# A COMPRESSED AIR SYSTEM MODEL FOR ANALYSIS AND DESIGN

A Thesis

by

JAMES TIRTOWIJOYO YOUNG

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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| Chair of Committee, | Michael B. Pate |
| Committee Member, | Partha P. Mukherjee |
| | Karen Vierow |
| Head of Department, | Andreas A. Polycarpou |

August 2016

Major Subject: Mechanical Engineering

**ABSTRACT**

The typical efficiency of a compressed air system ranges from 5% to 20%, making it one of the most expensive utilities in industrial plants, hence its name, "the fourth utility". It is certain that compressed air is an inefficient way to power any industrial machine; however with the large number of compressed air tools being used in factories or manufacturing facilities, it is uneconomical and impractical to replace all of the industrial tools with electric powered ones. In spite of its inefficiency, compressed air has many advantages that make it attractive and essential for industries. Some of the advantages of compressed air are that it can be easily stored and transported, tools are lightweight, cheap, and maintenance-free, and compressed air works well with different air temperatures, pressures and flow speeds. Finally, compressed air tools are spark-free, and high-torque.

Because the compressed air system is the largest consumer of electricity in industry, it is important to optimize the compressed air system and all of its components, to make sure that the system runs as energy efficient as possible. Moreover, a careful calculation of how each of the components works together as a single system is necessary to determine the system's performance, unnoticeable problems, and suitable energy improvements.

The main objective of this research is to model and calculate the compressed air system performance that includes determining the temperatures, pressures, flow rates, and leakages at each point in the system. Of special importance, the compressed air system

model using MATLAB will output the work and heat transfer rates mainly for the air compressor and the cooling system. The model is meant to be able to calculate several different unknown variables that are related to each of the components, depending on what values are known. Once the model and program are developed, they can be used to perform design and analysis for several different single compressed air systems. This design and analysis effort will allow an engineer to determine the energy consumption of components and output parameters, and to investigate possible changes to improve the system efficiencies.

# DEDICATION

I dedicate my research and thesis work to my family and many friends. A special feeling of gratitude to my loving Papa and Mama, Steven Tirtowidjojo and Rani Pandunata, whose good examples have taught me to work hard, and work smart for the things that I aspire to achieve. “If you want, you can,” that is what my father has always been telling me ever as far as I can remember, and also what keep me from ever giving up one I decided to start. I also thank my sister Katherine Tirtowijoyo Young, and my two brothers, Peter Tirtowijoyo Young and Andrew Tirtowijoyo Young, for all the support and encouragement. This work is also dedicated to my Akong and Ama who is always worrying about me, and call me every weekend to see how I have been doing, as well as to my Kungkung and Bobo who are always waiting for me to get home, and feed me with lots of delicious homemade cooking.

# ACKNOWLEDGEMENTS

I would first like to thank my thesis advisor Prof. Michael B. Pate of the Mechanical Engineering Department at Texas A&M University for consistently allowed this paper to be my own work, but steered me in the right the direction whenever he thought I needed it. Completing this Master of Science program was not even in my dream at first, but his valuable guidance has made this an enriching lifetime opportunity.

I would also like to thank my master thesis committee members: Dr. Karen Vierow, and Dr. Partha P. Mukherjee. Without their passionate participation and input, this research thesis could not have been successfully completed.

I would also like to acknowledge Dr. Timothy J. Jacobs, Prof. Harry A. Hogan, and Prof. Kumbakonam R. Rajagopal of the Mechanical Engineering Department at Texas A&M University. Their passion towards science, technology, and engineering has inspired me to never stop learning, and continue pursuing higher education. From them, I learned that the deeper you learn, and understand about a subject, the more you will realize that there are much more things to discover than what you thought there were.

Finally, I must express my very profound gratitude to my family and friends for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them. Thank you.

## NOMENCLATURE

### Roman Characters

| | | |
|---|---|---|
| $c_{\$}$ | Electricity cost | $[\frac{\$}{kW \cdot hr}]$ |
| $C$ | Clearance | [unitless] |
| $C_S$ | Sutherland's constant | [unitless] |
| $C_p$ | Specific heat (constant pressure) | $[\frac{BTU}{lbm \cdot R}]$ |
| $C_v$ | Specific heat (constant volume) | $[\frac{BTU}{lbm \cdot R}]$ |
| $C_{\$}$ | Electricity rate | [$] |
| $D$ | Diameter | $[in]$ |
| $E$ | Energy | $[BTU]$ |
| $f$ | Friction factor | [unitless] |
| $g$ | Acceleration due to gravity | $[\frac{ft}{sec^2}]$ |
| $h$ | Specific enthalpy | $[\frac{BTU}{lbm}]$ |
| $k$ | Heat constant ratio | [unitless] |
| $K_L$ | Loss coefficient | [unitless] |
| $L$ | Length | $[ft]$ |
| $L_c$ | Latent heat of condensation | $[\frac{BTU}{lbm}]$ |
| $m$ | Mass | $[lbm]$ |
| $\dot{m}$ | Mass flow rate | $[\frac{lbm}{sec}]$ |

| | | |
|---|---|---|
| $M$ | Molar mass | $[\frac{lbm}{lbmol}]$ |
| $n$ | Polytropic index | [unitless] |
| $P$ | Pressure | [$psia$ or $psig$] |
| $\dot{Q}$ | Heat transfer rate | $[\frac{BTU}{hr}]$ |
| $R_u$ | Universal gas constant | [$\frac{BTU}{lbmol \cdot R}$ or $\frac{lbf \cdot ft}{lbmol \cdot R}$] |
| $R$ | Specific gas constant | [$\frac{BTU}{lbm \cdot R}$ or $\frac{lbf \cdot ft}{lbm \cdot R}$] |
| $Re$ | Reynold's number | [unitless] |
| $t$ | Time period | [$sec$ or $min$ or $hr$ or $yr$] |
| $T$ | Temperature | [$R$ or °F] |
| $v$ | Specific volume | $[\frac{ft^3}{lbm}]$ |
| $V$ | Volume | [$ft^3$ or $SCF$] |
| $\dot{V}$ | Volume flow rate | [$\frac{ft^3}{sec}$ or $SCFM$ or $gpm$] |
| $\bar{V}$ | Velocity | $[\frac{ft}{sec}]$ |
| $\dot{w}$ | Specific work | $[\frac{BTU}{lbm}]$ |
| $\dot{W}$ | Work transfer rate or power | [$\frac{BTU}{hr}$ or $hp$ or $kW$] |
| $\dot{W}^*$ | Power per flow rate | $[\frac{hp}{100\ SCFM}]$ |
| $z$ | Altitude | $[ft]$ |
| $\%_{leak}$ | Leakage | [%] |

**Latin Characters**

| | | |
|---|---|---|
| $\varepsilon$ | Effectiveness | [%] |
| $\epsilon$ | Surface roughness | $[in]$ |
| $\eta$ | Efficiency | [%] |
| $\mu$ | Dynamic viscosity | $[\frac{lbf \cdot s}{ft^2}]$ |
| $\rho$ | Density | $[\frac{lbm}{ft^3}]$ |
| $\varphi$ | Relative humidity | [%] |
| $\omega$ | Humidity ratio | $[\frac{lbm_v}{lbm_a}]$ |

**Subscripts**

| | |
|---|---|
| 0 | Reference |
| $a$ | Dry air |
| $ac$ | Actual |
| $ann$ | Annual |
| $atm$ | Atmospheric |
| $cond$ | Condensation |
| $db$ | Dry bulb |
| $dew$ | Dew point |
| $eq$ | Equivalent |
| $f$ | Final |
| $i$ | Initial |

| | |
|---|---|
| $id$ | Ideal |
| $in$ | Inlet |
| $iso$ | Isothermal |
| $I$ | Intermediate |
| $L$ | Latent |
| $min$ | Minimum |
| $out$ | Outlet or exit |
| $pol$ | Polytropic |
| $s$ | Sonic |
| $sat$ | Saturation |
| $std$ | Standard |
| $S$ | Sensible |
| $v$ | Water vapor |
| $vol$ | Volumetric |
| $vs$ | Saturation vapor |

**Prefixes**

| | | |
|---|---|---|
| $\frac{d}{dt}$ | Rate of … change over time | $[\frac{\cdots}{sec}$ or $\frac{\cdots}{hr}]$ |
| $\frac{d}{dx}$ | Rate of … change over distance | $[\frac{\cdots}{ft}$ or $\frac{\cdots}{1000\ ft}]$ |
| $\Delta$ | Difference in … | |

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# CHAPTER I

# INTRODUCTION AND LITERATURE REVIEW

## Background

A brief explanation of the compressed air system and its components are explained in the following paragraphs. Additionally, the energy efficiency measures of the system, as well as the system controls are discussed at the end of this chapter.

## Introduction to Compressed Air

Compressed air is a form of stored potential energy that is used in most manufacturing and non-manufacturing industries. As the air compressor consumes electricity to compress air, about 91% of energy input will end up as losses, with most of this lost energy ending up as waste heat, which approximately accounts for 80% of the input energy as shown in Figure 1. With a generally low efficiency, compressed air is often used when it is dangerous or impractical to use electrical energy to directly supply power to tools and machineries. In addition, compressed air tools have the advantages of being lightweight, cheap, maintenance-free, and high-torque. There are many ways to improve the compressed air system efficiency, such as fixing of air leaks, lowering air pressure, adding more air storage capacity, as well as installing better compressors, dryers, and filters

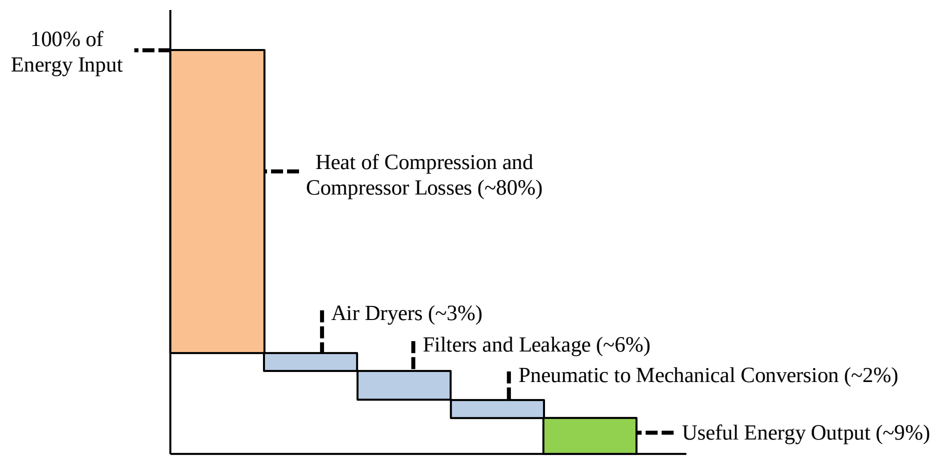



Figure 1. Compressed Air Energy Input and Useful Energy Output

### *Usage of Compressed Air*

The industrial and commercial application of compressed air is very diverse, and it is considered as the “fourth utility” at many industrial facilities. To improve and maintain peak compressed air system performance, both the supply and demand sides need to be evaluated carefully. A compressed air system that works efficiently will not only save electricity, but also decrease downtime, increase productivity, reduce maintenance, and improve product quality [1].

In industries, compressed air uses generally include powering pneumatic tools, packaging and automation equipment, and conveyors [2]. Some advantages that pneumatic tools have over electric motor-driven tools are, as mentioned earlier, smaller size, higher maneuverability, smoother power delivery, and higher durability on overloading. Compressed air powered tools are also safer as they produce less heat and no sparks. As listed in Table 1, there are many areas where compressed air system can be applied in manufacturing industrial sectors. Moreover, compressed air system is also a vital part in many non-manufacturing sectors; some examples are listed in Table 2.

Table 1. Industrial Sector Uses of Compressed Air [1]

| **Industry** | **Example Compressed Air Uses** |
|---|---|
| Apparel | Conveying, clamping, tool powering, controls and actuators, automated equipment |
| Automotive | Tool powering, stamping, control and actuators, forming, conveying |
| Chemicals | Conveying, controls and actuators |
| Food | Dehydration, bottling, controls and actuators, conveying, spraying coatings, cleaning, vacuum packing |
| Furniture | Air piston powering, tool powering, clamping, spraying, controls and actuators |
| General Manufacturing | Clamping, stamping, tool powering and cleaning, control and actuators |
| Lumber and Wood | Sawing, hoisting, clamping, pressure treatment, controls and actuators |
| Metals Fabrication | Assembly station powering, tool powering, controls and actuators, injection molding, spraying |
| Petroleum | Process gas compressing, controls and actuators |
| Primary Metals | Vacuum melting, controls and actuators, hoisting |
| Pulp and Paper | Conveying, controls and actuators |
| Rubber, and Plastics | Tool powering, clamping, controls and actuators, forming, mold press powering, injection molding |
| Stone, Clay, and Glass | Conveying, blending, mixing, controls and actuators, glass blowing and molding, cooling |
| Textiles | Agitating liquids, clamping, conveying, automated equipment, controls and actuators, loom jet weaving, spinning, texturizing |

Table 2. Non-Manufacturing Sector Use of Compressed Air [1]

| **Sector** | **Example Compressed Air Uses** |
|---|---|
| Agriculture | Farm equipment, materials handling, spraying of crops, dairy machines |
| Mining | Pneumatic tools, hoists, pumps, controls and actuators |
| Power Generation | Starting gas turbines, automatic control, emissions controls |
| Recreation | Amusement parks - air brakes |
| | Golf courses - seeding, fertilizing, sprinkler systems |
| | Hotels - elevators, sewage disposal |
| | Ski resorts - snow making |
| | Theaters - projector cleaning |
| | Underwater exploration - air tanks |
| Service Industries | Pneumatic tools, hoists, air brake systems, garment pressing machines, hospital respiration systems, climate control |
| Transportation | Pneumatic tools, hoists, air brake systems |
| Wastewater Treatment | Vacuum filters, conveying |

**Introduction to Industrial Compressed Air System**

Compressed air systems consist of a number of different components that can be classified into the supply and demand sides. The supply side includes the air filters, air compressors, aftercooler, receivers, and dryers. Air filters and air dryers are used to remove condensed moisture and contaminants from the compressed air system. To improve the system efficiency, the compressed air storage will be very useful. The accumulated condensed moisture can also be discharged through drains. Then, the demand side includes piping system and distribution, secondary storage and end use equipment. Distribution piping systems transport compressed air from the air compressor to the point of end use.

## Components of an Industrial Compressed Air System

A compressor is a machine that is used to increase the pressure of a gas. In this research work the compressed air system that will be discussed consist of a two-stage air compressor, an aftercooler, a receiver or storage tank, a dryer, as well as the piping, and regulator. A simple diagram showing how each of the component is going to be connected is shown in Figure 2 below.

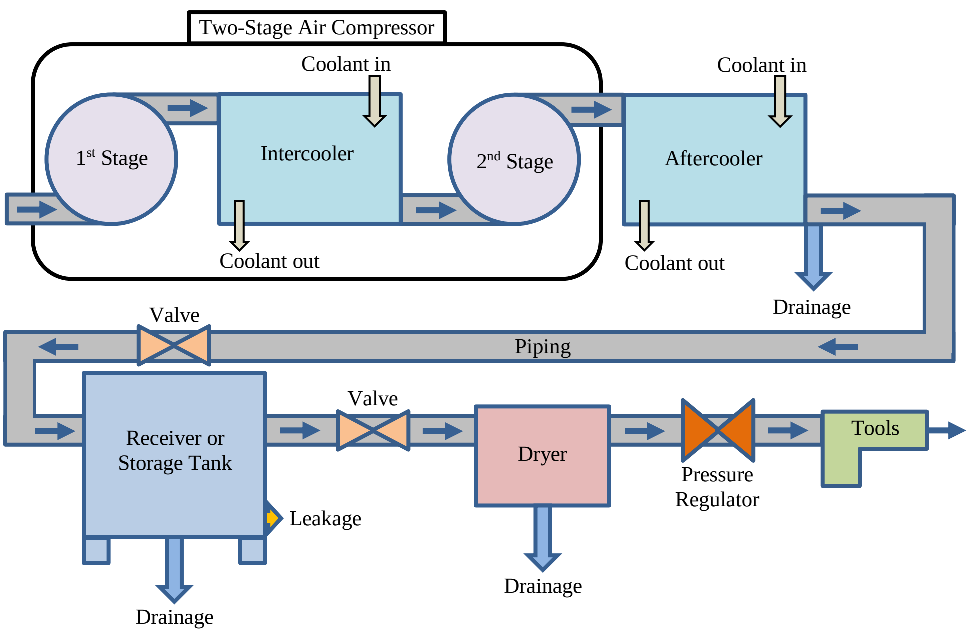



Figure 2. Compressed Air System Diagram

A compressed air system analysis can determine the ideal output of the system. The energy consumption of the air compressor can be calculated to determine the cost of operating the system. Then, the pressure drops caused by piping, valves, and other obstructions and friction within the distribution system, can be measured as well. With a component and system analysis based on mathematical models, the components that can be improved to increase the efficiency and save operational cost can be identified.

### *Air Inlet Filters*

An air inlet filter protects the compressed air system from atmospheric dirt, microorganisms, and other contaminants. They need to be replaced periodically to keep the system working efficiently. Finer or thicker filters would give a cleaner air, however it will increase the inlet filter pressure differential, which may reduce the air compressor output capacity and efficiency.

### *Air Compressors*

An air compressor converts electrical energy into potential energy stored in pressurized air. Most air compressor designs are available in either single-stage or multi-stage designs. The main advantage of utilizing a multi-stage air compressor over a single stage air compressor is that as the air temperature increases during compression, multi-stage compression allows the air to cool down between each stages, which saves the process work. Typical air compression process is isentropic, which means that no heat is being transferred into and out from the process. However, in a multi-stage compression, the partially compressed air will be cooled down after each stages. This cooling process between the stages is called intercooling, and it happens in a heat exchanger called the intercooler. The intercooling brings the compression process to isothermal process, which is a constant temperature condition relative to the initial temperature of the air. After intercooling the air, it will be further compressed in the next stage.

In this research work, the air compressor that will be discussed is a two-stage air compressor. A two-stage air compressor is a multi-stage air compressor with two compression stages and an intercooler in between.

### ***Compressed Air Coolers***

Approximately 80 percent of the electrical energy that a compressed air system consumes is converted into waste heat. As a result, industrial air compressors need a cooling mechanism to remove the heat from the system. The cooling unit is essentially a heat exchanger, where the coolant will receive the heat transferred out from the compressed air, and thus cools the compressed air temperature down. In general, the type of coolant that is being used is either air or water. The two type of cooling unit in a compressed air system are the intercooler, and the aftercooler.

#### *Intercoolers*

Most multi-stage compressors use intercoolers that remove the heat of compression between the stages of compression. Intercooler may affect the system efficiency as pressure dropping may occur within the intercooler. Furthermore, it is important to keep the flowing fluid from being cooled down past its dew point temperature, as doing so would result in condensation that may damage the compressor.

*Aftercoolers*

Aftercoolers are installed after the final stage of compression. Different from intercooler, the condensation that occurs as the air temperature reduces can be separated and drained out from the system. Therefore, with proper water separator and drainage system, the condensation will not damage the system.

### ***Compressed Air Separators, Traps, and Drains***

In the compressed air system, separator is an important device that separates the condensed moisture from the flowing air or gas. A separator normally is placed after the aftercooler. Next, a trap or drain will collect the separated water and remove the water from the compressed air system. A drain can be installed at many components of the compressed air system, such as the separators, filters, dryers and receivers.

### ***Compressed Air Receivers and Storages***

The purpose of a compressed air receiver is to provide store the compressed air and control system pressure by controlling the rate of pressure change in a system [1]. A compressed air receiver may also help cooling the compressed air, collecting the condensed moisture, as well as dampening the pressure pulsations.

### *Compressed Air Dryers*

Typically, the compressed air that leaves an aftercooler is saturated, and normally warmer than the ambient air. Cooling this saturated air further, which could happen along the distribution piping, may cause further condensation of moisture. At this point, any condensed moisture that is formed may corrode the piping system, and water contamination at point of end use. To prevent this problem, a compressed air dryer must be properly integrated to the compressed air system. Different types of compressed air dryers have different operating characteristics [1]. The dryer's rated capacity is also dependent of the gas inlet temperature and pressure. Fox example, as the gas inlet temperature increases or the gas inlet pressure decreases, the dryer's rated capacity decreases. Furthermore, it is important to notice that compressed air dryer may cause pressure drops, and affect the system efficiency. One of the primary purposes of the compressed air dryer is to reduce the dew point temperature of the air so that when the compressed air is reduced to atmospheric pressure and a lower temperature, condensation or frosting will not occur.

### *Pressure / Flow Controllers and Regulators*

The flow regulators are normally located at the output of the compressed air system. They control the discharged compressed air so that it would have the desired pressure and flow rate to operate the compressed air tools and equipment.

### *Filter Regulator Lubricator Devices*

A lubricator is normally installed near the point of end use to lubricate the compressed air tools and other equipment. The lubricator may be combined with a filter and a pressure regulator to make up what is commonly called a FRL (filter-regulator-lubricator) devices [2].

### *Fittings*

Fittings and couplings for air hoses need to be durable and airtight. A badly installed coupling may cause major pressure drops. To minimize the pressure difference, the hoses and connectors can be upsized. Figure 3 gives an indication of the approximate pressure drop for selected fittings in terms of equivalent length of straight pipe.

| Friction Loss Equivalent Length - feet of Straight Pipe (ft) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Fitting | Nominal Pipe Size (inches) | | | | | | | |
| | 0.50 | 0.75 | 1.00 | 1.50 | 2.00 | 3.00 | 4.00 | 6.00 |
| 90° Elbow | 1.5 | 2 | 2.5 | 4 | 5.7 | 7.9 | 12 | 18 |
| 45° Elbow | 0.8 | 1.1 | 1.4 | 2.1 | 2.6 | 4 | 5.1 | 8 |
| Gate valve | 0.3 | 0.4 | 0.6 | 1 | 1.5 | 3 | 4.5 | 6.5 |
| Tee Flow – Run | 1 | 1.4 | 1.7 | 2.7 | 4.3 | 6.2 | 8.3 | 12.5 |
| Tee Flow - Branch | 4 | 5 | 6 | 8 | 12 | 16 | 22 | 32.7 |
| Male /Female Adapter | 1 | 1.5 | 2 | 3.5 | 4.5 | 6.5 | 9 | 14 |

Figure 3. Friction Loss Equivalent Lengths [2]

### *Compressed Air Piping and Distribution System*

The air distribution system links different components of the compressed air system from the inlet to the point of end use. The system often consist of a network of piping lines, valves, drainages, and air hoses. Each piping, valves, drains, and air hoses contributes to the pressure drops of the compressed air flowing within the distribution system. As the pressure drop has to be kept minimum to increase the compressed air system efficiency, the network should also be kept as short as possible. Additionally, condensation may occur within the pipe as the outside temperature is usually lower than the temperature inside the pipes; therefore an air dryer is required to keep the pipes and valves dry to prevent corrosion.

## Energy Efficiency Measures of Industrial Compressed Air System

This section describes some measures to determine potential factors that could reduce the compressed air system efficiency, and to improve the energy efficiency of compressed air systems.

### *Compressed Air System Pressure Drop*

As the compressed air travels through the compressed air distribution system, any type of obstruction and roughness may cause the pressure of the air flow drops. Some items or components within the compressed air system that may cause pressure drops includes:

- Supply side: air filters, aftercoolers, moisture separators, air dryers, and filters.

- Demand side: air hoses, connections, filters, regulators, and lubricators (FRLs).

A well designed compressed air system should have a pressure drop less that ten percent of the air compressor's discharge pressure, which is measured from the receiver or storage tank output to the point of end use [1].

### ***Compressed Air System Leakage***

Air leakage can be a major cause to decreasing compressed air system efficiencies. Leaks may occur at any point of the compressed air system, however, the joints and connections are the locations where leaks are most likely to occur. Stopping leaks can be as simple as tightening a connection or as complicated as replacing defective equipment, such as fittings, pipe sections, joints, and drains [1].

# CHAPTER II

# PROBLEM DESCRIPTION

In CHAPTER III through CHAPTER X, various assumptions, variables, and unknown elements of each components that comprises the complete compressed air system will be defined, different mathematical models will be derived, and the solution processes for design and/or analysis variables will be posed. Explanations on how each of the calculated components works together will also be given.

For the simple compressed air system with a two-stage air compressor system, the temperatures, pressures, and flow rates of the moist air at each points or components are important variables for determining whether the system is working properly, with regards to the system efficiency, or if the components have been sized and designed optimally.

The scope of this work is to develop, formulate, and derive mathematical models of moist air and components of the compressed air system. Depending on the known and input variables, the model can be used for both analysis and design of a compressed air system, and each of its components. For example, if the compressor's flow rate and outlet pressure are known, then the model will be able to analyze the pressure drop reaching the tools through a given pipe length and diameter. Conversely, in a different case if the desired pressure drop is known, the model will be able to design and calculate the appropriate pipe diameter.

Additionally, the model will also be able to determine the required outlet pressure to power compressed air tools and equipment. With the calculated outlet pressure, the air compressor input power can be adjusted so that it would supply the correct amount of pressurized air, and prevent it from overworking, thus increasing and optimizing the compressed air system efficiency.

The MATLAB codes is described, and included in the Appendix pages of this thesis, where the APPENDIX A will cover the explanations of the functions that are used to handle the commonly used formulas applicable to the whole compressed air system analysis.

# CHAPTER III

# MOIST AIR DESCRIPTION, MODEL AND ANALYSIS

## Background

The air entering the compressed air system, usually at outdoor condition, is at a state defined by any combination of properties including pressure, temperature, density, and relative humidity. As the air passes through the compressed air system, its properties of pressure, temperature, density and humidity will keep changing depending on the process that the air experiences. The ideal gas assumption and equations apply to the compressed air – and along with the energy and mass conservation equation – are used to calculate the properties of the air at each point within the compressed air system.

Other important parameters associated with the compressed air such as the gas constants, specific heats, humidity ratios, specific humidities, and flow rates may or may not vary in the system. However, they must be determined to analyze the state of the air at each point (usually defined as the inlet and outlet of an important component) that is being processed. In this research work, moist air, which is a mixture of dry air and water vapor, will be the fluid of interest; however, other fluids such as liquid water and oil are present in some component.

Lastly, the discussion of the MATLAB codes for calculations related to this chapter can be found in the APPENDIX B. The example problems that will be discussed in this chapter will also be covered in the APPENDIX B.

**Ideal Gas and Moist Air Equation**

Equations (1) and (2) below are the ideal gas equations for dry air and water vapor. While, Equations (3) and (4) show how the specific gas constant is related to the molar mass and the universal gas constant.

$$P_a\, V = m_a\, R_a\, T\ , \tag{1}$$

$$P_v\, V = m_v\, R_v\, T\ , \tag{2}$$

$$R_a = \frac{R_u}{M_a}\,, \tag{3}$$

and

$$R_v = \frac{R_u}{M_v}\,, \tag{4}$$

where: • $V$ is the gas volume [$ft^3$],

• $T$ is the gas temperature [$R$ or °F],

• $P_a$ is the dry air partial pressure [$psia$],

• $P_v$ is the water vapor partial pressure [$psia$],

• $m_a$ is the mass of the dry air [$lbm$],

• $m_v$ is the mass of the water vapor [$lbm$],

• $M_a$ is the molar mass of the dry air [$\frac{lbm}{lbmol}$],

• $M_v$ is the molar mass of the water vapor [$\frac{lbm}{lbmol}$],

• $R_u$ is the universal gas constant [$\frac{BTU}{lbmol\cdot R}$ or $\frac{lbf\cdot ft}{lbmol\cdot R}$],

• $R_a$ is the specific gas constant of the dry air [$\frac{BTU}{lbm\cdot R}$ or $\frac{lbf\cdot ft}{lbm\cdot R}$], and

• $R_v$ is the specific gas constant of the water vapor [$\frac{BTU}{lbm \cdot R}$ or $\frac{lbf \cdot ft}{lbm \cdot R}$].

The molar mass of the dry air $M_a$, which consist of $78.09\%$ of nitrogen, $20.95\%$ of oxygen, and about $1\%$ of mixture of argon, carbon dioxide, and other gases, is equal to $28.97 \frac{lbm}{lbmol}$. The molar mass of the water vapor $M_v$, which is the gaseous phase of water, is equal to $18.02 \frac{lbm}{lbmol}$. Also, the universal gas constant $R_u$ is known to be $1{,}545 \frac{lbf \cdot ft}{lbmol \cdot R}$.

The ideal gas equation can also be written in terms of the gas specific volume $v$ as shown in Equations (5) and (6) below.

$$P_a \, v_a = R_a \, T \, , \tag{5}$$

and

$$P_v \, v_v = R_v \, T \, , \tag{6}$$

where: • $v_a$ is the dry-air specific volume [$\frac{ft^3}{lbm}$], and

• $v_v$ is the water-vapor specific volume [$\frac{ft^3}{lbm}$].

Also, the total pressure of the gas should equal to the sum of the partial pressure of the dry air $P_a$, and the water vapor $P_v$. Then, the following Equation (7) shows the relationship between the gas specific volume $v$, the gas density $\rho$, and the ideal gas equation.

$$v = \rho^{-1} = \frac{V}{m} = \frac{R \, T}{P} \, , \tag{7}$$

where: • $v$ is the gas specific volume [$\frac{ft^3}{lbm}$],

• $\rho$ is the gas density [$\frac{lbm}{ft^3}$],

• $m$ is the gas mass [$lbm$],

• $R$ is the gas constant [$\frac{BTU}{lbm \cdot R}$ or $\frac{lbf \cdot ft}{lbm \cdot R}$], and

• $P$ is the gas pressure [$psia$].

**Gas Constant and Specific Heat**

The gas constant $R$ is related to the gas specific heat for constant pressure $C_p$, and the gas specific heat for constant volume $C_v$, whose difference, as described in Equation (8), is the gas constant $R$.

$$R = C_p - C_v \, , \tag{8}$$

where: • $C_p$ is the gas specific heat for constant pressure [$\frac{BTU}{lbm \cdot R}$], and

• $C_v$ is the gas specific heat for constant volume [$\frac{BTU}{lbm \cdot R}$].

Another important parameter is the specific heat ratio $k$, which represents the ratio between the gas specific heat in constant pressure $C_p$, and the gas specific heat in constant volume $C_v$ as described in Equation (9).

$$k = \frac{C_p}{C_v} \, , \tag{9}$$

where $k$ is the gas specific heat ratio [unitless].

For dry air, the $k$ value is equal to 1.40, as shown on Table 3 below. Moist air would have various $k$ values since the presence of the water vapor mixed with the dry air could influence the property values of the moist air. For example, the $k$ value for steam where most of the process occurs in the wet region is 1.135, and the $k$ value for superheated steam is 1.33. Equations (10) and (11) below describe the additional relationship between $R$, $C_p$, $C_v$, and $k$.

$$C_p = \frac{k\,R}{k-1}, \tag{10}$$

and

$$C_v = \frac{R}{k-1} \tag{11}$$

In addition, some constant properties of dry air and water vapor are listed in Table 3 below.

Table 3. Constant Properties of Dry Air and Water Vapor

| Properties | Dry Air | Water Vapor |
|---|---|---|
| $M$ | $28.97\,\frac{lbm}{lbmol}$ | $18.02\,\frac{lbm}{lbmol}$ |
| $\rho$ | $0.075\,\frac{lbm}{ft^3}$ | $62.4\,\frac{lbm}{ft^3}$ |
| $C_p$ | $0.240\,\frac{BTU}{lbm\cdot R}$ | $0.446\,\frac{BTU}{lbm\cdot R}$ |
| $C_v$ | $0.172\,\frac{BTU}{lbm\cdot R}$ | $0.335\,\frac{BTU}{lbm\cdot R}$ |
| $R$ | $0.069\,\frac{BTU}{lbm\cdot R}\ or\ 53.33\,\frac{lbf\cdot ft}{lbm\cdot R}$ | $0.110\,\frac{BTU}{lbm\cdot R}\ or\ 85.78\,\frac{lbf\cdot ft}{lbm\cdot R}$ |
| $k$ | 1.40 | 1.33 |

**Humidity Ratio and Relative Humidity**

Another important property of the gas is the humidity ratio $\omega$, which is the ratio of the water vapor mass $m_v$ and the dry air mass $m_a$. It can be calculated by using Equation (12) that is shown below.

$$\omega = \frac{m_v}{m_a}, \tag{12}$$

where $\omega$ is the gas humidity ratio [unitless].

Using the ideal gas equation (Equations (1) and (2)), Equation (12) can be used to derive Equation (13).

$$\omega = \frac{M_v\, P_v}{M_a\, P_a} \tag{13}$$

Substituting the molar masses of the water vapor $M_v$ and the dry air $M_a$ with their respective numerical values results in Equation (14).

$$\omega = \frac{0.622\, P_v}{P - P_v} \tag{14}$$

With $\varphi$ as the gas relative humidity and $P_{vs}$ as the saturation vapor partial pressure, the partial pressure of water vapor $P_v$ can be calculated by using Equation (15) shown below.

$$P_v = \varphi\, P_{vs}\,, \tag{15}$$

where: • $\varphi$ is the gas relative humidity [%], and

• $P_{vs}$ is water vapor saturation partial pressure [$psia$].

Also, combining Equations (14) and (15) renders Equation (16).

$$\omega = \frac{0.622\, \varphi\, P_{vs}}{P - \varphi\, P_{vs}} \tag{16}$$

Moreover, at the saturation point where the gas relative humidity $\varphi$ is equal to $100\%$, the saturation pressure and temperature of the water vapor can be found by using Table 4 below.

Table 4. Saturated Steam Temperature - Pressure

| Temperature | Pressure (Saturation) | Temperature | Pressure (Saturation) | Temperature | Pressure (Saturation) |
|---|---|---|---|---|---|
| °F | $psia$ | °F | $psia$ | °F | $psia$ |
| 32 | 0.08871 | 110 | 1.2767 | 250 | 29.844 |
| 40 | 0.12173 | 120 | 1.6951 | 300 | 67.028 |
| 50 | 0.17812 | 130 | 2.2260 | 350 | 134.63 |
| 60 | 0.25638 | 140 | 2.8931 | 400 | 247.26 |
| 70 | 0.36334 | 150 | 3.7234 | 450 | 422.47 |
| 80 | 0.50745 | 170 | 5.9999 | 500 | 680.56 |
| 90 | 0.69904 | 200 | 11.538 | 600 | 1542.5 |
| 100 | 0.95052 | 212 | 14.709 | 700 | 3093.0 |

The relationship between the gas temperatures $T$, humidity ratios $\omega$, relative humidity $\varphi$, and other gas properties such as the gas wet bulb temperature, specific volume $v$, and enthalpy can also be observed in Figure 4 below. It is important to note that this psychometric chart only applies to the compressor inlet at atmospheric pressure.

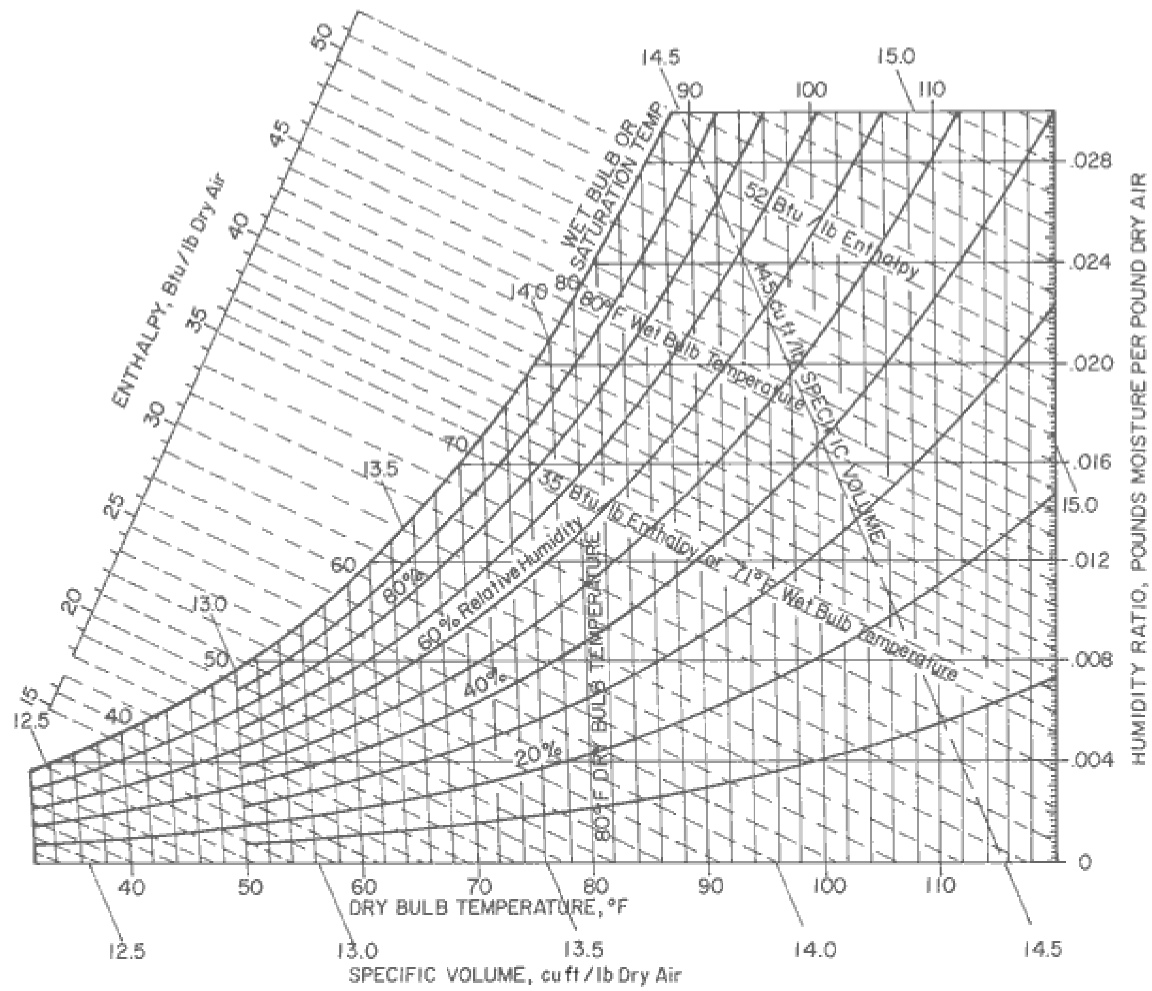



Figure 4. Moist Air Psychometric Chart [3]

It is important to know that the specific humidity ratio $\omega$ of the compressed air is always constant within the compressor because no condensation occurs, which if it did would damage the air compressor. However, in the aftercooler component, which is downstream of the compressor, condensation is allowed to form with the component having a separator and drain system that remove the condensed moisture out from the system. The condensation generated will be explained in the CHAPTER V.

## Volume and Mass Flow Rates

The volume flow rate of air is normally measured, assumed, or calculated, where the relationship between the gas volume flow rate $\dot{V}$, the pipe diameter $D$, and the gas flow velocity $\bar{V}$ is shown in the following Equation (17).

$$\dot{V} = \frac{\pi}{4} D^2 \, \bar{V} \, , \tag{17}$$

where: • $\dot{V}$ is the gas volume flow rate [$\frac{ft^3}{sec}$],

• $D$ is the pipe diameter $[in]$, and

• $\bar{V}$ is the gas velocity [$\frac{ft}{sec}$].

Rearranging Equation (17), the mass flow rate of the dry air can be calculated by using Equations (18) and (19).

$$\dot{m}_a = \frac{\pi}{4} D^2 \, \bar{V} \, \rho \, , \tag{18}$$

and

$$\dot{m}_a = \dot{V} \, \rho \, , \tag{19}$$

where $\dot{m}_a$ is the mass flow rate of the dry air [$\frac{lbm}{sec}$].

Furthermore, the mass flow rate for the water vapor can be calculated by using Equation (20).

$$\dot{m}_v = \omega \, \dot{m}_a \, , \tag{20}$$

where $\dot{m}_v$ is the mass flow rate for the water vapor [$\frac{lbm}{sec}$].

**Condensation of Water Vapor in the System**

Condensation of water vapor to liquid water can occur anywhere in a compressed air system, when moist air – a mixture of water vapor and dry air – is cooled down to its dew point temperature $T_{dew}$. However, the psychometric chart that is often used to find dew point temperatures, as shown in Figure 4, is only applicable for the inlet moist air, whose pressure is the atmospheric pressure condition. Therefore, once the inlet air enters the air compressor, the psychometric equations – rather than the psychometric chart – must be used to relate moist air properties such as the pressure $P$, temperature $T$, specific volume $v$, humidity ratio $\omega$, and relative humidity $\varphi$.

Regardless of the pressure or the location in the compressed air system, when the moist air temperature $T$ at any given values of the dry bulb temperature $T_{db}$, and the relative humidity $\varphi$ is cooled to its dew point temperature $T_{dew}$, the moist air will reach its saturation point, or in other words, its relative humidity $\varphi$ will become $100\%$. If the moist air temperature $T$ drops any further, then the water vapor contained within the moist air will condense into liquid water. Figure 5 below shows a plot of the dew point temperature $T_{dew}$ versus the dry bulb temperature $T_{db}$ for various values of relative humidity, assuming the moist air is at atmospheric pressure. It can be seen in Figure 5 that the dew point temperature $T_{dew}$ increases as the relative humidity $\varphi$ increases, for any given air temperature. For example, the dew point temperature $T_{dew}$ for moist air a typical room temperature of $70$°F is about $62$°F at $80\%$ relative humidity, and about $44$°F at $40\%$ relative humidity; meaning that at $80\%$ relative humidity, the moist air can be cooled down to $62$°F before condensation could occur, however at $40\%$ relative humidity, the moist air can be cooled down even further to 44°F before condensation could occur. Figure 5 below is based on the definition of the relative humidity, which is represented by Equation (15), $P_v = \varphi\, P_{vs}(T_{db})$ , where the water vapor saturation partial pressure $P_{vs}$ is in a function of the moist air dry bulb temperature $T_{db}$. Furthermore, the dew point is defined as the saturation temperature for the partial pressure of the water vapor in the moist air $P_v$. Again, it should be noted that Figure 5 only applies to the atmospheric pressure, mainly because the numerical values of the relative humidity $\varphi$ shown are only applicable to atmospheric pressure condition.

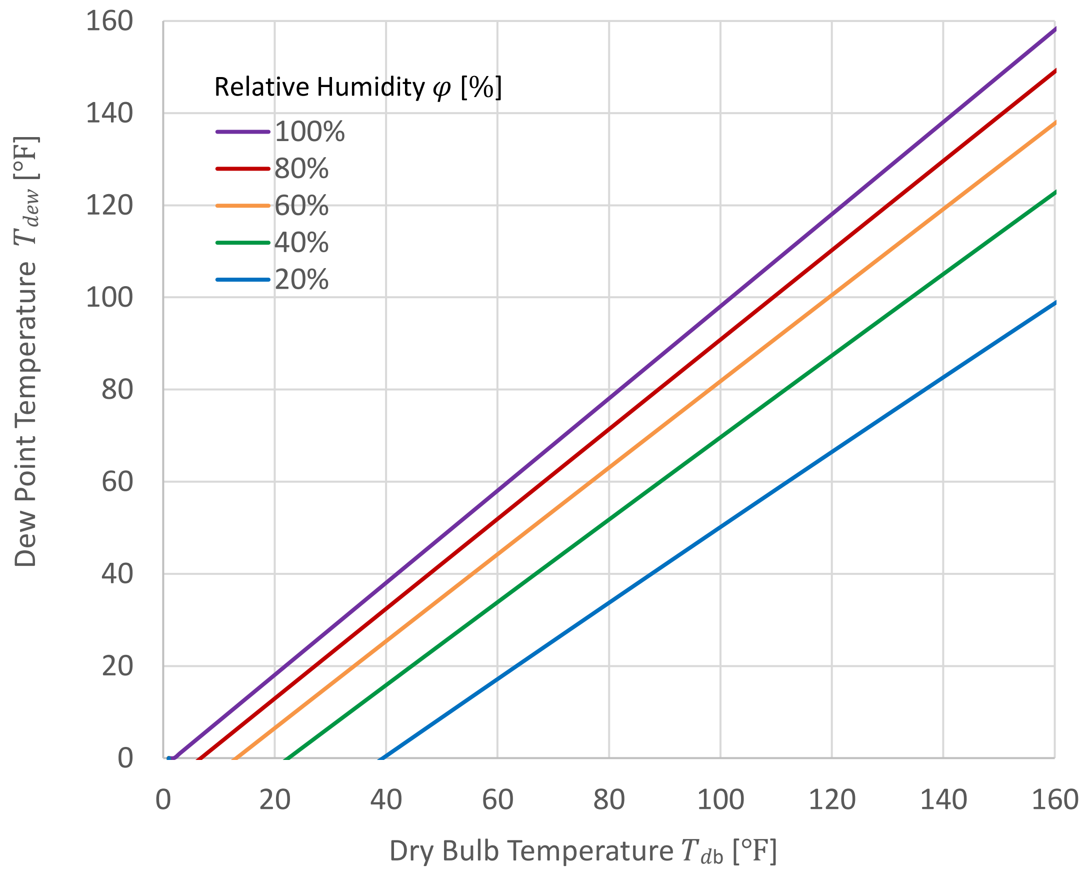



Figure 5. Dew Point Temperature Chart

The relationship between the dew point temperature $T_{dew}$, the air dry bulb temperature$T_{db}$, and the relative humidity $\varphi$, at atmospheric pressure condition (14.7 $psia$), can also be described by using Equations (21) or (22) or (23).

$$T_{dew} = \frac{564 + 405\left(\frac{17.6(T_{db} - 32)}{T_{db} + 405} + \ln(\varphi)\right)}{17.6 - \left(\frac{17.6(T_{db} - 32)}{T_{db} + 405} + \ln(\varphi)\right)}, \quad (21)$$

or

$$T_{db} = \frac{564 + 405\left(\frac{17.6(T_{dew} - 32)}{T_{dew} + 405} - \ln(\varphi)\right)}{17.6 - \left(\frac{17.6(T_{dew} - 32)}{T_{dew} + 405} - \ln(\varphi)\right)}, \quad (22)$$

or

$$\varphi = e^{\left(\frac{17.6(T_{dew} - 32)}{T_{dew} + 405} - \frac{17.6(T_{db} - 32)}{T_{db} + 405}\right)}, \quad (23)$$

where: • $\varphi$ is the relative humidity [%],

• $T_{dew}$ is the dew point temperature [°F], and

• $T_{db}$ is the air dry bulb temperature [°F].

