TERRAIN-ADAPTIVE CRUISE-CONTROL: A HUMAN-LIKE APPROACH

A Dissertation

by

NARAYANI VEDAM

Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Chair of Committee,	Shankar P. Bhattacharyya
Co-Chair of Committee,	Reza Langari
Committee Members,	Aniruddha Datta
	Ben Zoghi
Head of Department,	Miroslav M. Begovic

December 2015

Major Subject: Electrical Engineering

ABSTRACT

With rapid advancements in the field of autonomous vehicles, intelligent control systems and automated highway systems, the need for GPS based vehicle data has grown in importance. This has provided for a plethora of opportunities to improve upon the existing vehicular systems.

In this study, the use of GPS data for optimal regulation of vehicle speed is explored. A discrete dynamic programming algorithm with a model predictive control (MPC) scheme is employed. The objective function is formulated in such a way that the weighting gains vary adaptively based on the road slope. Unlike in the prevalent approaches, this eliminates the need for a preprocessing algorithm to ensure tracking along flat stretches of road.

Fuel savings of 0.48% along a downhill have been recorded. Also, the usage of brakes has been considerably reduced due to deceleration prior to descent. This is highly advantageous, particularly in the case of heavy-duty vehicles as they are prone to wearing of brake pad lining. Therefore, this method proves to be a simpler alternative to the existing methods, while incorporating the best attributes of a human driver and the tracking ability of a conventional controller.

NOMENCLATURE

AHS Automated Highway System

VHS Vehicle-Highway System

AT Automatic Transmission

CCC Conventional Cruise Control

CC Cruise Control

PI Proportional Integral

PID Proportional Integral Derivative

ICC Intelligent Cruise Control

ACC Adaptive Cruise Control

SCC Smart Cruise Control

PCC Predictive Cruise Control

LAC Look Ahead Control

MPC Model Predictive Control

TCO Total Cost of Ownership

HDVs Heavy Duty Vehicles

3D Three Dimensional

VSS Vehicle Speed Sensor

UI User Interface

ECM Electronic Control Module

ECC Expert Cruise Control

CVs Commercial Vehicles

ICE Internal Combustion Engine

SI Spark Ignition

TABLE OF CONTENTS

LIST OF FIGURES

FIGURE Page

LIST OF TABLES

1. INTRODUCTION

The economic challenges, constantly dwindling sources of fuel and more-than-ever stringent emission norms facing the entire automotive industry, manifest themselves as both - opportunities and challenges. In response, automotive researchers are constantly devising means that could cater to most or all of them. Among which, fuel efficiency and methods to improve the same have taken on a new urgency.

In the long-haul trucking industry, this is particularly important owing to the fact that fuel accounts for a major share of fleet operating costs. This industry, in particular, is a highly competitive one with small profit margins which greatly depend on the reduction of Total Cost of Ownership (TCO). For example, a 4% saving in fuel and AdBlue is approximately 2900 Euros/year and reduces the TCO by 1.6% [1]. As a consequence, customers buy a truck with the lowest TCO rating.

In order to boost sales, truck manufacturers are resorting to various measures, one among them being concerted efforts towards minimizing fuel consumption. Towards this end, many methods have been identified and executed. Of which, cruise-control systems, especially in Heavy-Duty Vehicles offer potential savings in fuel.

Also, from the study in [3] it is learnt that, Class-8 (GVW $\geq 33,000 kgs$) Commercial Vehicles consume nearly 68% of all Commercial Vehicle fuel used in the United States, even though they comprise less than 17% of the Commercial Vehicle fleet. Nearly 70 % of this consumption is said to occur during trips greater than 100 miles. Conventional Cruise Control is estimated to engage as much as 60% of the driving time of such vehicles. Therefore, an incremental reduction in fuel consumption during Cruise Control could translate into significant fuel savings.

Similarly, the Automated Highway System (AHS) and its potential benefits, has kept the automotive researchers engrossed for some time. It not only promises hands-off and feet off driving but also increased safety, improved fuel economy, and reduced congestion. Many vehicular and highway control systems that aid in achieving this have been identified and are as shown in Figure 1.1, among which Intelligent Cruise Control Systems find a mention.

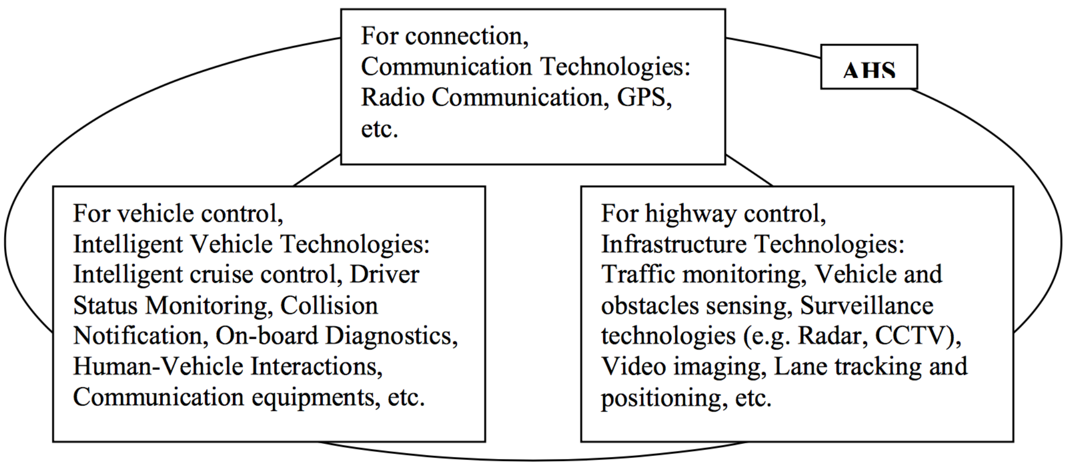

Figure 1.1: The Automated Highway System (AHS) Concept, Cheon S. (2003). An overview of automated highway systems (AHS) and the social and institutional challenges they face. [5]

With the increase in consumer acceptance of such technologies, there is a greater emphasis on the need to refine their design to make them user friendly and to conform with improved safety and emission norms. In subsequent sections, the evolution of cruise-control systems and their growing prominence across different class of vehicles is discussed in detail.

1.1 Speed Control in Automobiles

Cruise Control Systems are a convenient choice on long stretches of road like open highways, and among vehicles equipped with automatic transmission (AT). They serve to maintain steady accelerator pressure yielding constant vehicle speed and improved fuel economy. This also contributes to reduction in driver fatigue and increase in driving comfort.

1.1.1 Evolution of Speed Control

As early as in the 1900s, the technology that was used to control steam engines was adopted for speed control in automobiles [6]. This system contained all mechanical parts and could only hold the throttle at a fixed position. As an improvement, speed controllers with proportional feedback were used; this set the throttle at its maximum when the vehicle speed dropped by a predetermined amount. However, it is evident that such systems not only were ineffective in maintaining a set speed but also, did not assure safety or improved comfort to the operator [7].

In the early 1980s, rapid progress made in microprocessor technology revolutionized the design of vehicle systems, including that of cruise control. The design of Electronic Control Modules (ECMs) generously took to microprocessors. This eased assembly, pushed for component integration and increased reliability, thereby yielding cruise control systems which were easier to operate, robust, and efficient. Such conventional systems comprise a few basic components that include:

- Vehicle Speed Sensor (VSS)
- User Interface (UI)
- Electronic Control Module

- Throttle Acuator

These systems adopted a plethora of control techniques as a means to an end. From [7], it is learnt that the first improvement to the electronic controller was the use of various PID control schemes. With PID control, significant improvement in performance indices - rise time, settling time, steady-state error, and tracking was noticed as compared with the result of proportional feedback controllers of previous generations. To maximize the benefit of a PID controller, optimizing gains became necessary.

Fuzzy controllers, which were growing in popularity drew the attention of automotive researchers owing to their ease of implementation. Unlike in PID control, they offered flexibility in the number and choice of inputs, and their mapping to outputs could be achieved based on user experience and intuition.

The last decade or so, has witnessed rapid progress in speed control techniques. The availability of advanced and powerful micro-controllers have facilitated the adoption of various control methodologies. In Goodrich, M.A, a human-centred approach to determine throttle and brake actuation was proposed [8], while [9], [10], and [11] used a model-based lower-level controller whose control inputs were estimated using vehicle parameters and inverse dynamics. In [12], a gain-scheduling, linear quadratic controller for throttle and brake actuation using a linearized vehicle model was demonstrated. In [13], a model-free method of designing cruise control was laid out.

However, such conventional cruise control systems are deemed fit for use on long, flat stretches of road. On varying terrains, they lack the needed inputs and sophistication to make an optimal choice. Some obvious shortcomings are:

- They are oblivious to impending gradients on a route, rendering them fuel inefficient.

For example, a truck uses a lot of energy while going up a hill at a constant speed. But during descent, by virtue of its weight, energy is available. A lot of savings in fuel can be made if this could be accounted.

- They are unaware of curves along a route, requiring manual intervention.

  The cruise control set speed may not be optimal along curvatures, needing manual braking for safe traversal.

- They are matched to the highest possible gear to minimize fuel consumption, which works well only on relatively flat roads.

The next generation of cruise control systems came to be known as Intelligent Cruise Control (ICC). In addition to speed control, they were also capable of deceleration control and were fused with distance sensors as in Adaptive Cruise Control (ACC) Systems, vehicular data from various power-train sensors, and 3D maps and navigation systems as in Look-Ahead Control Systems. However, in literature, such systems are broadly classified to belong to, (1) Smart Cruise Control Systems (SCC): where reference speed is computed using current vehicle states, (2) Predictive Cruise Control Systems (PCC): where information from the road ahead is used in addition to the vehicle states to determine the reference speed [14].

1.1.2 Human Role in Speed Control

The vehicle and the driver constitute a complex feedback system. The behavior of the vehicle evokes certain reactions from the driver and vice-versa. From [15], it is clear that this 'man-machine' system mostly cannot be demarcated into purely 'mechanical' and purely 'human' components; Instead, they should be treated as a whole.

In [15], [16], [17], and [18] there is extensive discussion on various human driving characteristics, both in terms of limitations as well as attributes. The attributes include control behavior exhibited by drivers during regulatory tasks, driver preview utilization, and adaptation capabilities of drivers when confronted with altered vehicle dynamics and/or changing operating conditions.

