

FAULT ANALYSIS OF ELECTROMECHANICAL SYSTEMS USING  
INFORMATION ENTROPY CONCEPTS

A Thesis

by

RAVINDRA KRISHNA TANGIRALA

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2011

Major Subject: Mechanical Engineering

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## ABSTRACT

Fault Analysis of Electromechanical Systems Using  
Information Entropy Concepts. (August 2011)

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Fault analysis of mechanical and electromechanical systems has been a subject of considerable interest in the systems and control research community. Entropy, under its various formulations is an important variable, which is unrivaled when it comes to measuring order (or organization) and/or disorder (or disorganization). Researchers have successfully used entropy based concepts to solve various challenging problems in engineering, mathematics, meteorology, biotechnology, medicine, statistics etc. This research tries to analyze faults in electromechanical systems using information entropy concepts. The objectives of this research are to develop a method to evaluate signal entropy of a dynamical system using only input/output measurements, and to use this entropy measure to analyze faults within a dynamical system. Given discrete-time signals corresponding to the three-phase voltages and currents of an electromechanical system being monitored, the problem is to analyze whether or not this system is healthy.

The concepts of Shannon entropy and relative entropy come from the field of Information Theory. They measure the degree of uncertainty that exists in a system. The main idea behind this approach is that the system's dynamics may have regularities hidden in measurements that are not obvious to see. The Shannon entropy and relative entropy measures are calculated by using probability distribution functions (PDF) that are formed by sampling the time series currents and voltages of a system.

The system's health is monitored by, first, sampling the currents and voltages at certain time intervals, then generating the corresponding PDFs and, finally, calculating the information entropy measures. If the system dynamics are unchanged, or in other words, the system continues to be healthy, then the relative entropy measures will be consistently low or constant. But, if the system dynamics change due to damage, then the corresponding relative entropy and Shannon entropy measures will be increasing compared to the entropy of the system with less damage.

To my parents.

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I would also like to thank Dr. Won-Jong Kim and Dr. Aniruddha Datta for serving on my committee and being excellent teachers.

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Over the last two years, my peddamma, akka and bavagaru and my friends, Reeshav and Ayush, have made my stay in College Station bearable. I would like to thank them for their constant support and affection.

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## CHAPTER I

### INTRODUCTION

#### A. Research Motivation

Fault analysis of electromechanical systems has been a subject of considerable interest in the control research community. This is in response to the ever increasing requirements on the reliable operation of control systems, which are, in most cases, subject to a number of faults either in the internal closed loops or from external environmental agents. The failure of electromechanical systems often result in unrecoverable losses and, at times, lead to irreparable environmental damages. Therefore, effective fault analysis of electromechanical systems is of vital importance for the safe and successful operation of industrial plants.

Entropy, under its various formulations, is an important variable, which is unrivaled when it comes to measuring order and/or disorder. Entropy, be it of a dynamical system or a stochastic process or even as a source of information, has proved to be a very useful tool in various fields of science. Essentially, entropy quantifies randomness, chaos or uncertainty in a system/process/signal.

For example, to study the human vocal apparatus and the different voice disorders, researchers have developed several tools for diagnosing vocal tract pathologies. Traditionally, voice signals have been modeled as linear processes and the acoustic analysis tools were developed based on linear system theory. However, in recent years, there has been a significant development of signal processing based noninvasive techniques [1], [2] for diagnosing abnormal functioning vocal apparatus. Natural processes and systems, like the human vocal apparatus, seem to be unpredictable due to several

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reasons, viz. insufficient knowledge of system dynamics, environment induced sources of randomness etc. These issues are also common to most electromechanical systems. As a result, signal processing based online machine condition monitoring of electromechanical systems has been increasingly attracting attention from the research and engineering community worldwide over the past decade.

This thesis will present an accessible approach to answer some of the following questions:

*Could a simple variable, such as entropy, be used to track the mechanical damage of a system relative to a less damaged state of the same system?*

*Is it possible to assess damage in mechanical and electromechanical systems without the explicit use of a mathematical model?*

*And could such damage assessment be made using only the time series measurements?*

## B. Literature Review

Entropy has originated from the works of Ludwig Boltzmann and J. Willard Gibbs [3] on thermodynamics in the 1870s. Later, the concept of entropy was generalized and made into the cornerstone of information theory by Shannon [4]. Kolmogorov [5] and Sinai [6] extended its applications to dynamical systems and ergodic theory. In connection with chaos theory, Kolmogorov's theoretic entropy measure, together with the Lyapunov exponents [7], have been the main indicators of chaos in dynamical systems. Hausdorff dimension [8], volume growth and algorithmic complexity has also been used to link dynamics with chaos theory.

There are close parallels between the mathematical expressions for the thermodynamic entropy of a physical system established by Boltzmann and Gibbs; and the

information-theoretic Shannon entropy. Shannon, although not initially aware of this similarity, commented on it upon publishing his book[4].

The pioneering work of Lorenz [9] showed that because of extreme sensitivity of weather predictions to specific initial conditions, detailed forecasts are impossible beyond a certain time limit. This pioneering study led over the following decades to the extensive study of chaotic dynamics. (Eg.: Grassberger [10, 11]; Eckmann and Ruelle [12]). In 1971 Ruelle and Takens [13] came up with a new theory, based on the abstract concept of a strange attractor, for the onset of turbulence in fluids. Richard Kleeman [14] developed a new parameter of dynamical system predictability that measures the potential utility of predictions using relative entropy.

S. Pincus introduced the approximate entropy in order to measure the signal's complexity and regularity [15], which proved to be an invaluable tool to analyze biological signals, in particular, heart rate signals. Approximate entropy [16] was used to characterize the signals' spectral patterns in order to obtain a complexity measure of voice electroglottogram signals from healthy and larynx cancer patients.

J.M. Amigo *et al.* [17] proposed entropy for maps on finite sets (called discrete entropy) via the so-called permutation entropy, an alternative approach to measure-theoretic and topological entropy that was amenable to the methods of discrete mathematics.

Based on the traditional method of singular spectrum entropy, Yong Lu *et al.* [18] proposed a sliding singular spectrum entropy method for singularity detection and extraction of impaction signal.

Long Zhang *et al.* [19] developed a method, called multiscale entropy (MSE), taking into account multiple time scales, to extract features from faulty vibration signals. MSE in tandem with support vector machines (SVMs) were used to develop a intelligent hybrid fault diagnosis method.

Long Zhang *et al.* [20] also developed a bearing fault diagnosis method based on multi-scale entropy (MSE) and adaptive neuro-fuzzy inference system (ANFIS), in order to tackle the nonlinearity existing in bearing vibration and the uncertainty inherent in the diagnostic information. MSE refers to the calculation of entropies (e.g. appropriate entropy, sample entropy) across a sequence of scales, which takes into account not only the dynamic nonlinearity but also the interaction and coupling effects between mechanical components, thus providing much more information regarding machinery operating condition than traditional single scale-based entropy. ANFIS benefits from the decision-making under uncertainty enabled by fuzzy logic as well as from learning and adaptation that neural networks provide. MSE and ANFIS were employed for feature extraction and fault recognition, respectively.

He Zhengyou *et al.* [21] proposed wavelet entropy measures to signify the complexity of non-steady signal or system in both time and frequency domain. In order to meet the requirements of post-analysis on abundant wavelet transform result data and the need of information convergence, they defined wavelet entropy measure and proposed corresponding algorithms for several wavelet entropies, such as wavelet average entropy, wavelet time-frequency entropy, wavelet distance entropy etc. The physical meaning of these entropies were analyzed and their applications in electroencephalography (EEG) signal analysis, mechanical fault diagnosis, fault detection and classification in power system were discussed.

Chee Siong Teh *et al.* [22] proposed a hybrid system that integrates the SOM (Self Organizing Map) neural network, the kMER (kernel-based Maximum entropy learning Rule) algorithm and the Probabilistic Neural Network (PNN) for data visualization and classification. The hybrid SOM-kMER-PNN model achieved comparable classification rates to those from a number of existing classifiers and, at the same time, produced meaningful visualization of the data sets.

The article [23] by Gui-Bao Wang *et al.* estimated the uncertainty of possible failure events of redundancy systems based on the cross entropy (CE) method. Failure events of subsystems and components always result in the failure of engineering systems, hence optimal condition monitoring of a complex system is heavily dependent on the accuracy analysis of all the failure events of subsystems and components and their interaction effects. The CE method has been found to be a versatile tool for estimating probabilities of rare events in complex systems with the least bias beyond conditional constraints. They introduced the CE method for analyzing the system reliability and developed a numerical CE algorithm capable of estimating the uncertainty of failure modes in an M-dimensional redundancy system domain with moment constraints of order up to N.

Paulo Rogerio Scalassara *et al.* [1] analyzed the characteristics of the entropy rate curve and its asymptote to acquire knowledge about the evolution of pathological voice signals from patients with nodule in their vocal folds. Their analysis provided some differences between nodule and healthy signals, which they related to the level of evolution of the pathology. In [2], Paulo Rogerio Scalassara *et al.* characterized healthy and pathological voice signals with the aid of relative entropy measures. Using phase space reconstruction technique to select interesting regions of the signals, they showed that relative entropy was well suited due to its sensibility to uncertainties, since the pathologies were characterized by an increase in the signal complexity and unpredictability. Their results showed that the pathological groups had higher entropy values in accordance with certain vocal acoustic parameters.



### C. Problem Statement and Research Objectives

Most industrial systems exhibit nonlinear characteristics, and therefore, are difficult to model. Typically, only their input/output time series data are observable. Thus, it would be judicious to develop a measurement based method (over traditional model based methods) to analyze faults in dynamical systems.

Hence, the objectives of this research are as follows:

- (1) Develop a method to evaluate signal entropy of a dynamical system using only input/output measurements, and
- (2) Use this entropy measure to analyze faults within a dynamical system.

### D. Proposed Approach

Given discrete-time signals corresponding to the three-phase voltages and currents of an electromechanical system (being monitored), the problem is to analyze whether or not this system is healthy.

Linear models have been used for many years to describe the characteristics of electromechanical systems; however, this is inaccurate as there are several underlying nonlinearities that exist in the system dynamics. The use of entropy brings the possibility of treating these nonlinearities in the scope of dynamical systems and predicting their behavior over time with varying internal and/or external conditions.

The concept of Shannon's entropy [4] comes from the field of Information Theory. It measures the degree of uncertainty that exists in a system. The main idea is that the system's dynamics may have regularities hidden in measurements that are otherwise not obvious. This is related to the predictability of the system. Considering a random variable  $X$  that assumes values  $x \in \chi$  where  $\chi$  is a finite set, the Shannon entropy of a

variable  $X$ , is defined as follows (units in bits):

$$H(X) = - \sum_x p(x) \log_2(p(x)) \quad (1.1)$$

where  $p(x)$  is the probability that  $X$  is in the state  $x$  and  $p(0) \log_2(p(0)) = 0$ .