These Equations (21) through (23) are approximations using the August-Roche-Magnus formula [4] that has been modified from the original SI units to be applicable to the United States customary units. The following example calculation is for the moist air condensation with dew point temperature $T_{dew}$ of 44°F, and dry bulb temperature $T_{db}$ of 70°F, which produces a relative humidity $\varphi$ of 39.2% - close to 40% from the plot in Figure 5.

$$\varphi = e^{\left(\frac{17.6(T_{dew} - 32)}{T_{dew} + 405} - \frac{17.6(T_{db} - 32)}{T_{db} + 405}\right)}$$

$$\varphi = e^{\left(\frac{17.6(44°\text{F} - 32)}{44°\text{F} + 405} - \frac{17.6(70°\text{F} - 32)}{70°\text{F} + 405}\right)} = 39.2\%$$

From the above discussion, it can be seen how water vapor can be condensed when the dew point temperature $T_{dew}$ is reached as the moist air is cooled down from starting dry bulb temperature $T_{db}$ and relative humidity $\varphi$. However, this process is less obvious for the high pressure situation of the compressed air system, especially considering that the psychometric chart in Figure 4, and the dew point temperature chart in Figure 5 are not applicable.

To show that water vapor will immediately condense, or has the potential to condense throughout any components of the compressed air system, the entering moist air should be the starting point of the analysis. Rather than focusing on the relative humidity $\varphi$ that can change with the pressure, it is better to focus on the humidity ratio $\omega$, which remains constant with pressure changes, and only varies when the water vapor of the moist air is condensed.

Therefore, assuming that condensation is not occurring, meaning that the moist air temperature is above the dew point $T_{dew}$, then this specific humidity ratio of the moist air $\omega$ remains constant throughout the air compressor and the high pressure side of the compressed air system. Furthermore, with the humidity ratio $\omega$ being constant, the dew point temperature $T_{dew}$ is only a function of pressure and can be calculated by using equations introduced before. To illustrate how the dew point temperature $T_{dew}$ changes in the system, the air compressor can be divided into three locations to be focused on. Those three locations are the inlet, the intermediate, and the outlet regions of the air compressor; each with its own pressure condition, but has the same humidity ratio as the other locations. The intermediate pressure $P_I$ of the compressor, which is the compressed air pressure inside the intercooler, is equal to the square root of the product of the inlet pressure $P_{in}$ and the outlet pressure $P_{out}$ as shown in Equation (24).

$$P_I = \sqrt{P_{in} \cdot P_{out}}\,, \tag{24}$$

where: • $P_{in}$ is the inlet pressure into the air compressor [$psia$],

• $P_I$ is the intermediate pressure inside the air compressor [$psia$], and

• $P_{out}$ is the outlet pressure from the air compressor [$psia$].

As an example, with the inlet air being taken directly from the outdoor moist air, the inlet pressure $P_{in}$, temperature $T_{in}$, and relative humidity $\varphi$ can be assumed to be $14.7$ $psia$, $70$°F, and $50\%$, respectively. For the assumed dry bulb air temperature $T_{db}$ of $70$°F, the water vapor saturation partial pressure $P_{vs_{in}}$ would be $0.36334$ $psia$, as listed in the saturated steam table in Table 4. The water vapor partial pressure $P_{v_{in}}$ can be calculated using Equation (15) as follows:

$$P_{v_{in}} = \varphi_{in}\, P_{vs_{in}}$$

$$P_{v_{in}} = (50\%)(0.36334\, psia) = 0.182\, psia$$

With an atmospheric inlet pressure of $14.7$ $psia$, and if the outlet pressure $P_{out}$ is assumed to be $150$ $psia$, the intermediate pressure $P_I$ would be:

$$P_I = \sqrt{P_{in} \cdot P_{out}}$$

$$P_I = \sqrt{(14.7\, psia) \cdot (150\, psia)} = 47\, psia$$

The psychometric chart in Figure 4 can be used to find the inlet specific humidity ratio $\omega$ at the compressor, or it can be calculated using Equation (16) as follows:

$$\omega = \frac{0.622\ \varphi\ P_{vs}}{P_{in} - \varphi\ P_{vs}}$$

$$\omega = \frac{0.622(50\%)(0.36334\ psia)}{(14.7\ psia) - (50\%)(0.36334\ psia)} = 0.00778\frac{lbm_v}{lbm_a}$$

Again, this value of the inlet specific humidity ratio $\omega$ is assumed to be the same at all locations in the air compressor, regardless of the dry bulb temperature of the moist air being compressed. As such, the water vapor partial pressure $P_v$ at pressures higher than the atmospheric inlet pressure ($14.7\ psia$) can be calculated using Equation (14) as follows:

$$\omega = \frac{0.622\ P_v}{P - P_v},$$

and after rearranging, it became

$$P_v = \frac{\omega\ P}{\omega + 0.622}$$

Once the partial pressure $P_v$ is obtained, then its corresponding saturation temperature would be the dew point temperature $T_{dew}$, based on a condition that is reaching a saturation at $100\%$ relative humidity $\varphi$. Therefore, the partial pressure of the intermediate $P_{v_I}$ and the outlet $P_{v_{out}}$ sections can be calculated as follows:

$$P_{v_I} = \frac{\omega\ P_I}{\omega + 0.622}$$

$$P_{v_I} = \frac{\left(0.00778\frac{lbm_v}{lbm_a}\right)(47\ psia)}{\left(0.00778\frac{lbm_v}{lbm_a}\right) + 0.622} = 0.580\ psia\ ,$$

and

$$P_{v_{out}} = \frac{\omega\, P_{out}}{\omega + 0.622}$$

$$P_{v_{out}} = \frac{\left(0.00778\,\frac{lbm_v}{lbm_a}\right)(150\; psia)}{\left(0.00778\,\frac{lbm_v}{lbm_a}\right) + 0.622} = 1.854\; psia$$

Again, using the saturated steam table in Table 4 to find the saturation temperature corresponding to each partial pressures would results in the dew point temperature $T_{dew}$ at each locations as shown in the summary below:

| | | |
|---|---|---|
| Inlet section: | $P_{v_{in}} = 0.182\; psia$ | $T_{dew_{in}} = 50.5$ °F |
| Intermediate section: | $P_{v_I} = 0.580\; psia$ | $T_{dew_I} = 83.8$ °F |
| Outlet section: | $P_{v_{out}} = 1.854\; psia$ | $T_{dew_{out}} = 123.0$ °F |

This simple example of moist air entering an air compressor at a typical real world representation of inlet condition demonstrates quite vividly how the dew point temperature $T_{dew}$ changes with pressure in a compressed air system. Furthermore, the resulting dew point temperature $T_{dew}$ illustrates that at the low inlet atmospheric pressure, the 50.5 °F dew point temperature is not likely to produce condensation, but at higher pressures, where the dew point temperature ranges from 83.8 °F to 123.0 °F, condensation is more likely to occur, as the compressed air losses heat to the surrounding at 70 °F.

The fact that the dew point temperature $T_{dew}$ increases as moist air is compressed is one of the most important concepts in compressed air, namely that if the temperature of the high-pressure air goes below the dew point, then liquid water will be formed. Such event would create problems or forcing the system designs to accommodate this condensation. It should be noted that even though – in this example – the air enters the compressor at 70°F, the compression process will make the temperature of the exiting compressed air to be higher than its dew point temperature, which prevent condensation from occurring inside the air compressor. As a result, most of the condensation issues in a compressed air system occurs downstream of the compressor exit

Several important issues related to dew point temperature and condensation are summarized below. In some cases, more details will be presented in the various chapters that deal with components.

1. The intercooler between the 1st and the 2nd stages of the air compressor is used to decrease the compressed air temperature prior to entering the 2nd stage in order to increase the compressor efficiency. However, while being cooled, the air temperature must always be kept above the dew point temperature, which in this example is 84 °F, because going below this temperature will result in condensation, and the condensed liquid water could mix with the lubricating oil causing damage to the air compressor. Assuming that the intercooler is cooled by the 70 °F outside air, then it is certainly possible for the compressed air to go below its dew point temperature, therefore an appropriate intercooler design is needed to prevent the condensation.
2. The air exiting the 2nd stage air compressor is extremely hot, as it often exceed 300°F depending on the air compressor efficiency, the pressure ratio, and the 2nd stage air compressor inlet temperature. This exiting hot air immediately flows to the aftercooler where the temperature is expected to decrease below its dew point temperature, which in this example is 123 °F. Large amount of condensate will be formed in the aftercooler, and must be drained and discarded following appropriate environmental

regulations. As a final note, because the exiting air is saturated, its temperature is a dew point temperature, which is lower than the original 123 °F dew point temperature while also has lower specific humidity ratio $\omega$.

3. The saturated air (100% relative humidity) exiting the aftercooler is still warmer than the 70 °F surrounding air, and will be discharged to the accumulators, or to the system supply piping, where it cools further. For example, since the accumulator – or the receiver – metal walls are exposed to the outside room air temperature (typically 70 °F), then the entering air will be cooled further below its dew point temperature, causing water condensate to form in the accumulator. As a result, accumulators are designed to have water traps and drains installed at their base to capture and remove this condensate.
4. Not all of the compressed air leaving the aftercooler will be stored in the accumulator, where it cools down below its dew point temperature. Rather, depending on the supply and demand air flow rate balance, additional condensation may formed as the compressed air also flows directly to the supply piping system, whose pipe walls are exposed to the lower temperature environment, which is often the 70 °F conditioned space in a building. It is important to note that in this example the piping system is in a 70 °F conditioned space; however, piping can also run outdoor in a colder environment resulting in an even lower temperature. Regardless of the pipe wall exposure, as the compressed air is distributed throughout the system, its temperature continually falls below its dew point temperature, which follows the falling dry bulb temperature. To accommodate this water condensation build up in pipes, the

compressed air piping system should always be mounted at a downward angle to drain the liquid water to the traps and drains that are installed at a fixed piping-length interval, specified by the design guidelines.

5. The above four components, namely the intercooler, the aftercooler, the accumulator (or receiver), and the piping system, can all change the dew point temperature, along with the moist air specific humidity ratio, by producing condensation in the compressed air system. The fifth component, namely the drying system also alters the dew point temperature by using any one of several different techniques and approaches. Whereas the above components change the dew point temperature and the specific relative humidity ratio by condensation, the drying system usually lowers the dew point temperature by directly removing water vapor from the moist air. These technologies that remove water vapor without condensation are discussed elsewhere; however, one exception is that drying can also take place by using a refrigeration system that cools the air down causing condensation, and thus reducing the dew point temperature, even as low as the freezing point temperature of water. As will be discussed later, the reason for reducing the dew point temperature in a dryer is that when compressed air expands in a tool or other mechanical devices at the exit, then significant drops in temperature may result that can produce condensation in high-speed air motors, resulting in damage and failure.
6. The two other places that can reduce the dew point temperature due to pressure changes, and/or cause condensation due to the dry bulb temperature drops are the pressure regulators, and the operating tools or devices supplied by the air from the

compressed air system. The expansion of compressed air to a lower pressure can reduce the dew point temperature, just as pressure increase can increase the dew point temperature as well. Simultaneously, the expansion of compressed air can also reduce the air dry bulb temperature, and if the dry bulb temperature goes below the dew point temperature, then condensation will occur.

Because of the importance of the dew point temperature, which can result in condensation anywhere in a compressed air system, relative to the system's actual dry bulb temperature, tables and plots of the dew point temperature for different pressures are valuable for providing operating limits and guidelines. Using the same methodology used previously to determine the dew point temperatures (of $51°\mathrm{F}$, $84°\mathrm{F}$, and $123°\mathrm{F}$) for the three pressures (of $14.7\, psia$, $47\, psia$ and $150\, psia$) in the example, the plots of dew point temperature for different inlet temperatures and relative humidities can be created. As noted before, the basic assumption that must exist is that the specific humidity ratio at atmospheric pressure remains constant as the air pressure increases, which is reasonable as long as condensation does not occur.

Figure 6, Figure 7, and Figure 8 below show three different inlet temperature of low (50°F), medium (70°F), and high (90°F) that can be expected at the inlet of an air compressor. The inlet air temperature from a conditioned space is expected to be the medium temperature (70°F), while the two other temperatures could be the outdoor air temperatures representing two different seasons.

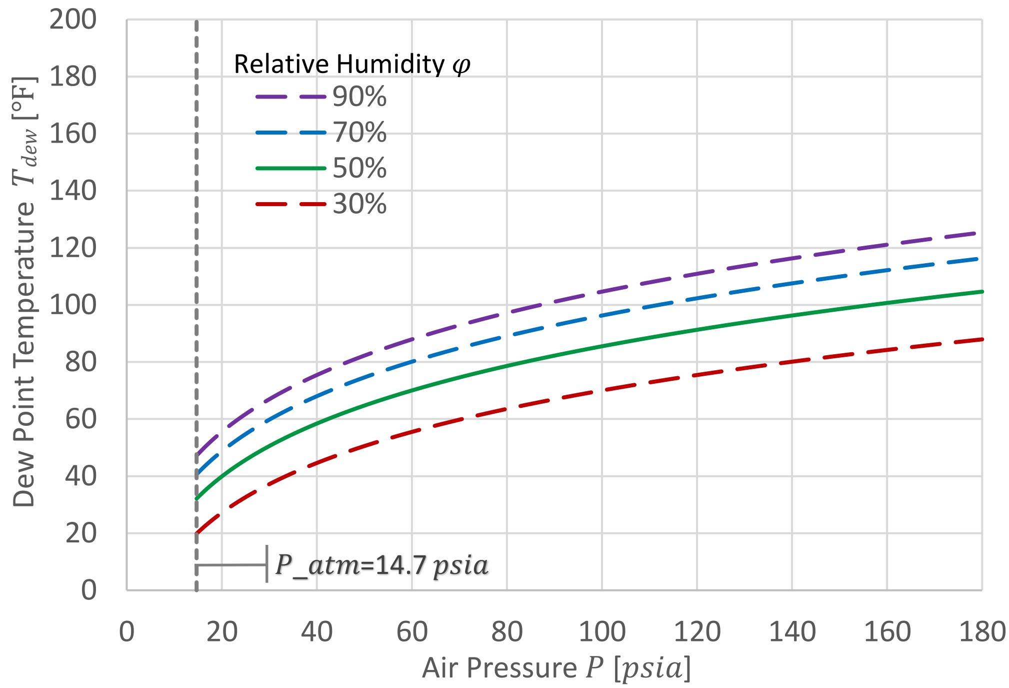



Figure 6. Dew Point Temperature Chart for Low Inlet Temperature (50°F)

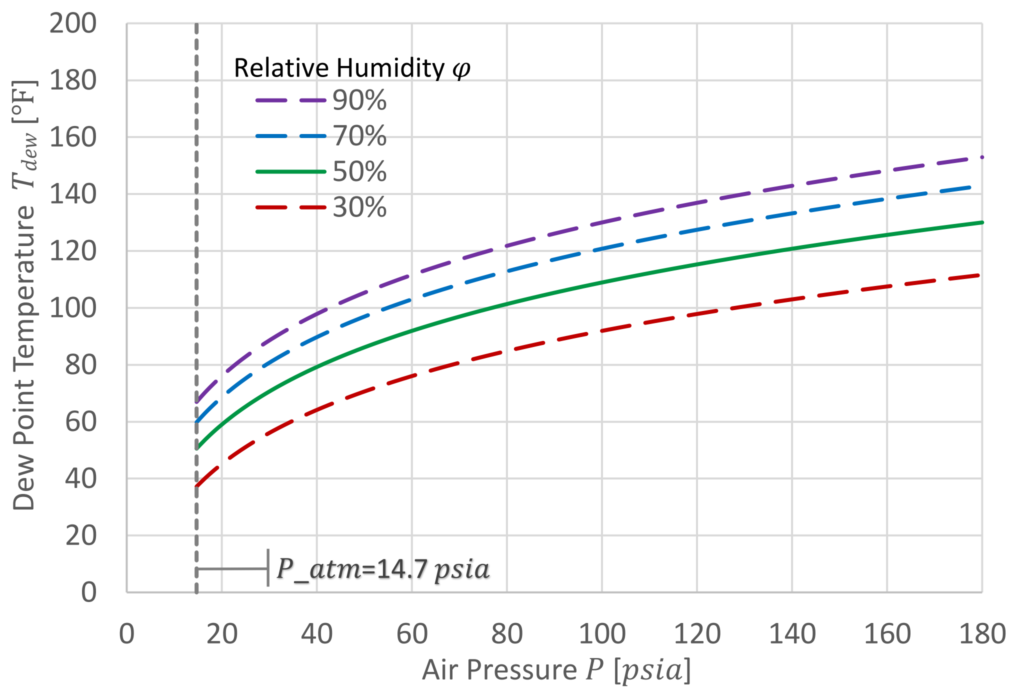



Figure 7. Dew Point Temperature Chart for Medium Inlet Temperature (70°F)

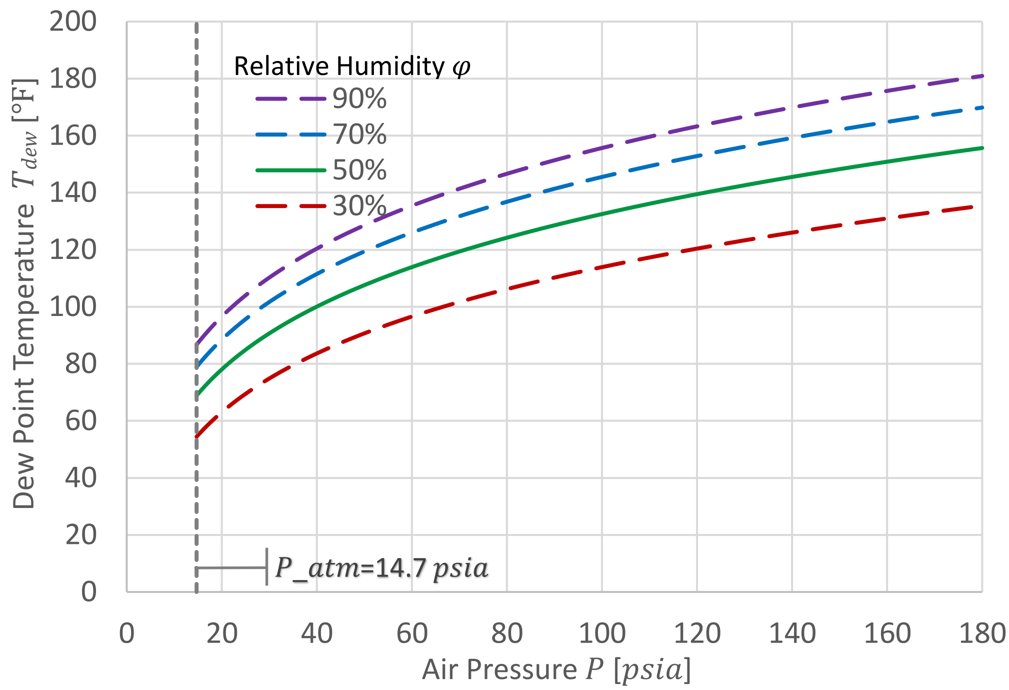



Figure 8. Dew Point Temperature Chart for High Inlet Temperature (90°F)

From the ideal gas equation, Equation (5), the air temperature and the air pressure is directly related, which means that in a system where there is no change in mass and volume, an increase in temperature would result in an increase in pressure. Therefore, as the air pressure increases, the dew point temperature decreases. On the other hand, the increase in the relative humidity would have the opposite effect, as it would increase the dew point temperature when the air temperature remains the same.

The condensation generated $\dot{V}_{cond}$ inside the aftercooler should be directly related to the volume flow rate of the compressed air $\dot{V}$, as well as the change in the humidity ratio $\Delta\omega$. The change in the humidity ratio $\Delta\omega$ is the decrease in humidity from the initial humidity ratio $\omega_i$ before condensation occurred to the final humidity ratio $\omega_f$ after the condensation occurred. Using the relationship between humidity ratio and the partial masses of the compressed air from Equation (20), the relationship between the change in the humidity ratio and the mass flow rate of the condensation generated can be derived as shown in Equation (25).

$$\Delta\omega = \omega_i - \omega_f = \frac{\dot{m}_v}{\dot{m}_a} - \frac{\dot{m}_v - \dot{m}_{cond}}{\dot{m}_a} = \frac{\dot{m}_{cond}}{\dot{m}_a} \tag{25}$$

where: • $\Delta\omega$ is the difference in the humidity ratio $[unitless]$,

• $\omega_i$ is the initial humidity ratio before condensation $[unitless]$,

• $\omega_f$ is the final humidity ratio after condensation $[unitless]$,

• $\dot{m}_a$ is the mass flow rate of dry air $[\frac{lbm}{sec}]$,

• $\dot{m}_v$ is the mass flow rate of water vapor $[\frac{lbm}{sec}]$, and

• $\dot{m}_{cond}$ is the mass flow rate of condensation generated $[\frac{lbm}{sec}]$.

Furthermore, using the relationship between the mass flow rate, the volume flow rate, and the density as described in Equation (19), the condensation generated $\dot{V}_{cond}$ can be related to the change in the humidity ratio $\Delta\omega$, the volume flow rate of the compressed air $\dot{V}$, and the gas density ratio $\frac{\rho_a}{\rho_v}$. With this relationship being shown in Equation (26) below.

$$\dot{V}_{cond} = \dot{V}\,\Delta\omega\,\frac{\rho_a}{\rho_v} \tag{26}$$

where: • $\dot{V}_{cond}$ is the condensation generated $[SCFM]$,

• $\dot{V}$ is the gas volume flow rate $[SCFM]$,

• $\Delta\omega$ is the difference (decrease) in the humidity ratio [$unitless$],

• $\rho_a$ is the density of dry air [$\frac{lbm}{ft^3}$], and

• $\rho_v$ is the density of water vapor [$\frac{lbm}{ft^3}$].

Since the density of dry air $\rho_a$ and water vapor $\rho_v$ at outdoor condition (at about 68°F) are known to be $0.075\,\frac{lbm}{ft^3}$ and $62.4\,\frac{lbm}{ft^3}$ respectively, the density ratio $\frac{\rho_a}{\rho_v}$ should be $0.0012$ or $1.2 \cdot 10^{-3}$. Therefore, Equation (26) can be written as Equation (27).

$$\dot{V}_{cond} = 0.0012\,\dot{V}\,\Delta\omega \tag{27}$$

<u>Example Problem III.a: Air Properties</u>

Objective: Solve for the specific volume $v_a$ of dry air at standard condition, which is a condition at temperature of 68°F, pressure of $14.7\ psia$, and relative humidity of $50\%$.

Assumption: The gas is an ideal gas.

Knowns: $R_a = 53.33\,\frac{lbf \cdot ft}{lbm \cdot R}$ $T = 68$°F $P = 14.7\ psia$

Find: • Specific volume $v_a$.

Solution:

From Equation (7),

$$v = \rho^{-1} = \frac{V}{m} = \frac{R\,T}{P}$$

Therefore, the specific volume $v$ is:

$$v_a = \frac{R_a\,T}{P_a}$$

$$v_a = \frac{R_a\,T}{P - \varphi\,P_{vs}}$$

The water vapor saturation partial pressure $P_{vs}$ at temperature of 68°F can be found using data interpolation of Table 4.

$$P_{vs} = 0.2564\text{ psia} + (\langle 0.3633 - 0.2564\rangle\text{ psia})\frac{(\langle 68 - 60\rangle°\text{F})}{(\langle 70 - 60\rangle°\text{F})} = 0.34195\,psia$$

$$v_a = \frac{\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(\langle 68 + 459.67\rangle R)}{\left(14.7\,psia \cdot 144\,\frac{in^2}{ft^2}\right) - (50\%)\left(0.34195\,psia \cdot 144\,\frac{in^2}{ft^2}\right)} = 13.42\,\frac{ft^3}{lbm}$$

Discussion: Since the specific volume $v_a$ is the multiplicative inverse of the density $\rho_a$, the multiplicative inverse of $13.42\,\frac{ft^3}{lbm}$ should equal to $0.0745\,\frac{lbm}{ft^3}$, which is true.

# CHAPTER IV

# AIR COMPRESSORS MODEL AND ANALYSIS

## Background

A two-stage air compressor package consists of a $1^{st}$-stage air compressor, an intercooler, and a $2^{nd}$-stage air compressor. In this chapter, only the air compressor parts of the package, which also includes an intercooler, will be discussed, modeled, and analyzed. The intercooler will be covered in the following chapter together with the aftercooler, as both of these components are heat exchangers.

In a two-stage air compressor, the air will undergo two separate, but similar, compression processes. For example, in the $1^{st}$ stage, the air is drawn in and compressed to an intermediate pressure. After that, the compressed air exits to the intercooler where it is cooled down before being compressed in the $2^{nd}$ stage. The air intercooling, which reduces the temperature of the air leaving the $1^{st}$ stage before entering the $2^{nd}$ stage, is what makes a two-stage air compressor more efficient at higher pressures. Some assumptions that can be made for this component is listed as follows:

1. The compressed air follows the ideal gas law,
2. Constant specific humidity ratio and mass flow rate in the two-stage air compressor,
3. There are no changes in both kinetic energy and potential energy, and
4. Compression process is adiabatic.

The $1^{st}$ stage air compressor that compresses air from its low inlet pressure is also called the low-pressure (LP) compressor. While the $2^{nd}$ stage air compressor is also called the high-pressure (HP) compressor because it pressurizes air from a higher intermediate pressure to a final pressure. Figure 9 shows the schematic of the air flow path, along with the interactions of the compressors and the coolers in a two-stage compressed air system.

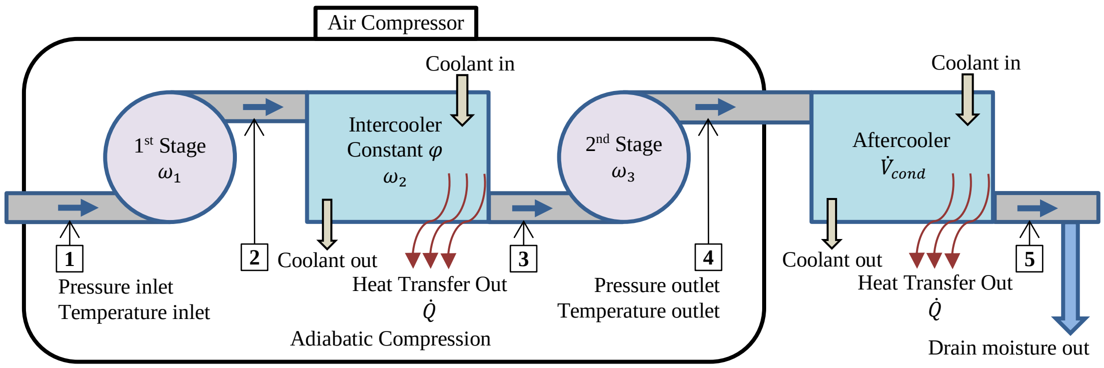



Figure 9. Compressed Air System Diagram (Two-Stage Air Compressor - Cooler)

As the first step in modeling and analyzing the compression process, Figure 10 shows one compression stage of the air compressor, which could be either the low-pressure or the high-pressure compression. Also shown in Figure 10 are the key parameters that are used in the air compressor modeling and analysis.

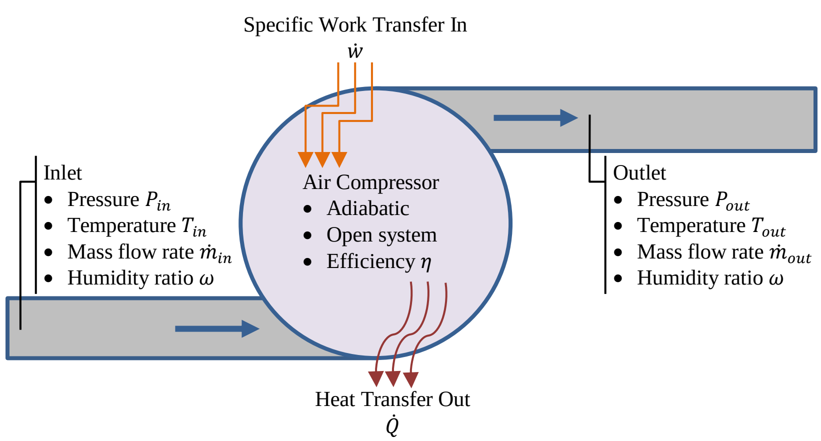



Figure 10. Compressed Air System Compressor Schematic and Process

Lastly, the discussion of the MATLAB codes for calculations related to this chapter can be found in the APPENDIX C. The example problems that will be discussed in this chapter will also be covered in the APPENDIX C.

**Steady-State Energy Balance**

To analyze the specific work of the air compression system, the steady-state energy rate balance in Equation (28) is the appropriate starting point. It should be noted that the subscript $in$ is for the inlet and the subscript $out$ is for the outlet or the exit.

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}\left(h + \frac{1}{2}\,\bar{V}^2 + g\,z\right)_{in} - \sum \dot{m}\left(h + \frac{1}{2}\,\bar{V}^2 + g\,z\right)_{out}, \quad (28)$$

where: • $\frac{dE}{dt}$ is the energy transfer rate [$\frac{BTU}{hr}$],

• $\dot{Q}$ is the heat transfer rate [$\frac{BTU}{hr}$],

• $\dot{W}$ is the work transfer rate [$\frac{BTU}{hr}$],

• $\dot{m}$ is the mass transfer rate [$\frac{lbm}{sec}$],

• $h$is the specific enthalpy [$\frac{BTU}{lbm}$],

• $\bar{V}$ is the inlet velocity [$\frac{ft}{sec}$],

• $g$ is the acceleration due to gravity [$\frac{ft}{sec^2}$], and

• $z$ is the position altitude [$ft$].

The work transfer rate $\dot{W}$ of an air compressor would be the amount of work that is received from the power supply, thus it will have a negative sign. Furthermore, since the process is steady-state and adiabatic, has constant mass flow rate, and has no changes in both kinetic energy and potential energy, Equation (28) can be simplified into Equation (29).

$$\dot{W} = \dot{m}\,(h_{out} - h_{in})\,, \tag{29}$$

where the changes in specific enthalpy can be described in terms of specific heat and change in temperature as shown in Equation (30) below.

$$h_{out} - h_{in} = C_p(T_{out} - T_{in})\,, \tag{30}$$

where: • $C_p$ is the gas specific heat for constant pressure [$\frac{BTU}{lbm \cdot R}$],

• $T_{out}$ is the gas outlet temperature [°F], and

• $T_{in}$ is the gas inlet temperature [°F].

Therefore, Equation (29) can be written as Equations (31) below

$$\dot{W} = \dot{m}\,C_p(T_{out} - T_{in})\,, \tag{31}$$

or

$$\dot{w} = C_p(T_{out} - T_{in})\,, \tag{32}$$

where $\dot{w}$ is the specific work [$\frac{BTU}{lbm}$].

## Open System Air Compression Processes

### *Polytropic and Isentropic Process*

An air compression process, which can be expressed as the product of the air pressure and its specific volume, is a polytropic process. For the compressed air system at ideal condition, which is an adiabatic and open system process, the specific work $\dot{w}_{id}$ can be derived from Equation (32) with ideal temperature difference $\left(T_{out_{id}} - T_{in}\right)$, which is described in Equation (33) below.

$$\dot{w}_{id} = C_p\left(T_{out_{id}} - T_{in}\right), \tag{33}$$

where: • $\dot{w}_{id}$ is the ideal specific work [$\frac{BTU}{lb}$],

• $T_{out_{id}}$ is the gas ideal outlet temperature [°F], and

• $T_{in}$ is the gas inlet temperature [°F].

The ideal outlet temperature $T_{out_{id}}$ will follow Equation (34) below.

$$T_{out_{id}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}, \tag{34}$$

where: • $P_{out}$ is the gas outlet pressure [$psia$],

• $P_{in}$ is the gas inlet pressure [$psia$], and

• $n$ is the gas polytropic index [unitless].

Also, the ideal outlet compressed air volume $V_{out_{id}}$ will depend on $P_{in}$, and $P_{out}$, as well as the $V_{in}$, which is the inlet air volume. Equation (35) can be used to calculate the $V_{out_{id}}$.

$$V_{out_{id}} = V_{in}\left(\frac{P_{in}}{P_{out}}\right)^{\frac{1}{n}}, \tag{35}$$

where: • $V_{out_{id}}$ is the gas ideal outlet volume [$ft^3$], and

• $V_{in}$ is the gas inlet volume [$ft^3$].

Furthermore, if the polytropic index $n$ of a compression process is equal to 1.0, then the process is called isothermal. If the polytropic index $n$ is equal to the heat capacity ratio $k$ of the gas, which is equal to 1.4 for dry air, the process is called the isentropic process.

### *Isothermal Process*

Next, for isothermal process, where the polytropic index $n$ is equal to 1.0, the specific work $\dot{w}_{iso}$ is in a function of the gas constant $R$, the temperature $T$, and the pressure ratio $\frac{P_{in}}{P_{out}}$. The polytropic index $n$ ranges from 1.0 for an ideal isothermal compression, where the process occurs slowly to keep the gas temperature constant, to 1.4 for an ideal adiabatic compression or an isentropic compression, where the process occurs rapidly to prevent any flow of energy from transferring into or out from the air compressor. Equation (36) below describes the specific work at isothermal process.

$$\dot{w}_{iso} = RT \ln \frac{P_{out}}{P_{in}}, \tag{36}$$

where: • $\dot{w}_{iso}$ is the isothermal specific work [$\frac{BTU}{lb}$],

• $R$ is the gas constant [$\frac{BTU}{lbm \cdot R}$], and

• $T$ is the gas constant temperature [°F].

<u>Example Problem IV.a: Exit Temperatures</u>

Objective: A polytropic process compressed air at standard condition to $100\,psia$. Solve for the exit temperature if $n = 1.0$ (isothermal compression), $n = 1.30$, and $n = 1.40$ (isentropic compression).

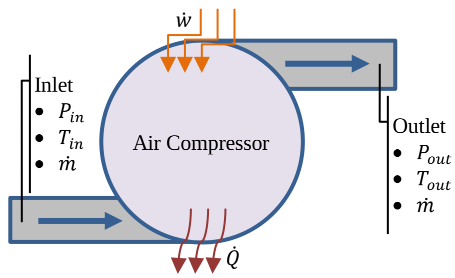
Assumption: The gas is an ideal gas.

The inlet condition of the gas is a standard condition.

Knowns: $T_{in} = 68°\text{F}$  $P_{in} = 14.7\ psia$  $P_{out} = 100psia$

Find: • The exit temperatures $T_{out\,id}$ for each $n$ values.

Solution:




The governing formula to calculate the exit temperature is:

$$T_{out\,id} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}$$

The exit temperature if $n = 1.0$ (isothermal compression) is:

$$T_{out\,id} = (\langle 68 + 459.67\rangle R)\left(\frac{100\ psia}{14.7\ psia}\right)^{\frac{1.0-1}{1.0}} = 527.67\ R = 68°\text{F}$$

The exit temperature if $n = 1.30$ is:

$$T_{out\,id} = (\langle 68 + 459.67\rangle R)\left(\frac{100\ psia}{14.7\ psia}\right)^{\frac{1.30-1}{1.30}} = 819.8\ R = 361.1°\text{F}$$

The exit temperature if $n = 1.40$ (isentropic compression) is:

$$T_{out\,id} = (\langle 68 + 459.67\rangle R)\left(\frac{100\ psia}{14.7\ psia}\right)^{\frac{1.40-1}{1.40}} = 910.9\ R = 452.2°\text{F}$$

Discussion: As the value of the polytropic index $n$ increases, the exit temperature also increases. The increase in $n$ limits the amount of heat that exits the system. When $n =$

1, the compression is isothermal, and on the other hand, when $n = 1.4$, the compression is isentropic.

### *Real Process*

The actual condition of the working system is normally different from the ideal condition. Equation (37) below describes the system specific work at actual condition $\dot{w}_{ac}$, which is in a function of $C_p$ and actual temperature difference $\left(T_{out_{ac}} - T_{in}\right)$.

$$\dot{w}_{ac} = C_p\left(T_{out_{ac}} - T_{in}\right), \tag{37}$$

where: • $\dot{w}_{ac}$ is the actual specific work [$\frac{BTU}{lb}$], and

• $T_{out_{ac}}$ is the gas actual outlet temperature [°F].

### *Two-Stage Process*

For two-stage air compressor, there will be an intermediate pressure $P_I$ across the intercooler, and it can be calculated by using Equation (38), which is a modification of Equation (24).

$$P_I = \sqrt{P_1 \cdot P_4}, \tag{38}$$

where: • $P_I$ is the intermediate pressure [$psia$],

• $P_1$ is the gas pressure into the air compressor package [$psia$], and

• $P_4$ is the gas pressure out from the air compressor package [$psia$].

$P_1$, and $P_4$ are the gas pressure at point 1, and point 4 in Figure 9. Note that the gas pressure at point 2, and point 3 are the same ($P_2 = P_3$), and are also called the intermediate pressure $P_I$.

**Polytropic and Isothermal Efficiency**

To describe the efficiency of a compressor, the actual specific work $\dot{w}_{ac}$ need to be related to the ideal specific work $\dot{w}_{id}$, which renders the efficiency value called the polytropic efficiency $\eta_{pol}$. The polytropic efficiency $\eta_{pol}$ of a compressor can be calculated as the ratio between the specific work at ideal condition, and the specific work at actual condition as described in Equation (39) below. In another case, if the compression is an isothermal process, the isothermal efficiency $\eta_{iso}$ can be calculated by using Equation (40).

$$\eta_{pol} = \frac{\dot{w}_{id}}{\dot{w}_{ac}}, \tag{39}$$

or

$$\eta_{iso} = \frac{\dot{w}_{iso}}{\dot{w}_{ac}}, \tag{40}$$

where: • $\eta_{pol}$ is the polytropic efficiency [%], and

• $\eta_{iso}$ is the isothermal efficiency [%].

Note that the polytropic efficiency $\eta_{pol}$, can also be addressed as the isentropic efficiency $\eta_{ise}$ when its polytropic index $n$ is equal to the heat capacity ratio $k$, or in other words, the process is isentropic.

Example Problem IV.b: Specific Work and Efficiency

Objective: Air at standard condition was compressed to 100 $psia$. If the compression process is an adiabatic process, and has an isentropic efficiency of 75%, solve for the actual specific work, the exit temperature, and the heat loss. In different case, if the compression process is an isothermal process, and has an isothermal efficiency of 75%, solve for the actual specific work, the exit temperature, and the heat loss. Additionally, if the compression process is an adiabatic and reversible process, solve for the isothermal efficiency $\eta_{iso}$.

Assumption: The gas or air is an ideal gas.

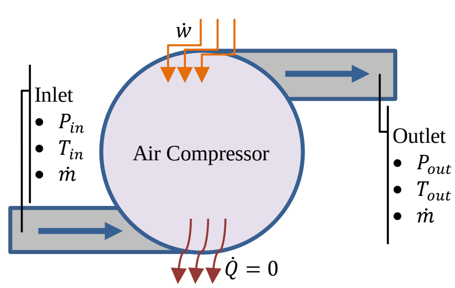
The inlet condition of the gas is a standard condition.

Knowns: $\eta_{ise} = 75\%$ $\eta_{iso} = 75\%$ $P_{out} = 100\ psia$

Find:
- The actual specific work $\dot{w}_{ac}$,
- the exit temperature $T_{out_{ac}}$,
- the heat loss $\dot{Q}$, and
- the isothermal efficiency $\eta_{iso}$.

Solution:

For adiabatic process:




The ideal specific work $\dot{w}_{id}$ is:

$$\dot{w}_{id} = C_p\left(T_{out_{id}} - T_{in}\right)$$

$$\dot{w}_{id} = \left(0.240\,\frac{BTU}{lbm \cdot R}\right)\left(\langle 452.2 - 68\rangle R\right) = 92.2\,\frac{BTU}{lbm}$$

The actual specific work $\dot{w}_{ac}$ is:

$$\eta_{pol} = \frac{\dot{w}_{id}}{\dot{w}_{ac}}$$

$$\dot{w}_{ac} = \frac{\dot{w}_{id}}{\eta_{ise}}$$

$$\dot{w}_{ac} = \frac{92.2\,\frac{BTU}{lbm}}{75\%} = 122.9\,\frac{BTU}{lbm}$$

The exit temperature $T_{out_{ac}}$ is:

$$\eta_{pol} = \frac{\dot{w}_{id}}{\dot{w}_{ac}}$$

$$\eta_{ise} = \frac{C_p\left(T_{out_{id}} - T_{in}\right)}{C_p\left(T_{out_{ac}} - T_{in}\right)}$$

$$T_{out_{ac}} = T_{in} + \frac{\left(T_{out_{id}} - T_{in}\right)}{\eta_{ise}}$$

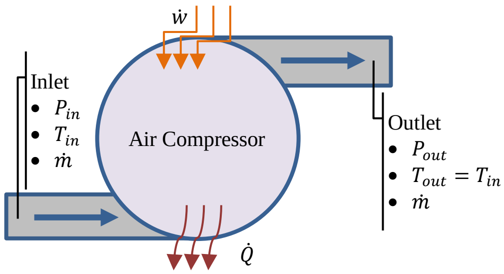
$$T_{out_{ac}} = (68°\text{F}) + \frac{(452.2°\text{F} - 68°\text{F})}{75\%} = 580.2°\text{F}$$

Then, the heat loss $\dot{Q}$ can also be calculated as the difference between the ideal and actual specific work.

$$\dot{Q} = \dot{w}_{ac} - \dot{w}_{id}$$

$$\dot{Q} = \left(122.9\,\frac{BTU}{lbm}\right) - \left(92.2\,\frac{BTU}{lbm}\right) = 30.7\,\frac{BTU}{lbm}$$

For isothermal process:




The isothermal specific work $\dot{w}_{iso}$ is:

$$\dot{w}_{iso} = R_a T \ \ln \frac{P_{out}}{P_{in}}$$

$$\dot{w}_{iso} = \left(0.069 \frac{BTU}{lbm \cdot R}\right)(527.67\ R)\left(\ln \frac{100\ psia}{14.7\ psia}\right) = 69.7 \frac{BTU}{lbm}$$

The actual specific work $\dot{w}_{ac}$ is

$$\eta_{iso} = \frac{\dot{w}_{iso}}{\dot{w}_{ac}}$$

$$\dot{w}_{ac} = \frac{\dot{w}_{iso}}{\eta_{iso}}$$

$$\dot{w}_{ac} = \frac{69.7 \frac{BTU}{lbm}}{75\%} = 92.9 \frac{BTU}{lbm}$$

The exit temperature $T_{out\,ac}$ is:

$$\dot{w}_{ac} = C_p\left(T_{out\,ac} - T_{in}\right)$$

$$T_{out\,ac} = T_{in} + \frac{\dot{w}_{ac}}{C_p}$$

$$T_{out\,ac} = (68°\text{F}) + \frac{92.9 \frac{BTU}{lbm}}{0.240 \frac{BTU}{lbm \cdot R}} = 455.1°\text{F}$$

Then, the heat loss $\dot{Q}$ can also be calculated as the difference between the isothermal and actual specific work.

$$\dot{Q} = \dot{w}_{ac} - \dot{w}_{iso}$$

$$\dot{Q} = \left(92.9\frac{BTU}{lbm}\right) - \left(69.7\frac{BTU}{lbm}\right) = 23.2\frac{BTU}{lbm}$$

Additionally, if the compression process is an adiabatic and reversible ($\eta = 100\%$) process, the isothermal efficiency $\eta_{iso}$ would be:

$$\eta_{iso} = \frac{\dot{w}_{iso}}{\dot{w}_{ac}}$$

$$\eta_{iso} = \frac{\dot{w}_{iso}}{\dot{w}_{id}}$$

$$\eta_{iso} = \frac{69.7\frac{BTU}{lbm}}{92.2\frac{BTU}{lbm}} = 75.6\%$$

Discussion: The temperature difference for the actual condition is always larger than the temperature difference for the ideal condition. It is more efficient to have the temperature difference to be as small as possible. The heat loss is the difference between the ideal and the actual specific work.

**Open System Air Compression Processes with P-v Diagrams**

The air compression process plotted in a $P - v$ (pressure – specific volume) diagram with isothermal process provides insight into how parameters change, along with the work transfer. Five scenarios including the process on a $P - v$ diagram, along with description and explanation are presented as a prelude to modeling and analysis. Of special note, because these are ideal processes, the area to the left of the curve represents the work of the compression process. Moreover, neglecting changes in both kinetic energy and potential energy of the compressed air, the ideal work of an open system air compressor can be seen on a $P - v$ diagram as $W = \int v \, dP$

### *One-Stage Polytropic Process*

Below is a $P - v$ diagram of a one-stage polytropic air compressor. In Figure 11, the polytropic air compression occurs at the curves between point 1 and point 2. The compression curve of this polytropic process should have a polytropic index $n$, which can be of any real numbers.

