PROBABILISTIC LIFE ASSESSMENT OF AN IMPELLER WITH DISCONTINUITIES

by

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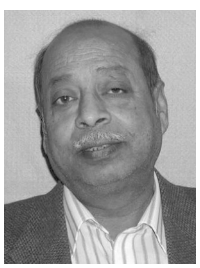

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ABSTRACT

This paper discusses and presents a probabilistic method to evaluate the useful life of an impeller with material imperfections. These discontinuities may be on the surface or subsurface. The method combines linear elastic fracture mechanics (LEFM) techniques with Monte Carlo simulation. Siddall (1983) provides a description of Monte Carlo technique in his book on probabilistic engineering design. An example is included in this paper to demonstrate the use of fracture mechanics and Monte Carlo techniques in estimating the reliability of an impeller. It will show the information required, and will present typical results that will

include stress contour plots, assumptions made relating the defects to a range of initial defect sizes, and plots of the probability of reaching a final critical defect size. A similar concept has been employed to assess reliability for various turbomachinery components (Singh, 1991; Jatla, 1990; Thacker, et al., 1990; Newell, et al., 1990; and Tipton, 1991).

INTRODUCTION

The evaluation of the life for rotating equipment components with surface or subsurface indications has been performed for years using fracture mechanics techniques. Typically, an indication of a defect has been discovered during a nondestructive test of a rotating component. Some conservative assumptions are made regarding the defect size and orientation, data are collected on the relevant material properties, the stress field in the area of the indication is estimated, and a calculation of the crack growth is undertaken using fracture mechanics techniques. The result is the number of stress cycles required for the defect to reach a critical size. This number of cycles is considered to be the useful life of the component.

Although this is a reasonable approach, it depends on a lot of assumptions and therefore, out of necessity, is very conservative. By applying ranges and distributions to the various parameters, one can perform a probabilistic evaluation of the defect growth and arrive at the probability of achieving a specific flaw size before the end of useful life is reached. Since, probabilistically, all of the conservative factors are not applied at the same time, this technique results in a more realistic evaluation of the useful life and, because the results include a plot of probability versus life, the risk associated with reaching any particular life can be found.

BRIEF THEORETICAL BACKGROUND

An indication of a defect is an abnormality found during a nondestructive test (NDT) of a structure or part of a structure. In this case an impeller or part of an impeller (disk, blade, or cover) is the subset of concern. The type of indication depends on the type of NDT being used. The indication would be a reflection of an input signal if ultrasonic inspection (UT) is used, a bleed-out if liquid penetrant inspection (LPI) is used, and a powder pattern if a magnetic particle inspection (MPI) is used. Likewise, the location, and to some extent the geometry, of the indication depends on the NDT being employed. Indications well below the surface of the part are revealed using UT, indications on or just below the surface can be detected using MPI, while only indications on the surface will be detected by LPI.

Once an indication is found, specific techniques are used to interpret the location, size, orientation, and severity (e.g., rounded or sharp) of the flaw the indication represents. Generally, UT indications are the most difficult to interpret since one is working with the reflection of an ultrasonic signal. On the other hand, a great deal of information about the flaw is revealed in an LPI indication because the bleed-out that forms the indication is from the indication itself.

Once an indication has been detected and interpreted, one needs to know something about the stresses around the indication before any kind of evaluation of the effect of the indication on impeller reliability can be made. For the case being described in this paper, stresses in the impeller were calculated using a finite element analysis (FEA).

Lastly, information on the relevant material properties of the impeller is required to determine the effect of the indication on reliability. The required properties are yield strength (0.2 percent), modulus of elasticity, ultimate tensile strength (UTS), and fracture toughness (K_{IC}). Along with these properties, information describing the crack growth rate under an alternating load will be required. The pertinent equation that describes the growth behavior of the material is discussed in the next section.

FRACTURE MECHANICS

The objective of this section is to provide and discuss briefly an integrated view of the theory of brittle fracture. This is important so that one can appreciate the theory to be used later in this paper. For a detailed discussion of what follows here, one should refer to Lawn and Wilshaw (1975). The treatment will emphasize the basic principles with the fracture model to be used in making a decision in the case of an impeller with reportable imperfections. In adopting this approach a balance is attempted with the following aims:

- Basic assumptions and also hypotheses are briefly analyzed to understand the basic setup of the theoretical framework.
- To discuss the diverse aspects of fracture theory to establish a physical basis for design of mechanical structure of practical importance, e.g., an impeller with indication.
- To justify the use of the theory of fracture mechanics in its own right.

