ANALYSIS OF OPERATIONS MANAGEMENT PROBLEMS IN CURRENCY AND FOOD SUPPLY CHAINS

A Dissertation

by

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ABSTRACT

This dissertation is constituted of three essays addressing operational issues belonging to two research domains including (i) Logistics and Supply Chain Management for Currency Supply Network and (ii) Facility Location and Capacity Optimization for Food Supply Chains. Minimum cost flow network model and mixed integer programming model (linear and nonlinear) are developed to analyze and optimize supply chains of banknotes, coins, and foods. Approaches developed are general and can easily be applied to other categories of supply chains under different settings around the world with appropriate modifications.

The first domain comprises two essays with different scopes and objectives, in which two unique monetary supply chains with distinctive operations and governmental regulations are analyzed from both supply-side and demand-side perspectives. In the first essay, in order to improve the efficiency of the central bank's currency network in a large country, currency vaults are upgraded by expanding their capacities, and the sourcing of the updated currency network is optimized. This is the first study that analyzes a country's overall currency network's operations from the supply-side perspective.

In the same domain, the second essay presents general models for analyzing the operational issues in the U.S. Coin Supply Chain. As the first study to view the U.S. Coin Supply Chain as a closed-loop/reverse supply chain, it investigates the supply chain from both supply-side and demand-side perspectives to increase efficiency and effectiveness in ordering, producing, packaging, distributing and managing inventory of coins. This essay provides efficient methods and guidelines for effectively managing the supply chain that can be implemented in practice.

Belonging to the second domain, the third essay optimizes a food supply chain to assure food safety and provides suggestions to government agencies and private companies concerning where to locate new irradiation facilities with appropriate capacities strategically, how to source the demand of U.S. hubs from the supply of Mexican growing regions through irradiation facilities tactically, and how to efficiently transport fresh fruits imported from Mexico to the U.S. operationally.

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1. INTRODUCTION

Supply chain management attracts unabated interests because of its high complexity and crucial impact on society. Obtaining the highest efficiency from supply chains requires thorough analysis and delicate design using operations management techniques. This dissertation is constituted of three essays addressing operational issues belonging to two research domains including (i) Logistics and Supply Chain Management for Currency Supply Network and (ii) Facility Location and Capacity Optimization for Food Supply Chains. Minimum cost flow network model and mixed integer programming model (linear and nonlinear) are developed to analyze and optimize supply chains of banknotes, coins, and foods for addressing operational issues of ordering, production, sourcing, distribution, transportation, forecasting, facility location, capacity planning, and inventory management.

Currency plays an essential role in commerce and trade. The operational problems in the currency supply chain can be classified into three different sectors (Geismar et al. 2016): (i) the supply-side, i.e., the parties who are in charge of supplying currency in the supply chain (the central bank system), (ii) the demand-side, i.e., the parties who request the currency (depository institutions and commercial/ individual customers), and (iii) the third-party logistics providers. Research in the operations management focuses on the management of banknotes from the demand-side perspective, whereas very few studies consider the supply-side operational problem. The first domain comprises two essays analyzing a large country's currency supply network from the supply-side perspective and the U.S. Coin Supply Chain from both supply-side and demand-side perspectives.

In the first essay, the effort was devoted to analyzing the supply-side problem of how to efficiently and effectively provide currency services to all financial institutions within a large country's central bank's currency supply network. Through its network of big vaults, regional vaults, and retail vaults, the central bank in this country provides currency to all branches which serve consumers and commerce. The two available managerial levers involve network and service design: choosing a subset of vaults to have their capacities increased (i.e., to be upgraded, a strategic problem) and changing each vault's or branch's source vault(s) from which it receives currency (a tactical problem). An optimization model and a heuristic algorithm are developed to efficiently select a few retail vaults to upgrade. To optimize the sourcing within the new currency network, a minimum cost flow network model is proposed to efficiently reduce the transportation cost further. This essay contributes to the literature by optimizing a currency supply chain from the supply-side perspective, which is new and has not been studied before in practice.

There are two types of physical currency circulated in the economy: banknotes and coins. Compared to banknotes, coins seem to be undervalued by the public because of their low-value denominations. However, the sustained growth of annual new coins production and circulating coins in the public's hands shows that the coin consumption in the U.S. remains strong. In today's high-technology world, it may be surprising that the new coin production at the U.S. Mint for circulation has been increasing steadily since FY 2007 (GAO 2013). In the meantime, the amount of coins in circulation has also risen rapidly in recent decades. In the U.S. Coin Supply Chain, the Federal Reserve System plays a crucial role as the central planner and provides banking services to depository institutions and the public. Although most consumer products are distributed in one direction, coins are recirculated/reused through the economy bi-directionally. The second essay models the coin supply chain as a closed-loop supply chain from both supply-side and demand-side perspectives to develop an optimal operating policy for increasing efficiency in ordering, producing, packaging, distributing and managing inventory of coins. The supply-side consists of parties (the U.S. Mint and the Federal Reserve System) in charge of supplying coins to the economy, whereas the demand-side parties (depository institutions and commercial/individual customers) request coins. From the supply-side perspective, the operations of Federal Reserve System are analyzed and improved by developing optimization models to minimize the total cost for the supply chain and to devise an efficient rolling horizon procedure and a robust planning approach for effective handling of the demand uncertainty. From the demand-side perspective, the currency supply planning problem of depository institutions is also investigated in the theoretical settings for improving their operational efficiency. The complexity of the demand-side problem under special context is also investigated. Both the supply-side problem and the demand-side problem are formulated as optimization problems using minimum cost flow network models with multi-products (coins of different denominations) and operational constraints. An extensive computational study is performed to answer managerially relevant questions in the context of improving the efficiency of the supply chain by strengthening the effectiveness of the coin supply operations and reducing the coin related cost of depository institutions.

The amount of fresh produce crossing the U.S. / Mexico border has increased exponentially since the implementation of the North American Free Trade Agreement in 1994: over $7.7 billion in fruits and vegetables are imported from Mexico to the U.S. each year (USDA 2015b). Imported fresh produce must be treated for pestilence and microbial pathogen contamination. This requirement protects the health of those who consume the produce and the viability of domestic crops that could be infested by pests or infected by those pathogens. Among various technologies, irradiation is favored by the U.S. Department of Agriculture. Electron beam irradiation is relatively new and has many advantages outweighing other irradiation technologies. The third essay, belonging to the second domain aims to increase food safety by providing guidelines for private industry in the U.S. and Mexico for selecting the most cost-efficient locations for the electron beam facilities specifically designed for phytosanitary treatment of fresh fruits and vegetables crossing the Texas/Mexico border. Specifically, a generalizable decision support system is developed for determining the optimum number of Electron beam facilities, their capacities, and their locations. This system also selects the optimal assignment for each truckload of fresh fruit from Mexican growing regions to the Electron beam facilities, and then to the U.S. hubs for distribution. In addition to the standard components of a cost function (transportation, operations, capacity, and building), the cost minimization objective in the nonlinear mathematical model takes into account the time spent waiting at the border crossing (exogenous) and the time lost because of congestion at the Electron beam facilities (endogenous) by using queuing approximations.

This dissertation relates to research areas such as facility location-allocation, scheduling, inventory management, and logistics and transportation optimization. It aims at shedding light on the operational issues to improve the efficiency and effectiveness of currency and food supply chains. This dissertation makes several contributions to the operations and supply chain management literature. The first essay is the first study to analyze the operations of a currency supply chain from the supply-side perspective, i.e., the distribution problem for providing currency services throughout a country. The second essay is the first study addressing operational issues within the Coin Supply Chain from both supply-side and demand-side perspectives. The third essay provides an effective importing and distribution planning tool that integrates multiple decisions for selecting sites for phytosanitary facilities along the Texas/Mexico border by considering new food safety technology and key economic factors that signal local growth and development. Overall, this dissertation contributes to the literature and provides valuable managerial insights to government agencies and private companies by analyzing untouched research problems and developing efficient and effective planning tools from multiple operational perspectives. Methodologies used in this dissertation are general enough to be applied to other categories of supply chains under different settings around the world with appropriate modifications.

The remainder of this dissertation is structured as follows. Chapter 2 presents methods to optimize a large country's currency supply network for its central bank. Chapter 3 presents a framework for analyzing both supply-side and demand-side problems in the Coin Supply Chain. Chapter 4 explores where to locate Electron beam facilities and how to optimally decide their capacities for food safety. Chapter 5 briefly concludes this dissertation.

2. OPTIMIZING LOGISTICS OPERATIONS IN A COUNTRY'S CURRENCY SUPPLY NETWORK

2.1 Introduction

We analyze the problem of providing currency services to all financial institutions within a large (top ten by GDP) country. The country's central bank (CB) manages currency delivery and retrieval services to all financial institutions' branches through a hierarchical network of vaults. Our objective is to minimize the cost for providing these services. The two available managerial levers involve network and service design: choosing a small subset of vaults to have their capacities increased (i.e., to be upgraded, a strategic problem) and changing each vault's or branch's source vault(s) from which it receives currency (a tactical problem). Optimizing the currency supply chain from the supplier's perspective is new to academic research and has not been analyzed in practice. Our approach has the potential to be applied to many other currency supply chains around the world, with appropriate modifications.

A major factor in the astounding growth in the financial services industry over the past sixty-five years (its share of corporate income in the U.S. doubled from 10% in 1947 to 20% in 2012 (Soltas 2013)) has been information technology, including wire transfers, electronic payment mechanisms (e.g., credit cards, debit cards, and online transactions), and electronic data interchange. Despite this, the financial service that is most important to the smooth functioning of day-to-day life for the majority of people is the distribution of currency. Currency is still the most widely used consumer payment mechanism worldwide. In many developing countries, especially in the rural areas, people only accept currency as the payment method.

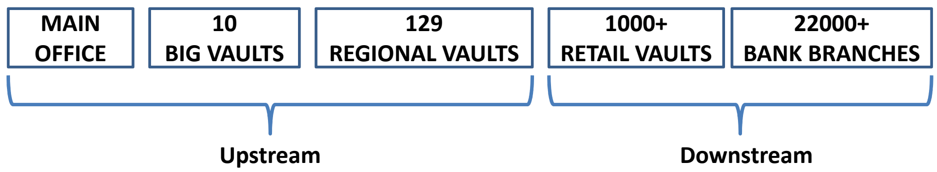
The value of currency in circulation has been increasing in most countries. In the U.S., the total face value of currency in circulation increased 87% from $601.2 billion in 1999 to $1127.1 billion in 2012 (Federal Reserve System 2013). Meanwhile, in Europe, despite a short period of decline in 2001, the value of euros in circulation increased from €341 billion in 1999 to €864 billion in 2012, a 153% jump (European Central Bank 2013). The amounts in developing countries are increasing at higher rates than in developed ones, partially due to higher inflation rates. For example, in China, the total value of currency in circulation has more than quadrupled from 1345 billion CNY in 1999 to 5465 billion CNY in 2012 (The People's Bank of China 2013). In Russia, the increase was an almost unbelievable 2316% during the past 13 years (Bank of Russia 2013). We next discuss the key players and the nature of demand in the currency network, the logistics network required to satisfy the demand, how changing the network will reduce the transportation cost, and the goals of this study.

2.1.1 Players in the Currency Network

The CB provides currency services to retail customers (both commercial and individual) through a supply chain with a standard arborescent structure, though the flows are less traditional. At one end of the chain are a small number of large vaults, and at the other are a large number of small bank branches. See Figure 2.1. The CB's ten *big vaults* distribute currency over the entire country through 129 *regional vaults* and over 1000 *retail vaults* to serve the country's commercial banks' over 22,000 *branches*. The big vaults also collect and destroy the unfit currency. The key services that the CB provides are to ship currency to wherever it is needed, to store it securely, and to sort currency by its fitness for recirculation (*fit-sorting*). The CB has a significant interest in reducing its transportation cost, but it does not compromise service levels when increasing efficiency.

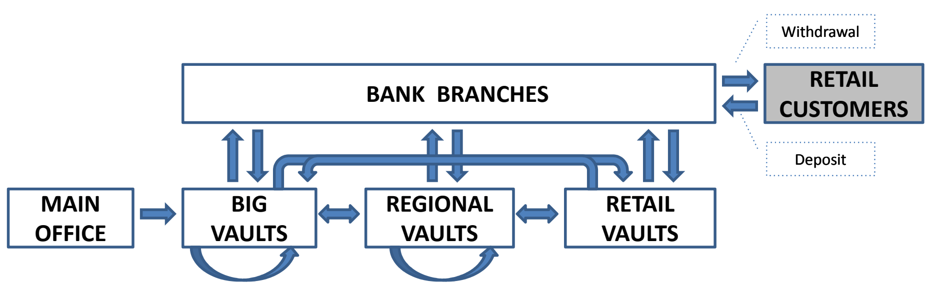
The CB has specified capacity thresholds that separate the classes of vaults, and

Figure 2.1: Currency Supply Chain Structure for This Country.



each class has its particular security protocols. Each location in the network is currently assigned a single supplier. The branches can be supplied by any type of vault; retail vaults can be supplied by big and regional vaults; regional vaults can be supplied by big vaults or other regional vaults; big vaults can only be supplied by other big vaults or by the central bank's main office. Retail customers' demands are satisfied only by branches. Thus, this supply chain is hierarchical, but the flows are not necessarily so. This distinguishes this environment by greatly increasing the number of potential delivery routes and, hence, the problem's complexity. Figure 2.2 illustrates the transaction types between entities in this country's currency supply network.

Figure 2.2: Currency Network Flows for This Country.



Since big and regional vaults act as aggregation points within the network, it is cost-effective for them to have high-speed (and high-cost) currency processing equipment to provide counting and fit-sorting services to the retail vaults and the branches. The resulting large quantities of cash imply that big and regional vaults have high levels of both security and insurance. In contrast, less-secure retail vaults have much less insurance and relatively small currency storage capacity. Each branch uses the same trip to deposit all currency received from customers to its supplier (who either fit-sorts it or forwards it to a vault that will do so) and to withdraw an appropriate amount to meet its retail customers' demands from the same supplier. Since retail vaults and branches have no capability for fit-sorting, for storage, or for subsequent recirculation, they are fully dependent on big or regional vaults for currency services.

The variety of flow directions has led to the classification and analysis of currency supply chains as closed-loop supply chains (Rajamani et al. 2006). However, the remanufacturing (fit-sorting) that is needed to recirculate currency happens at many regional vaults across the CB's network, rather than at one centralized location, as is done in most closed-loop supply chains. This exemplifies a responsive reverse supply chain that returns products to customers quickly (Guide et al. 2006). Previous studies of responsive reverse supply chains (e.g., Guide et al. 2005, Guide et al. 2008, Tagaras and C. Zikopoulos 2008) consider those that evaluate the quality of returns at collection facilities before sending them to a central location to be remanufactured, if appropriate. Ours goes beyond that by allowing for distributed remanufacturing (regional vaults remanufacture / fit-sort banknotes within the distribution process) so that banknotes are returned to the economy sooner. The reduction in transportation cost (no round-trip to the central location) compensates for the equipment purchase and the loss in economies of scale, though this loss is minor because fit-sorting has a near-constant return to scale (Bohn et al. 2001). This rapid return also allows there to be fewer banknotes in circulation without degrading service.

Additionally, Blackburn et al. (2004) and Guide et al. (2006) recommend such a structure when the marginal value of time (industry clock-speed or interest rates) is high or the proportion of high-quality returns is large. All previous studies of responsive reverse supply chains consider those that have a high marginal value of time (e.g., Guide et al. 2005, Guide et al. 2008, Tagaras and C. Zikopoulos 2008). This condition does not apply to currency (except in cases of high inflation), but the proportion of fit used banknotes is approximately 75%. Hence, the motivating features of this responsive reverse supply chain are new to academic research.

2.1.2 The Logistics Network

Currency travels within the CB's currency network using three transportation modes:

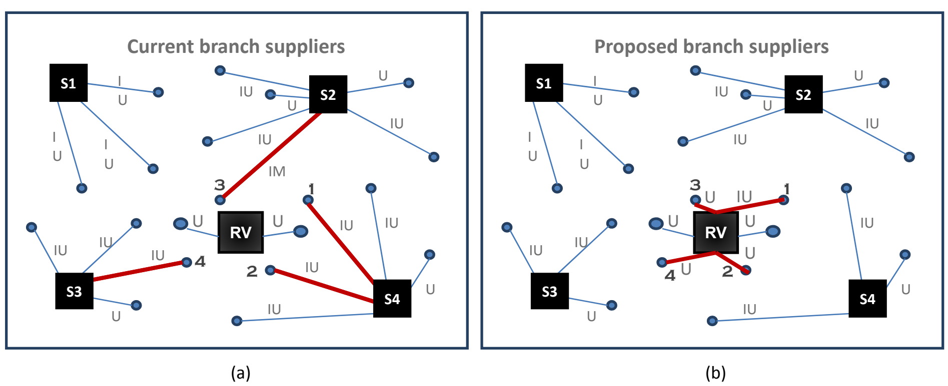
- Intermodal (IM) transportation is done by airplane, allowing larger order sizes (maximum of $2.75 million per airplane) over longer distances. The total shipment cost has three components: a fixed cost for use of the airplane, a cost per mile flown, and an insurance cost that is a percentage of the value shipped. This transportation mode is mainly used to supply vaults, either big, regional, or retail. In addition, IM transportation is also used to supply a few branches in remote areas that are not accessible by road.
- Interurban (IU) transportation is done by truck over distances larger than 50 miles and allows order sizes that are approximately 60% of IM's (maximum of $1.75 million per truck). Compare to the fixed cost of IM transportation, the cost of sending a truck is trivial, which is not considered in IU transportation. The total shipment cost has two components: a cost per mile driven and an insurance cost that is a percentage of the value shipped. This transportation mode is used to supply regional and retail vaults, in addition to branches.
- Urban (U) transportation is done by truck over distances shorter than 50 miles and allows order sizes that are approximately 25% of IM's. The total shipment cost has two components: a fixed cost per stop and a percentage of the value

shipped. The CB wants to create more regional vaults because this could increase the number of branches that are served by U transportation. This mode is more economical because the fixed cost per stop is low. This transportation mode is used only for supplying branches.

(U.S. Dollar amounts are used throughout to aid readers' intuition. The currency of the country being studied is kept confidential at the CB's request.)

Each trip with IM or IU transportation visits only one destination for supplying either a branch or a vault. Additionally, order sizes are assumed to depend not only on the transportation mode used, but also on the type of location being served. These modeling assumptions match the CB's current processes.

Figure 2.3: Current Situation and Proposed Situation with New Regional Vault.



Upgrading a retail vault can reduce the transportation cost by changing the transportation mode used to supply nearby branches. For instance, Figure 2.3 compares the current process for supplying branches in a particular area to the process with a new (upgraded) regional vault in that area. In these figures, the squares labeled $S1$, $S2$, $S3$, and $S4$ are existing big or regional vaults, whereas the square labeled RV is an existing retail vault serving two nearby branches and is a candidate for upgrade. Circles are branches that must be served by the vaults in the region. Figure 2.3(a) shows how these branches are currently supplied. Due to vault RV's insufficient capacity, it cannot supply branches 1, 2, 3, and 4, so they are served by vaults $S4$, $S4$, $S2$, and $S3$, respectively. Branch 3 is served by $S2$ via IM transportation, while branches 1, 2, and 4 are supplied through IU transportation.

If the capacity of RV is increased by upgrading it to regional vault status, then it will be able to serve branches 1, 2, 3, and 4. Branch 1 will still use interurban transportation, but branches 2, 3, and 4 will be served by urban transportation. See Figure 2.3(b). This scenario saves on the transportation cost but suffers an upgrade cost. This combination often results in net savings.

2.1.3 This Study's Goals

Since the CB seeks to increase the efficiency of its currency supply services, its mission is to reduce distribution costs by making the sourcing for all parties in the supply chain more efficient. This process will change the required capacities of most, if not all, vaults. Thus, some current retail vaults may have their capacities increased so that they are reclassified as regional vaults for security purposes. To analyze this complex facility location and transportation problem, in which each facility's capacity is a decision variable and there are a variety of available flows (both forward and backward), we decompose it into two subproblems: (i) Sourcing of bank branches for each state (or region) with possible upgrading of some retail vaults to regional vaults (downstream Problem 1) and (ii) Sourcing for the regional vaults in the entire resulting currency network (upstream Problem 2). We provide an analytical method for reducing the total cost by solving these two subproblems separately and quantifying the expected savings. More specifically, this paper's purpose is to answer the following questions:

1. Is there a robust selection of vaults to be upgraded in each region of the country,

i.e., a selection that would be effective over all typical, practical values of the demand for currency services that vaults may face, and over all reasonable cost structures for the CB?

2. What is the transportation cost saving of these upgrades for the CB?
3. What is the cost saving realized by allowing regional vaults to be supplied from multiple sources?

The outline of this paper follows. In Section 2.2, closely related studies are reviewed. Section 2.3 states the assumptions and proves that the problem is NP-hard. Therefore, we develop a mixed integer programming (MIP) model and a heuristic to solve Problem 1. Section 2.4 solves Problem 2 by developing a model to determine the optimal sourcing for all the regional vaults based on their net currency positions. Section 2.5 presents computational results for both Problem 1 and Problem 2. Discussion and conclusions are in Section 2.6.

2.2 Literature Review

First, we review studies for facility location problems. Next, related literature regarding transportation (intermodal transportation and multimodal transportation) is reviewed. Then, studies of currency supply chains from the demand side are discussed. Lastly, we briefly review inventory-location studies.

Discrete facility location problems have been extensively studied in the literature because of their theoretical interest and their importance in industry. There are four general formulations for deciding the location of facilities and allocating demand to them (Mirchandani and Francis 1990): the p-median problem, the uncapacitated facility location problem (UFLP), the p-center problem, and the quadratic assignment problem (QAP). All are NP-hard. Hakimi (1964) introduces the p-median problem, which became the *minimum location-allocation problem* (Kariv and Hakimi 1979b).

The p-median problem is used to locate p facilities relative to n customers such that the sum of the shortest demand-weighted distances between customers and facilities is minimized. Although the UFLP has a similar goal of locating facilities in order to minimize the demand-weighted distance, the UFLP contains costs for opening facilities and has no upper bound on the number of facilities that should be opened. The p-center problem is very similar to the p-median problem, except its objective is to minimize the maximum distance between any vertex and its nearest facility. The QAP minimizes the demand-weighted flow multiplied by the distance; it uses more information concerning flow and cost than the p-median problem does. Although the QAP is more realistic, it is theoretically harder to solve than the p-median problem, so it is not commonly studied in the literature.

The p-median problem is most similar to ours. Three primary types of heuristics—*Greedy*, *Alternating*, and *Vertex Substitution*—are the most widely used techniques for solving the p-median problem. Branch-and-bound, dual ascent, subgradient optimization, and surrogate relaxation techniques have been studied for solving the problem's IP formulation. Metaheuristics for approximate search algorithms currently dominate the research on the p-median problem; these include Genetic Algorithms, Hybrid Heuristic, Heuristic Concentration, Variable Neighborhood Search, Tabu Search, and Simulated Annealing Networks (Reese 2005).

The structure of our problem is different from the p-median problem in that we also include a fixed operating cost incurred for upgrading a vault and an incremental cost for each unit of capacity added to a vault. Moreover, there are different transportation modes, and each has its own maximum batch size. These new features imply that we cannot use the traditional approaches applied to the p-median problem to formulate and solve this specific facility location problem.

The integration of transportation and logistics systems continues to increase the complexity of efficient transportation. Studies of intermodal transportation (Southworth and Peterson 2000, Macharis and Bontekoning 2004, Janic 2007) and multimodal transportation (Rondinelli and Berry 2000, Van Nes 2002) address this problem. Intermodal and multimodal transportation integrate two or more modes (air, land, or sea). Similarly, currency traverses several segments as it moves from big vaults to branches, and each segment requires its own mode selection. However, our paper has some unique features that distinguish it from others. First, we consider two types of road transportation (IU or U), which, to our knowledge, has not been discussed in the literature. We also have the constraint that each location has its own load size limits for each mode, which affects the selection of retail vaults to be upgraded and the resulting sourcing.

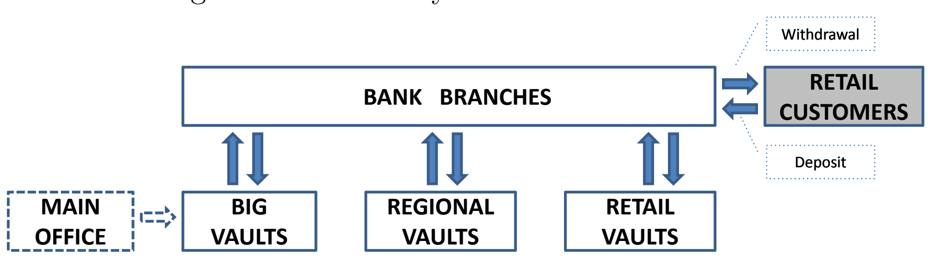
There are only a few papers that address optimizing currency supply chains. Rajamani et al. (2006) analyze the United States' currency supply chain within the framework of closed-loop supply chains and describe the Federal Reserve's new currency circulation policies. Hatzakis et al. (2010) provide an excellent review of research on operations in financial services. Geismar et al. (2007), Mehrotra et al. (2010), and Zhu et al. (2011) develop models to manage depository institutions' inventory and logistics under the U.S. Federal Reserve's new guidelines. These are considered *demand-side* studies because they address problems from the perspective of depository institutions. In contrast, this study address issues from the supply-side perspective considering the CB's supply operations. Our literature review suggests that this supply-side problem is new to academic research and that it has not been analyzed in practice. This paper demonstrates a modeling approach that not only analyzes a specific country's currency supply chain, but also has the potential to be applied to a variety of similar currency supply chains around the world, with appropriate modifications to suit the individual needs of each country.

Inventory-location models are another important stream of research related to location selection. Significant papers on this topic include Nozick and Turnquist (1998, 2001a, 2001b), Daskin et al. (2002), Shen et al. (2003), Yao et al. (2010) and Tancrez et al. (2012). We do not analyze these deeply because inventory cost is not a concern in the supply of currency. Because it all belongs to the CB (a non-profit public entity) as it flows through the network and because the CB considers only storage cost (no opportunity cost), the holding cost is constant and, thus, not a factor.

2.3 Problem 1

The currency supply from vaults to bank branches in each region (Problem 1) can be viewed as a Hub-and-Spoke transportation system (See Figure 2.3).

Figure 2.4: Currency Network for Problem 1.



Problem 1 determines the most efficient sourcing for each bank branch in a region. In doing so, it also selects retail vault locations to be upgraded so that the sum of the incremental operating cost of the new regional vaults, the total transportation cost, and the incremental capacity cost at all vaults is minimized. Here, the incremental operating cost is the extra operational expenditure (e.g., additional labor and security costs) required by regional vaults. The incremental capacity cost is for extra insurance and additional facility space, including its construction, which is amortized. The data obtained from the CB include the geographical currency network, branches' demands for currency services, the list of retail vaults that are candidates for upgrading, and the cost structures. Since upgrading retail vaults is the focus of this problem, we only consider currency flows between branches and vaults. See Figure 2.4. The upstream problem of supplying the vaults is addressed by Problem 2 (Section 2.4).

2.3.1 Model Assumptions

The service design decision to upgrade a retail vault into a regional vault has long-term implications since it often involves acquiring real estate and constructing larger facilities. Thus, seasonal variations in the demand are ignored. We, therefore, propose a single-period model, where a period is one month.

Demand Assumptions: The aggregated monthly demand for currency services from branches fluctuates significantly. Our baseline is the maximum monthly withdrawals and the maximum monthly deposits over a 12-month interval. Other values are used for sensitivity analysis in Subsection 2.5.2.

Capacity Assumptions: A vault's required capacity is highly correlated with its total monthly withdrawals. We, therefore, define the capacity of a vault as h times its total withdrawals per month, where h is a constant. The CB's risk of not satisfying demand weighs much more than the cost of holding too much currency inventory, so it adjusts the value of h to avoid stockouts. If a retail vault's computed required capacity exceeds a threshold limit c_ℓ (this value is specified by the CB), then it will be upgraded to a new regional vault.

We assume that big vaults have unlimited capacities.

Transportation Distance and Cost Assumptions: The actual land distances between all supply and demand nodes are not available and are expensive to obtain. Because we consider 7,000 facilities, about 25 million different pairs exist. In order to have a more manageable distance database, we base the distance calculation not on pairs of facilities, but on pairs of cities (recall that urban transportation does not charge per distance traveled). Furthermore, we only consider pairs of cities that belong to the same state or to neighboring states. The only exceptions are the cities containing the big vaults because the CB will occasionally ship currency from one big vault to another. Thus, a database containing no more than 4.5 million pairs of cities was enough to satisfy the CB's needs.

A mathematical formula based on latitude and longitude is used to compute the linear air distance between any two locations. We compute the average ratio of land/air for a sample by using known land distances obtained with Google Maps. Most land distances in our database are estimates, based on air distances and this multiplication factor. In general, road distances are best estimated as 1.30 times the air distances. For locations suggested by the model for upgrading, we recommend that all relevant distances be verified and any discrepancies be updated manually, then the model is rerun.

2.3.2 The MIP Model

Rather than using the distances explicitly in the mixed integer programming (MIP) formulation for Problem 1, we first compute the potential transportation cost for the entire period g_{ij} for each vault-branch pair (i, j). Clearly, this requires determining the best mode to use for each such pair. If the truck distance between i and j is no more than 50 miles, then U transportation is used, and the per unit of currency transportation cost for this pair is $g_{ij} = D_j \left(\frac{f_u}{l_j^U} + c_u \right)$. (A summary of the notation for the MIP can be found in Table 2.1 and Table 2.2.) Otherwise, either IU or U transportation is used, depending on which mode is more cost-effective. In this case, the monthly transportation cost for the pair per unit of currency is $g_{ij} = D_j \min\left\{\frac{f_a}{l_j^{IM}} + 2\frac{b_a}{l_j^{IM}}d_{ij}^a + c_a,\ 2\frac{b_t}{l_j^{IU}}d_{ij}^t + c_t\right\}$. (A formulation that determines transportation mode within the MIP is presented in Appendix A.1.) For implementation purposes, we let $\mathcal{R}$ be the subset of retail vaults that are candidates for upgrade. (The list for set $\mathcal{R}$ for each region is given by the CB.)

Table 2.1: Parameters Used to Formulate Problem 1.

M^{br}	The number of branches in the region.
M^B	The number of big vaults in the region.
M^{rv}	The number of regional vaults in the region.
N	The number of retail vaults in the region.
N_v	Total number of vaults: $N_v = M^B + M^{rv} + N$.
$\mathcal{R}$	The subset of retail vaults that are candidates for upgrade.
D_j^+	Withdrawal per period at branch j, $j = 1, \ldots, M^{br}$.
D_j	Maximum amount of currency per period transported between branch j and its suppliers. $D_j = \max\{D_j^+, D_j^-\}$ (D_j^- is deposit per period at branch j).
d_{ij}^t	The distance between vault i and branch j by land, $i = 1, \ldots, N_v$, $j = 1, \ldots, M^{br}$.
d_{ij}^a	The distance between vault i and branch j by air, $i = 1, \ldots, N_v$, $j = 1, \ldots, M^{br}$.
l_j^{IM}	The load size limit for branch j via intermodal (IM) mode, $j = 1, \ldots, M^{br}$.
l_j^{IU}	The load size limit for branch j via interurban (IU) mode, $j = 1, \ldots, M^{br}$.
l_j^U	The average per stop load size for sending cash to branch j via urban (U) mode, $j = 1, \ldots, M^{br}$.
F_i	The per period amortized operating cost of a new regional vault at location i, $i = 1, \ldots, N_v$. Set $F_i = 0$, $i = 1, \ldots, M^B + M^{rv}$.
c_o	The per unit cost of acquiring additional capacity for a vault.
f_a	The fixed costs of transporting one airplane load in intermodal (IM) mode.
f_u	The per stop fixed costs of transporting currency in urban (U) mode.
b_a	The per mile transportation costs of one airplane load via intermodal (IM) mode.
b_t	The per mile transportation costs of one truck load via interurban (IU) mode.
c_a	The insurance costs (per \$) of transporting by airplane via intermodal (IM) mode.
c_t	The insurance costs (per \$) of transporting by truck via interurban (IU) mode.
c_u	The insurance costs (per \$) of transporting by truck via urban (U) mode.
g_{ij}	The total transportation cost between vault i, $i = 1, \ldots, N_v$ and branch j, $j = 1, \ldots, M^{br}$.
C_i^e	The existing capacity (per period) of vault i, $i = 1, \ldots, N_v$.
h	The ratio of a vault's capacity to its monthly withdrawals.
c_ℓ	The capacity limit for a retail vault. If the capacity of a retail vault exceeds c_ℓ, then it becomes a regional vault.
J	The capacity limit for regional vaults. The value of J is given by the CB.

Problem 1:

$$\textbf{Minimize} \quad \Phi_1 = \sum_{i \in \mathcal{R}} F_i y_i + \sum_{i=1}^{N^v} \sum_{j=1}^{M^{br}} g_{ij} x_{ij} + c_o \sum_{i=1}^{N_v} C_i^a$$

Subject to

Table 2.2: Variables Used to Formulate Problem 1.

y_i	If $y_i = 1$, $i \in \mathcal{R}$, then retail vault i is upgraded to a new regional vault. If $y_i = 0$, then retail vault i is still open as a retail vault. We set $y_i = 0$ for $i \notin \mathcal{R}$, including big and existing regional vaults.
x_{ij}	If $x_{ij} = 1$, then the demand of branch j, $j = 1, \ldots, M^{br}$, is assigned to vault i, $i = 1, \ldots, N_v$. Otherwise, $x_{ij} = 0$.
C_i^p	The new capacity of vault i, $i = 1, \ldots, N_v$.
C_i^a	The capacity added to vault i.

$$\sum_{i=1}^{N_v} x_{ij} = 1,\ j = 1, 2, \ldots, M^{br} \tag{2.1}$$

$$C_i^p \geq h \sum_{j=1}^{M^{br}} x_{ij} D_j^+,\ i = 1, 2, \ldots, N_v \tag{2.2}$$

$$C_i^p \leq J y_i + c_\ell\ (1 - y_i),\ i \in \mathcal{R} \tag{2.3}$$

$$C_i^p \leq J,\ i = M^B + 1, \ldots, M^B + M^{rv} \tag{2.4}$$

$$C_i^a \geq C_i^p - C_i^e,\ i = 1, \ldots, N_v \tag{2.5}$$

$$C_i^p, C_i^a \geq 0,\ i = 1, 2, \ldots, N_v, j = 1, 2, \ldots, M^{br} \tag{2.6}$$

$$y_i, x_{ij} \in \{0, 1\},\ i = 1, 2, \ldots, N_v, j = 1, 2, \ldots, M^{br} \tag{2.7}$$

The objective function minimizes the total per period cost, which consists of the operating cost for new regional vaults, the per period transportation cost for supplying branches, and the incremental capacity cost for all vaults. Constraints (2.1) ensure that every branch has exactly one supplier vault. Constraints (2.2) compute the new capacity for all the vaults (big, regional, and retail) based on the new allocation of branches. Constraints (2.3) require that a retail vault becomes a new regional vault if its new capacity exceeds c_ℓ. Constraints (2.4) ensure that regional vaults' capacities do not exceed J. Constraints (2.5) calculate the added capacity for each vault. Constraints (2.6)-(2.7) are nonnegativity and binary constraints.

Theorem 1 *The decision problem corresponding to Problem 1 is strongly NP-complete.*

Proof: We use the 3-Satisfiability (3SAT) problem (Garey and Johnson 1979) for our reduction. The details are given in Appendix A.2.

Remark *The single period model presented for Problem 1 may specify fractional plane loads or partial truckloads, which are not economical and, hence, are not used. Instead, much like the process used when the standard EOQ model specifies a non-integer number of orders per year, here full planes or full trucks are dispatched as they are needed. Our MIP calculates one period's contribution to the total cost of this process.*

Remark *Because all currency flowing through the network belongs to the CB until it is delivered to a branch, and because the CB considers only storage cost and not opportunity cost, the inventory carrying cost is constant. Thus, we need not consider it in the cost minimization.*

We now consider whether some special cases for Problem 1 can be solved polynomially. We call these special cases with r specified retail vaults to be upgraded Problem P_1 and prove that Problem P_1 is NP-complete in the ordinary sense for any $r \geq 0$.

Theorem 2 *The decision problem corresponding to Problem P_1 with two specified retail vaults to be upgraded is NP-complete in the ordinary sense.*

Proof: We use the PARTITION problem for our reduction with details provided in Appendix A.3.

Theorem 3 *The decision problem corresponding to Problem P_1 in which no vaults may be upgraded is NP-complete in the ordinary sense.*

Proof: The construction of the decision probelm in the proof of Theorem 2 can easily be modified to prove this result. ■

The following corollary follows naturally from these two theorems.

Corollary 1 *The decision problem corresponding to Problem P_1 with r specified retail vaults to be upgraded is NP-complete in the ordinary sense for any $r \geq 0$.*

2.3.3 Approach to Solve Problem 1

Sequential Upgrades Heuristic (SUH)

Set $\Lambda_i = \emptyset$, for all $i \in \mathcal{R}$

For each branch j /* get current configuration */

  Let S_j be the original supplier of branch j

  Set $R_j = S_j$

  Add j to Λ_{R_j}

Next j

For each retail vault $i \in \mathcal{R}$

  For each branch j /* assign j to i if doing so reduces cost */

    If $g_{ij} < g_{R_j j}$ Then

      Remove j from Λ_{R_j}

      Add j to Λ_i

      Set $R_j = i$

  Next j

Next i

For each retail vault $i \in \mathcal{R}$

  If $h \sum_{j \in \Lambda_i} D_j^+ \leq c_\ell$ Then /* added demand does not cause upgrade of i */

    Assign all elements of Λ_i to i: $x_{ij} = 1, \forall j \in \Lambda_i$

  Else

    If the total transportation savings from assigning the branches in Λ_i to i would

        exceed $F_i + c_o[h \sum_{j \in \Lambda_i} D_j^+ - C_i^e]$ Then

      Upgrade vault i to regional status

      Assign the branches in Λ_i to i: $x_{ij} = 1, \forall j \in \Lambda_i$

    Else /* Do not upgrade, but reassign some branches */

      Renumber the elements of Λ_i as $1, \ldots, |\Lambda_i|$

      Define η as the element of $\{1, \ldots, |\Lambda_i|\}$ for which

        $h \sum_{j=1}^{\eta} D_j^+ \leq c_\ell < h \sum_{j=1}^{\eta+1} D_j^+$

      Assign the first η of the branches in Λ_i to i: $x_{ij} = 1$, for $j = 1, \ldots \eta$

      Assign branches $j = \eta + 1, \ldots |\Lambda_i|$ in Λ_i to their original supplier: $x_{S_j j} = 1$

    Exit If

  Exit If

Next i

SUH is a polynomial time heuristic with complexity $O(M^{br}|\mathcal{R}|)$. The SUH solution is compared to the MIP solution to evaluate its performance in Section 2.5.2.2. Experimental tests confirm that SUH can generate acceptable transportation savings and upgrade the correct retail vaults.

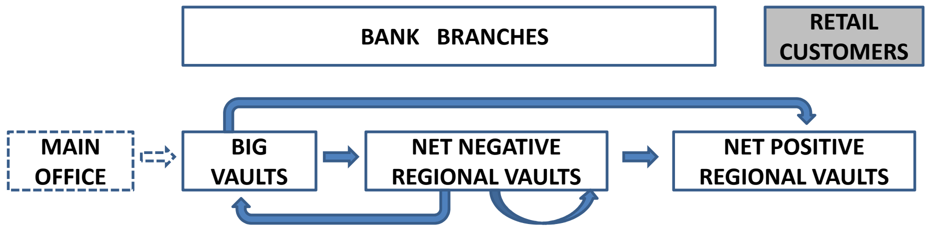
2.4 Problem 2

Having solved our strategic network design problem, we now consider the tactical problem of sourcing the regional vaults. In the CB's somewhat counterintuitive terminology, a regional vault that receives more in deposits than in withdrawals during a certain period is said to be *net negative*; otherwise, it is said to be *net positive*. We assume that big vaults always have enough cash to supply regional vaults and that net positive regional vaults can be supplied by either big vaults or net negative regional vaults. After the demand from net positive regional vaults is satisfied for the period, net negative regional vaults send their excess cash (if any) back to big vaults. The pertinent flows in this currency network are shown in Figure 2.5.

Since the road condition is poor between some pairs of vaults, a few net positive regional vaults cannot get cash directly from the closest big vault or net negative regional vault. This situation is modeled by assigning a very large number for the distance between any two vaults whose connecting road is in poor condition. If in addition other nearby net negative regional vaults have the insufficient surplus, then big or other regional vaults may send cash through the nearby net negative regional vaults to supply those net positive regional vaults. Thus, a net negative regional vault may satisfy a net positive vault's demand with currency it received from a big vault or another net negative regional vault.

A single-period model is also proposed for Problem 2. As before, a period is

Figure 2.5: Currency Network for Problem 2.



one month. The objective for Problem 2 is to optimize the sourcing in the currency network so that the total transportation cost for supplying all regional vaults is minimized. The following algorithm achieves this objective effectively and accurately. Step 1 allows the problem to be solved without first solving Problem 1. This is required because solutions to Problem 1 have not been implemented for all states of the country. Step 1 is omitted for states in which the solution of Problem 1 has been implemented. Step 3 is performed by a MIP (described in Appendix A.4) and a minimum cost flow formulation (described in Section 2.4.1).

