

**NOVEL EVALUATION METHODS FOR COMPLEX SYSTEMS VIA
ADAPTIVE SEQUENTIAL EXPLORATION OF VARIABLES INTERACTIONS**

A Dissertation

by

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ABSTRACT

The complex and coupled behavior of variables in the currently developing Generation IV reactors and Small Modular Reactors is becoming a major incentive to seek efficient design methods. This research develops and validates new methods to evaluate systems with various degrees of variables' interactions using basic knowledge in variables' directions of effect and an adaptive number of experiments. The methods replace the commonly used assumption of negligible interactions with a broader assumption of monotonic variables' effects. The assumption was evaluated using studies of other physical systems' regularities, and is expected to be significantly present in physical systems.

Four methods were developed and analyzed in this dissertation. Three of the introduced methods utilized an adaptive sequential spanning tree concept with a method specific criterion to construct piecewise multidimensional surfaces or subtrees. Each method then used a specific approach to project the results within the subtrees. The fourth method is an expansion to an existing method to explore any order of interactions through the introduction of a new domain of parameters. Three of the four methods significantly outperformed the common orthogonal arrays methods that rely on a uniform distribution of experiments in the design domain. Two of the three methods significantly outperformed the third method and were used in the dissertation's application. The strength of the applicable methods was demonstrated through their application to two examples from literature, each of which has a different degree of variables' monotonic behavior. The most applicable method of the two most effective

methods was used to decouple the effects of fourteen variables on six performance characteristics in the design of a Small Modular Reactor version of the Advanced Pressurized Water Reactor AP1000. The methods' application succeeded in finding the most important main effects and interactions of each performance characteristic. The performance of the methods' application to three performance characteristics was compared to the performance of fractional factorial designs. The methods were found to significantly reduce the projection error when the assumption of variables' monotonic behavior is valid.

DEDICATION

This work is dedicated to my beloved parents and family. You mean the world to me.

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NOMENCLATURE

| | |
|------------------|--|
| AOFAT | Adaptive One Factor at A Time |
| AP1000 | Advanced Pressurized Water Reactor -1000 MWe |
| ANN | Artificial Neural Network |
| ASIS | Asymmetric Synergistic Interaction Structure |
| BOF | Beginning Of Life |
| Cm | Centimeter |
| C | Conversion Matrix |
| cov | Covariance |
| α_d | Density Reactivity Coefficient |
| X | Design Matrix |
| ΔK_{eff} | Depletion Drop of Multiplication Factor |
| DBARs | Discrete Burnable Absorber Rods |
| EOL | End Of Life |
| e | Error |
| FFD | Fractional Factorial Design |
| α_{FT} | Fuel Temperature Reactivity Coefficient |
| g | Gram |
| I | Interaction |
| IFFD | Iterative Fractional Factorial Designs |
| J | Joul |

| | |
|-----------|---|
| K | Kelvin |
| Kg | Kilogram |
| LWSMR | Light Water Small Modular Reactor |
| ME | Main effect |
| MEr | Margin of Error |
| MCNP | Monte Carlo N-Particle Transport Code |
| MSIV | Monotonic Sparsely Interacting Variables |
| MHIV | Monotonic Highly Interacting Variables |
| MDO | Multidisciplinary Design Optimization |
| K_{eff} | Effective Multiplication Factor |
| MWth | MegaWatt Thermal Power |
| NRMSE | Normal Root Mean Square Error |
| N | Number of Variables |
| n | Total Number of Combinations |
| β | Parameter |
| RC | Range Convergence |
| OAT | One at A Time |
| OFAT | One Factor at A Time |
| Pa | Pascal |
| PIRT | Phenomenon Identification and Ranking Table |
| y | Performance Characteristic or Response |
| PB | Plackett Burman |

| | |
|----------|------------------------------|
| PPF | Power Peaking Factor |
| P | Probability |
| RFS | Ratio of Fast Spectrum |
| Res | Resolution |
| RMSE | Root Mean Square Error |
| SMR | Small Modular Reactor |
| x | Binary State 0 or 1 |
| σ | Standard Deviation |
| SMEr | Simultaneous Margin of Error |
| x | Variable |
| var | Variance |
| w | Weight |
| Z | Z score |

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I. INTRODUCTION

The development of nuclear reactors as a mean to utilize atomic fission energy in power generation was driven by simple yet innovative physical concepts. From the time of the first reactor, the Chicago Pile [1], to the currently developing Generation IV reactors, the complexity of reactors has increased and continues to increase in a rapid manner. In early stages of reactors development, the lack of computational power and, in some cases, the lack of sufficient knowledge of the physical behavior of various parts of the reactor did not generate what would be considered an optimal design today. However, the early designs are still the bases for the current running reactors, and similarities are apparent when comparing current designs to older designs [2, 3]. The occurrence of few major accidents and several minor accidents forced the consideration of scenarios and effects that were previously discarded. This was reflected in the design of new generations of reactors. The main drive for design changes of nuclear reactors was safety for a long period of time. In recent years, economics became another important drive for design changes as power utilities tried to extend the life of operating reactors. Existing designs were reviewed to consider the possibility of the extension. Both safety and economic factors did not cause significant divergence of reactors designs from their initial core designs. Design changes were determined through practice-driven performance optimization facilitated through experience buildup from several decades of operations of water cooled reactors.

The complex and coupled behavior of variables in the currently developing Generation IV reactors [4] and the increase of interest in Small Modular Reactors (SMRs) [5] are becoming a major incentive to seek efficient design methods. The application of modern design and optimization methods on Generation IV reactors and SMRs is receiving an increasing attention in recent years. The experience of earlier reactors' designs is being thoroughly reviewed to utilize it in the development of Generation IV reactors and SMRs (e.g. [6, 7]). A sample of the number of variables and performance characteristics involved in a typical reactor's design is shown in Figure 1. These design variables, along with others [8], vary significantly from one reactor to another. If two levels of variation are set for each of the shown 19 variables, 524,288 combinations are needed to determine all variables' effects and interactions. Performing such a large number of experiments is not practical when experiments are expensive. In nuclear reactors design, experiments, including computational ones, are often expensive.

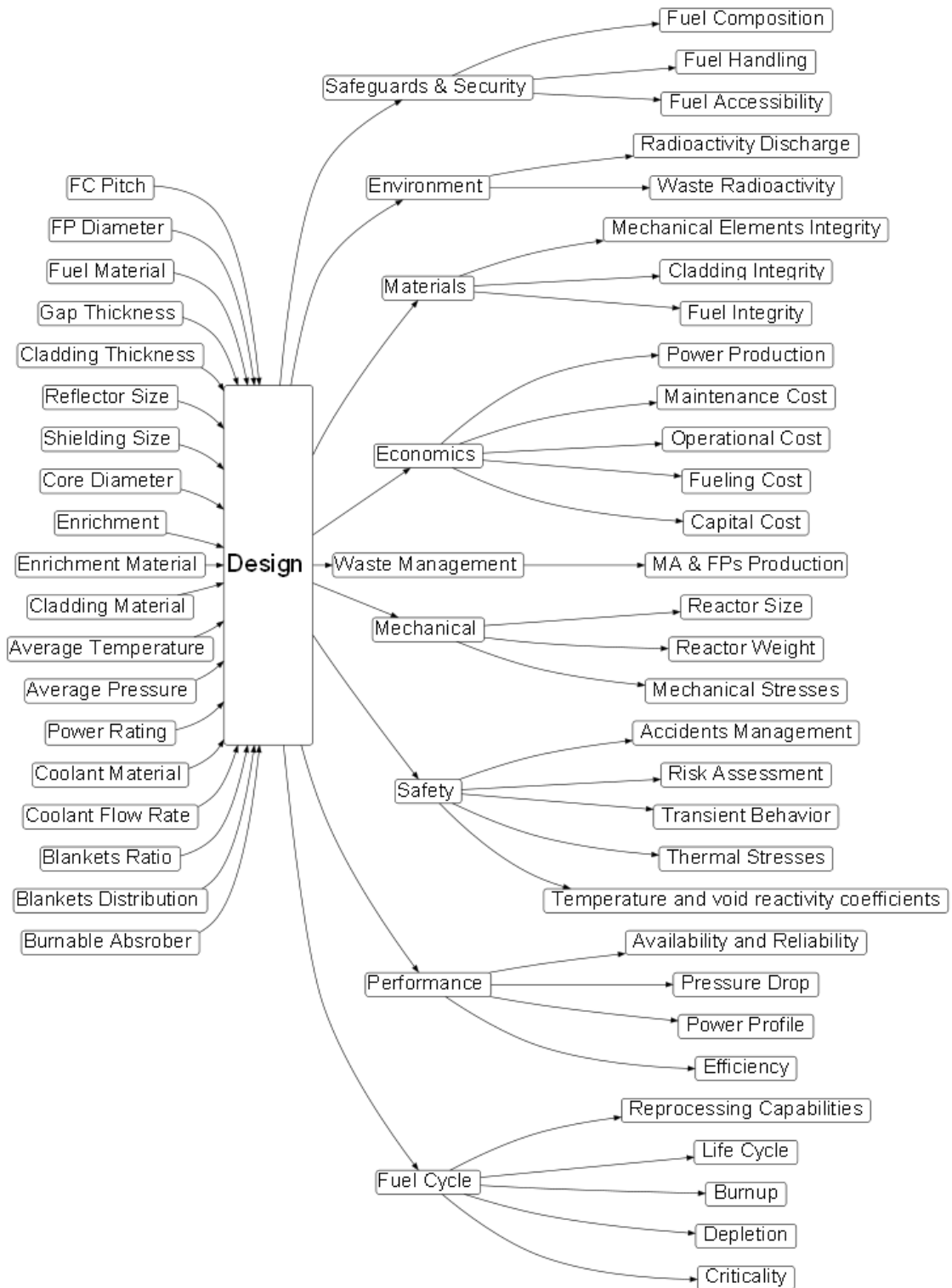


Figure 1. Example Variables and Performance Characteristics in the Design of Nuclear Reactors

Performances characteristics are sometimes optimized with respect to a set of variables [9-12]. It is often the case that the optimization method is not thoroughly described, a brute force approach is applied, or a comprehensive trial and error process is followed [13-17]. An example of a design flow chart for nuclear reactors is found in [18]. The coupling of variables in nuclear reactors design makes trial and error efforts very inefficient. An integrated approach that targets the coupling effect of several variables on several performance characteristics is not commonly applied in nuclear reactors design. Shortlisting variables affecting certain performance characteristics without a clear evaluation process is a commonly followed design approach. Experts' input dependent approaches, such as Phenomenon Identification and Ranking Tables (PIRTs), is another common approach (e.g. [19, 20]). It is also common to drive the sensitivity analysis process with a theory based modeling as performed in [21].

If a model is missing for a certain aspect of the design, experiments are needed to develop one. In nuclear reactors designs, most of the performed experiments are computational experiments. While computational experimentation is more economical than physical experimentation, it can become very expensive for complex systems. The design of experiments is a well-developed widely addressed subject in literature and books such as [22-24]. The subject presents several potential methods to decouple the performance characteristics of a nuclear reactors design. A variety of methods are investigated in this dissertation.

This dissertation develops new experiments design methods to guide the initial phase of the design process for a nuclear reactor. The methods apply an adaptive

approach for the development of approximation models between variables and performance characteristics. These models highlight the most contributing variables and interactions for each performance characteristic. From this, the number of variables considered in the detailed design process of each performance characteristic is significantly reduced and the optimization process became more economical. In the field of design of experiments, experiments performed for this purpose are defined as “Sensitivity Analysis” or “Screening” experiments, and are sometimes referred to as exploratory experiments. This is a topic that is addressed thoroughly in literature. Screening is directly related to sensitivity analysis. If the parameters found by screening are divided by the variables’ ranges, the resulting parameters are the sensitivity coefficients. In addition to finding important variables and interactions, these experiments have several benefits that can be found in [25]. Most of the sensitivity analysis and screening methods do not deal with nonlinear effects of isolated variables. Nonlinearity can be investigated in other response surface methodologies such as central composite designs (see [26] for details).

Different screening and sensitivity analysis methods utilize different assumptions. A review of existing sensitivity analysis and screening methods is presented in the “Existing Approaches” section. The methods of this dissertation will utilize the monotonic behavior assumption, which is present in reactors design and other engineering fields. In addition, the dissertation’s methods will assume knowledge of variables’ directions of effect on certain performance characteristics. For example, the magnitude of effect of changing fuel enrichment on the effective multiplication factor,

K_{eff} , might be unknown in the design of new reactors. However, there is a strong belief that increasing enrichment will always increase K_{eff} . This represents a monotonic or persistent positive relationship between the variable enrichment and K_{eff} . This knowledge can be used in the design of any reactor. In some cases, this relationship exists between some variables and components of the performance characteristic. For example, instead of dealing with K_{eff} , it might be simpler to deal with its main contributors, such as fission reaction rate, absorption reaction rate and leakage rate. The variables' directions of effect can often be predicted from common sense or knowledge that has developed over decades of experience.

The following section will evaluate the validity of the monotonic behavior assumption using existing studies of other regularities. Using this assumption, section III will present the shape for the response change after variables' alignment. Section III introduce four methods' concepts to benefit from this knowledge. The methods are explained in a detailed manner in section IV. Two examples of the application of the most applicable method on a regular and non-regular systems are presented in section V. Section VI applies the most applicable methods on the design of an SMR version of the AP1000. The conclusions of the application of these methods and the corresponding results are analyzed in section VII. Future development on the methods and application are presented in the section VIII.

1.1 Problem Statement

The design of new nuclear reactors, including Generation IV reactors, differs significantly from the conventional designs of water cooled reactors. The divergence of the design from the conventional domain into a new domain introduces several behavior uncertainties that could be comprehended through experimentation. The design of these experiments is challenging when experiments are expensive to conduct. The least experiments demanding model to evaluate the performance characteristics' dependence on a set of variables is a linear model. In a linear model, two end points of each variable can be used to define the link between the response and the variables. If variables' interactions are to be considered, the model becomes linear with respect to single variable changes only. If all interactions are to be explored, an exponentially increasing number of experiments is needed. If the number of variables considered is N and if all interactions are to be captured, the full factorial set of $n=2^N$ experiments will need to be performed. When N is large, the actual number of experiments performed, m , need to be much smaller than n . Since m is lower than the model full factorial number of unknowns, the system is underdetermined. Unlike over-determined systems with replicated experiments that apply regression methods [27], it is necessary in under-determined systems to either make valid assumptions to constrain the excessive degrees of freedom, or to use existing relevant knowledge of the system.

In reactors design, useful knowledge does exist and can be used to compensate for the missing degrees of freedom. The existing knowledge used in this dissertation will not be by physical behavior quantification, but rather a direction based knowledge. The majority of existing approaches uses an assumption on the significance of the interactions at various orders. Since there is no knowledge of whether this assumption is valid, the norm is to seek few orders and assume higher orders as negligible. This is inefficient since it assumes that the orders of interactions of all variables are equally important when, in reality, they are not. In addition, exploring very low order of interactions only would result in inaccurate results. Most systems tend to behave in a manner similar to the one shown in Figure 2. An adaptive method would mean that as experiments are performed, the large interactions of the tree to the left side are targeted first, followed by the second order interactions of the right side of the figure. The unknown smaller interactions in both sides are further explored as the experiments are performed. The majority of existing approaches assume a uniform distribution of experiments, or that all variables have the same importance. If second order interactions are assumed significant, the assumed system model will be similar to Figure 3. It is, thus, desired to develop a method to enable the experimenter to efficiently determine experiments that reveal important variables and interactions only.

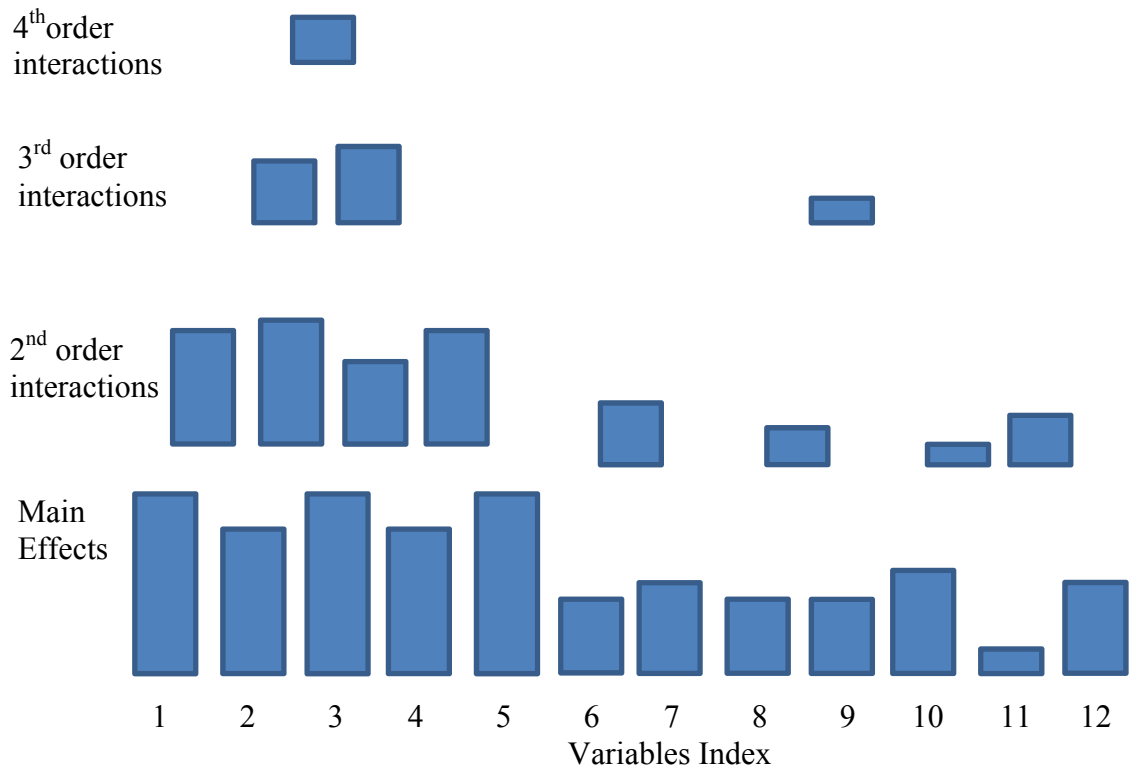


Figure 2. Example Realistic Importance of Main Effects and Interactions

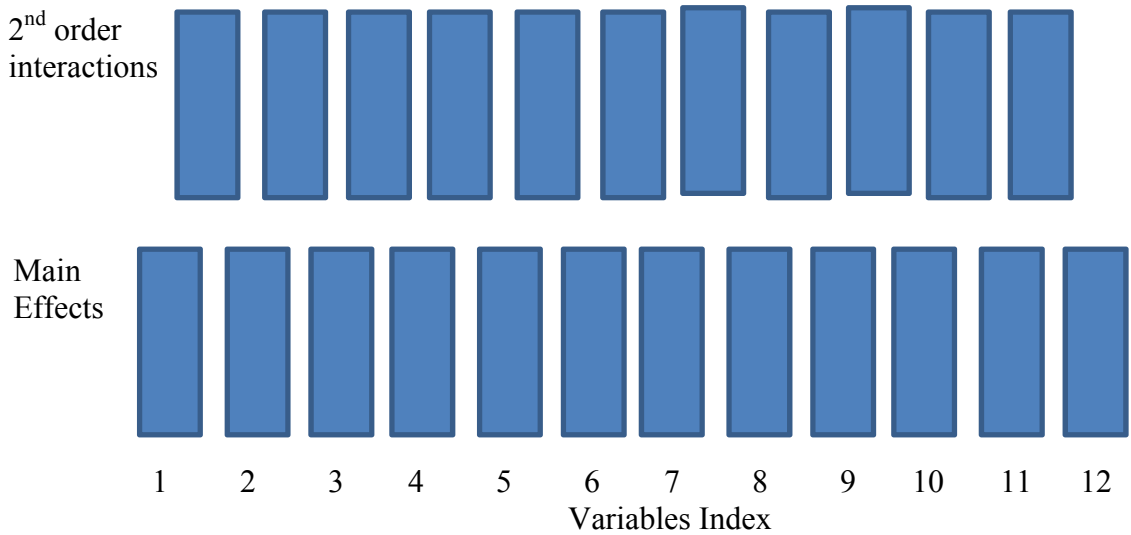


Figure 3. Example Assumed Importance of Main Effects and Second Order Interactions

1.2 Existing Approaches

If all variables' interactions of a system model are to be determined, a full factorial design is necessary. It is, however, impractical and often illogical to develop full factorial designs for problems with a large number of variables. Fractional Factorial Designs (FFDs) are orthogonal arrays that can reduce the number of experiments. However, FFDs can only be used if high order interactions are known to be negligible [25], since they are based on confounding main effects and various degrees of interactions. Plackett Burman (PB) introduced a new set of orthogonal arrays in 1946 [25, 28]. They differ from FFDs in the way the interactions are confounded. PB designs determine the main effects of N variables using $N+1$ experiments in a manner that distributes evenly the confounding interactions among the main effects. This reduces the error of the determined main effects. PB designs and FFDs are popular screening methods, but they both confound interactions with main effects, and provide accurate results only if the system has negligible interactions. Tagushi introduced orthogonal arrays to screen the most effective variables affecting a product's quality [29, 30]. Like other orthogonal arrays, they also confound effects.

Figure 4 demonstrates how FFDs confound interactions. The figure shows an example of all two state combinations of five variables. Each node represents one parameter of 32 parameters in total. There are two resolutions to screen this system through FFDs. One is using eight a Res III set of experiments and the other using a sixteen Res IV set of experiments. The degree of confounding parameters is usually described by resolution (Res). A low resolution design implies confounding parameters

of low order interactions. This reduces significantly the screening accuracy. Figure 4 shows the Res III FFD with the nodes of the same color as confounded. FFDs sum the nodes of the same color to generate eight summations. If high order interactions are neglected, this would represent neglecting nodes in the upper side of the tree. As the number of variables and the number of nodes increase, screening by a certain resolution sums an increasing number of nodes. Parameters evaluation based on these sums becomes questionable. In addition, confounding a large number of parameters makes it impossible to identify which interaction parameter is significant and which are not. For example, if green was judged as important and as a result variable 5 is defined as a significant variable, this finding might be due to the 13 interaction being important, which falsely caused variable 5 to appear important. In addition, even if variable 5 is classified as important, it is not possible to determine which of its interactions is significant since they confound with other variables. A simple heat transfer example is used in Appendix A to demonstrate how confounding main effects and interactions can cause inaccurate conclusions.

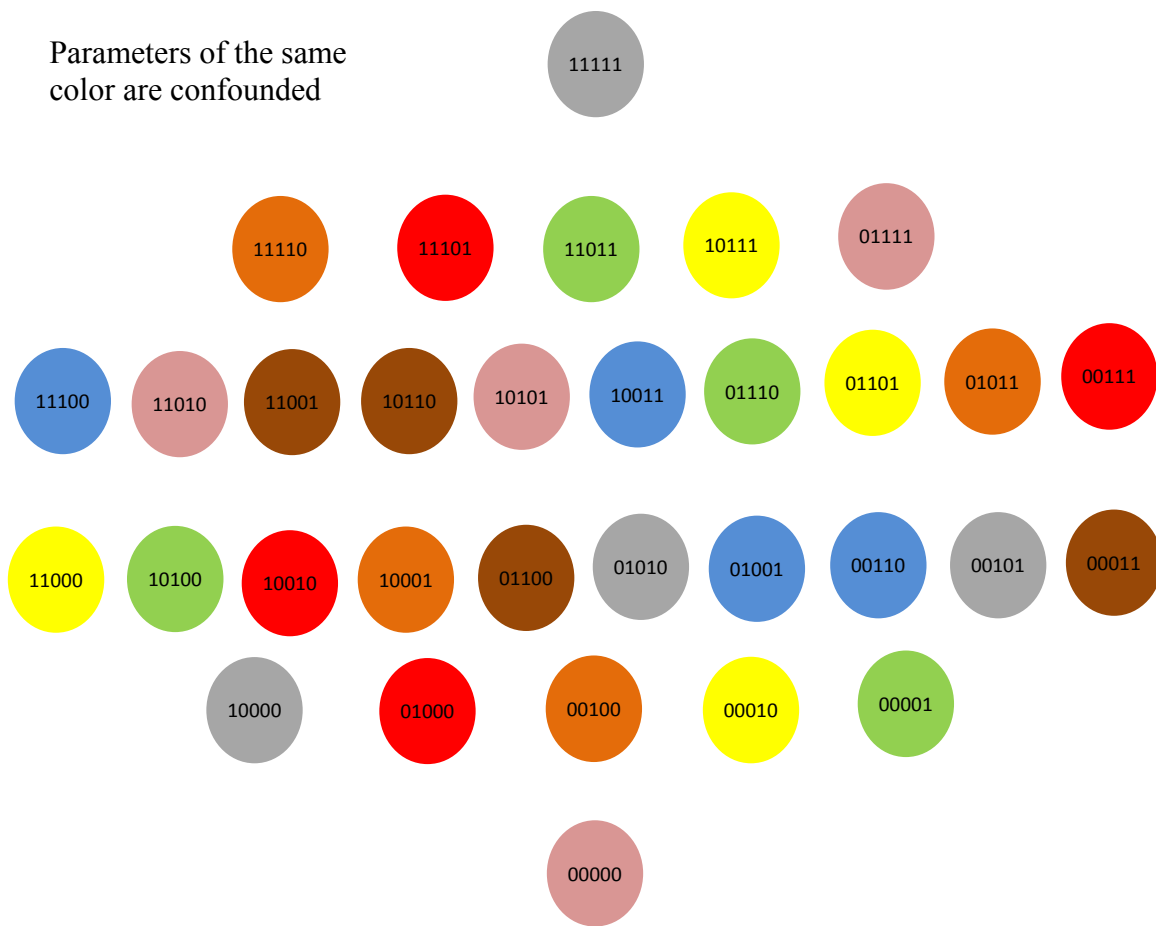


Figure 4. Res III Grouping of Confounded Parameters

From a mathematical prospective: all orthogonal arrays rely on projecting points in the unexplored domain using the explored points by means of interpolation or extrapolation. In order to provide the most accurate projection of unexplored domains, they rely on a uniform distribution of experiments over the whole domain of the design. Means to unfold confounding effects such as semifolding or mirror-image foldover designs also exist (see [25, 31] for details). However, the unfolding decision is based on an expert selecting specific interactions to unfold, and they are expensive in terms of the

number of experiments. Within the scope of the dissertations' problem, the limitation of the most common orthogonal arrays, FFDs, can be summarized as:

1. They do not provide information on whether the performed screening resolution is sufficient to produce an accurate model.
2. They are inefficient since they give equal importance to all variables and interaction. In the unfolding process, interactions deemed unimportant at the initial screening will also be unfolded as there is no possibility to unfold selected interactions.
3. They proceed in steps of power of 2 (8, 16, 32, 64, 128...), which makes it exponentially expensive to increase the resolution of screening. It is not possible to use a number of experiments between these steps except for certain semifolding matrices that usually require a large number of experiments such as Johns $\frac{3}{4}$ Designs.
4. Except for Res III to IV designs, it is not possible to increase the resolution of the design. If Res IV was deemed inaccurate, a new set of experiments are to be performed that slightly overlap with the lower resolution set of experiments.
5. They do not give the flexibility to economically add new variables to the model after screening is performed

Non-orthogonal arrays were developed to predict certain interactions provided that the experimenter knows which interactions to explore [32]. The method of [33] combined fractional factorial designs and orthogonal polynomials. To screen interacting variables, Cotter [34] developed a method that determines the summation of odd and even parameters for every variable. This requires a total of $2N+2$ experiments. This

method assumes that it is unlikely for a variable's odd and even parameters to confound. The method is mainly valid for screening important variables. It does not provide valuable information on parameters since they are confound with other interactions of the same parent variable. A high number of variables indicate that several second order interactions exist, and distinction of interacting variables' effects becomes not feasible. The method of [35] expanded on Cotter's method to unfold second order interactions, assuming third and fourth order interactions are negligible. The study determines the interactions to explore based on the assumption that either parents of an interaction must be active in order for an interaction to be active. This assumption is valid according to [36, 37], but this method's performance is sensitive to false negligence of an important interaction. One of this dissertation's methods expanded Cotters method to unfold the confounding effects of the combined or integrated parameters on all orders.

Group screening is a methodology that was developed to handle high number of weakly interacting variables [38]. These methods rely on placement of variables into groups and separately analyzing each group before analyzing cross groups interactions of important variables in each. Several group evaluation methods exist [38-46], and have similar assumptions and similar limitations. The common limitations are that these methods are dependent on the experimenter grouping process and on the strength of variables' interactions. A demonstration example of how confounding effects can hide active variables in group screening is shown in Appendix A. Three group screening methods are briefly introduced in this section for illustration. In the method of [43], the experimenter screens separate groups of design and noise. The method establishes a

main effect of groups. Once a group is defined as active, it is investigated for its variables' main effects and second order interactions. Only variables that interact within the group are investigated for interactions with other groups. The method aligns the directions of effect of variables in each group to reduce the confounding effects, thus reducing the probability of missing an important variable. The study of [44] analyzed the performance of [43] according to two criteria related to the number of experiments and the probability of missing an active effect. The method's limitation is that it does not always capture group to group variables' interactions if in-group interactions or main effects are classified as negligible due a local confounding behavior. It also requires a pre-knowledge of variables' importance as to group them in an efficient manner. If variables' importance was poorly judged, it would result in inaccurate results or inefficient number of experiments. Another group screening method that relies on the union of active factors from overlapping groups is introduced in [45]. The groups are selected randomly and adaptively. The method is efficient in determining important main effects. It does not target high order interactions and its efficiency in determining important variables is dependent on the grouping process. The study of [46] developed the Sequential Bifurcation (SB) method and provides a comparison of common group screening methods. The method relies on the spanning tree concept and the directional alignment of variables. It accounts for second order interactions and proposes to include second order nonlinearity of single variables. This adds N terms to the model, thus requires a middle point to be estimated for every variable states. The method makes the assumption that the variable's direction of effect is determined based on the main effect.

The method is not designed for highly interacting variables. The efficiency of this method is dependent on the distribution of the important variables on the groups.

One at A Time (OAT) ,also referred to as One Factor at A Time (OFAT), are sensitivity analysis and screening methods that are based on simply changing one variable at a time with all other variables fixed [47]. The experimenter evaluates the result before changing other variables. The study of [48] developed a trajectory based sensitivity analysis method with random starting points of experiments. Variables' effects are found by a mean and variance representing the main effect and confounded interactions with nonlinearity. The method was revised and improved by [49]. The study of [50] developed another OAT trajectory based experiments selection method that is in the order of N^2 , and explores second order interactions. Other OAT methods have been investigated too such as [51]. OAT methods' efficiencies and applicability remain a topic of discussion (see [52, 53][54][55]).

The Fourier Amplitude Sensitivity Test [56] is a sensitivity analysis method that is based on the fact that important variables behave in a periodic manner approximated by coefficients of the Fourier series. Its applicability in complex systems remains questionable. The method of Iterative Fractional Factorial Designs (IFFD) of [57], and evaluated in [58], uses a sets of different FFD experiments in an iterative manner to produce a sensitivity model.

