FLOW-INDUCED TURBOCOMPRESSOR AND PIPING NOISE AND VIBRATION PROBLEMS— IDENTIFICATION, DIAGNOSIS, AND SOLUTION

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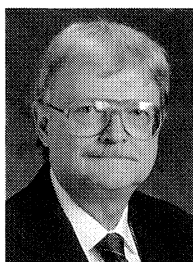and
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Mr. Jungbauer's interests have also extended into the positive displacement and centrifugal pump and compressor fields. He has been instrumental in identifying and solving failure mechanisms related to cavitation, impeller, and volute design, piping interaction, and skid and support flexibility.

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ABSTRACT

The problem of flow-induced noise and vibration in turbocompressors and piping systems is complex and not readily understood. These problems are difficult to identify and solutions may not be obvious. A working knowledge of vortex shedding and system acoustic response is necessary for solving such problems.

The discussion is concentrated on presenting a theoretical background of vortex generation and the excitation of system acoustic response. This is then coupled with identification and solution of both actual and potential problems using appropriate field data acquisition and analysis, simulation, and computational methods.

INTRODUCTION

The topic of flow-induced noise and vibration in turbocompressors and their associated piping systems has been gaining attention in the last 10 to 15 years. However, purely flow-induced problems are still not as well recognized as the more common compressor aerodynamics problems of surge, impeller stall, and diffuser stall.

Problems that are flow-induced are not directly related to compressor speed, head, or capacity. Instead, they are usually flow-velocity dependent and caused by a source or sources internal or external to the machine and are difficult to identify requiring a knowledge of vortex shedding and system acoustic resonance.

Typical symptoms of flow-induced acoustic problems are high machine noise levels, nonsynchronous shaft vibration, compressor casing or bearing housing vibration, and impeller fatigue damage. Severe vibration and accompanying noise of main gas piping, with excitation of shell resonances, and fatigue of small attached piping elements can also occur.

In attempting to diagnose flow-induced machinery problems in the operational stage, an understanding of certain identifying characteristics of flow-induced excitation is reviewed. Correlation of theoretical or analytical calculations or simulations with field data and experience is necessary for identification of the source of the problem and for derivation of an effective modification.

Similarly, in the design stage, computational and simulation techniques can be employed to predict and avoid the better understood flow-induced problems in piping systems. Modifications are then employed to ensure that problems will not occur during startup and normal operations.

Background

Simply put, most flow-induced problems associated with turbomachinery and piping systems are due to a coincidence of vortex shedding with an acoustic resonance of the same frequency. The result is an amplification of the dynamic pressure fluctuations induced by the vortex shedding. The amount of amplification is controlled by the acoustic damping or "Q" of the system.

The following discussion concentrates on an explanation of vortex shedding, acoustic length resonance, and acoustic radial or cross-mode resonances. In addition, an understanding of the

behavior of flow induced problems as a function of fluid flowing velocity is necessary.

Vortex Shedding Excitation

The relationship that gives the frequency distribution of vortex shedding as a function of fluid flow velocity and geometrical dimensions is shown in Equation (1).

$$f_s = N_s \frac{v}{d} \qquad (1)$$

It is known that obstructions or constrictions shed vortices efficiently at or near a Strouhal number of approximately 0.2. The vortex shedding or Strouhal frequency does not refer to a specific frequency, but to a band of energy centered about the Strouhal number of 0.2. The amplitude decreases at about three dB per octave above and below this frequency, as shown in Figure 1. However, depending upon the system obstruction geometry, flow field, etc., the Strouhal number can vary from 0.1 to more than 0.5. In actual systems, one dominant frequency will typically be excited at the acoustic resonant mode nearest to the frequency of Strouhal excitation.
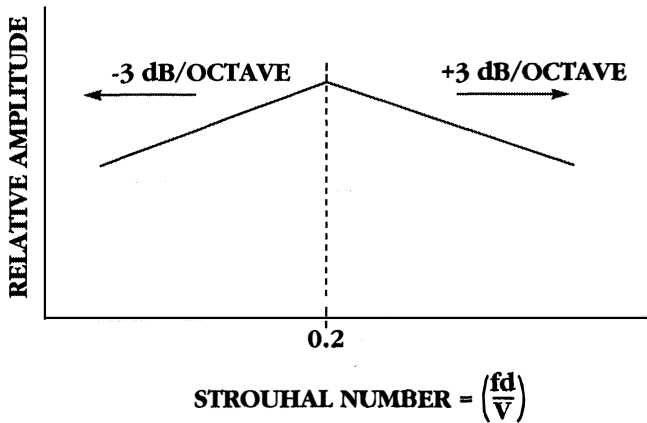

Figure 1. Energy Distribution of Strouhal Vortex Turbulence about the Preferred Strouhal Number $N_s = 0.2$.

It is apparent that flow-induced phenomena are dependent upon two factors. The first one is the existence of vortex generated energy of the proper frequency, and the second is the presence of an acoustic resonance of the same frequency.

