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• DETERMINING • *Rainfall-Runoff-Retention* • RELATIONSHIPS

TEXAS AGRICULTURAL EXPERIMENT STATION

R. D. LEWIS, DIRECTOR, COLLEGE STATION, TEXAS

in cooperation with the
UNITED STATES DEPARTMENT OF AGRICULTURE

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ABSTRACT

A method is presented for predicting daily runoff from native meadow watersheds in the Blacklands of Texas. The method is developed from daily rainfall (P), runoff (Q) and antecedent soil moisture (ASM) data from an experimental watershed. Evaluation of the constants in the equation $\frac{P}{P-Q} = a + bP$ by reference to ASM data resulted in a logical mathematical procedure that allows extension of the relationship beyond the range of experimental data.

ACKNOWLEDGMENTS

The authors acknowledge the valuable advice and leadership of L. L. Kelly and the aid and assistance of numerous others who have helped to make this publication possible.

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Determining Rainfall-Runoff-Retention Relationships

Monroe A. Hartman, Ralph W. Baird, James B. Pope and Walter G. Knisel*

For evaluation of flood prevention programs and the design of structures in upstream watersheds, it frequently is found that there are no specific data available on the runoff from the contributing watershed area and that the only available rainfall data are the daily totals at one or more rain gages in or near the watershed.

This study was made to develop a method of estimating the daily volumes of storm runoff that might be expected to occur, basing the estimates on these daily rainfall totals.

DATA

A 3-acre native grass watershed at the Blacklands Experimental Watershed near Riesel, Texas, was selected for the study. The soils of this small watershed are representative of the soils of the Blacklands area, being mostly deep, heavy soils which crack severely when they dry. The watershed, mowed annually for hay, has but few changes in hydrologic conditions due to cover or tillage. Thus the number of variables influencing the rainfall-runoff relationship was minimized.

The data used in the study were daily rainfall, daily runoff and periodic soil moisture samples during 1938-43 and 1949-57. Rainfall was measured by a recording gage adjacent to the watershed; runoff was measured with an H-3 rate measuring flume equipped with a float recorder; and soil moisture was measured by periodic sampling to a depth of 5 feet, using standard gravimetric methods. The moisture percentages for the samples at the selected depths were converted to inches of water above 18 percent, using bulk densities (grams per cubic centimeter) of 1.2 for the 0 to 6-inch depths; 1.3 for the 6 to 12-inch depths and 1.4 for the 12 to 36-inch depths (7).

Previous studies at the Experimental Watershed have indicated that the amount of moisture in the upper part of the soil profile immediately before the rain provides a good index for use in estimating the amount of water retained from any particular amount of rainfall (1). The index of antecedent soil moisture used in this study was the calculated amount of water above 18 percent, the approximate wilting point, in the upper 3 feet of the soil profile. The soil moisture prior to the day of each runoff-producing storm was computed by starting with the moisture content indicated by the most recent sampling data, adding any rainfall and subtracting any runoff occurring since the sampling, and subtracting the estimated soil moisture lost by evapotranspiration and deep percolation since the sampling (2). The estimate of evapotranspiration and deep percolation loss was based on other detailed studies which assumed uniform depletion between sampling dates (3).

The antecedent soil moisture index (ASM) is not necessarily the true quantitative amount of water stored in the profile (6). Rather, it is a value that lends itself to correlation with available potential water storage.

EXPLORATORY STUDIES

Various methods of correlating the daily rainfall, daily runoff and antecedent soil moisture index data were explored and tested. Some of the first predicting methods developed, though not efficient, provided valuable insight into the types of relationships among the variables. The more significant of these first studies will be described briefly.

A series of relationships was obtained between daily rainfall (P) and daily rainfall retained ($\frac{P-Q}{P}$ 100), grouping the data in accordance with the antecedent soil moisture (ASM) conditions prevailing before the storm (1). The general form of these estimating equations was:

$$\text{Log}\ \frac{P-Q}{P}\ 100 = a - bP \qquad (1)$$

when

$\frac{P-Q}{P}$ 100 = percent of daily rainfall retained,

Q = daily runoff, inches,
P = daily rainfall, inches,
a and b = constants.

