{(63000.) (7650.)}{4300} \text{ lb. in.} \\ &= 112082. \text{ lb. in.} \end{aligned}$$

Stress Concentration, $K_{t1} = K_{t2} = 2.5$
 Fretting Fatigue factor = 1.00 (no fretting)

Material Properties

$$\begin{aligned} \sigma_f &= 52500. \text{ psi} \\ \sigma_{yp} &= 85000. \text{ psi} \end{aligned}$$

Assume following values of fatigue strength adjustment factors

$$\begin{aligned} K_1 &= 0.89 \\ K_2 &= 0.8 \\ K_3 &= 0.75 \text{ (99.9\% Survival)} \end{aligned}$$

Then

$$\begin{aligned} \sigma'_f &= (0.89) (0.8) (0.75) (52500.) \text{ psi} \\ &= 28035. \text{ psi} \end{aligned}$$

Equation (6) is used to calculate size of shaft

$$d = 4.55 \text{ in}$$

Another calculation can be made using Equation (8) per API 671.

$$d = 4.43 \text{ in}$$

Diaphragm Type Coupling

The coupling chosen based on horsepower and speed has the following characteristics:

Spring-rate (K_B) = 8550 in-lb/degree
 Maximum misalignment = 0.25 degrees
 Moment factor for service factor of 1.3 - 0.004 (Using equation from APPENDIX)

Using equation (6), the diameter for factor of safety of 2 is given as

$$d = 4.3 \text{ in}$$

Calculation using Equation (8)

$$d = 3.44 \text{ in}$$

SENSITIVITY STUDIES

Utilizing material properties, Equation (6), and Equation (7), a sensitivity study has been performed. Detail is provided in Table 2 about cases which have been studied.

The object of this study is to compare:

- Diaphragm type vs gear type coupling.
- Shrunken-on vs integral hub construction.
- Fretting vs no fretting.

Table 2. Description of Case Studies.

Case	Coupling Type	Construction at Shaft End
1	Gear	Tapered, Key, Shrunken-on
2	Gear	Hydraulic Fit, Straight
3	Gear	Integral Hub
4	Diaphragm	Tapered, Key, Shrunken-on
5	Diaphragm	Hydraulic Fit, Straight
6	Diaphragm	Integral Hub

Results and Discussions

Two types of results are presented.

- For a given shaft size, the safety factor is calculated for a different magnitude of variable torque, in other words, for a different service factor.

- For a desired safety factor, shaft size is calculated for a different magnitude of variable torque.

Calculated results for case 1 through case 3 are plotted in Figure 5 and Figure 6. All three cases use gear type couplings but the method of attachment to the shaft ends is different for each case (Table 2). These results show the increase in reliability from shrunken-on keyed construction to hydraulic fit construction without key. The most reliable being the integral hub construction. The reason for integral construction to be the best in reliability is the absence of stress concentration due to keyway and absence of fretting by removing mating surfaces. Experience has shown that fretting should not occur in hydraulic fit construction. When it does, it is attributed to local imperfections either on the shaft surface and/or inside the coupling hub. One other feature to note is that the amount of reduction in the safety factor decreases with increased variable torque (Figure 5).