Behavior of Flow-Induced Problems

Two important characteristics of flow-induced problems are called "lock in, drop out," and hysteresis. The behavior of vibration, noise, and pulsations due to "lock in" is shown in Figure 2. As the flow increases, a specific frequency is excited over a small range of flow velocity. As velocity increases, the mode will "drop out" and cease to be excited until flow velocity increases again, and the vortex shedding frequency becomes coincident with a higher mode, which becomes excited and "locked in."

Flow-induced problems also exhibit hysteresis. This happens when "lock in" and "drop out" occur at different flow velocities, depending on whether the flow is increasing or decreasing, as shown in Figure 3.

Acoustic Length Resonance

Turbocompressor internal passages and piping systems are subject to vortex shedding-induced excitation. These discrete or distributed lengths and diameters define a halfwave resonant frequency as in Equation (2) and a quarterwave resonant frequency as in Equation (3).

$$f_{1/2\lambda} = N \frac{a}{2L} \qquad (2)$$

$$f_{1/4\lambda} = (2N-1) \frac{a}{4L} \qquad (3)$$

These "organ pipe" resonances are well known. The dynamic pressure mode shapes are dependent upon the boundary or end conditions. The acoustic mode shapes for the three basic boundary conditions of open-open, closed-closed, and open-closed are shown in Figure 4.
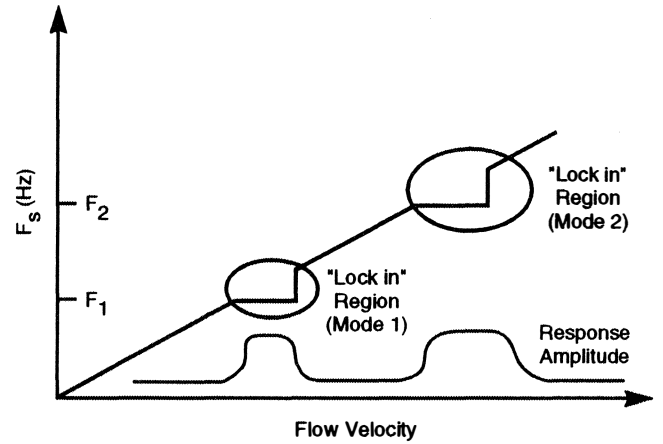

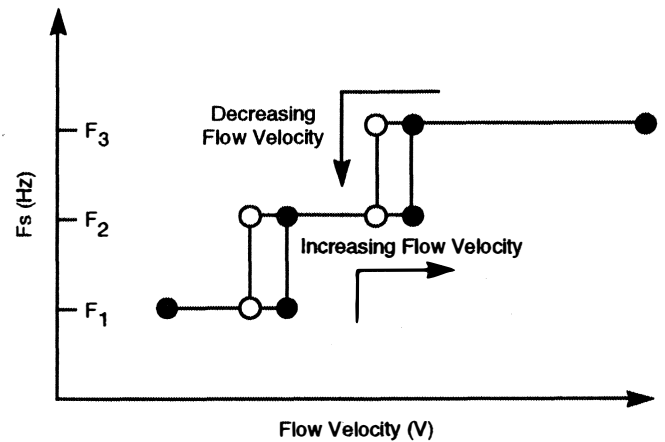*Figure 2. Typical Frequency Response in the Presence of Flow-Induced Excitation.*

Figure 3. Typical Response Characteristics in the Presence of Acoustic Modes Illustrating Hysteresis.
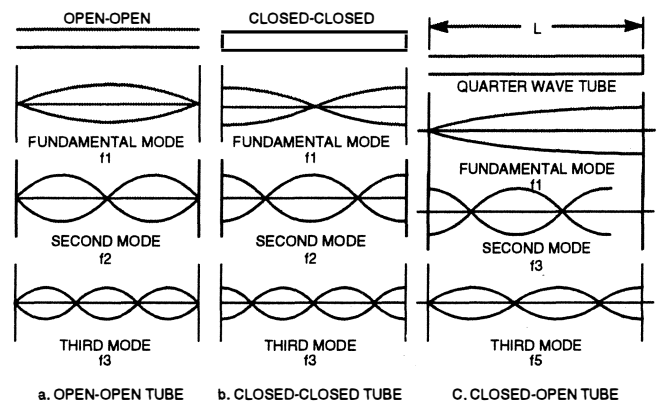

Figure 4. Dynamic Pressure Mode Shapes for Open-Open, Closed-Closed, and Open-Closed Tubes.