Each of the previously mentioned attributes is aided by a suite of sensors. A top to bottom ranking of the primary sensory channels used for driving by humans are:

1. Vision,

2. Vestibular and Kinesthetic,

3. Tactile, and

4. Auditory.

A survey of popular literature leads to a clear impression that - visual aspects of driving are the most important. From [16], [17], [18] it can be learnt that claims referring to driving as depending upon 90% of visual information are not uncommon. Their laboratory simulator studies also demonstrate that most humans can adequately control and navigate vehicles using only vision, even with distorted or inaccurate visual feedback information. So, when faced with different driving scenarios like varying terrain geometry, traffic, wind gusts, fog, . . . etc the capabilities of visual sensors and motion cues like linear/rotational acceleration sensed by vestibular and kinesthetic channels seem reasonable.

The above discussion indicates that a key characteristic feature of human drivers is the ability to look-ahead. The use of preview allows the human driver to not only provide an anticipatory control response, but also plan activities in response to developing situations. For instance, under varying road geometry, a typical driver's reaction would be to,

- ***Accelerate/Decelerate*** before (and/or) during an uphill
- ***Decelerate*** before (and/or) during a downhill
- Pick the right ***gear*** in advance

However, human performance is a variable and depends on a variety of parameters. Some of them being [15]:

- Driver experience,
- Familiarity of a route,
- Required processing time for sensed information,
- Information transmission time,
- Cognitive requirements to anticipate or predict ahead,
- Perceptions of acceleration and so on.

Thus, similar to automated systems, manual anticipatory control also has its share of shortcomings.

1.1.3 Look-Ahead Cruise Control

From the discussion thus far, it can be observed that conventional speed control and manual speed control on their own have some glaring shortcomings. Therefore, it is desirable to combine best human driving practices with a conventional cruise controller to yield reliable and robust anticipatory control. This provides for Look-Ahead/Predictive Cruise Control Systems (PCC).

Cruise Control used in Heavy Duty Vehicles (HDVs) today, are mostly based on PID control and are designed to track the operator set speed. An extension to this is cruise control that allows the vehicle to travel with a range of velocities before braking. This is to preserve kinetic energy when driving downhill [19]. However, since only the velocity error is fed back to the controller, anticipatory control which a human operator is capable of, is amiss.

Look-Ahead Controllers/Predictive Cruise Control use additional information about the situation ahead of the vehicle. In most cases, this information is the road slope. With this appended, the Look-Ahead Control/Predictive Cruise Control has shown to outperform Conventional Cruise Control.

1.1.3.1 Related Work

An early work by Schwarzkopf and Leipnik [20], formulates a feedback algorithm for driving a highway vehicle for minimum fuel consumption. The algorithm is derived from Pontryagin maximum principle to provide a mathematically optimal velocity profile, subject to the driver's choice of a trip time limit. Although the driver is allowed to choose a steady state velocity on a level road, it is modified on varying grades to ensure reduced fuel consumption.

In Hooker, *et al*. [21], a dynamic programming technique and a simulator based on a statistical model of the vehicle's behavior are used to solve the optimal control problem. The results show substantial fuel savings. Similarly, in [22], a dynamic programming approach is used to obtain solutions to a number of driving scenarios on short road sections. Inspired by these results, in [23] & [24], it is shown that constant speed is optimal on a constant road slope. However, this relies on the affine relation between fuel consumption and work produced.

In Huang, *et al.* [25], a gradient based Non Linear Programming (NLP) has been devised to solve for optimal throttle input, gear shifting, and velocity trajectory and to accelerate the convergence of the optimization problem.

Predictive Cruise Control [27] is a system that Daimler Chrysler has developed. In which, an optimal velocity profile for the vehicle up to a certain horizon is calculated using the road topography information. Fuel consumption, time and deviation from the reference velocity are weighed together in a cost function that is minimized over a finite horizon. The velocity profile is periodically communicated to the controller. A fuel saving of 5% was demonstrated for the selected vehicle and road. In [26], the solution of the optimal control problem is carried out by a combination of combinatorial search and a shooting algorithm. Also, in Hellstrom, *et al.* [28], a Predictive Cruise Controller is developed and discrete dynamic programming is used to numerically solve the optimal control problem.

Expert Cruise Control (ECC) [29], is an algorithm studied at Scania. Here, the entire route is classified into different terrain types - uphill, downhill and flat. Different control algorithms are employed based on the terrain. A fuel saving of up to 3.4% has been recorded.

In [30], a Model Predictive Control Scheme (MPC) is used to determine the velocity profile. Like in [27], the desired fuel injection amount, gear and brake level are all determined by minimizing a cost function with several addends over a horizon. Also, fuel saving of 2.5% has been recorded.

ECOROLL in [31], is a fuel efficient way of traveling down a hill. Here, Look Ahead data is used to predict the speed of the vehicle and thereby improve performance of the controller. It suggests a rule-based control strategy rather than a cost function. However, to determine a fair result, a cost function is used to regard both travel time and fuel consumption. A fuel saving of 3.4% was recorded.

1.1.4 Observations

From the methods discussed earlier and a few others in literature, some general observations have been made. They are tabulated in Table 1.1.

Particulars	Observation
Vehicle Type	*Mostly* Heavy-Duty Vehicles
Transmission	Automated Manual Transmission *Only*
Control Strategy	Model Predictive Control - Finite Horizon
Optimization Technique	*Mostly* Dynamic Programming
Implementation	Software Change
Relevant Products In Market	**Scania** - Opti-Cruise, Active Prediction, Eco-Roll, **Mercedes Benz** (Actros) - Eco-Cruising **Volvo** (FH) - I-See, I-See Extended, Eco-Roll
Others	1. Experiments with *unloaded* HDVs have not yielded any major fuel savings. 2. Considerable savings in fuel consumption observed only with *loaded* trucks along hilly terrain.

Table 1.1: Key Observations on Existing Approaches.

The existing methods have shown to yield promising results. As previously indicated, incremental fuel savings mean a great deal to not only the consumer but also the environment. Also, Dynamic Programming (DP), Non-Linear Programming (NLP) and several other optimization techniques have been employed which have proven to yield optimal results.

However, the use of non-linear programming algorithms for dynamic systems with both discrete and continuous parts, leads to great complexity. Therefore, dynamic programming algorithms for the computation of optimal trajectories is a more preferred approach.

The use of dynamic programming algorithm also has disadvantages. With increase in the range of input space and the number of inputs, the computational complexity increases manifold. This is also called the curse of dimensionality. To address this, existing methods use a pre-processing algorithm to shrink the input space. In this study, a single state is considered with alterations in the formulation of the objective function.

1.2 Thesis Objectives

The main aim of this study is to implement a controller whose regulatory actions,

- *resemble that of a conventional cruise control system along long-flat stretches of road, and*
- *mimic the best attributes of human driving along a varying terrain.*

These actions are chosen in such a way that the following requirements are met.

Drivability & Comfort - A reasonable change in velocity within a prescribed range, thereby reducing human intervention.

Fuel Economy - Ensuring that the power-train operates in the most fuel-efficient region at all times.

Simplicity - Needing minimal - computational resources and - implementation impact.

1.3 Thesis Outline

In the following section, the basic constituents of a vehicle, model predictive control scheme and dynamic programming algorithm are elaborated. In section 3, the proposed methodology is described in detail and in section 4 the simulation results are presented.

2. PRELIMINARIES

2.1 Power-Train

Automotive power train, is an important vehicular subsystem. It includes the engine, clutch, gearbox and the final drive. Unlike in passenger cars, the design of power train in commercial vehicles is highly customized to suit the application. This is to ensure an economical selection from the standpoint of cost, maintenance, fuel consumption and life of the vehicle. Therefore, to study changes in velocity of the vehicle, it is important to model these components accurately.

A conventional power-train and its constituents that will be discussed are as shown in Figure 2.1.

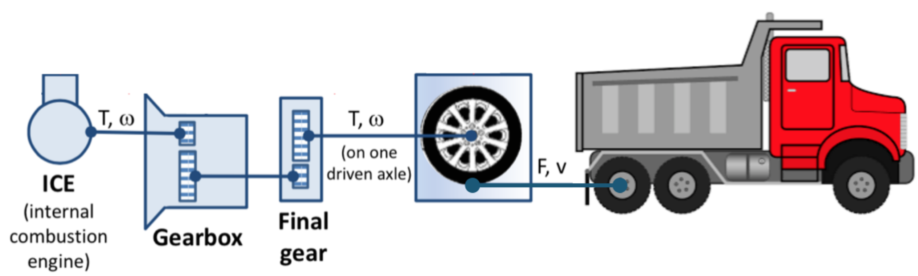

Figure 2.1: A Conventional Automotive Power-Train, Jacobson, B. (2012). Vehicle Dynamics. [37]

2.1.1 Prime Mover

2.1.1.1 Engine Model

Let the engine produced torque be T_e and the load on the engine due to clutch be T_c. Then, the resulting force-balance equation is,

$$J_e\ddot{\theta}_e = T_e - T_{loss} - T_c \tag{2.1}$$

where, J_e is the mass moment of inertia of the engine and θ_e is the flywheel angle. For the sake of simplicity, loss due to drag torque, coolant pump, and other accessories are ignored and the torque that the engine can deliver is only limited by its angular velocity.

The engine torque T_e obtained through polynomial approximation of the test measurements is given by [30],

$$T_e(P,N,G) = \begin{cases} a_eN + b_eP\delta_{max}(N) + c_e, & P > 0, G \neq 0 \\ a_dN + b_d, & P \leq 0, G \neq 0 \\ 0, & G = 0 \end{cases}$$

where,

a_e, b_e, c_e, a_d & b_d, are all constants,

δ_{max}, is the maximum fueling,

N, is the engine speed, and

G, is the gear position.

2.1.2 Transmission

The transmission includes all those components that deliver torque from the engine to the wheels. They include clutch, gearbox, propeller shaft, final drive, and drive shaft.