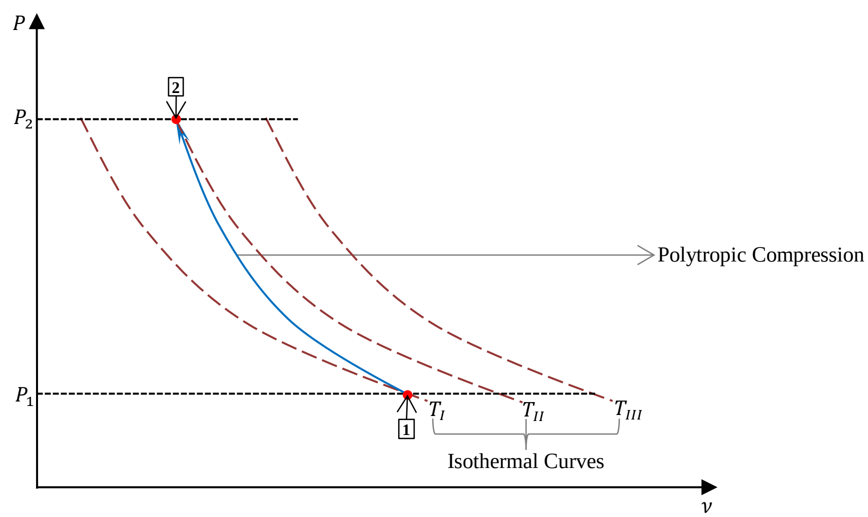



Figure 11. P-v Diagram of a Polytropic Air Compressor

### *Isothermal Process*

Figure 12 illustrates the $P - v$ relationship of an isothermal air compressor that has constant temperature. The isothermal air compression in Figure 12 occurs at the curves between point 1 and point 2. The compression curve of this isothermal process has a polytropic index $n$ equals to 1. An isothermal process means that there is heat transfer between the system and the surrounding environment. Moreover, since the temperature within the air compressor stays constant despite of the increase in the air pressure, it means that the isothermal process involves maximum cooling process.

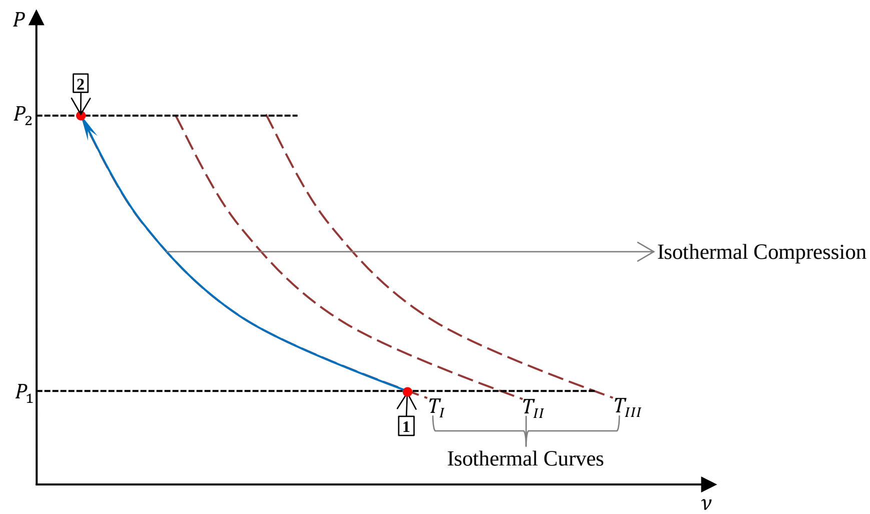



Figure 12. P-v Diagram of an Isothermal Air Compressor

***One-Stage Isentropic Process***

Figure 13 illustrates the $P - v$ relationship of a one-stage isentropic air compressor, which means that it has no cooling process, or in other words, it is an adiabatic and reversible process. The isentropic air compression in Figure 13 occurs at the curves between point 1 and point 2. The compression curve of this isentropic process has a polytropic index $n$ equals to the heat capacity ratio $k$ of the gas.

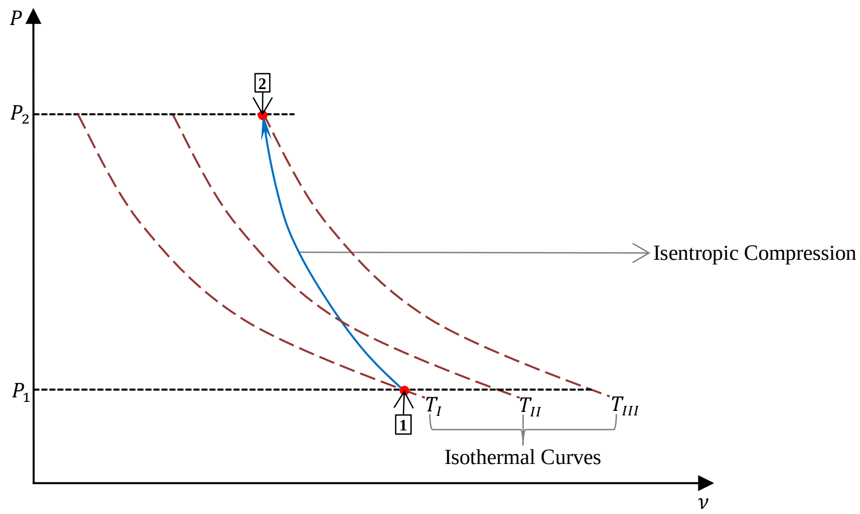



Figure 13. P-v Diagram of a One-Stage Isentropic Air Compressor

As the air is being compressed from $P_1$ to $P_2$, there will be an increase in temperature from $T_1$, which is the initial temperature $T_I$, to $T_2$, which is the one-stage isentropic process final temperature $T_{III}$.

For example, if the inlet pressure $P_1$ and temperature $T_1$ or $T_I$ are 14.7 $psia$ and 70°F, and the desired outlet pressure $P_2$ is 120 $psia$. Then, assuming the air is dry air, using Equation (34), the outlet temperature $T_{III}$ can be found as:

$$T_{out\,id} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}$$

$$T_{III} = T_I\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_{III} = (\langle 70°\text{F} + 459.67\rangle R)\left(\frac{120\ psia}{14.7\ psia}\right)^{\frac{1.4-1}{1.4}} = 505°\text{F}$$

Hence, the one-stage isentropic process final temperature $T_{III}$ is equal to 505°F.

### *One-Stage Real Process*

Figure 14 illustrates the $P - v$ relationship of a one-stage real process air compressor, which means that compressor efficiency may be less than 100%, which is always the case in reality. The real air compression in Figure 14 occurs at the curves between point 1 and point 2. The compression curve of this isentropic process has a polytropic index $n$ that is larger than the heat capacity ratio $k$ of the gas.

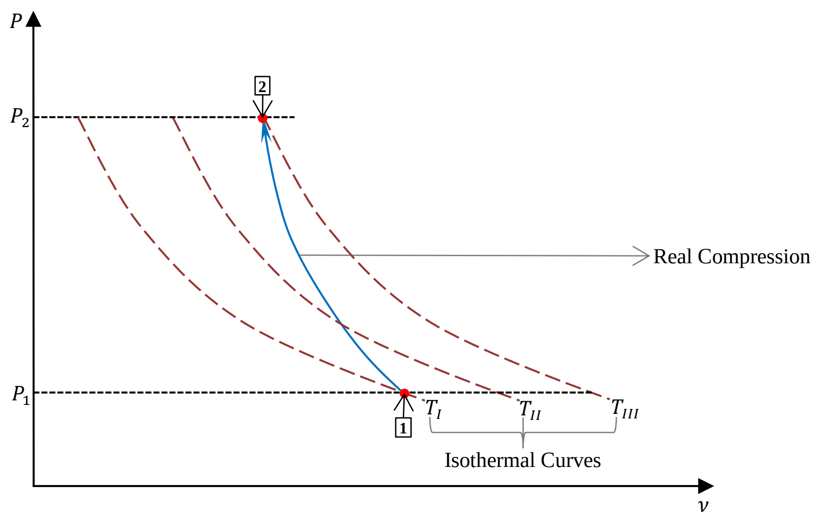



Figure 14. P-v Diagram of a One-Stage Real Air Compressor

The compression process is similar to that of the one-stage isentropic compression; however, the final temperature after the compression process should be a higher value.

For example, using the same condition as the previous scenario's example, where the final temperature $T_{III}$ is equal to 505°F, the final temperature $T_{III_{ac}}$ of the real air compression where the polytropic efficiency $\eta_{pol}$ equals to 80% can be found as:

$$\eta_{pol} = \frac{\dot{w}_{id}}{\dot{w}_{ac}}$$

$$\eta_{pol} = \frac{C_p\left(T_{out_{id}} - T_{in}\right)}{C_p\left(T_{out_{ac}} - T_{in}\right)}$$

$$\eta_{pol} = \frac{(T_{III} - T_I)}{\left(T_{III_{ac}} - T_I\right)}$$

$$T_{III_{ac}} = T_I + \frac{(T_{III} - T_I)}{\eta_{pol}}$$

$$T_{III_{ac}} = (70°\mathrm{F}) + \frac{(505°\mathrm{F} - 70°\mathrm{F})}{(80\%)} = 613°\mathrm{F}$$

Hence, the one-stage real process final temperature $T_{III_{ac}}$ is equal to $613°\mathrm{F}$, which is more than $100°\mathrm{F}$ higher than the final temperature of the one-stage isentropic process $T_{III}$.

### *Two-Stage Isentropic Process*

Figure 15 illustrates the $P - v$ relationship of a two-stage air compressor that represents the work saved (green area) as a result of two-stage compression with intercooling. The 1st stage and the 2nd stage of isentropic air compression in Figure 15 occurs at the curves between point 1 and point 2, and between point 3 and point 4, respectively. The intercooling occurs at the horizontal line between point 2 and point 3. The compression curve of this isentropic process has a polytropic index $n$ equals to the heat capacity ratio $k$ of the gas.

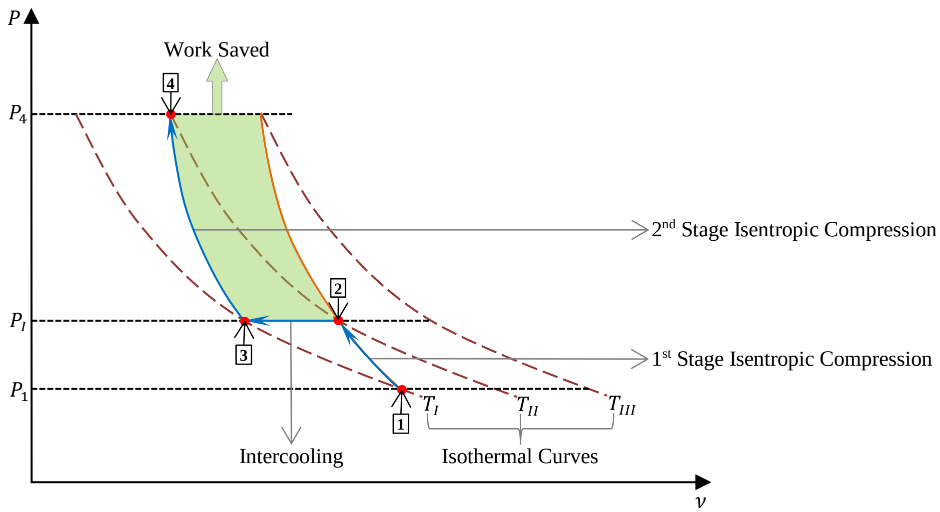



Figure 15. P-v Diagram of a Two-Stage Isentropic Air Compressor

As the air is being compressed from $P_1$ to $P_2$, there will be an increase in temperature from $T_1$, which is the initial temperature $T_I$, to $T_2$. Similarly, the air compression from $P_3$ to $P_4$ will increase the air temperature from $T_3$ to $T_4$. During the intercooling process, the air pressure stays constant, such that $P_2$ is the same as $P_3$; the pressure at this point is called the intermediate pressure $P_I$. Furthermore, because of the intercooling process, the air temperature after the 1st stage compression $T_2$ will decrease to the initial air temperature $T_I$ such that $T_3$ is the same as $T_I$. Also, the 2nd stage compression would increase the air temperature to $T_4$, which is the two-stage isentropic process final temperature $T_{II}$. The two-stage isentropic process final temperature $T_{II}$ is the same as the air temperature after the 1st stage compression, which makes $T_4$ the same as $T_2$.

For example, if the inlet pressure $P_1$ and temperature $T_1 = T_I$ are $14.7\ psia$ and $70°\text{F}$, and the desired outlet pressure $P_4$ is $120\ psia$. Then, assuming the air is dry air, using Equation (38), the intermediate pressure $P_I$ can be found as:

$$P_I = \sqrt{P_1 \cdot P_4}$$

$$P_I = \sqrt{(14.7\ psia) \cdot (120\ psia)} = 42\ psia$$

Next, using Equation (34), the outlet temperature $T_{II}$ can be found as:

$$T_{out_{id}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}$$

$$T_{II} = T_I\left(\frac{P_I}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_{II} = (\langle 70°\text{F} + 459.67\rangle R)\left(\frac{42\ psia}{14.7\ psia}\right)^{\frac{1.4-1}{1.4}} = 255°\text{F}$$

Hence, the two-stage isentropic process final temperature $T_{II}$ is equal to 255°F, which is close to half of the final temperature of the one-stage isentropic process $T_{III}$.

Example Problem IV.c: Intermediate Pressure and Total Specific Work

Objective: For a two-stage air compressor with $80\%$ isentropic efficiency $\eta_{ise}$ on an adiabatic compression ($k = 1.40$) on each stage, the exit pressure $P_4$ is found to be $125\ psig$, and the intercooler cools down the compressed air to $110°\text{F}$. The air inlet condition is also known to be in standard condition. Solve for the intermediate pressure $P_I$, and the actual exit temperature of each stage of the compression ($T_2$ and $T_4$). See Figure 15.

Assumption: The gas is an ideal gas.

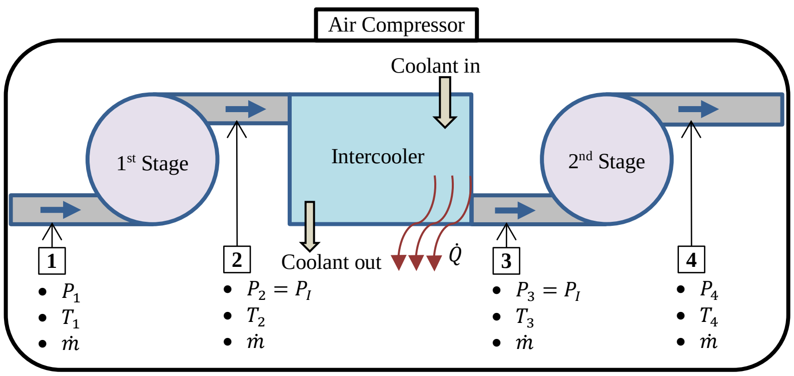
The inlet condition of the gas is a standard condition.

Knowns: $\eta_{ise} = 80\%$ $P_4 = 125\ psig$ $T_3 = 110°\text{F}$

Find: • The intermediate pressure $P_I$, and

• the actual exit temperature of each stages $T_2$ and $T_4$.

Solution:




The intermediate pressure $P_I$ is:

$$P_I = \sqrt{P_1 \cdot P_4}$$

$$P_I = \sqrt{(14.7\ psia) \cdot (\langle 125 + 14.7 \rangle\ psia)} = 45.32\ psia$$

The ideal exit temperature of each stages can be calculated by using this equation.

$$T_{out\,id} = T_{in} \left( \frac{P_{out}}{P_{in}} \right)^{\frac{n-1}{n}}$$

Such that

$$T_{2id} = T_1 \left( \frac{P_I}{P_1} \right)^{\frac{n-1}{n}},$$

and

$$T_{4_{id}} = T_3 \left(\frac{P_4}{P_I}\right)^{\frac{n-1}{n}}$$

Also, the actual exit temperature of each stages can be calculated by using this equation.

$$\eta_{pol} = \frac{\dot{w}_{id}}{\dot{w}_{ac}}$$

$$\eta_{ise} = \frac{C_p\left(T_{out_{id}} - T_{in}\right)}{C_p\left(T_{out_{ac}} - T_{in}\right)}$$

$$\eta_{ise} = \frac{\left(T_{out_{id}} - T_{in}\right)}{\left(T_{out_{ac}} - T_{in}\right)}$$

$$T_{out_{ac}} = T_{in} + \frac{\left(T_{out_{id}} - T_{in}\right)}{\eta_{ise}}$$

Therefore, the ideal exit temperature at point 2 $T_{2_{id}}$ is:

$$T_{2_{id}} = T_1 \left(\frac{P_I}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_{2_{id}} = (\langle 68 + 459.67\rangle R) \left(\frac{45.32\ psia}{14.7\ psia}\right)^{\frac{1.40-1}{1.40}} = 267.83°\mathrm{F}$$

Hence, the actual exit temperature at point 2 $T_{2_{ac}}$ is:

$$T_{2_{ac}} = T_1 + \frac{\left(T_{2_{id}} - T_1\right)}{\eta_{ise}}$$

$$T_{2_{ac}} = 68°\mathrm{F} + \frac{(267.83°\mathrm{F} - 68°\mathrm{F})}{80\%} = 317.79°\mathrm{F}$$

Then, the ideal exit temperature at point 4 $T_{4_{id}}$ is:

$$T_{4_{id}} = T_3 \left(\frac{P_4}{P_I}\right)^{\frac{n-1}{n}}$$

$$T_{4_{id}} = (\langle 110 + 459.67 \rangle R) \left( \frac{(\langle 125 + 14.7 \rangle \, psia)}{45.32 \, psia} \right)^{\frac{1.40-1}{1.40}} = 325.77°\text{F}$$

Hence, the actual exit temperature at point 4 $T_{4_{ac}}$ is:

$$T_{4_{ac}} = T_3 + \frac{\left(T_{4_{id}} - T_3\right)}{\eta_{ise}}$$

$$T_{4_{ac}} = 110°\text{F} + \frac{(325.77°\text{F} - 110°\text{F})}{80\%} = 379.71°\text{F}$$

Moreover, the total actual specific work can also be calculated by using the following equations:

$$\dot{w}_{ac} = C_p\left(T_{out_{ac}} - T_{in}\right)$$

Such that:

$$\dot{w}_{1st_{ac}} = C_p\left(T_{2_{ac}} - T_1\right),$$

and

$$\dot{w}_{2nd_{ac}} = C_p\left(T_{4_{ac}} - T_3\right)$$

Therefore, the total actual specific work at the 1st stage $\dot{w}_{1st_{ac}}$ is:

$$\dot{w}_{1st_{ac}} = C_p\left(T_{2_{ac}} - T_1\right)$$

$$\dot{w}_{1st_{ac}} = \left(0.240 \frac{BTU}{lbm \cdot R}\right) (\langle 317.79 - 68 \rangle R) = 59.95 \frac{BTU}{lbm}$$

And, the total actual specific work at the 2nd stage $\dot{w}_{2nd_{ac}}$ is:

$$\dot{w}_{2nd_{ac}} = C_p\left(T_{4_{ac}} - T_3\right)$$

$$\dot{w}_{2nd_{ac}} = \left(0.240 \frac{BTU}{lbm \cdot R}\right) (\langle 379.71°\text{F} - 110 \rangle R) = 64.73 \frac{BTU}{lbm}$$

Hence, the air compressor total actual specific work $\dot{w}_{total_{ac}}$ is:

$$\dot{w}_{total_{ac}} = \dot{w}_{1st_{ac}} + \dot{w}_{2nd_{ac}}$$

$$\dot{w}_{total_{ac}} = \left(59.95\frac{BTU}{lbm}\right) + \left(64.73\frac{BTU}{lbm}\right) = 124.68\frac{BTU}{lbm}$$

Discussion: The amount of work required by the 2nd stage air compressor is higher than the required work for the 1st stage air compressor. This means that higher temperature air requires more work for compression process. Furthermore, the lower the isentropic efficiency, the higher the exit temperature for each stages. This in turn requires a greater amount of work. The best two-stage air compressor would have an optimum intermediate pressure and also remove the most amount of heat by using the intercooler.

Example Problem IV.d: Compressor Analysis

Objective: A two-stage air compressor with a capacity of $400\,SCFM$ of standard condition air discharges to a $140\,psig$ header. With $90\%$ of isentropic efficiency and no condensation occurs in the intermediate stage, solve for the total power $\dot{W}$ and work transfer rate $\dot{w}$. See Figure 15.

Assumption: The gas is an ideal gas.

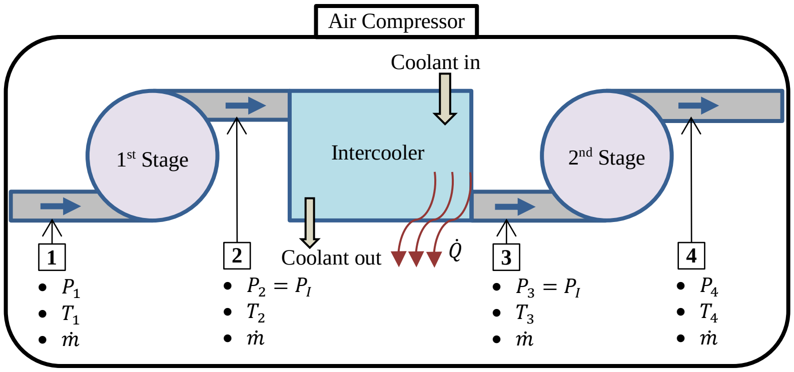
The inlet condition of the gas is a standard condition.

The compression is an adiabatic and polytropic process.

Knowns: $\dot{V} = 400\,SCFM$ $\quad P_4 = 140\,psig$ $\quad \eta_{ise} = 90\%$

Find:
- The total power $\dot{W}_{total}$, and
- the work transfer rate $\dot{w}_{total}$.

Solution:




First, the intermediate pressure $P_I$ need to be calculated.

$$P_I = \sqrt{P_1 \cdot P_4}$$

$$P_I = \sqrt{(14.7\ psia) \cdot (\langle 140 + 14.7 \rangle psia)} = 47.69\ psia$$

Second, the actual temperature at point 2 $T_{2_{ac}}$ can be calculated by finding the ideal temperature at the same point first.

$$T_{2_{id}} = T_1 \left(\frac{P_I}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_{2_{id}} = (\langle 68 + 459.67 \rangle R) \left(\frac{47.69\ psia}{14.7\ psia}\right)^{\frac{1.40-1}{1.40}} = 278.50°\text{F}$$

Hence,

$$T_{2_{ac}} = T_1 + \frac{(T_{2_{id}} - T_1)}{\eta_{ise}}$$

$$T_{2_{ac}} = 68°\text{F} + \frac{(278.50°\text{F} - 68°\text{F})}{90\%} = 301.89°\text{F}$$

Third, from the psychometric chart in Figure 4, the humidity ratio $\omega$ of the inlet air is $0.00725 \frac{lbm_{vapor}}{lbm_{air}}$. This number is the same all the way through the air compressor because there is no condensation. Therefore, the lowest pressure that the gas temperature can be cooled down is the saturation vapor pressure $P_{vs}$.

$$\omega = \frac{0.622\ \varphi\ P_{vs}}{P - \varphi\ P_{vs}}$$

$$P_{vs} = \frac{\omega\ P}{\varphi\ (0.622 + w)}$$

$$P_{vs} = \frac{(0.00725)(47.69\ psia)}{(100\%)(0.622 + 0.00725)} = 0.549\ psia$$

Interpolating the values from Table 4 would give the temperature $T_3$ that corresponds with the $P_{vs}$.

$$\frac{T_3 - 80°\text{F}}{(\langle 90 - 80\rangle°\text{F})} = \frac{(\langle 0.549 - 0.50745\rangle psia)}{(\langle 0.66904 - 0.50745\rangle psia)}$$

$$T_3 = (80°\text{F}) + \frac{(\langle 90 - 80\rangle°\text{F})(\langle 0.549 - 0.50745\rangle psia)}{(\langle 0.66904 - 0.50745\rangle psia)} = 82.57°\text{F}$$

Fourth, the actual temperature at point 4 $T_{4_{ac}}$ can be calculated by finding the ideal temperature at the same point.

$$T_{4_{id}} = T_3 \left(\frac{P_4}{P_I}\right)^{\frac{n-1}{n}}$$

$$T_{4_{id}} = (\langle 82.57 + 459.67\rangle R)\left(\frac{\langle 140 + 14.7\rangle psia}{47.69\ \ psia}\right)^{\frac{1.40-1}{1.40}} = 298.87°\text{F}$$

such that

$$T_{4_{ac}} = T_3 + \frac{\left(T_{4_{id}} - T_3\right)}{\eta_{ise}}$$

$$T_{4_{ac}} = 82.57°\text{F} + \frac{(298.87°\text{F} - 82.57°\text{F})}{90\%} = 322.91°\text{F}$$

Fifth, continuing to find the total power and work transfer rate $\dot{w}_{total_{ac}}$ as follows:

$$\dot{w}_{total_{ac}} = \dot{w}_{1st_{ac}} + \dot{w}_{2nd_{ac}}$$

$$\dot{w}_{total_{ac}} = C_p\left(T_{2_{ac}} - T_1\right) + C_p\left(T_{4_{ac}} - T_3\right)$$

$$\dot{w}_{total_{ac}} = C_p\left(T_{2_{ac}} + T_{4_{ac}} - T_1 - T_3\right)$$

$$\dot{w}_{total_{ac}} = \left(0.24\frac{BTU}{lbm \cdot R}\right)(\langle 301.89 + 322.91 - 68 - 82.57\rangle R) = 113.82\frac{BTU}{lbm}$$

and

$$\dot{W}_{total} = \dot{m}\,\dot{w}_{total_{ac}}$$

$$\dot{W}_{total} = \rho\,\dot{V}\,t\,\dot{w}_{total_{ac}}$$

$$\dot{W}_{total} = \left(0.075\frac{lbm}{ft^3}\right)(400\ SCFM)\left(113.82\frac{BTU}{lbm}\right)\cdot\frac{1}{60}\frac{min}{sec}\cdot 1.055\frac{kW \cdot sec}{BTU}$$

$$= 60.04\ kW$$

In addition, the total power needed in $hp$ per 100 $SCFM$ is:

$$\dot{W}_{total}{}^{*} = \frac{\dot{W}_{total}}{\dot{V}}$$

$$\dot{W}_{total}{}^{*} = \frac{60.04\ kW}{400\ SCFM}\cdot\frac{1}{0.746}\frac{hp}{kw} = 20.12\frac{hp}{100\ SCFM}$$

Discussion: In this two-stage air compressor, the work needed to operate the intercooler is not taken into account. Thus, the actual work needed to operate the entire system should be higher.

## Closed System Air Compression Processes with P-V Diagrams

The above compression processes are representations of open system compressions. However, one can also view the process from a closed system (constant mass) stand point, where a mass of air at low pressure is trapped in a large volume $V_1$ that is then decreased to a smaller volume $V_2$, causing the pressure to rise. Therefore, for one compression cycle in any stage of air compressor, the relationship between the pressure $P$ and the volume $V$ within the compression stage can be described in the $P - V$ diagram in Figure 16.

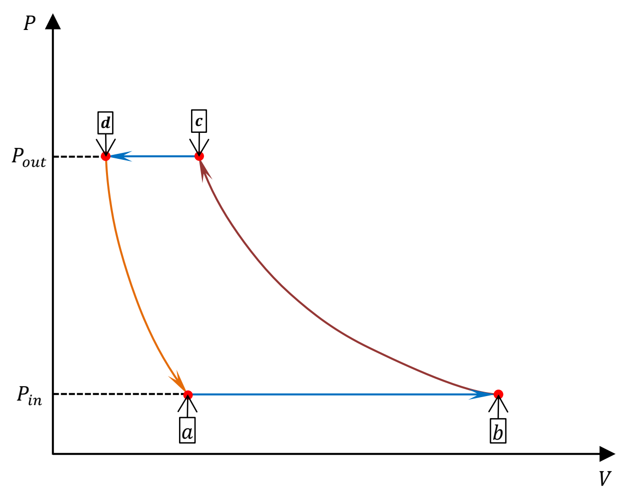



Figure 16. P-V Diagram of One Compression Cycle

One compression cycle can be described in four steps, and they are:

1. Point $a$ to point $b$, is when the air is drawn into the compression chamber. Here, the air experience volumetric expansion while, the pressure of the air stays constant.
2. Point $b$ to point $c$, is when the air compression occurs. The compression is an adiabatic process, where no heat transfers happen in the system.

3. Point $c$ to point $d$, is when an exit valve opens at point $c$ and the compressed air exits through the valve.
4. Point $d$ to point $a$, is when the compressed air left inside the compression chamber expands adiabatically as the pressure decreases. The volume of the chamber at point $d$, which is the point of maximum compression, is called the clearance volume, and is usually given in percentage of the actual displacement volume.

**Volumetric Efficiency**

Another important parameter of an air compressor is a volumetric efficiency $\eta_{vol}$. Volumetric efficiency determines what the pressure will be at the exit of the compressor. If there is a valve or other obstruction at the intake, the air pressure will be less than the atmospheric pressure during the intake, which means that there are less air molecules than there would be at atmospheric condition. The volumetric efficiency $\eta_{vol}$ can be described as shown in Equations (41) and (42), where the subscripts $a$, $b$, $c$, and $d$ refers to the points in Figure 16.

$$\eta_{vol} = \frac{(V_b - V_a)}{(V_b - V_d)}, \tag{41}$$

or

$$\eta_{vol} = 1 + C - C\left(\frac{P_c}{P_b}\right)^{\frac{1}{k}}, \tag{42}$$

where: • $V_a$, $V_b$, and $V_d$ are the gas volumes [$in^3$],

• $P_b$, and $P_c$ are the gas pressures [$psia$],

• $\eta_{vol}$ is the volumetric efficiency [%], and

• $C$ is the clearance [%].

The clearance $C$ is the ratio of the volume in the compression chamber at the maximum compression, which is called the clearance volume $V_d$, and the actual displaced volume. The clearance volume $V_d$ can be calculated by using Equation (43).

$$C = \frac{V_d}{V_b - V_a} \tag{43}$$

<u>Example Problem IV.e: Volumetric Efficiency and Clearance</u>

Objective: At a two-stage air compressor with 8% clearance $C$, and 1600 $rpm$ piston with a diameter $D$ of 4 $in$, and a length $L$ of 2.75 $in$ has an exit pressure of 100 $psig$. The system has adiabatic and isentropic compression processes. Solve for the piston volumes from point a through point b for both inner and outer piston cylinders of the low-pressure (LP), and the high-pressure (HP) stages. See Figure 16.

Assumption: The gas is an ideal gas.

The inlet condition of the gas is a standard condition.

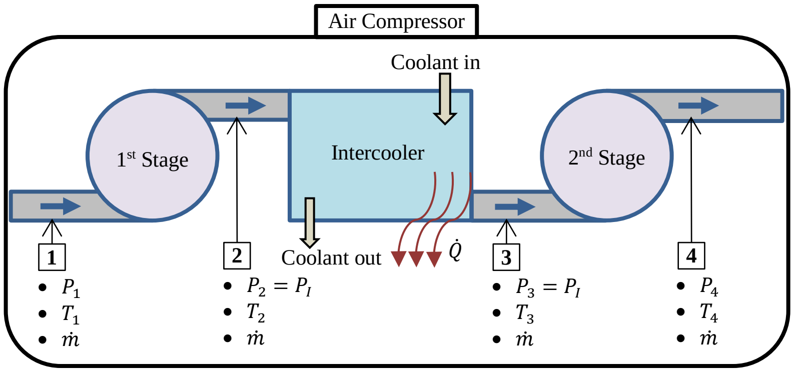
The compression is an adiabatic and polytropic process.

Knowns: $C = 8\%$ $D = 4\ in$ $L = 2.75\ in$ $P_4 = 100\ psig$

Find: • The piston volumes: $V_{a_1}$, $V_{b_1}$, $V_{c_1}$, $V_{d_1}$, $V_{a_2}$, $V_{b_2}$, $V_{c_2}$, $V_{d_2}$, and

• the volumetric efficiency: $\eta_{vol_1}$, and $\eta_{vol_2}$ (using two different methods).

Solution:




First, the intermediate pressure $P_I$ need to be calculated.

$$P_I = \sqrt{P_1 \cdot P_4}$$

$$P_I = \sqrt{(14.7\ psia) \cdot (\langle 100 + 14.7 \rangle psia)} = 41.06\ psia$$

Use a sub-subscript 1 for all variables from the 1st stage compression, and sub-subscript 2 for all variables from the 2nd stage compression.

$$P_1 = P_{a_1} = P_{b_1},$$

$$P_I = P_{c_1} = P_{d_1}, \text{ also}$$

$$P_I = P_{a_2} = P_{b_2}, \text{ and}$$

$$P_4 = P_{c_2} = P_{d_2}$$

The volume for all four points in the 1st stage compression.

Point $b$:

$$V_{b_1} = \frac{\pi\, D^2\, L}{4}$$

$$V_{b_1} = \frac{\pi\, (4\ in)^2 (2.75\ in)}{4} = 34.56\ in^3$$

Point $c$:

$$V_{c_1} = V_{b_1}\left(\frac{P_{b_1}}{P_{c_1}}\right)^{\frac{1}{k}}$$

$$V_{c_1} = (34.56\ in^3)\left(\frac{14.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}} = 16.59\ in^3$$

Point $d$:

$$V_{a_1} = V_{d_1}\left(\frac{P_{d_1}}{P_{a_1}}\right)^{\frac{1}{k}}$$

$$V_{a_1} = V_{d_1}\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}}$$

$$C = \frac{V_{d_1}}{V_{b_1} - V_{a_1}}$$

$$V_{d_1} + C\ V_{a_1} = C\ V_{b_1}$$

$$V_{d_1} + C\ V_{d_1}\left(\frac{P_{d_1}}{P_{a_1}}\right)^{\frac{1}{k}} = C\ V_{b_1}$$

$$V_{d_1} = \frac{C\ V_{b_1}}{1 + C\left(\frac{P_{d_1}}{P_{a_1}}\right)^{\frac{1}{k}}}$$

$$V_{d_1} = \frac{8\%\ (34.56\ in^3)}{1 + 8\%\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}}} = 2.37\ in^3$$

Point $a$:

$$V_{a_1} = V_{d_1}\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}}$$

$$V_{a_1} = (2.37\ in^3)\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}} = 4.94\ in^3$$

The volume for all four points in the 2nd stage compression.

Point $b$:

$$V_{b_2} = V_{c_1} - V_{d_1}$$

$$V_{b_2} = 16.59\ in^3 - 2.37\ in^3 = 14.22\ in^3$$

Point $c$:

$$V_{c_2} = V_{b_2}\left(\frac{P_{b_2}}{P_{c_2}}\right)^{\frac{1}{k}}$$

$$V_{c_2} = (14.22\ in^3)\left(\frac{41.06\ psia}{114.7\ psia}\right)^{\frac{1}{1.4}} = 6.83\ in^3$$

Point $d$:

$$V_{d_2} = \frac{C\ V_{b_2}}{1 + C\left(\frac{P_{d_2}}{P_{a_2}}\right)^{\frac{1}{k}}}$$

$$V_{d_2} = \frac{8\%\ (14.22\ in^3)}{1 + 8\%\left(\frac{114.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}}} = 0.98\ in^3$$

Point $a$:

$$V_{a_2} = V_{d_2}\left(\frac{P_{d_2}}{P_{a_2}}\right)^{\frac{1}{k}}$$

$$V_{a_2} = (0.98\ in^3)\left(\frac{114.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}} = 2.03\ in^3$$

Finally, the volumetric efficiency $\eta_{vol}$ for each stages can be calculated

For the 1st stage, using method 1.

$$\eta_{vol_1} = \frac{(V_{b_1} - V_{a_1})}{(V_{b_1} - V_{d_1})}$$

$$\eta_{vol_1} = \frac{(34.56\ in^3 - 4.94\ in^3)}{(34.56\ in^3 - 2.37\ in^3)} = 92.03\%$$

For the 1st stage, using method 2.

$$\eta_{vol_1} = 1 + C - C\left(\frac{P_{c_1}}{P_{b_1}}\right)^{\frac{1}{k}}$$

$$\eta_{vol_1} = 1 + 8\% - 8\%\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}} = 91.34\%$$

For the 2nd stage, using method 1.

$$\eta_{vol_2} = \frac{(V_{b_2} - V_{a_2})}{(V_{b_2} - V_{d_2})}$$

$$\eta_{vol_2} = \frac{(14.22\ in^3 - 2.03\ in^3)}{(14.22\ in^3 - 0.98\ in^3)} = 92.65\%$$

For the 2nd stage, using method 2.

$$\eta_{vol_2} = 1 + C - C\left(\frac{P_{c_2}}{P_{b_2}}\right)^{\frac{1}{k}}$$

$$\eta_{vol_2} = 1 + 8\% - 8\%\left(\frac{114.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}} = 91.34\%$$

Discussion: In this two-stage air compressor, the volumetric efficiency is high because the volume difference between state $a_1$ and $b_1$ is almost as big as the volume difference between state $d_1$ and $b_1$. The volume at the state $c_1$ and $d_1$ is very close to the volume at the state $b_2$ and $a_2$, respectively. This is because most of the air from the LP chamber transfers to the HP chamber. Decreasing the clearance would also increase the volumetric efficiency. Moreover, all of the volumetric efficiencies are similar, and in fact, they should be the same because the volume flow rates and the clearances are the same for the two stages. The ratio of pressures are also the same for both the 1$^{st}$ stage (LP) and the 2$^{nd}$ stage (HP) chambers.

# CHAPTER V

# COMPRESSED AIR COOLERS MODEL AND ANALYSIS

## Background

In a two-stage compressed air system, there are two units of compressed air coolers, the intercooler and the aftercooler. Both the intercooler and the aftercoolers follows the same working principle. The difference is that in intercooler, no condensation should ever occur because it may damage the air compressor. However, the aftercooler can be equipped with separator, traps, and drains which can remove any condensed humidity out from flowing further inside the compressed air system. Therefore, it is fine to have condensation forming inside the aftercooler, and in fact, typical aftercooler can produce from 30 $gallons$ to 300 $gallons$ of water per day, depending on the system pressure, and the air supply's temperature and relative humidity. Then, Figure 17 below describes a compressed air cooler with some key parameters.

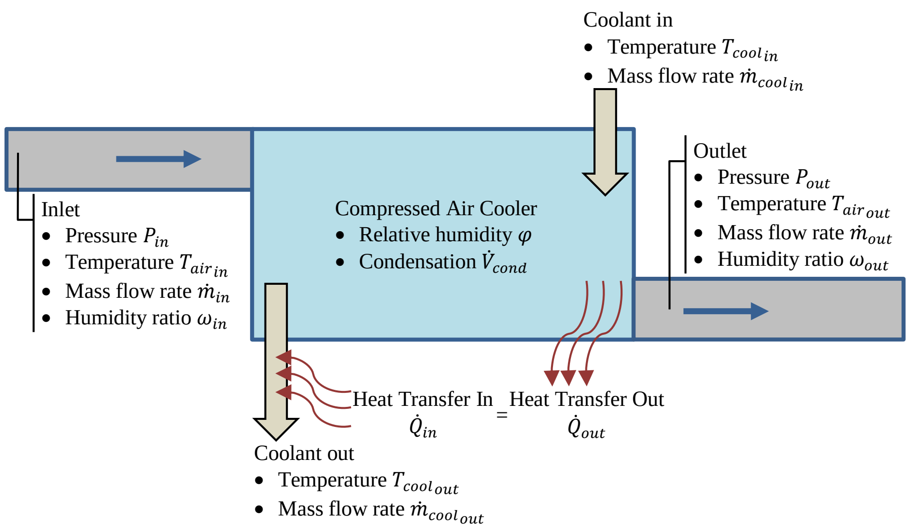



Figure 17. Compressed Air System Cooler Schematic and Process

Some assumptions that can be made for this component is listed as follows:

1. The compressed air follows the ideal gas law,
2. Constant relative humidity and mass flow rate at intercooler,
3. There are no changes in both kinetic energy and potential energy, and
4. Steady state process.

Lastly, the discussion of the MATLAB codes for calculations related to this chapter can be found in the APPENDIX D. The example problems that will be discussed in this chapter will also be covered in the APPENDIX D.

**Heat and Mass Transfer Rate**

To analyze the heat and mass transfer rate of the cooling system, the general energy rate balance for closed system in Equation (44), which is a derivation from the steady-state energy rate balance in Equation (28), should be the starting point.

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \,, \tag{44}$$

where: • $\frac{dE}{dt}$ is the energy transfer rate [$\frac{BTU}{hr}$],

• $\dot{Q}$ is the heat transfer rate [$\frac{BTU}{hr}$], and

• $\dot{W}$ is the work transfer rate [$\frac{BTU}{hr}$].

As the cooling system is a steady state system, and there is no work transfer applied on the system, the $\frac{dE}{dt}$ and $\sum \dot{W}$ should equal to zero. Moreover, since the cooling system consists of two different flows which keep exchanging heat, the heat transfer rate from the compressed air flow $\dot{Q}_{air}$ should equal to the heat transfer rate into the coolant flow $\dot{Q}_{cool}$. Thus, Equation (44) can be written as Equations (45) and (46) below.

$$0 = \left(\dot{Q}_{cool} - \dot{Q}_{air}\right) - 0\ , \tag{45}$$

or

$$\dot{Q}_{air} = \dot{Q}_{cool}\ , \tag{46}$$

where: • $\dot{Q}_{cool}$ is the coolant heat transfer rate [$\frac{BTU}{hr}$], and

• $\dot{Q}_{air}$ is the compressed air heat transfer rate [$\frac{BTU}{hr}$].

Now, there are two types of heat transfer process: the sensible heat transfer, and the latent heat transfer. In the cooling process, where condensation may occur, both heat transfer process may also occur, such that the compressed air heat transfer rate $\dot{Q}_{air}$ can be written as in Equation (47) below.

$$\dot{Q}_{air} = \dot{Q}_S + \dot{Q}_L\ , \tag{47}$$

where: • $\dot{Q}_S$ is the sensible heat transfer rate [$\frac{BTU}{hr}$], and

• $\dot{Q}_L$ is the latent heat transfer rate [$\frac{BTU}{hr}$].

First, in the sensible heat transfer, there will be no change of material phase, meaning that the gas will remain as gas with no condensation occurring. The sensible heat transfer rate $\dot{Q}_S$ at each coolers is related to the compressed air mass flow rate $\dot{m}_{air}$, which is described in Equation (18), the gas heat capacity $C_p$, and the compressed air temperature difference $\Delta T_{air}$ as shown in Equation (48) below.

$$\dot{Q}_S = \dot{m}_{air}\, C_p\, \Delta T_{air}\ , \tag{48}$$

where: • $\dot{m}_{air}$ is the compressed air mass flow rate [$\frac{lbm}{sec}$],

• $C_p$ is the heat capacity [$\frac{BTU}{lbm \cdot R}$], and

• $\Delta T_{air}$ is the compressed air temperature difference (decrease) [°F].

Using Equation (20), the compressed air mass flow rate $\dot{m}_{air}$ can also be written as Equation (49) below.

$$\dot{m}_{air} = \dot{m}_a + \dot{m}_v = \dot{m}_a + \omega\, \dot{m}_a = (1+\omega)\, \dot{m}_a\ , \tag{49}$$

where: • $\omega$ is the gas humidity ratio [unitless],

• $\dot{m}_a$ is the dry air mass flow rate [$\frac{lbm}{sec}$], and

• $\dot{m}_v$ is the water vapor mass flow rate [$\frac{lbm}{sec}$].

Second, in the latent heat transfer, there will be a change of phase while the temperature remains the same. Therefore, the latent heat transfer is independent of the fluid temperature. Moreover, the latent heat transfer only occurs when the gas temperature reached its dew point, or in other words, it occurs when the gas relative humidity $\varphi$ becomes $100\%$. The latent heat transfer rate $\dot{Q}_L$ at the aftercooler is related to the condensation generated mass flow rate $\dot{m}_{cond}$, which is described in Equation (20), and the water vapor latent heat of condensation $L_c$ as shown in Equation (50) below.

$$\dot{Q}_L = \dot{m}_{cond}\, L_c\ , \tag{50}$$

where: • $\dot{m}_{cond}$ is the condensation generated mass flow rate [$\frac{lbm}{sec}$], and

• $L_c$ is the water vapor latent heat of condensation [$\frac{BTU}{lbm}$].

The water (condensation generated) latent heat of condensation $L_c$ is a constant equals to $970.4\ \frac{BTU}{lbm}$.

Next, the coolant mass flow rate would have a heat transfer rate $\dot{Q}_{cool}$ that should be equal to the total heat transfer rate $\dot{Q}_{air}$ of the compressed air, as the energy is conserved. Also, the coolant heat transfer would be a sensible heat transfer since there should not be any matter phase change. If the coolant is water, the increase in temperature normally would not be high enough to evaporate the water, and if the coolant is outdoor air, any increase in temperature will not change its phase. Therefore, the coolant heat transfer rate $\dot{Q}_{cool}$ can be described by using Equation (51).

$$\dot{Q}_{cool} = \dot{m}_{cool}\, C_{p_{cool}}\, \Delta T_{cool}\ , \tag{51}$$

where: • $\dot{Q}_{cool}$ is the coolant heat transfer rate [$\frac{BTU}{hr}$],

• $\dot{m}_{cool}$ is the coolant mass flow rate [$\frac{lbm}{sec}$],

• $C_{p_{cool}}$ is the coolant heat capacity [$\frac{BTU}{lbm\cdot R}$], and

• $\Delta T_{cool}$ is the temperature difference (increase) [°F].

**Intercooler**

In the intercooler, the heat transfer or heat rejection process that occurs is only the sensible heat transfer, since there should not be any condensation in the two-stage air compressor package. Therefore, the latent heat transfer rate $\dot{Q}_L$ is equal to zero and the compressed air heat transfer rate is equal to the sensible heat transfer rate $\dot{Q}_S$.

In addition, the intercooler is rated in its effectiveness $\varepsilon$, which is a measure of how much the intercooler able to decrease the compressed air temperature in respect of the coolant inlet temperature. The intercooler effectiveness $\varepsilon$ is described in Equation (52) below.

$$\varepsilon = \frac{T_{in} - T_{out}}{T_{in} - T_{cool_{in}}}, \tag{52}$$

where: • $\varepsilon$ is the intercooler effectiveness [%],

• $T_{in}$ is the compressed air inlet temperature [°F],

• $T_{out}$ is the compressed air outlet temperature [°F], and

• $T_{cool_{in}}$ is the coolant inlet temperature [°F].