The two boundary conditions most likely to be excited in turbo-machinery and piping systems are the open-open and open-closed types. This is due to the fact that there is also a particle velocity associated with the acoustic pressure standing waves. This particle velocity component is 180 degrees out-of-phase with the pressure component. The velocity standing wave and pressure standing wave for the quarterwave length example is shown in Figure 5. For each type of acoustic pressure standing wave, velocity maxima are associated with pressure wave minima, and vice-versa. It then becomes apparent that any vortex-induced velocity variations at the open-ended boundary condition case can result in the excitation of significantly amplified pressure standing waves or "pulsations."
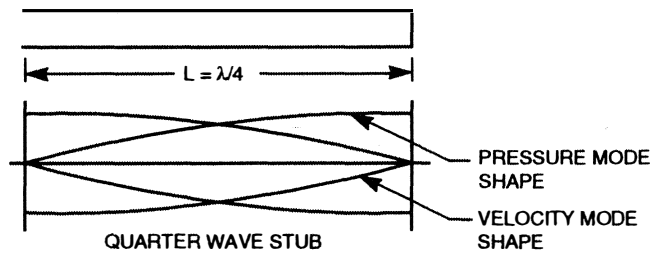

Figure 5. Pressure Mode Shape and Velocity Mode Shape in Quarterwave Stub.

Radial Acoustic Resonances

In addition to acoustic length resonances, there exists another type of resonance involving a three-dimensional acoustic resonance with standing acoustic pressure wave patterns that are also perpendicular to piping and flow passage axes. These resonances involve Bessel function solutions for the cylindrical wave equation and are defined by Equation (4):

$$f_{mn} = \alpha_{mn} \frac{a}{d} \qquad (4)$$

Where α_{mn} is obtained from Table 1.

Nodal patterns are presented in Figure 6. Radial acoustic cross-mode resonance standing waves not only rotate, but will propagate up and down the piping as a helical wave. The velocity of propagation is defined by Equation (5).

$$c_x = \frac{a}{\sqrt{1 - \frac{f_{mn}}{f}}} \qquad (5)$$

If $f < f_{mn}$, then the pulsation decays exponentially. If $f = f_{mn}$, the resonance is excited. Similarly, if $f > f_{mn}$, then the acoustic cross-mode propagates up and down the piping system in question.

CASE STUDIES—EXISTING FACILITIES

Case studies are now presented for specific types of flow-induced problems. These case histories serve to illustrate the excitation of two kinds of acoustic resonances in turbomachinery and piping systems.

Case 1

During startup of a gas processing plant, a turbocompressor train operated initially with high antisurge flows. Operation at these conditions resulted in noise levels of 130 dBA at and near the antisurge valve. The noise was a relatively pure tone, with lower level flow turbulence noise. The noise appeared to be originating in the valve. Another larger valve with a low noise trim was installed with little success in reducing or eliminating the problem.

In order to characterize the problem, pulsation and vibration acceleration data were taken at various test point locations shown in Figure 7. Data sets were obtained at different antisurge flowrates for each test point.
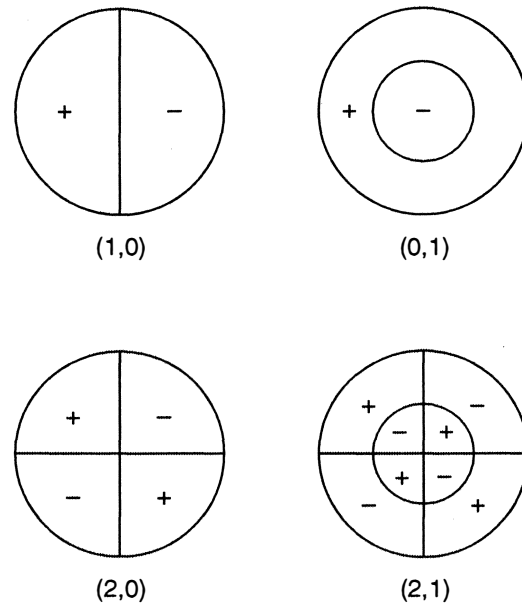

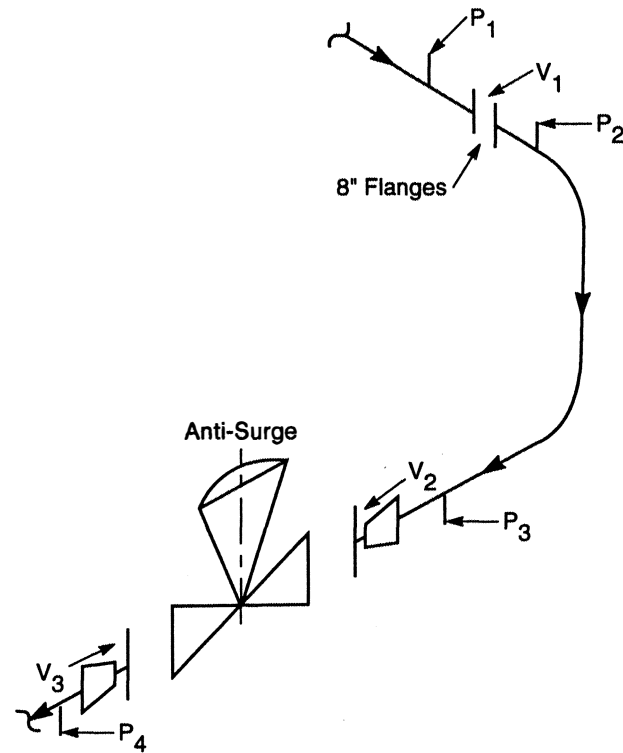
Figure 6. Nodal Patterns for Acoustic Radial Modes.