Correlation of the data for ASM's of 4.5 to 5.9, 5.0 to 6.9, 6.0 to 7.4, 7.0 to 8.0 and 7.5 to 8.5 inches were significant at the 1 percent level, and those with ASM of less than 5.0 or greater than 8.0 inches, at the 5 percent level.

Rainfall was then related directly to runoff by computing runoff for various values of rainfall in

*Respectively hydraulic engineer, project supervisor, soil conservationist and agricultural engineer, Blacklands Experimental Watershed, Soil and Water Conservation Research Division, Agricultural Research Service, U. S. Department of Agriculture, Riesel, Texas.

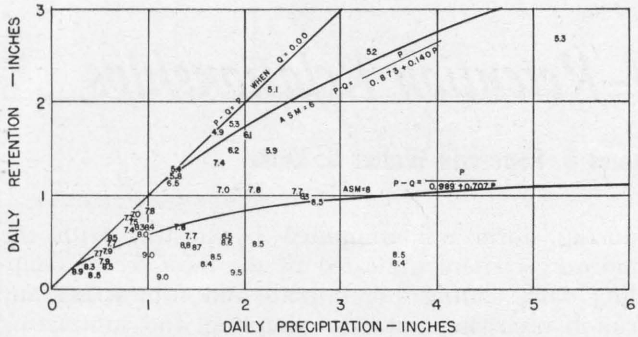


Figure 1. Daily precipitation and retention (P—Q) with antecedent soil moisture (ASM) index.

each of the estimating equations. The resulting P vs Q curves with average ASM as retention indices were extended as straight lines. These straight line extensions were parallel to the line of equal values when any increment of rainfall would give a zero or less increment of retention as computed from the rainfall-retention relationship.

Though this procedure appeared to provide fair estimates of runoff, it had objectionable features in that the entire P vs Q relationship could not be expressed by a derived formula; *i.e.,* beyond the range of data it was necessary to make a more or less arbitrary change in the slope of the curves. Various other equation forms were investigated in an effort to provide a form that would fit the data well and that would allow for a logical mathematical extension of the relationships beyond the range of the data.

Among the forms investigated was:

$$(Q - k)^2 - (P - h)^2 = a^2 \qquad (2)$$

in which Q and P are daily runoff and daily rainfall and k, h and a are constants. The constants were evaluated by reference to rainfall, runoff and soil moisture items within the range of data. This method did not produce completely reasonable results.

The equation:

$$\frac{P}{P-Q} = a + bP, \qquad (3)$$

also was investigated, evaluating the a and b constants by relating $\frac{P}{P-Q}$ to P for the several ASM indices. This is the straight line relationship of the hyperbolic function of rainfall (P) vs retention (P–Q) (4) (5), shown in Figure 1. This equation appeared reasonable, but the first methods for evaluating the constants was not precise, large differences in the lower ranges of the curves were reflected by extremely small differences in the a and b constants.

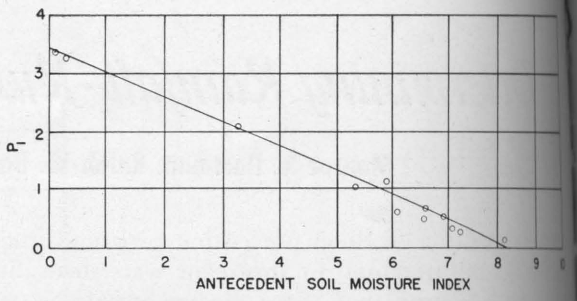


Figure 2. ASM vs P_1 for native grass meadow, Riesel, Texas, when ASM = antecedent soil moisture index for storms with less than 0.005 inch of runoff and P_1 = daily precipitation in inches.

DEVELOPMENT OF FINAL RELATIONSHIPS

It appeared from the exploratory studies that an equation of the form:

$$\frac{P}{P-Q} = a + bP \qquad (3)$$

would represent the data well and allow logical extension of the relationships beyond the range of data if the constants could be evaluated precisely. It also appeared that a most important factor in the relationships was the amount of daily rainfall that could be retained with no runoff, hereafter called (P_1).

