# AN ANALYSIS ON THE CORRELATES OF NUCLEAR PROLIFERATION AND NUCLEAR ENERGY

A Thesis

by

MEYAPPAN SUBBAIAH

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

| | |
|---|---|
| Chair of Committee, | Sunil Chirayath |
| Committee Members, | Paul Nelson |
| | Matthew Fuhrmann |
| Head of Department, | Yassin Hassan |

August 2016

Major Subject: Nuclear Engineering

ABSTRACT

The study of various indicators of nuclear proliferation actions by states can identify the associated level of risk. This study expands upon previous proliferation risk work by investigating the number of Enrichment and Reprocessing facilities a state has based on various historical indicators. These indicators include: (a) Gross Domestic Product (GDP) Per Capita, (b) Nuclear Electricity Production, (c) Possession of Nuclear Weapons, (d) Superpower Alliance, (e) Technical Capabilities, (f) number of Rival ENR facilities, and (g) number of ENR facilities held by a trading partner. ENR facilities are a vital part of the nuclear fuel cycle, regardless of intent be it civilian electricity production or weapons production. The number of ENR facilities is important to measure, as this provides information regarding a state's urgency and reasoning for a weapons program.

Data, from *A Spatial Model of Nuclear Technology Diffusion* by M. Fuhrmann and B. Tkach, is utilized to develop a predictive model. This dataset includes state data from 1945-2010, for 56 countries that had at least one operational research reactor. From the aforementioned indicators, both the number of Rival ENR facilities and number of ENR facilities held by a trading partner accounted for spatial clustering of nuclear weapons programs. Spatial clustering provided the opportunity to capture the dynamic nature of proliferation.

Bayesian networks were used as the investigative tool for this study. These networks are directed acyclic graphs that provide the ability to represent conditional dependence relationships between sets of random variables. This provides the ability to use information about the state of a random variable to infer additional information about the other random variable. Bayesian networks allow for a more visual approach to developing joint distributions of all important variables that model a

system. In most cases, there is a plethora of data for Bayesian networks to be constructed from. It is possible to inform these networks through expert judgement. However, due to the limited data available for nuclear weapons history, expert judgments are also required to ensure model specification.

From this study, it was evident that Bayesian networks were an appropriate tool to capture the dynamics of a potential proliferation threat and the level of proliferation risk. However, due to the complexity behind nuclear weapons programs there is always an opportunity for future work. The results from this study compared favorably to the historical data from Fuhrmann and Tkach, with some potential for better prediction accuracy. Refined models, with a higher validation rate with respect to historical data, can be used as a policy tool. These refined models will have the capability to forecast.

ACKNOWLEDGEMENTS

I would like to thank my advisor Dr. Chirayath for his support and guidance through the course of this work. The combined support from my advisor and committee members (Dr. Nelson and Dr. Furhmann) has been invaluable to the completion of my thesis. I would also like to extend thanks to Dr. Fuhrmann and Dr. Tkach for generously providing me with data to complete this research.

Secondly, I would like to express gratitude to the following Pacific Northwest National Laboratory staff: Valerie Finch, Amanda White and Garill Coles, for guiding me through the completion of this work. Val's expertise was vital to conducting the expert elicitation as well as developing models that captured the subject matter appropriately.

## NOMENCLATURE

| | |
|---|---|
| AVLIS | Atomic Vapor Laser Isotope Separation |
| CPT | Conditional Probability Table |
| EM | Expectation - Maximization |
| EMIS | Electromagnetic Isotope Separation |
| ENR | Enrichment and Reprocessing Facilities |
| F&T | Fuhrmann and Tkach datset |
| FP | Fisson Products |
| GD | Gradient Descent |
| GDP | Gross Domestic Product |
| HEU | Highly Enriched Uranium |
| IAEA | International Atomic Energy Agency |
| LEU | Low Enriched Uranium |
| MLIS | Molecular Laser Isotope Separation |
| NFC | Nuclear Fuel Cycle |
| NPT | Treaty on the Non-Proliferation of Nuclear Weapons |
| PUREX | Plutonium Uranium Redox Extraction |
| SNM | Special Nuclear Material |
| SWU | Separative Work Unit |
| THOREX | Thorium Extraction |
| TRU | Transuranics |
| UREX | Uranium Extraction |

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

TABLE Page

# 1. INTRODUCTION

The advent of the nuclear age brought with it both peaceful and non-peaceful nuclear applications. The spread of nuclear weapons technology began in the early stages of World War II. Currently, nine states have nuclear weapons and multiple other states possess the capabilities to start a nuclear weapons program. Only five states are recognized as nuclear weapons states per the Nuclear Non-Proliferation Treaty (NPT); United States (1945), Russia (1949), United Kingdom (1952), France (1960), and China (1964). There are four other states that possess nuclear weapons: India (1974), Israel (N/A), North Korea (2006) and Pakistan (1998). [1] Additional states that pursued nuclear weapons programs include Iran, Libya, South Africa and Syria. Besides accounting for current nuclear weapons states, it is also important to recognize when states initiated a nuclear weapons program, even if the program has since failed or ceased. There are thirteen cases identifying decisions to initiate a nuclear weapons programs predating 1975. According to Meyer, a nuclear weapons program initiation is defined by an explicit governmental decision. [2] In this modern age, technical capabilities are continuously improving. As a result the threat of nuclear weapons proliferation is at an all time high. However, various security protocols such as the NPT and Comprehensive Nuclear Test Ban Treaty and organizations such as the International Atomic Energy Agency have curbed major potential threats.

"There are indications because of new inventions, that 10, 15, or 20 nations will have a nuclear capacity, including Red China, by the end of the Presidential office in 1964. This is extremely serious...I think the fate not only of our own civilization, but I think the fate of world and the future of the human race, is involved in preventing a nuclear war." [3] From President Kennedy's remarks above, it was (and is) obvious that nuclear weapons will always be of concern to states on the global stage. The spread of nuclear weapons signaled the importance to globally emphasize the need for nuclear security and non-proliferation. Currently, nuclear weapons proliferation is the focal point of security concerns. [4] Nuclear weapons proliferation can severely impact strategic planning and have security implications regionally and globally [5]. Thus, it has become increasingly important to study nuclear security and nuclear proliferation risk.

## 1.1 Nuclear Non-Proliferation

Nuclear opportunity and Nuclear willingness are key to nuclear weapons proliferation. [4] Nuclear opportunity represents a state's capability to develop nuclear weapons, while nuclear willingness represents a state's motivation. Motivation can determine whether a state poses a credible threat, while technical capability measures whether a state can accomplish its goals. Therefore, it must be noted that proliferation decisions stem from a combination of technical capabilities and motivation(s).

There are two primary schools of thought regarding nuclear weapons proliferation, realist and idealist philosophies. Realism relies on the hypothesis that states acquire nuclear weapons because their security demands it [6]. The realist point of view focuses on the fact "friends today may become enemies tomorrow" [7]. Thus, based on this ideology, the driving factor for development is the technical capability of a state [8]. President Kennedy used the realist ideology, when predicting that 15-20 countries would have nuclear weapons by 1975 [9]. However, historical examples suggest that the realist ideology is a poor representation of proliferation decisions. President Kennedy's quote is a prime example of how this ideology over-predicts proliferation decisions. The failure of realism to explain proliferation, led to further thought being placed on idealism [10].

Idealism relies on the belief that proliferation decisions are dependent on multiple factors and not just dependent on security alone. Idealism represents the demand side of proliferation, while realism represents the supply side of proliferation [10] [6]. Idealism can be applied to three different levels: international, domestic and the individual level. Each level can impact proliferation decisions differently, thus highlighting how complex the proliferation decision process can be. These levels can also be used to assess whether a state is more likely to proliferate. Idealists stress the importance of meeting international standards by highlighting the importance of non-proliferation treaties. Idealism suggests that states should be punished if they do not abide by international norms, to prevent proliferation. However, there have been historical cases where such logic has backfired. It is also true that societies that promote openness are more likely to reject the idea of a weapons program [11]. While secretive societies are more likely to pursue a weapons program [7]. The individual level looks into trends of the leader of a state. Charles de Gaulle is an example of how individuals dictate proliferation decisions. Gaulle was very keen on developing a sense of independence and ensuring French sovereignty, based on the result of previous wars. [12] Therefore, Charles de Gaulle is one of a handful of leaders that highlight the importance of recognizing the individual level when considering non-proliferation theories.

It is also important to recognize the different types of proliferation, vertical and horizontal. Horizontal proliferation refers to non-nuclear states and their attempt to acquire nuclear weapons. Whereas vertical proliferation refers to nuclear weapons states attempting to increase their stockpiles and capabilities. The current status of the nuclear regime dictates that horizontal proliferation is of a higher concern than vertical proliferation. However, with the cold war serving as a prime example of vertical proliferation it is evident that both types hold equal importance in global security [13]. The majority of nuclear proliferation research aims to study the determinants of horizontal nuclear proliferation.

There have been varying arguments reasoning why horizontal proliferation is of higher importance. Some have pointed to the near absence of major war in the nuclear era as evidence that nuclear weapons proliferation is beneficial [14]. Authors like Mueller argue that nuclear weapons and the absence of major war is merely coincidental [15]. The arguments made by these authors sheds light on their stance between realism and idealism. Unfortunately, nuclear weapons proliferation is quite fickle, and requires individual case studies to understand the complexity behind proliferation decisions.

The chances of nuclear war should increase, with the increase of horizontal proliferation. By definition at least one state in a war should use nuclear weapon(s) for it to be deemed a "nuclear war". However the consequences of nuclear weapons may cause states to be less likely to engage in war. This would indirectly promote horizontal proliferation, directly asserting the concept of deterrence theory. It is interesting to note that an increase in nuclear stockpiles could deter other nations from proliferation.

## 1.2 Objectives and Motivations

The overall goal of this study is to estimate the number of Enrichment and Reprocessing (ENR) facilities a state has. This goal will be achieved through the following objectives:

1. Develop Bayesian network(s) that estimate the number of ENR facilities a state has at a specific point in time based on input parameters.

    (a) This network should reproduce historical examples and incorporate the potential to forecast.

(b) Develop multiple networks, by using different learning methods.

(c) Validate network(s) for the following historical examples:

i. Brazil,

ii. India,

iii. South Africa, and

iv. Sweden.

2. Conduct an expert elicitation to better understand the role of different indicators in the development of a state's ENR facilities.

3. Conduct a sensitivity analysis on the network(s) developed.

(a) Identify factors towards nuclear weapons proliferation and ensure they are modeled by the represented nodes.

(b) Identify whether certain factors in nuclear weapons proliferation are dependent on one another. Ensure that dependence is shown in the network.

(c) Determine the predicted effect of each node(s) on the dependent variable.

(d) Smooth data by merging yearly data into sets of $x$ (number) years. Determine the appropriate number of years to subset.

(e) Estimate the uncertainty in the number of ENR facilities' by varying the historical ENR facilities data set.

Each objective is key in achieving the overall goal. The first two objectives aim to develop the predictive nature of the model. The third objective studies sensitivities in the model, enabling the option for future use. These developed models can be further refined to forecast ENR developments, highlighting its use as a policy tool.

## 1.3 Previous Work

Previous work has been conducted on nuclear weapons proliferation determination. Before discussing previous work, it is important to recognize potential pitfalls when predicting proliferation. According to Montgomery and Sagan [16], there are five serious problems. First and foremost, there is ambiguity surrounding initiation and completion dates for nuclear programs. Next, methodologies and data sets are generally chosen for convenience instead of there relation to empirical questions asked in the study. Independent variables chosen for proliferation studies overlook important factors such as prestige, and bureaucracies. Additionally, findings can ignore data that is crucial to policy making. [16] A thorough review of the literature was used to provide background information whilst considering potential pitfalls.

Work done by Corey Freeman and Mike Mella (at Texas A&M) involved computational networks to assess proliferation determination [5] [10]. Freeman developed the original Bayesian network to test the following hypothesis, a state's motivations directly affect intention's, which in turn led to the proliferation pathways chosen. The Freeman network had flexibility as it tested for both states and non-state actors. Freeman's work found that motivations determined pathways, while capabilities affected the success rate. This work established the relative threat an adversary can pose [10]. Mella expanded upon the Freeman's network in a couple of ways. The network was refined to yield the most likely path a state would pursue in developing nuclear weapons. Additionally, the networks also included dual-use export controlled technologies to better assess state pathways [5]. This expanded network was tested for various historical examples. Both networks were successful in predicting proliferation pathways based on a number of different factors. However, both Freeman and Mella made multiple suggestions to improve upon their work respectively [5] [10].

Some suggestions include: conducting expert elicitations, including de-motivating factors, and testing more historical cases. A plethora of nuclear weapons proliferation studies provide insight on potential motivators, but do not seem to capture de-motivating factors [10]. The notion for recommending expert elicitations was to improve upon weighting factors used by Bayesian network software [5].

Other proliferation risk work utilized logistic models to asses both the nuclear opportunity and nuclear willingness factors. Papers in the *Journal of Conflict Resolution*, discuss potential indicators for nuclear proliferation [4]. Jo and Gartzke take a similar approach to the Singh and Way article, but two dependent variables are evaluated. The Jo and Gartzke study identifies whether a state has a program, and a weapon. Additionally, both the Jo and Gartzke and Singh and Way studies avoid defining classifying nodes but instead determine the dependent variables as a function of all listed independent variables. Finally, the Jo and Gartzke study continues to study the independent variables in groups based on its class (technical capability or motivation) [17].

# 2. NUCLEAR FUEL CYCLE

A review of the Nuclear Fuel Cycle (NFC) is made in this chapter for familiarization with the ENR facility definition used for this study. This study associates the development of ENR facilities with a states proliferation status. ENR facilities for this study are deemed to be sensitive technologies and correlates of nuclear weapons proliferation. Some states develop weapons capability, but avoid proliferating unless motivated to do so, more commonly known as *nuclear hedging* [18].

Special Nuclear Material (SNM) and source material of interest for proliferation include plutonium, uranium, and thorium. The NFC is the path followed by nuclear material during its use through a system of interconnected nuclear facilities. It starts with mining of ore and concludes with disposal of waste. All stages of the NFC can be seen in Figure 2.1. Additionally, it is important to note that there are two types of cycles, open and closed [19]. If spent fuel is not reprocessed then the cycle is classified as an open fuel cycle, whereas if spent fuel is recycled or reprocessed then it is classified as a closed fuel cycle.

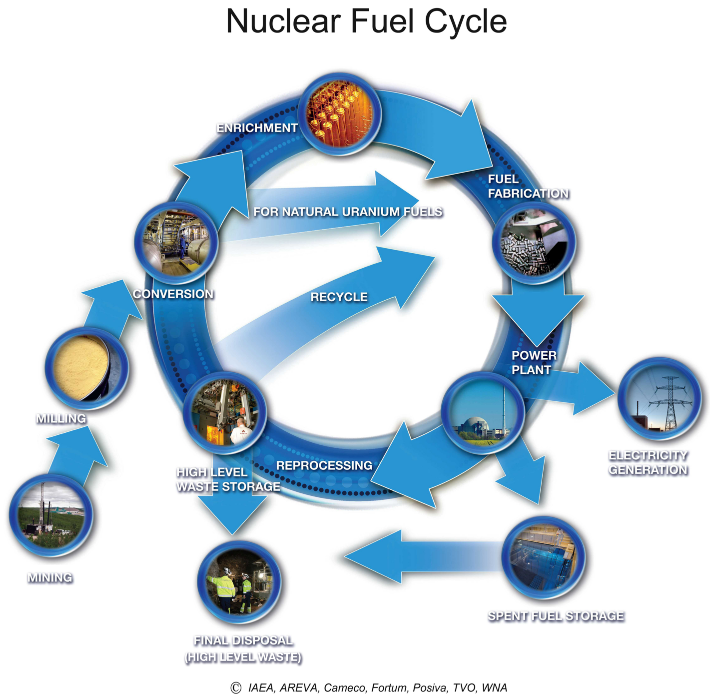


Figure 2.1: Stages of the Nuclear Fuel Cycle. Reprinted from [20]

The front-end of the NFC consists of five stages prior to fuel entering the reactor. The first stage is to mine and mill uranium and thorium from the ground. There are four different methods to mine: open-pit mining, underground mining, in-situ leaching, and by-product mining. Following the mining, material will be milled to extract uranium from mined ore. Generally speaking, mining and milling are considered one stage, since most facilities have the capability to extract and mill material. The resulting material, concentrated in $U_3O_8$ (yellowcake), is shipped to a conversion facility. Conversion facilities convert $U_3O_8$ to $UF_6$ gas, which is then shipped to a uranium enrichment facility.

Traditional nuclear fuel requires uranium that is slightly enriched in $^{235}U$. Natural uranium contains 99.28% of $^{238}U$, 0.711% of $^{235}U$, and 0.006% of $^{234}U$. Nuclear fuel used in light water reactors requires uranium to have a concentration varying between 2% to 5% of $^{235}U$. Enrichment process uses different isotope separation techniques to increases the concentration of $^{235}U$ relative to $^{238}U$. The most commonly used enrichment technologies are gaseous diffusion, gas centrifuge, and to a limited extent Atomic Vapor Laser Isotope Separation (AVLIS). Some other enrichment technologies include aerodynamic vortex tube, aerodynamic separation nozzle, chemical exchange, ion exchange, laser molecular separation, and electromagnetic isotope separation. Once the uranium is enriched, it is passed to a fuel fabrication facility. Fuel fabrication facilities take the enriched uranium and develop fuel assemblies. A fuel assembly is a column of ceramic fuel pellets made of uranium oxide or mixed-oxide and clad. Each assembly is sealed with zirconium alloy. Fuel fabrication marks the last stage of the front end of the fuel cycle.

The back-end of the NFC consists of all stages after fuel leaves the reactor (transmuter). Depending on the type of cycle (open or closed), the number of stages in the back end of the NFC can vary. In an open cycle, fuel from the reactor is stored in a spent fuel pool. After which, the fuel is taken from the spent fuel pool and stored in dry casks. These dry casks are sent to high level waste management facilities. In a closed cycle, the first stage in the back-end is fuel reprocessing. Reprocessing recovers uranium and plutonium isotopes from the used fuel. The second stage involves either uranium or plutonium conversion. Finally, the material is sent to either enrichment or fuel fabrication facilities, concluding the fuel cycle.

## 2.1 Weapons Development in Nuclear Fuel Cycle

The two most commonly known types of nuclear fission weapons are gun-type and implosion type weapons. Both of these require a mass of fissile material (enriched uranium or plutonium). This mass is assembled in such a way that it can start an uncontrolled nuclear chain reaction (supercritical mass). In gun-type weapons, a piece of sub-critical material is shot into another to initiate this reaction. Whereas, implosion type weapons compress the sub-critical material through the use of lenses, a spherical shell of high explosives. Let it be noted that gun-type weapons can use highly enriched, around 90% or higher, $^{235}U$ or $^{239}Pu$; implosion-type weapons can use either $^{239}Pu$ or $^{235}U$. Note that uranium based implosion-type weapons are difficult to manufacture. Nuclear weapons can be made out of any SNM, highly enriched uranium (HEU), $^{233}U$, $^{239}Pu$, and $^{241}Pu$. Therefore, peaceful applications of nuclear energy can shorten breakout time, thus increasing a state's capability to proliferate [21]. Breakout time is defined as the amount of time required to produce enough weapons-grade material for one weapon.

While the aforementioned are the most common types of nuclear weapons, there are a number of other categories of nuclear weapons. States pursuing a program will normally aim to achieve nuclear capability in the most feasible way possible. It is easier to achieve such a goal with traditional nuclear weapons as there is a known track record for these types of weapons (gun and implosion type).

### *2.1.1 Enrichment*

It is important to study enrichment facilities, as nuclear weapons require a higher enrichment level than nuclear fuel (less than 5% for power reactors). Enrichment is the process of increasing the ratio of $^{235}U$ to that of the $^{238}U$ isotope. $^{235}U$ has a higher spontaneous fission rate than $^{238}U$, hence the need to enrich uranium. As such, the number and capacity of enrichment facilities are vital to nuclear proliferation.

The capacity of an enrichment facility is measured in Separative Work Unit (SWU). The number of SWUs is the quantity directly related to the resources required to enrich material to a desired level. The main resource required to enrich material is electrical energy for isotope separation. Thus, SWUs are directly proportional to the energy consumed to enrich material.

This study accounts for all types of enrichment technologies used in pilot, laboratory or commercial scale plants. However, this study will focus on two of the most common enrichment technologies in gaseous diffusion and centrifuge technology.

Gaseous diffusion relies on the difference in molecular effusion rates of $^{235}UF_6$ and $^{238}UF_6$ through a thin barrier containing millions of pores. As a result, when the $UF_6$ molecules are kept at the same temperature, a kinetic energy comparison shows that $^{235}UF_6$ molecules are faster than $^{238}UF_6$ molecules. Thus, the separation of molecules is a result of the relative frequency with which molecules pass through a small hole, leaving material slightly enriched in $^{235}UF_6$. It is important to note that these plants consume about 2,300 - 3,000 kW-hr per SWU produced. Some operational concerns for gaseous diffusion plants are criticality issues, $UF_6$ leaks, and plugging of the diffusion barriers by solids [22].

Worldwide, roughly about 80% of enrichment is done using centrifuge technology. Centrifuges are more common due to a variety of factors that include: feasibility, an ease to build, and simple operation techniques. Centrifuge technology relies around the use of a rotating drum or cylinder, where the centrifugal force compresses heavier $^{238}UF_6$ gas molecules to the outer cylinder.

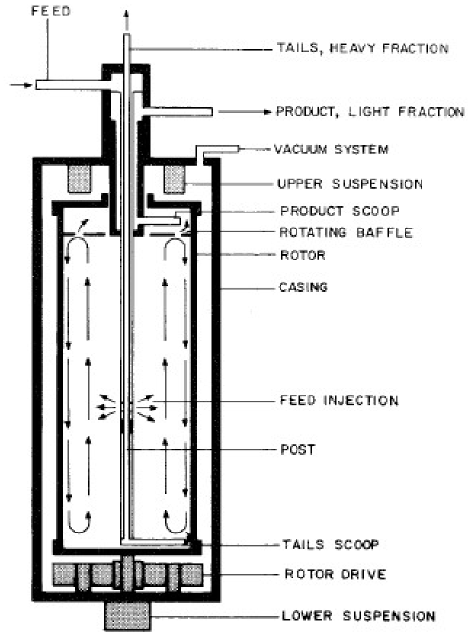


Figure 2.2: Counter-current Centrifuge Technology. Reprinted with permission from [23]

An example of a counter-current gas centrifuge can be seen in Figure 2.2. The $UF_6$ gas rotating inside the cylinder is subject to acceleration much greater than gravity. As a result, pressure at the outer radius is much greater than pressure at the center, causing a higher relative abundance of the heavier isotope to be pushed around the outer radius.

Some other advanced technologies include: AVLIS, Molecular Laser Isotope Separation (MLIS), and Electromagnetic Isotope Separation (EMIS).

### *2.1.2 Reprocessing*

Additionally, it is important to study used nuclear fuel reprocessing facilities because of the non-proliferation concerns associated with the plutonium products. Each one of the reprocessing methods has its own proliferation concern. The following three reprocessing methods are quite common, Plutonium Uranium Redox Extraction (PUREX), Uranium Extraction (UREX), and Thorium Extraction (THOREX).

