EQUATION OF STATE INFLUENCES ON COMPRESSOR PERFORMANCE DETERMINATION

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ABSTRACT

Accurate prediction of centrifugal compressor aero/thermodynamic performance is essential in establishment of new equipment acceptability and verification of compressor health during continuing operation. In many cases, this may be very dependent upon the ability to precisely calculate real gas properties through the use of an equation of state. Full load, ASME PTC-10 (1997) Type 1 hydrocarbon factory test data were used to compare equipment supplier test results against a number of different equation of state predictions across the relatively broad range of pressure and temperature conditions afforded by the test. In addition, a number of pressure-volume-temperature (PVT) databases for gas mixtures available in the open literature were used to evaluate equation of state accuracy in the prediction of compressibility factor. This included the equipment supplier's selected equation of state that acted as the baseline for predicted compressor performance comparisons. Thermodynamic properties such as enthalpy and entropy are shown to be related to the compressibility factor, so errors incurred in its calculation will also affect the accuracy of their prediction.

Results of this investigation demonstrated that some equations of state offer superior prediction of thermodynamic parameters and thus compressor performance when compared to others. Specific components of a gas mixture and operating pressure and temperature levels may affect a given equation of state's ability to accurately predict these properties. However, for a large number of industrially significant applications one or more of the equations of state investigated will yield acceptable results. In those cases where a more custom equation of state needs to be applied to a unique gas or mixture of gases, evaluation of actual PVT data for the particular application may be warranted.

INTRODUCTION

Centrifugal compressor aero/thermodynamic performance testing is an integral element of an overall equipment quality assurance program that also includes mechanical run testing of the machine. This performance testing is usually limited to Type 2, inert gas, reduced load testing as defined by ASME PTC-10 (1997). However, in very critical or severe applications the increased cost and schedule impacts of full load, full pressure Type 1 testing may be justified. A Type 1 test is normally completed after all Type 2 testing has been performed. It may include all compressor sections in a multisection train or may be limited to individual sections of a larger train of equipment. If the contract driver is also included in the testing program, the entire train may be tested at full load conditions to demonstrate acceptable driver operation. Although such full load tests are prescribed as performance tests, they are more commonly performed to provide mechanical and rotordynamic information under full load conditions. Aerodynamically induced forces that are capable of causing vibration issues may not be duplicated during the PTC-10 (1997) Type 2 performance or API 617 (2002) standard mechanical tests. Previous studies (Colby, 1987; Klein and Draughon, 1991) have concluded that Type 2 tests are very good predictors of compressor field performance under contract conditions when care is taken in establishing the Type 2 test conditions and in correlating these test results back to contract conditions.

The gas reinjection compressors associated with the Sanha Condensate Project for offshore Angola are an example of a severe application. They comprise four duplicate trains of gas turbine driven, two body, tandem compressors. The three intercooled compressor sections are responsible for raising the pressure of the 18.33 molecular weight natural gas from a suction pressure of approximately 270 psig (18.6 barg) to a discharge pressure above 6600 psig (455 barg). Alternate operating cases were also provided that reflected potential operation with a heavier molecular weight gas resulting from the shutdown or bypass of upstream "heavy end" separation and future field operation but the above conditions defined the certified operating point. A full load, full pressure string test was specified for one train of equipment to supplement the API 617 (2002) mechanical testing performed on each compressor body and the PTC-10 (1997) Type 2 testing that was completed on a single compressor body of each type. Figure 1 is a photograph of the string test setup at the equipment supplier's test facility.

The scope of the full load string test included a PTC-10 (1997) Type 1 test of the high stage compressor casing, a "modified" Type 1 test of the low pressure casing, and full load operation of the complete compression train to demonstrate acceptable operation of the gas turbine driver and connected speed increasing gear. The main objective of the Type 1 testing was to prove the mechanical integrity of the compressors and identify any fluid induced vibrations. Any refinements of the Type 2 testing established performance were considered of secondary importance. Results of preliminary load testing by the equipment supplier uncovered some mechanical and vibration issues that were subsequently addressed and will not be covered here. The witnessed full load

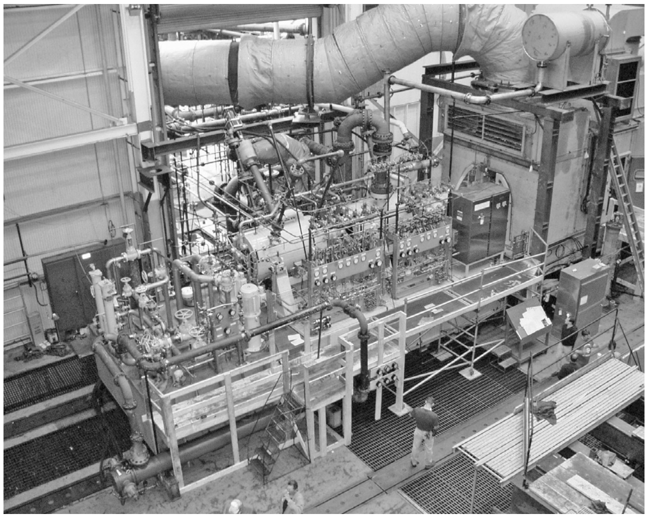

Figure 1. Factory String Test Setup. (Courtesy Dresser-Rand Company)

string testing did, however, also provide a substantial amount of performance data over the range of compressor operation that are seldom available prior to field operation. Figure 2 shows the contract operating points superimposed upon a pressure-temperature diagram for the gas mixture including the phase envelope. Compressor operation is confined to the superheated vapor and dense phase regions. Gas properties at relatively low pressures in the high temperature superheat region to the right of the phase envelope are expected to be very near to those of an ideal gas for most gas components. As the pressure increases, though, these properties tend to deviate from ideal and a real gas equation of state is needed to accurately predict various thermodynamic properties. Fluid behavior in the dense phase region above the phase envelope tends to demonstrate properties of both liquids and gases. For example, gas density is one of these properties with values approaching those of a liquid whereas in contrast the viscosity is much closer to that of a gas.

