RERATE OF CENTRIFUGAL PROCESS COMPRESSORS—WIDER IMPELLERS OR HIGHER SPEED OR SUCTION SIDE BOOSTING?

by

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ABSTRACT

Impeller exchange, speed increase, and supercharging are the usual means to increase the capacity of existing industrial turbo-compressors of the single shaft type. Replacing an existing rotor by a new one with wider impellers normally requires wider stationary flow channels as well and, therefore, necessitates some additional space that must be available within the compressor casing. Speed increase requires, besides the aerodynamics, careful consideration of rotordynamics and checking of mechanical components. Suction side boosting, on the other hand, leaves the existing compressor “almost” untouched since the booster forms a separate unit. One case study of a turbine driven hydrocarbon compressor in a fluid catalytic cracking (FCC) plant built in 1968, illustrates the complete exchange of the aero-package in combination with speed increase in two large steps to increase the volume flow from 100 percent to a spectacular value of 243 percent. The uprates included retrofitting bearings and shaft seals. In another example, a turbine driven air compressor in a terephthalic acid (PTA) plant built in 1980 is described, which was uprated to 164 percent of its original capacity by boosting.

INTRODUCTION

Quite a number of users wish to increase production as early as three, or as late as 30, years after commissioning their plant, even if they did not envision such an extension at the time of initial design. Two rerate questions are almost inevitable:

• Can the compressor be uprated without replacing the casing, its connecting pipework, and auxiliary equipment?
• Can the exchange of parts be accomplished during one of the short normal planned shutdowns?

Almost every compressor has the potential of being uprated by means of one of the described approaches. Experience has shown that all parts and new equipment to be exchanged can be ready at the beginning of the shutdown, including the installation of a booster compressor. Needless to say, rerates are lower in cost than new builds, although efficiencies may be somewhat lower than for a completely new compressor.

DEFINITION OF RERATES

A rerate is defined as the geometric modification of a compressor already in operation to change its aerothermodynamic behavior. The modification may cover flow, head, efficiency, and operating range and results in a performance curve change.

“Rerate” should not be confused with “retrofit,” which is defined as an exchange of functionally enhanced components, such as bearings, shaft seals, couplings, which improve the mechanical behavior, maintainability, operational safety, availability, etc., of a compressor already in operation. Retrofitting will not alter the pressure-volume curve. The term “revamp” is not used herein, because its definition appears to be blurred.

The majority of rerates encompass uprates (increased flow and/or head) rather than downrates (reduced flow and/or head). Accomplishing compressor uprates is a demanding engineering task, since the degrees of design freedom are less than designing new compressors from scratch.

INCREASE OF MASS FLOW

The options of increasing the mass flow through any flow channel (e.g., impeller, diffuser) are basically described by the continuity equation:

\[ \dot{m} = A \cdot w \cdot p = A w p \frac{1}{RT} \]  

A = Through flow area  
w = Average flow velocity  
p = Density  
\( p \) = Pressure  
R = Gas constant  
T = Temperature

There are three customary options:

• Equipping the compressor with wider impellers and diaphragms will enlarge area A.
• A higher rotational speed will increase velocity w.
• Installing an upstream booster will increase compressor suction pressure p.

In principle, a reduction of the inlet temperature would also increase the density. However, this is not common, because it is comparatively too expensive to install and operate a refrigerating system that achieves the same effect as a booster compressor. In the literature, only one example could be found where a favorable situation justified suction cooling: Runge [1] reports that for rerating an ammonia plant, the excess refrigerating capacity of the ammonia compressor was used to chill the inlet and the intercooler exit temperature of the syngas compressor, thus achieving an increase of mass flow at constant discharge pressure.
Reducing gas constant \( R \) (increasing molecular mass) excludes itself due to process reasons.

**NEW WIDER IMPELLERS AND DIAPHRAGMS**

The increase of the suction volume flow of an existing casing by installing wider impellers with wider stationary flow channels is feasible, if there is some additional space to accommodate them. It is often desired not to alter the casing with its inlet and discharge nozzles, scroll(s), bearings, bearing span, and rotational speed. The impeller flow path width can generally be increased through higher flow coefficients or through larger impeller diameters as described by the mass flow ratio of the uprated to the existing compressor (at constant suction density and constant speed).

The following equation reveals the relationship between the original and the rerated compressor:

\[
\frac{\dot{m}}{\dot{m}_0} = \frac{\varphi}{\varphi_0} \times \left( \frac{d_2}{d_{20}} \right)^3 \times \left[ \left( \frac{d_2}{d_{20}} \right)^2 - 1 \right]^{\frac{1}{S}}
\]

(2)

\( \varphi = \frac{V}{((\pi/4)d_2^2u_2)} \)

Flow coefficient

\( d_2 = \)

Impeller OD

\( u_2 = \)

Impeller tip speed

\( S = \frac{(\Delta h_p / h_p)}{((\Delta h / h_0))} \)

Performance curve slope

\( h_p = \)

Polytropic head

\( r_{d2} = \frac{(h_p / h_{d0} - 1)}{((d_2/d_{20})^2 - 1)} \)

Diameter mismatch factor

The subscript 0 = Existing compressor

The no subscript = Uprated compressor

The derivation of formula can be found in the APPENDIX.

The first term represents the extension of the flow coefficient related to the first stage flow coefficient of the existing machine (Figure 1). During the last 40 years, the maximum flow coefficient of centrifugal impellers has doubled from approximately 0.075 to 0.15, made possible by developing efficient backward leaning 3D-bladed impellers with high hub/tip ratios to be applied for multistage single shaft compressors. Therefore, older compressors still in operation, comparatively, have a very high uprating potential through higher flow coefficient impellers.

![Figure 1. Uprating by Using Higher Flow Coefficient Impeller, Schematic.](image)

Wider impellers can also be designed by just scaling up existing wheels, as described by the second term of the equation. The increased head caused by the higher tip speed is converted into more mass flow at specified head via the performance curve slope represented by the third term of the equation (Figure 2). This means that an actual increase of the flow coefficient is forcefully brought about by a volumetric overload at some efficiency sacrifice. The diameter mismatch factor states the fan law deviation, i.e., the overproportional head increase in relation to \( d_2^2 \) due to progressive aerodynamic stage mismatching, especially for multistage compressors with high Mach number impellers.

![Figure 2. Uprating by Using Scaledup Impeller, Schematic.](image)

Typical diameter mismatch factors are: 1.0 for low, 1.5 for medium, and up to 2.0 for high pressure ratio applications.