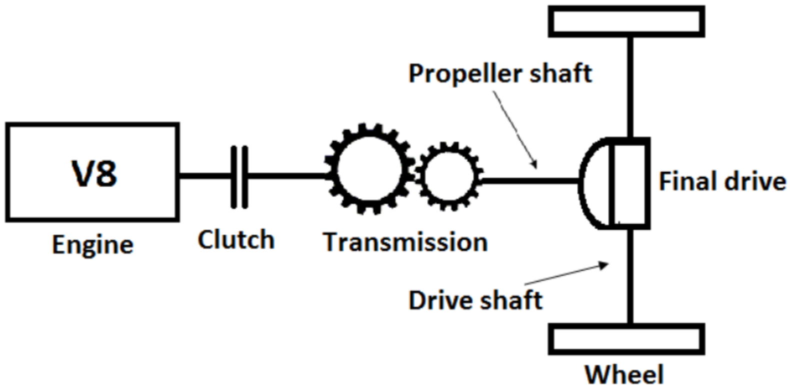

Figure 2.2: Drive -Train Components, Lechner G. *et al.* (1999). Automotive transmissions: fundamentals, selection, design and application.[38]

2.1.2.1 Transmission Model

A 12-speed manual gearbox with automated shifting is used. An appropriate gear is chosen based on vehicle speed and a driver request on the accelerator pedal. A host of assumptions are made and are as listed below.

- *Clutch*
  - The clutch is engaged and stiff at all times.
  - Neutral gear is not modeled.
  - Therefore, the relation between clutch torque (T_c) and transmission torque (T_t) is simply,

$$T_t = T_c. \tag{2.2}$$

- *Transmission Inertia* is not modeled.
- Any gear G, is mapped to a corresponding conversion ratio (i_t) and an efficiency (η_t) which is assumed a constant for all gears. With this, torque at the propeller

shaft (T_p) is given by,

$$T_p = i_t \eta_t T_t. \tag{2.3}$$

- The propeller shaft is assumed to be stiff and the torque at the final drive (T_f),

$$T_f = T_p. \tag{2.4}$$

- The final drive is modeled with a constant conversion ratio (i_f) and an efficiency (η_f). The drive torque (T_d) is then,

$$T_d = T_f. \tag{2.5}$$

- Also, it is assumed that the drive shafts are stiff and the vehicle is traveling with equal velocity on both wheels. Then, the torque at the wheels (T_w) is,

$$T_w = T_d. \tag{2.6}$$

2.2 Brakes

Vehicles are equipped with several systems that can either individually or in combination deliver the needed braking force. They include [37],

- Service Brake System

  Brake Pedal and/or Anti-lock Brake System (ABS)/Electronic Stability Control (ESC), which apply brake pads to brake disc/drum

- Parking Brake System

  Lever/Button when pressed apply brake pads to brake disc/drum normally on the rear axle

- Engine Brake System

  Negative propulsion generated at high speeds by the Internal Combustion Engine (ICE)

- Retarder/Auxiliary Brake for Heavy Duty Vehicles (HDVs) .

However, for the purpose of analyses the total brake force available at the wheels is modeled as a function of brake pedal position and is given by:

$$F_B = Brake\_Pedal\_Position * F_{B,max} \tag{2.7}$$

2.3 Longitudinal Vehicle Dynamics

To be able to design a controller for any system it is highly important to understand its underlying dynamics, and a vehicle is no exception. For the purpose of its analyses, it is a common practice to use a lumped parameter model. Such a model, treats distributed mechanical properties of mass (kg), stiffness (N/m), and damping (Ns/m) as concentrated at an imaginary location *i.e.*, the Center of Gravity (CG) of the rigid body.

The vehicle motion is typically described in terms of its velocities (*i.e.* forward, lateral, vertical, yaw, roll, and pitch) in the vehicle-fixed coordinate system as referenced to an inertial reference frame. This body-fixed coordinate system, moves with the vehicle which is assumed to be rigid [39]. Such a coordinate system is depicted in Figure 2.3. However, for our purpose it would suffice to consider the longitudinal dynamics of the vehicle alone.

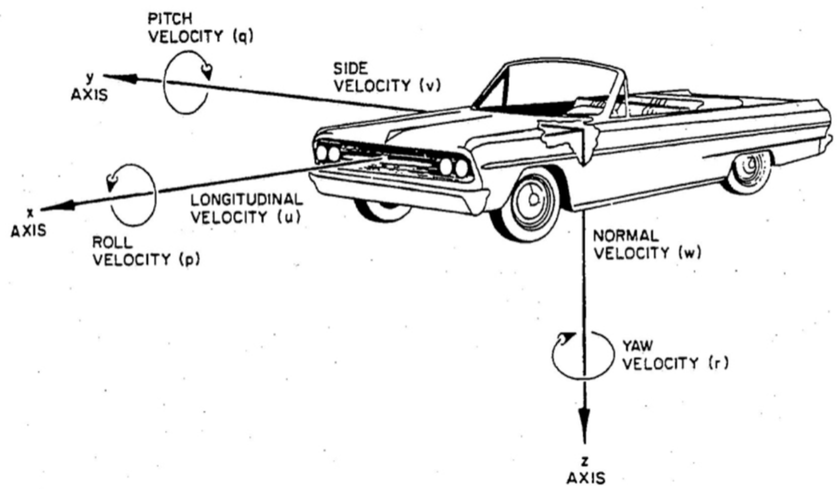

Figure 2.3: Vehicle-fixed Coordinate System, Ulsoy A. G. *et al.* (2012). Automotive control systems. [39]

Axis	Translational velocity	Angular displacement	Angular velocity	Force component	Moment component
x	u (forward)	ϕ	p or $\dot{\phi}$ (roll)	F_x	M_x
y	v (lateral)	θ	q or $\dot{\theta}$ (pitch)	F_y	M_y
z	w (vertical)	ψ	r or $\dot{\psi}$ (yaw)	F_z	M_z

Table 2.1: Vehicle-fixed Coordinate System: Symbols and Definitions, Ulsoy A. G. *et al.* (2012). Automotive control systems. [39]

2.3.1 Driving Resistance

From the previous discussion, it can be interpreted that higher speeds are possible with low transmission ratios, *i.e.* in higher gears. On extrapolating, an extremely high speed at a very low transmission ratio is plausible. However, something obviously stops the vehicle from attaining such high velocities, and the limit comes from driving resistance. They act opposite to the direction of movement of the vehicle, often determine the power needed for forward motion. They include:

1. Rolling Resistance (F_{roll}),
2. Air Resistance (F_a), and
3. Gradient Resistance (F_g).

2.3.1.1 Rolling Resistance

It is due to resistive forces acting on a rolling wheel. It is a function of velocity, wheel load, tyre pressure and tyre type.

$$F_{roll} = c_r * M * g * cos(\alpha) \tag{2.8}$$

where,

c_r , is coefficient of rolling resistance,

M , is mass of the vehicle (Kg),

g , acceleration due to gravity (m/s^2), and

α , road gradient (rad).

2.3.1.2 Air Resistance

Flow of air around and through the vehicle is very important for cooling and ventilation. However, there is a downside to it, in that, it offers some resistance to movement of the vehicle. This aerodynamic drag is a quadratic function of the longitudinal component of wind speed relative to the vehicle. For aerodynamic loads that resist forward motion of the vehicle the following equation is used.

$$F_a = \frac{1}{2} * \rho * c_d * A * v_x^2 \tag{2.9}$$

where,

ρ , is the density of air,

A , is the frontal area,

c_d , is the drag coefficient, and

v_x , is the flow rate.

The flow-rate $v_x = u + v_w$, is the sum of longitudinal velocity of the vehicle and longitudinal component of wind velocity. Most of the driving resistance calculations assume calm, *i.e.* $v_w = 0$ yielding, $v_x = v$.

2.3.1.3 Gradient Resistance

In addition to the above mentioned forces, there is one other opposing force which grows in significance when going up a hill. This is owing to grade of the hill or gravitational load on the vehicle and is given by,

$$F_g = M * g * sin(\alpha) \tag{2.10}$$

where,

M , is the mass of the vehicle (kg),

g , is the acceleration due to gravity (m/s^2), and

α , is the road grade (rad).

However, while on a downhill, this force is negative and propels the vehicle.

The total driving resistance (F_r) which is comprised of the above mentioned forces, is given by,

$$F_r = M * g * [c_r * cos(\alpha) + sin(\alpha)] + 0.5 * \rho * c_d * A * u^2. \tag{2.11}$$

2.3.2 Vehicle Model

The translational motion of a vehicle in the forward direction is largely influenced by forces acting towards and against it. This is characterized by the longitudinal vehicle model and is obtained on combining equations [2.1] - [2.12],

$$v = \dot{\theta}_w r_w, \tag{2.12}$$

$$\dot{v} = z_1\Big(z_3 T_e - F_{B,max} B - z_2 v^2 - z_6 sin(\alpha + z_7)\Big) \tag{2.13}$$

where,

$z_1 = \frac{r_w}{J_w + m r_w^2 + \eta_f i_f^2 \eta_t i_t^2 J_e}$,

$z_2 = 0.5 \rho c_w A r_w$,

$z_3 = \eta_t i_t \eta_f i_f$,

$z_4 = \frac{30 i_t i_f}{\pi r_w}$,

$z_5 = \frac{n_{cyl}}{n_r}$,

$z_6 = m g r_w \sqrt{(1 + c_r^2)}$,

$z_7 = \arctan(c_r)$, J_w, is the inertia at the wheels,

n_{cyl}, is the number of engine cylinders, and

n_r, is the number of crankshaft revolutions/stroke.

It can be observed that the longitudinal dynamics of the vehicle system as represented by [2.12] is non-linear in the forward velocity (u). Also, there are two factors critical to the design of a good speed controller: (1) Parametric uncertainty, the vehicle weight in particular, which is subject to change, and (2) External disturbances due to road grade and air drag.

Each of these topics has drawn the attention of automotive researchers for some time now. The literature even shows extensive work on dynamic estimation of vehicle weight, especially that of Heavy Duty Vehicles owing to potential fuel savings. Similar is the case with road geometry and the following section dwells on it in greater detail.

2.4 Fuel Consumption

The fuel consumed by the engine is a function of engine speed, accelerator pedal position and gear positions. It is given by [30],

$$\dot{m}_f(P, N, G) = \begin{cases} c_f P N \delta_{max}(N), & G \neq 0 \\ c_f N_{idle} \delta_{idle}, & G = 0 \end{cases}$$

where,

$c_f = \frac{n_{cyl}}{6e4 n_r}$,

n_r, is the number of crankshaft revolutions/stroke,

n_{cyl}, is the number of cylinders,

$\delta_{max} = a_\delta N^2 + b_\delta N + c_\delta$, is the maximum fueling,

N_{idle}, is idle engine speed, and

δ_{idle}, is idle fueling.