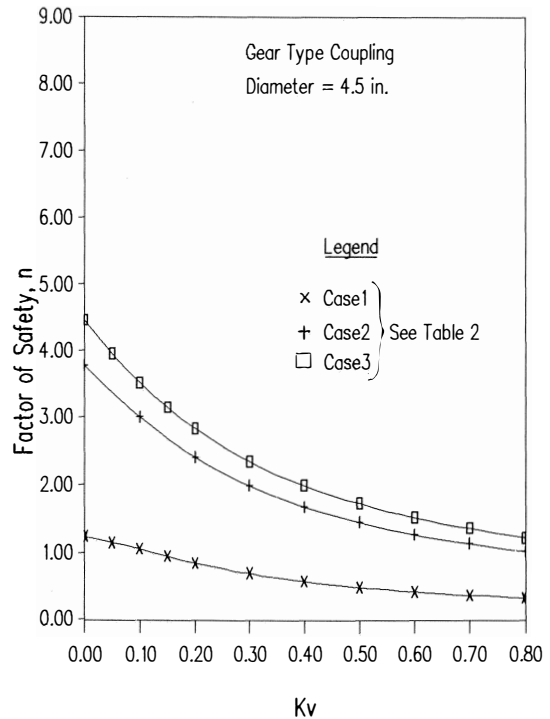


Figure 5. Plot of Factor of Safety versus Variable Torque for Case 1, Case 2, and Case 3.

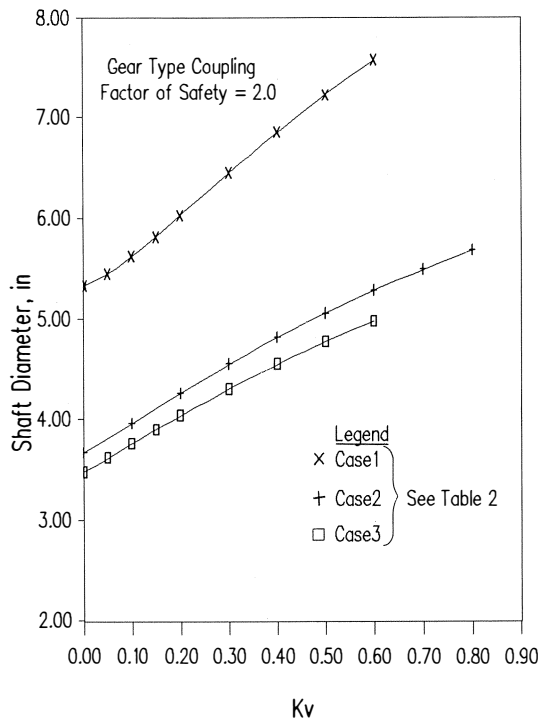


Figure 6. Plot of Shaft Diameter versus Variable Torque for Case 1, Case 2, and Case 3.

The results of case 4 through case 6 are shown in Figure 7 and 8. A diaphragm coupling has been used in these cases. Otherwise, all other parameters are the same as for case 1 through case 3, respectively. Again, the same conclusion can be made as for gear type couplings. The integral hub construction is the best,

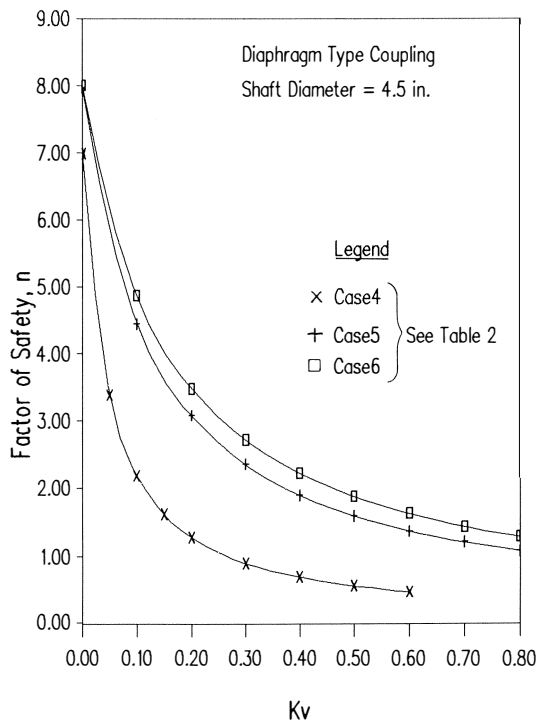


Figure 7. Plot of Factor of Safety versus Variable Torque for Case 4, Case 5, and Case 6.

followed by hydraulic fit and tapered type construction a third. Also, the behavior noted for the large value of variable torque in the case of a gear type coupling also applies for diaphragm types.