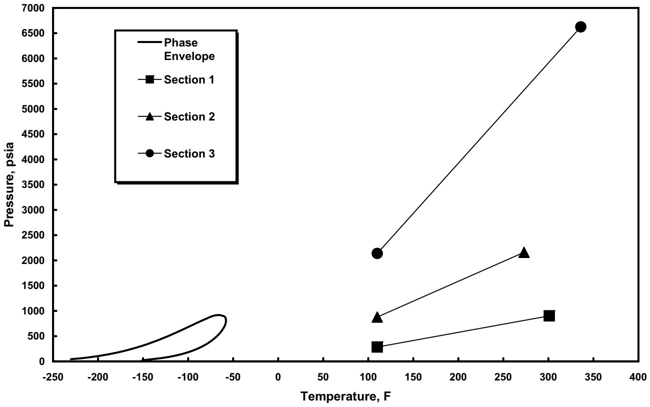

Figure 2. Contract Operating Points and Phase Envelope.

A number of equations of state have been developed over time in an attempt to fully describe the behavior of single and multi-component gases and liquids over a wide range of pressure and temperature conditions. While offering improved prediction of real gas properties relative to an ideal gas assumption, these equations of state provide differing levels of accuracy across the range of their application and with various gas mixtures. Many of the centrifugal compressor equipment suppliers utilize one or more of these equations of state to determine not only their predicted and "as-tested" behavior under contract conditions but also to calculate thermodynamic parameters during factory testing. Each supplier tends to have a preferred equation of state that they utilize to evaluate compressor performance. However, many are willing to use alternate relations if it is mutually agreed that the alternate equation of state and derived properties are more accurate for a specific application. The equation of state to be used for a given compression application should be established prior to equipment order placement to facilitate a consistent performance basis for the equipment and documentation.

Four commonly used equations of state were examined to compare calculated performance parameters relative to that provided by the equipment supplier for the Sanha Condensate Project Type 1 test results. They were selected based upon their known usage by one or more equipment suppliers or their widespread inclusion in one or more process simulation computer applications. The evaluated equations of state were the original Redlich-Kwong (R-K), the Benedict-Webb-Rubin-Starling (BWRSE) as modified by Lin and Hopke of Exxon Production Research, the Lee-Kesler (LKP) as modified by Plocker, and the Peng-Robinson (P-R). Baseline data provided by the equipment supplier from the Type 1 testing was based on the supplier recommended BWRSE equation of state. Relationships for gas phase enthalpy, entropy, and density (specific volume) derived from the individual equations of state were utilized in the corresponding ASME PTC-10 (1997) calculations to determine various compressor performance parameters. APPENDIX A provides a summary of the procedures outlined in PTC-10 for calculating the various performance parameters.

COMPARISON OF TEST RESULTS

Comparison of a selected number of compressor operating parameters was made between the equipment supplier reported baseline data based upon the BWRSE relations and the four investigated equations of state. Figure 3 presents this comparison on a percent difference basis between the supplier calculated polytropic head and that derived from the different equation of state relations for the first section of the compressor. Similar percentage deviation comparisons for polytropic efficiency, inlet volumetric flow rate, and section gas horsepower are provided in Figures 4, 5, and 6, respectively. The reduced pressure range for section one was 0.42 to 1.33 with a reduced temperature range of 1.54 to 2.05. These reduced parameters are useful for expressing comparable thermodynamic state conditions for different gases or gas mixtures and are an integral part of the principle of corresponding states. The reduced pressure is simply the ratio of the absolute pressure to the critical pressure of the single component gas or gas mixture. Reduced temperature is calculated in a similar fashion. Although there are a number of mixing rules, the most basic form of expressing the critical properties of a gas mixture also used here is simply the sum of the product of the component mole fraction and its critical property.

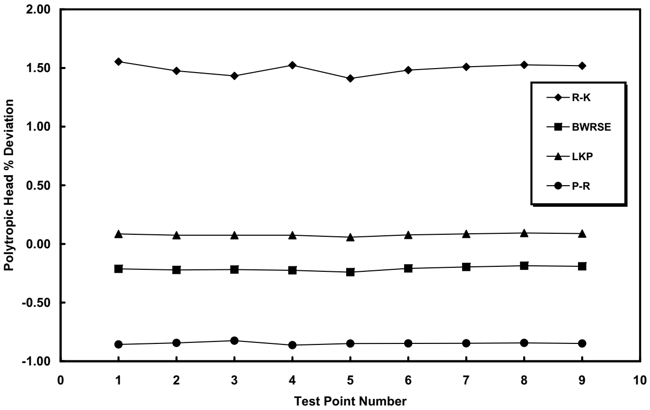

Figure 3. First Section Polytropic Head Deviation.

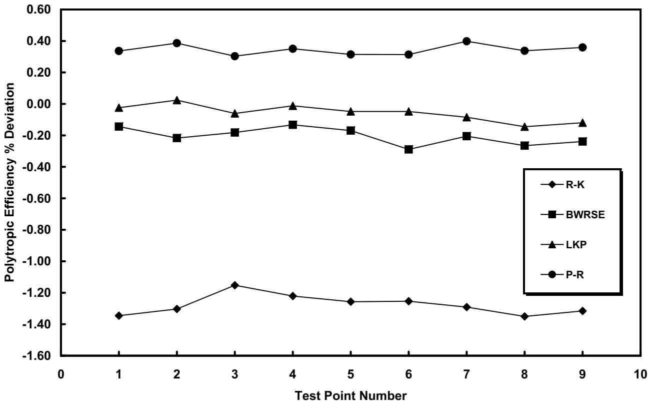

Figure 4. First Section Polytropic Efficiency Deviation.