Relationship of ASM to P_1

The amount of rainfall that can be retained before runoff begins (P_1) is related logically to available soil moisture storage and soil moisture storage is related to the ASM index. Therefore, P_1 should be related to the ASM for any day on which runoff from rainfall occurred.

The 12 daily rainfalls that produced measurable, but less than 0.005 inch of runoff, were correlated (Figure 2) with the ASM prior to rainfall, in a simple linear regression. This resulted in the equation:

$$P_1 = 3.37 - 0.41\text{ASM} \qquad (4)$$

The correlation coefficient is significant at the 1 percent level ($r = 0.993$).

To further verify the equation, all daily rainfall, runoff and soil moisture index data for all days on which runoff occurred were checked against the curve of Figure 2, entering the curve with the ASM value and noting where the P_1 factor fell. In this check, all daily storms that produced no runoff plotted below the line, indicating insufficient P to satisfy P_1; all daily storms with 0.005 to 0.01 inch of runoff plotted close to the line, but in most cases slightly above it, indicating that P_1 was slightly exceeded by P; and all daily storms greater than 0.01 inch plotted above the line, indicating that P more than satisfied P_1.

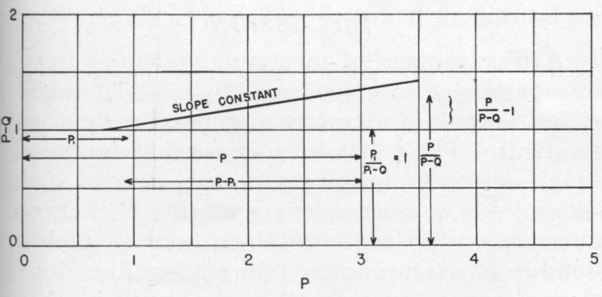


Figure 3. Schematic graph showing the factors used in determining the slope, $b = \dfrac{\dfrac{P}{P-Q} - 1}{P - P_1}$.

Evaluation of Constants

Equation (4) was used to compute the inches of rainfall when runoff began for each daily runoff of record with more than 0.01 inch of runoff. This, with the totals of daily rainfall and runoff, provided two points on an assumed straight line relationship of P vs $\dfrac{P}{P-Q}$ for each daily runoff event. These relationships are illustrated on the schematic chart in Figure 3. A b constant was then computed for each of the daily storms by solving:

$$b = \frac{\dfrac{P}{P-Q} - \dfrac{P_1}{P_1-Q}}{P - P_1} = \frac{\dfrac{P}{P-Q} - 1}{P - P_1} \qquad (5)$$

when

- b = slope constant,
- P = recorded rainfall for day, inches,
- Q = recorded runoff for day, inches,
- P_1 = rainfall when runoff begins and Q is zero, from equation (4).

The b constants, thus determined for all storms, were correlated with the associated ASM. This complex relationship is shown in Figure 4. The best relationship seemed to be a hyperbolic function between the reciprocal of the ASM and b. The straight line relationship is shown on Figure 5 (4) (5).

The plotting of Figure 5 shows a break in the relationship between the values of 0.12 and 0.14 for $\dfrac{1}{ASM}$. The value of 0.128 on this scale represents an ASM of about 7.8 inches, which is approximate field capacity. For ASM values exceeding this, there will be free water in the top 3 feet of profile and some seepage flow can be expected. It, therefore, appeared logical to develop two equations, one for