Algorithm: Top-Down

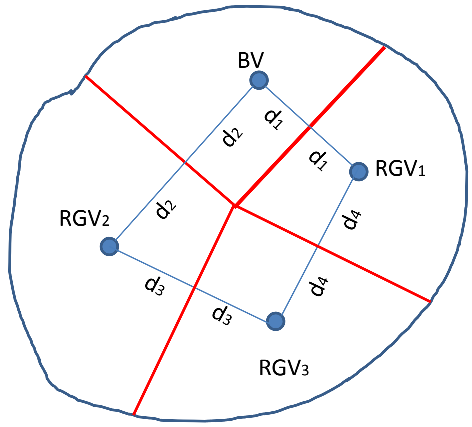
- **Step 1**: Use a geographical software package to divide the whole country into small areas so that there is only one big or regional vault in each area and that each boundary is equidistant from the two big or regional vaults it separates. The big vault or regional vault is automatically selected as the supplier of all retail vaults and branches in its area (see Figure 2.6).

- **Step 2**: Aggregate the maximum monthly demands for services from branches in each area to define the net position for each regional vault.

- **Step 3**: Based on the net position of in each regional vault, complete the optimal sourcing by minimizing the total transportation cost.

The CB's current contracts with three transportation companies state that each vault can be supplied by only one vault. The MIP models this situation for the whole

Figure 2.6: Example: Area Partition.



country's currency network. Our improved model uses a minimum cost flow (MCF) model that allows multiple suppliers for each vault and thereby reduces the CB's cost. Comparing these two models quantifies the value of changing the transportation contracts so that the CB can be fully informed when entering negotiations with its carriers. Additionally, MCF can be solved in polynomial time. In both models, the transportation modes for sourcing regional vaults can be intermodal (IM) or interurban (IU). Urban transportation is not considered in this problem because of the large quantities of cash and the long distance between any two vaults (big or regional). Distances are captured in the same way as in Problem 1.

Remark *Just like Problem 1, Problem 2 is a single period model whose solution may include partial loads (truck or plane). As before, full planes or full trucks are dispatched as soon as they are needed. Our solution calculates one period's contribution to the total cost.*

Remark *We have calculated each regional vault's net position based on each branch's maximum monthly demand. This planning problem could be solved for each month individually, based on forecasted demand, to update the sourcing more frequently.*

2.4.1 MCF Model

We now formulate a minimum cost flow (MCF) problem on a network to optimize sourcing for all regional vaults. Additional notation used to formulate the MCF model is in Table 2.3. See Appendix A.5 for the detailed IP formulation of the MCF model. The unit cost on arc (m, n) connecting any two nodes representing vaults m and n is $c_{mn} = \min\{2\frac{b_t}{Q_t}d^t_{mn} + c_t, \frac{1}{Q_a}(f_a + 2b_a d^a_{mn}) + c_a\}$, which chooses the less expensive transportation mode between IU and IM.

Table 2.3: Additional Notation Used to Formulate Problem 2 in the MCF Model: Parameters.

S_j^-	The amount of maximum monthly surplus (deposits minus withdrawals) in million dollars per period of net negative regional vault j.
S_k^+	The amount of maximum monthly deficit (withdrawals minus deposits) in million dollars per period of net positive regional vault k.
Q_a	Capacity of one airplane, by value of currency.
Q_t	Capacity of one truck, by value of currency.

We first describe the construction of the network model and then illustrate it with an example. We start with one node for each big or regional vault, then add a source node O and a sink node S, plus duplicate nodes for the big vaults to represent their receipt of excess cash from net negative regional vaults. Because big vaults are assumed to have infinite capacities, they could supply all regional vaults, if needed. A net negative regional vault can supply a net positive regional vault either by using its cash inventory or by being the transition node between either a big vault or another net negative regional vault and the net positive regional vault. We now describe each arc with its lower bound, upper bound, and unit cost for the flow on that arc. An example is shown in Figure 2.7.

- An arc from source node O to node B_i, where B_i represents a big vault. The lower bound, upper bound and unit cost of this arc are 0, $\sum_k S_k^+$, and 0.

- An arc from source node O to node N_j, where N_j represents a net negative regional vault. This arc represents the surplus at the net negative regional vault N_j. The lower bound, upper bound and unit cost of this arc are S_j^-, S_j^-, and 0.

- An arc from node B_i to node P_k, where P_k represents a net positive regional vault. The lower bound, upper bound and unit cost of this arc are 0, S_k^+, and $c_{ik} = \min\{2\frac{b_t}{Q_t}d_{ik}^t + c_t, \frac{1}{Q_a}(f_a + 2b_a d_{ik}^a) + c_a\}$, which is the unit cost for transporting cash from big vault i to net positive regional vault k.

- An arc from node N_j to node P_k. The lower bound, upper bound and unit cost of this arc are 0, S_k^+, and $c_{jk} = \min\{2\frac{b_t}{Q_t}d_{jk}^t + c_t, \frac{1}{Q_a}(f_a + 2b_a d_{jk}^a) + c_a\}$, which is the unit cost for transporting cash from net negative regional vault j to net positive regional vault k.

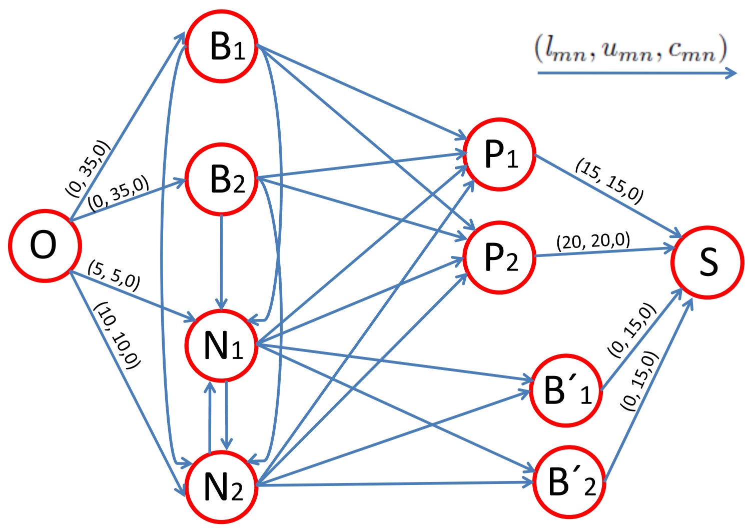
- An arc from node B_i to node N_j. The lower bound, upper bound and unit cost of this arc are 0, $\min\{J, \sum_k S_k^+\}$, and $c_{ij} = \min\{2\frac{b_t}{Q_t}d_{ij}^t + c_t, \frac{1}{Q_a}(f_a + 2b_a d_{ij}^a) + c_a\}$, which is the unit cost for transporting cash from big vault i to net negative regional vault j. Recall that J is the capacity limit for regional vaults.

- An arc from node N_j to node N_l. The lower bound, upper bound and unit cost of this arc are 0, $\min\{J, \sum_k S_k^+\}$, and $c_{jl} = \min\{2\frac{b_t}{Q_t}d_{jl}^t + c_t, \frac{1}{Q_a}(f_a + 2b_a d_{jl}^a) + c_a\}$, which is the unit cost for transporting cash from net negative vault j to net negative regional vault l.

- An arc from node N_j to node B_i'. The lower bound, upper bound and unit cost of this arc are 0, S_j^-, and $c_{ji} = \min\{2\frac{b_t}{Q_t}d_{ji}^t + c_t, \frac{1}{Q_a}(f_a + 2b_a d_{ji}^a) + c_a\}$, which is the unit cost for transporting excess cash from net negative vault j to big vault i.

- An arc from node P_k to sink node S. The lower bound, upper bound and unit cost of this arc are S_k^+, S_k^+, and 0. These bounds enforce P_k's deficit.
- An arc from node B_i' to sink node S. The lower bound, upper bound and unit cost of this arc are 0, $\sum_k S_k^-$, and 0.

Figure 2.7: Example for the MCF Model [l_{mn}: Lower Bound; u_{mn}: Upper Bound; c_{mn}: Unit Cost].



Example: There are 2 big vaults, 2 net negative regional vaults, and 2 net positive regional vaults. In this example, we let S_1^+, S_2^+, S_1^-, and S_2^- be 15, 20, 5 and 10, respectively (Figure 2.7).

For clarity, we assume that the cost structure makes it more economical for net negative regional vault 2 to be supplied by big vault 1 and for net negative regional vault 1 to be supplied by big vault 2. Additionally, net negative regional vaults 1 and 2 can supply each other. Consider the following five different types of feasible flows:

- **Type 1**: Big vault 1 or 2 is the only source for supplying net positive regional vaults. The net negative vaults return all of their cash to a big vault:
  - 15 units of flow move with sequence: $O \to B_1 \to P_1 \to S$.

- 20 units of flow move with sequence: $O \to B_1 \to P_2 \to S$.
- 5 units of flow move with sequence: $O \to N_1 \to B'_2 \to S$.
- 10 units of flow move with sequence: $O \to N_2 \to B'_1 \to S$.

- **Type 2**: All units of flow S_j^- from the net negative regional vaults go to the same net positive regional vault. The other net positive regional vault is supplied by a big vault:
  - 5 units of flow move with sequence: $O \to N_1 \to P_1 \to S$.
  - 10 units of flow move with sequence: $O \to N_2 \to P_1 \to S$.
  - 20 units of flow move with sequence: $O \to B_1 \to P_2 \to S$.
- **Type 3**: One net positive regional vault is supplied by a big vault and a net negative regional vault while another net negative vault sends its surplus back to a big vault. The other net positive regional vault is supplied by a big vault:
  - 5 units of flow move with sequence: $O \to B_1 \to P_1 \to S$.
  - 10 units of flow move with sequence: $O \to N_2 \to P_1 \to S$.
  - 20 units of flow move with sequence: $O \to B_2 \to P_2 \to S$.
  - 5 units of flow move with sequence: $O \to N_1 \to B'_2 \to S$.
- **Type 4**: Net negative regional vaults are supplied by big vaults, which then supply net positive regional vaults. This flow may happen when there is no road between big vaults and net positive regional vaults and the distance and demand do not justify IM transportation. This flow also indicates that the surplus at net negative regional vaults is not enough to satisfy the demand from net positive regional vaults:
  - 5 units of flow move with sequence: $O \to B_1 \to N_2 \to P_1 \to S$.

- 10 units of flow move with sequence: $O \to N_2 \to P_1 \to S$.
- 15 units of flow move with sequence: $O \to B_2 \to N_1 \to P_2 \to S$.
- 5 units of flow move with sequence: $O \to N_1 \to P_2 \to S$.

- **Type 5**: One net negative regional vault supplies the other, which then supplies a net positive regional vault. This flow path happens because there is no road between N_1 and P_1, N_2 has only 10 units of surplus, and no big vault supplies P_1.
  - 5 units of flow move with sequence: $O \to N_1 \to N_2 \to P_1 \to S$.
  - 10 units of flow move with sequence: $O \to N_2 \to P_1 \to S$.
  - 20 units of flow move with sequence: $O \to B_2 \to P_2 \to S$.

Clearly, solving the minimum cost flow problem constructed above is equivalent to optimizing the sourcing for Problem 2. Since all deposits and withdrawals in MCF are scaled as integers and all flows in the network have integer capacity, each feasible integer flow corresponds to a feasible sourcing for the currency network and vice versa.

2.5 Model Results and Validation

Our computational results provide insights for managers regarding increasing the number of regional vaults and changing the sourcing policy. We first briefly describe the primary objectives of our computational experiments.

2.5.1 Objectives of the Study

For Problem 1:

1. To test the effectiveness of the model solution for practical-sized problems.
2. To perform sensitivity analysis on the model solution.

For Problem 2:

1. To calculate the savings that the CB can realize by negotiating appropriate contracts with its transportation companies according to the solution given for Problem 2.

2. To measure the robustness of the MCF model solution under demand variations.

The MIP and MCF models were solved with CPLEX (version 12.4) on a Dell desktop with 2.66GHz CPU, 4.00GB of RAM, and 64-bit operating system. We use both data obtained directly from the CB and data randomly generated based on parameters from the CB's data.

2.5.2 Validation and Computational Study for Problem 1

To validate our MIP model for Problem 1, we first use the results for one unique state that has less-developed roads. This state represents the worst case with the most inefficient sourcing for branches and the highest attention from the CB. We collected the complete branch-level data for this state to quantify the potential savings from employing our model (Section 2.5.2.1). Section 2.5.2.2 uses ten randomly generated data sets to test the performance of the heuristic (SUH) and the MIP model.

2.5.2.1 Validation of One State

This state has 3 big vaults (BV: #1-3), 29 regional vaults (RGV: #4-32), 5 retail vaults (RTV: #33-37), and 130 branches. All five retail vaults are candidates to be upgraded to regional status. If a retail vault's capacity exceeds the threshold provided by the CB ($c_\ell = \$40$ million), it becomes a new regional vault. Big vaults and regional vaults' existing capacities are given by FiServ, a consulting firm (engaged by the CB) that aided our research. FiServ also recommended the assumption that the five candidate retail vaults have zero capacity before upgrading. After studying the correlation between existing storage capacity and the maximum monthly withdrawals, we chose $h = 2$ to calculate each vault's required capacity. For computing the transportation cost, we use $1.3 * air\_distance$ as the land distance. The baseline costs used to calculate the savings for Problem 1 were generated by FiServ, who estimated these costs by considering only the actual transportation cost incurred before the upgrades.

Our model selects retail vaults #33, #35, and #37 to be upgraded to new regional vaults, while the other two retail vaults (#34 and #36) remain open and function normally by serving branches (CPLEX solved the MIP in 4.51 seconds). These remaining retail vaults' capacities increase, but since they do not become regional vaults, the costs of their upgrades are small. Additionally, the capacities of two previously existing regional vaults are expanded to accommodate increases in the number of branches allocated to them. The resulting minimum capacity for each vault in this state is listed in Table 2.4.

The total cost calculated by the MIP model for this state is $1.563 million, including the operating costs for new regional vaults, the transportation costs for satisfying the maximum demand from branches, and the incremental capacity costs for the vaults. The transportation costs are 94% of the total cost, with IU transportation cost being 86% of the total transportation cost. FiServ confirms that after upgrading three retail vaults, the MIP model would generate actual savings of about $2 million, or 57.65%, compared to the original total cost observed by FiServ.

Observe how these savings occur by considering the ten branches that would be served by new regional vault #35. Originally, other regional vaults supplied five of these by intermodal (IM) transportation and the other five by interurban (IU)

Table 2.4: The Minimum Required Capacity of Vaults (in Million Dollars) [BV: Big Vaults. RGV: Regional Vaults. RTV: Retail Vaults]. Numbers in Parentheses are the Regional/Retail Vaults' Original Capacities Given By FiServ.

Vault No.	**Capacity of BV**	**Capacity of RGV**	**Capacity of New RGV**	**Capacity of RTV**
1	480			
2	400			
3	415			
4		130		
5		397.5(130)		
6		97.5		
7		20		
8		15		
9		45		
10		20		
11		20		
12		42.5		
13		20		
14		20		
15		55		
16		22.5(45)		
17		25		
18		65		
19		27.5		
20		62.5		
21		55		
22		81.3(40)		
23		47.5		
24		35		
25		12.5		
26		22.5		
27		27.5		
28		27.5		
29		15		
30		20		
31		32.5		
32		32.5		
33			50(0)	
34				33(0)
35			60.4(0)	
36				36.1(0)
37			111.7(0)	

transportation. However, after the upgrade, vault #35 supplies eight of the branches with interurban (IU) transportation and two with urban (U) transportation.

To perform sensitivity analysis, we first vary the demand without changing the upper bound on the retail vaults' capacities. The different values for demand are the average monthly demand, the maximum monthly demand, and the midpoint of these two values. Then, we vary both the demand and the capacity upper bound

c_ℓ. We also test the sensitivity of the selection of new regional vaults to the fixed upgrading cost.

Table 2.5 illustrates that the number of retail vaults upgraded in an optimal solution is indeed sensitive to demand. However, the objective values should not be misinterpreted to suggest that annual costs would be less with fewer upgraded vaults. We saw previously that transportation costs are over 90% of the total, so upgrading the number of retail vaults recommended under maximum demand is the best policy because it provides the flexibility to serve all possible demand profiles. Should the cost force upgrades to be spread over several periods, this analysis shows the preferred order for the upgrades.

Table 2.5: Sensitivity Analysis for Demand with $h = 2$, $c_\ell = \$40$ Million and F_i=\$0.022 Million.

Demand Type	**New RGV**	**Obj. Value (\$M)**
Ave. D. + Ave. W.	37	1.015
Mid. D. + Mid. W.	33, 37	1.300
Max. D. + Max. W.	33, 35, 37	1.563

Table 2.6: Sensitivity Analysis for Both Demand and c_ℓ (\$M) with $h = 2$ and $F_i = \$0.022$ Million.

Demand Type	c_ℓ **(\$M)**	**Total Cost (\$M)**	**New RGV**
Ave. D. + Ave. W.	30	1.035	33,37
	35	1.022	37
	40	1.015	37
	45	1.015	37
	50	1.015	37
Mid. D. + Mid. W.	30	1.301	33,35,37
	35	1.301	33,35,37
	40	1.298	33,37
	45	1.283	37
	50	1.278	37
Max. D. + Max. W.	30	1.585	33,34,35,36,37
	35	1.575	33,35,36,37
	40	1.564	33,35,37
	45	1.564	33,35,37
	50	1.549	37

The upper bound on retail vaults' capacity c_ℓ is another factor that significantly affects the model's results. The sensitivity analysis in Table 2.6 illustrates that \$40 million is a reasonable value for c_ℓ because it puts the number of new regional vaults within the limit (no more than three new regional vaults in each region) that the CB requires. In addition, the total cost and the number of new regional vaults decrease as c_ℓ increases, which indicates that the annual fixed operating cost F_i plays an important role in selecting the number of retail vaults to upgrade (F_i = \$0.022 million are provided by the CB). This is confirmed in Table 2.7 and Table 2.8, which show that the number of retail vaults upgraded is sensitive to F_i (its actual value is \$0.022 million). Table 2.7 and Table 2.8 also demonstrate that with maximum demand (deposits and withdrawals), $h = 2$, and c_ℓ = \$40 million, even if there is no upgrading cost, retail vault #36 should never be upgraded and retail vaults #37, #33, #35 are first three to be upgraded. In addition, Table 2.8 indicates that, as expected, the number of upgraded (new) regional vaults increases as the cost of upgrading F_i decreases and as c_ℓ decreases, even if both are changed concurrently.

Table 2.7: Sensitivity Analysis for F_i (\$M) with Maximum Demand, $h = 2$ and c_ℓ = \$40 Million.

Annual Oper. Cost (F_i)	**Total Cost (\$M)**	**New RGV**
0.05	1.593	37
0.035	1.583	33, 37
0.022	1.563	33, 35, 37
0.015	1.554	33, 35, 37
0.005	1.538	33, 35, 37
0.001	1.532	33, 34, 35, 37
0	1.531	33, 34, 35, 37

Examining the total cost when only one retail vault is upgraded (Table 2.9) also demonstrates that retail vault #36 should never be upgraded when using maximum demand, $h = 2$ and c_ℓ = \$40 million: the total cost in this case is higher than the baseline cost. In general, if the total cost after upgrading a particular retail vault is

Table 2.8: Sensitivity Analysis for both F_i (\$M) and c_ℓ (\$M) with Maximum Demand and $h = 2$.

Annual Oper. Cost (F_i)	**New RGV with** $c_\ell = 30$	**New RGV with** $c_\ell = 40$	**New RGV with** $c_\ell = 50$
0.05	33, 37	37	37
0.035	33, 35, 37	33, 37	37
0.022	33, 34, 35, 36, 37	33, 35, 37	37
0.015	33, 34, 35, 36, 37	33, 35, 37	33, 37
0.005	33, 34, 35, 36, 37	33, 35, 37	33, 35, 37
0.001	33, 34, 35, 36, 37	33, 34, 35, 37	33, 35, 37
0	33, 34, 35, 36, 37	33, 34, 35, 37	33, 34, 35, 37

higher than the baseline total cost with no upgrade, then this retail vault will never be upgraded in an optimal solution. Additionally, Table 2.9 verifies that if the CB is limited to upgrading only one retail vault at a time, then the order of upgrading these vaults is #37, #33, #35, #34.

Table 2.9: Upgrading Candidates Analysis with Maximum Demand, $h = 2$ and $c_\ell = \$40$ Million.

Condition	**Cost (\$M)**					
	Total	**Oper.**	**Trans.**	**IU Trans.**	**U Trans.**	**Add'l Capacity**
No Upgrading	1.795	0	1.737	1.539	0.198	0.058
Upgrade 37 Only	1.579	0.011	1.508	1.303	0.205	0.060
Upgrade 33 Only	1.682	0.011	1.611	1.411	0.200	0.060
Upgrade 35 Only	1.757	0.011	1.688	1.488	0.200	0.058
Upgrade 34 Only	1.775	0.011	1.705	1.505	0.200	0.059
Upgrade 36 Only	1.804	0.011	1.735	1.538	0.199	0.058

2.5.2.2 Performance Test of Sequential Upgrades Heuristic (SUH) and the MIP Model

We test the robustness of the heuristic solution of SUH and the optimal solution of the MIP model with ten sets of simulated data. The distance and demand values are randomly generated from uniform distributions based on the real data. These data sets are two to four times the size of the real data set to test computational efficiency. We use the same values as before for all the parameters.

The total costs and the percentage differences between the MIP and SUH are reported in Table 2.10. Because no baseline cost for the simulated data can be furnished by observation, we calculated it by running the MIP with no upgrading of retail vaults for each randomly generated data set. Thus, we are comparing optimal sourcing with no upgrades to optimal sourcing with upgrades. The resulting saving from the MIP model averages 7.39%. This percentage is significantly less than that achieved for the real data because in practice the sourcing without upgrades (the baseline) was not optimized. The average percentage increase in cost from using SUH rather than the MIP is 2.64%, which is reasonably small and acceptable.

Table 2.10: Total Cost (in Million Dollars) and Percentage of Saving Differences between the MIP model and SUH for Upgrading Retail Vaults for Randomly Generated Data.

ID	**Baseline Cost** (Base)	**Cost w/ Upgrades** (MIP)	**Upgrade Savings** (Base vs. MIP)	**Cost w/ Upgrades** (SUH)	**% Increase in Cost** (MIP vs. SUH)
1	6.28	5.92	5.73%	6.03	1.86%
2	6.65	6.28	5.56%	6.32	0.64%
3	9.68	8.70	10.12%	9.13	4.94%
4	7.14	6.59	7.70%	6.81	3.34%
5	5.61	5.07	9.63%	5.24	3.35%
6	7.90	7.32	7.34%	7.45	1.78%
7	7.44	7.02	5.65%	7.23	2.99%
8	10.26	9.46	7.80%	9.69	2.43%
9	8.18	7.80	4.65%	7.98	2.31%
10	11.04	9.97	9.69%	10.25	2.81%
AVE	8.018	7.413	7.39%	7.613	2.64%

In addition, both the MIP model and the SUH upgrade the same retail vaults to new regional vaults for all ten randomly generated data sets. Therefore, the numerical experiments demonstrate that SUH can solve Problem 1 to a near-optimal solution in polynomial time. Since Problem 1 is NP-hard, SUH becomes more attractive as the size of the data set becomes larger.

2.5.3 Computational Study for Problem 2: MIP and MCF Models

We consider the country's complete currency network for this problem: 10 big vaults (#1-10), 45 net negative regional vaults (#11-55) and 84 net positive regional vaults (#56-139). In addition to modeling the current situation by a MIP model, we test an alternative transportation contract structure with an MCF model. As in Problem 1, we first use the maximum monthly demand to test the MIP and MCF models. Then we vary demand to perform sensitivity analysis to measure the robustness of our MCF model.

CPLEX (version 12.4) is unable to solve the MIP model developed for the whole country in Appendix A.4 to optimality before exhausting memory. We obtained a near optimal solution by relaxing some binary variables to be continuous between 0 and 1. This result provides a lower bound that we use to evaluate the MCF model. This lower bound on the total transportation cost when each regional vault has a unique supplier is \$30.025 million, including the cost for sending excess cash back from net negative regional vaults to big vaults (\$8.325 million), the cost for satisfying the demand for net positive regional vaults (\$20.16 million), and the cost for sending cash to net negative regional vaults (\$1.54 million) to eventually satisfy the demand from net positive regional vaults. More than 90% of the transportation cost comes from the IU mode.

The MCF model results are qualitatively different from those of the MIP model. No net negative regional vault is supplied by big vaults, and none has excess currency to send back to a big vault. Net negative regional vaults do supply each other, and net positive regional vaults are supplied by both big vaults and net negative regional vaults. In the optimal sourcing, we have Type 1 flows (e.g., net positive regional vault #118 is only supplied by big vault #1), Type 2 flows (e.g., net positive regional vault

#139 is supplied by big vault #6 and net negative regional vault #35), and Type 5 flows (e.g., net negative regional vault #49 is supplied by net negative regional vault #34 and uses that to satisfy the demand from net positive regional vault #96). However, Type 3 and Type 4 flows are not in the optimal solution in this currency network because there is no flow between net negative regional vaults and big vaults. The optimal sourcing obtained from the MCF model is given in Table A.8 in Appendix A.6.

The total transportation cost for the MCF model is $20.566 million, almost all of which is the cost for satisfying the demand for net positive regional vaults ($19.348 million). Hence, the majority of the savings over the sole sourcing model comes from removing transportation between big vaults and net negative regional vaults. The transportation mode selection is similar to that of the MIP model: more than 90% of the transportation cost comes from the IU mode. The cost comparison for the MIP model and the MCF model is shown in Table 2.11: the MCF model saves 31.5% from the MIP model.

Table 2.11: Cost Comparison (in Million Dollars) for Optimizing Sourcing in Problem 2 [BV: Big Vaults. RGV-P: Net Positive Regional Vaults. RGV-N: Net Negative Regional Vaults].

Cost Type	**MIP Model Cost**	**MCF Model Cost**
Total Transportation Cost	30.025	20.566
Transportation Cost to BV	8.325	0
Transportation Cost to RGV-P	20.160	19.348
Transportation Cost to RGV-N	1.540	1.218

To test the robustness of our MCF model, we first find optimal sourcing schemes for several demand profiles. Then we use the sourcing that is optimal for one demand profile to supply a different demand profile. The cost when using a mismatched sourcing scheme is compared to the optimum for this demand. This difference shows the cost of a poor forecast or of not updating the sourcing scheme month-to-month.

Table A.9 and Table A.10 in Appendix A.6 illustrate the optimal sourcing obtained from the MCF model for the midpoint demand and the average demand, respectively. For the midpoint demand, four net negative regional vaults and twelve net positive regional vaults (total of 12.4% of regional vaults) change their sourcing in the optimal solution. If we reduce the demand further to the average values, five net negative regional vaults and sixteen net positive regional vaults (total of 16.28% of regional vaults) are sourced differently from the optimal maximum-demand solution.

Table 2.12: Costs (in Million Dollars) of Optimal and Non-Optimal Sourcing in the MCF Model.

Demand Type	**Cost with Optimal Sourcing**	**Type of Non-Optimal Sourcing**	**Cost with Non-Optimal Sourcing**	**Percentage Increase**
Maximum	20.566	n/a	n/a	n/a
Midpoint	15.094	Maximum	15.179	0.56%
Average	9.702	Maximum	9.944	2.5%
Average	9.702	Midpoint	9.731	0.3%

Table 2.12 shows the optimum costs for all three types of demand and illustrates the potential cost increase if we use sourcing that does not match the actual demand, e.g., we use the optimal sourcing for the maximum demand when the midpoint demand is realized. (We can only test when sourcing is greater than demand, as the opposite leads to infeasibility.) The average cost increase from a mismatch is 1.12%, which is evidence of a robust solution procedure.

2.5.4 Numerical Experiments for Problem 1 and Problem 2 Combined

To generalize the overall value and robustness of our methodology we now solve both Problem 1 and Problem 2 for ten randomly generated data sets. These are smaller (one-sixth to one-third of the size of the actual data set) for computational efficiency. We use the same values as before for all the parameters. The distance and demand values are randomly generated from uniform distributions based on the real data.

Table 2.13: Total Cost (in Million Dollars) and Savings from Upgrading Retail Vaults, Multi-sourcing, and Both, for Generated Data.

ID	**Baseline Cost** (Base)	**Cost w/ Upgrades** (MIPs)	**Upgrade Savings** (Base vs. MIPs)	**Multiple Sourcing Cost** (MCF)	**Overall Savings** (Base vs. MCF)	**Multiple Sourcing Savings** (MIPs vs. MCF)
1	28.54	27.00	5.40%	17.99	36.97%	33.37%
2	27.26	26.89	1.36%	18.18	33.31%	32.39%
3	31.00	29.74	4.06%	20.28	34.58%	31.81%
4	19.30	18.79	2.64%	9.62	50.16%	48.80%
5	20.80	19.63	5.63%	10.25	50.72%	47.78%
6	21.10	19.89	5.73%	10.50	50.24%	47.21%
7	17.51	17.13	2.17%	10.44	40.38%	39.05%
8	32.36	28.41	12.21%	18.35	43.29%	35.41%
9	17.43	16.13	7.46%	9.92	43.09%	38.50%
10	27.34	23.59	13.72%	16.98	37.89%	28.02%
AVE	24.26	22.72	6.04%	14.25	42.06%	38.24%

The total costs and percentages saved are reported in Table 2.13. Since these data sets are smaller, CPLEX can solve the MIP model for Problem 2 to optimality for each of them. The baseline cost is computed using the MIP models for both Problem 1 and Problem 2 with no upgrading of retail vaults and with single-sourcing of regional vaults. Compared to the baseline cost, the saving from allowing upgrades of retail vaults averages 6.04% (calculated with MIPs for both Problem 1 and Problem 2). Similarly, compared to the baseline cost, the average saving from solving Problem 1 with the MIP model and allowing multiple sourcing with the MCF model in Problem 2 is 42.06%. It follows that using multiple sourcing in Problem 2 provides an average additional benefit of 38.24% over sole sourcing for the randomly-generated data.

The numerical experiments demonstrate that the MIP model for Problem 1 and the MCF model for Problem 2, if supply contracts allow, provide significant savings within the currency network. The CB can use updated demand information annually and rerun the MIP model for Problem 1 to adjust the assignment of branches and the MCF model for Problem 2 to source the vaults.

2.6 Conclusion

This is the first study to address the supply side of a currency supply chain, i.e., the distribution problem for providing currency services throughout a country. We developed a decision support system that includes (i) a mixed integer programming model (and a heuristic algorithm for large problem instances) for determining the sourcing for bank branches and the optimum network and service configuration (downstream Problem 1) and (ii) a minimum cost flow formulation to specify sourcing for regional vaults (upstream Problem 2); these together optimize currency supply operations. The model captures the flow of currency and develops an effective integrated transportation plan over the planning horizon for vaults and branches, along with proposed vault upgrades. Furthermore, this is the first analysis of a responsive reverse supply chain that has distributed remanufacturing and a high proportion of high-quality returns. This combination allows for the variety of flows within the supply chain that connect vaults and branches in various combinations in both directions.

Problem 1 reduces the transportation costs by improving the sourcing of bank branches and by optimally selecting the retail vaults to upgrade into regional vaults. Implementing our recommendations for upgrading three retail vaults in one state and efficiently supplying the branches would result in a potential annual transportation cost savings of 57.65% (about $2 million). Whereas this most likely represents an upper bound on the improvement that could be realized in any other state, the results with simulated data suggest that 7.39% is a loose lower bound. Our solution to Problem 2 demonstrates that the potential savings from multi-sourcing regional vaults is greater than 31% (about $9.5 million nationwide) using the actual collected data and is about 38% using randomly-generated data. This demonstrates the potential utility, in particular, of altering the current logistics contracts and, in general, of having flexibility in a supply network. Interestingly, a test of multi-sourcing the branches in Problem 1 yielded almost no reduction in cost, which can be attributed to the branches having demands that are an order of magnitude less than those of regional vaults and to most branches being near only one supplying vault.

Lastly, this decision support system is a Hub-and-Spoke transportation system, so it is applicable to a wide variety of supply networks (see Aykin 1995, Alderighi et al. 2007, Almumur and Kara 2008, Bryan and O'Kelley 1999, Geismar et al. 2011, Van Buer et al. 1999, Pirkul and Schilling 1998, Racunica and Wynter 2005, Tarantilis and Kiranoudis 2007, Vidyarthi et al. 2013 for examples and analyses of Hub-and-Spoke networks). Our MIP formulation and heuristic SUH for Problem 1 are general enough to be used for many other currency supply chains world-wide. In addition, sourcing regional vaults across an entire country is a problem faced by many currency supply chains. Thus, our MCF formulation in Problem 2 can easily be adapted to model other currency supply chains. The resulting savings in these networks is expected to be significant, as demonstrated by our experiments with various randomly generated data sets.

The need to continually reduce transportation costs is a common thread across all supply chains. The basic problem that existed in our client's country is typical for currency supply chains in most countries. Furthermore, this problem has not been well-studied in the academic literature, nor has it been carefully addressed in practice. Consequently, there are no tools available in the market that address this issue for currency distribution. This paper demonstrates the use of a modeling approach that not only meets this need for this specific country, but also has the potential to be applied in a variety of similar currency supply chains around the world.

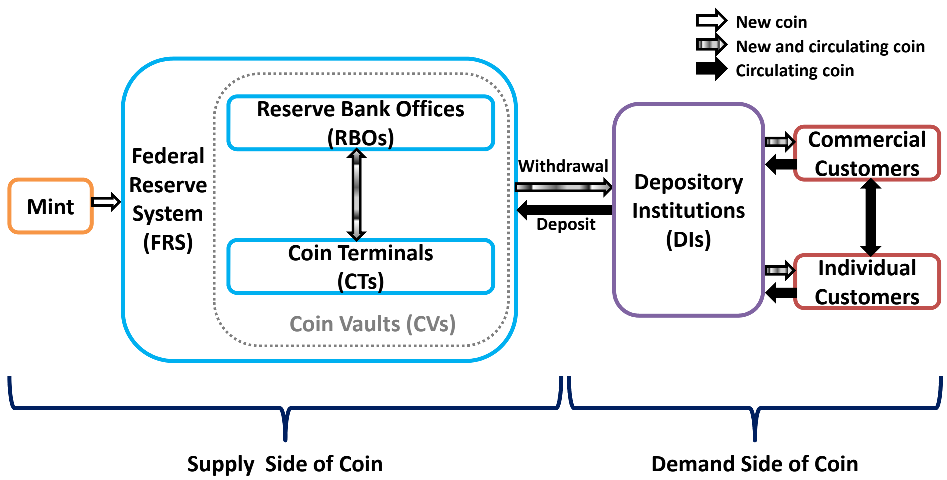
3. A FRAMEWORK FOR ANALYZING THE U.S. COIN SUPPLY CHAIN

3.1 Introduction

Currency plays an essential role in commerce and trade. Two types of physical currency are circulated in the economy: banknotes and coins. Compared to banknotes, coins seem to be undervalued by the public because of their low-value denominations. However, the sustained growth of annual new coins production and circulating coins in public's hands shows that the coin consumption in the U.S. remains strong. In today's high-technology world, it may be surprising that new coin production at the U.S. Mint (hereafter, simply Mint) for circulation has been increasing steadily since FY 2007 (GAO 2013), especially a dramatic increase began in 2011 (Mint 2015). In FY 2015, the Mint produced and delivered 16.2 billion coins to the Federal Reserve System (FRS)—the central banking system in the U.S.—for circulation, an increase of 23.9% compared to FY 2014 (Mint 2015). In the meanwhile, the amount of coins in circulation has also risen rapidly in recent decades. Coinstar reports over 77 trillion transactions per year in the U.S., a number that has gradually grown over years (Coinstar 2013). Based on the FRS's latest estimation, as of April 6, 2016, there was approximately $1.45 trillion of currency (coins and banknotes) in circulation in the U.S., of which $50 billion was in Federal Reserve coins (FRS 2016), an increase of about 25% compared to FY 2010.

Geismar et al. (2016) provide an excellent review of the recent work on operational issues concerning the currency supply chain. In this review paper, the operational problems in the currency supply chain are classified into three different domains: (i) the supply side, i.e., the parties who are in charge of supplying currency in the supply chain; (ii) the demand side, i.e., the parties who request the

Figure 3.1: U.S. Coin Supply Chain.



currency and act as intermediaries by providing it to customers (both individual and commercial); and (iii) the third-party logistics providers (3PLPs). According to our knowledge, there is no previous study addressing operational issues within a Coin Supply Chain. Although there are several studies in the literature having been focused on the management of banknotes from the demand side of operations, very a few studies consider the supply side operational problem (e.g., Zhu et al. 2015). Figure 3.1 shows the flow of coins in the U.S. Coin Supply Chain (hereafter, simply CSC), supply parties—the Mint and the FRS—who are in charge of supplying coins, demand parties—depository institutions (DIs), and commercial/individual customers—who request coins. 3PLPs act as intermediate distributors who contract with the FRS for transshipping coins and with DIs for processing and transporting coins to serve individual and commercial customers. This research is the first study analyzing the CSC and improving its efficiency and effectiveness for operations from both supply-side and demand-side perspectives.

Although most consumer products are distributed in one direction, coins are recirculated/reused through the economy. As in Rajamani et al. (2006), we view the

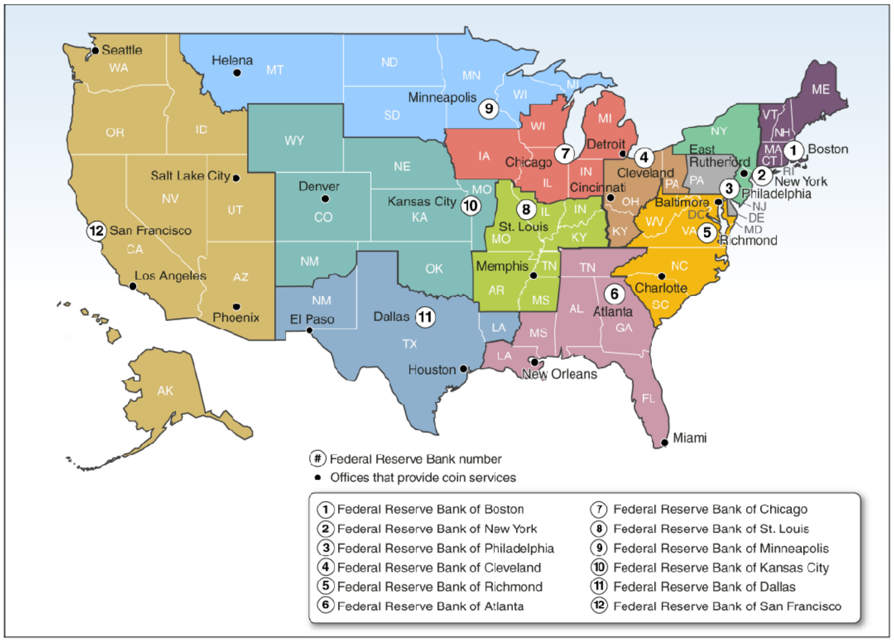
CSC as a closed-loop supply chain that is highly integrated with two channels consisting of forward and backward flows. We study the CSC in the context of improving the efficiency of its operations from both supply and demand sides. The FRS acts as the central planner that operates the two channels simultaneously. Figure 3.1 illustrates all the flow types between entities in the CSC. The forward coin flow in the CSC starts at the Mint that produces the new coins for circulation and ships them to the FRS. Next, the FRS combines the new coins with circulating coin inventory to satisfy the demand from DIs (such as saving banks, commercial banks, savings/loan associations, or credit unions). Then, DIs interact with commercial and individual customers directly to meet their needs. The backward coin flow is generated because of the deposits from retailers and public customers into DIs. The retail customers or businesses deposit/withdraw coins to/from a branch of a DI. The various branches of DIs send coins (deposited by the customers) to a designated branch, which in turn limits its own inventory level by depositing excess coins back to the FRS. Next, we describe the detailed structure of the CSC focusing on four functional components that impact the overall performance of the CSC.

3.1.1 U.S. Coin Supply Chain Structure

The CSC, similar to supply chains of banknotes, consists of 4-P (Player, Product, Process, and Policy) components. Major entities of the CSC are: (i) the Mint, (ii) the FRS, (iii) the DIs, (iv) the commercial and individual customers.

The Mint is not only the issuing authority for new coins, but it also plays a role in ensuring that the economy has a sufficient supply of coins. The Mint's highest priority is to cost effectively produce and supply required quantities of coins of all denominations required for the economy. The coins are produced at two Mint's facilities located in Denver and Philadelphia. After production, the Mint ships new coins to the FRS in quantities ordered by it. In exchange, the Mint receives the face value payment from the FRS.

Figure 3.2: Federal Reserve Districts and Reserve Bank Offices [Source: GAO (2013)].



The FRS consists of a Board of Governors and 12 Federal Reserve Banks, each of which is located in a Federal Reserve district (Figure 3.2). The Board is responsible for maintaining the stability of the U.S. financial markets and supervising the operations of the FRS's coin vaults (CVs). The CVs are located at 28 Reserve Bank offices (RBOs) and 170 coin terminals (CTs). Each CV is required to hold a minimum 2 weeks of *payable days* and a maximum 3 weeks of payable days in inventory. A *payable day* is defined as the amount of coin inventory needed to meet one day of expected payments to DIs (GAO 2013).