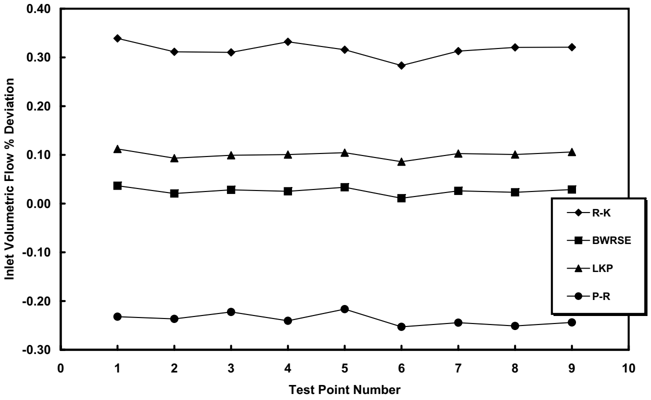

Figure 5. First Section Inlet Volumetric Flow Deviation.

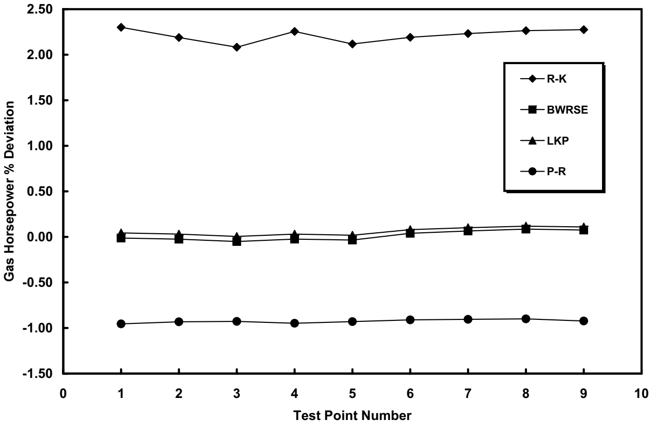

Figure 6. First Section Gas Horsepower Deviation.

An examination of the first section polytropic head comparison shows fairly good agreement between all of the equations of state with absolute deviations less than 1 percent overall for BWRSE, LKP, and P-R. The maximum deviation was noted from the R-K equation of state but it is within 1.5 percent. Similar tolerances are also displayed for the absolute deviations in polytropic efficiency with the R-K equation of state showing the maximum deviation within 1.5 percent. A comparison for inlet volumetric flow rate, however, demonstrates that all equations of state produce results within 0.5 percent of the baseline supplier data. In Figure 6, the plot of gas horsepower shows the highest deviation of the different parameters investigated. The BWRSE and LKP equations of state agree within 0.25 percent of the baseline data whereas the P-R equation of state is within 1 percent. The largest deviation from baseline is again shown to be the R-K equation of state with a maximum deviation below 2.5 percent.

First section results agree fairly closely with the supplier baseline data for all of the equations of state investigated. This is primarily due to the fact that conditions are relatively close to ideal with the operating points at relatively low pressures within the superheat region. Analysis of field test or factory Type 1 test results should agree closely to baseline data, regardless of equation of state used, whether they are extrapolated from Type 2 test data or "as-tested" performance characteristics derived using Type 2 test data with an assumed equation of state. Of course, the accuracy of process measurements and gas sampling can also affect the overall accuracy of the results when compared against the baseline.

Inspection of the same operating parameters for the second section of the compressor reveals greater deviation than that of the first section. Reduced pressure for the second section ranged from 1.30 to 3.20 and the reduced temperature ranged from 1.54 to 1.98. Figure 7 is a similar plot of percentage deviation of polytropic head for different equations of state relative to supplier baseline data. In this case, while BWRSE and LKP continue to show close agreement the P-R deviation has increased to approximately 2 percent and the R-K value has increased to nearly 6 percent. Polytropic efficiency (Figure 8) and inlet volumetric flow (Figure 9) deviations are closer to those observed for section one with slight increases for the P-R and R-K equations of state. Figure 10 is a plot of gas horsepower deviation. The deviation shown for the R-K equation of state is approximately 6.5 percent. Deviations for the BWRSE, LKP, and P-R equations of state are significantly less with P-R being approximately 1 percent and the other two less than this. Reduction of test data using the R-K equation of state for polytropic head and gas horsepower in this case would likely raise some questions and possibly lead to invalid conclusions regarding the performance of the machine in light of the standard API 617 (2002) performance tolerance of 4 percent on power for variable speed drives. Compressors with constant speed drives have tolerances on both head and power. Additional, more restrictive performance tolerances may also be included on these and other parameters in some cases.

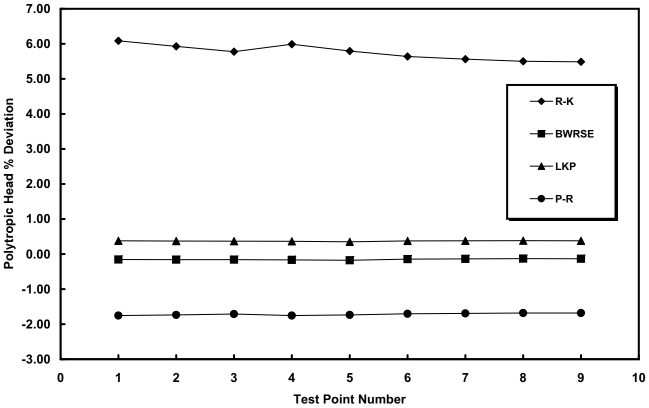

Figure 7. Second Section Polytropic Head Deviation.