In the aftercooler, where condensation may occurs, the heat transfer or heat rejection process may include both the sensible and the latent heat transfer. Therefore, the compressed air heat transfer rate is equal to the sum of the sensible heat transfer rate $\dot{Q}_S$ and the latent heat transfer rate $\dot{Q}_L$ as shown in Equation (47).

Figure 18 below shows the phase transition of both the compressed air and the coolant air inside the intercooler.

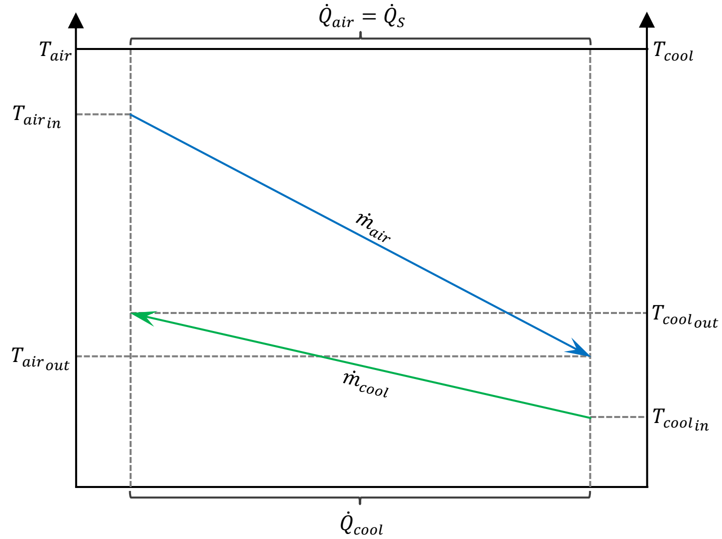



Figure 18. Phase Transition Diagram (Intercooler)

There is no phase transition for the compressed air inside the intercooler because any condensation that occurs would damage the air compressor system. Moreover, the coolant normally will not experience any phase transition. In case where the coolant used is water, the increase in its temperature will not be high enough to evaporate it. In other case where the coolant used is outdoor air, any increase in its temperature will not change its phase. Also, the slope of the line in the diagram is determined by the product of the fluid mass flow rate $\dot{m}$ and its heat capacity $C_p$.

Example Problem V.a: Intercooler Effectiveness

Objective: Air (the heat capacity is $0.240\frac{BTU}{lbm{\cdot}R}$) at 0% relative humidity and 250°F passes through an intercooler with effectiveness of 80%. If the coolant enters the intercooler at 70°F, solve for the exit temperature of the compressed air. If the air relative humidity is not 0%, solve for the maximum relative humidity.

Assumption: The gas is an ideal gas.

The gas is dry air.

No condensation occurs.

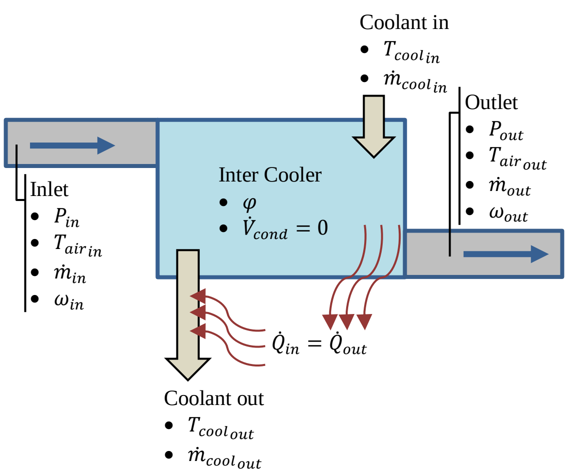
Knowns: $C_p = 0.240 \frac{BTU}{lbm \cdot R}$ $\varphi = 0\%$ $\varepsilon = 80\%$

$T_{in} = 250°\text{F}$ $T_{cool_{in}} = 70°\text{F}$

Find: • The exit temperature of the compressed air $T_{out}$, and

• the maximum allowable relative humidity $\varphi_{max}$.

Solution:




The equation to calculate the exit temperature of the compressed air $T_{out}$ can be derived from Equation (52).

$$\varepsilon = \frac{T_{in} - T_{out}}{T_{in} - T_{cool_{in}}}$$

$$T_{out} = T_{in} - \varepsilon \left(T_{in} - T_{cool_{in}}\right)$$

$$T_{out} = 250°\text{F} - 80\%\ (250°\text{F} - 70°\text{F}) = 106°\text{F}$$

The maximum allowable relative humidity $\varphi_{max}$ can be calculated using Equation (23) when the dew point temperature $T_{dew}$ is equal to the exit temperature $T_{out}$, and the temperature $T$ is the inlet temperature $T_{in}$.

$$\varphi = e^{(\frac{17.6(T_{dew} - 32)}{T_{dew} + 405} + \frac{17.6(T - 32)}{T + 405})}$$

$$\varphi_{max} = e^{(\frac{17.6(T_{out} - 32)}{T_{out} + 405} + \frac{17.6(T_{in} - 32)}{T_{in} + 405})}$$

$$\varphi_{max} = e^{(\frac{17.6(106°\text{F} - 32)}{106°\text{F} + 405} + \frac{17.6(250°\text{F} - 32)}{250°\text{F} + 405})} = 3.65\%$$

Discussion: In the intercooler where condensation is undesirable, keeping the cooled air above its dew point temperature is important. One way is to decrease the intercooler effectiveness so that the compressed air will not be cooled down too much.

**Aftercooler**

In the aftercooler, where condensation may occurs, the heat transfer or heat rejection process may include both the sensible and the latent heat transfer. Therefore, the compressed air heat transfer rate is equal to the sum of the sensible heat transfer rate $\dot{Q}_S$ and the latent heat transfer rate $\dot{Q}_L$ as shown in Equation (47).

Figure 19 below shows the phase transition of both the compressed air and the coolant air inside the aftercooler.

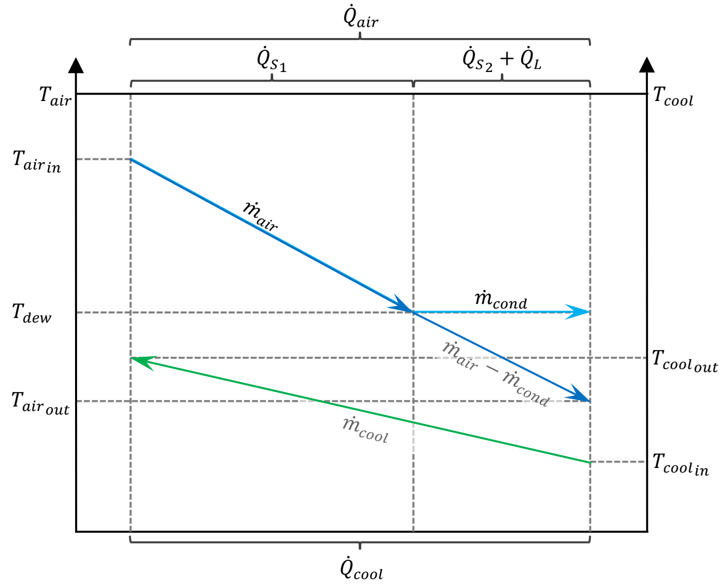



Figure 19. Phase Transition Diagram (Aftercooler)

The condensation starts when the temperature of the compressed air reaches its dew point temperature $T_{dew}$. As a fraction of the water vapor contained in the compressed air condenses, the mass of the compressed air decreases.

<u>Example Problem V.b: Condensation Generated</u>

Objective: Air (the heat capacity is $0.240\,\frac{BTU}{lbm\cdot R}$) at $10\%$ relative humidity and 250°F enters an air compressor that is rated at $1600\,SCFM$, and $125\,psig$. Then the compressed air exits the air compressor, passes through an aftercooler, and exits at 100°F. Inside the coolers, the 25 $gpm$ water coolant enters at 70°F and exits at 120°F. Solve for the mass flow rate of the condensation generated.

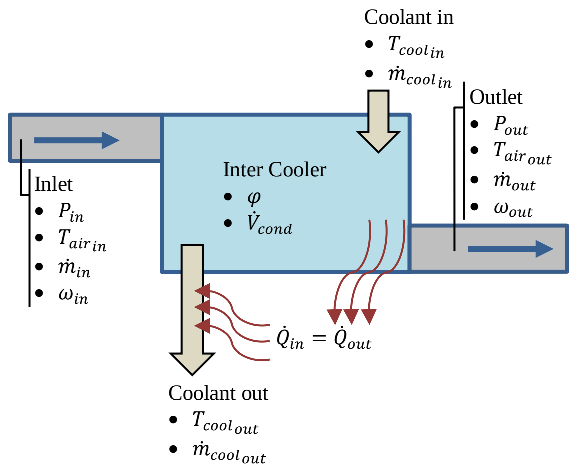
Assumption: The gas is an ideal gas.

Knowns: $C_p = 0.240\,\frac{BTU}{lbm\cdot R}$ $\quad \varphi = 10\%$

$$T_{air_{in}} = 250°\text{F} \qquad T_{air_{out}} = 100°\text{F}$$

Find: • The mass flow rate of the condensation generated $\dot{m}_{cond}$.

Solution:




First, the dew point temperature $T_{dew}$ of the compressed air need to be found to determine whether the condensation will occur or not. Using Equation (21)

$$T_{dew} = \frac{564 + 405\left(\frac{17.6\left(T_{air_{in}} - 32\right)}{T_{air_{in}} + 405} + \ln(\varphi)\right)}{17.6 - \left(\frac{17.6\left(T_{air_{in}} - 32\right)}{T_{air_{in}} + 405} + \ln(\varphi)\right)}$$

$$T_{dew} = \frac{564 + 405\left(\frac{17.6(250°\text{F} - 32)}{250°\text{F} + 405} + \ln(10\%)\right)}{17.6 - \left(\frac{17.6(250°\text{F} - 32)}{250°\text{F} + 405} + \ln(10\%)\right)} = 142.67°\text{F}$$

Since the $T_{out}$ is less than $T_{dew}$, condensation will occur.

Next, from the general energy rate balance for closed system in Equation (44), the heat transfer rate for both coolant and the compressed air can be observed.

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$\dot{Q}_{air} = \dot{Q}_{cool}$$

$$\dot{Q}_S + \dot{Q}_L = \dot{Q}_{cool}$$

The sensible heat transfer rate $\dot{Q}_S$ has to be divided into two parts. The first one is before the temperature reached the dew point temperature $\dot{Q}_{S_1}$, and the second one if for after the temperature reached the dew point temperature $\dot{Q}_{S_2}$. Therefore,

$$\dot{Q}_{S_1} + \dot{Q}_{S_2} + \dot{Q}_L = \dot{Q}_{cool}$$

$$(\dot{m})\ C_p\ \Delta T_{air_1}\ + (\dot{m} - \dot{m}_{cond})\ C_p\ \Delta T_{air_2}\ + \dot{m}_{cond}\ L_c = \dot{m}_{cool}\ C_{p_{cool}}\ \Delta T_{cool}$$

$$(\dot{m}_a + \dot{m}_v)\ C_p\ \Delta T_{air} + \dot{m}_{cond}\ \left( L_c - C_p\ \Delta T_{air_2}\right)\ = \dot{m}_{cool}\ C_{p_{cool}}\ \Delta T_{cool}$$

$$(\dot{m}_a + \omega\ \dot{m}_a)\ C_p\ \Delta T_{air} + \dot{m}_{cond}\ \left( L_c - C_p\ \Delta T_{air_2}\right)\ = \dot{m}_{cool}\ C_{p_{cool}}\ \Delta T_{cool}$$

$$\dot{m}_{cond}\ = \frac{\left(\dot{m}_{cool}\ C_{p_{cool}}\ \Delta T_{cool}\right) - \left((1+\omega)\ \dot{m}_a\ C_p\ \Delta T_{air}\right)}{\left( L_c - C_p\ \Delta T_{air_2}\right)}$$

Then, the coolant mass flow rate $\dot{m}_{cool}$ is:

$$\dot{m}_{cool} = \rho\ \dot{V}$$

$$\dot{m}_{cool} = \left(62.4\frac{lbm}{ft^3}\right)\left(25\ gpm \cdot \frac{1}{7.48}\frac{ft^3}{gal} \cdot \frac{1}{60}\frac{min}{sec}\right) = 3.48\frac{lbm}{sec}$$

Then, the compressed air mass flow rate $\dot{m}_a$ is:

$$\dot{m}_a = \rho\ \dot{V}$$

$$\dot{m}_a = \left(0.075\frac{lbm}{ft^3}\right)\left(1600\frac{ft^3}{min} \cdot \frac{1}{60}\frac{min}{sec}\right) = 2.00\frac{lbm}{sec}$$

Then, the humidity ratio $\omega$ is:

$$\omega = \frac{0.622\ \varphi\ P_{vs}}{P - \varphi\ P_{vs}}$$

Where the water vapor saturation partial pressure $P_{vs}$ found from Table 4 is $29.844\ psia$, therefore,

$$\omega = \frac{0.622\ (10\%)(29.844\ psia)}{\left((125 + 14.7)psia - (10\%)(29.844\ psia)\right)} = 0.0136$$

Thus, mass flow rate of the condensation generated $\dot{m}_{cond}$ will be:

$$\dot{m}_{cond} \ = \frac{\left(\dot{m}_{cool}\ C_{p_{cool}}\ \Delta T_{cool}\right) - \left((1+\omega)\ \dot{m}_a\ C_p\ \Delta T_{air}\right)}{\left(\ L_c - C_p\ \Delta T_{air_2}\right)}$$

Where

$$\dot{m}_{cool}\ C_{p_{cool}}\ \Delta T_{cool} = \left(3.48\frac{lbm}{sec}\right)\left(0.446\frac{BTU}{lbm \cdot R}\right)(\langle 120 - 70\rangle R)$$

$$= 77.51\frac{BTU}{sec},$$

$$(1+\omega)\ \dot{m}_a\ C_p\ \Delta T_{air} = (1 + .0136)\left(2.0\frac{lbm}{sec}\right)\left(0.24\frac{BTU}{lbm \cdot R}\right)(\langle 250 - 100\rangle R)$$

$$= 72.98\frac{BTU}{sec},$$

and

$$\left(\ L_c - C_p\ \Delta T_{air_2}\right) = \left(970.4\ \frac{BTU}{lbm}\right) - \left(0.240\frac{BTU}{lbm \cdot R}\right)(\langle 142.67 - 100\rangle R)$$

$$= 960.16\frac{BTU}{lbm}$$

Thus,

$$\dot{m}_{cond} \ = \frac{\left(77.513\frac{BTU}{sec}\right) - \left(72.978\frac{BTU}{sec}\right)}{\left(\ 960.159\frac{BTU}{lbm}\right)} \cdot 3600\frac{sec}{hr} = 17\ \frac{lbm}{hr}$$

Discussion: The cooling process in the aftercooler will produce condensation. Therefore, proper trap and drainage system need to be equipped into the aftercooler to remove the condensed moisture. The phase transition diagram for this problem can be seen below, where the temperature $T$ unit is the degree Fahrenheit °F, the heat transfer rate $\dot{Q}$ unit is the British thermal unit per second $\frac{BTU}{sec}$, and the mass flow rate $\dot{m}$ unit is the pound mass per second $\frac{lbm}{sec}$.

# CHAPTER VI

# COMPRESSED AIR RECEIVERS AND STORAGE MODEL AND ANALYSIS

## Background

The purpose of the compressed air receiver or storage is to store and deliver a specified pressure $P$ over a specified time period $t$. Some assumptions that can be made for this component is listed as follows:

1. The compressed air follows the ideal gas law, and
2. The receiver volume $V$, temperature $T$, inlet mass flow rate $\dot{m}_{in}$, and exit mass flow rate $\dot{m}_{ex}$ are constants.

A simple diagram of the receiver or storage tank is shown in Figure 20 below.

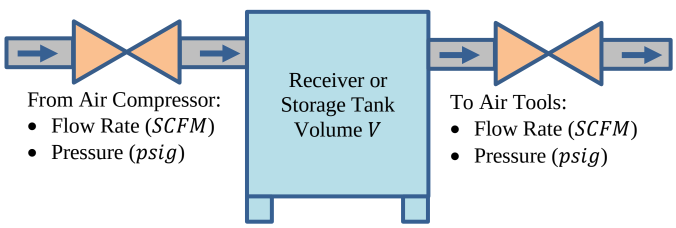



Figure 20. Compressed Air System Receiver Schematic and Process

There are three different cases where the receiver can work. The first one is called the charging condition; it is a condition when there is a constant mass flow rate into the receiver, and zero mass flow rate out from the receiver. The second case is called the discharging condition; it is a condition when there is zero mass flow rate into the receiver, and a constant mass flow rate out from the receiver, which is the opposite of the charging condition. The third case is the condition where the charging and the discharging happen simultaneously, meaning that there is a constant mass flow rate into the receiver, and a constant mass flow rate out from the receiver.

Lastly, the discussion of the MATLAB codes for calculations related to this chapter can be found in the APPENDIX E. The example problems that will be discussed in this chapter will also be covered in the APPENDIX E.

## Receiver Volume

To analyze the volume and mass of the receiver, the conservation of mass in Equation (53) should be the starting point.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \, , \tag{53}$$

where: • $\frac{dm}{dt}$ is the receiver mass change rate [$\frac{lbm}{sec}$],

• $\dot{m}_i$ is the inlet mass transfer rate [$\frac{lbm}{sec}$], and

• $\dot{m}_e$ is the outlet mass transfer rate [$\frac{lbm}{sec}$].

Then, integrating Equation (53) above will render Equation (54) below that shows the change of receiver mass $\Delta m$ over a period of time $t$.

$$\Delta m = (\dot{m}_{in} - \dot{m}_{out}) \, t \, , \tag{54}$$

where: • $\Delta m$ is the change of receiver mass [$lbm$], and

• $t$ is the time period [$sec$].

Substituting Equation (19) into Equation (54) above will shows the relationship between the change of receiver mass and both the inlet and the outlet volume flow rates and density as shown below in Equation (55).

$$\Delta m = \rho \left(\dot{V}_{in} - \dot{V}_{out}\right) t \, , \qquad (55)$$

where: • $\rho$ is the compressed air density [$\frac{lbm}{ft^3}$],

• $\dot{V}_{in}$ is the compressed air inlet volume flow rate [$\frac{ft^3}{sec}$], and

• $\dot{V}_{out}$ is the compressed air outlet volume flow rate [$\frac{ft^3}{sec}$].

The change of receiver mass can also be described as the difference between the receiver final mass, and the receiver initial mass as shown in Equations (56), (57), and (58) below.

$$\Delta m = m_f - m_i \qquad (56)$$

in which

$$m_i = \frac{P_i \, V}{R \, T} \, , \qquad (57)$$

and

$$m_f = \frac{P_f \, V}{R \, T} \, , \qquad (58)$$

where: • $m_i$ is the initial compressed air mass [$lbm$],

• $m_f$ is the final compressed air mass [$lbm$],

• $P_i$ is the initial compressed air pressure [$psia$],

• $P_f$ is the final compressed air pressure [$psia$],

• $V$ is the compressed air volume [$ft^3$ or $gallon$],

• $R$ is the ideal gas constant [$\frac{ft \cdot lbf}{lbm \cdot R}$], and

• $T$ is the receiver temperature [°F].

Thus, combining Equations (55), (56), (57), and (58) will result in Equations (59), (60), or (61) below.

$$\rho \left(\dot{V}_{in} - \dot{V}_{out}\right) t = \frac{\left(P_f - P_i\right) V}{R\ T}, \tag{59}$$

$$V = \frac{\rho \left(\dot{V}_{in} - \dot{V}_{out}\right) t\ R\ T}{P_f - P_i}, \tag{60}$$

or

$$P_f = P_i + \frac{\rho \left(\dot{V}_{in} - \dot{V}_{out}\right) t\ R\ T}{V}, \tag{61}$$

**Receiver Charging**

The first working scenario of a receiver is when there is only a constant inlet volume flow rate $\dot{V}_{in}$, and zero outlet volume flow rate $\dot{V}_{out}$. Such process is called the receiver charging process.

Example Problem VI.a: Receiver Charging

Objective: Solve for the time required to fully charge a partially filled $200\ gallon$ receiver at temperature of 70°F that is being supplied by a system that delivers $300\ SCFM$ of compressed dry air with density of $0.075 \frac{lbm}{ft^3}$ at $150\ psig$. The initial pressure of the receiver is $100\ psig$.

Assumption: The gas is an ideal gas.

The gas is dry air.

The temperature is constant.

The inlet mass flow rate is constant.

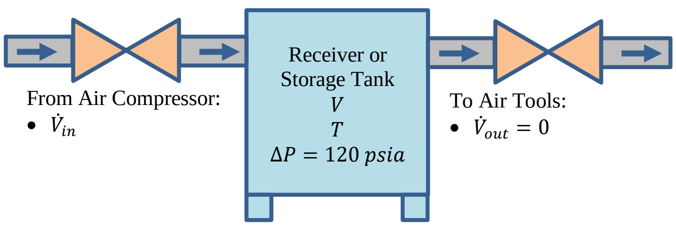
The exit mass flow rate is zero.

Knowns: $V = 200\ gallon$ $T = 70°\text{F}$ $\dot{V}_{in} = 300\ SCFM$

$P_i = 100\ psig$ $P_f = 150\ psig$ $\rho = 0.075 \frac{lbm}{ft^3}$

Find: • The time $t$ required to fully charge the receiver.

Solution:




Deriving the conservation of mass in Equation (53) renders Equation (60) which can be used to calculate the volume $V$, as well as the time $t$.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$m_f - m_i = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t$$

$$\frac{\left(P_f - P_i\right) V}{R\ T} = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t$$

Such that the volume $V$ and the time $t$ can be expressed as the two equations below.

$$V = \frac{\rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t\ R\ T}{P_f - P_i}$$

$$t = \frac{V\left(P_f - P_i\right)}{\rho\ \left(\dot{V}_{in} - \dot{V}_{out}\right)\ R_a\ T}$$

Therefore, the time $t$ required to fully charge the receiver is:

$$t = \frac{V\left(P_f - P_i\right)}{\rho\ \left(\dot{V}_{in} - 0\right)\ R_a\, T}$$

$$t = \frac{\left(200\ gal \cdot \frac{1}{7.48}\frac{ft^3}{gal}\right)\left(\left(150\frac{lbf}{in^2} - 100\frac{lbf}{in^2}\right)144\frac{in^2}{ft^2}\right)}{\left(0.075\frac{lbm}{ft^3}\right)\left(300\frac{ft^3}{min}\cdot\frac{1}{60}\frac{min}{sec}\right)\left(53.33\frac{lbf\cdot ft}{lbm\cdot R}\right)(529.67\ R)} = 18.2\ sec$$

Discussion: A $300\, SCFM$ air compressor takes about $18\, seconds$ to fully fill a $200\ gallon$ receiver or storage tank.

## Receiver Discharging

The second working scenario of a receiver is when there is zero constant inlet volume flow rate $\dot{V}_{in}$, and only a constant outlet volume flow rate $\dot{V}_{out}$. Such process is called the receiver discharging process.

Example Problem VI.b: Receiver Discharging

Objective: Solve for the size or the volume of the receiver needed to deliver air at a pressure of at least $100\, psig$ over $20\, seconds$ period, if it starts delivering air at $120\ psig$, and is supplied by a system that delivers $300\ SCFM$ of compressed dry air with density of $0.075\frac{lbm}{ft^3}$ at $120\ psig$. The receiver temperature is constant at 70°F.

Assumption: The gas is an ideal gas.

The gas is dry air.

The temperature is constant.

The inlet mass flow rate is zero.

The exit mass flow rate is constant.

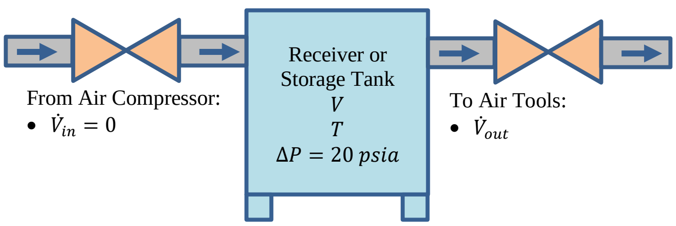
Knowns: $t = 20\ seconds$ $T = 70°\text{F}$ $\dot{V}_{out} = 300\ SCFM$

$P_i = 120\ psig$ $P_f = 100\ psig$ $\rho = 0.075 \frac{lbm}{ft^3}$

Find: • The volume $V$ of the receiver, and

• the initial and final mass ($m_i$ and $m_f$) of the air inside the receiver.

Solution:




Deriving the conservation of mass in Equation (53) renders Equation (60) which can be used to calculate the volume $V$.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$m_f - m_i = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t$$

$$\frac{\left(P_f - P_i\right) V}{R\ T} = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t$$

Such that the volume $V$ can be expressed as the equation below.

$$V = \frac{\rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t\ R\ T}{P_f - P_i}$$

Therefore, the volume $V$ of the receiver is:

$$V = \frac{\left(0.075\,\frac{lbm}{ft^3}\right)\left(0 - 300\,\frac{ft^3}{min}\cdot\frac{1}{60}\frac{min}{sec}\right)(20\,sec)\left(53.33\,\frac{lbf\cdot ft}{lbm\cdot R}\right)(529.67R)}{\left(\langle 100 - 120\rangle\frac{lbf}{in^2}\cdot 144\,\frac{in^2}{ft^2}\right)}$$

$$= 73.4\,ft^3 = 549\,gallons$$

Moreover, the initial and final mass of the compressed air inside the receiver are:

$$m_i = \frac{P_i\,V}{R_a\,T}$$

$$m_i = \frac{\left(\langle 120 + 14.7\rangle\frac{lbf}{in^2}\cdot 144\,\frac{in^2}{ft^2}\right)(73.4\,ft^3)}{\left(53.33\,\frac{lbf\cdot ft}{lbm\cdot R}\right)(70 + 459.67R)} = 50.5\,lbm$$

and

$$m_f = \frac{P_f\,V}{R_a\,T}$$

$$m_f = \frac{\left(\langle 100 + 14.7\rangle\frac{lbf}{in^2}\cdot 144\,\frac{in^2}{ft^2}\right)(73.4\,ft^3)}{\left(53.33\,\frac{lbf\cdot ft}{lbm\cdot R}\right)(70 + 459.67R)} = 43.0\,lbm$$

Discussion: The mass of the compressed air inside the receiver should be greater initially. The change of mass depends on the change in pressure.

<u>Example Problem VI.c: Combination</u>

Objective: Solve for the time that the air compressor need to fully recharge an operating $200\,gallon$ receiver at temperature of 70°F, if the air compressor delivers $100\,SCFM$ of compressed dry air with density of $0.075\,\frac{lbm}{ft^3}$ at $130\,psig$, and the receiver supplies 30 $SCFM$ of compressed dry air to the equipment at pressure of at least 110 $psig$. The recharging happens when the receiver pressure reaches 110 $psig$.

Assumption: The gas is an ideal gas.

The gas is dry air.

The temperature is constant.

The inlet mass flow rate is constant.

The exit mass flow rate is constant.

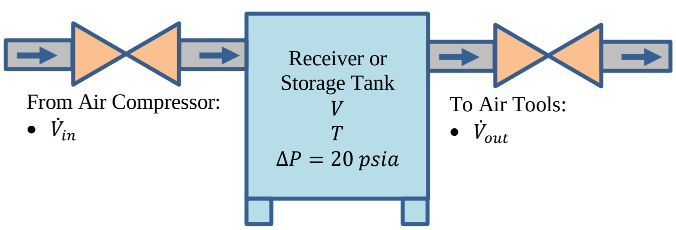
Knowns: $V = 200\ gallon$ $T = 70°\text{F}$ $\rho = 0.075 \frac{lbm}{ft^3}$

$P_i = 110\ psig$ $P_f = 130\ psig$

$\dot{V}_{in} = 100\ SCFM$ $\dot{V}_{out} = 30\ SCFM$

Find: • The time $t$ required to fully charge the receiver.

Solution:




Deriving the conservation of mass in Equation (53) renders Equation (60) which can be used to calculate the volume $V$, as well as the time $t$.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$m_f - m_i = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t$$

$$\frac{\left(P_f - P_i\right) V}{R\ T} = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t$$

Such that the volume $V$ and the time $t$ can be expressed as the two equations below.

$$V = \frac{\rho \left(\dot{V}_{in} - \dot{V}_{out}\right) t\ R\ T}{P_f - P_i}$$

$$t = \frac{V\left(P_f - P_i\right)}{\rho\ \left(\dot{V}_{in} - \dot{V}_{out}\right)\ R_a\ T}$$

Therefore, the time $t$ required to fully charge the receiver is:

$$t = \frac{\left(200\ gal \cdot \frac{1}{7.48} \frac{ft^3}{gal}\right) \left(\langle 130 - 110 \rangle \frac{lbf}{in^2} \cdot 144 \frac{in^2}{ft^2}\right)}{\left(0.075 \frac{lbm}{ft^3}\right)\left(\langle 100 - 30 \rangle \frac{ft^3}{min} \cdot \frac{1}{60} \frac{min}{sec}\right)\left(53.33 \frac{lbf \cdot ft}{lbm \cdot R}\right)(529.67R)}$$

$$= 31.2\ seconds$$

Discussion: A greater volume takes longer to recharge because the change of mass has to be greater. The time to recharge to receiver would also be greater when using the compressed air tools because the compressed air is being drawn out simultaneously.

CHAPTER VII

# PIPING MODEL AND ANALYSIS

## Background

In a compressed air system, piping is an important component because it connects all of the components together and thus may highly affect the fluid that flows through it. When the fluid or gas flows through the pipe, the inner surface of the pipe plays a huge role on how much the gas pressure may drop due to the friction. A diagram in Figure 21 below describes the fluid flow inside the pipe.

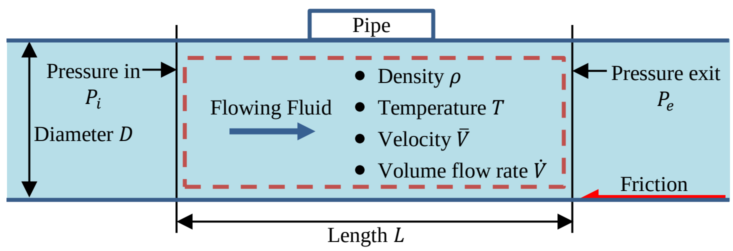



Figure 21. Pipe with Flowing Fluid Diagram

## Pressure Gradient and Pressure Drop

The pressure gradient $\frac{dP}{dx}$ is a physical quantity that describes the rate of change of pressure across particular length. It is an important quantity to analyze the amount of pressure drop within a particular location, and is described in Equation (62).

$$\frac{dP}{dx} = \frac{\rho\,\bar{V}^2\,f}{2\,D}, \tag{62}$$

where: • $\frac{dP}{dx}$ is the pressure gradient $[\frac{psi}{1000\,ft}]$,

• $\rho$ is the compressed air density [$\frac{lbm}{ft^3}$],

• $\bar{V}$ is the compressed air flow velocity [$\frac{ft}{sec}$],

• $f$ is the inner surface friction factor of the pipe [unitless], and

• $D$ is the inner diameter of the pipe [$in$].

For normal industrial piping, the typical values for air flow velocity is around $20\frac{ft}{s}$ to $30\frac{ft}{s}$. From Equation (62), the formula to calculate the pressure difference or pressure drop $\Delta P$ across a specific length $L$ can be derived into Equations (63) and (64).

$$\frac{\Delta P}{L} = \frac{\rho\, \bar{V}^2\, f}{2\, D}, \tag{63}$$

thus

$$\Delta P = \frac{\rho\, \bar{V}^2\, f\, L}{2\, D}, \tag{64}$$

where: • $\Delta P$ is the pressure drop [$psia$], and

• $L$ is the length [$ft$].

## Friction Factor and Reynold's Number

Next, the friction factor $f$ can be calculated by using the $Re$, the pipe inner diameter $D$, and the surface roughness $\epsilon$ as shown in Equation (65) below.

$$f = \frac{1.325}{\left[\ln\left(\frac{\epsilon}{3.7\, D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}, \tag{65}$$

where: • $\epsilon$ is the pipe inner surface roughness [$in$], and

• $Re$ is the Reynold's number [unitless].

Equation (65) only works for a turbulent flow where $3000 \leq Re \leq 10^8$, and relative pipe roughness within the range of $10^{-6} \leq \frac{\epsilon}{D} \leq 10^{-2}$. Moreover, for smooth pipe, the friction factor can be calculated by using Equation (66) for $Re < 50{,}000$, and Equation (67) for $Re \geq 50{,}000$ as well.

$$f = \frac{0.316}{Re^{0.25}} \tag{66}$$

and

$$f = \frac{0.184}{Re^{0.2}} \tag{67}$$

For laminar flow where, the Reynold's number is equal to or less than $2100$, the friction factor follows Equation (68).

$$f = \frac{64}{Re} \tag{68}$$

The list of surface roughness for a number of different materials can be found by using Moody diagram as shown in Figure 22 below.

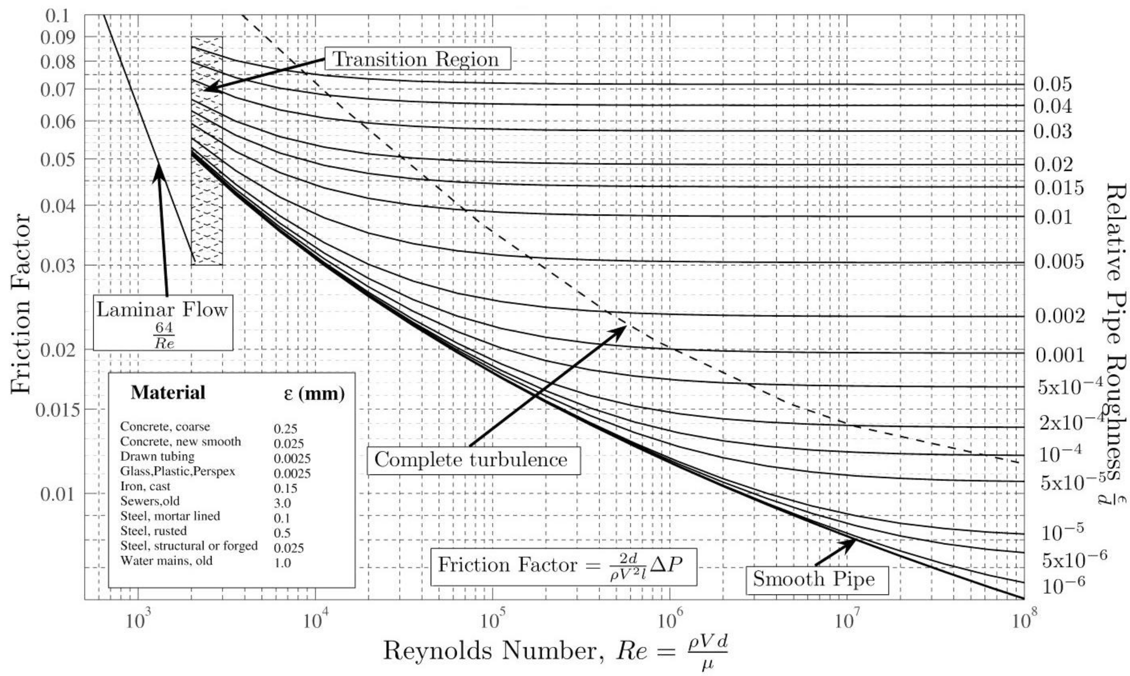



Figure 22. Moody Diagram [5]

Then, the formula to calculate the Reynold's number is shown in Equation (69) below.

$$Re = \frac{\rho \bar{V} D}{\mu}, \tag{69}$$

The gas dynamic viscosity can be found by using Equation (70) below.

$$\mu = \mu_0 \frac{0.555\, T_0 + C_S}{0.555\, T + C_S} \left(\frac{T}{T_0}\right)^{1.5}, \tag{70}$$

where: • $\mu$ is the gas dynamic viscosity $[\frac{lbf \cdot s}{ft^2}]$,

• $\mu_0$ is the gas reference viscosity $[\frac{lbf \cdot s}{ft^2}]$,

• $T$ is the gas temperature $[R]$,

• $T_0$ is the gas reference temperature $[R]$, and

• $C_S$ is the Sutherland's constant for the gas [unitless].

For air with the Sutherland's constant $C_S$ of $120$, the reference viscosity and the reference temperature are $3.816 \cdot 10^{-7}\ \frac{lbf \cdot s}{ft^2}$, and $524.07\ R$, or $65.4$°F respectively.

Example Problem VII.a: Friction Factor and Pressure Drop

Objective: For a $20 \frac{ft}{s}$ velocity in a $3\ inch$ diameter of $110\ psig$ compressed air distribution piping system with a smooth pipe, solve for the Reynold's number $Re$ of the flowing compressed air, the friction factor $f$ of the pipe, and the pressure gradient $\frac{dP}{dx}$ (in $\frac{psi}{1000\ ft}$) and the pressure drop in a $2000\ ft$ length.

Assumption: The gas is an ideal gas.

The temperature of the air equals to the temperature of the outside air.

The outside air is in standard condition.

Knowns: $\bar{V} = 20 \frac{ft}{s}$ $D = 3\ inch$ $L = 2000\ ft$ $P = 110\ psig$

Find:

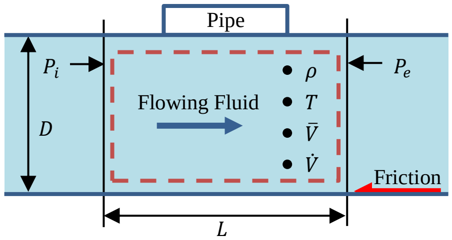
• The Reynold's number $Re$ of the air,

• the friction factor $f$ of the pipe,

• the pressure gradient $\frac{dP}{dx}$, and

• the pressure drop $\Delta P$.

Solution:




From Equation (7), the density $\rho$ of the air inside the pipe can be determined.

$$v = \rho^{-1} = \frac{V}{m} = \frac{R\,T}{P}$$

$$R\,T = \frac{P}{\rho}$$

Since $R$ is constant, and $T = T_{std}$, therefore:

$$R\,T_{std} = R\,T$$

$$\frac{P_{std}}{\rho_{std}} = \frac{P}{\rho}$$

$$\rho = P\,\frac{\rho_{std}}{P_{std}}$$

$$\rho = (\langle 110 + 14.7 \rangle psia)\frac{\left(0.075\,\frac{lbm}{ft^3}\right)}{(14.7\;psia)} = 0.636\,\frac{lbm}{ft^3}$$

The Reynold's number $Re$ of the flowing compressed air is:

$$Re = \frac{\rho\,\bar{V}\,D}{\mu}$$

$$Re = \frac{\left(0.636\,\frac{lbm}{ft^3}\right)\left(20\,\frac{ft}{s}\right)\left(3\;in \cdot \frac{1}{12}\frac{ft}{in}\right)}{\left(3.816 \cdot 10^{-7}\,\frac{lbf \cdot s}{ft^2} \cdot 32.2\,\frac{lbm \cdot ft}{lbf \cdot s^2}\right)} = 258{,}891$$

The Reynold's number shows that the flow is a turbulent flow since $Re \geq 3000$.

Then, the friction factor $f$ can be calculated as well as shown below.

$$f = \frac{0.184}{Re^{0.2}}$$

$$f = \frac{0.184}{258{,}891^{0.2}} = 0.0152$$

Then, if at the same compressed air distribution piping system, the pressure gradient $\frac{dP}{dx}$ (in $\frac{psi}{1000\ ft}$) and the pressure drop $\Delta P$ in a $2000\ ft$ length would be:

$$\frac{dP}{dx} = \frac{\rho\,\bar{V}^2\,f}{2\,D}$$

$$\frac{dP}{dx} = \frac{\left(0.636\,\frac{lbm}{ft^3}\right)\left(20\,\frac{ft}{s}\right)^2 (0.0152)}{2\,\left(3\ in \cdot \frac{1}{12}\frac{ft}{in}\right)} \cdot \frac{1}{32.2}\frac{lbf \cdot s^2}{lbm \cdot ft}$$

$$\frac{dP}{dx} = 0.240\,\frac{\frac{lbf}{ft^2}}{ft} \cdot \frac{1000 \cdot \frac{1}{144}\frac{ft^2}{in^2}}{1000} = 1.668\,\frac{psi}{1000\ ft}$$

and

$$\Delta P = \frac{\rho\,\bar{V}^2\,f\,L}{2\,D}$$

$$\Delta P = \frac{dP}{dx} L$$

$$\Delta P = 1.668\,\frac{psi}{1000\ ft} \cdot 2000\ ft = 3.337\ psia$$

Which means that the pressure drop is about $2.68\%$ of the initial pressure.

$$\frac{\Delta P}{P} \cdot 100\% = \frac{3.337\ psia}{(\langle 110 + 14.7 \rangle psia)} \cdot 100\% = 2.68\%$$

Discussion: There is another way to find the Reynold’s number, which is to use the Moody Diagram, which is shown in Figure 22. From the Moody Diagram or the equation for the friction factor, it can be seen that for greater values of Reynold's number, the friction factor is smaller. In other words, if the air density and velocity, and the pipe diameter increase, the friction factor will decrease, which makes sense because the viscosity is the internal friction of a fluid.

# CHAPTER VIII

# FLOW REGULATORS MODEL AND ANALYSIS

## Background

A regulator supplies air to a compressed air tools by dropping the system pressure $P_{sys}$ down to the tools required inlet pressure $P_{tool}$. Generally, the tools volume flow rate $\dot{V}_{tool}$ is given in a standard condition unit, therefore, a conversion to a system condition is needed to find the system volume flow rate $\dot{V}$. Equation (71) below shows how the conversion works.

$$\dot{V} = \dot{V}_{tool} \frac{P_{atm}}{P_{sys}} \tag{71}$$

A simple diagram describing the regulator component is shown in Figure 23 below.

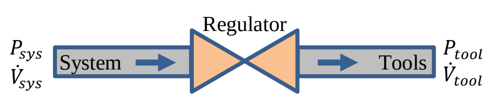



Figure 23. Compressed Air System Pipe – Regulator Schematic and Process

## Equivalent Length and Loss Coefficient

The diameter $D$ of the pipe size to the regulator can be found by using the relationship between the volume flow rate, pipe diameter, and the flow velocity as shown in the following Equation (72).

$$D = \sqrt{\frac{4\,\dot{V}}{\pi\,\overline{V}}}, \tag{72}$$

where: • $D$ is the pipe diameter $[in]$,

- $\dot{V}$ is the gas volume flow rate [$\frac{ft^3}{min}$ or $CFM$], and
- $\bar{V}$ is the gas flow velocitty [$\frac{ft}{sec}$].

Furthermore, the equivalent length $L_{eq}$ of the regulator can be found by using Equation (63), where the $L_{eq}$ is the same as the $L$, and $\Delta P$ is the pressure difference between the system pressure and the tools required inlet pressure. With that said, the Reynold's number and the friction factor $f$ should also be calculated. With all the obtained values, a loss coefficient $K_L$ can also be found by using Equation (73).

$$K_L = \frac{f\, L_{eq}}{D}\,, \tag{73}$$

where: • $K_L$ is the loss coefficient [unitless],

- $f$ is the friction factor [unitless], and
- $L_{eq}$ is the equivalent length [$ft$].

Some well-known values of typical loss coefficient $K_L$ for common hardware in a compressed air piping system are as follows:

a. Regular 90°, flanged elbow: $K_L = 0.3$

b. Branched flow, flanged tee: $K_L = 1.0$

c. Gate valve, fully open: $K_L = 0.15$

<u>Example Problem VIII.a: Pipe Diameter and Loss Coefficient</u>

Objective: Solve for the pipe diameter $D$ going to the regulator that supplies compressed air into a 50 $SCFM$ air tool by dropping the 120 $psig$ system pressure down to 80 $psig$ air tool inlet pressure.

Assumption: The gas is an ideal gas.

The pipe inner surface is smooth.

The temperature of the air equals to the temperature of the outside air.

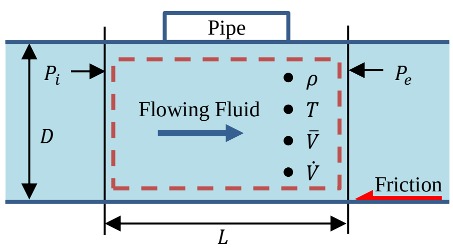
The outside air or the tool's outlet condition is in standard condition.

Knowns: $\dot{V}_{tool} = 50\, SCFM$ $\quad P_{min} = 80\, psig$ $\quad P_{sys} = 120\, psig$

Find: • The pipe diameter $D$ going to the regulator, and

• the loos coefficient $K_L$ of the regulator.