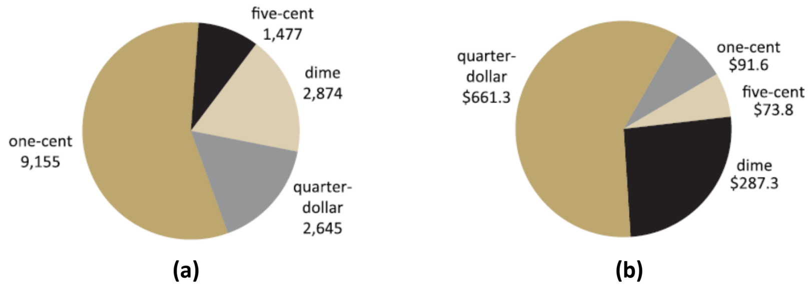
Each RBO is responsible for fulfilling the coin demand of DIs for an area within one of the FRS's 12 districts, either by ordering new coins from the Mint or by utilizing its coin inventory and/or coins in other CVs. After receiving new coins from the Mint, RBOs store coins either in their own on-site vaults or in CTs. According to GAO, RBOs held about 5% ($ 2.1 billion) of the circulating coin inventory, and the rest of 95% ($42 billion) was in general circulation in FY 2012 (GAO 2013). The CTs held about 50% in volume of RBOs' total coin inventory and the rest is held by RBOs' own on-site vaults. Approximately 15 armed carrier companies operate CTs, store RBOs' coin inventories for free, and earn revenue from the coin services (e.g., packaging coins) they provide to DIs. The FRS's national Cash Product Office (CPO) is the FRS' primary liaison with the Mint and is responsible for submitting a monthly consolidated coin order for RBOs and managing inventory levels for RBOs' own on-site vaults and CTs. From the coin supply perspective, the CPO's ordering process uses orders of new coins from the Mint, RBOs' coin inventories, and transfers of coins between the FRS's CVs to meet estimated DIs' demand. The transportation of coins between the Mint and the FRS's CVs is performed by 3PLPs.

DIs satisfy the demand of commercial and individual customers by either ordering/withdrawing coins from the nearest CV or withdrawing from their own coin inventory. DIs may also deposit excess coin received from their customers to a CV. Commercial and individual customers cannot interact with the Mint or FRS's CVs directly for their demands of circulating coins. Instead, they can only deposit or withdraw coins from DIs to meet their day-to-day coin requirements.

Next, we provide few facts about coins circulating in CSC. When new coins produced by the Mint reach public's hands, they are called circulating coins. Circulating coins contain the following two types: fit coins (suitable for transactions) and unfit coins (unsuitable for reuse and destroyed/reprocessed by the FRS). Unlike unfit banknotes that are deformed, torn, or defaced, unfit coins are scarce, and therefore are not considered in our study. However, we do consider reusing/recycling the coin deposits collected from the public to fulfill their demand in terms of coin withdrawals.

Hence, aggregated demand (deposits and withdrawals) is considered in this study to investigate the CSC.

Figure 3.3: (a) Shipments of Coins from the Mint to the FRS (Millions of Coins) and (b) Revenue by Denomination (Dollars in Millions) in FY 2015 [Source: U.S. Mint (2014a) and Mint (2015)].



Since FY 2013, circulating coins in the CSC for trade and commerce include four denominations: penny, nickel, dime, and quarter (circulating $1 coin production was suspended in early 2012). According to the Mint's 2014 and 2015 annual report, the Mint delivered 16.2 billion units of new coins to the FRS for circulation in FY 2015, a 23.9% increase from 13 billion FY 2014. Shipments increased for all denominations, especially for quarter and dime shipments, which is a continuing trend since FY 2011. The revenue from coin production was increased 42.3% amount to $1,114 million in FY 2015, driven by quarter revenue (59.3% of the total revenue) and dime revenue (25.8% of the total revenue). Figure 3.3 illustrates shipments of coins and revenues earned by the Mint. Although the production cost of both pennies and nickels exceeded their face value for the tenth consecutive fiscal year, all denominations' unit costs dropped in FY 2015 compared to FY 2014. Because of increased shipments, lower metal costs, and lower general and administrative costs, the Mint generated $540.9 million seigniorages by producing new coins for circulation in FY 2015, which is 87.1% higher than that in FY 2014 (U.S. Mint 2014a, 2015). *Seigniorage* is the difference between the face value of the coin and the cost to produce it.

3.1.2 Research Questions

This paper investigates the CSC from following operational perspectives: production and supply planning of coins for the FRS, coin inventory management at CVs, and coin transhipment between CVs, and inventory management for DIs. We devise some research questions concerning the overall performance of both supply and demand side parties. For the supply side, we address problems encountered in the central planner's (the FRS's) viewpoint of managing the CSC efficiently to supply coins to the economy. For the demand side, since DIs have great incentives in cost-effectively managing their coin inventory, we address issues facing DIs in the context of optimizing their operational cost to supply coins to the public.

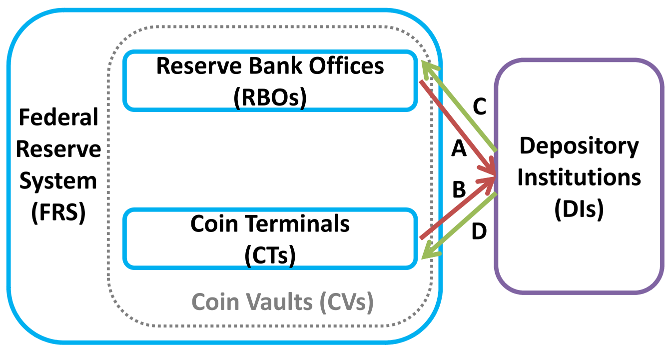
For the supply side, the FRS has implemented a centralized method in FY 2009 for managing coin supply across 12 FRS's districts and has established national inventory targets to track and measure the national coin inventory level (GAO 2008, 2013). Under the centralized approach, the CPO assumes the responsibility of reviewing and managing the inventory level at each CV (at RBO or CT) before placing a consolidated order to the Mint for new coin demand of each CV for the targeted month. The inventory management decisions at CVs are made by the CPO including setting individual lower and upper bounds on the coin inventory, and ordering/replenishing inventory. As a consequence of the centralized method, from FY 2009 to FY 2012, the combined inventory for all four denominations decreased 43%, the total annual coin management costs for RBOs increased by 69% (GAO 2013). Specifically, costs at individual RBO increased at rates ranging from 36% to 116%. These costs included RBOs' administration, coin handling, coin transshipment and coin inventory costs. This indicates that since the CPO does not monitor coin management costs by each RBO (instead focusing on managing coin costs from the national perspective), there are missing potential opportunities to improve the cost-effectiveness of coin-related operations across RBOs. The FRS's has an incentive to coordinate with the Mint and RBOs for efficiently monitoring coin costs for each CV to improve the cost-effectiveness of the CSC. Since new coin production cost is still a large portion of the total cost for the CSC, we develop optimization models to include the operational cost at both the FRS and the Mint in providing better supply planning methods in order to obtain additional efficiencies. In addition, we also examine whether the current inventory bounds for CVs set by the FRS are reasonable.

Moreover, since interrelated factors make it difficult to forecast long-term coin demand and a significant and unexpected fluctuation in coin demand could lead to increased coin inventory, an effective forecast and a planning system is crucial for the FRS to operate considering demand uncertainty. In other words, the FRS has the incentive to combine forecasts with continual tracking of demand and inventory levels, to adjust coin orders placed to the Mint on behalf of each RBO periodically, and to develop a robust planning system in the long run.

In addition, the FRS's current operating policy introduced as follows is not efficient enough to minimize the coin related cost, which brings the interest of analysis. When commercial and individual customers want more coins, DIs order more from the FRS (Croushore 2003), which then ship coins to DIs either from its RBOs' on-site vaults' inventory (arrow A in Figure 3.4) or directly from CTs (arrow B). However, if the public deposit more unpackaged coins into DIs, to reduce their own inventory cost and to get enough packaged coins to satisfy the public's demand, DIs may deposit the excess coins to the FRS, either directly to RBOs (arrow C) or through CTs (arrow D). For a given month, *net pay* that denotes the difference between the number of coins the FRS supplies to DIs and the number of coins the FRS receives from DIs for a year ($\textit{net pay} = (A + B) - (C + D)$), can be positive or negative.

In each year since 1993, the *net pay* for circulating coins is generally positive for all denominations (GAO 2008, GAO 2013). Since the new coin production cost results in inefficiency in the CSC, one important performance measure currently used by the FRS for the CSC is the ratio of the annual total production of new coins to net pay (denoted as β). Ideally $\beta \stackrel{def}{=} \frac{Annual\ Total\ Production\ of\ Coins}{Net\ Pay}$ should be equal or close to 1 (produce the exact amount needed to satisfy the public demand) indicating the system is efficient in managing and recirculating coins in the CSC. However, whether β is the right performance measure for the CSC needs to be investigated.

Figure 3.4: Net Pay.



Based on the above discussion, CVs can either order new coins from the Mint, or request transfers of circulating coins from other CVs, or use their own coin inventories to satisfy the demand. Therefore, regarding the supply side problem from the FRS's perspective, to satisfy the public's coin demand, there is an essential tradeoff between (i) minimizing the production and transportation cost for new coins paid by the Mint (or limiting β value close to one) and (ii) reducing the coin transportation and inventory management costs paid by the FRS using effective means. Hence, a comprehensive approach that can minimize the total cost for the FRS and the Mint is proposed for improving the efficiency of the CSC. In addition, in order to deal with uncertain demand, we propose two methods: (i) modifying and adjusting the coin orders and transhipment quantities periodically using a planning tool, and (ii) sticking to a robust solution that can handle uncertain demand with the total cost deviation within a reasonable range even in the worst case scenario. As regards to improving the performance for the supply parties (the Mint and the FRS), we plan to answer the following research questions.

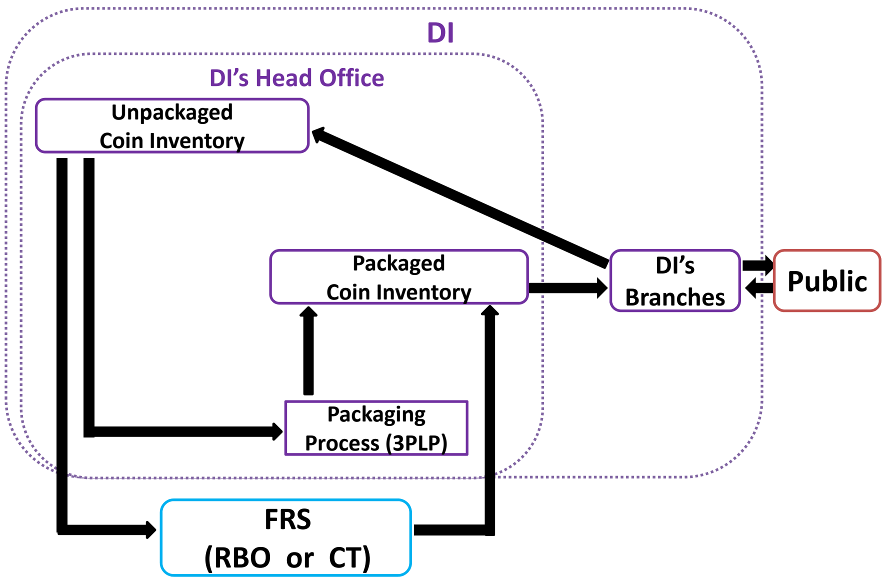
- Are current inventory bounds for CVs set by the FRS reasonable? If not, what would be the best method to set those bounds for inventories held at CVs?
- How should the FRS plan the production, distribution and management of inventories of all denominations of coins to minimize the total cost for the CSC?
- Is β a good performance measure for managing the CSC efficiently?
- To conquer the variation in coin demand, how can we develop a procedure to determine/adjust coin orders and transhipment quantities periodically and efficiently for the FRS? How can we develop a robust planning system for the FRS to manage coin circulation in the present of demand uncertainty?

In the demand side problem, commercial and individual customers deposit unpacked coins and withdraw packaged coins (rolls of one denomination of coins) at DIs. DIs send unpacked coins either to the FRS or to local 3PLPs for rolling/processing into packaged coins for their customers (Figure 3.5). To improve the performance of inventory management at DIs, we deal with the following research questions.

- How does a DI cost-effectively manage coin inventory and plan to process unpacked coins at the FRS or 3PLPs to obtain packaged coins?

- How does a DI cost-effectively manage coin inventory and plan to process unpacked coins under the 3PLPs' volume discount scheme for packaging?

Figure 3.5: DIs' Unpackaged and Packaged Coin Inventory Management Process.



In the supply side (respectively, demand side) problem, we assume that the demand of each denomination of coins at CVs (respectively, DIs) is independent and uncorrelated with each other. The reason behind this assumption can be described as follows. In the demand side problem, the demand for each denomination at a DI (say quarter) for a period occurs from its customers (commercial/individual) demand for quarters due to the cash transactions at their commercial stores. For example, in a cash transaction at a commercial store, a consumer gives \$10 for items purchased for a total \$8.43, the cashier must return the remaining change of \$1.57 (a random variable). The way the cashier is trained to return this change with a combination having the minimum number of coins: one dollar note, two quarters, one five-cent, and two pennies. In other words, the cashier rarely replaces a change for a quarter with two dimes and one five-cent. A large number of such cash transactions occur in a day. The daily aggregate demand for quarters occurs in this way at a commercial customer who in turn, requests quarters from a DI. Thus, we can reasonably assume that the daily aggregate demand at a DI for quarters is fairly independent due to the convenient way the changes are returned to the consumer. The DI, in turn, requests quarters at a CV to satisfy its customers. Similarly, in the supply side problem, when a DI withdraws coins (e.g., quarters) from a CV, it cannot replace the demand for each quarter with other denominations (e.g., two dimes and one five-cent).

The outline of this paper is as follows. Section 3.2 reviews closely related literature. The supply side operational problems are analyzed in Section 3.3. In this section, mathematical models are developed to minimize the total cost for the CSC and to investigate the effectiveness of using *net pay* by the FRS as the performance measure. A rolling horizon procedure based on the FRS's current operational policy and a robust planning system are developed to deal with demand uncertainty more effectively. In Section 3.4, the computational study of the supply side problem is presented. The *demand side Problem* is studied in the theoretical settings for improving DIs operational efficiency in Section 3.5. Discussion and conclusions are presented in Section 3.6.

3.2 Literature Review

Coins remain an important component of the transactions even in economies that have experienced a significant growth in checks, credit, debit and smart cards, and electronic transactions. However, the nature of the logistics and physical distribution of the CSC is still largely untouched in the published literature. To the best of our knowledge, this is the first paper analyzing the operations related to coin production, distribution, recycling, and inventory management, not only nationally, but also regionally, from both supply side and demand side for a CSC. We first review studies related to the CSC and position our research with respect to them. Then, we discuss the similarities between banknote supply chains and the CSC. Contributions of related research regarding the closed-loop banknote supply chains and banknote inventory management are highlighted as well.

3.2.1 Coin Supply Chain

As valuable as bank notes, even in our increasingly digital economy, coins play an essential role in commerce and trade. The production of coins is expensive and the shortage of coins to satisfy the public's demand are even more costly to banks and businesses (GAO 2013; Gadsby 1996). The rate at which coins fail to recycle through the economy is called as "lost coins." Mints produce vastly more coins than needed and the loss rate is much higher for coins of smaller value (Gadsby 1996; Goldin 1985). When it comes to the CSC, only a few papers discuss the coin denominational system (Cramer 1983; Sumner 1993; Telser 1995; Van Hove and Heyndels 1996). Many studies have made attempts to derive a superior coin denominational policy so that fewer coins per transaction are required (Bounie 2007; Kelber 2003; Tschoegl 1997). Others study the numbers of coins in circulation of various types (Cainiello 1982). All of these papers discuss the efficiency of alternative coin denominational systems, in which efficiency is defined as the average number of coins required for purchasing all the goods in the economy.

Besides the coin denominational system, research is scarce for operational problems in CSCs (Geismar et al. 2016). There are several potential opportunities for exploration of research in this domain. The supply side problem in this study focuses on the FRS's national perspective in analyzing the production, distribution, recirculation and inventory management of coins in the CSC. In addition, from the regional perspective, the demand side problem addresses issues related to the coin packaging operations and inventory management for DIs.

3.2.2 Banknote Supply Chain

Although most consumer products are distributed in only one direction, banknotes and coins are recirculated/reused through our economy. Rajamani et al. (2006) propose a useful framework of the closed-loop banknote supply chain where both forward distribution and reverse logistics are considered. They analyze the closed-loop structure of U.S. banknote supply chains and describe the FRS's new currency circulation policies. Conceptually, this structure is similar to the supply chain of false-failure returns (Ferguson et al. 2006). As considered in Rajamani et al. (2006) that analyzing the U.S. banknote supply chain, we also view the coin supply as a closed-loop supply chain. By applying inventory control principles to the banknote supply chains context, Geismar et al. (2007) extends Rajamani et al. (2006) and sheds light on how banks in the U.S. should cope with recent recirculation policy changes by the FRS. Although the research focused on closed-loop supply chains typically includes some kind of remanufacturing (Savaskan et al. 2004; Guide et al. 2006a, 2006b; Ketzenberg et al. 2006), there are studies focusing on the closed-loop service network (Kusumastuti et al. 2008). These supply chains have features resembling the coin and banknote supply chains, which can be considered as a special case of a closed-loop supply chain where products (i.e. coins and banknotes) are recycled and processed for redistribution.

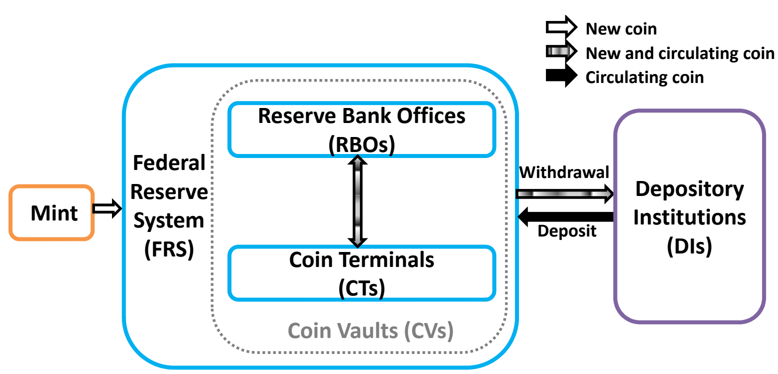
The specific banknote supply chain for different countries (Europe, Austria, and China) are also discussed in the literature (Schautzer 2007; the European Central Bank 2010; Carlin 2004; Smith et al. 2008). Each of these countries has a system of logistics that fits its individual circumstances. In Europe, the European Central Bank (ECB) and the national central banks (NCBs) of all 28 European Union (EU) Member States work together to determine banknote demand, production, and distribution across the Eurozone. Carlin (2004) shows similar operations (processing and distribution) of banknote supply chain in Australia. China's nationwide logistics network for Renminbi is discussed and analyzed by Smith et al.(2008), in which Chinese banknote supply chain's key problems such as production costs, inventory levels, transportation, and storage security are investigated. Zhu et al. (2015) study the supply side problem on behalf of a central bank in an international country focusing on managing the currency supply under security concerns.

The studies reviewed above deal with the structure, issue, and policy of the banknote supply chain. Some other papers develop models to manage currency supply, demand, and distribution under the Federal Reserve's new currency circulation policies recently implemented in the U.S. considering incentives for different parties in a banknote supply chain. Dawande et al. (2010) address a strategic approach to the problem of banknote recirculation from the perspective of DIs. This problem has been addressed by the FRS in its attempt to minimize the social cost of providing banknote to the public. They also develop a new coordinating mechanism to show that the introduction of banknote recirculation incentives for private banks reduces returns (i.e., deposits) to the FRS. They show that in general, the recirculation fee may not induce DIs to behave in a socially optimal manner. Hatzakis et al. (2010) provide an excellent review of the operational aspect of financial services. Mehrotra et al. (2010) derive effective operating policies for managing the day-to-day inventory of fit and used banknote for a DI. Later, Mehrotra et al. (2012) address the problem of pricing the transportation and other services offered to banks by a 3PLP. The banknote inventory management problem also has been studied by Zhu et al. (2011), which discuss the reuse of banknote to manage the inventory for medium-size DIs under the new Federal Reserve Policy. Two models are developed by Zhu et al. (2011) for various situations to improve the operations of a medium-sized DI by managing the banknote inventory.

3.3 Supply Side Problem

Here we provide a framework and formulation of a supply planning problem (namely, Problem I) for the FRS. The FRS must manage the supply of coins of different denominations at CVs in order to minimize the total cost of production, transportation, and inventory holding. The FRS spends a significant fraction of its annual budget on managing its coin supply to DIs for the CSC. The reasons for the significant spending are due to (i) inefficient the coin inventory management and (ii) the cost of coin production at the Mint. In order to fulfill the coin demand of DIs, the CPO places monthly orders for new coins to the Mint on behalf of CVs. For simplicity, we also view the Mint's two facilities (in Denver and Philadelphia) as CVs. Thus, there are 200 CVs in total distributing coins across 12 FRS's districts. Each CV has a detailed record of withdrawals or deposits of each denomination of coins ordered from or deposited by DIs. Note that the transportation of coins among the Mint, the FRS, and DIs is done in bags.

Figure 3.6: Framework of *Supply Side Problem*.



We assume that we can use the historical data to obtain a relatively accurate forecast of the demand over the next 12 months. Recall that we also assume that the demand of each denomination of coins at CVs is independent and uncorrelated with each other. A coin supply and distribution model for managing coin production and circulation for a single denomination (i.e., quarter) is developed in this section. The models for other denominations are similar. Figure 3.6 illustrates the framework of Problem I. Notations used in Problem I are in Table 3.1.

Table 3.1: Parameters and Variables Used in Problem I.

	Parameters
T	Number of periods (months) in the planning horizon, $T = 12$.
N	The number of CVs, where $N = 200$. The Mint is also viewed as a CV with $j = 0$ and there is no demand at $j = 0$.
D_j^t	The mean aggregated deposits of coin (in bags) during period t at CV j, $t = 1, 2, \ldots, T$ at CV j, $j = 1, 2, \ldots, N$.
W_j^t	The mean aggregated withdrawals of coin (in bags) during period t at CV j, $t = 1, 2, \ldots, T$ at CV j, $j = 1, 2, \ldots, N$.
B_j^t	$B_j^t = D_j^t - W_j^t$ for period t at CV j, $t = 1, 2, \ldots, T$, $j = 1, 2, \ldots, N$.
h_j	The per bag per period coin holding cost incurred at the CV j.
a	The per bag production and ordering cost at the Mint.
b_{ij}	The per bag transportation cost between CV i and CV j. $i, j = 0, 1, 2, \ldots, N$.
U_j	The upper bound of the inventory level at CV j. Set it as three weeks of payable days.
O_j	The lower bound of the inventory level at CV j. We set $O_j = 0 \ \forall j$.
	Variables
x_{0j}^t	The amount of coins (in bags) shipped from the mint facility 0 (either M_1 or M_2) that is closer to CV j at the beginning of period t, $j = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$. $x_{oj}^t = min\{x_{M_1 j}^t, x_{M_2 j}^t\}$.
x_{ij}^t	The amount of coins (in bags) shipped from CV i to CV j at the beginning of period t, $i, j = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$.
I_j^t	The coin inventory level(in bags) at the end of period t at CV j, $t = 0, 1, \ldots, T$, $j = 1, 2, \ldots, N$. For each CV j, I_j^t is the inventory level right after receiving $\sum_{i \neq j} x_{ij}^t$ bags and shipping out $\sum_{i \neq j \, and \, i \neq 0} x_{ji}^t$ bags of coins.

3.3.1 Coin Inventory Management in Practice

Before developing a mathematical model to solve Problem I, we briefly discuss the current coin ordering and inventory management in practice, which crucially impacts our model. We have obtained this information from GAO's 2013 report and our interaction with the Fed officials.

The CPO has implemented a centralized approach to inventory management in 2009. In order to track and measure the coin inventory, it has established the national upper and lower inventory targets for all four denominations and the upper and lower inventory levels at each CV. In order to maintain sufficient supply of coins, the CPO reviews the daily inventories at CVs (denoted CV j, $j = 1, 2, \ldots, N$). Since the coin supply at each CV differs depending on that CV's typical volume of coin payments and receipts, each CV is required to hold a minimum 2 weeks of "payable days" and a maximum 3 weeks of "payable days" in inventory, where *payable day* is the amount of coin inventory needed to meet one day of expected payments to depository institutions (GAO 2013).

The CPO transfers excess coins from one CV to another CV where coin deficits occur. Before performing such transfers, it considers future expected demand, seasonal shifts etc. in the demand of the CVs having excess. These transfers are known as interbank transfers which incur the transportation cost to the FRS. If there is an insufficient supply of coins to meet demand via interbank transfers, the CPO orders new coins from the Mint.

Currently, although the CPO reviews and places a monthly order to the U.S. Mint, the supply planning is done in 2-month rolling horizon fashion. In other words, the CPO revises and uses a 2-month forecast of expected demand of coins of each denomination and then places a monthly order to the Mint about 2 months prior to the expected delivery. For every 2 months passing by, demand is realized and the CPO may revise the monthly order quantity based on updated 2-month realized demand information. In order to facilitate the planning of coin production at the Mint, the CPO provides estimates of projected demand and new coin orders for up to the next 12 months.

Each CV j at time t (denoted as CV_j^t), $t = 1, 2, \ldots, T$, $j = 1, 2, \ldots, N$, has two random events defined by two random variables, $\tilde{D}_j$ (deposit of coins) and $\tilde{W}_j$ (withdrawal of coins) with mean aggregated values D_j and W_j, respectively. Here each time period t is a month since the inventory planning is done monthly basis. We assume that two random variables, $\tilde{D}_j$ and $\tilde{W}_j$ have certain probability distributions that can be determined by the historical data. Let w_j be *payable day* at CV j which is the mean daily withdrawal of coins by DIs. Thus, the mean withdrawal, $W_j = 4 \times 5 \times w_j$ assuming that month is equivalent to 4 weeks and each week consists of 5 working days. Currently, the inventory planning is done based on only the withdrawal of coins ($\tilde{W}_j$) during time t, even though the deposit of coins ($\tilde{D}_j$) also occur simultaneously at time t. The CPO does not want to take a risk of relying on deposits to satisfy the withdrawals in the same period. This is achieved by holding a minimum of 2 weeks of "payable days" of inventory as safety stock.

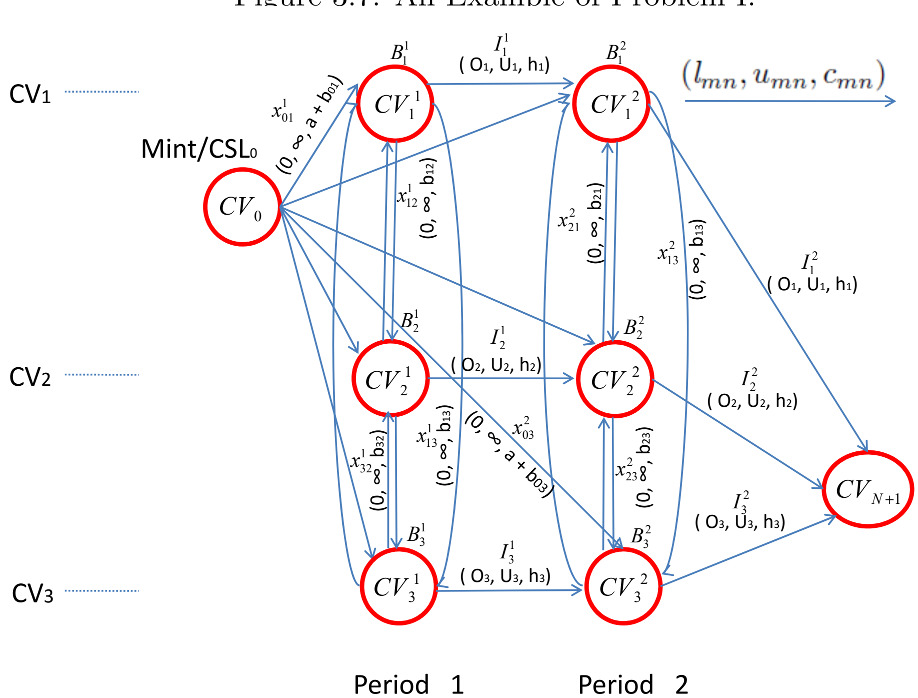
The periodic review inventory model is used in many practical environments. For example, Hewlett-Packard team has successfully implemented the inventory model in its distribution centers (Lee et al. 1993). The periodic review model is also suitable for managing coin inventories at each location since the upper and lower inventory levels at each location need to be established optimally (see Appendix B.1). Currently, a form of periodic review inventory model is now used at each CV j. For CV, the CPO has established that the upper bound and lower bound for that CV's inventory are 3 weeks of "payable days" and 2 weeks of "payable days", respectively. Since our analysis first assume deterministic demand in Problem I, in which we plan the inventory levels for all CVs above their safety stock levels (2 weeks of "payable days" in inventory in each location). Thus, for each CV j in our deterministic model in section below, we set the lower bound, $O_j = 0$ and vary the upper bound, U_j as 1, 2, or 3 weeks of "payable days") for Problem I to test whether the current bounds set by the CPO are appropriate. The detailed computational testing is done in Section 3.4.1. The review period R is one month ($R = 4$ weeks). A request for an order can be fulfilled in one week, i.e., the lead time, $L = 1$.

3.3.2 Network Flow Model for Problem I: One Denomination

We first assume that there is no uncertainty either in the deposits ($\tilde{D}_j$) or withdrawals ($\tilde{W}_j$) in order to gain insights into the problem. Later in Section 3.3.4 and Section 3.3.5, we will deal with uncertainty by proposing a procedure and developing a robust solution system, respectively. Since the demand of each denomination of coins at CVs is assumed to be independent and uncorrelated with each other, we elaborate our approach considering only one denomination (i.e., quarters). In this section, we propose a multi-period network approach using a minimum cost flow (MCF) model to optimize the coin ordering, transshipping, and inventory holding process between the Mint and the FRS. Specifically, for specified denomination in each period, we solve the flow in the network using an MCF model. Recall that notation needed to model Problem I is in Table 3.1.

We first describe the construction of the network model and then illustrate it with an example. In the construction, we start with a supplying node CV_0 (the Mint). Although circulating coins are produced at the Mint's two facilities: Philadelphia and Denver, since for each CV j, one of the two Mints is closer, without loss of generality, we use CV_0 stands for both facilities. Then, we add nodes CV_j^t for CV j in period t. Each node CV_j^t has B_j^t, surplus (or deficit) of coin (in bags) during period t at CV j, $t = 1, 2, \ldots, T$, $j = 1, 2, \ldots, N$. D_j^t and W_j^t are the mean aggregated deposits and mean aggregated withdrawals, respectively, in period t at CV j. Note that each CV has multiple nodes for different periods. Then we add a sink node CV_{N+1} standing for the total coin inventory left after the last period T coming from all CVs, except for CV_0 (the Mint). We now describe each arc with its lower bound, upper bound, and unit cost for the flow on that arc. An example for Problem I of one denomination is shown in Figure 3.7 considering a 2-period (month) rolling horizon.

Figure 3.7: An Example of Problem I.



- An arc from node CV_0 to CV_j^t, $j = 1, 2, \ldots, N$, representing the new coins produced and shipped from the Mint (both mint facilities: M_1 and M_2) to CVs, where CV_j^t represents CV j in period t. The amount of coin flow on this arc, x_{oj}^t, is defined as $\forall t, j,\ x_{oj}^t = min\{x_{M_1j}^t, x_{M_2j}^t\}$, where the index 0 represents the Mint's facility that is closer to CV j. The lower bound, upper bound and unit cost of this arc are 0, ∞, and $a + b_{0j}$. We also set the initial coin inventory level at each CV as its lower bound: $I_j^0 = O_j$.

- An arc from node CV_j^{t-1} to node CV_j^t, $j = 1, 2, \ldots, N$, representing the coin inventory carried from period $t-1$ to period t. The lower bound, upper bound and unit cost of this arc are O_j, U_j, and h_j.

- An arc between each pair of node CV_i^t and node CV_j^t, $i, j = 1, 2, \ldots, N$ representing the coin transshipments among different CVs in period t. The lower bound, upper bound and unit cost of this arc are 0, ∞, and b_{ij}.

- An arc from node CV_j^T, $j = 1, 2, \ldots, N$, to the sink node CV_{N+1} representing the ending coin inventory after the last period T at CV j. The lower bound, upper bound and unit cost of this arc are O_j, U_j, and h_j.

The mixed integer programming (MIP) formulation for Problem I is presented below.

Problem I:

The objective function minimizes the total cost for the Mint and the FRS, which includes the new coin production and ordering cost, circulating coin transportation cost between the Mint and the FRS, and the circulating coin inventory cost for the FRS. We compute below the average coin inventory for each period. The beginning coin inventory level at period t is I_j^{t-1} and the coin inventory level at the end of period t is $I_j^t = I_j^{t-1} + B_j^t + \sum_{i=0,\, i\neq j}^N x_{ij}^t - \sum_{i=1, i\neq j}^N x_{ji}^t$. The average coin inventory level during period t is $\frac{1}{2}(I_j^{t-1} + I_j^t)$. Thus, the total inventory cost during the planning horizon is $\sum_{j=1}^N h_j \cdot (\sum_{t=1}^{T-1} I_j^t + \frac{1}{2} I_j^0 + \frac{1}{2} I_j^T)$.

$$\begin{aligned}
\textbf{Min} \;\; \Pi(x_{ij}^t, I_j^t) = \sum_{t=1}^{T}\sum_{j=1}^{N} a \cdot x_{0j}^t \;\; &+ \;\; \sum_{t=1}^{T} \sum_{i=0,\, i\neq j}^{N} \sum_{j=1}^{N} b_{ij} \cdot x_{ij}^t \\
&+ \;\; \sum_{j=1}^{N} h_j \cdot (\sum_{t=1}^{T-1} I_j^t + \frac{1}{2} I_j^0 + \frac{1}{2} I_j^T)
\end{aligned}$$

Subject to:

Constraints (3.1) are the coin flow balance equations that compute the coin inventory level at the end of each period t for each CV j, $j = 1, 2, \ldots, N$.

$$I_j^t \;\; = \;\; I_j^{t-1} + B_j^t + \sum_{i=0,\, i\neq j}^{N} x_{ij}^t - \sum_{i=1,\, i\neq j}^{N} x_{ji}^t, \quad \forall t, j \tag{3.1}$$

Constraints (3.2) limit the coin inventory level at each CV between its upper and lower bound.

$$O_j \;\leq\; I_j^t \;\leq\; U_j, \quad 1 \leq t \leq T, \forall j \tag{3.2}$$

Constraints (3.3) are non-negativity constrains.

$$I_j^t, x_{ij}^t \geq 0, \;\; \forall i, j, t \tag{3.3}$$

Property 1 *Since $\forall B_j^t$ and $\forall U_j$ are integer valued in the MIP formulation of Problem I, all the variables in every basic feasible solution (including an optimal one) also have integer values.*

Note that all deposits and withdrawals in MCF are counted in bags and all flows in the network have integer capacity. Since each feasible integer flow corresponds to a feasible shipment of coins in Problem I and vice versa, we have the following result.

Lemma 1 *Solving the minimum cost flow problem constructed above is equivalent to optimizing the FRS's coin ordering and inventory management process.*

Proof: Since each node represents a specific location (either mint facility location or CV) with each arc standing for the corresponding movement of coin flows (new coin transportation, circulating coin transshipment, or circulating coin inventory carried across periods), each feasible integer flow corresponds to a feasible sourcing for the FRS's coin ordering and inventory management process and vice versa. Thus, the optimal solution to the minimum cost flow problem constructed above is equivalent to the optimal process for the FRS for Problem I. This result holds in general. ∎

3.3.3 Net Pay Analysis

Recall that currently, the FRS uses β (the ratio of the annual total production of new coins to net pay) to measure the performance of the CSC. Ideally, the FRS wants β to be equal or close to 1. To check if β is a good performance measure, we develop a MIP model with the objective of minimizing the difference between β and 1. This problem is referred as Problem I_N. We use the same notation in Table 3.1. The MIP formulation for Problem I_N is presented as below.

MIP Formulation for Problem I_N:

The objective function minimizes the difference between the annual total production of new coins and net pay. Note that net pay is equal to $\sum_{t=1}^{T}\sum_{j=1}^{N}(W_j^t - D_j^t) = -\sum_{t=1}^{T}\sum_{j=1}^{N} B_j^t$. Since net pay is known, we only need to minimize the annual new coin production quantity. With the minimum objective value, β would be close to 1.

Problem I_N:

$$\textbf{Min} \ \sum_{t=1}^{T}\sum_{j=1}^{N} x_{0j}^t$$

Subject to:

Constraints (3.4) compute the new coin production quantity for period t.

$$x_{0j}^t \ = \ I_j^t - I_j^{t-1} - B_j^t - \sum_{i=1,\, i\neq j}^{N} (x_{ij}^t - x_{ji}^t), \ \ \forall t, j \tag{3.4}$$

Constraints (3.5) enforce that $\beta \geq 1$.

$$\sum_{t=1}^{T}\sum_{j=1}^{N} x_{0j}^t \ \geq \ -\sum_{t=1}^{T}\sum_{j=1}^{N} B_j^t \tag{3.5}$$

Constraints (3.6) restrict the coin inventory level at each CV between its upper and lower bound.

$$O_j \ \leq \ I_j^t \ \leq \ U_j, \ \ 1 \leq t \leq T, \forall j \tag{3.6}$$

Constraints (3.7) are non-negativity constrains.

$$x_{0j}^t, I_j^t, x_{ij}^t \geq 0, \ \ \forall i, j, t \tag{3.7}$$

Suppose β is a good performance measure for the CSC, then the optimal solution to Problem I_N should also give a very low total cost Π_N^*. In other words, the difference between Π_N^* for Problem I_N and the minimum total cost $\Pi^*(x_{ij}^t, I_j^t)$ for Problem I should be equal or close to 0. We test this through a comprehensive computational study in Section 3.4.1. We show below that the minimum total cost obtained from Problem I_N should be at least as large as the minimum total cost for *Problem* I.

Lemma 2 $\Pi^*(x_{ij}^t, I_j^t) \leq \Pi_N^*$.

Proof: Let $(\hat{x}_{ij}^t, \hat{I}_j^t)$ be an optimal solution toProblem I_N. Since $(\hat{x}_{ij}^t, \hat{I}_j^t)$ is also a feasible solution to *Problem* I, we have $\Pi^*(x_{ij}^t, I_j^t) \leq \Pi^*(\hat{x}_{ij}^t, \hat{I}_j^t) = \Pi_N^*$. ■

3.3.4 Demand Uncertainty: Procedure Rolling Horizon

In order to handle the variation in actual demand (withdrawal and deposit), the FRS uses a two-month rolling operating policy (GAO 2013). Based on the FRS's current operating policy and our MIP and MCF models for Problem I, we now develop a procedure with the rolling-horizon of a two-month period to solve Problem I (called the Procedure Rolling Horizon). The FRS can use this procedure in practice to handle the variation in actual demand effectively by incorporating the updated demand when it becomes available. Computational studies for Procedure Rolling Horizon are presented in Section 3.4.2 assuming that the coin demand is normally distributed.

3.3.5 Demand Uncertainty: μ-Robust Solution

In order to understand the impact of variation in demand on the total cost, Π, we introduce uncertainty into both deposits and withdrawals in the following manner. We define $\tilde{D}_j^t$ and $\tilde{W}_j^t$ as two random demand variables with mean aggregated values

D_j^t and W_j^t. We also define $\tilde{B}_j^t = \tilde{D}_j^t - \tilde{W}_j^t$ as the random variable for the net demand in period t at CV j. Clearly, $\tilde{B}_j^t$ has the mean aggregated value: $B_j^t = D_j^t - W_j^t$.

Procedure Rolling Horizon(PRH):

$min\_cost\_PRH = 0$

Solve Problem I for periods 1 to 12 with initial inventories $I_j^0 = 0$ and mean net demand $\bar{B}_j^t$.

Label the transhipment quantity solutions for period 1 and 2 as x_{ij}^{1*} and x_{ij}^{2*}.

Collect realized net demand for period 1 and 2: $B_j^{r1} = D_j^{r1} - W_j^{r1}$ and $B_j^{r2} = D_j^{r2} - W_j^{r2}$.

Compute updated inventories for period 1 (I_j^{u1}) and 2 (I_j^{u2}) using I_j^0, B_j^{r1}, B_j^{r1}, x_{ij}^{1*} and x_{ij}^{2*}.

If $I_j^{u1} < 0$ (or $I_j^{u2} < 0$), set $I_j^{u1} = 0$ (or $I_j^{u2} = 0$).

Record the total amount (defined as $M_{t_1t_2}$) required to bring I_j^{u1} and I_j^{u2} to zero.

Else

Use B_j^{r1}, B_j^{r2}, x_{ij}^{1*} and x_{ij}^{2*} to compute I_j^{u1} and I_j^{u2}.

Exit If

Compute the total cost for period 1 and 2 using I_j^0, I_j^{u1}, I_j^{u2}, x_{ij}^{1*} and x_{ij}^{2*} as $min\_cost\_(t_1t_2)$.

Update $min\_cost\_PRH = min\_cost\_(t_1t_2)$.

Update the mean demand for period 3 as $\bar{B^{u3}}_j = \bar{B^3}_j + M_{t_1t_2}$.

Solve Problem I for periods 3 to 14 with $\bar{B^{u3}}_j$, $\bar{B}_j^4$, ..., and $\bar{B}_j^{14}$.