Nearly all materials have a tendency to fracture when stressed beyond a critical level. It seems perfectly reasonable to assume that fracture strength is an inherent material property. The idea of a stress-limit is attractive in engineering design; one simply needs to ensure that the maximum stress level in the component does not exceed the stress limit. With an increase in engineering failures, the concept of a critical applied stress approach was questioned. It is easy to see that the inadequacy of the critical stress criterion lay in its empirical nature. For the notion that a solid should break at a characteristic stress level, however intuitively appealing, is not based on sound physical principles. For instance, how is the applied stress transmitted to the inner regions where fracture actually takes place? What is the mechanism of failure? Griffith (1920) in his classical paper considered an isolated crack and formulated a criterion in terms of fundamental energy theorems of classical mechanics and thermodynamics. A worth mentioning work of Inglis (1913) presented the analysis of stress of an elliptical hole in a uniform stress field. He showed that the local stresses about a sharp notch or corner could rise to a level several times that of the applied stress. The Inglis equations gave the first real clue to the mechanism of fracture; the limiting case of an infinitesimally narrow ellipse could be considered to represent a crack. The Inglis equation yields the value of an "elastic stress concentration factor." The equation showed that this factor, though considerably larger than unity, depends on the shape of the hole rather than the size. If this analysis is indeed to be applicable to a crack system, then why in practice do large cracks tend to propagate more easily than small ones. Such behavior violates the size-independence property of the stress concentration factor. What then is the physical significance of the radius of curvature at the crack tip of a real crack? Griffith (1920) set up a model based in terms of a reversible thermodynamics process. He simply argued that in the configuration that minimized the total free energy of the system, the crack would then be in a state of equilibrium, and thus on the verge of extension. An expression of total energy of the system was developed that contained individual energy terms. The change in the energy was also examined as a result of crack formation. The outer boundary of a cracked body will undergo some displacement such that the applied load will do some work W_L. Second, the potential strain energy U_E stored in the body will be sensitive to change in system geometry. Third, the surfaces generated by a growing crack require the expenditure of free surface energy U_S.

The total energy of the static system is thus given by:

$$U = \left(-W_L + U_E\right) + U_S \tag{1}$$

The first composite term favors crack extension, while the second opposes it. This is the Griffith energy-balance concept. The formal equilibrium requirement is:

$$dU / dc = 0, \tag{2}$$

where c represents the size of the crack.

The Griffith (1920) concept provided a starting point for fracture mechanics problems when the applied forces are conservative. He took advantage of Inglis' (1913) analysis by considering a very narrow elliptical crack (minor axis dimension to be nearly zero) and applied it to a material obeying Hooke's law at all stress levels, i.e., linear elasticity. In the case of a constant applied stress during crack extension according to the linear theory of elasticity, Lawn and Wilshaw (1975), W_L equals to $2U_E$.

$$U = -U_E + U_S \tag{3}$$

Figure 1 is the plot of the energy relation shown in Equation (3). This shows the energy balance and also shows the crack size at equilibrium.

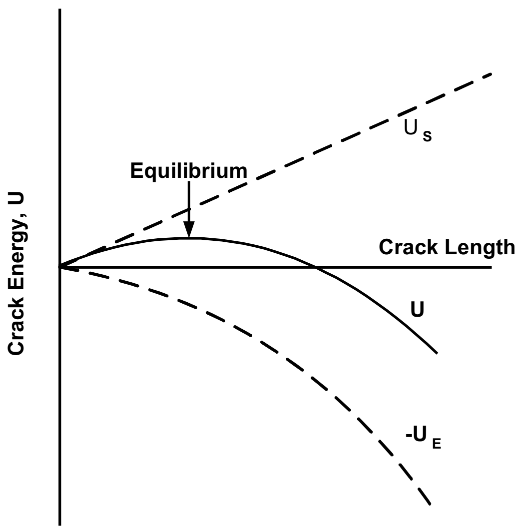

Figure 1. Energy Balance of Griffith.

Applied loading on the system provides the driving force for the propagation of the crack in the solid. The existence of a crack, considered in conjunction with the fundamental Griffith energy-balance concept, provides the basis of the powerful tool in the brittle fracture theory, the vast field of fracture mechanics.

Irwin (1958) provided the impetus of the formulation of fracture mechanics that is expressed in terms of continuum approximation. It retains the macroscopic, or thermodynamic, viewpoint of crack propagation. Let us consider an isotropic linear elastic continuum, with an arbitrary applied load at the boundary with an internal crack. The problem is reduced to determine the stress and strain fields. The basic question is its role in the crack extension. The work of Inglis provides insight in the intensity of the field due to boundary condition or applied load at the boundary. The distribution of the field is governed by the stress-free crack surfaces in the vicinity of the crack tip. Irwin's work showed that the mechanical energy released during an incremental extension of a crack is independent of loading configuration and a generalized crack-extension force,

$$G = -d\left(-W_L + U_E\right) / dc \tag{4}$$

per unit width of crack front.

Since the above term does not depend on the loading type, the equation reduces to:

$$G = -\left(dU_E / dc\right) \tag{5}$$

This is the strain-energy-release rate per unit width of crack front. A method of determining G is a crack field analysis through the theory of elasticity. The surfaces of a crack are the dominating influence of the stress distribution near and around the crack tip. The remote boundaries and loading forces affect the intensity of the local stress field. The stress fields near the crack tips are considered to be of three basic types. Each one of these is associated with a local mode of deformation. These are shown in Figure 2. Mode I is called opening mode, Mode II is known as sliding mode, and Mode III is termed as tearing mode. Of all three modes, Mode I is the most pertinent for brittle crack propagation.