Solution:




From Equation (7), the volume flow rate $\dot{V}_{sys}$ of the air inside the pipe can be determined.

$$v = \rho^{-1} = \frac{V}{m} = \frac{R\,T}{P}$$

$$V\,P = m\,R\,T$$

Since $m$, and $R$ are constant, and $T = T_{std} = T_{tool}$, therefore:

$$m\,R\,T_{sys} = m\,R\,T_{tool}$$

$$V_{sys}\,P_{sys} = V_{tool}\,P_{tool}$$

$$\frac{V_{sys}}{V_{tool}} = \frac{P_{tool}}{P_{sys}}$$

Also, the ratio of the volume should equal to the ratio of the volume flow rate, therefore:

$$\frac{V_{sys}}{V_{tool}} = \frac{\dot{V}_{sys}}{\dot{V}_{tool}} = \frac{P_{tool}}{P_{sys}}$$

$$\dot{V}_{sys} = \dot{V}_{tool}\frac{P_{tool}}{P_{sys}}$$

$$\dot{V}_{sys} = (50\ SCFM)\frac{14.7\ psia}{(\langle 120 + 14.7\rangle psia)} = 5.46\frac{ft^3}{min}$$

Therefore, the pipe diameter $D$ can be calculates as:

$$D = \sqrt{\frac{4\ \dot{V}}{\pi\ \bar{V}}}$$

$$D = \sqrt{\frac{4\left(5.46\frac{ft^3}{min}\right)}{\pi\left(30\frac{ft}{sec}\cdot 60\frac{sec}{min}\right)}}\cdot 12\frac{in}{ft} = 0.746\ in$$

Note that the compressed air velocity used in the calculation above is $30\frac{ft}{s}$, which is the typical maximum air velocity. From Equation (7), the density $\rho$ of the air inside the pipe can be determined.

$$v = \rho^{-1} = \frac{V}{m} = \frac{R\ T}{P}$$

$$R\ T = \frac{P}{\rho}$$

Since $R$ is constant, and $T = T_{std} = T_{tool}$, therefore:

$$R\ T_{std} = R\ T$$

$$\frac{P_{std}}{\rho_{std}} = \frac{P}{\rho}$$

$$\rho = P\ \frac{\rho_{std}}{P_{std}}$$

$$\rho = (\langle 120 + 14.7 \rangle psia) \frac{\left(0.075 \frac{lbm}{ft^3}\right)}{(14.7\ psia)} = 0.687 \frac{lbm}{ft^3}$$

Then the loss coefficient $K_L$ can be calculated as follows: First calculate the Reynold's number $Re$:

$$Re = \frac{\rho\ \bar{V}\ D}{\mu}$$

$$Re = \frac{\left(0.687 \frac{lbm}{ft^3}\right)\left(30 \frac{ft}{s}\right)\left(0.746\ in \cdot \frac{1}{12} \frac{ft}{in}\right)}{\left(3.816 \cdot 10^{-7}\ \frac{lbf \cdot s}{ft^2} \cdot 32.2 \frac{lbm \cdot ft}{lbf \cdot s^2}\right)} = 104{,}310$$

Then, find the friction factor $f$:

$$f = \frac{0.184}{Re^{0.2}}$$

$$f = \frac{0.184}{104{,}310^{0.2}} = 0.0182$$

Next, the equivalent length $L_{eq}$ can be calculated by using this equation:

$$\frac{\Delta P}{L} = \frac{\rho\ \bar{V}^2\ f}{2\ D}$$

$$L_{eq} = \frac{2\ D\ \Delta P}{\rho\ \bar{V}^2\ f}$$

$$L_{eq} = \frac{2\left(0.746\ in \cdot \frac{1}{12} \frac{ft}{in}\right)\left(\langle 120 - 80 \rangle psia \cdot 144 \frac{in^2}{ft^2}\right)}{\left(0.687 \frac{lbm}{ft^3}\right)\left(30 \frac{ft}{s}\right)^2 (0.0182)} \cdot 32.2 \frac{lbm \cdot ft}{lbf \cdot s^2}$$

$$= 2048.5\ ft$$

Finally, the loss coefficient $K_L$ is:

$$K_L = \frac{f\ L_{eq}}{D}$$

$$K_L = \frac{(0.0182)(2048.5\ ft)}{\left(0.746\ in \cdot \frac{1}{12}\frac{ft}{in}\right)} = 599.7$$

Discussion: To find the size of the pipe going to the regulator, the tools volume flow rate has to be converted from the standard condition to a pressure of $120\ psig$. Then the desired pressure drop can be determined as the difference between the system pressure and the compressed air tool pressure. If the compressed air velocity or pressure were decreased, the pipe diameter would have to be larger to handle the same flow rate. The tube exiting the regulator also has to be larger since the pressure has to be reduced.

# CHAPTER IX

# COMPRESSED AIR SYSTEM LEAKAGE MODEL AND ANALYSIS

## Background

Leakage in an industrial compressed air system can significantly reduce the system efficiency and at the end cost a lot of money. For a system with some leaks, the increase in pressure discharge would increase both the mass flow rate and the volume flow rate. However, it would not change the velocity of the air leaking out. Leakage could be significantly reduced by using the necessary discharge pressure for the tools if not a little higher. A diagram describing leakage from a component of a compressed air system is shown below in Figure 24.

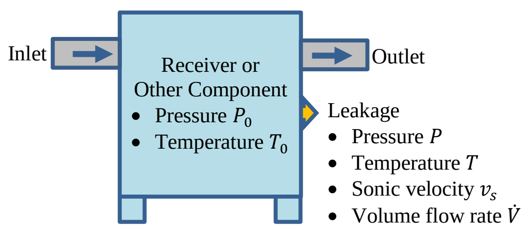



Figure 24. Compressed Air System Leakage Diagram

In analyzing the leakage, an assumption that the leak or hole cause an isentropic expansion, where the leaks are frictionless and there is no transfer of heat or of matter, is made. Moreover, as the gas flow through a small area, its linear velocity must increase until it reaches the speed of sound or sonic velocity $v_s$. In addition, for a choked flow, the ratio of the pressure and the temperature at the exit plane ($P$ and $T$ respectively) to the ones in the system or reservoir ($P_0$ and $T_0$ respectively) should follow Equations (74) and (75).

$$\frac{P}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}, \tag{74}$$

and

$$\frac{T}{T_0} = \left(\frac{2}{k+1}\right), \tag{75}$$

where: • $P$ is the gas pressure at the exit plane [$psia$],

• $P_0$ is the gas pressure in the system [$psia$],

• $T$ is the gas temperature at the exit plane [°F],

• $T_0$ is the gas temperature in the system [°F], and

• $k$ is the gas heat capacity ratio [unitless].

In case where the fluid is dry air, the heat capacity ratio $k$ should equal to 1.4, and therefore $\frac{P}{P_0} = 0.528$, and $\frac{T}{T_0} = 0.833$.

**Sonic Velocity and Flow Rates**

The compressed air that flows out through a small hole or leak would reach a maximum velocity called the sonic velocity. The sonic velocity $\bar{V}_s$ can be determined by using the following Equation (76).

$$\bar{V}_s = \sqrt{k\ R\ T}, \tag{76}$$

where: • $\bar{V}_s$ is the gas sonic velocity [$\frac{ft}{sec}$], and

• $R$ is the specific gas constant [$\frac{lbf \cdot ft}{lbm \cdot R}$].

Next, the leakage mass flow rate and volume flow rate can be calculated by using Equations (18) and (17). However, for the volume flow rate, the unit that is normally used is the standard cubic feet per minute ($SCFM$), thus a conversion as shown in Equation (77) is needed.

$$\dot{V}_{SCFM} = \dot{V}\ \frac{\rho}{\rho_{SCFM}}, \tag{77}$$

where $\rho$ is the gas density at the exit plane, and $\rho_{SCFM}$ is the gas density at standard condition, which normally is equal to $0.075\ \frac{lbm}{ft^3}$.

Example Problem IX.a: Sonic Velocity and Flow Rates

Objective: If there is a $\frac{1}{4}\,inch$ leak hole at a receiver (the gas is dry air, so $k$ is 1.4 ) where the component pressure $P_0$ and temperature $T_0$ are $100\,psig$ and 80 °F respectively, solve for the pressure $P$ and the temperature $T$ at the exit plane, the sonic velocity $v_s$, and both the volume flow rate $\dot{V}$ and the mass flow rate $\dot{m}$ of the air leaking out.

Assumption: The gas is an ideal gas.

Isentropic expansion through the leakage.

Knowns: $D = \frac{1}{4}\,inch$ $k = 1.4$ $P_0 = 100\,psig$ $T_0 = 80$ °F

Find:
- The pressure $P$ of the air at the exit plane,
- the temperature $T$ of the air at the exit plane,
- the sonic velocity $v_s$ of the air,
- the volume flow rate $\dot{V}$ of the air, and

• the mass flow rate $\dot{m}$ of the air.

Solution:

The pressure $P$, and the temperature $T$ of the air at the exit plane is:

$$\frac{P}{P_0} = 0.528$$

$$P = 0.528\, P_0$$

$$P = 0.528(\langle 100 + 14.7 \rangle psia) = 60.56\, psia$$

and

$$\frac{T}{T_0} = 0.833$$

$$T = 0.833\, T_0$$

$$T = 0.833(80\ °\mathrm{F}) = 66.64\ °\mathrm{F}$$

Next, the sonic velocity $\bar{V}_s$ of the air leaking out is:

$$\bar{V}_s = \sqrt{k\, R_a\, T}$$

$$\bar{V}_s = \sqrt{(1.4)\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R} \cdot 32.2\,\frac{lbm \cdot ft}{lbf \cdot s^2}\right)(66.64 + 459.67R)} = 1{,}123.8\,\frac{ft}{sec}$$

At the exit plane, the density $\rho$ of the leaked gas would be:

$$\rho = \frac{P}{R_a\, T}$$

$$\rho = \frac{\left(60.56\, psia \cdot 144\,\frac{in^2}{ft^2}\right)}{\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(66.64 + 459.67R)} = 0.311\,\frac{lbm}{ft^3}$$

Then, both the volume flow rate $\dot{V}$ and the mass flow rate $\dot{m}$ would be:

$$\dot{V} = \frac{\pi}{4} D^2 \, \bar{V}_s$$

$$\dot{V} = \frac{\pi}{4}\left(\frac{1}{4} in \cdot \frac{1}{12}\frac{ft}{in}\right)^2 \left(1{,}123.8\frac{ft}{sec} \cdot 60\frac{sec}{min}\right) = 22.99\frac{ft^3}{min}$$

or

$$\dot{V}_{SCFM} = \dot{V} \, \frac{\rho}{\rho_{SCFM}}$$

$$\dot{V}_{SCFM} = \left(22.99\frac{ft^3}{min}\right)\frac{\left(0.311\frac{lbm}{ft^3}\right)}{\left(0.075\frac{lbm}{ft^3}\right)} = 95.31 \, SCFM$$

and

$$\dot{m} = \rho \, \dot{V}$$

$$\dot{m} = \left(0.311\frac{lbm}{ft^3}\right)\left(22.99\frac{ft^3}{min}\right) = 7.148\frac{lbm}{min}$$

Discussion: The compressed air that flows out through a small hole or leak would reach a maximum velocity called the sonic velocity. In adiabatic condition, the compressed air the point when the velocity of the air leaking out reaches sonic velocity is the point where the choking occurs.

# CHAPTER X

# COMPRESSED AIR TOOLS MODEL AND ANALYSIS

## Background

There are many different kinds of compressed air tools, or pneumatic tools. They consist of any equipment that is driven by compressed air. The compressed air can be directly supplied by the compressed air system, or supplied from compressed air storage tanks or cylinders. In general, it is important to know the required air pressure (psi or bar) and air consumption ($CFM$ or $SCFM$) to operate the tools. With these two values, the compressed air system can be adjusted to supply the right amount of compressed air and increase its efficiency.

## Compressor Size

To calculate the size $\dot{W}$ of air compressor that is required to power a compressed air tool, Equation (78) can be used.

$$\dot{W} = \dot{W}^* \, \dot{V}_{tool} \, (1 + \%_{leak}) \, , \tag{78}$$

where: • $\dot{W}$ is the power [$hp$ or $kW$],

• $\dot{W}^*$ is the consumption over a unit of flow rate [$\frac{hp}{100\ SCFM}$],

• $\dot{V}_{tool}$ is the air flow rate required [$SCFM$], and

• $\%_{leak}$ is the percentage of the leaks [%].

Then, Equation (79) can be used to find the amount of energy or work $W$ required to operate the compressed air tool over a period of time $t$.

$$W = \dot{W}\ t\ (1 + \%_{leak})\ , \tag{79}$$

where: • $W$ is the energy or work required [$hp \cdot hr$ or $kW \cdot hr$], and

• $t$ is the time period [$hour$ or $year$].

Lastly, Equation (80) can be used to calculate the volume $V$ of compressed air that the tool used over a period of time $t$.

$$V = \dot{V}_{tool}\ t\ (1 + \%_{leak})\ , \tag{80}$$

where $V$ is the gas volume [$SCF$].

Example Problem X.a: Compressor Size Requirement

Objective: A typical compressed air hand tool uses 25 $SCFM$ of compressed air during its operation. Solve for the size $\dot{W}$ of air compressor required to power 100 hand tools if the air compressor consumes 20 $hp$ for every 100 $SCFM$ of air compressed. If the tools is operated continuously for a year, solve for the energy or work $W$ (in $kW \cdot hr$) required, as well as the volume of compressed air (in standard cubic feet or $SCF$) that is used. Note that in this system, 30% of the total compressed air capacity is wasted by leaks.

Assumption: The gas is an ideal gas.

Knowns: $\dot{V}_{tool} = 25\ SCFM$ $\quad\quad$ $\dot{W}^* = 20\,\frac{hp}{100\ SCFM}$

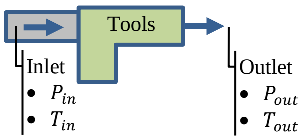
$t = 1\ year$ $\quad\quad$ $\%_{leak} = 30\%$

Find: • The size or power $\dot{W}$ of air compressor required,

• the work $W$ (in $kW \cdot hr$) required, and

• the volume of compressed air (in $SCF$) used.

Solution:




The size $\dot{W}$ of air compressor required for a single tool without the leak is:

$$\dot{W} = \dot{W}^* \, \dot{V}_{tool} \, (1 + \%_{leak})$$

$$\dot{W} = \left(20 \frac{hp}{100 \, SCFM}\right)(25 \, SCFM)(1 + 0\%) = 5 \, hp$$

Then, the size $\dot{W}_{100}$ of air compressor required for 100 tools without the leak would be:

$$\dot{W}_{100} = 100 \, \dot{W}$$

$$\dot{W}_{100} = 100 \, (5 \, hp) = 500 \, hp$$

Then, the size of air compressor required for 100 tools with the leak would be:

$$\dot{W}_{100+leak} = \dot{W}_{100} \, (1 + \%_{leak})$$

$$\dot{W}_{100+leak} = (500 \, hp)(1 + 30\%) = 650 \, hp$$

The work $W_{100}$ (in $kW \cdot hr$) required is:

$$W_{100} = \dot{W}_{100} \, t$$

$$W_{100} = \left(500 \, hp \cdot 0.746 \frac{kw}{hp}\right)\left(1 \, yr \cdot 365 \frac{day}{yr} \cdot 24 \frac{hr}{day}\right) = 3{,}267{,}500 \, kW \cdot hr$$

or

$$W_{100} = 3.27 \, GW \cdot hr$$

Then, the work (in $kW \cdot hr$) required with the leak would be:

$$W_{100+leak} = \dot{W}_{100+leak}\ t$$

$$W_{100+leak} = \left(650\ hp \cdot 0.746 \frac{kw}{hp}\right)\left(1\ yr \cdot 365 \frac{day}{yr} \cdot 24 \frac{hr}{day}\right)$$

$$= 4{,}247{,}700\ kW \cdot hr$$

or

$$W_{100+leak} = 4.25\ GW \cdot hr$$

The volume $V_{100}$ of compressed air (in $SCF$) used is:

$$V_{100} = 100\ \dot{V}_{tool}\ t\ (1 + \%_{leak})$$

$$V_{100} = 100\ (25\ SCFM)\left(1\ yr \cdot 365 \frac{day}{yr} \cdot 24 \frac{hr}{day} \cdot 60 \frac{min}{hr}\right)(1 + 0\%)$$

$$= 1{,}314{,}000{,}000\ SCF$$

or

$$V_{100} = 1.31\ billion\ SCF$$

Then, the volume $V_{100+leak}$ of compressed air (in $SCF$) used with the leak would be:

$$V_{100+leak} = V_{100}\ (1 + \%_{leak})$$

$$V_{100+leak} = (1{,}314{,}000{,}000\ SCF)(1 + 30\%) = 1{,}708{,}200{,}000\ SCF$$

or

$$V_{100+leak} = 1.71\ billion\ SCF$$

Discussion: In designing or choosing a suitable air compressor to power some compressed air tools and equipment, it is better to get an air compressor that is capable of supply more power than what the tools actually need. The purpose is to anticipate any pressure drops that may happen as the compressed air is distributed to the compressed air tools. Leakage is one example of the things that may cause the pressure drop.

# CHAPTER XI

# COMPRESSED AIR SYSTEM ECONOMIC ANALYSIS

## Background

Operating an industrial compressed air system is an expensive operation as it consumes significant amounts of electricity. In fact, it is often called the $4^{th}$ utility, and of all forms of utility, it is the largest in terms of cost. The distribution process of compressed air also requires costly equipment. While maintaining compressed air components to optimize the system efficiency can also be complicated and expensive. The annual electricity charges for a compressed air system can be as expensive as the initial cost of the air compressor itself. In addition, the annual maintenance costs can reach more than 10% of the initial cost of the system. Therefore, it is important to understand the annual compressed air system costs, and how the system can be economically optimized.

## Electricity Cost

Equation (81) can be used to calculate the electricity cost $C_\$$ over a period of time $t$.

$$C_\$ = \frac{\dot{W}\ t\ c_\$}{\eta}, \tag{81}$$

where: • $C_\$$ is the electricity cost [\$],

• $\dot{W}$ is the power consumption of the air compressor [$hp$ or $kW$],

• $t$ is the time period [$sec$],

• $c_\$$ is the rate of the electricity [$\frac{\$}{kW \cdot hr}$], and

• $\eta$ is the air compressor efficiency [%].

<u>Example Problem XI.a: Electricity Cost</u>

Objective: Solve for the annual electricity cost $C_\$$ for a $100\ hp$ air compressor that operates continuously. The efficiency $\eta$ of the air compressor is assumed to be $80\%$. The rate of the electricity $c_\$$ is $\frac{\$0.15}{kW \cdot hr}$. Find the annual cost of operating the compressor, along with the cost of leakage if $30\%$ of the total compressed air capacity is wasted by leakage.

Assumption: The gas is an ideal gas.

Knowns: $\dot{W} = 100\ hp$ $\quad \eta = 80\%$ $\quad t = 1\ year$

$c_\$ = \frac{\$0.15}{kW \cdot hr}$ $\quad \%_{leak} = 30\%$

Find: • The annual electricity cost $C_{\$_{ann}}$, and

• the annual cost of the leakage $C_{\$_{ann_{leak}}}$.

Solution:

The annual electricity cost $C_{\$_{ann}}$ is:

$$C_{\$_{ann}} = \frac{\dot{W}\ t\ c_\$}{\eta}$$

$$C_{\$_{ann}} = \frac{\left(100\ hp \cdot 0.746 \frac{kW}{hp}\right)\left(1\ yr \cdot 365 \frac{day}{yr} \cdot 24 \frac{hr}{day}\right)\left(\frac{\$0.15}{kW \cdot hr}\right)}{80\%}$$

$$= \$122{,}530$$

Then the annual cost of the leakage $C_{\$_{ann_{leak}}}$ is:

$$C_{\$_{ann_{leak}}} = C_{\$_{ann}} \%_{leak}$$

$$C_{\$_{ann_{leak}}} = (\$122{,}530)(30\%) = \$36{,}760$$

Discussion: Typical $100\ hp$ air compressor may cost around $\$30{,}000$ to $\$50{,}000$, depending on the type of the compressor and manufacturer. This calculation shows how the annual electricity cost can be higher than the cost of the air compressor itself.

# CHAPTER XII

# COMPRESSED AIR SYSTEM DESIGN AND ANALYSIS

## Background

The characteristic of moist air, before and after compression, have been presented in CHAPTER III along with equations that govern these characteristics. Of equal importance, the main components making up a compressed air system have been described, and mathematically modeled in CHAPTER IV through CHAPTER X by using conservation of mass and energy equations, along with other relationships from the fields of thermodynamics, fluid dynamics, and heat transfer.

All of the equations, relationships, and mathematical models presented in the above aforementioned chapters were used in a series of example problems imbedded in each of the chapters. These example problems are important because they not only show the usefulness of the equations presented, but they also show how units are manipulated and converted for each parameters, including the dependent variable that is the subject of each example problem calculations.

The mathematical models presented herein can be used for design and analysis of any of the components that make up the overall compressed air system. This chapter demonstrates using these models for design and analysis by introducing a series of plots for moist air properties, and for each components comprising the overall system. The organization herein consists of a series of sections with each section representing CHAPTER III through CHAPTER XI. Additionally, each section, which is titled after each chapter of interest, contains the same information for moist air and each components, with their information being grouped under the following organization:

- Chapter Number and Subject and/or Component Title,
- Design and/or Analysis Objective,
- Functional Relationships,
- Example Problem Summary Calculation with Units,
- Sample Design and/or Analysis (from Plots), and
- Plots.

**CHAPTER III – Moist Air Description, Model and Analysis**

- Design and/or Analysis Objective
  - Determine the dew point temperature $T_{dew}$ at air compressor's inlet and outlet pressures $P$, given outdoor condition of temperature $T$, and relative humidity $\varphi$.
  - Determine specific humidity ratio $\omega$ entering the air compressor.
  - Determine air specific volume $v$ at air compressor.
- Functional Relationships
  - III.a: $T_{dew} = f(T, \varphi)$
    - $T_{dew} = \frac{564 + 405\left(\frac{17.6(T-32)}{T+405} + \ln(\varphi)\right)}{17.6 - \left(\frac{17.6(T-32)}{T+405} + \ln(\varphi)\right)}$
  - III.b: $\omega = f(T, \varphi)$
    - $\omega = \frac{0.622\,\varphi\left(0.0886 \cdot 10^{\left(\frac{7.5\,(T-32)}{T+395.14}\right)}\right)}{P - \varphi\left(0.0886 \cdot 10^{\left(\frac{7.5\,(T-32)}{T+395.14}\right)}\right)}$, where $P$ is 14.7 $psia$

- III.c, III.d, and III.e: $T_{dew} = f(P, \varphi)$ for three outdoor temperatures $T$
  - $T_{dew} = \frac{240 + 395.14\ \ln\left(\frac{\omega P}{0.0886\ (\omega + 0.622)}\right)}{7.5 - \ln\left(\frac{\omega P}{0.0886\ (\omega + 0.622)}\right)}$, where $\omega = \frac{0.622\ \varphi\ P_{sat}}{P - \varphi\ P_{sat}}$, for $P_{sat}$ at $T$ equals to 50°F, 70°F, or 90°F
- III.f: $v_a = f(T, \varphi)$
  - $v_a = \frac{R_a\ T}{P - \varphi\left(0.0886 \cdot 10^{\left(\frac{7.5\ (T-32)}{T+395.14}\right)}\right)}$, where $P$ is 14.7 $psia$, and $R_a$ is 53.33 $\frac{lbf \cdot ft}{lbm \cdot R}$

- Example Problem Summary Calculation
  - III.a: Air Properties

$$v_a = \frac{R_a\ T}{P_a} = \frac{R_a\ T}{P - \varphi\ P_{vs}}$$

Where

$$P_{vs} = 0.256\text{psia} + (\langle 0.363 - 0.256 \rangle \text{psia}) \frac{(68 - 60)\text{°F}}{(70 - 60)\text{°F}} = 0.342\ psia$$

$$v_a = \frac{\left(53.33 \frac{lbf \cdot ft}{lbm \cdot R}\right)(\langle 68 + 459.67 \rangle R)}{\left(14.7\ psia \cdot 144 \frac{in^2}{ft^2}\right) - (50\%)\left(0.34195\ psia \cdot 144 \frac{in^2}{ft^2}\right)} = 13.42 \frac{ft^3}{lbm}$$

- Sample Design or Analysis (from plots)
  - Plot III.a - Air Compressor Inlet Dew Point Temperature Chart

    If $T = 75$°F, and $\varphi = 60\%$, then $T_{dew} = 58.3$°F
  - Plot III.b - Specific Humidity Ratio Chart

    If $T = 75$°F, and $\varphi = 50\%$, then $\omega = 0.00923$
  - Plot III.c - Air Compressor Exit Dew Point Temperature for Inlet Temperature of 50°F

If $T = 50°F$, $P = 120\ psia$, and $\varphi = 70\%$, then $T_{dew} = 102.3°F$

- Plot III.d - Air Compressor Exit Dew Point Temperature for Inlet Temperature of 70°F

  If $T = 70°F$, $P = 120\ psia$, and $\varphi = 70\%$, then $T_{dew} = 127.4°F$

- Plot III.e - Air Compressor Exit Dew Point Temperature for Inlet Temperature of 90°F

  If $T = 90°F$, $P = 120\ psia$, and $\varphi = 70\%$, then $T_{dew} = 152.8°F$

- Plot III.f - Air Compressor Inlet Specific Volume

  If $T = 70°F$, and $\varphi = 50\%$, then $v_a = 13.5 \frac{ft^3}{lbm}$

- Plots

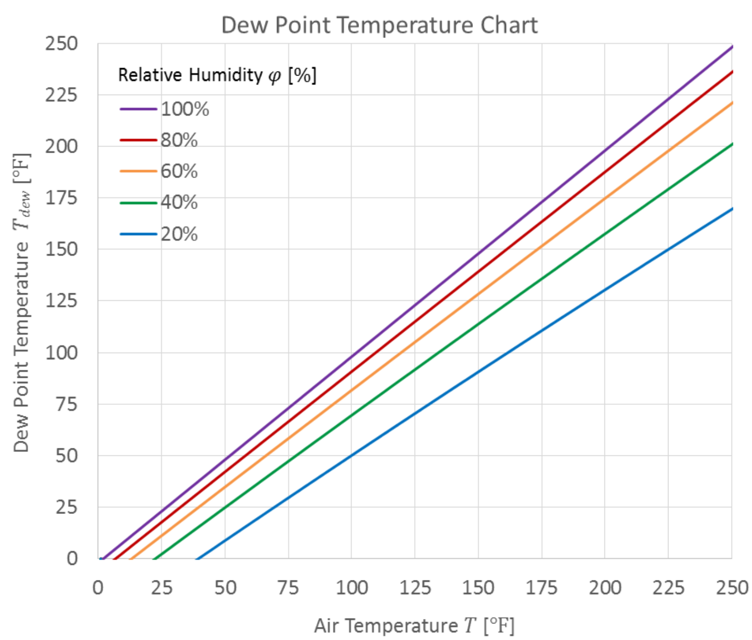



Figure 25. Plot III.a - Air Compressor Inlet Dew Point Temperature Chart

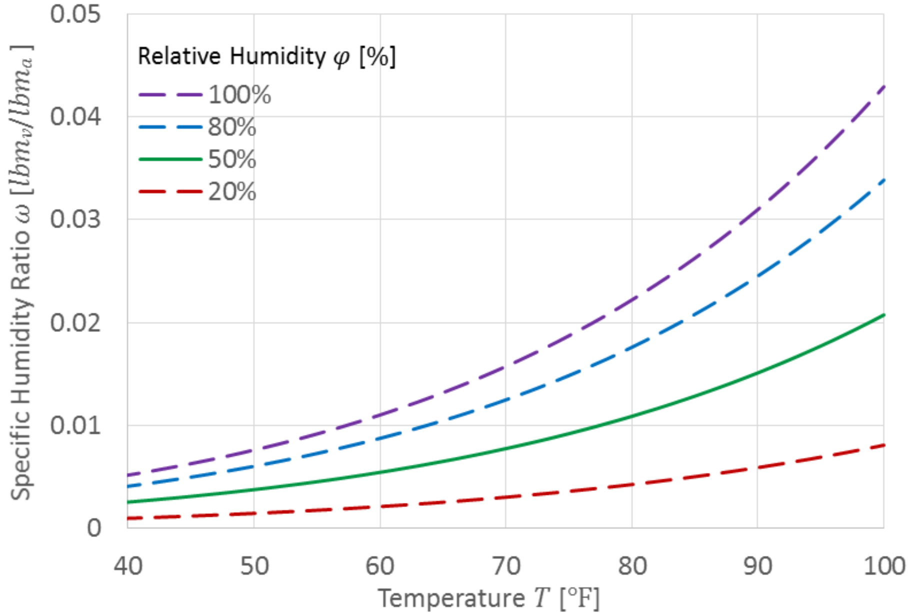



Figure 26. Plot III.b - Specific Humidity Ratio Chart

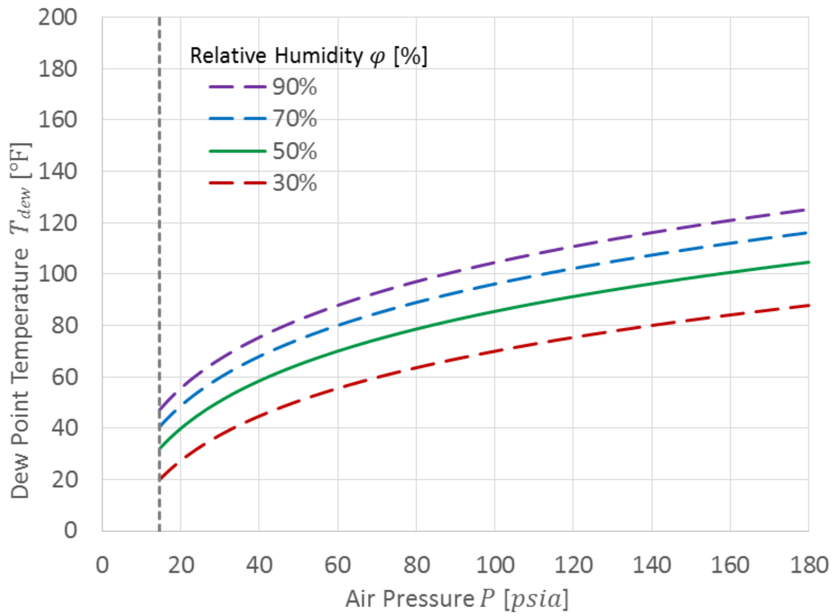



Figure 27. Plot III.c - Air Compressor Exit Dew Point Temperature for Inlet Temperature of 50°F

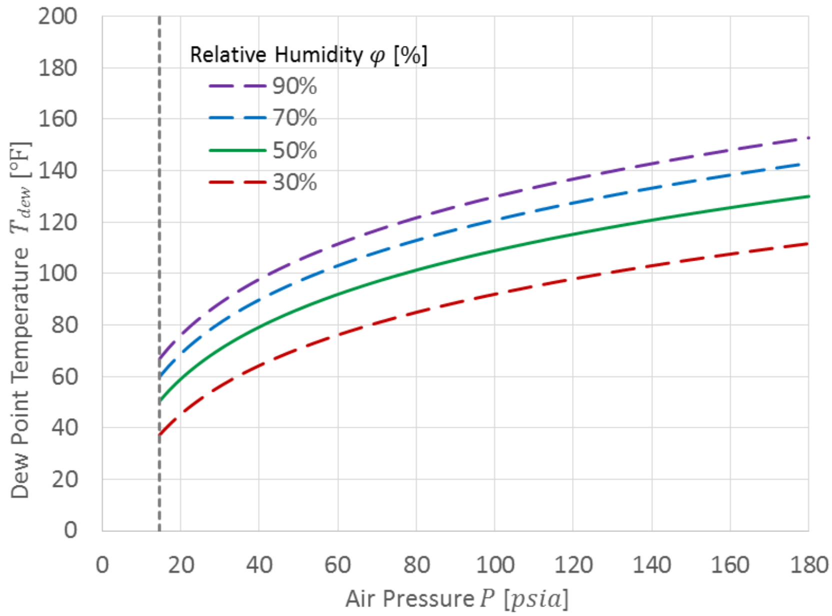



Figure 28. Plot III.d - Air Compressor Exit Dew Point Temperature for Inlet Temperature of 70°F

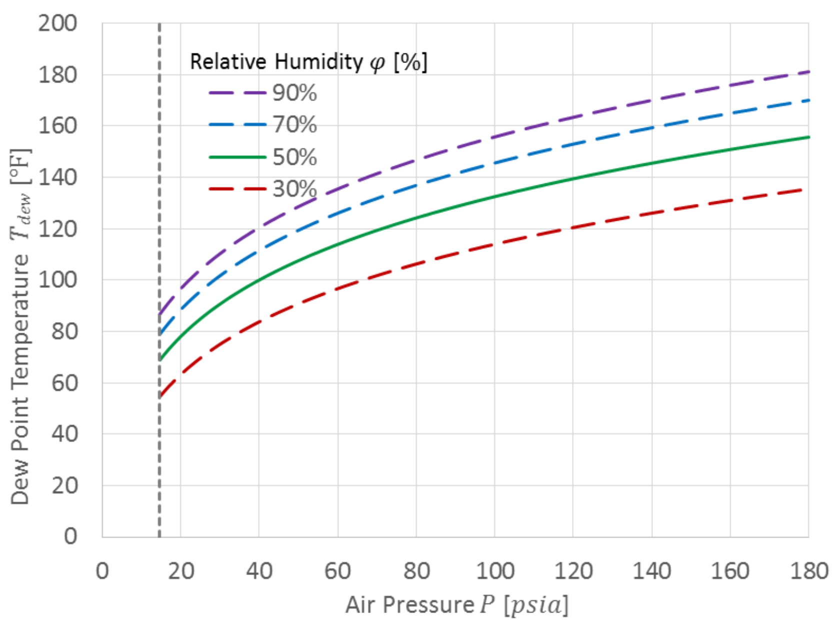



Figure 29. Plot III.e - Air Compressor Exit Dew Point Temperature for Inlet Temperature of 90°F

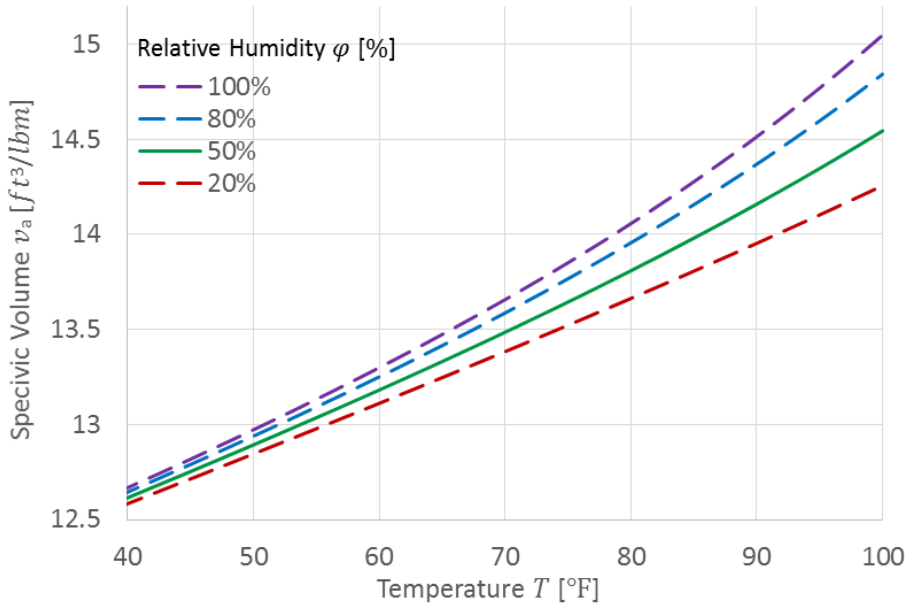



Figure 30. Plot III.f - Air Compressor Inlet Specific Volume

**CHAPTER IV – Air Compressor Model and Analysis**

- Design and/or Analysis Objective
    - Determine the exit temperature $T_{out}$ of the compressed air from air compressor.
    - Determine the specific work $\dot{w}$, efficiency $\eta$, and exit pressure $P$ and temperature $T$, given the pressure ratio $\frac{P_{out}}{P_{in}}$ for either isothermal, isentropic, and real compression processes.
    - Determine the exit vapor pressure $P_v$, and saturation temperature $T_{sat}$.
- Functional Relationships
    - IV.a: $T_{out} = f(n, P_{out})$

- $T_{out_{id}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}$, where $T_{in}$, and $P_{in}$ are equal to 68°F or $527.67\ R$, and $14.7\ psia$

- IV.b: $\dot{w}_{ac} = f(\frac{P_{out}}{P_{in}}, \eta_{ise})$

  - $\dot{w}_{ac} = \frac{C_p\ T_{in}\left(\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}} - 1\right)}{\eta_{ise}}$, where $C_p$, $T_{in}$, and $n$ are equal to $0.240\,\frac{BTU}{lbm\cdot R}$, 68°F, and 1.4

- IV.c: $\dot{w} = f(\frac{P_{out}}{P_{in}}, k)$

  - $\dot{w}_{iso} = R_a T\ \ln\frac{P_{out}}{P_{in}}$, where $R_a$, and $T$ are equal to $0.069\,\frac{BTU}{lbm\cdot R}$, and 68°F $psia$, for isothermal process

  - $\dot{w}_{id} = C_p\ T_{in}\left(\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}} - 1\right)$, where $C_p$, $T_{in}$, and $n$ are equal to $0.240\,\frac{BTU}{lbm\cdot R}$, 68°F, and 1.4 (for isentropic process) or 1.5 (for real process)

- IV.d: $\dot{w} = f(\frac{P_{out}}{P_{in}})$

  - $\dot{w} = C_p T_{in}\left(\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}} - 1\right)$, where $C_p$, $T_{in}$, and $n$ are equal to $0.240\,\frac{BTU}{lbm\cdot R}$, 68°F, and 1.4, for one-stage isentropic process

  - $\dot{w} = 2\ C_p T_{in}\left(\left(\sqrt{\frac{P_{out}}{P_{in}}}\right)^{\frac{n-1}{n}} - 1\right)$, where $C_p$, $T_{in}$, and $n$ are equal to $0.240\,\frac{BTU}{lbm\cdot R}$, 68°F, and 1.4, for two-stage isentropic process

  - $\dot{w} = C_p T_{in}\left(\left(\sqrt{\frac{P_{out}}{P_{in}}}\right)^{\frac{n-1}{n}} - 1\right)$, for half of the two-stage isentropic process

  - $\dot{w}_{iso} = R_a T \ \ln\frac{P_{out}}{P_{in}}$, where $R_a$, and $T$ are equal to $0.069\,\frac{BTU}{lbm \cdot R}$, and 68°F, for isothermal process

- IV.e: $\eta = f(\frac{P_{out}}{P_{in}}, n_1, n_2)$

  - $\eta = \frac{\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n_1-1}{n_1}} -1}{\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n_2-1}{n_2}} -1}$, where $n_1, n_2$ are equal to 1.0, 1.4 (for isentropic efficiency), or 1.4, 1.5 (for real process efficiency)

- IV.f: $P = f(\frac{P_{out}}{P_{in}})$

  - $P_I = P_{in}\sqrt{\frac{P_{out}}{P_{in}}}$ or $P_{out} = P_{in}\left(\frac{P_{out}}{P_{in}}\right)$, where $P_{in}$ is equal to 14.7 $psia$

- IV.g: $T = f(\frac{P_{out}}{P_{in}})$

  - $T_I = T_{in}\left(\sqrt{\frac{P_{out}}{P_{in}}}\right)^{\frac{n-1}{n}}$ or $T_{out} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}$, where $T_{in}$, and $n$ are equal to 68°F, and 1.4, for isentropic process

- IV.h: $P_{v_{out}} = f(P_{out}, \omega)$

  - $P_{v_{out}} = \frac{P_{out}\,\omega}{0.622 + \omega}$

- IV.i: $T_{sat} = f(P_{out}, \omega)$

  - $T_{sat} = \frac{240+395.14\ \ln\left(\frac{P_{out}\,\omega}{0.0886\,(0.622+\omega)}\right)}{7.5 - \ln\left(\frac{P_{out}\,\omega}{0.0886\,(0.622+\omega)}\right)}$

- Example Problem Summary Calculation

- IV.a: Exit Temperatures $T_{out_{id}}$

  $$T_{out_{id}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{n-1}{n}}$$

  $n = 1.0$ (isothermal compression)

  $$T_{out_{id}} = (\langle 68 + 459.67\rangle R)\left(\frac{100\ psia}{14.7\ psia}\right)^{\frac{1.0-1}{1.0}} = 527.67\ R = 68°\text{F}$$

  $n = 1.40$ (isentropic compression)

  $$T_{out_{id}} = (\langle 68 + 459.67\rangle R)\left(\frac{100\ psia}{14.7\ psia}\right)^{\frac{1.40-1}{1.40}} = 910.9\ R = 452.2°\text{F}$$

- IV.b: Specific Work and Efficiency

  Isentropic:

  The actual specific work $\dot{w}_{ac}$

  $$\dot{w}_{ac} = \frac{\dot{w}_{id}}{\eta_{ise}} = \frac{C_p\left(T_{out_{id}} - T_{in}\right)}{\eta_{ise}}$$

  $$\dot{w}_{ac} = \frac{\left(0.240\frac{BTU}{lbm \cdot R}\right)(\langle 452.2 - 68\rangle R)}{75\%} = 122.9\frac{BTU}{lbm}$$

  The exit temperature $T_{out_{ac}}$

  $$T_{out_{ac}} = T_{in} + \frac{\left(T_{out_{id}} - T_{in}\right)}{\eta_{ise}}$$

  $$T_{out_{ac}} = (68°\text{F}) + \frac{(452.2°\text{F} - 68°\text{F})}{75\%} = 580.2°\text{F}$$

  The heat loss $\dot{Q}$

  $$\dot{Q} = \dot{w}_{ac} - \dot{w}_{id}$$

$$\dot{Q} = \left(122.9\frac{BTU}{lbm}\right) - \left(92.2\frac{BTU}{lbm}\right) = 30.7\frac{BTU}{lbm}$$

Isothermal:

The actual specific work $\dot{w}_{ac}$

$$\dot{w}_{ac} = \frac{\dot{w}_{iso}}{\eta_{iso}} = \frac{R_a T \ \ln\frac{P_{out}}{P_{in}}}{\eta_{iso}}$$

$$\dot{w}_{ac} = \frac{\left(0.069\frac{BTU}{lbm \cdot R}\right)(527.67\ R)\left(\ln\frac{100\ psia}{14.7\ psia}\right)}{75\%} = 92.9\frac{BTU}{lbm}$$

The exit temperature $T_{out_{ac}}$

$$T_{out_{ac}} = T_{in} + \frac{\dot{w}_{ac}}{C_p}$$

$$T_{out_{ac}} = (68°\text{F}) + \frac{92.9\frac{BTU}{lbm}}{0.240\frac{BTU}{lbm \cdot R}} = 455.1°\text{F}$$

The heat loss $\dot{Q}$

$$\dot{Q} = \dot{w}_{ac} - \dot{w}_{iso}$$

$$\dot{Q} = \left(92.9\frac{BTU}{lbm}\right) - \left(69.7\frac{BTU}{lbm}\right) = 23.2\frac{BTU}{lbm}$$

The isothermal efficiency $\eta_{iso}$

$$\eta_{iso} = \frac{\dot{w}_{iso}}{\dot{w}_{ac}}$$

$$\eta_{iso} = \frac{69.7\frac{BTU}{lbm}}{92.2\frac{BTU}{lbm}} = 75.6\%$$

- IV.c: Intermediate Pressure and Total Specific Work

The intermediate pressure $P_I$

$$P_I = \sqrt{P_1 \cdot P_4}$$

$$P_I = \sqrt{(14.7\ psia) \cdot (\langle 125 + 14.7 \rangle\ psia)} = 45.32\ psia$$

The ideal and actual exit temperature of each stages See Figure 15.

$$T_{out_{id}} = T_{in} \left( \frac{P_{out}}{P_{in}} \right)^{\frac{n-1}{n}}$$

$$T_{out_{ac}} = T_{in} + \frac{\left( T_{out_{id}} - T_{in} \right)}{\eta_{ise}}$$

Point 2 (Outlet of the 1st Stage compressor)

$$T_{2_{id}} = (\langle 68 + 459.67 \rangle R) \left( \frac{45.32\ psia}{14.7\ psia} \right)^{\frac{1.40-1}{1.40}} = 267.83\text{°F}$$

$$T_{2_{ac}} = 68\text{°F} + \frac{(267.83\text{°F} - 68\text{°F})}{80\%} = 317.79\text{°F}$$

Point 4 (Outlet of the 2nd Stage compressor)

$$T_{4_{id}} = (\langle 110 + 459.67 \rangle R) \left( \frac{\langle 125 + 14.7 \rangle\ psia}{45.32\ psia} \right)^{\frac{1.40-1}{1.40}} = 325.77\text{°F}$$

$$T_{4_{ac}} = 110\text{°F} + \frac{(325.77\text{°F} - 110\text{°F})}{80\%} = 379.71\text{°F}$$

The total actual specific work $\dot{w}_{total_{ac}}$

$$\dot{w}_{total_{ac}} = \dot{w}_{1st_{ac}} + \dot{w}_{2nd_{ac}} = C_p\left(T_{2_{ac}} - T_1\right) + C_p\left(T_{4_{ac}} - T_3\right)$$

$$\dot{w}_{total_{ac}} = \left( 59.95 \frac{BTU}{lbm} \right) + \left( 64.73 \frac{BTU}{lbm} \right) = 124.68 \frac{BTU}{lbm}$$

- IV.d: Compressor Analysis

The total actual specific work $\dot{w}_{total_{ac}}$

$$\dot{w}_{total_{ac}} = \dot{w}_{1st_{ac}} + \dot{w}_{2nd_{ac}} = C_p\left(T_{2_{ac}} + T_{4_{ac}} - T_1 - T_3\right)$$

$$\dot{w}_{total_{ac}} = \left(0.24\frac{BTU}{lbm \cdot R}\right)(\langle 301.89 + 322.91 - 68 - 82.57\rangle R)$$

$$= 113.82\frac{BTU}{lbm}$$

The total power $\dot{W}_{total}$

$$\dot{W}_{total} = \rho\,\dot{V}\,t\,\dot{w}_{total_{ac}}$$

$$\dot{W}_{total} = \left(0.075\frac{lbm}{ft^3}\right)(400\,SCFM)\left(113.82\frac{BTU}{lbm}\right)\cdot\frac{1.055}{60}\frac{kW \cdot min}{BTU}$$

$$= 60.04\,kW$$

- IV.e: Volumetric Efficiency and Clearance

Subscripts $a$, $b$, $c$, and $d$ on all variables represent $a$, $b$, $c$, and $d$ of a compression cycle. See Figure 16. Sub-subscript 1 represents variables from the 1st stage compression, and sub-subscript 2 represents variables from the 2nd stage compression.