Label the transhipment quantity solutions for period 3 and 4 as x_{ij}^{3*} and x_{ij}^{4*}.

Collect realized net demand for period 3 and 4: $B_j^{r3} = D_j^{r3} - W_j^{r3}$ and $B_j^{r4} = D_j^{r4} - W_j^{r4}$.

Update realized net demand for period 3 as $B_j^{ur3} = B_j^{r3} + M_{t_1t_2}$.

Compute updated inventories for period 3 (I_j^{u3}) and 4 (I_j^{u4}) using I_j^{u2}, B_j^{ur3}, B_j^{r4}, x_{ij}^{3*} and x_{ij}^{4*}.

If $I_j^{u3} < 0$ (or $I_j^{u4} < 0$), set $I_j^{u3} = 0$ (or $I_j^{u4} = 0$).

Record the total amount (defined as $M_{t_3t_4}$) required to bring I_j^{u3} and I_j^{u4} to zero.

Else

Use B_j^{ur3}, B_j^{r4}, x_{ij}^{3*} and x_{ij}^{4*} to compute I_j^{u3} and I_j^{u4}.

Exit If

Compute the total cost for period 3 and 4 using I_j^{u2}, I_j^{u3}, I_j^{u4}, x_{ij}^{3*} and x_{ij}^{4*} as $min\_cost\_(t_3t_4)$.

Update $min\_cost\_PRH = min\_cost\_(t_1t_2) + min\_cost\_(t_3t_4)$.

Repeat steps for computing $min\_cost\_(t_3t_4)$ for the following periods until $min\_cost\_(t_{11}t_{12})$ is obtained.

Update $min\_cost\_PRH = min\_cost\_(t_1t_2) + ... + min\_cost\_(t_{11}t_{12})$.

For each CV j during the planning horizon T, we assume that the demand distribution is normal (later in Section 3.4.2, we perform computational experiments using normal distributions with varying levels of uncertainties) and randomly generate K sample paths representing possible realizations of B_j^{tk}, where $B_j^{tk} = D_j^{tk} - W_j^{tk}$. Considering all paths for all N CVs, we will have K scenarios. Let $V_j^{t_s}$ be the value of B_j^{tk} at each CV j in each period t ($t = 1, 2, \ldots, T$) for a given scenario s, where $s = 1, 2, \ldots, K$. Let Π^{s*} be the optimal objective value for the optimal solution, $x_{ij}^{t_s*}$. Our problem ($MinCost - R1$) of obtaining an optimal solution, $x_{ij}^{t_s*}$, for a given scenario s can be stated as follows.

Problem $MinCost - R1$:

$$\textbf{Min } \Pi^s(x_{ij}^{t_s}, I_j^{t_s}) = \sum_{t=1}^{T}\sum_{j=1}^{N} a \cdot x_{0j}^{t_s} + \sum_{t=1}^{T}\sum_{i=0,\, i\neq j}^{N}\sum_{j=1}^{N} b_{ij} \cdot x_{ij}^{t_s} + \sum_{j=1}^{N} h_j \cdot (\sum_{t=1}^{T-1} I_j^{t_s} + \frac{1}{2} I_j^{0_s} + \frac{1}{2} I_j^{T_s})$$

Subject to

$$I_j^{t_s} = I_j^{t_s-1} + V_j^{t_s} + \sum_{i=0,\, i\neq j}^{N} x_{ij}^{t_s} - \sum_{i=1,\, i\neq j}^{N} x_{ji}^{t_s}, \quad \forall t, j, s \tag{3.8}$$

$$0 \leq I_j^{t_s} \leq U_j, \quad 1 \leq t \leq T, \forall t, j, s \tag{3.9}$$

$$I_j^{t_s}, x_{ij}^{t_s} \geq 0 \textit{ and integer}, \quad \forall i, t, j, s \tag{3.10}$$

Let ρ^s be the occurrence probability of scenario s. In order to minimize the expected cost, we find below an optimal feasible solution (x_{ij}^{t*}) whose corresponding objective value should not deviate from a μ fraction of the optimal objective value Π^{s*} for any scenarios s. We archive this in the following model:

Problem $MinCost - R2$:

$$\textbf{Min } \Psi = \sum_{s=1}^{K} \rho^s \Pi^s(x_{ij}^{t}, I_j^{t_s})$$

Subject to

$$\Pi^s(x_{ij}^{t}, I_j^{t_s}) = \sum_{t=1}^{T}\sum_{j=1}^{N} a \cdot x_{0j}^{t} + \sum_{t=1}^{T}\sum_{i=0,\, i\neq j}^{N}\sum_{j=1}^{N} b_{ij} \cdot x_{ij}^{t}$$

$$
\begin{aligned}
& + \sum_{j=1}^{N} h_j \cdot (\sum_{t=1}^{T-1} I_j^{t_s} + \frac{1}{2} I_j^{0_s} + \frac{1}{2} I_j^{T_s}) && (3.11)\\
I_j^{t_s} &= I_j^{t_s-1} + V_j^{t_s} + \sum_{i=0,\, i\neq j}^{N} x_{ij}^t - \sum_{i=1,\, i\neq j}^{N} x_{ji}^t,\ \forall t, j, s && (3.12)\\
0 &\leq I_j^{t_s} \leq U_j, \quad 1 \leq t \leq T, \forall t, j, s && (3.13)\\
\Pi^s(x_{ij}^t, I_j^{t_s}) &\leq (1+\mu)\Pi^{s*}, \quad \forall s && (3.14)\\
x_{ij}^t, I_j^{t_s} &\geq 0 \textit{ and integer}, \quad \forall i, t, j, s && (3.15)
\end{aligned}
$$

Relative Regret Limit: μ

We may find a robust solution that minimizes the expected cost, $\sum_s \rho^s \Pi^s(x_{ij}^t, I_j^{t_s})$, which depends on the relative regret limit μ. If we include all possible sample paths K, we can solve problem Problem $MinCost-R2$ very precisely. In practice, K could be a very large number. We can also find the upper bound (μ_u) and the lower bound (μ_l) of relative regret limit μ. By setting the value of μ in the range ($\mu_u \leq \mu \leq \mu_l$) we may find a series of robust solutions. If μ is large (respectively, small), the feasible region of Problem $MinCost-R2$ is large (respectively, small). In addition, if μ is very large, Problem $MinCost - R2$ becomes a stochastic programming problem. Note that when μ increases, the feasible solution region increases. Hence, the expected cost value decreases.

Lemma 3 *If all possible sample paths K for representing uncertainty are included, the lower bound for relative regret limit μ_l can be obtained by solving μ for Problem $MinCost - R3$.*

Proof: The solution Π^{s*} is obtained by solving Problem $MinCost - R1$ for each sample path s. The constraints (3.19) of Problem $MinCost - R3$ assure that the solution obtained does not deviate more than μ fraction any of Π^{s*} values of sample paths. Since the objective is to minimize μ, we obtain a lower bound μ_l for μ. ■

Problem $MinCost - R3$:

$$\textbf{Min} \quad \mu$$

Subject to

$$\begin{aligned}
\Pi^s(x_{ij}^t, I_j^{t_s}) &= \sum_{t=1}^{T}\sum_{j=1}^{N} a \cdot x_{0j}^t + \sum_{t=1}^{T}\sum_{i=0,\, i\neq j}^{N}\sum_{j=1}^{N} b_{ij} \cdot x_{ij}^t \\
&+ \sum_{j=1}^{N} h_j \cdot (\sum_{t=1}^{T-1} I_j^{t_s} + \frac{1}{2}I_j^{0_s} + \frac{1}{2}I_j^{T_s}) && (3.16) \\
I_j^{t_s} &= I_j^{t_s-1} + V_j^{t_s} + \sum_{i=0,\, i\neq j}^{N} x_{ij}^t - \sum_{i=1,\, i\neq j}^{N} x_{ji}^t, \ \forall t, j, s && (3.17) \\
0 &\le I_j^{t_s} \ \le \ U_j, \quad 1 \le t \le T, \forall t, j, s && (3.18) \\
\Pi^s(x_{ij}^t, I_j^{t_s}) &\le (1+\mu)\Pi^{s*}, \quad \forall s && (3.19) \\
x_{ij}^t, I_j^{t_s} &\ge 0 \ and \ integer, \ \ \forall i, t, j, s && (3.20)
\end{aligned}$$

By relaxing constraints (3.14) in Problem $MinCost - R2$ (i.e., assuming a very large μ), we obtain the minimum expected cost Ψ_{min} by running the following model. Note that this model is a stochastic programming version of Problem $MinCost - R2$.

Problem $MinCost - R4:$

$$\textbf{Min} \ \Psi \ = \ \sum_{s=1}^{K} \rho^s \Pi^s(x_{ij}^t, I_j^{t_s})$$

Subject to

$$\begin{aligned}
\Pi^s(x_{ij}^t, I_j^{t_s}) &= \sum_{t=1}^{T}\sum_{j=1}^{N} a \cdot x_{0j}^t + \sum_{t=1}^{T}\sum_{i=0,\, i\neq j}^{N}\sum_{j=1}^{N} b_{ij} \cdot x_{ij}^t \\
&+ \sum_{j=1}^{N} h_j \cdot (\sum_{t=1}^{T-1} I_j^{t_s} + \frac{1}{2}I_j^{0_s} + \frac{1}{2}I_j^{T_s}) && (3.21) \\
I_j^{t_s} &= I_j^{t_s-1} + V_j^{t_s} + \sum_{i=0,\, i\neq j}^{N} x_{ij}^t - \sum_{i=1,\, i\neq j}^{N} x_{ji}^t, \ \forall t, j, s && (3.22) \\
0 &\le I_j^{t_s} \ \le \ U_j, \quad 1 \le t \le T, \forall t, j, s && (3.23) \\
x_{ij}^t, I_j^{t_s} &\ge 0 \ and \ integer, \ \ \forall i, t, j, s && (3.24)
\end{aligned}$$

In the above model, since we relax constraints (3.14) of Problem $MinCost - R2$, there also exists a value for μ_u. However, we would like to the smallest value of μ_u. The following model finding the smallest upper bound, μ_u has the same objective function value, Ψ_{min}.

Lemma 4 *If all possible sample paths K for representing uncertainty are included, the upper bound for relative regret limit μ_u can be obtained by solving μ for Problem $MinCost - R5$.*

Proof: In Problem $MinCost - R4$, since we relax constraints (3.14) of Problem $MinCost - R2$, there also exists an upper bound μ_u for μ. However, we would like to the smallest value of μ_u. The following Problem $MinCost - R5$ finding the smallest upper bound, μ_u has the same objective function value, Ψ_{min}. ■

Problem $MinCost - R5:$

$$\textbf{Min} \quad \mu$$

Subject to

$$\begin{aligned}
\Psi_{min} &\geq \sum_{s=1}^{K} \rho^s \Pi^s(x_{ij}^t, I_j^{t_s}) && (3.25)\\
\Pi^s(x_{ij}^t, I_j^{t_s}) &= \sum_{t=1}^{T}\sum_{j=1}^{N} a \cdot x_{0j}^t + \sum_{t=1}^{T}\sum_{i=0,\, i\neq j}^{N}\sum_{j=1}^{N} b_{ij}\cdot x_{ij}^t \\
&+ \sum_{j=1}^{N} h_j \cdot (\sum_{t=1}^{T-1} I_j^{t_s} + \frac{1}{2} I_j^{0_s} + \frac{1}{2} I_j^{T_s}) && (3.26)\\
I_j^{t_s} &= I_j^{t_s-1} + V_j^{t_s} + \sum_{i=0,\, i\neq j}^{N} x_{ij}^t - \sum_{i=1,\, i\neq j}^{N} x_{ji}^t,\ \forall t, j, s && (3.27)\\
0 &\leq I_j^{t_s} \leq U_j, \quad 1 \leq t \leq T, t, j, s && (3.28)\\
\Pi^s(x_{ij}^t, I_j^{t_s}) &\leq (1+\mu)\Pi^{s*}, \quad \forall s && (3.29)\\
x_{ij}^t, I_j^{t_s} &\geq 0 \ \textit{and integer}, \quad \forall i, t, j, s && (3.30)
\end{aligned}$$

The above solution μ_u has the same minimum expected cost, within which, Ψ_{min} assures that the relative regret value in the worst case scenarios to be as the smallest as possible.

Remark 1 *Note that if μ is large (respectively, small) the feasible region of Problem $MinCost - R2$ is large (respectively, small). Thus, the expected cost Ψ decreases as the value of μ increases in problem $MinCost - R2$. When μ is small, the solution is good (respectively, poor) for the worst case scenario (respectively, the overall scenarios) and vice versa. Thus, the smaller (respectively, larger) value for μ leads to a better worst case (respectively, expected) performance and vice versa.*

Remark 2 *Bottleneck scenarios: Given the solution for Problem $MinCost - R2$ for a μ value, we may calculate the α^s values for each scenario s as follows. $\alpha^s = \frac{\Pi^s(x_{ij}^t, I_j^{ts}) - \Pi^{s*}}{\Pi^{s*}}$. The α^s values for some scenarios may be close or equal to the μ value. Such scenarios are called "bottleneck scenarios" as they are the worst case scenarios for the corresponding μ value. We need to study such scenarios here as they hinder the expected performance (i.e., the improvement of the objective value of Problem $MinCost - R2$).*

Next, we consider Problem $MinCost - R2$ with μ-Robust Solution. Note that with given regret limit μ and the occurrence probability of scenario s: ρ^s, the objective function Ψ has a lower bound of $\sum_s \rho^s \Pi^{s*}$ and a upper bound of $\sum_s \rho^s (1+\mu)\Pi^{s*}$. Therefore, we have the following results.

Property 2 *For Problem $MinCost - R2$, with given regret limit μ and the occurrence probability of scenario s: ρ^s, the objective function Ψ has a lower bound of $\sum_s \rho^s \Pi^{s*}$ and a upper bound of $\sum_s \rho^s (1+\mu)\Pi^{s*}$, respectively.*

Property 3 *If $\forall V_j^{t_s}$, $\forall L_j$, and $\forall U_j$ are integer valued, any feasible μ-robust solutions including an optimal one, to the MIP formulation of Problem $MinCost - R2$ should also have integer valued variables.*

3.4 Computational Study for the Supply Side Problem

The key objectives of our computational studies are as follows: (i) evaluating the effectiveness of the current operating policy of FRS (i.e., using β as the performance measure for the CSC), and (ii) testing the effectiveness of Procedure Rolling Horizon and μ-Robust Solution for conquering demand uncertainty. For the first objective, we run both Problem I and Problem I_N using the FRS's real data available in the public domain, and compare their β values and total costs. For the second objective, based on the real data from FRS, we run randomly generated instances and sample paths for Procedure Rolling Horizon and μ-Robust Solution, respectively, and compare their performances.

We consider one denomination (quarter), 13 CVs (12 Reserve Banks located in the center of each Fed district and the Mint), and a planning horizon of 12 months. We collect the distance data among 13 CVs (including the Mint) using Google Maps. We use both (i) the three-year demand data (2010-2012 national monthly deposits and withdrawals) obtained directly from GAO's 2013 report, and (ii) the randomly generated demand data assuming a normal distribution of coin demand based on the real demand data of FRS in 2010, 2011 and 2012. The monthly deposits and withdrawals for quarter for each Fed district are estimated using the real population data in each Fed district and the proportion of quarter in circulation (about 47.85% of all denominations).

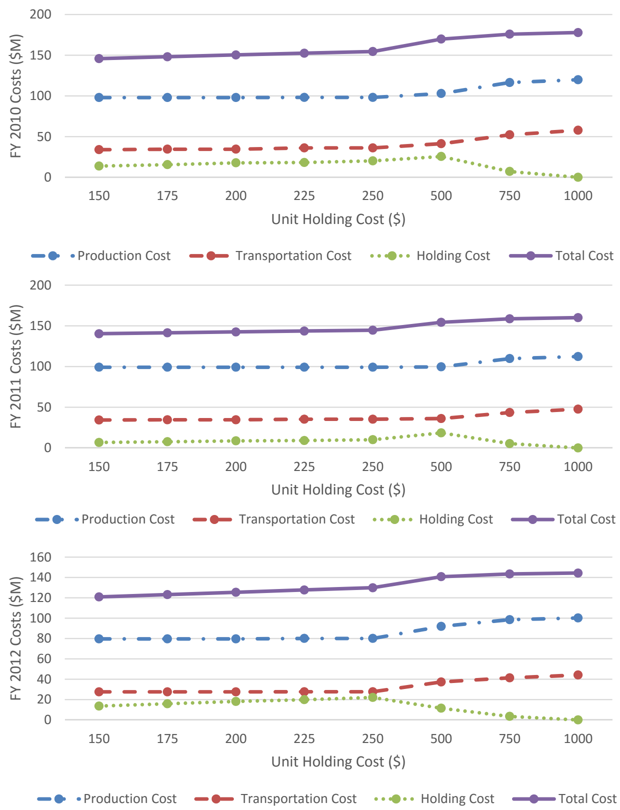
The values of other parameters are estimated based on the real cost and weight (50 lbs. per bag of quarters). The unit transportation cost between any two locations is estimated using FedEx 2-Day rate, which is the most commonly used mode of transportation in this setting. Since it costs the Mint \$0.0895 of produce one quarter and there are 4000 quarters in a bag (\$1000 face value of one bag), the per bag production cost can be easily calculated (Mint 2014b). Regarding the holding cost, it is set as between 15% and 100% of the face value of one bag of quarters to perform sensitivity analysis in Section 3.4.1, and as 20% of the face value of one bag of quarters to test performances of Procedure Rolling Horizon and μ-Robust Solution. We assume that all Fed districts have the same unit holding cost (h). The MIP and the MCF models are solved using CPLEX (version 12.6.1) and Visual Studio (version Express 2012) on a Lenovo desktop with 3.60 GHz CPU, 16 GB RAM, and 64-bit operating system.

3.4.1 Performance Testing for Problem I and Problem I_N

Before proceeding to the testing for Problem I and Problem I_N, we estimate the lower and upper bounds of inventories for each CV in each period using the three-year withdrawals (2010-2012) of FRS. The initial inventory level I_j^0 for each CV and the inventory lower bound O_j for all months of each CV are set as zero. Before the computational run, we know that the inventory upper bound U_j and the unit holding cost may be the two crucial cost drivers that have great impacts on the total cost. Although Problem I theoretically optimizes the operations of FRS such that the total cost for the CSC is minimized, we still want to check whether current inventory upper bound for each CV set by the FRS is reasonable. Hence, we first run both Problem I and Problem I_N with varying inventory upper bound U_j and varying unit holding cost h to check how different cost components might change. Since Problem I is focusing on cost minimization, the major results presented below for this problem are related to only the cost. Detailed cost components for both Problem I and Problem I_N (for 2010, 2011 and 2012) are provided in the Appendix B.3.

The cost components for increasing unit holding cost and varying inventory upper bound (one, two or three weeks of payable days) for Problem I with three-year

Figure 3.8: Cost Components for Problem I When $O_j = 0$ and $U_j = 1, 2, 3$ Weeks of Payable Days for 2010, 2011 and 2012.



demand data are shown in Figure 3.8. Interestingly, for varying inventory upper bounds, for each year's demand and the same unit holding cost, the solution and each type of cost for Problem I remain the same. This is because when U_j equal to 3 weeks of payable days, the optimal inventory solutions are within the 1 week of payable days range. This result answers our first research question and confirms that the current inventory upper bound U_j (1 week of payable days above the safety stock level for each CV) set by the CPO is reasonable, which allows the FRS to continue using the current inventory upper bound to manage the inventory for any individual CV. This result is summarized in the following observation.

Observation 1 *The inventory upper bound U_j does not have any effect on the opti-*

mal solution for Problem I, and therefore, the current inventory upper bound set by the CPO is reasonable.

A lower bound which is a safety stock S_j can be determined by the periodic review model as described in Appendix B.1.

We also observe that when unit inventory holding cost increases, the total inventory holding cost for Problem I increases until a point, and then starts decreasing. One reason is that, with certain unit holding cost (when $h < \$500$), the inventory level remains the same to minimize the total cost. Thus, within low unit holding cost range, the total inventory holding cost for Problem I increases with increasing unit holding cost. However, when h becomes too large, to minimize the total cost, inventory levels decreases dramatically, especially when $h = 1000$, because no inventory is being held in this case. This result is summarized in an observation below.

Observation 2 *The holding cost is concave with increasing unit holding cost h.*

In addition, when the unit inventory holding cost increases, for Problem I, both new coin production cost and transportation cost increase. This indicates that, when h becomes larger, to satisfy the demand with minimum total cost, more new coins are produced and more circulating coin transshipments are made. This result suggests that the FRS should select the appropriate level of the unit holding cost for CVs. Based on the total cost curve, it is reasonable for the manager of each RBO to keep the unit holding cost between \$150 and \$250 for a low total cost. This result is summarized in the following observation.

Observation 3 *For Problem I, both new coin production cost and transportation cost increase with increasing unit holding cost h, and therefore the holding cost needs to be managed properly.*

Next, for each year's demand data set, we run Problem I and Problem I_N to compare the β values and the total costs obtained from these problems. Results of β values and total cost ratio are shown in Tables 3.2, 3.3, and 3.4, respectively, for 2010, 2011, and 2012. Results show that Problem I not only minimizes the total cost, but also gives low β value, especially for $h \leq 250$. Some of the other interesting results are summarized in the following remark (the details are provided in the Appendix).

Table 3.2: Computational Results for Varying Inventory Upper Bound for Problem I and Problem I_N When $O_j = 0$ With 2010 Demand Data.

Inventory Upper Bound U_j	**Unit Holding Cost (\$)** h	**Critical Ratio Problem I** β_I	**Critical Ratio Problem I_N** β_{I_N}	**Total Cost (\$M) Problem I** $\Pi^*(x_{ij}^t, I_j^t)$	**Total Cost (\$M) Problem I_N** $\Pi_N^*(x_{ij}^t, I_j^t)$	**Total Cost Ratio** $\frac{\Pi_N^*(x_{ij}^t, I_j^t)}{\Pi^*(x_{ij}^t, I_j^t)}$
1 Weeks of Payable Days	150	1.1626	1.1626	145.79	305.9	2.0982
	175	1.1626	1.1626	148.09	308.63	2.0841
	200	1.1626	1.1626	150.32	311.35	2.0712
	225	1.1645	1.1626	152.46	314.08	2.0601
	250	1.1645	1.1626	154.48	316.8	2.0508
	500	1.2213	1.1626	169.83	344.04	2.0258
	750	1.3821	1.1626	175.88	371.29	2.111
	1000	1.4227	1.1626	177.78	398.53	2.2417
2 Weeks of Payable Days	150	1.1626	1.1626	145.79	322.37	2.2112
	175	1.1626	1.1626	148.09	324.75	2.1929
	200	1.1626	1.1626	150.32	327.13	2.1762
	225	1.1645	1.1626	152.46	329.51	2.1613
	250	1.1645	1.1626	154.48	331.89	2.1484
	500	1.2213	1.1626	169.83	355.7	2.0258
	750	1.3821	1.1626	175.88	379.5	2.111
	1000	1.4227	1.1626	177.78	403.3	2.2417
3 Weeks of Payable Days	150	1.1626	1.1626	145.79	328.14	2.2508
	175	1.1626	1.1626	148.09	330.52	2.2319
	200	1.1626	1.1626	150.32	332.9	2.2146
	225	1.1645	1.1626	152.46	335.28	2.1991
	250	1.1645	1.1626	154.48	337.66	2.1858
	500	1.2213	1.1626	169.83	361.47	2.1284
	750	1.3821	1.1626	175.88	385.27	2.1905
	1000	1.4227	1.1626	177.78	409.07	2.301

Remark 3 *For Problem I_N, when unit inventory holding cost increases, both new coin production cost and transportation cost remain the same and the inventory holding cost increases. When inventory upper bound increases from 1 week of payable days to 2 weeks of payable days, the transportation cost increases and the holding cost decreases. However, when inventory upper bound increases from 2 weeks of payable days to 3 weeks of payable days, although the transportation cost keeps increasing, the holding cost does not change. In addition, there is no direct relation between inventory upper bound and transportation cost (may increase or decrease), and between*

Table 3.3: Computational Results for Varying Inventory Upper Bound for Problem I and Problem I_N When $O_j = 0$ With 2011 Demand Data.

Inventory Upper Bound U_j	**Unit Holding Cost ($)** h	**Critical Ratio Problem I** β_I	**Critical Ratio Problem I_N** β_{I_N}	**Total Cost ($M) Problem I** $\Pi^*(x_{ij}^t, I_j^t)$	**Total Cost ($M) Problem I_N** $\Pi_N^*(x_{ij}^t, I_j^t)$	**Total Cost Ratio** $\frac{\Pi_N^*(x_{ij}^t, I_j^t)}{\Pi^*(x_{ij}^t, I_j^t)}$
1 Weeks of Payable Days	150	1.0643	1.0643	140.34	287.8	2.0507
	175	1.0643	1.0643	141.47	288.9	2.0421
	200	1.0643	1.0643	142.56	290	2.0342
	225	1.0643	1.0643	143.62	291.1	2.0269
	250	1.0643	1.0643	144.63	292.21	2.0204
	500	1.0696	1.0643	154.36	303.21	1.9643
	750	1.1783	1.0643	158.68	314.22	1.9802
	1000	1.2058	1.0643	160.1	325.22	2.0314
2 Weeks of Payable Days	150	1.0643	1.0643	140.34	290.39	2.0692
	175	1.0643	1.0643	141.47	291.31	2.0592
	200	1.0643	1.0643	142.56	292.23	2.0499
	225	1.0643	1.0643	143.62	293.15	2.0412
	250	1.0643	1.0643	144.63	294.07	2.0333
	500	1.0696	1.0643	154.36	303.28	1.9648
	750	1.1783	1.0643	158.68	312.49	1.9693
	1000	1.2058	1.0643	160.1	321.7	2.0094
3 Weeks of Payable Days	150	1.0643	1.0643	140.34	293.34	2.0902
	175	1.0643	1.0643	141.47	294.26	2.08
	200	1.0643	1.0643	142.56	295.18	2.0706
	225	1.0643	1.0643	143.62	296.1	2.0617
	250	1.0643	1.0643	144.63	297.02	2.0537
	500	1.0696	1.0643	154.36	306.23	1.9839
	750	1.1783	1.0643	158.68	315.44	1.9879
	1000	1.2058	1.0643	160.1	324.65	2.0278

inventory upper bound and holding cost (may decrease or stay the same).

Since the average total cost ratio for all cases for three years of data is 2.0423, the total cost for Problem I_N is approximately twice as that for Problem I. Therefore, β is clearly not a good performance measure even though the FRS can use it to minimize the new coin production quantity. Instead, it is better for the FRS to use the total cost as the performance measure for the CSC in order to minimize the total cost for the CSC and limit the new coin production quantity simultaneously.

In addition, we also observe that Problem I performs better when h is small from the perspective of giving low new coin production quantity. This is consistent with Observation 3, which indicates that when h is small, the FRS may request CVs to hold more coins to satisfy the demand, so fewer new coins are needed to be produced. However, when h is large, holding too many coins is too costly and it is better for the FRS to order more new coins from the Mint to satisfy the demand with minimum total cost, which makes β large. This result and some other results are summarized

Table 3.4: Computational Results for Varying Inventory Upper Bound for Problem I and Problem I_N When $O_j = 0$ With 2012 Demand Data.

Inventory Upper Bound U_j	**Unit Holding Cost ($)** h	**Critical Ratio Problem I** β_I	**Critical Ratio Problem I_N** β_{I_N}	**Total Cost ($M) Problem I** $\Pi^*(x_{ij}^t, I_j^t)$	**Total Cost ($M) Problem I_N** $\Pi_N^*(x_{ij}^t, I_j^t)$	**Total Cost Ratio** $\frac{\Pi_N^*(x_{ij}^t, I_j^t)}{\Pi^*(x_{ij}^t, I_j^t)}$
1 Weeks of Payable Days	150	1	1	120.9	219.6	1.8164
	175	1	1	123.17	221.87	1.8013
	200	1	1	125.44	224.14	1.7868
	225	1.0057	1	127.71	226.42	1.7729
	250	1.0057	1	129.92	228.69	1.7602
	500	1.1546	1	140.81	251.42	1.7855
	750	1.2372	1	143.43	274.15	1.9114
	1000	1.2581	1	144.35	296.88	2.0567
2 Weeks of Payable Days	150	1	1	120.9	240.22	1.9869
	175	1	1	123.17	242.49	1.9687
	200	1	1	125.44	244.76	1.9512
	225	1.0057	1	127.71	247.04	1.9344
	250	1.0057	1	129.92	249.31	1.919
	500	1.1546	1	140.81	272.04	1.932
	750	1.2372	1	143.43	294.77	2.0551
	1000	1.2581	1	144.35	317.5	2.1995
3 Weeks of Payable Days	150	1	1	120.9	239.51	1.9811
	175	1	1	123.17	241.78	1.963
	200	1	1	125.44	244.05	1.9456
	225	1.0057	1	127.71	246.33	1.9288
	250	1.0057	1	129.92	248.6	1.9135
	500	1.1546	1	140.81	271.33	1.9269
	750	1.2372	1	143.43	294.06	2.0502
	1000	1.2581	1	144.35	316.79	2.1946

in an observation and in a remark below.

Observation 4 *Problem I achieves the same β values for low unit inventory holding cost, but when the unit inventory holding cost is high, Problem I has larger β values than Problem I_N.*

Remark 4 *For any inventory upper bound, with increasing unit holding cost, the total cost ratio decreases first and then start increasing. For the same unit holding cost, the total cost ratio for high inventory upper bound is higher than that for the low inventory upper bound.*

3.4.2 Performance Testing of Procedure Rolling Horizon and μ-Robust Solution

In this subsection, we examine performances of Procedure Rolling Horizon and μ-Robust Solution assuming normally distributed demand. The mean demand is estimated as the average of the FRS's three-year demand data (2010-2012) for each CV in each period. We assume that the standard deviation σ_j^t of demand for each

CV j in each period t is within a certain percentage range of the mean demand. Specifically, we assume $6\sigma_j^t$ can be equal to 4%, 6% or 8% of the mean demand. Then, we use the mean and the standard deviation of demand for each CV in each period to randomly generate normally distributed demand data for testing. To evaluate the performance of Procedure Rolling Horizon, 15 instances in total (5 instances for each case of σ_j^t with varying percentages of the mean demand) are tested. To evaluate μ-Robust Solution, for each case of $6\sigma_j^t$ with the varying percentages of mean demand, we first test 50 sample paths and then extend it to 100 sample paths.

Results for Procedure Rolling Horizon and μ-Robust Solution are presented in Tables 3.5 and 3.6, respectively. In Table 3.5, we define C_{RH} as the total cost for 12-month rolling horizon obtained from Procedure Rolling Horizon. Further, we define C^* as the total cost assuming that the realized demand for 12-month rolling horizon for all CVs are known, which serves as a benchmark. Here, $GAP \stackrel{def}{=} \frac{100 \cdot |C_{RH} - C^*|}{C^*}\%$. In the μ-Robust Solution, μ represents how the cost generated from the robust solution deviates from the minimum cost if we optimize for each sample path. Therefore, we can compare GAP in the Procedure Rolling Horizon to μ in the μ-Robust Solution to analyze their performances.

Table 3.5: Performance Testing for Problem I Using Procedure Rolling Horizon(PRH) With Normally Distributed Demand Data.

Range of $6\sigma_j^t$	**Min GAP**	**Max GAP**	**Average GAP**
4% of the mean demand	8.78%	9.16%	8.97%
6% of the mean demand	9.16%	9.76%	9.35%
8% of the mean demand	9.59%	10.04%	9.84%

As shown in Tables 3.5 and 3.6, with more demand variation (i.e., increase in standard deviation for demand), the solution of Procedure Rolling Horizon is less sensitive compared to that of μ-Robust. Hence, Procedure Rolling Horizon is an effective approach for the FRS to deal with uncertain demand since the total cost difference is less than 10% of the actually realized demand scenarios even in the

Table 3.6: Performance Testing for Problem I Using μ-Robust Solution With Normally Distributed Demand Data.

Range of $6\sigma_j^t$	**Number of Sample Paths**	μ
4% of the mean demand	50	4.36%
	100	4.83%
6% of the mean demand	50	6.38%
	100	7.15%
8% of the mean demand	50	7.54%
	100	8.96%

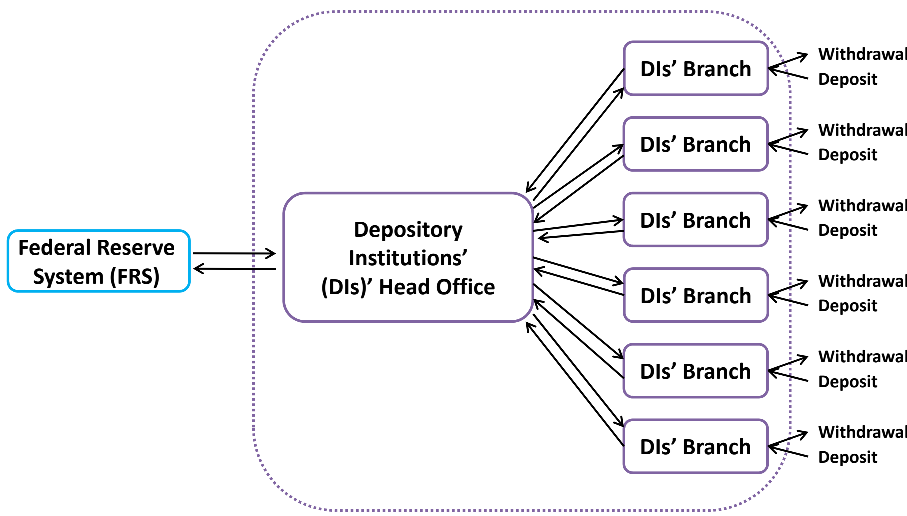
present of high uncertainty in demand (i.e., 8% deviation from the mean). However, our results also illustrate that the average GAP for Procedure Rolling Horizon for each value of σ_j^t is higher than that for μ-Robust Solution. The difference is more significant when the standard deviation is not large (i.e., 4% deviation from the mean). Overall, the μ-Robust Solution can effectively deal with uncertain demand: it generates a cost that is less than 9% different from the optimal even in the worst case scenarios with high uncertainty in demand (i.e., 8% deviation from the mean).

In summary, our computational study suggests that even with a large number of randomly generated sample paths, μ is still much smaller compared to the GAP for Procedure Rolling Horizon. Hence, although Procedure Rolling Horizon is effective for the FRS to deal with demand uncertainty, μ-Robust Solution performs better. In addition, μ-Robust Solution system is perhaps easier to implement, because it does not require updating and resolving the problem in multiple periods. Clearly, a trade-off in implementing the μ-Robust Solution is that it require more computation upfront. However, given its benefits, we recommend FRS to prefer μ-Robust Solution over the Procedure Rolling Horizon.

3.5 *Problem II*: Demand Side Problem

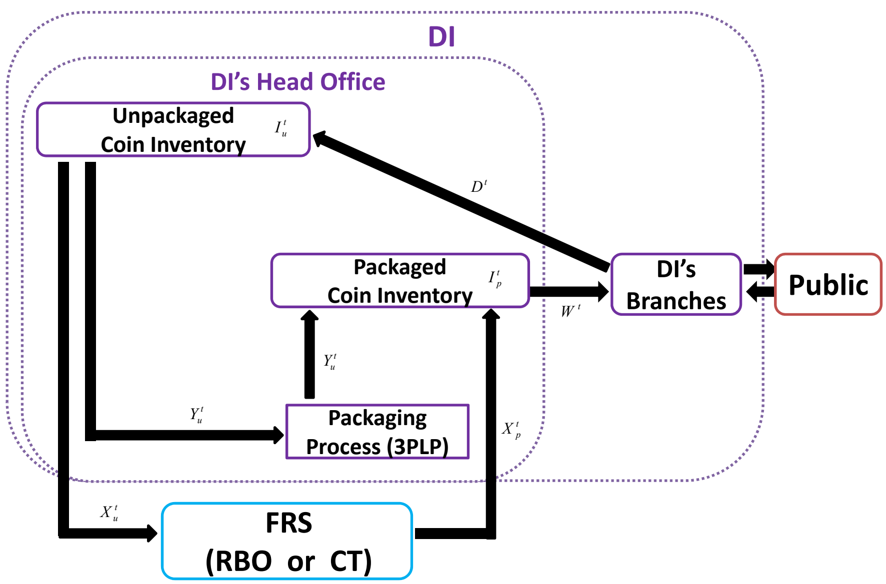
DIs have the following options for the circulating coins received from the customers: (i) deposit those coins at the FRS and withdraw packaged coins, (ii) send those coins to a 3PLP for packaging into rolls of coins. Customers at a DI withdraw coins in the form of packaged coins (i.e., rolls of coins). DIs have to pay an additional packaging fee at the FRS to receive each roll of packaged coins in addition to its face value. This packaging fee is higher for a DI than the cost of locally packaging coins with the aid of a 3PLP. From a DI's point of view, we study the question: in one the FRS's region, based on the forecast of this DI's aggregate demand (for each denomination of coins) at all of its branches, how would a DI manage its unpackaged and packaged coins facing the above two options? Note that the DI deals with only one CV (RBO or CT) in the region. Figure 3.9 illustrates the general framework of *Problem II*.

Figure 3.9: Framework of *Problem II*.



The specific structure of *Problem II* is showed in Figure 3.10. After receiving coins from commercial and individual customers, each DI's branch combines various denominations of coins in bags and deposits bags of circulating coins (each bag contains multi-denomination coins) back to the DI. The DI's branches send the received coins to the DI's head office's unpackaged coin inventory, which can be sent to the FRS (either an RBO or a CT) or its own packaging process to roll coins. The DI packaging process is operated by a local third party logistic provider (3PLP). Since the packaging fee charged by 3PLPs is lower than that charged by the FRS, each DI has the incentive to hire the 3PLP to package its unpackaged coins into packaged coins. After the packaging process either in the FRS or in the DI, coins are repackaged and each bag of packaged coins contains only one denomination. Next, packaged coins from the DI's packaged coin inventory and from the FRS are combined in the DI's packaged coin inventory to satisfy the demand of this DI's branches. Since DIs' ordering cycle is much shorter than RBOs', we assume that on average, each DI's ordering and supply cycle is one week. Thus, we consider a weekly packaging horizon for each DI.

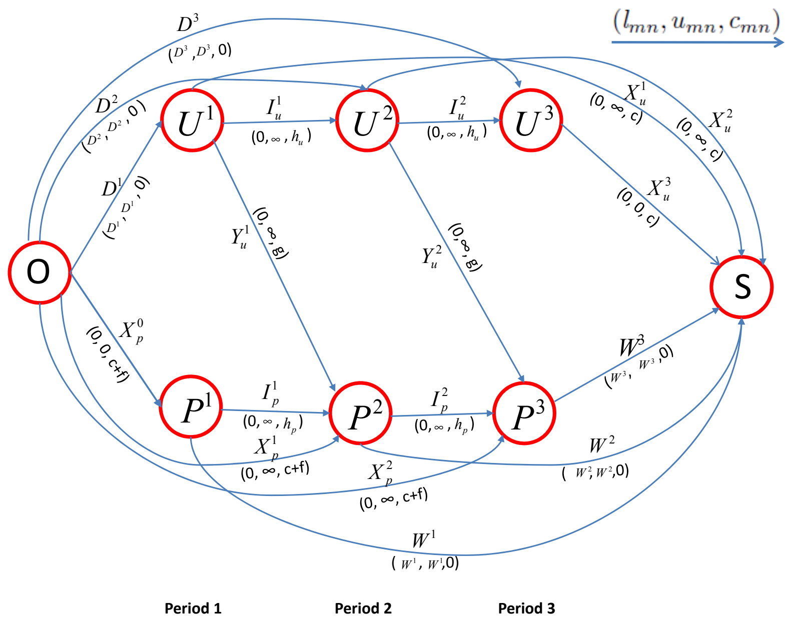
Figure 3.10: Structure of DIs' Ordering and Packaging Process in *Problem II*.



3.5.1 Network Flow Model for Problem II *of One Denomination*

We first describe the construction of the minimum cost flow (MCF) network model and then illustrate it with an example. In the construction, we start with one DI containing one node U for the unpackaged coin inventory and one node P for the packaged coin inventory plus duplicate nodes for the unpackaged and packaged coin inventory in different ordering and receiving periods. A source node O is included for providing the deposits from the DI's branches and packaged coins supplied by the FRS. In the end, a sink node S is also added for receiving withdrawals of the DI's branches and unpackaged coins send from the DI to the FRS for packaging. Now, we complete the whole network for a DI's coin ordering, packaging, holding, and supply. Notation needed for *Problem II* is in Table 3.7. An example for one DI is shown in Figure 3.11 considering a 3-period rolling horizon.

Figure 3.11: An Example of *Problem II* for One DI.



We now describe each arc with its lower bound, upper bound, and unit cost for the flow on that arc. We assume that there is no beginning and ending period coin inventory, neither unpackaged nor packaged, left in the DI. In other words, $I_u^0 = I_p^0 = I_u^T = I_p^T = 0$. We also assume that at the beginning of period 1 (end of period 0) and at the end of period T, neither the FRS nor the 3PLP receives unpackaged coins from the DI or sent packaged coins to the DI. Thus, $X_u^0 = X_u^T = X_p^0 = X_p^T = Y_u^0 = Y_u^T = 0$. Note that in the readily, each DI needs to keep a certain amount of

Table 3.7: Parameters and Variables for Formulating the MCF Model of *Problem II*.