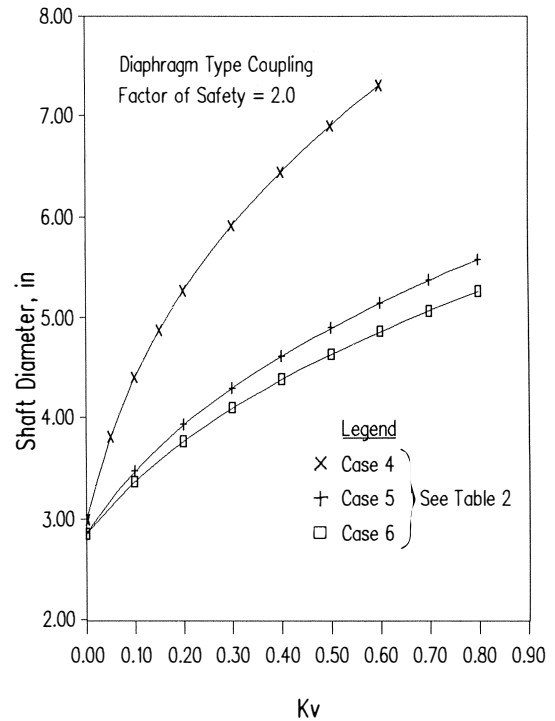


Figure 8. Plot of Shaft Diameter versus Variable Torque for Case 4, Case 5, and Case 6.

Results of case 2 and case 5 are plotted in Figure 9 and Figure 10. Case 2 used gear type coupling with hydraulic fit construction, while case 5 used diaphragm coupling with hydraulic fit construction. A diaphragm type coupling is better than a gear type coupling for the whole range of variable torque. The difference between these two types is very pronounced at low values of variable torque. However, at larger values of variable torque, the difference diminishes.

For low value of variable torque, the influence of the misalignment moment (which is large for gear type coupling) is pronounced. For larger values of variable torque, the effect of the misalignment moment is very small compared to the variable torque.

SUMMARY

It is evident that design of the shaft end at the coupling is very complex and the application of fatigue theory in design is mandatory. There is not a universally accepted design method for shaft ends. A method has been presented in Equation (6) and Equation (7) which captures many important factors that must be considered in shaft end design, and this use has been demonstrated by example.

The proposed method is based on von Mises criteria, and this method considers alternating bending stress and variable torsional stress. It utilizes “service factor” to estimate alternating as well as mean torque. This is different than the traditional use of service factor in sizing of shaft end. Influence of coupling type, as shown by Bloch (1), has been included in the design procedure. A modification in the moment factor due to misalign-

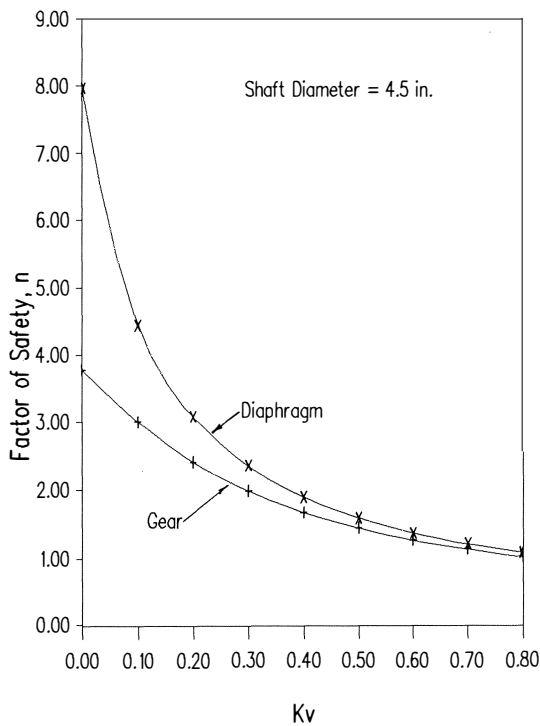


Figure 9. Comparing Factor of Safety of Gear Type and Diaphragm Type Coupling for Different Variable Torque.