The volumes $V$

$$V_{b_1} = \frac{\pi\,D^2\,L}{4}$$

$$V_{b_1} = \frac{\pi\,(4\,in)^2(2.75\,in)}{4} = 34.56\,in^3$$

$$V_{c_1} = V_{b_1}\left(\frac{P_{b_1}}{P_{c_1}}\right)^{\frac{1}{k}}$$

$$V_{c_1} = (34.56\ in^3)\left(\frac{14.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}} = 16.59\ in^3$$

$$V_{d_1} = \frac{C\ V_{b_1}}{1 + C\left(\frac{P_{d_1}}{P_{a_1}}\right)^{\frac{1}{k}}}$$

$$V_{d_1} = \frac{8\%\ (34.56\ in^3)}{1 + 8\%\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}}} = 2.37\ in^3$$

$$V_{a_1} = V_{d_1}\left(\frac{P_{d_1}}{P_{a_1}}\right)^{\frac{1}{k}}$$

$$V_{a_1} = (2.37\ in^3)\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}} = 4.94\ in^3$$

$$V_{b_2} = V_{c_1} - V_{d_1}$$

$$V_{b_2} = 16.59\ in^3 - 2.37\ in^3 = 14.22\ in^3$$

$$V_{c_2} = V_{b_2}\left(\frac{P_{b_2}}{P_{c_2}}\right)^{\frac{1}{k}}$$

$$V_{c_2} = (14.22\ in^3)\left(\frac{41.06\ psia}{114.7\ psia}\right)^{\frac{1}{1.4}} = 6.83\ in^3$$

$$V_{d_2} = \frac{C\ V_{b_2}}{1 + C\left(\frac{P_{d_2}}{P_{a_2}}\right)^{\frac{1}{k}}}$$

$$V_{d_2} = \frac{8\%\ (14.22\ in^3)}{1 + 8\%\left(\frac{114.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}}} = 0.98\ in^3$$

$$V_{a_2} = V_{d_2}\left(\frac{P_{d_2}}{P_{a_2}}\right)^{\frac{1}{k}}$$

$$V_{a_2} = (0.98\ in^3)\left(\frac{114.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}} = 2.03\ in^3$$

The volumetric efficiency $\eta_{vol}$ (Method 1)

$$\eta_{vol_1} = \frac{\left(V_{b_1} - V_{a_1}\right)}{\left(V_{b_1} - V_{d_1}\right)}$$

$$\eta_{vol_1} = \frac{(34.56\ in^3 - 4.94\ in^3)}{(34.56\ in^3 - 2.37\ in^3)} = 92.03\%$$

$$\eta_{vol_2} = \frac{\left(V_{b_2} - V_{a_2}\right)}{\left(V_{b_2} - V_{d_2}\right)}$$

$$\eta_{vol_2} = \frac{(14.22\ in^3 - 2.03\ in^3)}{(14.22\ in^3 - 0.98\ in^3)} = 92.65\%$$

The volumetric efficiency $\eta_{vol}$ (Method 2)

$$\eta_{vol_1} = 1 + C - C\left(\frac{P_{c_1}}{P_{b_1}}\right)^{\frac{1}{k}}$$

$$\eta_{vol_1} = 1 + 8\% - 8\%\left(\frac{41.06\ psia}{14.7\ psia}\right)^{\frac{1}{1.4}} = 91.34\%$$

$$\eta_{vol_2} = 1 + C - C\left(\frac{P_{c_2}}{P_{b_2}}\right)^{\frac{1}{k}}$$

$$\eta_{vol_2} = 1 + 8\% - 8\%\left(\frac{114.7\ psia}{41.06\ psia}\right)^{\frac{1}{1.4}} = 91.34\%$$

- Sample Design or Analysis (from plots)

- Plot IV.a - Air Compressor Outlet Temperature for Polytropic Process

  If $n = 1.4$, and $P_{out} = 100\ psia$, then $T_{out} = 452.2$°F

- Plot IV.b - Air Compressor Actual Specific Work for Isentropic Process

  If $\frac{P_{out}}{P_{in}} = 5$, and $\eta_{ise} = 60\%$, then $\dot{w}_{ac} = 123.0\,\frac{BTU}{lbm}$

- Plot IV.c - Air Compressor Ideal Specific Work

  If $\frac{P_{out}}{P_{in}} = 5$, and $k = 1.5$, then $\dot{w} = 89.7\,\frac{BTU}{lbm}$

- Plot IV.d - Air Compressor Specific Work

  If $\frac{P_{out}}{P_{in}} = 5$, for two-stage isentropic process, then $\dot{w} = 65.3\,\frac{BTU}{lbm}$

- Plot IV.e - Air Compressor Efficiency

  If $\frac{P_{out}}{P_{in}} = 5$, and $n_1, n_2 = 1.4, 1.5$, then $\eta = 82\%$

- Plot IV.f - Compressed Air Exit Pressure

  If $\frac{P_{out}}{P_{in}} = 5$, for intermediate pressure, then $P = 32.9\ psia$

- Plot IV.g - Compressed Air Exit Temperature

  If $\frac{P_{out}}{P_{in}} = 5$, for intermediate temperature, then $T = 204.1$°F

- Plot IV.h - Air Compressor Outlet Vapor Pressure

  If $P_{out} = 100\ psia$, and $\omega = 0.02$, then $P_{v_{out}} = 3.14\ psia$

- Plot IV.i - Air Compressor Outlet Saturation Temperature

  If $P_{out} = 100\ psia$, and $\omega = 0.02$, then $T_{sat} = 143.2$°F

- Plots

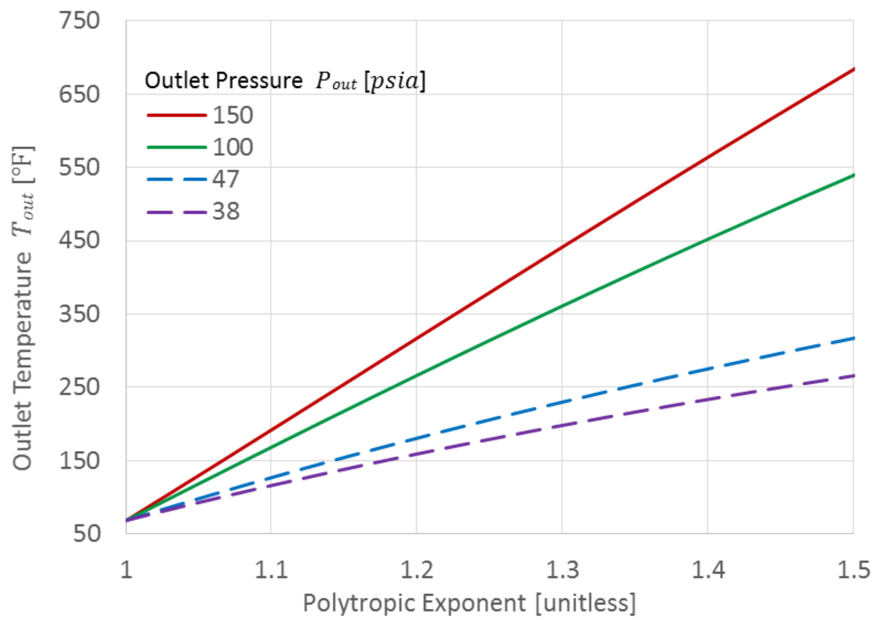



Figure 31. Plot IV.a - Air Compressor Outlet Temperature for Polytropic Process

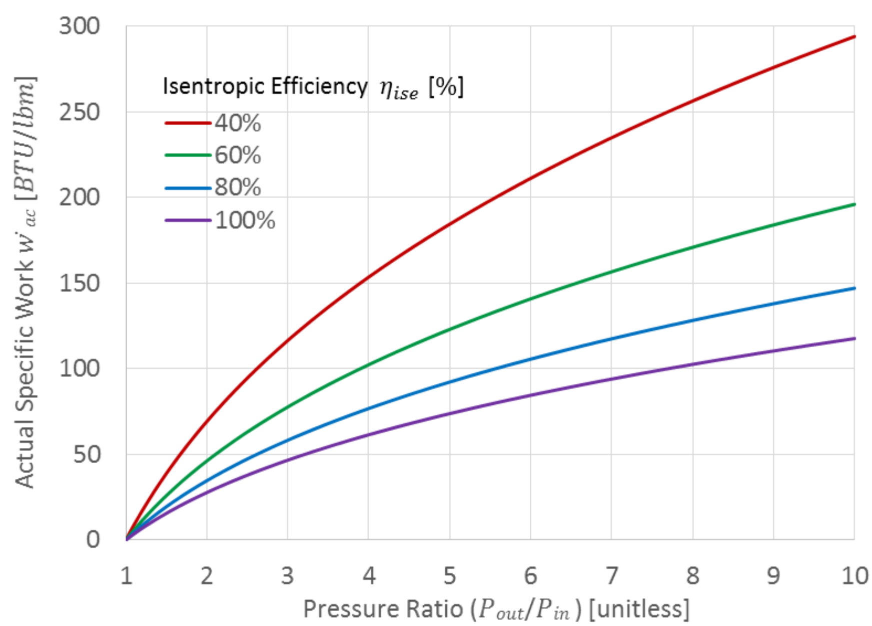



Figure 32. Plot IV.b - Air Compressor Actual Specific Work for Isentropic Process

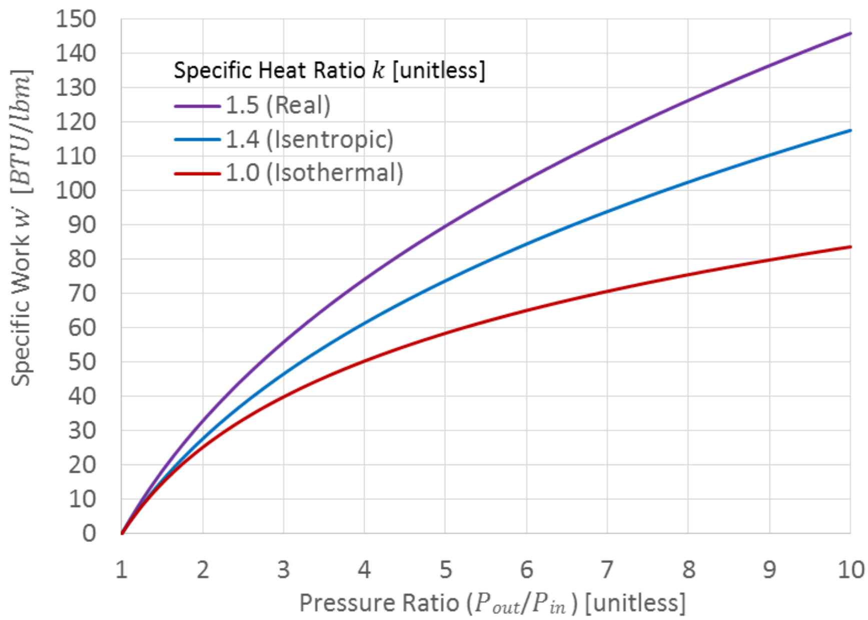



Figure 33. Plot IV.c - Air Compressor Ideal Specific Work

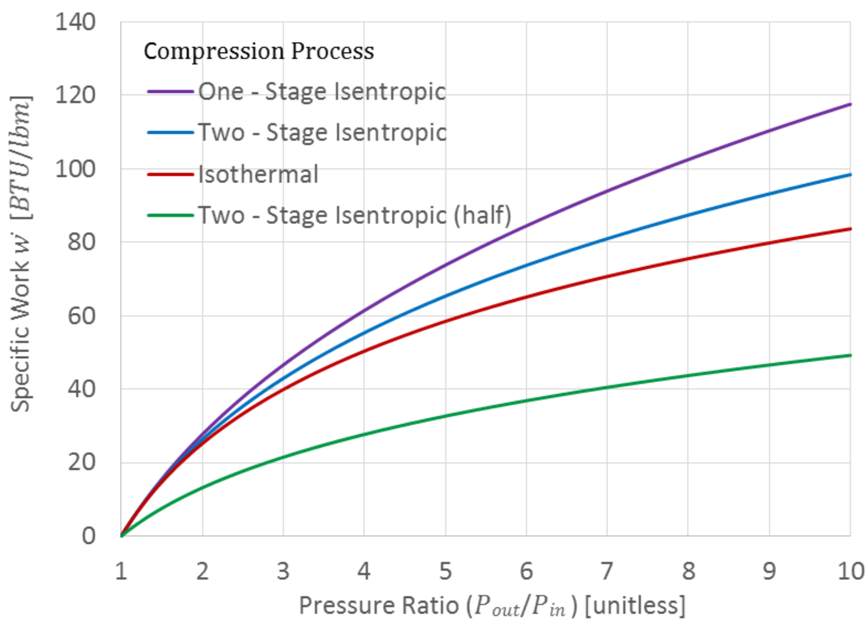



Figure 34. Plot IV.d - Air Compressor Specific Work

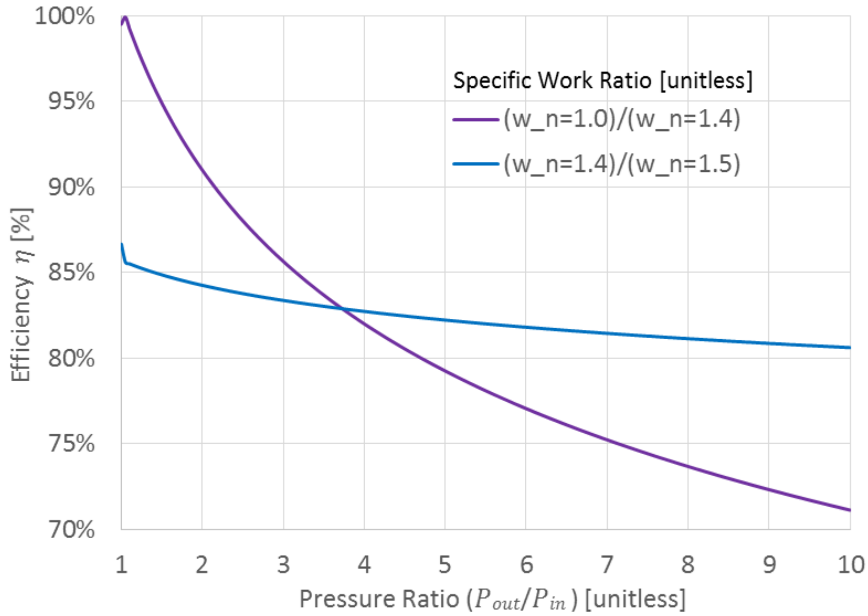



Figure 35. Plot IV.e - Air Compressor Efficiency

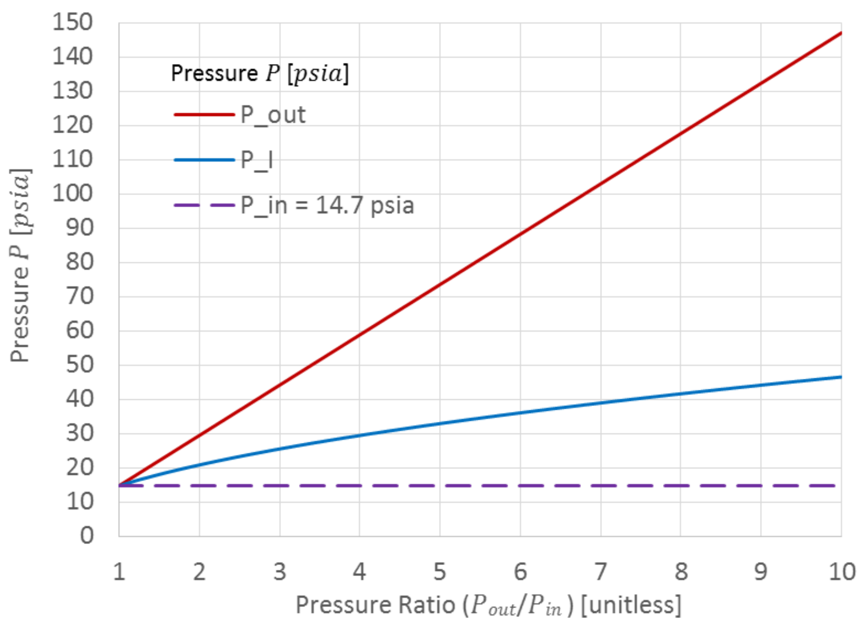



Figure 36. Plot IV.f - Compressed Air Exit Pressure

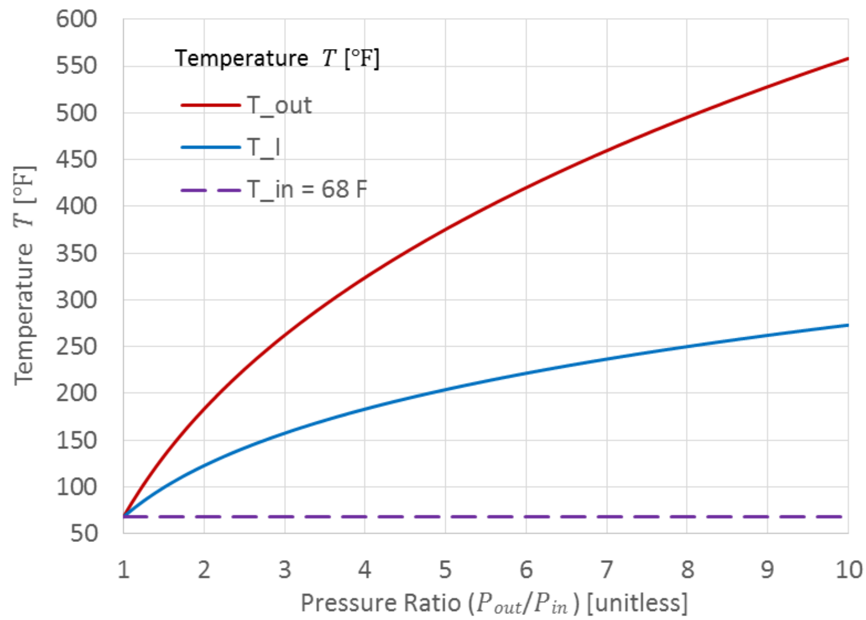



Figure 37. Plot IV.g - Compressed Air Exit Temperature

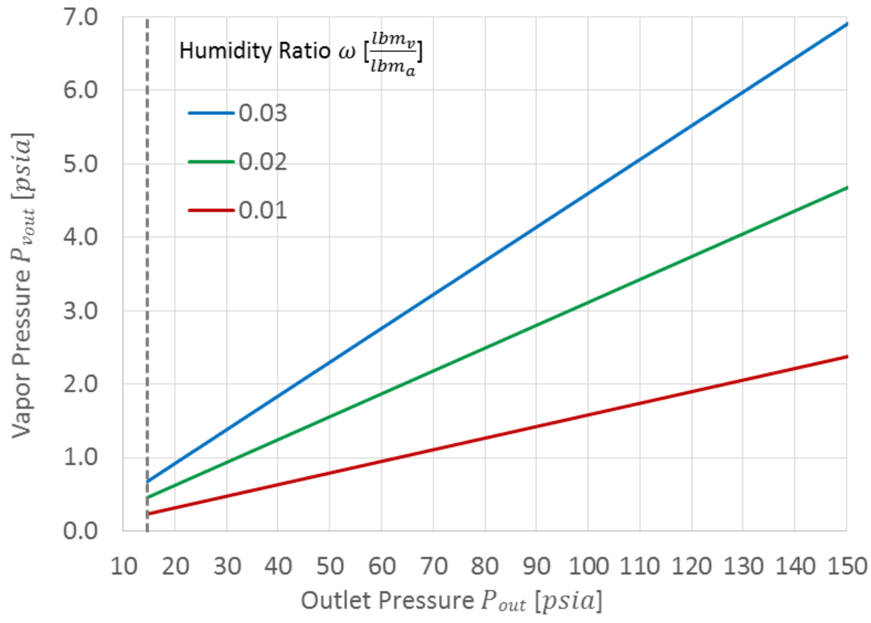



Figure 38. Plot IV.h - Air Compressor Outlet Vapor Pressure

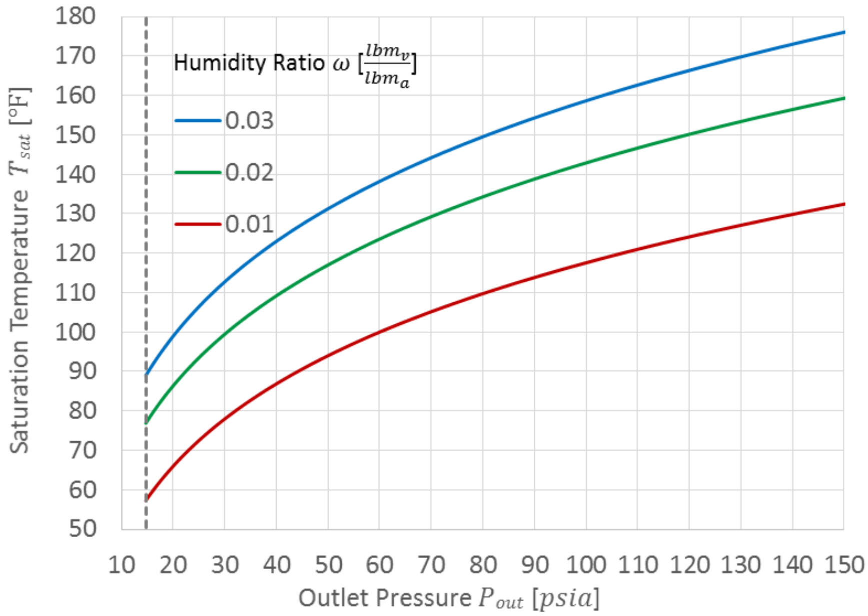



Figure 39. Plot IV.i - Air Compressor Outlet Saturation Temperature

## CHAPTER V – Compressed Air Coolers Model and Analysis

- Design and/or Analysis Objective
  - Determine the intermediate vapor pressure $P_v$, and saturation temperature $T_{sat}$.
  - Determine the saturation heat transfer rate $\dot{Q}_{sat}$.
- Functional Relationships
  - V.a: $P_{v_I} = f(P_I, \omega)$
    - $P_{v_{out}} = \frac{P_I\,\omega}{0.622 + \omega}$
  - V.b:$T_{sat} = f(P_I, \omega)$
    - $T_{sat} = \frac{240+395.14\ \ln\left(\frac{P_I\,\omega}{0.0886\,(0.622\,+\,\omega)}\right)}{7.5-\ln\left(\frac{P_I\,\omega}{0.0886\,(0.622\,+\,\omega)}\right)}$
  - V.c: $\dot{Q}_{sat} = f(P_I, \omega)$

- $\dot{Q}_{sat} = C_p \left( T_{in} \left(\frac{P_I}{P_{in}}\right)^{\frac{n-1}{n}} - \frac{240+395.14\ \ln\left(\frac{P_I\,\omega}{0.0886\,(0.622\,+\,\omega)}\right)}{7.5-\ \ln\left(\frac{P_I\,\omega}{0.0886\,(0.622\,+\,\omega)}\right)}\right)$, where $C_p$, $T_{in}$, and $n$ are equal to $0.240\,\frac{BTU}{lbm{\cdot}R}$, 68°F, and 1.4

- Example Problem Summary Calculation
  - V.a: Intercooler Effectiveness

    The exit temperature of the compressed air $T_{out}$

    $$T_{out} = T_{in} - \varepsilon\left(T_{in} - T_{cool_{in}}\right)$$

    $$T_{out} = 250°\text{F} - 80\%\ (250°\text{F} - 70°\text{F}) = 106°\text{F}$$

    The maximum allowable relative humidity $\varphi_{max}$

    $$\varphi_{max} = e^{\left(\frac{17.6(T_{out}\,-\,32)}{T_{out}\,+\,405} + \frac{17.6(T_{in}\,-\,32)}{T_{in}\,+\,405}\right)}$$

    $$\varphi_{max} = e^{\left(\frac{17.6(106°\text{F}\,-\,32)}{106°\text{F}\,+\,405} + \frac{17.6(250°\text{F}\,-\,32)}{250°\text{F}\,+\,405}\right)} = 3.65\%$$

  - V.b: Condensation Generated

    Find the dew point temperature $T_{dew}$

    $$T_{dew} = \frac{564 + 405\left(\frac{17.6\left(T_{air_{in}} - 32\right)}{T_{air_{in}} + 405} + \ln(\varphi)\right)}{17.6 - \left(\frac{17.6\left(T_{air_{in}} - 32\right)}{T_{air_{in}} + 405} + \ln(\varphi)\right)}$$

    $$T_{dew} = \frac{564 + 405\left(\frac{17.6(250°\text{F} - 32)}{250°\text{F} + 405} + \ln(10\%)\right)}{17.6 - \left(\frac{17.6(250°\text{F} - 32)}{250°\text{F} + 405} + \ln(10\%)\right)} = 142.67°\text{F}$$

    Use the general energy rate balance for closed system in Equation (44)

    $$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

$$\dot{Q}_{air} = \dot{Q}_{cool}$$

$$\dot{Q}_{S_1} + \dot{Q}_{S_2} + \dot{Q}_L = \dot{Q}_{cool}$$

Thus,

$$\dot{m}_{cond} = \frac{\left(\dot{m}_{cool}\, C_{p_{cool}}\, \Delta T_{cool}\right) - \left((1+\omega)\, \dot{m}_a\, C_p\, \Delta T_{air}\right)}{\left(L_c - C_p\, \Delta T_{air_2}\right)}$$

$$\dot{m}_{cond} = \frac{\left((\rho\, \dot{V})_{cool}\, C_{p_{cool}}\, \Delta T_{cool}\right) - \left((1+\omega)\, (\rho\, \dot{V})_a\, C_p\, \Delta T_{air}\right)}{\left(L_c - C_p\, \Delta T_{air_2}\right)}$$

And find the humidity ratio $\omega$ using the water vapor saturation partial pressure $P_{vs}$ found from Table 4.

$$\omega = \frac{0.622\, \varphi\, P_{vs}}{P - \varphi\, P_{vs}}$$

$$\omega = \frac{0.622\ (10\%)(29.844\, psia)}{\left((125+14.7) psia - (10\%)(29.844\, psia)\right)} = 0.0136$$

Therefore,

$$\dot{m}_{cond} = \frac{\left(77.513\, \frac{BTU}{sec}\right) - \left(72.978\, \frac{BTU}{sec}\right)}{\left(960.159\, \frac{BTU}{lbm}\right)} \cdot 3600\, \frac{sec}{hr} = 17\ \frac{lbm}{hr}$$

- Sample Design or Analysis (from plots)
  - Plot V.a - Air Compressor Intermediate Vapor Pressure

    If $P_I = 40\, psia$, and $\omega = 0.02$, then $P_{v_{out}} = 1.25\, psia$
  - Plot V.b - Air Compressor Intermediate Saturation Temperature

    If $P_I = 40\, psia$, and $\omega = 0.02$, then $T_{sat} = 109.3$°F
  - Plot V.c - Air Compressor Saturation Heat Transfer Rate

If $P_I = 40\ psia$, and $\omega = 0.02$, then $\dot{Q}_{sat} = 32.0 \frac{BTU}{lbm}$

- Plots

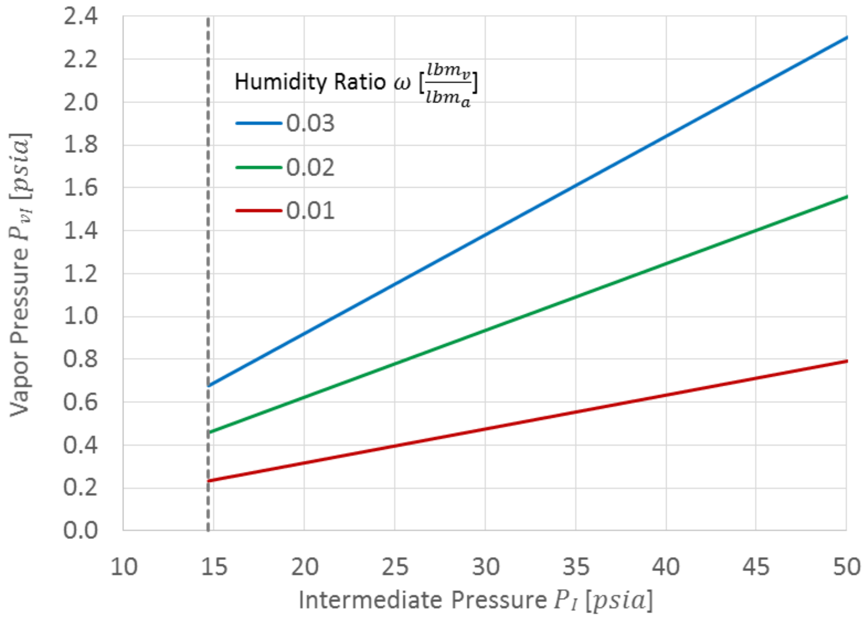



Figure 40. Plot V.a - Air Compressor Intermediate Vapor Pressure

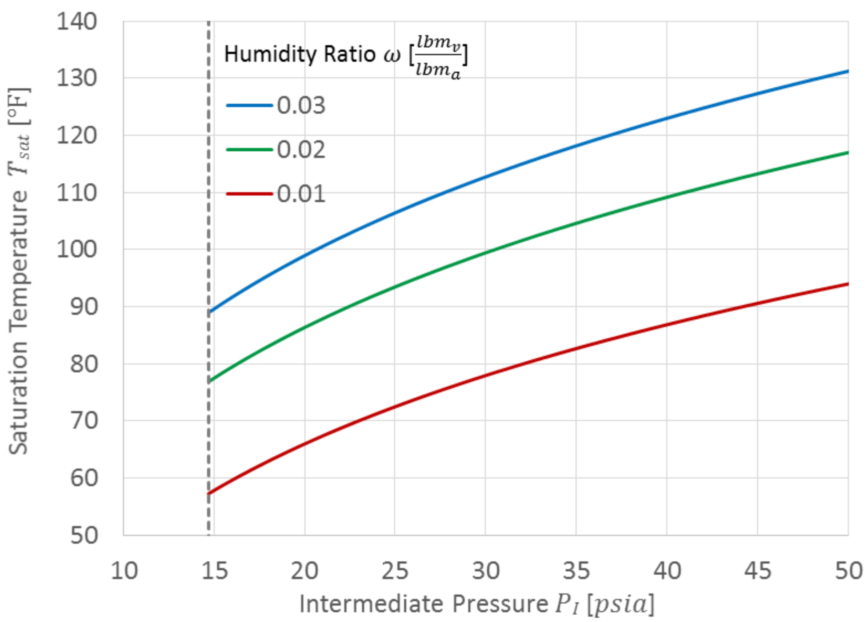



Figure 41. Plot V.b - Air Compressor Intermediate Saturation Temperature

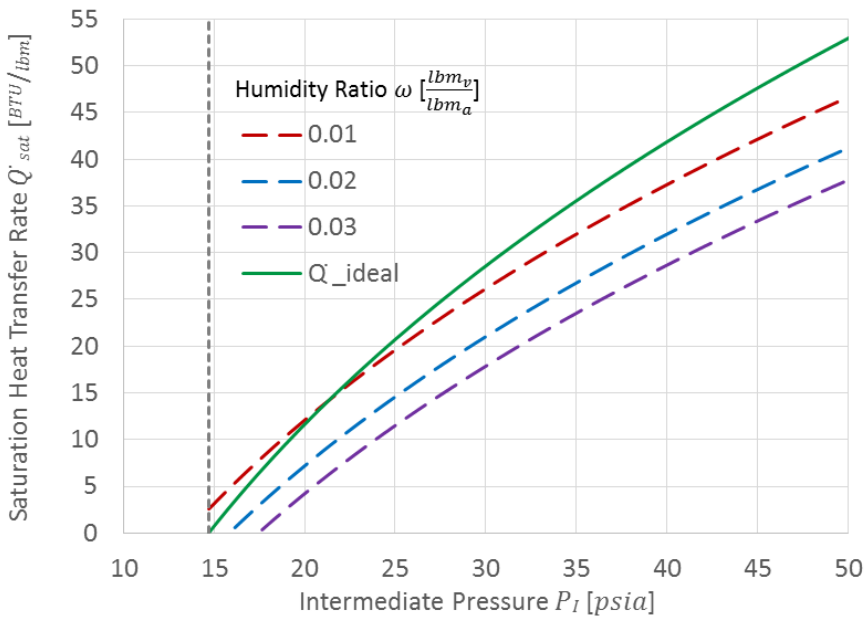



Figure 42. Plot V.c - Air Compressor Saturation Heat Transfer Rate

**CHAPTER VI – Compressed Air Receiver and Storage Model and Analysis**

- Design and/or Analysis Objective
  - Determine the Volume $V$ of a receiver needed to discharge a compressed air for a certain amount of time $t$ at a certain volume flow rate $\dot{V}$.
  - Determine the time $t$ to charge a receiver with a certain volume $V$ at a certain volume flow rate $\dot{V}$.
- Functional Relationships
  - VI.a, and VI.g: $V = f(P_i, t)$
    - $V = \frac{\rho\, (\dot{V}_{in} - \dot{V}_{out})\, t\, R_a\, T}{P_f - P_i}$, where $\rho$, $R_a$, and $T$ are equal to $0.075\, \frac{lbm}{ft^3}$, $53.33\, \frac{lbf \cdot ft}{lbm \cdot R}$, and $529.67\ R$

- VI.b, and VI.h: $t = f(P_i, V)$
  - $t = \frac{V\left(P_f - P_i\right)}{\rho\ (\dot{V}_{in} - \dot{V}_{out})\ R_a\, T}$, where $\rho$, $R_a$, and $T$ are equal to $0.075\,\frac{lbm}{ft^3}$, $53.33\,\frac{lbf \cdot ft}{lbm \cdot R}$, and $529.67\ R$
- VI.c, and VI.d: $V = f(t, \dot{V})$
  - $V = \frac{\rho\ (\dot{V} - 0)\ t\ R_a\, T}{P_f - P_i}$, where $\rho$, $R_a$, and $T$ are equal to $0.075\,\frac{lbm}{ft^3}$, $53.33\,\frac{lbf \cdot ft}{lbm \cdot R}$, and $529.67\ R$, then $P_f$, and $P_i$ are equal to $150\ psia$, and $100\ psia$ or $120\ psia$
- VI.e, and VI.f: $t = f(\dot{V}, V)$
  - $t = \frac{V\left(P_f - P_i\right)}{\rho\ (\dot{V} - 0)\ R_a\, T}$, where $\rho$, $R_a$, and $T$ are equal to $0.075\,\frac{lbm}{ft^3}$, $53.33\,\frac{lbf \cdot ft}{lbm \cdot R}$, and $529.67\ R$, then $P_f$, and $P_i$ are equal to $150\ psia$, and $100\ psia$ or $120\ psia$

- Example Problem Summary Calculation
  - VI.a: Receiver Charging

    The time $t$ required to fully charge the receiver

$$t = \frac{V\left(P_f - P_i\right)}{\rho\ \left(\dot{V}_{in} - 0\right)\ R_a\, T}$$

$$t = \frac{\left(200\ gal \cdot \frac{1}{7.48}\frac{ft^3}{gal}\right)\left(\left(150\,\frac{lbf}{in^2} - 100\,\frac{lbf}{in^2}\right)144\,\frac{in^2}{ft^2}\right)}{\left(0.075\,\frac{lbm}{ft^3}\right)\left(300\,\frac{ft^3}{min} \cdot \frac{1}{60}\frac{min}{sec}\right)\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(529.67\ R)}$$

$$= 18.2\ sec$$

  - VI.b: Receiver Discharging

    The volume $V$ of the receiver

$$V = \frac{\rho\left(\dot{V}_{in} - \dot{V}_{out}\right) t\, R\, T}{P_f - P_i}$$

$$V = \frac{\left(0.075\,\frac{lbm}{ft^3}\right)\left(0 - \frac{300}{60}\frac{ft^3}{sec}\right)(20\ sec)\left(53.3\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(529.67R)}{\left(\langle 100 - 120\rangle\,\frac{lbf}{in^2} \cdot 144\,\frac{in^2}{ft^2}\right)}$$

$$= 73.4\ ft^3 = 549\ gallons$$

The initial and final mass of the compressed air inside the receiver

$$m_i = \frac{P_i\, V}{R_a\, T}$$

$$m_i = \frac{\left(\langle 120 + 14.7\rangle\,\frac{lbf}{in^2} \cdot 144\,\frac{in^2}{ft^2}\right)(73.4\ ft^3)}{\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(70 + 459.67R)} = 50.5\ lbm$$

$$m_f = \frac{P_f\, V}{R_a\, T}$$

$$m_f = \frac{\left(\langle 100 + 14.7\rangle\,\frac{lbf}{in^2} \cdot 144\,\frac{in^2}{ft^2}\right)(73.4\ ft^3)}{\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(70 + 459.67R)} = 43.0\ lbm$$

- VI.c: Combination

The time $t$ required to fully charge the receiver

$$t = \frac{V\left(P_f - P_i\right)}{\rho\ \left(\dot{V}_{in} - \dot{V}_{out}\right)\ R_a\, T}$$

$$t = \frac{\left(200\ gal \cdot \frac{1}{7.48}\frac{ft^3}{gal}\right)\left(\langle 130 - 110\rangle\,\frac{lbf}{in^2} \cdot 144\,\frac{in^2}{ft^2}\right)}{\left(0.075\,\frac{lbm}{ft^3}\right)\left(\frac{\langle 100 - 30\rangle}{60}\frac{ft^3}{sec}\right)\left(53.33\,\frac{lbf \cdot ft}{lbm \cdot R}\right)(529.67R)}$$

$$= 31.2\ seconds$$

- Sample Design or Analysis (from plots)
  - Plot VI.a - Receiver Charging Volume

    If $P_i = 120\ psia$, and $t = 20\ seconds$, then $V = 610\ gallons$
  - Plot VI.b - Receiver Charging Time

    If $P_i = 120\ psia$, and $V = 500\ gallons$, then $t = 16.4\ seconds$
  - Plot VI.c - Receiver Charging Capacity (Initial Pressure 100 psig)

    If $t = 20\ seconds$, and $\dot{V} = 500\ SCFM$, then $V = 366\ gallons$
  - Plot VI.d - Receiver Charging Capacity (Initial Pressure 120 psig)

    If $t = 20\ seconds$, and $\dot{V} = 200\ SCFM$, then $V = 244\ gallons$
  - Plot VI.e - Receiver Charging Speed (Initial Pressure 100 psig)

    If $\dot{V} = 500\ SCFM$, and $V = 400\ gallons$, then $t = 21.9\ seconds$
  - Plot VI.f - Receiver Charging Speed (Initial Pressure 120 psig)

    If $\dot{V} = 500\ SCFM$, and $V = 400\ gallons$, then $t = 13.1\ seconds$
  - Plot VI.g - Receiver Discharging Volume

    If $P_i = 130\ psia$, and $t = 20\ seconds$, then $V = 610\ gallons$
  - Plot VI.h - Receiver Discharging Time

    If $P_i = 130\ psia$, and $V = 500\ gallons$, then $t = 16.4\ seconds$

- Plots

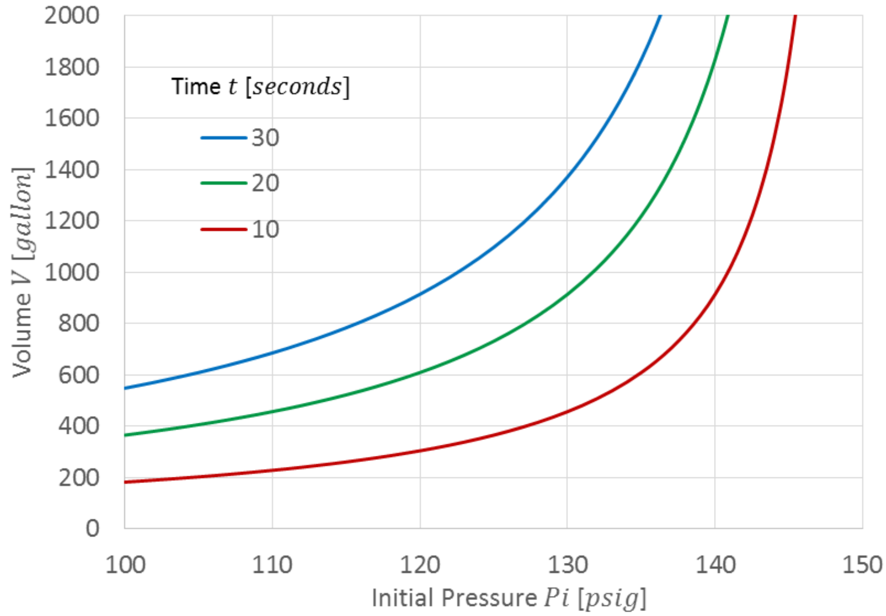



Figure 43. Plot VI.a - Receiver Charging Volume

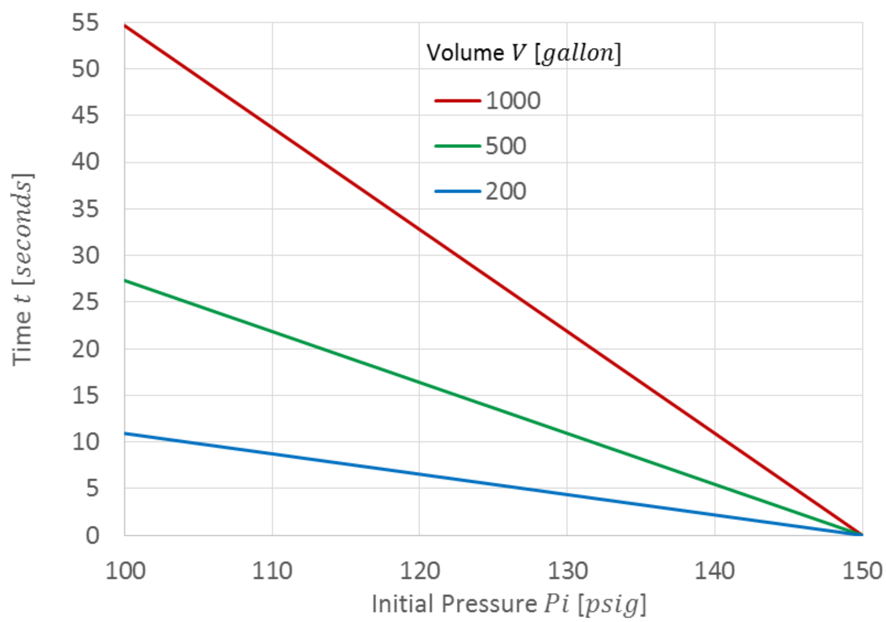



Figure 44. Plot VI.b - Receiver Charging Time

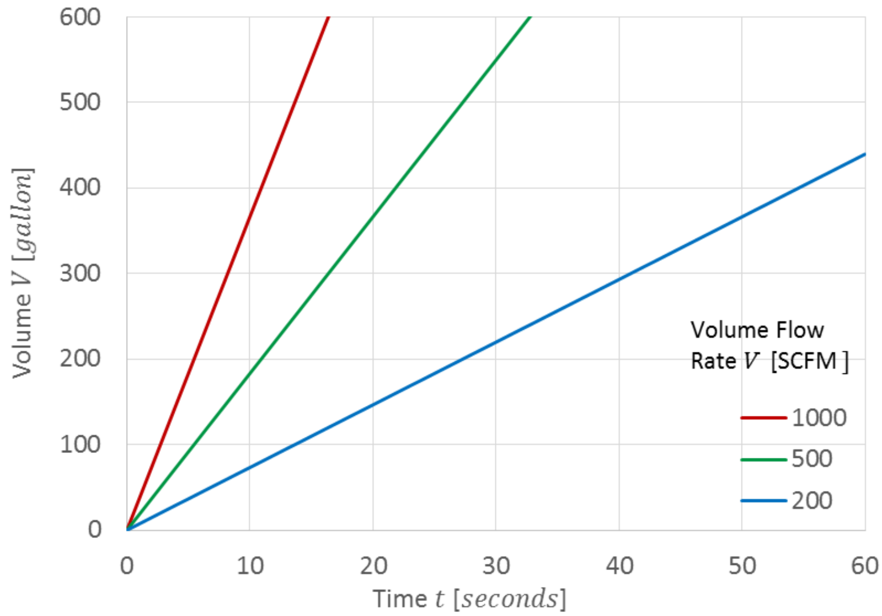



Figure 45. Plot VI.c - Receiver Charging Capacity (Initial Pressure 100 psig)

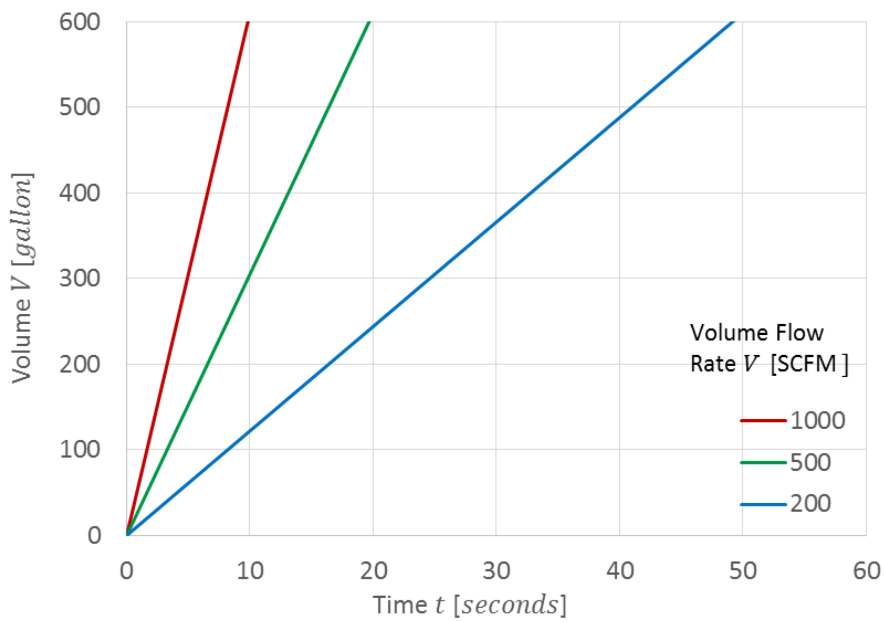



Figure 46. Plot VI.d - Receiver Charging Capacity (Initial Pressure 120 psig)