To compensate for the lack of sufficient information on physical systems behavior, studies were conducted to evaluate certain regularities in the behavior of systems as to find common physical patterns. Recent studies [36, 37] analyzed four

systems regularities: sparsity, hierarchy, heredity and asymmetric synergistic interaction structure. The statistical results of [37] were used to develop a Bayesian approach in predicting active main effects and interactions.

Fields other than screening and sensitivity analysis have been investigated for applicability to solve the dissertation's problem. Designing a set of experiments is analogous to the mathematical problem of multidimensional nonlinear surface fitting. A sample of points, or experiments, is used to define the behavior of the surface, or response. Several examples of curve fitting methods can be found in literature [59-61]. Artificial Intelligence and specifically Artificial Neural Networks (ANNs) can be used to solve the dissertation's problem. ANNs require the experimenter to define the size of the network and the functions to use. ANNs' applicability was tested by modeling one performance characteristic, K_{eff} , of this dissertation's case study. The ANN was found to be significantly less accurate than FFDs.

Instead of focusing on variables, it is also possible to drive the experimentation process towards a certain response point or region. This process is referred to as optimization. Optimization efficiency is very dependent on the complexity of the system. If the system is too complex, it can be broken into subcomponents using Multidisciplinary Design Optimization (MDO) methods. MDO studies the means to optimize coupled systems by optimizing the whole system or optimizing the coupled subsystems separately then accounting for the interactions between subsystems. The study of [62] reviews existing MDO approaches and discusses their performances. There are several optimizations methods in literature [63]. Several of these are gradient based

[64]. Others are based on the divide and conquer concept, such as the branch and bound optimization that is utilized in [65]. The concept of branch and bound is to split the domain of variables according to a certain criterion into smaller subdomains. The bound of the optimization criteria in every subdomain is then found. This concept has been utilized in the dissertation's methods development.

Genetic algorithms are one of the most common optimization methods. They optimize systems with respect to certain targets using observations made from experiments performed using various combination of variables' states. They start from the no-knowledge state and builds knowledge of the performance characteristic, not the variables, towards a desired value as more experiments are performed. In genetic algorithms, the result is a set of variables' states or ranges that achieves the desired performance characteristic to a certain degree. These algorithms have been applied in nuclear reactors designs [66] and in fuel shuffling and depletion analysis [67-70]. Since these algorithms start from no knowledge, they are not as efficient as the dissertation's methods, which assume knowledge of directions of effect. In addition, these methods do not introduce an understanding of how systems behave. They rather deal with systems as a black box. If the optimization value of the performance characteristic is changed, these algorithms have to be performed again utilizing only a portion of the performed experiments. Genetic algorithms converge on the performance characteristics. Screening and sensitivity analysis methods converge on the variables' parameters.

Predictive modeling, dimensionality reduction and pattern recognition methods such as [71-73] were explored to solve the problem of underdetermined systems. These

methods are mostly non-deterministic, but can explain extremely large number of variables. They could be thought of as encoding methods that reduces the size of a large set of data into a small one. They reduce the dimensionality of the system to a defined size based on the assumption that, underneath the complex, high dimensional system lays a low dimensional model that can represent the high dimensional system with an acceptable error. These approaches are mostly probabilistic and driven by historical data, thus are inapplicable to solve the dissertation's problem.

1.3 Objectives

The target of the dissertation's methods is to overcome the limitations of the existing methods' assumptions in order to evaluate systems with various degrees of interactions in an efficient manner. The method should not neglect high order interactions or make any assumptions on the strength of high order interactions. The results of the application of the methods should reduce optimization efforts through the identification of important variables and interactions. The detailed objectives of the method can be listed as:

- Utilization of various degrees of knowledge of the directions of variables' effects on a performance characteristic
- Efficient selection of experiments to determine only significant performance-relevant main effects and interactions in a system with large number of interacting variables
- Adaptive modeling and identification of significant performance-relevant main effects and interactions with a quantified estimated response uncertainty

In terms of application, the methods should be applied to the design of a sample of variables and performance characteristics of a Light Water Small Modular Reactor (LWSMR) to understand the coupling behavior of variables.

1.4 Assumptions

Two main assumptions and few minor assumptions are used in this dissertation. The first assumption is the knowledge of the direction of variables effects on performance characteristics. This is the key input for the methods introduced in this dissertation. The methods utilize this knowledge in aligning the variables to reduce the confounding effects. The second assumption is that the directions of effect of important variables, also referred to as active variables, tends to be persistent. This is considered a phenomenon, referred to as monotonic, that has been seldom investigated in literature. The validity of this assumption is evaluated in a later section. Absolute confidence of the monotonic behavior is not necessary, since the assumption is evaluated through a process to predict the performance of the model within every part of design domain.

As often followed in screening and sensitivity analysis experiments, a linear model is considered sufficient to generate an approximation of the response behavior, since a linear model will require the minimum number of experiments. The model is only considered linear with respect to every variable when all other variables are fixed. It will allow nonlinearity due to variables' interactions.

Since an adaptive sequential method is desired, it should be possible to conduct experiments one at a time. The design ranges of variables should be defined while the

performance characteristic's domain remains boundless. As was mentioned earlier, screening and sensitivity analysis methods limit the variables' domain while optimization methods limit the performance characteristics domain. In summary, the assumptions made in the introduced methods are:

- Various degrees of knowledge of the variables' directions of effect on the performance characteristics
- Monotonic effects of active variables on the performance characteristics
- Sequential experimentation
- Finite variables' design domain
- Limited nonlinearity of single variables

1.5 Approximation Model and Design Domains

The model used in the dissertation's methods is a linear model in every variable. This model requires a minimum of two states for every variable. A linear model is considered sufficient at this stage of the design, since, in an interacting system, the interactions nonlinearity is more significant than single variable's nonlinearity. The linear model of a response or performance characteristic is approximated using Taylor series[74] as:

$$\begin{aligned}
y(x_1, x_2, x_3, \dots) &= y(x_{1,R}, x_{2,R}, x_{3,R}, \dots) + \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_1} \Big|_{x_{2,R}, x_{3,R}, \dots} (x_1 - x_{1,R}) \\
&+ \frac{\partial Y(x_1, x_2, x_3, \dots)}{\partial x_2} \Big|_{x_{1,R}, x_{3,R}, \dots} (x_2 - x_{2,R}) + \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_3} \Big|_{x_{1,R}, x_{2,R}, \dots} (x_3 - x_{3,R}) + \dots \\
&+ \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2} \Big|_{x_{3,R}, \dots} (x_1 - x_{1,R})(x_2 - x_{2,R}) + \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_1 \partial x_3} \Big|_{x_{2,R}, \dots} (x_1 - x_{1,R})(x_3 - x_{3,R}) \quad (1) \\
&+ \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_2 \partial x_3} \Big|_{x_{1,R}, \dots} (x_2 - x_{2,R})(x_3 - x_{3,R}) + \dots \\
&+ \frac{\partial^3 Y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} (x_1 - x_{1,R})(x_2 - x_{2,R})(x_3 - x_{3,R}) + \dots + e
\end{aligned}$$

where x_1, x_2, x_3, \dots are the states of variables in a defined domain and $x_{1,R}, x_{2,R}, x_{3,R}, \dots$ are reference points for the model in the same domain. Since a linear model is assumed with respect to every variable, the second and higher order derivatives of single variables were omitted from the model.

If the experimental error can be found, it can be used in choosing a threshold for importance as will be demonstrated at a later stage. These errors can play a very important role in small response changes as they can significantly alter the shape of the model. The impact of the experimental error on the response model development has been addressed in literature and will not be addressed in this dissertation. It is expected that the response projection error will dominate over the experimental error. The model of equation 1 can be rewritten as:

$$E[y(x_1, x_2, x_3, \dots, x_n)] = \beta_0 + \sum_{i=1}^N \beta x_i + \sum_{i=1}^N \sum_{j>i}^N \beta_{ij} x_i x_j + \sum_{i=1}^N \sum_{j>i}^N \sum_{k>j}^N \beta_{ijk} x_i x_j x_k + \dots \quad (2)$$

β s are the parameters of the models, and are equal to

$$\beta_0 = \left[\begin{array}{c} y(x_{1,R}, x_{2,R}, x_{3,R}, \dots) - \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_1} \Big|_{x_{2,R}, x_{3,R}, \dots} x_{1,R} \\ - \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_2} \Big|_{x_{1,R}, x_{3,R}, \dots} x_{2,R} - \frac{\partial Y(x_1, x_2, x_3, \dots)}{\partial x_3} \Big|_{x_{1,R}, x_{2,R}, \dots} x_{3,R} - \dots \\ - \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_1 \partial x_2} \Big|_{x_{3,R}, \dots} x_{1,R} x_{2,R} - \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_1 \partial x_3} \Big|_{x_{2,R}, x_{4,R}, \dots} x_{1,R} x_{3,R} \\ - \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_2 \partial x_3} \Big|_{x_{1,R}, x_{4,R}, \dots} x_{2,R} x_{3,R} - \dots \end{array} \right] \quad (3)$$

$$\beta_1 = \left[\begin{array}{c} \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_1} \Big|_{x_{2,R}, x_{3,R}, \dots} - \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2} \Big|_{x_{3,R}, \dots} x_{2,R} \\ - \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_1 \partial x_3} \Big|_{x_{2,R}, \dots} x_{3,R} - \dots + \frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} x_{2,R} x_{3,R} + \dots \end{array} \right] \quad (4)$$

$$\beta_2 = \left[\begin{array}{c} \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_2} \Big|_{x_{1,R}, x_{3,R}, \dots} - \frac{\partial^2 y(x, x_2, \dots)}{\partial x_1 \partial x_2} \Big|_{x_{3,R}, \dots} x_{1,R} \\ - \frac{\partial^2 y(x_2, x_3, \dots)}{\partial x_2 \partial x_3} \Big|_{x_{1,R}, \dots} x_{3,R} - \dots + \frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} x_{1,R} x_{3,R} + \dots \end{array} \right] \quad (5)$$

$$\beta_3 = \left[\begin{array}{c} \frac{\partial y(x_1, x_2, x_3, \dots)}{\partial x_3} \Big|_{x_{1,R}, x_{2,R}, \dots} - \frac{\partial^2 y(x_1, x_3, \dots)}{\partial x_1 \partial x_3} \Big|_{x_{2,R}, \dots} x_{1,R} \\ - \frac{\partial^2 y(x_2, x_3, \dots)}{\partial x_2 \partial x_3} \Big|_{x_{1,R}, \dots} x_{2,R} - \dots + \frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} x_{1,R} x_{2,R} + \dots \end{array} \right] \quad (6)$$

$$\beta_{12} = \left[\begin{array}{c} \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2} \Big|_{x_{3,R}, \dots} - \frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} x_{3,R} + \dots \end{array} \right] \quad (7)$$

$$\beta_{13} = \left[\begin{array}{c} \frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_1 \partial x_3} \Big|_{x_{2,R}, \dots} - \frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} x_{2,R} + \dots \end{array} \right] \quad (8)$$

$$\beta_{23} = \left[\frac{\partial^2 y(x_1, x_2, \dots)}{\partial x_2 \partial x_3} \Big|_{x_{1,R}, \dots} - \frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} x_{1,R} + \dots \right] \quad (9)$$

$$\beta_{123} = \left[\frac{\partial^3 y(x_1, x_2, \dots)}{\partial x_1 \partial x_2 \partial x_3} \Big|_{x_{4,R}, \dots} + \dots \right] \quad (10)$$

The target of the dissertation's methods is to find the most influential parameters that will explain the relationship between the response y and the variables x_1, x_2, x_3, \dots of equation 2. Using full factorial designs will require $n = 2^N$ experiments to determine all n parameters. Every experiment will provide a result for a unique combination of variables' states. In a matrix format, this can be expressed as:

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = X\beta = \begin{bmatrix} x_{1,1} & x_{2,1} & \cdots & x_{n,1} \\ x_{1,2} & x_{2,2} & \cdots & x_{n,2} \\ \vdots & \vdots & \cdots & x_{n,3} \\ x_{1,n} & x_{2,n} & \cdots & x_{n,n} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} \quad (11)$$

X is the design matrix and defines all state combinations of the variables. β_n is the interaction parameter of all variables ($\beta_{12345\dots N}$). Two domains were investigated for the variables x_1, x_2, x_3, \dots in this dissertation. The 0,1 domain, $x \in \{0,1\}$, transforms the variables' range into a minimum state of zero and a maximum of one. The -1,1 domain, $x \in \{-1,1\}$, transforms the variables' range into a minimum state of negative one and a maximum of one. The design matrices of the two domains are:

$$X_{1,0} = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 1 \\ 1 & 0 & \dots & 1 & 0 \\ 1 & 0 & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix}, X_{1,-1} = \begin{bmatrix} 1 & -1 & \dots & -1 & -1 \\ 1 & -1 & \dots & -1 & 1 \\ 1 & -1 & \dots & 1 & -1 \\ 1 & -1 & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix} \quad (12, 13)$$

The parameters have different interpretation depending on the design matrix domain. In the -1,1 design domain, the parameters represent half the main effects and interactions. Since every variable varies from -1 to 1, the actual change of the response due to variation of one variable is twice the value of its parameters, thus a parameter value is half the main effect or interactions. The relationship between parameters in the two design domains is:

$$\beta_{1,-1} = X_{1,-1}^{-1} Y = X_{1,-1}^{-1} \underbrace{X_{1,0} X_{1,0}^{-1}}_I Y = X_{1,-1}^{-1} X_{1,0} \beta_{1,0} \quad (14)$$

A brief explanation of the two design domains' relationship in addition to the effect of the response variance on the parameters of each of the two domains can be found in Appendix B. The most important finding to take from the appendix is the advantage of the -1,1 design matrix domain over the 0,1 design domain. It is that it provides an equal value and error weight to all experiments results. If the covariances of the response results are assumed as zero then:

$$\beta_{k,1,-1} = \frac{1}{n} \sum_{i=1}^n S_i y_i \Rightarrow \delta(\beta_{k,1,-1})^2 = \left(\frac{1}{n}\right)^2 \sum_{i=1}^n \delta(y_i)^2 \quad (15)$$

where S_i is the sign of the parameter $\beta_{k,1,-1}$ effect on the response. If all the response errors are assumed of the same magnitude then:

$$\delta(\beta_{k,1,-1})^2 = \left(\frac{1}{n}\right)^2 \sum_{i=1}^n \delta(y_i)^2 = \left(\frac{1}{n}\right)^2 n \delta(y)^2 = \left(\frac{1}{n}\right) \delta(y)^2 \quad (16)$$

Since only a small portion (m) of the total number of experiments is performed, the response error can be categorized into an experimental error, $\delta_{Exp}(y_i)$ and an experimental and projection error, $\delta_{Exp+Projection}(y_i)$. The parameters' variance is, then, defined as:

$$\delta(\beta_{k,1,-1})^2 = \left(\frac{1}{n}\right)^2 \left[\sum_{i=1}^m \delta_{Exp}(y_i)^2 + \sum_{i=m}^n \delta(y_i)_{Exp+Projection}^2 \right] \quad (17)$$

If all the response errors are assumed of the same magnitude and

$$\delta(y_i)_{Exp+Projection}^2 \gg \delta(y_i)_{Exp}^2 \text{ then}$$

$$\delta(\beta_{k,1,-1})^2 = \left(\frac{n-m}{n^2}\right) \delta(y)_{Exp+Projection}^2 \quad (18)$$

This shows that as the number of parameters, n, exponentially increases by the increase of the number of variables N, the parameter's variance dependence on the response's variance weakens. The equation also shows that as $n \rightarrow \infty$, the effect of incrementing m decreases and the main contributor for reduction of $\delta(\beta_{k,1,-1})^2$ is the reduction of $\delta(y)_{Exp+Projection}^2$. If $m \ll n$, then there is a need to establish a covariance of the response, y, through introducing an assumption to overcome the weak dependence of $\delta(\beta_{k,1,-1})^2$ and $\delta(y)_{Exp+Projection}^2$. The assumption used, in this dissertation, is the monotonic variables' behavior assumption. The validity of this assumption is addressed in the following section.

II. VALIDITY OF VARIABLES MONOTONIC BEHAVIOR ASSUMPTION

This section will demonstrate the validity of the monotonic variables' behavior assumption from the findings of previous studies [36, 37] that analyzed physical systems regularities. The studies used the Lenth method [75] to classify variables' effects from un-replicated experiments as active or inactive, representing important or unimportant variables. Other methods can be used to assess the variables' effects [76-78][25], and a review of several methods is available at [79]. Four systems regularities were analyzed in the studies. These are sparsity, hierarchy, heredity and Asymmetric Synergistic Interaction Structure (ASIS). Sparsity indicates that the number of significant effects and interactions on a physical system response is usually small. Hierarchy indicates that the ratio of active effects or interactions to all effects or interactions of the same order tends to be larger than the next order ratio of active interactions to all interactions. It also indicates a larger average magnitude of an effect or interaction than the average magnitude of the next order interactions. Heredity indicates that the probability of an active interaction increases as the number of its active parent variables increase. ASIS indicates that second order interactions tend to follow a certain directional pattern with respect to their parent variables. The studies concluded that sparsity, hierarchy, heredity and ASIS are significantly present in active variables of physical systems. This section will evaluate whether a monotonic behavior is significantly present in active variables if these regularities are significantly present.

If a consequence relationship of active variables on monotonic variables is proven then there are two scenarios for each variable as shown in Figure 5. If the

variable is found active, the assumption of monotonic behavior is very likely to be valid. If the variable is found inactive, the variable effect is weak on the performance characteristic. The variable monotonic behavior assumption is, thus, invalid, but since the variable is not important, it can only cause small monotonic violation. Thus, the wrong assumption will not severely affect the accuracy of the projection. The mathematical condition for positive monotonic behavior of variable i in the $-1,1$ design domain is derived in Appendix C, and is found as:

$$\beta_{i,1,-1} + \sum_{j=1, j \neq i}^N s_j \beta_{ij,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N s_{jk} \beta_{ijk,1,-1} + \sum_{l>k, l \neq i}^N \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N s_{jkl} \beta_{ijkl,1,-1} + \dots > 0 \quad (19)$$

for all combinations of $s_j, s_{jk}, s_{jkl}, \dots$ where $s_j = x_j, s_{jk} = x_j x_k, s_{jkl} = x_j x_k x_l, \dots$. The two extreme design domain states are when all variables are at their high state or all variables are at their low state. If all variables are at their low state, the monotonic behavior requires that

$$\beta_{i,1,-1} - \sum_{j=1, j \neq i}^N \beta_{ij,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijk,1,-1} - \sum_{l>k, l \neq i}^N \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijkl,1,-1} + \dots > 0, \quad (20)$$

which can be written as:

$$Odd(\beta_{i,1,-1}) - Even(\beta_{i,1,-1}) > 0 \quad (21)$$

Odd and *Even* represent functions that sum all odd or even parameters of the variable in a similar manner to [34]. These functions will be referred to as the ‘‘Integrated Odd Parameters’’ and the ‘‘Integrated Even Parameters’’ in this dissertation. If all variables are at their high state, the monotonic behavior requires that

$$Odd(\beta_{i,1,-1}) + Even(\beta_{i,1,-1}) > 0 \quad (22)$$

In the following sections, the relationship between each of the four mentioned regularities of active variables and the monotonic behavior of these variables will be investigated.

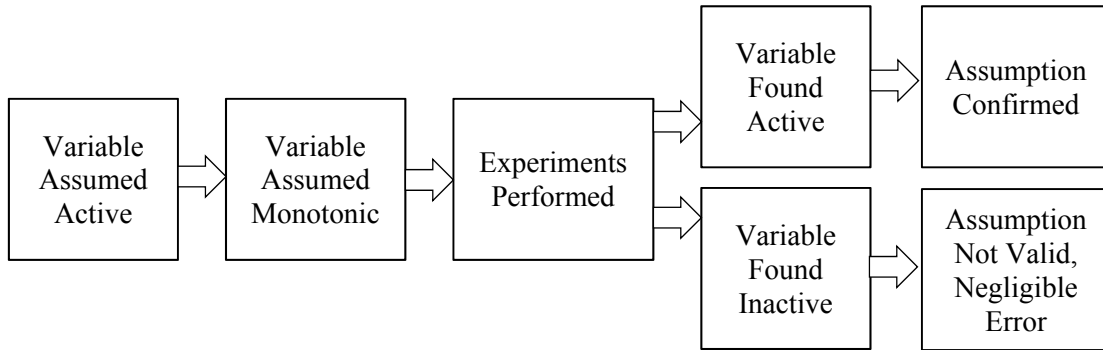


Figure 5. Validation Scenarios of the Monotonic Variables Assumption

II.1 Sparsity Effect on Variables Monotonic Behavior

Sparsity implies that the number of active effects affecting any performance characteristic is a small portion of the total effects. Both of [36] and [37] found that this behavior is significantly present. Sparsity implies that a big portion of the variables will not be active or monotonic, but these variables are not of concern since their effect on the performance characteristic is weak.

II.2 Heredity Effect on Variables Monotonic Behavior

Heredity implies that if an interaction is active, its parent variables and interactions are likely to be active too. Both of [36] and [37] found that this behavior is

significantly present. Due to the nature of inequality 19, and as is demonstrated in inequality 21 and inequality 22, second order interactions are the main players in satisfying the monotonic behavior inequality. It is, thus, expected that any active interaction for an inactive variable presents a variable that is non-monotonic and has a significant change of direction due to the active interaction. This presents a scenario that is not probable (around 4.5% by [36] and 2.5% by [37]). In addition, since one of the parents is active, the active interaction will be captured when exploring the active parent. The scenario of two inactive parents of a second order active interactions is further improbable (0.48% by [36] and 1.3% by [37]), and will cause both variables to be non-monotonic. Unless the interaction in this case is confounded in another interaction, it is also likely that this interaction will be captured too. Since both of the previous scenarios, representing non-monotonic behavior, are weakly present in systems, most of the active second order interactions are for active variables with strong main effects. This indicates that heredity supports that active variables in physical systems will satisfy inequality 21 and 22, and are monotonic.

II.3 Hierarchy Effect on Variables Monotonic Behavior

Hierarchy implies that as the order of active interactions increase, its average magnitude tends to be lower than its active parents. This means that for any variable i , $\beta_{i,1,-1} > \beta_{ij,1,-1}$. It also implies that the number of active interactions from the total number of interactions of the same order decrease as the order increase. Since inequality 19 has several second order interactions to every first order parameter, it is not

guaranteed that $\beta_{i,1,-1} > \sum_{j=1, j \neq i}^N \beta_{ij,1,-1}$ when neglecting higher orders. However, hierarchy

and sparsity indicate that the number of active $\beta_{ij,1,-1}$ is not high, and heredity analysis found that the probability of an active third order interaction, if the parent second order interaction is active, is relatively high. Including third order interactions in inequality 19 increases the probability of satisfying it, since it requires that

$\beta_{i,1,-1} + \sum_{k>j, j=1, j \neq i}^N \beta_{ijk,1,-1} > \sum_{j=1, j \neq i}^N \beta_{ij,1,-1}$ when neglecting higher orders. It is, thus, concluded

that the presence of hierarchy supports that active variables in physical systems will satisfy inequality 21 and 22, and are monotonic.

II.4 Asymmetric Synergistic Interaction Structure Effect on Variables Monotonic Behavior

ASIS introduced by [36] implies that the active synergistic second order interactions tend to have a sign that is the result of the multiplication of the signs of their parent variables. The paper found that 80% of active interactions obeyed the inequality

$\beta_{i,1,-1} \beta_{j,1,-1} \beta_{ij,1,-1} > 0$. Since this is significantly present, the alignment of variables' directions of effect will cause the interactions to accumulate, thus reducing the

confounding effect of interactions. This behavior means that $\beta_{i,1,-1} > \sum_{j=1, j \neq i}^N \beta_{ij,1,-1}$ is harder

to achieve since most parameters $\beta_{ij,1,-1}$ will have the same sign. It also indicates that the most challenging scenario of inequality 19 is when all or most of the variables are at

their low state. On the other hand, this regularity provides valuable knowledge on the expected shape of the response as explained in the following section.

III. CONCEPT: DIRECTIONAL KNOWLEDGE AND MONOTONIC BEHAVIOR UTILIZATION

In this section, the concept of knowledge of the variables' directions of effect and the variables' monotonic behavior will be utilized in the response shape development. The concept behind four new methods will be explained. To visually comprehend the idea behind the introduced methods, the multivariable model development will be explained by a non-linear single variable response model. A diamond tree structure will be introduced to graphically comprehend the concept in the multivariable space.

III.1 Response Shape after Variables Alignment

The approximation model, of N variables with $n=2^N$ parameters, was introduced earlier as:

$$E[y(x_1, x_2, x_3, \dots, x_n)] = \sum_{i=1}^N \beta_{i,1,-1} x_i + \sum_{i=1}^N \sum_{j>i}^N \beta_{ij,1,-1} x_i x_j + \sum_{i=1}^N \sum_{j>i}^N \sum_{k>j}^N \beta_{ijk,1,-1} x_i x_j x_k + \dots \quad (23)$$

, and is a combination of intersecting splines with N degrees of freedom in each. Each of these splines can be expressed as:

$$E[y(x)] = \sum_{i=0}^N \beta_{n,1,-1} x^i \quad (24)$$

The regularities explained in the previous section, especially the ASIS, are used in this section to predict that, if the variables are aligned in the positive direction, the drop at the maximum end of the response is relatively large, and the increase at the lower end is small, thus:

$$\beta_{i,1,-1} + \sum_{j=1, j \neq i}^N \beta_{ij,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijk,1,-1} + \dots \gg \beta_{i,1,-1} - \sum_{j=1, j \neq i}^N \beta_{ij,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijk,1,-1} - \dots \quad (25)$$

This is due to the fact that if the active main effect has $\beta_{i,1,-1} > 0$ by alignment,

$\sum_{j=1, j \neq i}^N \beta_{ij,1,-1}$ is maximized, since $\beta_{ij,1,-1} > 0$ in most of the active interactions. The slope at

the top end of the multidimensional surface is expected to decrease till the other end of the multidimensional surface is reached. In the 1-D nonlinear splines space, the expected response drop due to a variable i drop at various other variables' states is similar to the examples in Figure 6. However, non-monotonic behavior of variables will cause the response drop to depart from the general shape of Figure 6. The multidimensional surface will have $n!$ possibilities to drop from the high state of all variables to the low state of all variables. Each possibility presents a different drop path that is not smooth, because the variable will drop by a different magnitude at different states of other variables. The curvature of the spline increases as the degree of interaction of the dropped variable increases. For every variable, there are $(n-1)!$ paths or splines that are similar to the examples of Figure 6.

Since the shape of the response can be predicted for monotonic variables, the concepts of spanning tree, branch and bound and the Newton's Raphson method are utilized in the next sections to find an accurate approximation of the $n!$ splines with a relatively small number of experiments. The word "tree" is used to describe the set of experiments' results of all variables' combinations. Assuming that all variables are monotonic and aligned, it is possible to predict the increase of experiments' results in a

diamond tree as shown in Figure 7. Each binary number represents the states of the variables in one experiment, or one node. Though the tree uses the 0,1 notation in its nodes to represent the variables' states, the parameters found using the dissertation's methods are all -1,1 domain parameters. The 0,1 notation is only used to present variables' states, since even though $\beta_{1,-1} \neq \beta_{1,0}$, the response $y_{x=1,-1}$ is the same as $y_{x=1,0}$, and the variable physical state of $x=1,-1$ is the same as $x=1,0$. The 0,1 notation is used, from here onwards, for compact presentation of variables' states.

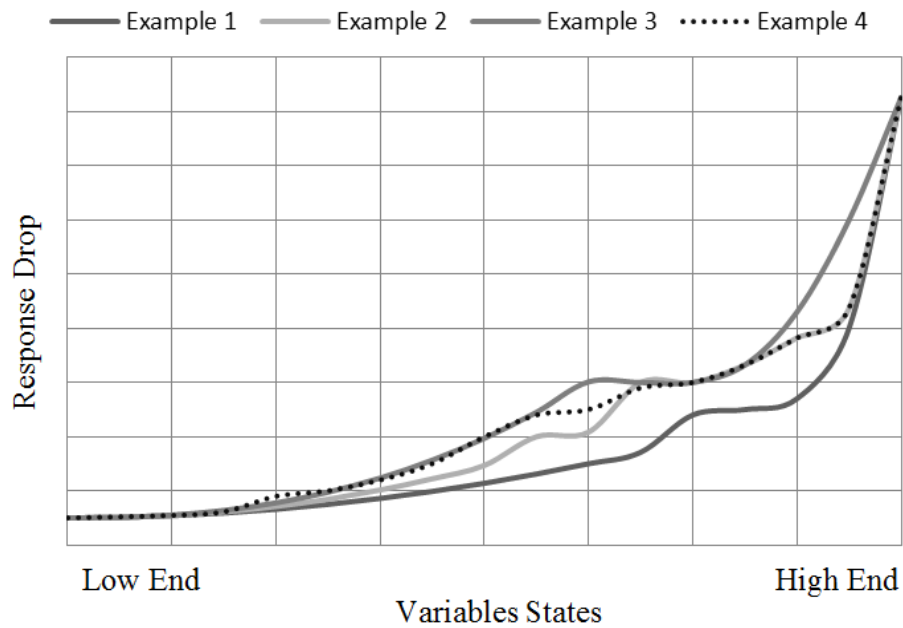


Figure 6. Examples of Response Drop by One Variable Drop at Various States of Variables

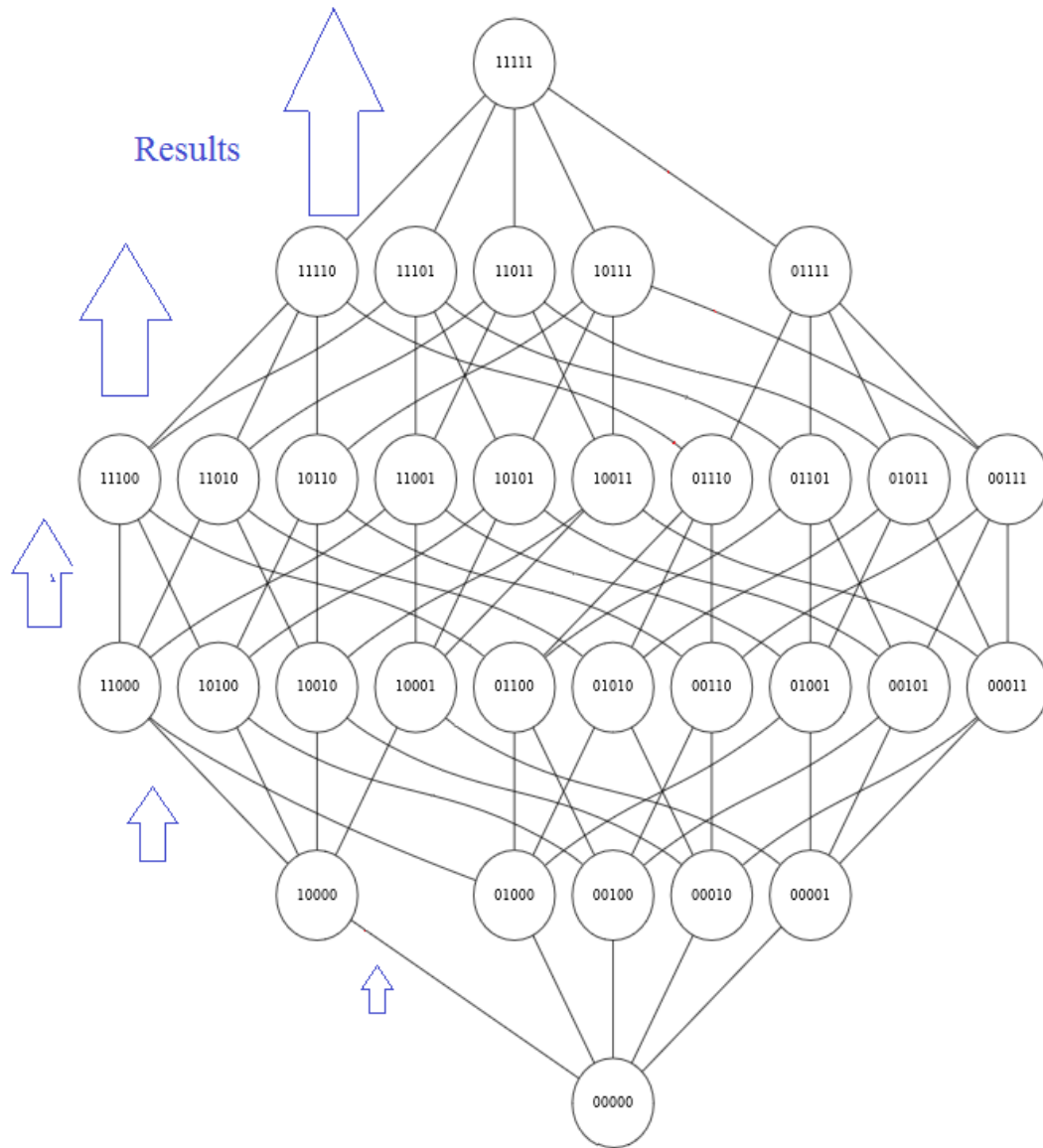


Figure 7. Five Variables Diamond Tree Representation of Experiments

III.2 Monotonic Sparsely Interacting Variables (MSIV) Method Concept

This method groups the variables into interacting and non-interacting variables based on an initial evaluation of the strength of the integrated even parameters of every

variable. In the interacting variables' group, or subtree, the summation of the integrated odd parameters of order R is compared with the integrated even parameters of the parent order $R-1$. If the difference is small, then all interactions are explained in the interacting variables' subtree, otherwise a cross interaction exist with the non-interacting variables' subtree, and the non-interacting variables' subtree is replicated with the binary states of cross interacting variables. In addition, if the integrated even parameters of order R are important, the higher order interactions, $R+1$, are explored. Since this process requires an expensive exploration of level by level of interactions in the interacting variables' subtree, this method is only recommended when there are a small number of interacting variables. The non-interacting variables' subtree is analyzed through any of the existing low resolution screening or sensitivity analysis methods, such as FFDs. The exploration path of this method is shown in Figure 8.