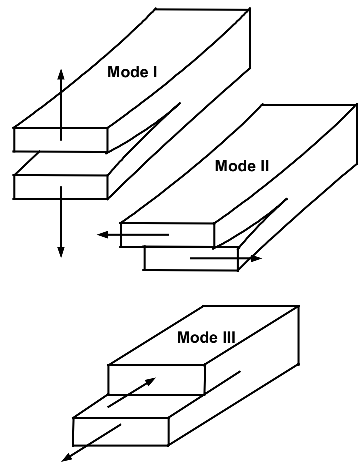

Figure 2. Modes of Fracture.

The investigation to find the stress distribution near the crack tip is the important topic of the theory of elasticity. Westergaard (1939) and Muskhelishvili (1933) developed an analytical technique for the case of plane-crack geometry. The analysis is simplified based on the sharp slit approximation of a crack (Figure 3). The tip of the crack is assumed to be perfectly sharp in the unstressed condition and the fracture surfaces are free of traction, i.e., there is no force normal to the crack surfaces at all stages of loading. The differences between the continuum model and the limiting elliptical hole model are more mathematical than physical. The stress field at the crack tip takes a simple solution of stress field near the tip (the distances from the tip are small compared to the characteristic dimensions of the crack system).

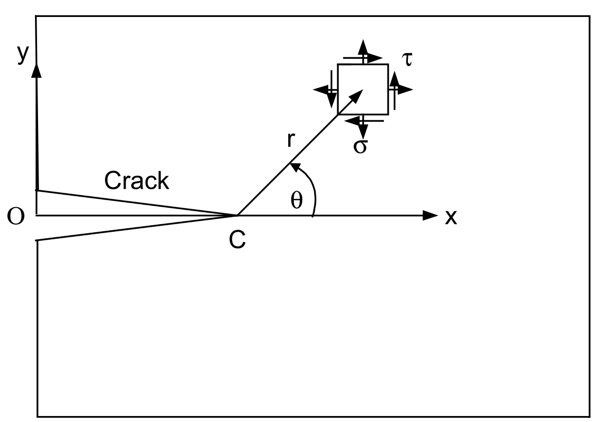

Figure 3. Stress Analysis at the Crack Tip.

The solution for the opening mode is given as:

$$\sigma = K_I \left(2\pi r\right)^{-1/2} f(\theta) \tag{6}$$

$$\tau = K_{II}(2\pi r)^{-1/2} f(\theta) \quad (7)$$

$$\tau = K_{III}(2\pi r)^{-1/2} f(\theta) \quad (8)$$

The stress intensity factors (K_I, K_{II}, K_{III}) define the characteristics of the stress field and its strength at the crack tip. These depend only on the loading and crack geometry. Other terms in the expression depend only on the spatial coordinates about the tip and determine the distribution of the field. Physically the stress intensity factor is the load transmittal near the crack tip as a crack is introduced in the elastic body. Thus, it can also be regarded as the mathematical expression that describes the redistribution of load paths for transmitting load past a crack.

Now let us consider Equation (5). If there is no displacement of the load point for the cracked body, then there is no work done by the imposed forces. The energy that is available for extension is the strain energy released by the extension. Tada, et al. (1973), discussed the energy concept of the strain energy released rate and they provided the relation between G and K. These can be stated as follows:

Total energy rate, G, is given as follows in terms of energy release rate for each mode.

$$G = G_I + G_{II} + G_{III} \quad (9)$$

$$G_I = (K_I)^2 / E \quad (10)$$

$$G_{II} = (K_{II})^2 / E \quad (11)$$

$$G_{III} = (1+\nu)(K_{III})^2 / E \quad (12)$$

Fracture mechanics thus is a technique that combines effects of the geometry, stresses, and material properties to estimate the structural reliability in the presence of a crack-like defect. The primary difference between a fatigue approach to design and a fracture mechanics approach is that fracture mechanics assumes that a sharp crack already exists in the part. Using the flaw size, shape and orientation, and the stresses around the flaw, one can calculate a crack tip stress intensity factor (K_I). The value of stress intensity factor, K_I, then can be compared to the fracture mechanics properties of the material obtained from the test. In other words, K_I is analogous to the stress in a part due to loading, and K_{IC} is analogous to the stress at fracture in a simple tension test of a material. The units for the crack tip stress intensity factor (K_I) and material fracture toughness K_{IC} are in terms of stress times the square root of crack length, e.g., ksi*(in)$^{1/2}$.

Based on the practical assumption that any plastic deformation at the crack tip is negligible in comparison to the crack's dimension, the theory and the concept of the linear elastic fracture mechanics (LEFM) can be applied. That is, where the amount of crack tip plasticity is very small compared to the size of the crack and the cracked component. LEFM is relatively easy to apply and it is generally applicable to the types of flaws, stresses, and geometries encountered in turbomachinery.