2.5 Road Gradient

In this study, short, artificial road sections are used to evaluate the behavior of the controller. Of all its characteristics, road gradient (α) is an important geometric characteristic which is relevant to the problem of speed regulation in vehicles. It is defined as the ratio of the difference in altitudes at any two points on the road to the difference in their positions. This is illustrated in Figure 2.4.

$$\alpha = \frac{Vertical\ Projection}{Horizontal\ Projection} \tag{2.14}$$

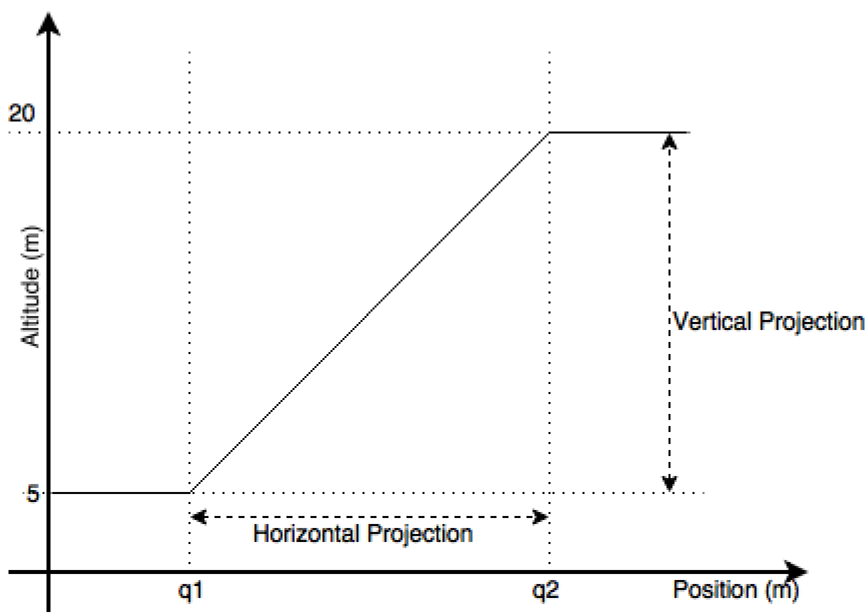

Figure 2.4: Road Grade(α)

The road grade can either be positive or negative to denote ascending and descending road sections. During an ascent, by virtue of its weight, the vehicle experiences a positive gradient resistance (F_g). However, during descent, the gradient resistance (F_g) aids vehicular motion.

The work in [40]-[44] propose various methods for real-time determination of road grade. The predicted road grade is used for the estimation of several vehicular parameters and control. However, in this study, it is assumed that the vehicle is equipped with a sufficiently accurate road grade database and a navigation system for localization.

2.6 Control Methodology

2.6.1 Model Predictive Control

An accurate model of the system and a suitable cost function formulation are two basic requirements for a typical Model Predictive Control approach. The main principle of Model Predictive Control is to compute a control signal that minimizes the objective function. The objective function is formulated as a function of system states. A typical controller uses the following methodology:

- The possible outputs for a fixed prediction horizon are computed using the model of the system.
- The formulated criterion is optimized with respect to the control input.
- The first instance of the control input alone is considered and the rest are discarded.
- At the next sampling instance, the above steps are repeated sequentially.

For reasonable predictions, the model employed should be sufficiently accurate and computationally tangible. Therefore, the model together with the cost function are instrumental in determining the algorithm employed for optimization.

From the discussion in the previous sections, it can be observed that the resulting model of the system is hybrid; a dynamic system with continuous and discrete parts. The velocity, acceleration, and brake levels constitute the continuous part of the system. However, the gear signal is discrete. In essence, the system is a non-linear hybrid system.

With such a system, all optimization algorithms render great complexity. Since dynamic programming handles constraints on the state and control spaces with ease, discretization of the system facilitates formulation of this problem as a shortest path problem. Therefore, a MPC scheme with dynamic programming is employed.

In studies [24], [28] & [30], a pre-processing algorithm is employed to minimize the search space. For every step, a feasible set of velocities are computed and the minimum and maximum permitted velocities are updated at each stage. This is done to ensure that the velocity is always greater than the set value. As an alteration, in this approach, the weights in the objective function are modified based on the knowledge of the disturbance.

2.6.2 Dynamic Programming

Consider a discrete dynamic system given by,

$$x_{k+1} = f_k(x_k, u_k, w_k),\ k = 0, 1, ..., N-1 \tag{2.15}$$

where,

$x_k \in X_k$, is the state

$u_k \in U_k$, is the control input, and

$w_k \in W_k$, is the disturbance.

A sequence of functions $\{\mu_0, \mu_1, \mu_2, ..., \mu_{N-1}\}$ that map a state x_k to a control action u_k, is called a policy π.

$$u_k = \mu_k(x_k) \tag{2.16}$$

The set Π contains all policies that satisfy,

$$\mu_k(x_k) \in U_k, \forall x_k, k \tag{2.17}$$

Each admissible policy $\pi \in \Pi$, is evaluated using a function β which determines its cost. This is also called the cost function.

Now consider an initial state x_0 and a policy $\pi \in \Pi$, the associated cost is,

$$J_\pi(x_0) = \beta_N(x_N) + \sum_{k=0}^{N-1} \beta_k(x_k, u_k, w_k) \tag{2.18}$$

The policy that minimizes the above objective is called the optimal policy π^*.

2.6.2.1 Principle Of Optimality

Consider the system in equation [2.15]. Let π^* be an optimal policy. Let us assume that π^* has been used upto stage i. Then at state x_i, the problem is to minimize the cost-to-go from i to N [30].

$$J^*(x_i) = \beta_N(x_N) + \sum_{k=i}^{N-1} \beta_k(x_k, u_k, w_k) \tag{2.19}$$

For which, the truncated policy $\{\mu_i^*, \mu_{i+1}^*, ..., \mu_{N-1}^*\}$ remains optimal. In essence, an optimal policy to a problem is comprised of optimal solutions to its sub-problems.

2.6.2.2 Dynamic Programming Algorithm

On assuming that the disturbance in equation [2.15] is known, the system is deterministic. With a finite state space, such a system can be represented by a directed graph. The arcs in the graph are indicative of transitions between states in successive stages and there is a cost $c_k^{i,j}$ associated with every such transition. This is illustrated in Figure 2.5 below.

The dynamic programming algorithm is adopted from [45] and is as follows.

Let,

$c_k^{i,j}$, be the cost associated with the transition from $x_i \in X_k$ to $x_j \in X_{k+1}$,

c_N^i, be the terminal cost associated with state $x_i \in X_N$.

Then,

1. $J_N(x_i) = c_N^i$, $x_i \in X_N$

2. Let $k = N - 1$, then

3. $J_k(x_i) = \min\limits_{x_j \in X_{k+1}} \left(c_k^{i,j} + J_{k+1}(x_j) \right)$

4. If $k \geq 0$, repeat steps $1, 2, 3$ for $k = N - 2, N - 3, ..., 1$.

5. The optimal cost at x_0 is $J^*(x_0)$.

6. The corresponding policy is the optimal policy π^*.

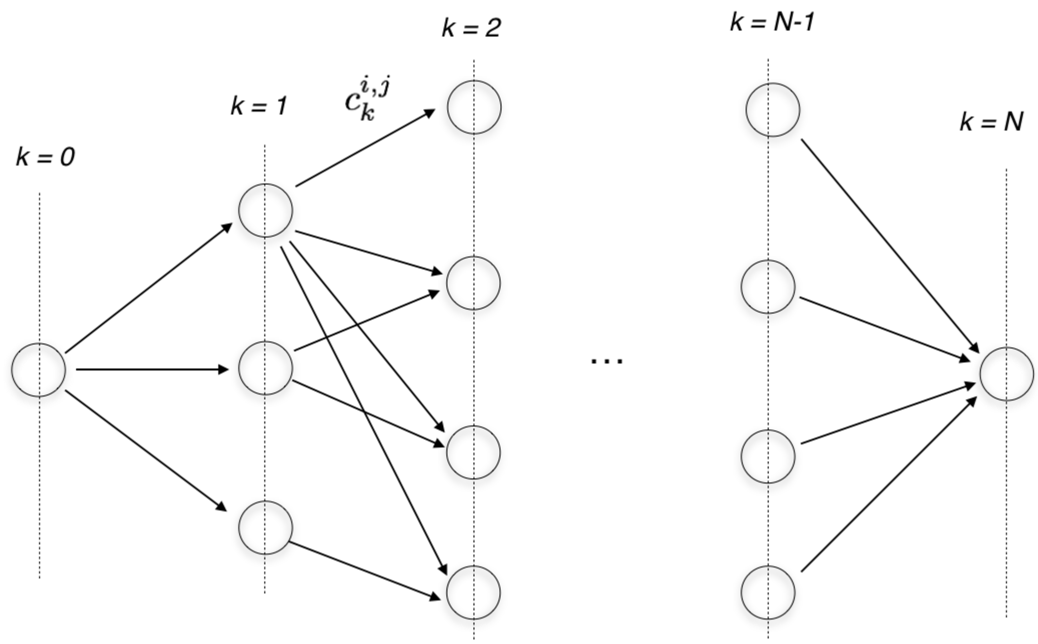

Figure 2.5: Evolution of States in a Deterministic System

3. PROPOSED METHODOLOGY

In this section, the formulation of the cost function, the discretization of the model, inputs, outputs, constraints and the control algorithm are discussed in detail. The differences between existing methods and the proposed approach are also highlighted.

A comparison of a conventional cruise control system with a look-ahead controller is shown in Figure 3.1. From the Figure, it can be observed that in a predictive controller the vehicle takes cognizance of its position with respect to its surrounding. This is achieved with the aid of an on-board navigation system and a road slope database of sufficient accuracy. For the sake of simplicity, it is assumed that the route traversed by the vehicle during a trip is known in advance and the total time of travel is set by the driver.