The advantage of this method is that it only applies a sufficient Res III evaluation method on the non-interacting variables' subtrees, and is able to determine the degree of exploration to perform in the interacting variables' subtree. The method's disadvantage is that it is not applicable for systems with a large number of interacting variables, because it becomes exponentially expensive as the number of variables in the interacting variables' subtree increases.

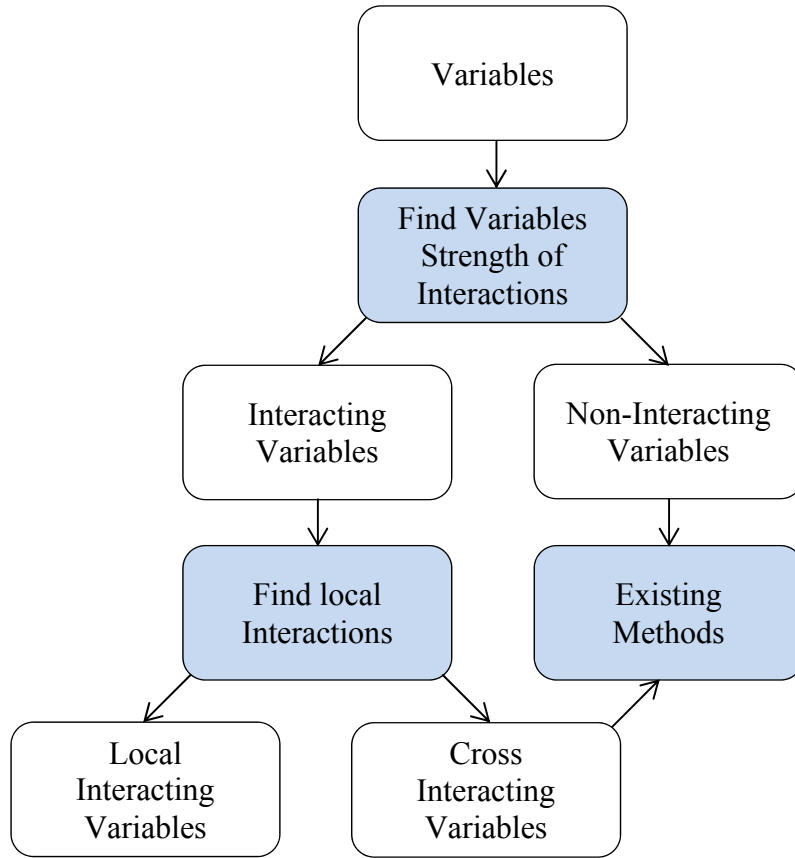


Figure 8. Exploration Path of the MSIV Method

III.3 Monotonic Highly Interacting Variables (MHIV) Method Concept

This method recursively groups the variables into interacting and non-interacting variables based on a local evaluation of the strength of the integrated even parameters of each variable. If a variable is non-interacting, the drop or increase at each end of a tree or subtree is equal. This is analogous to two equal slopes at two end points of a 1-D spline when applying the Newton’s Raphson method. The grouping is initially applied to all variables. The non-interacting variables are isolated as they are found, and the

interacting variables are further explored. For every interacting variables' subtree, the most interacting variable is used to span the subtree into two subtrees at the two states of that variable. The child interacting variables' subtrees are then analyzed in a similar manner to the parent subtree, but with fewer variables.

The exploration path of this method is shown in Figure 9. The spanning of interacting variables' subtrees continues to a point where no further interacting variables remain in the subtrees (referred to as leaf subtrees). In the leaf subtrees, the local and inherited integrated odd parameters of non-interacting variables are used to project the response results through a non-interacting variables' model. Since two nodes are performed in every subtree, the projection can be done by either of the top or bottom end nodes of the tree. The other end node can be used for the subtree's projection error evaluation. This is analogous to projecting the two ends of a spline using Newton's Raphson method and finding that each end projection intersects with the other. If the subtree has a very high error, but no interacting variables, then interactions were missed in that subtree due to confounding parameters, and further subtree's exploration is needed.

The advantage of this method is that it drives the exploration process by the strength of interactions, and provides a mean to determine the accuracy of the results within every subtree.

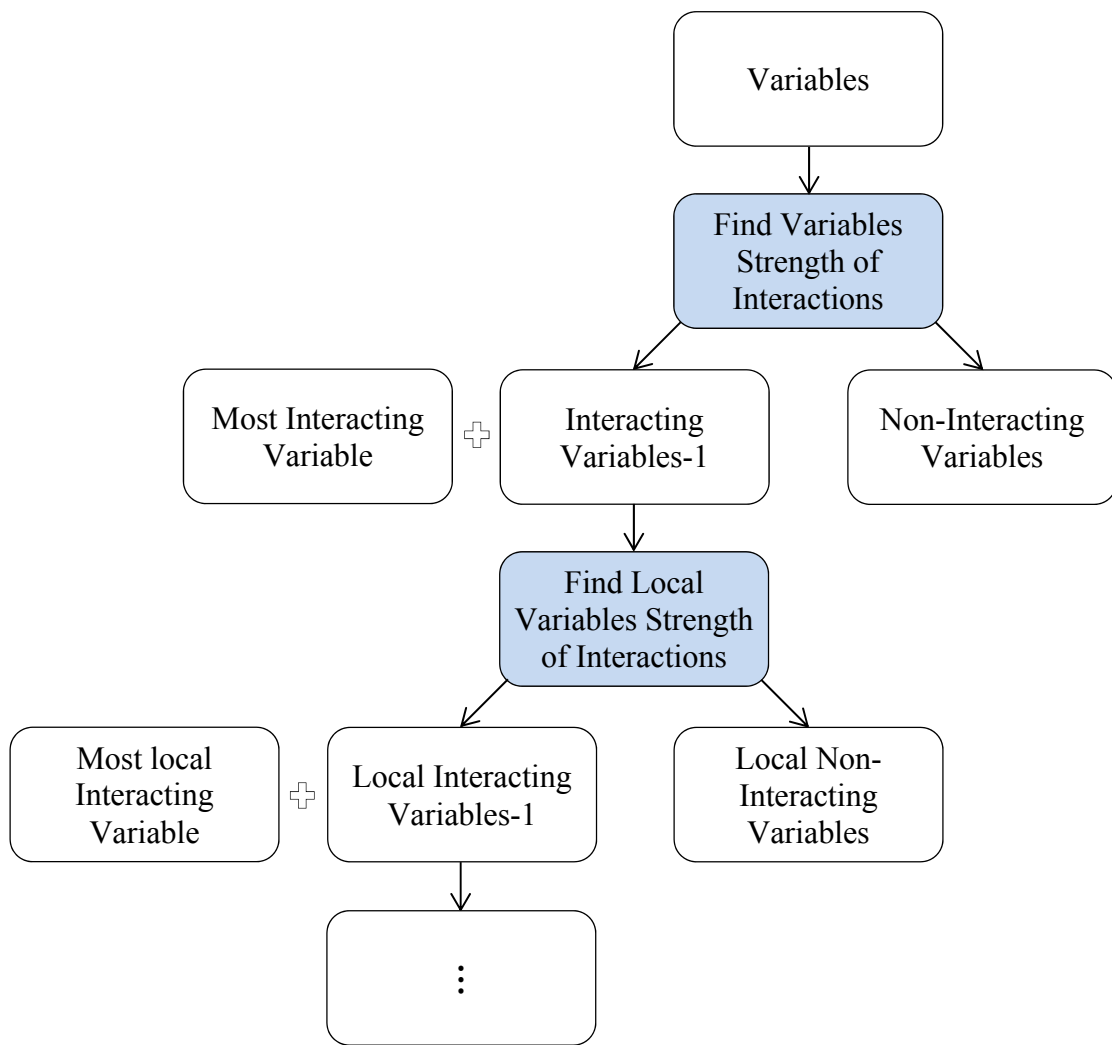


Figure 9. Exploration Path of the MHIV Method

III.4 Range Convergence (RC) Method Concept

This method utilizes the branch and bound concept. The method recursively groups the variables into important and non-important variables' subtrees based on an initial evaluation of the strength of the integrated parameters, representing summed even

and odd parameters, of each variable. This integrated value is equivalent to the result drop of each variable at the top end of a tree, thus there is no strict need in this method to determine the increase of the variables at the lower end of the tree. On the other hand, if the variables' integrated odd and even parameters are needed, for the experimenter to determine the strength of interactions, the top and bottom end of the tree are explored.

The experimenter will continue to perform the RC method if variables are strongly interacting. Otherwise, the experimenter can perform the SB method, since it is designed for weakly interacting variables. In this case, the experimenter will utilize the knowledge gained through the initial RC evaluation steps in the variables' grouping of the SB.

If the RC method is applied, the grouping is initially applied to all variables. For every important variable subtree, the most important local variable is used to span the subtree into two subtrees at the two states of that variable. The child variables' subtrees are then analyzed in a similar manner to the parent subtree, but with fewer variables. The exploration path of this method is shown in Figure 10. The spanning of important variables' subtrees continues to a point where no further important variables remain in the subtrees, when the span of the subtrees falls below a certain threshold, or when the maximum number of experiments is reached. The span of every node in the leaf subtrees is found by enforcing that every node has to be smaller than any performed higher connected node and higher than any performed lower connected node. This establishes a range for every node. The projected response for every node is set as the middle point of the node range.

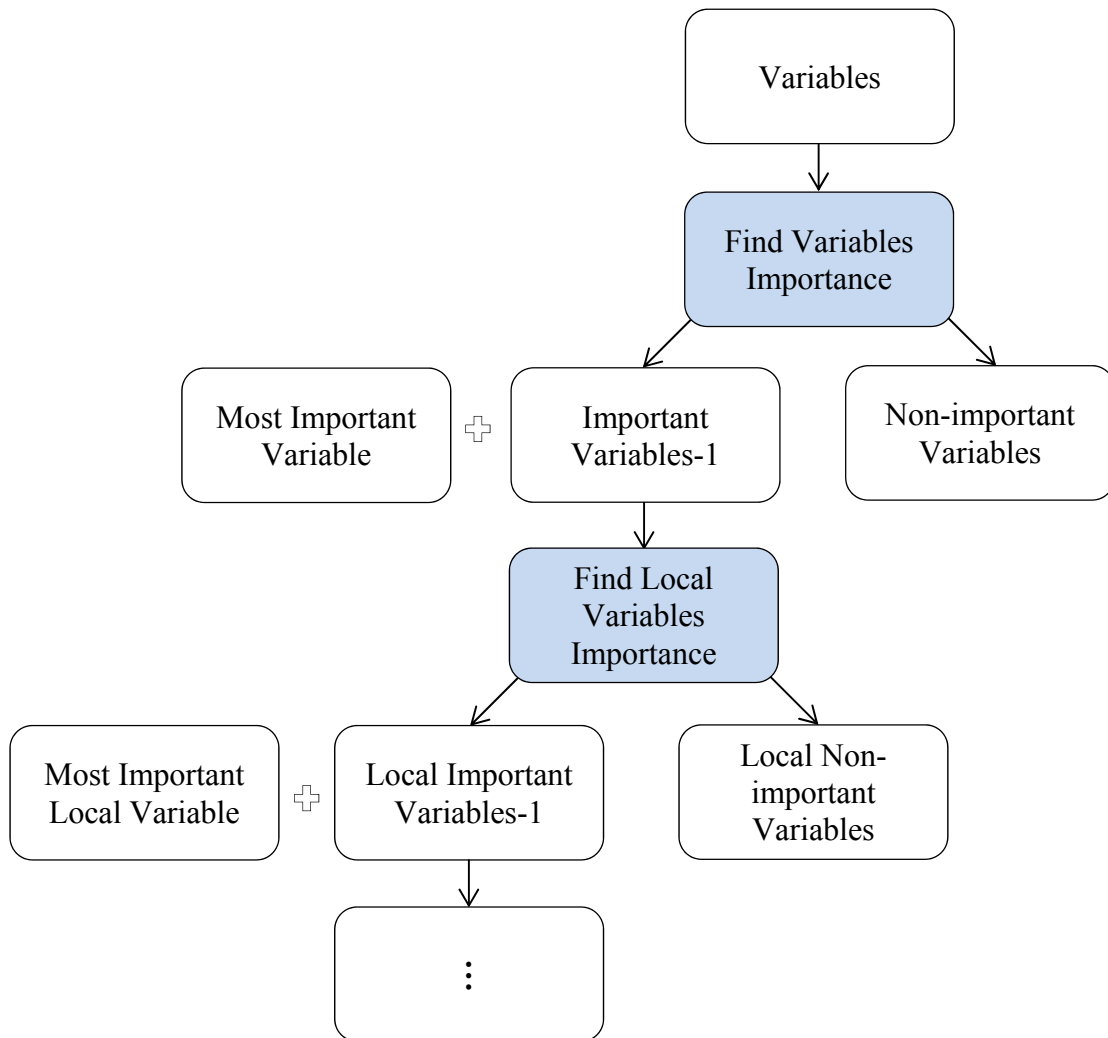


Figure 10. Exploration Path of the RC Method

This method has some similarities to the SB method of [46]. Unlike SB, this method utilizes the response shape finding of Figure 7, and is not limited to second order interactions. Instead of splitting the tree from a middle node, this method drives the exploration process by the importance of variables, thus efficiently reducing the span of

the nodes. The span of nodes provides a way to determine the accuracy of the results within every subtree. The disadvantage of the RC method is that it is weak when applied to systems with weakly interacting variables, since the skewed shape of Figure 6 becomes closer to a line. The RC method is designed for systems with strongly interacting variables, and thus, complements the SB method that is designed for systems with weakly interacting variables.

III.5 Hierarchical Integrated Parameters (HIP) Method Concept

This method hierarchically and reclusively explores the important interactions by finding the integrated odd and even parameters of every interaction to a point where the integrated even parameters of all interactions are not important. This method is developed as an expansion to the method of [34]. The method of [35] expanded [34] to unfold second order interactions. This method expands [35] and [34] to determine any order of interaction. This can be accomplished by a hierarchical structure of exploration, i.e. $\beta_{ijk,1,-1}$ can only be unfolded if $\beta_{ij,1,-1}$, $\beta_{jk,1,-1}$, $\beta_{ik,1,-1}$ and their parents are unfolded first. A new domain of parameters (α) is introduced for this method (see Appendix D for an example of the domain conversion process). The exploration path of this method is shown in Figure 11.

This method was found to result in high response projection error especially in its middle zone of the tree. For the four variables' example of Appendix D, the design matrix was found as:

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & -2 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 4 & 4 & 0 \\ 1 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 4 \\ 1 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 4 \\ 0 & 1 & 2 & -2 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 4 & -4 & 0 \\ 1 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & 2 & -2 & 0 & 0 & 0 & 0 & 4 & -4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \alpha \quad (26)$$

The 4 and -4 multiplication affecting the high end of the integrated parameters, which is usually assumed as zero, increases the projection error of this method. This is analogous to projecting the shape of 1-D spline by averaging the slope at its two ends. As a result, this method is very economical in screening important variables and interactions, but not efficient in response's projection or parameters' determination. Its response's projection efficiency improves as the sparsity of effects increases and high orders interactions weakens.

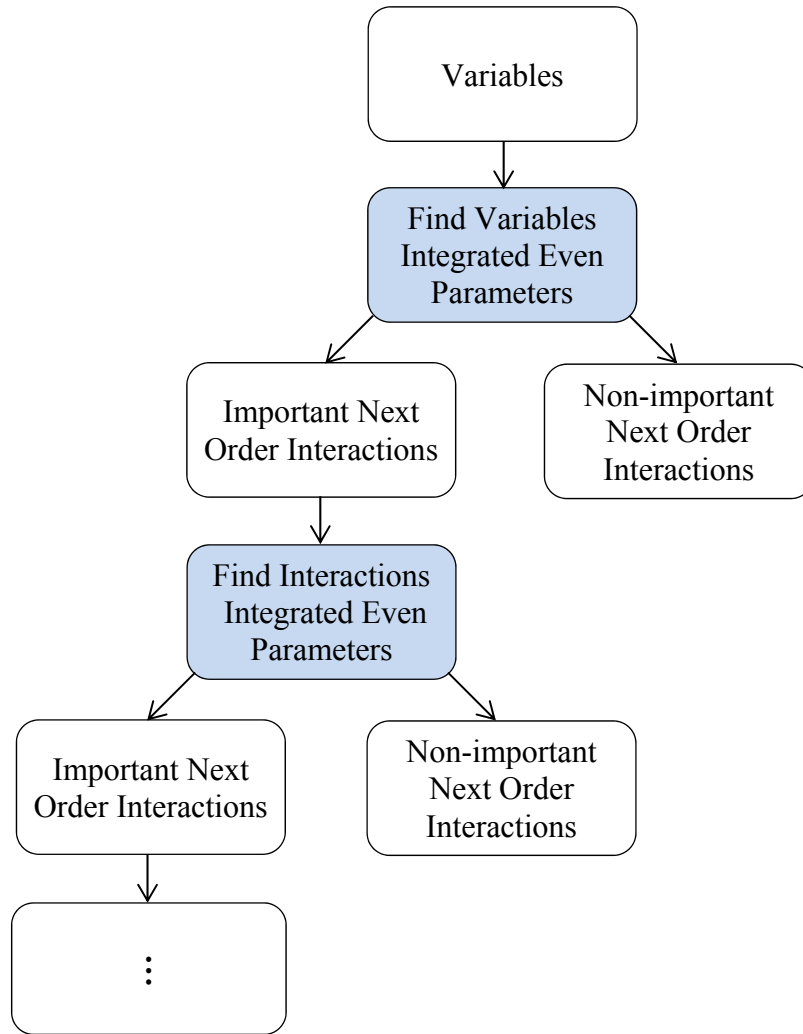


Figure 11. Exploration Path of the HIP Method

IV. METHODS*

In this section, the concepts introduced in the previous section are utilized to produce a flow chart of the steps for each method. The methods description will utilize a diamond tree structure of experiments similar to the example shown in Figure 7, where the tree nodes represent an experiment's result at a unique state of variables. The methods will utilize the assumption that variables are monotonic. The following sections will explain the common then specific steps of the four methods.

IV.1 Common Steps of All Methods

The first four steps of the four methods in addition to the last validation step are the same in all methods, so they will be explained in the following sections.

IV.1.1 Performance Characteristic Directional Alignment

This step is optional, except in the top end exploration scenario of the RC method. It can be performed before or after the next step depending on the degree of knowledge about the performance characteristic. The step is performed in this dissertation to better present the methods results. This step aligns the low end of the performance characteristic with the potential saturation plateau, if one is expected to exist, thus

*Some definitions and formulas in this section are from: Cotter, S. C., A screening design for factorial experiments with interactions, *Biometrika*, 1979, vol. 66,2, pp. 317-320, by permission of Oxford University Press

$$\left. \frac{dy}{dx_i} \right|_{X=111\dots 1} \gg \left. \frac{dy}{dx_i} \right|_{X=000\dots 0} \quad (27)$$

If the performance characteristic is to be reversed, this can be achieved by simply inverting the performance characteristic or subtracting it from a reference point. The reversal process is usually dependent on the performance characteristic.

IV.1.2 Variables Directional Alignment

This step is the main input of the experimenter and represents the added value to the evaluation process. If variables are aligned in one direction, it is expected that regularities will reduce the confounded interactions as was explained in section III. A table is established of all variables and their expected directions of effect on the performance characteristic. An example is shown in Table 1. The justification of the directional decision making could be shown in another table such as Table 2, and is used as a reference to why the signs of the directions of effect are selected. The positive sign corresponds to the alignment of the variable high state to 1 and the low state to -1. The negative corresponds to the alignment of the high state to -1 and the low state to 1. The equal sign indicates that the variable state has no effect on the performance characteristic. A question mark indicates an unknown directional relationship of the variables and the performance characteristic. In some cases, the initial directional judgment is found to be wrong, indicated by two signs with the correct sign in brackets. If a variable is assigned a sign and found to be reversed, the assignment of -1 (or 1) to

low and 1 (or -1) to high is reversed to -1 (or 1) to high and 1 (or -1) to low. Whenever a wrongly aligned active variable is found, its effect has to be reversed.

Table 1
Variables' Directions of Effect on the Performance Characteristic

| Variable Index | Variable | Direction of Effect on the Performance Characteristic |
|----------------|----------|---|
| 1 | Name 1 | + |
| 2 | Name 2 | = |
| 3 | Name 3 | - |

Table 2
Justification of the Variables' Directions of Effect on the Performance Characteristic

| Variable Index | Variable | Justification | Direction of Effect on the Performance Characteristic |
|----------------|----------|--|---|
| 1 | Name 1 | Justification of why variable 1 affects the performance characteristic in this direction | ? |
| 2 | Name 2 | Justification of why variable 2 affects the performance characteristic in this direction | + |
| 3 | Name 3 | Justification of why variable 3 affects the performance characteristic in this direction | +(-) |

IV.1.3 Variables Integrated Parameters Determination

In section III, the integrated odd and even parameters were introduced. This step will apply the mathematical representation of these definitions using the definitions of [34]. The first order integrated odd and even parameters are defined as:

$$Odd(\beta_{i,1,-1}) = \beta_{i,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijk,1,-1} + \dots \quad (28)$$

$$Even(\beta_{i,1,-1}) = \sum_{j=1, j \neq i}^N \beta_{ij,1,-1} + \sum_{l>k, l \neq i}^N \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijkl,1,-1} + \dots \quad (29)$$

and the variables' importance is defined as:

$$imp(\beta_{i,1,-1}) = |Odd(\beta_{i,1,-1})| + |Even(\beta_{i,1,-1})| \quad (30)$$

The integrated odd and even parameters for every variable are dependent on four experiments only, two at the top end of the tree and another two at the bottom end. The integrated odd and even parameters are determined using:

$$Odd(\beta_{i,1,-1}) = 0.25 \left(\frac{y_{x=1111\dots1} - y_{x=1111\dots1, xi=0}}{2[Odd(\beta_{i,1,-1}) + Even(\beta_{i,1,-1})]} \right) + 0.25 \left(\frac{y_{x=0000\dots0, xi=1} - y_{x=0000\dots0}}{2[Odd(\beta_{i,1,-1}) - Even(\beta_{i,1,-1})]} \right) \quad (31)$$

$$Even(\beta_{i,1,-1}) = 0.25 \left(\frac{y_{x=1111\dots1} - y_{x=1111\dots1, xi=0}}{2[Odd(\beta_{i,1,-1}) + Even(\beta_{i,1,-1})]} \right) - 0.25 \left(\frac{y_{x=0000\dots0, xi=1} - y_{x=0000\dots0}}{2[Odd(\beta_{i,1,-1}) - Even(\beta_{i,1,-1})]} \right) \quad (32)$$

This represents the top node and the node right below it with variable i set at its low state and the bottom node and the node right above it with variable i set at its high state. This process will be referred to as the “one down and one up” process later in the dissertation. For N variables, 2N+2 experiments are needed to determine the integrated odd and even parameter of all main effects.

If the absolute function used in [34], and shown in equation 30 is dropped, the importance of each variable becomes only dependent on two experiments at the top end of the tree, and is determined using:

$$imp(\beta_{i,1,-1}) = 0.5 \left(\frac{y_{x=1111\dots1} - y_{x=1111\dots1,xi=0}}{2[Odd(\beta_{i,1,-1}) + Even(\beta_{i,1,-1})]} \right) \quad (33)$$

Otherwise, four experiments are needed to determine the integrated odd and even parameters, and the absolutes of both values are summed.

IV.1.4 Variables Categorization

In this step, depending on the applied method, the variables are sorted by the strength of interactions in a bar chart similar to the example shown in Figure 12 or sorted by the variables' importance as shown in Figure 13. In addition, a threshold for the strength of integrated even parameters or importance is defined in this step to categorize important integrated even parameters or important variables. Depending on the method used, this step will isolate the non-interacting variables, the non-important variables, or the non-important interactions.

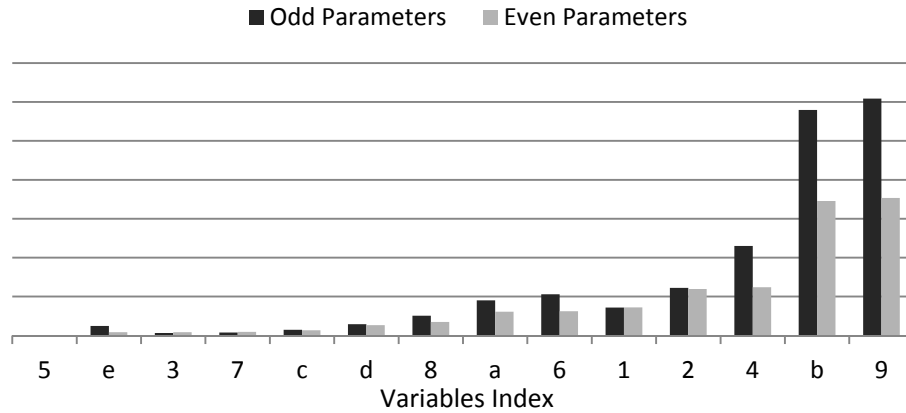


Figure 12. Example Integrated Odd and Even Parameters Sorted by the Strength of Interactions

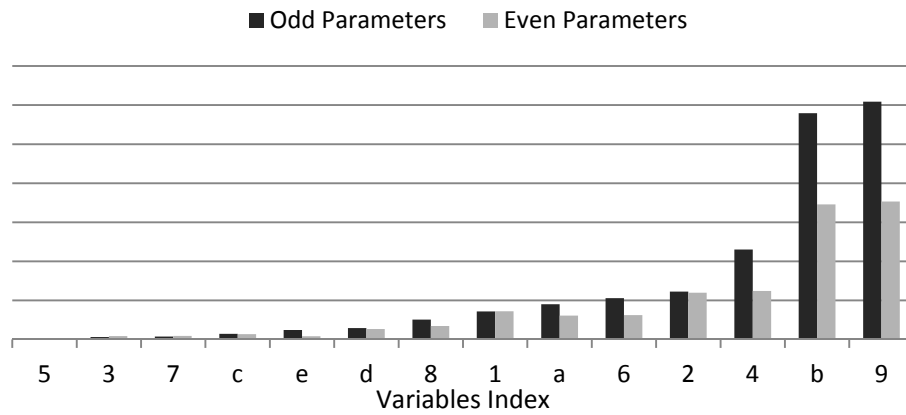


Figure 13. Example Integrated Odd and Even Parameters Sorted by Importance

IV.1.5 Validation

The validation of the results, found by any of the methods of this dissertation, is performed using a set of validation experiments. Though the parameters are targeted in the methods, validation is done through the performance characteristic results, since the

parameters of the validation set of experiments are unknown. The Root Mean Square Error (RMSE, and referred to as Root Mean Square Deviation in [80]) of the response is used for validation, and is defined as

$$RMSE = \sqrt{\frac{\sum_{i=1}^S \hat{y}_i - y_i}{S}} \quad (34)$$

where S is the number of validation points, \hat{y}_i is the projected performance characteristic, and y_i is the result of the performed performance characteristic experiment at the same state of variables as \hat{y}_i . The selection of the validation set is problem dependent since the experimenter might bias the validation toward certain regions of interest. In this dissertation, a FFD set of experiments is selected for a uniform distribution of validation points.

In the following sections, the specific flow chart and steps of the four methods of the dissertation are explained.

IV.2 Monotonic Sparsely Interacting Variables Method

The key of this method is the separation of interacting and non-interacting variables into separate subtrees, the exploration of interactions of the interacting variables, and then the exploration of cross interactions of interacting variables with non-interacting variables. The flow chart of the proposed method is shown in Figure 14.