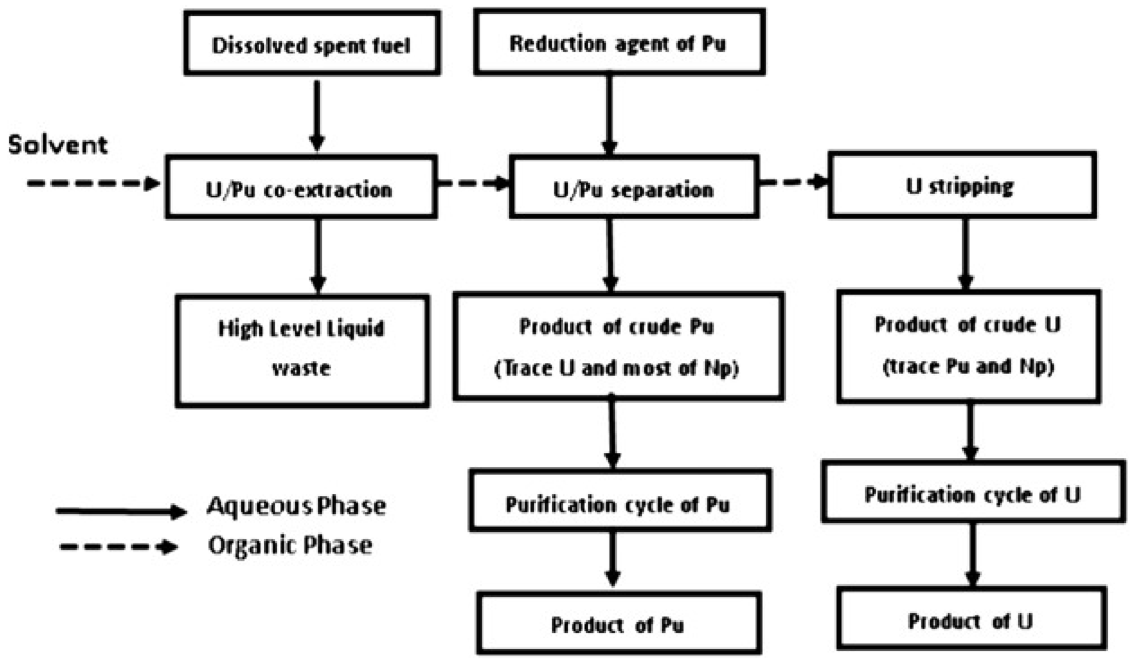


Figure 2.3: PUREX Process Reprinted with permission from [24]

First let us examine the PUREX process. As seen in Figure 2.3, this process produces two separate streams of material (U and Pu). Organizations like the IAEA use in-depth study of these reprocessing methods to appropriately implement safeguards, attempting to avoid weapons proliferation or diversion of material.

Initially, the used fuel is prepared for dissolution by separating the fuel matrix from the cladding. After which, the fuel is dissolved into an aqueous solution (fuel dissolution). The next step is to prepare dissolved feeds by adjusting valence and acidity for maximum separation. The next step is to remove decontaminants such as Fission Products (FP) and Transuranics (TRU). The plutonium is separated from the uranium, thus developing two separate process streams. Following this, both the U and Pu are purified in their respective streams. Pu production in this process makes it a proliferation concern. Potential contamination is the only setback the Pu stream faces. Otherwise, the Pu produced from the stream is weapons usable, specifically when the used fuel is discharged at a very low burn-up.

The UREX process recovers 99.9% of Uranium and 95% Technetium from the spent fuel. Both of these materials are recored in separate product streams. The UREX process uses similar tools to the PUREX process, but doesn't recover pure plutonium. This difference occurs through a modification in the front end of the process.

The THOREX process can produce either Pu or U depending on the core configuration used. The THOREX process is extremely similar to the PUREX process; two product streams (Pu & U vs. Th & U). Details of the THOREX process is shown in Figure 2.4 The general thorium fuel cycle requires that fast reactors use a seed-blanket configuration, as seen in Figure 2.5.

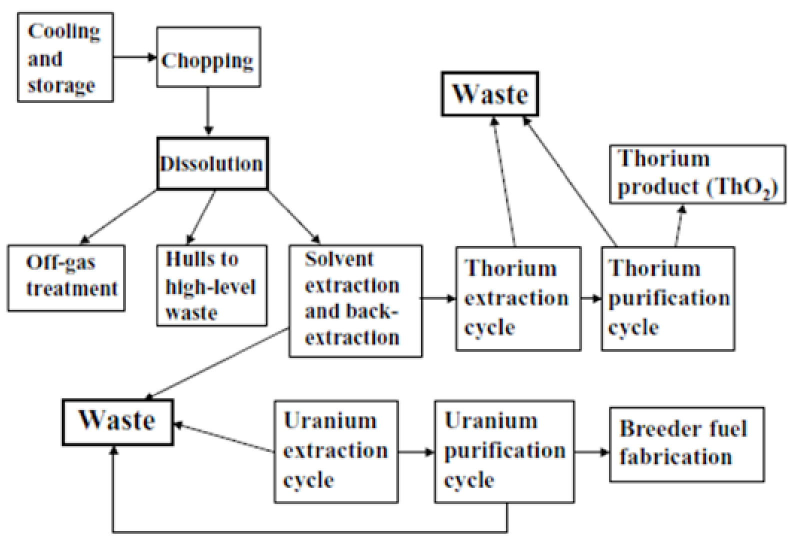


Figure 2.4: THOREX Process Reprinted with permission from [25]

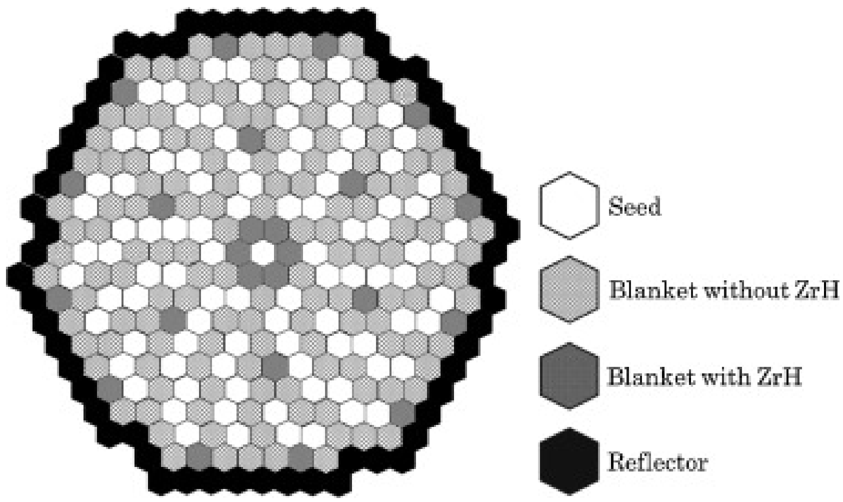


Figure 2.5: Seed-Blanket Core Reprinted with permission from [26]

Based on the seed-blanket configuration, the reactor can produce U, Pu or Th. The traditional fast reactor configuration can be seen in Table 2.1. When the blanket region comprises of a mixed oxide fuel containing $ThO_2$ and $UO_2$ (Scenario #3 in Table 2.1), regardless of the seed material the fuel will transmute into the following $^{233}U$, $^{235}U$ and $^{239}Pu$. The separation of $^{239}Pu$ from the uranium is a rather straightforward process, however proliferators will run into issues when attempting to separate $^{233}U$ and $^{235}U$ as this requires isotope separation and not element separation. The preferential scenario is to use a blanket region that comprises of $^{233}UO_2$ and $ThO_2$ (India's Goal in Table 2.1). In this case, the fuel will transmute and form an abundance of $^{233}U$.

Table 2.1: Fast Reactor Core Configurations

| Scenario | Fuel | | Product |
|---|---|---|---|
| | Seed | Blanket | |
| Traditional Fast Reactor | $UO_2$ | Depleted $UO_2$ | $UO_2$ + Pu |
| Scenario #2 | $UO_2$ | $ThO_2$ | $ThO_2 + {}^{233}U$ |
| Scenario #3 | ${}^{233}UO_2$ or ${}^{235}UO_2$ or $PuO_2$ | Depleted $UO_2 + ThO_2$ | ${}^{233}U + {}^{235}U$ + Pu |
| India's Goal | ${}^{233}UO_2$ | $ThO_2$ | $Th + {}^{233}U$ |

As discussed earlier, the listed reprocessing and enrichment methods can produce significant amounts of SNM, to be used in weapons. Hence, the objective of estimating the number of ENR facilities a state can possess based on indicators selected in this study is very important. In the following two chapters the methodology and the tool for estimation are described.

# 3. BAYESIAN NETWORK ANALYSIS

Bayesian networks were used as the predictive model for this investigation. An overview of Bayesian methodologies should provide insight on how objectives were met. Bayesian networks are graphical models that represent conditional dependence relationships between a set of random variables such that information about the state of one random variable can be used to infer additional information about other variables. Bayesian networks can be constructed from data and expert judgment, allowing for comparison and cross-checking of independent results.

Previous work on nuclear weapons proliferation and their indicators have used different methodologies. For instance, *Singh and Way* studied correlates of nuclear weapons proliferation with **Event History Models** and **Multinomial Logistic Regressions** [4]. *Jo and Gartzke* studied the effect of determinants on nuclear weapons programs and possession through **Probit Regression Analysis** [17]. Lastly, *Kroenig* employs **Rare Events Logistic Regression** to study the correlates of sensitive nuclear assistance [27]. Previous work highlights the use of various **regression analysis** types to study nuclear weapons proliferation.

However, work done by *Freeman* [10] and *Mella* [5] indicate the potential to use **Bayesian Networks** as an analysis tool. Freeman identifies the need for decision making tools and the ability for models to capture the dynamic nature of proliferation [10]. Both Freeman and Mella studied proliferation pathways, for non-state actors and state actors. Additional work done by *Elmore* [28] highlights the ability to scientifically define physical realities, such as steps required to acquire SNM, in Bayesian networks. Evolving Bayesian networks allow continuous updates on new applicable proliferation technologies [28]. The overall goal is to measure the number of ENR facilities. The varying dynamic factors that affect ENR development prompted the use of **Bayesian Networks** as an analysis tool.

## 3.1 Bayes Theorem

To understand Bayes' theorem, take the following two independent events, H and E. There is an initial probability for event H, P(H), based on a prior belief about H. Using P(E) the revised probability of H is represented as P(H|E). Based on this a conditional probability, P(H|E) can be represented as:

$$P(H|E) = \frac{P(H \cap E)}{P(E)} \tag{3.1}$$

The previous equation determines the probability of H occurring given E occurred. In this case, $P(H \cap E)$ represents the probability of both events occurring. Similarly,

$$P(E|H) = \frac{P(E \cap H)}{P(H)} \tag{3.2}$$

The probability of the intersection of these events are identical. Additionally, the probability of E is equal to probability of the intersection of H and E plus the probability of the complement of H ($H_c$) and E. [5]. The probability of the complement of H ($H_c$) is $1 - P(H)$. With some algebraic rearrangement, the generic Bayes' theorem becomes:

$$P(H|E) = \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|Hc)P(Hc)} \tag{3.3}$$

The above identity theorem establishes the basis of Bayesian networks.

## 3.2 Bayesian Network(s)

Bayesian networks are probabilistic directed acyclic graphs that rely on Bayes theorem to represent probabilistic relationships. All of the edges in a graph are directed, and there are no cycles [29]. Bayesian networks represent joint probability models among given variables [30].

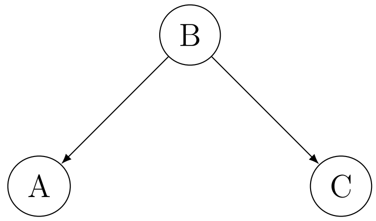

Figure 3.1: Bayesian Network

Figure 3.1 represents a Bayesian network with the following set of Edges: E = $(B, A), (B, C)$. It is important to recognize that there are no undirected edges and no cycles. A cycle is where in a graph after leaving one vertex and following the direction of the edges, there is a way to cycle back to the initial vertex. The joint probability distribution for this network (Figure 3.1) is as follows:

$$P(A, B, C) = P(A|B) * P(B) * P(C|B). \tag{3.4}$$

Since A and C are conditionally independent, results in $P(A|B, C) = P(A|B)$ and $P(C|A, B) = P(C|B)$. This allows for the simplification of the joint probability distribution. $P(A, B, C)$ represents the joint probability distribution for all nodes in the network (Figure 3.1). Equation 3.4 can be simplified using Bayes' theorem, resulting in:

$$P(A, B, C) = P(A|B) * P(B) * P(C|B) \tag{3.5}$$

$$= \frac{P(B|A) * P(A)}{P(B)} * P(B) * P(C|B) \tag{3.6}$$

$$= P(A) * P(B|A) * P(C|B) \tag{3.7}$$

It is important to note that edges in Bayesian networks are connections. As a result, it must be recognized that a joint probability distribution represented by one set of edges can equally be represented by another set of edges. Therefore equation 3.7 results in the following Bayesian network.

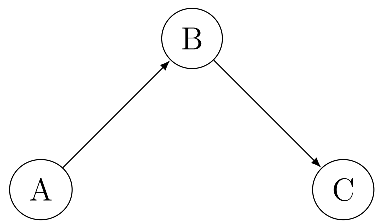

Figure 3.2: Bayesian Network with Reverse Link

Therefore, it can be seen that the joint probability distribution for Figure 3.1 and Figure 3.2 are identical [31]. From this basic example, a general bayesian network with nodes $\mathbf{X} = X_1,...,X_n$, has the following the joint probability distribution:

$$P(\mathbf{X}) = \prod_{i=1}^{n} P(X_i|parents(X_i)) \tag{3.8}$$

Characteristics of such networks include:

- A set of variables identifying important factors,
- Direct dependencies between variables are represented by directed edges (links) between the corresponding nodes,
- Each variable has a finite set of mutually exclusive states, and
- Each variable $A$ with parents $B_1, ...., B_n$, will have an corresponding conditional probability table. [32]

One of the largest challenges when developing probabilistic models is the high number of combination of results (joint distributions). Therefore, Bayesian networks were introduced to avoid further problems. The conditional dependency of one node to another saves computational time. Instead of storing all possible configurations, Bayesian networks only require that all possible combinations of states between related parent and child nodes to be stored [32].

### 3.3 Bayesian Learning Methods

Once models are developed, a dataset must be identified as the training dataset. This dataset is trained on the model, and allows the model to develop predictive relationships. After which, a testing dataset can be used to test predictions. However, in some cases the training dataset doesn't encapsulate all variables. In such circumstances, Netica provides two different learning methods, to be used for "hidden" variables. "Hidden" variables are defined as those for which there are no observations, but are believed to be vital to the developed model. The following two methods are provided; (i) Expectation - Maximization (EM) and (ii) Gradient Descent (GD).

There are four different types of learning problems that are usually faced when using Bayesian networks. The learning problems are a result of the combination of possibilities between the network structure and the status of the data. A network that has a complete dataset and a known structure only requires statistical parameter estimation. A network with an unknown structure and complete dataset requires discrete optimization over structures. If the network has a known structure and an incomplete dataset, it requires parametric optimization (EM, GD methods). Finally, a network with an unknown structure and incomplete data requires combined algorithms such as structural EM and mixture models. [33]

It is important to note that it is common practice to have unique training and testing datasets. However, the dearth of data for nuclear weapons programs forces this study to use the same training and testing datasets. Chapter 6 will be dedicated to identifying the accuracy of results.

### *3.3.1 Expectation - Maximization*

The EM algorithm is deterministic and can be applied to problems that can be considered incomplete data problems. It is an iterative process that utilizes several predictive distributions.

The contents of this paragraph are paraphrased from *Approximation Methods for Efficient Learning of Bayesian Networks* by Riggselsen [34]. Refer to Riggselsen for additional details. The EM algorithm consists of two specifics steps, E-step and the M-step. The E-step predicts missing values given the current best estimate. Following that, the M-step calculates the parameter estimate using the statistics form of the E-step and now inputs that as the new best estimate. This process is repeated to produce a sequence of statistical values. Eventually, this iterative process will converge towards the true parameter. The convergence time is dependent upon the number of missing data variables. EM methods are used extensively for parameter learning and as such would be very useful to develop further models for this study.

### *3.3.2 Gradient Descent*

Gradient Descent is an algorithm used to train a model with the given observation data. GD methods are more often used to tune a Bayesian network such that certain nodes represent the anticipated probability. The GD method relies on back propagation, where local calculations are used to calculate the gradient of error as a function of the identified parameters (variables). This method is an iterative method that converges when the gradient is as close to 0 as possible.

It is important to note that gradient descent methods are most commonly used with neural networks. However, Ramachandran and Mooney devised a method to incorporate back-propagation methods into Bayesian networks [35]. The proposed learning technique suggested to first learn conditional probabilities on Bayesian networks. After which, the Bayesian network should be mapped to a multi-layered neural network. This step is required as the GD method was developed to be used in neural networks. Once the network is mapped to a neural network, that network must be trained with the GD method. After this, the trained neural network can be transformed into a Bayesian network. Luckily, a lot of Bayesian network software is designed to provide this method intrinsically. [35]

The provided gradient descent option via Netica software, utilizes a "conjugate gradient descent" method to maximize the probability of the data by adjusting the conditional probability tables. This algorithm will generally converge faster than EM learning, this however should not be of concern for this study. [36]

# 4. DATA AND MODEL DEVELOPMENT

## 4.1 Dataset

The forthcoming paper *A Spatial Model of Nuclear Technology Diffusion* by Fuhrmann and Tkach captures global patterns of nuclear diffusion from 1950 to 2000, which proves to be a useful dataset for this study. [37] Geo-spatial modeling aims to capture the concept of geo-spatial contagion, the inter-dependence between countries. Previous studies have recognized that international diffusion influences the spread of nuclear technology. The Fuhrmann & Tkach study is not focusing on spatial contagion occurring with explicit militarization of a nuclear program. Instead, it studies spatial contagion in relation to a perceived nuclear threat. A perceived nuclear threat can be defined in a multitude of ways. A common example is obtaining peaceful dual-use technology, thus enabling states to build nuclear weapons in the event of a crisis, better known as nuclear hedging.

The accompanying dataset to the Fuhrmann and Tkach paper was chosen for this study as it captured potential indicators of weapons proliferation. These indicators include both motivations and technical capabilities. The dependent variable (Advanced Nuclear Plants) was taken from the Nuclear Latency (NL) dataset. In this case, Advanced Nuclear Plants measures the number of operating ENR facilities a state has in a given year. The NL dataset has information on all ENR facilities in the world from 1939 to 2010.

Table 4.1 summarizes the dataset. It lists countries, the number of years a country was considered, and the number of ENR facilities per state. Each country is represented in the dataset based on the existence of an operational research reactor. This data shows that over 55% of countries considered have not developed ENR facilities.

Table 4.1: Summary of Furhmann & Tkach Dataset

| State | Frequency | Number of Advanced Nuclear Facilities | State | Frequency | Number of Advanced Nuclear Facilities |
|---|---|---|---|---|---|
| Algeria | 9 | 1 | Argentina | 40 | 4 |
| Australia | 28 | 2 | Austria | 38 | 0 |
| Belarus | 7 | 0 | Belgium | 42 | 1 |
| Brazil | 41 | 7 | Bulgaria | 37 | 0 |
| Canada | 49 | 3 | Chile | 24 | 0 |
| China | 40 | 13 | Colombia | 33 | 0 |
| Czechoslovakia | 31 | 1 | Democratic Republic of Korea | 33 | 3 |
| Denmark | 41 | 0 | Egypt | 37 | 3 |
| Finland | 36 | 0 | France | 50 | 19 |
| Georgia | 7 | 0 | German Democratic Republic | 29 | 0 |
| German Federal Republic | 29 | 0 | Germany | 8 | 6 |
| Ghana | 33 | 0 | Greece | 39 | 0 |
| Hungary | 39 | 0 | India | 42 | 11 |
| Iran | 38 | 10 | Iraq | 31 | 9 |
| Israel | 39 | 4 | Italy | 39 | 4 |
| Jamaica | 14 | 0 | Japan | 41 | 9 |
| Korea, Republic of | 39 | 0 | Latvia | 7 | 0 |
| Libya | 17 | 3 | Lithuania | 7 | 0 |
| Mexico | 30 | 0 | Netherlands | 43 | 5 |
| Norway | 47 | 1 | Pakistan | 37 | 8 |
| Peru | 20 | 0 | Poland | 40 | 0 |
| Portugal | 39 | 0 | Romania | 41 | 1 |
| Russia | 54 | 31 | South Africa | 34 | 5 |
| Spain | 40 | 4 | Sweden | 44 | 1 |
| Switzerland | 43 | 0 | Taiwan | 37 | 3 |
| Ukraine | 7 | 0 | United Kingdom | 51 | 19 |
| Uruguay | 20 | 0 | USA | 54 | 46 |
| Venezuela | 38 | 0 | Yugoslavia | 19 | 4 |

## 4.2 Procedures

Based on the literature review, the following factors were considered as pertinent to developing ENR facilities [37]:

1. Technical Capability
   (a) GDP per Capita
   (b) Nuclear Weapons Arsenal (Binary)
   (c) Nuclear Electricity Production
2. Motivation
   (a) Super Power Alliance (Binary)
   (b) Number of Disputes
   (c) Number of ENR Facilities by Rival States
3. Number of ENR Facilities by Trading Partners

After identifying these factors, it is vital to develop two different models. The first model (basic model) determines the relevancy of the identified factors, while the second model (tiered model) ensures that a structured approach is used to develop relationships between certain factors.

The predictive networks(s) developed were simulated through Bayesian networks. These Bayesian networks were developed using Netica. After eight nodes are developed, for the basic model, they are linked together with the use of edges. The result is seen in Figure 4.5, notice that the states of each node have equivalent probabilities. At this stage, it is necessary to train (provide) the model with data. Model training enables the model to transform into a predictive model. It must also be noted that the historical data was adjusted to fit the discrete constraints of each node, more information can be found in Section 4.4.1 and Table(s) A.1 to A.4. After the model is trained with historical data, the resultant model is seen in Figure 4.1.

The tiered model requires a similar approach, where ten nodes are developed and linked together with the use of edges. However, with the tiered model there are no available data for two nodes (Tech Capability and Motivation). The identified learning methods in Section 3.3 were used in addition to the data, thus enabling the model with predictive capabilities.

Models were ready for use after being developed and trained with relevant datasets. Netica offers a Java API, which can be used to run large case files. In this situation, a case file would represent the dataset to be tested on the developed model. Case files must contain data for nodes in the network, and represent node values as discretized in the network. One row in a case file represents a specific year for a state, this dataset has 2241 rows of data. Using the API and R, the Netica model was modified for each row of data. Once the model is modified, each node is set to a specific state specified by the data. The dependent nodes probability vector changes as the inputs to the models change.

The probability vector represents the probability of having a certain number of ENR facilities. Referencing Table A.3, weighted means are found for each state. The probability vector and weighted means are multiplied together, resulting in a vector of values. The summation of these vector values are rounded resulting in the overall estimate of the number of ENR facilities a state has in that year.

Plots were developed that compare the historical data versus the predictive model results on a time series x-axis. These plots are developed for each state, to better visualize the results of the simulations from the Bayesian networks.

## 4.3 Indicators

As previously identified, the dependent variable is the number of ENR facilities a state holds. Exactly ten variables were identified, two of which are classifying variables, which will be used in a more complex model. The independent variables account for Nuclear Electricity production, GDP Per Capita, Disputes, Superpower Alliance and Nuclear Arsenal. Additionally, the average number of ENR plants possessed by a states rivals and its nuclear trading partners are two variables that were used from Fuhrmann and Tkachs work that identify inter-dependence between countries. The classifying variables are technical capabilities and motivations.

After identifying variables, the following topics were studied: key concepts influencing the phenomena of interest (variables) and the relationship amongst these variables. This ensures that the variables are related and have an impact on measuring the overall dependent variable.

## 4.4 Model Development

Bayesian networks were used as the predictive model for this study. Three different models were developed. The first model (basic model) aims to validate that the independent variables can appropriately measure the number of ENR facilities a state would have.

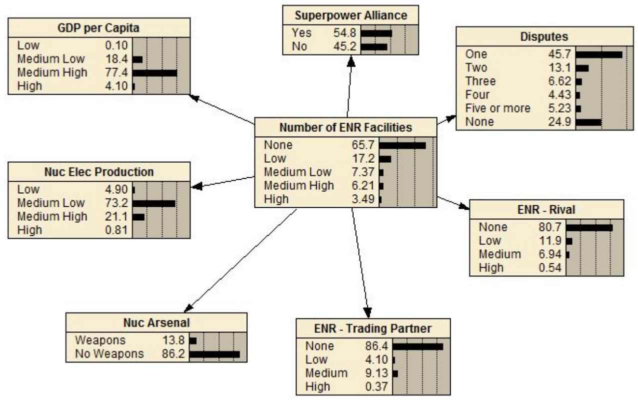


Figure 4.1: Basic Model (Bayesian Network)

The basic model assumes that all predictor variables are conditionally independent from one another. Note how there are no edges between the predictor variables and the classifying variables, which are omitted as it detracts from assessing variable independence.