Operation of the third section of the compressor reaches well into the dense phase region of the phase diagram with reduced pressure ranging from 3.16 to 9.80 and a reduced temperature range of 1.54 to 2.15. Although the deviation of the BWRSE and LKP equations of state remain well within an acceptable range for the calculated parameters examined, deviations from baseline for the P-R and R-K equations of state increased beyond those observed for the second section. Figure 11 shows that while the BWRSE and LKP deviations in polytropic head remain within 0.5 percent of the baseline, the P-R deviation almost doubled to nearly 4 percent and the R-K difference increased to approximately 11 percent. Figures 12 and 13 demonstrate lower overall deviations

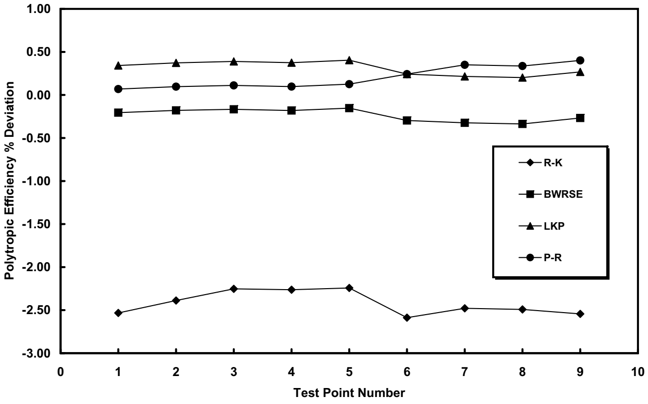

Figure 8. Second Section Polytropic Efficiency Deviation.

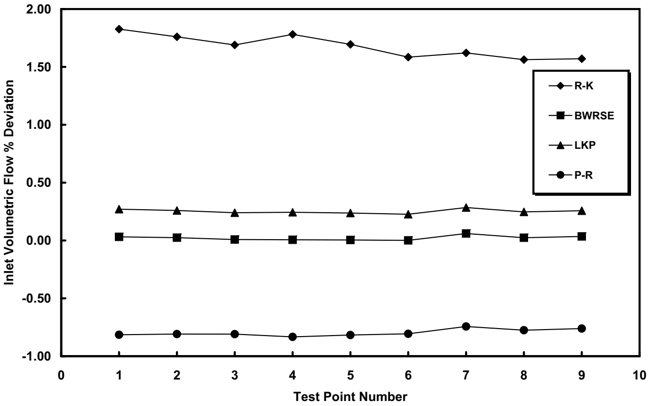

Figure 9. Second Section Inlet Volumetric Flow Deviation.

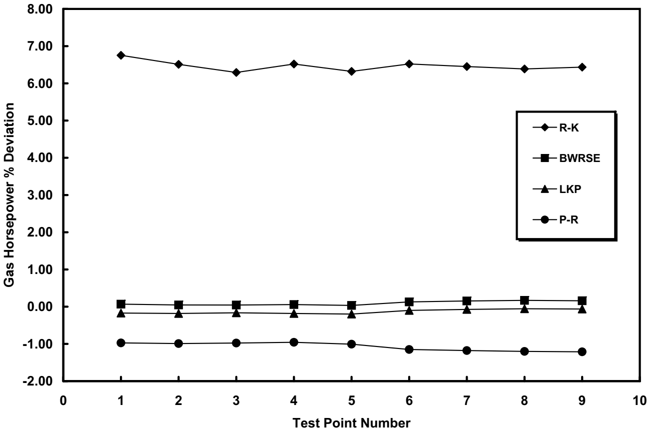

Figure 10. Second Section Gas Horsepower Deviation.

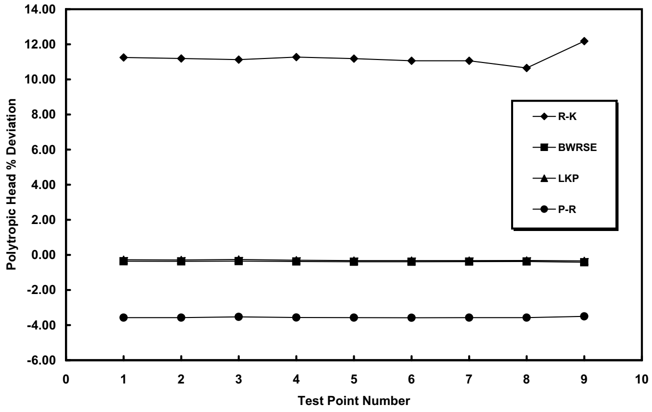

Figure 11. Third Section Polytropic Head Deviation.

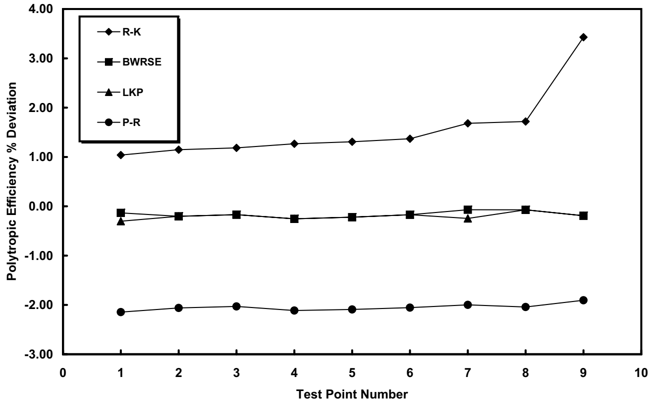

Figure 12. Third Section Polytropic Efficiency Deviation.

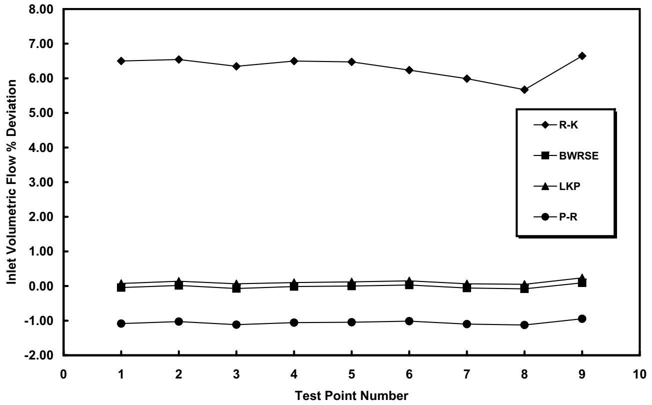

Figure 13. Third Section Inlet Volumetric Flow Deviation.

between the different equations of state and baseline data for polytropic efficiency and inlet volumetric flow, respectively. In general, the polytropic efficiencies are within 2 percent with the largest deviations again being attributed to the R-K and P-R equations of state. Inlet flow deviations are relatively low for all equations of state except R-K, which displays a difference of approximately 6 percent. A comparison of gas horsepower deviations is presented in Figure 14. All values are within 1 percent of the baseline data with the exception of the R-K equation of state with an overall difference over 6 percent.