Typical performance curve slopes are: 0.5 for low Mach number compressors (flat), 1.5 for average (normal), and up to 3.0 for high Mach number multistage applications (steep).

As can be seen from Equation (2), each percent flow coefficient increase yields one percent mass flow raise, and each percent impeller diameter increase results in some five percent mass flow raise. So even if, due to technological reasons, higher flow coefficient impellers are not available, an efficient uprating is feasible through scaleup of existing impellers.

The consequences of installing wider impellers in an existing casing are as follows:

- Impeller stress increases due to wider flow channels. Separation margin to yield strength of wheel material should be checked.
- Shaft diameter may have to be increased to offset higher impeller weights to assure critical speed separation margin and stability against subsynchronous vibrations. Since larger shafts tend to decrease the flow area, the impeller eye diameters have to be increased accordingly.
- If the rerate power input exceeds the maximum power capacity of existing shafts and power train components, shaft journals, couplings, gear, and driver should be checked and uprated or replaced if necessary.
- Higher efficiencies inherent with higher flow coefficient stages are reduced by higher losses in the unchanged piping, inlet nozzles, casing openings, inlet duct(s), and scroll(s), so that the original efficiency level can usually be maintained.
- Larger impeller diameters reduce diffuser ratios, thus diminishing efficiencies.

**HIGHER ROTATIONAL SPEED**

In the case of a motor drive, the volume flow can, in many cases, be uprated by increasing the rotational speed through exchanging the gear elements. This can be less complicated and less costly for the compressor. In the case of a turbine drive however, generally a new turbine rotor and guide blade carrier(s) are required. The compressor casing with nozzles, scroll(s), bearings, and bearing span remain constant.
The mass flow ratio of the uprated compressor referenced to the existing compressor is:

\[
\frac{\dot{m}}{\dot{m}_0} = \frac{N}{N_0} \times \left( \frac{\left( \frac{N}{N_0} \right)^2 - 1}{(S - 1)} \right)^{F_N/S} + 1
\]  

(3)

\[S = \frac{(\Delta h_{\text{rev}}/\dot{m}_0)}{(\Delta h_{\text{rev}}/\dot{m}_0)} \]

Performance curve slope

\[F_N = \frac{(h_0/h_{rev}-1)}{((N/N_0)^2-1)} \]

Speed mismatch factor

The first term is the speed increase of the rerated compressor at constant flow coefficient, and the second term is, again, the increased head caused by the higher speed and converted into increased mass flow at constant head via the curve slope (Figure 3). This also means that an actual flow coefficient raise takes place by volumetrically overloading the stage at some efficiency decrease.

Figure 3. Uprating by Using Higher Rotational Speed, Schematic.

Although theoretically different, the curve slope is similar for the increased impeller diameter. The same is true for the speed mismatch factor representing the overproportional head increase in relation to \(N^2\) due to stage mismatching. Each percent speed increase yields some three percent mass flow increase.

Consequences of a speed increase are as follows:
- A higher machine Mach number brings about a higher reduction of volume flows from the first to the last compressor stage leading to an aerodynamic stage mismatch for constant pressure ratio, which necessarily reduces the efficiency (Figure 3). The higher the Mach number, the more reduction in the efficiency.
- The operating flow range decreases since surge and choke point (maximum flow at zero head) approach best efficiency point.
- The impeller stress increases. Separation margin to impeller material yield strength should be checked.
- Rotordynamics should be checked with regard to critical speed separation margin and stability.
- Impeller and coupling shrink fits should be checked.
- New overspeed test may be required.
- If the rerate power input exceeds the maximum power capacity of existing shafts and power train components, shaft journals, couplings, gear, and driver should be checked and uprated if necessary.

ADDITIONAL LOSSES

For both wider impellers and a speed increase, the friction losses caused by higher flow velocities in the casing nozzles, casing openings, and stationary ducts will increase.

The inlet loss from the inlet flange via the plenum inlet up to the eye of the first impeller, referenced to the gas power of the pertinent downstream stage, is described as follows:

\[P_L = \frac{1}{(1-\zeta(\frac{2}{k_2})M_{a}^{2})^{k_2-1}} \]

(4)

\[\Pi = \frac{P_L}{P_{\text{gas}}} \]

Pressure ratio of single stage downstream of nozzle

\[\zeta = 3.3...4.5 \]

Loss coefficient, function of Reynolds number

\[k_2 = \frac{1}{\kappa} \]

Isentropic volume exponent

\[M_{a} = \frac{c_s}{a_0} \]

Sonic velocity

\[c_s = \text{Flange velocity} \]

\[a_0 = \text{Sonic velocity} \]

A derivation of the formula can be found in the APPENDIX; results are shown in Figure 4.

Figure 4. Compressor Inlet Loss.

CASE STUDY 1—WIDER IMPPELLERS AND SPEED INCREASE COMBINED

In 1968, the author's company was awarded a contract by a major oil company in Germany for a turbine driven five-stage hydrocarbon compressor in a fluid cat cracking plant. Its design specifications are shown in Table 1, first column, designated FCC 68; suction volume flow 6220 m³/h. In 1986, this compressor was uprated to 200 percent of the original volume flow, i.e., 12,500 m³/h, second column, designated FCC 86. In 1989, another uprate to 243 percent, i.e., 15,088 m³/h, third column, designated FCC 89 was required. The uprate pressure ratio was approximately constant; however, due to molar mass changes the head requirements rose by five percent for the first rerate and fell by eight percent for the second rerate.

How was this extreme uprating of an existing casing accomplished? Since an increase of impeller diameters was not deemed appropriate due to the short diffusers, a combination of flow coefficient and speed change was the only way.

First Rerate

Since the maximum flow coefficient, which is regarded as a technological limit, increased considerably between 1968 and
1986 as a result of concentrated R&D activities, the greatest uprate potential was applying the ratio of flow coefficients $\psi / \psi_0$ (first term in Equation (2)), which amounted to a whopping 162 percent. The second largest potential was speed increase. In spite of the low impeller material yield strength (630 N/mm² (90 ksi)), there was enough margin for a hefty speed raise of 14 percent bringing the flow to 200 percent (Figure 5).