Figure 4 is a typical plot of the test data showing crack growth per stress cycle (da/dN) versus the stress intensity range (ΔK_I). It can be seen that below a certain value of ΔK_I, it seems that the crack will not grow. This is called the threshold level of ΔK_I denoted as ΔKth. On the right side end of the plot the crack growth rate is nearly infinite. This means that with just one load application the crack becomes unstable or the crack goes to rupture. This value of stress intensity factor is greater than the value of the fracture toughness, K_{IC} , as defined by the American Society for Testing and Materials (ASTM) test procedure E399. It is observed that there are two asymptotes of this curve. This plot shows that ΔK_{th} is essentially the vertical asymptote on the left end of the crack growth line. ASTM procedure defines K_{IC} such that it is left to the other asymptote that is in the right end of the curve. This is a conservative value in such a way that the velocity of the crack growth does not reach infinite magnitude. Also, note that the crack growth rate increases with increasing ΔK_I. And, since K_I is a function of crack size (refer to the equations below) the crack growth rate increases as the crack grows if the stress intensity range levels remain the same.

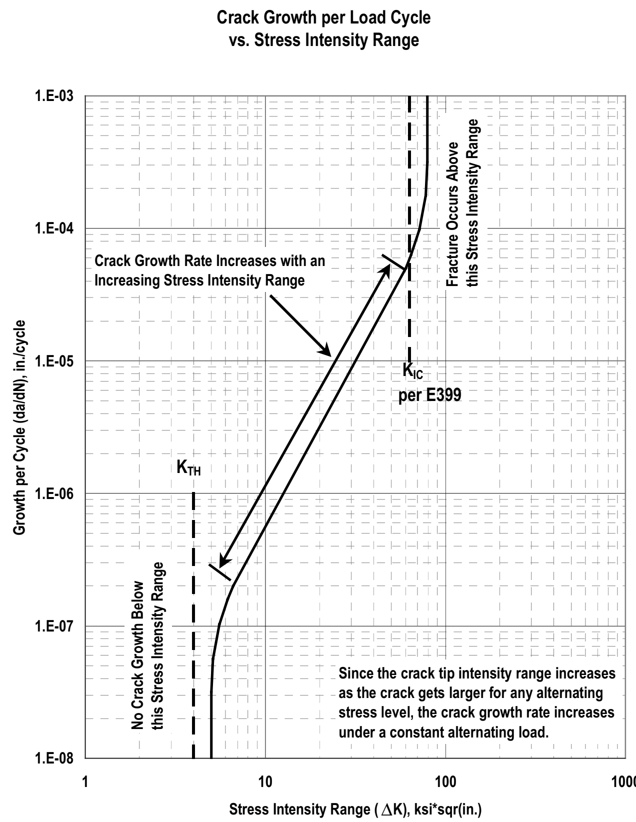

Figure 4. A Typical Crack Growth Data.

An expression for the middle portion of the crack growth rate (called slow crack growth regime) versus stress intensity range is expressed as follows:

$$da / dN = m(\Delta K)^b \quad (13)$$

or

$$\log(da / dN) = \log(m) + b \log(\Delta K) \quad (14)$$

The plot of the above expression on a log-log plot is a straight line with b as the slope of the straight line. These properties are obtained by a fracture mechanics type test on the material of interest. Typical data of many materials can be found in Boyer (1986). Similar equations generated from the material test data have been used to assess reliability of many mechanical components for many years.

DETERMINISTIC EVALUATION

A deterministic evaluation of the effect of a crack-like indication on the reliability of an impeller can be accomplished by following the steps below:

- *Step 1*—Estimation of the location, size, and orientation of the defect is achieved by any of the methods described in the earlier section.

• *Step 2*—For a crack-like defect the far field stresses are estimated by many methods. For the example included in this paper finite element technique is used to estimate stresses. For Mode I type brittle fracture, the maximum principal stresses have been used for both the steady and alternating loads.

• *Step 3*—Material properties are obtained by appropriate fracture mechanics type tests.

• *Step 4*—An analytical expression of the stress intensity factor at the crack tip is mostly available in a fracture mechanics book, e.g., Hertzberg (1976) and also in a handbook, Tada, et al. (1973). To simply demonstrate the use of the probabilistic method, a conservative expression for the crack tip stress intensity factor of a surface crack with a circular cross section is (remember, a is ½ the crack width):

$$K_I = \left(1.21\pi a\right)^{1/2}\left(\sigma\right) \tag{15}$$

In case of alternating stress, the stress intensity range expression can be written as follows:

$$\Delta K_I = \left(1.21\pi a\right)^{1/2}\left(\Delta\sigma\right) \tag{16}$$

• *Step 5*—Calculate the critical defect size (a_c). This is the flaw size that, when reached, the crack will "run" causing an immediate fracture in the impeller. This is simply a rearrangement of the expression in Equation (15):

$$a_c = \left(K_{IC} / \left(\sigma\right)\right)^2 / \left(1.21\pi\right) \tag{17}$$