Figure 7. Antisurge Piping in Vicinity of Control Valve.

The results are presented in Figure 8. The data confirmed the presence of high pressure pulsations of 240 psi peak-to-peak at 1800 Hz near the eight-inch flanges, located at a distance upstream of the antisurge valve. Data taken near the antisurge valve gave much lower pulsation amplitudes. Acceleration data gave highest amplitudes at the eight-inch flanges. Valve acceleration data showed much lower amplitudes at 1800 Hz.

Using the information gained from these tests, the upstream piping was determined to be the source of the noise at 1800 Hz. The piping was disassembled and found to have an aerodynamic source of excitation, i.e., vortex shedding. The source was found to be a gap between the flanges.
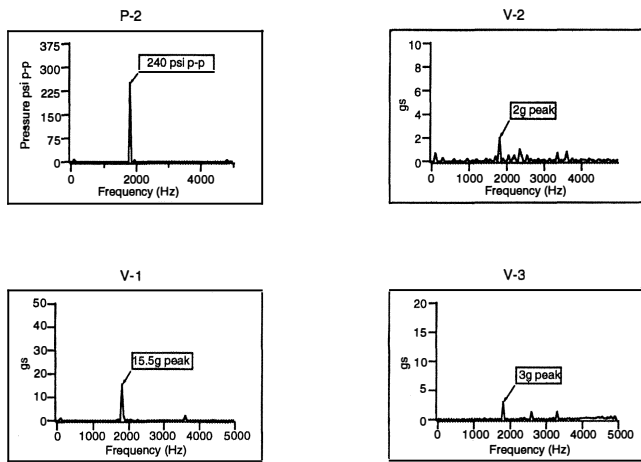
Figure 8. Antisurge Flow = 65 Percent, Pulsation and Vibration Data.

The vortex shedding frequency was calculated using the dimensions obtained from the piping and the antisurge gas flow velocities. Based upon the flange gap width and depth, calculations were in close agreement with theory obtained for aircraft wind tunnel tests.

The acoustic cross-modes were then calculated. The piping inside diameter was 7.625 in and gas speed of sound was 1935 ft/sec. Using Equation (4) with $\alpha_{mn} = \alpha_{1,0} = 0.5861$ from Table 1, gives:

$$f_{1,0} = \alpha_{1,0} \frac{a}{d}$$

$$f_{1,0} = 0.5861 \frac{1935 \text{ ft/sec}}{0.635 \text{ ft}} \qquad (6)$$

$$f_{1,0} = 1786 \text{ Hz}$$

These calculations confirmed the coincidence of vortex shedding with an acoustic cross-mode for the range of flow velocities tested at antisurge conditions.

Table 1. Characteristic Values of α_{mn} for Cylindrical Pipe Solutions.

m/n	0	1	2	3	4
0	0.0000	1.2197	2.2331	3.2383	4.2411
1	0.5861	1.6970	2.7140	3.7261	4.7312
2	0.9722	2.1346	3.1734	4.1923	5.2036
3	1.3373	2.5513	3.6115	4.6428	5.6624
4	1.6926	2.9547	4.0368	5.0815	6.1103
5	2.0421	3.3486	4.4523	5.5108	6.5494
6	2.3877	3.7353	4.8600	5.9325	6.9811
7	2.7304	4.1165	5.2615	6.3477	7.4065
8	3.0709	4.4931	5.6576	6.7574	7.8264

Subsequently, the upstream piping was modified to alter the vortex shedding characteristics. This involved elimination of the flange gap by gasket material. The unit was restarted and antisurge flows varied over the normal operating range. During these tests, no noise, pulsation, or vibration was measured at or near 1800 Hz.

Case 2

Three pipeline compressors were restaged to increase unit flowrates. During startup, high compressor noise and suction piping vibration occurred at the inlet of all three compressors. The noise and vibration occurred at 240 Hz.