After assessing conditional independence, a more advanced model was developed (hereafter known as the tiered model). This model assumes conditional dependence of certain independent variables on the classifying variables: Technical Capability and Motivations. This model adds complexity by acknowledging the decisions and factors that go into constructing ENR facilities. As discussed before, *Nuclear Opportunity* and *Nuclear Willingness* influence weapons proliferation decisions [4]; these two categories roughly translate to the classifying nodes in the tiered model. Various literature has confirmed the complexities of nuclear weapons proliferation, hence the development of a tiered model. The variables GDP Per Capita, Nuclear Electricity Production, Nuclear Arsenal, and ENR Facilities-Trading Partners influence a states technical capability, while ENR Facilities  Rival, Disputes, and Superpower Alliance influence a states motivation.

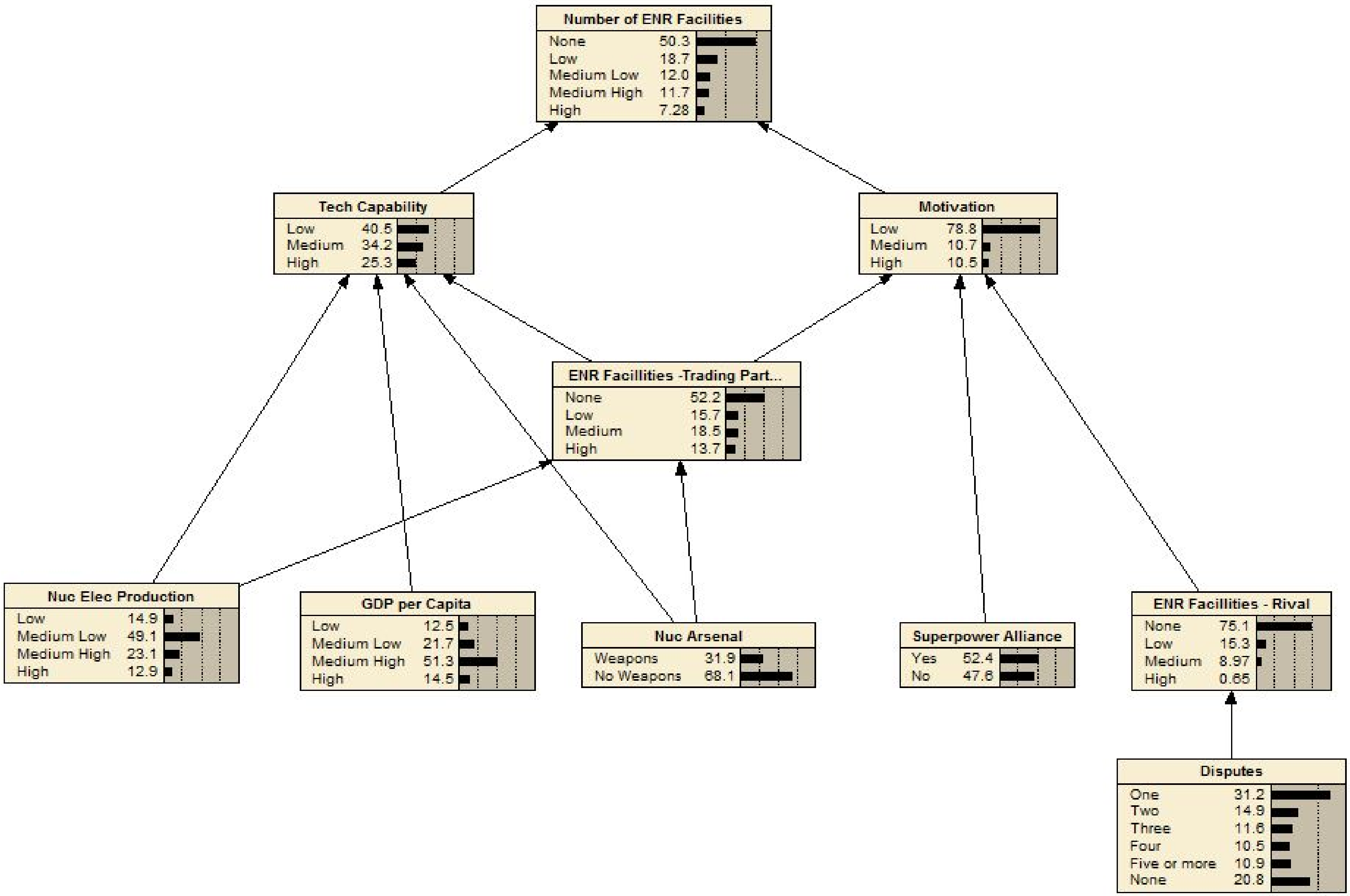


Figure 4.2: Tiered Model (Bayesian Network)

As seen in Figure 4.1, the network contains eight nodes each of which has a number of states (ranging from two states to five states). When developing such a network, two key steps are required: node discretization and network training. Network training provides initial conditional probability tables (CPT) which are used to allow the model to estimate or predict the dependent variable. This initial CPT is developed through the distribution of data for each node. The data provided must have column names that match the node names in the network. For further details on Bayesian network, please see Section 3.

### *4.4.1 State Development*

In a Bayesian network, each variable has a discrete set of possible states. Regardless of the type of variable (continuous or numeric) used in a Bayesian network, it is necessary to define a mapping from the natural variable domain to a set of chosen discrete states (e.g. High/Medium/Low). In order to choose appropriate states, the data for each variable was plotted as a histogram. Each histogram used a different discrimination techniques. The even spacing technique developed five even breaks in the data, forcing there to be exactly five states in all of the nodes except for the Nuclear Arsenal and Superpower Alliance node. The fixed spacing technique developed breaks based on user choice. We will later realize that user choice is too arbitrary. The quantile method develops quantiles using the data provided [38].

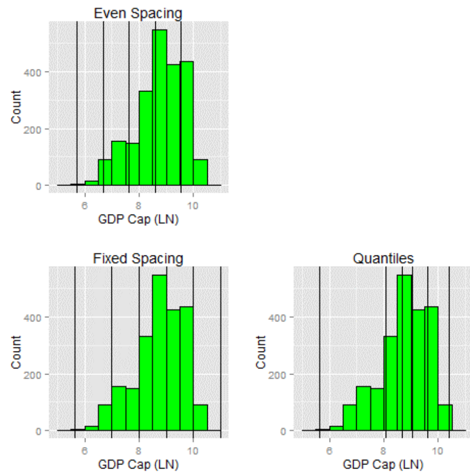

Figure 4.3: State Discrimination for the GDP Cap (LN)

In the Figure 4.3, the aforementioned histograms are plotted for the logged values of the GDP per capita variable in the dataset. From these plots, it is clear that none of these breaks are intuitive. After reviewing similar histograms for all independent variables, it was concluded that state discretization would occur based on natural breaks in the data. Natural breaks in the data were defined based on the expected occurrences. For instance, for data to be categorized as high it should not appear to often in the data. Histograms were used to identify the location of these natural breaks. A figure of final discretization for a few variables can be seen in Table A.1.

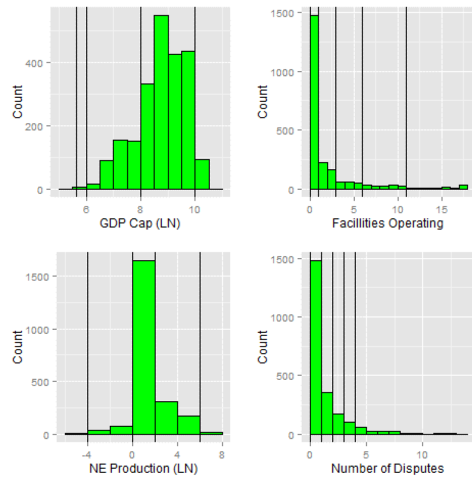


Figure 4.4: Final State Discrimination

### *4.4.2 Model Training*

A model is only partially developed after nodes and the number of states for each are identified, commonly known as an untrained model (Figure 4.5). This model captures an equivalent probability between states for each node, which is extremely rare. Model training can use a variety of different algorithms, the most common algorithms include expectation-maximization, gradient descent, and generalized probability distribution. Further details for the algorithms can be found in section 3.

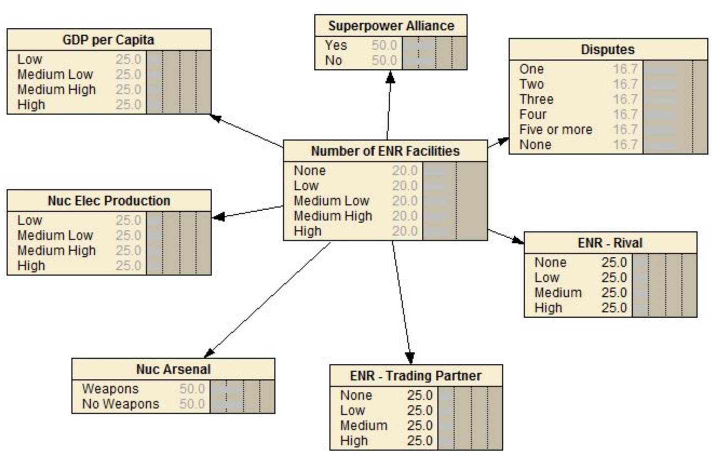


Figure 4.5: Untrained Basic Model

Depending on the model structure, it may or may not be required that additional CPTs are developed for missing data. The basic model did not require additional CPTs, instead marginal probabilities were developed by training the dataset on the network. In the case of the tiered model, there is no data for the classifying nodes. This can be mitigated through the use of specific training methods (EM). Another option would be to conduct an expert elicitation and aggregate the results. Such a process will require the development of a CPT.

In the tiered model, technical capability depends on GDP per Capita, Nuclear Electricity production, Nuclear arsenal and the number of ENR facilities held by Trading Partners. This classifying node is a bit more complex, as no single parent node overwhelms the other (See Table A.5). For the development of this CPT, GDP per Capita and the number of ENR facilities held by Trading Partners were considered to be more influential than other nodes. As their relative strength increased, so did a states technical capability. Table A.5 affirms that the two overwhelming parent nodes are GDP per Capita and the number of ENR facilities held by Trading Partners. Additionally, if a state were to also possess a nuclear weapons arsenal, then their technical capability probability is representative of a higher technical capability. The technical capability probability for a state with a weapons arsenal is not skewed to represent a low technical capability, but instead is dependent on the other parent nodes [38]

As a result of the multiple training methods and training through expert elicitation, there are three different tiered models. A correlation statistic will be calculated to determine the most accurate method for this study.

# 5. SIMULATIONS

## 5.1 Countries Chosen

The following historical examples were chosen to validate the developed networks: Brazil, India, South Africa, and Sweden. Each of these countries were chosen to highlight different aspects of nuclear weapons proliferation. These case studies were examined to ensure historical validation.

### *5.1.1 South Africa*

South Africa was always a country of importance to the United States and United Kingdom as a source of uranium ore. In 1940, the South African Atomic Energy Board was formed. In the 1950's more distinct plans were made for nuclear science research in South Africa ranging from the development of a research reactor to allowing South African scientists to visit U.S Atomic Energy Commission facilities [39] [40]. The first South African research reactor (Safari I) went operational in 1965. Safari I is still in operation, but as of 2005 was converted from HEU fuel to LEU fuel [41].


Figure 5.1: South African ENR Facilities

As per Fuhrmann and Tkach, it was found that South Africa has five different

ENR facilities. The Fuhrmann and Tkach study considers South Africa in the dataset after 1965, as per the definition of when the first research reactor is operational. The following five ENR facilities were identified:

1. Valindaba (Laser)
2. Valindaba Y-Plant
3. Valindaba Z-Plant
4. Valindaba Z-Plant at Laboratory Enrichment Facility
5. Hot Cell Complex, Pelindaba Nuclear Research Center.

Note that the Valindaba Z-Plant includes two facilities a semi-commercial enrichment plant, and a laboratory enrichment plant. The laboratory facility was operational between 1967-1988, while the commercial facility was operation from 1986-1995. Both facilities used aerodynamic isotope separation for enrichment, the lab scale facility was developed initially to determine feasibility. Location of all ENR plants can be seen in Figure 5.1. For further information on ENR facilities, please refer to Fuhrmann and Tkach[37].

South Africa is the only example of a state voluntarily dismantling a nuclear weapons program. It is essential for this case to be tested as it would determine whether the nuclear arsenal variable is necessary and functional in the developed predictive models.

### *5.1.2 Brazil*

Brazil is another interesting case, as it sought nuclear capabilities to rival Argentina and to gain international prestige. Brazil's nuclear program began in a very similar fashion to the South African's, as they signed a mining agreement with the United States in 1945. After which, subsequent agreements were signed to transfer nuclear technology to Brazil. The United State's "Atoms for Peace" program paved the way for Brazil to obtain its first research reactor (IEA-R1), in 1957 [42]. In 1975, the Brazilian-German deal was signed, ensuring Brazil's purchase of eight nuclear power reactors, and pilot-scale technology (plutonium and uranium reprocessing) from West Germany. This deal was estimated to be between $10-$15 million dollars [43]. This deal signified Brazil's interest in developing latent capability.

The pilot-scale facility purchased from West Germany was very similar to the Trombay facility in India. It's irradiated fuel throughput, based on fuel burn-up, could have produced enough plutonium for up-to half a dozen nuclear weapons a year. The purchased enrichment facility had the capability to produce HEU for several nuclear weapons per year [44]. Additionally, Brazil's refusal to sign the NPT signaled that the German deal facilitated Brazil's nuclear weapons development. This is further confirmed when Meyer shows that the Brazil's nuclear propensity jumps from 0.1 in 1974 to 0.2 in 1975, as per the definition of nuclear propensity this is seen as a substantial increase [2].

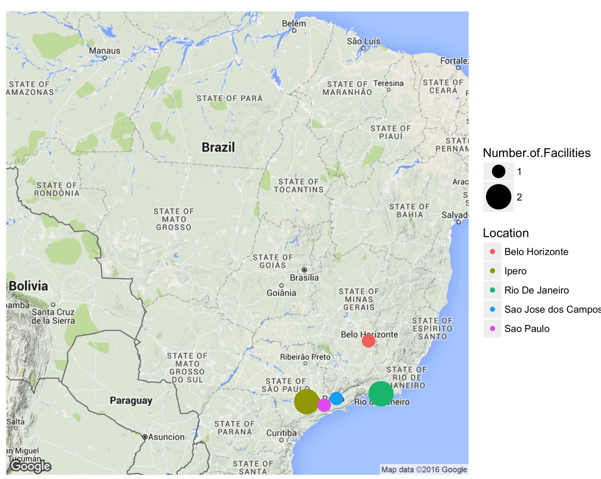


Figure 5.2: Brazilian ENR Facilities

Fuhrmann and Tkach identified seven different ENR facilities in Brazil. Brazil was considered in the dataset after 1957, when the first research reactor became operational. These seven facilities were identified:

1. Aerospace Technical Center (Institute of Advanced Studies)
2. BRF Enrichment - Aramar Demonstration Center
3. BRN Enrichment (Aramar Isotopic Enrichment Lab)
4. INB Resende - Enrichment Facility
5. INB Resende - Enrichment Facility
6. IPEN - Reprocessing
7. Pilot Enrichment Plant (INB Resende)

Note that once again, a single location (INB Resende) is counted multiple times. In this case, the three different facilities are a commercial centrifuge plant, an aerodynamic isotope separation pilot plant (1979-1989), and an aerodynamic isotope separation pilot plant (1990-1994).

The Brazilian case study will lend insight to the importance that rivals play in proliferation decisions. It will also study the importance of a state's desire to be considered an international power or as a "regional superpower".

### *5.1.3 India*

India obtained nuclear weapons as means of deterrence against China and to protect against their rival Pakistan. Jawarhlal Nehru, the first prime minister of India would take it upon himself to found the non-aligned movement and advocate nuclear disarmament. However, Nehru refused to rule out the nuclear option for India. In 1948, India would pass the Atomic Energy Act. Following this, in 1955 India would construct the Apsara research reactor with British assistance. In 1956, Canada would agree to supply India with a research reactor (named CIRUS) with higher power output (40 MW). The Canadian reactor was used to produce weapons-grade plutonium, signaling Indian interest in a nuclear weapons program. Subsequently, in

May of 1974 India would successfully test its first nuclear bomb, famously known as the Smiling Buddha.

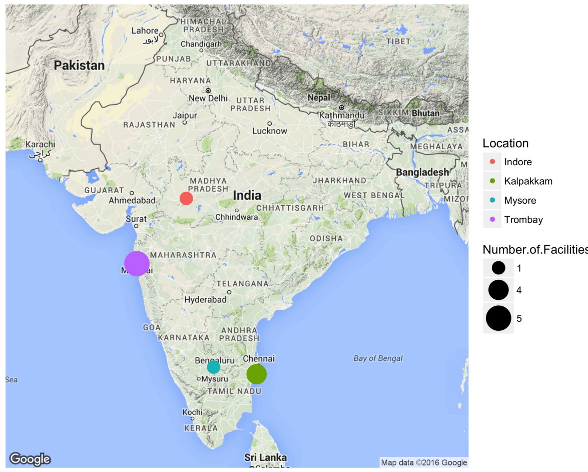

Figure 5.3: Indian ENR Facilities

Fuhrmann and Tkach identified eleven different ENR facilities in India. India was considered in the dataset after 1955, when the first research reactor became operation. The following facilities were identified:

1. BARC, Trombay (Pilot)
2. BARC, Trombay (Commercial)
3. BARC, Laser Enrichment Plant
4. BARC, Trombay (Reprocessing)
5. BARC, PREFRE (Reprocessing)
6. CAT, Laser Enrichment Plant
7. FRFRP (Reprocessing)
8. KARP, Reprocessing
9. KARP, Laboratory
10. Lead Facility, (Reprocessing)
11. Materials Plant, (Enrichment)

As identified before, certain locations have multiple facilities. The Indian nuclear complex was structured in such a manner that a certain location would posses multiple fuel cycle capabilities (i.e fuel fabrication, enrichment, reprocessing, and etc).

The Indian case required two proliferation decisions in 1965 and 1972. The former was reversed in 1966 thus requiring a second decision. Additionally, both proliferation decisions highlight the convergence of motivation and a pre-existing technical capability. It also recognizes that regardless of technical capability, when motivations change it could lead to decision reversals [2]. There are multiple driving factors towards the heightened development of the Indian nuclear weapons program. The primary factor for Indian proliferation was the Chinese nuclear weapon test in 1964. Border conflicts between the two nations, preempted India to develop a nuclear deterrent. At the time, development of nuclear weapons gave India an upper-hand against Pakistan. It is important to recognize that Pakistan received aid from the Chinese to further develop their nuclear weapons program. Besides countering rivals, India sought nuclear weapons to be recognized as a "regional superpower" in Asia.

#### *5.1.4 Sweden*

Sweden explored nuclear weapons immediately following World War II, to posses a form of deterrence against a looming Soviet Union and to maintain political nonalignment. In its infancy, the Swedish program was a clandestine program within the Swedish National Defence Research Institute (FOA).

The Swedish program identified local sources of uranium, which were later confirmed as one of the richest in the western world by both the UK and USA. The Swede's focused on nuclear energy production, with plutonium production as a byproduct of the system. In 1954, the Swedish nuclear weapons development was discussed openly, as a result of the first reactor (R-1) going operational.

Between 1959 to 1962, Sweden and its armed forces regime did not feel compelled to acquire nuclear weapons. This occurred due to multiple reasons, Sigvard Eklund becoming the Secretary General of the IAEA, Foreign Minister Unden continuously promoted international disarmament, and multiple reports by the FOA had shown the increased costs of a nuclear weapons program developed internally. Instead, cooperating with the USA would significantly decrease the cost, but would place restrictions such as foreign inspections. Eventually in 1968, Sweden would sign the NPT and close the opportunity for a nuclear option [45].

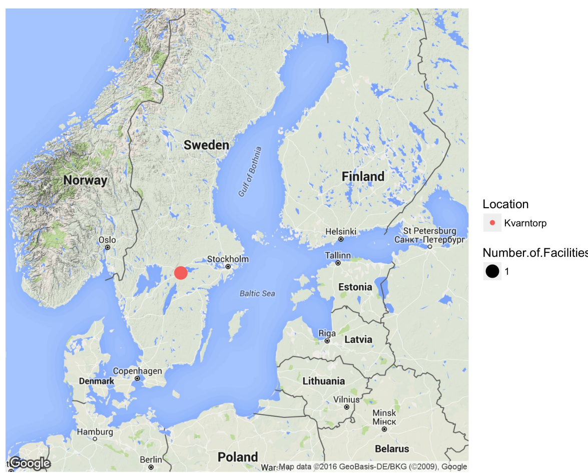


Figure 5.4: Swedish ENR Facilities

Fuhrmann and Tkach identified a lone ENR facility in Sweden.

1. Pilot Plutonium Reprocessing Plant.

This facility was a pilot spent fuel reprocessing plant. This pilot plant began construction in 1946. It was estimated to be operation between 1946 to 1968. Multiple different sources cite varying dates for when Sweden acquired nuclear latency. However, all sources cite the development of R-1 and the enrichment plant as indicators of developing nuclear latency.

## 5.2 Model Results

In the following section, we will examine the different models developed with respect to the countries discussed above. The countries selected had key characteristics relating to geo-spatial contagion.

When provided with inputs for a given state in a year, all the developed models "backcast" in time the number of ENR facilities a given state would be expected to have. The result is a probability vector, which corresponds to the estimated number of ENR facilities a state should have in a particular year, based on the inputs provided. This probability vector is transformed into a single numeric value, by a weighted sums approach for each vector value.

### *5.2.1 Basic Model*

The basic model (as seen in Figure 4.1), represents the simplest model, where all predictor variables are independent of one another. In the following results, it is important to identify trends, instead of the accuracy of results. Identifying trends allows for cross-checking of various nodes, it is easily identified when studying binary nodes such as: nuclear arsenal and superpower alliance.

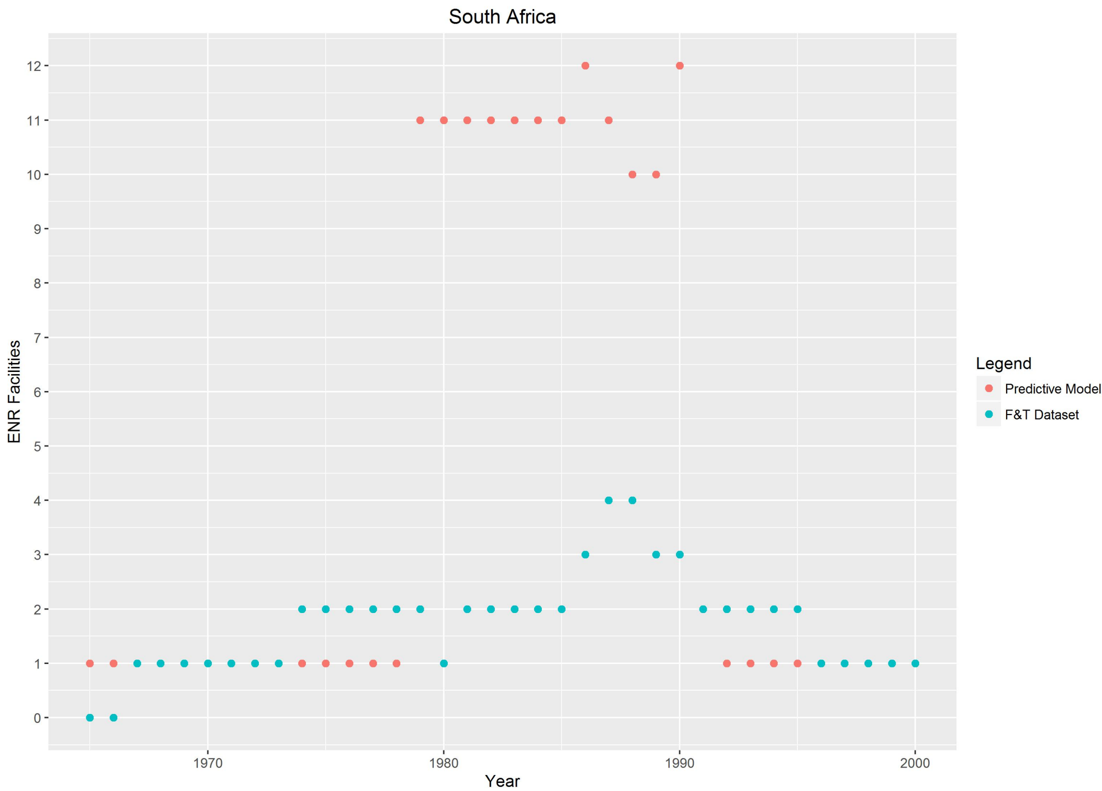

Figure 5.5: Geo-spatial results for RSA (Basic)

Results obtained from the basic model for South Africa highlight the extreme change in the proliferation status of a state. As discussed earlier, South Africa voluntarily dismantled their nuclear weapons program in 1989. In 1991, South Africa would sign the NPT. The predictive model estimates drop drastically from 12 facilities to 1, between 1990 and 1991. In comparison, there is a similar drop from 4 facilities to 1, in the Fuhrmann and Tkach dataset, between 1988 to 1995. Therefore, the results above affirm that the model accurately accounts for whether a state has a nuclear arsenal in a given year or not.