The Kullback-Leibler divergence or relative entropy measure is an important parameter which incorporates the concept of signal entropy. It is a non-symmetric measure that compares two different probability distributions functions  $p$  and  $q$ . Particularly, it measures the inefficiency to code samples from  $p$  when using a code based on  $q$ . This inefficiency is represented as the number of extra bits necessary to implement the codification based on this misplaced distribution. The relative entropy,  $D$ , is given by the following equation:

$$D(p||q) = \sum_{x \in \mathcal{X}} p(x) \log_2 \left( \frac{p(x)}{q(x)} \right) \quad (1.2)$$

where  $0.\log(0/q) = 0$  and  $p.\log(p/0) = \infty$ .

Relative entropy measure exhibits some distance characteristics like being positive and equal to zero if the probability distribution functions involved are equal. However, it is not a real distance because  $D(p||q)$  is different from  $D(q||p)$ . Assume that a probability distribution function (PDF),  $q(x)$ , is formed by sampling the time series currents and voltages of a healthy system. With  $q(x)$  as reference, the system's health can be monitored at any instant by sampling the currents and voltages around that time and generating a corresponding PDFs  $p_i(x)$ ,  $i = 1, 2, 3...$  If the system dynamics remain the same, in other words, the system remains healthy, then the relative entropy measure,  $D(p_i||q)$ , will be found to be minimal. But, if the system dynamics change due to some mal-functioning, then the corresponding  $D(p_i||q)$  value will be found to be relatively higher. Similarly, the Shannon entropy measures,  $H_{p_i}(X)$ , will be found to be very close to  $H_q(X)$ , if the system is healthy. But,  $H_{p_i}(X)$  will be

higher for faulty systems, as there is more disorder in such systems.

#### E. Research Contributions

The main contribution of this research is the development of a method for the analysis of faults in electromechanical systems using the concepts of information entropy. Firstly, this study will validate the theory by applying the proposed approach to a simple academic problem. Secondly, this study will also apply the proposed fault diagnosis method to two real world data sets that correspond to an induction motor. One data set is obtained by observing an induction motor in a controlled environment and the other is a blind data set obtained from actual field operation.

#### F. Thesis Organization

The remaining parts of this thesis are organized as follows. In Chapter II, fault detection in electromechanical systems is presented. The different fault categories and the various fault detection methods are discussed. Chapter III gives a theoretical background on entropy. It describes, in detail, the concept of entropy, the theory of thermodynamic entropy and the various types of information entropy measures. The relationship between the thermodynamic entropy and information entropy is also discussed. Also discussed in Chapter III are the methods to evaluate relative entropy and Shannon entropy, along with some of their applications. To validate the proposed fault analysis approach, in Chapter IV, the proposed theory is applied, first, to a simple academic problem and then to two real world data sets, corresponding to an induction motor. The results of the proposed approach are also compared to ones obtained using other methods. In Chapter V, a summary and conclusions of this research, along with some directions for future work are briefly described.

## CHAPTER II

## FAULT DETECTION IN ELECTROMECHANICAL SYSTEMS

The reliability of electromechanical systems, in particular electric motors, have been studied for many years. In this chapter, some of the most common motor faults are presented. The failures of electric motors are typically grouped into mechanical and electrical faults. Failures happen for a wide variety of reasons, viz, inadequate lubrication, unbalanced loading, vibrations and misalignment etc. Most mechanical failures occur gradually. They show certain warning signs which intensify over time. Surveys [24, 25] show that bearing failures cause nearly half of all failures, while stator winding failures account for about 15% to 35%. Rotor and shaft failures are under 10% of all failures in electric motors (Fig. 1).

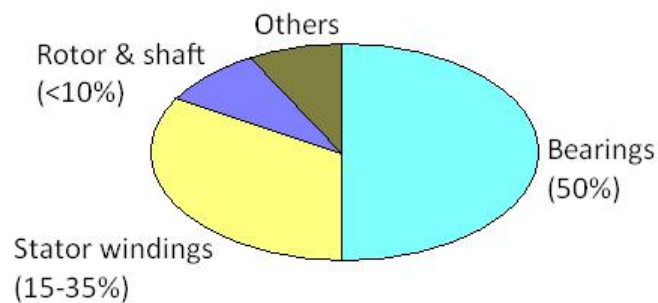


Fig. 1. Failure rate of electric motors

## A. Most Common Faults in Electric Motors

### 1. Bearing Faults

Rolling element bearings consist of inner raceways, outer raceways and rolling elements rotating between them. Bearing faults can occur due to fatigue sometimes even under normal balanced operation condition with good shaft alignment. It can also occur due to improper lubrication, installation errors and contamination (Fig. 2).

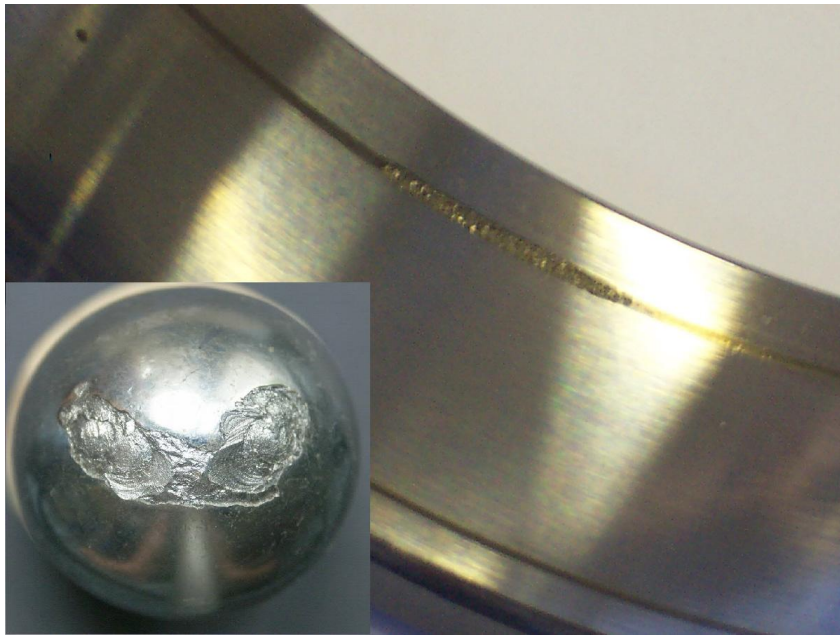


Fig. 2. Bearing fault

Typically, bearing failures cause an increase in the level of vibration and noise. Studies show that when defects exist in bearings, they generate certain characteristic frequencies which can be observed in the vibration signals. Many publications [26] have discussed the use of these feature frequencies to identify defects in a bearing assembly. Some of the prominent frequencies are: the ball pass outer raceway frequency

( $F_{BPO}$ ), the ball pass inner raceway frequency ( $F_{BPI}$ ) and the ball spin frequency ( $F_B$ ). Sometimes good bearing condition may have  $F_{BPO}$ ,  $F_{BPI}$  and its harmonics, however their amplitudes are insignificant.  $F_B$ ,  $F_{BPI}$ ,  $F_{BPO}$  frequencies can be calculated using the bearings geometric dimensions. These frequencies are expressed as follows:

$$F_{BPO} = \frac{N_B}{2} F_S \left( 1 - \frac{D_B \cos \theta}{D_C} \right) \quad (2.1)$$

$$F_{BPI} = \frac{N_B}{2} F_S \left( 1 + \frac{D_B \cos \theta}{D_C} \right) \quad (2.2)$$

$$F_B = \frac{D_C}{2D_B} F_S \left( 1 - \left( \frac{D_B \cos \theta}{D_C} \right)^2 \right) \quad (2.3)$$

In the above equations,  $D_B$  is the ball diameter;  $D_C$  is the bearing pitch diameter and  $\theta$  is the contact angle of the bearing.

Schoen *et al.* [27] have shown that stator current monitoring can be used to detect bearing faults in induction motors. Line current spectral components are predicted at frequencies of:

$$F_{bng} = [F_e \pm mF_v] \quad (2.4)$$

where  $F_v$  is one of the characteristic vibration frequencies,  $F_e$  is the supply frequency and  $m = 1, 2, 3, \dots$

Although the magnitudes of these harmonic components are often small compared to the other spectral components, they fall at locations that are different from those of the supply and machine inherent slot harmonics. This phenomenon makes it feasible to distinguish between a healthy and a faulty condition.

## 2. Stator Winding Faults

Stator winding faults constitute about 15-35% of all electric motor faults. These faults often lead to the damage of the stator coils. They are caused by short circuits

between a phase winding and the ground or between two phases due to insulation breakdown (Fig. 3). This breakdown leads to an unbalance in the stator which can be observed as changes in the current and vibration spectra. Kliman *et al.* [28] observed that an increase in the motor phase currents results in shifts in both positive and negative sequence currents.

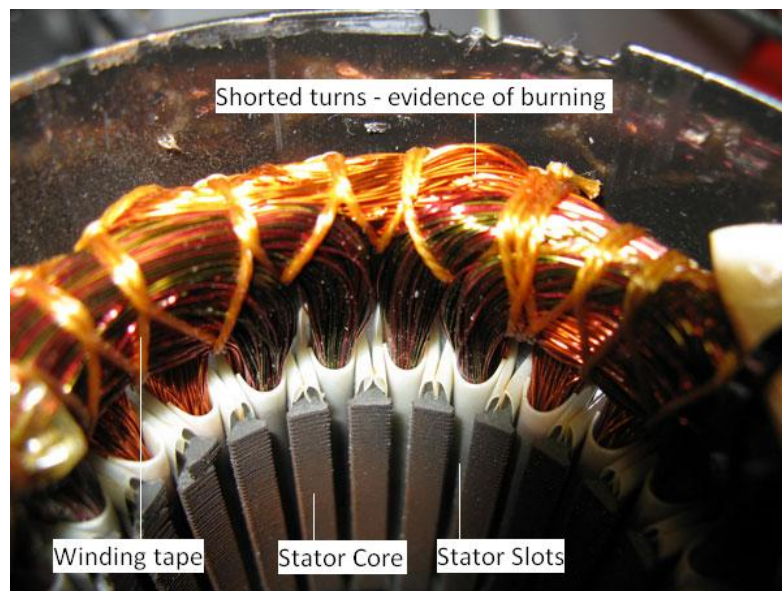


Fig. 3. Stator winding fault

### 3. Broken Rotor Bars and Unbalanced Rotor

Broken rotor bars (Fig. 4) cause fault vibrations, which can be seen around the fundamental rotor frequency as sidebands. These sideband frequencies are given by:

$$f_b = (1 + 2ks)f \quad (2.5)$$

where  $k = 1, 2, 3, \dots$ ,  $s$  is the motor slip and  $f$  is the rotor frequency.

