ALTERNATIVE ESTIMATION PROCEDURES FOR EVENT-HISTORY ANALYSIS: A MONTE CARLO STUDY*

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1. Introduction

Tuma and Hannan (1978) have argued that analysis of event-histories offers substantial advantages for causal inference about change in discrete dependent variables. Their methodology involves the formal specification of continuous-time models of transition rates. The parameters of such models can be estimated by several alternative procedures: ordinary least squares, Kaplan-Meier least squares, maximum likelihood, and partial likelihood. In this paper we consider the relative merits of these techniques. We begin with a short discussion of the model and these various estimators. We then report Monte Carlo experiments that compare the two estimators with the best asymptotic properties. These experiments explore the issues of censoring, collinearity, misspecification, and confounding disturbances.

2. The Model

We assume that the investigator is interested in qualitative dependent variables (referred to generally as events) such as collective violence, mergers or divorces. For causal analysis, this investigator will want to determine the association between the values of certain exogenous variables for a unit and that unit's propensity to experience events (event rate). For example, a political sociologist may want to test the hypothesis that the rate of collective violence in nation-states increases with the power of the state when the level of economic development is held constant. If data containing the exact timing of events is available across units, then event-history analysis is appropriate.

Event-history analysis assumes an underlying model of event occurrence.

To show this model we denote by t the waiting time until an event and by T the corresponding random variable. We define the survivor function G(t) by

$$G(t) = \Pr\{T \geq t\} \tag{1}$$

and the event rate (or hazard) by

$$r(t) = \lim_{\Delta t \to 0+} \frac{\Pr\{t \leq T < t+\Delta t \mid t \leq T\}}{\Delta t} \tag{2}$$

where Δt is an increment of time.

Cox (1972) shows that by the product law of probability we can write G(t) as the product integral

$$\exp\left\{ -\int_0^t r(u)du \right\} \tag{3}$$

for the continuous time case, and for discrete time the survival function is given by

$$\prod_{r_s < t} (1-r_s) \tag{4}$$

where $r_s = pr(T=t \mid T \geq t)$.

For causal analysis, we introduce a vector of exogenous variables X_i for each of the i sample units and an unknown disturbance g(t) which affects all sample units equally. We let the rate depend on the exogenous variables in a log-linear way. Specifically, we write

$$r_i(t) = g(t)e^{\beta X_i} \tag{5}$$

where β is a vector of unknown coefficients used to assess the strength of the relationships.

If we return to the example of the political sociologist, we can show a complete specification of the model. We let X_1 represent state power and X_2 the level of economic development. We then define the rate of collective violence for country i as

$$r_i(t) = g(t)\exp\{\beta_0+\beta_1 X_{1i}+\beta_2 X_{2i}\} \tag{6}$$

where β_0 is a constant and β_1 and β_2 measure the effects of X_1 and X_2 respectively upon the rate. Thus, the proposed hypothesis would be supported if the estimate of β_1 is positive and significant.

One complicating factor of models such as (6) is that parameter estimation is not straightforward. Several techniques might be used but the better are not in the repertoire of most sociologists. The remainder of this paper explores selected issues involved in choosing between these alternative estimators. For clarity, the discussion focuses on the specification in (6), although our remarks apply to the more general model in (5) as well.

3. Estimation Procedures

The process described by the model might be estimated by any of several observable dependent variables. The most elementary procedure consists of employing a dummy variable that is 1 if an event occurs during a specificed observation period and 0 otherwise. A second approach, called event-count analysis by Tuma et al. (1979), uses the number of events occurring during the observation period as the dependent variable. Finally, an alternative approach which Tuma and Hannan (1978) label event-history analysis, uses t, the waiting time between events. In this paper, we restrict our attention to this final approach.

Ordinary Least Squares. Sociologists might be tempted to use familiar least squares estimation for event-history analysis. The motivation proceeds as follows. We first take the conditional expectation of t with respect to the rate when the X's are log-linear independent variables:

$$E(t|r(t)) = \exp\,(\beta_0^*+\beta_1^*X_1 + \beta_2^*X_2) \tag{7}$$

We next use a disturbance ε to remove the operator E such that

$$t = \exp\,(\beta_0^*+\beta_1^*X_1 + \beta_2^*X_2) + \varepsilon \tag{8}$$

or

$$\ln t = \beta_0^*+\beta_1^*X_1 + \beta_2^*X_2 + \varepsilon^* \;. \tag{9}$$

The problem is to find the relationship between the parameters β^* in (9) and the coefficients β of the hazard (6). However, from (7) it follows that

$$\frac{1}{r(t)} = \exp\ (\beta_0^*+\beta_1^*X_1 + \beta_2^*X_2) \tag{10}$$

and solving for r(t) yields

$$r(t) = \exp\ (-\beta_0^*-\beta_1^*X_1-\beta_2^*X_2)\ . \tag{11}$$

Comparing (12) with the hazard rate (6) makes it obvious that $\beta_i=-\beta_i^*$. A least squares estimation of this model then would use (9) for estimation and subsequently solve for the coefficients of interest.

While this estimation procedure for rate models is relatively straightforward, it is not recommended. Two critical problems arise in the application. First, the error term ε^* is nonnormal and hence complicates estimation. Prentice (1973) shows that a transformation can be used to overcome this obstacle; however, it is rather cumbersome and tedious. The second deficiency of ordinary least squares estimators for these models is that they cannot satisfactorily handle the censoring problem. When a sample unit does not experience an event, the technique does not provide advice regarding the appropriate estimation procedure. This deficiency severely restricts the research contexts where ordinary least squares estimators might be used with confidence. Consequently, we do not advocate use of this estimator for event-history analysis.

Miller's Kaplan-Meier Least Squares. Miller (1976) has proposed a modified least squares estimator based on the Kaplan and Meier (1958) survivor function estimator. This procedure also uses the specification in (9) for estimation. However, we must first order the N sample units by waiting time in order of increasing magnitude and denote these observed cases with the index j, e.g. $t_{(j)}$. Then the parameter estimates are obtained by minimizing the weighted sum of squares

$$\sum_j w_j(\beta) \quad (\ln t_{(j)} - \beta_0^* - \beta_1^* X_{1j} - \beta_2^* X_{2j})^2 \tag{12}$$

where $t_{(j)}$ takes on the value of the largest waiting time if censored and the weight $w_j(\beta)$ is derived from the Kaplan-Meier estimator $\hat{F}_{KM}$ as follows

$$w_j(\beta) = d\hat{F}_{KM}(t_j) = \prod_{t_{(k)}<t} \left(\frac{n-k}{n-k+1}\right)\left(\frac{m_j}{n-j+1}\right) \tag{13}$$

where m_j is the number of uncensored observations tied with $t_{(j)}$. For a more detailed explanation of this procedure, see Miller (1976).

For these models, Kaplan-Meier least squares estimation is preferable to ordinary least squares. Kaplan-Meier estimators are asymptotically consistent and normally distributed under fairly general conditions (Kaplan and Meier, 1958; Efron, 1967). The lack of parametric assumptions is also appealing. Despite these advantages, there is a serious practical obstacle to use of these estimators in empirical research: minimization of (12) is not straightforward. Multiple minima might be located at discontinuous rather than continuous points. To avoid this, a tedious and costly grid search must be utilized. Miller (1976) reports that such searching is impractical even for a model with only two exogenous variables. Extensions to more realistic models are also quite difficult. Although Miller (1976) proposes a method for extensions to nonlinear models, it is, we believe, more laborious than necessary. For these reasons we do not think sociologists will find Miller's Kaplan-Meier estimators practical for the estimation of rate models.

Maximum Likelihood. In previous research with rate models (Hannan et al., 1977) we have employed maximum likelihood estimators (MLE). When data on the exact timing of events is available, the likelihood function may be written as

$$L = \prod_{i=1}^{N} \left(r_i(t) - e^{-r_i(t)t} \right)^{z} \left(e^{-r_i(t)t} \right)^{(1-z)} \tag{14}$$

where z is an indicator variable of unity if an event is observed and zero otherwise. The likelihood (14) cannot be solved explicitly for the parameters of interest but they are found instead by iteratively maximizing L.

Maximum likelihood estimators are asymptotically consistent and normally distributed under fairly weak regularity conditions (see Dhrymes, 1970). The MLE technique also has been shown to have very good small sample properties when applied to time-independent data (Tuma and Hannan, 1978; Fennell, Tuma and Hannan, 1977). Moreover, the procedure is easily generalized to handle the more complicated models likely to arise in sociological studies of event histories (Tuma and Hannan, 1978). Thus, when the model can be specified with confidence, the MLE method can be of great value to sociologists.