It even goes to show that the nuclear arsenal node is a large driving factor for estimations. Realistically speaking, when a state makes a decision to voluntary dismantle, the dismantlement process takes a few years and as a result the decommissioning of other nuclear facilities cane take anywhere from two to ten years. The results show that the model does not have the capability to capture the entire dismantlement process, which should be considered for future work.

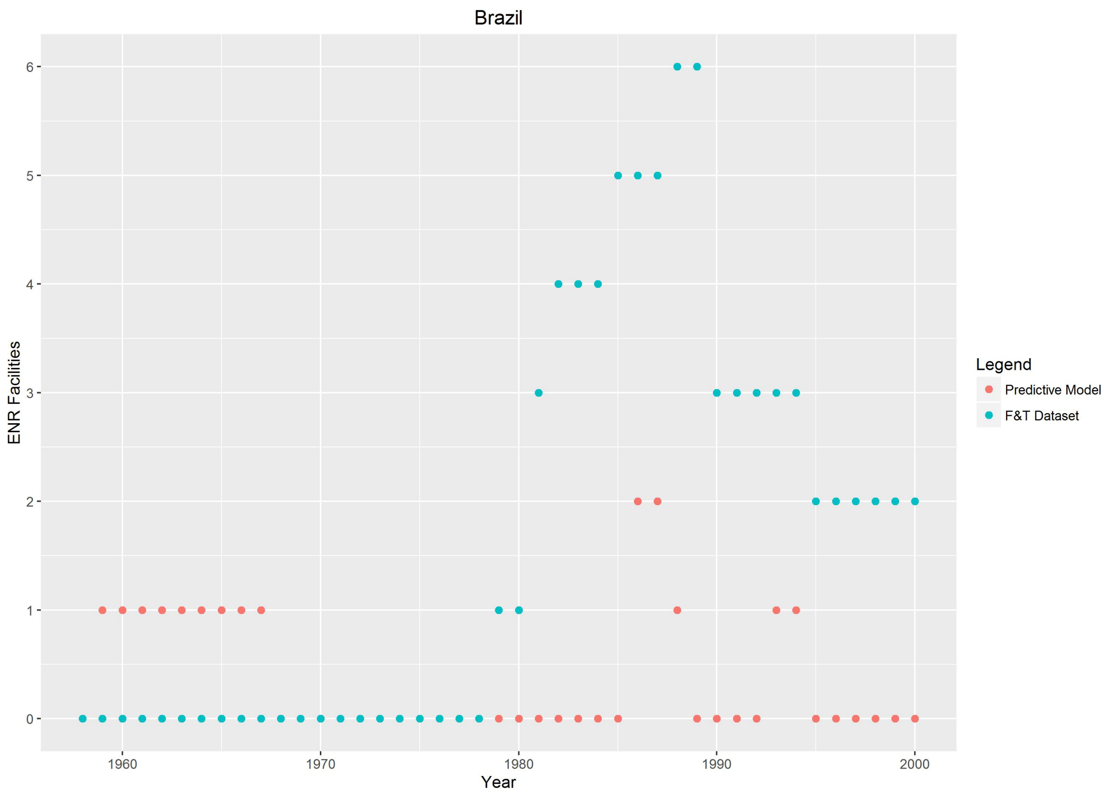

Figure 5.6: Geo-spatial results for Brazil (Basic)

The Brazilian-German deal signed in 1975, signals an interest in nuclear weapons. This interest in nuclear weapons should inherently affect GDP per capita and nuclear electricity production. It can be seen in both datasets, following 1975 the number of ENR facilities increase. there seems to be other factors that cause for a delay and underestimation in the results provided by the basic model. These factors will be addressed when studying the tiered model.

Additionally, the Brazilian case also highlights a rivalry between Brazil and Argentina. However, this is unlike the Indian case as the rivalry does not result in confrontation. Argentina furthered its nuclear research in 1983, at which time both datasets show that Brazilian efforts increased through the development of ENR facilities. In 1991, both Argentina and Brazil renounced their nuclear rivalry, at which point it can be seen that the number of ENR facilities decreases.

The Brazilian case study affirms that the basic model has the capability to manage complex situations, where multiple factors play into a state's nuclear weapons program. In this specific case, technical capability factors, a rivalry, and the need to be a regional superpower are all highlighted in the estimations provided.

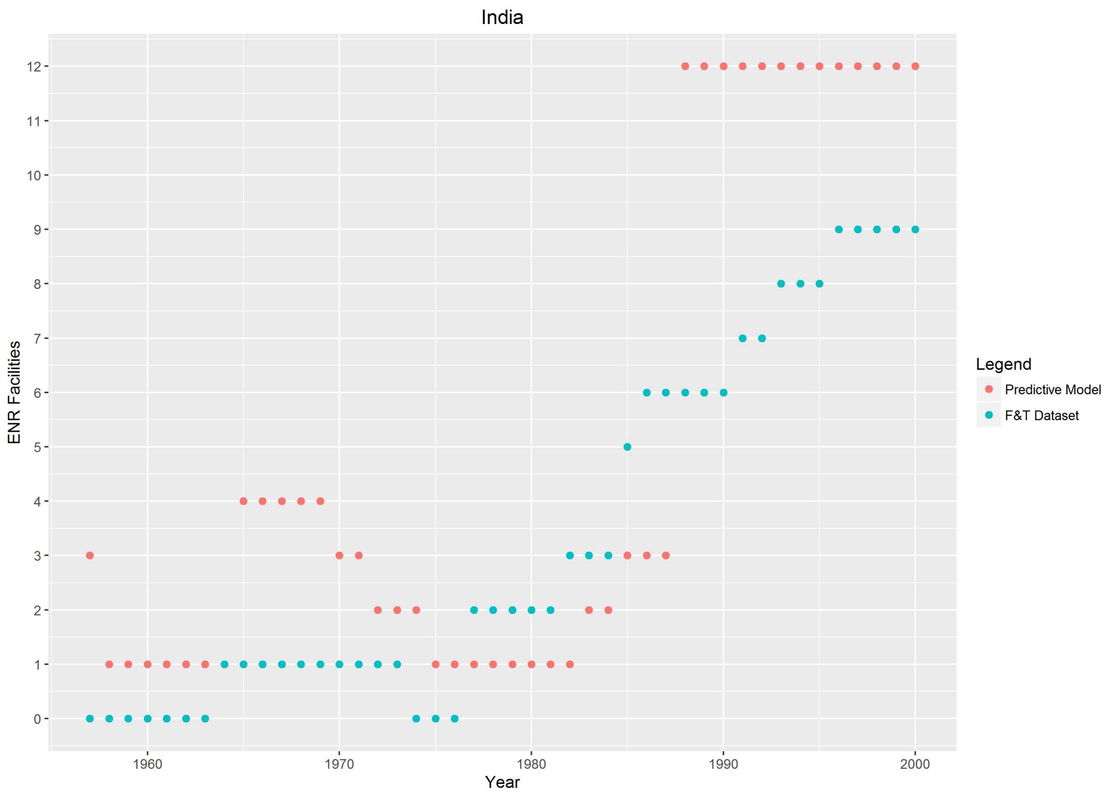


Figure 5.7: Geo-spatial results for India (Basic)

It is important to note that India made two proliferation decisions, one in 1965 and another in 1972. The reversal of the first decision in 1966 has a delayed effect on the estimation results from 1966 to 1975. Following which, India went into the

"the Emergency" phase from 1975 to 1977. It is also important to note, that even though the number of ENR facilities dropped in both the predictive model as well as the Fuhrmann and Tkach dataset, India conducted its first nuclear test (Smiling Buddha) in 1974. The Indian case is important to study as it encapsulates the effect of rivals. China successfully tested its first nuclear weapon in 1964, leading to the second proliferation decision by India. In fact, the predictive model captures this and suggests that the number of ENR facilities should increase. In 1972, Pakistan initialized its weapons program. After which, it can be seen that both the predictive model and the Fuhrman and Tkach dataset begin trending upwards. Therefore, the Indian estimation results identify trends based on rivals and disputes both of which are represented as nodes in the basic model. However, the current model does not have a node with the capability of capturing proliferation decisions.

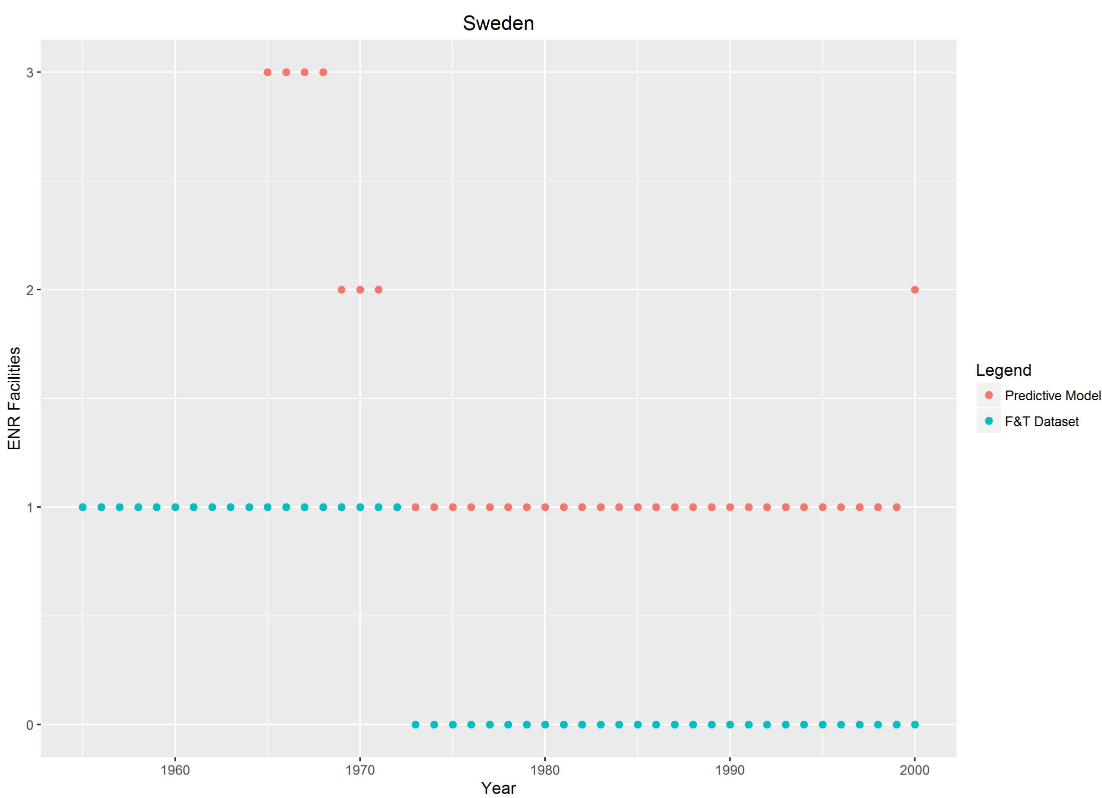


Figure 5.8: Geo-spatial results for Sweden (Basic)

Sweden is a unique case for proliferation in the European region. Sweden had no interest in becoming a regional superpower, instead the proliferation option was considered as a means of deterrence against the Soviet Union and other potential disputes. It is also important to note, that Sweden's development of nuclear facilities greatly impacted their economy as entities were partially owned by the state. Therefore an interest in nuclear weapons and developing capabilities in the early to mid 60's dictated the increase of the results from the predictive model. This interest directly correlates to higher economic capacities, a driving factor in the basic model.

There is no particular method to capture the motivation behind proliferation decisions. However, the factors represented in the basic model (as seen in Figure 4.1) can potentially be associated with a state's motivation to proliferate. One factor in the basic model captures the signing of the NPT. The expected number of Swedish ENR facilities drops around 1968, Sweden officially agrees to the NPT. In most cases, the signing of the NPT could be linked with acquiring a superpower alliance.

After examination of the results of all states represented in the data, the trends in the basic model estimations, be it increasing or decreasing, are consistent with trends found in the dataset provided by Fuhrmann and Tkach. After ensuring that the model consistently depicted trends, the next step is to develop a model that has the capability to match trends consistently but also make more accurate estimates.

### *5.2.2 Tiered Model: EM Learning Method*

In order to develop accuracy for the estimates while retaining trends, it was necessary to introduce a hierarchy system. This hierarchy system developed can be seen in Figure 4.2. This new model, hereafter known as the tiered model, grouped together the independent predictors into two overarching categories: motivations and technical capability. After extensive literature review, and examining results from the basic model it seems evident that proliferation decisions are based on the convergence of motivation and technical capability. There are exceptions however, when a state's motivation is strong enough, it can dictate the development of technical capability further leading to the development of a nuclear weapons program.

With the introduction of two new variables, technical capability and motivation, it is necessary to provide data to the model. Providing data to the model allows the bayesian network to become a predictive tool. In this case, there is no data for these nodes as there is no way to distinctly measure a state's motivation or technical capability. Therefore, when data is missing bayesian networks can be equipped with two different learning methods, Expectation-Maximization and gradient descent learning. In this section, the model was trained with the EM learning method. Later on certain statistical methods will be used to analyze and determine which method is more appropriate.

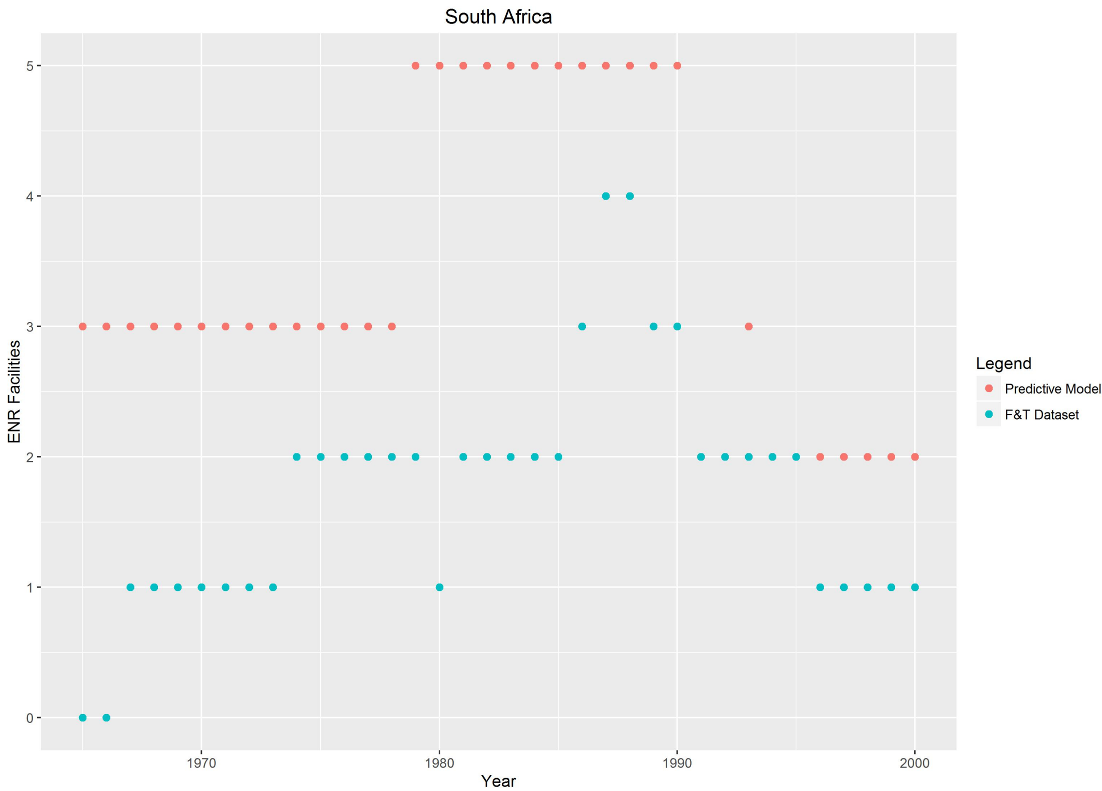

Figure 5.9: Geo-spatial results for RSA (Tiered-EM)

From Figure 5.9, the EM method does not change the fact that the model captures the presence of the nuclear arsenal node.

The EM method does yield slightly different results compared to those shown with the basic model in Figure 5.1, specifically magnitude and trends. With regards to trends, the EM method alleviates the random one year discrepancies, notice in Figure 5.1 that the estimated number of ENR facilities varies slightly from 1985 to 1990. These one year discrepancies are unwanted as they are most likely anomalies. It must be noted that the network does not produce an estimate in the form of an exact number. Instead, it produces a probability vector that must be deciphered and converted to represent a whole number.

At this point, to yield a better discussion on the estimations provided it is important to study the inputs provided, see Figure 5.10. These inputs will provide insight on what variable drives the model to make certain estimations.

From Figure 5.10, it is immediately obvious that the nuclear arsenal node is a driving factor. This affirms that ENR facilities are required for a state to maintain a sustainable nuclear weapons program. Note that between the late 70's and 1990, the major driving factors were a combination of the GDP per Capita and nuclear electricity production. Therefore, in this case it seems that the model places a higher emphasis on the true technical capabilities than the motivational profile.

However, the actual development of ENR Facilities (shown in blue) follows the trend displayed by the number of disputes over time. Such a trend is expected as the South African's built nuclear weapons to develop deterrence from a potential emerging Soviet threat. Therefore the motivational profile consisted of the need for a deterrence (due to disputes), need for regional prestige, and a growing insecurity of neighboring countries. It is important to note that alongside voluntary dismantlement the motivational profile decreased in 1988 as the South African's signed a cease-fire with Cuba and Angola. Thus prompting a decrease in the number of ENR facilities.

Thus for this case, the actual development of ENR facilities was dependent upon the motivational profile. Whereas the predictive model seems to indicate that the technical capabilities drive the development of ENR facilities.

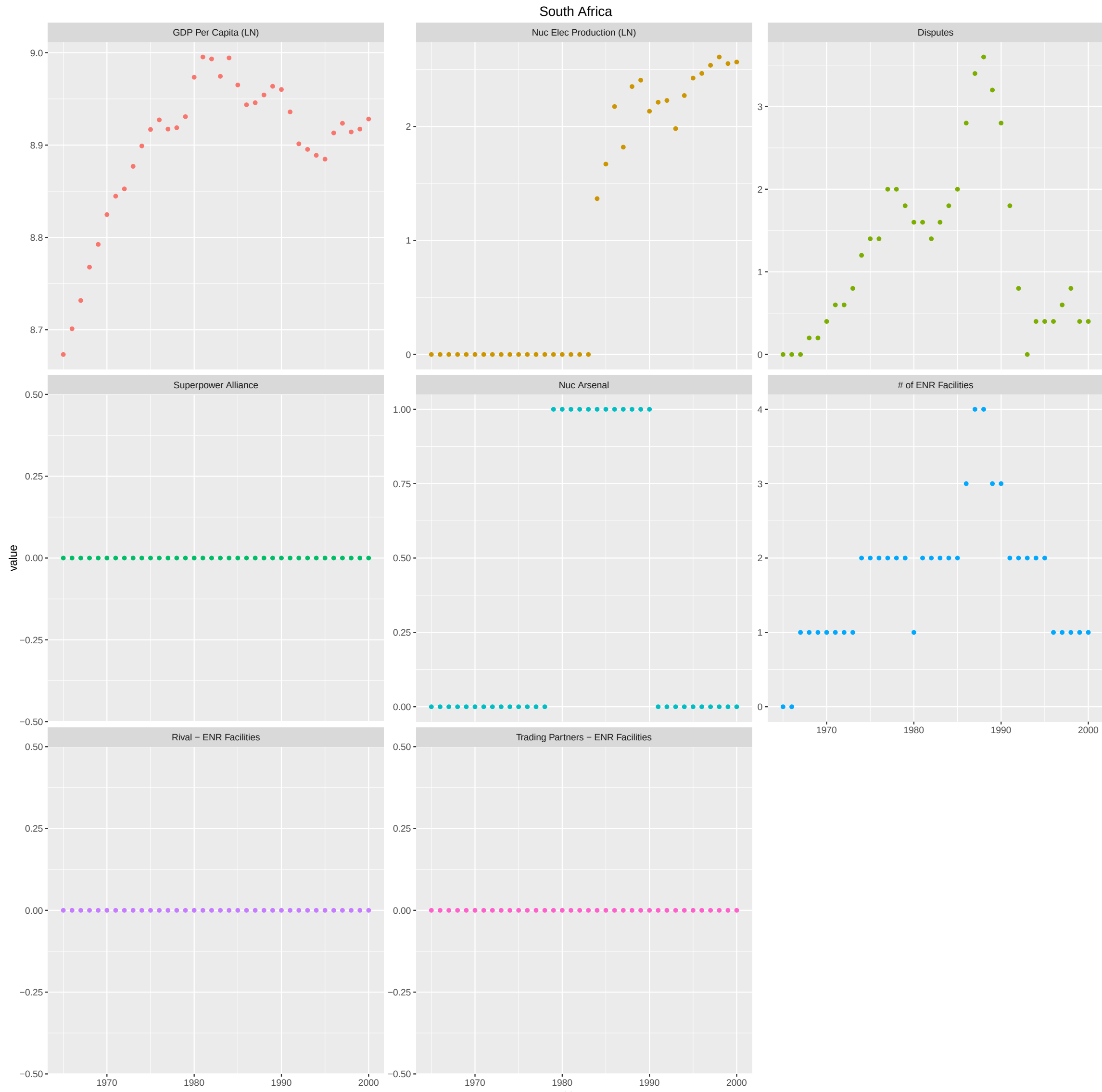


Figure 5.10: Data for South Africa

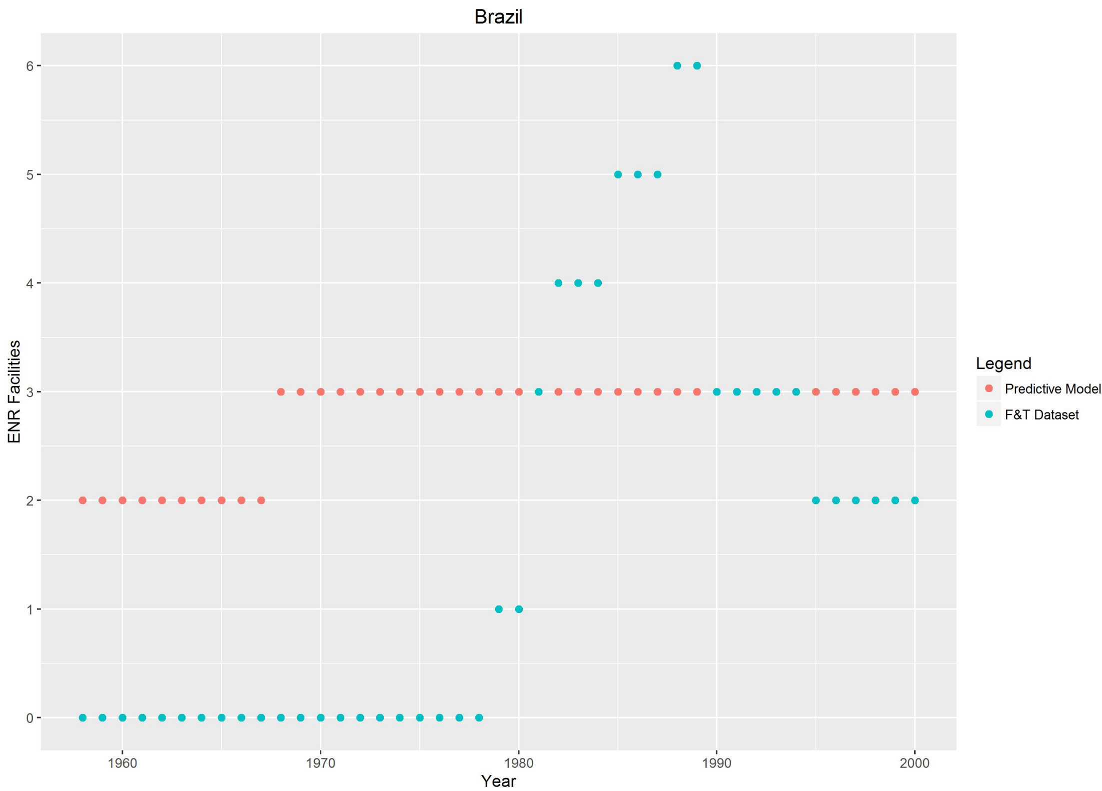

Figure 5.11: Geo-spatial results for Brazil (Tiered-EM)

In comparison to Figure 5.6, the EM method results (Figure 5.11) successfully mitigates the one year discrepancies shown in the basic model. It seems that in this case the EM algorithm smooths the results, however this could be a result of the inputs provided.