The amplitudes and the locations of these sidebands depend upon the physical position of the broken rotor bars, the operating speeds and loads. Typically, when an electric motor is healthy, no sidebands are visible. However, when the motor is operating under no load, sidebands are not detectable in the vibration spectra whether a rotor bar is broken or not. The reason for this is because the slip is too small [29]. When the load on the motor is increased, the sidebands start to appear at the expected locations. Motor current analysis has shown that the current spectra and the vibration spectra for broken rotor bars are very similar.

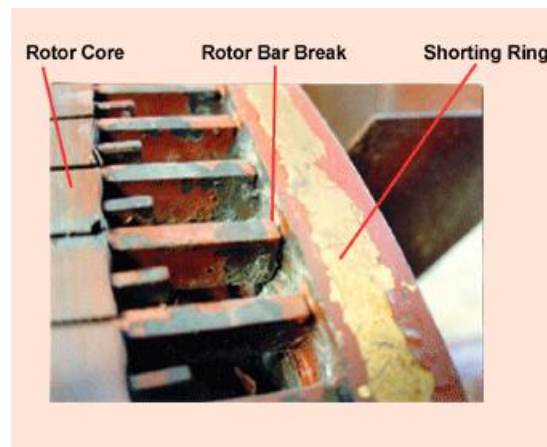


Fig. 4. Broken rotor bar

## B. Methods Used for Fault Detection

Some of the most common methods for fault detection of electromechanical systems are discussed in this section. They can be broadly grouped into the following two categories:

- (1) Model based Fault Detection



## (2) Data driven and Knowledge based Fault Detection

### 1. Model Based Fault Detection

In the early 1980's, model based fault detection methods constituted the main stream of research. Depending on whether the system dynamics were represented using state space models or input-output models, most model based fault detection methods were grouped as: Observer based fault detection methods [30] and System Identification based fault detection methods [31]. Also, to combine the best features of these two approaches, there was another group of fault detection methods called Adaptive Observer based fault detection methods, which use parameter tuning principles from model reference adaptive control to directly estimate fault parameters online for effective fault diagnosis [32, 33]. However, model based fault detection methods use mathematical models to estimate the system's state and parameters, and are typically applicable to low dimensional systems only.

### 2. Data Driven and Knowledge Based Fault Detection

Data driven and knowledge based fault detection methods can deal with high dimensional data. They are broadly grouped as:

- (i) Signal based fault detection methods
- (ii) Multi-variable statistics based fault detection methods
- (iii) Knowledge based fault detection methods

#### a. Signal Based Fault Detection Methods

The first group of data driven fault detection methods are signal based. Signal based fault detection methods use signal processing methods consisting of correlation functions, signal model identification, signal parity checks, and spectral analysis using

fast Fourier transformation and wavelet transformation. These are similar to signal detection and trend detection for important variables using the available data. The key idea is that unexpected changes in the magnitude, phase and/or frequencies of the important signals can be regarded as the faults in the system.

#### b. Multi-variable Statistics Based Fault Detection Methods

The key concept in principal component analysis (PCA) is to reduce a high dimensional data volume into a lower dimensional space containing most of the useful information of the original data. PCA has been widely used as a standard technique for data analysis and process abnormality identification in several industrial processes. A set of PCA components should be determined for the healthy data set and then fault detection can be performed by checking whether or not the new incoming data lies in the space spanned by the healthy principal components. PCA has the advantage of being able to detect process faults in large multivariate data sets. However, PCA assumes that the data is a multivariate Gaussian distributed, which limits its application in complex industrial systems that exhibit either time-varying or non-Gaussian or nonlinear characteristics.

#### c. Knowledge Based Fault Detection Methods

Using continuously accumulated process knowledge, knowledge based fault detection can be applied to perform online monitoring for industrial processes. Knowledge based fault detection includes cause-effect analysis and expert systems. The cause-effect analysis uses symbolic graphs or fault trees to detect faults. While expert systems attempt to mimic the reasoning of human experts for fault detection and classification. To make best use of the gained knowledge, a combination of fuzzy logic and neural networks has also been used for knowledge based fault detection.

Neural network based fault detection methods use the available mapping between the process variables and the faults to identify the system faults. Neural networks have also been used to carry out residual signal analysis and classify the residual signals into healthy and unhealthy categories.

## CHAPTER III

### SIGNAL ENTROPY ESTIMATION

The concept of entropy was developed in response to the observation that a certain amount of functional energy released from combustion reactions was always lost to dissipation or friction and thus couldn't be transformed into useful work. Early engines powered by heat like Savery's and Newcomen's engines were found to be highly inefficient. A lot of useful energy was dissipated or lost as immeasurable randomness. Physicists investigated this loss of energy, which has resulted in the concept of entropy. In the early 1800s, Clausius set forth the concept of a thermodynamic system and later argued that in any irreversible process some heat  $\delta Q$  was dissipated across the system boundary. Clausius developed this idea of lost energy, and coined the term entropy. Since then, the concept of entropy has found invaluable applications in various fields of science and technology.

#### A. Thermodynamic Entropy

In 1803, mathematician Lazare Carnot published a work entitled *Fundamental Principles of Equilibrium and Movement*. He discussed the efficiency of fundamental machines, such as pulleys and inclined planes. His 's theorem states that in any machine the accelerations and shocks of the moving parts represent losses of moment of activity. From this theorem he drew the inference that perpetual motion was impossible. This loss of moment of activity was the first-ever rudimentary statement of the second law of thermodynamics [34].

In 1854, Clausius first developed the concepts of interior work and exterior work. He discussed the three categories into which heat  $Q$  may be divided:

1. Heat employed in increasing the heat actually existing in the body.
2. Heat employed in producing the interior work.
3. Heat employed in producing the exterior work.

He stated [35] that if two non-permanent, mutually replaceable, transformations are equivalent, then the generation of the quantity of heat  $Q$  from work at the temperature  $T$ , has the value:  $Q/T$

The quantity of heat  $Q$  from the temperature  $T_1$  to the temperature  $T_2$ , has the value:  $Q(1/T_2 - 1/T_1)$

In modern terminology, we think of this equivalence-value as “entropy”, symbolized by  $S$ . Thus, we can calculate the entropy change  $\Delta S$  for the quantity of heat  $Q$  from the temperature  $T_1$ , as follows (Fig. 5):

$$\Delta S = Q \left( \frac{1}{T_2} - \frac{1}{T_1} \right) \quad (3.1)$$

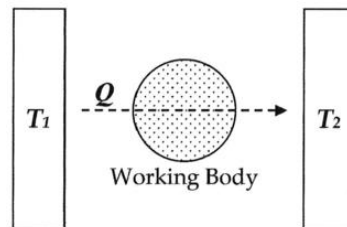


Fig. 5. Carnot's heat engine

In 1862, Clausius stated that the algebraic sum of all the transformations occurring in a cyclical process can only be positive, or, as an extreme case, equal to nothing. Quantitatively, the mathematical expression for this theorem can be understood as follows. Let  $\delta Q$  be an infinitesimal element of the heat given up by the body

during its own changes, and  $T$  the absolute temperature of the body at the moment of giving up this heat, then:

$$\int \frac{\delta Q}{T} \geq 0 \quad (3.2)$$

This was an early formulation of the second law of thermodynamics and the original concept of entropy. In 1876, physicist Gibbs, based on the work of Clausius, proposed that the measurement of “available energy”  $\Delta G$  in a thermodynamic system could be mathematically accounted for by subtracting the “energy loss”  $T\Delta S$  from total energy change of the system  $\Delta H$ . These concepts were later developed further by James Maxwell and Max Planck.

$$\Delta H = \Delta G + T\Delta S \quad (3.3)$$

In 1877, Ludwig Boltzmann formulated the alternative definition of entropy  $S$  defined as:

$$S = k_B \ln \Omega \quad (3.4)$$

In the above equation,  $k_B$  is Boltzmann’s constant and  $\Omega$  is the number of micro states consistent with the given macro state.

Boltzmann defines entropy as a measure of statistical disorder. This concept was later advanced by Gibbs. It is now regarded as the cornerstones of statistical mechanics. Statistical mechanics provides a molecular-level interpretation of macroscopic thermodynamic quantities like work, heat, free energy and also entropy. It also enables thermodynamic properties of materials to be related to the spectroscopic data of individual molecules. This ability to make macroscopic predictions based on microscopic properties is the main advantage of statistical mechanics over classical thermodynamics. Both these theories are governed by the second law of thermodynamics using the concept of entropy. However, entropy in thermodynamics can

only be calculated empirically, whereas in statistical mechanics, it is a function of the distribution of the system on its micro-states.

## B. Information Entropy

An analog to thermodynamic entropy is information entropy. In 1948, while working at Bell Telephone Laboratories Claude Shannon set out to quantify the statistical nature of “lost information” in phone-line signals. Shannon developed a general concept of information entropy, which later was considered a cornerstone of information theory. He wasn’t aware of the similarity between his definition of entropy and earlier works in classical thermodynamics. Later in 1948, Shannon published his now famous paper *A Mathematical Theory of Communication*, in which he devoted a section to Choice, Uncertainty, and Entropy [4]. Shannon introduces an information entropy function  $H$ , as:

$$H = -K \sum_{i=1}^k p(i) \log(p(i)) \quad (3.5)$$

Shannon states that “any quantity of such form has an important role in information theory as a measure of information and uncertainty.” Over the last few decades, people have found that these concepts of entropy are exactly the same. Shannon’s information entropy is a much more general concept than statistical thermodynamic entropy. Information entropy is present whenever there are unknown quantities that can be described only by their corresponding probability distribution functions. In the sections that follow, an expression of the form  $p \log p$  is considered by convention to be equal to zero whenever  $p = 0$ .