IV.2.1 Next Order Integrated Parameters Determination

In this step, the next order odd and even parameters of variables in the interacting variables' subtree are found. Appendix D demonstrates the approach developed to determine the integrated parameters of any order of interactions using the new α domain. If four variables are considered, the integrated odd and even parameters are found using:

$$\begin{matrix} \alpha_{0,Odd} \\ \alpha_{0,Even} \\ \alpha_{1,Odd} \\ \alpha_{1,Even} \\ \alpha_{2,Odd} \\ \alpha_{2,Even} \\ \alpha_{3,Odd} \\ \alpha_{3,Even} \\ \alpha_{4,Odd} \\ \alpha_{4,Even} \\ \alpha_{12,Odd} \\ \alpha_{12,Even} \\ \alpha_{13,Odd} \\ \alpha_{13,Even} \\ \alpha_{14,Odd} \\ \alpha_{14,Even} \end{matrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 1/4 \\ 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 1/4 \\ -1/4 & 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 0 & 0 & 1/4 \\ 1/4 & 0 & 0 & 0 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 0 & 0 & 1/4 \\ -1/4 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 1/4 \\ 1/4 & 0 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 1/4 \\ -1/4 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 1/4 \\ 1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 1/4 \\ 1/8 & 0 & 0 & 1/8 & -1/8 & 0 & 0 & -1/8 & -1/8 & 0 & 0 & -1/8 & 1/8 & 0 & 0 & 1/8 \\ -1/8 & 0 & 0 & 1/8 & 1/8 & 0 & 0 & -1/8 & 1/8 & 0 & 0 & -1/8 & -1/8 & 0 & 0 & 1/8 \\ 1/8 & 0 & -1/8 & 0 & 0 & 1/8 & 0 & -1/8 & -1/8 & 0 & 1/8 & 0 & 0 & -1/8 & 0 & 1/8 \\ -1/8 & 0 & 1/8 & 0 & 0 & 1/8 & 0 & -1/8 & 1/8 & 0 & -1/8 & 0 & 0 & -1/8 & 0 & 1/8 \\ 1/8 & -1/8 & 0 & 0 & 0 & 0 & 1/8 & -1/8 & -1/8 & 1/8 & 0 & 0 & 0 & 0 & -1/8 & 1/8 \\ -1/8 & 1/8 & 0 & 0 & 0 & 0 & 1/8 & -1/8 & 1/8 & -1/8 & 0 & 0 & 0 & 0 & -1/8 & 1/8 \end{bmatrix} Y \quad (35)$$

For every second order interaction, eight experiments need to be performed. However, six of the experiments already have been performed for the parent main effects. As a result, only two experiments are needed. The same applies to any order of interactions, thus two experiments are needed to find the integrated odd and even parameters of any interaction, provided that the parent main effects and interactions have been explored.

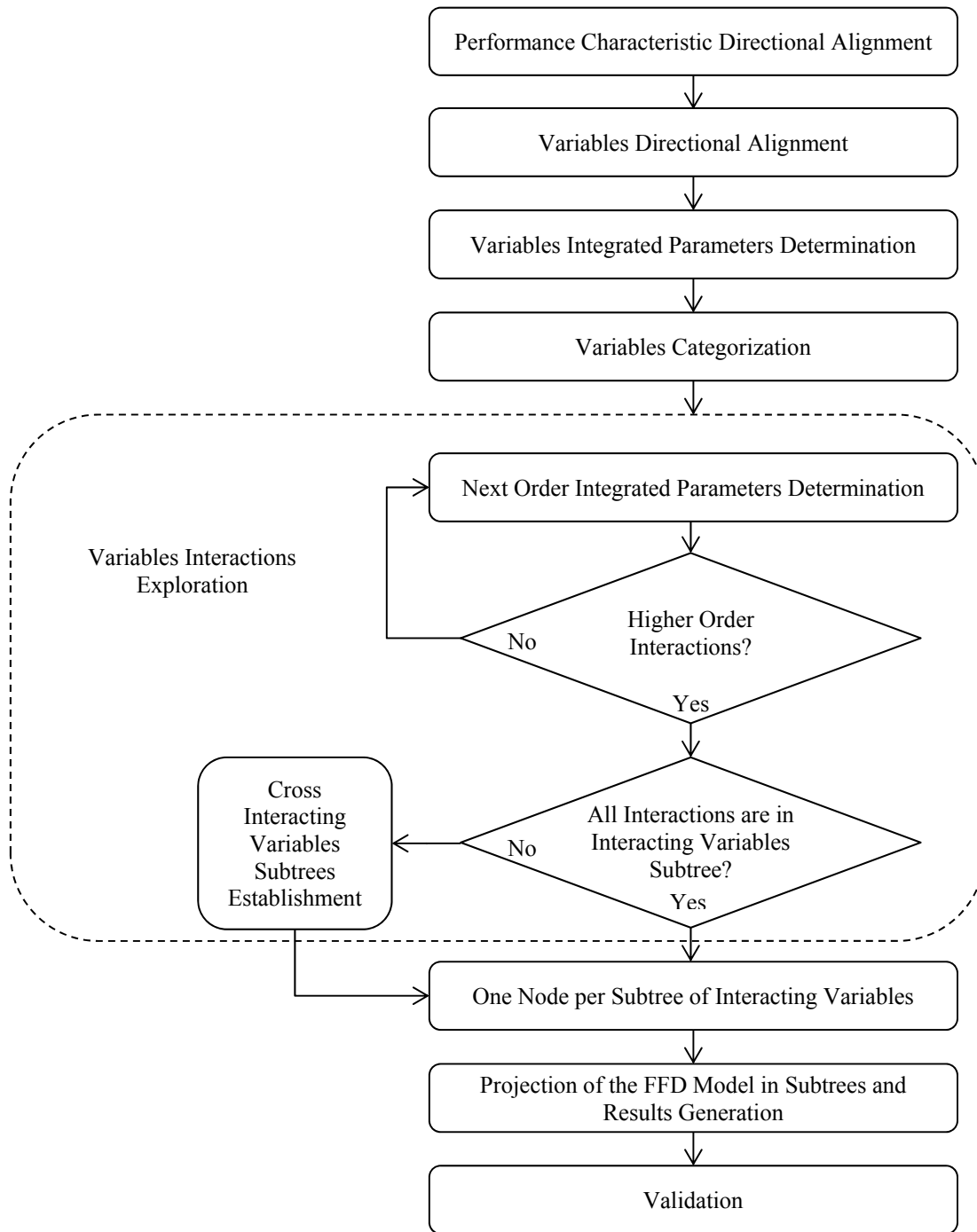


Figure 14. Flow Chart of the MSIV Method

IV.2.2 Evaluation of Interactions in Interacting Variables Subtree

After the next order integrated parameters in the interacting variables' subtree are found, two checks are performed. The first check is to determine if any of the integrated even parameters is important. If an important integrated even parameters is found, its next order integrated parameters are determined in the same manner explained in the previous step. Thus, if $Even(\beta_{R,1,-1}) > \text{threshold}$, $Even(\beta_{R+1,1,-1})$ and $Odd(\beta_{R+1,1,-1})$ are to be determined.

The second check is to find the difference between the summation of integrated odd parameters of order R interactions and the integrated even parameters of its parent order R-1 interaction. If $Even(\beta_{R-1,1,-1}) = \sum Odd(\beta_{R,1,-1})$, then all interactions of R-1 are in the interacting variables' subtree. If $Even(\beta_{R-1,1,-1}) \neq \sum Odd(\beta_{R,1,-1})$, then some interactions of R-1 are with the non-interacting variables' subtree. A table similar to the one shown in Table 3 is used to check the equality. If the equality is not satisfied for a number of interactions, the interacting variables' tree is split into groups representing all states of these cross interacting variables as will be shown in the following step.

Table 3
Check Table for Interactions of Interacting Variables with Non-Interacting Variables

| Index | $Even(\beta_{R-1,1,-1})$ | $\sum Odd(\beta_{R,1,-1})$ | Difference |
|-------|--------------------------|----------------------------|------------------|
| 1 | Value 1 | Value 4 | Value 4- Value 1 |
| 2 | Value 2 | Value 5 | Value 5- Value 2 |
| 3 | Value 3 | Value 6 | Value 6- Value 3 |

If both of the above checks show that no higher order interactions are expected, and that no interacting variables exist outside the interacting variables' subtree, one node per subtree is performed to find the offset of every subtree

IV.2.3 Cross Interacting Variables Subtrees Establishment

In this step, subtrees are established for all combinations of interacting variables that are not fully interacting within the interacting variables' subtree. As a result, 2^L subtrees are established where L is the number of interacting variables that are not fully interacting within the interacting variables' subtree.

IV.2.4 One Node per Subtree of Interacting Variables

For every subtree that has been created, either a FFD is performed or the FFD model from another subtree in the same group is applied. If the FFD model from another subtree is used, it is necessary to perform at least one experiment in every subtree to determine the offset. The offset for every subtree is found in the following step, since the needed FFD parameters will be found then. The offset is found using:

$$\begin{aligned}
 y(x_1, x_2, x_3, \dots, x_S) = y &= X \left[\beta_{0,1,-1} + \sum_{i=1}^S \beta_{i,1,-1,FFD} \right] = \left[\underbrace{x_0}_{=1} \beta_{0,1,-1} + \sum_{i=1}^S x_{i,1,-1} \beta_{i,1,-1,FFD} \right] \\
 &= \left[\beta_{0,1,-1} + \sum_{i=1}^S x_{i,1,-1} \beta_{i,1,-1,FFD} \right] \Rightarrow \beta_{0,1,-1} = \left[y - \sum_{i=1}^S x_{i,1,-1} \beta_{i,1,-1,FFD} \right]
 \end{aligned} \tag{36}$$

$\beta_{i,1,-1,FFD}$ is a FFD parameter, and y is the result of the performed node per subtree of this step.

IV.2.5 Projection of the FFD Model in Subtrees and Results Generation

In this step, the subtrees are grouped according to the binary combinations of interacting variables that cross interact with the non-interacting variables. For example, if two variables were found to interact with the non-interacting variables, then four groups are established for the 00,01,10,11 states of these two variables. Within these groups, several interacting variables' subtrees exist, but only one FFD per group is needed. For every cross interacting variables' group, one low Res FFD is performed. The FFD is performed at the high end of the main tree to reduce confounded interactions. The interaction of any non-interacting variable j with an interacting variable i is found using:

$$\beta_{ij,1,-1X=111\dots1} - \beta_{ij,1,-1,X=1\dots1,xi=0} = \beta_{ij,1,-1} - (-\beta_{ij,1,-1}) = 2\beta_{ij,1,-1} \Rightarrow \beta_{ij,1,-1} = (\beta_{ij,1,-1X=111\dots1} - \beta_{ij,1,-1X=1\dots1,xi=0})/2 \quad (37)$$

The fact that two interacting variables are cross interacting with the non-interacting variables' subtree does not imply that the second order interaction of these two variables is also cross interacting. The group 00 of the example above does not require a separate FFD, unless the interaction of the two variables has an integrated even parameter that is not fully explained in the interacting variables' subtree. Once all cross interactions are found, the equality of $Even(\beta_{R-1,1,-1}) = \sum Odd_{(R,1,-1)}$ can be checked and should be almost satisfied for all variables. After the FFD models have been developed for all subtrees, the results of every node are found using

$$y(x_1, x_2, x_3, \dots, x_S) = \left[\beta_{0,1,-1} + \sum_{i=1}^S x_{i,1,-1} \beta_{i,1,-1,FFD} \right] \quad (38)$$

IV.3 Monotonic Highly Interacting Variables Method

The key of this method is the recursive isolation of non-interacting variables from subtrees of interacting variables. The flow chart of the proposed method is shown in Figure 15.

IV.3.1 Interacting Variables Subtrees Establishment

After the variables were categorized into interacting and non-interacting in the previous step, subtrees are established based on the binary state combinations of the most interacting variables. There are two approaches to perform this. The first approach is to create subtrees of one variable at a time. This approach requires a large number of experiments batches, but is more efficient in terms of the total number of experiments. The second approach is to unfold few variables at a time, which is less efficient in terms of the total number of experiments, but requires a smaller number of experiments batches. If two variables are selected for constructing a subtrees, $4=2^2$ subtrees are constructed with the two variables' states at one of the four combinations. The end nodes are performed to bound every subtree. If two interacting variables are being unfolded, eight experiments are to be performed representing the end nodes of the four subtrees. Six of these experiments are performed at the previous integrated parameters determination step, so only two experiments are actually performed in this step.

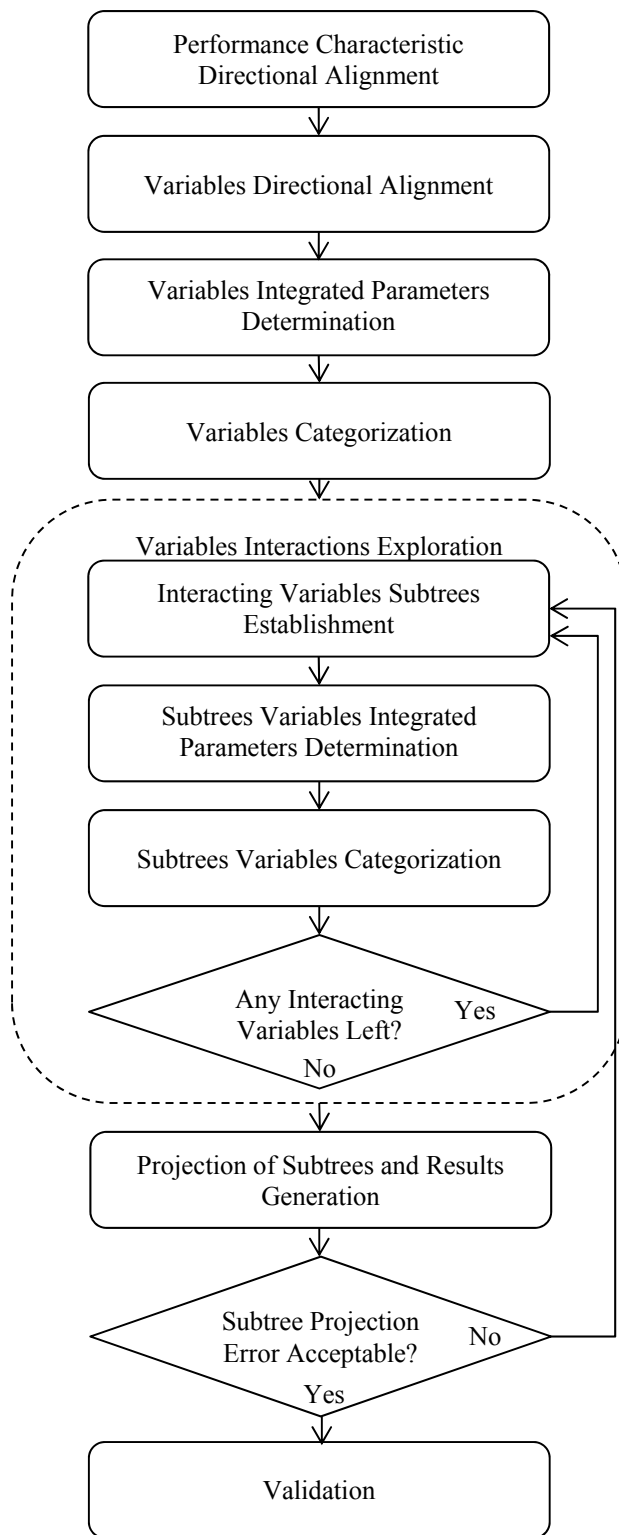


Figure 15. Flow Chart of the MHIV Method

IV.3.2 Subtrees Variables Integrated Parameters Determination

This step determines the integrated odd and even parameters of the variables in a similar manner to the previous step “Variables Integrated Parameters Determination” except it is applied to the local variables of the interacting variables’ subtrees.

IV.3.3 Subtrees Variables Categorization and Evaluation

This section combines two steps. The first step evaluates the integrated odd and even parameters of the variables in a similar manner to the previous step “Variables Categorization” except it applies to local variables of the interacting variables’ subtrees. In the second step, the integrated even parameters are checked against the integrated even parameters threshold to determine if any local variable in the interacting variables’ subtree require subtrees creation. If further subtrees are to be created, the previous step of “Interacting Variables Subtrees Establishment” is performed. For every non-interacting variable i , it is expected that:

$$\begin{aligned}
 & \beta_{i,1,-1} + \underbrace{\sum_{j=1, j \neq i}^N \beta_{ij,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijk,1,-1} + \dots}_{\approx 0} \\
 & \approx \beta_{i,1,-1} - \underbrace{\sum_{j=1, j \neq i}^N \beta_{ij,1,-1} + \sum_{l>k, l \neq i}^N \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N \beta_{ijkl,1,-1} - \dots}_{\approx 0}
 \end{aligned} \tag{39}$$

Thus, the first order parameter of the variable is set to its integrated odd parameters, and the integrated even parameters is assumed as zero.

$$\beta_{i,1,-1} = \text{Odd}(\beta_{i,1,-1}) \tag{40}$$

$$\beta_{ij,1,-1} = \text{Even}(\beta_{ij,1,-1}) = 0 \quad (41)$$

The main effects' parameters of the non-interacting variables are cascaded to all child subtrees of the subtree where they are found.

IV.3.4 Projection of Subtrees and Result Generation

At this point, subtrees were unfolded to an extent where interactions fell below the threshold defined earlier and the first order parameters of all variables in the subtrees are set to the integrated odd parameters from parent subtrees. In these leaf subtrees, the response model used is the simple non-interacting variables' model:

$$y(x_1, x_2, x_3, \dots, x_Q) = \beta_{0,1,-1} + \sum_{i=1}^Q \beta_{i,1,-1} x_{i,1,-1} \quad (42)$$

where Q is the number of variables of the leaf subtree. For every subtree, two nodes were performed as part of the previous steps, thus two offset values $\beta_{0,1,-1,H}$, $\beta_{0,1,-1,L}$ can be determined by projecting each end of the subtree. The offset parameter of the subtree is the average offset of the two end offsets. Thus:

$$\beta_{0,1,-1,L} = \left[y_{x=0000\dots0} + \sum_{i=1}^Q \beta_{i,1,-1} \right] \quad (43)$$

$$\beta_{0,1,-1,H} = \left[y_{x=1111\dots1} - \sum_{i=1}^Q \beta_{i,1,-1} \right] \quad (44)$$

$$\beta_{0,1,-1} = \beta_{0,1,-1,mean} = (\beta_{0,1,-1,H} + \beta_{0,1,-1,L})/2 = (y_{x=1111\dots1} + y_{x=0000\dots0})/2 \quad (45)$$

IV.3.5 Projection Results Evaluation

In order to verify that the confounding parameters did not cause false non-interacting variables' classification, $\beta_{0,1,-1,H}$ and $\beta_{0,1,-1,L}$ are compared. Ideally, if all interactions within the subtree are indeed zero, the two found values $\beta_{0,1,-1,H}$ and $\beta_{0,1,-1,L}$ from the projection of each side of the subtree should be the same. The offset value should be identical regardless of which node is used. The difference of the two offsets is an indication of the validity of the negligible interactions assumption within the subtree.

The error is found using

$$\beta_{0,1,-1,Error} = \left| \beta_{0,1,-1,H} - \beta_{0,1,-1,L} \right| / 2 \quad (46)$$

If a subtree offset has very high error, but no interacting variables, then one or more interactions of the subtree were not captured due to confounding parameters in that subtree, thus further subtree spanning is needed.

IV.4 Range Convergence Method

The key of this method is the recursive isolation of important variables from variables' subtrees, thus creating leaf subtrees with a small span. The range of every node in the subtree is determined based on lowest higher connected node and highest lower connected node of performed experiments. The flow chart of the proposed method is shown in Figure 16.

IV.4.1 Important Variables Subtrees Establishment

In this step, the variable that has the highest importance is selected as the most influential variables to use for subtrees' creation. Creating subtrees using the most important variables ensures minimum to almost no overlapping of subtrees in addition to maximum span reduction.

The number of variables to use to branch a tree or subtree into child subtrees is selected depending on the number of experiments that are to be performed. There are two approaches to perform this. The first approach is to create two subtrees of the most important variable at a time. The second is to create subtrees of multiple important variables at a time. The process of subtrees' establishment was previously explained in the "Interacting Variables Subtrees Establishment" step of the MHIV method.

IV.4.2 Subtrees Nodes Ranges Update

After subtrees are established, the results of the end nodes of every subtree are cascaded to the subtree's nodes. Assuming, the variables are aligned in the positive direction, the upper end of the potential range of each node, $R_{i,High}$, is equal to the lowest result of a connected performed node, j , above that node. The lower end of the potential range of each node, $R_{i,Low}$, is equal to the highest result of a connected performed node, j , above that node.

$$R_{i,High} = \min(y_j) \text{ if } AND(X_i, X_j) = X_i \quad (47)$$

$$R_{i,Low} = \max(y_j) \text{ if } OR(X_i, X_j) = X_i \quad (48)$$

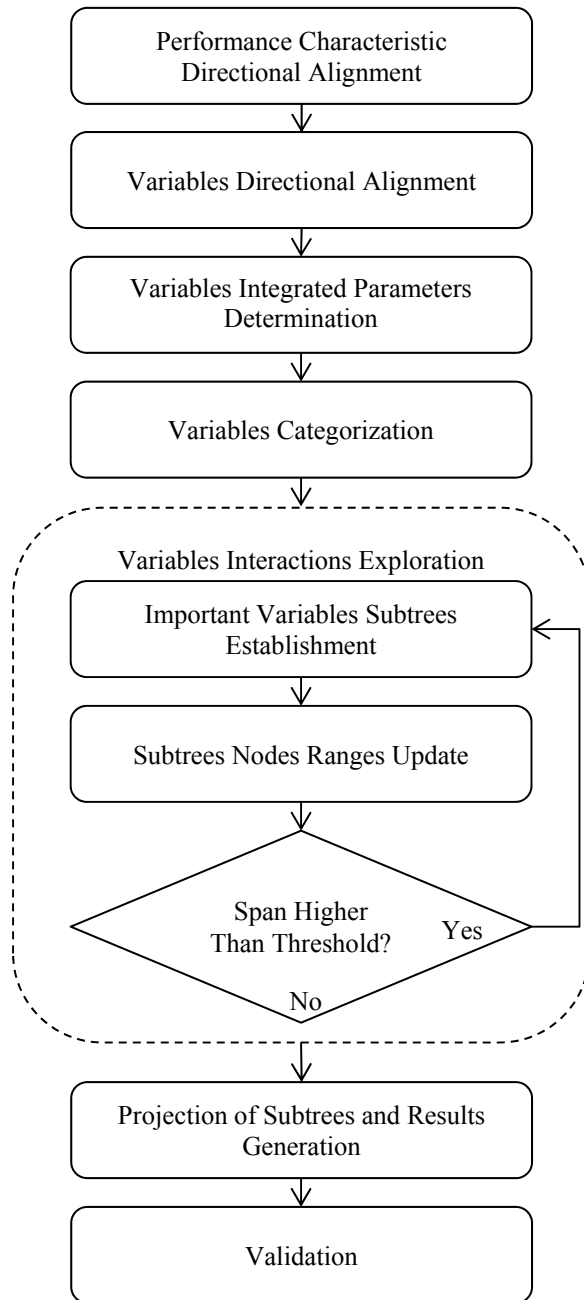


Figure 16. Flow Chart of the RC Method

IV.4.3 Span Check

Unless the maximum number of allowed experiments is reached, the single nodes' spans or total nodes' span in the subtree are found using:

$$\text{Node Span} = (R_{i,High} - R_{i,Low}) \quad (49)$$

$$\text{Total Subtree Span} = \sum_{i=1}^S (R_{i,High} - R_{i,Low}), \quad (50)$$

and are compared against the threshold of allowed single or total nodes span. It is also possible to track the span of the parameters that are found by the span of the response nodes. If the span is smaller than the threshold, the subtree is not further spanned. If it is higher than the threshold, the next most important variables are explored and the previous step of "Important Variables Subtrees Establishment" is performed. Instead of using variables' importance strength as a criterion for subtrees' unfolding, other experiment's path selection processes were investigated, and are explained in Appendix E.

IV.4.4 Projection of Subtrees and Result Generation

Once all subtrees have reached a span that does not exceed the threshold of the span, the experimentation stops and each node's value is projected to fall in the middle of its range.

$$\hat{y}_i = (R_{i,High} + R_{i,Low})/2 \quad (51)$$

where $R_{i,High}$ and $R_{i,Low}$ are the boundaries of every node. The middle point selection biases the parameters convergence towards the actual parameter. Important variables' parameters tend to converge fast toward their values unlike noise and weak variables.

IV.5 Hierarchical Integrated Parameters Method

The key of this method is the hierarchical exploration of important integrated even parameters of the main variables' tree. This method assumes that if the value of the integrated even parameters of any order of interactions is small, the integrated even parameters for that order can be equated to zero. The flow chart of the proposed method is shown in Figure 17. A new domain of parameters is introduced for this method. The relationship of the new domain α to the domain of β is:

$$\alpha = C\beta \tag{52}$$

where α is a vector of odd and even parameters for $n/2$ parameters. C is a domain conversion matrix between the two domains. The new parameters can be found using

$$Y = X\beta = XC^{-1}\alpha \Rightarrow \alpha = (XC^{-1})^{-1}Y \tag{53}$$

An example of the new domain conversion process can be found in Appendix D.

IV.5.1 Next Order Interactions Determination

In this step, the next order integrated odd and even parameters of every important variable or interaction in the main variables' tree are found. This step is similar to the previous step of "Next Order Interactions Determination" of the MSIV method except it is applied to the main tree only.

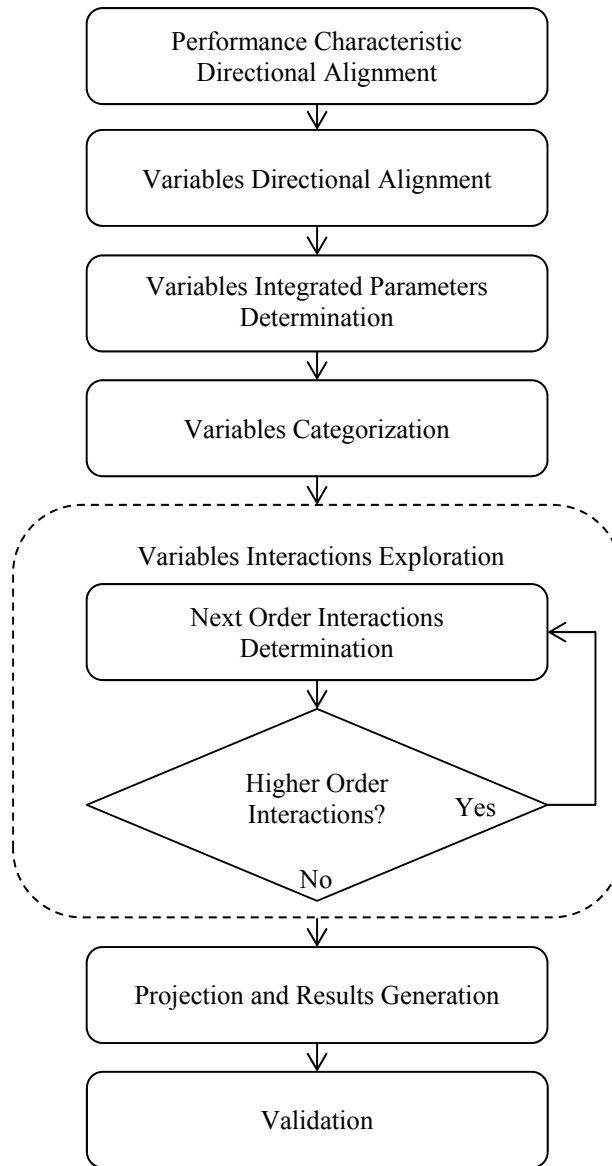


Figure 17. Flow Chart of the HIP Method

IV.5.2 Evaluation of Interactions in Variables Main Tree

After the next order interactions in the variables' main tree are found, the integrated even parameters of every order, $Even(\beta_{R,1,-1})$, is checked against the

threshold. This step is similar to the first check of the previous step of “Evaluation of Interactions in Interacting Variables Subtree” of the MSIV method except it is applied to the main tree only. If $Even(\beta_{R,1,-1})$ is found to be below the threshold, then no further high order interactions containing R are explored. If $Even(\beta_{R,1,-1})$ is found to exceed the threshold then the previous step of “Next Order Interaction Determination” is performed on the next order assuming a heredity compliant behavior.

IV.5.3 Projection and Results Generation

Once all interactions have been unfolded to an extent where they have no integrated even parameters, the integrated parameters of all order interactions are used to project the response in the main tree using these parameters. The integrated offset parameters of the tree are found by each end of the tree using the first two rows of α .

V. EXAMPLE APPLICATION

Out of the four introduced methods, the two methods of MHIV and MSIV are applied in the examples and application of this dissertation since they were both found to produce the most accurate results. In this section, the MHIV method is used on two examples from literature. Both examples variables were found highly interacting, so the method of MSIV was not applicable. An example application of the RC Method is shown in Appendix F for the method demonstration.

V.1 Method Application to Regular System

In this section, the MHIV method is tested on a system that conforms with the monotonic assumption to a high extent. The example used is the study of the “Analysis of earth-moving systems using discrete-event simulation” [81]. The method steps will be explained and the results will be compared with the results of a FFD using a close number of experiments. The study [81] performed a full factorial set of experiments on two performance characteristic. This example will analyze one of the performance characteristics for demonstration of the method.

V.1.1 Performance Characteristic Directional Alignment

The performance characteristic analyzed in this example is the production rate. It is limited by a value of zero. Thus, a saturation plateau is expected at the lower end of the response, if the variables are aligned in the same direction of effect. The alignment of the response is sustained as high is proportional to performance characteristic high.

V.1.2 Variables Directional Alignment

Since this is a demonstrative example, no experimenter input is needed, and the variables of the example are aligned to produce the lowest value of the response at the variables' states of low. This is satisfied by reversing the unaligned variable's signs. Accordingly, variables two to six of the paper are reversed.

V.1.3 Variables Integrated Parameters Determination

In this step, the variables "one down and one up" process is applied. Since 6 variables are explored, fourteen experiments are performed at this stage. The integrated odd and even parameters of all variables are shown in Table 4.

Table 4
Integrated Odd and Even Parameters in the Main Tree of Production

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 96.8 | 31.9 | 64.3 | 32.4 |
| 2 | 73.5 | 1.9 | 37.7 | 35.8 |
| 3 | 65.6 | 0.9 | 33.2 | 32.3 |
| 4 | -1.4 | 0.1 | -0.6 | -0.8 |
| 5 | 124.5 | 39.7 | 82.1 | 42.4 |
| 6 | 0.4 | 0.3 | 0.3 | 0.1 |

V.1.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of interaction and presented in Figure 18. It is clear from the magnitudes of the

integrated even parameters that four of the variables are interacting. It is possible to fully unfold the four variables by performing sixteen experiments and determining the full factorial design of the four variables. It is also possible to apply adaptive non-interacting variables' isolation within the four variables to produce the results with a smaller number of experiments. It is important to note that if the variables were analyzed using FFD, the interactions behavior would not have been so easy to judge. Instead of performing a full factorial design on the four interacting variables, the MHIV method is applied.

The value of the integrated even parameters threshold is the experimenter decision. As the selected threshold is lowered, the number of experiments increases. In this example a threshold of 3 is set. Accordingly, it is possible to categorize variables 1,2,3 and 5 (variables' indices will be referred to in a combined format, 1235, in the dissertation) as interacting and variables 46 as non-interacting. The non-interacting variables are not further explored. They are assumed to have first order parameters that are equal to the integrated odd parameters, highlighted in grey in Table 4, and zero even parameters.