	Parameters
T	Number of periods in the planning horizon.
D^t	The deposits of coin (in bags) during period t at the DI, $t = 1, 2, \ldots, T$.
W^t	The withdrawals of coin (in bags) during period t at the DI, $t = 1, 2, \ldots, T$, $k = 1, 2, \ldots, K$.
h_u	The per bag per period holding cost for the unpackaged coin inventory.
h_p	The per bag per period holding cost for the packaged coin inventory.
c	The per bag transportation and handling cost incurred for the DI for making deposits (or withdrawals) of coins at the FRS.
f	The per bag fee paid by the DI (in addition to the face value) to the FRS for purchasing packaged coins.
g	The per bag packaging fee paid by the DI to the 3PLP.
	Variables
X_u^t	The amount of unpackaged coins (in bags) shipped from the DI to the FRS at the end of period t, $t = 0, 1, 2, \ldots, T$.
X_p^t	The amount of packaged coins (in bags) shipped from the FRS to the DI at the end of period t, $t = 0, 1, 2, \ldots, T$.
Y_u^t	The amount of unpackaged coins (in bags) shipped from the DI to the 3PLP for packaging at the end of period t, $t = 0, 1, 2, \ldots, T$.
I_u^t	The unpackaged coin inventory (in bags) at the end of period t at the DI, $t = 0, 1, \ldots, T$. For each period t, I_u^t is the unpackaged coin inventory level right after the DI ships out X_u^t to the FRS and Y_u^t to the 3PLP.
I_p^t	The packaged coin inventory (in bags) at the end of period t at the DI, $t = 0, 1, \ldots, T$. For each period t, I_p^t is the packaged coin inventory level right after the DI receives X_p^t and Y_u^t bags of packaged coins from the FRS and from the 3PLP.

packaged coins as safety stock. Here, we assume that this safety stock is zero, which will not affect the analysis.

- An arc from source node O to node U^t, where U^t represents the unpackaged coin inventory at the end of period t. The lower bound, upper bound and unit cost of this arc of deposits from the DI's branches are D^t, D^t, and 0.

- An arc from source node O to node P^t, where P^t represents the packaged coin inventory at the end of period t. For $t = 0$ and $t = T$, there is no flow on this arc. For $0 < t < T$, the lower bound, upper bound and unit cost of this arc are 0, ∞, and $c + f$.

- An arc from node U^{t-1} to node P^t represents a flow from the DI's unpackaged coin inventory to its own packaging process. For $t = 0$ and $t = T$, there is no flow on this arc. For $0 < t < T$, the lower bound, upper bound and unit cost of this arc are 0, ∞, and g. Note that in the reality, $g < (c + f)$.

- An arc from node U^{t-1} to node U^t represents a flow of unpackaged coin inventory carried from on period to the next. The lower bound, upper bound and unit cost of this arc are 0, ∞, and h_u. Note that since $I_u^0 = I_u^T = 0$, they are not included in the network.

- An arc from node U^t to the sink node S represents a flow of unpackaged coin inventory sent to the FRS for packaging at the end of period t. For $t = 0$ and $t = T$, there is no flow on this arc. For $0 < t < T$, the lower bound, upper bound and unit cost of this arc are 0, ∞, and c.

- An arc from node P^{t-1} to node P^t represents a flow of packaged coin inventory carried from on period to the next. The lower bound, upper bound and unit cost of this arc are 0, ∞, and h_p. Note that since $I_p^0 = I_p^T = 0$, they are not included in the network.

- An arc from node P^t to the sink node S represents a flow of packaged coin withdrawal from the DI's branches. The lower bound, upper bound and unit cost of this arc are W^t, W^t, and 0.

The objective of the MCF model is to send $\sum_t W^t$ from source node O to sink node S at a minimum cost. The MIP formulation for *Problem II* is presented below.

MIP formulation for *Problem II*

The objective function minimizes the total cost for one specified DI, which includes coin packaging cost (either paid to the FRS or paid to the 3PLP), circulating coin transportation and handling cost between the DI and the FRS, and the packaged and unpackaged coin inventory holding cost. The last two terms compute the packaged and unpackaged coin inventory holding cost. Note that when considering the average packaged coin inventory during the period j, the beginning packaged coin inventory level is I_p^{t-1} and the instantaneous packaged coin inventory level right before the end of period j is $I_p^{t-1} - W^t$. Thus, the average packaged coin inventory level during the period j is equal to the half of the sum of the beginning inventory and the instantaneous ending inventory. Similarly, for the average unpackaged coin inventory during the period j, the beginning unpackaged coin inventory level is I_u^{t-1} and the instantaneous unpackaged coin inventory level right before the end of period j is $I_u^{t-1} + D^t$. Thus, the average unpackaged coin inventory level during period j is equal to the half of the sum of these two.

Problem II:

$$\begin{aligned}\textbf{Min} \quad \sum_{t=1}^{T-1} f \cdot X_p^t \quad &+ \quad \sum_{t=1}^{T-1} g \cdot Y_u^t + c \cdot (\sum_{t=1}^{T-1} X_p^t + \sum_{t=1}^{T-1} X_u^t) \\ &+ \quad h_u \cdot \sum_{t=1}^{T} (I_u^{t-1} + \frac{D^t}{2}) + h_p \cdot \sum_{t=1}^{T} (I_p^{t-1} - \frac{W^t}{2})\end{aligned}$$

Subject to:

Constraints (3.31)-(3.32) are the coin flow balance equations.

$$I_p^t = I_p^{t-1} - W^t + X_p^t + Y_u^t, \quad \forall t \tag{3.31}$$

$$I_u^t = I_u^{t-1} + D^t - X_u^t - Y_u^t, \quad \forall t \tag{3.32}$$

Constraints (3.33) enforce the initial inventory level and the inventory level in the end of the planning horizon to be zero.

$$I_p^0 \quad = \quad I_u^0 = I_p^T = I_u^T = 0 \tag{3.33}$$

Constraints (3.34) are non-negativity constrains.

$$\textit{All variables} \quad \textit{are} \quad \textit{nonnegative} \ \& \ \textit{ingeter}, \ \ \forall t \tag{3.34}$$

Lemma 5 *Solving the minimum cost flow problem constructed above is equivalent to optimizing DIs' coin ordering, packaging, holding and supply process.*

Proof: Since each feasible integer flow corresponds to a feasible sourcing for DIs' coin receiving, transportation, and inventory management process and vice versa, the optimal solution to the minimum cost flow problem constructed above is equivalent to the optimal process for DIs' for *Problem II*. This result holds in general. ■

Property 4 *Since the flows in MCF are integers with integer lower and upper capacity limits, each feasible integer flow corresponds to a feasible shipment of coins for* Problem II *and vice versa.*

3.5.2 Problem II *with "Volume Discount" for Packaging

Here the 3PLP offers the DI a "volume discount" contract for packaging. That is, the packaging fee charged by the 3PLP decreases when the volume of unpacked coins, v bags per period, sent to the 3PLP reaches a threshold τ. Specifically, if $v < \tau$, the DI pays 3PLPs of g per bag. Otherwise if $v \geq \tau$, each bag will be charged at a lower packaging fee, $\bar{g}$, where $\bar{g} < g$. This is also known as *all-units discount* scheme. Problem II can be stated as follows: given mean deposits and mean withdrawals, D^t and W^t, $t = 1, 2, \ldots, T$, at a DI, find the decision $\sigma = (X_p^t, X_u^t, Y_u^t | t = 1, 2, \ldots, T)$ that minimizes the total cost, Φ^σ under the "volume discount" contract. We show below that *Problem II* with " *volume discount*" is NP-hard. We also investigate the impact of "*volume discount*" on the DI's the total cost.

Theorem 1 *The decision problem corresponding to* Problem II *with* "volume discount" *is NP-complete.*

Proof: The proof is placed in Appendix B.3.

3.6 Conclusion

This is the first study to address operational issues within a CSC from both the supply side and the demand side. We view the CSC as a closed-loop supply chain with integrated bidirected flows and to investigate the CSC from optimization perspectives to increase efficiency and effectiveness in ordering, producing, packaging, distributing and managing inventory of coins. For all network mathematical models developed in this study, the bidirected flows of coins are captured and an effective integrated plan for the management of coin supply over the planning horizon is developed.

For the supply side Problem I, the FRS's current operation and its performance measure (net pay) are analyzed. A decision support model (based on minimum cost flow formulation) is developed from the FRS's perspective to minimize the total cost for the CSC with reasonably low new coin production quantity. This is possibly because optimal solutions effectively use inter-bank transshipment of coins and deposits of circulating coins collected from the public to meet the demand partially. We show that focusing solely on the current performance measure "net pay" for coin supply planning could be costly. In addition, we develop a 2-month Rolling Horizon procedure based on the FRS's current operating policy. Our computational results indicate that the performance of the procedure is remarkable in the sense that the total cost difference is less than 10% of the actually realized demand scenarios even in the present of high uncertainty in demand (8% deviation from the mean). To effectively conquer the demand uncertainty, a μ-Robust Solution planning system is developed to solve the stochastic version of Problem I with uncertain demand. Our computational study suggests that even with a large number of randomly generated sample paths, the performance of the μ-Robust solution is acceptable and reasonable. The robust solution generates the cost that is less than 9% different from the worst case scenarios even with high uncertainty in demand (8% deviation from the mean). Therefore, it will perform much better in the actual application. The future research may focus on adopting the μ-Robust solution in Rolling Horizon procedure and evaluating its performance.

Problem II investigates the demand side problem from DIs' perspective. A minimum cost flow model is developed to assist each DI to efficiently manage the circulation of its packaged and unpacked coin inventory. We also investigate the complexity of *Problem II* in the presence of "*volume discount*" offered by the local 3PLP to the DI for packaging and prove that it is NP-hard.

The need and desire to continually reduce operating costs are common threads across the coin supply chains around the world. Other coin supply chains may also face similar operational issues encountered at the FRS and DIs. There does not exist academic literature dealing with operations management issues covered in this study. Despite the increase in the use of electronic payment mechanism, the growth of usage of physical banknotes and coins are increasing around the world. This paper demonstrates the use of a modeling approach that not only meets the need for effective and efficient management of coin supply in the U.S., but also has the potential to be applied to a variety of similar coin supply chains around the world.

4. LOCATION AND CAPACITY OPTIMIZATION FOR ELECTRON BEAM FACILITY FOR PHYTOSANITARY TREATMENT OF MEXICAN IMPORT COMMODITIES

4.1 Introduction

We analyze the problem of providing electron beam (eBeam) irradiation services for fresh produce imported from Mexico to the U.S. through the Texas/Mexico border. With increasingly restrictive regulations or complete prohibition on the use of many chemical fumigants (such as methyl bromide) for pest control, irradiation is becoming a necessary phytosanitary treatment technology to meet strict import standards. Technologies commonly used for irradiation include gamma rays (often generated by radioactive cobalt-60), X-rays, and eBeam. Irradiation has practical benefits when integrated within an established system for the safe handling and distribution of food. Thus, interest in using it for pathogen control and maturation inhibition has increased globally. This interest is demonstrated by the International Plant Protection Convention (IPPC), which established International Standard for Phytosanitary Measures No. 18 (ISPM 18) to provide guidelines for the use of irradiation in phytosanitary treatments to control pests in fresh produce.

The volume of irradiated imported produce entering the U.S. has grown by almost 4000% (from 262,000 kg to 10,119,500 kg) between 2007 and 2014 (Jeffers 2015). The gamma ray irradiation facilities in Mexico still account for the majority of this produce. However, the amount of Mexican commodities treated at the eBeam facility in College Station, Texas, has increased by about 175% between 2014 and 2015 (Wall 2015).

Many technologies have been used in the U.S. to prevent the accidental introduction of pests and pathogens into cultivated and wild plants, including drying, smoking, salting, heat treatments (hot air, hot water dip, and steam), cold treatments (refrigeration), and chemical fumigation (phosphine gas, methyl bromide). Food irradiation is the latest technology to be used for this purpose. A relatively low dose of radiation can be effective in pest and pathogen control, thereby helping in food-borne disease prevention. A summary of the advantages and the disadvantages of different phytosanitary treatments can be found in Table 4.1 (Hallman 2007, 2011; Neven 2010).

Table 4.1: Comparison of Phytosanitary Treatments

Methods	**Technologies**	**Advantages**	**Disadvantages**
Cold Treatment	Freezing	1) Most widely used disinfestation treatment 2) Easy to manage in logistics 3) Wide tolerance range 4) Low cost 5) Organic certified	1) Not effective for all types of pests 2) Long treatment duration
Heat Treatment	Hot air Hot water dip Vapor heat	1) Effective in disinfection for fresh fruits and vegetables 2) Short treatment duration 3) Can be performed on a large scale and in flow-through systems 4) Low cost 5) Organic certified	1) Not tolerated well by temperate fruits 2) Moderate difficulty in logistics 3) Difficult to maintain pulp temperatures
Chemical Fumigation	Phosphine gas Methyl bromide	1) Effective in pest and pathogen control 2) Easy to manage in logistics 3) Short treatment duration 4) Low cost	1) Detectable chemical residues 2) Import regulation limitation 3) Can be used only for some fresh fruits and vegetables 4) Not organic certified
Irradiation	Gamma rays (cobalt-60) X-rays EBeam	1) Effective in pest and pathogen control 2) The most widely tolerated phytosanitary treatment for fruits 3) Can be applied after packing and palletizing 4) Short treatment duration	1) Moderate cost 2) Moderate difficulty in logistics 3) Not organic certified

The Food and Drug Administration (FDA), the U.S. Department of Agriculture

(USDA), the U.S. Centers for Disease Control and Prevention (CDC), and the World Health Organization (WHO) have all accepted the safety of irradiated foods. The USDA Animal and Plant Health Inspection Service (USDA-APHIS) recommends that irradiation of fresh fruits and vegetables be considered for import permits from other countries. Increasingly in recent years, more kinds of food consumed in the U.S. are undergoing irradiation treatment to eliminate risks associated with microbial contamination. More than 250 million pounds of irradiated food is consumed annually in the U.S. (Eustice 2014). Specifically, approximately 18 million pounds of frozen and fresh ground beef, 8 million pounds of produce, and 175 million pounds of spices are irradiated annually to control food-borne pathogens and to remove infestations (Pillai and Shayanfar 2015).

4.1.1 Food Irradiation Process

Food irradiation is the process of exposing specific foods to a carefully controlled amount of energy delivered as gamma rays (cobalt-60), X-rays, or electrons. The advantages and the disadvantages of these technologies are summarized in Table 4.2.

There is a long and successful history of using gamma rays (cobalt-60) for food irradiation for phytosanitary treatment. However, gamma ray technology suffers from key drawbacks in processing as well as other challenges associated with transportation, storage, disposal, and safeguarding a radioactive substance. Given the global security climate, the chances of large gamma ray irradiation facilities being approved are low (National Research Council 2008). Currently, the largest gamma ray irradiation facility in North America for food treatment is in Matehuala, Mexico.

Similar to gamma rays, a major advantage of using X-rays is that entire pallets can be irradiated because this technology has high penetrating depth (60 - 400 cm), depending upon the energy used (Curry et al. 2000). X-ray technology overcomes

Table 4.2: Comparison of Irradiation Technology

Technologies	Advantages	Disadvantages
Gamma rays (cobalt-60)	1) Well-established system 2) Widely used phytosanitary treatment 3) Low operating cost 4) High penetrating power	1) Replenishment of cobalt-60 2) Low throughput 3) High acquisition cost of cobalt-60 4) Slow dose rate 5) Moderate difficulty in logistics 6) Difficulty in getting the approval
X-rays	1) High penetrating power 2) Uses commercial electricity	1) Low throughput 2) Slow dose rate 3) High capital investment cost 4) Low energy utilization efficiency 5) Moderate difficulty in logistics
EBeam	1) High throughput 2) High dose rate 3) Low capital investment and operating costs 4) Uses commercial electricity 5) High energy utilization efficiency 6) Easy to manage in logistics	1) Low penetrating power 2) Only suitable for certain dimensions of packaging

some of the challenges associated with gamma rays by using commercial electricity to generate the sanitizing beam. However, there are still great disadvantages to the use of X-ray technology for phytosanitary treatment (Table 4.2). The only X-ray facility that is dedicated to phytosanitation is located in Kona, Hawaii.

EBeam technology, also known as beam pasteurization or electronic pasteurization, is a non-thermal treatment of food products and ingredients using electron beam linear accelerators to convert commercial electricity to highly energetic electrons. Compared to gamma rays and X-rays, the greatest advantages for eBeam are the high throughput (ten times more efficient than the generation of X-rays), the high dose rate (at least ten times higher than that of gamma rays or X-rays), the low capital investment and operating costs (much lower capital equipment cost than gamma rays or X-rays; and at least ten times lower operating cost than X-rays, but slightly more than gamma rays). The major disadvantage of eBeam technology is its short penetration depth. The single 10 MeV eBeam, which is typical for phytosanitary treatment, can barely reach a depth of approximately 5 cm in water or of 10 cm at a density of 0.5 g/cm^3 (Diehl 1999). Thus, eBeam is not suitable for bulky food packaging such as large crates used for shipping fruit. Single retail-ready packages of low density fruits and vegetables (i.e., mango, guava, sweet lime, tangerine, and manzano pepper) are currently treated in two large eBeam facilities in the U.S. in Sioux City, Iowa, and College Station, Texas. EBeam technology has the following characteristics:

- Green, chemical free technology: no chemicals are used for preservation of fresh foods.

- No environmental issues: eBeam has none of the environmental issues associated with radioactive waste, nor does it have challenges related to costs, transportation, storage, containment, and disposal.

- Use of commercial electricity: no replenishment of the irradiation source is needed.

- No loss of nutrients or compromise of food quality: eBeam does not change the temperature and does not alter the appearance, taste, or chemical makeup of the food product or its packaging.

- Effective: eBeam reduces or eliminates pathogens and pests, depending on the dose that is delivered.

- High throughput process: eBeam processing is at least ten times faster than conventional radioactive isotopes because it is based on electricity and not natural isotope decay.

- Easy to use and control: eBeam irradiation can be applied after packaging, and the whole process is automatically controlled.

- Cost-effective: depending on the commercial processing contract type, eBeam costs 5 to 8 cents to irradiate one pound of food, which is lower than using X-rays or gamma rays (approximately 10 cents per pound).

Table 4.3: Foods Currently Permitted to be EBeam Irradiated Under FDA's Regulations. [Source: FDA 2015. Regulatory Report: Irradiation of Food Packaging Materials].

Food	**Purpose**	**Max. Allowable Dose**
Fresh, non-heated processed pork	Control of trichinella spiralis	1 kGy
Fresh fruits and vegetables	**Pathogen control, maturation inhibition**	**1 kGy**
All foods	Arthropod disinfection	1 kGy
Dry or dehydrated enzyme preparations	Microbial disinfection	10 kGy
Dry or dehydrated spices/seasonings	Microbial disinfection	30 kGy
Fresh or frozen, uncooked poultry products	Pathogen control	3 kGy
Refrigerated, uncooked meat products	Pathogen control	4.5 kGy
Frozen uncooked meat products	Pathogen control	7 kGy
Fresh shell eggs	Control of salmonella	3.0 kGy
Seeds for sprouting	Control of microbial pathogens	8.0 kGy
Fresh or frozen molluscan shellfish	Control of vibrio species and other foodborne pathogens	5.5 kGy
Fresh iceberg lettuce and fresh spinach	Control of food-borne pathogens, and extension of shelf-life	4.0 kGy

Table 4.3 shows the foods that may be eBeam irradiated in the U.S. Recent studies find that doses of up to 1 kilogray (kGy) do not affect the quality or sensory attributes of the fruits (Shayanfar et al. 2015, Smith et al. 2015a, Smith et al. 2015b). Doses greater than 1 kGy significantly decrease ascorbic acid (vitamin C) concentrations during fruit storage, but do not affect overall antioxidant concentrations.

4.1.2 Fruit Imports across the Texas/Mexico Border

Food and agricultural trade between the U.S. and Mexico grew quickly after the North American Free Trade Agreement (NAFTA). According to the Office of the

United States Trade Representative (USTR) and USDA, U.S. agricultural imports from Mexico have experienced growth from \$2.8 billion in 1994 to \$17.7 billion in 2013, among which fruit and vegetables are the two leading categories. As the U.S.'s largest fruit and vegetables supplier, Mexico's annual exports of fresh fruits and vegetables to the U.S. also continues to increase (more than tripled) during the NAFTA period, approaching \$7.7 billion (\$9.4 billion including juice) in 2013 (USDA 2015b). This represents 20% of Mexico's fresh fruits and vegetables production and 36% of the U.S.'s imported fresh fruits and vegetables. Texas is the leading trade state overall, accounting for about 40% of the total U.S. trade with Mexico. Hence, at a conservative estimate, \$3 billion in fresh fruits and vegetables were imported from Mexico to the U.S. through Texas in 2013, and this number has been increasing steadily.

Mexico supplies over 60% of the mangoes imported into the U.S., which is the world's largest importer of mangoes: 307,855 tons in 2013. The peak importing season for mangoes and guavas is from October to February (20 weeks). Total fresh citrus imports by the U.S. from Mexico, November through October in 2013/2014, were 42,558 tons, much greater than the imports for the same period in 2012/2013 (USDA 2014). In the last 5 years, with Mexico gaining year-round access to the U.S. avocado market, the U.S. has become a net importer of avocados, with imports averaging over 70% of avocado supplies available for domestic consumption (USDA 2012b).

Over 95% of the Mexican agricultural products that cross into the U.S. use nine principal inland ports of entry. Figure 4.1 shows these. The top crossing for agricultural products is Nuevo Laredo/Laredo, which currently accounts for over a third of all imports. Colombia/Laredo is the second, followed by Ciudad Juarez/El Paso and Reynosa/McAllen. Latest estimates on the number of truckloads of fruits and vegetables imported from Mexico via border crossing points along the Mexico/Texas border are around 15,000 (each truckload carries approximately 40,000 lbs.) per month (USDA-AMS, 2015).

Figure 4.1: Border Crossing Points. [Source: USDA, GAIN Report (2015)]

To protect domestic agriculture from invasive pests and diseases, government regulations require that four commodities (mango, guavas, citrus, and avocado)—if untreated—can enter the U.S. only via specific parts of the Texas / Mexico border (USDA 2012a) and cannot cross or enter particular regions (called prohibited movement areas). After the treatment, either in Mexico or the U.S., treated commodities can travel anywhere in the U.S. Since this paper seeks to develop a decision support system for treating fresh fruits imported from Mexican growing regions and shipped to Texas hubs (San Antonio, Dallas, and Houston), we only consider border crossing points on the Texas/Mexico border east of El Paso.

This study focuses on these four commodities because they have the largest import quantities and strict prohibited movement areas. We consider the prohibited movement area for untreated mangoes and guavas (Figure 4.2) in our model because it is only slightly larger than those for citrus and avocadoes. Thus, in this study untreated fruits may enter the U.S. from Mexico only through Laredo or Eagle Pass.

Fruits treated in Mexico may cross the border anywhere from Brownsville to Eagle Pass.

Figure 4.2: Transit Corridor for Untreated Guavas and Mangoes from Mexico [Source: USDA. 2012a. Manual for Agricultural Clearance].

4.1.3 Problem Statement

Given potential locations in Mexico and the U.S., our problem is to determine the optimum number of facilities, to find the locations for the eBeam facilities, to determine the number of eBeam service lines (i.e., the capacity) for each facility, and to assign truckloads of fresh fruits leaving Mexican growing regions to eBeam facilities and to hubs. The objective is to minimize the total weekly cost, which includes (i) amortized fixed setup cost for opening each eBeam facility, (ii) capacity and operating costs for running eBeam machines (and corresponding service lines), (iii) processing cost for treating fruits and vegetables, (iv) transportation cost for moving fruits from growing regions in Mexico to hubs in Texas via the eBeam facilities, (v) queuing delay cost at the eBeam facilities, and (vi) border delay cost.

The paper is organized as follows. The next section reviews the closely related food supply chain and facility location literature. Section 4.3 states the problem and the assumptions. A network model and a heuristic used to solve the general problem are developed in Section 4.4. Section 4.5 describes the computational study, performs sensitivity analysis, and validates the queuing approximation used in the heuristic. Section 4.6 describes the factor analysis that corroborates the quantitative results and the impact of these factors on the cost parameter values. Section 4.7 concludes the paper and provides future research directions.

4.2 Literature Review

This study interacts with several streams of literature that include food safety, food supply chains, agricultural trade, facility location, network flow, queueing theory, and transportation. Thus, the study contributes to the literature by connecting multiple areas of research. For the sake of brevity, this section reviews the two most closely related streams of literature that impact our methodology, and it positions our research with respect to them: (i) food supply chains for fresh produce (including the U.S./Mexico agricultural trade) and (ii) facility location problems.

4.2.1 Food Supply Chains for Fresh Produce

Although supply chain management has been intensively studied in the literature, food supply chains for fresh produce (i.e., fruits, flowers, and vegetables) have been studied only in the last decade. Fresh fruit supply chains are competitive and dynamic with many uncertainties related to fruit safety treatment, foodbone disease, production, and demand, which add complexity in trade, logistics, and transportation. Soto-Silva et al. (2015) review the literature on operations research models that apply to fruit supply chains (FSC). They conclude that only a few papers are focused on supply chain structures and that there is a lack of holistic approaches for the design and management of FSC. We review a few recent studies that investigate and analyze food supply chains for fresh produce based on the decision type (planting, harvesting, production, distribution, or inventory), the decision level (operational, tactical, or strategic), and the modeling approach.

A few studies address issues for the upstream stages of the FSC. From the operational level, Ampatzidis et al. (2014) apply queueing theory (an $M/M/s$ queueing system in one fruit harvesting location) to a machine repair model and use a simulation to analyze the performance of the fruit harvesting process. Their paper aims at finding the optimal resource allocation, e.g., machines and labor, in a harvest of fresh fruits. Our study has the similarity of considering an $M/D/s$ queueing system with FCFS at each eBeam facility, but multiple decisions (i.e., facility location-allocation, capacity optimization, transportation, and distribution with import and regulation constraints) are made through an integrated nonlinear mathematical model and a heuristic under a very different setting. Cittadini et al. (2008) analyze the consequences of different strategic and tactical options (planting, harvesting, and production) of an FSC in Argentina. They develop a dynamic multi-objective LP model to allocate production activities to different land units, while optimizing two objective functions: (1) maximization of the present value of cumulative financial results (main objective of growers), and (2) maximization of cumulative farm labor (objective of policy makers that want to generate employment opportunities).

Studies that address downstream decisions are more closely related to this work. Blanco et al. (2005) only considers the production decision in a tactical planning and profit-oriented model of a packaging plant in an FSC in Argentina. In their mixed integer linear programming (MILP) model, the costs (including the raw material purchase, storage, and labor costs) and the revenue are captured in the objective to maximize the profit for a typical facility given maximum processing and storage capacities. From both operational and tactical levels, Masini et al. (2007) extend the previous literature by integrating multiple downstream decisions (production, distribution, and inventory). They propose a linear programming (LP) model to maximize the total net profit of an Argentinean fruit company and to make multiple decisions, such as the quantity to produce for each type of fruit, the cold storage capacity required for each third-party provider, and the quantity of fruit to be distributed to each party in the system. Tsao (2013) analyzes a multi-echelon supply network for fresh food from tactical and strategic perspectives. The paper applies non-linear optimization to fresh food supply chain design with the objective of maximizing the total network profit. The key decisions include the locations of food distributors, the assignments of retailers to food distributors, and the freshness-keeping effort of each food distributor. However, distribution factors embedded in our unique FSC, such as queueing delay at the eBeam facilities, border crossing delay, and transportation constraints due to the prohibited movement area, are not touched in Blanco et al. (2005), Masini et al. (2007), or Tsao (2013).

Some studies address exports, competition, communication, and information sharing. Ortmann et al. (2006) analyze the South African fresh fruit export supply chain and develop two graph theoretic and linear programming (LP) models to solve for two extreme seasonal export scenarios in foreign markets. Their models seek to maximize the throughput of fresh fruit via the existing export infrastructure and to minimize the overall transportation cost from storage sites to ports. The paper focuses on distribution from tactical and strategic perspectives; however, it does not consider location and capacity optimization of the processing facilities as our study does. A group of firms supplying fresh produce to the same market and competing in an oligopolistic manner is investigated by Yu and Nagurney (2013) using a game theoretic model. Each food supplier seeks to maximize its own profit by determining its optimal product flows throughout its supply chain. Ketzenberg et al. (2015) use a simulation to evaluate the value of information in a sustainable food supply chain in which each retailer seeks to find the optimal replenishment policy so that its long-run average per period expected cost is minimized. Compared to these studies, our paper contributes to the literature of food supply chains from a very different perspective by considering an integrated model that incorporates many issues regarding food treatment and distribution.

Other works related to Mexico/U.S. trade for agricultural produce focus on impacts of the North American Free Trade Agreement (NAFTA). Espinosa and Noyola (1997) discuss emerging patterns in U.S./Mexico trade by comparing circumstances before and after NAFTA. Málaga et al. (2001) examine the relative contribution of NAFTA and other factors that have important effects on U.S./Mexico agricultural trade in general and on the fresh vegetable trade in particular. They use an econometric simulation model for markets in the U.S. and Mexico to simultaneously determine the supplies, demands, prices, and trade of five fresh vegetables accounting for 80% of the U.S. fresh vegetable imports. Ackerman et al. (2003) conclude that U.S. corn exports to Mexico increased because of trade liberalization, so Mexico lost a significant share of its domestic corn market to the U.S. For other types of fresh produce, Mexico has improved production, investments, and marketing to substantially increase fresh fruit and vegetable exports to the U.S. The strong export growth of Mexican fresh produce also results from the successful phytosanitary negotiations between the U.S. and Mexico arising from NAFTA, especially for avocado (Huang and Huang 2007).

Although our study considers the U.S./Mexico trade for fresh produce, our focus is not the impacts of NAFTA. Our study develops a generalizable decision support system to allocate the eBeam facilities for irradiating fresh produce, to decide the required capacity for each eBeam facility, and to optimize the assignment for each truckload of fruit imported from Mexico to the U.S. To our knowledge, there is no recent study using operations research methodologies to solve the FSC problem covering all three decision levels (operational, tactical, and strategic) as is done in this paper. To introduce eBeam as the new phytosanitary treatment technology, this study determines not only tactical and operational decisions (transportation and assignment of truckloads of fruits), but also strategic decisions (locating the eBeam facilities and optimizing their capacities for irradiating fruits). Thus, it considers the overall structure of the FSC covering Mexico and Texas, including new food safety technology, multiple commodities, transportation restrictions due to prohibited movement areas in Texas, regulation requirements, and delays at border crossing points and at the eBeam facilities. In addition, this study's comprehensive objective includes the amortized fixed setup cost for opening each eBeam facility, the capacity and operating costs for running the eBeam facilities, the eBeam processing cost for irradiating untreated fruits, the transportation and border delay costs for moving fruits from Mexican growing regions to Texas hubs, and the queuing delay cost at the eBeam facilities for processing.

4.2.2 Facility Location Problem

There are four general problems that decide the location of facilities and also allocate demand to them (Mirchandani and Francis 1990). These are the p-median problem (Hakimi 1964), the uncapacitated facility location problem (UFLP), the p-center problem, and the quadratic assignment problem (QAP). The p-median problem, the UFLP, and the p-center problem are NP-hard and thus do not admit polynomial-time algorithms to find an optimal solution (Kariv and Hakimi 1979a, 1979b). Although the QAP is more realistic and uses more information, since it is theoretically harder to solve than the p-median problem, it is not commonly studied in the literature. This study has similarities with the typical facility location-allocation problem, but it makes wider decisions that incorporate more realistic practical issues and constraints by considering the economic, social, and governmental factors related to food safety.

Some studies investigate the application of the traditional facility location problem under other settings. ReVelle and Laporte (1996) describe two alternative formulations for a plant location problem, where one objective is to minimize cost and the second objective is to maximize the demand that can be served by a plant within a certain time limit. Snyder and Daskin (2005) develop a facility location model with the objective of choosing locations that are both inexpensive and reliable. The expected transportation cost after failures of facilities are included in their cost minimization objective function to account for reliability. Caro et al. (2012) investigate process allocation related to facility location in the food processing industry. They add penalty costs to the objective function to capture production yield uncertainty and make decisions such as where to open processes, which products should be assigned to which processes, and which markets to supply from which processes. Our study differs by considering many food safety, import, regulation, infrastructure, queueing, transportation, and routing issues that influence the choice of locations. Factors such as transportation, maximum allowed utilization rate, and costs of resources, which could affect the eBeam facilities' capacity limitation or associated social cost for the U.S. and Mexico, are also analyzed using a factor rating system. Other studies that analyze facility location problems by identifying and evaluating major factors in selecting an industrial location include Blair and Premus (1987), Yang and Lee (1997), MacCarthy and Atthirawong (2003), and Chou et al. (2008).

When multiple commodities are produced and distributed through the supply chain network, the facility location and capacity determination issues become complex. A few studies analyze the multi-commodity capacitated facility location problem and present mathematical modeling frameworks for strategic supply chain planning (Pirkul and Jayaraman 1998, Canel et al. 2001, Melo et al. 2006). We develop a heuristic using a minimum cost flow (MCF) network model to solve a multi-commodity problem efficiently by solving it for each commodity independently and then aggregating the solutions. We demonstrate that this simplification does not reduce the accuracy of the calculation. This type of methodology has not been used previously in the context of food treatment and distribution.

4.3 The General Problem Description

Given the minimum and maximum allowed utilization rates at an eBeam facility, the deterministic service rate per service line, each commodity's demand at each hub, and the capacity of each growing region for each commodity, the general problem (hereafter called GP) focuses on selecting the most cost-effective locations for the eBeam facilities and on how many service lines each new eBeam facility requires (i.e., the capacity), while considering the border crossing delays and the queuing delays at the eBeam facilities. The optimal assignment for each truckload of fruit from Mexican growing regions to the eBeam facilities and then to U.S. hubs for distribution is also determined.

Mexico supplies the U.S. fruit market throughout the year but ships heavily during the winter months (December to April). Thus, we focus on the peak importing season during which four commodities, mangoes, guavas, citrus, and avocado, are imported and irradiated each week.

There are eight potential locations for the eBeam facilities (Figure 4.3): four

Figure 4.3: Eight Potential Locations (Highlighted) for Establishing the EBeam facilities.

Mexico cities (Nuevo Laredo, Reynosa, Matamoros, and Matehuala), and four Texas cities, including the major hubs (San Antonio, Dallas, and Houston) and Laredo, which is on the Texas/Mexico border. We also consider five Texas/Mexico border crossing points (Eagle Pass/Piedras Negras, Laredo/Nuevo Laredo, Laredo/Colombia, McAllen/Reynosa, and Brownsville/Matamoros in Figure 4.1) and seven major growing regions in Mexico (Sinaloa, Nayarit, Jalisco, Michoacan, Guerrero, Oaxaca, and Chiapas). Border crossing points together with the prohibited movement areas for untreated mangoes and guavas (Figure 4.2) are used to decide the routes to transport fruits. Details of how we estimate the aggregated operating cost, which includes the labor cost, land cost, holding cost, insurance cost, tax cost, and export & import regulation costs, are in Section 4.6.

The following assumptions are made to develop a one-week mathematical model to formulate GP. Parameters and variables are defined in Table 4.4 and Table 4.5.

- **A1:** Demand from Texas hubs during the peak importing season are the same for each week.

- **A2:** Each commodity is transported independently in full truckloads.

- **A3:** Each truckload of a particular commodity has the same weight. These weights differ between commodities.

- **A4:** There is a known exogenous border crossing delay at each border crossing point.

- **A5:** There is a multi-commodity $M/D/s$ queuing system at each eBeam facility. All the truckloads (for all the commodities) form a single queue in each eBeam facility and are processed following the first-in-first-out (FIFO) rule.

- **A6:** There is a minimum allowed utilization rate $\underline{\rho}$ and a maximum allowed utilization rate $\bar{\rho}$ at all the eBeam facilities.

- **A7:** Demand is always satisfied: $\sum_i P_i^k \geq \sum_h M_h^k$.

The objective function of the mathematical model minimizes the total weekly cost, including the setup cost of opening new eBeam facilities, the operating cost at the eBeam facilities, the eBeam processing cost, the transportation cost, the border delay cost, and the queuing delay cost at the eBeam facilities.

$$
\begin{aligned}
\textbf{Minimize } \Phi = \sum_{i=1}^{I} f_i \cdot F_i \quad &+ \quad \sum_{i=1}^{I} o_i \cdot s_i + \sum_{i=1}^{I} p_i \cdot \lambda_i + \sum_{k=1}^{K}\sum_{h=1}^{H} \beta_h^k \\
&+ \quad \sum_{k=1}^{K}\sum_{e=1}^{E} \gamma_e^k + \sum_{i=1}^{I} bL_i
\end{aligned}
$$

Subject to:

Constraints (4.1-4.2) ensure that service lines and commodity flows are assigned only to the facilities that have been opened. If $e = 0$, there is no border entry on this

Table 4.4: Parameters Used to Formulate the General Problem.

	Parameters
i	$1, 2, \ldots, I$ as potential locations to build an eBeam facility.
j	$1, 2, \ldots, J$ as growing regions in Mexico.
k	$1, 2, \ldots, K$ as commodity types.
e	$0, 1, 2, \ldots, E$ as border crossing points.
h	$1, 2, \ldots, H$ as hubs in Texas for importing fruits.
M_h^k	Demand in truckloads per week for commodity k at hub h.
P_j^k	The capacity in truckloads per week of commodity k at growing region j in Mexico.
$\hat{d}_{jei}$	The truckload distance (miles) between growing region j and potential location i through border crossing point e. If $e = 0$, there is no border entry on this route.
$\check{d}_{ieh}$	The truckload distance (miles) between potential location i and hub h through border crossing point e. If $e = 0$, there is no border entry on this route.
t_e	The average waiting time per truckload at border crossing point e. $t_0 = 0$.
$\underline{\rho}$	The minimum allowed utilization rate at each eBeam facility.
$\bar{\rho}$	The maximum allowed utilization rate at each eBeam facility.
ℓ^k	The weight in pounds of one truckload of commodity k. $\ell^0 = \min_k\{\ell^k\}$.
μ	The service rate per service line (truckloads per week) of commodity k in terms of standardized truckloads with ℓ^0 pounds.
f_i	The amortized single-week setup cost, proportional to the total fixed cost of opening an eBeam facility at location i, including the land cost, the cost to build an eBeam facility, and the export & import regulation cost.
o_i	The single-week per service line operating cost at location i. This cost includes the labor cost, the holding cost, the insurance cost, and the tax cost.
p_i	The eBeam processing cost for radiating one standardized truckload of ℓ^0 pounds of commodities at location i.
u	The fixed cost of dispatching a truckload.
a	The per-mile per-truckload transportation cost.
b	The unit time delay cost for one truckload at the eBeam facilities and border crossing points.
V	The assumed upper bound on the total number of service lines required to satisfy the total demand.
A	A very small positive number used in computation. Set $A = 10^{-6}$.

route.

$$s_i \leq V \cdot F_i, \ \forall i. \tag{4.1}$$

Table 4.5: Variables Used to Formulate the General Problem.