## 1. Shannon Entropy

The Shannon entropy,  $H$ , of a discrete random variable  $X$  is a measure of the amount of uncertainty associated with the value of  $X$ . Consider the task of transmitting  $n$ -bits of information. If these bits are known ahead of the actual transmission at the receiving end to be a certain value with absolute probability, then basic logic dictates that no new information has been transmitted. However, if each bit of information is independent and equally probable to be either a 0 or a 1, then on transmission we say  $n$ -bits of information has been transmitted. Between these two extremes cases, Shannon quantified information as follows. If  $\chi$  is the set of all messages  $\{x_1, x_2, x_3, \dots, x_n\}$  that  $X$  could be, and  $p(x)$  is the probability of  $X$  for some given  $x \in \chi$ , then the entropy of  $X$  is defined as:

$$H(X) = E_X [I(x)] = - \sum_{x \in \chi} p(x) \log(p(x)) \quad (3.6)$$

Here,  $I(x)$  is the self-information, which is the entropy contribution of an individual message, and  $E_X$  is the expected value. The property of entropy is that it is maximized when all the messages in the message space are equal probability  $p(x) = \frac{1}{n}$ .

## 2. Relative Entropy

The Kullback-Leibler divergence or relative entropy measure is a non-symmetric measure that compares two different probability distributions functions  $p$  and  $q$ . Particularly, it measures the inefficiency to code samples from  $p$  when using a code based on  $q$ . This inefficiency is represented as the number of extra bits necessary to implement the codification based on this misplaced distribution. The relative entropy,  $D$ , is given by the following equation:

$$D(p||q) = \sum_x -p(x) \log(q(x)) - (-p(x) \log(p(x))) = \sum_x p(x) \log\left(\frac{p(x)}{q(x)}\right) \quad (3.7)$$



Relative entropy measure exhibits some distance characteristics like being positive and equal to zero if the probability distribution functions involved are equal. However, it is not a real distance because  $D(p||q)$  is different from  $D(q||p)$ . (making it a semi-quasi metric).

### 3. Joint Entropy

The joint entropy of two discrete random variables  $X$  and  $Y$  is simply the entropy of their pairing:  $(X, Y)$ . If  $X$  and  $Y$  are independent, then their joint entropy is the sum of their individual entropies. Like for example, if  $(X, Y)$  represents the position of a chess piece —  $X$  the row and  $Y$  the column, then the joint entropy of the row of the piece and the column of the piece will be the entropy of the position of that piece.

$$H(X, Y) = - \sum_{x,y} p(x, y) \log(p(x, y)) \quad (3.8)$$

### 4. Conditional Entropy

The conditional entropy or conditional uncertainty of  $X$  given random variable  $Y$  is defined as the average conditional entropy over  $Y$ :

$$H(X|Y) = H(X, Y) - H(Y) = \sum_{x,y} p(x, y) \log\left(\frac{p(x, y)}{p(y)}\right) \quad (3.9)$$

Proper care should be taken not to confuse these two definitions of conditional entropy, because entropy can be conditioned on a random variable or on that random variable being a certain value.

### 5. Mutual Information

Mutual information measures the amount of information that can be obtained about one random variable by observing another random variable. It is very important in

communication as it is used to maximize the amount of information shared between sent and received signals. The mutual information of  $X$  relative to  $Y$  is given by:

$$I(X; Y) = H(X) + H(Y) - H(X, Y) = \sum_{x,y} p(x, y) \log \left( \frac{p(x, y)}{p(x)p(y)} \right) \quad (3.10)$$

## 6. Cross Entropy

The cross entropy between two probability distributions measures the average number of bits that are needed to identify an event from a set of possible events. If a coding scheme is used based on a given probability distribution  $q$ , rather than the “real” distribution  $p$ . The cross entropy for two distributions  $p$  and  $q$  over the same probability space is defined as follows:

$$H(p, q) = - \sum_x p(x) \log(q(x)) \quad (3.11)$$

### C. Relation Between Thermodynamic Entropy and Information Entropy

The Second Law of Thermodynamics states that a closed system will always move toward a state of increasing disorder. Now to show the relationship between thermodynamic entropy and information entropy, consider this simple example of a vial, where the gate is closed only once at a point when there are no molecules in one portion of it (Fig. 6). The atoms’ natural Brownian motion is moving them about and by the use of a well-timed gate, it is possible to capture them in a state of lower entropy and prevent them from returning to a state of higher entropy. The entropy of the gas in the vial has been reduced, storing potential energy in the form of air pressure. If the amount of work into which that air pressure can be transformed exceeds the entropy generated in closing the gate, then it appears we have violated the second law. But according to Leoszilard, this missing entropy could be accounted

for as entropy used in generating information. The observer would need to know the positions and velocities of the atoms to know where and when to drop the gate. So the observer had to logically recognize the opportunity to strategically drop the gate, which meant they had to generate entropy in the form of information. This is a very rudimentary demonstration of how thermodynamic entropy could be related to information. A detailed experimental demonstration of this idea is presented in [36] by Toyabe *et al.* They describe an experiment where information was converted into energy by exploiting the Brownian motion of atoms. It involved using the vibrations of an atom and observations of its changing position to let it naturally work its way up a sine wave, increasing its potential energy, which could, theoretically, be used to perform work when it vibrates back down the wave.

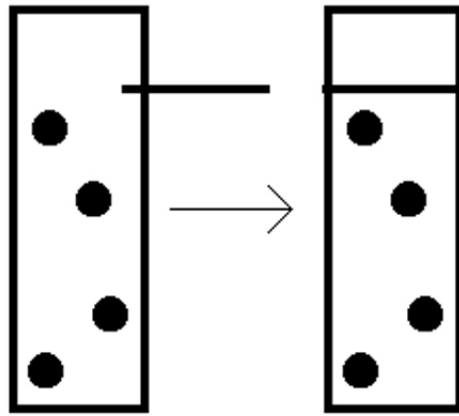


Fig. 6. A simple gated tube to increase the air pressure

#### D. Entropy Estimation

Various entropy measures presented in section 3.2, can be used to quantify the chaotic nature of dynamical systems. The change in entropy has a direct relationship with system faults. Healthy systems have lower entropies than faulty ones. In order to estimate the signal entropy, an algorithm was developed using the considerations presented in [37], which are based on a simple histogram algorithm. In [37], the author presents several examples that evaluate the algorithm showing its reliability. The principle of the method is to try to estimate the probability distribution function (PDF) of the signals under study. This can be performed by dividing discrete-time signals into  $N$  number of equally sized bins, essentially, quantizing the signals (Fig. 7). The occurrences of the signal's points in each bin,  $j$ , are summed, as  $p_j$  and  $q_j$ . Then, the approximate Shannon and relative entropies of the signals are estimated using the following formulae:

$$H_p(X) = \sum_{j=1}^N p_j \log_2(p_j) \quad (3.12)$$

$$D(p||q) = \sum_{j=1}^N p_j \log_2\left(\frac{p_j}{q_j}\right) \quad (3.13)$$

Assume that a PDF,  $q(x)$ , is formed by sampling the input/output time series data of a healthy system. Now with  $q(x)$  as reference, the system's health can be monitored at any instant by sampling the input/output signals and generating corresponding PDFs  $p_i(x)$ ,  $i = 1, 2, 3...$

If the system remains healthy, then the relative entropy measure,  $D(p_i||q)$ , will be found to be minimal. But if the system dynamics change due to some mal-functioning, then the corresponding  $D(p_i||q)$  value will be found to be relatively higher.

Similarly, the Shannon entropy measures,  $H_{p_i}(X)$ , will be found to be very close

to  $H_q(X)$ , if the system is healthy. But  $H_{p_i}(X)$  will be found to be higher for faulty systems.

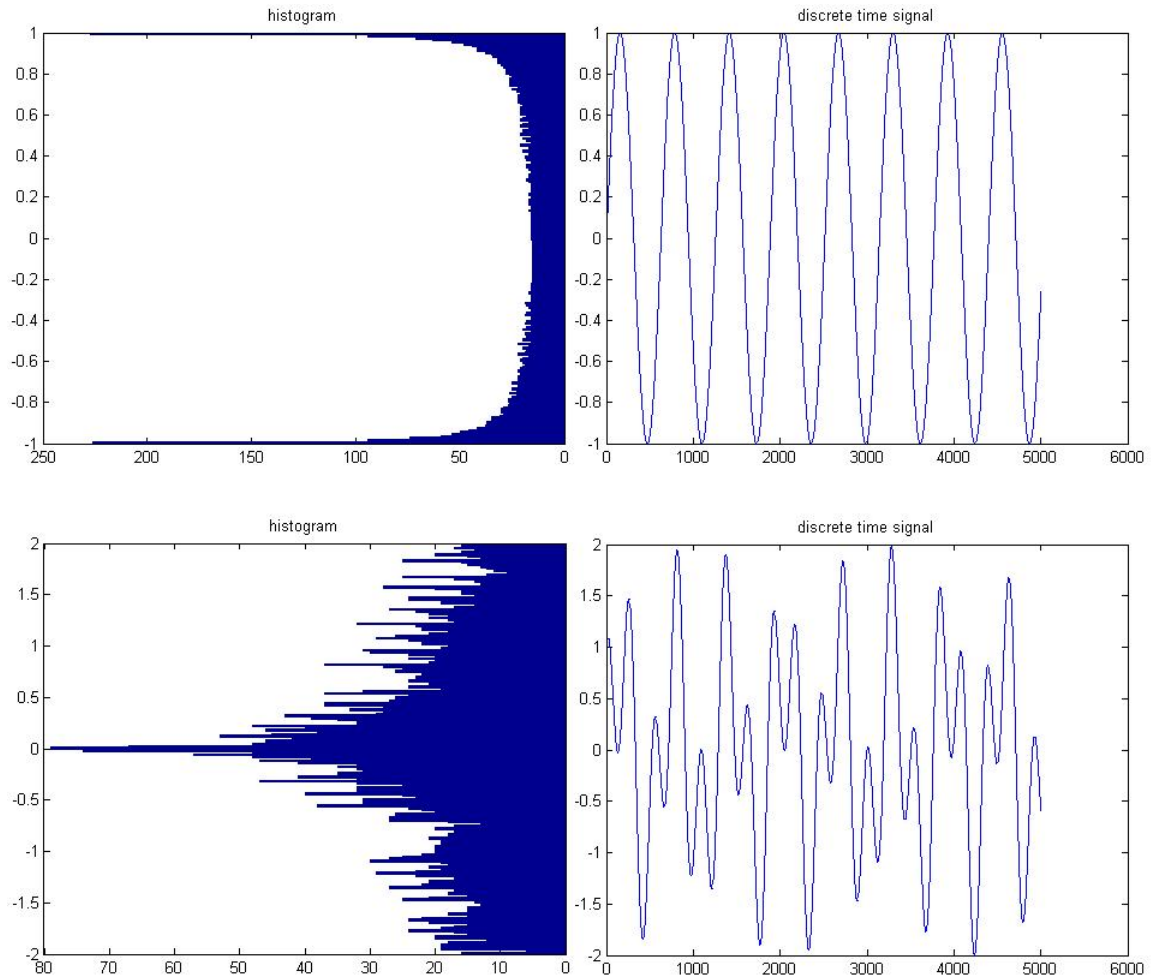


Fig. 7. Examples of discrete time signals and their corresponding histograms (PDF)

## CHAPTER IV

### RESULTS

#### A. Validation of the Proposed Approach

##### 1. A Simple Academic Problem

Typically, attempts to model “mechanical damage” require an increasing number of degrees-of-freedom (DOFs). For example, if a 2 DOF model of a healthy, rotating shaft is used then a 4 DOF model might be needed to model the same shaft with some “damage”, such as a crack. This results in the following observation:

Increased Damage  $\leftrightarrow$  Increase in DOFs  $\leftrightarrow$  Increase in number of states  $\leftrightarrow$  Higher entropy.