- The complete internals, including rotor and diaphragms, were completely replaced within the short plant shutdown time.
- No attempt was made to salvage any of the old components. It was decided that this possible cost savings was not a reasonable risk compared to potential losses for each day's production.
- The original compressor had enough internal voids to accommodate wider impellers and wider stationary flow paths.
- The diffuser ratios could be increased from 1.41 to 1.55, benefiting the efficiency.
- The scroll through flow area could be increased by 25 percent.
- The specification not only called for 200 percent flow of the original design, but also for increased operational flexibility. This was accomplished by a so-called volumetric overload design that resulted in an efficiency penalty of five points at the design point of 12,500 m³/h. Thus, the turndown could be improved from 30 percent to more than 40 percent.
- The increased turndown was responsible for the efficiency degradation from 0.76 to 0.74. If the turndown could have been maintained at 30 percent, the efficiency would have gone up to 0.79.
- The flow coefficient increase from 0.053 to 0.094 was possible through a first stage impeller with backward curved (twisted) 3D-blades. These were developed around 1970 and had accumulated a good record of experience by 1986.
- The stages used for FCC 86 represent the scalable fixed geometry component system, the so-called “standard impeller family” introduced in 1971 and described in detail by the author [2]. As can be seen, this comprehensive system with its flexibility can be successfully used for extreme rerates as well, without any compromises with the philosophy of the system.
- The inlet loss from the suction flange to the impeller eye per Equation (4) increased from two percent (FCC 68) to seven percent (FCC 86), due to the considerably increased velocity. Therefore, the difference of five percent had to be charged to the first stage. In a similar way, the additional losses in the scroll and the adjoining diffuser and discharge nozzle were charged to the last stage.
- Since the maximum continuous speed increased by 17 percent, the shaft diameter had to be increased in order to maintain the so-called shaft stiffness ratio, i.e., the first rigid support critical over maximum speed $N_{1cr} / N_{max}$, which can be a cause for the onset of subsynchronous vibrations.
- In spite of the higher rotational speed, the actual impeller stress at maximum continuous speed is acceptable.
- The maximum turbine power increased from 1300 to 2500 kW. This required a thickening of the shaft diameter under the coupling and an increase of the compressor journal bearing diameter from 70 mm to 85 mm.
- Doubling power output necessitated a new rotor and guide blade carrier for the back pressure reaction type turbine with partial admission control stage (Figures 8 and 9).
- Consequences of higher steam consumption and speed:
  - The number of stages was reduced from 25 to 20.
  - Longer, more efficient blades with better profiles were used.
- Increasing the first stage exit pressure (wheel chamber pressure) permitted a more favorable load distribution between the less efficient control stage and the highly efficient reaction type stages.
- The end result of these changes was an increase of the turbine efficiency by seven percent, offsetting part of the increased compressor power requirements.

### Table 1. Case Study 1: Technical Data Original Compressor, First and Second Rerate.

<table>
<thead>
<tr>
<th>Specification</th>
<th>FCC 68</th>
<th>FCC 86</th>
<th>FCC 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass flow</td>
<td>$kg/h$</td>
<td>16445</td>
<td>3288</td>
</tr>
<tr>
<td>Volume flow</td>
<td>$m^3/h$</td>
<td>0.220</td>
<td>12969</td>
</tr>
<tr>
<td>Cut</td>
<td>$\beta_c$</td>
<td>28.05</td>
<td>28.05</td>
</tr>
<tr>
<td>Weight mass</td>
<td>$m$</td>
<td>56.8</td>
<td>36.54</td>
</tr>
<tr>
<td>Stress press/σ</td>
<td>$\sigma_t$</td>
<td>24.74</td>
<td>20.058</td>
</tr>
<tr>
<td>Discharge press</td>
<td>$p_k$</td>
<td>3.76</td>
<td>3.84</td>
</tr>
<tr>
<td>Pressure ratio</td>
<td>$\psi / \psi_0$</td>
<td>5.08</td>
<td>5.53</td>
</tr>
</tbody>
</table>

### Figure 5. Case Study 1, Schematic. (a: wider impellers, b: speed increase, c: slope effect, d: wider impellers, e: speed reduction.)

As can be seen from Table 1 and from a comparison of the cross sectional drawings for FCC 68 and FCC 86 (Figures 6 and 7), the prerequisites and consequences of this large flow increase were the following:

- The inlet loss from the suction flange to the impeller eye per Equation (4) increased from two percent (FCC 68) to seven percent (FCC 86), due to the considerably increased velocity. Therefore, the difference of five percent had to be charged to the first stage. In a similar way, the additional losses in the scroll and the adjoining diffuser and discharge nozzle were charged to the last stage.
- Since the maximum continuous speed increased by 17 percent, the shaft diameter had to be increased in order to maintain the so-called shaft stiffness ratio, i.e., the first rigid support critical over maximum speed $N_{1cr} / N_{max}$, which can be a cause for the onset of subsynchronous vibrations.
- In spite of the higher rotational speed, the actual impeller stress at maximum continuous speed is acceptable.
- The maximum turbine power increased from 1300 to 2500 kW. This required a thickening of the shaft diameter under the coupling and an increase of the compressor journal bearing diameter from 70 mm to 85 mm.
- Doubling power output necessitated a new rotor and guide blade carrier for the back pressure reaction type turbine with partial admission control stage (Figures 8 and 9).
- Consequences of higher steam consumption and speed:
  - The number of stages was reduced from 25 to 20.
  - Longer, more efficient blades with better profiles were used.
- Increasing the first stage exit pressure (wheel chamber pressure) permitted a more favorable load distribution between the less efficient control stage and the highly efficient reaction type stages.
- The end result of these changes was an increase of the turbine efficiency by seven percent, offsetting part of the increased compressor power requirements.
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Second Rerate

In 1989, the same compressor got another uprate (FCC 89); this time by 22 percent, so that the total uprate accumulated to an unprecedented 243 percent of the original volume flow. However, the head dropped by 12 percent, so that seven percent was chopped off the original head because the gas became more dense. By 1989, another extension of flow coefficients to more than 0.12 was available and already well established, which led to the application of such a high flow coefficient first stage impeller being the prerequisite for this extreme capacity uprate.