	Variables
F_i	If $F_i = 1$, then potential location i is selected to build an eBeam facility. Otherwise, $F_i = 0$.
x^k_{jei}	The actual number of truckloads of commodity k sent from Mexican growing region j to the eBeam facility i through border crossing point e. Thus, the number of standardized truckloads of commodity k with minimum weight ℓ^0 is $\frac{\ell^k}{\ell^0} \cdot x^k_{jei}$
y^k_{ieh}	The actual number of truckloads of commodity k sent from the eBeam facility i to Texas hub h through border crossing point e.
s_i	the number of service lines required in eBeam facility i.
P_b	The steady-state probability that all service lines are busy.
λ_i	The mean arrival rate at eBeam facility i.
L^q_i	The average length of the queue in the number of truckloads with ℓ^0 pounds for all the commodities at location i.
L_i	The average number of truckloads with ℓ^0 pounds for all the commodities at location i.
β^k_h	The transportation cost to satisfy the demand of commodity k at Texas hub h.
γ^k_e	The border delay cost at border crossing point e for commodity k.

$$\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{e=0}^{E} x^k_{jei} \leq \sum_{k=1}^{K}\sum_{h=1}^{H} M^k_h \cdot F_i, \ \forall i. \tag{4.2}$$

Constraints (4.3-4.4) are flow balancing equations. Constraints (4.3) indicate that the inbound flow must equal the outbound flow for each eBeam facility. Constraints (4.4) ensure that the total of the outflows from each growing region in Mexico does not exceed its capacity.

$$\sum_{j=1}^{J}\sum_{e=0}^{E} x^k_{jei} = \sum_{e=0}^{E}\sum_{h}^{H} y^k_{ieh}, \ \forall i, \forall k \tag{4.3}$$

$$\sum_{i=1}^{I}\sum_{e=0}^{E} x^k_{jei} \leq P^k_j, \ \forall j, \ \ \forall k \tag{4.4}$$

Constraints (4.5) calculate the aggregated arrival rate in standardized truckloads (truckloads with weight ℓ^0) for eBeam facility i based on the arrival rate for each commodity k. Since the service rate of an eBeam machine is given in the number of truckloads with a standard weight ℓ^0, we normalize the aggregate arrival rate into standardized truckloads.

$$\lambda_i \;=\; \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{e=0}^{E}\frac{\ell^k}{\ell^0}\cdot x_{jei}^k, \quad \forall i \tag{4.5}$$

Constraints (4.6-4.8) calculate the utilization rate for eBeam facility i and restrict the utilization rate to be between the minimum and the maximum allowed utilization rates. Note that Constraints (4.6) introduce a very small positive number $A = 10^{-6}$ to both denominator and numerator to allow for locations for which $F_i = s_i = 0$ in the optimal solution.

$$\rho_i \;=\; \left(\frac{\lambda_i + A}{s_i\cdot\mu + A}\right)\cdot F_i, \quad \forall i \tag{4.6}$$

$$\rho_i \;\leq\; \bar{\rho}, \quad \forall i \tag{4.7}$$

$$\rho_i \;\geq\; \underline{\rho}\cdot F_i, \quad \forall i \tag{4.8}$$

Constraints (4.9) compute the approximate average length of the queue at eBeam facility i by using the formulas for an $M/M/s$ queue. Based on Kingman's law of congestion, when the utilization rate is moderate, the approximate average length of the queue for $M/D/s$ queue is close to the average queue length for $M/M/s$ queue. This is true especially when there are large number of service lines in one location, which results in moderate utilization rate. Thus, we may use the length of $M/M/s$ queues to approximate the length of $M/D/s$ queues in this study, especially when the utilization rate is moderate. Define $f(\rho_i) = \frac{P_b\cdot\rho_i}{(1-\rho_i)}$, where P_b is the steady-state probability that all service lines are busy. P_b depends on ρ_i and s_i. The value of P_b can be found in tables of queueing values in Operations Research textbooks, for example, Winston and Goldberg (2004). Constraints (4.10) calculate the approximate average number of truckloads in eBeam facility i.

$$L_i^q \;=\; F_i\cdot f(\rho_i), \quad \forall i \tag{4.9}$$

$$L_i = L_i^q + \frac{\lambda_i}{\mu}, \quad \forall i \tag{4.10}$$

Constraints (4.11-4.12) compute the transportation cost and the border delay cost for all commodities. Specifically, in Constraints (4.11), the transportation cost includes the fixed cost of dispatching a truckload and the single trip distance cost. The distance calculations respect the prohibited regions and the routes to the border crossing points.

$$\begin{aligned} \beta_h^k &= u \cdot M_h^k \\ &+ a \cdot \left(\sum_{i=1}^{I} \sum_{j}^{J} \sum_{e=0}^{E} x_{jei}^k \cdot \hat{d}_{jei}^k + \sum_{i=1}^{I} \sum_{e=0}^{E} y_{ieh}^k \cdot \check{d}_{ieh}^k \right), \forall h, \forall k \end{aligned} \tag{4.11}$$

$$\gamma_e^k = b \cdot t_e \cdot \left(\sum_{j=1}^{J} \sum_{i=1}^{I} x_{jei}^k + \sum_{i=1}^{I} \sum_{h=1}^{H} y_{ieh}^k \right), \ \forall e, \forall k \tag{4.12}$$

Constraints (4.13-4.15) are non-negativity, binary, and integrality constraints.

$$All\ variables \geq 0, \forall i, \forall j, \forall k, \forall e, \forall h \tag{4.13}$$

$$F_i \in \{0, 1\}, \ \forall i \tag{4.14}$$

$$s_i \quad integer, \ \forall i \tag{4.15}$$

This model is nonlinear because of the multi-commodity $M/D/s$ queuing system at each eBeam facility, which is represented in Constraints (4.6) and Constraints (4.9). Therefore, we propose an approach to simplify the model and develop efficient solution procedures to solve GP in Section 4.4. Before proceeding to that approach, we show that the single-commodity version of GP is strongly NP-hard.

Theorem 2 *Assuming that the queuing delay cost at eBeam facilities is zero, the single-commodity version of GP selecting at most Q eBeam facility locations is strongly NP-hard.*

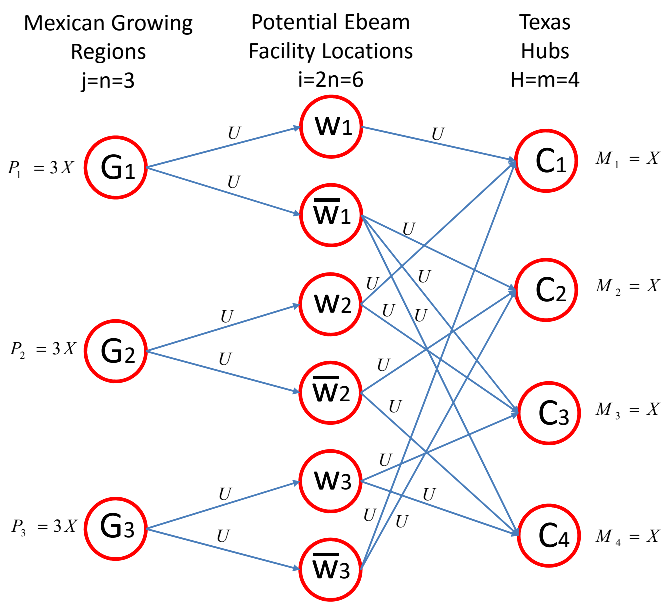
Proof: We use the 3-Satisfiability (3SAT) problem (Garey and Johnson 1979) for our reduction.

<u>3-SATISFIABILITY (3SAT)</u>

<u>INSTANCE</u>: A set $\hat{W} = \{w_1, w_2, \ldots, w_n\}$ of boolean variables and a collection $C = \{C_1 \cap C_2 \cap \ldots \cap C_m\}$ of clauses over $\hat{W}$, each of which is a disjunction of literals, $w_1, \bar{w}_1, w_2, \bar{w}_2, \ldots, w_n, \bar{w}_n$ such that $|C_j| = 3$ for $1 \leq j \leq m$ and $\bar{w}_i = 1 - w_i$ for $1 \leq i \leq n$.

<u>SOLUTION</u>: Find an assignment of either a TRUE (1) or a FALSE (0) value to each variable in $\{w_1, w_2, \ldots, w_n\}$, such that the expression C evaluates to TRUE (1).

Figure 4.4: A Graphical Representation of the Decision Problem for the Example with $n = 3$ and $m = 4$.



Given an arbitrary instance of 3SAT, we provide the construction of the instance of our decision problem corresponding to GP using the following example. Refer to Figure 4.4.

Example: $n = 3$ and $m = 4$. $\hat{W} = \{w_1, w_2, w_3\}$; $C = \{C_1 \cap C_2 \cap C_3 \cap C_4\}$, where $C_1 = w_1 \cup w_2 \cup \bar{w}_3$, $C_2 = \bar{w}_1 \cup \bar{w}_2 \cup \bar{w}_3$, $C_3 = \bar{w}_1 \cup w_2 \cup w_3$, and $C_4 = \bar{w}_1 \cup \bar{w}_2 \cup w_3$.

- $j = 1$, $j = 2$, denote growing regions G_1, G_2, respectively, and so on. The number of growing regions in Mexico is $J = n$.

- Each variable in the set $W = \{w_1, \bar{w}_1, w_2, \bar{w}_2, \ldots, w_n, \bar{w}_n\}$ corresponds to a distinct potential location to build a processing facility, denoted as $i = 1, 2, \ldots, 2n-1, 2n$, in that order, respectively. Note that $i = 1$, $i = 2$, $i = 3$, $i = 4$, denote locations w_1, $\bar{w}_1$, w_2, $\bar{w}_2$, respectively, and so on. The number of potential locations to build processing facilities is $I = 2n$.

- Each clause in the set $C = \{C_1 \cap C_2 \cap \ldots \cap C_m\}$ corresponds to a distinct hub in Texas for importing fruits, denoted as $h = 1, 2, \ldots, m$, in that order. The number of hubs in Texas for importing fruits is $H = m$.

- $Q = n$, $a = u = 1$, $b = 0$, $t_e = 0$, $K = 1$ (single commodity). A truckload is ℓ^0 pounds. $p_i = 1$, $o_i = f_i = 0$, $i = 1, 2, \ldots, 2n$.

- P_j is the capacity in truckloads per week of a commodity at growing region j in Mexico. $P_j = 3X$, $j = 1, 2, \ldots, n-1, n$, where X is large number.

- M_h is the demand in truckloads per week for a commodity at hub h. $M_h = X$, $h = 1, 2, \ldots, m$.

- Note that $\hat{d}_{jei}$ is the distance between growing region j and potential location i through border crossing point e. For a pair $(j, 2i)$ and $(j, 2i-1)$, $d_{jei} = U$, $i = 1, 2, \ldots, n$, if $j = i$; otherwise $d_{jei} = 2U$, where U is large number (See Figure 4.4).

- Note that $\check{d}_{ieh}$ is the truckload distance between potential location i and hub h through border crossing point e. For a pair (i, h), $\check{d}_{ieh} = U$, if $i \in W$ belongs to clause C_h, otherwise $\check{d}_{ieh} = 2U$, $1 \leq i \leq 2n$, and $1 \leq h \leq m$ (See Figure 4.4).

- $\underline{\rho} = 0$, the minimum allowed utilization rate at each processing facility.

- $\bar{\rho} = 0.75$, the maximum allowed utilization rate at each processing facility.

- $\mu = 4X$, the service rate per service line (truckloads per week) of commodity k in terms of standardized truckloads with ℓ^0 pounds.

For the instance of GP constructed above, we consider the following question:

<u>Decision Problem</u>: Does there exist a solution for the single-commodity version of GP with total cost $\Phi \leq 2mX(U+1)$?

The decision problem is clearly in NP. Also, it can be easily verified that the construction of our decision problem from the 3SAT instance can be done in polynomial time. We now show that the decision problem has an affirmative answer if and only if the 3SAT instance is satisfiable.

<u>If part:</u> Suppose the instance of 3SAT is satisfiable. Then, there is an assignment of either w_i or $\bar{w}_i$ that is TRUE (1), such that the expression C evaluates to TRUE (1). We illustrate the solution corresponding to the following truth assignment for the example: $\bar{w}_1 = w_2 = w_3 = 1$: $2X$ truckloads, X truckloads, and X truckloads of fruit are assigned from Mexican growing region G_1 to potential location $\bar{w}_1$, from Mexican growing region G_2 to potential location w_2, and from Mexican growing region G_3 to potential location w_3, respectively. The number of service lines for each location ($\bar{w}_1$, w_2, and w_3) is set equal to one. Note that the utilization requirement ($\underline{\rho} \leq \rho \leq \bar{\rho}$) is satisfied since $\mu = 4X$ and at most $3X$ truckloads will be received at any processing facility. Hub $C_1 = w_1 \cup w_2 \cup \bar{w}_3$ receives X truckloads from w_2, hub $C_2 = \bar{w}_1 \cup \bar{w}_2 \cup \bar{w}_3$ receives X truckloads from $\bar{w}_1$, and $C_3 = \bar{w}_1 \cup w_2 \cup w_3$ receives X truckloads from $\bar{w}_1$, and $C_4 = \bar{w}_1 \cup \bar{w}_2 \cup w_3$ receives X truckloads from w_3.

Thus, for the solution above, the decision problem has zero setup cost, zero operating cost, mX processing cost, $mX + 2mXU$ transportation cost, zero border delay cost, and zero queueing delay cost. Hence, the proposed solution gives the total cost $\Phi = 2mX(U+1)$. For the example, $\Phi = 8X(U+1)$ as $m = 4$. $\square$

Only if part: Suppose there exists a solution s to the decision problem with $\Phi_s \leq 2mX(U+1)$. Let $\Phi_s = \Phi_s^f + \Phi_s^o + \Phi_s^p + \Phi_s^t + \Phi_s^b + \Phi_s^q$, where Φ_s^f, Φ_s^o, Φ_s^p, Φ_s^t, Φ_s^b, and Φ_s^q are the fixed setup cost, the operating cost, the processing cost, the transportation cost, the border delay cost, and the queueing delay cost components of Φ_s, respectively. We now show that there exits a satisfiable truth assignment for the 3SAT instance by the following claims.

Claim 1: In solution s, $\Phi_s^f = \Phi_s^o = \Phi_s^b = \Phi_s^q = 0$ and $\Phi_s^p = mX$.

Proof of Claim 1: Since each potential processing location has zero setup and operating costs, $f_i = o_i = 0$, the average waiting time per truckload at border crossing points is zero ($t_e = 0$), and the unit time delay cost per truckload at eBeam facilities and border crossing points is zero ($b = 0$), it follows that we have $\Phi_s^f = \Phi_s^o = \Phi_s^b = \Phi_s^q = 0$. As the total demand at hubs is equal to mX and the processing cost is $p_i = 1$ for all potential eBeam facility locations, $\Phi_s^p = mX$. $\square$

Claim 2: In solution s, $\Phi_s^t \leq mX(2U+1)$.

Proof of Claim 2: Note that $\Phi_s^f = \Phi_s^o = \Phi_s^b = \Phi_s^q = 0$ and $\Phi_s^p = mX$ (Claim 1). Since $\Phi_s \leq 2mX(U+1)$, we must have $\Phi_s^t \leq mX(2U+1)$. $\square$

Claim 3: There must exist a satisfiable truth assignment for the 3SAT instance.

Proof of Claim 3: Since the total demand at hubs equals mX trucks and $u = 1$, the fixed cost of dispatching trucks is mX. Since the minimum distance between Mexican growing regions and potential eBeam facility locations is U (respectively, the minimum distance between hubs and potential eBeam facility locations is U), we must have $\Phi_s^t = mX(2U+1)$. This implies that there exists a minimum distance U between a hub and the eBeam facility supplying this hub (respectively, a minimum distance U between a Mexican growing region and the eBeam facility processing its supply). Thus, at most one location from a pair $(w_i, \bar{w}_i)$, $i = 1, 2, \ldots, n$, must be selected for a processing facility since $Q = n$. This in turn implies that there is a truth assignment of variables in W belonging to clause C_h. Thus, there exits a satisfiable truth assignment for the 3SAT instance. In solution s, the number of service lines for any selected location is set equal to one. Note that the utilization requirement ($\underline{\rho} \leq \rho \leq \bar{\rho}$) is satisfied since $\mu = 4X$ and at most $3X$ truckloads will be received at any selected location of the processing facility. This completes the proof.

■

4.4 Approach to Solve the General Problem

To solve GP efficiently, we develop a heuristic (namely H_{GP}) using a minimum cost flow (MCF) network model. Specifically, we let Q denote the number of facilities to be opened from $\bar{Q}$ candidate locations for $Q = 1, 2, ..., \bar{Q}$. Thus, there are $\sum_{Q=1}^{\bar{Q}} \begin{pmatrix} \bar{Q} \\ Q \end{pmatrix}$ ways of choosing facility locations. Our heuristic procedure considers all possible combinations of potential locations. This section develops a minimum cost flow (MCF) model that is used for each combination and then discusses the queueing system.

4.4.1 The MCF Model

For each Q value and each set of Q potential locations, an MCF model is developed for solving a special case of GP with no capacity limitation and no queueing delay at the Q locations. We then use this MCF model to devise a heuristic procedure H_{GP} (in Section 4.4.2) to solve GP. The MCF model simply represents a single commodity's flow in the highway transportation system with processing requirements at the eBeam facilities. Therefore, since we ignore capacities of the eBeam facilities, the MCF model can be run for each commodity independently to decide the best routes and to compute the transportation and processing cost for each commodity. Summing these individual costs yields the total transportation and processing cost for all commodities in this simplified system.

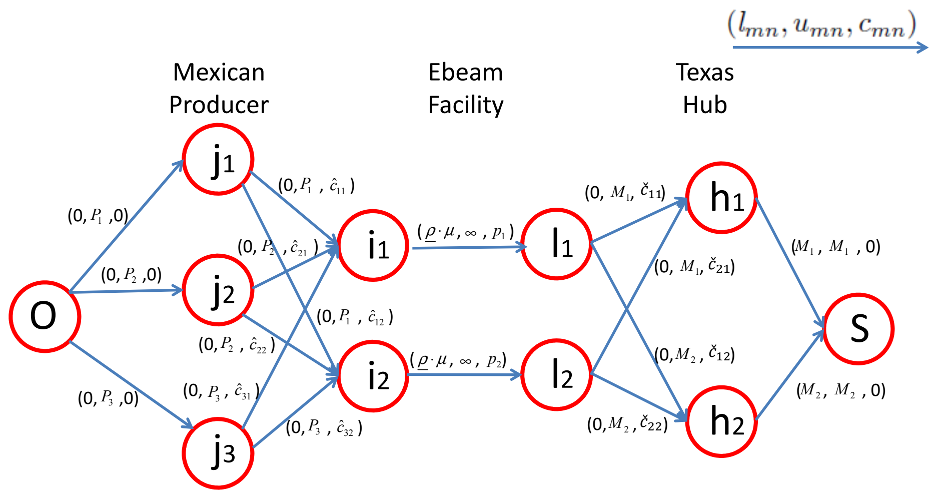
Table 4.6: Additional Parameters and Variables Used in the MCF Model.

	Parameters
Q	Number of locations to select.
$\bar{Q}$	Total number of potential locations.
$\hat{c}_{ji}$	The per truckload cost of transporting from growing region j to eBeam facility i. This cost includes the transportation cost calculated from the transportation time for the best route between j and i, and the border delay cost if facility i is in Texas.
$\check{c}_{ih}$	The per truckload cost of transporting from eBeam facility i to Texas hub h. This cost includes the transportation cost calculated from the transportation time for the best route between i and h, and the border delay cost if facility i is in Mexico.
	Variables
x_{ji}	The flow of truckloads on arc (j, i) sent from Mexican growing region j to eBeam facility i.
y_{ih}	The flow of truckloads on arc (i, h) sent from eBeam facility i to hub h.
λ_{ik}	The arrival rate for commodity k at eBeam facility i.
λ'_{ik}	The arrival rate for commodity k in terms of multiples of truckloads with weight $\ell^0 = \min_k\{\ell^k\}$ at eBeam facility i. $\lambda'_{ik} = \lambda_{ik}\frac{\ell^k}{\ell^0}$.
$\underline{s}_i$	The minimum number of service lines required in eBeam facility i to satisfy demand and to maintain a utilization rate below the maximum allowed utilization rate.

We illustrate the MCF model through an example with $Q = 2$, three growing regions, and two hubs (Figure 4.5). Additional notations are in Table 4.6. Because we run the MCF model for each commodity independently, we drop superscript k for the demand and capacity parameters (M_h^k and P_j^k).

The upper bound on an arc from the source node O to the growing region j models

Figure 4.5: Minimum Cost Flow Problem Example [O: Source; j: Mexican growing regions; i: eBeam facilities; l: Dummy eBeam facilities; h: Texas Hubs; S: Sink].



the capacity at growing region j. The unit flow costs on the arcs linking the growing region j and the eBeam facility i include the transportation cost for one truckload and the border delay cost (if appropriate) for one truckload. We create dummy eBeam facility nodes l to assess an eBeam processing cost p_i for the truckloads of the commodity flowing though the eBeam facilities. The lower bound $\underline{\rho} \cdot \mu$ on arc (i, l) enforces the minimum utilization rate at the eBeam facility. Similar to the flow on arc (j, i), the arc (l, h) linking dummy eBeam facility l and Texas hub h has a unit flow cost including the unit transportation cost and the border delay cost (if appropriate) for one truckload. The lower bound on the flow from Texas hub h to the sink node S represents the demand at Texas hub h.

Theorem 3 *Assuming that the queuing delay cost at the eBeam facilities is zero and the processing capacity is unlimited, the single-commodity version of GP with Q given eBeam facility locations can be solved polynomially.*

Proof: Since each node in the MCF model presented above represents a specific location (growing region in Mexico, eBeam facility, or hub in Texas) with each arc standing for the corresponding movement of a single commodity, each feasible integer flow in the MCF model corresponds to a feasible solution for GP and vice versa. Therefore, the optimal solution for the MCF model is equivalent to the optimal commodity distribution for GP. The complexity of solving the MCF model is $O(n^4 \log n)$, where $n = J + H + 2Q + 2$ is the number of nodes in the network (Ahuja et al. 1993). ■

The linear programming formulation for the MCF model corresponding to Theorem 3 is presented below. The objective function minimizes the total cost, including the transportation cost and the processing cost. The transportation costs are computed in terms of each commodity's actual number of truckloads, and the processing cost on arcs (i, l) are computed according to the weight of a truckload by adjusting p_i.

$$\textbf{Minimize} \quad \sum_i \sum_j \hat{c}_{ji} x_{ji} + \sum_i p_i \sum_j x_{ji} + \sum_i \sum_h \check{c}_{ih} y_{ih}$$

Subject to:

Constraints (4.16) enforce that for eBeam facility i, the inbound flow equals the outbound flow.

$$\sum_j x_{ji} = \sum_h y_{ih}, \forall i \tag{4.16}$$

Constraints (4.17) enforce the supply capacity for each growing region j in Mexico.

$$\sum_i x_{ji} \leq P_j, \forall j \tag{4.17}$$

Constraints (4.18) guarantee that each Texas hub's demand is met.

$$\sum_i y_{ih} \geq M_h, \forall h \tag{4.18}$$

Constraints (4.19) guarantee that each eBeam facility's utilization rate is above the minimum allowed utilization rate.

$$\sum_j x_{ji} \geq \underline{\rho} \cdot \mu, \forall i \tag{4.19}$$

Constraints (4.20) are non-negativity constraints.

$$x_{ji}, y_{ih} \geq 0, \forall i, \forall j, \forall h \tag{4.20}$$

Since there is no interaction among the commodities when the queuing delay cost is zero and the processing capacity is unlimited at the eBeam facilities, for each Q and each set of Q potential locations, each commodity's cost can be calculated in an individual MCF (Theorem 3). Then, the results are combined to calculate the multi-commodity transportation and processing costs for each set of Q potential locations. Therefore, for each Q, the set of eBeam facilities with the minimum cost and the corresponding routes from growing regions to hubs can be determined. Thus, we state the following result.

Corollary 1 *Assuming that the queuing delay cost at the eBeam facilities is zero and that the processing capacity is unlimited, the multi-commodity version of GP with Q given eBeam facility locations can be solved polynomially by solving for each commodity individually and then aggregating the solutions.*

4.4.2 Queueing System

After a set of Q eBeam facilities is addressed by the MCF model constructed above, at each facility i, Heuristic H_{GP} uses an $M/D/s$ queuing model to assign the minimum number of service lines $\underline{s}_i$ required so that the maximum acceptable utilization $\bar{\rho}$ is not exceeded. The utilization at each eBeam facility is computed based on the aggregated arrival rate obtained from all of the commodities' MCF results. Specifically, the number of truckloads, λ_{ik}, for each commodity k is converted into the number of truckloads with weight $\ell^0 = \min_k\{\ell^k\}$ ($\lambda'_{ik} = \lambda_{ik}\frac{\ell^k}{\ell^0}$), and the λ'_{ik}s are summed together to compute the aggregated arrival rate, $\lambda_i = \sum_k \lambda'_{ik}$, for truckloads with weight ℓ^0 for all commodities at each location. This aggregated arrival rate determines the number of service lines needed to satisfy the utilization rate requirement.

Next, H_{GP} calculates the total cost for each facility, which includes the facility setup cost, the service line operating cost, and the queuing delay cost. H_{GP} starts with $\underline{s}_i$ (minimum number of service lines at facility i), then successively adds one service line to the facility until doing so no longer reduces the total cost. There is a tradeoff between the queuing delay cost and the service line operating cost. Increasing s_i from $\underline{s}_i$ to $\underline{s}_i + 1$ will decrease the total cost if the resulting queuing savings exceeds the operating cost of the extra service line. Continued increases in s_i may also decrease the overall cost, but with diminishing returns. This process takes advantage of the convex cost function and is demonstrated in Section 4.5.

The queueing delay cost at eBeam facility i is calculated for all commodities as follows. Based on the aggregated arrival rate λ_i and the average number of truckloads waiting in the queue L_i^q calculated by (4.9), the average number of truckloads in eBeam facility i is $L_i = L_i^q + \frac{\lambda_i}{\mu}$. The expected sojourn time at eBeam facility i is $W_i = \frac{L_i}{\lambda_i}$. Thus, the total queuing delay cost for the λ_i truckloads served by eBeam facility i is $b\lambda_i W_i = bL_i$. This queuing approximation is validated in Section 4.5.2.

Summing costs for all the facilities in this set of Q potential locations and adding the result to the cost of the MCF model produces the total cost for this set of potential locations. This process is repeated for all combinations of Q facilities to find the set of locations with the minimum total cost for each Q. In the end, the best value over all $Q \in \{1, ..., \bar{Q}\}$ is chosen to achieve the minimum cost. Since there are only eight potential candidate locations ($\bar{Q} = 8$), we can explore all possible combinations of the eBeam facilities to obtain the best solution in reasonable time. Therefore, Heuristic H_{GP} allows the use of network models to solve GP efficiently.

Heuristic H_{GP}:

$min\_cost\_GP = \infty$

For $Q = 1$ to $\bar{Q}$

    $min\_cost\_Q = \infty$

    For each possible set of Q eBeam facility locations from $\bar{Q}$ potential eBeam facility locations

        Solve the MCF model independently for each commodity k

        Label the sum of these solutions min_cost_MCF

        For each facility i in this set of Q eBeam facility locations

            $min\_cost\_i = \infty$

            Based on the aggregated arrival rate and the maximum allowed utilization rate,

                  assign $\underline{s}_i$ service lines to eBeam facility i

            For $s_i = \underline{s}_i$ to ∞

                Compute the cost including the setup cost, the capacity and operating cost,

                    and the queuing delay cost for eBeam facility i. Label solution $cost\_i$

                If $min\_cost\_i > cost\_i$ Then

                  $min\_cost\_i = cost\_i$

                Else

                  Exit For s_i

            Next s_i

        Next eBeam facility i

        If $min\_cost\_Q > min\_cost\_MCF + \sum_i min\_cost\_i$ Then

          $min\_cost\_Q = min\_cost\_MCF + \sum_i min\_cost\_i$

    Next set of Q eBeam facility locations

    If $min\_cost\_GP > min\_cost\_Q$ Then

      $min\_cost\_GP = min\_cost\_Q$

Next Q

The complexity of solving an integer MCF problem with $n = J+H+2Q+2$ nodes in the network is $O(n^4 \log n)$. We run MCF models separately for K commodities and there are $\sum_{Q=1}^{\bar{Q}} \begin{pmatrix} \bar{Q} \\ Q \end{pmatrix}$ ways in total of choosing facility locations. Therefore, the complexity for H_{GP} is given by Lemma 6.

Lemma 6 *The overall complexity for H_{GP} is*

$$O\left(\sum_{Q=1}^{\bar{Q}} \begin{pmatrix} \bar{Q} \\ Q \end{pmatrix} Kn^4 \log n\right),$$

where $n = J + H + 2Q + 2$ and K is the total number of commodities.

4.5 Computational Study

The distance, demand, and cost parameter values are based on actual data (Table 4.7). Heuristic H_{GP} selects the best locations for the eBeam facilities, assigns trucks to eBeam facilities and to hubs while respecting prohibited areas for transporting the four commodities, and determines the number of eBeam machines (service lines) at each facility. Tests are run for $Q = 1, 2, 3, 4$.

Nuevo Laredo is the best overall choice for $Q = 1$ (Table 4.8). The minimum number of service lines is $s_i = 13$, but adding another service line ($s_i = 14$) reduces the total cost by 0.02% to \$23.35M. An additional service line ($s_i = 15$) does not improve the total cost. If a U.S. location must be chosen, then San Antonio is the best choice (total cost \$23.69M), also with 14 service lines. Building a facility in the U.S. rather than Mexico results in a cost increase of \$0.34M. Although the weekly cost of choosing San Antonio is only 1.46% higher than choosing Nuevo Laredo, the total annual cost difference would be \$6.8M for 20 peak importing weeks, which is significant.

Nuevo Laredo and Matehuala are the best locations to build two eBeam facilities, with minimum total cost of \$23.46M (Table 4.9). These two eBeam facilities use 15 service lines in total: 12 service lines in Nuevo Laredo and 3 service lines in

Table 4.7: Parameter Values.

	Parameters
ℓ^k	The weight in pounds of one truckload of commodity k. Select mangoes for $\ell^0 = \min_k\{\ell^k\} = 40{,}000$ lb.
M_h^k	Demand in truckloads per week for commodity k at hub h. For mangoes, the total demand is 15,393 truckloads per year (770 truckloads per week for 20 peak importing weeks). Use $M_1 : M_2 : M_3 = 3 : 4 : 3$ for mangoes. For guavas, citrus, and avocado, the demands are 1 truckload per week, 1308 truckloads per week, and 1405 truckloads per week, respectively, for 20 peak importing weeks annually. Use $M_1 = M_3 = 0$, $M_2 = 1$ for guavas and $M_1 : M_2 : M_3 = 2 : 3 : 2$ for citrus and avocado.
P_j^k	The capacity in truckloads per week of commodity k at Mexico location j. For mangoes, the total annual supply capacity in Mexico is 80,752 truckloads. We assume that each region has a weekly supply of 577 truckloads. For guavas, we assume that Michoacan has a weekly supply of 1 truckload. For citrus, we assume that Jalisco and Michoacan have a weekly supply of 2000 and 8000 truckloads, respectively. For avocado, we assume that Michoacan has a weekly supply of 3627 truckloads.
$\underline{\rho}$	The minimum allowed utilization rate: 40%.
$\bar{\rho}$	The maximum allowed utilization rate: 80%.
μ	The service rate per line: $\mu = 400$ standard truckloads per line per week.
f_i	The amortized setup cost at location i: \$69,500 per week.
o_i	The weekly capacity and operating cost per service line at location i: \$11,750 per week in the U.S., \$10,000 per week in Mexico.
p_i	The eBeam processing cost: \$4000 per truckload.
b	The unit time cost for queuing delay: \$100,000 per week per truckload.
$\hat{c}_{ji}$, $\check{c}_{ih}$	The unit cost on arc (j, i) and arc (i, h). Let t be the total traveling time, including the transportation time and the border delay time. Labor costs determine the values for $\hat{c}_{ji}$ and $\check{c}_{ih}$. If the eBeam facility is in Mexico, $\hat{c}_{ji} = 2.3t$, $\check{c}_{ih} = 2.5t$. If the eBeam facility is in Texas, $\hat{c}_{ji} = 2.5t$, $\check{c}_{ih} = 2.5t$.

Matehuala. The optimal number of service lines is less for $Q = 1$ because the pooling of demand into one set of service lines yields operational efficiency.

Choosing one facility in Mexico and one facility in the U.S. has a total cost of \$23.47M, resulting in a total cost increase of \$0.01M (0.04%). It also requires 15 service lines in total: 12 built in Nuevo Laredo and 3 built in Houston.

If two locations have to be selected in the U.S., the best combination is San

Table 4.8: Computational Study for One EBeam Facility (The best combination is highlighted).

	One In Mexico		
Best Location	**Number of Service lines**	**Queueing Cost** ($M)	**Total Cost** ($M)
Nuevo Laredo	13	0.8970	23.3555
Nuevo Laredo	**14**	**0.8825**	**23.3510**
Nuevo Laredo	15	0.8760	23.3545
	One In The U.S.		
Best Location	**Number of Service lines**	**Queueing Cost** ($M)	**Total Cost** ($M)
San Antonio	13	0.8970	23.6977
San Antonio	**14**	**0.8825**	**23.6949**
San Antonio	15	0.8760	23.7002

Antonio and Houston, with the total cost of $23.72M. Compared to the best option for building two facilities in Mexico, building in San Antonio and Houston brings additional cost of $0.25M, a 1.07% increase. Because of how the total demand is split into two locations based on the MCF model, San Antonio and Houston operate efficiently with 16 lines, while Nuevo Laredo and Matehuala only need 15 lines.

Nuevo Laredo, Matehuala, and Houston are the best locations to build three eBeam facilities, with 17 service lines in total: 11 in Nuevo Laredo, 3 in Matehuala and 3 in Houston (Table 4.10). The minimum total cost is $23.58M. The detailed computational results for other combinations when choosing three locations are in Table C.1 in Appendix C.1.

For $Q = 4$, the best choice has three facilities in Mexico and one facility in the U.S.; 18 service lines in total are needed: 9 in Nuevo Laredo, 3 in Reynosa, 3 in Matehuala, and 3 in Houston. The total cost is $23.71M (Table 4.11). Detailed computational results for other combinations when choosing four locations are presented in Table C.2 in Appendix C.2.

Overall, Nuevo Laredo is the best location for $Q = 1, 2, 3, 4$. Matehuala takes

Table 4.9: Computational Study for Two EBeam Facilities (The best combination is highlighted).

Best Locations	Number of Service lines	Queueing Cost ($M)	Total Cost ($M)
	Two In Mexico		
Nuevo Laredo Matehuala	11 3	0.9182	23.4734
Nuevo Laredo Matehuala	**12 3**	**0.8963**	**23.4615**
Nuevo Laredo Matehuala	13 3	0.8870	23.4622
	One In Mexico One In The U.S.		
Nuevo Laredo Houston	11 3	0.9182	23.4802
Nuevo Laredo Houston	**12 3**	**0.8963**	**23.4683**
Nuevo Laredo Houston	13 3	0.8870	23.4690
	Two In The U.S.		
San Antonio Houston	10 5	0.9037	23.7203
San Antonio Houston	**11 5**	**0.8919**	**23.7202**
San Antonio Houston	12 5	0.8872	23.7273

second place and Houston is right behind Matehuala. When selecting only one location in the U.S., San Antonio is the best option (Table 4.8), but when selecting more than one location, Houston surpasses it. This is because if Nuevo Laredo is selected, supplying the Houston demand either from Houston itself or directly from Nuevo Laredo is more efficient than supplying it from San Antonio. That no optimal solution for $Q \leq 3$ employs the Reynosa/McAllen crossing is significant because it is currently the third-most used crossing east of El Paso (Section 4.1.2), second if the analysis combines Nuevo Laredo/Laredo and Colombia/Laredo.

Because the MCF model minimizes the transportation costs while ignoring the

Table 4.10: The best Combination for Building Three EBeam Facilities (the instance with the best number of service lines is highlighted).

Best Locations	Number of Service lines	Queueing Cost ($M)	Total Cost ($M)
Nuevo Laredo	10		
Matehuala	3		
Houston	3	0.9121	23.5809
Nuevo Laredo	**11**		
Matehuala	**3**		
Houston	**3**	**0.8985**	**23.5773**
Nuevo Laredo	12		
Matehuala	3		
Houston	3	0.8931	23.5819

Table 4.11: The best Combination for Building Four EBeam Facilities (the instance with the best number of service lines is highlighted).

Best Locations	Number of Service lines	Queueing Cost ($M)	Total Cost ($M)
Nuevo Laredo	**8**		
Reynosa	**3**		
Matehuala	**3**		
Houston	**3**	**0.9314**	**23.7215**
Nuevo Laredo	**9**		
Reynosa	**3**		
Matehuala	**3**		
Houston	**3**	**0.9108**	**23.7109**
Nuevo Laredo	**10**		
Reynosa	**3**		
Matehuala	**3**		
Houston	**3**	**0.9033**	**23.7134**

queueing delay cost, the MCF result is a loose lower bound on the total cost. The total cost is on average 5.54% (maximum 6.15%) higher than the MCF result (Table 4.8 - Table 4.11) for all the problems tested. This shows that H_{GP} is effective for solving GP.

Table 4.12 illustrates that as Q increases, so does queueing delay cost (because of less pooling), setup cost (cost to open a facility), capacity and operating cost (less pooling implies more service lines), and transportation cost. Processing cost, obviously, remains constant because it is determined by total demand.

In general, one would expect that having more eBeam facilities would not increase

Table 4.12: Computational Result for the General Problem (The Best Option is highlighted).

$Q = 1$								
Best Location	**# of Service Lines**	**Util. Rates**	**Que. Delay Cost ($M)**	**Setup Cost ($M)**	**Cap. & Oper. Cost ($M)**	**Proc. Cost ($M)**	**Trans. & Bor. Del. Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	14	0.6220	0.8825	0.0695	0.1400	13.9320	8.3270	23.3510

$Q = 2$								
Best Locations	**# of Service Lines**	**Util. Rates**	**Que. Delay Cost ($M)**	**Setup Cost ($M)**	**Cap. & Oper. Cost ($M)**	**Proc. Cost ($M)**	**Trans. & Bor. Del. Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	12	0.6256						
Matehuala	3	0.4000	0.8963	0.1390	0.1500	13.9320	8.3442	23.4615

$Q = 3$								
Best Locations	**# of Service Lines**	**Util. Rates**	**Que. Delay Cost ($M)**	**Setup Cost ($M)**	**Cap. & Oper. Cost ($M)**	**Proc. Cost ($M)**	**Trans. & Bor. Del. Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	11	0.5734						
Matehuala	3	0.4000						
Houston	3	0.4000	0.8985	0.2085	0.1753	13.9320	8.3630	23.5773

$Q = 4$								
Best Locations	**# of Service Lines**	**Util. Rates**	**Que. Delay Cost ($M)**	**Setup Cost ($M)**	**Cap. & Oper. Cost ($M)**	**Proc. Cost ($M)**	**Trans. & Bor. Del. Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	9	0.5675						
Reynosa	3	0.4000						
Matehuala	3	0.4000						
Houston	3	0.4000	0.9108	0.2780	0.1853	13.9320	8.4049	23.7109

transportation costs. Table 4.12 shows that this does not hold. There are two reasons for this. First, all traffic goes through Nuevo Laredo/Laredo in an optimal solution no matter how many facilities are open. This is largely a result of the transportation infrastructure around the border and the locations of growing regions (Mexican Pacific Coast) and of the Texas hubs (all are easily accessible from Laredo). Combining this with the minimum utilization requirement in (4.8) and (4.19), which ensures that trucks go to each open facility, leads to the transportation cost being increased when a second facility is added at Matehuala. Second, even though opening a third facility in Houston does not increase the number of trucks traveling there (the amount needed to satisfy the minimum utilization requirement is less than the Houston hubs demand), trucks destined for eBeam treatment in the U.S. must have U.S. drivers for the entire trip from the growing region to the eBeam facility to the hub. Trucks carrying fruit treated by Mexican facilities use Mexican drivers from the growing region to the eBeam facility, and U.S drivers from the eBeam facility to the hub. Thus, opening facilities in the U.S. leads to higher transportation costs.

4.5.1 Sensitivity Analysis

To perform sensitivity analysis, we first vary the maximum utilization rate $\bar{\rho}$ without changing other parameter values. Table 4.13 illustrates that the selection of locations is insensitive to variations in the maximum utilization rate for $Q = 1, 2, 3$. The sensitivity for $Q = 4$ is minimal. Table 4.14 shows similar results for unit transportation costs.

Table 4.13: Sensitivity Analysis Result: Best Locations for $\bar{\rho}$ with $\mu = 400$ Truckloads.

	$Q = 1$	$Q = 2$	$Q = 3$	$Q = 4$
$\bar{\rho} = 0.6$	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Reynosa Matehuala Houston
$\bar{\rho} = 0.7$	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Reynosa Matehuala Houston
$\bar{\rho} = 0.8$	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Reynosa Matehuala Houston
$\bar{\rho} = 0.9$	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Matehuala San Antonio Houston

4.5.2 Validation of Queueing Approximation

The proposed approximation for the average length of queue for any eBeam facility with multiple service lines is validated by a simulation model using Matlab 2015. For the single-location ($Q = 1$) $M/D/s$ queueing system, the number of service lines in the simulation model is chosen between 11 and 20 to ensure that the utilization rate is below the maximum allowed level, and the arrival rate is set equal to the total of the expected weekly demands for each commodity. We executed 100 replications of 20 weeks for each number of service lines. The cells highlighted in Table 4.15 show

Table 4.14: Sensitivity Analysis Result: Best Locations for The Unit Transportation Cost $\hat{c}_{ji}$ and $\check{c}_{ih}$ with $\bar{\rho} = 0.8$ and $\mu = 400$ Truckloads.

	$Q = 1$	$Q = 2$	$Q = 3$	$Q = 4$
Low cost	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Reynosa Matehuala Houston
Med cost	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Reynosa Matehuala Houston
High cost	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Matehuala San Antonio Houston
Very high cost	Nuevo Laredo	Nuevo Laredo Matehuala	Nuevo Laredo Matehuala Houston	Nuevo Laredo Matehuala San Antonio Houston

that the average length of queue computed using the queueing approximation in Section 4.4.2 falls within the 95% confidence interval built from the simulation results for $s = 14, ..., 20$. Thus, our queuing approximation works well for the single-location $M/D/s$ queueing system when the utilization rate is below 0.65.

Table 4.15: Average Length of Queue in Truckloads in One eBeam Facility Using Queueing Approximation and Simulation (Highlighted cells are within 95% confidence interval).

# of Serv. Lines	**Util. Rate**	**Queue Length (L_i) From Queueing Approximation**	**Queue Length (L_i) From Simulation**
11	0.7916	10.1108	12.438 ±0.78
12	0.7256	9.2986	11.0921±0.72
13	0.6698	8.9697	10.0656 ±0.68
14	0.6220	8.8250	9.2821 ±0.65
15	0.5805	8.7596	8.8995 ±0.59
16	0.5442	8.7301	8.7673 ±0.49
17	0.5122	8.7170	8.7264 ±0.43
18	0.4838	8.7114	8.7137 ±0.41
19	0.4583	8.7090	8.7096 ±0.38
20	0.4354	8.7081	8.7082 ±0.39

Next, to validate our queueing approximation for a multi-location $M/D/s$ queueing system with a small number of service lines ($s < 10$) in each location, we rerun the simulation for $Q = 4$. We use the utilization rates and the best two combinations of service lines (Table C.2 in Appendix B) obtained from Heuristic H_{GP}. We again executed 100 replications of 20 weeks for each combination. The results from this simulation (Table 4.16) are consistent with those for the first simulation (Table 4.15). The cells highlighted in Table 4.16 show that all tested samples fall within the 95% confidence interval built from the simulation results. Thus, our queuing approximation works well for moderate utilization rates in multi-location $M/D/s$ queueing systems.