Partial Likelihood. Cox (1972, 1975) developed a procedure to estimate rate models in which the disturbance function g(t) in (5) is unknown, but uniform in the population. His partial likelihood procedure is appealing because it offers a general nonparametric alternative to MLE. To use his procedure we must again order the j waiting times in order of increasing magnitude. Then the partial likelihood function for (5) is defined as

$$L = \prod_{j=1}^{J} \left\{ r_j(t) \Big/ \left[\sum_{k \in R(t_j)} r_k(t) \right] \right\} \tag{15}$$

where J is the number of uncensored observations, and $R(t_j)$ is the risk set of those units which have not experienced an event after t_j. The parameters in (15) are estimated like those in any likelihood function.[1]

Partial likelihood estimators (PLE) are asymptotically consistent and normally distributed (Efron, 1977). Little is yet known about their small sample properties. We think the method is potentially fruitful for sociological research because it is fairly easy to implement and fairly general. Still, we have no basis upon which to compare PLE with MLE; both estimators have good properties in the probability limit. For this reason, we turn next to a series of Monte Carlo experiments. These experiments compare the PL and ML estimators under various identical conditions in the hope of developing a preliminary guide to decisions regarding the use of these alternative estimators.

4. Small Sample Properties

We study first the small sample properties of partial likelihood estimators in a model with $g(t)=1$. We examine this estimator under conditions of three levels of censoring (uncensored, 60% censored, 80% censored) and two levels of collinearity between the exogenous variables ($\rho=0.0$, $\rho=0.5$). We have previously studied MLE under identical conditions and use these results for a comparison (Fennell, Tuma and Hannan, 1977).

The simulation is structured as follows. We set g(t) in (6) equal to 1, thereby removing noise from the model. We set the parameters $\beta_0, \beta_1, \beta_2$ to be -4, -1 and 1 respectively. We generate pseudo-random normal deviates to represent the X variables and compute a time value for each sample unit.[2] We repeat this process 100 times for each sample of size 100.

Results. In Table 1 we report the Monte Carlo findings for the case in which $\rho=0.0$. Throughout our results we report both the mean bias and percent-bias of estimators over the 100 samples. We also report the variance and mean squared error (MSE).

We discuss first the behavior of PLE across censoring levels. As Table 1 shows, when the level of censoring increases, the quality of partial likelihood estimates deteriorates. Both the bias and variance of the estimator increase slightly as the level of censoring increases. Figure 1 graphs these results. The curves in that figure give the frequency distributions of the estimator as interpolated from a histogram with intervals of .2. By comparing the curves in the figure, it is easy to see the effect of censoring on PLE; increased censoring shifts the central tendency (bias) of the estimator and increases dispersion (variance). Nonetheless, we think the estimator performs well considering that at the highest censoring level, 80% of the information on the timing of events has been lost.

The lower half of Table 1 reports findings for the MLE from our previous study using identical data. The direction of the bias in both estimators follows the same pattern at each censoring level; both estimators tend to overestimate the magnitude (absolute value) of the parameter. In causal analysis, this means the null hypothesis of no relationship will be rejected at times when it should be accepted. With uncensored data, MLE is less biased and has a lower variance than PLE. As Figure 2 shows, these differences are substantial. However, the relative performance of PLE improves with higher levels of censoring. At the 60% censoring level (see Figure 3), it is almost impossible to distinguish between the two estimators. This improved performance of PLE continues to the point that at 80% censoring, the bias is actually smaller than for MLE. Despite this improvement in bias, however, PLE remains less efficient than MLE at all censoring levels. Table 1 shows that the difference is substantial enough to allow MLE to outperform PLE in terms of MSE under all conditions. In general, then, MLE appears to be the better small sample estimator when $g(t)=1$, that is when there is no time dependence.

In Table 2, we show PLE's and MLE's performances when the correlation between the exogenous variables is 0.5. As we can see from this table, the estimators perform very similarly. Both display a tendency to underestimate parameters for uncensored data and to overestimate parameters when censoring occurs. Despite these similarities, the quality of MLE retains a slight advantage in this experiment; its bias, variance and MSE are all lower than those for PLE.

We can also compare these findings to those in Table 1 without collinearity between the exogenous variables. As we can see by such a comparison, estimator performance is altered, although only slightly. The directional pattern of the bias remains the same. However, the magnitude of the bias is increased for low levels of censoring and actually decreased for high levels of censoring. Nonetheless, the variance and MSE only slightly change from the case in which $\rho=0.0$. We do not think these differences deserve much attention.

5. Normal, Lognormal and Uniform Exogenous Variables

Sociologists often have data that is not normally distributed, e.g., income conforms more closely to a lognormal distribution. For this reason, we study the behavior of PLE and MLE when the exogenous variables have a normal, lognormal and uniform distribution.

We again use the model in (3) and fix g(t)=1. We rescale the parameters $\beta_0, \beta_1, \beta_2$ to be -.4, -.1, .1 respectively. We then generated three data sets as before, each using as exogenous variables pseudo-random deviates drawn exclusively from either a normal N(0,1), lognormal $\Lambda(0,1)$ or uniform U(0,1) distribution. We impose the censoring schemes used above and in each case draw values of the X's so that X_1 and X_2 are not correlated.

Results. Table 3 reports the Monte Carlo results for the model with normal exogenous variables and the rescaled parameters. For the most part these findings are similar to those in Table 1. MLE again outperforms PLE although the two are remarkably congruent, especially at the highest censoring level. The quality of the estimates remains good, although the percent-bias of β_1 rises because of rescaling. We do find several differences from Table 1 though. First, the directional pattern of the bias is lost. Estimators no longer consistently overestimate the effects of variables. Second, the impact of censoring is less severe. As censoring increases we can no longer predict a consequent increase in the absolute value of the bias. Thus the mean squared error of the estimates increases only slightly with censoring.

Table 4, which reports the quality of estimators for the same model with lognormal exogenous variables, contains no surprises. The size and direction of the bias conform closely to those for the model with normal variables (especially Table 1) in all cases. Again MLE slightly outperforms PLE, although both are high in quality. These findings indicate the insensitivity of both estimation procedures to normal and lognormal parametric forms of the exogenous variables in the model. This robustness is encouraging.

Table 5 reports findings of uniform exogenous variables. The quality of both PLE and MLE is noticeably poorer in this case. Both the bias and percent-bias increase substantially over the two cases just considered. However, the most dramatic shift in estimator quality is that the variance of the estimates increases substantially. Further, the direction of the bias differs from previous patterns. Both PLE and MLE are now consistently upwardly biased except for the uncensored positive parameter. Frankly, we find these results somewhat baffling. In a similar though smaller study, Keeley (1975) found MLE yielded good quality estimates with uniform exogenous variables. We are uncertain why these results differ from his or from those for our previous models. However, we are consoled by the fact that uniformly distributed variables are rare in sociological research.

6. Random Gamma Disturbance

Event-history data are sometimes contaminated by disturbances that are ignored. For this reason, we next consider the quality of both PLE and MLE when noise is contained in the data and is ignored. First we consider a random disturbance that affects each sample unit differently. That is, we substitute a random gamma-distributed disturbance for g(t) in (6). We simulate data as before, except a pseudo-random deviate is drawn for each individual and inserted in the place of g(t) before the waiting time is computed.[3] Parameter estimates are then obtained by ignoring the simulated disturbance.

Results. Table 6 presents findings for the behavior of both estimators when the gamma-distributed disturbance is present. As this table shows, the disturbance does not eliminate the overall high quality of the estimates yielded by both techniques. In all instances, the bias, variance and mean squared error remain low. Several new patterns are also noteworthy. First, the disturbance breaks any previous patterns concerning the direction of the bias and leaves instead an apparently random situation. Second, MLE again outperforms PLE. Third, as censoring increases, the two estimators differ less. Figures 4 and 5 show this convergence clearly. In Figure 4 we show the distribution of the estimates for the uncensored data. The ML estimator is better, especially in terms of bias. In contrast, similar curves for the 80% censored data (see Figure 5) show the two estimators to be virtually indistinguishable. Thus while MLE outperforms PLE in all cases, these differences are quite small at high levels of censoring. Moreover, the high degree of robustness of both estimators is surprising in view of the random disturbance.

7. Time-Dependent Disturbances

Event-history data are more likely to contain time-dependent disturbances than simple random disturbances. This problem has been discussed in numerous substantive contexts. For examples of time-dependence in labor mobility studies, see Spilerman (1977), in family marital events see Glick and Norton (1971), and in organizational structure see Stinchcombe (1965).

These substantive concerns aside, we are interested in the case of time-dependent disturbances for methodological reasons. While we have extensively studied MLE with Monte Carlo methods, we have not explored time-dependence in small samples. It is important to know MLE's quality for a correctly specified model and for a misspecified model that ignores time-dependence. Moreover, the comparison with PLE is especially critical for this case since the partial likelihood procedure was developed precisely for estimation in this situation.