In Table 1 and on the cross sectional drawing (Figure 10), the following facts are revealed:

- The number of stages could be reduced from five to four.
- This, in turn, was a prerequisite for increasing the first stage flow coefficient from 0.086 to 0.119, since the axial stage pitch increased substantially (the first stage axial space of FCC 89 alone occupies one and a half stages of FCC 86).
- Needless to say, this new radical change required the replacement of the rotor and diaphragms with the casing being untouched.
- No old superfluous aerodynamic components were salvaged.
- The scroll was the only flow path component to be retained.
- A large turndown was not required anymore, so that the design point came close to the best efficiency point.
- Standard impeller family components were applied exclusively. The second impeller had to be blade-trimmed at the exit. The first two impellers have 3D-blades.
- Inlet and exit losses due to the higher velocities in the flanges, casing openings, inlet, and exit ducts were charged to the first and last stage and reduced their efficiencies by nine percentage points each.
- In spite of these additional losses, the original compressor efficiency could be raised by two points.
- The rotor weight was reduced and the mean shaft diameter increased slightly, thus increasing the rotor stiffness ratio to 0.45, safeguarding against subsynchronous vibration.
- The wider impellers have an inherently higher stress level; nevertheless, the actual stress at maximum continuous speed is still acceptable at 73 percent of the material yield strength.
- Although the power consumption increased, the maximum turbine power was still sufficient. Thus, the turbine did not need another rerate.

It is thought to be a significant achievement that, in spite of the massive capacity increase, the efficiency suffered a minor loss of only two points for the first rerate and actually gained two points for the second rerate. The almost constant efficiency line vs flow coefficient for the described rerates is compared (Figure 11) with the efficiencies that could have been attained with completely new, but much more expensive compressors.

Performance maps of FCC 68, FCC 86, and FCC 89, displaying polytropic head, polytropic efficiency, and power input vs actual volume flow are shown in Figure 12.

The first rerate incorporated a retrofit from floating ring oil shaft seals to mechanical contact seals to minimize inner seal oil leakage, which had to be disposed of (Figures 6 and 7).
The second rerate comprised a retrofit to dry gas seals in order to totally eliminate the seal oil system (Figure 10).

SUPERCHARGING WITH A BOOSTER COMPRESSOR

In certain cases, the application of a single stage booster upstream of the existing compressor is advantageous compared to new impellers or a speed increase. The advantages are:

- The flow increasing potential is as high as 100 percent and more, even if the compressor is already equipped with high flow coefficient impellers.
- The main compressor remains (nearly) unchanged and can be operated with almost constant power input.

The approximate mass flow increase of the main compressor with an aftercooled booster in operation is:

\[
\frac{\dot{m}}{\dot{m}_0} = \Pi_B \frac{T_{x0}}{T_R} \left( \frac{V}{V_0} \right)^{MC}
\]  \hspace{1cm} (5)

Approximate mass flow increase of the main compressor with the booster in operation (and no cooler between the booster and the main compressor):

\[
\frac{\dot{m}}{\dot{m}_0} = \Pi_B^{k\eta_{MB}} \left( \frac{V}{V_0} \right)^{MC}
\]  \hspace{1cm} (6)

Booster pressure ratio:

\[
\Pi_B = [s(k-1)M_{u2}^2+1]^{k-1}
\]  \hspace{1cm} (7)

The approximate suction volume flow ratio of the main compressor with the aftercooled booster in operation:

\[
\left( \frac{\dot{V}}{V_0} \right)^{MC} = \frac{P_k}{P_{k0}} \left[ \frac{c + \frac{T_R}{T_{50}}}{(1+c)\Pi_B} \right]^{k-1} \left( \frac{\eta_{MC}}{\eta_{MB}} \right)^{k-1}
\]  \hspace{1cm} (8)

The approximate suction volume flow ratio of the main compressor with the booster in operation (and no intercoolers) is:

\[
\left( \frac{\dot{V}}{V_0} \right)^{MC} = \frac{P_k}{P_{k0}} \frac{\Pi B^{k-1}}{(\Pi B)^{k-1}}
\]  \hspace{1cm} (9)

\begin{align*}
s & = \text{Work input factor booster} \\
k & = \text{Isentropic exponent} \\
M_{u2} & = \text{Tip speed Mach number booster} \\
\eta_{MB} & = \text{Polytropic efficiency booster} \\
T_{x0} & = \text{Suction temperature booster} \\
T_R & = \text{Recooling temperature} \\
V_0 & = \text{Original suction volume flow main compressor} \\
V & = \text{Suction volume flow main compressor with booster in operation} \\
\dot{m}_0 & = \text{Original mass flow} \\
\dot{m} & = \text{Uprated mass flow} \\
c & = \text{Original number of intercoolers} \\
\Pi & = \text{Total pressure ratio including booster} \\
\eta_{MC} & = \text{Original polytropic efficiency main compressor} \\
\eta_{MB} & = \text{Polytropic efficiency main compressor with booster in operation} \\
P_k & = \text{Power input main compressor with operating booster} \\
P_{k0} & = \text{Original power input main compressor} \\
\text{subscr. } B & = \text{Booster} \\
\text{subscr. } MC & = \text{Main compressor}
\end{align*}

Derivations of equations can be found in the APPENDIX.

As can be seen from the previous equations, there are many parameters that influence the booster induced mass flow increase. Simplified calculation results are displayed (Figures 13 and 14) for the conditions indicated on the figures. These curves also contain lines of constant main compressor volume ratios, indicating how...
much the suction volume flow necessarily has to change when the booster is put in operation. Higher pressure ratios require volumetric part load. Lower pressure ratios overload operation of the main compressor. Overload is feasible, because main compressor head reduces! For H2-rich gas applications, the volumetric part load. Much the suction volume flow has to change when the booster is put in operation. Higher pressure ratios overload the main compressor. Overload is feasible, because main compressor head reduces! For H2-rich gas applications, the volumetric part load. Lower pressure ratios overload operation of the main compressor.

- Maximum mass flow increase by boosting reduces sharply with increasing pressure ratio.
- Full increasing potential can only be utilized if the booster operates at high Mach numbers.
- Compressors for light gases (e.g., H2) with inherently low Mach numbers (i.e., very low pressure ratios) can be boosted less.

Note: The maximum attainable booster Mach number is subjected to the maximum impeller tip speed: M = u/\sqrt{kZ RT} with u = 280 to 320 m/s. Example: A compressor with a 1.4:1 pressure ratio and a maximum booster Mach number of 0.3 can only be uprated to 117 percent.

At constant overall pressure ratio the main compressor excess head must be eliminated. This can be accomplished by:

- Removal of impeller(s)
- Impeller blade trimming
- Volumetric overload operation
- Speed reduction

The latter is easy with a turbine drive. However, a motor drive will require the replacement of the gear box or, if possible, the replacement of the gear elements. For a turbine drive, in the event of a booster trip, the variable speed main compressor can stay in operation at constant speed, however, has to be shutdown in case of a booster trip, since it is no longer able to develop the original head.