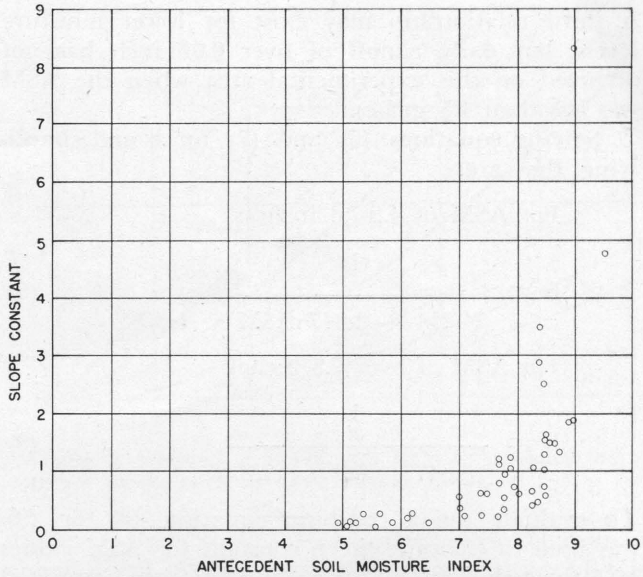


Figure 4. ASM vs b for native grass meadow, Riesel, Texas, when ASM = antecedent soil moisture index for storms with more than 0.01 inch of runoff and slope constant = b as computed by equation (5).

ASM values of 4.9 to 7.8 inches and one for values exceeding 7.8 inches. The lines as fitted by the least squares method (correlations significant at the 1 percent level) are shown in Figure 5, and the equations are:

For ASM of 4.9-7.8 inches, ($r = 0.768$)

$$\frac{1/ASM}{b} = -2.847 + 24.214\frac{1}{ASM} \qquad (6)$$

For ASM of over 7.8 inches, ($r = 0.556$)

$$\frac{1/ASM}{b} = -0.904 + 8.647\frac{1}{ASM} \qquad (7)$$

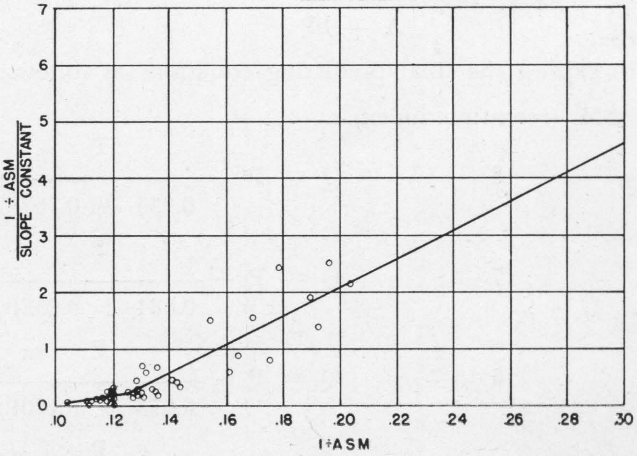


Figure 5. $1/ASM$ vs $\dfrac{1/ASM}{b}$ for native grass meadow at Riesel, Texas, when ASM = antecedent soil moisture index for storms with more than 0.01 inch of runoff and slope constant = b as computed by equation (5).

A third relationship may exist for lower moisture levels, but daily runoff of over 0.01 inch has not occurred on this experimental area when the ASM was less than 4.5 inches.

Solving equations (6) and (7) for b and simplifying, they are:

For ASM of 4.9-7.8 inches:

$$b = \frac{1}{24.214 - 2.847 \text{ ASM}} \qquad (8)$$

For ASM of over 7.8 inches:

$$b = \frac{1}{8.647 - 0.904 \text{ ASM}} \qquad (9)$$

To evaluate the a constant, equation (8) or (9) was used to compute the b constant for ASM values of 5 through 8. The a constant was then computed for the same ASM values from the y intercept equation, illustrated in Figure 3. (The y intercept of a line is equal to the ordinate of a known point minus the product of the abscissa of the known point and the slope of the line.)

$$a = \frac{P_1}{P_1 - Q} - bP_1 = 1 - bP_1 \qquad (10)$$

when

a = intercept constant
b = slope constant
P_1 = inches of rainfall when runoff begins
Q = the zero runoff associated with P_1.