Our simulation proceeds in basically the same way as before. In this experiment however, we set $g(t)=e^{\gamma t}$. We explore two levels of time-dependence, weak and strong. Weak time dependence has $\gamma=.1$ and strong time dependence $\gamma=1$. Two ML estimators and one PL estimator are obtained. The first ML estimator is the same one used in the above studies; for this model it represents a misspecified form in which the investigator ignores the time-dependence. The second ML estimator is for a correctly specified model containing γ as well as β_0, β_1 and β_2. The PL estimator is the specification in (15), used throughout this paper.

Results. Tables 7 and 8 report the bias and percent-bias of the estimates from this experiment. We discuss these tables jointly. For the weakly time-dependent model, the quality of the estimates yielded by both PLE and MLE (correctly specified) is high. Under all levels of censoring, the bias of MLE remains fairly low. Furthermore, ML yields estimates of the time dependent parameter γ with an exceptionally small bias. For PLE the results are even better. The estimates have in all but three instances a bias less than 5%. In addition, we note that the direction of the bias for both estimators is similar and congruent with the pattern in the time-independent results given in Table 1.

The results for the strongly time-dependent model exhibit similar patterns but with greater exaggeration. Thus the direction of the bias remains the same but is more pronounced since the size of the bias increases. Nonetheless, the magnitude of the bias does not change enough to merit reclassification of the estimates as other than high in quality.

In contrast, the performance of the misspecified MLE is less satisfactory. The bias of the estimates is high enough to warrant considerable concern, especially for the uncensored case. In addition, the pattern of the bias has changed. It is important to note, however, that the bias of the misspecified MLE is low at the highest censoring levels. In fact, in most cases it is considerably lower than the fully specified estimators at the comparable censoring level.

Table 9 gives the variance of the estimates for this model. We notice, as usual, that estimator variance increases with censoring in all situations. Again PLE outperforms the properly specified MLE for this time-dependent model, although the overall quality of both estimators is good. The increase in time dependence apparently causes increased variance and hence loss of efficiency, but this effect is slight. The most surprising result of Table 9, however, concerns the misspecified ML estimator. At all censoring levels and for each parameter, this estimator gives more efficient estimates than either of the other two techniques.

We summarize these experiments with both a descriptive statistic and plots of the distribution of the estimators. Table 10, which presents the mean squared error, again shows the high quality of both PLE and the correctly specified MLE under all levels of censoring. Further, the ML estimator of γ, the parameter describing time dependence is quite good. The quality of estimators deteriorate only slightly, as time dependence increases in strength. In general, both techniques yield high quality estimates for this model with PLE consistently giving slightly more efficient and less biased estimates.

The performance of the misspecified MLE relative to the other two estimators is best displayed by Figures 6 and 7. In Figure 6, we notice that for the uncensored data, the misspecified MLE is extremely biased even though it remains efficient. But with 80% censored data (see Figure 7), it is difficult to distinguish between estimators. Moreover, the higher peak of the misspecified MLE in Figure 7 suggests that it is actually the best estimator in this context. The mean squared error in Table 10 reenforce this impression. Thus without advocating the use of a misspecified estimator, we must at least remark in conclusion that for these models misspecification of time dependence is much less severe than misspecification of the vector of exogenous variables, especially with highly censored data (see Fennell, Tuma and Hannan, 1977).

8. Conclusion

We began this study in search of the most appropriate estimators to be used in event-history analysis. We discussed two least squares procedures but dismissed them on theoretical and pragmatic grounds. This left two likelihood procedures from which to choose. We had previously advocated and studied ML estimators of such models. However, the partial likelihood procedure offered an attractive nonparametric alternative. Unfortunately, statistical theory offered little information concerning small sample properties. So we embarked on a series of Monte Carlo experiments designed to compare the procedures under a variety of conditions. Our primary finding is that the two procedures yield remarkably similar estimates, especially at high levels of censoring. Thus we find it difficult to recommend unequivocally one technique over the other. Instead our results suggest the MLE is slightly superior for time-independent data with or without random disturbances. In contrast, PLE performs slightly better when the rate is time-dependent. As with all Monte Carlo studies these findings have limited scope as we have considered only a single general model and a limited number of combinations of parameters. Nonetheless, we have found no evidence that these estimators perform poorly in moderately small samples.

FOOTNOTES

[1]The constant g(t) can also be estimated (see Oakes 1972). However, since our concern here is strictly with causal analysis, we ignore this issue.

[2]A "fast normal random deviate generator" was used to produce single-precision pseudo-normal (0,1) random numbers. This method follows Marsaglia's rectangle-wedge-tail algorithmn as described in Knuth (1969). The Marsaglia method uses the following distribution:

$$F(x) = \sqrt{\frac{2}{\Pi}} \int_0^X e^{-v^2/2}\, dv \quad x \geq 0$$

which gives the distribution of the absolute value of a normal deviate. The time of a change t was generated as follows

$$t = \frac{-\ln(U(0,1))}{\exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}$$

where U(0,1) represents a uniformly distributed variable on the range 0 to 1 and X_1 and X_2 are standard normal deviates.

[3]The gamma distributed deviate is generated with a rejection technique due to Johnk (1964) and developed by Phillips and Beightler (1972). The simulated distribution is

$$f(\gamma;\, a,b) = \frac{\gamma^{a-1}}{\Gamma(a)b^{a}} \exp(-\gamma/b)$$

where γ, a and b are always positive. We chose the parameters a and b so that the disturbance has mean 1 and variance 1/3.

REFERENCES

Cox, D. R.

1972 "Regression models and life tables." Journal of Royal Statistical Society B34: 187-202.

1975 "Partial likelihood." Biometrika 62: 269-276.

Dhrymes, P.

1970 Econometrics: Statistical Foundations and Application. New York: Harper and Row.

Efron, B.

1967 "The two sample problems with censoring data." Proceedings of the Fifth Berkeley Symposium 4: 831-853.

1977 "The efficiency of Cox's likelihood function for censored data." Journal of the American Statistical Association 72: 557-565.

Fennell, M., N. B. Tuma and M. T. Hannan

1977 "Quality of maximum likelihood estimates of parameters in a log-linear rate model." Technical Report #59, Laboratory for Social Research, Stanford University.

Glick, P. C. and A. J. Norton

1971 "Frequency, duration and probability of marriage and divorce." Journal of Marriage and the Family 33 (May): 307-317.

Hannan, M. T. and N. B. Tuma

1979 "Methods for temporal analysis." Annual Reviews in Sociology. Forthcoming.

Hannan, M. T., N. B. Tuma and L. P. Groeneveld

1977 "Income and marital events: evidence from an income maintenance experiment." American Journal of Sociology 82, 6 (May): 1186-1211.

Johnk, M. D.

1964 "Erzung von betaverteiler und gammaverteiler zufallzahlen." Metrika 8(2).

Kaplan, E. L. and P. Meier

1958 "Nonparametric estimation from incomplete observations." Journal of the American Statistical Association 53: 57-81.

Keeley, M. C.

1975 "A Monte Carlo test of Nancy Tuma's maximum likelihood 'RATE' program." Stanford Research Institute, Menlo Park, Calif.

Knuth, D. E.

1969 The Art of Computer Programming. Vol. II. Reading, Mass.: Addison-Wesley.

Miller, R. G.

1976 "Least squares regression with censored data." Biometrika 63: 449-464.

Oakes, D.

1972 "Discussion of Professor Cox's paper." Journal of the Royal Statistical Society 34B: 208.

Phillips, D. T. and C. S. Beightler

1972 "Procedure for generating gamma variates with non-integer parameter sets." Journal of Statistical Computation and Simulation 1: 197-208.

Prentice, R. L.

1973 "Exponential survivals with censoring and explanatory variables." Biometrika 60: 279-288.

Spilerman, S.

1977 "Careers, labor, market structure and socioeconomic achievement." American Journal of Sociology 83: 551-593.

Stinchcombe, A.

1965 "Social structure and organization." In James G. March (ed.), Handbook of Organizations. Chicago: Rand McNally.

Tuma, N. B. and M. T. Hannan

1978 "Approaches to the censoring problem in the analysis of event histories." In K. Schuessler (ed.), Sociological Methodology 1979. San Francisco: Jossey-Bass.

Tuma, N. B., M. T. Hannan and L. P. Groeneveld

1979 "Dynamic analysis of event histories." American Journal of Sociology 84. In press.

Table 1. Quality of Estimators: Log-Linear Rate Model with Normally-Distributed Exogenous Variables (ρ=0.0; N=100; No. of Samples=100).