Although all four equations of state investigated yielded what would be considered acceptable performance parameters for reduced pressures below 1.50, the BWRSE and LKP equations of state provided superior agreement when compared to the supplier baseline data. This superior behavior of the BWRSE and LKP equations of state was also evident at higher pressures where the R-K and P-R equation of state deviations increased. The differences that may be noted between the supplier BWRSE results and those presented in this study are likely due to small differences in equation constants and binary interaction coefficients. However, since the supplier data were also based upon the BWRSE equation of state another obvious question to be answered is the accuracy of the BWRSE along with the other three equations of state in predicting actual gas properties.

EQUATION OF STATE COMPARISON

The accuracy of an equation of state in predicting real gas thermodynamic properties can best be demonstrated by comparing predicted values of compressibility against experimental PVT data.

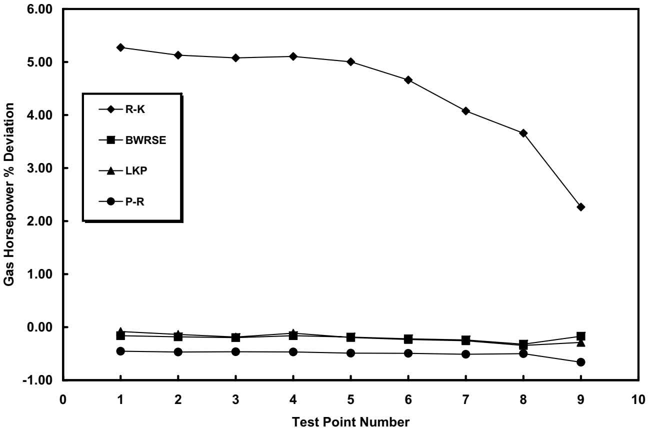

Figure 14. Third Section Gas Horsepower Deviation.

Not only does the compressibility factor allow for determination of real gas density but derivatives of the compressibility function provide the ability to determine deviations from ideal gas values for other thermodynamic properties such as enthalpy and entropy, which are critical in calculating actual compressor performance. These deviations are often referred to as residual or departure functions. In fact, all real gas thermodynamic parameters can be derived from relationships for the ideal, isobaric (constant pressure) specific heat capacity and the equation of state. APPENDIX B provides a derivation of real gas enthalpy and entropy relations from the isobaric heat capacity and a real gas equation of state.

Calculated compressibility factors from the four different equations of state investigated were compared against empirical values obtained from a number of PVT databases. These databases represent approximately 1200 individual measurements in 19 different data sets covering a reduced pressure range of 0.05 to 50.5 and a reduced temperature range of 0.78 to 3.10. Not all of the data sets cover the entire referenced range of reduced pressures or temperatures nor include the same gas composition. All of the data contained in these databases are available in the literature with the exception of two data sets (Hwang, et al., 1997; Brugge, et al., 1997; Hacura, et al., 1988; Staby and Mollerup, 1991). Figure 15 presents the results from one of these data sets (Staby and Mollerup, 1991) covering a reduced pressure range from slightly above zero to nearly 14. The gas is a mixture of methane, ethane, and nitrogen at a constant temperature of 161°F. A significant deviation is apparent for the R-K equation of state above a reduced pressure of 2. The P-R equation of state displays a percentage deviation within 3 percent of measured compressibility up to a reduced pressure of 4 but then increases almost linearly with pressure above that point to a maximum deviation of approximately 9 percent. The BWRSE and LKP equations of state demonstrate close agreement with the measured data across the entire reduced pressure range with deviations within 2 percent.

Results from another data set are provided in Figure 16 that reflects a range of both reduced pressure and temperature. These data are from an actual natural gas mixture composed of approximately 80 percent methane, 10 percent carbon dioxide, 1 percent nitrogen, and the remaining components varying percentages of ethane and higher molecular weight paraffinic components. The temperature range covers approximately 70°F to 300°F and a pressure range from 500 psig to 5000 psig, which represents conditions for a fairly broad range of compressor applications, particularly in the upstream oil and gas industry. Detailed inspection of this figure demonstrates overall trends similar to those presented for the other data set. The R-K equation of state displays some significant deviations at reduced pressures above 2. The P-R equation of state shows more reasonable agreement with empirical data below reduced pressures of 5 but this deviation increases for increasing pressures at lower values of reduced temperature. The BWRSE and LKP equations of state continue to show fairly uniform deviations within 2 percent across the range of reduced pressures and temperatures. Figure 16 also provides insight into the behavior of the departure functions in addition to the compressibility factor. The derivations in APPENDIX B illustrate that the departure of both the enthalpy and entropy from ideal conditions are functions of the derivative of the compressibility with respect to temperature at constant pressure. Changes in the percentage deviation of compressibility as the reduced temperature changes for a given reduced pressure are significant for the R-K equation of state, reducing somewhat for the P-R equation of state, and being relatively constant for the BWRSE and LKP equations of state. This reduced and more consistent change in deviation coupled with the more accurate prediction of compressibility factor should lead to more accurate estimations of the departure functions, enthalpy, entropy, and other thermodynamic parameters.