• *Step 6*—Estimate how many cycles it will take for the initial crack (a_i) to reach the critical defect size (a_c). This will be called the "useful cycles." Generally, an iterative calculation is employed for calculating cycles. Equation (13) and Equation (16) can be used to obtain a closed form solution for the number of cycles. It should be noted that as the defect grows, the ΔK at the tip increases.

$$N = \left(a_f^p - a_i^p\right) / \left(\left(1.21\pi\right)^{0.5b} pm\left(\Delta\sigma\right)^b\right) \tag{18}$$

And the final crack length after N cycles is given by:

$$a_f = \left(a_i^p + \left(1.21\pi\right)^{0.5b} pm\left(\Delta\sigma\right)^b N\right)^{1/p} \tag{19}$$

where $p = (2.0 - b)/b$.

• *Step 7*—Finally, the value of cycles to fracture is compared with the expected number of cycles the impeller will experience during its life. If the estimated number of cycles to fracture is greater than the expected number of operational cycles the compressor will experience, the crack is acceptable and no further action is required. If the estimated number of cycles to fracture is less than the expected number of cycles the compressor will experience during its normal operating life, the crack is unacceptable and some action must be taken.

In as much as the deterministic evaluation is relatively easy to apply and interpret, and is conservative, it is a reasonable approach to take when trying to estimate the effect of an indication on impeller reliability. However, because it boils down to the comparison of a calculated number against a preset criterion it really reveals very little about reliability. This is most evident when the comparison being made is "borderline."

PROBABLISTIC EVALUATION

The reliability of an impeller in the presence of a crack-like defect can be estimated using fracture mechanics as discussed in the previous section. However, there are uncertainties or randomness in the parameters that can influence the reliability. For example, there is randomness in the material properties and there is always randomness associated with the assessment of the size of the defects. Singh (1991) used fracture mechanics in combination with probabilistic concepts to estimate reliability of a weld repaired steam turbine rotor. His view was that "one cannot argue that input parameters have variability, however small it might be and that response or performance of the component thus becomes random." Life extension, remaining life assessment, and fitness-for-service concepts have evolved to keep plants running beyond design life. This has been achieved either by reassessing the design and/or repairing as needed. Methods using probabilistic concepts have been used to estimate reliability of many structures, e.g., turbopump blades (Thacker, et al., 1990), dynamic analysis pressurized ducts (Deb Chaudhury, et al., 1990), turbine blades (Singh, 1991), and turbine bladed disk design (Singh and Ewins, 1988; Singh, 1992). The theory of reliability described in detail can be found in the book by Melchers (1987).

In essence, the basic premise of a probabilistic evaluation is that with inherent variations in flaws, in stress levels, and in properties of the material, it is extremely rare that any particular set of values will occur at once. If one can determine the probability of occurrence of any parameters that influences safe life of the equipment, then one can use this information to calculate the cycles for the initial crack to reach a predetermined size. In the end one will know not only the information about cycles to failure, but also will know what the probability of reaching a specific number of cycles is.

Obviously, the more information one has on the statistical characteristics of the parameters used in estimating the cycles to fracture, the better will be the estimate of the probability of reaching a specific number of cycles before fracture. Unfortunately the analysis gets more complicated with an increasing number of parameters to be considered.

Mathematically, the response of a structure will depend on the interaction of imposed stress (S) and components' resistance (R). The deterministic method defines a margin by the ratio R/S. There are uncertainties in the values of both S and R and these are represented in statistical terms by a probability density function (PDF). In the probabilistic terms the reliability is estimated as follows:

$$\begin{aligned} &P_s = 1 - P_f = 1 - P\left(R < or = S\right) \\ &or \\ &P_s = P\left(R > or = S\right) \end{aligned} \tag{20}$$

where P_f is the probability of failure. Equation (20) provides the expression of the probability of success when the resistance, R, of the material is equal to or greater than the applied stress, S.

In general terms the above equation takes the following form:

$$P_s = P\left(H\left(R,S\right) > or = 0\right) \tag{21}$$

where H(R,S) is the limit function and defines the relationship between R and S. In other words, the requirement of the reliability is expressed through the function H. To understand the limit function, H, let us examine the following relationship as if R is yield strength of the material and S is the imposed stress.

$$H\left(R,S\right) = 0.67 * R - S \tag{22}$$

The above relation states that the limit function, H, is equal to 67 percent of yield strength less the imposed stress. For reliable operation H must be positive, a deterministic statement.

The probabilistic aspect of the problem can be stated as follows:

$$P_{fs} = P\left(\left(0.67 * R - S\right) > or = 0\right) \tag{23}$$

Probability density functions for R and S are shown in Figure 5. A vertical line is also drawn at the value of 0.67R. Mean value of S (applied stress) is less than 0.67R (e.g., 67 percent of the yield strength of the material) and it satisfies the condition for safe design. However, when consideration is given to the possible uncertainties in R and S, it becomes clear that there is still a possibility of S being larger than R. Hence, there exists a chance of failure due to the statistical nature of R and S. Deterministic approach indicates success.