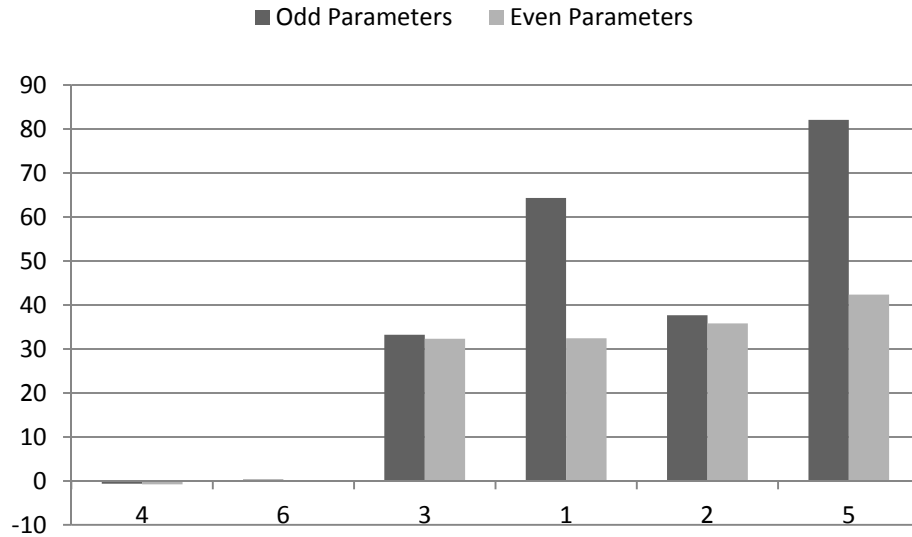


Figure 18. Integrated Odd and Even Parameters in the Main Tree of Production

V.1.5 Variables Interactions Exploration

To better present the flow of the method, several steps are combined in this section. These are the steps in the “Variables Interactions Exploration” box of Figure 15. Since the numbers of experiments are to be minimized, one variable with the highest integrated even parameters is selected at each step of subtree’s establishment. This requires unfolding variable 5 first. The result of unfolding 5 and determining the integrated even and odd parameters of variables 1, 2 and 3 within every subtree is shown in Figure 19.

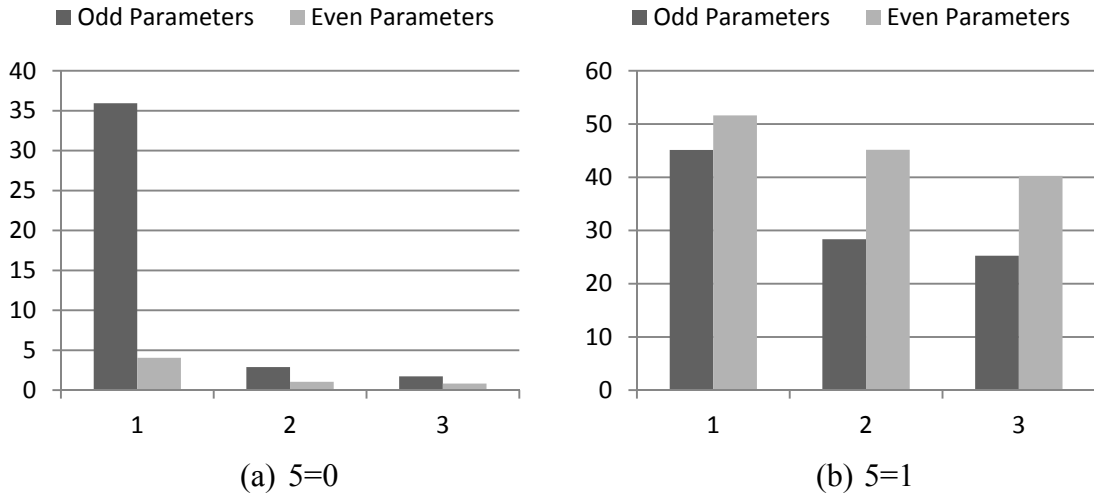


Figure 19. Integrated Odd and Even Parameters of 123 in Subtrees $5=x$ of Production

Within subtree $5=0$, only variable 1 is unfolded, thus creating two subtrees $1=x$ of $5=0$. As for subtree $5=1$, variable 1 is selected to establish the next level subtree since it is the variable with the highest integrated even parameters. The integrated odd and even parameters of variables 23 in subtrees $1=x$ of $5=1$ are found next. The results are shown in Figure 20. Only subtree $1=1$ of $5=1$ is to be further unfolded by establishing subtrees $23=xx$ of $1=1$ of tree $5=1$. The flow of the experiments followed in this example is shown in Figure 21.

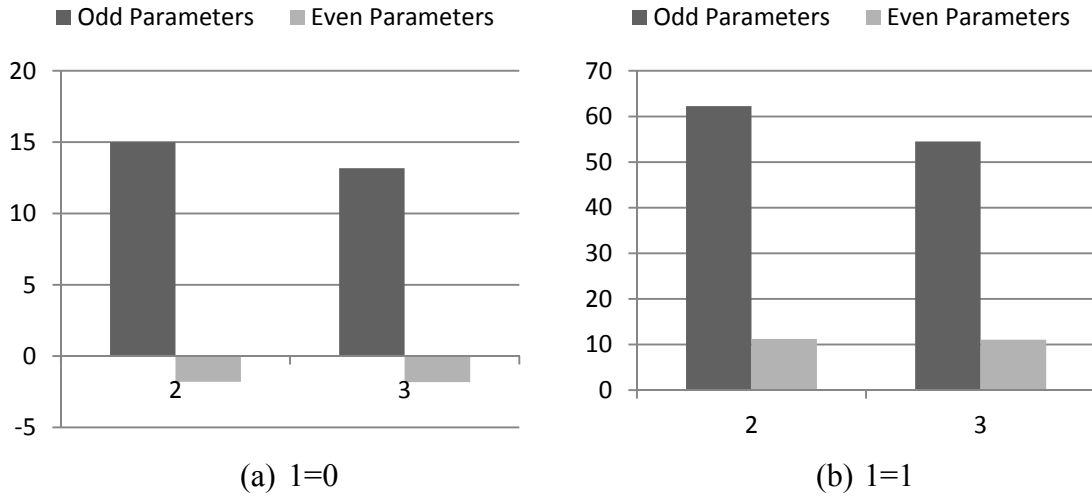


Figure 20. Integrated Odd and Even Parameters of 23 in Subtrees $1=x$ of $5=1$ of Production

V.1.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the two end offsets using by equation 45. The value of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46. In this example, $\beta_{0,1,-1,Error}$ in subtree $1=0$ of $5=0$ was found high, as a result, subtrees $2=x$ was created. The values of $\beta_{0,1,-1,Error}$ in all subtrees are shown in Table 5.

Table 5
Offset Errors in All Subtrees of Production

| Subtree 5123 | 000x | 001x | 010x | 011x | 10xx | 1100 | 1101 | 1110 | 1111 |
|------------------------|------|-------|------|------|------|-------|-------|-------|-------|
| States | 000x | 001x | 010x | 011x | 10xx | 1100 | 1101 | 1110 | 1111 |
| $\beta_{0,1,-1,Error}$ | 0 | -0.75 | 5.25 | 2.1 | 2.65 | 2.375 | 4.125 | 4.525 | 6.325 |

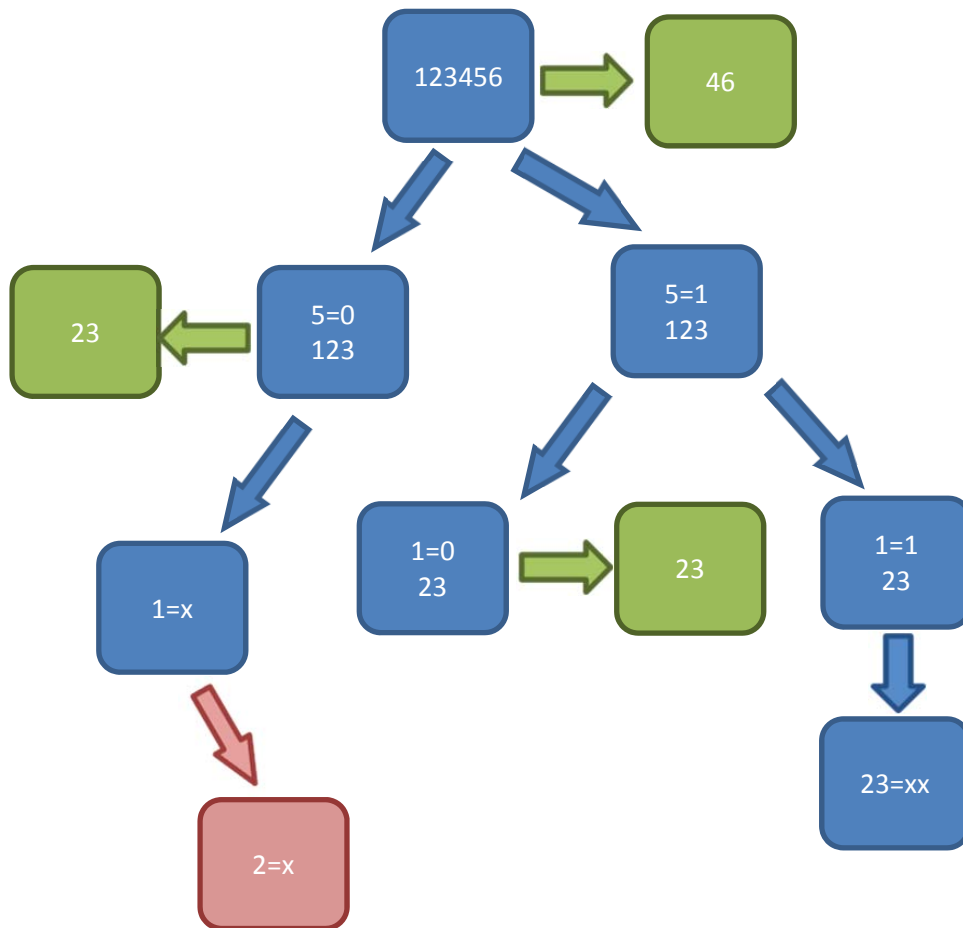


Figure 21. Flow Chart of the Recursive Exploration of Production Variables' Interactions

The resulting parameters of all subtrees are shown in Table 6. The parameters in every subtree along with the offset are used to project the performance characteristic in

that subtree. The parameters resulting from the MHIV method projection are shown in Table 7.

Table 6
Parameters in All Subtrees of Production

| Parameter Index | 5=0,12=xx | 5=1,1=0 | 5=1,1=1,23=xx |
|-----------------|-----------|---------|---------------|
| 1 | 0 | 0 | 0 |
| 2 | 0 | 15 | 0 |
| 3 | 1.725 | 13.175 | 0 |
| 4 | -0.625 | -0.625 | -0.625 |
| 5 | 0 | 0 | 0 |
| 6 | 0.3 | 0.3 | 0.3 |

V.1.7 Method Validation

In total, 28 experiments were performed to find the model of the production. In this section, the model projection results are evaluated with respect to the full factorial results of the paper then the results are compared to the performance of the closest FFD, which is a 32 res VI. The errors of the projected production using the MHIV method are shown in Figure 22. The errors of the projected production using the 32 Res VI FFD are shown in Figure 23. The RMSE of the projected results using the MHIV method is found as 2.24 while the 32 FFD RMSE is found as 6.54. Thus, the MHIV method projection is significantly more accurate using a smaller number of 28 experiments.

Table 7
Parameters of Production

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '0' | 137.090 | '234' | -0.114 |
| '1' | 41.057 | '235' | 2.615 |
| '2' | 21.373 | '236' | -0.114 |
| '3' | 18.232 | '245' | 0.278 |
| '4' | 0.405 | '246' | -0.082 |
| '5' | 58.459 | '256' | 0.278 |
| '6' | 1.330 | '345' | 0.091 |
| '12' | 12.963 | '346' | 0.059 |
| '13' | 10.760 | '356' | 0.091 |
| '14' | 0.477 | '456' | 0.048 |
| '15' | 4.487 | '1234' | 0.052 |
| '16' | 0.580 | '1235' | 3.118 |
| '23' | 2.454 | '1236' | 0.155 |
| '24' | -0.055 | '1245' | 0.125 |
| '25' | 17.852 | '1246' | -0.345 |
| '26' | 0.048 | '1256' | 0.228 |
| '34' | 0.077 | '1345' | 0.094 |
| '35' | 16.205 | '1346' | -0.054 |
| '36' | 0.180 | '1356' | 0.197 |
| '45' | 0.500 | '1456' | -0.502 |
| '46' | -0.035 | '2345' | -0.100 |
| '56' | 0.603 | '2346' | -0.573 |
| '123' | 2.857 | '2356' | 0.003 |
| '124' | 0.092 | '2456' | -0.193 |
| '125' | 10.930 | '3456' | -0.334 |
| '126' | 0.092 | '12345' | 0.066 |
| '134' | 0.080 | '12346' | -0.079 |
| '135' | 10.171 | '12356' | 0.066 |
| '136' | 0.080 | '12456' | -0.262 |
| '145' | 0.059 | '13456' | -0.552 |
| '146' | -0.391 | '23456' | -0.321 |
| '156' | 0.059 | '123456' | 0.091 |

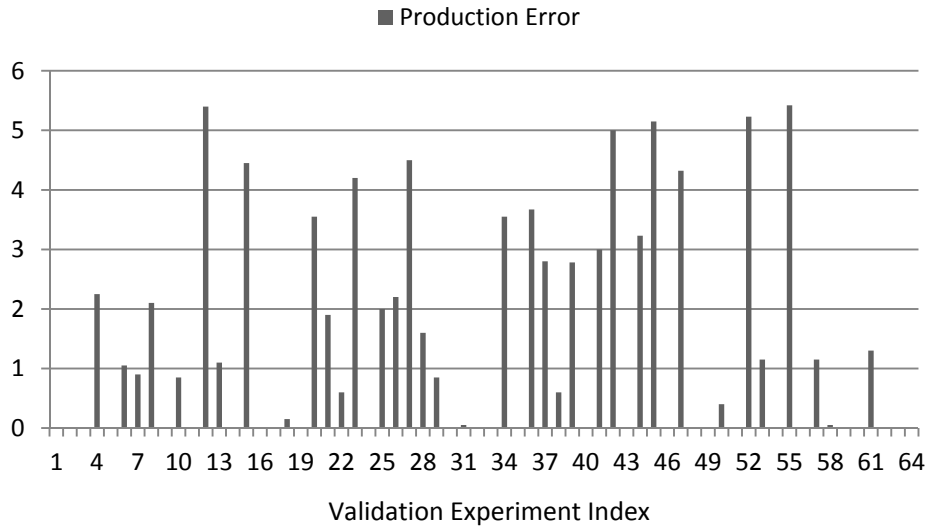


Figure 22. Production’s Projection Error of the MHIV Method with Respect to the Performed Experiments of the Full Factorial Design

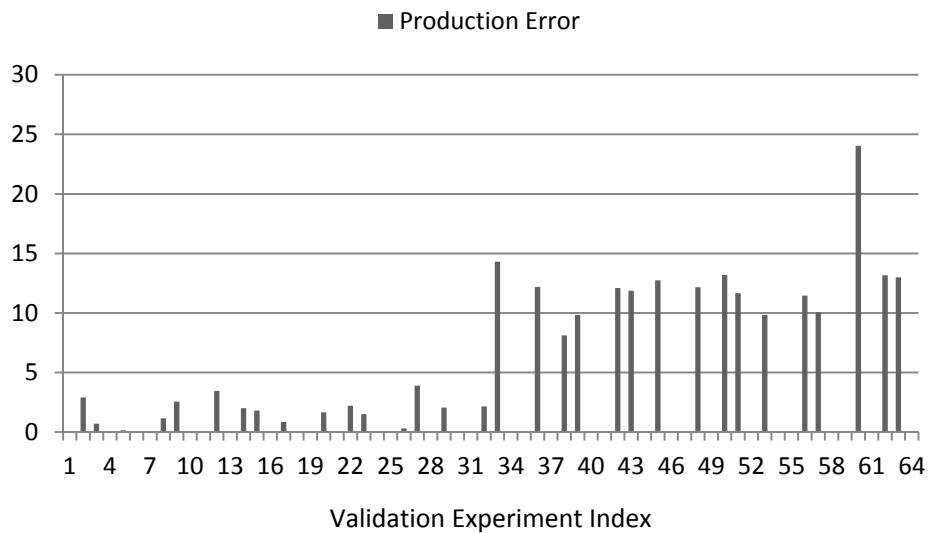


Figure 23. Production’s Projection Error of the 32 Res VI FFD with Respect to the Performed Experiments of the Full Factorial Design

V.1.8 Conclusions

Since the FFD's experiments rely on a uniform distribution of points, it failed to perform at the high end of the response as it did at the low end. This explains the bias of the errors toward one end in Figure 23. The sparsity of interactions is not accounted for by the FFD, since it applies equal importance to all variables. The MHIV method was able after few steps to isolate two variables and target the remaining four. It needed an additional 12 experiments to explore these four variables. A total of 28 experiments generated a very accurate model compared to the FFD. The method performed poorly in subtrees like subtree 1=0 of 5=0 due to the non-monotonic behavior of variables in these subtrees. Fortunately these poorly behaving subtrees were detected by $\beta_{0,Error}$.

V.2 Method Application to Non-Regular System

In this section, the MHIV method is tested on a system that does not fully conform to the monotonic assumption. The example used is the study of the "Parameters Contributing to Power Loss in Disengaged Wet Clutches" [82]. The method steps will be explained and the results will be compared with the results of a FFD of the same number of experiments. The study of [82] performed a full factorial set of experiments analyzing one performance characteristic.

V.2.1 Performance Characteristic Alignment

The performance characteristic analyzed in this example is the Parasitic Drag Torque. It is limited by a value of zero. Thus, a saturation plateau is expected at the

lower end of the response. The alignment of the response is sustained as high is proportional to performance characteristic high. This example will show that even if the monotonic behavior is not always present, the assumption of monotonic behavior and application of the MHIV method will generate more accurate results than a FFD. This is explained by the fact that for most of the time, this assumption is valid. If the variables are all highly monotonic, the results are very accurate. If the variables are less monotonic, the results are less accurate.

V.2.2 Variables Directional Alignment

Since this is a demonstrative example, no experimenter input is needed, and the variables of the example are aligned to produce the highest value of the response at the high states of variables. Accordingly, variables three, four, six, and seven of the paper are reversed. Though the variables are aligned, the monotonic behavior is not achieved for all variables of this example due to the variables' non-regular interactions. In fact, the violation percentage ratio of the variables' monotonic behavior, shown in Table 8, indicates that only one of the variables is always monotonic.

Table 8
Validity of Variables' Monotonic Effect on the Parasitic Drag Torque

| Variable | Ratio of Non-Monotonic Links to Total Number of Links (%) | Ratio of the Non-Monotonic Behavior Magnitude to the Span of the Response (%) |
|----------|---|---|
| 1 | 12.5 | 13.3 |
| 2 | 54.7 | 9.9 |
| 3 | 0 | 0 |
| 4 | 31.3 | 30.4 |
| 5 | 15.6 | 10.5 |
| 6 | 9.4 | 5.5 |
| 7 | 4.7 | 3.3 |

V.2.3 Variables Integrated Parameters Determination

In this step the “one down and one up” process is applied. Since seven variables are explored, sixteen experiments are performed at this stage. The integrated odd and even parameters of all variables are shown in Table 9.

Table 9
Integrated Odd and Even Parameters in the Main Tree of the Parasitic Drag Torque

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 5.500 | -0.250 | 2.625 | 2.875 |
| 2 | 4.000 | 0.050 | 2.025 | 1.975 |
| 3 | 6.300 | 0.600 | 3.450 | 2.850 |
| 4 | 4.100 | 0.200 | 2.150 | 1.950 |
| 5 | 4.700 | 0.700 | 2.700 | 2.000 |
| 6 | 5.000 | 0.200 | 2.600 | 2.400 |
| 7 | 4.250 | 0.100 | 2.175 | 2.075 |

V.2.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of the integrated even parameters and presented in Figure 24. It is concluded from the magnitudes of the integrated even parameters that all of the variables are interacting. The threshold of integrated even parameters is set at 1. Since no non-interacting variables are found at this tree, none of the variables are isolated at this level.

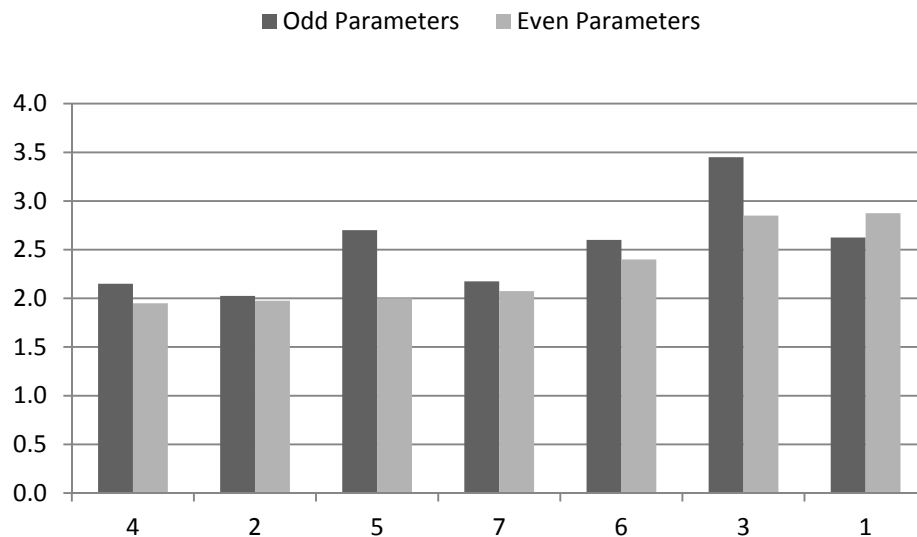


Figure 24. Integrated Odd and Even Parameters in the Main Tree of the Parasitic Drag Torque

V.2.5 Variables Interactions Exploration

Since the numbers of experiments are to be minimized, one variable with the highest integrated even parameters is selected at each step of subtree establishment. This requires unfolding variable 1 first. The result of determining the integrated odd and even

parameters of all other variables in subtrees $1=x$ is shown in Figure 25. Within subtree $1=0$, only variable 4 is to be unfolded, thus two subtrees of $4=x$ of $1=0$ are created. As for subtree $1=1$, all variables are interacting. Subtrees of the most interacting variable, 3, are created. In subtrees $3=x$ of $1=1$, the integrated odd and even parameters of 24567 are found, and are shown in Figure 26. Subtree $3=0$ of $1=1$ has one interacting variable, 5, so subtrees $5=x$ of $3=0$ of $1=1$ are created. Subtree $3=1$ of $1=1$ has all of the five variables as interacting, so the most interacting variable, 6, is selected to create the next level subtrees. The integrated odd and even parameters of 2457 in subtrees $6=x$ of $3=1$ of $1=1$ are found, and shown in Figure 27. Within subtree $6=0$ of $3=1$ of $1=1$, no variable is interacting. Subtree $6=1$ of $3=1$ of $1=1$ has four interacting variables, so the most interacting variable, 2, is unfolded by creating subtrees $2=x$ of $6=1$ of $3=1$ of $1=1$. The results of the integrated odd and even parameters of 457 within subtrees $2=x$ of $6=1$ of $3=1$ of $1=1$ are shown in Figure 28.

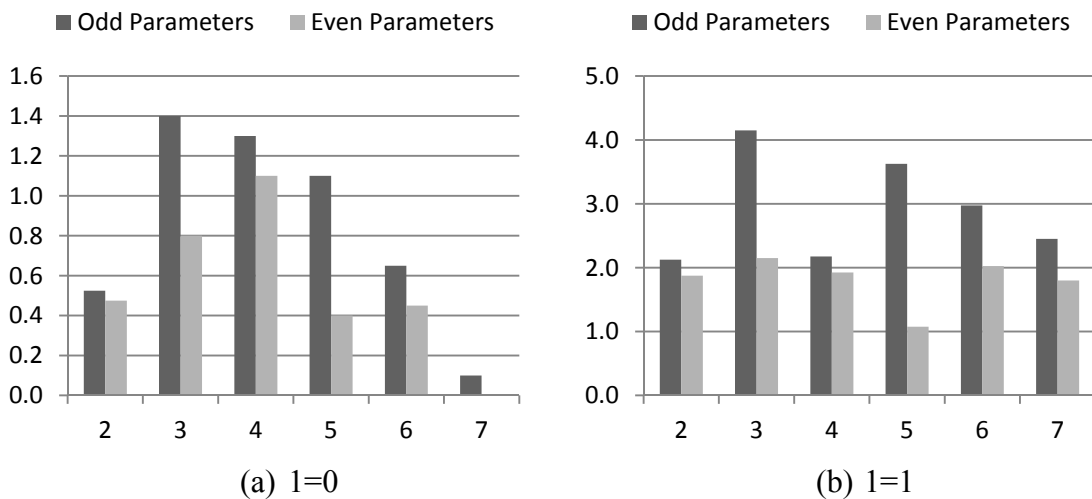


Figure 25. Integrated Odd and Even Parameters of 234567 in Subtrees $1=x$ of the Parasitic Drag Torque

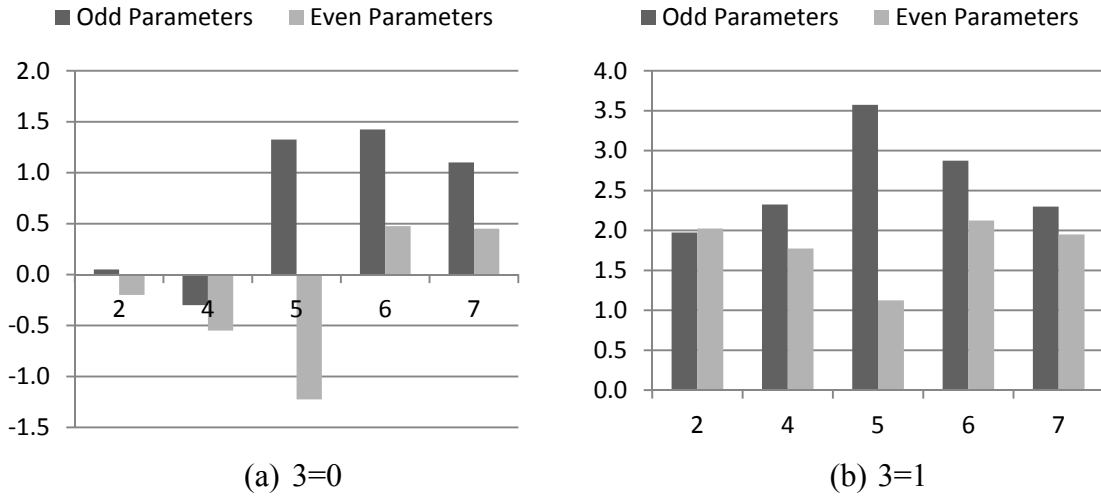


Figure 26. Integrated Odd and Even Parameters of 24567 in Subtrees 3=x of 1=1 of the Parasitic Drag Torque

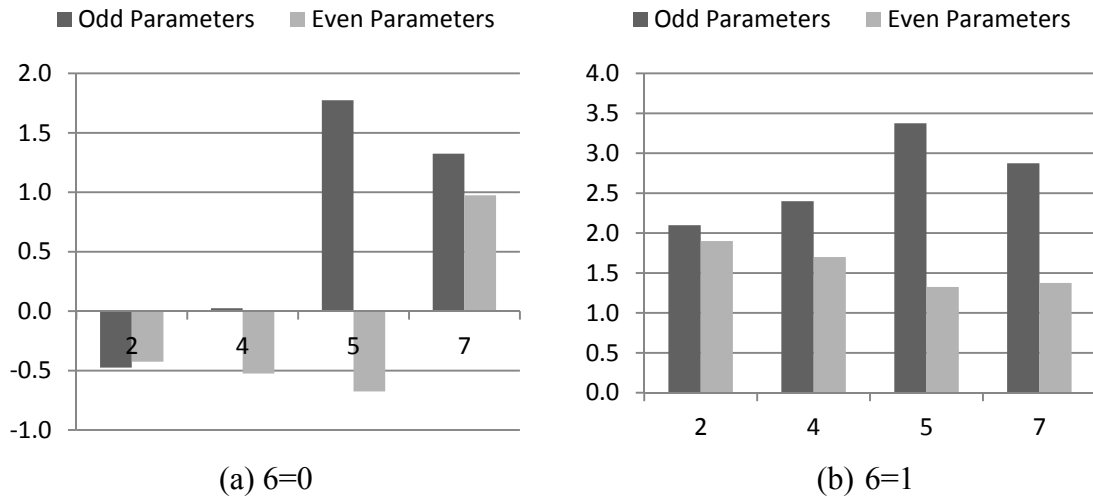


Figure 27. Integrated Odd and Even Parameters of 2457 in Subtrees 6=x of 3=1 of 1=1 of the Parasitic Drag Torque

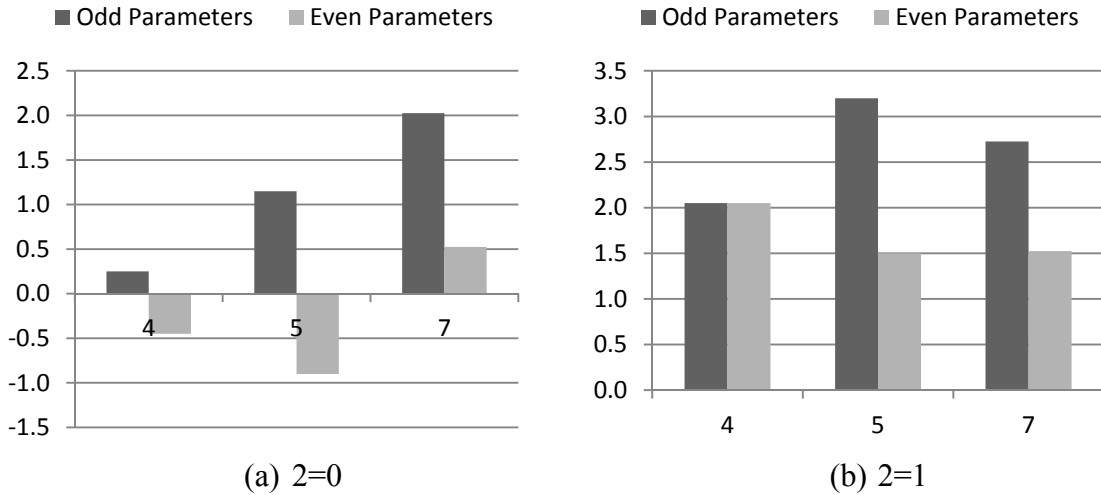


Figure 28. Integrated Odd and Even Parameters of 457 in Subtrees 2=x of 6=1 of 3=1 of 1=1 of the Parasitic Drag Torque

Subtree 2=0 of 6=1 of 3=1 of 1=1 has no interacting variables. Subtree 2=1 has the three variables as interacting. However, all nodes of 2=1 of 6=1 of 3=1 of 1=1 are found, since the exploration of three variables in an eight nodes subtree represents finding all nodes. As a result, no further subtrees are spanned. The flow chart of the experiments performed to explore interactions of this example is shown in Figure 29.