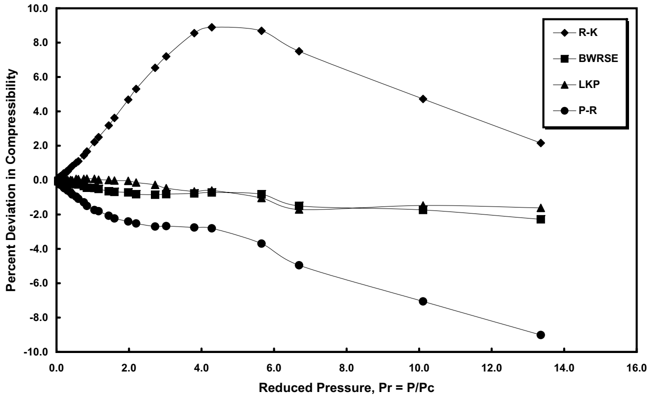

Figure 15. Compressibility Factor Deviation at Reduced Temperature of 1.63 and Acentricity of 0.036.

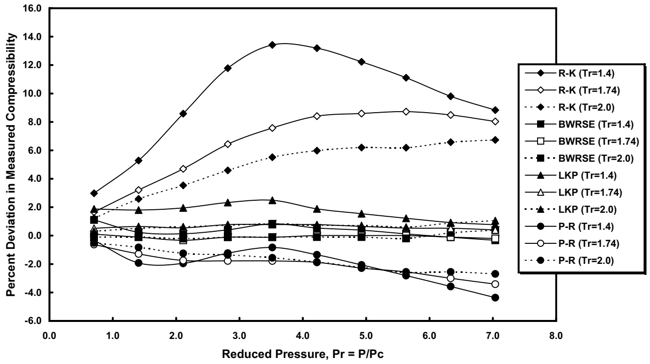

Figure 16. Compressibility Factor Deviation at Reduced Temperatures and Acentricity of 0.045.

Although these two evaluations have provided valuable insight into the accuracy of the four different equations of state as applied to compressor performance determination, a more general comparison of results from all 19 data sets is desirable. The principle of corresponding states is based on the assumption that different gases and mixtures of gases behave the same at a common reduced pressure and reduced temperature. However, this requires that all gases are comparable in geometric structure and polarity. Simple, nonpolar, spherical molecules act as the simplest molecular structure that can be imagined. Both increasingly complex, nonspherical structure and rising charge polarity lead to molecular interactions that reduce the validity of the principle of corresponding states and, potentially, the accuracy of equation of state predictions.

One parameter that accounts for nonspherical structure and, to a lesser extent, the polarity is the acentric factor. More complex molecular structure and higher polarity results in increasing acentric factors. Monatomic noble gases such as helium have an acentric factor of zero. The geometrically uniform tetrahedral molecular structure of methane also exhibits a relatively small acentric factor value. Increasing molecular weight and more complex structure will generally result in higher acentric factors. These factors for a number of common gases are provided in Table 1. Dipole moments for each of these compounds are also provided that provide a relative measure of the molecular polarity. Larger dipole moments are indicative of increased polarity. The acentric factor for a gas mixture is similarly calculated as the sum of the products of mole fraction and component acentric factor. The mixture acentric factors for the contract gas and test gas are 0.025 and 0.026, respectively.

Table 1. Selected Gas Properties Including Acentric Factor and Dipole Moments.

Compound	Critical Pressure (psia)	Critical Temperature (F)	Acentric Factor	Dipole Moment (Debyes)
Methane	667.0	-116.66	0.0108	0.0
Ethane	707.8	90.07	0.0972	0.0
Propane	615.0	205.92	0.1515	0.0
i-Butane	527.9	274.41	0.1852	0.1
n-Butane	548.8	305.51	0.1981	0.0
n-Pentane	488.1	385.7	0.2510	0.0
n-Hexane	439.5	451.8	0.2990	0.0
Ethylene	731.0	48.54	0.0860	0.0
Propylene	676.6	198.31	0.1404	0.4
Nitrogen	492.8	-232.49	0.0370	0.0
Carbon Dioxide	1069.5	87.73	0.2667	0.0
Hydrogen Sulfide	1300.0	212.40	0.0948	0.9
Hydrogen	187.5	-400.3	-0.2216	0.0
Oxygen	731.4	-181.41	0.0216	0.0
Helium	32.99	-450.31	0.0000	0.0
Water	3200.1	705.11	0.3445	1.8

Mixture acentric factors were calculated for the 19 different data sets and absolute deviations between the actual and calculated compressibilities were obtained for the individual measurements covering the varying reduced pressure and temperature ranges represented by each data set. An average absolute deviation was then derived for each data set that represented the accuracy of the given equation of state in predicting the compressibility factor afforded by the data set range of reduced pressure and temperature. The root mean squared deviation of each data set was also calculated to determine the distribution of the error. Gas components contained in these mixtures included nitrogen, carbon dioxide, and paraffinic hydrocarbons from methane through hexane in varying percentages. Figure 17 presents a summary of these average absolute deviations versus mixture acentric factor for all of the data sets. The BWRSE and LKP equations of state clearly provide lower deviation from measured PVT data across the range of acentric factors when compared to R-K and P-R, although the LKP equation of state displays increased deviation at higher mixture acentric factors comparable to those exhibited by the P-R equation of state. The only deviations above 2 percent demonstrated by the BWRSE equation of state were found on an approximate 60 percent to 40 percent carbon dioxide/nitrogen mixture at reduced pressures between 10 and 40 (Hacura, et al., 1988).

Although the analysis provided for acentric factor cannot be considered definitive for all gases, it should be valid for the gases and mixtures of the gases examined. This includes light hydrocarbon systems and mixtures containing carbon dioxide and nitrogen in modest concentrations. Any extrapolations to other gases should be accompanied by experimental PVT data analysis to ensure maximum accuracy of the chosen equation of state or identification of a more appropriate relation to be used.

CONCLUSIONS

The ability to accurately determine centrifugal compressor thermodynamic performance under shop test and field operating conditions is essential in establishing the initial acceptability of a machine along with the continuing evaluation of compressor health. Current emphasis on continuous machinery monitoring systems is extending solely beyond the assessment of mechanical operating parameters and vibration to also include a machine's aero/thermodynamic performance. This necessitates periodic comparison of operating parameters against a performance baseline that should be established through factory proof testing or field testing of the compressors in new or newly refurbished condition. Deviations in these baseline performance parameters may then provide insight into problems such as excessive internal recirculation, fouling, and partial blockage. The use of performance related information in combination with vibration and mechanical data clearly provides enhanced machinery diagnostic capabilities.