To verify this observation, a simple 2 DOF and 4 DOF torsional spring-damper model of a shaft is considered, as shown in Fig. 8. Assuming a set of initial conditions, the signal entropy (both Shannon and relative entropies) for one of the 2 DOF model states was computed using the approach proposed in the previous section. Later, the same set of initial conditions were applied and the signal entropy of the same state was computed again for the 4 DOF model. The transient responses and the probability distributions for the 2-DOF and 4-DOF models are summarized in Fig. 9.

As shown in Tables I and II, the Shannon and relative entropy measures for the 2-DOF model were found to be lower than those of the 4-DOF model. The same trends were observed even when disturbances were applied to these models thereby indicating that as the number of DOFs of a system increases, the entropy of its measured signals increases too.

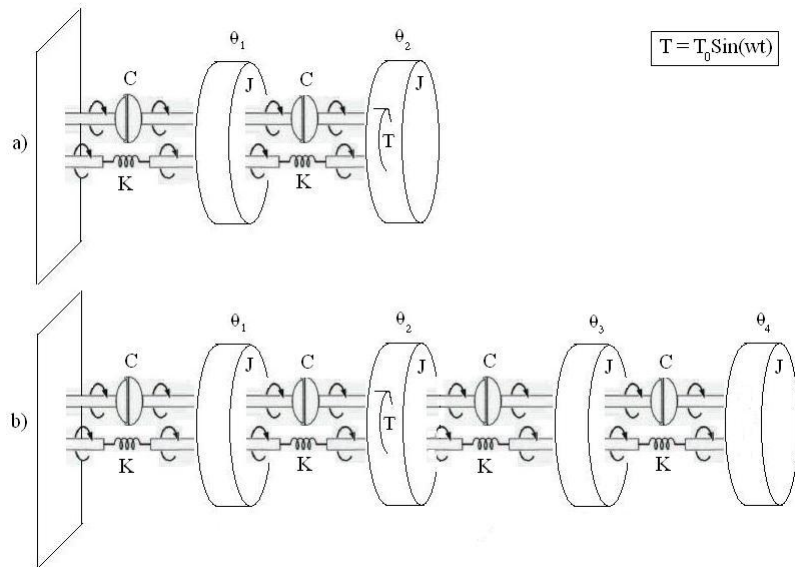


Fig. 8. 2DOF vs 4DOF torsional spring-mass-damper system

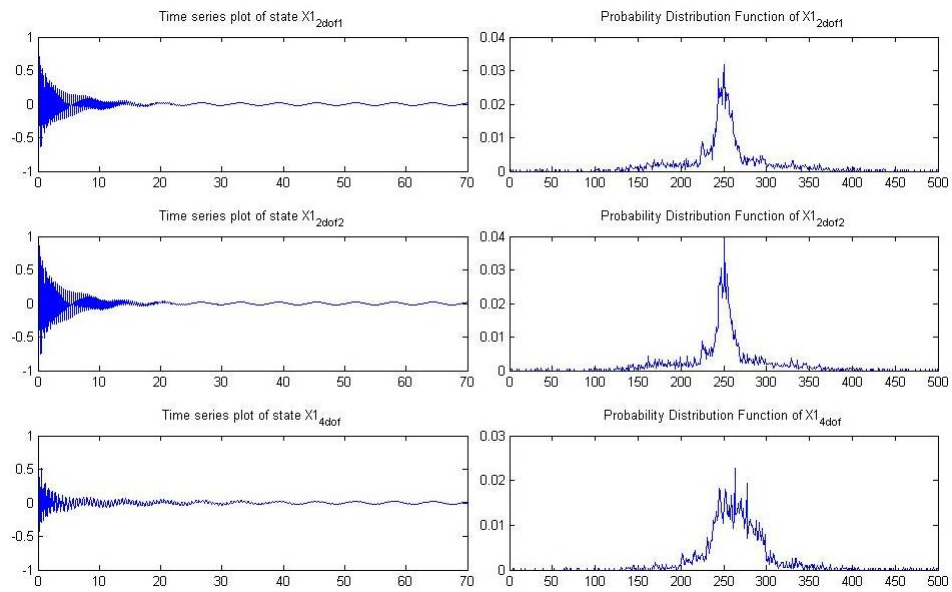


Fig. 9. Time series plots and PDF plots of 2-DOF and 4DOF systems

Table I. Shannon Entropy Measures - Academic Problem

Shannon entropy measures of a state for	Value
2-DOF (model 1)	6.4681
2-DOF (model 2)	6.4281
4-DOF	6.7507

Table II. Relative Entropy Measures - Academic Problem

Relative entropy measures of a state for	Value
2-DOF (model 1) w.r.t 2-DOF (model 2)	0.0826
4-DOF w.r.t 2-DOF (model 1)	0.4640
4-DOF w.r.t 2-DOF (model 2)	0.4943

## 2. A Controlled Laboratory Experiment

For the purpose of validating the proposed fault analysis approach, a test-bed consisting of a 7.5HP, 3 Phase, 208V induction motor (Fig. 10) (3KX07G manufactured by Dayton) was loaded with a synchronous generator and the rotor bearings were damaged in incremental step, as follows [38]:

1. The test-bed was run for about 60 hours and healthy motor data was collected (Fig. 11);
2. Then a shaft current was injected for the next 5 hours. This was done in order to damage the rotor bearing by inducing roughness bearing fault.
3. The shaft current was then removed and faulty motor data was collected for several hours;



4. The above two steps were repeated three more times and three different sets of faulty motor data were generated.

The electrical signals (3-phase voltages and currents) were sampled by an A/D converter (Fig. 10) and then transferred by a DSP to a PC. The magnitude of the samples vary from  $-15,000$  to  $+15,000$  (ADC readout). The signal was filtered to remove the fundamental frequency (60Hz) from all the samples (Fig. 12).

Choosing an appropriate number of steps to quantize the samples, histograms of the filtered samples were generated using MATLAB. Using these histograms, the corresponding probability distribution functions (PDF) were generated (Figs. 13, 14, 15, 16 & 17). PDFs of known healthy samples (Fig. 13) were called  $q(x)$ , while the remaining samples (Figs. 14, 15, 16 & 17) were named  $p_i(x)$ . The Shannon entropy and relative entropy of these samples were computed using equations 3.12 and 3.13.

Observations: The first obvious observation was that the PDF of the healthy signals was uni-modal while the PDF of the faulty signals were bi-modal. Secondly, the separation between the two modes in the PDFs of faulty signals increases as the extent of bearing damage increases (as can be seen from the PDFs of faulty set 1 to set 4).

The average Shannon entropy of the healthy signals (Table III) was 7.8436 and their maximum value was 7.8971. For the different sets of faulty signals, the average Shannon entropy values observed were 7.8853, 7.8608, 7.8601 and 7.9150. This change (of 0.21% to 0.91%) though noticeable was not as sensitive as the change in relative entropy between a healthy and faulty signal.

The average relative entropy of healthy samples (Table IV) was found to be 0.0153, while the relative entropies of the faulty signals were on an average 894.8% higher. The average relative entropy (Fig. 18) of the different sets of faulty data were

0.1668, 0.1340, 0.1210 and 0.1595. The percentage change in relative entropy of the healthy samples (Figs. 19, 20) varied from -36.24% to +201.22%, while that of the faulty ones varied from 390.94% to 1521.9%.

This experiment clearly shows, the effectiveness of signal entropy measures, in particular relative entropy measure, for analyzing bearing faults in induction motors. The next experiment will show the effectiveness of this approach for analyzing failures in electric motors using actual field data.