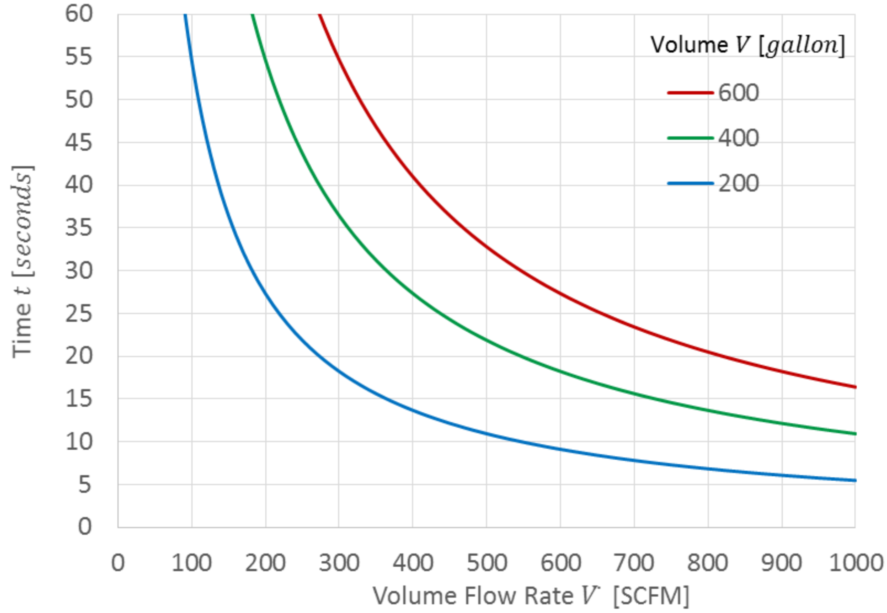



Figure 47. Plot VI.e - Receiver Charging Speed (Initial Pressure 100 psig)

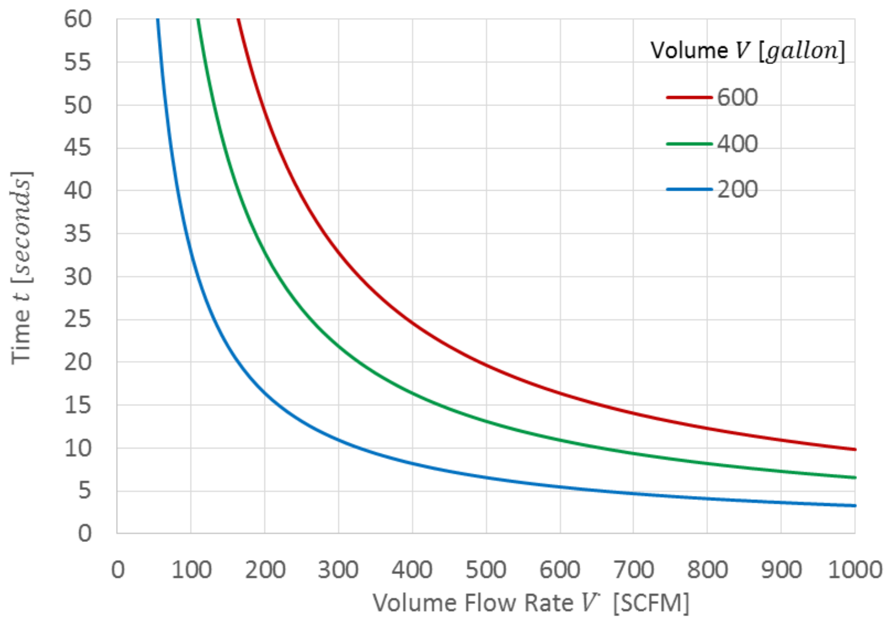



Figure 48. Plot VI.f - Receiver Charging Speed (Initial Pressure 120 psig)

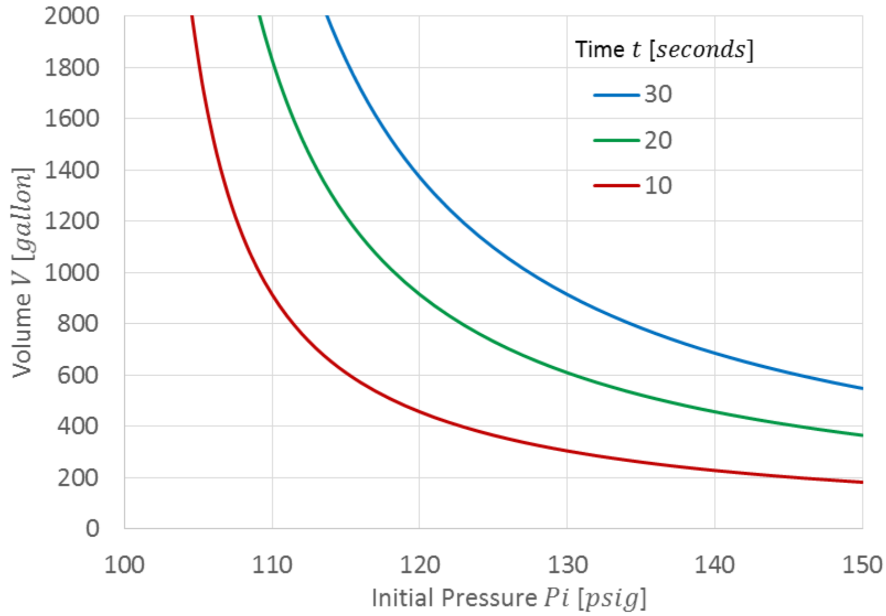



Figure 49. Plot VI.g - Receiver Discharging Volume

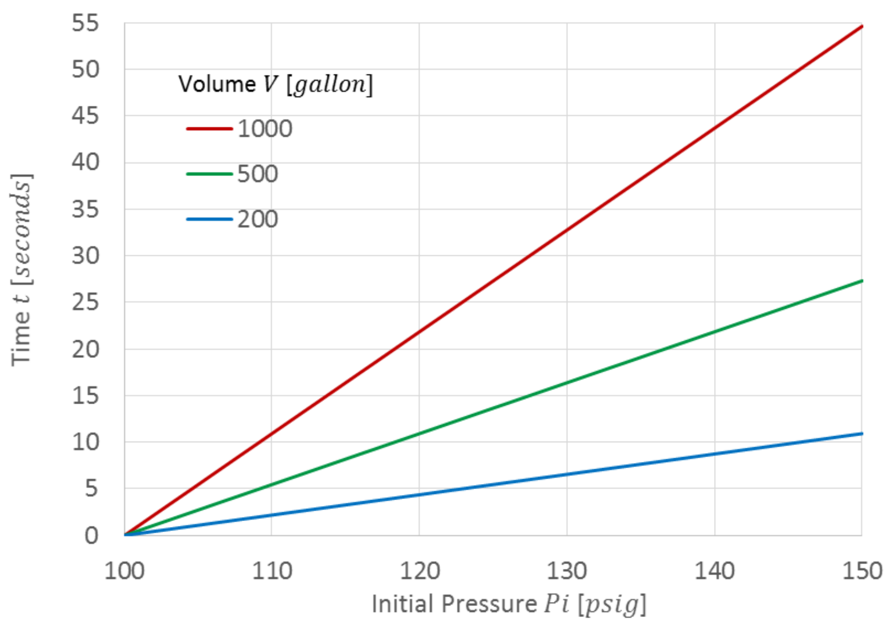



Figure 50. Plot VI.h - Receiver Discharging Time

## CHAPTER VII – Piping Model and Analysis

- Design and/or Analysis Objective
  - Determine the pressure gradient $\frac{dP}{dx}$ of a compressed air flowing inside a pipe with a certain diameter $D$ at a certain volume flow rate $\dot{V}$.
  - Determine diameter $D$ of a pipe necessary to achieve a certain pressure gradient $\frac{dP}{dx}$ of a flowing compressed air at a certain volume flow rate $\dot{V}$.
- Functional Relationships
  - VII.a: $\frac{dP}{dx} = f(\dot{V}_{SCFM}, D)$
    - $\frac{dP}{dx} = 3.866 \cdot 10^{-5} \left(\rho_{SCFM}\left(\frac{P}{P_{SCFM}}\right)\right)^{0.8} \mu^{0.2} \left(\dot{V}_{SCFM}\left(\frac{P_{SCFM}}{P}\right)\right)^{1.8} D^{-4.8}$,

      where $\rho$, and $\mu$ are equal to $0.687\frac{lbm}{ft^3}$, and $3.816 \cdot 10^{-7} \frac{lbf \cdot s}{ft^2}$, and also $\rho_{SCFM}$, $P_{SCFM}$, and $P$ are equal to $0.075\frac{lbm}{ft^3}$, 14.7 $psia$, and 120 $psia$
  - VII.b: $D = f(\dot{V}_{SCFM}, \frac{dP}{dx})$
    - $D = \left(3.866 \cdot 10^{-5} \left(\rho_{SCFM}\left(\frac{P}{P_{SCFM}}\right)\right)^{0.8} \mu^{0.2} \left(\dot{V}_{SCFM}\left(\frac{P_{SCFM}}{P}\right)\right)^{1.8} \left(\frac{dP}{dx}\right)^{-1}\right)^{\frac{1}{4.8}}$

      , where $\rho$, and $\mu$ are equal to $0.687\frac{lbm}{ft^3}$, and $3.816 \cdot 10^{-7} \frac{lbf \cdot s}{ft^2}$, and also $\rho_{SCFM}$, $P_{SCFM}$, and $P$ are equal to $0.075\frac{lbm}{ft^3}$, 14.7 $psia$, and 120 $psia$
- Example Problem Summary Calculation
  - VII.a: Friction Factor and Pressure Drop

    The Reynold's number $Re$

$$Re = \frac{\rho \, \bar{V} \, D}{\mu}$$

$$Re = \frac{\left(0.636 \frac{lbm}{ft^3}\right)\left(20 \frac{ft}{s}\right)\left(3 \; in \cdot \frac{1}{12} \frac{ft}{in}\right)}{\left(3.816 \cdot 10^{-7} \; \frac{lbf \cdot s}{ft^2} \cdot 32.2 \frac{lbm \cdot ft}{lbf \cdot s^2}\right)} = 258{,}891$$

The friction factor $f$ for turbulent flow ($Re \geq 3000$) in smooth pipe

$$f = \frac{0.184}{Re^{0.2}}$$

$$f = \frac{0.184}{258{,}891^{0.2}} = 0.0152$$

The pressure gradient $\frac{dP}{dx}$

$$\frac{dP}{dx} = \frac{\rho \, \bar{V}^2 \, f}{2 \, D}$$

$$\frac{dP}{dx} = \frac{\left(0.636 \frac{lbm}{ft^3}\right)\left(20 \frac{ft}{s}\right)^2 (0.0152)}{2 \, \left(3 \; in \cdot \frac{1}{12} \frac{ft}{in}\right)} \cdot \frac{1}{32.2} \frac{lbf \cdot s^2}{lbm \cdot ft} \cdot \frac{1}{144} \frac{ft^2}{in^2}$$

$$= 1.668 \frac{psi}{1000 \; ft}$$

The pressure drop $\Delta P$

$$\Delta P = \frac{\rho \, \bar{V}^2 \, f \, L}{2 \, D}$$

$$\Delta P = \frac{dP}{dx} L$$

$$\Delta P = 1.668 \frac{psi}{1000 \; ft} \cdot 2000 \; ft = 3.337 \; psia$$

Which means that the pressure drop is about 2.68% of the initial pressure.

$$\frac{\Delta P}{P}\cdot 100\% = \frac{3.337\ psia}{(\langle 110 + 14.7\rangle psia)}\cdot 100\% = 2.68\%$$

- Sample Design or Analysis (from plots)
  - Plot VII.a - Pressure Gradient of Compressed Air Inside a Pipe

    If $\dot{V}_{SCFM} = 400\ SCFM$, and $D = 3\ inch$, then $\frac{dP}{dx} = 1.04\frac{psi}{1000\ ft}$

  - Plot VII.b - Pipe Diameter

    If $\dot{V}_{SCFM} = 400\ SCFM$, and $\frac{dP}{dx} = 1.0\frac{psi}{1000\ ft}$, then $D = 3.02\ inch$

- Plots

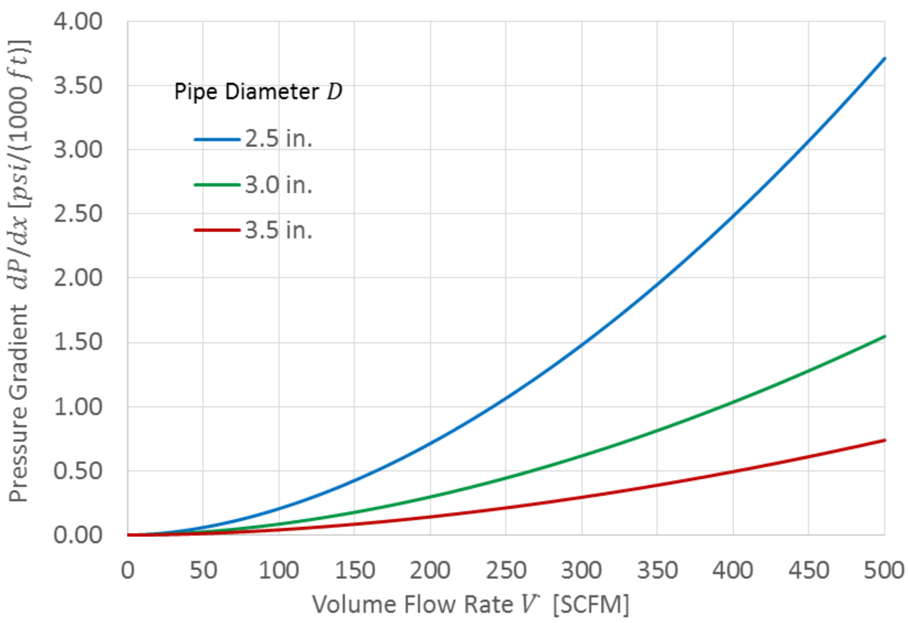



Figure 51. Plot VII.a - Pressure Gradient of Compressed Air Inside a Pipe

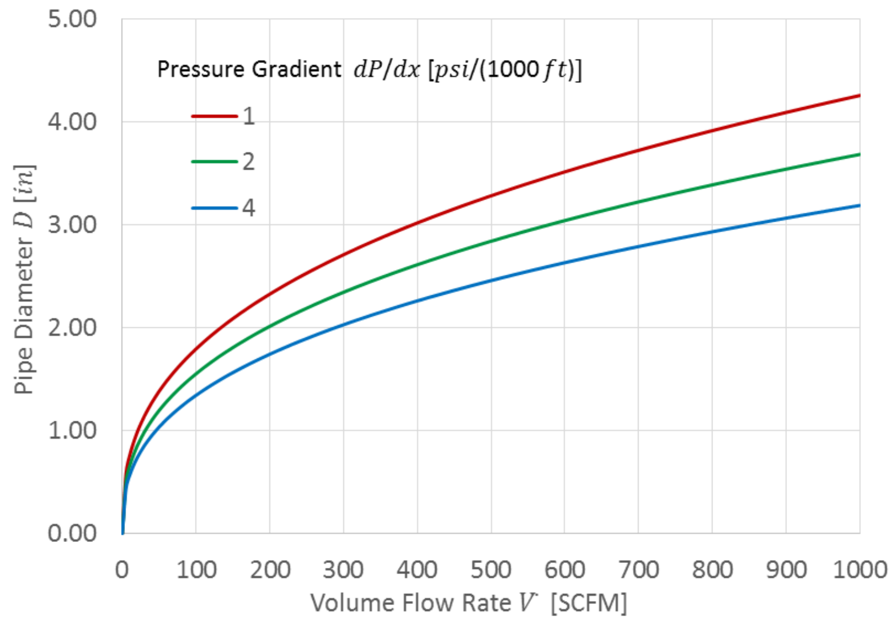



Figure 52. Plot VII.b - Pipe Diameter

**CHAPTER VIII – Flow Regulators Model and Analysis**

- Design and/or Analysis Objective
    - Determine the pressure gradient $\frac{dP}{dx}$, given the compressed air pressure $P$, velocity $\bar{V}$, and the loss coefficient $K_L$.
- Functional Relationships
    - VIII.a, and VIII.c: $\Delta P = f(P, K_L)$
        - $\Delta P = 2.2 \cdot 10^{-4}\ P\ K_L$
    - VIII.b: $\Delta P = f(\bar{V}, K_L)$
        - $\Delta P = 6.6 \cdot 10^{-5}\ \bar{V}^2\ K_L$
- Example Problem Summary Calculation
    - VIII.a: Pipe Diameter and Loss Coefficient

The pipe diameter $D$

$$D = \sqrt{\frac{4\,\dot{V}}{\pi\,\bar{V}}}$$

$$D = \sqrt{\frac{4\left(5.46\,\frac{ft^3}{min}\right)}{\pi\left(30\,\frac{ft}{sec}\cdot 60\,\frac{sec}{min}\right)}}\cdot 12\,\frac{in}{ft} = 0.746\ in$$

The Reynold's number $Re$

$$Re = \frac{\rho\,\bar{V}\,D}{\mu}$$

$$Re = \frac{\left(0.687\,\frac{lbm}{ft^3}\right)\left(30\,\frac{ft}{s}\right)\left(0.746\ in\cdot\frac{1}{12}\frac{ft}{in}\right)}{\left(3.816\cdot 10^{-7}\ \frac{lbf\cdot s}{ft^2}\cdot 32.2\,\frac{lbm\cdot ft}{lbf\cdot s^2}\right)} = 104{,}310$$

The friction factor $f$ for turbulent flow ($Re \geq 3000$) in smooth pipe

$$f = \frac{0.184}{Re^{0.2}}$$

$$f = \frac{0.184}{104{,}310^{0.2}} = 0.0182$$

The equivalent length $L_{eq}$

$$L_{eq} = \frac{2\,D\,\Delta P}{\rho\,\bar{V}^2\,f}$$

$$L_{eq} = \frac{2\left(0.746\ in\cdot\frac{1}{12}\frac{ft}{in}\right)\left(\langle 120-80\rangle\,\frac{lbf}{in^2}\cdot 144\,\frac{in^2}{ft^2}\right)}{\left(0.687\,\frac{lbm}{ft^3}\right)\left(30\,\frac{ft}{s}\right)^2 (0.0182)}\cdot 32.2\,\frac{lbm\cdot ft}{lbf\cdot s^2}$$

$$= 2048.5\ ft$$

The loss coefficient $K_L$

$$K_L = \frac{f\ L_{eq}}{D}$$

$$K_L = \frac{(0.0182)(2048.5\ ft)}{\left(0.746\ in \cdot \frac{1}{12}\frac{ft}{in}\right)} = 600$$

- Sample Design or Analysis (from plots)
  - Plot VIII.a - Pressure Gradient of Compressed Air Inside a Flow Regulator 1

    If $P = 120\ psia$, and $K_L = 0.5$, then $\Delta P = 0.013\ psia$
  - Plot VIII.b - Pressure Gradient of Compressed Air Inside a Flow Regulator 2

    If $\bar{V} = 25\ \frac{ft}{s}$, and $K_L = 0.5$, then $\Delta P = 0.021\ psia$
  - Plot VIII.c - Pressure Gradient of Compressed Air Inside a Flow Regulator 3

    If $K_L = 500$, and $P = 100\ psia$, then $\Delta P = 11.0\ psia$

- Plots

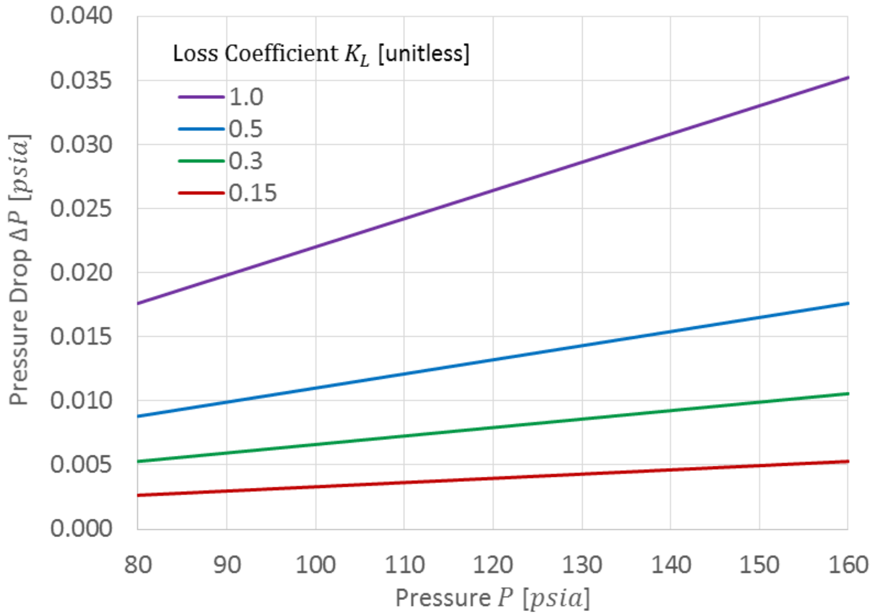



Figure 53. Plot VIII.a - Pressure Gradient of Compressed Air Inside a Flow Regulator 1

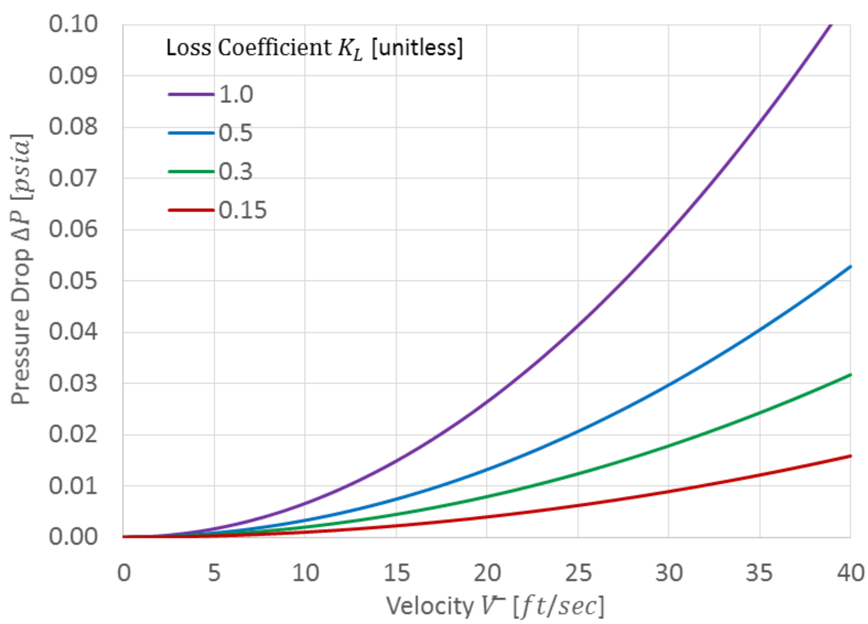



Figure 54. Plot VIII.b - Pressure Gradient of Compressed Air Inside a Flow Regulator 2

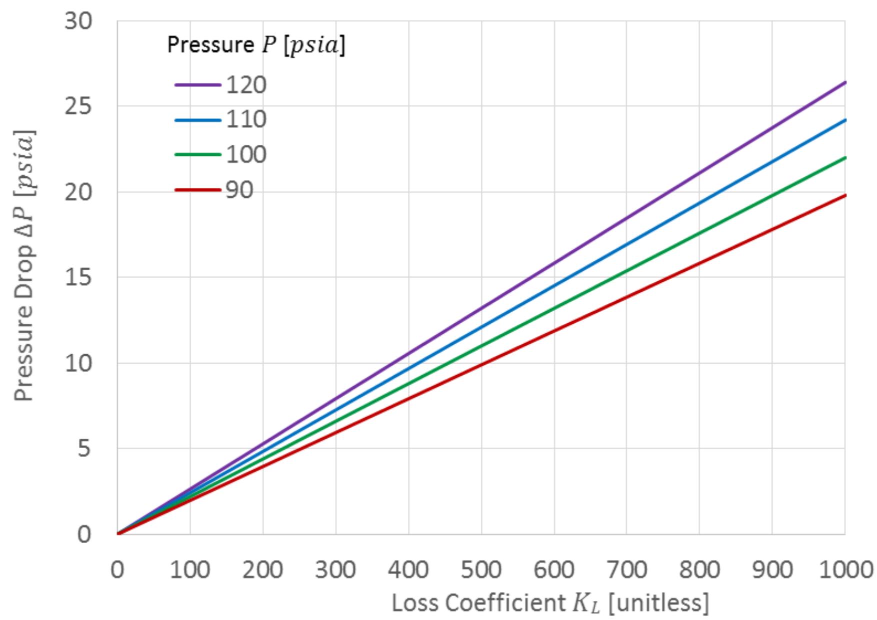



Figure 55. Plot VIII.c - Pressure Gradient of Compressed Air Inside a Flow Regulator 3

## CHAPTER IX – Compressed Air System Leakage Model and Analysis

- Design and/or Analysis Objective
    - Determine the sonic velocity $\bar{V}_s$ of a leaking compressed air.
    - Determine both the volume and mass flow rates ($\dot{V}$ and $\dot{m}$) of a leaking air.
- Functional Relationships
    - IX.a: $\bar{V}_s = f(T)$
        - $\bar{V}_s = \sqrt{k\ R_a\ T}$, where $k$, and $R_a$ are equal to 1.4, and 53.33 $\frac{lbf \cdot ft}{lbm \cdot R}$
    - IX.b: $\dot{V}_{SCFM} = f(D, P)$
        - $\dot{V}_{SCFM} = \frac{\pi \sqrt{k}\ D^2 P}{3.332\ \sqrt{R_a\ T}\ \rho_{SCFM}}$, where $k$, $R_a$, $T$, and $\rho_{SCFM}$ are equal to 1.4, 53.33 $\frac{lbf \cdot ft}{lbm \cdot R}$, 68°F, and 0.075 $\frac{lbm}{ft^3}$

- IX.c: $\dot{m} = f(D, P)$
  - $\dot{m} = \frac{\pi \sqrt{k}\, D^2 P}{3.332\, \sqrt{R_a\, T}}$, where $k$, $R_a$, and $T$ are equal to 1.4, 53.33 $\frac{lbf \cdot ft}{lbm \cdot R}$, and 68°F

- Example Problem Summary Calculation
  - IX.a: Sonic Velocity and Flow Rates

The pressure $P$, and the temperature $T$ of the air at the exit plane

$$P = 0.528\, P_0$$

$$P = 0.528(\langle 100 + 14.7 \rangle psia) = 60.56\, psia$$

$$T = 0.833\, T_0$$

$$T = 0.833(80\ °\mathrm{F}) = 66.64\ °\mathrm{F}$$

The sonic velocity $\bar{V}_s$ of the air leaking out

$$\bar{V}_s = \sqrt{k\, R_a\, T}$$

$$\bar{V}_s = \sqrt{(1.4)\left(53.33 \frac{lbf \cdot ft}{lbm \cdot R} \cdot 32.2 \frac{lbm \cdot ft}{lbf \cdot s^2}\right)(66.64 + 459.67R)}$$

$$= 1{,}123.8 \frac{ft}{sec}$$

The volume flow rate $\dot{V}$

$$\dot{V} = \frac{\pi}{4} D^2\, \bar{V}_s$$

$$\dot{V} = \frac{\pi}{4}\left(\frac{1}{4} in \cdot \frac{1}{12} \frac{ft}{in}\right)^2 \left(1{,}123.8 \frac{ft}{sec} \cdot 60 \frac{sec}{min}\right) = 22.99 \frac{ft^3}{min}$$

$$= 95.31\, SCFM$$

The mass flow rate $\dot{m}$

$$\dot{m} = \rho\, \dot{V}$$

$$\dot{m} = \left(0.311\frac{lbm}{ft^3}\right)\left(22.99\frac{ft^3}{min}\right) = 7.148\frac{lbm}{min}$$

- Sample Design or Analysis (from plots)
  - Plot IX.a - Leaking Compressed Air Sonic Velocity

    If $T = 80$°F, then $\bar{V}_s = 1123.8\frac{ft}{sec}$

  - Plot IX.b - Leaking Compressed Air Volume Flow Rate

    If $D = {}^1\!/_8\ inch$, and $P = 125\ psig$, then $\dot{V}_{SCFM} = 29.33\ SCFM$

  - Plot IX.c - Leaking Compressed Air Mass Flow Rate

    If $D = {}^1\!/_8\ inch$, and $P = 125\ psig$, then $\dot{m} = 2.20\frac{lbm}{min}$

- Plots

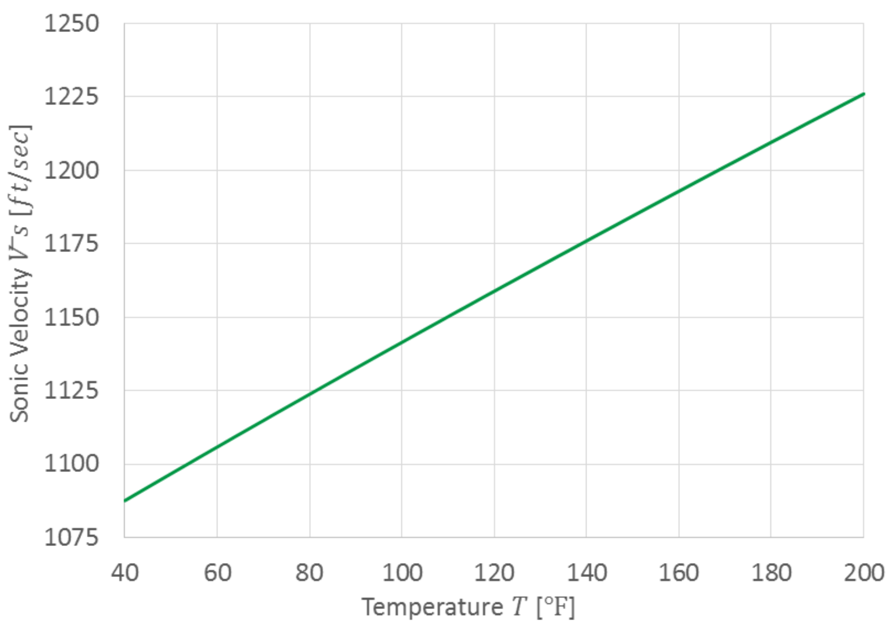



Figure 56. Plot IX.a - Leaking Compressed Air Sonic Velocity

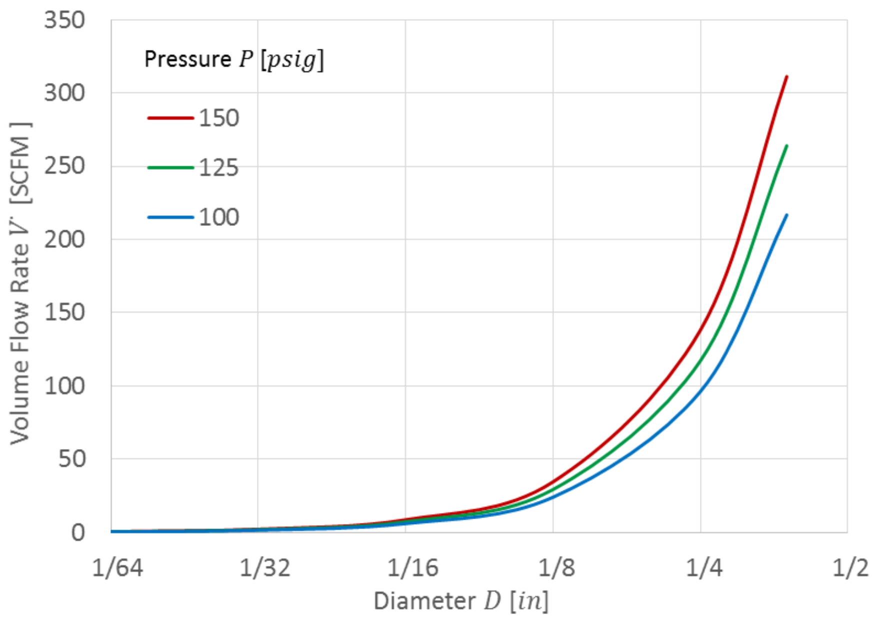



Figure 57. Plot IX.b - Leaking Compressed Air Volume Flow Rate

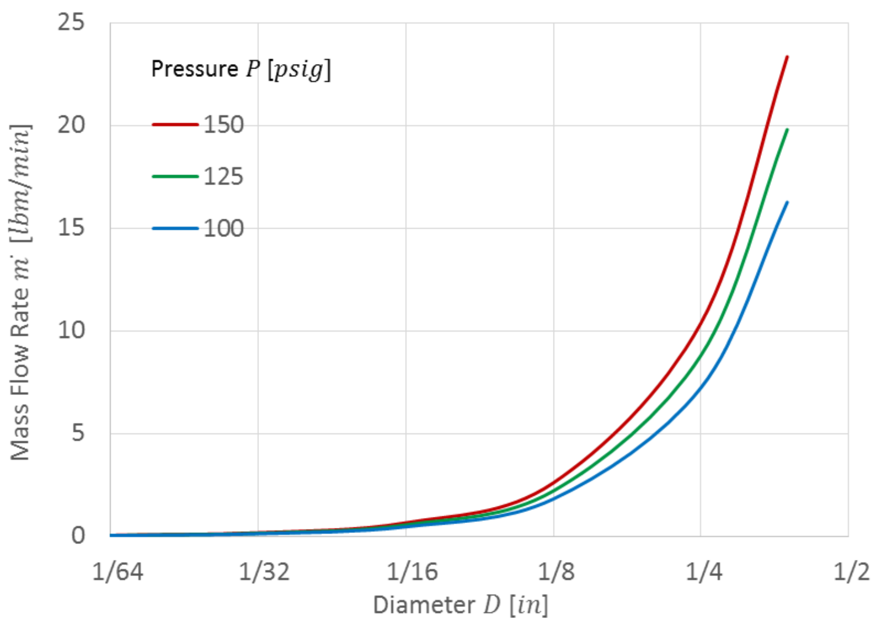



Figure 58. Plot IX.c - Leaking Compressed Air Mass Flow Rate

**CHAPTER X – Compressed Air Tools Model and Analysis**

- Design and/or Analysis Objective
  - Determine the exit temperature $T_{out}$ of air from a compressed air tool.
  - Determine specific work $\dot{w}$ from an expanding compressed air to a compressed air tool
- Functional Relationships
  - X.a: $T_{out} = f(P_{in}, \eta)$
    - $T_{out} = T_{in}\left(1 - \eta\left(1 - \left(\frac{P_{in}}{P_{out}}\right)^{\frac{k-1}{k}}\right)\right)$, where $T_{in}$, $P_{out}$, and $k$ are equal to 68°F or $527.67\ R$, $14.7\ psia$, and 1.4
  - X.b: $\dot{w} = f(P_{in}, \eta)$
    - $\dot{w} = C_p\, T_{in}\, \eta\left(1 - \left(\frac{P_{in}}{P_{out}}\right)^{\frac{k-1}{k}}\right)$, where $C_p$, $T_{in}$, $P_{out}$, and $k$ are equal to $0.240\frac{BTU}{lbm\cdot R}$, 68°F or $527.67\ R$, $14.7\ psia$, and 1.4
- Example Problem Summary Calculation
  - X.a: Compressor Size Requirement

    The size or power $\dot{W}$ of air compressor required

    $$\dot{W} = \dot{W}^*\, \dot{V}_{tool}\,(1 + \%_{leak})$$

    $$\dot{W} = \left(20\frac{hp}{100\ SCFM}\right)(25\ SCFM)(1 + 0\%) = 5\ hp$$

    For 100 unit compressed air tools

    $$\dot{W}_{100} = 100\ \dot{W}$$

$$\dot{W}_{100} = 100\ (5\ hp) = 500\ hp$$

With 30% leak

$$\dot{W}_{100+leak} = \dot{W}_{100}\ (1 + \%_{leak})$$

$$\dot{W}_{100+leak} = (500\ hp)(1 + 30\%) = 650\ hp$$

The work (in $kW \cdot hr$) required with the leak

$$W_{100+leak} = \dot{W}_{100+leak}\ t$$

$$W_{100+leak} = \left(650\ hp \cdot 0.746 \frac{kw}{hp}\right)\left(1\ yr \cdot 365 \frac{day}{yr} \cdot 24 \frac{hr}{day}\right)$$

$$= 4{,}247{,}700\ kW \cdot hr = 4.25\ GW \cdot hr$$

The volume $V_{100+leak}$ of compressed air (in $SCF$) used with the leak

$$V_{100+leak} = V_{100}\ (1 + \%_{leak})$$

$$V_{100+leak} = 100\ (25\ SCFM)\left(1\ yr \cdot 525600 \frac{min}{yr}\right)(1 + 30\%)$$

$$= 1.71\ billion\ SCF$$

- Sample Design or Analysis (from plots)
  - Plot X.a - Exit Temperature from a Compressed Air Tool

    If $P_{in} = 60\ psia$, and $\eta = 50\ \%$, then $T_{out} = -19.1$°F
  - Plot X.b - Specific Work from an Expanding Compressed Air

    If $P_{in} = 60\ psia$, and $\eta = 50\ \%$, then $\dot{w} = 20.9 \frac{BTU}{lbm}$

- Plots

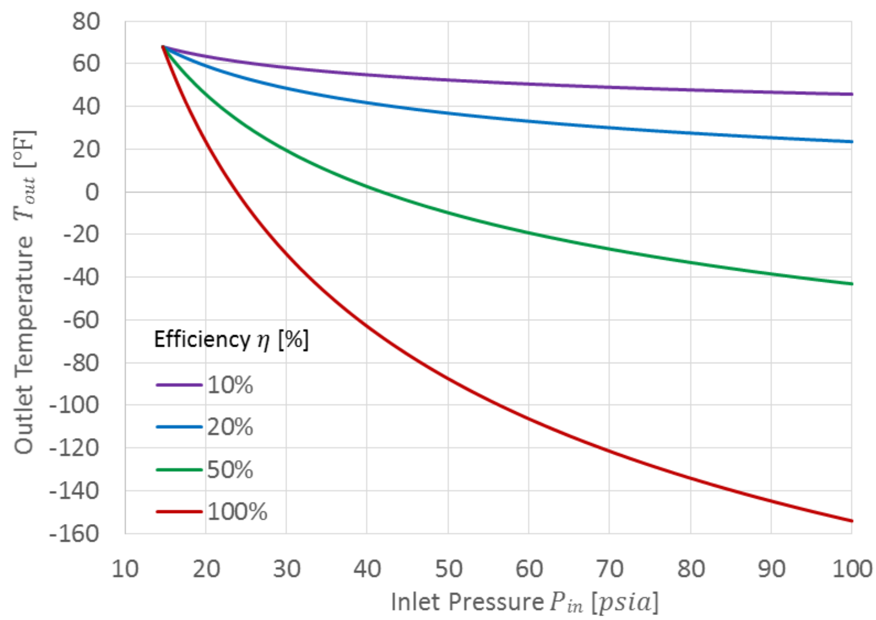



Figure 59. Plot X.a - Exit Temperature from a Compressed Air Tool

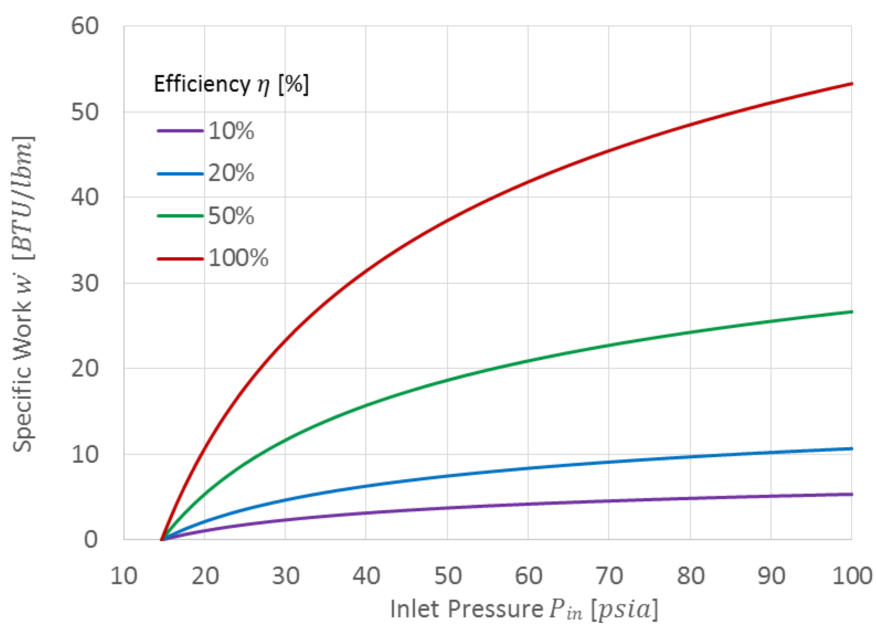



Figure 60. Plot X.b - Specific Work from an Expanding Compressed Air

# CHAPTER XIII

# CONCLUSIONS

The basic components of a compressed air system, along with how moist air behaves, and its properties such as, pressure, temperature, and relative humidity changes, are discussed and analyzed by using theories derived herein, and a number of example problems. These Example Problems, along with the MATLAB models, and component behavior plots in CHAPTER XII will enable engineers to understand the fundamentals of compressed air system analysis and design.

For compressed air system design and analysis, this thesis provides a clear explanation of the relevant and important specifications, and operations of each of the main components of a compressed air system. Of special importance, general energy and mass conservation equations are applied for each components so that energy transfer can be determined along with moist air properties of temperature, pressure, and humidity at each point in the system. Moreover, the step-by-step explanation, accompanied by the MATLAB codes enable engineers to accurately analyze and determine the performance and efficiencies of either the system or any components.

The compressed air system components described and modeled herein are the two-stage air compressor, with the intercooler, the aftercooler, the accumulator or receiver, the air dryers, the distribution piping and components, the regulators, and the compressed air tools, along with the unwanted air leakage. In addition to deriving equations and applying assumptions, which make up the system and component models, example problems, and design and analysis plots (in CHAPTER XII) were formulated for each components, along with moist air and leakage analysis.

## REFERENCES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[3] Northeast Regional Poultry Research Project, "Psychrometric Chart Use," in *Biomeasurements and Experimental Techinques for Avian Species*, Storrs, University of Connecticut, 2004.

[4] O. Alduchov and R. Eskridge, "Improved Magnus` form approximation of saturation vapor pressure," Department of Energy, 1997.

[5] S. Beck and R. Collins, "Moody diagram. Lines created using Swami and Jaine formula. Plot created on Matlab.," Donebythesecondlaw, 2008.

# APPENDIX A

# FUNCTION DEVELOPMENT

In order to provide consistent, and efficient codification, separate functions have been developed, each of which represents one of the thermodynamic principles. This way, when the main code from one of the chapters needs data from those principles, any of those functions can be called as many times as needed. The following sections explain each of the functions and the input data that they need, as well as the output data that they produce.