V.2.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by equation 45. The value of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46. In this example, $\beta_{0,1,-1,Error}$ values in subtrees 4 of 1=0 and 5 of 3=0 of 1=1 were found high, as a result, they were spanned into subtrees 36=xx and 4=x subsequently. The values of $\beta_{0,1,-1,Error}$ in the subtrees are shown in Table 10. The

resulting parameters of all subtrees are shown in Table 11. The parameters of every subtree along with the offset are used to project the performance characteristic in that subtree. The parameters resulting from the MHIV method projection are shown in Table 12.

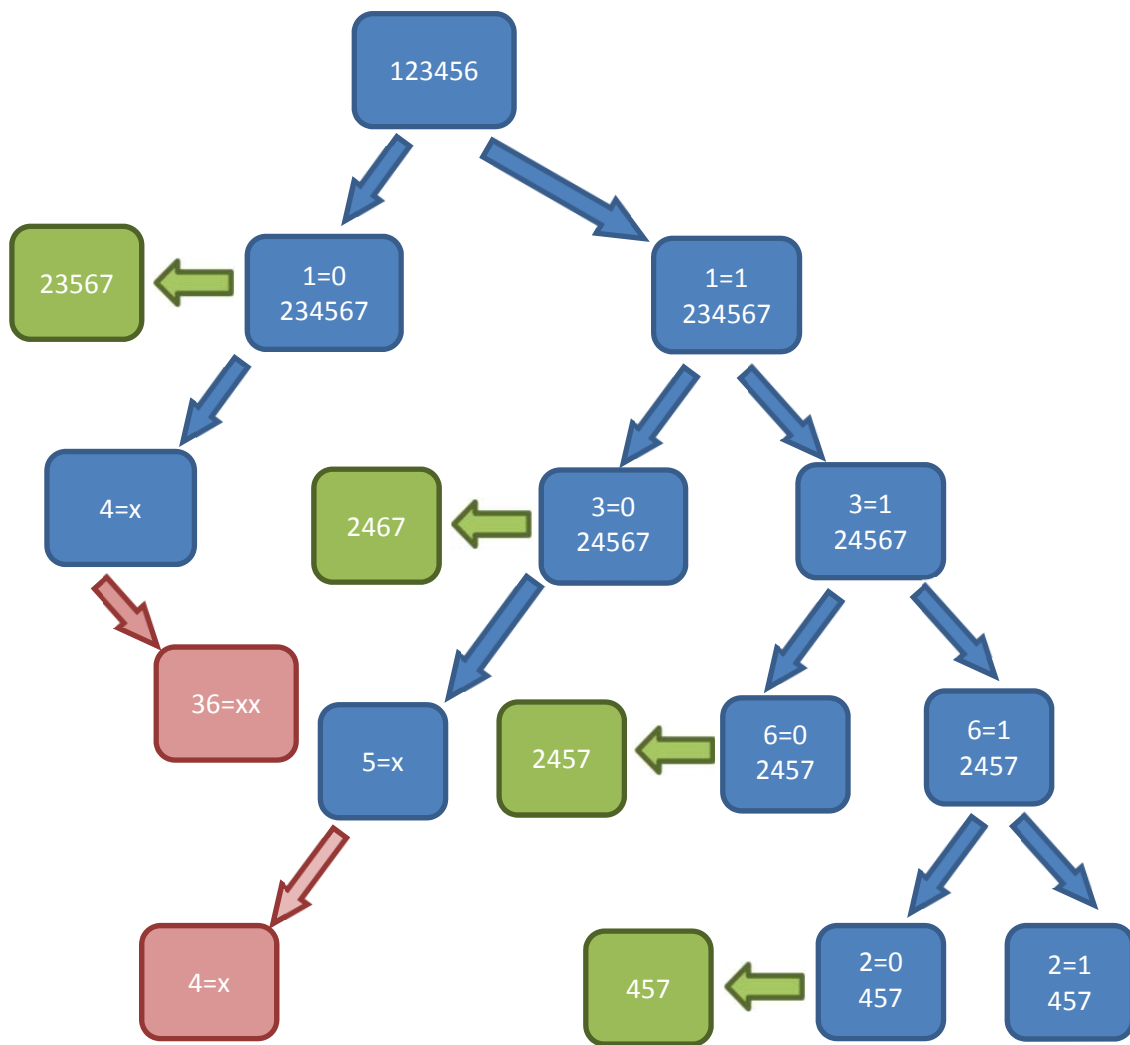


Figure 29. Flow Chart of the Recursive Exploration of the Parasitic Drag Torque Variables' Interactions

Table 10
Offset Errors in All Subtrees of Parasitic Drag Torque

| Subtree 1234567 States | $\beta_{0,1,-1,Error}$ |
|------------------------|------------------------|
| 0x00x0x | -0.875 |
| 0x00x1x | -1.425 |
| 0x01x0x | -1.325 |
| 0x01x1x | -1.125 |
| 0x10x0x | -0.925 |
| 0x10x1x | -1.525 |
| 0x11x0x | -0.525 |
| 0x11x1x | 0.275 |
| 1x000xx | -0.425 |
| 1x001xx | -1.525 |
| 1x010xx | -0.175 |
| 1x011xx | 0.025 |
| 1x1xx0x | -0.6 |
| 101xx1x | -1.125 |
| 111xx1x | 0 |

Table 11
Parameters in All Subtrees of Parasitic Drag Torque

| Parameter Index | 1=0, 346=xxx | 1=1, 3=0, 45=xx | 1=1, 3=1, 6=0 | 1=1, 3=1, 6=1, 2=0 | 1=1, 3=1, 6=1, 2=1 |
|-----------------|-----------------|-----------------------|---------------------|-----------------------------|-----------------------------|
| 1 | 0 | 0 | 0 | | |
| 2 | 0.525 | 0.05 | -0.475 | | |
| 3 | 0 | 0 | 0 | | |
| 4 | 0 | 0 | 0.025 | | |
| 5 | 1.1 | 0 | 1.775 | | |
| 6 | 0 | 1.425 | 0 | | |
| 7 | 0.1 | 1.1 | 1.325 | | |

No Projection
(All Nodes Performed)

Table 12
Parameters of Parasitic Drag Torque

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '0' | 4.8594 | '1234' | 0.0215 |
| '1' | 1.5414 | '1235' | 0.0973 |
| '2' | 0.1086 | '1236' | 0.1707 |
| '3' | 1.4637 | '1237' | 0.0809 |
| '4' | 0.1609 | '1245' | 0.1438 |
| '5' | 1.0414 | '1246' | 0.1023 |
| '6' | 0.7375 | '1247' | 0.0422 |
| '7' | 0.5086 | '1256' | 0.1516 |
| '12' | -0.1742 | '1257' | 0.1430 |
| '13' | 0.5606 | '1267' | -0.0625 |
| '14' | -0.4422 | '1345' | 0.1801 |
| '15' | 0.1836 | '1346' | -0.0027 |
| '16' | 0.5594 | '1347' | 0.0746 |
| '17' | 0.6508 | '1356' | -0.0160 |
| '23' | 0.0574 | '1357' | 0.0723 |
| '24' | 0.0898 | '1367' | 0.0301 |
| '25' | 0.0289 | '1456' | 0.0477 |
| '26' | 0.1656 | '1457' | 0.1156 |
| '27' | 0.0578 | '1467' | 0.0211 |
| '34' | 0.3996 | '1567' | -0.0078 |
| '35' | 0.1699 | '2345' | 0.1324 |
| '36' | 0.0293 | '2346' | 0.1465 |
| '37' | 0.1152 | '2347' | 0.0340 |
| '45' | -0.2734 | '2356' | 0.1684 |
| '46' | 0.2852 | '2357' | 0.0668 |
| '47' | 0.1945 | '2367' | -0.0176 |
| '56' | -0.0148 | '2456' | 0.1219 |
| '57' | 0.0555 | '2457' | 0.0250 |
| '67' | 0.0766 | '2467' | 0.0586 |
| '123' | -0.0293 | '2567' | 0.0148 |
| '124' | 0.0031 | '3456' | 0.0590 |
| '125' | -0.0078 | '3457' | 0.0746 |
| '126' | 0.1477 | '3467' | 0.0355 |
| '127' | 0.0711 | '3567' | -0.0207 |
| '134' | -0.0184 | '4567' | 0.1047 |
| '135' | 0.0926 | '12345' | 0.0957 |
| '136' | -0.0137 | '12346' | 0.1035 |

Table 12 Continued

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '137' | 0.0879 | '12347' | 0.0473 |
| '145' | -0.3508 | '12356' | 0.1316 |
| '146' | -0.0578 | '12357' | 0.0020 |
| '147' | 0.1672 | '12367' | -0.0043 |
| '156' | -0.0234 | '12456' | 0.0852 |
| '157' | 0.0781 | '12457' | -0.0398 |
| '167' | 0.1180 | '12467' | 0.0719 |
| '234' | 0.0855 | '12567' | 0.0188 |
| '235' | 0.1066 | '13456' | 0.0254 |
| '236' | 0.1785 | '13457' | 0.0973 |
| '237' | 0.0902 | '13467' | 0.0520 |
| '245' | 0.1531 | '13567' | 0.0020 |
| '246' | 0.1602 | '14567' | 0.1273 |
| '247' | 0.0516 | '23456' | 0.1262 |
| '256' | 0.1609 | '23457' | 0.0645 |
| '257' | -0.0617 | '23467' | -0.0105 |
| '267' | -0.0531 | '23567' | 0.0434 |
| '345' | 0.2441 | '24567' | 0.1211 |
| '346' | 0.0754 | '34567' | 0.0574 |
| '347' | 0.1387 | '123456' | 0.1168 |
| '356' | -0.0082 | '123457' | -0.0371 |
| '357' | 0.0816 | '123467' | -0.0199 |
| '367' | 0.0379 | '123567' | -0.0020 |
| '456' | 0.1055 | '124567' | 0.0258 |
| '457' | 0.1250 | '134567' | 0.0480 |
| '467' | 0.0789 | '234567' | 0.0246 |
| '567' | 0.0016 | '1234567' | 0.0035 |

V.2.7 Method Validation

In total, 62 experiments were performed to produce the model presented in the previous section. In this section, the results are evaluated with respect to the results of the full factorial design of the paper, then the results are compared to the performance of

the closest FFD, which is a 64 Res VII FFD. The errors of the projected Parasitic Drag Torque using the MHIV method are shown in Figure 30. The errors of the projected Parasitic Drag Torque using the 64 FFD are shown in Figure 31. The RMSE of the projected results using the MHIV method is found as 0.7339 while the RMSE of the projected results using the 64 FFD is found as 0.8978. Thus, the MHIV method produced more accurate results with a similar number of 62 experiments, even though the system variables are moderately non-monotonic.

V.2.8 Conclusions

Since the FFD experiments rely on a uniform distribution of points, it failed again to perform at the high end of the response. This explains the bias of the errors toward one end in Figure 31. Both methods required a high number of experiments for a reasonable accuracy. The MHIV method was able, from the first few steps, to determine that exploring the response will require a high number of experiments. This is due to the high number of important integrated even parameters with respect to the defined threshold. FFDs do not give such information, so if a low Res FFD was performed, such as sixteen Res IV experiments, the results would have been misleading.

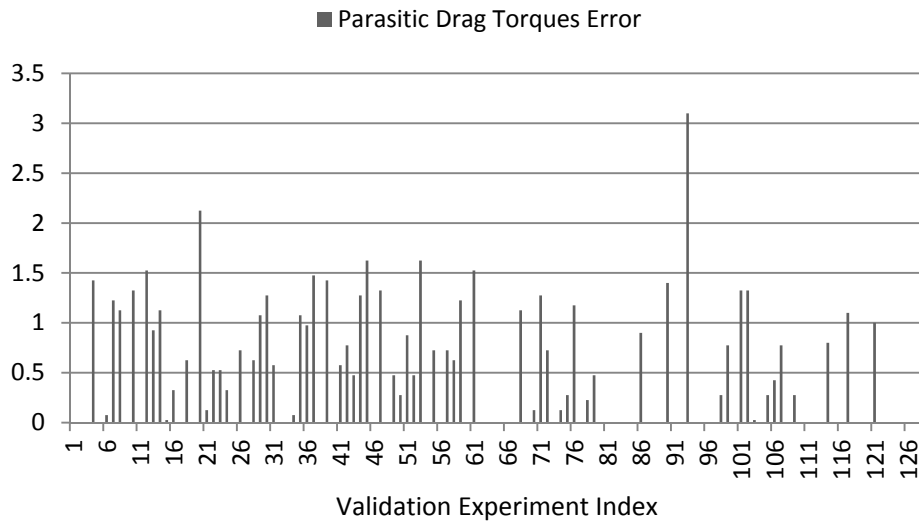


Figure 30. Parasitic Drag Torque’s Projection Error of the MHIV Method with Respect to the Performed Experiments of the Full Factorial Design

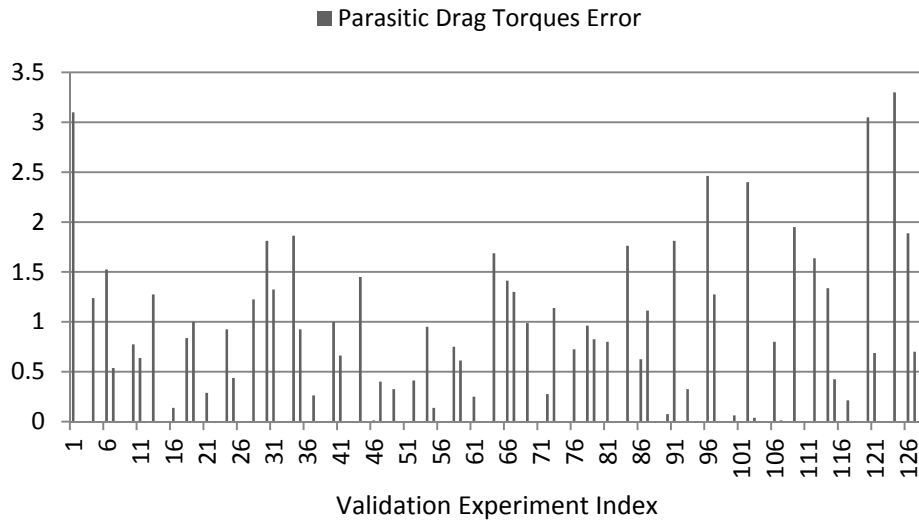


Figure 31. Parasitic Drag Torque’s Projection Error of the 32 Res VII FFD with Respect to the Performed Experiments of the Full Factorial Design

VI. CASE STUDY: NEUTRONICS EVALUATION OF AN SMR VERSION OF THE ADVANCED PRESSURIZED WATER REACTOR AP1000

VI.1 Problem Statement

The complex nature of the developing LWSMRs requires application of efficient design methods. The reference design used in developing LWSMRs is the current water cooled reactors. The reason behind this is the proven safety behavior of this type of reactors, which significantly impact the regulatory aspects of the SMRs' licensing. The problem used to demonstrate the capabilities of the dissertation's methods is to scale down a water cooled reactor. In specific, the AP1000 design [2] is scaled down with a proposed reduction of power between 100 MWth and 300 MWth. The scaling down of such a reactor will influence the design performance characteristics, and constitutes a complex problem. There are tens if not hundreds of performance characteristics to consider while designing a reactor. Examples are:

- Fuel cycle: criticality, fuel front end requirements, depletion, burnup, life cycle and reprocessing capabilities.
- Performance: efficiency, power profile, power peaking, pressure drop, availability and reliability.
- Safety: temperature and void reactivity coefficients, thermal stresses, transient behavior, risk assessment and accidents management.
- Mechanical: reactor's size, mass and mechanical stresses.
- Waste management: minor actinides & fission products production.

- Reactor economics: capital cost, fueling cost, operational cost, maintenance cost and power production.
- Materials: fuel integrity, cladding integrity and mechanical elements integrity.
- Environment: radioactivity discharge and spent fuel radioactivity.
- Safeguards & security: fuel accessibility, fuel handling and fuel composition.

Designing these performance characteristics to fall within desired acceptable ranges is a significant challenge, especially, because they interact and often conflict. A simpler way to design such a system is to apply screening or sensitivity analysis methods to find the set of most important variables and interactions on each of the performance characteristic.

In this case study, two of the dissertation's methods are applied to six performance characteristics that are determined from computational experiments. The Beginning Of Life (BOL) effective multiplication factor (K_{eff}), the BOL Flux Spectrum in Fuel and Cladding (will be referred to as RFS for Ratio of Fast Spectrum) , the BOL fuel temperature reactivity coefficient (α_{FT}), the BOL water density reactivity coefficient (α_d), the power peaking factor (PPF) and the depletion drop of the effective multiplication factor (ΔK_{eff}) are considered as the performance characteristics of interest.

The aim of this case study is to determine the behavior of the performance characteristics with respect to fourteen variables in order to determine their most important variables and interactions, and to develop a good understanding of the system's behavior. Table 13 lists the variables of interest along with their design limits. Since one of the main objectives of SMRs is to reduce the size and power of the core, this was reflected in the selection of the design ranges. Two ends are set for each of the fourteen variables as shown in Table 13. If the methods of this dissertation, or any other method, are not utilized, 6×2^{14} experiments are needed to unfold all interactions of all orders. The example will demonstrate how a relatively small number of experiments (few hundreds) are sufficient to unfold the most important variables and interactions provided that the variables' directions of effect on the performance characteristics are known. Demonstrative figures of a sample of the variables' states are shown in Figure 32 to Figure 42. For abbreviation purposes, the variables' indices of Table 13 will be used from here onwards instead of using the full variables' names.

Table 13

Design Variables of the Case Study and Their Proposed Ranges

| Index | Variables | Min [*] | Max [*] | AP1000 ^{**} |
|----------------|--|-------------------------|-------------------------|--------------------------|
| Core Level | | | | |
| 1 | No Of Active Core Assemblies (cm) | 37 | 73 | 157 |
| 2 | Active Fuel Height (cm) | 85.0 | 256.0 | 426.7 |
| 3 | Core Radial Surrounding Water (cm) | 24.96 | 49.91 | 49.91 |
| 4 | Enrichments (%) | 2.35, 3.4, 4.45 | 3.4, 4.45, 4.95 | 2.35, 3.4, 4.45 |
| 5 | Power (MWth) | 100 | 300 | 3415 |
| 6 | Moderator Density (g/cm ³) | 0.716 (305C 15.5MPa) | 0.813 (243C 7.2 MPa) | 0.716 (305C 15.5 MPa) |
| 7 | Fuel Temperature (K) | 600 | 900 | 293.15 (cold) |
| Assembly Level | | | | |
| 8 | No of Rods per Assembly | 11x11 | 13x13 | 17x17 |
| 9 | Rod Pitch (cm) | 1.051 | 1.260 | 1.260 |
| 10 | No of DBARs per Assembly | 9 | 21 | 25 |
| Fuel Pin Level | | | | |
| 11 | IFAB and Fuel Pellet Diameter (cm) | 0.680 | 0.819 | 0.819 |
| 12 | IFAB and Fuel Gap Thickness (cm) | 0.0065 | 0.011 | 0.008 |
| 13 | IFAB and Fuel Clad Thickness (cm) | 0.046 | 0.057 | 0.057 |
| 14 | Discrete Burnable Absorber Area (cm ²) | 0.094 | 0.187 | 0.375 |

* Rounded to the number of shown decimals

** Based on actual and estimated values

Table 14

Performance Characteristics of the Case Study and their Desired Values

| Performance Characteristics | Desired Values |
|---|--------------------------------|
| BOL Effective Multiplication factor | Between 1.15 and 1.2 |
| Flux Spectrum in Fuel and Cladding | As Thermal as possible |
| BOL Fuel Temperature Reactivity Coefficient | Negative and high in magnitude |
| BOL Water Density Reactivity Coefficient | Positive and high in magnitude |
| Power Peaking Factor | As close as possible to one |
| Depletion Drop of Effective Multiplication Factor | BOL Keff-1 |

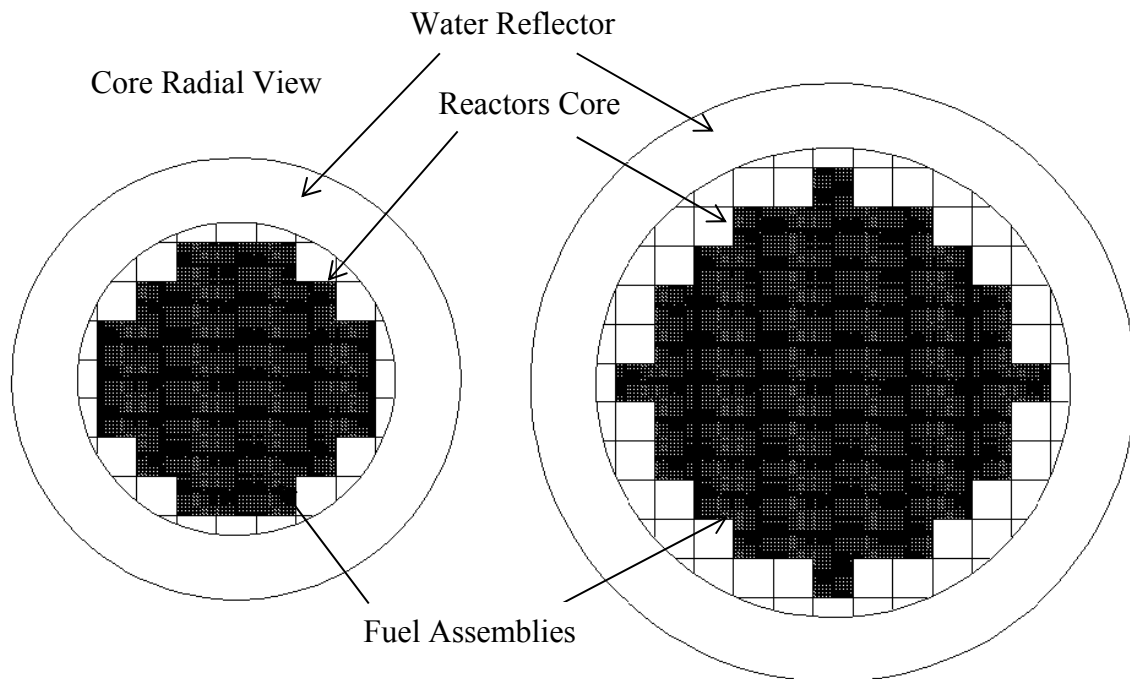


Figure 32. Variable 1 “No of Fuel Assemblies” States

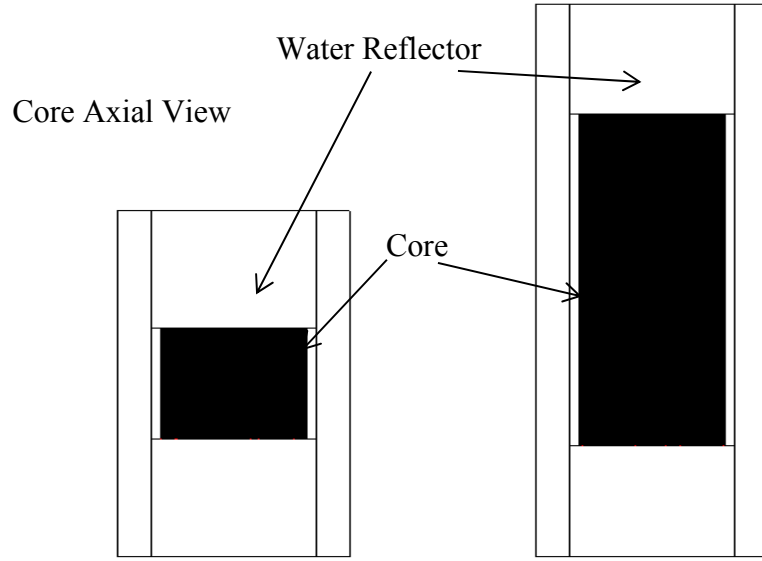


Figure 33. Variable 2 “Active Fuel Height” States

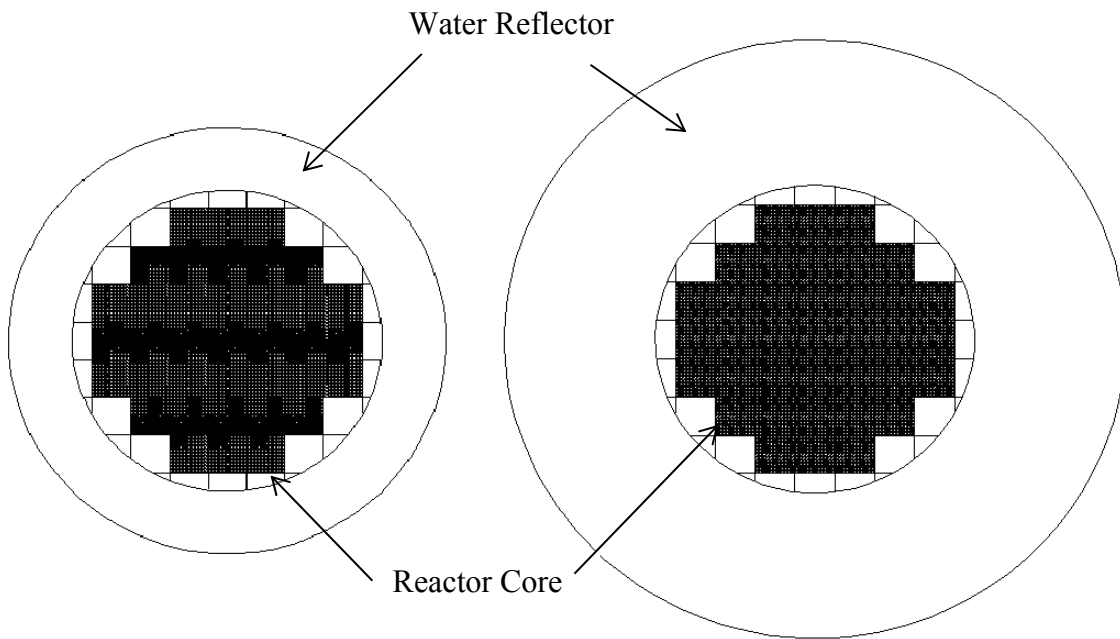


Figure 34. Variable 3 “Core Radial Surrounding Water” States

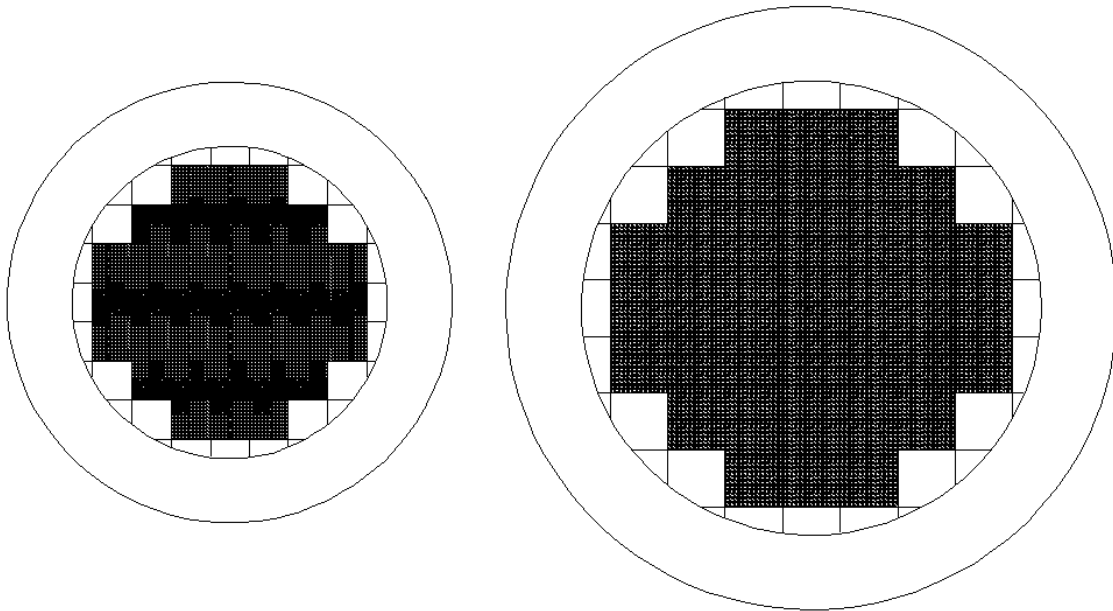


Figure 35. Variable 8 “No of Rods per Assembly” Core View of States

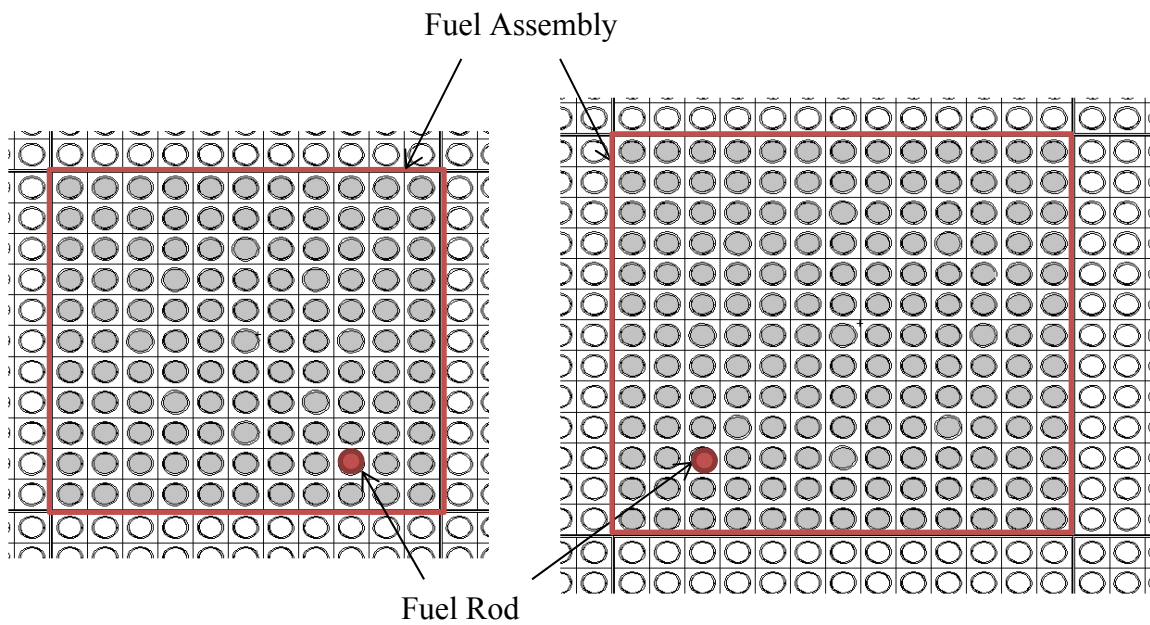


Figure 36. Variable 8 “No of Rods per Assembly” Assembly View of States

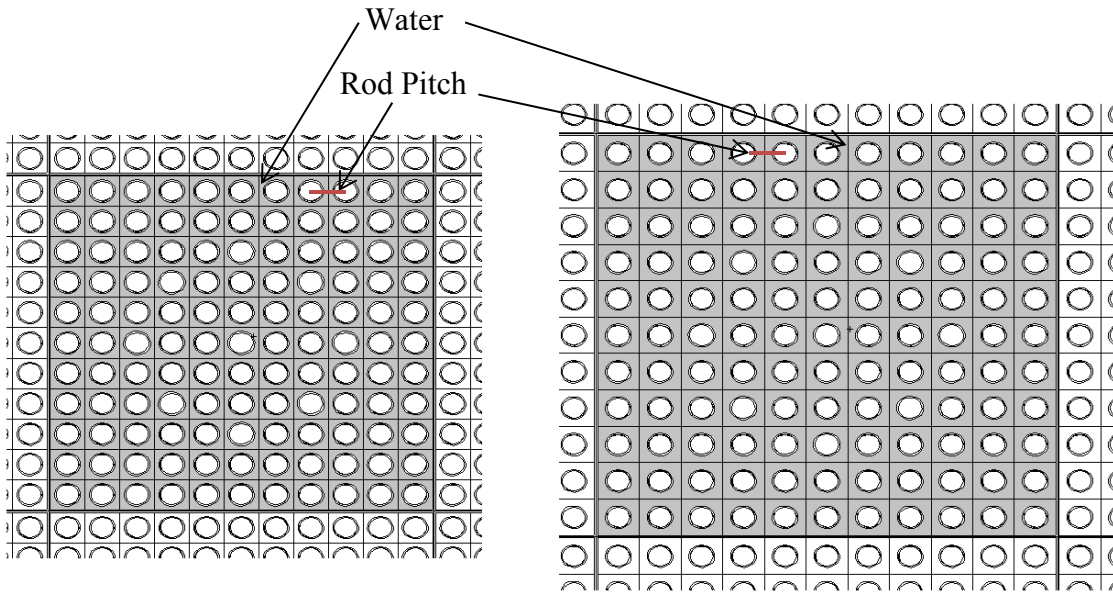


Figure 37. Variable 9 “Rod Pitch” States

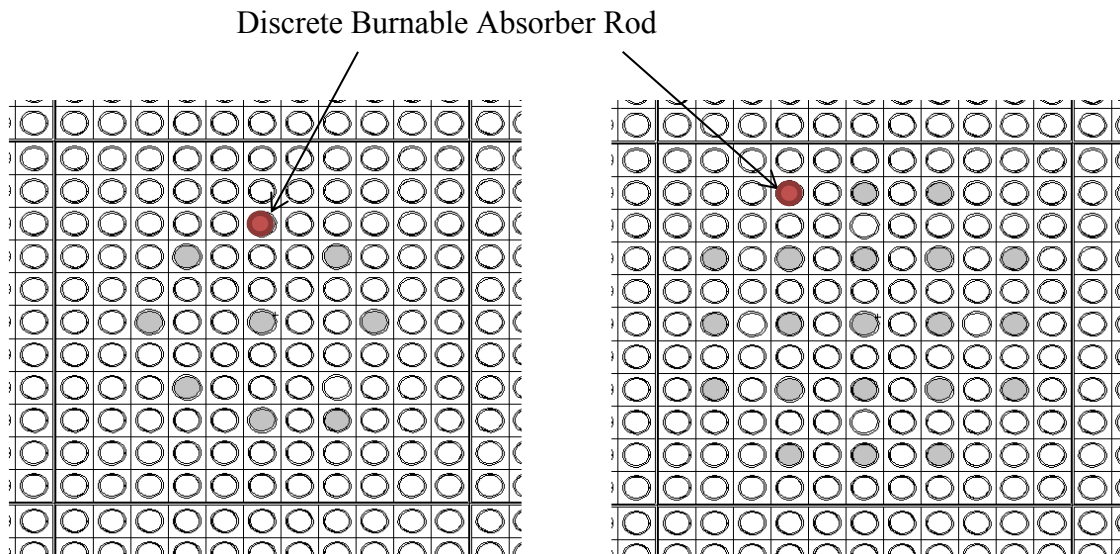


Figure 38. Variable ‘a’ “No of Discrete Burnable Absorber Rods per Assembly” States