Thus there were values of the a and b constants for several ASM's to substitute in basic equation (3). Equation (3) was rearranged to the form:

$$Q = P - \frac{P}{A + bP} \qquad (11)$$

to express the final predicting equations as follows:

ASM (retention index)	
8	$Q = P - \frac{P}{0.936 + 0.707P}$
7	$Q = P - \frac{P}{0.884 + 0.232P}$
6	$Q = P - \frac{P}{0.873 + 0.140P}$
5	$Q = P - \frac{P}{0.868 + 0.100P}$

Figure 6 was developed from the above equation by substituting several values of P.

DISCUSSION

A disadvantage of the rainfall-retention relationship expressed in equations (3) or (11) is the involved procedure necessary to determine the a and b constants. The equation does provide, however, for equal weight to be given to each item of data in determining averages; for expressing the effects of interrelationship of a and b as well as their relationship to soil moisture; and for a logical and mathematical expression that allows for extension beyond the range of available data.

The relationships expressed in the equation are compatible with the physical interpretations. This can be demonstrated by rearrangement and combining of the equations.

If equation (10) is substituted for the a constant in equation (11):

$$Q = P - \frac{P}{(1 - bP_1) + bP} = \frac{P(P - P_1)}{1/b + (P - P_1)} \qquad (12)$$

If now, $P_1 = 0$ (runoff begins when rainfall begins), equation (12) will be:

$$Q = \frac{P^2}{1/b + P} \qquad (13)$$

In this equation, as 1/b approaches zero, which is the condition of a saturated profile, Q will approach P. That is, runoff will begin when rainfall begins and abstractions approach zero. This would be the condition also on a tin roof.

Further, equation (13) can be arranged as:

$$P - Q = \frac{1/b \; Q}{P} \qquad (14)$$

Now as Q approaches P, the ratio $\frac{Q}{P}$ will approach 1, and P − Q will approach 1/b. As 1/b represents a direct function of the ASM (see equations 8 and 9), this means that the retention (P−Q) is directly related to the ASM.

Again, equation (12) also can be rearranged as:

$$\frac{Q}{P - P_1} = \frac{P - Q}{1/b} \qquad (15)$$

Now if P_1 is zero, that is, if no initial abstractions occur:

$$\frac{Q}{P} = \frac{P - Q}{1/b} \qquad (16)$$

and the ratio of rainfall to runoff is equal to the ratio of retention to the factor 1/b. Therefore, the parameter *b* represents a storage factor or an infiltration index.

Rainfall amount and available storage in the soil immediately preceding the rain are two major factors influencing the amount of runoff from agricultural lands in the Blacklands of Texas. The amount of water in the soil at the time of a rain, provided other hydrologic factors remain constant, is a measure of the available storage and the transmission rates of the soil profile.

The scatter of the points (in Figure 5) is a measure of the effect of all other factors that influence the rainfall-runoff relationship other than daily rainfall and antecedent soil moisture in the top 3 feet of soil. The dominant factors not considered may be rainfall intensity, surface soil condition and permeability below 3 feet. The latter factor possibly could be measured by a moisture index for the 4 and 5 feet of soil. Use of other factors probably would increase the accuracy of estimates, but also would require data that frequently are not available.

The rainfall vs runoff curves developed by the method described have given good results where tested. As data for other land uses are analyzed, it should be possible to estimate with fair accuracy the effects of various practices on runoff. In areas with other soils and crops, different factors may have dominant effects on surface runoff.

For the Blacklands soils, there is a rapid change in the rainfall and runoff relationship as the soil approaches the extreme wet or dry condition. At these