		Bias	%-Bias	Variance	Mean Squared Error
		PARTIAL LIKELIHOOD ESTIMATES			
	Uncensored				
$\beta_0 = -4$	60% Censored	- - NOT ESTIMATED - -			
	80% Censored				
	Uncensored	-.024	-2.4%	.020	.021
$\beta_1 = -1$	60% Censored	-.026	-2.6%	.030	.031
	80% Censored	-.064	-6.4%	.053	.068
	Uncensored	.002	.2%	.017	.017
$\beta_2 = 1$	60% Censored	.004	.4%	.029	.029
	80% Censored	.042	4.2%	.056	.058
		MAXIMUM LIKELIHOOD ESTIMATES			
	Uncensored	.013	.3%	.011	.011
$\beta_0 = -4$	60% Censored	-.008	-.2%	.071	.033
	80% Censored	-.085	-2.1%	.228	.107
	Uncensored	-.014	-1.4%	.013	.013
$\beta_1 = -1$	60% Censored	-.027	-2.7%	.027	.028
	80% Censored	-.066	-6.6%	.055	.059
	Uncensored	-.007	-.7%	.013	.013
$\beta_2 = 1$	60% Censored	.002	.2%	.026	.026
	80% Censored	.045	4.5%	.052	.054

Table 2. Quality of Estimates: Log-Linear Rate Model with Normally-Distributed Exogenous Variables (N=100; No. of Samples=100).

		PARTIAL LIKELIHOOD ESTIMATES (ρ=.5)			
		Bias	%-Bias	Variance	Mean Squared Error
	Uncensored	-- NOT ESTIMATED --			
β_0 = -1	60% Censored				
	80% Censored				
	Uncensored	-.027	-2.7%	.021	.021
β_1 = -1	60% Censored	-.029	-2.9%	.040	.041
	80% Censored	-.038	-3.8%	.050	.051
	Uncensored	.010	1.0%	.021	.021
β_2 = 1	60% Censored	.010	1.0%	.038	.038
	80% Censored	.021	2.1%	.053	.053
		MAXIMUM LIKELIHOOD ESTIMATES (ρ=.5)			
	Uncensored	.013	.3%	.011	.011
β_0 = -.4	60% Censored	-.013	-.3%	.029	.029
	80% Censored	-.024	-.6%	.049	.059
	Uncensored	-.010	-1.0%	.014	.014
β_1 = -.1	60% Censored	-.027	-2.7%	.036	.037
	80% Censored	-.030	-3.0%	.045	.046
	Uncensored	-.008	-.8%	.017	.017
β_2 = .1	60% Censored	.008	.8%	.035	.035
	80% Censored	.016	1.6%	.051	.051

Table 3. Quality of Estimates: Log-Linear Rate Model with Normally-Distributed Exogenous Variables (ρ=0.0; N=100; No. of Samples=100).

		PARTIAL LIKELIHOOD ESTIMATES			
		Bias	%-Bias	Variance	Mean Squared Error
$\beta_0 = -.4$	Uncensored	-- NOT ESTIMATED --			
	60% Censored				
	80% Censored				
$\beta_1 = -.1$	Uncensored	-.026	-26.0%	.015	.016
	60% Censored	-.026	-26.0%	.019	.020
	80% Censored	-.024	-24.0%	.020	.021
$\beta_2 = .1$	Uncensored	-.008	-8.0%	.018	.018
	60% Censored	.001	1.0%	.025	.025
	80% Censored	-.001	-1.0%	.026	.026
		MAXIMUM LIKELIHOOD ESTIMATES			
$\beta_0 = -.4$	Uncensored	.016	4.0%	.011	.011
	60% Censored	-0-	-0-	.020	.020
	80% Censored	-0-	-0-	.021	.021
$\beta_1 = -.1$	Uncensored	-.019	-19.0%	.014	.014
	60% Censored	-.025	-25.0%	.018	.019
	80% Censored	-.023	-23.0%	.019	.020
$\beta_2 = .1$	Uncensored	-.008	-8.0%	.016	.016
	60% Censored	.002	2.0%	.025	.025
	80% Censored	-.001	-1.0%	.026	.026

Table 4. Quality of Estimates: Log-Linear Rate Model with Log Normally-Distributed Exogenous Variables (ρ=0.0; N=100; No. of Samples=100).

		PARTIAL LIKELIHOOD ESTIMATES			
		Bias	%-Bias	Variance	Mean Squared Error
	Uncensored	-- NOT ESTIMATED --			
β_0 = -.4	40% Censored				
	50% Censored				
	Uncensored	-.009	-9.0%	.004	.004
β_1 = -.1	40% Censored	-.017	-17.0%	.010	.010
	50% Censored	-.022	-22.0%	.012	.012
	Uncensored	.006	6.0%	.004	.004
β_2 = .1	40% Censored	.005	5.0%	.006	.006
	50% Censored	.006	6.0%	.006	.006
		MAXIMUM LIKELIHOOD ESTIMATES			
	Uncensored	.001	.3%	.033	.033
β_0 = -.4	40% Censored	.019	4.8%	.056	.056
	50% Censored	.020	5.0%	.061	.061
	Uncensored	-0-	-0-	.003	.003
β_1 = -.1	40% Censored	-.017	-17.0%	.009	.009
	50% Censored	-.021	-21.0%	.011	.011
	Uncensored	.007	7.0%	.004	.004
β_2 = .1	40% Censored	.005	5.0%	.006	.006
	50% Censored	.007	7.0%	.006	.006

Table 5. Quality of Estimates: Log-Linear Rate Model with Uniformally-Distributed Exogenous Variables (ρ=0.0; N=100; No. of Samples=100).

		Bias	%-Bias	Variance	Mean Squared Error
		PARTIAL LIKELIHOOD ESTIMATES			
	Uncensored				
β_0 = -.4	40% Censored	-- NOT ESTIMATED --			
	50% Censored				
	Uncensored	.042	42.0%	.143	.145
β_1 = -.1	40% Censored	.029	29.0%	.224	.225
	50% Censored	.022	27.0%	.225	.225
	Uncensored	-.014	-14.0%	.142	.142
β_2 = .1	40% Censored	.023	23.0%	.211	.212
	50% Censored	.038	38.0%	.210	.211
		MAXIMUM LIKELIHOOD ESTIMATES			
	Uncensored	.001	.3%	.093	.093
β_0 = -.4	40% Censored	-.022	-5.5%	.159	.159
	50% Censored	-.023	-5.8%	.155	.156
	Uncensored	.038	38.0%	.129	.130
β_1 = -.1	40% Censored	.025	25.0%	.225	.226
	50% Censored	.018	18.0%	.224	.224
	Uncensored	-.030	-30.0%	.124	.125
β_2 = .1	40% Censored	.022	-22.0%	.210	.210
	50% Censored	.037	-37.0%	.210	.211

Table 6. Quality of Estimates: Log-Linear Rate Model with Normally-Distributed Exogenous Variables and Random Gamma Disturbance in the Rate (ρ=0.0; N=100; No. of Samples=100).

		Bias	%-Bias	Variance	Mean Squared Error
		PARTIAL LIKELIHOOD ESTIMATES			
	Uncensored				
β_0 = -4	60% Censored	-- NOT ESTIMATED --			
	80% Censored				
	Uncensored	.168	16.8%	.027	.055
β_1 = -1	60% Censored	.078	7.8%	.034	.040
	80% Censored	-.013	-1.3%	.067	.067
	Uncensored	-.193	-19.3%	.023	.060
β_2 = 1	60% Censored	-.110	-11.0%	.031	.043
	80% Censored	-.062	-6.2%	.053	.057
		MAXIMUM LIKELIHOOD ESTIMATES			
	Uncensored	-.375	-3.4%	.022	.163
β_0 = -4	60% Censored	-.127	-3.2%	.039	.055
	80% Censored	-.147	-3.7%	.089	.111
	Uncensored	-.020	-2.0%	.029	.029
β_1 = -1	60% Censored	.044	4.4%	.032	.034
	80% Censored	-.030	-3.0%	.068	.069
	Uncensored	-.019	-1.9%	.029	.029
β_2 = 1	60% Censored	-.078	-7.8%	.032	.038
	80% Censored	-.042	-4.2%	.054	.056

Table 7. Bias of Estimates: Time Dependent Log-Linear Rate Model with Normally-Distributed Exogenous Variables (ρ=0.0; No. of Samples=100).