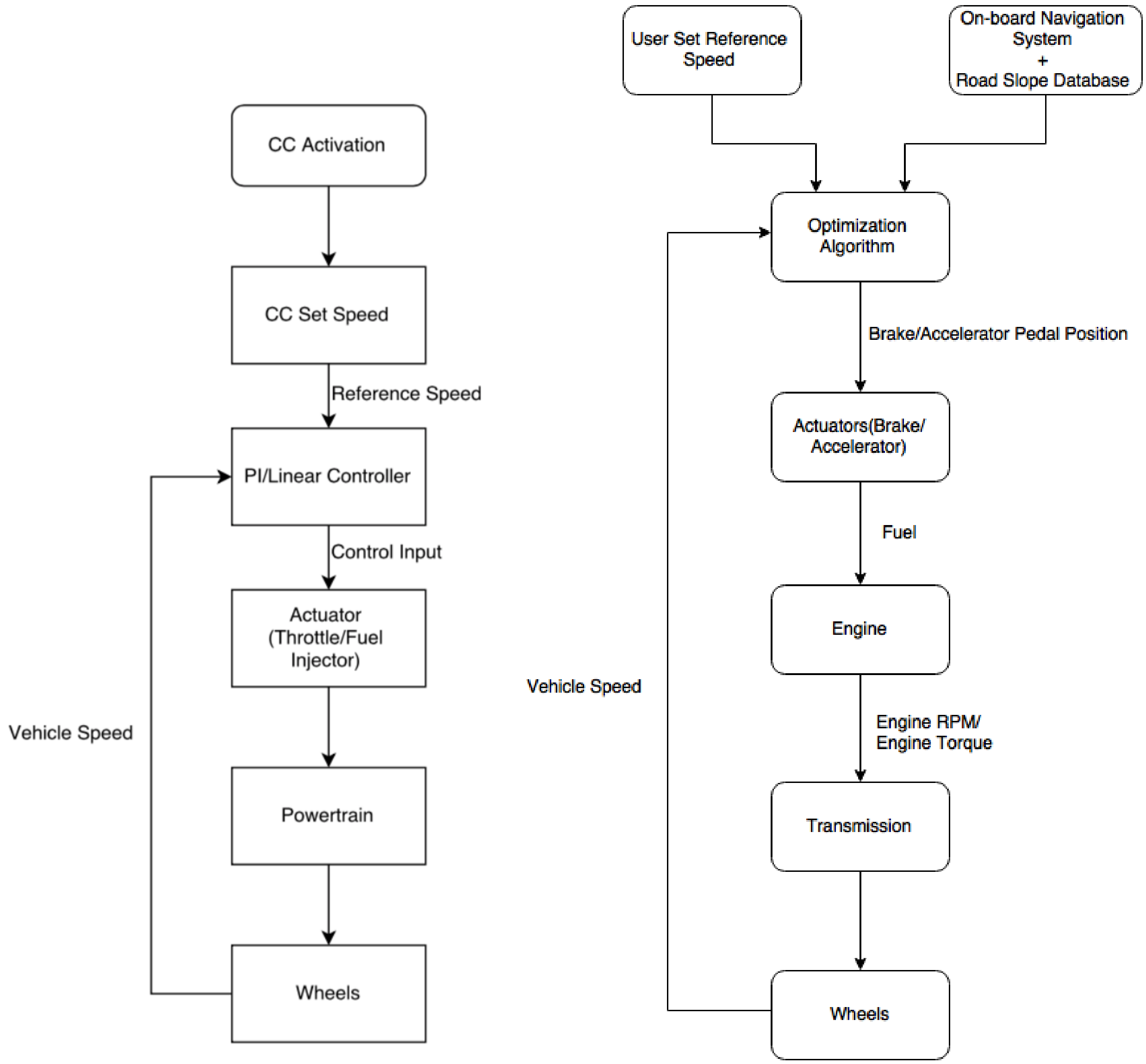

Figure 3.1: Conventional Cruise Control and Predictive Cruise Control Systems

3.1 Model

Consider the drive line model given by the equation [2.13]. The velocity constitutes the only state of the system and is given by,

$$\dot{v} = f_1(v, u, \alpha) \tag{3.1}$$

The accelerator and brake pedal are inputs to the system and the corresponding input vector is,

$$u = [P \ B]^T \tag{3.2}$$

The outputs include velocity and fuel flow,

$$y_1 = v, \tag{3.3}$$

$$y_2 = f_2(v, u, \alpha) \tag{3.4}$$

The fuel consumption model is as described in section [2.3]. The road slope data are dependent on the position of the vehicle which necessitates a change in coordinates,

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{1}{v}\frac{dv}{ds} \tag{3.5}$$

3.1.1 Discretization

To limit the number of cost computations at each stage, it is important to discretize the state space. The accelerator and brake pedal inputs are not discretized, instead, given the start and finish states at each stage they are computed using inverse dynamics of the system using equation [3.7].

Let N be the number of stages, each of length Sm. Each stage is further divided into M equal parts. Then, the step-size h, is given by $\frac{S}{M}$. During each stage S, the control inputs and the disturbance are assumed to be a constant.

Using Euler's method of numerical integration, the system model represented by equation [2.13] is discretized with step length h. This yields,

$$v_{k+1} = v_k + \frac{h}{v_k} f_1(v_k, u_k, \alpha_k), \;\; k = 0, 1, 2, ..., M, \;\; v_k > 0 \tag{3.6}$$

$$m_{f,k+1} = m_{f,k} + \frac{h}{v_k} f_2(v_k, u_k, \alpha_k), \;\; k = 0, 1, 2, ..., M, \;\; v_k > 0 \tag{3.7}$$

3.1.2 State Space & Constraint

The velocity v is restricted between a minimum value, $v_{min} = v_{ref} - 5$ and a maximum value, $v_{max} = v_{ref} + 5$.

$$v_{min} \leq v \leq v_{max} \tag{3.8}$$

Since the vehicle may not be able to achieve the lower limit at all times, it is treated as a soft constraint. However, presuming that the vehicle is equipped with an effective brake system the violation of the upper bound is highly penalized. The entire state space is discretized in steps of γ and values in between are treated with suitable interpolation.

3.1.3 Inputs & Constraints

The accelerator pedal (P) and brake pedal (B) inputs are not discretized. They are computed using the system equation [3.7] and the drive line model in equation[2.13]. Given the current state v_k at stage k and state v_{k+1} at stage $(k+1)$ the accelerator pedal input is determined with $B = 0$,

$$P = \frac{v_{k+1} - v_k - Tz_1z_3a_eN - Tz_1z_3c_e + Tc_2v_k^2 + Tz_1z_6sin(\alpha + z_7)}{Tz_1z_3b_e\delta_{max}}, \;\; B = 0 \tag{3.9}$$

All feasible transitions are those for which $P \in [0,1]$. In violation of this condition and when $|v_{k+1} - v_k| < \delta$, a feasible brake pedal input B is sought with $P = 0$.

$$B = \frac{-v_{k+1} + v_k + Tz_1z_3a_eN + Tz_1z_3z_e - Tz_1z_2v_o^2 - Tz_1z_6sin(\alpha + z_7)}{Tz_1k_B}, \;\; P = 0 \tag{3.10}$$

Here, $T = \frac{h}{v_k}$. A feasible transition is one for which $B \in [0,1]$. If there exists no valid P and B, then the cost of the corresponding transition is set to an extremely high value. However, the control inputs are set to the last known feasible value.

3.2 Objective Function

The main distinction of this method from the ones presented in [24], [28] & [30] is in the formulation of the cost function. Consider the cost function β given by,

$$\beta = [L_1(\alpha), L_2(\alpha), L_3(\alpha), L_4(\alpha)] \begin{bmatrix} m_{f,k} \\ e_k^2 \\ |v_k - v_{k+1}| \\ B_k \end{bmatrix} \tag{3.11}$$

where,

$L_1(\alpha)$, is the weighting factor associated with fuel consumption $m_{f,k}$.

$L_2(\alpha)$, is the weighting factor associated with deviations in speed from the reference value, $e_k = |v_{ref} - v_k|$.

$L_3(\alpha)$, is the weighting factor associated with rate of change in velocity $|v_k - v_{k+1}|$.

$L_4(\alpha)$, is the weighting factor associated with usage of brakes B_k.

All deviations in velocity are considered and penalised. To ensure comfortable driving experience, the rate of change in velocity (acceleration) is also incorporated in the objective function. It can be observed that the weighting factors are dependent on the disturbance α. By varying these gains based on the terrain, different objectives are met. These objectives are drawn from the desired attributes of both conventional speed control and human driving behavior. They are as summarized in Table [3.1].

Terrain	Conventional Control	Human Behavior
Flat	Constant Speed Tracking	-
Before Ascent	-	Acceleration
	-	Appropriate Gear Choice
On An Uphill	-	Acceleration
Before Descent	-	Deceleration
On A Downhill	-	Coasting

Table 3.1: Desired Regulatory Behavior of Conventional Cruise Control & Humans

To be able to realize these regulatory actions using an optimal controller, the penalties - L_1, L_2, L_3 & L_4 are varied in accordance with the terrain. This distinguishes the proposed method from other approaches in the literature. They use a fixed-weight objective function which necessitates the determination of a feasible transition. This is needed to ensure a terminating state with a speed greater than the initial start speed.

3.3 Control Algorithm

Let us consider a N stage problem as represented in Figure 3.2 below. For the purpose of illustration, let q be the position of the vehicle.