Table 4.16: Average Length of Queue in Truckloads in Four eBeam Facilities Using Queueing Approximation and Simulation (Highlighted cells are within 95% confidence interval).

Combination Number	# of Serv. Lines	Util. Rate	Queue Length (L_i) From Queueing Approximation	Queue Length (L_i) From Simulation
1	9	0.5675	5.2253	5.3317 ±0.25
	3	0.4000	1.2941	1.3210 ±0.11
	3	0.4000	1.2941	1.3135 ±0.09
	3	0.4000	1.2941	1.3181 ±0.10
2	3	0.4000	1.2941	1.3304 ±0.15
	6	0.5033	3.1231	3.2211 ±0.23
	6	0.4917	3.0398	3.1298 ±0.21
	3	0.5125	1.8008	1.8543 ±0.16

4.6 Factor Rating System and Cost Estimation

The selection of locations for the eBeam facilities is driven by transportation, resources (labor, materials, and utilities), taxes, and regulations. This section discusses these factors to develop a factor rating system (FRS) for the eBeam facility location alternatives and to analyze each factor's impact on the setup cost, the operating cost, and the unit transportation costs.

Table 4.17: Factor Rating System for Eight Potential Locations.

Factor	Trans. Conv.	Constr. Cost	Labor Cost	Trade Activity	Electr. Rate	Tax Rate	Border Delay	Total Points
Point Range	0 to 300	0 to 200	0 to 100	0 to 100	0 to 60	0 to 30	0 to 50	0 to 840
Nuevo Laredo	260	120	90	90	55	25	5	645
Houston	250	120	65	80	45	20	30	610
Matehuala	170	190	85	45	55	25	30	600
San Antonio	250	120	60	55	45	20	30	580
Reynosa	185	120	85	70	55	25	40	580
Laredo	245	120	60	85	45	20	5	580
Dallas	250	120	65	45	45	20	30	575
Matamoros	180	120	80	50	55	25	45	555

We collect data from the U.S. Census Bureau, the U.S. Bureau of Labor Statistics, the Bureau of Transportation Statistics, the U.S. Energy Information Administration, and the U.S. Customs and Border Protection (CBP). We focus on border entry/crossing data, North American transborder freight data, labor force data for both Mexico and the U.S., U.S. exports and imports trade activity, industrial electricity rates for Texas and Mexico, and border wait times data. The detailed results of the FRS (Table 4.17) for the eight potential locations indicate that Nuevo Laredo is the best location to build an eBeam facility. It has the highest score for almost all factors. Houston's high ratings in transportation convenience and trade activity place it above other locations in the U.S. Matehuala, San Antonio, Reynosa, and Laredo rank behind the two leaders. The results of this analysis are very similar to the results of Section 4.5.

To estimate the fixed setup cost, we collect the estimated construction costs for all potential locations from the National Center for Electron Beam Research. The approximate construction costs indicate that it is slightly less expensive to build an eBeam facility in Mexico than in the U.S. However, because this difference is so small, we use the same fixed setup cost for each. The only exception is Matehuala, which already has a gamma ray radiation facility that can be converted to an eBeam facility at a lower cost.

The border crossing delay time at each border crossing point depends on the number of truckloads crossing it and its number of Verification and Inspection Points (VIPs). Nuevo Laredo is strategically positioned at the convergence of several highways, railroads, and bridges. It is considered to be Mexico's most important inland port for exporting agricultural products to the U.S. (USDA 2015a). Between 2013 and 2015, Nuevo Laredo/Laredo, even though it has only five VIPs, accounts for over two thirds of all imports (truckloads of commodities) from Mexico. Colombia/Laredo has four VIPs, Ciudad Juarez/El Paso has nine VIPs, and Reynosa/McAllen has seven VIPs (USDA 2015a). Thus, the average border waiting time is longest at Nuevo Laredo/Laredo. The border delay time estimates are collected from CBP Border Wait Times Data (2016) and Avetisyan et al. (2015).

Our MCF model decides to which eBeam facility and to which Texas hub each truck is assigned. The actual route for each truckload of fruit is determined by on-line mapping software, such as Google Maps, while respecting the prohibited movement areas for untreated commodities. The transportation cost between two locations in the MCF model is estimated based on these routes.

The per-line capacity and operating cost at each location are estimated using the Bureau of Labor Statistics Labor Force Data (2014), including the average revenue, the employment rate, and the annual tax report for the eight potential locations. As expected, the overall employment rate, the average labor cost, and the tax rate are all lower in Mexico than in the U.S. In addition, the average industrial electricity rate in Texas is \$0.0557/kWh and in Mexico is \$0.0502/kWh. We calculate the capacity and operating cost by aggregating amortized setup cost, labor cost, tax, and electricity consumption. The resulting capacity and operating cost in Mexico is set lower than that in the U.S. (see Table 4.7).

4.7 Conclusion

The amount of fresh produce crossing the U.S. / Mexico border has increased exponentially since the implementation of the North American Free Trade Agreement in 1994. Imported fresh produce must be treated for pestilence and microbial pathogen contamination. This requirement protects the health of those who consume the produce and the viability of domestic crops that could be infested by pests or infected by those pathogens. Among the various technologies that have been used for this function—freezing, heat treatments, chemical fumigation, and irradiation—electron beam (eBeam) irradiation is relatively new and has many advantages, including high throughput, high dose rate, low capital investment, and low operating costs. Furthermore, it requires only commercial electricity (rather than, for example, gamma ray radiation from cobalt-60), which it uses efficiently.

This study provides guidelines for private industry in the U.S. and Mexico to select the most cost-efficient locations for the eBeam facilities specifically designed for phytosanitary treatment of fresh fruits and vegetables crossing the Texas/Mexico border. It also determines how many service lines (one eBeam machine per line) each facility should have. The produce is grown in seven Mexican states and is shipped to three hubs in Texas. Thus, our algorithm assigns each truck leaving a growing region to an eBeam facility and to a hub so that costs are minimized. The study incorporates the unique characteristics of the problem, such as eBeam irradiation technology, multiple commodities, prohibited movement areas in Texas, regulation and infrastructure issues, delays at border crossing points, and queueing delays at the eBeam facilities. To capture all factors that a manager must consider, the cost objective includes the fixed set-up cost for building each eBeam facility; each facility's operating cost, which depends on the number of service lines selected; the transportation, processing, and border delay costs; and the queuing delay cost, which is determined by the number of service lines and the number of truckloads assigned to each facility.

We developed a generalizable decision support system that uses a heuristic that is based on a minimum cost flow model (MCF). This polynomial-time heuristic considers all possible combinations of locations for the eBeam facilities (from eight candidate locations). For each such combination, the MCF optimizes the transportation, processing, and border delay costs. For each facility within this combination, the heuristic determines the number of service lines that minimizes the operating and queueing delay costs.

Both the computational study and the factor rating analysis suggest that Nuevo Laredo (Mexico) is the best location to build an eBeam facility overall. This is because all traffic is routed to the Nuevo Laredo / Laredo border crossing, and operating a facility in Mexico is less expensive. In addition to Nuevo Laredo, Matehuala (Mexico) is also highly rated. Houston and San Antonio are the two best locations for building eBeam facilities in the U.S. Overall, if building no more than two eBeam facilities, selecting locations in Mexico has lower total cost than selecting them in the U.S.

In summary, our analysis provides an effective importing and distribution planning tool that integrates multiple decisions for selecting sites for phytosanitary facilities along the Texas/Mexico border by considering new food safety technology and key economic factors that signal local growth and development. The methodology developed in our problem is general enough to be applicable to a wide variety of food distribution networks in other countries or to similar contexts with minor modifications.

5. CONCLUSIONS

This dissertation is motivated by critical and emerging issues within special sectors of supply chains of products managed by the government agencies and private industries. It develops methodologies to improve efficiency and effectiveness of the operations for supply chains of banknotes, coins, and foods and provides meaningful managerial insights for the decision maker. In this context, three methodologies are used (i) linear and nonlinear mixed integer programming models for achieving optimal solutions, (ii) minimum cost flow network optimization models for transforming the intractable formulations into solvable sub-problems, and (iii) heuristic algorithms for efficiently finding the near-optimal solutions using network models. Essays in this dissertation develop conceptual frameworks of the currency supply chain, optimize the performance of the currency supply chain from both supply-side and demand-side perspectives, and analyze economic factors and managerial constraints that impact the overall operational efficiency of the food supply chain.

This dissertation makes several contributions to the literature by answering untouched research questions and analyzing practically motivated research issues. In Chapter 2, motivated by an urging need of the central bank in a large county, a supply-side problem is analyzed to optimize currency supply operations. This supply-side problem has not been well studied in the academic literature, nor carefully addressed in practice. In Chapter 3, the framework of U.S. Coin Supply Chain is developed to analyze the operational performances from both supply-side and demand-side perspectives and to deliver practically usable planning methodologies / tools for the central planner and the depository institutions. This essay demonstrates the use of a modeling approach that not only meets the need for effective and efficient management of coin supply in the U.S., but also has the potential to be applied to a variety of similar coin supply chains around the world. Chapter 4 deals with emerging food safety issues in the logistics of supplying fresh produce from Mexico to the U.S. This research is inspired by emerging need of eBeam irradiation facilities and a sophisticated import and distribution planning system along the U.S. / Mexico border. This essays proposes potential locations for new eBeam facilities, suggests the capacities of eBeam facilities, and optimizes the route for treating fresh fruits and vegetables crossing the Texas/Mexico border. The methodology developed in this chapter is general enough to be applicable to a wide variety of food distribution networks in other countries or to similar contexts with minor modifications.

This dissertation opens up several opportunities for future research. Methodologies developed in this dissertation are general enough to be applied to other categories of supply chains under different settings around the world with appropriate modifications. In addition, each individual essay can be extended from both analytical and empirical perspectives to further explore interesting managerially relevant research problems.

Firstly, the general intractable problem in Chapter 2 is split into two sub-problems (a downstream problem and an upstream problem) to solve the problem sequentially for reaching the near-optimal solutions for minimizing the total operational cost in a country's currency supply chain. This general optimization problem for improving the efficiency of the overall currency supply network could be modeled as a single non-linear mixed integer programming model considering all the managerial constraints. This problem may be solved by using the relaxation of integer constraints and alteration of nonlinear constraints or by developing efficient heuristic algorithms. Another extension of Chapter 2 is to collect a complete data set including all the supply, demand, transaction, and sourcing data to conduct an empirical study of economic factors that impact the overall performance of the currency supply chain.

Secondly, an immediate extension of Chapter 3 could be the development of the rolling horizon procedure incorporating the robust planning approach presented in this essay. This procedure may be compared with the traditional rolling horizon procedure presented in Chapter 3. Moreover, a comprehensive mathematical model may be developed covering all the parties (supply-side parties and demand-side parties) in order to minimize the overall cost for the entire coin supply chain. Currently, the objectives of the Federal Reserve System and the coin terminals are not aligned. A mechanism design may be developed to coordinate the objectives of parties involved in the supply side of the coin supply chain. This, in turn, may help the Federal Reserve System to develop policies to increase the efficiency of coin recirculation in the economy. In addition, demand-side parties may collaborate with a third party logistic provider to reduce the operational cost of coin supply to the public from the demand-side perspective. An interesting further study may be a mechanism design using game theory for exploring this cooperation and competition among different depositary institutions.

Finally, further work may be done to improve the heuristic algorithms in Chapter 4 in order to efficiently solve large size problems with substantive potential locations. Thus, the methodologies specifically developed for this food supply chain may be more generalizable to solve large size problems efficiently. To extensively extend approaches/methodologies of this dissertation to other supply chains under different settings, further studies may be performed by incorporating multiple objectives in the model and considering different requirements from perspectives of various parties. Another immediate extension of Chapter 4 may be the introduction of the demand uncertainty into the model by extending to multiple periods. The methodology similar to the robust planning approach presented in Chapter 3 may be applied to tackle this complex problem. Undoubtedly, all of these extensions would require significant, non-trivial modifications to our models. However, these difficulties will not stop me from performing further research, but only inspire and encourage me to step forward and explore more.

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APPENDIX A

SUPPLEMENT TO CHAPTER 2

A.1 Structured MIP Formulation of Problem 1

Table A.1: Additional Parameters Used for the Structured MIP Formulation of Problem 1.

L_1	Twice of the largest distance between any vault and any branch in the region of interest (this number varies for different regions).
L_2	The largest possible transportation cost to satisfy a branch's demand in the region of interest.

Table A.2: Additional Variables Used for the Structured MIP Formulation of Problem 1.

IM_j	If $IM_j = 1$, then intermodal (IM) transportation satisfies the demand at branch j.
IU_j	If $IU_j = 1$, then interurban (IU) transportation satisfies the demand at branch j.
U_j	If $U_j = 1$, then urban (U) transportation satisfies the demand at branch j.
e_j	The cost of transportation for branch j.
e_j^{IM}	The cost of intermodal (IM) transportation for branch j.
e_j^{IU}	The cost of interurban (IU) transportation for branch j.
e_j^{U}	The cost of urban (U) transportation for branch j.
C_i^+	Total withdrawal per period from vault i.

Problem 1 MIP:

The objective function minimizes the total per period cost, which consists of the operating cost for new regional vaults, the transportation cost for branches, and the incremental capacity cost for vaults.

$$\textbf{Minimize } \Phi = \sum_{i \in A} F_i y_i + \sum_{j=1}^{M^{br}} e_j + c_o \sum_{i=1}^{N_v} C_i^a$$

Subject to:

Constraints (A.1) enforce that every branch has exactly one supplier vault.

$$\sum_{i=1}^{N_v} x_{ij} = 1,\ j = 1, \ldots, M^{br} \tag{A.1}$$

Constraints (A.2)-(A.4) decide if U transportation is used for a branch. If the land distance between vault i and branch j is less than 50 miles, then j uses U transportation.

$$IM_j + IU_j + U_j = 1,\ j = 1, 2, \ldots, M^{br} \tag{A.2}$$

$$\sum_{i=1}^{N_v} x_{ij} d_{ij}^t \le 50U_j + L_1 (1 - U_j),\ j = 1, \ldots, M^{br} \tag{A.3}$$

$$\sum_{i=1}^{N_v} x_{ij} d_{ij}^t \ge 50(1 - U_j),\ j = 1, \ldots, M^{br} \tag{A.4}$$

Constraints (A.5)-(A.8) compute the transportation cost for branch j for different transportation modes. If the land distance is greater than 50 miles, then the transportation mode, either IU or IM, is decided by which costs less.

$$e_j^{IM} \ge \frac{D_j}{l_j^{IM}} f_a + \frac{D_j}{l_j^{IM}} b_a \sum_{i=1}^{N_v} 2d_{ij}^a x_{ij} + c_a D_j - L_2(1 - IM_j), j = 1, \ldots, M^{br} \tag{A.5}$$

$$e_j^{IU} \ge \frac{D_j}{l_j^{IU}} b_t \sum_{i=1}^{N_v} 2d_{ij}^t x_{ij} + c_t D_j - L_2(1 - IU_j), j = 1, \ldots, M^{br} \tag{A.6}$$

$$e_j^{U} \ge \frac{D_j}{l_j^{U}} f_u + c_u D_j - L_2(1 - U_j), j = 1, \ldots, M^{br} \tag{A.7}$$

$$e_j \ge e_j^{IM} + e_j^{IU} + e_j^{U}, j = 1, \ldots, M^{br} \tag{A.8}$$

Constraints (A.9) compute the total amount branches withdraw from vault i.

$$C_i^+ \ge \sum_{j=1}^{M^{br}} x_{ij} D_j^+,\ i = 1, \ldots, N_v \tag{A.9}$$

Constraints (A.10)-(A.13) compute the new capacity and added capacity for each vault based on the new allocation of branches.

$$C_i^p \ge hC_i^+,\ i = 1, 2, \ldots, N_v \tag{A.10}$$

$$C_i^p \le Jy_i + c_\ell (1 - y_i),\ i \in A \tag{A.11}$$

$$C_i^p \le J, \quad i = M^B + 1, \ldots, M^B + M^{rv} \tag{A.12}$$

$$C_i^a \geq C_i^p - C_i^e,\ i = 1, \ldots, N_v \tag{A.13}$$

Constraints (A.14) and (A.15) are non-negativity and binary constraints, respectively.

$$e_j, e_j^{IM}, e_j^{IU}, e_j^{U}, C_i^p, C_i^a, C_i^+ \geq 0, i = 1, \ldots, N_v, j = 1, \ldots, M^{br} \tag{A.14}$$

$$y_i, x_{ij}, IM_j, IU_j, U_j \in \{0, 1\}, i = 1, \ldots, N_v, j = 1, \ldots, M^{br} \tag{A.15}$$

A.2 Analysis of Problem 1

Proof of Theorem 1: We use the 3-Satisfiability (3SAT) problem (Garey and Johnson 1979) for our reduction.

<u>3-SATISFIABILITY (3SAT)</u>

<u>INSTANCE</u>: A set $Z = \{z_1, z_2, \ldots, z_p\}$ of boolean variables and a collection $W = \{W_1 \cap W_2 \cap \ldots \cap W_k\}$ of clauses over Z, each of which is a disjunction of literals, $z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots, z_p, \bar{z}_p$ such that $|W_i| = 3$ for $1 \leq i \leq k$ and $\bar{z}_j = 1 - z_j$ for $1 \leq j \leq p$.

<u>SOLUTION</u>: Find an assignment of either a TRUE (1) or a FALSE (0) value to each variable in $\{z_1, z_2, \ldots, z_p\}$, such that the expression W evaluates to TRUE (1).

Given an arbitrary instance of 3SAT, we construct the following instance of Problem P.

- The parameter values related to estimating the transportation cost are as follows: $c_u = 0$, $c_t = 0$, $c_a = 1$, $f_a = 4$, $f_u = 0$, $b_a = 1$, $b_t = 1$, $l_j^U = p$, $l_j^{IU} = 2p$, and $l_j^{IM} = 4p$.
- The capacity limit for the retail vault is $c_\ell = pL - 1$ and $h = 1$. The unit cost of incremental capacity at the vaults is $c_o = 0$. The fixed cost of upgrading the retail vaults z_i, (or $\bar{z}_i$) to a regional vault, $F_i = 1$, $i = 1, 2, \ldots, 2p$.
- If the distance is less than 1 then the transportation mode is urban (U), otherwise it is inter-urban (IU) or inter-modal (IM).
- Each variable in the set $\{z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots, z_p, \bar{z}_p\}$ corresponds to a distinct retail vault, i.e., $N = 2p$. There is one big vault BV and one regional vault RGV. That is, $M^B = 1$, $M^{rv} = 1$, and $N_v = N + M^{br} + M^{rv} = 2p + 2$.
- A branch is attached to each retail vault in $\{z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots, z_p, \bar{z}_p\}$. See Table A.3 for details.

- Each clause in $\{W_1, W_2, \ldots W_k\}$ corresponds to a distinct branch. Currently branch W_j, where $j = 1, 2, \ldots, k$, is assigned to the regional vault RGV since there is not enough capacity at any of the retail vaults in $\{z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots, z_p, \bar{z}_p\}$. The distance between W_j and RGV is 1. The transportation mode between W_j and RGV is inter-urban and the transportation cost for a period is L. See Table A.3 for details.

- There are p dummy branches, $\{U_1, U_2, \ldots U_p\}$ and one big vault, BV. Currently branch U_j, where $j = 1, 2, \ldots, p$, is assigned to big vault BV since there is not enough capacity at any of the retail vaults in $\{z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots, z_p, \bar{z}_p\}$. The distance between U_j and BV is 1. The transportation mode between U_j and BV is inter-urban and the transportation cost for a period is L. See Table A.3 for details.

- There is one branch near BV that is served by BV. There is one branch near RGV that is served by RGV. See Table A.3 for details. There is no other financial intuition present in this problem instance.

- The number of branches is $M^{br} = k + 3p + 2$.

- The distance between the clause node, W_j, where $j = 1, 2, \ldots, k$ and a variable node in W_j is 0, and other distances are 1. The transportation cost per period (urban mode) from a clause node, W_j, $j = 1, 2, \ldots, k$ to a variable node in W_j is therefore 0; from W_j to other nodes it is L (the cost corresponds to the inter-urban mode).

- The distance between node U_j, where $j = 1, 2, \ldots, p$ and the node z_j, (or $\bar{z}_j$) is 0, and other distances are 1. The transportation cost per period (urban mode) from U_j, $j = 1, 2, \ldots, p$ to node z_j (or $\bar{z}_j$) is therefore 0; from U_j to other nodes it is L (the cost corresponds to the inter-urban mode).

Table A.3: Current Transportation Cost. If the Distance is Less Than 1, Then the Transportation Mode is U, and Otherwise is IU or IM.

Bank Branch j j	Current Supply Point i	Distance from the Current Supply Point	Load Size Load Size	Withdrawals per Period (D_j^+) Deposits per Period (D_j^-)	Transportation Cost (IU)
W_j	RV	$d_{ij}^t = 1$, $d_{ij}^a = 1$	$l_j^{IU} = 2p$	$D_j^+ = pL$, $D_j^- = 0$	$\frac{1}{l_j^{IU}} b_t.D_j.(2d_{ij})$ $= L$
U_j	BV	$d_{ij}^t = 1$, $d_{ij}^a = 1$	$l_j^{IU} = 2p$	$D_j^+ = pL$ $D_j^- = 0$	$\frac{1}{l_j^{IU}} b_t.D_j.(2d_{ij})$ $= L$
branches at z_j, $\bar{z}_j$	z_j, $\bar{z}_j$	$d_{ij}^t = 0$	-	$D_j^+ = pL - 1$, $D_j^- = 0$	0
Bank Branch near BV	BV	$d_{ij}^t = 0$	-	$D_j^+ = pL$, $D_j^- = 0$	0
Bank Branch near RV	RV	$d_{ij}^t = 0$	-	$D_j^+ = pL$, $D_j^- = 0$	0

- Before proceeding with the proof, we provide the construction of the instance of Problem P using the following example.
- **Example:** $p = 3$ and $k = 4$. $Z = \{z_1, z_2, z_3\}$; $W = \{W_1 \cap W_2 \cap W_3 \cap W_4\}$, where $W_1 = (z_1 \cup z_2 \cup \bar{z}_3)$, $W_2 = (\bar{z}_1 \cup \bar{z}_2 \cup \bar{z}_3)$, $W_3 = (\bar{z}_1 \cup z_2 \cup z_3)$, and $W_4 = (\bar{z}_1 \cup \bar{z}_2 \cup z_3)$. Each variable in the set $\{z_1, \bar{z}_1, z_2, \bar{z}_2, z_3, \bar{z}_3\}$ corresponds to a retail vault. The current total transportation cost is $(p + k)L$. The distance is 1 for all edges that are not shown in the network. Refer to Figure A.1.

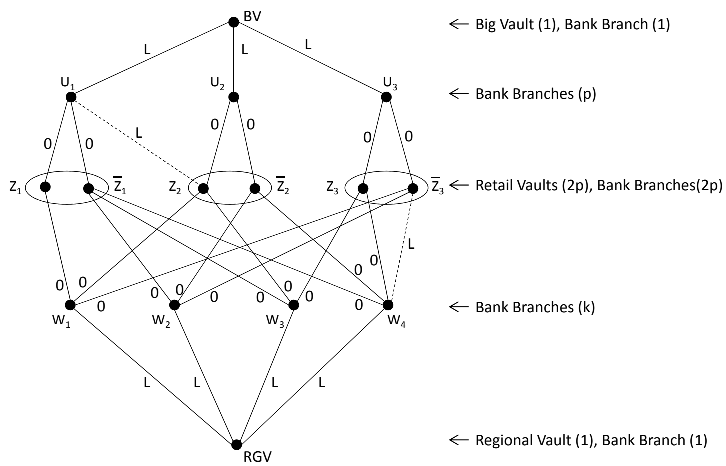
Let $L = 2p$. For the instance of Problem P constructed above, we consider the following question:

<u>Decision Problem</u>: Does there exist a feasible solution with total cost $\Phi \leq p$?

The decision problem is clearly in NP. It can also be easily verified that the construction of our decision problem from the 3SAT instance can be performed in polynomial time. We now show that the decision problem has an affirmative answer if and only if the 3SAT instance is satisfiable.

<u>*If part:*</u> Suppose the instance of 3SAT is satisfiable. Let z_i (or $\bar{z}_i$) represent the

Figure A.1: Problem Instance Corresponding to the Example Problem with $p = 3$, $k = 4$, $W_1 = \{z_1, z_2, \bar{z}_3\}$, $W_2 = \{\bar{z}_1, \bar{z}_2, \bar{z}_3\}$, $W_3 = \{\bar{z}_1, z_2, z_3\}$, $W_4 = \{\bar{z}_1, \bar{z}_2, z_3\}$, Solution $z_1 = 1$, $\bar{z}_2 = 1$, and $z_3 = 1$. The Distance is 1 for All Edges That Are Not Shown in the Network.



TRUE variables in a truth assignment. These vaults then correspond to the truth assignment, z_i (or $\bar{z}_i$), where $i = 1, 2, \ldots, p$, will be upgraded. A total of p retail vaults will therefore be upgraded with the total cost of p. branch W_j is assigned to a variable node (a retail vault) corresponding the truth assignment in W_j. The transportation cost for W_j accordingly corresponds with this truth assignment which is 0 (the urban mode). Similarly, branch U_j is assigned to a variable node (a retail vault) corresponding to the truth assignment of variable z_j (or $\bar{z}_j$). The transportation cost for U_j therefore corresponds this truth assignment which is 0 (the urban mode), and $\Phi = p$.

Only if part: Suppose there exists a solution to the decision problem with $\Phi \leq p$. The current total transportation cost, $(p + k)L$ must be eliminated since $L = 2p$. branches U_j, where $j = 1, 2, \ldots, p$ and W_j, $j = 1, 2, \ldots, k$ must therefore be assigned to one of the upgraded vaults in $\{z_1, \bar{z}_1, z_2, \bar{z}_2, \ldots, z_p, \bar{z}_p\}$. Since the currency demand for each branch U_j, where $j = 1, 2, \ldots, p$ must be done via the urban transportation, exactly one vault form $\{z_j, \bar{z}_j\}$ must be upgraded. The total upgrade cost of retail vaults is therefore p and $\Phi = p$. Since $\Phi = p$, branch W_j must be assigned to a variable node (a retail vault) with the transportation cost 0 (urban mode), and otherwise $\Phi > L > p$. A satisfiable assignment corresponding to $\Phi = p$ for the 3SAT instance is now immediate. This completes the proof. ■

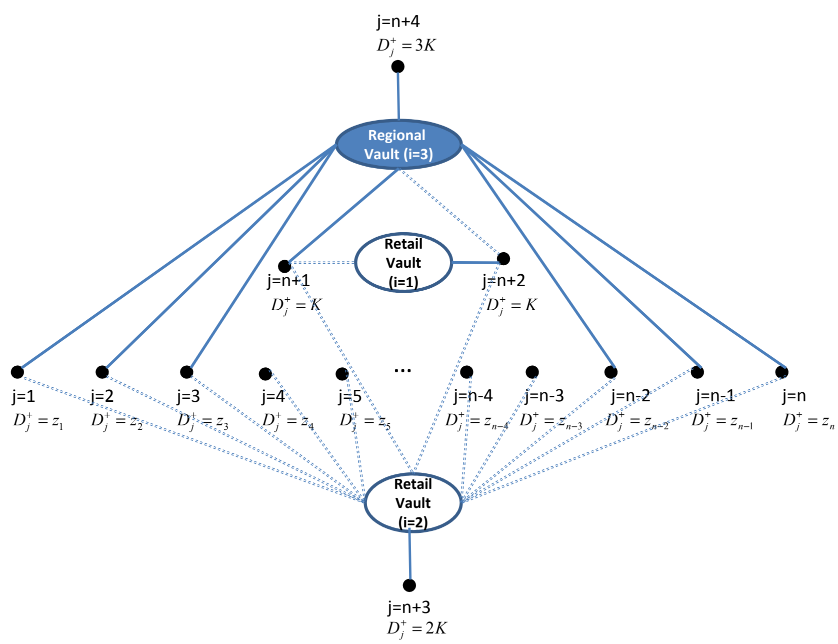
A.3 Proof of Theorem 2 for Problem P_1

Consider an arbitrary instance of PARTITION (Garey and Johnson 1979):

<u>PARTITION</u>:

Given an integer number $2K$, a set of n positive integers $Z = \{z_1, z_2, \ldots, z_{n-1}, z_n\}$ and $\sum_{z_j \in Z} z_j = 2K$, is there a partition of Z into two disjoint subsets Z_1 and Z_2 such that $Z = Z_1 \cup Z_2$ and $\sum_{z_j \in Z_1} z_j = \sum_{z_j \in Z_2} z_j = K$?

Figure A.2: Problem Instance Corresponding to the Example.



- Before proceeding with the proof, given an instance of PARTITION, we provide the construction of the instance of Problem P_1 using the following example. Refer to Figure A.2.

- There are one regional vault, two specified retail vaults, and $n + 4$ branches in the network. That is $M^{rv} = 1$, $r = 2$, $M^{br} = n + 4$, and $M^B = 0$. In the current assignment, branches $j = 1, 2, \ldots, n$, $n+1$, and $j = n+4$ are supplied

Table A.4: Transportation Cost g_{ij} between Vault i and Branch j.

$i \setminus j$	$j=1$	$j=2$	$\dots$	$j=n$	$j=n+1$	$j=n+2$	$j=n+3$	$j=n+4$
$i=1$	$z_1(2X+1)$	$z_2(2X+1)$	$\dots$	$z_n(2X+1)$	KX	KX	$2K5X$	$3K5X$
$i=2$	$z_1 2X$	$z_2 2X$	$\dots$	$z_n 2X$	$K4X$	$K4X$	$2KX$	$3K9X$
$i=3$	$z_1 6X$	$z_2 6X$	$\dots$	$z_n 6X$	$K4X$	$K4X$	$2K9X$	$3KX$

by the regional vault $i = 3$; branch $j = n + 2$ is supplied by retail vault $i = 1$; branch $j = n + 3$ is supplied by retail vault $i = 2$.

- $Z = \{z_1, z_2, \dots, z_{n-1}, z_n\}$. Each variable in the set $Z = \{z_1, z_2, \dots, z_{n-1}, z_n\}$ corresponds to a branch. The demand values of D_j^- and D_j^+, $j = 1, 2, \dots, n+4$, are given as follows.

  $D_j^- = 0, \forall j.$

  $D_j^+ = z_j, j = 1, 2, \dots, n;\ \ D_j^+ = K, j = n+1, n+2;\ \ D_{n+3}^+ = 2K;\ \ D_{n+4}^+ = 3K.$

- For two specified retail vaults $(i = 1, 2)$ and the existing regional vault $(i = 3)$, their existing capacities are $C_1^e = K$, $C_2^e = 2K$, and $C_3^e = 6K$, respectively.

- Values of the transportation cost parameter g_{ij} are set in Table A.4 with $X \geq 1$ by selecting d_{ij}^a and d_{ij}^t accordingly for given value of c_u, c_t, c_a, f_a, f_u, b_a, b_t, l_j^U, l_j^{IU}, and l_j^{IM}, where X is an integer number.

- The capacity limit for the retail vault is $c_\ell = 3K$ and $h = 1$. The unit cost of incremental capacity at the vaults is $c_o = 1$.

- For the current assignment, costs incurred to vault 1, 2, 3 are KX, $2KX$, and $19KX$, respectively. So, the total cost is $22KX$ currently.

For the instance of Problem P_1 constructed above, we consider the following question:

<u>DECISION PROBLEM</u>: Does there exist a assignment σ with two specified retail vaults being upgraded such that the total cost $\Phi_1^\sigma \leq 11KX + 4K$?

The decision problem is clearly in class NP. Also, it is easy to verify that the construction of the decision problem can be done in polynomial time. We now show that there exists a assignment σ with two specified retail vaults being upgraded such that $\Phi_1^\sigma \leq 11KX + 4K$ if and only if there exists a solution to the Partition problem.

<u>*If part:*</u> Suppose there exists a partition of Z into two disjoint subsets Z_1 and Z_2 such that $Z = Z_1 \cup Z_2$ and $\sum_{z_j \in Z_1} z_j = \sum_{z_j \in Z_2} z_j = K$. With $|B| = 2$, assignment σ specifies the assignment to one regional vault and two upgraded regional vaults.

Note that both retail vault 1 and 2 are upgraded, consider the following assignment σ: $x_{1j} = 1$, $j = z_i \in Z_1, n+1, n+2$; $\quad x_{2j} = 1$, $j = z_i \in Z_2, n+3$; $\quad x_{3j} = 1$, $j = n+4$.

In σ, all three vaults have new capacities of $3K$: $C_1^p = C_2^p = C_3^p = 3K$. Therefore, both retail vault 1 and 2 are upgraded to regional vaults with incremental capacity costs $2K$ and K. There is no incremental capacity cost for the original regional vault 3. So, the total incremental capacity cost is $3K$ in σ. Since in σ, branches $j = z_j \in Z_1, n+1, n+2$ are assigned to new regional vault 1, branches $j = z_j \in Z_2, n+3$ are assigned to new regional vault 2, and branches $j = n+4$ is assigned to the original regional vault 3, the total transportation cost is $11KX + K$ ($4KX + K$ for vault 1, $4KX$ for vault 2, and $3KX$ for vault 3). Thus, the assignment σ gives the total cost $\Phi_1^\sigma = (11KX + K) + 3K = 11KX + 4K$.

<u>*Only if part:*</u> Suppose there exists a assignment σ with two specified retail vaults being upgraded such that the total cost $\Phi_1^\sigma \leq 11KX + 4K$. We first show that if the total cost $\Phi_1^\sigma \leq 11KX + 4K$ both retail vaults must be upgraded.

Claim 1: The retail vault $i = 1$ is only upgraded, then $\Phi_1^\sigma > 11KX + 4K$.

Proof: The following assignment σ is optimal: $x_{1j} = 1$, $j = z_i \in Z, n+1, n+2$; $x_{2j} = 1$, $j = n+3$; $x_{3j} = 1$, $j = n+4$.

In σ, three vaults have new capacities: $C_1^p = 4K; C_2^p = 2K; C_3^p = 3K$. Thus, retail vault $i = 1$ is upgraded to new regional vault with incremental capacity costs $3K$. There is no incremental capacity cost for vault $i = 2$ and $i = 3$. Since in assignment σ_2, branches $j = z_i \in Z, n+1, n+2$ are assigned to new regional vault $i = 1$, branches $j = n+3$ is assigned to new regional vault $i = 2$, and branches $j = n+4$ is assigned to the original regional vault $i = 3$, the total transportation cost is $11KX + 2K$ ($6KX + 2K$ for vault $i = 1$, $2KX$ for vault $i = 2$, and $3KX$ for vault 3). Thus, with $X \geq 1$, the total cost is $\Phi_1^\sigma = 11KX + 2K + 3K > 11KX + 4K$.

Claim 2: The retail vault $i = 2$ is only upgraded, then $\Phi_1^\sigma > 11KX + 4K$.

Proof: The following assignment σ is optimal: $x_{1j} = 1$, $j = n+1$; $x_{2j} = 1$, $j = z_i \in Z, n+3$; $x_{3j} = 1$, $j = n+2, n+4$.

In σ, three vaults have new capacities: $C_1^p = K; C_2^p = 4K; C_3^p = 4K$. Thus, retail vault $i = 2$ is upgraded to new regional vault with incremental capacity costs $2K$. There is no incremental capacity cost for vaults $i = 1$ and $i = 3$. So, the total incremental capacity cost is $2K$ in σ. Since in σ, branches $j = n+1$ is assigned to new regional vault $i = 1$, branches $j = z_i \in Z, n+3$ are assigned to new regional vault $i = 2$, and branches $j = n+2, n+4$ are assigned to the original regional vault $i = 3$, the total transportation cost is $14KX$ (KX for vault $i = 1$, $6KX$ for vault $i = 2$, and $7KX$ for vault $i = 3$). Thus, with $X \geq 1$, the total cost is $\Phi_1^\sigma = 14KX + 2K > 11KX + 4K$.

Claim 3: Both retail vaults $i = 1$ and $i = 2$ must be upgraded.

Proof: Note that the total cost is $22KX$ without any upgrades of vaults. Since $\Phi_1^\sigma \leq 11KX + 4K$, the result follows from Claims 1 and 2.

Claim 4: All branches $j = z_i \in Z$ cannot be assigned to vault $i = 1$.

Proof: Suppose all branches $j = z_i \in Z$ are assigned to vault $i = 1$. To find the minimum cost solution all other branches must be assigned to the vaults having minimum g_{ij}. Thus we the following assignment σ: $x_{1j} = 1$, $j = z_i \in Z, n+1, n+2$; $x_{2j} = 1$, $j = n+3$; $\quad x_{3j} = 1$, $j = n+4$; with $\Phi_1^\sigma = 11KX + 5K > 11KX + 4K$. This contradicts with the fact that $\Phi_1^\sigma \leq 11KX + 4K$.

Claim 5: All branches $j = z_i \in Z$ cannot be assigned to vault $i = 2$.

Proof: Suppose all branches $j = z_i \in Z$ are assigned to vault $i = 2$. To find the minimum cost solution all other branches must be assigned to the vaults having minimum g_{ij}. Thus we the following assignment σ: $x_{1j} = 1$, $j = n+1$; $\quad x_{2j} = 1$, $j = z_i \in Z, n+3$; $\quad x_{3j} = 1$, $j = n+2, n+4$; with $\Phi_1^\sigma = 14KX + 2K > 11KX + 4K$. This contradicts with the fact that $\Phi_1^\sigma \leq 11KX + 4K$.

As a consequence of Claims 4 and 5, we assume that Z_1 (respectively, Z_2) is a subset of branches $z_j \in Z$ are assigned to vault $i = 1$ (respectively, $i = 2$). Since a branch can only be assigned to one vault, sets Z_1 and Z_2 are disjoint sets. In the minimum cost solution, a branch $z_j \in Z$ must be assigned to either vault $i = 1$ or $i = 2$, we have $Z = Z_1 \cup Z_2$.

Claim 6: $\sum_{z_j \in Z_1} z_j = \sum_{z_j \in Z_2} z_j = K$ and there exists a solution to PARTITION Problem.

Proof: In order to obtain minimum cost solution we have the following assignment σ: $x_{1j} = 1$, $j = z_j \in Z_1, n+1, n+2$; $\quad x_{2j} = 1$, $j = z_j \in Z_2$, $j = n+3$; $x_{3j} = 1$, $j = n+4$. Since both vaults must be upgraded and $c_\ell = 3K$, we must have $\sum_{z_j \in Z_1} z_j = \sum_{z_j \in Z_2} z_j = K$. Thus, $\Phi_1^\sigma = 11KX + 4K$ and there exists a solution to PARTITION Problem. ■

A.4 Formulation of MIP for Subproblem 2

Table A.5: Additional Notation Used to Formulate Subproblem 2 in the MIP Model: Parameters.

M^N	The number of net negative regional vaults in the region.
M^P	The number of net positive regional vaults in the region.
M^V	Total number of big vaults and regional vaults.
Q_a	Capacity of one airplane, by value of currency.
Q_t	Capacity of one truck, by value of currency.
L_3^k	The largest possible transportation cost to satisfy regional vault k's demand: $L_3^k = S_k^+ \max\left\{2\frac{b_t}{Q_t}\max_{i,j}\{d_{ik}^t, d_{jk}^t\} + c_t, \frac{f_a}{Q_a} + 2\frac{b_a}{Q_a}\max_{i,j}\{d_{ik}^a, d_{jk}^a\} + c_a\right\}$

Subproblem 2 MIP:

The objective function minimizes the total per period transportation cost for vaults, which includes sending excess cash back from net negative regional vault j to its closest big vault, sending cash to net negative regional vaults j to eventually satisfy net positive regional vaults' demands, and satisfying the demand for net positive regional vault k from either a big or a net negative regional vault.

$$\textbf{Minimize} \quad \sum_j(\alpha_j^{IU} + \alpha_j^{IM} + \gamma_j^{IU} + \gamma_j^{IM}) + \sum_k(\beta_k^{IU} + \beta_k^{IM})$$

Subject to:

Constraints (A.16)-(A.17) are balance equations that equate the inflow and outflow for net positive regional vaults and net negative regional vaults.

$$\sum_i x_{ik} + \sum_j y_{jk} = S_k^+, \ \forall k \tag{A.16}$$

$$S_j^- + \sum_i z_{ij} + \sum_{l \neq j} v_{lj} = \sum_k y_{jk} + \sum_{m \neq j} v_{jm} + \sum_i g_{ji}, \ \forall j \tag{A.17}$$

Constraints (A.18)-(A.22) are flow upper bound constraints that set values for binary variables used to enforce sole sourcing. Specifically, (A.18)-(A.19) are for inflow at net positive regional vaults while (A.20)-(A.22) are for outflow and inflow at net

Table A.6: Additional Notation Used to Formulate Subproblem 2 in the MIP Model: Variables.