Pressure pulsation and vibration data were obtained at the test points presented in Figure 9. Flowrates were varied in order to define the pressure pulsation and vibration behavior.
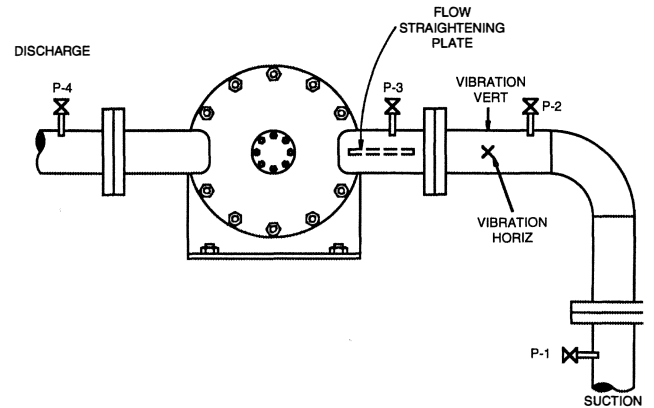

Figure 9. Pulsation and Vibration Test Point Locations for a Pipeline Compressor.

The pressure pulsation data obtained for the tests are presented in Figure 10. These data revealed highest pulsation levels occurred at test point P-3, located at the midpoint of the inlet flow splitter passage of the compressor. The pressure pulsations were first excited or became "locked in" at an inlet flow velocity of approximately 63 to 64 ft/sec. The pulsation amplitude increased with flow velocity. Vibration was identical in behavior to pressure pulsation.
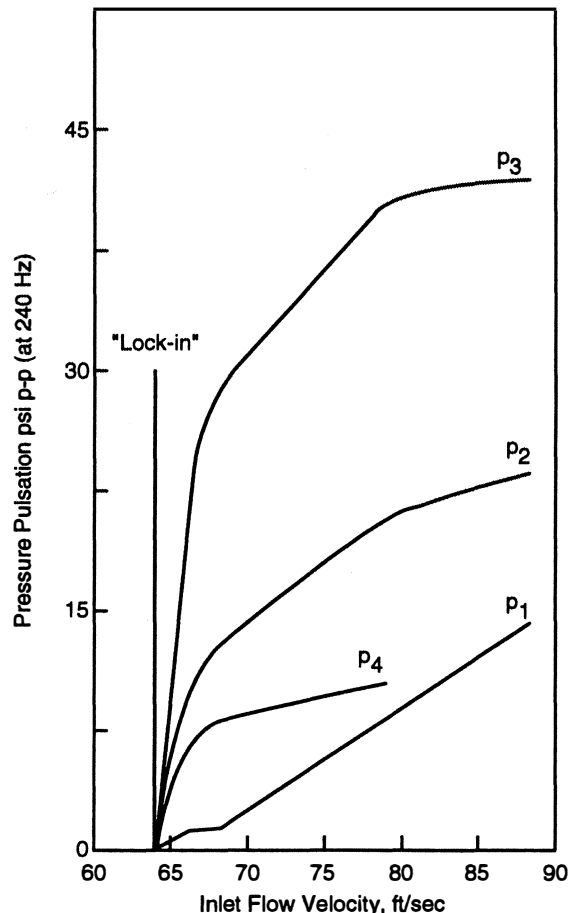

Figure 10. Pressure Pulsation vs Suction Flow Velocity.

Vortex shedding calculations were performed using Equation (1). The results are as follows:

$$f_s = N_s \frac{v}{d}$$

$$f_s = 0.4 \, \frac{80 \text{ ft/sec}}{0.125 \text{ ft}} \tag{7}$$

$$f_s = 256 \text{ Hz}$$

The calculated frequency of 256 Hz is in approximate agreement with theory, given a Strouhal number of 0.4 and plate thickness of approximately 1.5 in and flow velocity of 80 ft/sec. The Strouhal number of 0.4 was selected based upon empirical data obtained from previous studies and can actually vary from approximately 0.35 to 0.42. Exact agreement, however, can be obtained using a Strouhal number of 0.375. Acoustic resonant frequency calculations of the flow splitter passage, using the length of the passage with end correction factors and the gas velocity of sound, defined the fundamental halfwave resonance for an open-open configuration at or near 240 Hz and presented in Figure 11. Acoustic length resonance calculations were performed using Equation (2). However, the passage was converging between the inlet and outlet ends of the passage, with the leading edge scalloped to resemble a half-moon shape. The passage length was approximately 2.2 ft, with end connection factors of four inches for each end. Calculations were as follows:

$$f_{1/2\lambda} = N \frac{a}{2L}$$

$$f_{1/2\lambda} = 1 \, \frac{1375 \text{ ft/sec}}{2(2.2 \text{ ft} + 0.67 \text{ ft})} \tag{8}$$

$$f_{1/2\lambda} = 239 \text{ Hz}$$

This calculation was in good agreement with measured data for the frequency of acoustic response. Therefore, field data and calculations confirmed the coincidence of an acoustic resonance of the passages and vortex shedding of approximately the same frequency.
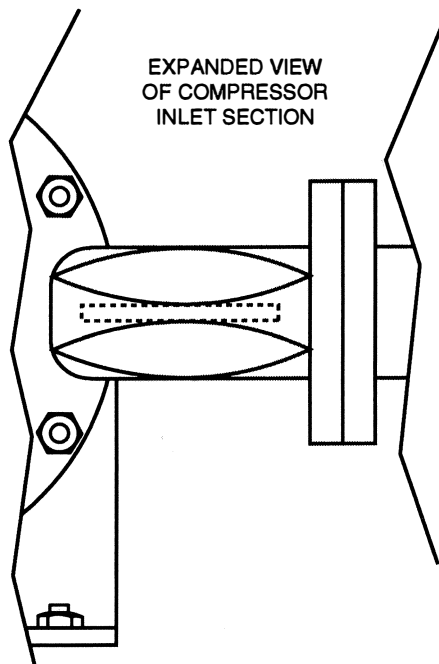

Figure 11. Acoustic Standing Waves in Flow Splitter Passage at 240 Hz.