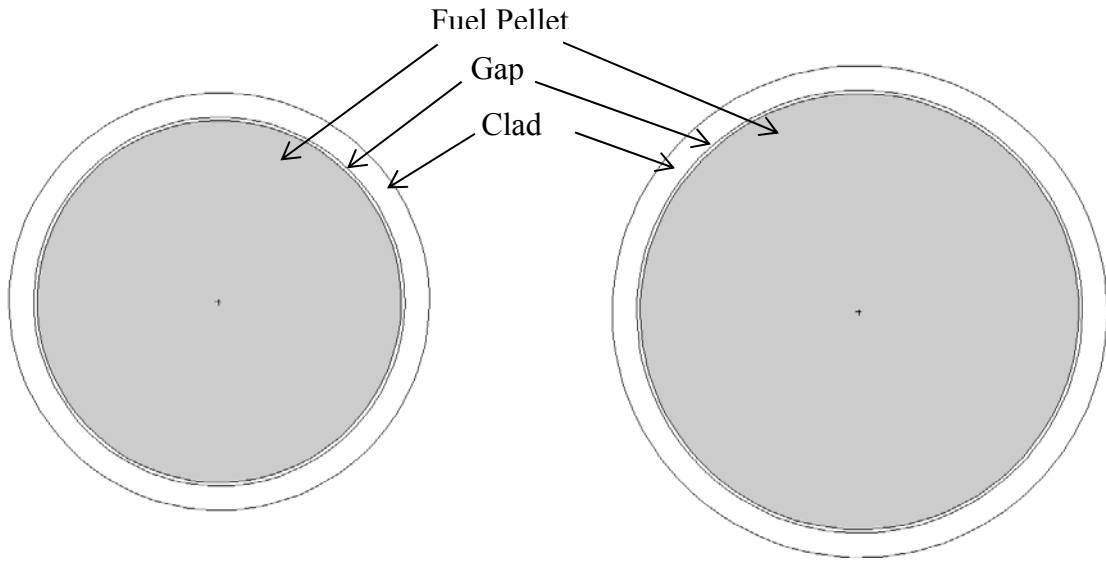


Figure 39. Variable 'b' "IFAB and Fuel Pellet Diameter" States

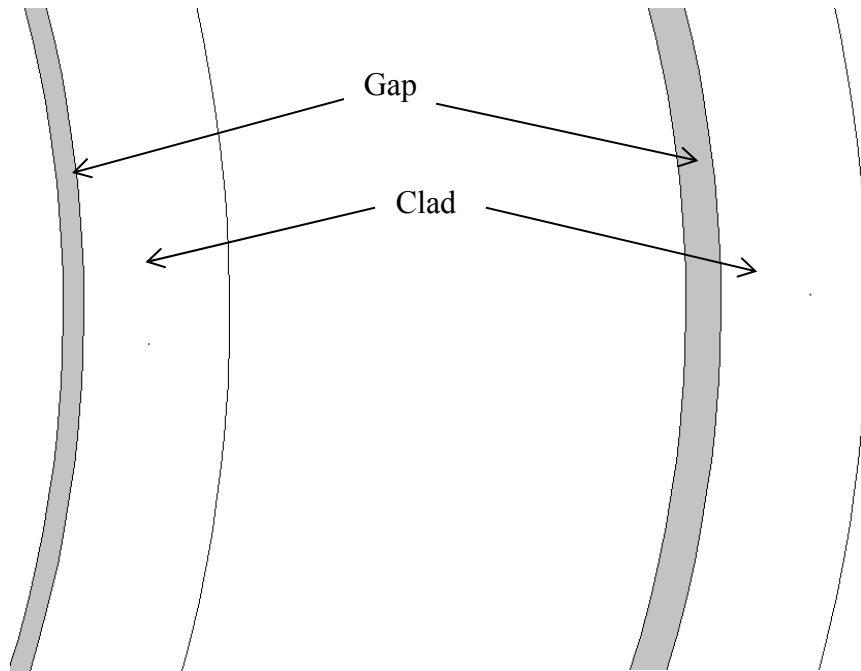


Figure 40. Variable 'c' "IFAB and Fuel Gap Thickness" States

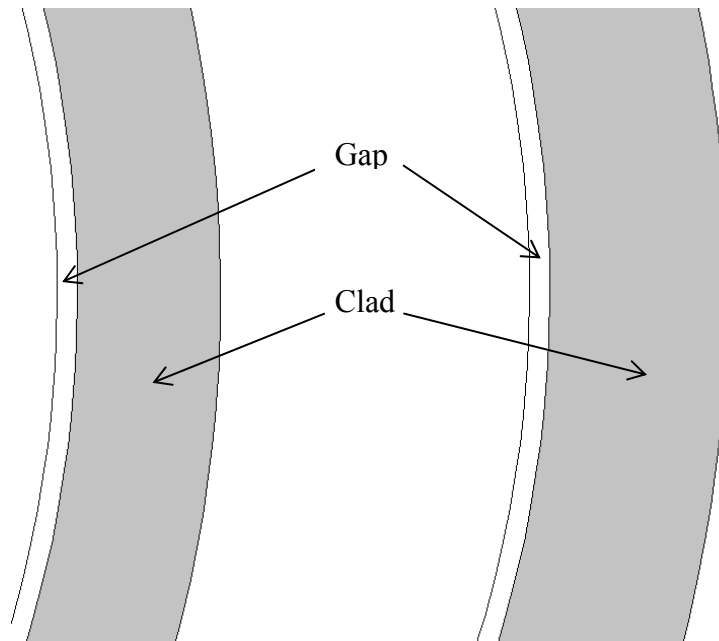


Figure 41. Variable 'd' "IFAB and Fuel Clad Thickness" States

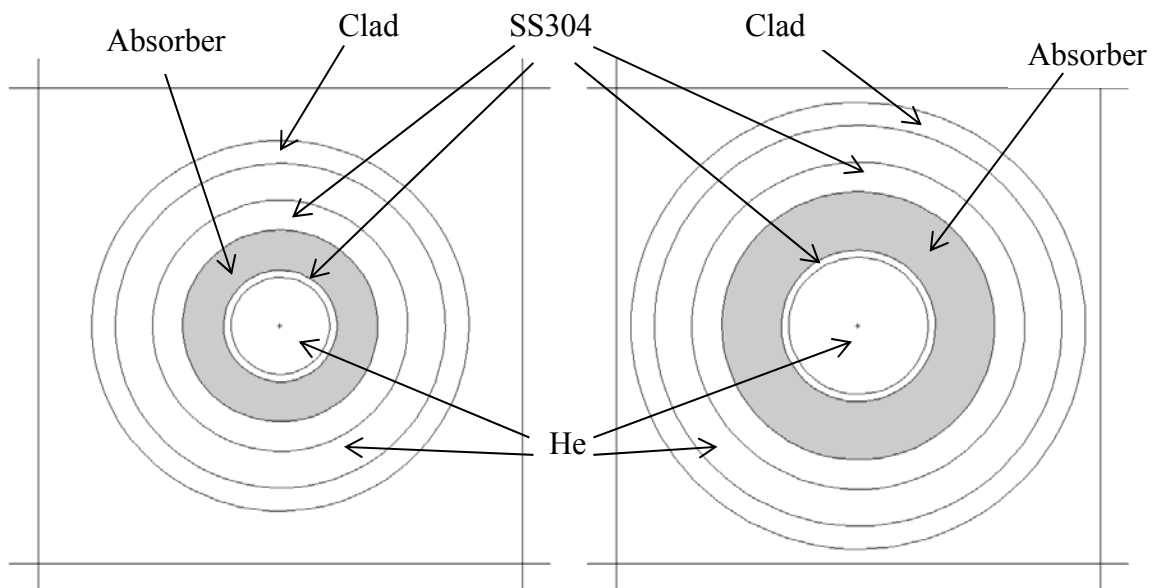


Figure 42. Variable 'e' "Discrete Burnable Absorber Area" States

VI.2 Analysis and Results

In this section, the dissertation's methods steps are applied and results are presented. The package used to simulate the reactor's design is the Monte Carlo N-Particle transport code (MCNP5) [83]. The number of particles used for each Monte Carlo experiment is dependent on the sensitivity of the performance characteristic, and are shown in Table 15.

Table 15
Experiments' Number of Particles and Errors' Order of Magnitude of the Six Performance Characteristics

| Performance Characteristic | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|--|-----------|-------|------------|---------------|------|------------------|
| Number of Particles in millions | 2 | 2 | 2 | 32 | 4 | 1 |
| Performance Characteristic Standard Deviation order of Magnitude | 0.000x | 0.00x | 0.000x | 0.0001x | 0.0x | 0.000x |

Two of the dissertation's methods were used to evaluate the system. These are the MSIV and MHIV methods. The steps applied were introduced in Figure 14 and Figure 15. The RC method was not used in this application, but was tested on one of the performance characteristics, RFS, for results comparison. The methods' results of three performance characteristics are compared to the results from a 128 Res IV FFD for validation. The high Res IV FFDs are generated using the Franklin-Bailey algorithm implemented in MATLAB [84]. The remaining three performance characteristics are validated using a FFD set of sixteen experiments.

VI.2.1 Multiplication Factor Analysis

In this section, the BOL of the effective multiplication factor K_{eff} is analyzed. K_{eff} is a measurement of the criticality of the core(see [85] for criticality definition). The aim for K_{eff} is to be in the range of 1.15 and 1.2 to sustain enough excess reactivity for the life of the core. However, the defined variables' ranges are expected to cause the effective multiplication factor to exceed this range at both ends.

VI.2.1.1 Performance Characteristic Directional Alignment

The effective multiplication factor is theoretically limited by a zero at its low end, and the number of neutrons per fission (around 2.4) at its high end. However, it is impossible that the design will approach this value. Instead, it is expected that K_{eff} is limited by the under-to-over moderation plateau at its high end, if the design falls in the under moderation region. At the low end, a plateau is expected as K_{eff} approaches zero. This might requires that the direction of K_{eff} be reversed to ensure the alignment of the high end of the variables' tree with the highest slope of the multidimensional surface of K_{eff} . For now, the direction of K_{eff} is held in its current state, till the level of saturation is found at both ends, in a later step.

VI.2.1.2 Variables Directional Alignment

The directional dependence of K_{eff} on the variables is determined at this stage. Variables are categorized into known monotonic variables and variables with unknown direction of effect. Power is a passive variable, since it does not affect K_{eff} . The variables with the highest monotonic behavior confidence are determined first, and are shown in Table 16. The directional dependence is briefly justified in Table 17. The unknown variables' directions of effect, marked by a question mark, was found by performing low accuracy Monte Carlo experiments to develop a preliminary sense of the directions of effect. According to the table, the maximum K_{eff} occurs at the variables' states of 1111x10110000, thus the variables are aligned to all high at this combination.

VI.2.1.3 Variables Integrated Parameters Determination

In this step, the “one down and one up” process is performed. Since 13 variables are explored, 28 experiments are performed at this stage. The results of the performed experiments are shown in Appendix G. The corresponding integrated odd and even parameters of all variables are shown in Table 18. All results of K_{eff} will be shown in units of 10^{-3} from here onwards.

Table 16Variables' Directions of Effect on K_{eff}

| Variable Index | Variable | Direction of Effect on K_{eff} |
|----------------|---------------------------------|-------------------------------------|
| 1 | No of Fuel Assemblies | + |
| 2 | Active Fuel Height | + |
| 3 | Core Radial Surrounding Water | + |
| 4 | Enrichments | + |
| 5 | Power | = |
| 6 | Moderator Density | + |
| 7 | Fuel Temperature Lib | - |
| 8 | No of Rods per Assembly | + |
| 9 | Rod Pitch | + |
| 10 (a) | No of DBARs per Assembly | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | - |
| 12 (c) | IFAB and Fuel Gap Thickness | - |
| 13 (d) | IFAB and Fuel Clad Thickness | - |
| 14 (e) | Discrete Burnable Absorber Area | - |

Table 17Justification of Variables' Directions of Effect on K_{eff}

| Index | Variable | Justification | Direction of Effect on K_{eff} |
|--------|---------------------------------|---|----------------------------------|
| 1 | No of Fuel Assemblies | <ul style="list-style-type: none"> ▪ Larger core causes lower leakage ratio to other reaction rates | + |
| 2 | Active Fuel Height | <ul style="list-style-type: none"> ▪ Larger core causes lower leakage ratio to other reaction rates | + |
| 3 | Core Radial Surrounding Water | <ul style="list-style-type: none"> ▪ Higher reflection by surrounding water ▪ Higher capture in surrounding water | +-(+)* |
| 4 | Enrichments (%) | <ul style="list-style-type: none"> ▪ Higher ratio of fission to absorption in fuel | + |
| 5 | Power | <ul style="list-style-type: none"> ▪ No effect | = |
| 6 | Moderator Density | <ul style="list-style-type: none"> ▪ Higher moderation ▪ Higher water capture | +-(+)* |
| 7 | Fuel Temperature Lib | <ul style="list-style-type: none"> ▪ Higher absorption in the epithermal energy region | - |
| 8 | No of Rods per Assembly | <ul style="list-style-type: none"> ▪ Lower ratio of absorber to fuel | + |
| 9 | Rod Pitch | <ul style="list-style-type: none"> ▪ Higher moderation ▪ Higher water capture | +-(+)* |
| 10 (a) | No of DBARs per Assembly | <ul style="list-style-type: none"> ▪ Higher ratio of absorber to fuel | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | <ul style="list-style-type: none"> ▪ More fissile material ▪ Lower water capture ▪ Lower water moderation | ++(-) |
| 12 (c) | IFAB and Fuel Gap Thickness | <ul style="list-style-type: none"> ▪ Lower water moderation ▪ Lower water capture | -+(-)* |
| 13 (d) | IFAB and Fuel Clad Thickness | <ul style="list-style-type: none"> ▪ Lower water capture but higher, clad, stronger, capture ▪ Lower water moderation | -- |
| 14 (e) | Discrete Burnable Absorber Area | <ul style="list-style-type: none"> ▪ Higher ratio of absorber to fuel | - |

*These variables can be non-monotonic, but assuming the design will fall mostly in the under moderation region, they are assumed to follow the moderation effect

Table 18Integrated Odd and Even Parameters in the Main Tree of K_{eff}

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 18.2500 | 23.9800 | 21.1150 | -2.8650 |
| 2 | 22.9500 | 17.0750 | 20.0125 | 2.9375 |
| 3 | 0.0000 | -0.3500 | -0.1750 | 0.1750 |
| 4 | 43.7500 | 28.2050 | 35.9775 | 7.7725 |
| 5 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 7.9500 | 16.0900 | 12.0200 | -4.0700 |
| 7 | 4.0500 | 3.4750 | 3.7625 | 0.2875 |
| 8 | 19.3500 | 23.1200 | 21.2350 | -1.8850 |
| 9 | 36.4500 | 75.0900 | 55.7700 | -19.3200 |
| a | 26.9500 | 24.2150 | 25.5825 | 1.3675 |
| b | 12.2000 | 25.4300 | 18.8150 | -6.6150 |
| c | 0.5500 | 3.2600 | 1.9050 | -1.3550 |
| d | 2.4000 | 8.1150 | 5.2575 | -2.8575 |
| e | 7.1500 | 26.2000 | 16.6750 | -9.5250 |

VI.2.1.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of the integrated even parameters and presented in Figure 43. It is clear from the magnitudes of the integrated even parameters that most of the variables are non-interacting. If the threshold of integrated even parameters is set at 3, it is possible to categorize variables 469be as important and interacting, variables 128a as important and non-interacting, and variables 37cd as non-important and non-interacting. Due to the sparsity of interactions with respect to this performance characteristic, the MSIV method

can be applied. However, for comparison purposes, both of the MHIV and MSIV methods are applied to this performance characteristic.

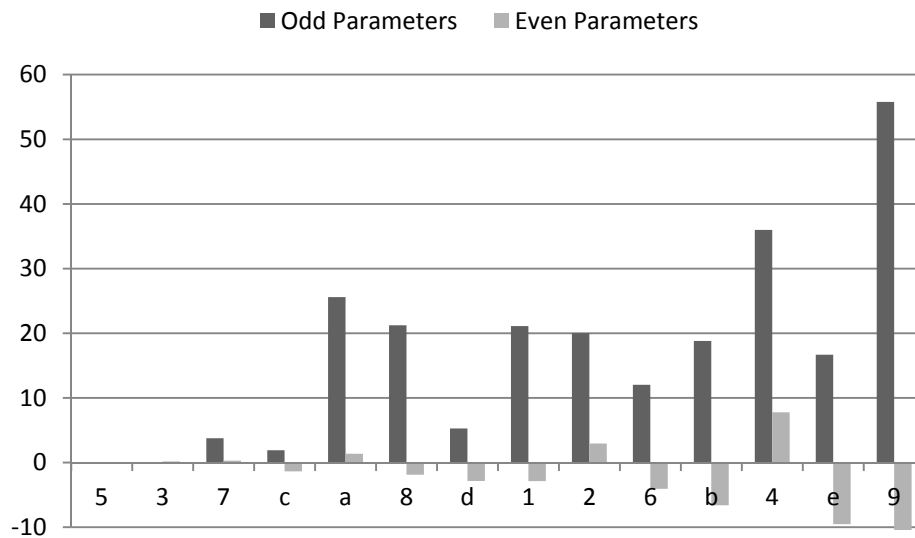


Figure 43. Integrated Odd and Even Parameters in the Main Tree of K_{eff}

VI.2.1.5 Monotonic Highly Interacting Variables Method Application

VI.2.1.5.1 Variables Interactions Exploration

Variables 469be are selected as the most interacting variables. Since the interactions of the variables are not accumulative, the exploration process of this method on this performance characteristic will not be very comprehensive, but it is applied for comparison with the MSIV. The three most interacting variables are 49e. Eight subtrees are established representing all binary states of 49e (49e=xxx). Within each of these trees, the integrated odd and even parameters of 6b are found. In total, four experiments

per subtree are performed. However, the top and bottom subtrees already have two relevant experiments performed in each, thus the total number of experiments performed at this stage is 28 (4x7). The resulting odd and even parameters of variables 6b are listed in Table 19, and shown in Figure 44. Checking the integrated even parameters against the set threshold of 3, the variables highlighted in grey in Table 19 of subtrees 49e=xxx are not further explored and their integrated odd parameters are used as the first order parameters in 49e spanned subtrees.

Variable b is interacting in all 49e subtrees while variable 6 is only interacting in 49e=010,011,110,111, thus subtrees b=x are created for 49e= 000,001,100,101 and 6b=xx for the rest. In total, 71 experiments were performed. The results of all performed experiments are shown in Appendix G.

Table 19
Integrated Odd and Even Parameters of 6b in Subtrees 49e=xxx of K_{eff}

| 49e= | 000 | 001 | 010 | 011 |
|--------|---------|---------|---------|---------|
| 6 Odd | 14.8700 | 14.7825 | 10.5225 | 10.6000 |
| 6 Even | -1.2200 | -2.0825 | -4.0725 | -5.0500 |
| b Odd | 35.6150 | 35.5050 | -1.0725 | 0.1500 |
| b Even | 10.1850 | 8.5450 | 4.3725 | 3.9000 |
| 49e= | 100 | 101 | 110 | 111 |
| 6 Odd | 16.2175 | 16.4225 | 12.8250 | 12.4250 |
| 6 Even | -1.3675 | -1.9225 | -3.9250 | -4.4750 |
| b Odd | 41.0350 | 42.1200 | 6.2250 | 7.2500 |
| b Even | 9.5150 | 9.1300 | 5.6250 | 4.9500 |

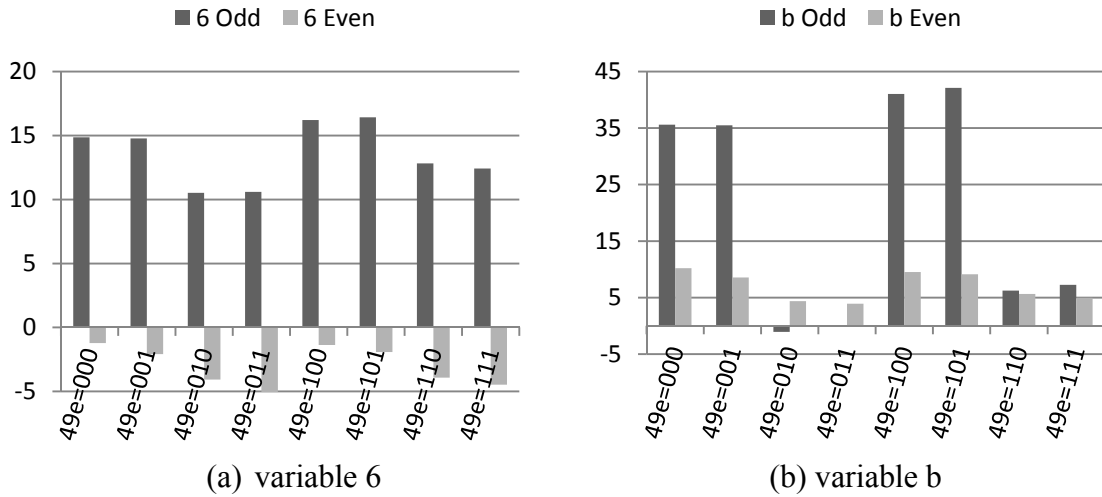


Figure 44. Integrated Odd and Even Parameters of 6b in Subtrees 49e=xxx of K_{eff}

VI.2.1.5.2 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by each end of the subtree using equation 45. The values of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46, and are shown in Table 20. The parameters of all subtrees are shown in Table 21. The performance characteristic is projected in all subtrees, then the global model parameters are found. The highest thirty parameters are shown in Table 22 and Figure 45.

Table 20Offset Errors in All Subtrees of K_{eff}

| | | | | | | | |
|-------------------------|----------|--------|----------|--------|-------|--------|---------|
| Subtree 49e6b States | 000x0 | 000x1 | 001x0 | 001x1 | 01000 | 01001 | 01010 |
| $\beta_{0,1,-1,Error}$ | 5.71 | 26.08 | -10.1525 | 6.9375 | 5.99 | 19.985 | 3.095 |
| Subtree 49e6b States | 01011 | 01100 | 01101 | 01110 | 01111 | 100x0 | 100x1 |
| $\beta_{0,1,-1,Error}$ | 15.095 | -9.27 | 4.93 | -12.97 | -0.62 | 6.8575 | 25.8875 |
| Subtree 49e6b States | 101x0 | 101x1 | 11000 | 11001 | 11010 | 11011 | 11100 |
| $\beta_{0,1,-1,Error}$ | -11.7275 | 6.5325 | 5.83 | 20.98 | 1.88 | 15.58 | -9.87 |
| Subtree 49e6b States | 11101 | 11110 | 11111 | | | | |
| $\beta_{0,1,-1,Error}$ | 5.13 | -13.72 | -0.67 | | | | |

Table 21Parameters in All Subtrees of K_{eff}

| Parameter Index | 49e=000, b=x | 49e=001, b=x | 49e=010,011, 110,111, 6b=xx | 49e=100, b=x | 49e=101, b=x |
|--------------------|-----------------|-----------------|-----------------------------------|-----------------|-----------------|
| 1 | 21.1150 | 21.1150 | 21.1150 | 21.1150 | 21.1150 |
| 2 | 20.0125 | 20.0125 | 20.0125 | 20.0125 | 20.0125 |
| 3 | -0.1750 | -0.1750 | -0.1750 | -0.1750 | -0.1750 |
| 4 | | | | | |
| 5 | 00.000 | 00.000 | 00.000 | 00.000 | 00.000 |
| 6 | 14.8700 | 14.7825 | | 16.2175 | 16.4225 |
| 7 | 3.7625 | 3.7625 | 3.7625 | 3.7625 | 3.7625 |
| 8 | 21.2350 | 21.2350 | 21.2350 | 21.2350 | 21.2350 |
| 9 | | | | | |
| a | 25.5825 | 25.5825 | 25.5825 | 25.5825 | 25.5825 |
| b | | | | | |
| c | 1.9050 | 1.9050 | 1.9050 | 1.9050 | 1.9050 |
| d | 5.2575 | 5.2575 | 5.2575 | 5.2575 | 5.2575 |
| e | | | | | |

Table 22
 Highest Thirty Parameters of K_{eff} Using the MHIV Method

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | -100.9700 | 'd' | 3.2848 |
| 'b' | 96.1290 | '1b' | 3.2604 |
| '4' | 38.8860 | '69' | 3.1484 |
| '9b' | -32.1080 | '6b' | -3.1191 |
| '6' | -17.6650 | '9ab' | 2.9177 |
| 'a' | -14.4400 | 'c' | 2.8972 |
| '2' | 10.5630 | '19b' | -2.2948 |
| '49' | -8.0825 | '89b' | -2.1764 |
| '8' | 8.0290 | '24' | 1.9152 |
| '4b' | 7.9489 | '9d' | -1.7549 |
| 'ab' | -6.3008 | '7' | -1.4745 |
| '1' | 5.6937 | '26' | -1.4058 |
| '29' | -5.6475 | '49b' | -1.404 |
| '2b' | 5.5902 | '3' | 1.2672 |
| '9a' | 5.5679 | '46' | -1.2427 |
| 'e' | 4.8722 | '249' | -1.1793 |
| '19' | -4.6412 | '24b' | 1.086 |
| '8b' | 4.1482 | 'bd' | 1.0115 |
| '89' | -3.8978 | '269' | 0.80853 |
| '29b' | -3.3063 | '26b' | -0.79712 |

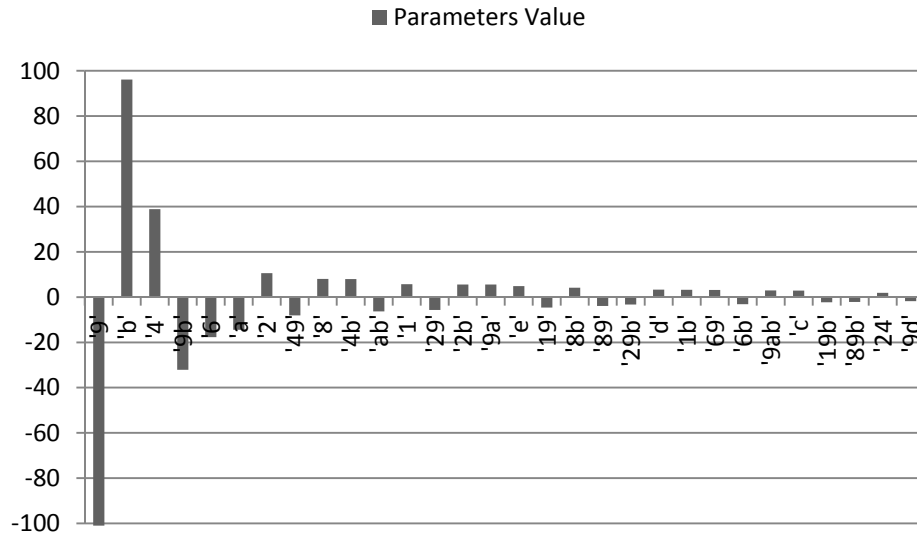


Figure 45. Highest Thirty Parameters of K_{eff} Using the MHIV Method

VI.2.1.6 Monotonic Sparsely Interacting Variables Method Application

VI.2.1.6.1 Variables Interactions Exploration

Since the five variables of 469be are selected as the interacting variables, a subtree of non-interacting variables is established at every combination, or node, of the 32 nodes interacting variables' subtree. The interacting variables' subtree is evaluated at this stage. The initial test is to determine the ten second order interactions by performing ten experiments at the second level of the top end of the subtree and another ten at the second level of the bottom end. The resulting integrated odd and even parameters are shown in Table 23. Table 24 compares the variables' integrated even parameters, found in a previous step, with the summation of integrated odd parameters of the second order interactions, found in this step. If all interactions are explained in the interacting variables' subtree, the two values should be identical. This indicates that the variables

are only interacting in the interacting variables' subtree. Unlike variables 469, Table 24 shows that variables b and e are not fully interacting in the interacting variables' subtree. It is, thus, decided to explore b and e interactions with the non-interacting variables. It is also found from Table 23 that none of the second order integrated even parameters is above the threshold of 3, thus no higher order interactions are explored.