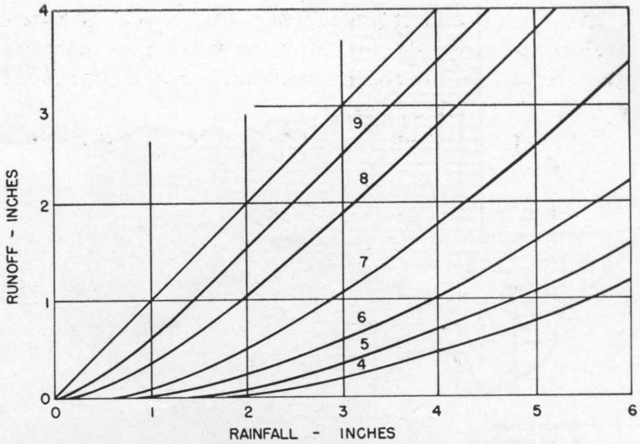
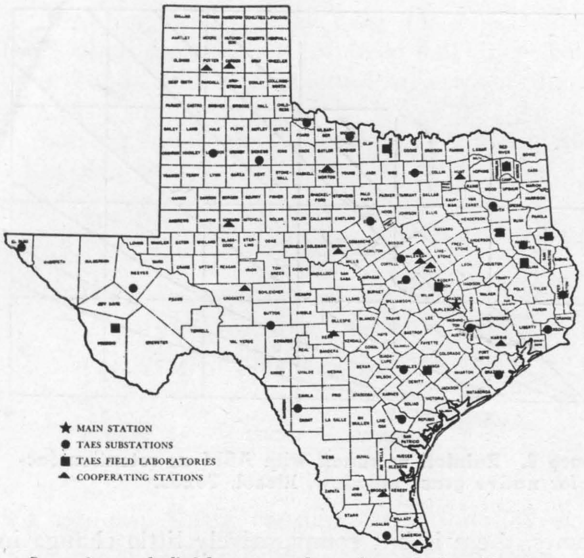


Figure 6. Rainfall vs runoff with ASM as retention factor for native grass meadow, Riesel, Texas.

extremes, there is but comparatively little change in runoff for a given change in soil moisture. This is shown by the space between the retention index lines in Figure 6. There is a relatively large change in runoff amounts for relatively small changes in soil moisture between 7 and 8 inches. Below 7 inches of soil moisture, drying cracks are beginning to be visible, allowing for rapid transmission of excess rainfall to lower levels. The field soil moisture capacity is approximately 8 inches and at this level the intake and transmission rate of the soil is very low.

Additional work is underway along the lines described to determine the rainfall-retention relationship for other cover and land use conditions from daily rainfall, runoff and soil moisture data.



Location of field research units of the Texas Agricultural Experiment Station and cooperating agencies

State-wide Research

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The Texas Agricultural Experiment Station is the public agricultural research agency of the State of Texas, and is one of ten parts of the Texas A&M College System

ORGANIZATION

IN THE MAIN STATION, with headquarters at College Station, are 16 subject-matter departments, 2 service departments, 3 regulatory services and the administrative staff. Located out in the major agricultural areas of Texas are 21 substations and 9 field laboratories. In addition, there are 14 cooperating stations owned by other agencies. Cooperating agencies include the Texas Forest Service, Game and Fish Commission of Texas, Texas Prison System, U. S. Department of Agriculture, University of Texas, Texas Technological College, Texas College of Arts and Industries and the King Ranch. Some experiments are conducted on farms and ranches and in rural homes.

OPERATION

THE TEXAS STATION is conducting about 400 active research projects, grouped in 25 programs, which include all phases of agriculture in Texas. Among these are:

Conservation and improvement of soil	Beef cattle
Conservation and use of water	Dairy cattle
Grasses and legumes	Sheep and goats
Grain crops	Swine
Cotton and other fiber crops	Chickens and turkeys
Vegetable crops	Animal diseases and parasites
Citrus and other subtropical fruits	Fish and game
Fruits and nuts	Farm and ranch engineering
Oil seed crops	Farm and ranch business
Ornamental plants	Marketing agricultural products
Brush and weeds	Rural home economics
Insects	Rural agricultural economics

Plant diseases

Two additional programs are maintenance and upkeep, and central services.

Research results are carried to Texas farmers, ranchmen and homemakers by county agents and specialists of the Texas Agricultural Extension Service

AGRICULTURAL RESEARCH seeks the WHATS, the WHYS, the WHENS, the WHERES and the HOWS of hundreds of problems which confront operators of farms and ranches, and the many industries depending on or serving agriculture. Workers of the Main Station and the field units of the Texas Agricultural Experiment Station seek diligently to find solutions to these problems.

Today's Research Is Tommorrow's Progress