Figure 5.12 depicts the change in inputs over time. The predictive model seems to stay pretty stagnant. The predictive model values technical capabilities such as GDP Per Capita and nuclear electricity production. It believes that the Brazilian case was strong enough to have two ENR facilites as soon as the first research reactor went operational in 1957. However, it is possible for a state to be considered in this study after ENR facilities have been developed. The predictive model recognizes the erratic nature of the nuclear electricity production and the number of disputes and suggests that an average number of facilities (mostly 3) would suffice for Brazilian needs.

The actual development of ENR facilities was first dependent upon GDP Per Capita up until 1989. As the GDP Per Capita increased, so did the number of ENR facilities. This corresponds with the Brazil and German agreement. Additionally, economies tend to see an boosts following a proliferation decision. After 1989, the erratic nature of nuclear electricity production and the decreased number of disputes caused a drop off in the number of ENR facilities held by Brazil. The drop off in facility count also goes hand in hand with the nuclear cooperation agreement between Brazil and Argentina in 1991. The result of this agreement was the creation of the Brazilian-Argentine Agency for Accounting and Control of Nuclear Materials (ABACC).

Thus in this case, the actual number of facilities (F&T model) correspond with technical capabilities as well as the motivational profile. This example shows the complexity of a proliferation decision, and how it can be affected by varying factors over time. The predictive model also represents a combination of technical capabilities and the motivational profile. However, the erratic nature of inputs caused the estimation to be an average value.

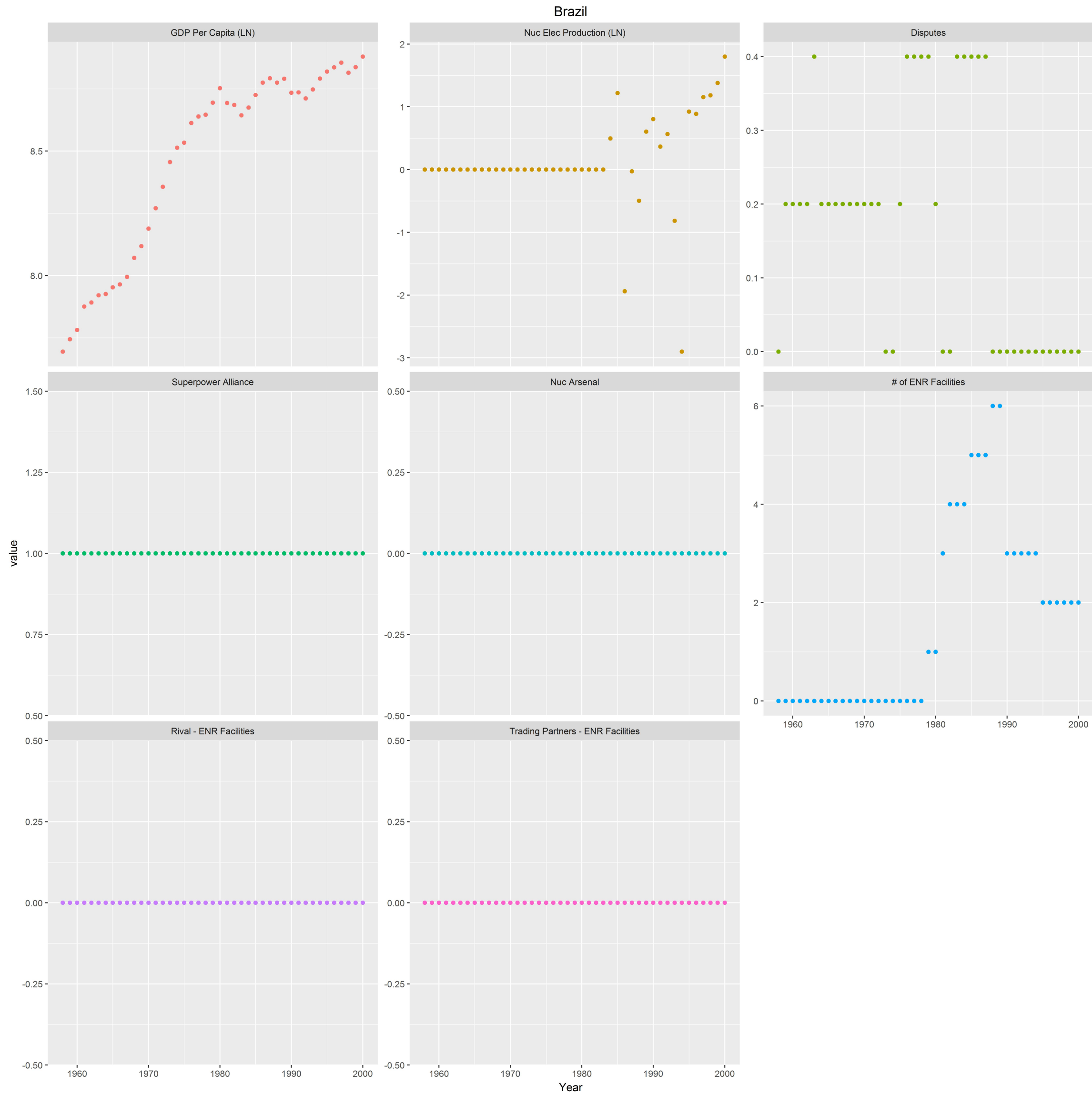


Figure 5.12: Data for Brazil

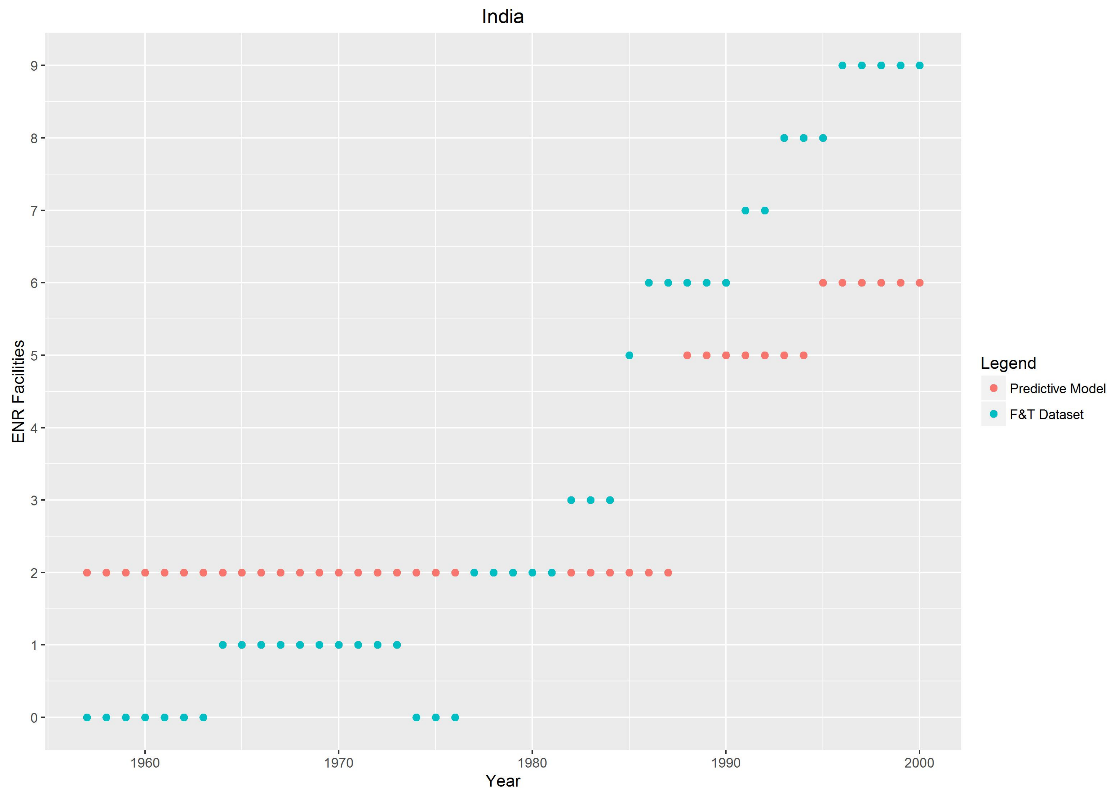

Figure 5.13: Geo-spatial results for India (Tiered-EM)

The basic model (Figure 5.7) captures the two proliferation decisions in 1965 and 1972. However, this is not immediately eminent when studying results from the EM method, as seen in Figure 5.13.

The predictive model does not capture proliferation decisions, however there is no variable included in the model to measure decisions. It does however estimate a total of two ENR facilities from the late 50's to mid 80's. This occurs because the technical capability is relatively low in comparison to the number of disputes during this time frame. Following the second proliferation decision, India ramped up its technical capability in order to develop ENR facilities. After, declaring that India was a nuclear power the predictive model estimates a growth in facilities. This growth corresponds to the maintenance and growth of the program, but can't be confirmed without explicitly developing a variable to capture this motivation. Such work is out of the scope of this study. Therefore, the predictive model results delineates three phases associated with ENR facility development: initial weapons program development (50's - 80's), weapons declaration (88-94), and growth of weapons program (95-2000).

The actual development of ENR facilities corresponds well with disputes, representing the motivational profile. As per the historical case study provided earlier, India's driving factor to proliferate occurred due to its rival neighbors. Initially, the number of disputes were quite high from the late 50's to 1965, but the technical capability did not follow. As a result, the development of ENR facilities was held in check. It is important to recognize the increase in disputes following the Indo-Pakistani war in 1971, alongside an increase in technical capabilities. With these increases, and the second proliferation decision the development of ENR facilities increased from 1972 onwards. Thus once again, the development of ENR facilities does not rely on either technical capabilities or motivations but instead a combination of both.

For this specific case, the predictive model does not capture certain aspects such as proliferation decisions. This results from not specifying all potential variables, which varies from case to case. As such compromises are made as to which variables are key to recognizing ENR development. However, the predictive model does highlight the three key phases behind India's nuclear weapons program. Even though the predictive model does not match the actual development of ENR facilities, it highlights a different and more stable route to developing India's weapons program. Once again, the combination of technical capability and motivations are the driving factor behind the predictive model.

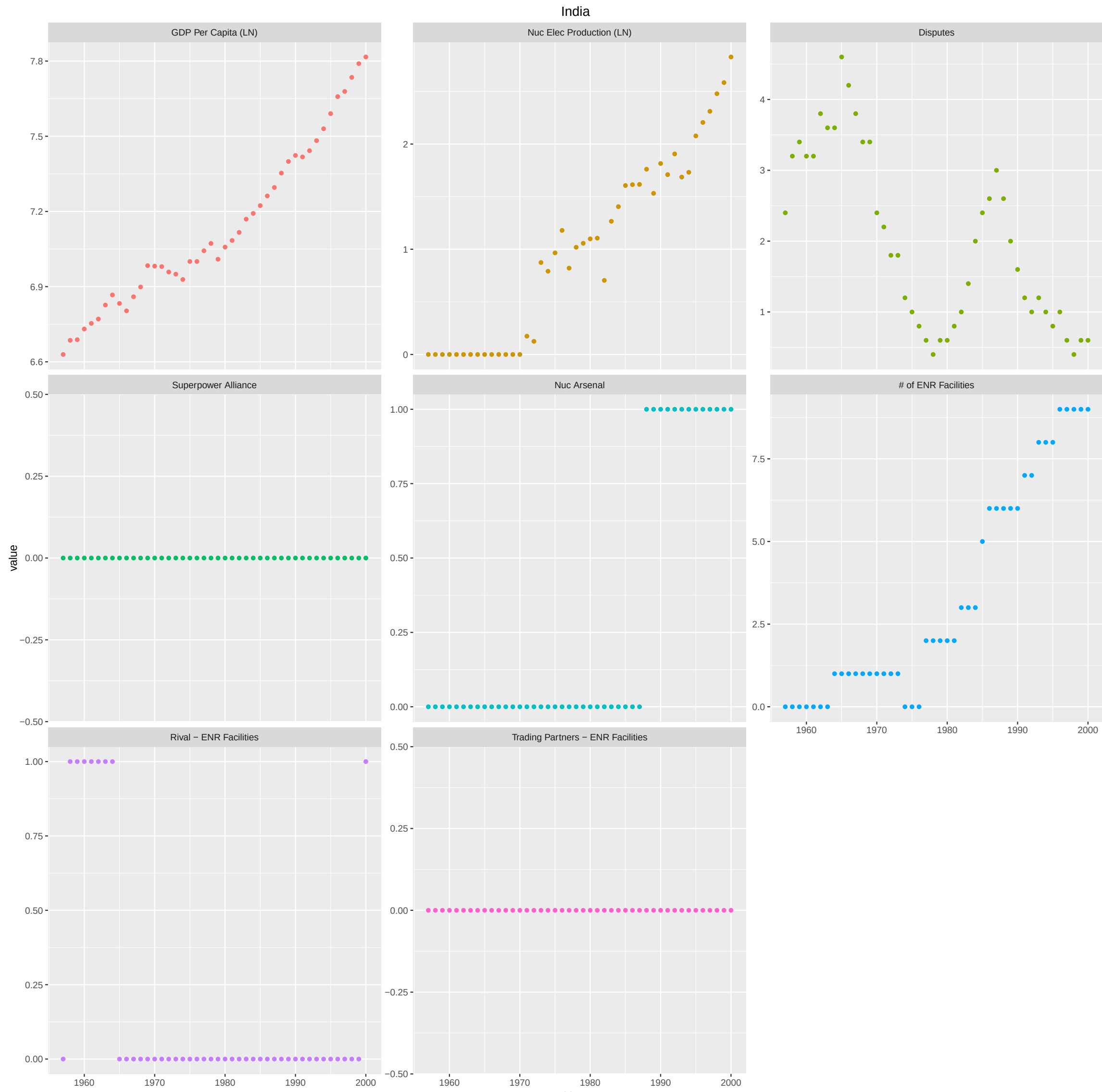

Figure 5.14: Data for India

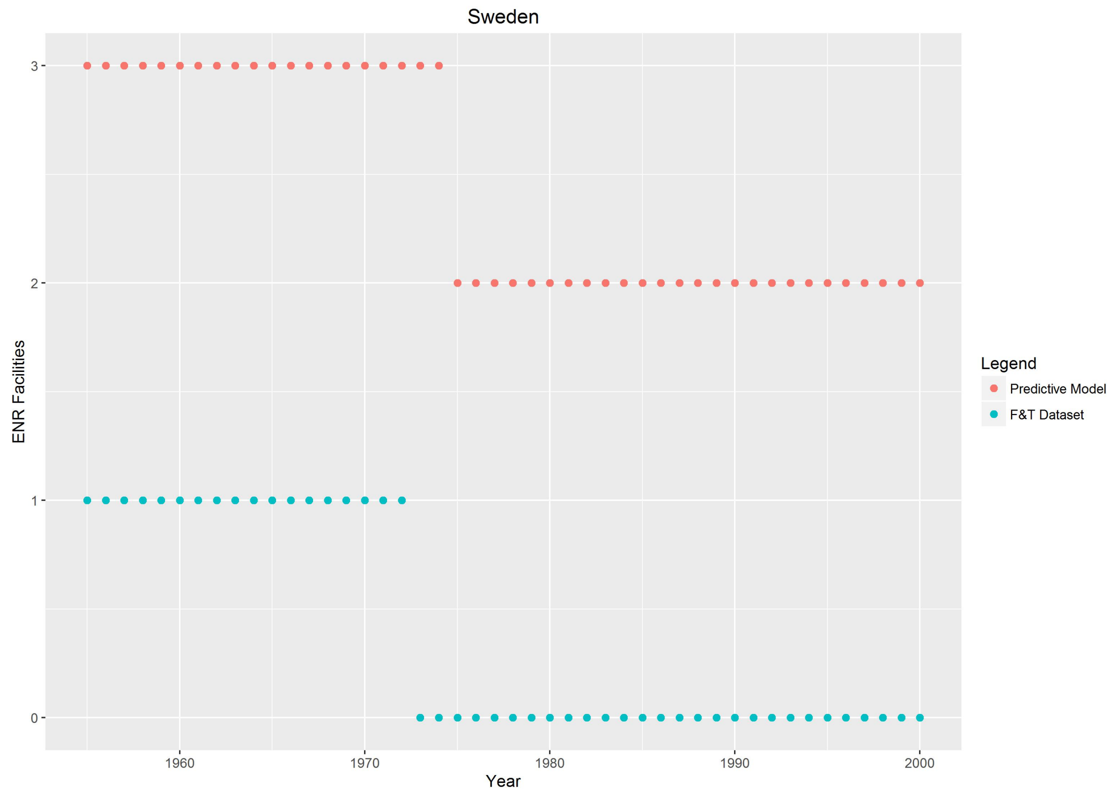


Figure 5.15: Geo-spatial results for Sweden (Tiered-EM)

The results from the EM case are far better than those found using the basic model (see Figure 5.8). The results found in the predictive model with the EM algorithm do not seem to differ much from the actual development except for sheer magnitude.

Sweden was highly motivated to invested nuclear weapons research for deterrence purposes. The technical capabilities for Sweden don't seem to be driving factors for either the results from the predictive model or the actual development of ENR facilities. Once Sweden signed the NPT, they reached a deal with the USA to provide materials that would kick start their civilian nuclear program. The drop in the ENR facilities around 1972 occurs from the singing of the NPT. Nuclear electricity production as well as the number of disputes Sweden had both decreased as a result.

It is actually interesting to see this drop in disputes, as Sweden prescribed to political non-alignment. Note that the NPT signing did not have an impact on the GDP Per capita, in fact Sweden's economy continued to grow as a result of further development of a civilian nuclear program.

The predictive model uses the initial strength of the technical capabilities to suggest that Sweden had the capability to have three ENR facilities. However, in reality only one ENR facility was operational. Besides the difference in the magnitude, the trends from the predictive model are identical to that of the actual development of ENR facilities in Sweden.

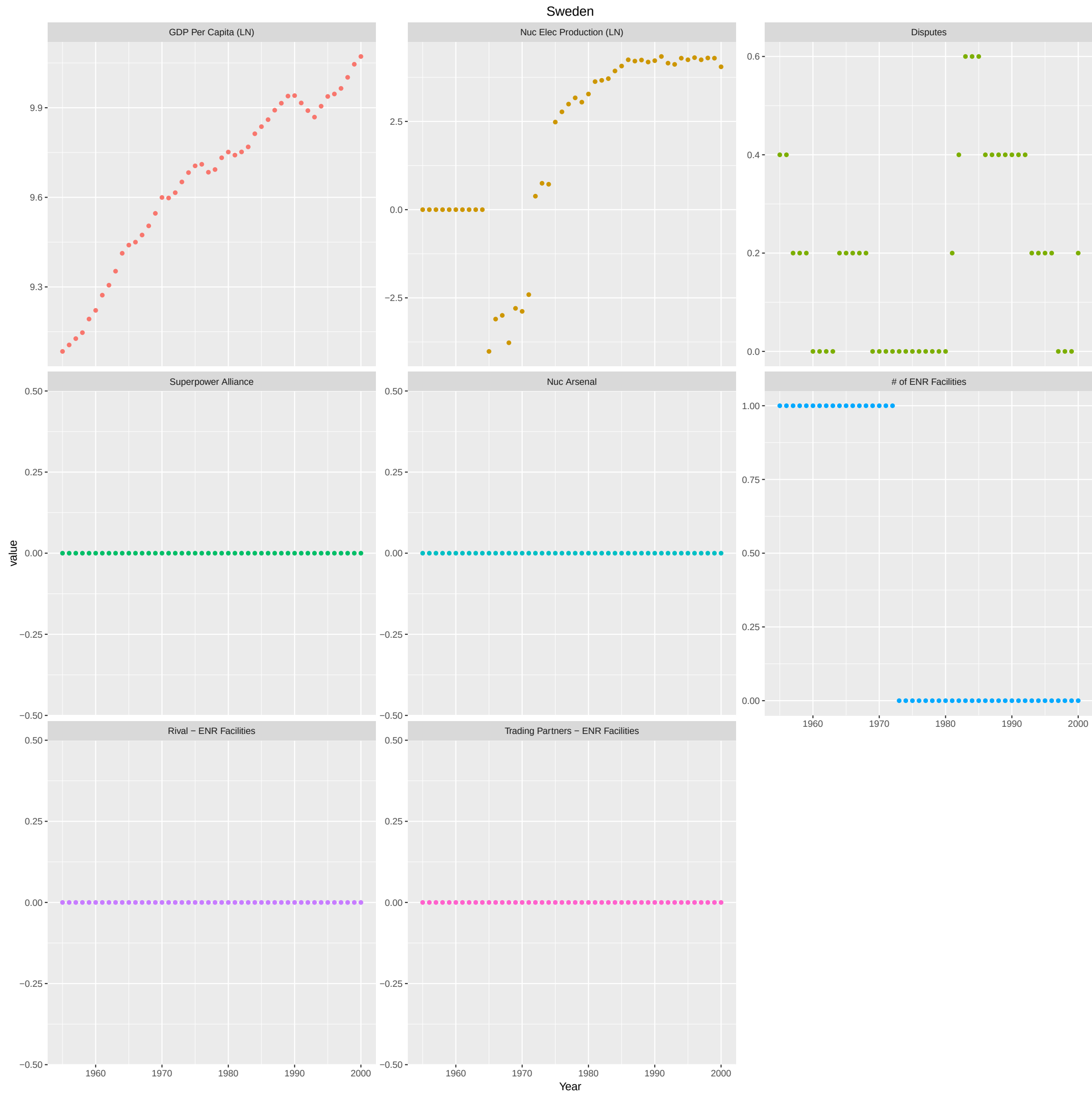


Figure 5.16: Data for Sweden

### 5.2.3 Tiered Model: Sensitivity Analysis

A sensitivity analysis was performed to determine the effect data uncertainty would have on the developed Bayesian networks. In order to assess data uncertainty, a 25% decreased was placed upon the dataset provided. The 25% decrease was specifically placed on the dependent variable (number of ENR facilities) before it was discriminated (see Table A.3). After the dataset was altered, the same procedure was followed to train and produce estimates from the tiered model. The models in this case were also trained with the EM learning method as it was shown to be the best

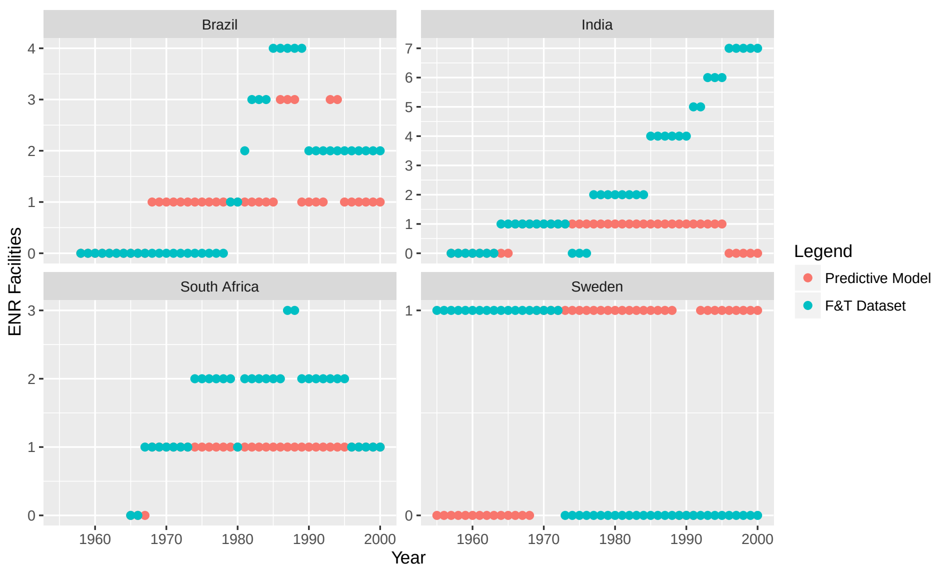

Figure 5.17: Sensitivity Results for all Cases (Tiered-EM)

The Brazilian case tends to follow a very similar trend to that of the actual number of ENR facilities Brazil had. Additionally, this 25% decrease in the actual number is reflected in the predictive model as well, the magnitude of the predictive model results are not larger than those of the actual number in any of the case studies. This sensitivity analysis seems to have affected the predictive nature of the model. Notice that the model does not capture the nuclear arsenal node, and the voluntary dismantlement of nuclear weapons. Errors in the dataset can affect the driving factors in the predictive model.