		Misspecified MLE	Maximum Likelihood Estimator		Partial Likelihood Estimator	
		N=100	N=50	N=100	N=50	N=100
		MODEL WITH WEAK TIME DEPENDENCE				
β_0 = -4	Uncensored	1.364	NOT ESTIMATED	-.042	NOT ESTIMATED	
	60% Censored	.645		-.058		
	80% Censored	.234		-.173		
β_1 = -1	Uncensored	.511		-.041	-.026	-.026
	60% Censored	.080		-.053	-.043	-.034
	80% Censored	-.024		-.097	-.147	-.079
β_2 = 1	Uncensored	-.514		.037	.032	.019
	60% Censored	-.093		.030	.046	.014
	80% Censored	-.006		.057	.129	.039
γ = .1	Uncensored			.004		
	60% Censored			.005		
	80% Censored			.023		
		MODEL WITH STRONG TIME DEPENDENCE				
β_0 = -4	Uncensored	2.806	-.134	-.085	NOT ESTIMATED	
	60% Censored	2.022	-.223	-.115		
	80% Censored	1.129	-.783	-.325		
β_1 = -1	Uncensored	.709	-.047	-.042	-.026	-.028
	60% Censored	.269	-.069	-.056	-.041	-.040
	80% Censored	-.018	-.274	-.145	-.240	-.131
β_2 = 1	Uncensored	-.713	.062	.040	.032	.015
	60% Censored	-.283	.068	.045	.039	.017
	80% Censored	-.050	.243	.104	.200	.057
γ = 1	Uncensored		.047	.028		
	60% Censored		.077	.036		
	80% Censored		.234	.093		

Table 8. Percent-Bias of Estimates: Time Dependent Log-Linear Rate Model with Normally-Distributed Exogenous Variables (ρ=0.0; No. of Samples=100).

		Misspecified MLE	Maximum Likelihood Estimator		Partial Likelihood Estimator	
		N=100	N=50	N=100	N=50	N=100
MODEL WITH WEAK TIME DEPENDENCE						
$\beta_0 = -4$	Uncensored	34.1%	NOT ESTIMATED	-1.1%	NOT ESTIMATED	
	60% Censored	16.1%		-1.5%		
	80% Censored	5.9%		-4.3%		
$\beta_1 = -1$	Uncensored	51.1%		-4.1%	-2.6%	-2.6%
	60% Censored	8.0%		-5.3%	-4.3%	-3.4%
	80% Censored	-2.4%		-9.7%	-14.7%	-7.9%
$\beta_2 = 1$	Uncensored	-51.4%		3.7%	3.2%	1.9%
	60% Censored	-9.3%		3.0%	4.6%	1.4%
	80% Censored	-.6%		5.7%	12.9%	3.9%
$\gamma = .1$	Uncensored			4.0%		
	60% Censored			5.0%		
	80% Censored			23.0%		
MODEL WITH STRONG TIME DEPENDENCE						
$\beta_0 = -4$	Uncensored	70.2%	-3.4%	-2.1%	NOT ESTIMATED	
	60% Censored	50.1%	-5.6%	-2.9%		
	80% Censored	28.2%	-19.6%	-8.1%		
$\beta_1 = -1$	Uncensored	70.9%	-4.7%	-4.2%	-2.6%	-2.8%
	60% Censored	26.9%	-6.9%	-5.6%	-4.1%	-4.0%
	80% Censored	-1.8%	-27.4%	-14.5%	-24.0%	-13.1%
$\beta_2 = 1$	Uncensored	-71.3%	6.2%	4.0%	3.2%	1.5%
	60% Censored	-28.3%	6.8%	4.5%	3.9%	1.7%
	80% Censored	-5.0%	24.3%	10.4%	20.0%	5.7%
$\gamma = 1$	Uncensored		4.7%	2.8%		
	60% Censored		7.7%	3.6%		
	80% Censored		23.4%	9.3%		

Table 9. Variance of Estimates: Time Dependent Log-Linear Rate Model with Normally-Distributed Exogenous Variables (ρ=0.0; No. of Samples=100).

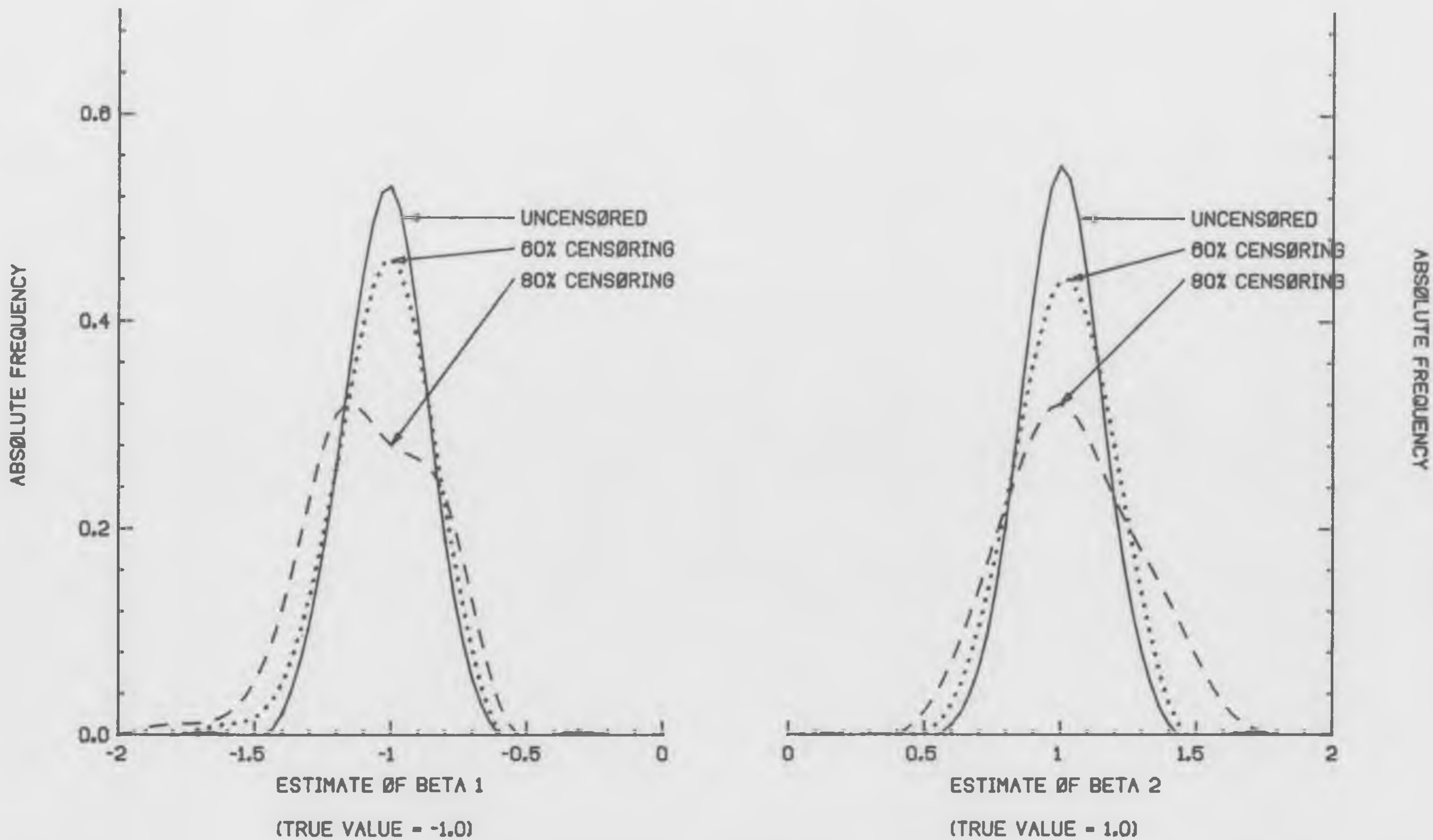
		Misspecified MLE	Maximum Likelihood Estimator		Partial Likelihood Estimator	
		N=100	N=50	N=100	N=50	N=100
MODEL WITH WEAK TIME DEPENDENCE						
$\beta_0 = -4$	Uncensored	.003	NOT ESTIMATED	.055	NOT ESTIMATED	
	60% Censored	.036		.177		
	80% Censored	.120		.292		
$\beta_1 = -1$	Uncensored	.004		.018	.047	.020
	60% Censored	.018		.032	.095	.030
	80% Censored	.054		.063	.216	.063
$\beta_2 = 1$	Uncensored	.005		.016	.031	.016
	60% Censored	.028		.038	.076	.037
	80% Censored	.060		.074	.184	.069
$\gamma = .1$	Uncensored			-0-		
	60% Censored			.002		
	80% Censored			.010		
MODEL WITH STRONG TIME DEPENDENCE						
$\beta_0 = -4$	Uncensored	.001	.184	.132	NOT ESTIMATED	
	60% Censored	.023	.424	.237		
	80% Censored	.154	2.074	.531		
$\beta_1 = -1$	Uncensored	.002	.044	.021	.047	.020
	60% Censored	.012	.088	.030	.082	.030
	80% Censored	.064	.503	.083	.455	.088
$\beta_2 = 1$	Uncensored	.002	.033	.016	.031	.017
	60% Censored	.016	.079	.036	.070	.033
	80% Censored	.069	.375	.102	.337	.091
$\gamma = 1$	Uncensored		.015	.010		
	60% Censored		.064	.028		
	80% Censored		.525	.184		

Table 10. Mean Squared Error of Estimates: Time Dependent Log-Linear Rate Model with Normally-Distributed Exogenous Variables (ρ=0.0; No. of Samples=100).