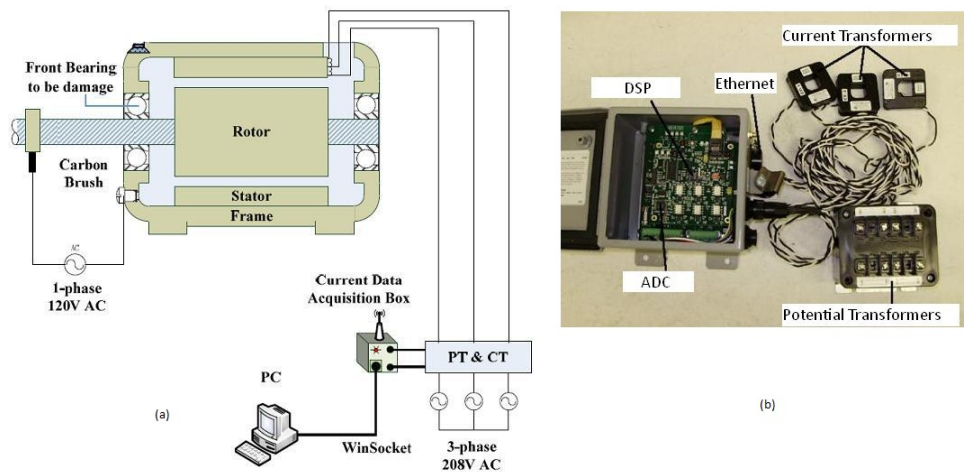


Fig. 10. Test-bed

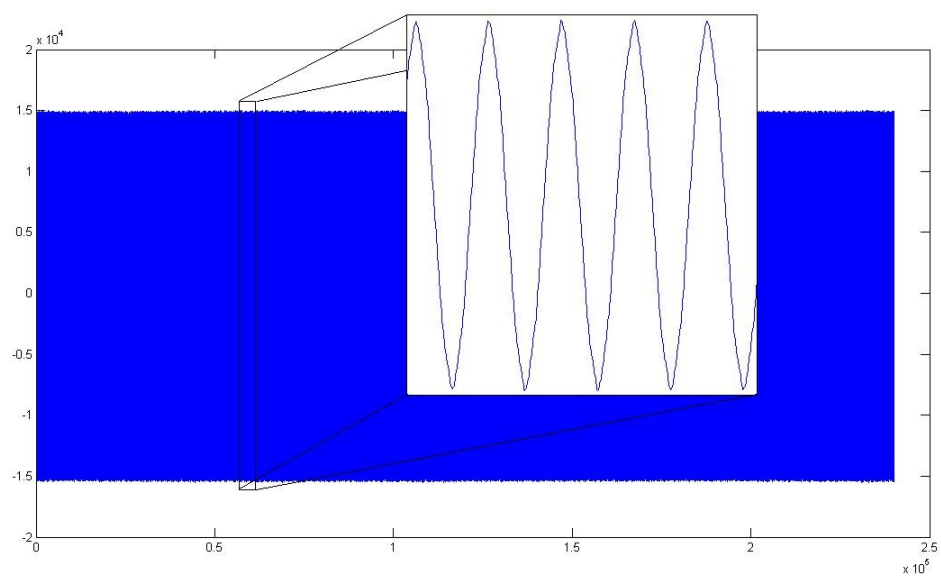


Fig. 11. Healthy signal - time series plot

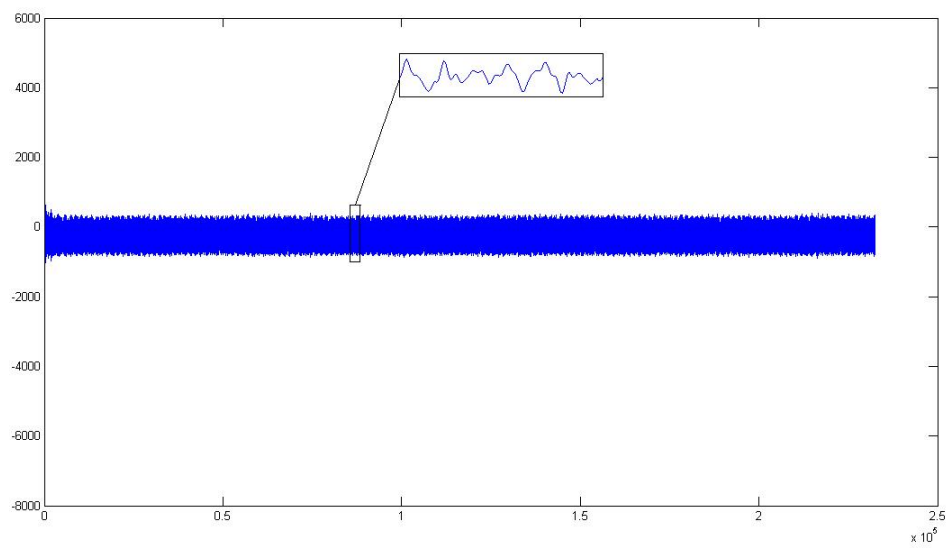


Fig. 12. Filtered signal - time series plot

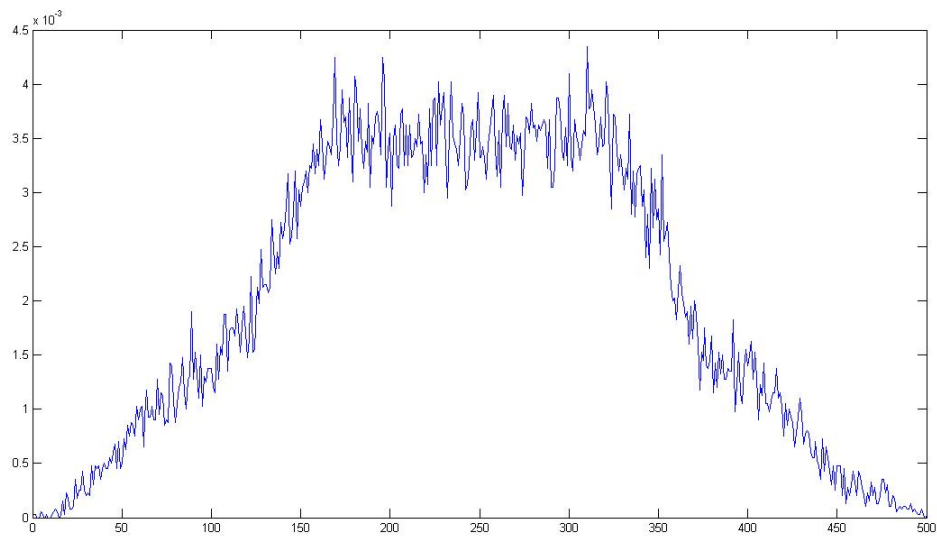


Fig. 13. Probability distribution function of the healthy signal

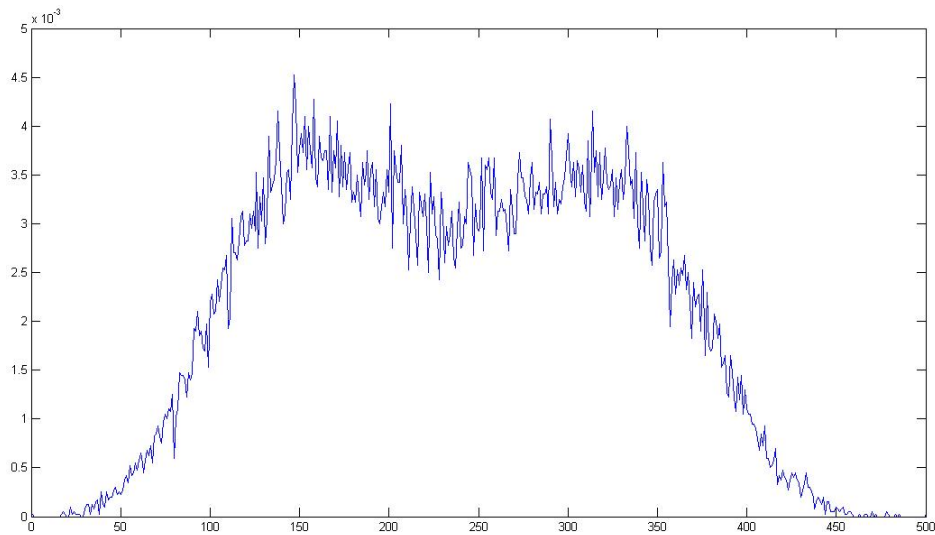


Fig. 14. Probability distribution function of faulty sample from set 1

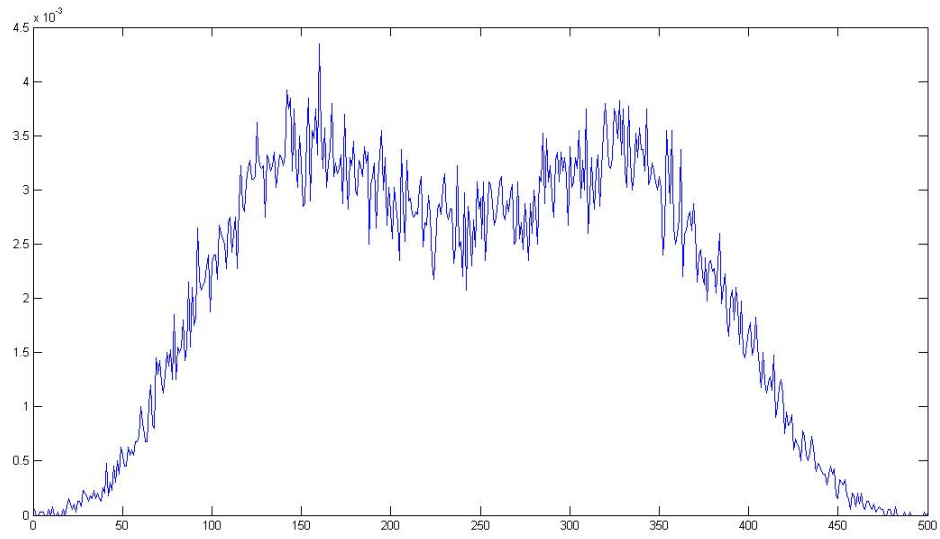


Fig. 15. Probability distribution function of faulty sample from set 2

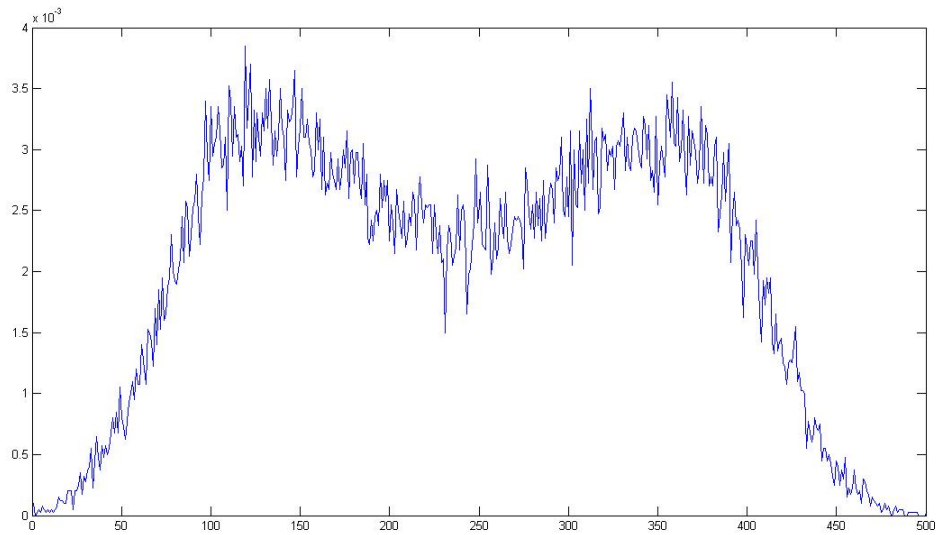


Fig. 16. Probability distribution function of faulty sample from set 3

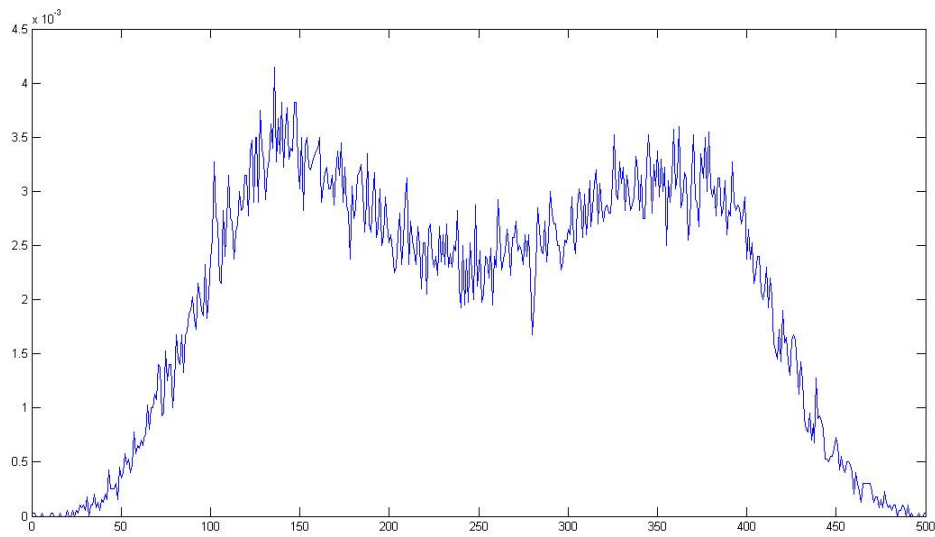


Fig. 17. Probability distribution function of faulty sample from set 4

Table III. Shannon Entropy Measures - Controlled Data Set

Samples	Average Shannon Entropy	Maximum Shannon Entropy
Healthy signal	7.8436	7.8971
Faulty signal-1	7.8853	8.0261
Faulty signal-2	7.8608	8.0260
Faulty signal-3	7.8601	8.0074
Faulty signal-4	7.9150	8.0514