## Ideal Gas Equation Function (PVTRo.m)

First, this function automatizes Equations (1) and (2). The code starts with an `if` loop which helps with a basic validation for the input value. This function uses a string input so this validation will make sure that the string asks only for calculation of the pressure `P`, volume `V`, temperature `T`, mass `m` or density `Ro`, thus,

```
function y = PVTRo(x)
if
~strcmp(x,'P')&&~strcmp(x,'V')&&~strcmp(x,'T')&&~strcmp(x,'m')&&~
strcmp(x,'Ro')
    error('Input must be P, V, T, m, or Ro.')
```

After the validation, the code defines the value for the ideal gas constant `R`, and it uses a `switch` loop to choose between cases that are the five parameters to be calculated. Each one of these five cases will ask for consistent inputs depending on the case that is initially selected.

```
else
```

```
Ru = 1545; % units lbf.ft.(lbmol.R)^-1
Ma = 28.97; % units lbm.lbmol^-1
R = Ru/Ma;
switch x
    case 'P'
        vol = 'Volume? (Insert value in ft^3): ';
        temp = 'Temperature? (Insert value in fahrenheit): ';
        mass = 'Mass of dry air? (Insert value in lbm): ';
        V = input(vol);
        T = input(temp)+459.67;
        m = input(mass);
        P = m*R*T/(V*144);
        Ro = m/V;
    case 'V'
        pre = 'Pressure? (Insert value in psia): ';
        temp = 'Temperature? (Insert value in fahrenheit): ';
        mass = 'Mass of dry air? (Insert value in lbm): ';
        P = input(pre)*144;
        T = input(temp)+459.67;
        m = input(mass);
        V = m*R*T/P;
        Ro = m/V;
    case 'T'
        pre = 'Pressure? (Insert value in psia): ';
        vol = 'Volume? (Insert value in ft^3): ';
        mass = 'Mass of dry air? (Insert value in lbm): ';
        P = input(pre)*144;
        V = input(vol);
        m = input(mass);
        T = P*V/(m*R);
        Ro = m/V;
    case 'm'
        pre = 'Pressure? (Insert value in psia): ';
        vol = 'Volume? (Insert value in ft^3): ';
        temp = 'Temperature? (Insert value in fahrenheit): ';
        P = input(pre)*144;
        V = input(vol);
        T = input(temp)+459.67;
        m = P*V/(R*T);
        Ro = m/V;
    case 'Ro'
        pre = 'Pressure? (Insert value in psia): ';
        temp = 'Temperature? (Insert value in fahrenheit): ';
        P = input(pre)*144;
        T = input(temp)+459.67;
        V = 0;
        m = 0;
        Ro = P/(R*T);
End
```

Finally, from each of these five cases, five values will be earned. In order to be able to manage those results effectively, those answers have to be stored in a vector `y`, thus when this function is called into a main program, the right parameter from the vector should be carefully chosen for our calculation.

```
end
y = [P,V,T,m,Ro];
end
```

**Receiver Calculator (Recv.m)**

First, this function automatizes Equations (59), (60), and (61), used to calculate the volume of a receiver tank. The code starts with an `if` loop which helps with a basic validation for the input value. This function uses a string input so this validation will make sure that the string asks only for calculation of the pressure `P`, volume `V`, temperature `T`, flow rate `Vdot`, or time `t`, thus,

```
function y = Recv(x)
if
~strcmp(x,'dP')&&~strcmp(x,'V')&&~strcmp(x,'T')&&~strcmp(x,'Vdot'
)&&~strcmp(x,'t')
    error('Input must be P, V, T, Vdot, or t.')
```

After the validation, the code defines the value for the ideal gas constant `R` and the density of dry air `Ro`, and it uses a `switch` loop to choose between cases that are the six parameters to be calculated. Each one of these six cases will ask for consistent inputs depending on the case that is initially selected.

```
else
    Ru = 1545; % units lbf.ft.(lbmol.R)^-1
    Ma = 28.97; % units lbm.lbmol^-1
    Ro = 0.075; % units lbm.ft^-3
    R = Ru/Ma;
    switch x
        case 'dP'
            vol = 'Volume? (Insert value in gal): ';
            temp = 'Temperature? (Insert value in fahrenheit): ';
            flow = 'Flow rate? (Insert value in SCFM): ';
            tim = 'Time? (Insert value in seconds): ';
```

```
            V = input(vol)/7.48;
            T = input(temp)+459.67;
            Vdot = input(flow)/60;
            t = input(tim);
            dP = Ro*Vdot*t*R*T/(V*144);
        case 'V'
            flow = 'Flow rate? (Insert value in SCFM): ';
            tim = 'Time? (Insert value in seconds): ';
            temp = 'Temperature? (Insert value in fahrenheit): ';
            pre = 'Pressure change? (Insert value in psia): ';
            Vdot = input(flow)/60;
            t = input(tim);
            T = input(temp)+459.67;
            dP = input(pre)*144;
            V = Ro*Vdot*t*R*T*7.48/dP;
        case 'T'
            vol = 'Volume? (Insert value in gal): ';
            pre = 'Pressure change? (Insert value in psia): ';
            flow = 'Flow rate? (Insert value in SCFM): ';
            tim = 'Time? (Insert value in seconds): ';
            P = input(pre)*144;
            V = input(vol)/7.48;
            Vdot = input(flow)/60;
            t = input(tim);
            T = V*dP/(Ro*Vdot*t*R);
        case 'Vdot'
            vol = 'Volume? (Insert value in gal): ';
            pre = 'Pressure change? (Insert value in psia): ';
            tim = 'Time? (Insert value in seconds): ';
            temp = 'Temperature? (Insert value in fahrenheit): ';
            dP = input(pre)*144;
            V = input(vol)/7.48;
            t = input(tim);
            T = input(temp)+459.67;
            Vdot = V*dP*60/(Ro*t*R*T);
        case 't'
            vol = 'Volume? (Insert value in gal): ';
            pre = 'Pressure change? (Insert value in psia): ';
            flow = 'Flow rate? (Insert value in SCFM): ';
            temp = 'Temperature? (Insert value in fahrenheit): ';
            dP = input(pre)*144;
            V = input(vol)/7.48;
            Vdot = input(flow)/60;
            T = input(temp)+459.67;
            t = V*dP/(Ro*Vdot*R*T);
    end
```

Finally, from each of these six cases, six values will be earned. In order to be able to manage those results effectively, those answers have to be stored in a vector y, thus when this function is called into a main program, the right parameter from the vector should be carefully chosen for our calculation.

```
end
y = [P,V,T,m,Ro];
end
```

**Volume Flow Rate Function (VAv.m)**

First, this function automatizes Equation (17), used to calculate the mass and volume flow rate in a pipe. The code starts with an `if` loop which helps with a basic validation for the input value. This function uses a string input so this validation will make sure that the string asks only for calculation of the flow rate `Vdot`, diameter of the pipe `D`, or velocityy `v`, thus,

```
function y = VAv(x)
if ~strcmp(x,'Vdot')&&~strcmp(x,'D')&&~strcmp(x,'v')
    error('Input must be Vdot, D, or v')
```

After the validation, the code uses a `switch` loop to choose between cases that are the three parameters to be calculated. Each one of these three cases will ask for consistent inputs depending on the case that is initially selected.

```
else
    switch x
        case 'Vdot'
            diam = 'Diameter? (Insert value in inches): ';
            vel = 'Velocity? (Insert value in ft/sec): ';
            D = input(diam)/12;
            v = input(vel);
            Vdot = pi*D^2*v/4;
        case 'D'
            flow = 'Flow rate? (Insert value in SCFM): ';
            vel = 'Velocity? (Insert value in ft/sec): ';
            Vdot = input(flow)/60;
            v = input(vel);
            D = 2*(Vdot/(pi*v))^(1/2);
        case 'v'
            flow = 'Flow rate? (Insert value in SCFM): ';
```

```
            diam = 'Diameter? (Insert value in inches): ';
            Vdot = input(flow)/60;
            D = input(diam)/12;
            v = (4*Vdot)/(pi*D^2);
    end
```

Finally, from each of these three cases, three values will be earned. In order to be able to manage those results effectively, those answers have to be stored in a vector `y`, thus when this function is called into a main program, the right parameter from the vector should be carefully chosen for our calculation.

```
end
y = [Vdot,D,v];
end
```

**Reynolds Number Function (Re.m)**

First, this function automatizes Equation (69), used to calculate the Reynold's number of a flow. The code is pretty simple and uses also Equation (70) to calculate the gas dynamic viscosity. It needs numerical input values for density `Ro`, velocity `v`, diameter of the pipe `D`, and temperature of the fluid `T`. Finally, this function is called inside the function `Pgrad`, which calculates the pressure gradient and will be explained next.

```
function y = Re(Ro,v,D,T)
C = 120;
u0 = 3.816e-7;
T0 = 524.07;
u = u0*((0.555*T0+C)/(0.555*T+C))*(T/T0)^1.5;
y = Ro*v*D/(u*32.2*12);
end
```

**Pressure Gradient Function (Pgrad.m)**

First, this function automatizes Equation (62), used to calculate the pressure gradient of a flow. It needs numerical input values density `Ro`, velocity `v`, diameter of the pipe D, and temperature of the fluid T. This code starts using the function Re for calculating the Reynold's number of the flow, and then it asks for the surface roughness eps,

```
function y = Pgrad(Ro,v,D,T)
Ry = Re(Ro,v,D,T);
eps = 'Surface roughness? (Use Moody Diagram): ';
ee = input(eps);
```

These codes below also uses Equation (65), (66), and (68) to calculate the friction factor as a function of the Reynold's number, as follows.

```
if 3000 <= Ry && Ry <= 10e8
    sp = 'Smooth pipe? (Y/N): ';
    switch sp
        case 'Y'
            f = 0.184/Ry^.2;
        case 'N'
            f = 1.325/(log(ee/(3.7*D)+5.74/Ry^0.9))^2;
    end
elseif Ry <= 2100
    f = 64/Ry;
else
    f1 = 64/2100;
    f2 = 1.325/(log(ee/(3.7*D)+5.74/3000^0.9))^2;
    f = f1+(f2-f1)*(Ry-2100)/900;
end
y = Ro*f*v^2/(2*D);
end
```

### Outlet Pressure and Temperature for Compression Function (TTPP.m)

First, this function automatizes Equation (34), used to calculate either the Outlet Temperature T2 or Pressure P2 in a polytrophic compression. It needs numerical input values for both the inlet pressure P1, and inlet temperature T1, as well as either the outlet pressure or temperature PT. This function uses both a string x and numerical input values: PT, P1, T1, and k. Therefore it needs a validation that will make sure that the string asks only for calculation of the outlet pressure P2, or final temperature T2. As the string x that is assigned to the variable PT is replaced by the input, which can be either P2, or T2, the function will calculate either the final pressure (if P2 replaces x) or final temperature (if T2 replaces x), using the inlet pressure P1, inlet volume V1, and polytropic constant k.

```
function y = TTPP(x,PT,P1,T1,k)
if ~strcmp(x,'P2')&&~strcmp(x,'T2')
    error('Input must be P2, or T2.')
```

After the validation, the code uses a switch loop to choose between cases that are the two parameters to be calculated. Each one of these two cases will ask for consistent inputs depending on the case that is initially selected. This code uses Equation (34) to calculate either the outlet pressure P2 or temperature T2, as follows.

```
else
    if strcmp(x,'P2')
        T2 = PT;
    else
        P2 = PT;
    end
    switch x
        case 'P2'
            P2 = P1*(k/(k-1))*log(T2/T1);
        case 'T2'
            T2 = T1*(P2/P1)^(k/(k-1));
    end
end
y = [P1,P2,T1,T2];
end
```

**Polytropic Process Function (VVPP.m)**

First, this function automatizes Equation (35), used to calculate either the outlet volume or pressure in a polytrophic process. It needs numerical input values for both the initial pressure P1, and initial volume V1, as well as either the final pressure or volume

PV. This function uses both a string x and numerical input values: PV, P1, V1, and k. Therefore it needs a validation that will make sure that the string asks only for calculation of the final pressure P2, or final volume V2. As the string x that is assigned to the variable PV is replaced by the input, which can be either P2, or V2, the function will calculate either the final pressure (if P2 replaces x) or final volume (if V2 replaces x), using the initial pressure P1, initial volume V1, and polytropic constant k.

```
function y = VVPP(x,PV,P1,V1,k)
if ~strcmp(x,'P2')&&~strcmp(x,'V2')
    error('Input must be P2, or V2.')
```

After the validation, the code uses a switch loop to choose between cases that are the two parameters to be calculated. Each one of these two cases will ask for consistent inputs depending on the case that is initially selected. This code uses Equation (35) to calculate either the final pressure P2 or volume V2, as follows.

```
else
    if strcmp(x,'P2')
        V2 = PV;
    else
        P2 = PV;
    end
    switch x
        case 'P2'
            P2 = P1/(k*log(V2/V1));
        case 'V2'
            V2 = V1*(P1/P2)^(1/k);
    end
end
y = [P1,P2,V1,V2];
end
```

**Heat Rate Function (Heat.m)**

First, this function automatizes Equation (29), used to calculate the heat rate in any thermodynamic process. The code starts with an if loop which helps with a basic validation for the input value. This function uses a string input so this validation will make sure that the string asks only for calculation of the heat rate "`Qdot`", flow rate "`Vdot`", outlet temperature "`To`", inlet temperature "`Ti`", or specific work "`Wdot`", thus,

```
function y = Heat(x,Ro,Cp)
if
~strcmp(x,'Qdot')&&~strcmp(x,'Vdot')&&~strcmp(x,'To')&&~strcmp(x,
'Ti')&&~strcmp(x,'Wdot')
    error('Input must be Qdot, Vdot, To, Ti, or Wdot.')
```

After the validation, the code uses a `switch` loop to choose between cases, that are the five parameters to be calculated. Each one of these five cases will ask for consistent inputs depending on the case that is initially selected.

```
else
    switch x
        case 'Qdot'
            flow = 'Flow rate? (Insert value in SCFM): ';
            Tout = 'Temperature out? (Insert value in rankine):';
            Tin = 'Temperature in? (Insert value in rankine): ';
            Vdot = input(flow);
            To = input(Tout);
            Ti = input(Tin);
            dT = To-Ti;
            Qdot = Ro*Vdot*Cp*dT*60;
            Wdot = Cp*dT;
        case 'Vdot'
            heat = 'Heat rate? (Insert value in BTU.hr^-1): ';
            Tout = 'Temperature out? (Insert value in rankine):';
            Tin = 'Temperature in? (Insert value in rankine): ';
            Qdot = input(heat);
            To = input(Tout);
            Ti = input(Tin);
            dT = To-Ti;
            Vdot = Qdot/(Ro*Cp*dT*60);
            Wdot = Cp*dT;
        case 'To'
            heat = 'Heat rate? (Insert value in BTU.hr^-1): ';
            flow = 'Flow rate? (Insert value in SCFM): ';
            Tin = 'Temperature in? (Insert value in rankine): ';
            Qdot = input(heat);
            Vdot = input(flow);
            Ti = input(Tin);
            To = Ti + Qdot/(Ro*Cp*Vdot*60);
            dT = To-Ti;
            Wdot = Cp*dT;
        case 'Ti'
```

```
                heat = 'Heat rate? (Insert value in BTU.hr^-1): ';
                flow = 'Flow rate? (Insert value in SCFM): ';
                Tout = 'Temperature out? (Insert value in rankine):';
                Qdot = input(heat);
                Vdot = input(flow);
                To = input(Tout);
                Ti = To - Qdot/(Ro*Cp*Vdot*60);
                dT = To-Ti;
                Wdot = Cp*dT;
            case 'Wdot'
                Tout = 'Temperature out? (Insert value in rankine):';
                Tin = 'Temperature in? (Insert value in rankine): ';
                Qdot = 0;
                Vdot = 0;
                To = input(Tout);
                Ti = input(Tin);
                dT = To-Ti;
                Wdot = Cp*dT;
        end
```

Finally, from each of these five cases, five values will be earned. In order to be able to manage those results effectively, those answers have to be stored in a vector `y`, thus when this function is called into a main program, the right parameter from the vector should be carefully chosen for our calculation.

```
end
y = [Qdot,Wdot,Vdot,To,Ti,dT];
end
```

**Humidity Ratio (Hum.m)**

First, this function automatizes Equation (16), used to calculate the humidity ratio in any compression process. The code starts with an `if` loop which helps with a basic validation for the input value. This function uses a string input so this validation will make sure that the string asks only for calculation of the humidity ratio `w`, saturation pressure `Pvs`, pressure `P`, or relative humidity `Phi`, thus,

```
function y = Hum(x)
```

```
if
~strcmp(x,'w')&&~strcmp(x,'Pvs')&&~strcmp(x,'P')&&~strcmp(x,'phi'
)
    error('Input must be w, Pvs, P, or Phi.')
```

After the validation, the code uses a `switch` loop to choose between cases, that are the four parameters to be calculated. Each one of these four cases will ask for consistent inputs depending on the case that is initially selected.

```
else
    switch x
        case 'w'
            relh = 'Relative humidity? (Insert value in %): ';
            pres = 'Presure? (Insert value in psia): ';
            satp = 'Saturation Pressure?(Insert value in psia):';
            phi = input(relh);
            P = input(pres);
            Pvs = input(satp);
            w = 0.622*phi*Pvs/(P-phi*Pvs);
        case 'Pvs'
            hrat = 'Humidity ratio';
            pres = 'Presure? (Insert value in psia): ';
            relh = 'Relative humidity? (Insert value in %): ';
            w = input(hrat);
            P = input(pres);
            phi = input(relh);
            Pvs = w*P/(phi*(0.622+w));
        case 'P'
            hrat = 'Humidity ratio';
            relh = 'Relative humidity? (Insert value in %): ';
            satp = 'Saturation Pressure?(Insert value in psia):';
            w = input(hrat);
            phi = input(relh);
            Pvs = input(satp);
            P = (0.622+w)*phi*Pvs/w;
        case 'phi'
            hrat = 'Humidity ratio';
            pres = 'Presure? (Insert value in psia): ';
            satp = 'Saturation Pressure?(Insert value in psia):';
            w = input(hrat);
            P = input(pres);
            Pvs = input(satp);
            phi = w*P/(Pvs*(0.622+w));
    end
```

Finally, from each of these four cases, four values will be earned. In order to be able to manage those results effectively, those answers have to be stored in a vector `y`, thus when this function is called into a main program, the right parameter from the vector should be carefully chosen for our calculation.

```
end
y = [w,Pvs,P,phi];
end
```

# APPENDIX B

# CHAPTER III MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start by defining some constants that are useful for further calculations.

```
%Constants used for calculations
Ru = 1545; % units lbf.ft.(lbmol.R)^-1
Ma = 28.97; % units lbm.lbmol^-1
Ro = 0.075; % units lbm.ft^-3
R = Ru/Ma;
```

## Example Problem III.a: Air Properties

The code for the example problem is as simple as just printing information regarding the given problem, and calling the function `PVTRo` specifying the density as the unknown, to calculate the specific volume of the dry air in analysis. After this, print the inverse of the fifth element of the answers vector, which corresponds to the density.

```
%Example Problem a
disp('Air properties')
disp('Solve for the Specific Volume of dry air at standard
conditions.')
pre = 'Pressure? (Insert value in psia): ';
relh = 'Relative humidity? (Insert value in %): ';
pres = 'Pressure of saturation? (Insert value in psia): ';
P = input(pre);
phi = input(relh);
Pvs = input(pres);
Pa = P-phi*Pvs;
disp(['The pressure of dry air is ',num2str(vs),' psia.'])
s1 = PVTRo('Ro');
vs = 1/s1(5);
disp(['The specific volume is ',num2str(vs),' ft^3.lbm^-1.'])
```

In order to provide a plot for above calculation, the remaining values from the answers vector should be taken from the function PVTRo,

```
%Plots
T = s1(3);
m = 20; %Chose the mass amount as desired.
```

Now, define three plots based on the obtained answer from the above.

```
%Ideal gas equation.
T1 = T*.75; % units rankine
T2 = T; % units rankine
T3 = T*1.25; % units rankine
```

Then, define the domain of the plots, and proceed to define the plots, as well as their labels and information that will be shown.

```
% Define the array of volume in ft^3
V = 0 : .01*5 : vs*m;
% Calculate the pressure disribution
P1 = m*R*T1./(V*144);
P2 = m*R*T2./(V*144);
P3 = m*R*T3./(V*144);
% Plot the three graphs on the one axis
figure
plot(V, P1, 'r-', 'LineWidth', 2)
hold
plot(V,P2, 'g-', 'LineWidth', 2)
plot(V,P3, 'b-', 'LineWidth', 2)
grid on;
legend([num2str(T1),' rankine.'],[num2str(T2),'
rankine.'],[num2str(T3),' rankine.']);
xlabel('V', 'FontSize', 20);
ylabel('P', 'FontSize', 20);
title('Pressure vs. Temperature','FontSize', 20);
```

## APPENDIX C

## CHAPTER IV MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start defining some constants that are useful for further calculations.

```
%Constants used for calculations
Cpa = 0.24; % units BTU.(lbm.R)^-1)
Cpw = 1.00; % units BTU.(lbm.R)^-1)
Roa = 0.075; % units lbm.ft^-3
Row = 62.40; % units lbm.ft^-3
Pstd = 14.7; % units psia
Tstd = 68; % units fahrenheit
Ru = 1545; % units lbf.ft.(lbmol.R)^-1
Ma = 28.97; % units lbm.lbmol^-1
R = Ru/Ma;
```

**Example Problem IV.a: Exit Temperatures**

The code for the first example problem is as simple as just printing information regarding the given problem, and calling the function `TTPP` specifying the outlet temperature as the unknown, and the values for the outlet pressure, inlet temperature, inlet pressure, and the polytropic constant. After this, print the outlet temperature for each one of the values of the polytropic constant provided.

```
% Example Problem a
disp('Exit Temperatures')
disp('Solve for the exit temperature, if k is 1, 1.3, and 1.4.')
ipre = 'Inlet Pressure? (Insert value in psia): ';
opre = 'Outlet Pressure? (Insert value in psia): ';
itemp = 'Inlet Temperature? (Insert value in fahrenheit): ';
P1 = input(ipre);
P2 = input(opre);
T1 = input(itemp)+459.67;
```

```
k = [1,1.3,1.4];
y1 = TTPP('T2',P2,P1,T1,k(1));
y2 = TTPP('T2',P2,P1,T1,k(2));
y3 = TTPP('T2',P2,P1,T1,k(3));
disp(['The outlet temperature for k = ',num2str(k(1)),' is
',num2str(y1(4)-459.67),' fahrenheit.'])
disp(['The outlet temperature for k = ',num2str(k(2)),' is
',num2str(y2(4)-459.67),' fahrenheit.'])
disp(['The outlet temperature for k = ',num2str(k(3)),' is
',num2str(y3(4)-459.67),' fahrenheit.'])
```

**Example Problem IV.b: Specific Work and Efficiency**

The code for the second example problem is as just printing information regarding the given problem, and calling the function `Heat` specifying the specific work as the unknown, and the values for the density of dry air `Roa`, and the specific heat at constant pressure `Cpa`.

```
%Example Problem b
disp('Specific Work and Efficiency')
disp('Solve for the actual specific work, exit temperature, and
heat loss.')
nise = 'Isentropic Efficiency? (Insert value in %): ';
niso = 'Isothermal Efficiency? (Insert value in %): ';
ipre = 'Inlet Pressure? (Insert value in psia): ';
opre = 'Outlet Pressure? (Insert value in psia): ';
P1 = input(ipre);
P2 = input(opre);
Nis = input(nise)/100;
Nit = input(niso)/100;
disp('For adiabatic process:')
y5 = Heat('Wdot',Roa,Cpa);
Wid = y5(2);
```

In addition, calculate all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
disp(['The ideal specific work is ',num2str(Wid),' BTU.lbm^-1.'])
Wacis = Wid/Nis;
disp(['The actual specific work is ',num2str(Wacis),' BTU.lbm^-
1.'])
Toid = y5(4);
Tiid = y5(5);
dTid = y5(6);
Toac = Tiid+dTid/Nis;
```

```
disp(['The exit temperature is ',num2str(Toac),' fahrenheit.'])
Qdotid = Wacis - Wid;
disp(['The heat loss is ',num2str(Qdotid),' BTU.lbm^-1.'])
disp('For isothermal process:')
Tit = Tiid;
Wit = R*Tit*log(P2/P1);
disp(['The isothermal specific work is ',num2str(Wit),' BTU.lbm^-
1.'])
Wacit = Wit/Nit;
disp(['The actual isothermal specific work is ',num2str(Wacit),'
BTU.lbm^-1.'])
Qdotit = Wacit - Wit;
disp(['The isothermal heat loss is ',num2str(Qdotit),' BTU.lbm^-
1.'])
Niti = Wit/Wid;
disp(['The isothermal efficiency is ',num2str(Niti*100),'%.'])
```

**Example Problem IV.c: Intermediate Pressure and Total Specific Work**

Here is the code for the third example problem. It is necessary to print information regarding the given problem. Then, call the function `TTPP` specifying the outlet temperature as the unknown, and the values for the outlet pressure, inlet temperature, inlet pressure, and the polytropic constant. Also, call the function `Heat` specifying the specific work as the unknown, and the values for the density of dry air `Roa`, and the specific heat at constant pressure `Cpa`, as many times as needed. In addition, calculate all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
%Example Problem c
disp('Intermediate Pressure and Total Specific Work')
disp('Solve for the intermediate pressure, and the actual exit
temperature of each stage of compression.')
nise = 'Isentropic Efficiency? (Insert value in %): ';
ipre = 'Inlet Pressure P1? (Insert value in psia): ';
opre = 'Outlet Pressure P4? (Insert value in psia): ';
itemp = 'Inlet Temperature T1? (Insert value in fahrenheit): ';
initemp = 'Intermediate Inlet Temperature T3? (Insert value in
fahrenheit): ';
Nis = input(nise)/100;
P1 = input(ipre);
```

```
P4 = input(opre);
T1 = input(itemp)+459.67;
T3 = input(initemp)+459.67;
PI = (P1*P4)^.5;
disp(['The intermediate pressure is ',num2str(PI),' psia.'])
y5 = TTPP('T2',PI,P1,T1,k(3));
T2id = y5(4);
disp(['The ideal exit temperature at 2 is ',num2str(T2id),'
fahrenheit.'])
T2ac = T1 + (T2id-T1)/Nis;
disp(['The actual exit temperature at 2 is ',num2str(T2ac),'
fahrenheit.'])
y6 = TTPP('T2',P4,PI,T3,k(3));
T4id = y6(4);
disp(['The ideal exit temperature at 4 is ',num2str(T4id),'
fahrenheit.'])
T4ac = T3 + (T4id-T3)/Nis;
disp(['The actual exit temperature at 4 is ',num2str(T4ac),'
fahrenheit.'])
y7 = Heat('Wdot',Roa,Cpa);
W1ac = y7(2);
disp(['The total actual specific work at 1st stage is
',num2str(W1ac),' BTU.lbm^-1.'])
y8 = Heat('Wdot',Roa,Cpa);
W2ac = y8(2);
disp(['The total actual specific work at 2nd stage is
',num2str(W2ac),' BTU.lbm^-1.'])
Wtac = W1ac + W2ac;
disp(['The total actual specific work for the compressor is
',num2str(Wtac),' BTU.lbm^-1.'])
```

**Example Problem IV.d: Compressor Analysis**

Here is the code for the fourth example problem. It is necessary to print information regarding the given problem. Then call the function `TTPP` specifying the outlet temperature as the unknown, and the values for the outlet pressure, inlet temperature, inlet pressure, and the polytropic constant. Also, call the function `Hum` specifying the pressure of saturation as unknown. Finally call the function `Heat` specifying the specific work as the unknown, and the values for the density of dry air `Roa`, and the specific heat at constant pressure `Cpa`, as many times as needed. In addition, calculate all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
%Example Problem d
disp('Compressor Analysis')
disp('Solve for the total power and the work transfer rate.')
nise = 'Isentropic Efficiency? (Insert value in %): ';
ipre = 'Inlet Pressure P1? (Insert value in psia): ';
opre = 'Outlet Pressure P4? (Insert value in psia): ';
itemp = 'Inlet Temperature T1? (Insert value in fahrenheit): ';
Nis = input(nise)/100;
P1 = input(ipre);
P4 = input(opre);
T1 = input(itemp)+459.67;
PI = (P1*P4)^.5;
disp(['The intermediate pressure is ',num2str(PI),' psia.'])
y5 = TTPP('T2',PI,P1,T1,k(3));
T2id = y5(4);
disp(['The ideal exit temperature at 2 is ',num2str(T2id),'
fahrenheit.'])
T2ac = T1 + (T2id-T1)/Nis;
disp(['The actual exit temperature at 2 is ',num2str(T2ac),'
fahrenheit.'])
y9 = Hum('Pvs');
Pvs = y9(2);
initemp = 'Interpolated Intermediate Inlet Temperature T3?
(Insert value in fahrenheit): ';
T3 = input(initemp)+459.67;
y10 = TTPP('T2',P4,PI,T3,k(3));
T4id = y10(4);
disp(['The ideal exit temperature at 4 is ',num2str(T4id),'
fahrenheit.'])
T4ac = T3 + (T4id-T3)/Nis;
disp(['The actual exit temperature at 4 is ',num2str(T4ac),'
fahrenheit.'])
y11 = Heat('Wdot',Roa,Cpa);
W1ac = y11(2);
disp(['The total actual specific work at 1st stage is
',num2str(W1ac),' BTU.lbm^-1.'])
y12 = Heat('Wdot',Roa,Cpa);
W2ac = y12(2);
disp(['The total actual specific work at 2nd stage is
',num2str(W2ac),' BTU.lbm^-1.'])
Wtac = W1ac + W2ac;
disp(['The total actual specific work for the compressor is
',num2str(Wtac),' BTU.lbm^-1.'])
```

**Example Problem IV.e: Volumetric Efficiency and Clearance**

Here is the code for the fifth example problem. It is necessary to print information regarding the given problem. Then call several times the function VVPP specifying the outlet volume as the unknown, and the values for the outlet pressure, inlet volume, inlet pressure, and the polytropic constant, and calculate all the required data using the formulas defined before. Finally, present the results in the same order as the example problem previously defined.

```
%Example Problem e
disp('Volumetric Efficiency and Clearance')
disp('Solve for the piston volumes.')
dia = 'Piston Diameter? (Insert value in inches): ';
len = 'Piston Length? (Insert value in inches): ';
cl = 'Piston Diameter? (Insert value in inches): ';
ipre = 'Inlet Pressure P1? (Insert value in psia): ';
opre = 'Outlet Pressure P4? (Insert value in psia): ';
itemp = 'Inlet Temperature T1? (Insert value in fahrenheit): ';
D = input(dia);
L = input(len);
C = input(cl)/100;
P1 = input(ipre);
Pa1 = P1;
Pb1 = P1;
P4 = input(opre);
Pc2 = P4;
Pd2 = P4;
T1 = input(itemp)+459.67;
PI = (P1*P4)^.5;
Pc1 = PI;
Pd1 = PI;
Pa2 = PI;
Pb2 = PI;
disp(['The intermediate pressure is ',num2str(PI),' psia.'])
disp(['Pressure Pa1 = ',num2str(Pa1),' psia.'])
disp(['Pressure Pb1 = ',num2str(Pb1),' psia.'])
disp(['Pressure Pc1 = ',num2str(Pc1),' psia.'])
disp(['Pressure Pd1 = ',num2str(Pd1),' psia.'])
disp(['Pressure Pa2 = ',num2str(Pa2),' psia.'])
disp(['Pressure Pb2 = ',num2str(Pb2),' psia.'])
disp(['Pressure Pc2 = ',num2str(Pc2),' psia.'])
disp(['Pressure Pd2 = ',num2str(Pd2),' psia.'])
Vb1 = pi*D^2*L/4;
disp(['The volume at 1st compression at b is ',num2str(Vb1),'
in^3.'])
y13 = VVPP('V2',Pc1,Pb1,Vb1,k(3));
Vc1 = y13(4);
Vd1 = C*Vb1/(1+C(Pd1/Pa1)^(1/k(3)));
```

```
y14 = VVPP('V2',Pa1,Pd1,Vd1,k(3));
Va1 = y14(4);
Vb2 = Vc1-Vd1;
y15 = VVPP('V2',Pc2,Pb2,Vb2,k(3));
Vc2 = y15(4);
Vd2 = C*Vb2/(1+C(Pd2/Pa2)^(1/k(3)));
y16 = VVPP('V2',Pa2,Pd2,Vd2,k(3));
Va2 = y16(4);
disp(['Volume Va1 = ',num2str(Va1),' psia.'])
disp(['Volume Vb1 = ',num2str(Vb1),' psia.'])
disp(['Volume Vc1 = ',num2str(Vc1),' psia.'])
disp(['Volume Vd1 = ',num2str(Vd1),' psia.'])
disp(['Volume Va2 = ',num2str(Va2),' psia.'])
disp(['Volume Vb2 = ',num2str(Vb2),' psia.'])
disp(['Volume Vc2 = ',num2str(Vc2),' psia.'])
disp(['Volume Vd2 = ',num2str(Vd2),' psia.'])
Nv11 = (Vb1-Va1)/(Vb1-Vd1);
Nv12 = 1+C-C(Pc1/Pb1)^(1/k(3));
disp(['The Volumetric Efficiency for 1st stage - method 1 is
',num2str(Nv11*100),'%.'])
disp(['The Volumetric Efficiency for 1st stage - method 2 is
',num2str(Nv12*100),'%.'])
Nv21 = (Vb2-Va2)/(Vb2-Vd2);
Nv22 = 1+C-C(Pc2/Pb2)^(1/k(3));
disp(['The Volumetric Efficiency for 2st stage - method 1 is
',num2str(Nv21*100),'%.'])
disp(['The Volumetric Efficiency for 2st stage - method 2 is
',num2str(Nv22*100),'%.'])
```

# APPENDIX D

# CHAPTER V MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start defining some constants that are useful for further calculations.

```
%Constants used for calculations
Cpa = 0.24; % units BTU.(lbm.R)^-1)
Cpw = 1.00; % units BTU.(lbm.R)^-1)
Roa = 0.075; % units lbm.ft^-3
Row = 62.40; % units lbm.ft^-3
```

## Example Problem V.a: Intercooler Effectiveness

Then, the code for the example problem is as simple as just printing information regarding the given problem, calculating all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
%Example Problem a
disp('Intercooler Efectiveness')
disp('Solve for the maximum relative humidity.')
tempai = 'Initial temperature of air? (Insert value in
fahrenheit): ';
tempci = 'Initial temperature of coolant? (Insert value in
fahrenheit): ';
eff = 'Intercooler effectiveness? (Insert value in %): ';
Tai = input(tempai);
Tci = input(tempci);
ee = input(eff);
Tao = Tai-ee*(Tai-Tci);
disp(['The exit temperature of the air is ',num2str(Tao),'
fahrenheit.'])
phi = exp(17.6*(Tao-32)/(Tao+405)+17.6*(Tai-32)/(Tai+405));
```

```
disp(['The maximum allowable relative humidity is
',num2str(phi*100),'%.'])
```

**Example Problem V.b: Condensation Generated**

Here is the code for the fourth example problem It is necessary to print information regarding the given problem. Then calculate the dew temperature with the equations defined above, and call the function `Hum` specifying the pressure of saturation as unknown. Finally calculate all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
%Example Problem b
disp('Condensantion Generated')
disp('Solve for the mass flow rate of condensation generated.')
tempai = 'Initial temperature of air? (Insert value in
fahrenheit): ';
tempao = 'Exit temperature of air? (Insert value in fahrenheit):
';
tempci = 'Initial temperature of coolant? (Insert value in
fahrenheit): ';
tempco = 'Exit temperature of coolant? (Insert value in
fahrenheit): ';
relh = 'Relative humidity? (Insert value in %): ';
flow = 'Flow rate? (Insert value in SCFM): ';
Tai = input(tempai);
Tao = input(tempao);
Tci = input(tempci);
Tco = input(tempco);
phi = input(relh);
Vdot = input(flow)/60;
Tdew = (564+405*(17.6*(Tai-32)/(Tai+405)+log(phi)))/(17.6-
(17.6*(Tai-32)/(Tai+405)+log(phi)));
disp(['The dew point temperature is ',num2str(Tdew),'
fahrenheit.'])
mdotc = Row*Vot;
disp(['The coolant mass flow rate is ',num2str(mdotc),' lbm.sec^-
1.'])
mdota = Roa*Vot;
disp(['The air mass flow rate is ',num2str(mdota),' lbm.sec^-
1.'])
y1 = Hum('w');
w = y1(1);
disp(['The humidity ratio is ',num2str(w),'.'])
mdotw = (mdotc*Cpw*(Tco-Tci)-(1+w)*mdota*Cpa*(Tao-Tai))/(Lc-
Cpa*(Tao-Tai));
```

```
disp(['The mass flow rate of the condensation generated is
',num2str(mdotw),' lbm.sec^-1.'])
```

# APPENDIX E

# CHAPTER VI MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start defining some constants that are useful for further calculations.

```
%Constants used for calculations
Ru = 1545; % units lbf.ft.(lbmol.R)^-1
Ma = 28.97; % units lbm.lbmol^-1
Ro = 0.075; % units lbm.ft^-3
R = Ru/Ma;
```

**Example Problem VI.a: Receiver Charging**

Then, the code for the first example problem is as simple as just printing information regarding the given problem, and calling the function `Recv` specifying the time as the unknown, to calculate the time needed to fill the receiver tank. After this, print the results for the fifth element of the answers vector from `Recv`, which corresponds to time.

```
%Example Problem a
disp('Receiver charging')
disp('Calculate the time to charge a 200 gallon receiver')
s1 = Recv('t');
t1 = s1(5);
disp(['The time to fill the receiver is ',num2str(t1),' sec.'])
```

In order to provide a plot for above calculation get the remaining values from the answers vector from the function `Recv`,

```
%Charging plots
```

```
dP1 = s1(1);
V1 = s1(2);
T1 = s1(3);
Vdot1 = s1(4);
```

Now, define three plots based on the answer that was obtained above.

```
%Modified ideal gas equation for calculating the size of a
receiver.
Vdot11 = Vdot1*.75; % units SCFM
Vdot21 = Vdot1; % units SCFM
Vdot31 = Vdot1*1.25; % units SCFM
```

Then, define the domain of the plots and proceed to define the plots, as well as their labels and information that will be shown.

```
% Define the array of time in seconds
t = 0 : .01*5 : 20;
% Calculate the pressure disribution
dP11 = Ro*Vdot11*R*T1*t/(V1*144);
dP21 = Ro*Vdot21*R*T1*t/(V1*144);
dP31 = Ro*Vdot31*R*T1*t/(V1*144);
% Plot the three graphs on the one axis
figure
plot(t, dP11, 'r-', 'LineWidth', 2)
hold
plot(t,dP21, 'g-', 'LineWidth', 2)
plot(t,dP31, 'b-', 'LineWidth', 2)
grid on;
legend([num2str(Vdot11*60),' SCFM.'],[num2str(Vdot21*60),'
SCFM.'],[num2str(Vdot31*60),' SCFM.']);
xlabel('t', 'FontSize', 20);
ylabel('dP', 'FontSize', 20);
title('Pressure Change vs. Time','FontSize', 20);
```

**Example Problem VI.b: Receiver Discharging**

Then, the code for the second example problem is as simple as just printing information regarding the given problem, and calling the function `Recv` specifying the volume as the unknown, to calculate the volume of the receiver tank. After this, print the results for the second element of the answers vector from `Recv`, which corresponds to volume.

```
%Problem Calculation b
disp('Receiver discharging')
disp('Calculate the receiver size.')
s2 = Recv('V');
V2 = s2(2);
disp(['The volume of the receiver is ',num2str(V2),' gal.'])
```

Additionally, in order to calculate the initial and final mass inside the receiver, the function `PVTRo` is used, selecting the fourth element of the answers vector as the result.

```
disp('Receiver discharging')
disp('Calculate the initial and final mass inside the receiver.')
s3 = PVTRo('m');
m1 = s3(4);
disp(['The initial mass inside the receiver is ',num2str(m1),'
lbm.'])
s4 = PVTRo('m');
m2 = s4(4);
disp(['The final mass inside the receiver is',num2str(m2,'lbm.'])
```

In order to provide a plot for above calculation, get the remaining values from the answers vector from the function `Recv`,

```
%Discharging plots
dP2 = s2(1);
T2 = s2(3);
Vdot2 = s2(4);
```

Now, define three plots based on the answer that was obtained above.

```
%Modified ideal gas equation for calculating the size of a
receiver.
Vdot12 = Vdot2*.75; % units SCFM
Vdot22 = Vdot2; % units SCFM
Vdot32 = Vdot2*1.25; % units SCFM
```

Then, define the domain of the plots and proceed to define the plots, as well as their labels and information that will be shown.

```
% Define the array of time in seconds
t = 0 : .01*5 : 20;
% Calculate the pressure disribution
dP12 = dP2/144-Ro*Vdot12*R*T2*t/(V2*144);
dP22 = dP2/144-Ro*Vdot22*R*T2*t/(V2*144);
dP32 = dP2/144-Ro*Vdot32*R*T2*t/(V2*144);
```

```
% Plot the three graphs on the one axis
figure
plot(t, dP12, 'r-', 'LineWidth', 2)
hold
plot(t,dP22, 'g-', 'LineWidth', 2)
plot(t,dP32, 'b-', 'LineWidth', 2)
grid on;
legend([num2str(Vdot12*60),' SCFM.'],[num2str(Vdot22*60),'
SCFM.'],[num2str(Vdot32*60),' SCFM.']);
xlabel('t', 'FontSize', 20);
ylabel('dP', 'FontSize', 20);
title('Pressure Change vs. Time','FontSize', 20);
```

**Example Problem VI.c: Combination**

Then, the code for the third example problem is as simple as just printing information regarding the given problem, and calling the function `Recv` specifying the volume as the unknown, to calculate the volume of the receiver tank. After this, print the results for the second element of the answers vector from `Recv`, which corresponds to volume.

```
%Example Problem c
disp('Combination')
disp('Calculate the time to charge a 200 gallon receiver while
using it.')
s5 = Recv('t');
t2 = s5(5);
disp(['The time to fill the receiver is ',num2str(t2),' sec.'])
```

## APPENDIX F

## CHAPTER VII MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start defining some constants that are useful for further calculations.

```
%Constants used for calculations
Cpa = 0.24; % units BTU.(lbm.R)^-1)
Cpw = 1.00; % units BTU.(lbm.R)^-1)
Roa = 0.075; % units lbm.ft^-3
Row = 62.40; % units lbm.ft^-3
Pstd = 14.7; % units psia
```

### Example Problem VII.a: Friction Factor and Pressure Drop

Then, the code for the example problem is as simple as just printing information regarding the given problem, calling the function `Pgrad` to calculate the pressure gradient, the friction factor and the Reynolds number, and finally calculating all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
%Example Problem a
disp('Friction factor and pressure drop')
disp('Solve for the Reynolds number, the friction factor, and the
pressure gradient.')
pre = 'Pressure? (Insert value in psig): ';
dia = 'Diameter? (Insert value in inches): ';
len = 'Length? (Insert value in ft): ';
vel = 'Velocity? (Insert value in ft/sec): ';
temp = 'Temperature? (Insert value in fahrenheit): ';
P = input(pre)+14.7;
D = input(dia);
L = input(len);
```

```
ve = input(vel);
T = input(temp)+459.67;
Ro = P*Roa/Pstd;
disp(['The density is ',num2str(Ro),' lbm.ft^-3.'])
y1 = Pgrad(Ro,ve,D,T);
Pgrad = y1(1);
f = y1(2);
Ry = y1(3);
disp(['The Reynolds Number is ',num2str(Ry),'.'])
disp(['The friction factor is ',num2str(f),'.'])
disp(['The pressure gradient is ',num2str(Pgrad),' psia.ft^-1.'])
dP = Pgrad*L;
disp(['The pressure drop is ',num2str(dP),' psia.'])
```

## APPENDIX G

## CHAPTER VIII MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start defining some constants that are useful for further calculations.

```
%Constants used for calculations
Cpa = 0.24; % units BTU.(lbm.R)^-1)
Cpw = 1.00; % units BTU.(lbm.R)^-1)
Roa = 0.075; % units lbm.ft^-3
Row = 62.40; % units lbm.ft^-3
Pstd = 14.7; % units psia
Tstd = 68; % units fahrenheit
```

### Example Problem VIII.a: Pipe Diameter and Loss Coefficient

Then, the code for the example problem is as simple as just printing information regarding the given problem, calling the function `VAv` specifying the diameter of the pipe as the unknown, calling the function `Pgrad` to calculate the pressure gradient, the friction factor and the Reynolds number, and finally calculating all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
%Example Calculation a
disp('Pipe Diameter and Loss Coefficient')
disp('Solve for the pipe diameter.')
flt = 'Flow rate of the tool? (Insert value in ft^3.min^-1): ';
pret = 'Pressure of the tool? (Insert value in psig): ';
prem = 'Minimum of the tool? (Insert value in psig): ';
pres = 'Pressure of the system? (Insert value in psig): ';
vel = 'Velocity? (Insert value in ft/sec): ';
Vdt = input(flt);
```

```
Pt = input(pret)+14.7;
Ps = input(pres)+14.7;
Pm = input(prem)+14.7;
ve = input(vel);
Vds = Vdt*Pt/Ps;
y1 = VAv('D');
D = y1(2);
Ro = Ps*Roa/Pstd;
y2 = Pgrad(Ro,ve,D,T);
Pgrad = y2(1);
f = y2(2);
Ry = y2(3);
disp(['The Reynolds Number is ',num2str(Ry),'.'])
disp(['The friction factor is ',num2str(f),'.'])
disp(['The pressure gradient is ',num2str(Pgrad),' psia.ft^-1.'])
Leq = (Ps-Pm)/Pgrad;
disp(['The equivalent length is ',num2str(Leq),' ft.'])
Kl = f*Leq/D;
disp(['The loss coefficient is ',num2str(Kl),'.'])
```

# APPENDIX H

# CHAPTER IX MODEL

First, the codes that carry the example problems need to be defined. In the following lines, the method to call the functions from APPENDIX A, and to produce the plots in order to process the input data will be explained. Now, start defining some constants that are useful for further calculations.

```
%Constants used for calculations
Cpa = 0.24; % units BTU.(lbm.R)^-1)
Cpw = 1.00; % units BTU.(lbm.R)^-1)
Roa = 0.075; % units lbm.ft^-3
Row = 62.40; % units lbm.ft^-3
Pstd = 14.7; % units psia
Tstd = 68; % units fahrenheit
k = 1.4; % isentropic expansion
Ru = 1545; % units lbf.ft.(lbmol.R)^-1
Ma = 28.97; % units lbm.lbmol^-1
R = Ru/Ma;
```

## Example Problem IX.a: Sonic Velocity and Flow Rates

Then, the code for the example problem is as simple as just printing information regarding the given problem, calling the function `PVTRo` specifying the density as the unknown, calling the function `VAv` specifying the diameter of the pipe as the unknown, and finally calculating all the required data using the formulas defined before, and present the results in the same order as the example problem previously defined.

```
disp('Sonic Velocity and Flow Rates')
disp('Solve for the pressure and temperature at exit plane,sonic
velocity, volume and mass flow rates.')
preo = 'Pressure of the component? (Insert value in psig): ';
temo = 'Temperature of the component? (Insert value in
fahrenheit): ';
Po = input(preo)+14.7;
```

```
To = input(temo);
P = ((2/(k+1))^(k/(k-1)))*Po;
disp(['The Pressure at the exit plane is ',num2str(P),' psia.'])
T = (2/(k+1))*To;
disp(['The Temperature at the exit plane is ',num2str(T),'
fahrenheit.'])
Vs = (k*R*32.2*T)^.5*60;
disp(['The Sonic Velocity is ',num2str(Vs),' ft.min^-1.'])
y1 = PVTRo('Ro');
Ro = y1(5);
disp(['The Density of the leaked gas is ',num2str(Ro),' lbm.ft^-
3.'])
y2 = VAv('Vdot');
Vdot = y2(1)*60;
disp(['The Volume Flow Rate is ',num2str(Vdot),' SCFM.'])
Vdots = Vdot*Ro/Roa;
disp(['The Volume Flow Rate is ',num2str(Vdots),' SCFM.'])
mdot = Ro*Vdot;
disp(['The Mass Flow Rate is ',num2str(mdot),' lbm.min^-1.'])
```