All head lowering measures widen the operating range, thus adding more flexibility for the joint operation with the booster.

Note: The maximum attainable booster Mach number is subjected to the maximum impeller tip speed: M = u/\sqrt{kZ RT} with u = 280 to 320 m/s. Example: A compressor with a 1.4:1 pressure ratio and a maximum booster Mach number of 0.3 can only be uprated to 117 percent.

Figure 13. Uprating by Suction Boosting, Pressure Ratios 2 to 40, Booster with Aftercooler.

Figure 14. Uprating by Suction Boosting, Pressure Ratios 1.2 to 1.8, No Intercoolers.

CASE STUDY 2—SUCTION SIDE BOOSTING OF A TWO-CASING AIR COMPRESSOR

In 1979, the author’s company was awarded a contract by a major petrochemical company in the Far East for two two-casing steam and expansion turbine driven 17 MW air compressors in a terephthalic acid plant (PTA), whose design specifications are shown in Table 2, first column. In 1994, one of the turbo trains was uprated to 164 percent volume flow, corresponding to 148 percent mass flow by means of a single stage integrally geared compressor boosting the main compressor suction pressure to 2.0 bar. Booster data are shown in the third column and main compressor data jointly operating with the booster are exhibited in the second column.

Table 2. Case Study 2: Technical Data Original Compressor and with Booster.

<table>
<thead>
<tr>
<th>Drive</th>
<th>Single-shaft, 2-casing, no gear</th>
<th>Single-shaft, 2-casing, integral gear</th>
<th>Single-shaft, 2-casing, single-stage</th>
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<tbody>
<tr>
<td>Number of stages</td>
<td>6, 4, 6</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Impeller diameter</td>
<td>880 - 515, 880 - 515</td>
<td>880 - 515</td>
<td>880 - 515</td>
</tr>
<tr>
<td>Flow coeff. std u2</td>
<td>0.128, 0.108</td>
<td>0.11</td>
<td>0.11</td>
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<tr>
<td>Tip speed</td>
<td>u, m/s</td>
<td>335, 304</td>
<td>335, 304</td>
</tr>
<tr>
<td>Mach no. 1st stn</td>
<td>M, -</td>
<td>0.96, 0.95</td>
<td>0.96, 0.95</td>
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<tr>
<td>Pol. head</td>
<td>h, kJ/kg</td>
<td>349, 270</td>
<td>349, 270</td>
</tr>
<tr>
<td>Pol. efficiency</td>
<td>ηp, %</td>
<td>0.85, 0.77</td>
<td>0.85, 0.77</td>
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<tr>
<td>Speed</td>
<td>N, /min</td>
<td>7250, 6600</td>
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<td>Power input</td>
<td>P, kW</td>
<td>13,275, 4250</td>
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<tr>
<td>Max drive power</td>
<td>Pmax, kW</td>
<td>17,500, 4850</td>
<td>17,500, 4850</td>
</tr>
<tr>
<td>Surge at p1 = coast</td>
<td>Vc, m³/h</td>
<td>63,000, 25,000</td>
<td>63,000, 25,000</td>
</tr>
<tr>
<td>Turndown</td>
<td>q, %</td>
<td>33, 23</td>
<td>33, 23</td>
</tr>
<tr>
<td>Mass flow ratio</td>
<td>m/mo</td>
<td>1.0, 1.44</td>
<td>1.0, 1.44</td>
</tr>
<tr>
<td>Volume flow ratio</td>
<td>V/VN</td>
<td>1.0, 0.77</td>
<td>1.0, 0.77</td>
</tr>
<tr>
<td>Head ratio</td>
<td>h/p, %</td>
<td>1.0, 0.72</td>
<td>1.0, 0.72</td>
</tr>
<tr>
<td>Speed ratio</td>
<td>N/N</td>
<td>1.0, 0.91</td>
<td>1.0, 0.91</td>
</tr>
<tr>
<td>Power input ratio</td>
<td>P/Pmax</td>
<td>1.0, 1.13</td>
<td>1.0, 1.13</td>
</tr>
<tr>
<td>Per input ratio</td>
<td>P/Pmax</td>
<td>1.0, 1.45</td>
<td>1.0, 1.45</td>
</tr>
</tbody>
</table>

In 1979, the author’s company was awarded a contract by a major petrochemical company in the Far East for two two-casing steam and expansion turbine driven 17 MW air compressors...
• For such an uprate, it was, from the very beginning, a common understanding between the user and the manufacturer that the capital cost for new rotors and stationary internals for LP, HP compressor, steam, and expansion turbine (i.e., a total of eight rotors including spares and four stationary aeropackages) would be comparatively expensive, and disassembly and reassembly time would be excessively long causing high losses of production.

• A speed increase was out of the question for such a high uprate, due to the high tip speeds used in the original designs.

• The planned PTA train rerate lent itself for suction side boosting, since:
  • It required no geometrical changes to the original compressor itself due to its turbine drive.
  • Erection of the skid mounted booster with cooler and the new piping system could be accomplished without disturbing normal train operation.
  • The integration of the additional pipes, valves, control instrumentation, and monitoring system and the booster commissioning and tying to the main compressor would increase the normal plant shutdown time insignificantly.
  • Space for placing the booster, cooler, and pipes was available.
  • It was the least costly solution.
  • It offered the possibility, in case of a booster trip, to continue operating the main compressor at the original capacity and specified discharge pressure, as if the booster had never existed, and would not interrupt production.

From Table 2 and performance maps Figures 15 (base: mass flow) and 16 (base: volume flow), the following is deduced for the operating mode “booster + main compressor”:

• The specified mass flow increase requires raising the main compressor suction pressure from 1.0 bar to 1.95 bar, in order to keep the main compressor power input below the maximum driver power (mass-for-head-swap). The power input of 15,040 kW remains well below the maximum power. The difference between the volume increase (164 percent) and mass increase (148 percent) derives from a redefinition of suction conditions (lower pressure, higher temperature, higher relative humidity).

• The main compressor speed dropped to 91 percent of the original speed.

• The booster pressure ratio of 2.05 requires a high head/high Mach number machine, which can only be achieved with an unshrouded impeller. So, the single stage booster is responsible for 22 percent of the total head, three times the main compressor average stage head.

• The main compressor suction volume flow necessarily has to drop to 71,880 m³/h or 77 percent of the original value, also depicted by the flow coefficient drop from 0.128 to 0.108. This small flow can be handled without blowoff, since the surge limit at the reduced speed falls to a comfortable 55,000 m³/h at constant discharge pressure vs 63,000 m³/h at 100 percent speed.