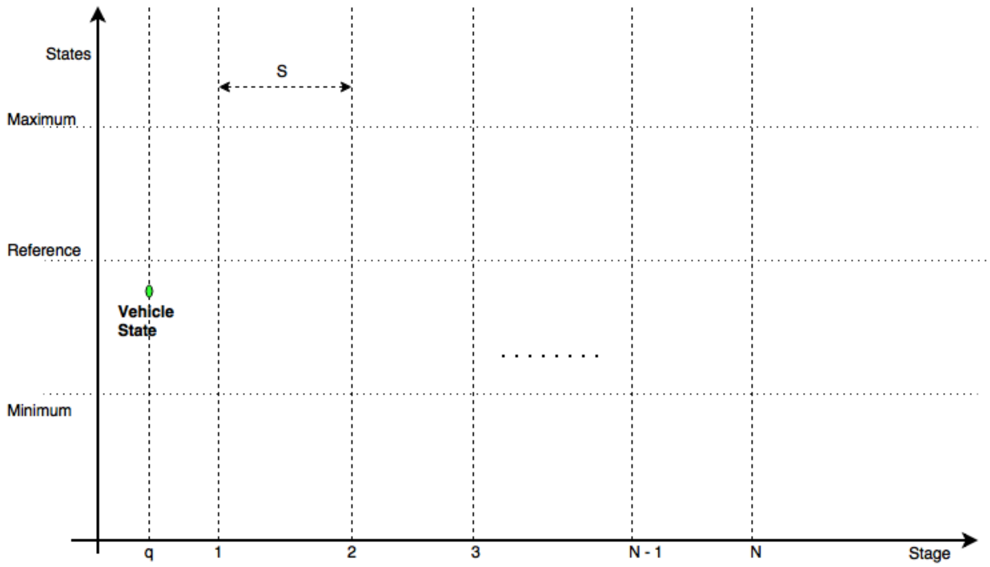

Figure 3.2: N Stage Problem

Then, the model predictive control scheme entails sequential execution of the following steps.

1. Given the vehicle position at q m and the corresponding state v_q, the optimal policy π^* for the interval $[q + S, q + 2S]$ is computed. The time available for the same being the interval $[q, S + q)$.

2. Only the first value of the optimal policy π^*, say $\mu_{q+S}(v_q)$ is stored while the rest are discarded.

3. At $(q + S)$m, the optimal control input $\mu_{q+S}(v_q)$ is applied.

4. During the interval S, the corresponding input $\mu_{q+S}(v_q)$ and the disturbance α are assumed to be a constant.

5. The steps 1-4 are repeated once every Sm.

The optimal policy π^* is generated using a dynamic programming algorithm. This includes:

1. Set the terminal cost $J_N(v_i) = 0$, $\forall\ v_{i,N} \in X_N$.

2. Take one step backward, $k = N - 1$.

3. Then, the optimal cost of a feasible transition from state $v_i \in X_k$ is $J_k(v_i) = \min_{v_j \in X_{k+1}} \{\beta_k^{i,j} + J_{k+1}(v_j)\}$

4. $\forall k > 0$, repeat the above step.

5. The set of control actions that yield an optimal cost constitute the optimal policy π^*.

The system dynamics and a MPC scheme with dynamic programming algorithm, are implemented in Julia. The parameters used for simulation are as tabulated in Table 4.1. A prediction horizon of 1500m is chosen with a step size of $S = h = 50$m. Simulations which were carried out with a step-size of 25m did not indicate any significant improvement in performance. Therefore, there are 30 stages in each grid with a step-size of 50m each.

Parameter	Description	Value
S, h	Step size	50m
N	Number of steps	30
γ	State Space Discretization	0.1
v_{max}	Upper bound on velocity	90 km/h
v_{min}	Lower bound on velocity	80 km/h
v_{ref}	Reference velocity	85 km/h

Table 4.1: Simulation Parameters

The values of the penalties in the objective function are determined iteratively. The ones that yield a desired performance over the entire course of simulation are chosen. The selected values are as in Table 4.2

Parameter	Uphill	Downhill	Flat stretch	Fixed-Weights
L_1	1	2.5	1	2
L_2	2	1	5	5
L_3	15	15	15	15
L_4	0	100	0	100

Table 4.2: Penalties Associated with the Objective Function

The simulation is carried out using three different controllers:

1. PI controller,
2. Optimal controller with a fixed-weight objective function, and
3. Optimal controller with a varying-weight objective function.

Each of these controllers are evaluated on an artificial stretch of road. Uphill and downhill scenarios are treated separately. The simulation results and observations are presented subsequently on a case-by-case basis.

To be able to quantify the savings in fuel,

$$\Delta_{fuel}(\%) = \frac{m_{f,VW} - m_{f,PI}}{m_{f,PI}} \times 100 \tag{4.1}$$

4.1 Uphill

An artificial road segment of length 2500m with an ascent which is 500m long is considered. A road grade of 3% is chosen. The performance of the three controllers in this scenario are discussed subsequently.

4.1.1 PI vs Varying-Weights Control

The performance of a PI controller on an uphill, in comparison with the performance of a varying-weights controller is as shown in Figure 4.1 below. In the case of a PI controller, during ascent, a significant drop in vehicle speed is observed. In some cases, this leads to a down-shift in gears which could cause the speed to drop even further. In contrast, in case of a varying-weights controller, the momentum gained through acceleration prior to an ascent holds the vehicle speed in good stead. Also observed is the cumulative savings in fuel of -0.44% on a 2.5km stretch.

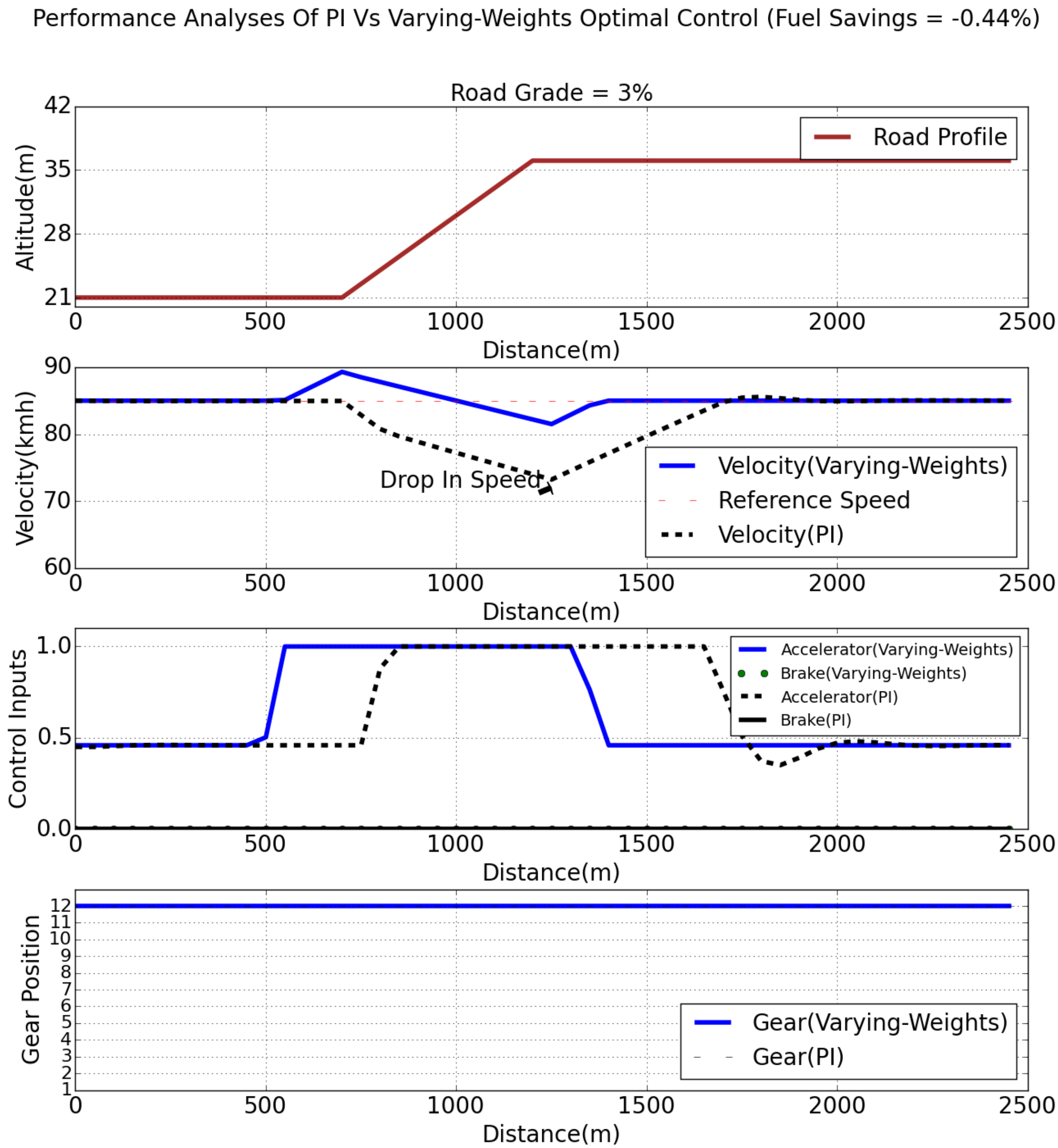

Figure 4.1: PI vs Varying-Weights Optimal Control

4.1.2 Fixed-Weights vs Varying-Weights Optimal Control

These two controllers are compared to illustrate the decline in speed towards the terminal states. In Figure 4.2, it can be observed that there is a sharp decline in speed towards the end of the stage. This necessitates a preprocessing algorithm which can provide a set of feasible transitions. These transitions always result in speeds greater than the initial speed.

The decline in speed is because there are no restrictions/cost associated with the states at the terminal stage. Due to this, the algorithm seeks a state which yields an optimal transition. This always happens to be the lowest allowed speed (if it can be achieved in the same gear) as the amount of fueling needed is the least. However in the varying weights case, the emphasis on fueling(L_1) along a straight road is not as significant as the the one on deviation in velocity from the reference value(L_2). This always ensures speeds as close to the reference speed as possible.