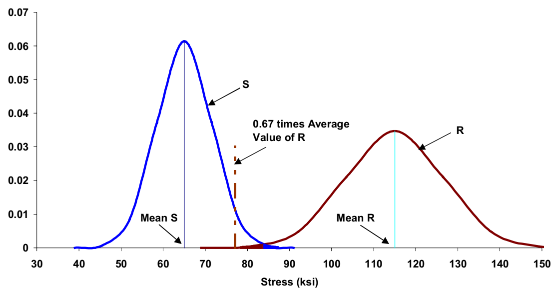

Figure 5. Description of $P_f = P((0.67*R - S) < or = 0)$.

The calculation gets very involved. To mitigate this complexity, techniques have been developed that perform many calculations (often thousands) rapidly or at least automatically using information from PDFs to select the parameters. In Monte Carlo simulation, for example, as the analysis proceeds a picture of the probability of reaching a particular cycle to fracture value immerges.

Using the results of the Monte Carlo simulation, the level of risk associated with any given cycle to failure value can be established. Based on this information, better decisions can be made regarding the risks of operating with a suspected defect and also the risks associated with reaching a particular number of load cycles can be ascertained as well.

EXAMPLE

Application of the theories of the deterministic and the probabilistic aspects of reliability is now discussed. The example is presented by discussing the steps to evaluate the reliability of an impeller that contains crack-like defects. A typical process that can be followed is described in the flow diagram shown in Figure 6.

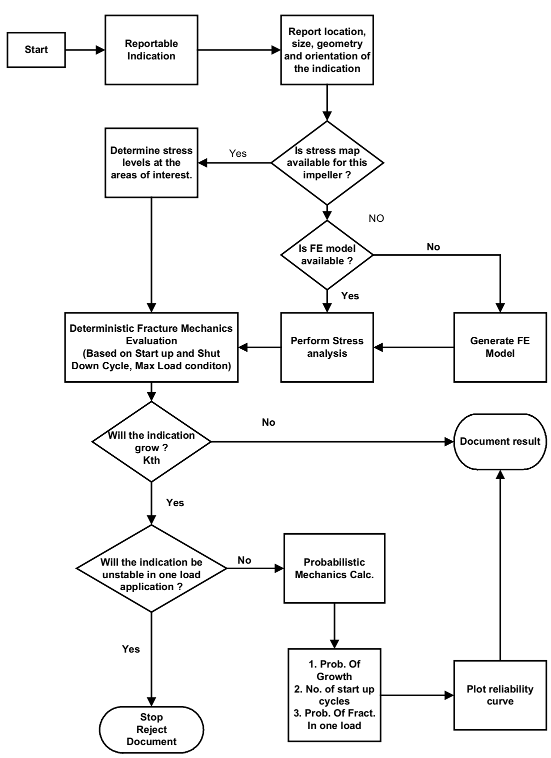

Figure 6. Flow Diagram of the Process.

Through the established NDT procedure, the location, size, geometry, and the orientation of the defects are determined. The stress distribution in the impeller (disk, cover, or blade) is either available by previous design work or it should be determined through a finite element analysis as described in the later section. A deterministic fracture mechanics calculation is performed to determine if with one load application the defect will become unstable. This means that the defect will run to failure. If such is the case, the impeller either should be retired or reworked if the operational use cannot be adjusted. Next, an estimate should be made to see if the defect will grow at all. If the result of the estimation is that the defect will grow then the number of load cycles (after which the defect will become unstable) should be estimated. In both of these situations a probabilistic type of analysis should be performed to establish reliability. Similar analysis is performed in case of multiple defects. Various methods can be employed to assess the stress intensity factors that depend on the size and distribution of multiple defects. One can take a more conservative approach to consider many close defects as one. Expressions of stress intensity factor for many different idealized types of multiple defects are also available (Tada, et al., 1973).

STRESS ANALYSIS

For the stress analysis, an impeller wheel was analyzed using 3-D finite element modeling. The model is shown in Figure 7. One blade sector of the impeller is analyzed using 3-D solid brick (eight-node) elements utilizing a proprietary computer software program. The shaft is included in the model so that the impeller to shaft contact pressures, which are used to calculate the torque transmission capability of the fits, can be determined as well. Opposing nodes on the "cut" surfaces of the model (impeller and shaft) are coupled in the global cylindrical coordinate system to maintain cyclic symmetry.

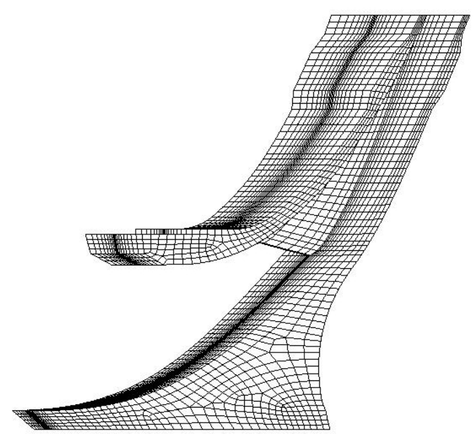

Figure 7. FEA Model of an Impeller.