• Related to the new rated point, the train has a phenomenal turndown of 59 percent.

• Main compressor performance maps with and without booster are shown in Figures 15 and 16. Surge limit shift is clearly illustrated. The efficiency drops some two percentage points with boosting, because at nine percent lower speed the latter stages tend to volumetrically overload, thus sacrificing efficiency through aerodynamic mismatching.

• As can be seen in Figure 16, comparing heads of both operating modes, the mismatch, on the other hand, benefits the surge limit, thus enabling a safe surge-free operation with the booster in operation.

• The overall efficiency is kept approximately constant, since the booster compensates for the efficiency drop.

• The single stage booster is a compact integrally geared design with axial inlet. The common base frame for the compressor, gear, and motor houses the complete oil system, which enables an easy installation on a table foundation or on the ground beside the main compressor (Figure 17). The normally unshrouded overhung impeller is mounted on the extended pinion shaft. Impeller blade channels are five-axis-NC-milled from the solid hub forging, even for such large impellers. The axial inlet assures a high efficiency and can be equipped with adjustable inlet guide vanes that serve as an efficient means to adapt both compressors to each other during seasonal changes of operating conditions and to bring about load sharing if required. The booster shaft can be equipped with a choice of a carbon ring seal, a mechanical contact seal, or a dry gas seal, depending on the gas handled (Figure 18).

• The booster, with its aftercooler, was placed on a ground level foundation beside the main compressor.

• Refer to Figure 19 for a schematic of the piping arrangement for the booster and main compressor. During normal operation, the booster/main compressor hookup valve V1 is open, the main compressor atmospheric suction valve V2 and the booster blowoff valve V3 are closed. During the sequential startup of the two compressors, valve positions are vice versa.

• In 1996, the expansion turbine was uprated to 150 percent mass flow and power output through a new rotor and new nozzles.

• Another very similar example of an air compressor in an FCC Plant is shown in Figure 20, whose volume flow was increased by 83 percent through boosting (a photo of the booster as per case study 2 is not available).

Advantages and disadvantages are summarized in Table 3 of the three rerate modes.

Table 3. Advantages and Disadvantages of the Three Rerate Modes.

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wider Impellers</td>
<td>Estimation Space must be available</td>
</tr>
<tr>
<td>Speed Increase</td>
<td>New rotor, diaphragms</td>
</tr>
<tr>
<td>Boosting</td>
<td>New turbine rotor, stator</td>
</tr>
<tr>
<td></td>
<td>Motor, gear, cooler(s) uprate or new</td>
</tr>
<tr>
<td></td>
<td>New compressor, skid</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Casing unchanged</td>
<td>Limited uprate potential</td>
</tr>
<tr>
<td>Drive turbine casing unchanged</td>
<td>New turbine rotor, stator</td>
</tr>
<tr>
<td>Older compressors high spurial potential</td>
<td>Motor, gear, cooler(s) uprate or new</td>
</tr>
<tr>
<td></td>
<td>Check foundation, skid</td>
</tr>
<tr>
<td></td>
<td>High pressure ratio: limited uprates (steep performance curves)</td>
</tr>
<tr>
<td></td>
<td>No uprates of original speed is maximum</td>
</tr>
</tbody>
</table>

ANTICIPATING RERATES

The specification for a new compressor should be carefully scrutinized with regard to any anticipated capacity increases. If there are any plans to increase the flow sometime in the future, even if it is only a faint idea, it is highly advisable to take that into account when specifying and ordering the new compressor. Not only nozzles, shaft diameters, and bearings, but also the flow volutes, which are often the bottlenecks, should have appropriate reserves for these future extensions. These provisions do not normally carry an additional price tag. However, it would also be advisable in such a case that couplings, gears, and main drives be selected on the basis of the required extra power reserve normally exceeding the API 617 minimum reserve of 10 percent. This, of course, does increase the capital costs, but to a much smaller extent than later on, when these items have to be uprated or even totally replaced.
RERATE OF CENTRIFUGAL PROCESS COMPRESSORS—
WIDER IMPELLERS OR HIGHER SPEED OR SUCTION SIDE BOOSTING?

In order to offset the efficiency reduction normally brought about by uprating existing compressors, basically two options are applicable and in quite a number of cases feasible:

- Channel surface texture enhancement
- Application of rub-resistant labyrinth seal material
Figure 20. Example: Main Compressor with Booster and Intercooler.

Enhancement of Flow Channel Surface Texture

As can be seen from Figure 21, decreasing the flow channel roughness height increases the efficiency. This effect is more pronounced for narrower flow passages and higher Mach numbers. Cutting the roughness height in half will increase the efficiency by approximately one to three points.

Surface smoothing is carried out conventionally and also by chemical or electropolishing. Needless to mention that compressors handling gases with contaminating constituents do not lend themselves for such a treatment.

Efficiency Increase through Rub-Resistant Labyrinth Seals

Leakage flows occur in all compressor labyrinth seals: at impeller cover disks, interstage shaft sections, buffer gas seals, and at the balance piston. These parasitic flows, which have to be continuously compressed, do not contribute to the process mass flow. They account for three percent to, as high as, 12 percent of the overall power depending on flow coefficient level, pressure ratio, and impeller arrangement.

Especially compressors with narrow impellers arranged inline can be made more efficient if the conventional steel labyrinth seals are replaced by plastic seals with clearances of only 30 percent to 40 percent of steel seals. This "plastic engineering" is introduced more and more into today’s process compressors.

There are basically two types of plastic seal materials:

- Abradable materials, like polytetrafluoroethylene (PTFE) combined with mica and silicon-aluminum (AlSi) combined with polyester (PE), permit rubbing of rotating parts by giving way to intruding tips through abrasion, i.e., adapting their geometry locally. Labyrinth tips are of conventional material (e.g., steel).

- Rub-resistant materials forming the labyrinth tips, like polyamide-imide (PAI) and poly-ether-ether-ketone (PEEK), tolerate rubbing by deforming locally and resuming nearly the former shape.

AlSi+PE can be used on stationary as well as on rotating parts; all others are used as stationary parts. An example is shown in Figure 22 of an impeller cover disk seal and an interstage shaft seal.

The overall energy savings attainable by applying rub-resistant labyrinth seals is between two and eight percent and varies with compressor size, impeller flow coefficient, impeller arrangement, and pressure ratio.

CONCLUSIONS

Quite an array of options are available for centrifugal compressor rerating, which is defined as modification of a compressor already in operation in order to change the aerothermodynamic performance.