Aerodynamic modifications were made to the flow splitter plate. This involved tapering of the trailing edge to an included angle of less than 30 degrees as shown in Figure 12. The modifications eliminated vortex shedding and hence, excitation of the acoustic resonance based upon the observations of the user. The flow splitter plate could have also been shortened, raising the frequency of the acoustic resonance. However, if this modification had been employed, there was a possibility of re-exciting the passage acoustic resonance at a higher flowrate.
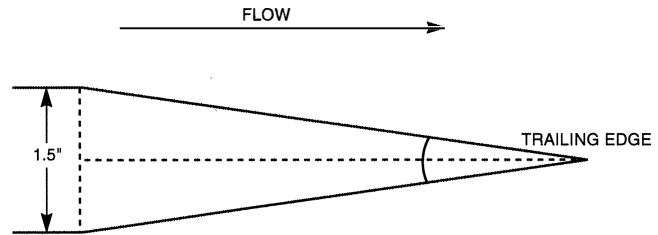

Figure 12. Flow Splitter Trailing Edge Modifications.

DESIGN STUDIES—NEW INSTALLATIONS

Historically, flow past a branch line is the most common source of low frequency (zero to 100 Hz) pulsation-induced piping vibration in centrifugal compressor installations. Normally low in amplitude, these pulsations can, when amplified by acoustic resonance, result in significant shaking forces and associated branch piping vibration. The frequency of the pulsation is controlled by the acoustics of the branch piping, often a recycle or antisurge line.

The process of vortex formation is illustrated in Figure 13. The vortex shedding frequency can be predicted for perpendicular side branch piping with a relatively high degree of accuracy. The frequency is dependent upon the velocity (v) in the gas piping and the diameter (d) of the branch line, as shown in Equation (1). The acoustic natural frequencies of the system can be obtained by simple calculations for this and other similar configurations. The quarterwave mode shape has a pressure maximum at the closed valve, or stub end, and a pressure minimum at the open end. The quarterwave mode shape can be calculated using Equation (3). The frequency is dependent upon the piping length (L) and the velocity of sound in the system.
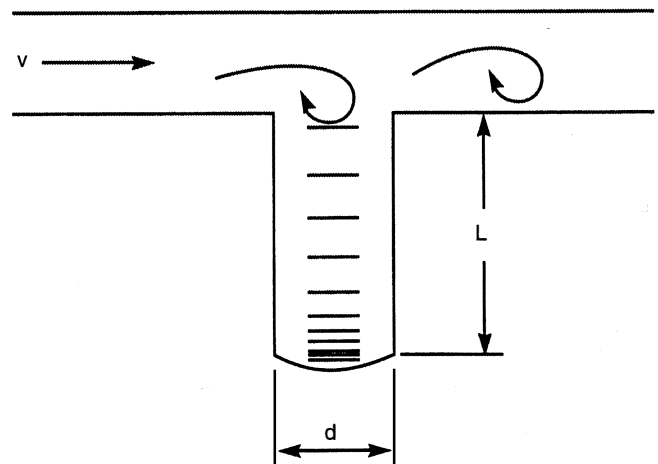

Figure 13. Simple Perpendicular Side Branch With Vortex Formation.

Problematical vortices do not form when there is gas flow through the side branch pipe, since flow entering or exiting the side branch tends to alter the boundary conditions necessary for vortex

formation. When a valve in the branch piping is closed, however, vortex formation will occur, and the configuration should be analyzed for a potential buildup of pulsation.

Acoustical Simulation

Complex branches require acoustical simulation techniques to determine the piping response frequencies. A complex configuration is one that contains multiple diameters, multiple branches, or both. The discharge side of a combination surge control and recycle system has two major branches and several different pipe diameters (Figure 14). Simple calculations based on piping length are not sufficient to accurately predict the acoustic frequencies of this type of configuration.
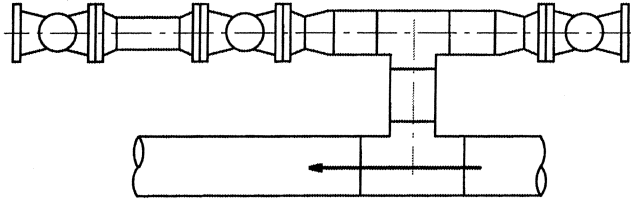

Figure 14. Complex Perpendicular Side Branch.