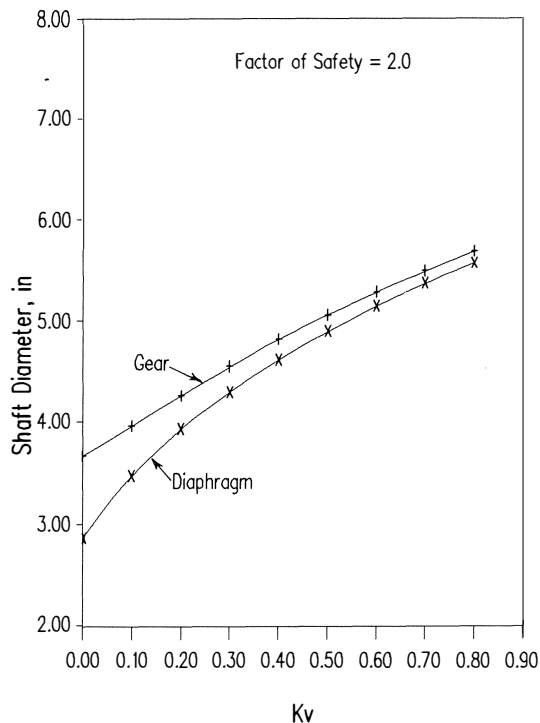


Figure 10. Comparing Shaft Diameter for Gear Type and Diaphragm Type Coupling for Different Variable Torque.

ment has been made so that service factor can be used. An attempt has been made to include the influence of fretting fatigue.

The study shows that from a fatigue point of view, a diaphragm type coupling is better than a gear type coupling. This difference

is small when the variable torque reaches a value of about 50 percent maximum of the continuous torque or more.

While all methods of shaft-end construction can be used, the best from a reliability concern is integral hub construction. Various applications, accessories and economics may prevent an integral hub design, but other design type might be adequate.

APPENDIX

Expressions for moment factor used in Equation (6) through Equation (8) are given as follows:

For Gear Type Coupling

$$M_f = \left[\left(\frac{x}{D_p} \right)^2 + (\mu + \sin \alpha)^2 \right]^{1/2} \quad [1] \quad (A-1)$$

where

- x face width of gear, inches
- D_p Pitch Diameter, inches
- μ Coefficient of Friction in Gear Teeth
- α Maximum misalignment-angle in degrees.

For Diaphragm Type Coupling

$$M_f = \left[\left(\frac{K_B \cdot \alpha}{T_o} \right)^2 + \sin^2 \alpha \right]^{1/2} \quad [1] \quad (A-2)$$

Modified M_f to include service factor

$$M_f = \left[\left(\frac{2 K_B \cdot \alpha}{(SF+1) \cdot T_o} \right)^2 + \sin^2 \alpha \right]^{1/2} \quad (A-3)$$

where

- K_B Spring rate of coupling, in-lb/degree
- SF Service factor
- T_o Maximum Continuous Torque, in-lb

REFERENCES

1. Bloch, H.P., "Less Costly Turbogroup Uprates through Optimized Coupling Selection," *Proceedings of the Fourth Turbomachinery Symposium*, Turbomachinery Laboratory, Department of Mechanical Engineering, Texas A&M University, College Station, Texas (1975).
2. Thoma, F.A., "Mechanical Drive Systems and Their Service Factors," AGMA 159.03 (1969).
3. Lipson, C., and Sheth, N.J., "Statistical Design and Analysis of Engineering Experiments," New York: McGraw Hill Book Company, pp 282-288 (1973).
4. Waterhouse, R.B., "Fretting Corrosion," New York: Pergamon Press (1972).
5. Waterhouse R.B., "Fretting Fatigue," London: Applied Science Publishers, Limited (1981).

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