		Misspecified MLE	Maximum Likelihood Estimator		Partial Likelihood Estimator	
		N=100	N=50	N=100	N=50	N=100
MODEL WITH WEAK TIME DEPENDENCE						
$\beta_0 = -4$	Uncensored	1.863	NOT ESTIMATED	.057	NOT ESTIMATED	
	60% Censored	.452		.180		
	80% Censored	.175		.332		
$\beta_1 = -1$	Uncensored	.265		.020	.048	.021
	60% Censored	.024		.035	.097	.031
	80% Censored	.055		.072	.238	.069
$\beta_2 = 1$	Uncensored	.269		.017	.032	.016
	60% Censored	.037		.039	.078	.037
	80% Censored	.060		.077	.201	.071
$\gamma = .1$	Uncensored			-0-		
	60% Censored			.002		
	80% Censored			.011		
MODEL WITH STRONG TIME DEPENDENCE						
$\beta_0 = -4$	Uncensored	7.875	.202	.139	NOT ESTIMATED	
	60% Censored	4.111	.474	.240		
	80% Censored	1.429	2.687	.607		
$\beta_1 = -1$	Uncensored	.505	.046	.023	.048	.021
	60% Censored	.084	.093	.033	.084	.032
	80% Censored	.064	.578	.104	.513	.105
$\beta_2 = 1$	Uncensored	.510	.037	.018	.032	.017
	60% Censored	.096	.084	.038	.072	.033
	80% Censored	.072	.428	.113	.377	.094
$\gamma = 1$	Uncensored		.017	.010		
	60% Censored		.070	.029		
	80% Censored		.580	.193		

FIGURE 1

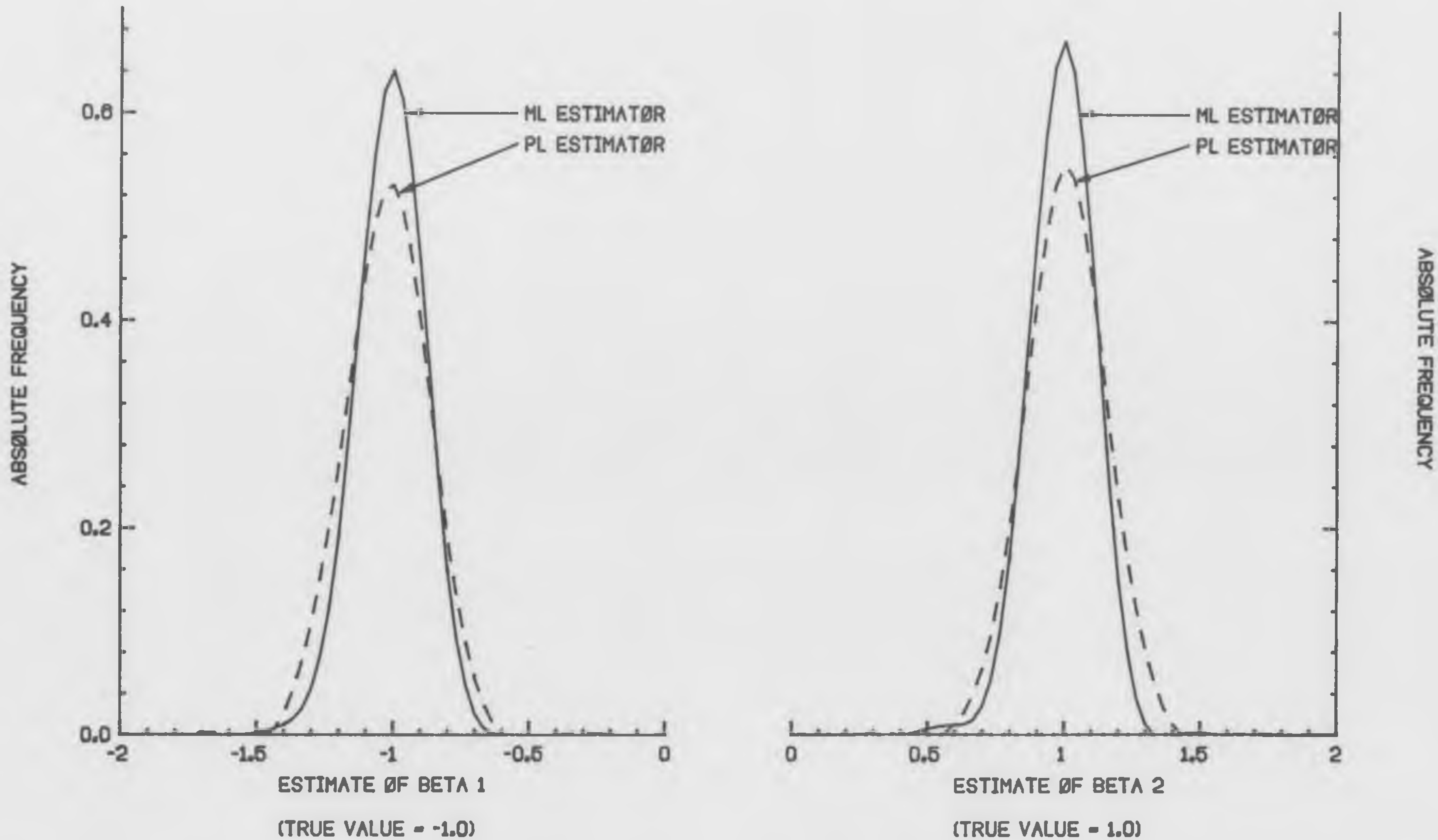
EFFECT ØF CENSØRING ØN PARTIAL LIKELIHØØD ESTIMATES



RHØ=0.0; N=100; NØ. ØF SAMPLES=100

FIGURE 2

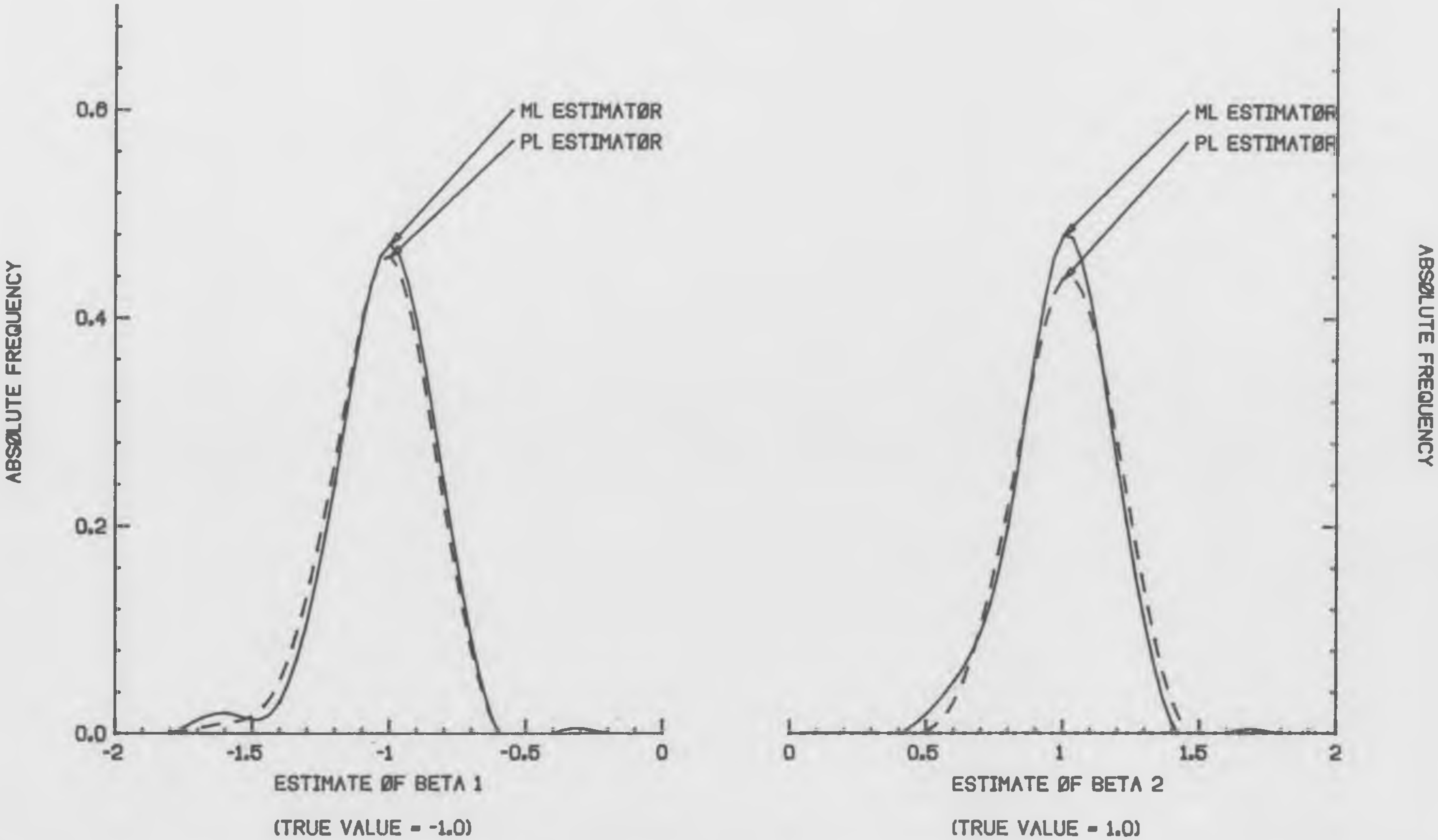
CØMPARISØN ØF ALTERNATIVE ESTIMATØRS FØR UNCENSØRED DATA