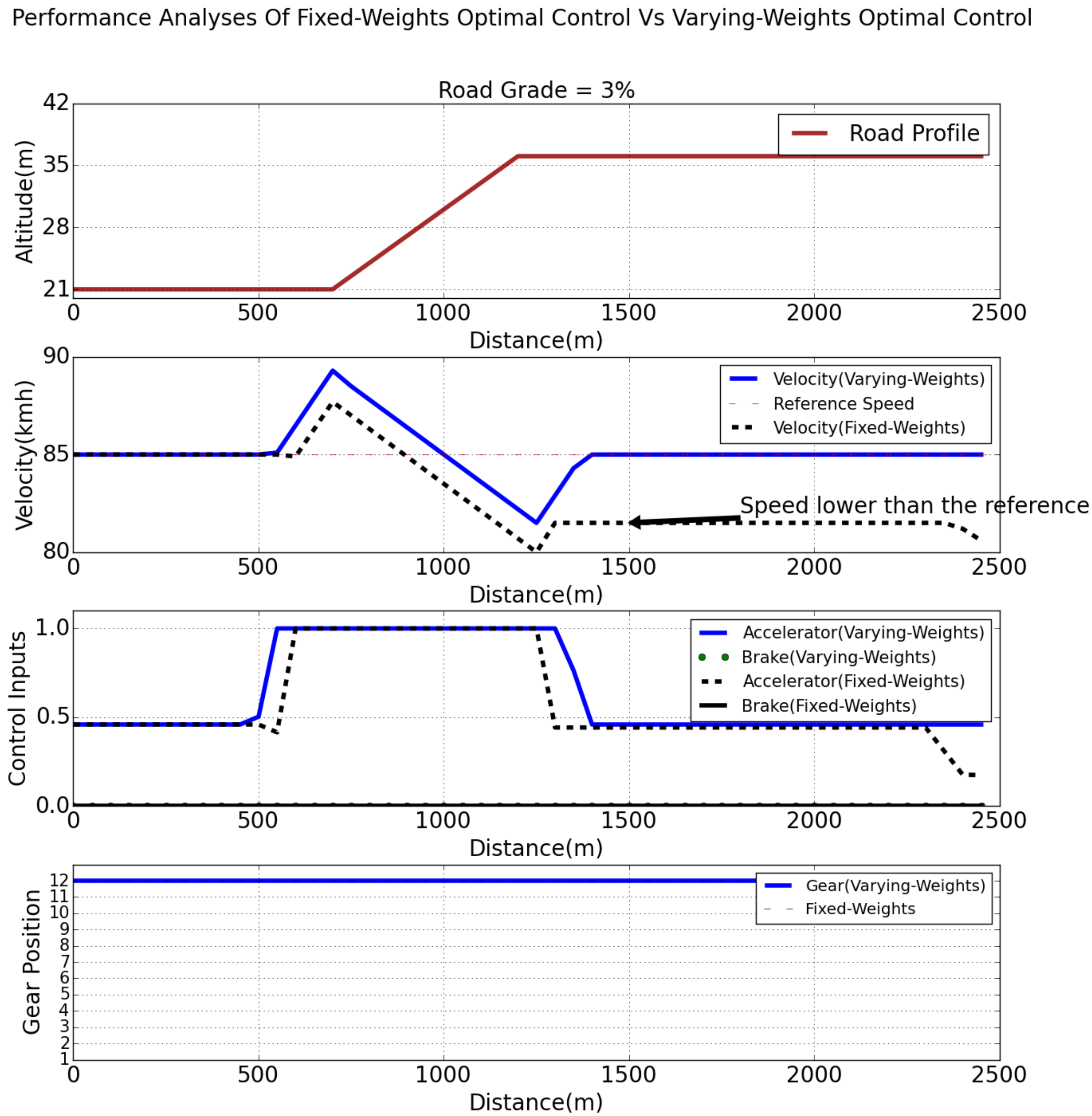

Figure 4.2: Fixed-Weights Optimal Control vs Varying-Weights Optimal Control

4.2 Downhill

An artificial road of length 2500m with a 500m stretch of descent is considered. A road grade of -3% is chosen. The performance of the three controllers in this scenario are discussed subsequently.

4.2.1 PI vs Varying-Weights Optimal Control

The performance of a PI controller on a downhill in comparison with the performance of a varying-weights controller is as shown in Figure 4.3 below. By virtue of its weight, the vehicle gathers momentum during descent. This necessitates the use of brakes as shown in the Figure. However, in case of varying-weights optimal control, there is significant reduction in speed while approaching a descent. This lowers the need for braking, which is highly desirable in the case of heavy vehicles. Also, even in this case a savings in fuel of -0.48% is recorded.

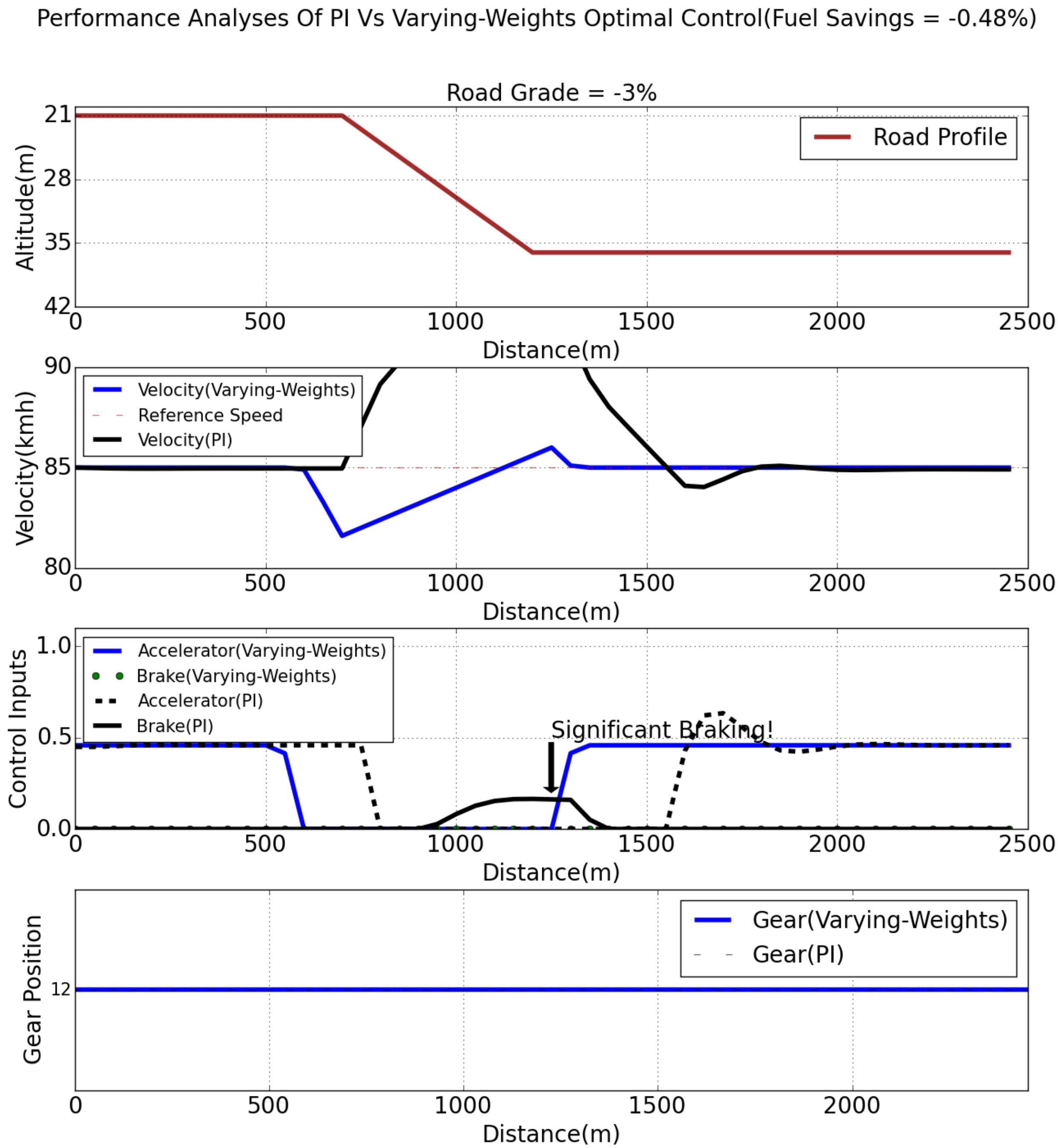

Figure 4.3: PI vs Varying-Weights Optimal Control

4.2.2 Fixed-Weights vs Varying-Weights Optimal Control

Again, these two controllers have been compared to illustrate the decline in speed towards the terminal states. In Figure 4.4, the variations in speed is more pronounced than in Figure **??**.