- Method 1—Wider impellers, i.e., increase of the flow channel through flow area that can be brought about by either higher flow coefficient impellers or impeller scaleup.
• Method 2—Increasing the flow path gas velocity, which is achieved by raising the rotational speed.
• Method 3—Increasing the suction density, which is attainable by placing a separate booster compressor upstream of the existing machine.

The potential of all three approaches was demonstrated by means of two case studies from the daily life of an OEM.

In the first study, concerning a hydrocarbon compressor in an FCC plant built in 1968, Method 1 was combined with Method 2 to obtain 200 percent of the original flowrate in a first step and 243 percent of the original flowrate in a second step. Step 1 involved a complete replacement of the compressor rotor, including diaphragms, and the drive turbine rotor including the guide blade carrier. Step 2 comprised another new compressor rotor and diaphragms. The stepup of flow coefficients from 0.05 to 0.09 to 0.12 reflected the R&D efforts over the decades and demonstrated clearly that a 20-year-old compressor has an enormous uprate potential, possibly without changing speed. However, an uprate of magnitude described required a speed increase as well, 14 percent for Step 1 and eight percent for Step 2, related to the original speed. The inevitable head rise as a side effect of speed increase was converted to flow increase via the performance curve slope.

The first flow extension of 62 percent was achieved by a flow coefficient increase and 38 percent by a speed increase. The final extension to 243 percent flow would have rarely been feasible without the elimination of one stage made possible by a specified head reduction.

In general, an older compressor (say, 20 years) has an uprate potential by flow coefficient of at least 50 percent and maximally around 100 percent. In most cases, the uprate potential by speed increase is at least around 20 percent. Impeller scaleup normally yields around 20 percent, also.

Newer compressors (say, five years), equipped with high flow coefficient impellers with the maximum possible diameter in the particular casing and running at the highest permissible speed can hardly be uprated by wider impellers or speedup.

Even if the compressor is already congested, jampacked, and tightly overcrowded, suction side boosting can be applied to bring about a substantial uprate. This is what the second case study is all about. It concerns a 17 MW two-casing air compressor train in a PTA plant built in 1980. It was uprated to 148 percent mass flow by boosting with a single stage compressor. The suction pressure of the existing machine was raised from one bar to 1.95 bar, reducing its head to 77 percent, its speed to 91 percent, and its suction volume flow to 77 percent, and increasing its power input to 113 percent, well below maximum turbine power output. Geometrically, the main compressor remained unchanged, hence with the booster not in operation, it can instantly resume the old operating mode at atmospheric suction and specified discharge pressure. Boosting was, by far, the lowest cost solution compared to new rotors and/or speedup.

In general, the uprate potential through suction side boosting depends on many parameters, especially on the overall pressure ratio and the feasible tip speed Mach number of the booster. For pressure ratios of two up to 10, uprates to between 160 percent and 220 percent are possible. For pressure ratios up to 40, 130 percent to 150 percent are possible. Low pressure ratio compressors (e.g., H₂-rich gases) have a lower uprate potential of between 110 to 140 percent, depending on how much the booster tip speed Mach number can be maximized, e.g., through high yield strength impeller material.

In spite of additional losses occurring for any rerate, be it in the nozzles or by aerodynamic mismatch or by part load or overload operation, the efficiency level can, in most cases, be maintained. Levels of completely new compressors designed from scratch for the high flowrates must necessarily be higher than for rerated compressors. Efficiencies can be improved through by flow channel surface smoothing and replacement of conventional interstage seals with rub-resistant labyrinth seal material.

When new compressors are specified and ordered, future uprates should be anticipated from the very beginning, even if a rerate is only a remote idea. Every compressor casing ought to have some space reserves internally, in the casing openings or some oversized shaft diameters and couplings. Every gear box should have the possibility of accommodating a new gear wheel set suitable for higher speed and power, in order to rerate at a later date with wider impellers and higher power inputs.

Compressor rerating is a favorable alternative to increase the production since the efficiency is maintained. The costs are lower than for new builds and the hardware exchanges can be accomplished in little more than the routine plant shutdown period.

ACKNOWLEDGEMENT

The author is greatly indebted to his former colleague and friend James H. Hudson for polishing the author's mediocre English syntax.

APPENDIX

Derivation of Equation (2)

Starting point is the flow coefficient:

\[ \phi = \frac{V}{(\pi/4)d_{2}^{2}u_{2}} \]

With \( u_{2} = \pi N_{d_{2}} \) and \( \phi = \frac{V}{\rho_{s}} \) and rearranging:

\[ \frac{m}{\pi^{2} \rho_{s} N_{d_{2}}^{2}} = \frac{\phi}{\rho_{s} \times (d_{2}/d_{20})^{3}} \]

Since the flow benefits from the diameter induced head increase, the flow ratio is to be corrected by the slope effect:

\[ S = \left( \frac{h_{s} / h_{p}}{h_{s}} - 1 \right) / \left( \frac{\rho_{s} / \rho_{0}}{\rho_{0}} - 1 \right) \]

Since for multistage compressors with compressible flow (i.e., medium to high Mach numbers) the head ratio is \( h_{s} / h_{p} > (d_{2}/d_{20})^{2} \), a mismatching correction factor has to be accounted for:

\[ F_{d2} = \left( h_{s} / h_{p} - 1 \right) / \left( (d_{2}/d_{20})^{2} - 1 \right) \]

So the final equation is:

\[ \frac{\bar{m}}{\bar{m}_{0}} = \frac{\phi}{\phi_{0}} \times \left( \frac{d_{2}}{d_{20}} \right)^{3} \times \left[ \left( \frac{d_{2}}{d_{20}} \right)^{2} - 1 \right] F_{d2} / S + 1 \]

Derivation of Equation (4)

The inlet loss from the radial flange to the impeller eye, referenced to the gas power of the single stage downstream of the nozzle is:

\[ P_{L} / P_{gas} = (h_{s} - h_{0}) / h_{s} \]

\( h_{s} = \) Isentropic stage head including loss
\( h_{0} = \) Isentropic stage head without loss

\[ P_{L} / P_{gas} = \frac{\frac{P_{d}}{P_{s}}^{(k-1)/k}}{\frac{P_{d}}{P_{s}} - \frac{\Delta P}{P_{s}}} - 1 \]

\[ P_{d} = \text{Total stage discharge pressure} \]
\[ P_{s} = \text{Total suction pressure at inlet flange} \]