More specifically, this sensitivity analysis highlights a complication. When placing this 25% decrease on the dependent variable, this affects the numerical value but does not affect the categorical value. For example, the 25% decrease on a base of 10 facilities would translate in to 8 facilities, 7.5 to be exact. A state that has 6 to 11 facilities are both categorized in the medium-high range (refer to Table A.3). The above example falls in the same categorical range. Future work should consider a different approach for an uncertainty analysis.

Thus it is extremely important to select or develop a dataset appropriately. When selecting a dataset ensure that it has been cross validated with multiple sources or has been used extensively.

#### *5.2.4 Tiered Model: Smoothed Results*

Central limits (median,mode,mean) were used to smooth the data in three year time frames based on the type of variable. The data was smoothed to test for inconsistencies such as those one-year discrepancies found in the results of the basic models (Figure 5.6 to 5.8).

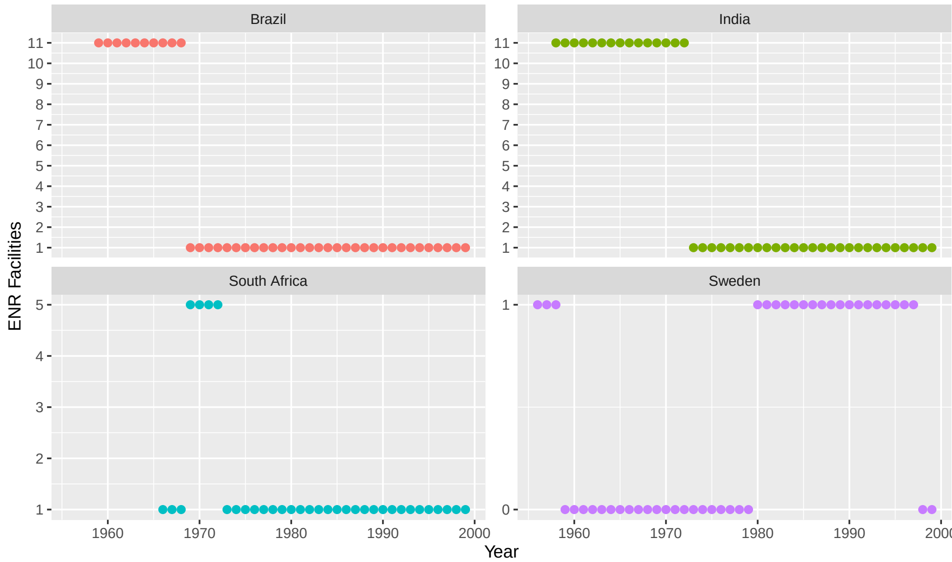


Figure 5.18: Smoothed Results for all Cases (Tiered-EM)

Immediately, we see some glaring errors. Both the Brazilian and Indian cases produce identical results. Neither result matches the historical implications as well as the actual number of ENR facilities these states had. However, these results do make sense. When data is smoothed the inherent characteristics of the data change, especially when each variable is smoothed with a different central limit. For example, the dependent variable (number of ENR facilities) was smoothed based on the mode of 3 year subsets; while one of the independent variables (GDP Per Capita) was smoothed based on the mean of 3 year subsets.

Therefore different central limits applied to smooth each variable introduce uncertainties in the model. These uncertainties seem to mask the predictive nature of the model and affect the results obtained. Overall it is unnecessary to smooth data, especially with studies associated with proliferation risk. This type of smoothing can mask the intended effect expected from a model.

### 5.2.5 Tiered Model: Expert Elicitation Results

An expert elicitation was conducted, as another method to obtain data for nodes without data. In the Fuhrmann and Tkach dataset, both the Technical Capability and Motivation nodes do not have data. Therefore, results from the expert elicitation were used to develop conditional probabilities tables for the model. The expert elicitation resulted in two CPTs, one for the Technical Capability node and another for the Motivation node. A detailed explanation of the expert elicitation process can be found in Appendix C.

After these CPTs were developed, the exact same procedure was used to create visualizations for each state. Figure 5.19 compares results from the predictive model (based on the expert elicitation) and the historical data.

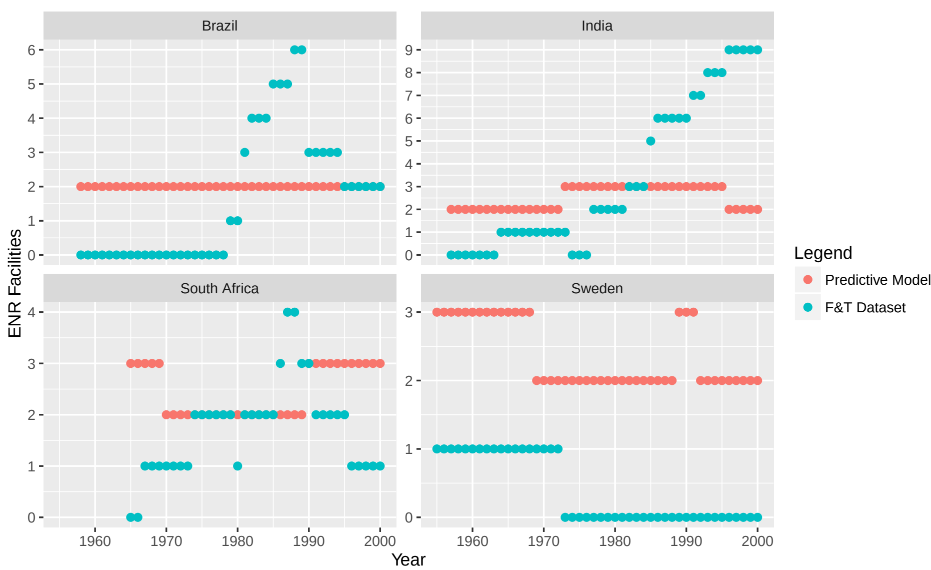


Figure 5.19: Expert Elicitation results for all cases

From Figure 5.19, it seems that the predictive model is much more conservative than the historical data. This result is expected as knowledge based experts understand both the failures and successes as a result of a weapons program. Experts as a result will have more conservative answers to the elicitation.

The expert elicitation based model does not capture the nuclear arsenal node, which is highlighted in the South African case. In 1989, South Africa voluntarily dismantled nuclear weapons. With regards to the Swedish case, the predictive model follows an identical trend except between 1989 to 1991. This occurs due to the combined increase in the following independent variables for Sweden: GDP Per Capita, Nuclear electricity production and the number of disputes. These indicators affect both the technical capability and motivation nodes, which could be a possible reason for the increase in the estimated number of ENR facilities between 1989 to 1991 for Sweden. From the Indian case, it seems that motivations are just as much of a driving factor as technical capabilities. The decrease in disputes between 1970 and 1980 seem to outweigh the technical capabilities when providing estimates. Note, in Figure 5.13 the technical capabilities drive the estimation far more than any other nodes.

From Figure 5.19, it seems that experts are more conservative in their approach to assessing nuclear proliferation. Unfortunately, for these cases it does not seem to match the historical data. A more extensive expert elicitation could mitigate issues such as not capturing the nuclear arsenal node.

# 6. FURTHER ANALYSIS

There are a few analysis steps required to ensure model specification, which incorporates structure and variable definition. After which, model verification is required to determine its fit with the data. The additional analysis complements the case studies as it evaluates specific statistics to determine model fit with the data.

First and foremost, it is important that model specification is studied. More specifically, with Bayesian networks (Figure 4.1 and 4.2) it is important to assess confidence between existing and non-existing links. The network structure was assessed, with the use of a methodology previously developed by PNNL scientists. This methodology results in graphical figures that depict dependence or independence of Bayesian network nodes [46] [47].

Once model specification is ensured, the next step is model verification. Each model was assessed based on its fit to the data, with the use of the correlation statistic. There are occurrences where prediction accuracy is not a good measure for model fit. This occurs because of drastically different dynamic relationships between nodes for each tested state. As a result the correlation statistics is used to capture how accurate the predictive model results are. the correlation coefficient is calculated both on the model and state levels, between the predictive results and "truth" data, in this case the Fuhrmann and Tkach dataset. Even though other metrics could be used, the correlation coefficient provides insight on how well predictive models estimated the number of ENR facilities over time (with respect to trends and magnitude). The prediction accuracy is also reported for each model for reference purposes.

## 6.1 Model Analysis

Model specification is required for the tiered model, as it introduces links between independent variables and also incorporates the use of two intermediate nodes

(Technical Capability and Motivations). Please refer to Figure 4.2 for the Tiered model, regardless of the learning method used. Note that when testing variable relationships, it is important to test those that are specified in the model as well as those that don't exist. The following variable relationships were tested:

- Disputes and Rivals,
- Disputes and Superpower Alliance,
- GDP Per Capita and Nuclear Electricity Production,
- GDP Per Capita and Superpower Alliance,
- Nuclear Electricity Production and Nuclear Arsenal,
- Nuclear Electricity Production and Trading Partner(s),
- Nuclear Arsenal and GDP Per Capita, and
- Nuclear Arsenal and Trading Partner(s).

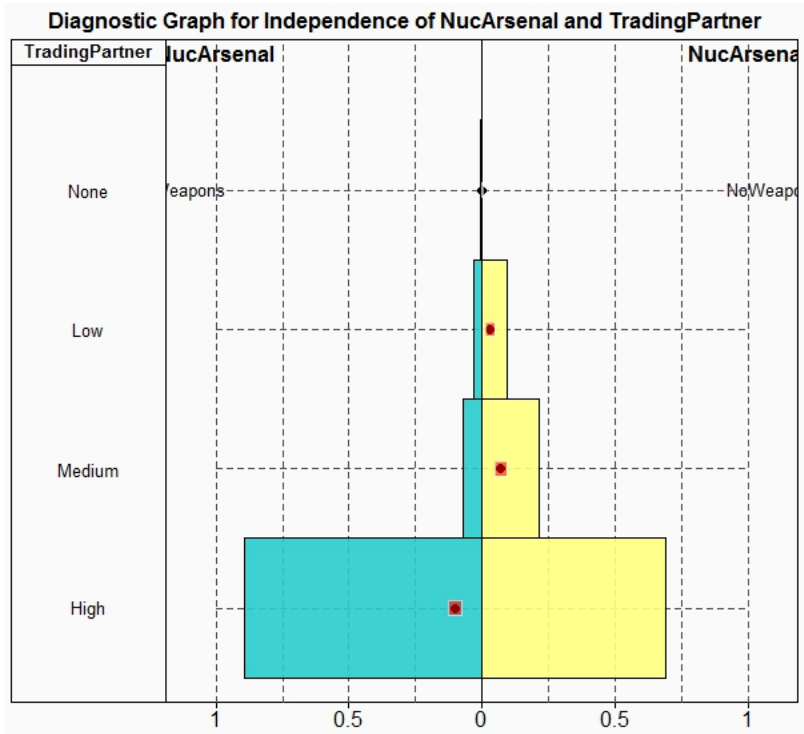

Figure 6.1: Diagnostic Graph for Nuclear Arsenal and Trading Partner

Figure 6.1 depicts the interdependence between Nuclear Arsenal and Trading partners. Based on literature review, prior to model development, these two nodes were to be dependent upon one another. It suggests that the presence of a Nuclear Arsenal would affect the trading partners a state would have.

To decipher Figure 6.1 it is important to recognize the null hypothesis along with characteristics of the diagnostic graph. The null hypothesis is that the two variables tested are independent of one another. These diagnostic plots show pairwise comparisons of the empirical probability two states of one variable given a particular state of another variable. In Figure 6.1, each row corresponds to a specific state of the Trading Partner node. The corresponding yellow and blue bars represent the observed states of the Nuclear Arsenal variable (No Weapons v. Weapons, respectively). The length of each bar is dependent upon the number of times these combination of states occurs in the provided dataset.

If two variables are independent, each pair of bars are to be symmetric about x intercept. Alongside the yellow and blue bars, each row has a point estimate indicated with a black point. These point estimates have an accompanying confidence interval, represented by a red or green color over the black point. Confidence intervals will be green, when they intersect the x-intercept and red otherwise. If a confidence interval does not intersect with the x-intercept, then the variables are dependent. Thus with this background information, reviewing Figure 6.1 shows that these two variables are dependent. Beside the "none" state for Trading Partner, confidence intervals for all other states do not intersect the x-intercept. As a result, this test rejects the null hypothesis.

To find all of the diagnostic graphs, see Appendix B. Figure 6.2 shows the diagnostic dependence between Nuclear Electricity Production and Trading Partners.

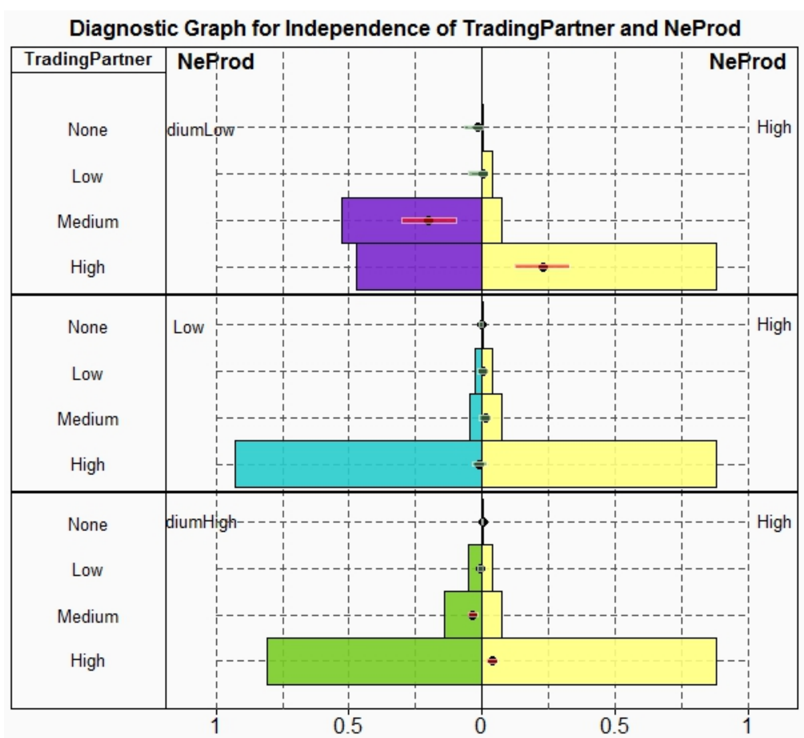


Figure 6.2: Diagnostic Graph for Nuclear Electricity Production and Trading Partner

For this case, the graph observes all four states for Nuclear Electricity Production with respect to the four states of the Trading Partner node. The code used is instructed to develop an even number of graphs on both sides of the axis, hence why the high node is repeated three times. This repetition has no bearing on analysis. From Figure 6.2, the pairwise combination between "Low" and "High" states for Nuclear Electricity production depict that these two variables are independent. However to fail to reject the null hypothesis, this must occur for all state pairings which it doesn't. Therefore, in this case these variables are also dependent. The network, seen in Figure 4.2, shows that Nuclear Electricity Production feeds into the Trading Partner node.

For further dependence analysis, see Appendix B (Figures B.1 to B.6).

There are a few metrics presented that will ensure model verification. As stated above, the correlation statistics and the predication accuracy will be studied to ensure the model represents the provided dataset to the best of its ability. Besides assessing the model with respect to results, the components of the models must also be studied.

The correlation coefficient represents a quantitative measure of dependent between a set of values. The correlation value identifies the strength of a linear relationship between two chosen variables. In this case, the two sets of values are results from the predictive model and the actual number of ENR facilities from the Fuhrman and Tkach dataset. This coefficient will be tested for each model to identify the best model. This correlation coefficient ranges from -1 to 1. If the coefficient is 1, then the there is a very strong linear relationship between the two variables chosen. If the coefficient is 0, then there is no linear relationship between the two, and finally if the coefficient is -1, then there is a negative linear relationship between the two. Prediction accuracy is a percentage value that is displayed as a decimal in Table 6.1.

The higher the decimal value the more accurate the model in guessing exact values per year.

Table 6.1: Model Type Analysis

| Model Type | Correlation | Prediction Accuracy |
|---|---|---|
| Basic Model | 0.45 | 0.39 |
| Tiered Model (EM) | 0.78 | 0.06 |
| Tiered Model (Gradient) | 0.50 | 0.14 |
| Expert Elicitation Model | 0.40 | 0.05 |

The Basic Model provides the most accurate results, according to the prediction accuracy reported. The tiered model, regardless of learning method, introduces two intermediate (classifier) nodes for which there is no definite data. As a result, the EM and Gradient descent methods were used to train the model appropriately. These intermediate nodes introduces some uncertainty as the conditional probability tables developed are not from definite data. They are based on the parent nodes and an expected convergence term. Another option was to use expert knowledge to develop the necessary CPTs for the intermediate nodes. The correlation statistic however deems this to be poor in comparison to other methods. It must be noted that the experts consulted seem to take a conservative approach to proliferation, which probably affected the results in comparison to the historical data.

The correlation values shown in Table 6.1 represent the overall comparison between the estimated results and the actual number of ENR facilities. However, to get a better understanding of the strength of these estimations histograms were developed. The data for the histograms represents correlation coefficients and prediction accuracy per state. The red line on the correlation coefficient histogram represents the median value.

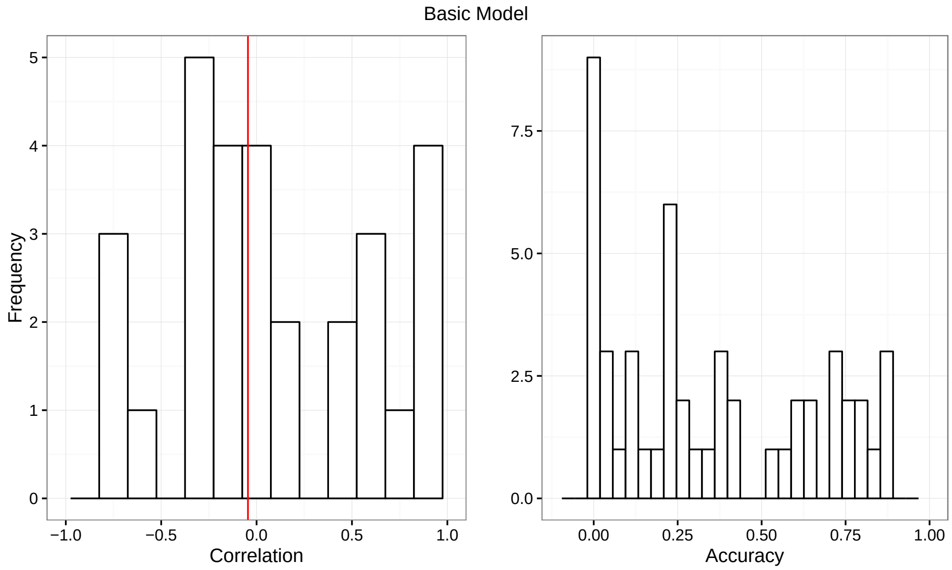


Figure 6.3: Data for Basic Model

From Figure 6.3, the basic model has respectable predication accuracy results. However, the median correlation coefficient sits at about -0.04. This suggest that there is no relationship between the sets of data. It is important to recognize the purpose of the basic model. It was used to determine whether the appropriate variables were identified to measure the number of ENR facilities.

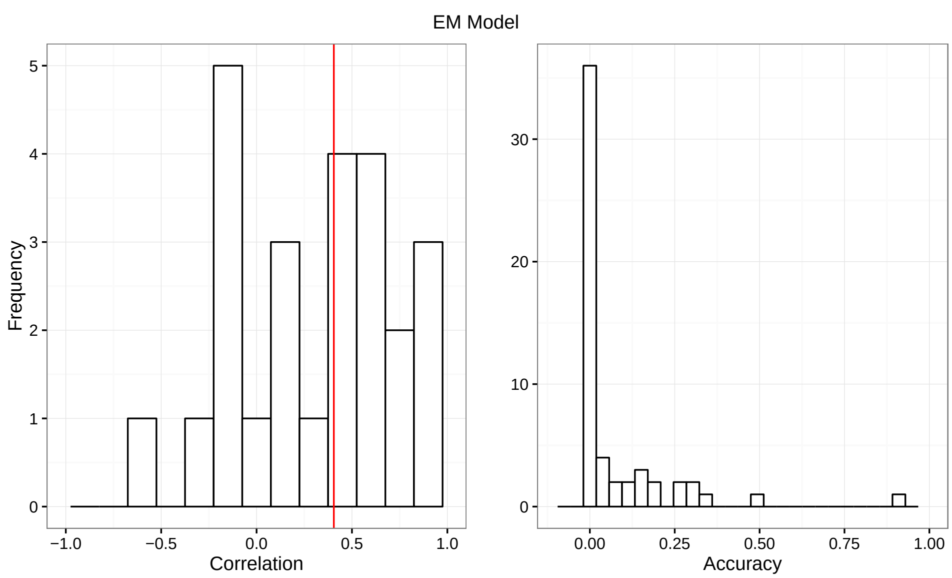


Figure 6.4: Data for EM Model

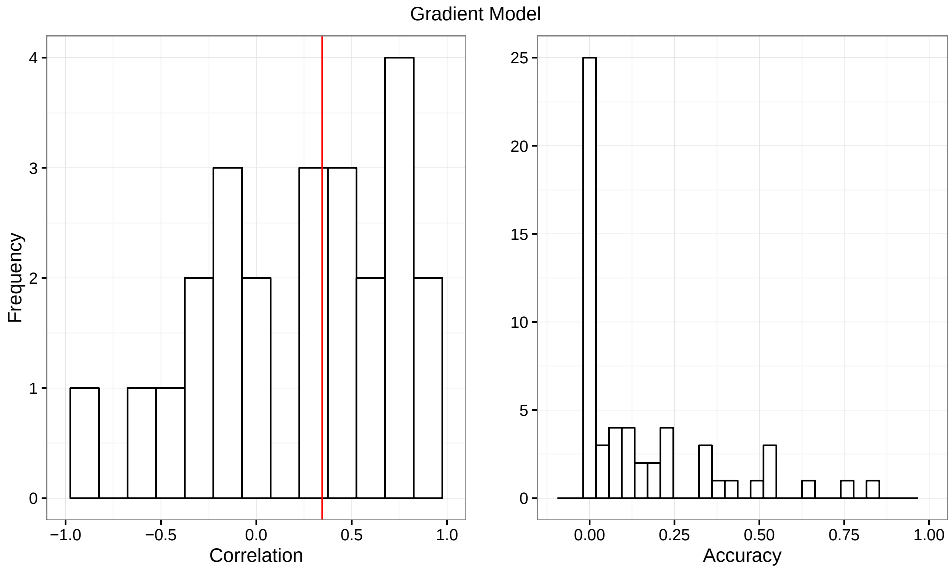


Figure 6.5: Data for Gradient Model

Figures 6.4 and 6.5 allow for comparison between the two learning methods. Note that the EM learning method is primarily used when data is missing, as is this case with the intermediate nodes. The gradient descent method is primarily used to fine tune models after they are initially trained. From the right side of each figure, we see that gradient descent model has better prediction accuracy results. However, prediction accuracy can not be used as an appropriate measure to assess the model as both have over 20 cases with a 0% accuracy. The left side of each figure indicates that the median correlation coefficient is slightly higher for the EM model than it is for the gradient descent model. The overall correlation coefficient for the EM model (as shown in Table 6.1) is 0.78, while the median correlation coefficient is 0.40. The gradient descent model has a overall correlation coefficient of 0.50, while the median correlation coefficient is 0.35. It is important to study both results as the individual correlation coefficient can be found to be NA, since the actual data suggested that there were no ENR facilities for that state. This is not a calculation error but instead just a result of the correlation calculation and the data provided.