Once determined, the acoustic frequencies of the system are compared to the calculated range of Strouhal frequencies, which vary according to flow velocity. The acoustic resonant frequencies vary with changes in operating pressure, temperature, and gas composition. A side branch typically generates a band of excitation frequencies in a centrifugal compressor installation as discussed previously. However, only one frequency is usually excited at any given time, depending upon the gas velocity in the main line, and the amplitude of pulsation is most severe when the Strouhal frequency corresponds to the acoustic resonant frequency of the branch pipe, which results in acoustic amplification. There can be additional frequencies excited, depending upon the complexity of the piping system. These frequencies are usually near the predominant frequency.

The acoustic response peaks and the band of excitation produced by vortex shedding are compared for a perpendicular side branch in Figure 15. Acoustic responses occur at 25 Hz and 75 Hz based upon Equation (3) for a quarterwave resonance.

$$f_{1/4\lambda} = (2N-1) \frac{a}{4L}$$

$$f_{1/4\lambda} = (2N-1) \frac{1450 \text{ ft/sec}}{4(14.5 \text{ ft})} = 25 \text{ Hz where N} = 1 \qquad (9)$$

$$f_{1/4\lambda} = (2N-1) \frac{1450 \text{ ft/sec}}{4(14.5 \text{ ft})} = 75 \text{ Hz where N} = 2$$

In reality, the acoustic responses are subject to variations in the velocity of sound so that a range of frequencies (plus and minus five or 10 percent) usually must be considered. Strouhal excitation, based upon a 10-inch branch intersection, ranges from 20 Hz to 30 Hz, because of the expected range in gas flow velocities over the operating envelope of the system. Coincidence occurs at 25 Hz in this case, indicating a potential pulsation-induced vibration problem.

Piping Modifications

Making the change to a tee with a larger branch diameter significantly lowers the range of Strouhal frequencies. Use of a 16-inch tee in lieu of a 10-inch tee at the branch intersection, now places the excitation between 13 Hz and 20 Hz when flow velocity changes are considered, well below the acoustic responses in the system. This change to the piping is beneficial because it eliminates the possibility of a coincidence of pulsation energy in the system with an acoustic resonance.
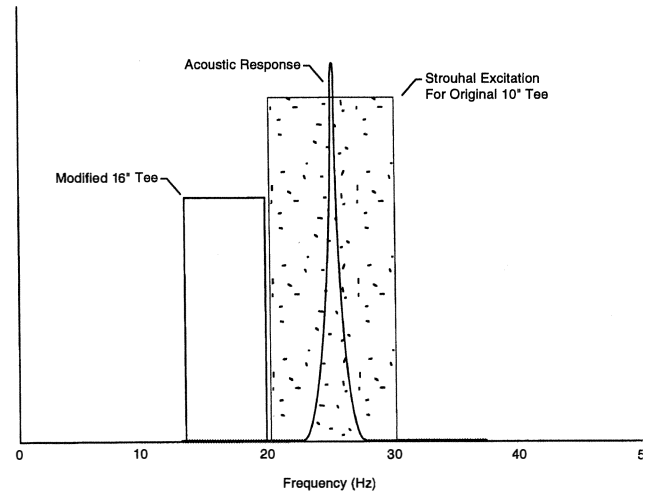

Figure 15. Interference Diagram with Modification that Removes Coincidence.

Other types of piping modifications also successfully reduce o eliminate coincidence in the system, such as moving the valve ii the line, which changes the effective pipe length. A longer distanc to the valve lowers the acoustic response frequency, whereas ; shorter length of pipe raises it. The Strouhal range is reduced b; restricting the unit flow range until the system is moved out o coincidence. A safe range of operation that does not requir making any modifications to the piping system at all is ofte identified in this manner.

Typically, an increase in branch diameter (up to the size of th main gas line) is an acceptable modification. A reducer back to th original line size is welded directly to the branch tee in most cases as shown in Figure 16. Solutions of this kind are neither difficul nor costly to install. Problems develop, however, when full siz tees and other practical modifications do not completely remov the coincidence. This most frequently occurs in very long brancl lines that have numerous acoustic responses in the frequenc spectrum of interest. The pulsation remaining in the system cai cause undesirable piping vibration. There is one exception. A Strouhal frequencies of less than 10 Hz, experience has shown tha piping modifications are generally not needed. The velocity of th flow and the energy of the corresponding vortex are not sufficien to sustain a cyclic excitation at such low frequencies. With n source of cyclic excitation, there is no amplification of th acoustical energy reflecting back from the closed end of th branch.
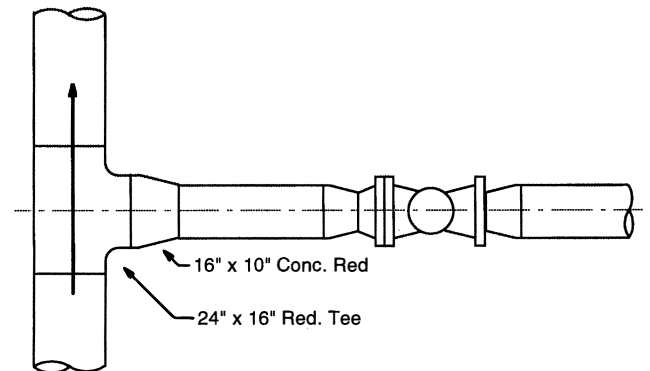

Figure 16. Typical Piping Modification.