The total pressure loss is:
\[
\Delta p = \frac{\zeta P_s c_s^2}{2Z_s R T_s} = \frac{\zeta P_s c_s^2}{2Z_s R T_s}
\]

\[
\frac{\Delta p}{P_s} = \frac{k v}{2} M_{cs}^2
\]

\(k_v\) = Isentropic volume exponent

\(Z_s\) = Compressibility factor at suction

\(R\) = Gas constant

\(T_s\) = Suction temperature

\(\zeta\) = Loss coefficient

\(M_{cs}\) = Nozzle Mach number

Final formula:

\[
P_L = \frac{P_{gas}}{\Pi} = \left(\frac{\Pi}{1 - \frac{1}{k_v} \left(M_{cs}^2 \right)^{k_v - 1}}\right) - 1
\]

Derivation of Equations (5), (6), (7)

Original mass flow of main compressor:

\(\dot{m}_0 = V_0 P_s / Z_s R T_{s0}\)

\(P_{s0}\) = Original suction pressure

\(T_{s0}\) = Original suction temperature = suction temperature booster

Increased mass flow of main compressor through boosting:

\(\dot{m} = V_p / Z_s R T_s \)

\(P_s\) = Boosted suction pressure main compressor

\(T_s\) = Suction temperature main compressor with booster

Mass flow ratio through boosting (Z = const, R = const):

\(\frac{\dot{m}}{\dot{m}_0} = \frac{(p/p_{s0})(T_{s0}/T_s)(\dot{v}/\dot{v}_0)_{MC}}{\Pi_{B}}\)

With aftercooled booster:

\(T_{s0}/T_s = T_{s0}/T_R\)

\(T_R\) = Recooling temperature

\[
\frac{\dot{m}}{\dot{m}_0} = \frac{T_R}{T_{s0}} \left(\frac{\dot{v}}{\dot{v}_0}\right)_{MC}
\]

No cooler between booster and main compressor:

\[
T_{s0}/T_s = -\frac{1}{k_{n_{pa}}} \frac{k_{n_{pa}}}{\Pi_{B}}
\]

Mass flow ratio, no cooler between booster and main compressor (Equation (6)):

\[
\frac{\dot{m}}{\dot{m}_0} = \frac{\Pi_{B}}{k_{n_{pa}}} \left(\frac{\dot{v}}{\dot{v}_0}\right)_{MC} = \Pi_{B} \left(\frac{\dot{v}}{\dot{v}_0}\right)_{MC}
\]

Booster pressure ratio:

\[
\Pi_B = \left[\frac{h_{pB}(k-1)}{k Z_{s0} R T_{s0} \eta_{pB}} + 1\right]^{k_{n_{pa}}} / k_{n_{pa}}
\]

\(h_{pB}\) = Booster polytropic head

\(Z_{s0}\) = Suction compressibility factor

\(h_{p}/\eta_{p} = \Delta h = u_2^2\) = Enthalpy difference (= Euler head)

\(u_2\) = Impeller tip speed

\(s\) = Work input coefficient

\(M_{u2}^2\) = \(u_2^2/(kZ_{s0} R T_{s0})\)

\(M_{u2}\) = Tip speed Mach number

Booster pressure ratio (Equation (7)):}

\[
\Pi_B = \left[\frac{s(k-1) M_{u2}^2 + 1}{k_{n_{pa}}^{k_{n_{pa}}}}\right]^{k_{n_{pa}}}
\]

Derivation of Equations (8) and (9)

Boosting not only increases the mass flow, but, at the same time, reduces the main compressor head. In order to keep its power consumption approximately constant (mass flow for head swap), the inlet volume flow of the main compressor necessarily has to change (in case study 2 to 77 percent, Table 2).

Approximate polytropic head of an intercooled compressor:

\[
h_p = T_s\left(1+c\right) T_{s0}\left(\frac{Z_s}{Z_{s0}}\right)_{MC} R \left(\frac{1+c}{k_{n_{pa}}^{k_{n_{pa}}}} - 1\right)
\]

\(c\) = Number of intercoolers

Original gas power input of main compressor:

\[
P_0 = \dot{m}_0 \eta_{p0} = \dot{m}_0 T_{s0}\left(1+c\right) T_{t0}/T_{s0}\ Z_s R k_{n_{pa}} \left(\frac{k_{n_{pa}}^{k_{n_{pa}}}}{k-1}\right)
\]

Gas power input of main compressor with booster in operation:

\[
P = \dot{m} T_{R} \left(1+c\right) Z_s R k_{n_{pa}} \left(\frac{k_{n_{pa}}^{k_{n_{pa}}}}{k-1}\right)
\]

Main compressor power with booster, referenced to original power:

\[
P_k = \frac{P_k}{P_{k0}} = \dot{m}_0 T_{s0}\left(1+c\right) T_{t0}/T_{s0}\ Z_s R k R_{k_{n_{pa}}^{k_{n_{pa}}}} \left(\frac{k_{n_{pa}}^{k_{n_{pa}}}}{k-1}\right)
\]

\(P_k\) = Power input at coupling (including mechanical losses)

Combining with Equation (A-6):

\[
\frac{\dot{m}}{\dot{m}_0} = \frac{T_R}{T_{s0}} \left(\frac{\dot{v}}{\dot{v}_0}\right)_{MC}
\]

\[
\frac{P_k}{P_{k0}} = \frac{\dot{m}_0 T_R}{\dot{m}_0 T_{s0}} \left(1+c\right) T_{t0}/T_{s0}\ Z_s R k R_{k_{n_{pa}}^{k_{n_{pa}}}} \left(\frac{k_{n_{pa}}^{k_{n_{pa}}}}{k-1}\right)
\]

Approximate volume ratio of main compressor with aftercooled booster in operation (Equation (8)):

\[
\left(\frac{\dot{v}}{\dot{v}_0}\right)_{MC} = \frac{1+c}{1+c} \frac{T_R}{T_{s0}} \left(\frac{k_{n_{pa}}^{k_{n_{pa}}}}{k-1}\right)
\]
Approximate volume ratio of main compressor with booster in operation and no intercoolers (small pressure ratios, H₂-rich gases Equation (9)):

\[
\left( \frac{V}{V_0} \right)_{MC} = \frac{P_k}{P_{k0}} \frac{k-1}{\Pi^{k_{\Pi0}}} - 1 \left[ \frac{k-1}{\Pi^{k_{\Pi0}}} - 1 \right] \quad (A-18)
\]

REFERENCES