For the state specific correlation values and prediction accuracy, please refer to Table 6.2. Note that the Indian case had the best correlation coefficient, while the South African case had the best predication accuracy, regardless of model type. It is also noteworthy to recognize the drastically different correlation coefficient results for the Swedish case.

Therefore these results confirm that the EM method is the learning method of choice when developing advanced models such as the tiered model. This analysis provides confidence in the EM method results, shown in Section 5.

Table 6.2: Case Specific Analysis

| Country | Basic Model | | EM Model | | Gradient Model | |
|---|---|---|---|---|---|---|
| | Correlation | Prediction Accuracy | Correlation | Prediction Accuracy | Correlation | Prediction Accuracy |
| Brazil | 0.12 | 0.28 | 0.52 | 0.19 | 0.52 | 0.09 |
| India | 0.88 | 0.02 | 0.90 | 0.11 | 0.87 | 0.22 |
| Sweden | 0.46 | 0.24 | 0.53 | 0.00 | 0.23 | 0.24 |
| South Africa | 0.64 | 0.36 | 0.61 | 0.28 | 0.68 | 0.33 |

# 7. CONCLUSIONS

Different Bayesian network models were developed to estimate the correlates of nuclear proliferation and nuclear energy. This study measured correlates as the number of Enrichment and Reprocessing facilities a state has. The work on geospatial factors and correlates in Fuhrmann and Tkach complemented this study.

Results from the most refined model, in this case the tiered model with an EM learning method, produced estimations for the number of ENR facilities a given state has. These estimations compared reasonably well with the historical data, based on correlation coefficients and predication accuracy metrics. Estimates from other models did not perform as well as the tiered model with an EM learning method. However, these models all produced similar trends to those found in the historical data categorized by the Fuhrmann and Tkach for nearly all countries during 1945-2010.

It is possible to further refine these models such that there is better agreement between estimates and the historical data. This falls in line with the common understanding that proliferation decisions are quite complex and must be studied on a case by case basis. This limits potential for research to verify historical data and potentially forecast the future.

Based on the results from this study, learning methods associated with Bayesian networks have shown to effectively estimate proliferation indicators. The learning methods were much more favorable than the results of the expert elicitation. One of the many pitfalls with expert elicitation is the potential bias that can be introduced. An unforeseen bias in this elicitation process was the conservative approach shown by experts. This conservative approach taken by experts resulted in drastically different estimates than seen in the other models. A more detailed expert elicitation could avoid such bias.

From the results and analysis seen in Chapters 5 and 6, the overall goal was met. This specific study used the number of ENR facilities as a proliferation indicator. The models did fairly well to represent independent variables related to nuclear history, as long as these variables were accompanied with data. There are algorithms that can be used to account for missing data, this is only recommended when there are a few missing variables. It is highly recommended to develop models for which data is present or easily accessible. In addition the majority of the objectives were also fulfilled. However, the results from the sensitivity analysis suggest that future work needs to be considered, for this specific objective. This study also provided insight on the use of Bayesian Networks in nuclear weapons proliferation research. From the work conducted, Bayesian Networks seem to be an appropriate tool to measure nuclear weapons proliferation especially considering the dynamic nature of both the NFC and nuclear weapons proliferation.

## 7.1 Future Work

With these types of studies there are several possible areas for future work. The first obvious realization is to further specify the networks developed. In order to do this, the appropriate dataset must also be chosen. The model can only be further specified if data exists. If it is feasible, data verification is recommended. However, this is not feasible in most cases as datasets will be upwards of 2,000 lines or more.

Even though the Bayesian network software (Netica) utilized provided inherent cross-validation methods, it is important to have large datasets. These large datasets can be split into testing and training data, which can allow the user to preselect the cross-validation method. This could prevent the need for a learning method algorithm such as the EM or gradient descent methods discussed in this study. Another option to prevent the need for learning method algorithms is the use of expert elicitations. A larger sample size for the expert elicitation should produce highly correlated results with the historical dataset. Larger sample sizes should remove any bias such as the conservative proliferation estimates seen in Figure 5.19.

Alongside this, it would be pivotal to estimate the total ENR capacity a state has. This type of measurement would delineate the difference between 10 laboratory facilities and 3 commercial facilities. In order to complete this exact study with capacity instead of the sheer magnitude, a dataset would need to be developed. Unfortunately, capacity data for ENR facilities is not widely accessible for states currently or historically as this may or may not be confidential information.

A more rigorous uncertainty quantification would provide more insight to the model. The discrete states in each node created some issues when carrying out an uncertainty test. With the current model, an impactful uncertainty analysis would require testing a 50% decrease on the dependent variable.

The current Bayesian Network has the ability to forecast. However, there are some flaws such as the sensitivity and the need to account for various other indicators. A more refined model will have the capability to forecast more accurately and the potential to be used as a policy tool.

## REFERENCES


[1] J Cirincione, J. B. Wolfsthal, and M. Rajkumar. *Deadly Arsenals (Tracking Weapons of Mass Destruction).* A Carnegie Endowment book. Carnegie Endowment for International Peace, 2002.

[2] S. M. Meyer. *The Dynamics of Nuclear Proliferation*. Political science: International relations. University of Chicago Press, 1986.

[3] Senator John F. Kennedy United States. *Third Nixon-Kennedy Presidential Debate*. October 13, 1960.

[4] S. Singh and C. Way. The Correlates of Nuclear Proliferation: A quantitative test. *Journal of Conflict Resolution*, 48(6):859–885, 2004.

[5] M. R. Mella. Pathways Analysis for State Proliferators. Masters of Science, Texas A&M University, May 2011.

[6] M. Subbaiah and W. Charlton. Effect of Grouping Motivations on State Proliferator Pathways. In *Transactions of American Nuclear Society for 2014 American Nuclear Society Winter Meeting*, pages 1028–1031, Anaheim, CA.

[7] J. E. C. Hymans. Theories of Nuclear Proliferation The State of the Field. *The Nonproliferation Review*, 13(3):455–465, Nov 2010.

[8] S. Sagan. Why Do States Build Nuclear Weapons?: Three Models in Search of a Bomb. *International Security*, 21(3):54–86, 1996.

[9] R. J. Einhorn. Identifying Nuclear Aspirants and Their Pathways to The Bomb. *The Nonproliferation Review*, 13(3):491–499, 2006.

[10] C. R. Freeman. Bayesian Network Analysis of Nuclear Acquisitions . Master of Science, Texas A&M University, August 2008.

[11] E. Solingen. The Political Economy of Nuclear Restraint. *International Security*, 19(2):126–169, 1994.

[12] M. Fuhrmann and M. C. Horowitz. When Leaders Matter: Rebel Experience and Nuclear Proliferation. *The Journal of Politics*, 77(1):72–87, Jan 2015.

[13] E. Gartzke. Nuclear Proliferation Dynamics and Conventional Conflict. *Unpublished manuscript*, 2010.

[14] K. N. Waltz. Nuclear Myths and Political Realities. 84(3):731–745, 1990.

[15] J. Mueller. The Essential Irrelevance of Nuclear Weapons: Stability in the Postwar War. 12(2):55–79, 1988.

[16] A. Montgomery and S. Sagan. The Perils of Predicting Proliferation. *Journal of Conflict Resolution*, 53(2):302–328, April 2009.

[17] E. Gartkze and D-J Jo. Determinants of Nuclear Weapons Proliferation. *Journal of Conflict Resolution*, 51(1):167–194, February 2007.

[18] R. K. C. Hersman and R. Peters. NUCLEAR U-TURNS. *The Nonproliferation Review*, 13(3):539–553, 2006.

[19] S. S. Chirayath. Introduction to Nuclear Fuel Cycles and Safeguards. Lecture, 2015.

[20] International Atomic Energy Agency. Nuclear Fuel Cycle & Materials.

[21] W. K. H. Panofsky. Capability versus intent: The latent threat of nuclear proliferation. *Bulletin of the Atomic Scientists*, 2007.

[22] R.G Cochran and N. Tsoulfanidis. *The Nuclear Fuel Cycle: Analysis and Management*. 1999.

[23] G. Kessler. *Sustainable and Safe Nuclear Fission Energy: Technology and Safety of Fast and Thermal Nuclear Reactors*. Power Systems. Springer Berlin Heidelberg, 2012.

[24] W. Jianchen and C. Jing. Separation of plutonium and neptunium from uranium by N,N-dimethyl-3-oxa-glutaramic acid (DMOGA) as a complexant in the {PUREX} process. *Hydrometallurgy*, 157:44 – 49, 2015.

[25] K. J. Moody, P. M. Grant, and I. D. Hutcheon. *Nuclear Forensic Analysis*. CRC Press, 2005.

[26] T. Someya and A. Yamaji. Core design of a high breeding fast reactor cooled by supercritical pressure light water. *Nuclear Engineering and Design*, 296:30–37, 1 2016.

[27] M. Kroenig. Exporting the Bomb: Why States Provide Sensitive Nuclear Assistance. *American Political Science Review*, 103:113–133, 2 2009.

[28] R. A. Elmore and W. S. Charlton. Nonproliferation informatics: Employing Bayesian analysis, agent based modeling, and information theory for dynamic proliferation pathway studies. In *Intelligence and Security Informatics (ISI), 2015 IEEE International Conference on*, pages 43–48, May 2015.

[29] R. E. Neapolitan. *Probabilistic Reasoning in Expert Systems: Theory and Algorithms*. Wiley, 1990.

[30] T. Koski and J. M. Noble. *Bayesian Networks An Introduction*. Wiley, 2009.

[31] T. A. Stephenson. An Introduction To Bayesian Network Theory and Usage. Research report, Dalle Molle Institute for Perceptual Artificial Intelligence .

[32] F. V. Jensen. *Bayesian Networks and Decisions Graphs*. Springer, 2001.

[33] G. Castillo. Bayesian Network Classifiers. Lecture, 2011.

[34] C. Riggelsen. *Approximation Methods for Efficient Learning of Bayesian Networks*. IOS Press, Amsterdam, The Netherlands, The Netherlands.

[35] S Ramachandran and R. J. Mooney. Revising bayesian network parameters using backpropagation. In *Proceedings of the International Conference on Neural Networks (ICNN-96), Special Session on Knowledge-Based Artificial Neural Networks*, pages 82–87, Washington DC, June.

[36] Norsys Software Corp. *Netica's Manual/Help Guide*. Norsys.

[37] M. Fuhrmann and B. Tkach. A Spatial Model of Nuclear Technology Diffusion.

2015.

[38] G. Coles, A White, V Finch, and M. Subbaiah. Deliverable for Subtask 3. b of Agreement No. B5841, "Correlates of Sensitive Tecnologies" - Draft Report PNNL-24748 . 2015.

[39] J. E. Spence. The Republic of South Africa: Proliferation and the Politics of 'Outward Movement'. In Lawrence M. R. and Larus J., editors, *Nuclear proliferation, phase II*, pages 214–215. University of Kansas Press for the National Security Education Program.

[40] K. L. Adelman and A. W. Knight. Can South Africa go nuclear? *Orbis*, 23(3):640–645.

[41] C.S.B Piani. SAFARI-I: Achieving Converstion to LEU A Local Challenge. Technical report, South African Nuclear Energy Corporation (Necsa).

[42] Comisso Nacional de Energia Nuclear. Cronologia da Energia Nuclear no Brasil. Technical report.

[43] W. Courtney. Brazil and Argentina. In J.A. Yager, editor, *Nonproliferation and U.S Foreign Policy*. Brookings.

[44] E. W. Lefever. *Nuclear Arms in the Third World: U.S Policy Dilemma*. Brookings, 1979.

[45] T. Jonter. The Swedish Plans to Acquire Nuclear Weapons, 1945-1968: An Analysis of the Technical Preparations. *Science & Global Security*, 18(2):61–86, 2010.

[46] P. Whitney, A. White, S. Walsh, A. Dalton, and A. Brothers. Bayesian networks for social modeling . In *Social Computing, Behavioral-Cultural Modeling and Prediction*.

[47] S.J Walsh and Whitney. P. A Graphical Approach to Diagnosing the Validity of the Conditional Independence Assumptions of a Bayesian Network Given Data.

# APPENDIX A

# ADDITIONAL MODEL DEVELOPMENT

## A.1 STATE DISCRETIZATION

Table A.1: Data Discrimination Part 1

| | GDP Per Capita | Nuclear Electricity Production |
|---|---|---|
| Low | $x <= 5.64$ | -6.21 $<= x < 0$ |
| Medium-Low | $5.64 < x < 8$ | $0 <= x < 2$ |
| Medium-High | $8 <= x < 10$ | $2 <= x < 6$ |
| High | $10 <= x$ | $6 <= x$ |

Table A.2: Data Discrimination Part 2

| | ENR Facilities held by Trading Partners | ENR Facilities held by Rivals |
|---|---|---|
| None | $x = 0$ | $x = 0$ |
| Low | $1 < x < 25$ | $1 <= x < 15$ |
| Medium | $25 <= x < 60$ | $15 <= x < 27$ |
| High | $60 <= x$ | $27 <= x$ |

Table A.3: Dependent Variable Discrimination

| | Number of ENR Facilities |
|---|---|
| None | $x = 0$ |
| Low | $1 <= x < 3$ |
| Medium-Low | $3 <= x < 6$ |
| Medium-High | $6 <= x < 11$ |
| High | $11 <= x$ |

Table A.4: Binary Variable Discrimination

| | Super Power Alliance | Nuclear Arsenal |
|---|---|---|
| Yes | $x = 1$ | $x = 1$ |
| No | $x = 0$ | $x = 0$ |

Table A.5: CPT for Technical Capability

| Trading Partners | GDP Per Capita | Nuclear Elec Production | Nuclear Arsenal | Technical Capability | | |
|---|---|---|---|---|---|---|
| | | | | Low | Medium | High |
| None | Low | Low | Weapons | 41.6669 | 36.6665 | 21.6666 |
| None | Low | Low | No Weapons | 56.6668 | 26.1666 | 17.1666 |
| None | Low | Medium-Low | Weapons | 36.6669 | 41.6665 | 21.6666 |
| None | Low | Medium-Low | No Weapons | 78.9693 | 15.4514 | 5.57923 |
| None | Low | Medium-High | Weapons | 24.1667 | 49.1667 | 26.6667 |
| None | Low | Medium-High | No Weapons | 46.6667 | 31.6667 | 21.6667 |
| None | Low | High | Weapons | 21.6667 | 56.6667 | 21.6667 |
| None | Low | High | No Weapons | 36.6667 | 26.6667 | 36.6667 |
| None | Medium-Low | Low | Weapons | 26.9614 | 32.5483 | 40.4903 |
| None | Medium-Low | Low | No Weapons | 51.058 | 34.3454 | 14.5965 |
| None | Medium-Low | Medium-Low | Weapons | 20.8313 | 36.7818 | 42.3869 |
| None | Medium-Low | Medium-Low | No Weapons | 67.6652 | 26.0564 | 6.27838 |
| None | Medium-Low | Medium-High | Weapons | 5.64101 | 36.7495 | 57.6095 |

| | | | | | | |
|---|---|---|---|---|---|---|
| None | Medium-Low | Medium-High | No Weapons | 44.1667 | 34.1667 | 21.6667 |
| None | Medium-Low | High | Weapons | 21.6667 | 54.1667 | 24.1667 |
| None | Medium-Low | High | No Weapons | 34.1667 | 34.1667 | 31.6667 |
| None | Medium-High | Low | Weapons | 18.4484 | 20.0073 | 61.5443 |
| None | Medium-High | Low | No Weapons | 45.2268 | 42.0033 | 12.7699 |
| None | Medium-High | Medium-Low | Weapons | 15.1008 | 39.5449 | 45.3543 |
| None | Medium-High | Medium-Low | No Weapons | 55.9988 | 30.715 | 13.2862 |
| None | Medium-High | Medium-High | Weapons | 6.50257 | 36.3525 | 57.1449 |
| None | Medium-High | Medium-High | No Weapons | 58.6354 | 33.9959 | 7.36869 |
| None | Medium-High | High | Weapons | 21.6667 | 49.1667 | 29.1667 |
| None | Medium-High | High | No Weapons | 31.6667 | 31.6667 | 36.6667 |
| None | High | Low | Weapons | 33.1667 | 33.1667 | 33.6667 |
| None | High | Low | No Weapons | 41.6667 | 34.1667 | 24.1667 |
| None | High | Medium-Low | Weapons | 29.1667 | 41.6667 | 29.1667 |
| None | High | Medium-Low | No Weapons | 64.2685 | 24.4584 | 11.2731 |
| None | High | Medium-High | Weapons | 21.6667 | 41.6667 | 36.6667 |
| None | High | Medium-High | No Weapons | 56.5239 | 31.076 | 12.4001 |
| None | High | High | Weapons | 5.62013 | 39.5197 | 54.8602 |

| | | | | | | |
|---|---|---|---|---|---|---|
| None | High | High | No Weapons | 31.6667 | 31.6667 | 36.6667 |
| Low | Low | Low | Weapons | 39.1667 | 39.1667 | 21.6667 |
| Low | Low | Low | No Weapons | 59.1667 | 23.6667 | 17.1667 |
| Low | Low | Medium-Low | Weapons | 34.1667 | 41.6667 | 24.1667 |
| Low | Low | Medium-Low | No Weapons | 46.6667 | 31.6667 | 21.6667 |
| Low | Low | Medium-High | Weapons | 21.6667 | 51.6667 | 26.6667 |
| Low | Low | Medium-High | No Weapons | 44.1667 | 34.1667 | 21.6667 |
| Low | Low | High | Weapons | 21.6667 | 51.6667 | 26.6667 |
| Low | Low | High | No Weapons | 36.6667 | 31.6667 | 31.6667 |
| Low | Medium-Low | Low | Weapons | 36.6667 | 39.1667 | 24.1667 |
| Low | Medium-Low | Low | No Weapons | 51.6667 | 26.6667 | 21.6667 |
| Low | Medium-Low | Medium-Low | Weapons | 31.6667 | 41.6667 | 26.6667 |
| Low | Medium-Low | Medium-Low | No Weapons | 55.6805 | 37.5526 | 6.76693 |
| Low | Medium-Low | Medium-High | Weapons | 21.6667 | 44.1667 | 34.1667 |
| Low | Medium-Low | Medium-High | No Weapons | 44.1667 | 31.6667 | 24.1667 |
| Low | Medium-Low | High | Weapons | 21.6667 | 51.6667 | 26.6667 |
| Low | Medium-Low | High | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| Low | Medium-High | Low | Weapons | 22.7548 | 35.1215 | 42.1237 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Low | Medium-High | Low | No Weapons | 44.1667 | 31.6667 | 24.1667 |
| Low | Medium-High | Medium-Low | Weapons | 13.2408 | 36.837 | 49.9222 |
| Low | Medium-High | Medium-Low | No Weapons | 56.1468 | 32.8916 | 10.9616 |
| Low | Medium-High | Medium-High | Weapons | 5.46722 | 23.1091 | 71.4237 |
| Low | Medium-High | Medium-High | No Weapons | 62.2589 | 32.1348 | 5.60637 |
| Low | Medium-High | High | Weapons | 21.6667 | 46.6667 | 31.6667 |
| Low | Medium-High | High | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| Low | High | Low | Weapons | 29.1667 | 36.6667 | 34.1667 |
| Low | High | Low | No Weapons | 39.1667 | 36.6667 | 24.1667 |
| Low | High | Medium-Low | Weapons | 26.6667 | 41.6667 | 31.6667 |
| Low | High | Medium-Low | No Weapons | 39.1667 | 31.6667 | 29.1667 |
| Low | High | Medium-High | Weapons | 21.6667 | 36.6667 | 41.6667 |
| Low | High | Medium-High | No Weapons | 61.8232 | 29.2126 | 8.96422 |
| Low | High | High | Weapons | 21.6667 | 46.6667 | 31.6667 |
| Low | High | High | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| Medium | Low | Low | Weapons | 36.6667 | 39.1667 | 24.1667 |
| Medium | Low | Low | No Weapons | 56.6667 | 24.1667 | 19.1667 |
| Medium | Low | Medium-Low | Weapons | 31.6667 | 41.6667 | 26.6667 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Medium | Low | Medium-Low | No Weapons | 44.1667 | 31.6667 | 24.1667 |
| Medium | Low | Medium-High | Weapons | 21.6667 | 46.6667 | 31.6667 |
| Medium | Low | Medium-High | No Weapons | 41.6667 | 34.1667 | 24.1667 |
| Medium | Low | High | Weapons | 21.6667 | 44.1667 | 34.1667 |
| Medium | Low | High | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| Medium | Medium-Low | Low | Weapons | 36.6667 | 36.6667 | 26.6667 |
| Medium | Medium-Low | Low | No Weapons | 41.6667 | 31.6667 | 26.6667 |
| Medium | Medium-Low | Medium-Low | Weapons | 31.6667 | 36.6667 | 31.6667 |
| Medium | Medium-Low | Medium-Low | No Weapons | 41.6667 | 34.1667 | 24.1667 |
| Medium | Medium-Low | Medium-High | Weapons | 21.6667 | 41.6667 | 36.6667 |
| Medium | Medium-Low | Medium-High | No Weapons | 44.1667 | 31.6667 | 24.1667 |
| Medium | Medium-Low | High | Weapons | 21.6667 | 46.6667 | 31.6667 |
| Medium | Medium-Low | High | No Weapons | 26.6667 | 41.6667 | 31.6667 |
| Medium | Medium-High | Low | Weapons | 31.6667 | 31.6667 | 36.6667 |
| Medium | Medium-High | Low | No Weapons | 55.4359 | 33.3438 | 11.2203 |
| Medium | Medium-High | Medium-Low | Weapons | 11.2678 | 43.2715 | 45.4608 |
| Medium | Medium-High | Medium-Low | No Weapons | 58.445 | 28.9084 | 12.6466 |
| Medium | Medium-High | Medium-High | Weapons | 5.95907 | 19.119 | 74.9219 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Medium | Medium-High | Medium-High | No Weapons | 48.5147 | 39.1882 | 12.2971 |
| Medium | Medium-High | High | Weapons | 21.6667 | 41.6667 | 36.6667 |
| Medium | Medium-High | High | No Weapons | 31.6667 | 31.6667 | 36.6667 |
| Medium | High | Low | Weapons | 26.6667 | 39.1667 | 34.1667 |
| Medium | High | Low | No Weapons | 36.6667 | 39.1667 | 24.1667 |
| Medium | High | Medium-Low | Weapons | 26.6667 | 41.6667 | 31.6667 |
| Medium | High | Medium-Low | No Weapons | 28.9438 | 52.1813 | 18.8749 |
| Medium | High | Medium-High | Weapons | 6.4416 | 20.8257 | 72.7327 |
| Medium | High | Medium-High | No Weapons | 60.7596 | 26.2204 | 13.0201 |
| Medium | High | High | Weapons | 7.52629 | 39.2742 | 53.1996 |
| Medium | High | High | No Weapons | 26.6667 | 36.6667 | 36.6667 |
| High | Low | Low | Weapons | 31.6667 | 41.6667 | 26.6667 |
| High | Low | Low | No Weapons | 51.6667 | 26.6667 | 21.6667 |
| High | Low | Medium-Low | Weapons | 31.6667 | 36.6667 | 31.6667 |
| High | Low | Medium-Low | No Weapons | 41.6667 | 31.6667 | 26.6667 |
| High | Low | Medium-High | Weapons | 21.6667 | 41.6667 | 36.6667 |
| High | Low | Medium-High | No Weapons | 41.6667 | 31.6667 | 26.6667 |
| High | Low | High | Weapons | 21.6667 | 41.6667 | 36.6667 |

| | | | | | | |
|---|---|---|---|---|---|---|
| High | Low | High | No Weapons | 29.1667 | 39.1667 | 31.6667 |
| High | Medium-Low | Low | Weapons | 34.1667 | 39.1667 | 26.6667 |
| High | Medium-Low | Low | No Weapons | 39.1667 | 34.1667 | 26.6667 |
| High | Medium-Low | Medium-Low | Weapons | 29.1667 | 39.1667 | 31.6667 |
| High | Medium-Low | Medium-Low | No Weapons | 39.1667 | 36.6667 | 24.1667 |
| High | Medium-Low | Medium-High | Weapons | 26.6667 | 39.1667 | 34.1667 |
| High | Medium-Low | Medium-High | No Weapons | 41.6667 | 34.1667 | 24.1667 |
| High | Medium-Low | High | Weapons | 21.6667 | 44.1667 | 34.1667 |
| High | Medium-Low | High | No Weapons | 24.1667 | 44.1667 | 31.6667 |
| High | Medium-High | Low | Weapons | 26.6667 | 31.6667 | 41.6667 |
| High | Medium-High | Low | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| High | Medium-High | Medium-Low | Weapons | 24.1667 | 36.6667 | 39.1667 |
| High | Medium-High | Medium-Low | No Weapons | 37.2156 | 45.2321 | 17.5523 |
| High | Medium-High | Medium-High | Weapons | 21.6667 | 29.1667 | 49.1667 |
| High | Medium-High | Medium-High | No Weapons | 34.1667 | 36.6667 | 29.1667 |
| High | Medium-High | High | Weapons | 21.6667 | 36.6667 | 41.6667 |
| High | Medium-High | High | No Weapons | 26.6667 | 34.1667 | 39.1667 |
| High | High | Low | Weapons | 21.6667 | 39.1667 | 39.1667 |

| | | | | | | |
|---|---|---|---|---|---|---|
| High | High | Low | No Weapons | 31.6667 | 39.1667 | 29.1667 |
| High | High | Medium-Low | Weapons | 21.6667 | 36.6667 | 41.6667 |
| High | High | Medium-Low | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| High | High | Medium-High | Weapons | 21.6667 | 31.6667 | 46.6667 |
| High | High | Medium-High | No Weapons | 31.6667 | 36.6667 | 31.6667 |
| High | High | High | Weapons | 21.6667 | 26.6667 | 51.6667 |
| High | High | High | No Weapons | 26.6667 | 31.6667 | 41.6667 |