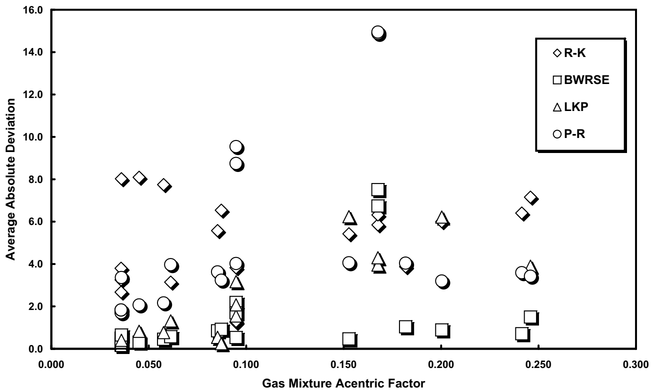

Figure 17. Compressibility Factor Absolute Average Deviation

This evaluation of equation of state influence on compressor performance has demonstrated that there may be differences in calculated compressor performance depending upon the equation of state selected. Such deviations are capable of causing significant error in calculating initial predicted and factory tested compressor performance as applied to actual field conditions and in field performance test comparisons. These differences tend to be more pronounced among the individual relations as reduced pressures increase above unity. Of the four equations of state examined, clearly the BWRSE and LKP equations provide superior predictions over the P-R and R-K equations of state for a large number of gases and gas mixtures across a wide range of reduced pressures and temperatures. Although this evaluation is limited to relatively simple, nonpolar molecules it is valid for a large number of industrial applications particularly those involving light hydrocarbons. Gases that do not fall within these limitations may require more customized or alternate equations of state and potential evaluation of specific PVT data to ensure performance prediction accuracy.

A number of important points should be considered when selecting an equation of state that will be applied in the determination of compressor performance. This must include an assessment of the capabilities and needs of both the user and the equipment supplier with regards to initial evaluation of compressor performance and long-term condition monitoring plans. The equipment supplier and end-user should:

- Collectively determine the most appropriate equation of state to be used for a specific compressor application during equipment selection and prior to order placement. The chosen equation of state should be utilized for evaluating both predicted and factory "as-tested" performance.

- Establish baseline performance data from factory or initial field test data. These baseline data will be used to identify compressor performance deviations resulting from subsequent field performance tests.

- Maintain the use of the original equation of state employed to determine compressor performance in subsequent field performance tests regardless of assumed equation accuracy. Errors in thermodynamic parameter prediction should remain consistent unless operating parameters are modified significantly.

- Reestablish baseline performance after rerates or equipment overhauls with field performance tests. Such a comparison of prior and post-overhaul data may also be used to verify the adequacy of the machinery overhaul. The equation of state utilized to determine thermodynamic properties may be modified at this time as long as sufficient field data are gathered to adequately define the compressor characteristic and, in variable speed cases, the compressor map across the anticipated speed range.

NOMENCLATURE

n = Isentropic or polytropic volume exponent
P = Absolute pressure
T = Absolute temperature
H, h = Enthalpy
S = Entropy
W = Compressor work or head
f = Schultz correction factor
GHP = Gas horsepower
m = Mass flow rate
ν = Specific volume
μ = Head coefficient
η = Efficiency
U = Impeller tip speed (APPENDIX A), internal energy (APPENDIX B)
V = Gas volume
R = Universal gas constant
Cp = Gas specific heat
Z = Gas compressibility factor

Subscripts

s = Suction conditions
d = Discharge conditions
id = Discharge conditions at constant entropy with suction conditions
is = Isentropic
p = Polytropic
P = Constant pressure
T = Constant temperature
o = Standard ideal conditions

APPENDIX A—
ASME PTC-10 EQUATIONS

The governing industry standard for determining centrifugal compressor performance, particularly in the United States, is ASME PTC-10 (1997), "Performance Test Code on Compressors and Exhausters." The ISO equivalent of this standard is ISO 5389 (1992) with essentially the same equations and calculation methodology. ASME PTC-10 (1997) defines the minimum requirements for compressor testing including setup and instrumentation, computation of results, and uncertainty analysis. It is applicable for both inert gas, Type 2 tests such as might be encountered in a factory testing program, as well as full load testing according to Type 1 requirements that might be applied to factory full load testing or field testing.

Compressor performance is obtained through the measurement of a limited number of readily available operating parameters. Stagnation pressure and temperature at both suction and discharge conditions are required along with the compressor operating speed. If only static pressures and temperatures are available, methods are provided to convert these to stagnation conditions. The flow rate is determined by some type of flow meter that typically uses temperature, static pressure, and differential pressure measurement at the meter. Gas composition and barometric pressure are also required. Once these parameters have been determined, the following physical and thermodynamic parameters can be derived and compressor performance established. There are two fundamental thermodynamic models that are most commonly used to describe compressor performance; the isentropic process and the polytropic process. Although simpler to manipulate, the isentropic process is more limited in application due to its restricted accuracy about an initially defined pressure ratio. The polytropic process can be used across varying compression ratios with greater accuracy. A majority of process compression equipment suppliers currently use the polytropic relations.

Irrespective of the thermodynamic model used, isentropic discharge conditions must be estimated and used in a number of the calculations. These isentropic discharge conditions are determined by setting the discharge pressure equal to the actual discharge pressure and varying the temperature to achieve the calculated entropy constant with the suction conditions. This is typically an iterative calculation utilizing relations derived from an equation of state.