A stress map of the maximum first principal stress in the disk of the impeller at maximum continuous operating speed (MCOS) is shown in Figure 8. The maximum first principal stress of 37 ksi occurs at the radial location where the blades meet the disk.

Materials' Properties

For the purpose of this analysis the following typical values of material properties have been used.

- Minimum 0.2 percent yield strength (YS) = 115 ksi
- Minimum ultimate tensile strength (UTS) = 140 ksi
- Minimum elongation = 14 percent
- Maximum hardness = Rc 38
- Fracture toughness (K_{IC}) = 80 ksi $(in)^{1/2}$

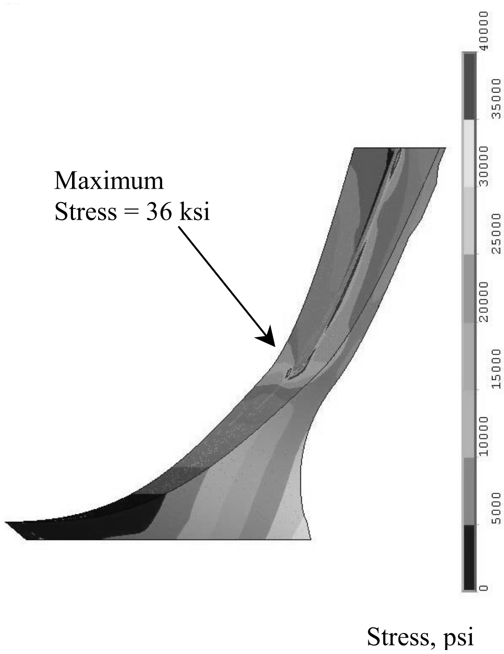

Figure 8. Maximum Principal Stress in Impeller Disk.

- Threshold (ΔK_{th}) = 8 ksi $(in)^{1/2}$
- For the crack growth in Equation (13):
  m = 4.3 × 10^{-12} and
  b = 4.36

RESULTS

Results for deterministic as well as probabilistic calculations are presented next. It is worth mentioning that the mention of UT inspection implies a defect of subsurface in nature. The method is equally applicable to estimate on surface and subsurface indications as long as an appropriate expression for stress intensity factor is used together with the stresses near the indications.

Deterministic LEFM Calculations

An indication is detected during a UT inspection of an impeller forging. The size of the indication (a_i) is determined conservatively to be 0.25 inch. A stress map of the impeller shows that the maximum stress (S) at the location of the indication is 37 ksi.

Using the relationships given in Equation (16), the stress intensity factor at the crack tip for the size (0.25 inch) of the indication is calculated to be 36 ksi $(in)^{1/2}$. This value is compared to the threshold value for the material. It is greater than the ΔK_{th} value of 8 ksi $(in)^{1/2}$; therefore one concludes that the defect will grow. However, it is less than the fracture toughness (K_{IC}) value of 80 ksi $(in)^{1/2}$ so the crack will not become unstable under one application of load.

The next step is to calculate the critical defect size for the material with a fracture toughness (K_{IC}) value of 80 ksi $(in)^{1/2}$ under an applied stress of 37 ksi. Using Equation (17), the critical flaw size (a_c) is 1.23 inch.

The next step is to calculate the number of load cycles that can be applied before the defect becomes unstable. Equation (18) is used to estimate the number of load cycles. The number of cycles required to grow from the a_i of 0.25 inch to the a_c of 1.23 inch is calculated to be 4200 cycles, assuming each cycle goes from zero stress to full stress (37 ksi) and back to zero.

Probabilistic Calculations

Three basic conclusions are drawn based on the results of the deterministic analysis. These are:

1. The existing defect (0.25 inch in size) will not become unstable under one application of load (37 ksi),
2. The existing defect will grow during startup cycle and the critical defect size is 1.23 inch, and
3. It will take about 4200 startup cycles for the existing defect (0.25 inch) to grow to the critical size (1.23 inch).

Now let us examine the last two conclusions probabilistically due to the fact that there are uncertainties in the values of defect size, material properties, and the applied stress. Table 1 lists the uncertainties in the value of defect size, properties, and stress.

Table 1. Details of the Statistical Characteristics.

	Distribution Type	Mean Value	Standard Deviation	Lowest Value	Largest Value
Stress	Normal, Bounded	37 KSI	2.0 KSI	27KSI	47KSI
Starting Defect Size	Normal, Bounded	0.25 in	0.0125in	0.1875	0.3125
Value of b (equation 13)	Normal, Bounded	4.36	0.218	3.27	5.45
Value of m (equation 13)	Lognormal, Bounded	4.30E-12	2.15E-13	3.23E-12	5.38E-12

The values of different parameters as listed in Table 1 have been used in the example to calculate the probability of the critical defect size to be greater than the stated value. Depending on the material, the method of defect measurement and the method for estimating stresses will have different values for the statistical parameter. These values have been used to demonstrate the method. The Monte Carlo simulation technique has been used for the probability calculation. The results of the analysis are plotted in Figure 9.