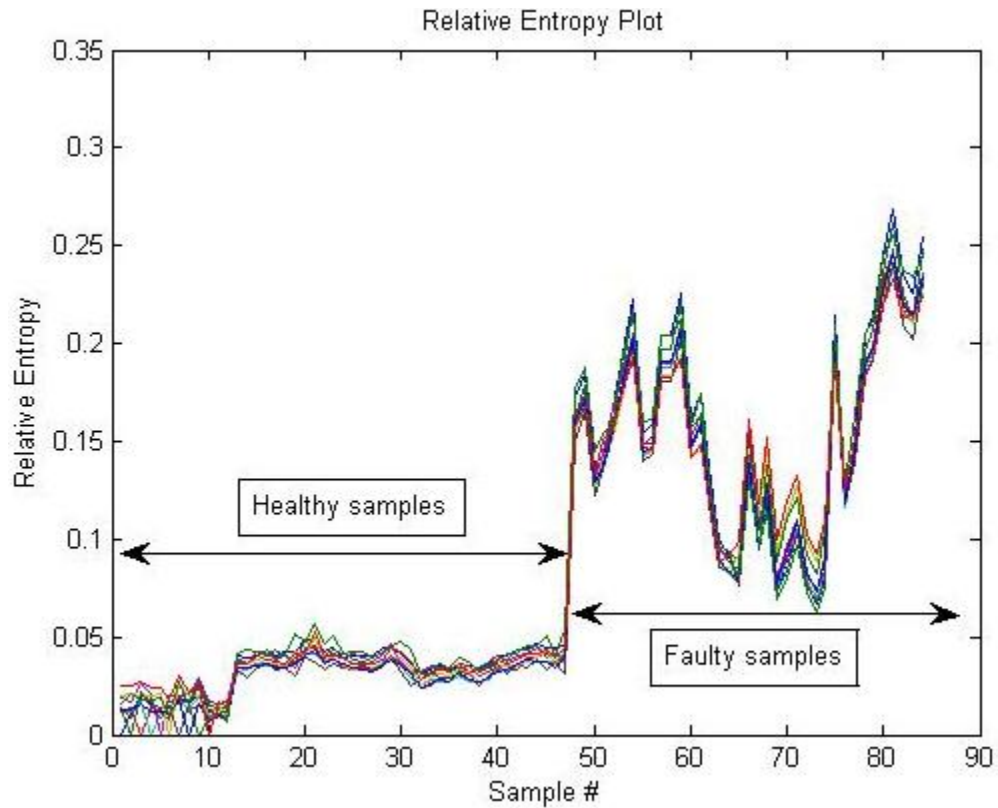


Fig. 18. Relative entropy plot

Table IV. Relative Entropy Measures - Controlled Data Set

Samples	Average Relative Entropy	Maximum Relative Entropy
Healthy signal	0.0153	0.0461
Faulty signal-1	0.1668	0.2061
Faulty signal-2	0.1340	0.2079
Faulty signal-3	0.1210	0.2001
Faulty signal-4	0.1595	0.2484

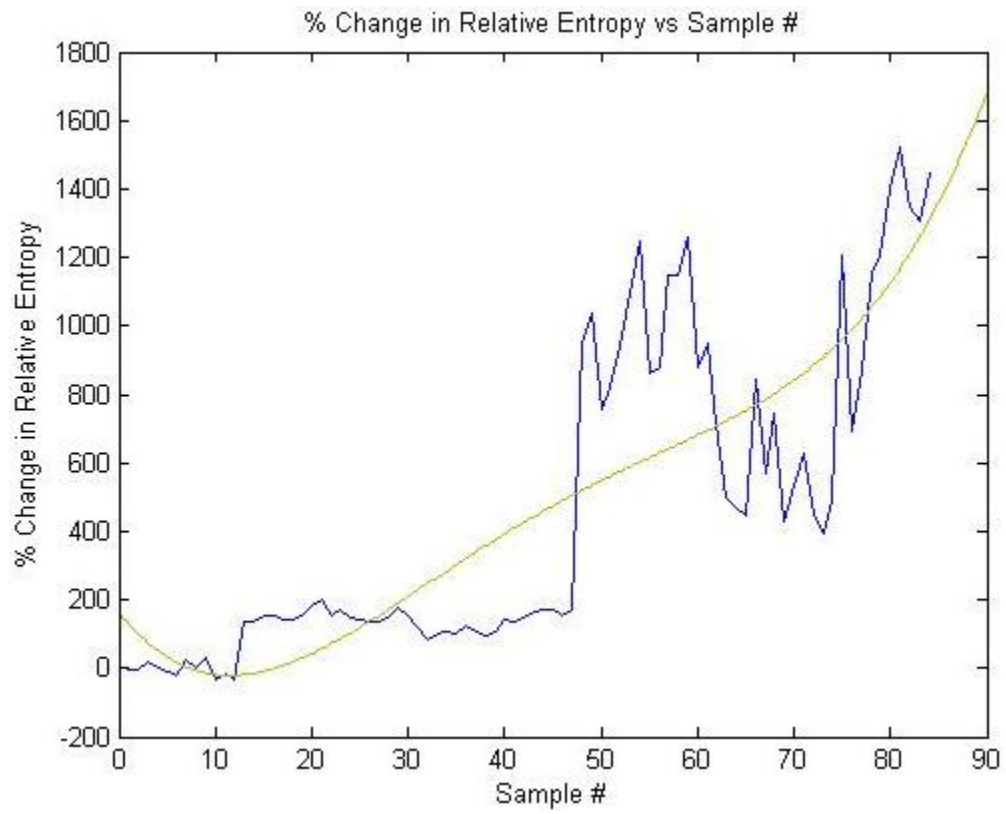


Fig. 19. Percentage change in relative entropy



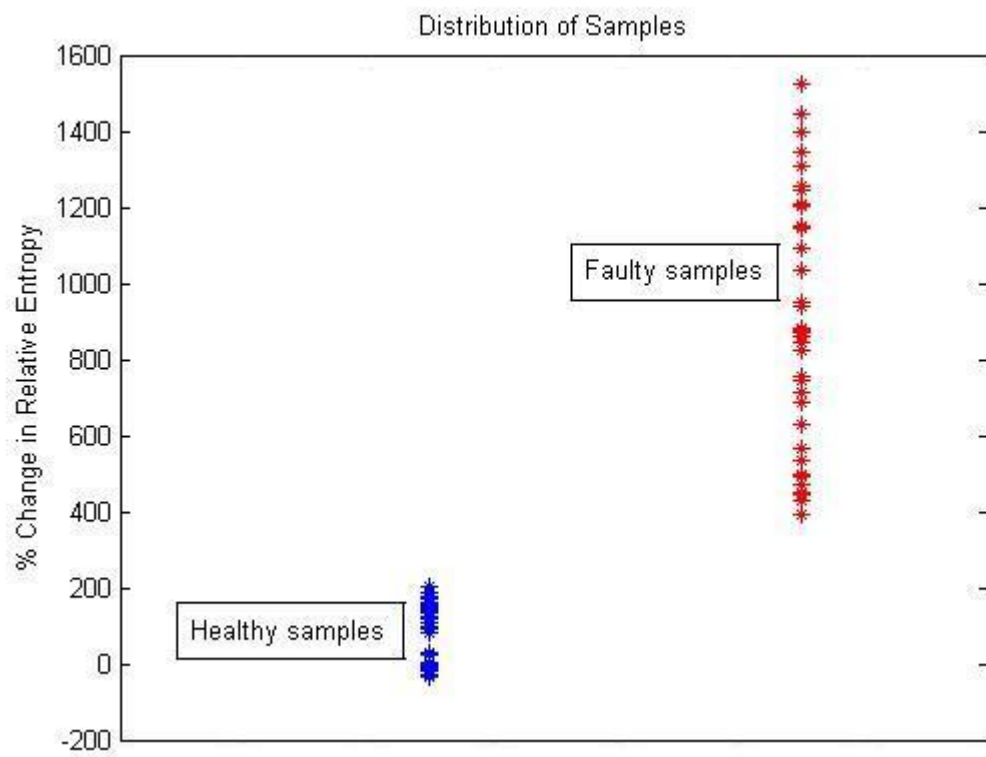


Fig. 20. Relative entropy - Distribution of samples

### 3. Field Data Test

To further validate the proposed fault analysis approach, it was also applied to a set of field data. The data was that of a motor driving a compressor. The bearings of the compressor were known to have failed on 09/08/2010. A repair was attempted and the same motor-compressor system was found to be operational for a little while before failing again on 02/08/2011. The two cases were considered separately and the proposed fault analysis approach was applied; to see if the results would match, what was already known about the system's status at those times. Fig. 21, Fig. 22 and Fig. 23 show that precisely. Therefore, the validity of the proposed fault analysis approach has been verified using both controlled experimental data and actual field data.