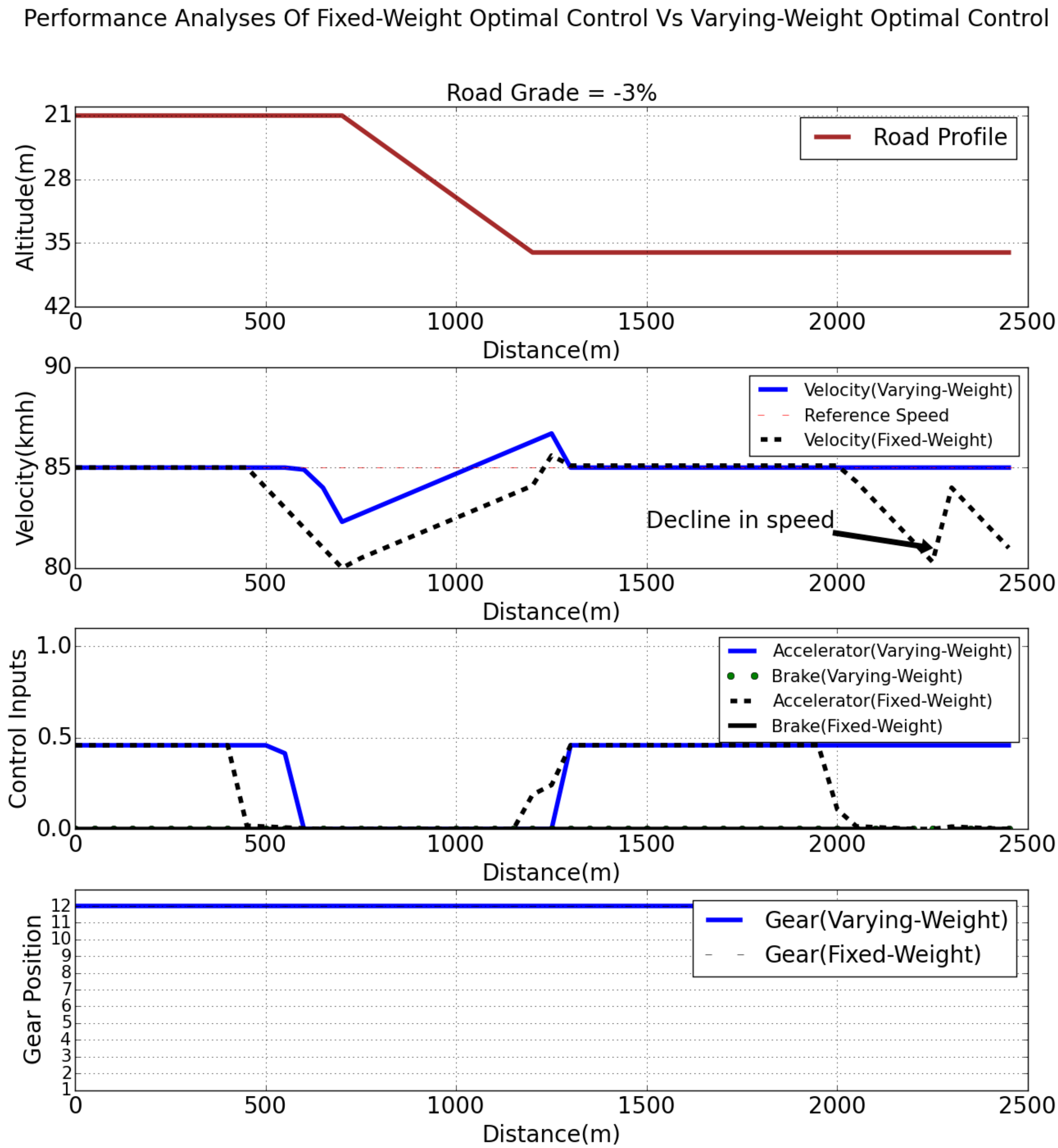

Figure 4.4: Fixed-Weights Optimal Control vs Varying-Weights Optimal Control

5. CONCLUSION

The optimal controller with varying penalty parameters provide for a relatively simpler implementation of the MPC scheme with dynamic programming algorithm. This is because, it eliminates the need to restrict the feasible transitions at every stage to ensure that the speed remains above the reference value.

As indicated in the previous section, it ensures velocity tracking on flat stretches of road, right amount of acceleration prior to an ascent and needed deceleration before descent. This provides a right mix of human driving practises and tracking ability. In addition, in comparison with a PI controller, fuel savings of -0.44% & -0.48% have been recorded in uphill and downhill scenarios. During descent, the need for braking is minimized unlike in a PI controller. This proves to be highly beneficial in case of heavy duty vehicles as their brake pads are prone to frequent wear owing to their weight.

REFERENCES

[1] Kock, P., Ordys, A., Collier, G., & Weller, R. (2013). Intelligent Predictive Cruise Control Application Analysis for Commercial Vehicles based on a Commercial Vehicles Usage Study. SAE International Journal of Commercial Vehicles, 6(2013-01-9022), 598-603.

[2] Kock, P., Welfers, H. J., Passenberg, B., Gnatzig, S., Stursberg, O., & Ordys, A. O. (2008). Saving energy through predictive control of longitudinal dynamics of heavy trucks. VDI Berichte, 53-68.

[3] Lattemann, F., Neiss, K., Terwen, S., & Connolly, T. (2004). The predictive cruise control - a system to reduce fuel consumption of heavy duty trucks (No. 2004-01-2616). SAE Technical paper.

[4] Gaines, L., Vyas, A., & Anderson, J. (2006). Estimation of fuel use by idling commercial trucks. Transportation Research Record: Journal of the Transportation Research Board, (1983), 91-98.

[5] Cheon, S. (2003). An overview of automated highway systems (AHS) and the social and institutional challenges they face. University of California Transportation Center.

[6] Cruise Control. Retrieved from *https://en.wikipedia.org/wiki/Cruise_control*.

[7] Shaout, A., & Jarrah, M. A. (1997). Cruise control technology review. Computers & electrical engineering, 23(4), 259-271.

[8] Goodrich, M., & Boer, E. R. (2003). Model-based human-centered task automation: a case study in ACC system design. Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on, 33(3), 325-336.

[9] Moon, S., Moon, I., & Yi, K. (2009). Design, tuning, and evaluation of a full-range adaptive cruise control system with collision avoidance. Control Engineering Practice, 17(4), 442-455.

[10] Liang, C. Y.,& Peng, H. (1999). Optimal adaptive cruise control with guaranteed string stability. Vehicle System Dynamics, 32(4-5), 313-330.

[11] Moon, S., & Yi, K. (2008). Human driving data-based design of a vehicle adaptive cruise control algorithm. Vehicle System Dynamics, 46(8), 661-690.

[12] Shakouri, P., Ordys, A., Laila, D. S., & Askari, M. (2011). Adaptive cruise control system: Comparing gain-scheduling PI and LQ controllers. tc, 2(1), 1.

[13] Vedam, N., Diaz-Rodriguez, I., & Bhattacharyya, S. P. (2014, October). A novel approach to the design of controllers in an automotive cruise-control system. In Industrial Electronics Society, IECON 2014-40th Annual Conference of the IEEE (pp. 2927-2932). IEEE.

[14] Ivens, T., Hofman, T., & Pesgens, M. (2010). Predictive cruise control for heavy duty vehicles. Master traineeship report Eindhoven University of Technology, he Netherlands.

[15] Macadam, C. C. (2003). Understanding and modeling the human driver. Vehicle System Dynamics, 40(1-3), 101-134.

[16] Peacock, B. (1993). Automotive ergonomics (pp. 237-320). W. Karwowski (Ed.). London; Washington: Taylor & Francis.

[17] Riemersma, J. B. J. (1987). Visual cues in straight road driving (No. HS-041 029).

[18] Schieber, F. (1994). Recent developments in vision, aging and driving. Transportation Research Institute, 1988-1994.

[19] Kozica, E. (2005). Look ahead cruise control: road slope estimation and control sensitivity.

[20] Schwarzkopf, A. B., & Leipnik, R. B. (1977). Control of highway vehicles for minimum fuel consumption over varying terrain. Transportation Research, 11(4), 279-286.

[21] Hooker, J. N. (1988). Optimal driving for single-vehicle fuel economy. Transportation Research Part A: General, 22(3), 183-201.

[22] Monastyrsky, V. V., & Golownykh, I. M. (1993). Rapid computation of optimal control for vehicles. Transportation Research Part B: Methodological, 27(3), 219-227.

[23] Chang, D. J., & Morlok, E. K. (2005). Vehicle speed profiles to minimize work and fuel consumption. Journal of transportation engineering, 131(3), 173-182.

[24] Frberg, A., Hellstrm, E., & Nielsen, L. (2006). Explicit fuel optimal speed profiles for heavy trucks on a set of topographic road profiles (No. 2006-01-1071). SAE Technical Paper.

[25] Huang, W., Bevly, D. M., Schnick, S., & Li, X. (2008, October). Using 3D road geometry to optimize heavy truck fuel efficiency. In Intelligent Transportation Systems, 2008. ITSC 2008. 11th International IEEE Conference on (pp. 334-339). IEEE.

[26] Terwen, S., Back, M., & Krebs, V. (2003). Predictive powertrain control for heavy duty trucks. In Modeling and Control of Economic Systems 2001:(SME 2001): a Proceedings Volume from the 10th IFAC Symposium, Klagenfurt, Austria, 6-8 September 2001 (p. 105). Elsevier.

[27] Lattemann, F., Neiss, K., Terwen, S., & Connolly, T. (2004). The predictive cruise controla system to reduce fuel consumption of heavy duty trucks (No. 2004-01-2616). SAE Technical paper.

[28] Hellstrm, E., Ivarsson, M., slund, J., & Nielsen, L. (2009). Look-ahead control for heavy trucks to minimize trip time and fuel consumption. Control Engineering Practice, 17(2), 245-254.

[29] Wingren, A. (2005). Fordonsreglering med framfrhllning.

[30] Hellstrm, E. (2005). Explicit use of road topography for model predictive cruise control in heavy trucks.

[31] Abdul-Rasool, M. (2011). The impact of ecoroll on fuel consumption-using look ahead.

[32] Steve Nelson. Automotive Market & Industry Update. Retrieved from *http://www.freescale.com/files/ftf_2010/Americas/WBNR_FTF10_AUT_F0747_PDF.pdf*

[33] Rathmann, S., & Fischerkeller, R. (2007). Latest Trends In Automotive Electronic Systems-Highway Meets Off-Highway?. Agricultural Engineering International: CIGR Journal.

[34] Under the Hood: Silicon in autos driving patent plans. Retrieved from *http://www.eetimes.com/document.asp?doc_id=1271565*

[35] Chakraborty, S., Lukasiewycz, M., Buckl, C., Fahmy, S., Chang, N., Park, S., ... & Adlkofer, H. (2012, March). Embedded systems and software challenges in electric vehicles. In Proceedings of the Conference on Design, Automation and Test in Europe (pp. 424-429). EDA Consortium.

[36] Bellman, R. (1957). Dynamic programming. Princeton, Princeton University Press, 1957.

[37] Jacobson, B. (2012). Vehicle Dynamics. Compendium for Course MMF062. Chalmers University of Technology.

[38] Lechner, G., Naunheimer, H., & Ryborz, J. (1999). Automotive transmissions: fundamentals, selection, design and application. Springer Science & Business Media.

[39] Ulsoy, A. G., Peng, H., & akmakci, M. (2012). Automotive control systems. Cambridge University Press.

[40] Sahlholm, P., & Johansson, K. H. (2010). Road grade estimation for look-ahead vehicle control using multiple measurement runs. Control Engineering Practice, 18(11), 1328-1341.

[41] Sahlholm, P., Gattami, A., & Johansson, K. H. (2011). Piecewise linear road grade estimation. In SAE 2011 World Congress and Exhibition; Detroit, MI; United States; 12 April 2011 through 12 April 2011. SAE international.

[42] WANG, H., SHI, Z., & REN, Z. (2013). GPS/IMU. Journal of Computational Information Systems, 9(18), 7207-7214.

[43] Bae, H. S., Ryu, J., & Gerdes, J. C. (2001, August). Road grade and vehicle parameter estimation for longitudinal control using GPS. In Proceedings of the IEEE Conference on Intelligent Transportation Systems (pp. 25-29).

[44] Wood, E., Burton, E., Duran, A., & Gonder, J. (2014). Appending High-Resolution Elevation Data to GPS Speed Traces for Vehicle Energy Modeling and Simulation. National Renew Energy Lab.

[45] Bertsekas, D. P., & Bertsekas, D. P. (1995). Dynamic programming and optimal control (Vol. 1, No. 2). Belmont, MA: Athena Scientific.