Figure 9. Probability Versus Critical Defect Size.

It can be seen from Figure 9 that the critical defect size can vary from 0.80 inch to about 2.00 inches.

The next set of calculations is performed for the probability of the number of startup cycles that can safely be applied before the growing defect reaches the critical value, a_c. Again the Monte Carlo simulation technique has been employed for the analysis.

Figure 10 shows the results of this analysis. Again, examination of the result shows that the probability of safely achieving startup cycles decreases with the increasing number of cycles.

Figure 11 shows the same data but up to 5000 cycles of load application. It amplifies the probability of success for 4200 cyles of load application as estimated in the deterministic analysis. This will help in understanding the statements made in the summary section.

SUMMARY

Comparison of the results of the deterministic and the probabilistic analyses is very valuable and points toward the need of a probabilistic type of analysis. For example, a critical defect size is estimated to be 1.23 inches through the deterministic analysis.

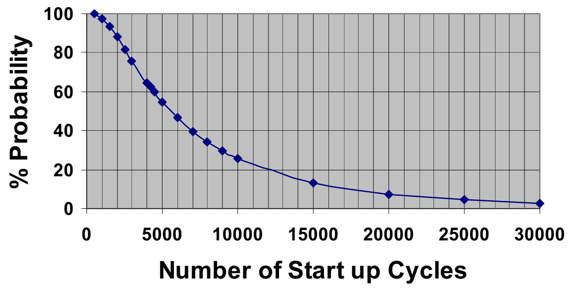

Figure 10. Probability Versus Startup Cycle.

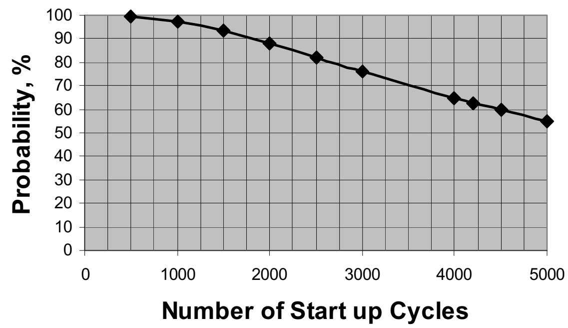

Figure 11. Probability Versus Startup Cycle (Up to 5000 Cycles).

However, after considering the uncertainties in the properties and the applied stress, the critical defect size is estimated to be from 0.80 inch to 2.00 inches with various probability of occurrence. Also the number of startup cycles that can be applied before the growing defect reaches the critical size is estimated to be 4200 cycles. The result of the probabilistic calculation points to this varying from about 400 cycles to more than 30,000 cycles with various levels of probabilities.

A deterministic life assessment of an impeller with defects can yield some guidance on the acceptability of the impeller for service and, therefore, will remain a useful tool in the evaluation of these impellers. However, because it does not address the possible variations in geometry, material properties, and defect characteristics, a deterministic life assessment must be very conservative. As more load cycles are applied the probability of success decreases or the reliability decreases when a discontinuity is present as it is evident from a probabilistic type analysis. For this reason, the deterministic approach falls short in its ability to make any real predictions as to the actual expected life of the impeller containing discontinuities and reveals nothing about the probability of reaching any particular life.

A probabilistic life assessment, on the other hand, because it brings in the possible variations in geometry, material properties, and defects, can provide a more realistic evaluation of the expected life of the impeller and, at the same time, will identify the risk associated with achieving any particular life. This makes probability life assessment particularly valuable in evaluating the risks involved in the continued operation of impellers with indication of cracks discovered during inspections conducted during shutdowns.

Of course, a probabilistic life assessment is only as good as the probability density functions used. Care must be taken to ensure that the data used in the analysis reflect the actual probabilities of the various factors.

The calculated number of load cycles and the associated probability of achieving them provide a logical basis for setting inspection intervals for successful operation.

NOMENCLATURE

a,c	=	Crack dimensions for LEFM
a_c	=	Critical crack dimension
a_i	=	Starting crack dimension
a_f	=	Final crack dimension
a_{th}	=	Crack size that will not grow
b	=	Exponent in the growth relation
G	=	Strain-energy-release rate
H(R,S)	=	Joint probability density function
K_{IC}	=	Fracture toughness
K_I	=	Stress intensity factor
m	=	Constant in the growth equation
N	=	Total number of cycles
P_f	=	Probability of failure
R	=	Resistance of material
S	=	Stress or load
U_E	=	Potential strain energy
U_S	=	Free surface energy
W_L	=	Work done
ΔK	=	Stress Intensity range
ΔK_{th}	=	Threshold value
$\Delta\sigma$	=	Stress range
σ	=	Far field stress

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