## A.3 GRADIENT METHOD RESULTS

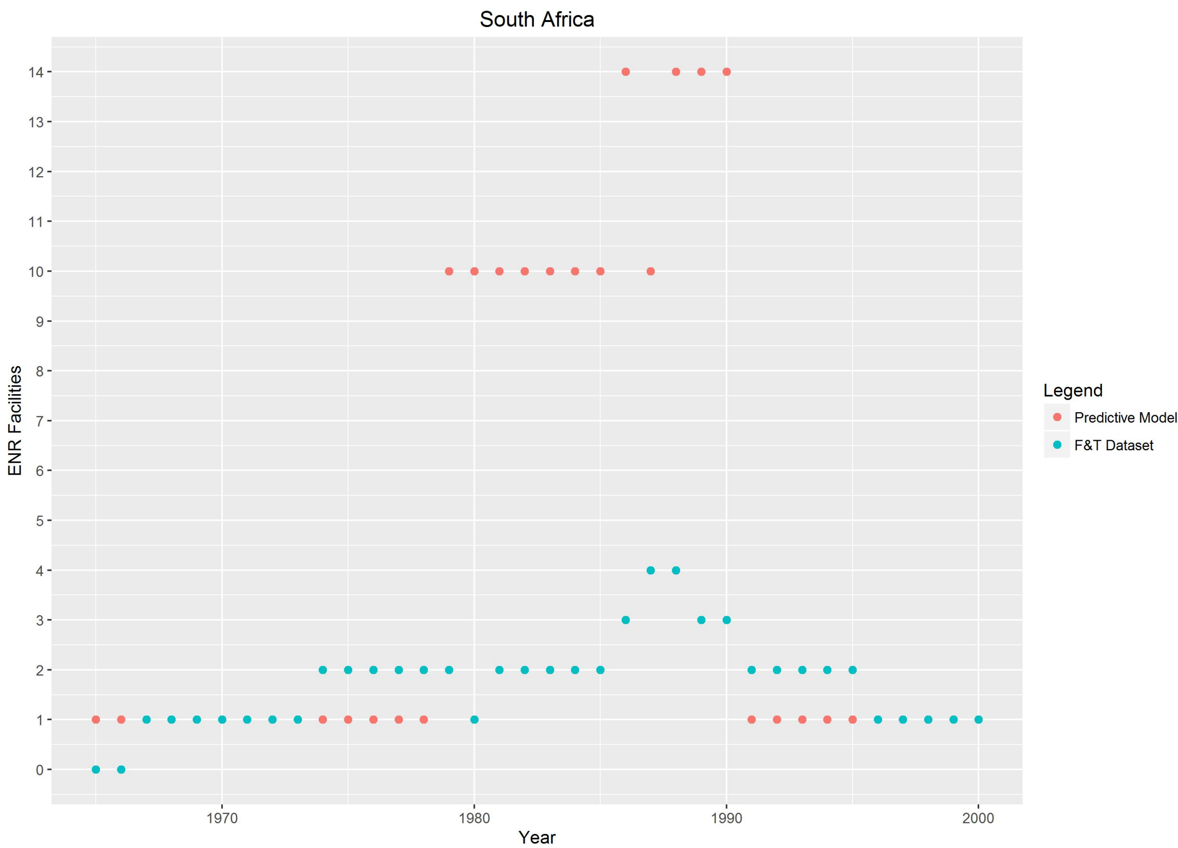


Figure A.1: Geo-spatial results for RSA (Tiered-Gradient)

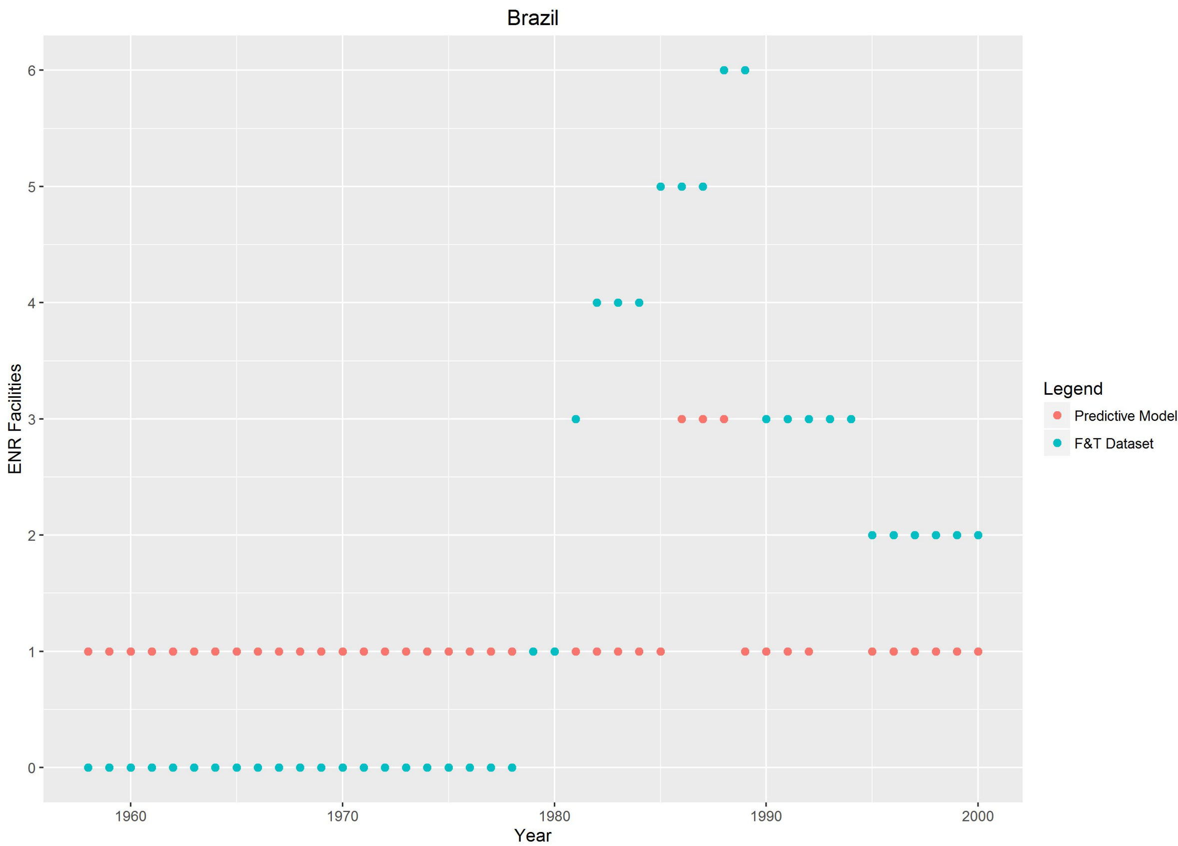

Figure A.2: Geo-spatial results for Brazil (Tiered-Gradient)

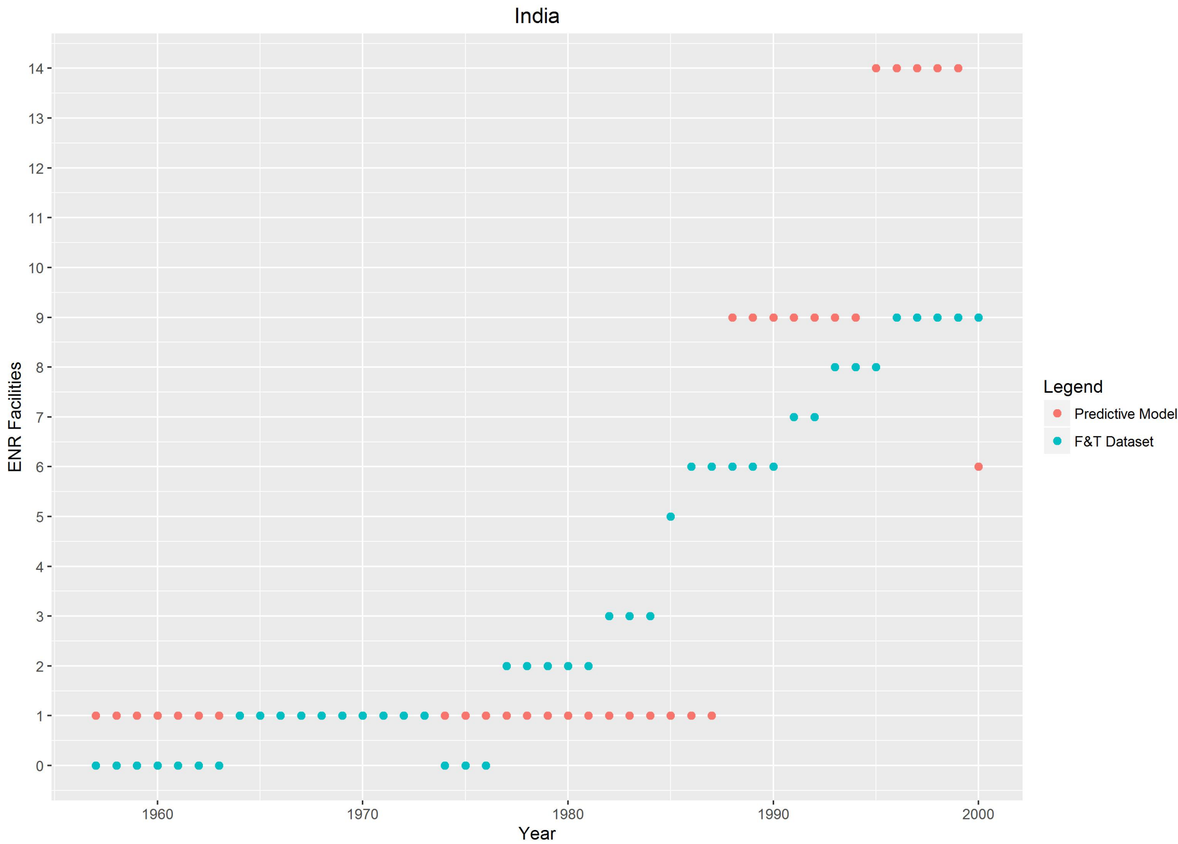


Figure A.3: Geo-spatial results for India (Tiered-Gradient)

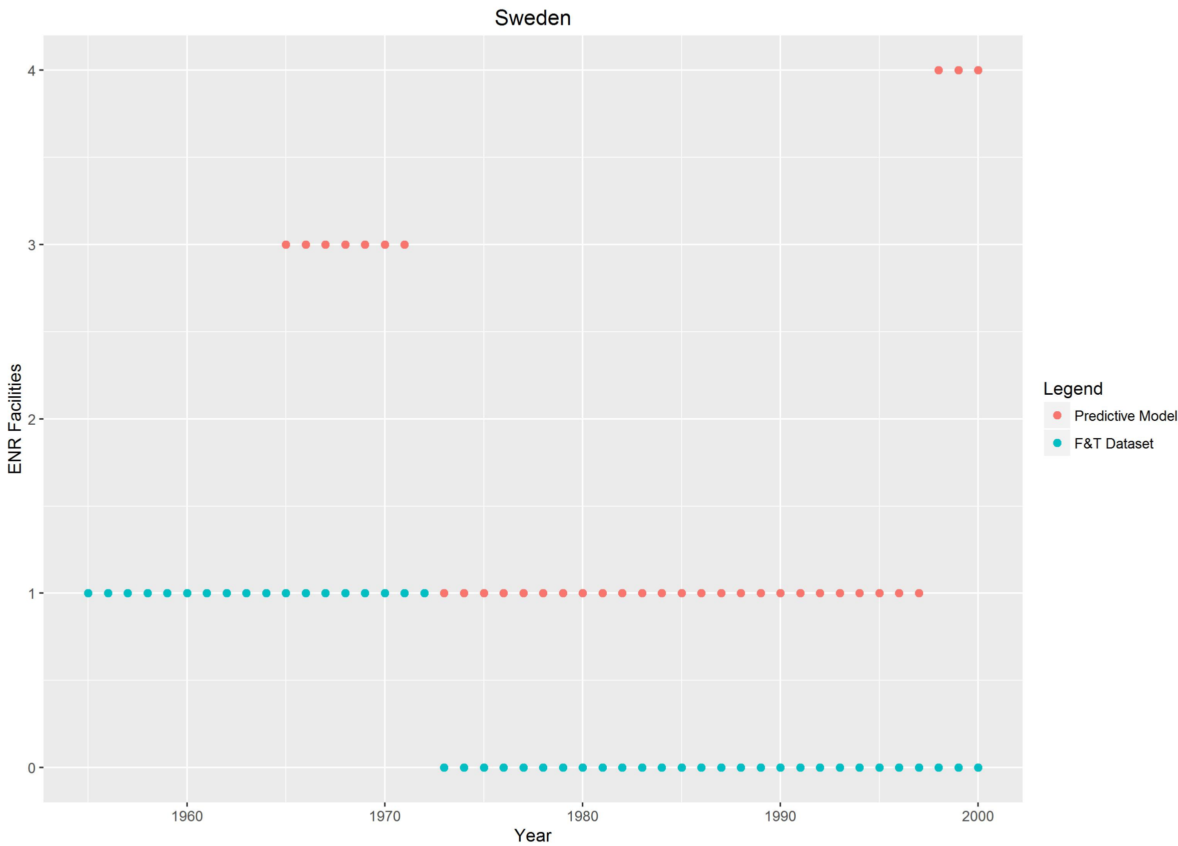

Figure A.4: Geo-spatial results for Sweden (Tiered-Gradient)

APPENDIX B

# CONDITIONAL INDEPENDENCE

## B.1 DIAGNOSTICS GRAPHS

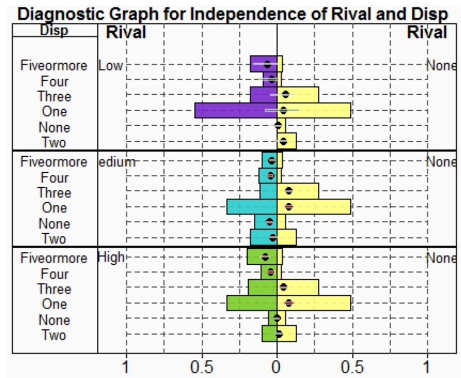

Figure B.1: Diagnostic Graph for Disputes and Rivals

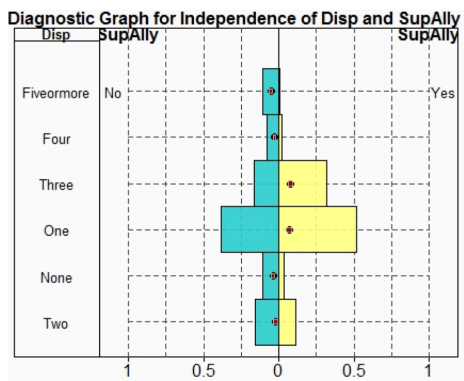

Figure B.2: Diagnostic Graph for Disputes and Superpower Alliance

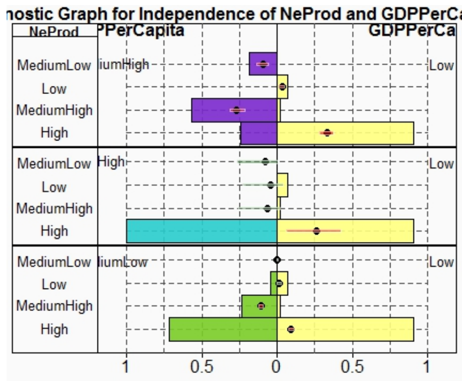


Figure B.3: Diagnostic Graph for GDP Per Capita and Nuclear Electricity Production

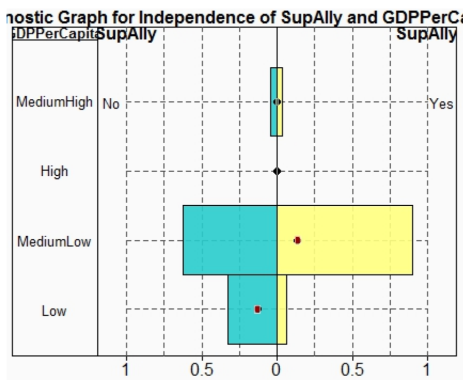


Figure B.4: Diagnostic Graph for GDP Per Capita and Superpower Alliance

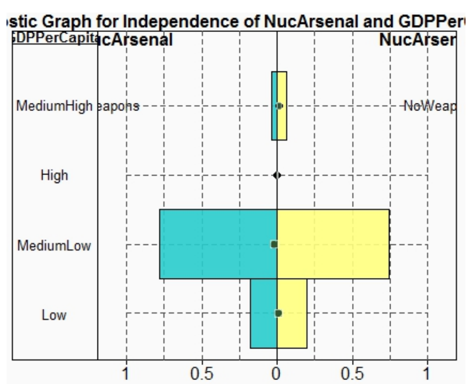


Figure B.5: Diagnostic Graph for Nuclear Arsenal and GDP Per Capita

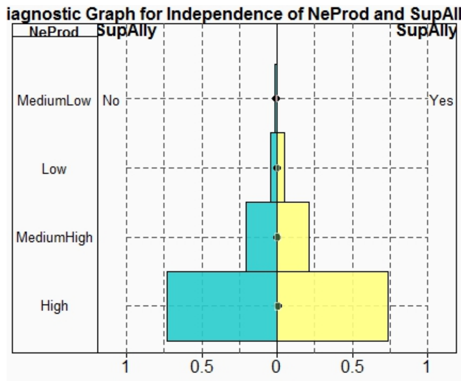

Figure B.6: Diagnostic Graph for Nuclear Electricity Production and Superpower Alliance

# APPENDIX C

# EXPERT ELICITATION

## C.1 METHODOLOGY

The expert elicitation asked 18 questions regarding the indicators and structure of the tiered model. Definitions of the indicators were provided, they are as follows:

- **ENR**: Enrichment and Reprocessing (facilities)
- **ENR facilities held by Trading Partners**: The total number of ENR facilities held by all trading partners of a state.
- **ENR facilities held by Rivals**: The total number of ENR facilities held by all rivals of a state.
- **Superpower Alliance**: A political alliance with a superpower. States under a nuclear umbrella may value ENR technology less because they can rely on other states for protection.
- **Nuclear Arsenal**: States with a nuclear weapons program and more than one nuclear weapon.
- **Technical Capability**: Represents a states measure of technical expertise, monetary funds, and resources specifically geared towards the construction of ENR facilities.
- **Motivation**: Represents a states desire or ambition to construct ENR facilities.
- **# Of Disputes**: The number of disputes a State has with other recognized States. A dispute is defined as threatening, displaying or using force against

other state.

Following definitions a list of questions, on a five point likert scale, were developed. These questions aimed to understand an experts thought process on nuclear proliferation. An example can be seen below:

2. Which state do you think is more likely to be motivated to develop their own ENR facilities: (1) a State ***with*** a superpower alliance (2) a State ***without*** a superpower alliance

| | | | | |
|---|---|---|---|---|
| State WITH a superpower alliance is much more motivated | State WITH a superpower alliance is somewhat more motivated | Equally motivated | **State WITHOUT a superpower alliance is somewhat more motivated** | State WITHOUT a superpower alliance is much more motivated |

Figure C.1: Example Question from the Expert Elicitation

Based on Figure C.1, the expert has selected the boldfaced and underlined option as the response to the question. This question aims to gage the effect superpower alliance has on motivations. The tiered model in Figure 4.2 highlights the edge between these two nodes. Similar to this question, other questions are asked to assess all the links represented in Figure 4.2 for both the technical capability and motivation nodes.

The result of each response is coded in an alpha value for that specific node. Alpha values are coded to be between 0 and 1. For example, in Figure C.1, the response has an associated alpha value of 0.7. After all the alpha values are collected, CPTs are developed. The alpha values between all the states were averaged. Note that the mode and median of these values resulted in the same value, this might occur due to the small sample size. A very basic example of how the alpha values were used to develop a CPT can be seen in Table C.1.

Table C.1: Example Conditional Probability Table

| | Motivation | |
|---|---|---|
| Superpower Alliance | High | Low |
| TRUE | alpha | 1-alpha |
| FALSE | 1-alpha | alpha |

CPTs for the technical capability and motivation nodes were developed by developing similar tables. These tables had three to four parent nodes on the left hand side of the table dictating, very similar to that of Table A.5. These CPTs were formed based on the alpha values derived from the expert elicitation. Following development, the CPTs were trained on the Bayesian networks to add an expert elicitation flair to the predictive nature of the network.

This concluded the model development with the use of an expert elicitation.

## C.2 FURTHER ANALYSIS

Figure C.2 identifies the correlation and predication accuracy plots for the expert elicitation method.

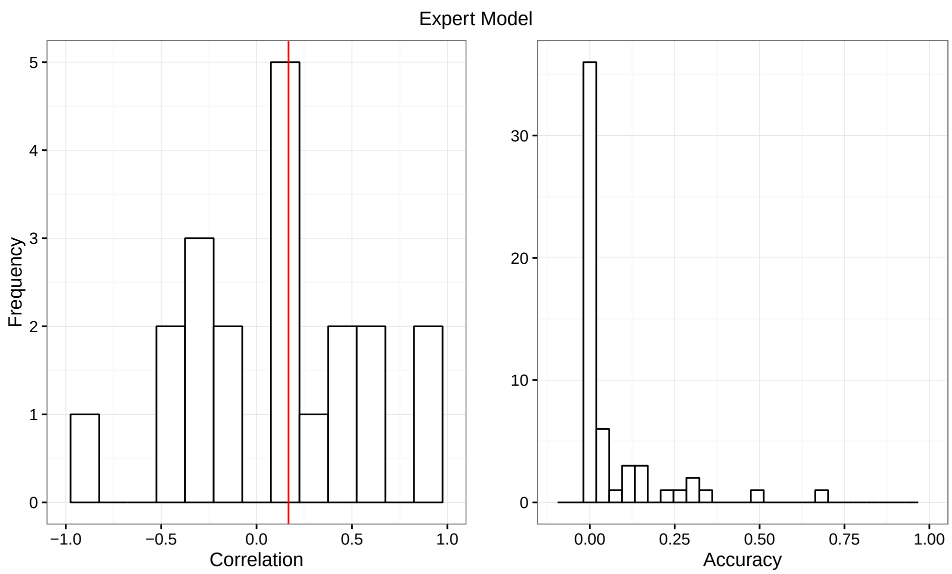


Figure C.2: Data for Expert Elicitation Model