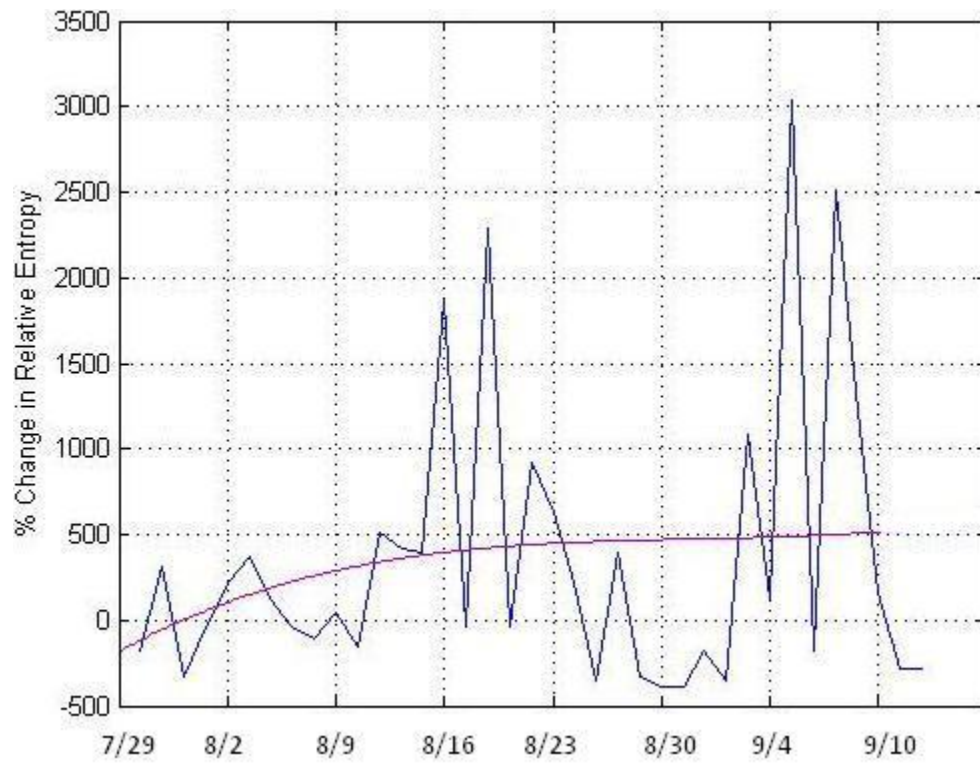


Fig. 21. Percentage change in relative entropy - Field data set 1

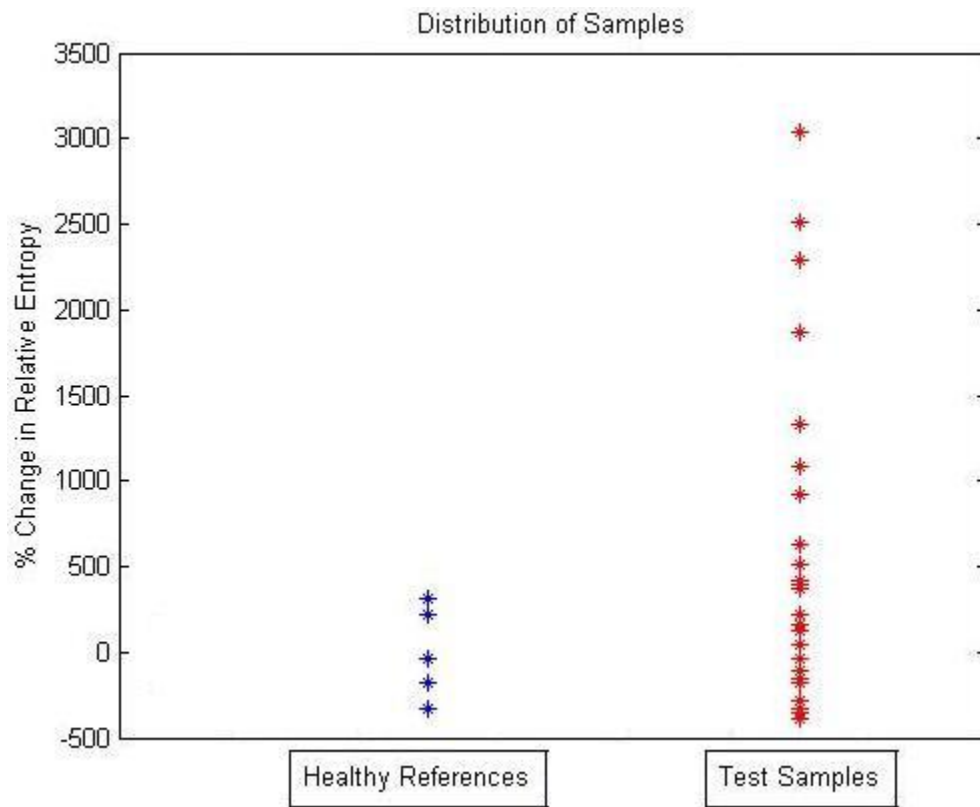


Fig. 22. Relative entropy - Distribution of samples of field data set 1

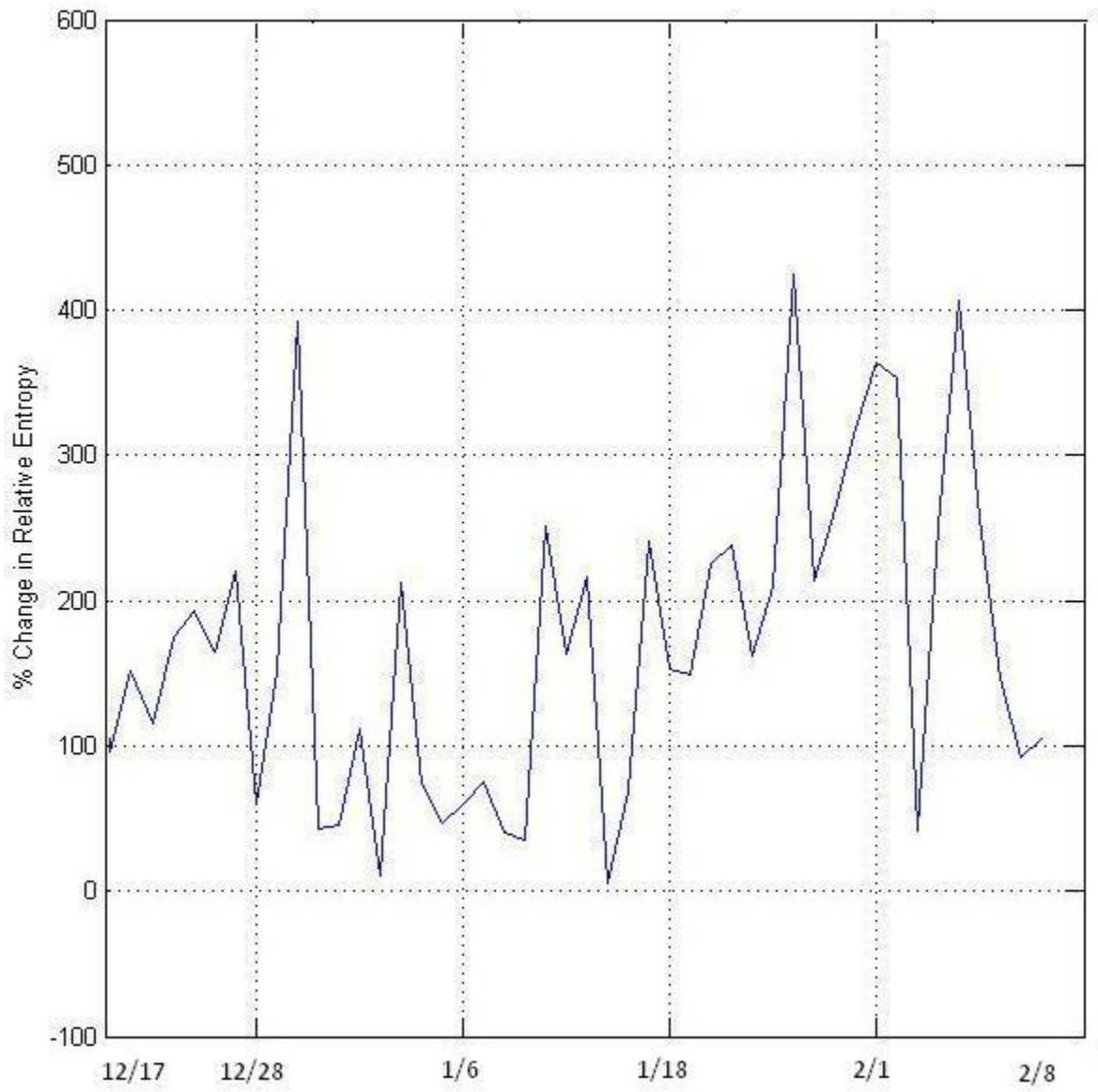


Fig. 23. Percentage change in relative entropy - Field data set 2

## CHAPTER V

### SUMMARY AND CONCLUSIONS

In this chapter, the work of this research is summarized. The conclusions of this research and some suggestions for future work are also presented.

#### A. Summary of Research

Fault analysis of electromechanical systems has been a very important research topic in mechanical engineering. The wide spread use of entropy in various fields of science has proved to be an important tool to measure order and/or disorder. Researchers from meteorology, biotechnology, medicine, mathematics, engineering, statistics etc have successfully used entropy based concepts to solve several challenging problems. Using this as motivation, this research tries to analyze faults in electromechanical systems using information entropy concepts. The objectives of the research were, to develop a method to evaluate signal entropy of a dynamical system using only input/output measurements, and to use this entropy measure to anticipate faults within a dynamical system. Given discrete-time signals corresponding to the three-phase voltages and currents of an electromechanical system being monitored, the problem was to detect whether or not this system was healthy. The concepts of Shannon entropy and relative entropy come from the field of Information Theory. They measure the degree of uncertainty that exists in a system. The main idea was that the system's dynamics may have regularities hidden in measurements that were not obvious to see. Probability distribution functions (PDF) were formed by sampling the time series currents and voltages of a healthy system. The system's health was monitored by sampling the currents and voltages at certain time intervals and generating corresponding PDFs. If the system dynamics were to remain the

same, in other words, the system continued to be healthy, then the relative entropy measures, were found to be minimal and the Shannon entropy measures were found to be close to that of the healthy system. But if the system dynamics were found to change due to some mal-functioning, then the corresponding relative entropy and Shannon entropy measures were found to be relatively higher than the healthy ones. Thus, using the proposed approach, fault analysis of electromechanical systems was carried out.

## B. Conclusions

The conclusions drawn from this research are summarized as follows:

- The proposed fault analysis approach can clearly distinguish a healthy system from a faulty one using Shannon and relative entropy measures.
- The experimental results show that relative entropy measures are much more sensitive to faults than Shannon entropy measures.

## C. Suggestions for Future Work

- The experiments in this research consider the loading of electric motors to be constant. To be more practical, an ideal fault analysis method should be load independent. Further research is needed to study the effectiveness of the proposed approach to varying loads.
- Although bearing faults are the most common cause of failures in electric motors, other failures, like stator windings faults, broken rotor bars etc, cannot be ignored. Further research is needed to study the effectiveness of the proposed approach in analyzing these failures in electric motors.

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