Mechanical Analysis

System safety relies upon adequate protection from pipin; fatigue failures. Where pulsation cannot be totally eliminated

other measures are taken to design a system free from damaging vibrations. The piping should not be mechanically resonant to the system excitation forces. This requires a knowledge of the piping mechanical natural frequencies. Calculations are made to assure that there is sufficient separation from residual pulsation frequencies. The techniques used to do this may be either simple or complex.

Simplified tools, such as spreadsheets or nomograms, can be used to calculate the lowest natural frequency of steel beams, and corrections can be made for the presence of concentrated weights, such as valves, in the line. Then piping restraints can be added to the system to raise the mechanical frequency above the frequency of residual pulsation in the system. Unnecessary bends should be eliminated from the system, since they provide strong coupling points between pulsation forces and the mechanical system. Piping restraints can also be placed at or near each bend so that the design of the system becomes one of deciding the maximum allowable spacing for straight runs of pipe.

Removing elbows and adding restraints must be consistent with system thermal design requirements. In cases where thermal flexibility or other requirements preclude the addition of pipe restraints at optimum locations, the resulting vibration and stresses can be predicted using structural computer programs, based upon three dimensional piping systems. The accuracy of this type of analysis is highly dependent upon modelling the stiffness characteristics of the piping restraints and their attached structures. Similar stiffness characteristics should be maintained in the piping systems that are constructed for the predictions to be valid. Because of the wide variation in fabrication and installation tolerances, this method is not always successful. Experience has shown that it is better to remove damaging pulsation energy from the system, than it is to attempt to control resultant vibration by restraining the piping.

Compressor System Design

In a centrifugal compressor installation, the number of relevant side branch configurations that need analysis at the design stage varies widely. It depends upon the number of compressors and the complexity of the station piping system. For a single compressor addition to a typical natural gas transmission station, 20 or more different configurations often will be checked. Some of these configurations will undergo redesign if they reveal a potential problem. With multiple suction or discharge headers, valve positions (open or closed) and corresponding flowrates for several operating scenarios usually must be evaluated. An analysis of this kind can be completed in approximately one week, including development and approval of piping modifications.

If reciprocating compressors operate in the same piping circuit as the centrifugal compressor, the effects of pulsation generated by these units generally should be assessed. In many cases, the additional acoustical simulation and analysis that are required for an installation are conducted simultaneously with the flow-induced pulsation analysis. Both centrifugal piping vibration and performance problems have been caused by reciprocating compressors. If too much pulsation energy enters the centrifugal system, modifications can be made to the piping to shift acoustic resonant frequencies away from excitation frequencies to protect it from the possible damaging pulsations of reciprocating compressors.

CONCLUSIONS

The solution to flow-induced turbomachinery and piping noise and vibration problems can be accomplished by appropriate field tests, calculations, and predictive simulations in the operating or design stages. However, it requires an understanding of the underlying cause and effect relationships for diagnosis and solution.

Solutions of problems in the operational stages must be based upon confirmation of appropriate mechanical or aerodynamic modifications. Mechanical modifications refer to those changes that result in an alteration of the acoustic response characteristics so that coincidence with vortex shedding does not occur.

Aerodynamic modifications are more subtle. These entail elimination of the source of vortex shedding. Proper judgment for selection of a modification is, therefore, critical to ensure troublefree operations.

NOMENCLATURE

a	=	Velocity of sound in gas, ft/sec
c_x	=	Helical wave velocity of propagation, ft/sec
d	=	Basic hydraulic (geometric) diameter of obstruction or constriction (pipe or passage diameter), ft
f	=	Excitation frequency, Hz
f_{mn}	=	Cross-mode frequency, Hz
f_s	=	Preferred vortex generation frequency, Hz
$f_{1/4\lambda}$	=	Quarterwave acoustic resonant frequency, Hz
$f_{1/2\lambda}$	=	Halfwave acoustic resonant frequency, Hz
L	=	Piping length, ft
m	=	Number of nodal diameters
α_{mn}	=	Bessel function determined from Table 1
n	=	Number of nodal circles
N	=	Integer number = 1, 2, 3, . . .
N_s	=	Strouhal number (dimensionless)
v	=	Flow velocity, ft/sec

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