The various parameters that may be calculated are:

Isentropic Exponent:

$$n_{is} = \frac{\ln \frac{P_d}{P_s}}{\ln \frac{v_s}{v_{di}}} \tag{A-1}$$

Schultz Polytropic Head Correction Factor:

$$f = \frac{h_{di} - h_s}{\left(\frac{n_{is}}{n_{is}-1}\right)\left(P_d v_{di} - P_s v_s\right)} \tag{A-2}$$

Isentropic Head:

$$W_{is} = \left(h_{di} - h_s\right) = \left(\frac{n_{is}}{n_{is}-1}\right) f P_s v_s \left[\left(\frac{P_d}{P_s}\right)^{\frac{n_{is}-1}{n_{is}}} - 1\right] \tag{A-3}$$

Isentropic Head Coefficient:

$$\mu_{is} = \frac{W_{is}}{\sum U^2} \tag{A-4}$$

Isentropic Efficiency:

$$\eta_{is} = \frac{\left(h_{di} - h_s\right)}{\left(h_d - h_s\right)} = \frac{W_{is}}{\left(h_d - h_s\right)} \tag{A-5}$$

Polytropic Exponent:

$$n_p = \frac{\ln \frac{P_d}{P_s}}{\ln \frac{v_s}{v_d}} \tag{A-6}$$

Polytropic Head:

$$W_p = \left(\frac{n_p}{n_p - 1}\right) f P_s v_s \left[\left(\frac{P_d}{P_s}\right)^{\frac{n_p-1}{n_p}} - 1\right] \tag{A-7}$$

Polytropic Head Coefficient:

$$\mu_p = \frac{W_p}{\sum U^2} \tag{A-8}$$

Polytropic Efficiency:

$$\eta_p = \frac{W_p}{\left(h_d - h_s\right)} \tag{A-9}$$

Gas Horsepower:

$$GHP = \frac{\dot{m}W_{is}}{\eta_{is}} = \frac{\dot{m}W_p}{\eta_p} \qquad \text{(A-10)}$$

APPENDIX B— DERIVATION OF REAL GAS DEPARTURE FUNCTIONS

Real gas thermodynamic properties such as enthalpy and entropy can be derived from a relation for the isobaric specific heat capacity as a function of temperature at standard pressure conditions and an equation of state expressed in terms of the compressibility. Taking enthalpy as a first example and assuming that the enthalpy is a function of both temperature and pressure:

$$H = H(T,P) \qquad \text{(B-1)}$$

Enthalpy is also defined as:

$$H = U + PV \qquad \text{(B-2)}$$

with its derivative being:

$$dH = dU + d(PV) = dU + PdV + VdP \qquad \text{(B-3)}$$

Reference to any thermodynamics textbook will also show that the derivative of the internal energy can be defined as:

$$dU = TdS - PdV \qquad \text{(B-4)}$$

Combining the above two relations yields:

$$dH = TdS + VdP \qquad \text{(B-5)}$$

Along an isotherm, this may be expressed as:

$$\left(\frac{\partial H}{\partial P}\right)_T = T\left(\frac{\partial S}{\partial P}\right)_T + V \qquad \text{(B-6)}$$

Again, reference to any thermodynamics text will show that the Maxwell relations give:

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P \qquad \text{(B-7)}$$

Substitution results in the following relation:

$$\left(\frac{\partial H}{\partial P}\right)_T = \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right] \qquad \text{(B-8)}$$

Next, writing the total derivative of the enthalpy as:

$$dH = \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP \qquad \text{(B-9)}$$

but, by definition, the isobaric specific heat capacity is defined as:

$$Cp_0 = \left(\frac{\partial H}{\partial T}\right)_P \qquad \text{(B-10)}$$

Substitution of both of these terms into the enthalpy total derivative yields:

$$dH = Cp_0 dT + \left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right] dP \qquad \text{(B-11)}$$

For an ideal gas, PV = RT, the second term is found to be equal to zero:

$$\left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]_{ideal} = 0 \qquad \text{(B-12)}$$

This simplifies the relation for enthalpy to the familiar form:

$$dH = Cp_0 dT \qquad \text{(B-13)}$$

However for a real gas, PV = ZRT, the second term is not equal to zero. This term is often referred to as the residual or departure function. It is related to the derivative of the compressibility and thus may be determined from an equation of state.

$$\left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]_{real} = \frac{-RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P \qquad \text{(B-14)}$$

This results in the more general form of enthalpy for a real gas, which is shown to be a function of both pressure and temperature.

$$dH = Cp_0 dT - \frac{RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P dP \qquad \text{(B-15)}$$

Integration of this equation to determine a change in enthalpy between two states is then accomplished along an isobar for the first term and then an isotherm for the second term. The total change in enthalpy between two state points is given by:

$$\Delta H = \int Cp_0 dT - \int \frac{RT^2}{P}\left(\frac{\partial Z}{\partial T}\right)_P dP \qquad \text{(B-16)}$$

A similar analysis for the entropy can be derived that results in the following relationships for the total derivative of entropy given that it is a function of temperature and pressure:

$$dS = \frac{Cp_0}{T} dT - \left(\frac{\partial V}{\partial T}\right)_P dP \qquad \text{(B-17)}$$

Which, for an ideal gas reduces to:

$$dS = \frac{Cp_0}{T} dT - \frac{R}{P} dP \qquad \text{(B-18)}$$

and, for a real gas:

$$dS = \frac{Cp_0}{T} dT - \left[\frac{ZR}{P} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P\right] dP \qquad \text{(B-19)}$$

Finally, integrating to obtain the entropy difference between two state points:

$$\Delta S = \int \frac{Cp_0}{T} dT - \int \left[\frac{ZR}{P} + \frac{RT}{P}\left(\frac{\partial Z}{\partial T}\right)_P\right] dP \qquad \text{(B-20)}$$

Once again, the residual function is demonstrated to be a function of the compressibility that can be derived directly from an equation of state. The integration of both equations to determine the change in enthalpy and entropy is accomplished by isobarically integrating the temperature dependent first term and isothermally integrating the pressure dependent second term.

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