RHØ=0.0; N=100; NØ. ØF SAMPLES=100

FIGURE 3

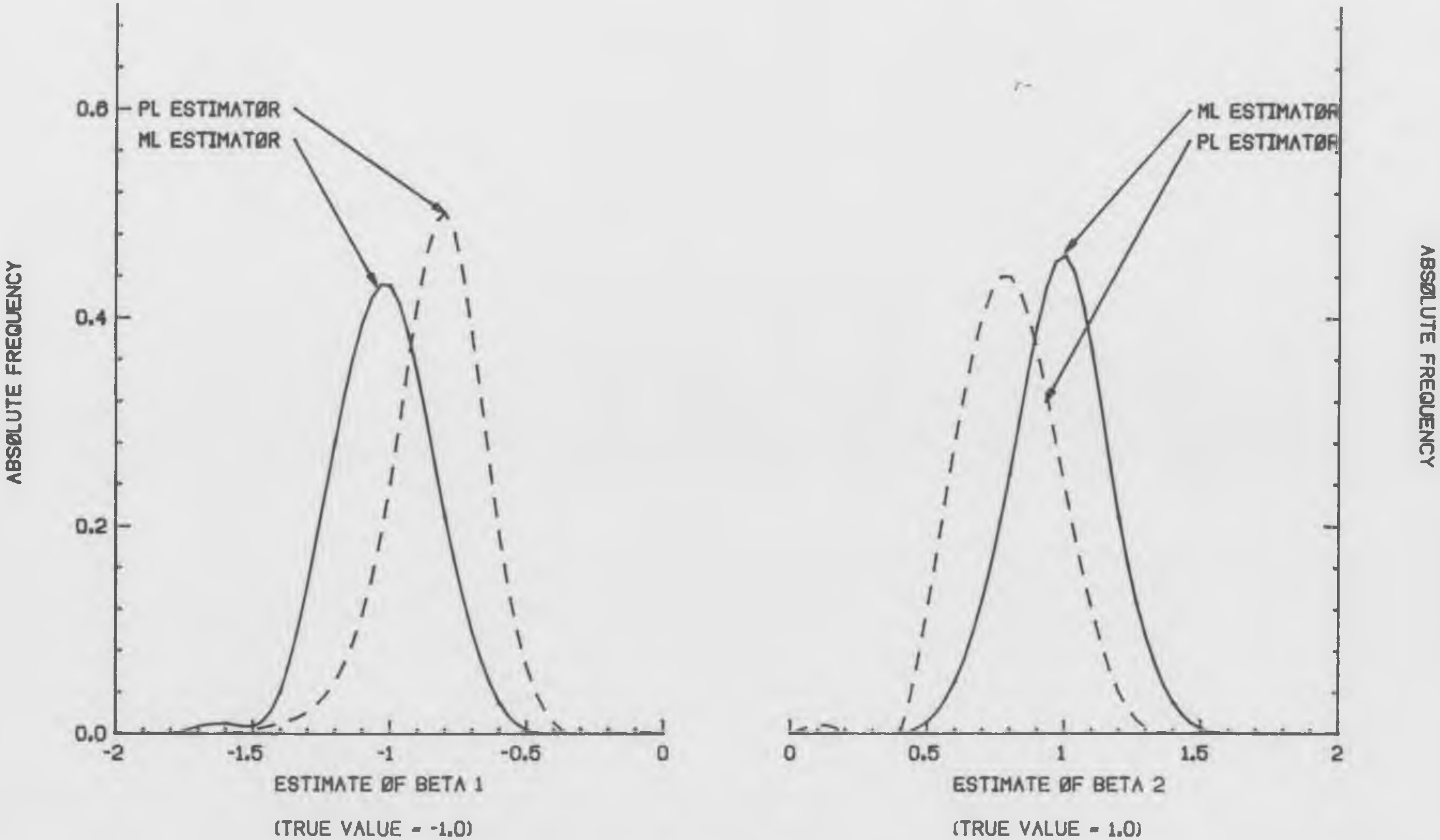
CØMPARISØN ØF ALTERNARIVE ESTIMATØRS FØR 60% CENSØRED DATA



RHØ=0.0; N=100; NØ. ØF SAMPLES=100

FIGURE 4

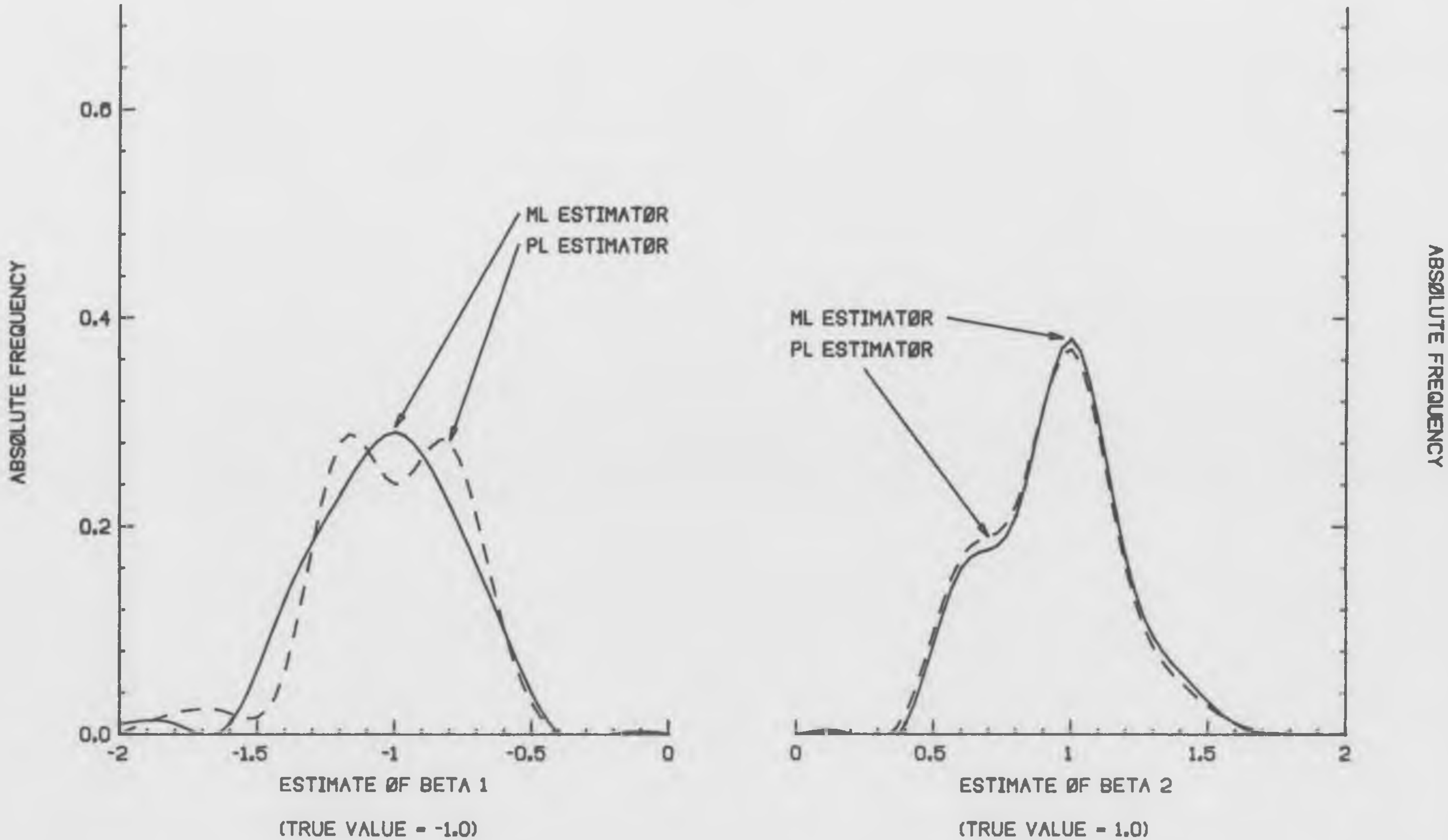
EFFECT ØF SIMULATED DISTURBANCE ØN ESTIMATES ØF UNCENSØRED DATA