	Indices
i	$1, 2, \ldots, M^B$ as big vaults.
j, l, m	$M^B + 1, \ldots, M^B + M^N$ as net negative regional vaults.
k	$M^B + M^N + 1, \ldots, M^V$ as net positive regional vaults.
p, q	$1, \ldots, M^V$ as any vault.
	Destination: Big Vaults
α_j^{IU}	The interurban (IU) transportation expense for shipping excess cash from net negative regional vault j back to a big vault.
α_j^{IM}	The intermodal (IM) transportation expense for shipping excess cash from net negative regional vault j back to a big vault.
g_{ji}	The amount of currency sent back from net negative regional vault j to big vault i.
B_{ji}^N	If $B_{ji}^N = 1$, then currency is sent back from net negative regional vault j to big vault i. Otherwise, $B_{ji}^N = 0$.
B_j^{IU}	If $B_j^{IU} = 1$, then net negative regional vault j uses interurban (IU) mode to send excess cash back to big vaults.
B_j^{IM}	If $B_j^{IM} = 1$, then net negative regional vault j uses intermodal (IM) mode to send excess cash back to big vaults.
	Destination: Net Positive Regional Vaults
β_k^{IU}	The interurban (IU) transportation cost for supplying net positive regional vault k from either a big or a net negative regional vault.
β_k^{IM}	The intermodal (IM) transportation cost for supplying net positive regional vault k from either a big or a net negative regional vault.
x_{ik}	The amount of currency sent from big vault i to net positive regional vault k.
y_{jk}	The amount of currency sent from net negative regional vault j to net positive regional vault k.
P_{ik}^B	If $P_{ik}^B = 1$, then big vault i is the supplier for net positive regional vault k. Otherwise, $P_{ik}^B = 0$.
P_{jk}^N	If $P_{jk}^N = 1$, then net negative regional vault j is the supplier for net positive regional vault k. Otherwise, $P_{jk}^N = 0$.
P_k^{IU}	If $P_k^{IU} = 1$, then net positive regional vault k receives cash by interurban (IU) mode.
P_k^{IM}	If $P_k^{IM} = 1$, then net positive regional vault k receives cash by intermodal (IM) mode.
	Destination: Net Negative Regional Vaults
γ_j^{IU}	The interurban (IU) transportation cost for supplying net negative regional vault j from either a big or a net negative regional vault.
γ_j^{IM}	The intermodal (IM) transportation cost for supplying net negative regional vault j from either a big or a net negative regional vault.
z_{ij}	The amount of currency sent from big vault i to net negative regional vault j.
v_{lj}	The amount of currency sent from regional vault l to regional vault j, where both of indexes are net negative regional vaults.
N_{ij}^B	If $N_{ij}^B = 1$, then big vault i is the supplier for net negative regional vault j. Otherwise, $N_{ij}^B = 0$.
N_{lj}^N	If $N_{lj}^N = 1$, then net negative regional vault l is the supplier for net negative regional vault j. Otherwise, $N_{lj}^N = 0$.
N_j^{IU}	If $N_j^{IU} = 1$, then net negative regional vault j obtains cash by interurban (IU) mode.
N_j^{IM}	If $N_j^{IM} = 1$, then net negative regional vault j obtains cash by intermodal (IM) mode.

negative regional vaults.

$$x_{ik} \leq S_k^+ P_{ik}^B, \quad \forall i, \forall k \tag{A.18}$$

$$y_{jk} \leq S_k^+ P_{jk}^N, \quad \forall j, \forall k \tag{A.19}$$

$$g_{ji} \leq J B_{ji}^N, \quad \forall i \forall j \tag{A.20}$$

$$z_{ij} \leq J N_{ij}^B, \quad \forall i \forall j \tag{A.21}$$

$$v_{lj} \leq JN_{lj}^{N}, \quad \forall l \forall j \tag{A.22}$$

Constraints (A.23)-(A.24) enforce that each regional vault can have no more than one supplier (one big vault or one net negative regional vault).

$$\sum_{i} P_{ik}^{B} + \sum_{j} P_{jk}^{N} = 1, \ \forall k \tag{A.23}$$

$$\sum_{i} N_{ij}^{B} + \sum_{l} N_{lj}^{N} \leq 1, \ \forall j \tag{A.24}$$

Constraints (A.25)-(A.27) select the transportation mode (IM or IU) for sending excess cash back from net negative regional vault j to big vault i. It is clear that each net negative regional vault will send excess cash back to no more than one big vault. The transportation expense for this flow is also captured.

$$B_{j}^{IU} + B_{j}^{IM} = \sum_{i} B_{ji}^{N}, \quad \forall j \tag{A.25}$$

$$\alpha_{j}^{IU} \geq \frac{b_t}{Q_t} \sum_{i} 2d_{ji}^{t} \ g_{ji} + c_t \sum_{i} g_{ji} - L_3(1 - B_{j}^{IU}), \quad \forall j \tag{A.26}$$

$$\begin{aligned} \alpha_{j}^{IM} \geq & \frac{f_a}{Q_a} \sum_{i} g_{ji} + \frac{b_a}{Q_a} \sum_{i} 2d_{ji}^{a} \ g_{ji} \\ & + c_a \sum_{i} g_{ji} - L_3(1 - B_{j}^{IM}), \quad \forall j \end{aligned} \tag{A.27}$$

Constraints (A.28)-(A.30) select the transportation mode for sending cash to net positive regional vault k from either big vault i or net negative regional vault j. Round trip transportation cost is also calculated in this set of constraints.

$$P_{k}^{IU} + P_{k}^{IM} = 1, \quad \forall k \tag{A.28}$$

$$\begin{aligned} \beta_{k}^{IU} \geq & \frac{b_t}{Q_t} S_{k}^{+} (\sum_{i} 2d_{ik}^{t} P_{ik}^{B} + \sum_{j} 2d_{jk}^{t} P_{jk}^{N}) \\ & + c_t S_{k}^{+} - L_3^{k}(1 - P_{k}^{IU}), \forall k \end{aligned} \tag{A.29}$$

$$\begin{aligned} \beta_{k}^{IM} \geq \frac{f_a}{Q_a} S_{k}^{+} & + \frac{b_a}{Q_a} S_{k}^{+} (\sum_{i} 2d_{ik}^{a} P_{ik}^{B} + \sum_{j} 2d_{jk}^{a} P_{jk}^{N}) \\ & + c_a S_{k}^{+} - L_3^{k}(1 - P_{k}^{IM}), \forall k \end{aligned} \tag{A.30}$$

Constraints (A.31)-(A.33) compute the transportation cost of sending cash from either big vault i or net negative regional vault l to net negative regional vault j, in order to eventually satisfy the demand from net positive regional vaults.

$$N_j^{IU} + N_j^{IM} = \sum_i N_{ij}^B + \sum_l N_{lj}^N, \quad \forall j \tag{A.31}$$

$$\begin{aligned} \gamma_j^{IU} \geq {} & \frac{b_t}{Q_t}(\sum_i 2d_{ij}^t z_{ij} + \sum_l 2d_{lj}^t v_{lj}) \\ & + c_t(\sum_i z_{ij} + \sum_l v_{lj}) - L_3(1 - N_j^{IU}), \quad \forall j \end{aligned} \tag{A.32}$$

$$\begin{aligned} \gamma_j^{IM} \geq {} & \frac{f_a}{Q_a}(\sum_i z_{ij} + \sum_l v_{lj}) + \frac{b_a}{Q_a}(\sum_i 2d_{ij}^a z_{ij} + \sum_l 2d_{lj}^a v_{lj}) \\ & + c_a(\sum_i z_{ij} + \sum_l v_{lj}) - L_3(1 - N_j^{IM}), \ \forall j \end{aligned} \tag{A.33}$$

Constraints (A.34)-(A.36) are non-negativity and binary constraints.

$$\textit{All variables nonnegative}, \qquad \forall i, \forall j, \forall k, \forall l, \forall m \tag{A.34}$$

$$B_{ji}^N, P_{ik}^B, P_{jk}^N, N_{ij}^B, N_{lj}^N \in \{0,1\}, \quad \forall i, \forall j, \forall k, \forall l \tag{A.35}$$

$$B_j^{IU}, B_j^{IM}, P_k^{IU}, P_k^{IM}, N_j^{IU}, N_j^{IM} \in \{0,1\}, \quad \forall i, \forall j, \forall k, \forall l \tag{A.36}$$

A.5 Formulation of MCF for Subproblem 2

$$\textbf{Minimize} \quad cB + cP + cN$$

S.t.

$$\sum_i x_{ik} + \sum_j y_{jk} = S_k^+, \ \forall k \tag{A.37}$$

$$S_j^- + \sum_i z_{ij} + \sum_{l \neq j} v_{lj} = \sum_k y_{jk} + \sum_{m \neq j} v_{jm} + \sum_i g_{ji}, \ \forall j \tag{A.38}$$

$$x_{ik} \leq S_k^+, \quad \forall i, \forall k \tag{A.39}$$

$$y_{jk} \leq S_k^+, \quad \forall j, \forall k \tag{A.40}$$

$$z_{ij} \leq S_i, \quad \forall i \forall j \tag{A.41}$$

$$g_{ji} \leq J, \quad \forall i \forall j \tag{A.42}$$

$$v_{lj} \leq J, \quad \forall l \forall j \tag{A.43}$$

$$cB = \sum_j \sum_i c_{ji} g_{ji}, \ \forall i \forall j \tag{A.44}$$

$$cP = \sum_i \sum_k c_{ik} x_{ik} + \sum_j \sum_k c_{jk} y_{jk}, \ \forall i \forall j \forall k \tag{A.45}$$

$$cN = \sum_i \sum_j c_{ij} z_{ij} + \sum_l \sum_{l \neq j} c_{lj} v_{lj}, \ \forall i \forall j \forall l \forall m \tag{A.46}$$

$$All \quad variables \quad nonnegative, \ \forall i, \forall j, \forall k, \forall l, \forall m \tag{A.47}$$

The objective function minimizes the total transportation cost for vaults which includes the part for sending excess cash back from net negative regional vault j to big vault i, the part for satisfying the demand for net positive regional vault k and the part for sending cash to net negative regional vaults j to eventually satisfy net positive regional vaults' demand. Constraints (A.37)-(A.38) are flow balance equations for each regional vault. Constraints (A.39)-(A.43) are flow upper bound constraints. Constraints (A.44)-(A.46) compute the transportation cost for each part in the objective function. Constraints (A.47) are non-negativity constraints for all

Table A.7: Notations Used to Formulate Subproblem 2 in the MCF Model.

	Parameters
i	$1, 2, \ldots, M^B$ as big vaults.
j, l, m	$M^B+1, \ldots, M^B+M^N$ as net negative regional vaults.
k	$M^B+M^N+1, \ldots, M^V$ as net positive regional vaults.
p, q	$1, \ldots, M^V$ as any vault.
M^B	The number of big vaults in the region.
M^N	The number of net negative regional vaults.
M^P	The number of net positive regional vaults.
M^V	Total number of big vaults and regional vaults. $M^V = M^B + M^N + M^P$. Set $i = 1, 2, \ldots, M^B$ as big vault; $j, l = M^B + 1, \ldots, M^B + M^N$ as net negative regional vaults. $k = M^B + M^N + 1, \ldots, M^V$ as net positive regional vaults.
c_{pq}	The unit cost of flow sent from vault p to q.
J	The capacity limit for regional vaults. The value of J is given by CB.
	Variables
x_{ik}	The amount of flow send from big vault i to net positive regional vault k.
y_{jk}	The amount of flow send from net negative regional vault j to net positive regional vault k.
z_{ij}	The amount of flow send from big vault i to net negative regional vault j.
g_{ji}	The amount of flow send back from net negative regional vault j to big vault i.
v_{lj}	The amount of flow send from net negative regional vault l to net negative regional vault j.
cB	The transportation cost for shipping excess cash from net negative regional vault j back to big vaults.
cP	The transportation cost for satisfying the demand from net positive regional vault k.
cN	The transportation cost for satisfying the demand from net negative regional vault j.

the variables.

The minimum cost flow problem can be solved in polynomial time using existing polynomial time linear programming algorithms. The resulting algorithms will be polynomial, but not strongly polynomial, i.e., their complexities will based on the number of bits needed to represent the integral flow in the currency network and the number of nodes in the currency network.

A.6 Sourcing for Regional Vaults

Table A.8: Sourcing for Regional Vaults (#11-139) with the Maximum Demand in the MCF Model [The First Five Rows Represent Net Negative Regional Vaults; Net Positive Regional Vaults Are Represented in the Following Rows].

Supplier−Recipient	Supplier−Recipient	Supplier−Recipient	Supplier−Recipient
37−11	13,52−12	14−15	47−16
19−20	12−24	28−26	23,48−29
18−31	11−32	21,49−34	27−36
16−45	44−47	24,29−48	34−49
42−51	15,40−55		
52−56	17,55−57	19−58	45−59
21−60	11−61	10−62	26−63
11−64	13−65	23,50−66	7,25−67
6−68	37,39−69	2−70	39,45−71
1−72	8−73	5−74	2−75
6−76	48,53−77	2−78	6−79
11,17,20,26−80	26−81	6−82	10−83
6−84	13−85	11−86	25−87
10−88	6−89	6−90	10−91
9−92	16−93	2,41−94	24−95
49−96	11−97	6−98	13,30−99
2−100	11−101	31,34−102	2,27−103
14,38−104	14−105	3−106	29−107
11−108	2−109	8−110	3−111
3,33−112	36−113	34−114	9−115
2−116	7,54−117	1−118	2−119
7−120	20−121	42,51−122	2−123
3−124	36−125	6−126	11−127
10,22,32−128	7−129	1−130	3,43−131
1−132	10−133	2−134	7−135
17,46−136	10−137	36−138	6,35−139

Table A.9: Sourcing for Regional Vaults (#11-139) with the Midpoint Demand in the MCF Model [The First Five Rows Represent Net Negative Regional Vaults; Net Positive Regional Vaults Are Represented in the Following Rows].

Supplier−Recipient	Supplier−Recipient	Supplier−Recipient	Supplier−Recipient
37−11	13,52−12	14−15	47−16
19−20	12,26,39−24	28−26	23,48−29
18,55−31	11−32	21,49−34	27−36
16,17−45	44−47	24−48	34−49
42−51	15,40−55		
52−56	17−57	19−58	45−59
21−60	11−61	22−62	26−63
11−64	13−65	23,50−66	25−67
6−68	39−69	2−70	39,45−71
1−72	8−73	5−74	2,27−75
6−76	6,48,53−77	2−78	6−79
20−80	26−81	6−82	10−83
6−84	13−85	11−86	25−87
10,22−88	6−89	6−90	10−91
9−92	16−93	2,41−94	24−95
49−96	11−97	6−98	13,30−99
2−100	11−101	31,34−102	27−103
14,38−104	14−105	3−106	29−107
11,20−108	2−109	8−110	3−111
3,33−112	36−113	34−114	9−115
2−116	7,25,54−117	1−118	2−119
7−120	20−121	42,51−122	2−123
3−124	36−125	6−126	11−127
22,32−128	7−129	1−130	3,43−131
1−132	10−133	2−134	7−135
17,46−136	10−137	36−138	6,35−139

Table A.10: Sourcing for Regional Vaults (#11-139) with the Average Demand in the MCF Model [The First Two Rows Represent Net Negative Regional Vaults; Net Positive Regional Vaults Are Represented in the Following Rows].

Supplier−Recipient	**Supplier−Recipient**	**Supplier−Recipient**	**Supplier−Recipient**
37−11	13,52−12	14−15	47−16
19−20	20−21	12,26,39−24	28−26
23,48−29	18,55−31	11−32	21,49−34
27−36	16,17−45	44−47	24,29−48
42−51	15,40−55		
52−56	17−57	19−58	45−59
21−60	11−61	22−62	26−63
11−64	13−65	23,50−66	25−67
6−68	39−69	27−70	39,45−71
1−72	8−73	5−74	27−75
6−76	29,48,53−77	2−78	6−79
20−80	26−81	6−82	10−83
6−84	13−85	11−86	25,31−87
22−88	6−89	6−90	10−91
9−92	16−93	2,41−94	24−95
32,49−96	11−97	6−98	30−99
2−100	11−101	31−102	2,27−103
14,38−104	14,30−105	3−106	29−107
20−108	2−109	8−110	3−111
3,33−112	36−113	34−114	9−115
2−116	7,25,54−117	1−118	2−119
7−120	20−121	42,51−122	2−123
3−124	36−125	6−126	11−127
22,32−128	7−129	1−130	3,43−131
1−132	10−133	2−134	7−135
17,46−136	10,22−137	36−138	6,35−139

APPENDIX B

SUPPLEMENT TO CHAPTER 3

B.1 Periodic Review Model

Periodic Review Model Without Seasonality for CV j is (E_j, R).

E_j = Order-Up-To limit of coin inventory at j.

$E_j = 5w_j(L+R) + z\sqrt{5}\sigma_j\sqrt{L+R}$.

z = service level factor.

$5w_j(R+T)$ = Mean demand during $(R+L)$.

$\sqrt{5}\sigma_j\sqrt{L+R}$ = Standard deviation of the demand during time $(R+L)$ for random variable $\tilde{W}_j$.

w_j = Mean daily withdrawals at CV j (*payable day* at CV j).

$5w_j$ = Mean weekly withdrawals at CV j assuming that a week consists of 5 working days.

σ_j = Daily standard deviation of the demand at CV j.

$\sqrt{5}\sigma_j$ = Weekly standard deviation.

$S_j = z\sqrt{5}\sigma_j\sqrt{L+R}$ = Safety stock at j.

Determination of Z_j^t

At time t, the realization of the random variables, $\tilde{D}_j$, $\tilde{W}_j$ are $\hat{D}_j^t$, $\hat{W}_j^t$, respectively. Let $\hat{I}_j^t$ be the realized coin inventory level at the end of period t at CV j. Note that $\hat{I}_j^t$ can be higher than the Order-Up-To limit because of deposits from DIs. According to Base Stock Model, we define $Z_j^t = \hat{I}_j^t - E_j$. If $Z_j^t > 0$, it means surplus of coins $(\hat{I}_j^t - E_j)$ can be transshipped. If $Z_j^t < 0$, it means deficit of coins $(E_j - \hat{I}_j^t)$ must be ordered.

B.2 Computational Results for Problem I and Problem I_N

Table B.1: Cost Components for Problem I When $O_j = 0$ and $U_j = 1, 2, 3$ Weeks of Payable Days With 2010 Demand Data.

Unit Holding Cost h ($)	**Prod. Cost ($M)**	**Trans. Cost ($M)**	**Holding Cost ($M)**	**Total Cost ($M)**
150	97.99	33.96	13.85	145.79
175	97.99	34.53	15.57	148.09
200	97.99	34.53	17.8	150.32
225	98.15	36.13	18.18	152.46
250	98.15	36.13	20.21	154.48
500	102.93	41.32	25.58	169.83
750	116.49	52.21	7.18	175.88
1000	119.91	57.86	0	177.78

Table B.2: Cost Components for Problem I_N When $O_j = 0$ and $U_j = 1$ Weeks of Payable Days With 2010 Demand Data.

Unit Holding Cost h ($)	**Prod. Cost ($M)**	**Trans. Cost ($M)**	**Holding Cost ($M)**	**Total Cost ($M)**
150	97.99	191.57	16.35	305.9
175	97.99	191.57	19.07	308.63
200	97.99	191.57	21.79	311.35
225	97.99	191.57	24.52	314.08
250	97.99	191.57	27.24	316.8
500	97.99	191.57	54.49	344.04
750	97.99	191.57	81.73	371.29
1000	97.99	191.57	108.97	398.53

Table B.3: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 2$ Weeks of Payable Days With 2010 Demand Data.

Unit Holding Cost h ($)	**Prod. Cost ($M)**	**Trans. Cost ($M)**	**Holding Cost ($M)**	**Total Cost ($M)**
150	97.99	210.1	14.28	322.37
175	97.99	210.1	16.66	324.75
200	97.99	210.1	19.04	327.13
225	97.99	210.1	21.42	329.51
250	97.99	210.1	23.8	331.89
500	97.99	210.1	47.61	355.7
750	97.99	210.1	71.41	379.5
1000	97.99	210.1	95.21	403.3

Table B.4: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 3$ Weeks of Payable Days With 2010 Demand Data.

Unit Holding Cost h (\$)	**Prod. Cost (\$M)**	**Trans. Cost (\$M)**	**Holding Cost (\$M)**	**Total Cost (\$M)**
150	97.99	215.87	14.28	328.14
175	97.99	215.87	16.66	330.52
200	97.99	215.87	19.04	332.9
225	97.99	215.87	21.42	335.28
250	97.99	215.87	23.8	337.66
500	97.99	215.87	47.61	361.47
750	97.99	215.87	71.41	385.27
1000	97.99	215.87	95.21	409.07

Table B.5: Cost Components for Problem I When $O_j = 0$ and $U_j = 1, 2, 3$ Weeks of Payable Days With 2011 Demand Data.

Unit Holding Cost h (\$)	**Prod. Cost (\$M)**	**Trans. Cost (\$M)**	**Holding Cost (\$M)**	**Total Cost (\$M)**
150	99.19	34.37	6.78	140.34
175	99.19	34.62	7.65	141.46
200	99.19	34.62	8.75	142.56
225	99.19	35.3	9.13	143.62
250	99.19	35.3	10.14	144.63
500	99.68	36.13	18.54	154.35
750	109.81	43.5	5.37	158.68
1000	112.38	47.72	0	160.1

Table B.6: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 1$ Weeks of Payable Days With 2011 Demand Data.

Unit Holding Cost h (\$)	**Prod. Cost (\$M)**	**Trans. Cost (\$M)**	**Holding Cost (\$M)**	**Total Cost (\$M)**
150	99.19	182.01	6.6	287.8
175	99.19	182.01	7.7	288.9
200	99.19	182.01	8.8	290
225	99.19	182.01	9.9	291.1
250	99.19	182.01	11.01	292.21
500	99.19	182.01	22.01	303.21
750	99.19	182.01	33.02	314.22
1000	99.19	182.01	44.02	325.22

Table B.7: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 2$ Weeks of Payable Days With 2011 Demand Data.

Unit Holding Cost h **($)**	**Prod. Cost ($M)**	**Trans. Cost ($M)**	**Holding Cost ($M)**	**Total Cost ($M)**
150	99.19	185.67	5.53	290.39
175	99.19	185.67	6.45	291.31
200	99.19	185.67	7.37	292.23
225	99.19	185.67	8.29	293.15
250	99.19	185.67	9.21	294.07
500	99.19	185.67	18.42	303.28
750	99.19	185.67	27.63	312.49
1000	99.19	185.67	36.84	321.7

Table B.8: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 3$ Weeks of Payable Days With 2011 Demand Data.

Unit Holding Cost h **($)**	**Prod. Cost ($M)**	**Trans. Cost ($M)**	**Holding Cost ($M)**	**Total Cost ($M)**
150	99.19	188.62	5.53	293.34
175	99.19	188.62	6.45	294.26
200	99.19	188.62	7.37	295.18
225	99.19	188.62	8.29	296.1
250	99.19	188.62	9.21	297.02
500	99.19	188.62	18.42	306.23
750	99.19	188.62	27.63	315.44
1000	99.19	188.62	36.84	324.65

Table B.9: Cost Components for Problem I When $O_j = 0$ and $U_j = 1, 2, 3$ Weeks of Payable Days With 2012 Demand Data.

Unit Holding Cost h **($)**	**Prod. Cost ($M)**	**Trans. Cost ($M)**	**Holding Cost ($M)**	**Total Cost ($M)**
150	79.66	27.6	13.64	120.9
175	79.66	27.6	15.91	123.17
200	79.66	27.6	18.18	125.44
225	80.11	27.71	19.89	127.71
250	80.11	27.71	22.1	129.92
500	91.97	37.33	11.51	140.81
750	98.55	41.39	3.49	143.43
1000	100.21	44.14	0	144.35

Table B.10: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 1$ Weeks of Payable Days With 2012 Demand Data.

Unit Holding Cost h (\$)	**Prod. Cost (\$M)**	**Trans. Cost (\$M)**	**Holding Cost (\$M)**	**Total Cost (\$M)**
150	79.66	126.3	13.64	219.6
175	79.66	126.3	15.91	221.87
200	79.66	126.3	18.18	224.14
225	79.66	126.3	20.46	226.42
250	79.66	126.3	22.73	228.69
500	79.66	126.3	45.46	251.42
750	79.66	126.3	68.19	274.15
1000	79.66	126.3	90.92	296.88

Table B.11: Cost Components forProblem I_N When $O_j = 0$ and $U_j = 2$ Weeks of Payable Days With 2012 Demand Data.

Unit Holding Cost h (\$)	**Prod. Cost (\$M)**	**Trans. Cost (\$M)**	**Holding Cost (\$M)**	**Total Cost (\$M)**
150	79.66	146.92	13.64	240.22
175	79.66	146.92	15.91	242.49
200	79.66	146.92	18.18	244.76
225	79.66	146.92	20.46	247.04
250	79.66	146.92	22.73	249.31
500	79.66	146.92	45.46	272.04
750	79.66	146.92	68.19	294.77
1000	79.66	146.92	90.92	317.5

Table B.12: Cost Components for Problem I_N When $O_j = 0$ and $U_j = 3$ Weeks of Payable Days With 2012 Demand Data.

Unit Holding Cost h (\$)	**Prod. Cost (\$M)**	**Trans. Cost (\$M)**	**Holding Cost (\$M)**	**Total Cost (\$M)**
150	79.66	146.21	13.64	239.51
175	79.66	146.21	15.91	241.78
200	79.66	146.21	18.18	244.05
225	79.66	146.21	20.46	246.33
250	79.66	146.21	22.73	248.6
500	79.66	146.21	45.46	271.33
750	79.66	146.21	68.19	294.06
1000	79.66	146.21	90.92	316.79

B.3 Proof of Theorem 1

Consider an arbitrary instance of Equal Cardinality Partition:

<u>Equal Cardinality Partition</u>: Given an integer number $2K$, a set of $2n$ positive integers $Z = \{z_1, z_2, \ldots, z_{2n-1}, z_{2n}\}$ and $\sum_{z_i \in Z} z_i = 2K$, does there exist a partition of Z into two disjoint subsets Z_1 and Z_2 such that $Z = Z_1 \cup Z_2$, $\sum_{z_i \in Z_1} z_i = \sum_{z_j \in Z_2} z_j = K$, and $|Z_1| = |Z_2| = n$?

The optimization *Problem II* with " *volume discount*" can be restated as the following decision problem.

<u>Decision Problem</u>: Given deposits and withdrawals, D^t and W^t, $t = 1, 2, \ldots, T$, does there exist a decision $\sigma = (X_p^t, X_u^t, Y_u^t | t = 1, 2, \ldots, T)$ for the DI such that the total cost $\Phi^\sigma \leq \frac{3(n+1)K}{2} + (n+1)K(g + \bar{g} + c + f)$?

<u>Construction:</u> We construct an instance of *Problem II* with " *volume discount*" from Equal Cardinality Partition with $T = 2n + 2$. We choose the values such that $(c + f) > g > \bar{g}$. Without loss of generality, we may assume that K is even number.

Table B.13: Withdrawal and Deposit at the DI.

Demand \ Period	$t = 1$	$t = 2$	$t = 3$	$\ldots$
Withdrawal	$W^1 = 0$	$W^2 = K + z_1$	$W^3 = K + z_2$	$\ldots$
Deposit	$D^1 = K + z_1$	$D^2 = K + z_2$	$D^3 = K + z_3$	$\ldots$
Demand \ Period	$\ldots$	$t = 2n$	$t = 2n + 1$	$t = 2n + 2$
Withdrawal	$\ldots$	$W^{2n} = K + z_{2n-1}$	$W^{2n+1} = K + z_{2n}$	$W^{2n+2} = (n + 1)K$
Deposit	$\ldots$	$D^{2n} = K + z_{2n}$	$D^{2n+1} = 0$	$D^{2n+2} = 0$

- Withdrawal and Deposit values (D^t, W^t, $t = 1, 2, \ldots, 2n + 2$), are given in Table B.13.

- Discount related parameters are set as follows: $\tau = (n+1)K$, $g = 4$, and $\bar{g} = 1$.

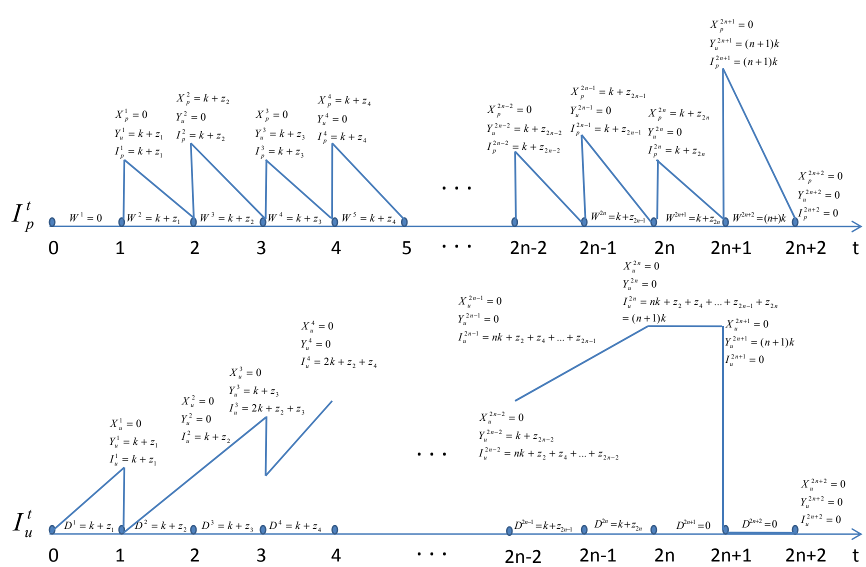
- Values of the other parameters are set as follows: $h_p = 1$, $h_u = 0$, $c = 4$, and $f = 1$.

Table B.14: The Decision σ in the If Part.

Flow \ Period	$t=1$	$t=2$	$t=3$	$t=4$	...	...
X_p^t	0	$K+z_2$	0	$K+z_4$	...	...
Y_u^t	$K+z_1$	0	$K+z_3$	0	...	...
X_u^t	0	0	0	0	...	...
Flow \ Period	$t=2n-3$	$t=2n-2$	$t=2n-1$	$t=2n$	$t=2n+1$	$t=2n+2$
X_p^t	0	0	$K+z_{2n-1}$	$K+z_{2n}$	0	0
Y_u^t	$K+z_{2n-3}$	$K+z_{2n-2}$	0	0	$(n+1)K$	0
X_u^t	0	0	0	0	0	0

It is easy to verify that the construction of the decision problem can be done in polynomial time. The decision problem is clearly in class NP. We now prove that there exists a decision $\sigma = (X_p^t, X_u^t, Y_u^t | t = 1, 2, \ldots, T)$ for the DI such that the total cost $\Phi^\sigma \leq \frac{3(n+1)K}{2} + (n+1)K(g + \bar{g} + c + f)$ if and only if there exists a solution to the Equal Cardinality Partition problem.

Figure B.1: Construction of the Instance for *Problem II*.



If part: Suppose there exists a Equal Cardinality Partition of Z into two disjoint subsets Z_1 and Z_2 such that $Z = Z_1 \cup Z_2$, $\sum_{z_j \in Z_1} z_j = \sum_{z_j \in Z_2} z_j = K$, and $|Z_1| = |Z_2| = n$, where $Z_1 = \{z_1, z_3, \ldots, z_{2n-2}\}$ and $Z_2 = \{z_2, z_3, \ldots, z_{2n-1}, z_{2n}\}$. The decision σ is fully specified in Table B.14. Note that we assign values of variables, Y_u^t (respectively, X_p^t) correspond to Z_1 (respectively, Z_2) as shown in Table B.14.

The inventory status of coins corresponding to σ is shown in Figure B.1. The cost components associated with σ can be estimated as follows: the inventory cost in σ is $\frac{3(n+1)K}{2}$; the coin packaging cost paid to 3PLPs is $(n+1)K(g+\bar{g})$; and the transportation cost plus the fee paid for obtaining the packaged coins from the FRS is $(n+1)K(c+f)$. Thus the total cost $\Phi^{\sigma} = \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$. Thus, there exists σ such that the total cost $\Phi^{\sigma} \leq \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$.

Only if part: Suppose there exists a decision σ for the DI such that the total cost $\Phi^{\sigma} \leq \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$. We need to prove that there must exist a Equal Cardinality Partition. Let us examine the decision σ_0 with minimum cost. Note that $\Phi^{\sigma_0} = \Phi_1^{\sigma_0} + \Phi_2^{\sigma_0} + \Phi_3^{\sigma_0}$ consists of three components: the inventory holding cost at the DI, $\Phi_1^{\sigma_0}$; the coin packaging cost paid to the 3PLP, $\Phi_2^{\sigma_0}$; and the transportation cost plus the fee paid for obtaining the packaged coins from the FRS, $\Phi_3^{\sigma_0}$.

- *Claim II.1:* $\Phi_1^{\sigma_0} \geq \frac{3(n+1)K}{2}$.

  Proof: Since $h_u = 0$, the inventory holding cost for σ_0 is zero. The demand for withdrawal of packaged coins ($W^1 = 0$, $W^2 = K + z_1$, $W^3 = K + z_2$, $\ldots$, $W^{2n} = K + z_{2n-1}$, $W^{2n+1} = K + z_{2n}$, $W^{2n+2} = (n+1)K$) must be satisfied. Thus we must maintained minimum inventory at each period such that $I_p^0 \geq 0$, $I_p^1 \geq K + z_1$, $I_p^2 \geq K + z_2$, $\ldots$, $I_p^{2n-1} \geq K + z_{2n-1}$, $I_p^{2n} \geq K + z_{2n}$, $I_p^{2n+1} \geq (n+1)K$, and $I_p^{2n+2} \geq 0$. Thus, $\Phi_1^{\sigma_0} = \sum_{t=0}^{2n+2} \frac{I_p^t}{2} \geq \frac{3(n+1)K}{2}$.

- *Claim II.2:* $\Phi_2^{\sigma_0} + \Phi_3^{\sigma_0} \geq (n+1)K(g+\bar{g}+c+f)$.

  Proof: Note that $\sum_{t=1}^{2n+2} D^t = 2(n+1)K$ and $\sum_{t=1}^{2n+2} W^t = 3(n+1)K$. Since $c = 4$, $f = 1$, $g = 4$, and $\bar{g} = 1$, it economical that all deposits $\sum_{t=1}^{2n+2} D^t = 2(n+1)K$ must be packaged at the 3PLP to satisfied a part of the withdrawal amount to $2(n+1)K$ in which the withdrawal amount $(n+1)K$ corresponds to period $2n+2$ meets the threshold value, $\tau = (n+1)K$. Thus we have $\Phi_2^{\sigma_0} \geq (n+1)K(g+\bar{g})$.

Consequently the remaining withdrawal amount $(n+1)K$ must be satisfied by purchasing packaged coins from FRS and transporting to the DI. Hence we have must have $\Phi_3^{\sigma_0} \geq (n+1)K(c+f)$.

- *Claim II.3:* $\Phi^{\sigma_0} = \Phi_1^{\sigma_0} + \Phi_2^{\sigma_0} + \Phi_3^{\sigma_0} \geq \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$.

  Proof: Follows from Claims II.1 and II.2.

- *Claim II.4:* $\sigma = \sigma_0$ and $\Phi^{\sigma_0} = \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$.

  Proof: Since there exists a decision σ such that the total cost $\Phi^{\sigma} \leq \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$ and $\Phi^{\sigma_0} \geq \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$. $\sigma = \sigma_0$ and σ_0 must be a minimum cost solution with $\Phi^{\sigma_0} = \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$.

Now we can characterize σ_0 with $\Phi^{\sigma_0} = \frac{3(n+1)K}{2} + (n+1)K(g+\bar{g}+c+f)$. As consequence of above claims, we observe the following facts regarding σ_0: (i) No inventory of packaged coins is carried from one period to the next (Claim II.1), (ii) Exactly $(n+1)K$ amount of deposits are packaged to satisfy the withdrawals during the periods 2 to $2n+1$ (Claim II.2), (iii) Exactly $(n+1)K$ amount of deposits are packaged at the discount rate $\bar{g} = 1$ to satisfy the withdrawal during period $2n+2$ (Claim II.2), (iv) Exactly $(n+1)K$ amount of withdrawals during the periods 2 to $2n+1$ are satisfied by purchasing packaged coins from FRS and transporting to the DI (Claim II.2).

Since no inventory of packaged coins is carried from one period to the next and the demand for withdrawals during the periods 2 to $2n+1$ must be satisfied, we must have either $(Y_u^t = K + z_t,\ X_p^t = 0)$ or $(Y_u^t = 0,\ X_p^t = K + z_t)$, $t = 1, 2, \ldots, 2n$. We let the positive variables, Y_u^t (respectively, positive variables X_p^t) belong to $\bar{Z}_1$ (respectively, $\bar{Z}_2$). From Claim II.2, we have $\sum_{Y_u^t \in \bar{Z}_1} Y_u^t = (n+1)K$ (respectively, $\sum_{X_p^t \in \bar{Z}_2} X_p^t = (n+1)K$). This implies that $|\bar{Z}_1| = |\bar{Z}_2| = n$ and there exists a solution to Equal Cardinality Partition. This completes the proof. ∎

APPENDIX C

SUPPLEMENT TO CHAPTER 4

C.1 Computational Results for Building Three EBeam Facilities

For $Q = 3$, four cases are analzyed (Table C.1). The best option, as discussed in Section 4.5 (Table 4.10), is to choose two facilities in Mexico and one facility in the U.S. with 17 service lines in total: 11 in Nuevo Laredo, 3 in Matehuala, and 3 in Houston. The total cost is \$23.5773 M.

To build three facilities in Mexico, the best three locations are Nuevo Laredo with 11 service lines, Reynosa with 3 service lines, and Matehuala with 3 service lines. The total number of service lines required is 17, and the total cost is \$23.5833 M.

To build one facility in Mexico and two facilities in the U.S., 17 service lines are required: 11 in Nuevo Laredo, 3 in San Antonio, and 3 in Houston. For this case, the total cost is \$23.6033 M.

If management decides to build three facilities only in the U.S., San Antonio needs 5 service lines, Dallas need 7 service lines, and Houston needs 5 service lines. The total cost for this case is \$23.7761 M.

Table C.1: Computational Study for Three EBeam Facilities (The best combination is highlighted).

Two In Mexico One In The U.S.			
Best Locations	**Number of Service lines**	**Queueing Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	10		
Matehuala	3		
Houston	3	0.9121	23.5809
Nuevo Laredo	**11**		
Matehuala	**3**		
Houston	**3**	**0.8985**	**23.5773**
Nuevo Laredo	12		
Matehuala	3		
Houston	3	0.8931	23.5819

Three In Mexico			
Best Locations	**Number of Service lines**	**Queueing Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	10		
Reynosa	3		
Matehuala	3	0.9121	23.5869
Nuevo Laredo	**11**		
Reynosa	**3**		
Matehuala	**3**	**0.8985**	**23.5833**
Nuevo Laredo	12		
Reynosa	3		
Matehuala	3	0.8931	23.5880

One In Mexico Two In The U.S.			
Best Locations	**Number of Service lines**	**Queueing Cost ($M)**	**Total Cost ($M)**
Nuevo Laredo	10		
San Antonio	3		
Houston	3	0.9121	23.6069
Nuevo Laredo	**11**		
San Antonio	**3**		
Houston	**3**	**0.8985**	**23.6033**
Nuevo Laredo	12		
San Antonio	3		
Houston	3	0.8931	23.6079

Three In The U.S.			
Best Locations	**Number of Service lines**	**Queueing Cost ($M)**	**Total Cost ($M)**
San Antonio	5		
Dallas	6		
Houston	5	0.9313	23.7875
San Antonio	**5**		
Dallas	**7**		
Houston	**5**	**0.9081**	**23.7761**
San Antonio	5		
Dallas	8		
Houston	5	0.9010	23.7807

C.2 Computational Results for Building Four EBeam Facilities

For $Q = 4$, five cases are also analyzed (Table C.2). The best option is discussed in Section 4.5 (Table 4.11). Choosing three facilities in Mexico and one facility in the U.S. is the best, with 18 service lines in total: 9 in Nuevo Laredo, 3 in Reynosa, 3 in Matehuala, and 3 in Houston. The queuing delay cost for the best option is \$0.9108 M, and the total cost is \$23.7109 M. The solutions for other cases are also shown in Table C.2. Clearly, selecting all four locations in the U.S. is the worst case.

Table C.2: Computational Study for Four EBeam Facilities (The best combination is highlighted).

	Three In Mexico One In The U.S.		
Best Locations	**Number of Service lines**	**Queueing Cost** (\$M)	**Total Cost** (\$M)
Nuevo Laredo	9		
Reynosa	3		
Matehuala	3		
Houston	3	0.9108	23.7109
	Four In Mexico		
Best Locations	**Number of Service lines**	**Queueing Cost** (\$M)	**Total Cost** (\$M)
Nuevo Laredo	9		
Reynosa	3		
Matamoros	3		
Matehuala	3	0.9108	23.7117
	Two In Mexico Two In The U.S.		
Best Locations	**Number of Service lines**	**Queueing Cost** (\$M)	**Total Cost** (\$M)
Nuevo Laredo	**9**		
Matehuala	**3**		
San Antonio	**3**		
Houston	**3**	**0.9108**	**23.7123**
	Three In The U.S. One In Mexico		
Best Locations	**Number of Service lines**	**Queueing Cost** (\$M)	**Total Cost** (\$M)
Nuevo Laredo	9		
San Antonio	3		
Dallas	3		
Houston	3	0.9108	23.7235
	Four In The U.S.		
Best Locations	**Number of Service lines**	**Queueing Cost** (\$M)	**Total Cost** (\$M)
Laredo	3		
San Antonio	6		
Dallas	6		
Houston	3	0.9258	23.8965