RHØ=0.0; N=100; NØ. ØF SAMPLES=100

FIGURE 5

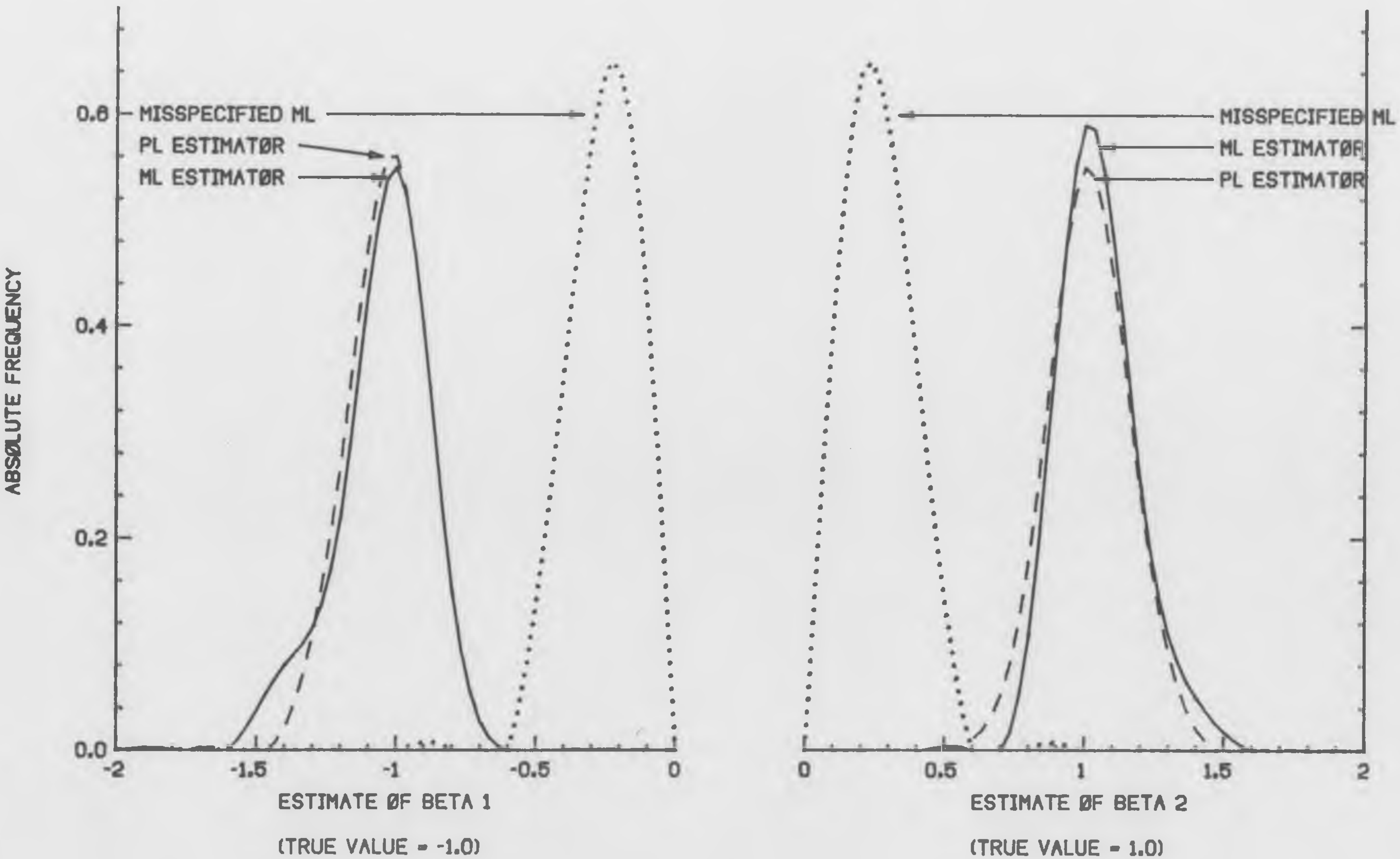
EFFECT ØF SIMULATED DISTURBANCE ØN ESTIMATES ØF 80% CENSØRED DATA



RHØ=0.0; N=100; NØ. ØF SAMPLES=100

FIGURE 6

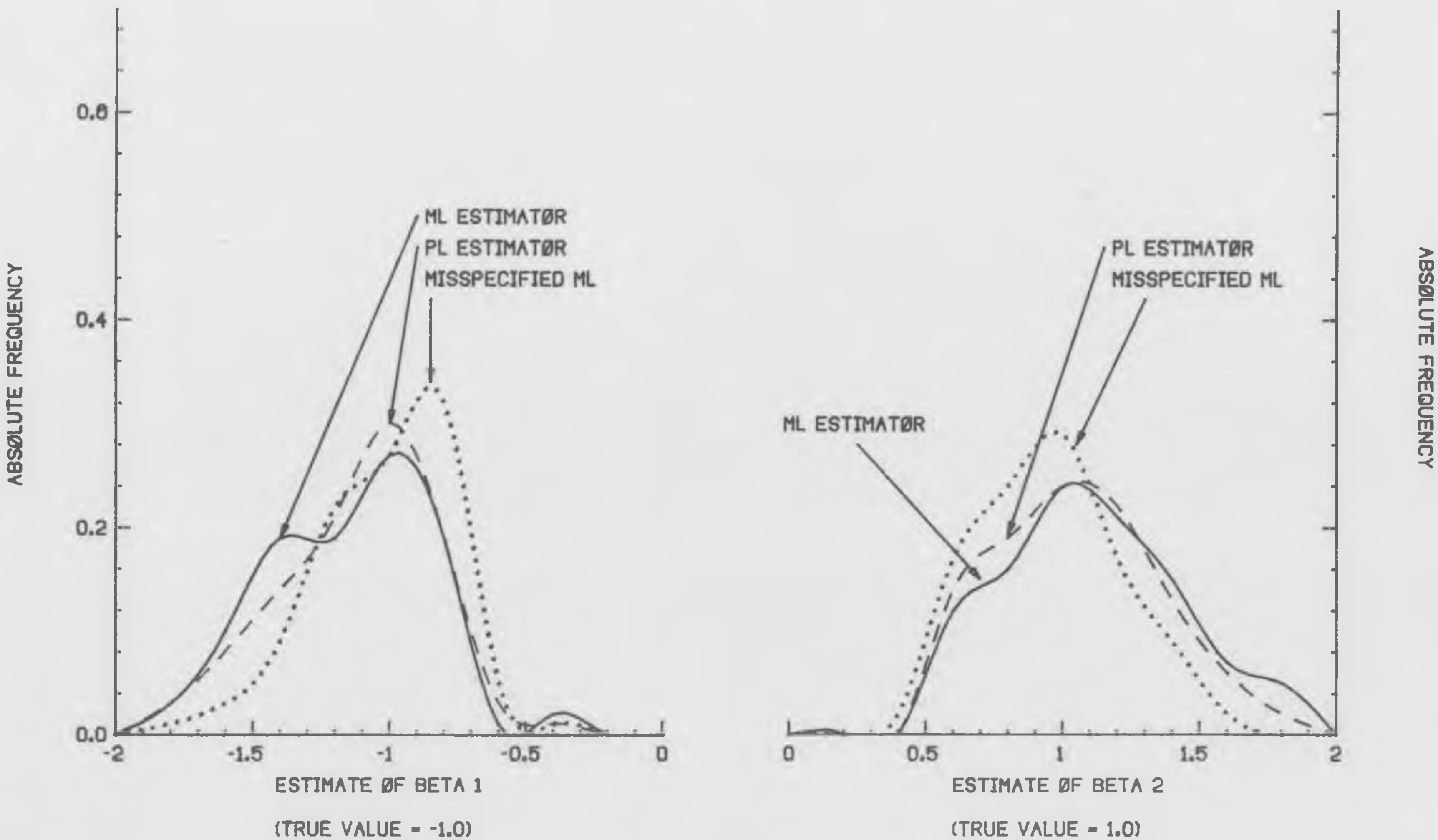
CØMPARISØN ØF ESTIMATØRS FØR TIME-DEPENDENT UNCENSØRED DATA



RHØ=0.0; N=100; NØ. ØF SAMPLES=100

FIGURE 7

CØMPARISØN ØF ESTIMATØRS FØR TIME-DEPENDENT 80% CENSØRED DATA



RHØ=0.0; N=100; NØ. ØF SAMPLES=100