Table 23

Integrated Odd and Even Parameters of the Second Order Interactions of 469be of K_{eff}

| Interaction | Integrated Odd Parameters | Integrated Even Parameters |
|-------------|---------------------------|----------------------------|
| '46' | 1.0288 | -0.1713 |
| '49' | 3.6763 | -0.2988 |
| '4b' | 3.5363 | -0.5388 |
| '4e' | -0.0225 | 0.0525 |
| '69' | -2.0113 | 1.2638 |
| '6b' | -1.2500 | 1.3000 |
| '6e' | -0.1038 | 0.3713 |
| '9b' | -17.4738 | 2.0513 |
| '9e' | -1.6350 | -0.7850 |
| 'be' | 0.5525 | 0.3775 |

Table 24

Evaluation of Second Order Interactions of 469be of K_{eff}

| | 1 st Order Integrated Even Parameters | 2nd Order Summation of Integrated Odd Parameters | Difference |
|---|--|--|------------|
| 4 | 7.7725 | 8.2188 | 0.4463 |
| 6 | -4.0700 | -2.3363 | 1.7338 |
| 9 | -19.3200 | -17.4438 | 1.8763 |
| b | -6.6150 | -14.6000 | -8.0200 |
| e | -9.5250 | -1.2088 | 8.3163 |

VI.2.1.6.2 One Node per Subtree of Interacting Variables

In order to project a FFD of the non-interacting variables on all interacting variables' nodes, one node per subtree needs to be known to find the offset β_0 . However, the twelve experiments out of the 28 representing the “one down and one up” process of Table 18, in addition to the twenty second order interactions' experiments of Table 23, are the 32 needed nodes. Thus, no additional experiments are needed. The performed experiments are shown in Appendix G.

VI.2.1.6.3 Projection of the FFD model in Subtrees and Results Generation

In this step, a Res III FFD set of experiments are performed on the non-interacting variables at the top end of the interacting variables' subtree (469be=11111) in addition to subtrees b=0 and e=0 (469be =11101, and 469be= 11110). The results of the performed experiments are shown in Appendix G. The first order parameters, of the nine non-interacting variables, in the three subtrees are shown in Table 25. The interactions of a non-interacting variable i with variables b and e are found by:

$$\beta_{469be=11111,i} - \beta_{469be=11101,i} = \beta_{bi} - (-\beta_{bi}) = 2\beta_{bi} \Rightarrow \beta_{bi} = (\beta_{469be=11111,i} - \beta_{469be=11101,i})/2 \quad (54)$$

$$\beta_{469be=11111,i} - \beta_{469be=11110,i} = \beta_{ei} - (-\beta_{ei}) = 2\beta_{ei} \Rightarrow \beta_{ei} = (\beta_{469be=11111,i} - \beta_{469be=11110,i})/2 \quad (55)$$

The interactions of the non-interacting variables with b and e are shown in Table 25. The summations of b and e set of interactions are -7.475 and 6.7625. These values are close to the unexplained difference of -8.0200 and 8.31625 of Table 24. It is thus concluded, that with these three FFD sets, most of the important interactions have been

unfolded. The remaining confounded parameters or unexplained interactions are summarized in Table 26.

Since a FFD for each of the three non-interacting variables' subtrees (469be=xxx11, 469be=xxx01 and 469be=xxx10) was performed, the FFD parameters used for a non-interacting variable i are $\beta_i = \beta_{i,FFD,be=11}$ for subtrees 469be=xxx11, $\beta_i = \beta_{i,FFD,be=10}$ for subtrees 469be =xxx10, $\beta_i = \beta_{i,FFD,be=01}$ for subtrees 469be =xxx01 and $\beta_i = \beta_{i,FFD,be=01} + (\beta_{i,FFD,be=10} - \beta_{i,FFD,be=11})$ for subtrees 469be=xxx00. The value of $(\beta_{i,FFD,be=11} - \beta_{i,FFD,be=11})$ is the integrated parameters of e with the non-interacting variables. The integrated parameters of 'be' was found below the threshold, so it was omitted from the cross interactions' evaluation.

Table 25
Second Order Interactions of Non-Interacting Variables with Variables b and e of K_{eff}

| Index | e down parameters | b down parameters | Top Node parameters | Interactions of b and Non-Interacting Variables | Interactions of e and Non-Interacting Variables |
|-------|-------------------|-------------------|---------------------|---|---|
| 1 | 20.1500 | 20.4438 | 20.7188 | 0.1375 | 0.2844 |
| 2 | 21.3750 | 21.2563 | 21.4688 | 0.1063 | 0.0469 |
| 3 | 0.1625 | -0.1563 | -0.0312 | 0.0625 | -0.0969 |
| 5 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 4.1125 | 3.1313 | 3.1063 | -0.0125 | -0.5031 |
| 8 | 18.3500 | 26.7813 | 23.2938 | -1.7438 | 2.4719 |
| a | 16.4750 | 39.7313 | 27.9813 | -5.8750 | 5.7531 |
| c | 1.2875 | 0.8938 | 0.4813 | -0.2063 | -0.4031 |
| d | 3.7625 | 2.0688 | 2.1813 | 0.0563 | -0.7906 |

Table 26Confounded Parameters in the First Order Parameters of K_{eff}

| Index | Confounded Parameters or Unexplained Interactions |
|-------|---|
| 1 | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| 2 | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| 3 | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| 4 | 0.44625 of unexplained interactions |
| 5 | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| 6 | 1.73375 of unexplained interactions |
| 7 | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| 8 | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| 9 | 1.87625 of unexplained interactions |
| a | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| b | -0.545 of unexplained interactions |
| c | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| d | 2nd order with non-interacting and interacting variables , 3rd order with b and e |
| e | 1.55375 of unexplained interactions |

The offset β_0 is found using the performed node in every non-interacting variables' subtree from the previous step. In total, 98 experiments are performed to apply this method. The FFD parameters are used to project the performance characteristic in all non-interacting variables subtrees, then the global model parameters are found. The highest thirty parameters are shown in Table 27 and Figure 46.

Table 27
Highest Thirty Parameters of K_{eff} Using the MSIV Method

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | 0.05802 | '69' | -0.00177 |
| '4' | 0.035431 | '9e' | 0.001535 |
| 'a' | -0.0256 | '69b' | 0.001064 |
| '8' | 0.021254 | '6b' | 0.001064 |
| '1' | 0.021134 | '46' | 0.000873 |
| 'b' | -0.02109 | 'be' | 0.000407 |
| '2' | 0.020032 | '49b' | -0.00028 |
| '9b' | 0.017495 | '9be' | 0.000178 |
| 'e' | -0.0165 | '49be' | -0.00016 |
| '6' | 0.013816 | '46b' | -0.00015 |
| 'd' | -0.00528 | '469b' | -0.00015 |
| '7' | -0.00378 | '4be' | 0.000147 |
| '49' | 0.003407 | '69be' | -0.00014 |
| '4b' | -0.00329 | '6be' | -0.00013 |
| 'c' | -0.00192 | '469' | 0.000132 |

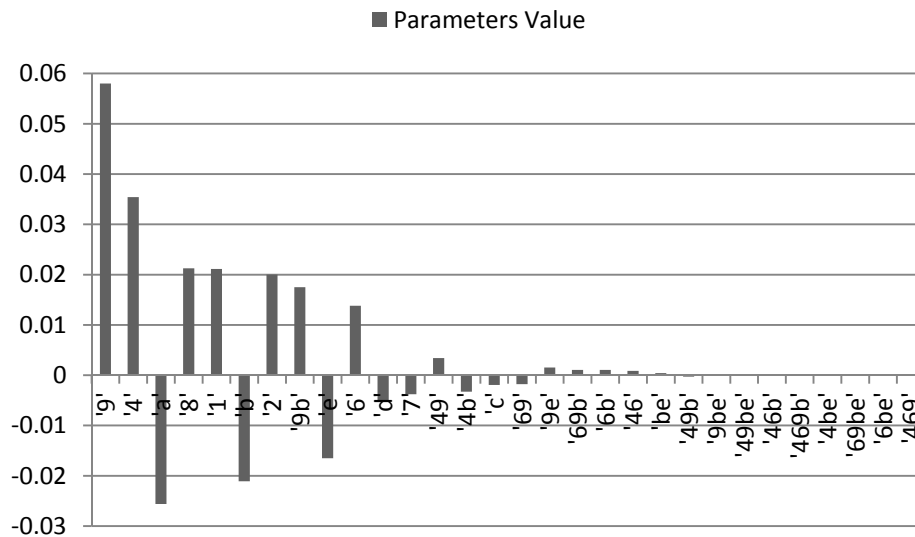


Figure 46. Highest Thirty Parameters of K_{eff} Using the MSIV Method

VI.2.1.7 Validation

In this section, it is desired to evaluate and compare the MHIV and MSIV methods with a 128 Res IV FFD set of experiments. The validation is performed by comparing each method's projected results with the performed experiments of the other method. The errors of the projected K_{eff} using MHIV and MSIV are shown in Figure 47 and Figure 48. The RMSEs for all three methods are found using:

$$RMSE_{MHIV} = \sqrt{\frac{\sum_{i=1}^{128} (K_{eff,MHIV,i} - K_{eff,FFD,Performed,i})^2}{128}} \quad (56)$$

$$RMSE_{MSIV} = \sqrt{\frac{\sum_{i=1}^{128} (K_{eff,MSIV,i} - K_{eff,FFD,Performed,i})^2}{128}} \quad (57)$$

$$RMSE_{FF} = \sqrt{\frac{\sum_{i=1}^{98} (K_{eff,FFD,i} - K_{eff,MSIV,Performed,i})^2}{98}} \quad (58)$$

The numbers of experiments performed using the MHIV and MSIV methods are 71 and 98. The RMSEs of K_{eff} , using both methods, are 11.924 and 10.325. These are 2.4% and 2.1% of the projected response range, referred to as NRMSE, and 3.97 and 3.44 times the threshold. The 128 FFD projection RMSE was found as 10.8349. Thus, the dissertation's methods seem to be slightly more accurate with a smaller number of experiments in the case of the MSIV method and slightly less accurate with a much smaller number of experiments in the case of the MHIV method.

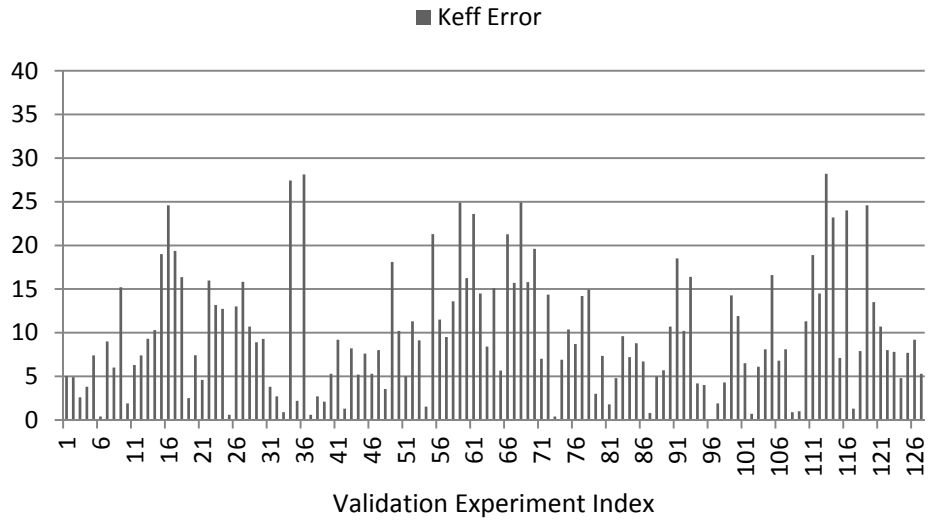


Figure 47. K_{eff} 's Projection Error of the MHIV Method with Respect to the 128 Performed Experiments of the FFD

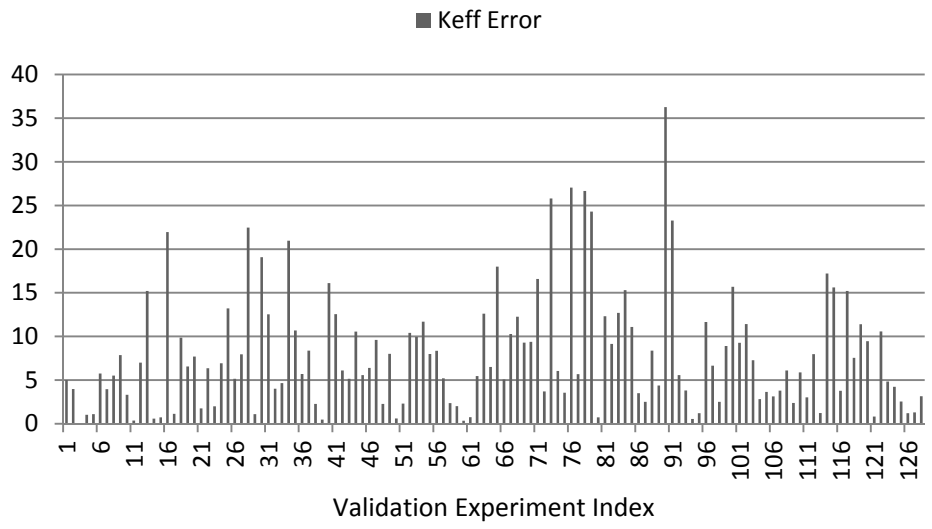


Figure 48. K_{eff} 's Projection Error of the MSIV Method with Respect to the 128 Performed Experiments of the FFD

In order to further evaluate the methods, a 256 Res V FFD set of experiments was performed. The RMSEs of the projected K_{eff} by the two methods from the 256 FFD performed experiments were found as 12.711 and 9.747. The RMSE of the 128 FFD projection from the 256 FFD performed experiments was found as 12.354. This demonstrates that, in comparison to the 128 FFD, the dissertation's MSIV method required 75% the number of experiment, and resulted in around 25% error reduction. The MHIV method achieved a similar error to the 128 FFD with 55% the number of experiments. This example does not demonstrate the full strength of the introduced methods since variables were found to not significantly interact in their effect on K_{eff} . Thus, the FFDs' assumption of weak interactions was relatively valid in the first place.

Since ANNs can also be used to develop models of systems behavior, an ANN was also developed in Appendix F for modeling this performance characteristic using the 128 FFD set of experiments. It was found to be much less accurate than the 128 FFD.

VI.2.2 Flux Spectrum Analysis

In this section, the energy spectrum of the flux in the fuel and cladding is analyzed. The target for the spectrum is to be as thermal as possible to reduce the effect of neutrons fluence on the fuel and cladding materials' structure.

VI.2.2.1 Performance Characteristic Directional Alignment

Since the operational energy spectrum of water reactors is in the thermal region, it is expected that a saturation behavior will occur as the spectrum is increasingly

thermalized. Since the saturation plateau is be aligned with the bottom end of the performance characteristic, the performance characteristic is defined as reversely proportional to the thermal end of the spectrum. Thus:

$$RFS = \frac{\int_{E_{th}}^{E_{max}} \sum_{m=1}^4 \varphi(E,m)dE}{\int_0^{E_{th}} \sum_{m=1}^4 \varphi(E,m)dE} \quad (59)$$

RFS is the Ratio of the Fast flux Spectrum over the thermal flux spectrum. The index m is used to sum the spectrum of three fuel materials with different enrichments, and one cladding material. E_{th} is the threshold energy used to separate the thermal and fast spectrums. This was chosen as 0.05eV, since this value was found to maximize the sensitivity of the ratio on design changes. High E_{th} values produce very low ratios with magnitudes that are of the same order as the result's error. E_{max} is defined as 20 MeV.

VI.2.2.2 Variables Directional Alignment

The directional dependence of the RFS on the variables is determined in this step. Variables are categorized into monotonic variables, and variables with unknown direction of effect. Power is a passive variable since it does not affect the spectrum. The variables with the highest monotonic behavior confidence are determined, and are shown in Table 28. The directional dependence is briefly justified in Table 29. According to the table, the maximum RFS occurs at the variables' states of 1111x001011111, thus the variables are aligned to all high at this combination.

VI.2.2.3 Variables Integrated Parameters Determination

Since thirteen variables are explored, 28 experiments are performed at this stage. The results of the performed experiments are shown in Appendix G. The corresponding integrated odd and even parameters of all variables are shown in Table 30.

Table 28
Variables' Directions of Effect on RFS

| Variable Index | Variable | Direction of Effect on RFS |
|----------------|---------------------------------|----------------------------|
| 1 | No of Fuel Assemblies | + |
| 2 | Active Fuel Height | + |
| 3 | Core Radial Surrounding Water | + |
| 4 | Enrichments | + |
| 5 | Power | = |
| 6 | Moderator Density | - |
| 7 | Fuel Temperature Lib | - |
| 8 | No of Rods per Assembly | + |
| 9 | Rod Pitch | - |
| 10 (a) | No of DBARs per Assembly | + |
| 11 (b) | IFAB and Fuel Pellet Diameter | + |
| 12 (c) | IFAB and Fuel Gap Thickness | + |
| 13 (d) | IFAB and Fuel Clad Thickness | + |
| 14 (e) | Discrete Burnable Absorber Area | + |

Table 29

Justification of Variables' Directions of Effect on RFS

| Index | Variable | Justification | Direction of Effect on RFS |
|--------|---------------------------------|--|----------------------------|
| 1 | No of Fuel Assemblies | <ul style="list-style-type: none"> ▪ Larger core causes higher fuel ratio to surrounding radial water | + |
| 2 | Active Fuel Height | <ul style="list-style-type: none"> ▪ Larger core causes higher fuel ratio to surrounding axial water | + |
| 3 | Core Radial Surrounding Water | <ul style="list-style-type: none"> ▪ Lower fuel ratio to surrounding radial water ▪ Higher fission at the high enrichment edge of the core | -+(+) |
| 4 | Enrichments | <ul style="list-style-type: none"> ▪ Higher production of fast neutrons and absorption of thermal neutrons | + |
| 5 | Power | <ul style="list-style-type: none"> ▪ No effect | = |
| 6 | Moderator Density | <ul style="list-style-type: none"> ▪ Higher moderation ▪ Higher water capture | -+(-)* |
| 7 | Fuel Temperature Lib | <ul style="list-style-type: none"> ▪ Higher absorption in the epithermal energy region | - |
| 8 | No of Rods per Assembly | <ul style="list-style-type: none"> ▪ Lower absorber ratio to fuel | + |
| 9 | Rod Pitch | <ul style="list-style-type: none"> ▪ Higher moderation ▪ Higher water capture | -+(-)* |
| 10 (a) | No of DBARs per Assembly | <ul style="list-style-type: none"> ▪ Higher absorption of neutrons especially thermal neutrons | + |
| 11 (b) | IFAB and Fuel Pellet Diameter | <ul style="list-style-type: none"> ▪ Lower moderation ▪ Lower water capture | +-(+)* |
| 12 (c) | IFAB and Fuel Gap Thickness | <ul style="list-style-type: none"> ▪ Lower moderation ▪ Lower water capture | +-(+)* |
| 13 (d) | IFAB and Fuel Clad Thickness | <ul style="list-style-type: none"> ▪ Lower moderation ▪ Lower water capture | +-(+)* |
| 14 (e) | Discrete Burnable Absorber Area | <ul style="list-style-type: none"> ▪ Higher absorption of neutrons especially thermal neutrons | + |

*These variables can be non-monotonic, but assuming the design will fall mostly in the under moderation region, they are assumed to follow the moderation effect

Table 30
Integrated Odd and Even Parameters in the Main Tree of RFS

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 0.028779 | -7.76E-05 | 0.014351 | 0.014428 |
| 2 | 0.048397 | 0.000571 | 0.024484 | 0.023913 |
| 3 | 0.002996 | -0.00045 | 0.001272 | 0.001724 |
| 4 | 0.070861 | 0.021237 | 0.046049 | 0.024812 |
| 5 | 0 | 0 | 0 | 0 |
| 6 | 0.033665 | 0.008697 | 0.021181 | 0.012484 |
| 7 | 0.003323 | -0.00034 | 0.001493 | 0.001829 |
| 8 | 0.017099 | 0.003249 | 0.010174 | 0.006925 |
| 9 | 0.192387 | 0.0511 | 0.121744 | 0.070643 |
| a | 0.030326 | 0.005809 | 0.018068 | 0.012258 |
| b | 0.185029 | 0.046741 | 0.115885 | 0.069144 |
| c | 0.005596 | 0.000207 | 0.002902 | 0.002695 |
| d | 0.011166 | 0.000506 | 0.005836 | 0.00533 |
| e | 0.006568 | 0.00321 | 0.004889 | 0.001679 |

VI.2.2.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of the integrated even parameters and presented in Figure 49. It is concluded from the magnitudes of the integrated even parameters that most of the variables are interacting. It is also clear that there is a significant step in the magnitudes of the integrated even parameters of variables 9 and b. Unlike the effective multiplication factor, it is not possible to clearly isolate the interacting and non-interacting variables of this performance characteristic. If a threshold of integrated even parameters is set at 0.0027 (slightly higher than the integrated even parameters of variable c), it is possible to categorize variables 9b as most interacting, variables 12468ad as interacting and

variables 357ce as non-interacting. The non-interacting variables are not further explored.

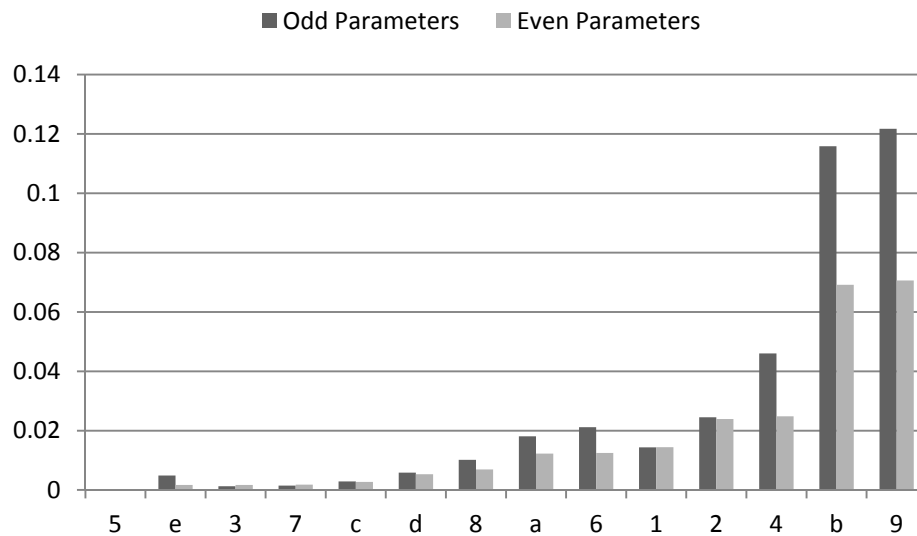


Figure 49. Integrated Odd and Even Parameters in the Main Tree of RFS

VI.2.2.5 Variables Interactions Exploration

Since two variables are selected as the most interacting variables, four subtrees are to be established representing all binary states of $9b=xx$. In each of these subtrees, the integrated odd and even parameters of each of 12468ad are found. In total fourteen experiments per subtree are performed. However, the top and bottom subtrees already have seven relevant experiments performed in each. Thus, the total number of experiments performed at this stage is 42. The resulting integrated odd and even parameters of variables 12468ad are listed in Table 31 and Table 32, and shown in

Figure 50. Checking the integrated even parameters against the set threshold of 0.0027, the variables highlighted in grey in Table 31 and Table 32 of subtrees 9b=xx are not further explored, and their integrated odd parameters are used as the first order parameters in 9b spanned subtrees

Table 31
Integrated Odd Parameters of 12468ad in Subtrees 9b=xx of RFS

| Variable | 9b=00 | 9b=01 | 9b=10 | 9b=11 |
|----------|----------|----------|----------|----------|
| 1 | 0.000113 | 0.002032 | 0.004792 | 0.019913 |
| 2 | 0.002658 | 0.007645 | 0.007903 | 0.030085 |
| 4 | 0.024478 | 0.037738 | 0.03833 | 0.058645 |
| 6 | 0.01141 | 0.018838 | 0.01817 | 0.025813 |
| 8 | 0.002185 | 0.006117 | 0.005615 | 0.01822 |
| a | 0.005515 | 0.01227 | 0.010803 | 0.029195 |
| d | 0.0007 | 0.0024 | 0.003885 | 0.00789 |

Table 32
Integrated Even Parameters of 12468ad in Subtrees 9b=xx of RFS

| Variable | 9b=00 | 9b=01 | 9b=10 | 9b=11 |
|----------|----------|----------|----------|----------|
| 1 | 0.000188 | 0.001362 | 0.001663 | 0.008837 |
| 2 | 0.002087 | 0.00472 | 0.006193 | 0.018315 |
| 4 | 0.003237 | 0.007232 | 0.007675 | 0.012205 |
| 6 | 0.00271 | 0.004618 | 0.00503 | 0.007838 |
| 8 | -0.00107 | -0.0012 | -0.00114 | -0.00112 |
| a | -0.0003 | -0.00058 | 0.000623 | 0.001105 |
| d | 0.000195 | 0.00023 | 0.002005 | 0.00326 |

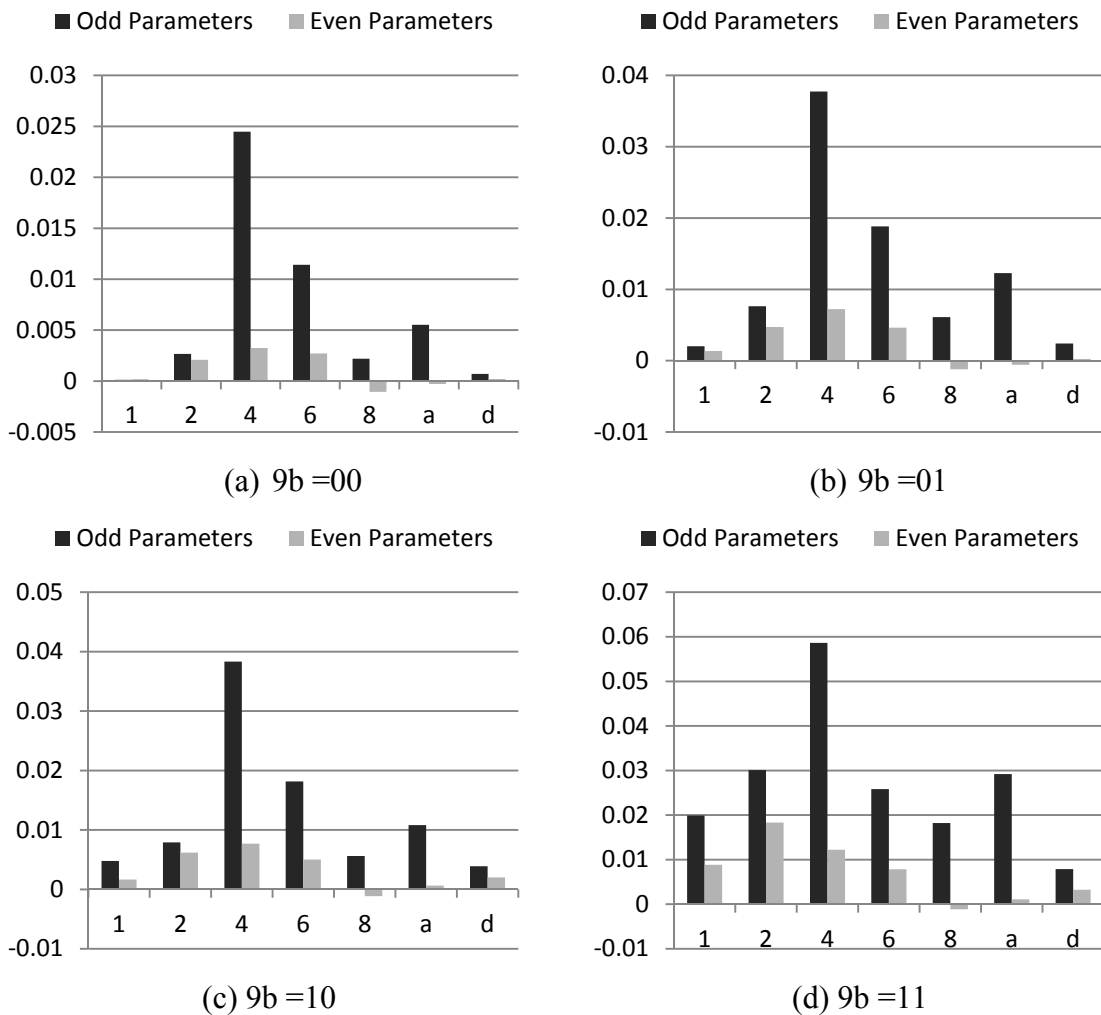


Figure 50. Integrated Odd and Even Parameters of 12468ad in Subtrees $9b=xx$ of RFS

Within subtree $9b=00$, variables 4 and 6 are unfolded, thus establishing subtrees $46=xx$ in subtree $9b=00$. Within each of the 46 subtrees all variables are non-interacting. The same is followed for $9b=01$ and $9b=10$, but with the three variables 246 requiring eight experiments each. Subtrees $9b=11$ has five variables that are interacting. Instead of establishing 32 subtrees, it decided to establish subtrees of the most interacting three

variables 124 of 9b=11. In the subtrees 124=xxx of 9b=11, variables 6 and d integrated odd and even parameters are found. The results are shown in Table 33 and Figure 51. It is concluded from the figure, that variables 6 and d are not considered interacting in all subtrees of 124 except 124=110 and 124=111. In these two subtrees, variables 6 and d are unfolded establishing subtrees 6d=xx of 124=110 of 9b=11 and 6d=xx of 124=111 of 9b=11. The results of all performed experiments are shown in Appendix G.

Table 33
Integrated Odd and Even Parameters of 6d in Subtrees 124=xxx of 9b=11 of RFS

| | 124=000 | 124=001 | 124=010 | 124=011 | 124=100 | 124=101 |
|--------|----------|----------|----------|-----------|----------|----------|
| 6 Odd | 0.0185 | 0.019463 | 0.022963 | 0.024285 | 0.020638 | 0.023528 |
| 6 Even | 0.000525 | 0.000132 | 0.002128 | -3.50E-05 | -0.00031 | -0.00188 |
| d Odd | 0.004965 | 0.003245 | 0.006978 | 0.005882 | 0.005895 | 0.004497 |
| d Even | 0.000335 | -0.0008 | 0.001048 | 0.001218 | 0.000895 | -0.00095 |
| | 124=110 | 124=111 | | | | |
| 6 Odd | 0.030058 | 0.03094 | | | | |
| 6 Even | 0.003648 | 0.00271 | | | | |
| d Odd | 0.010705 | 0.008055 | | | | |
| d Even | 0.003345 | 0.003095 | | | | |

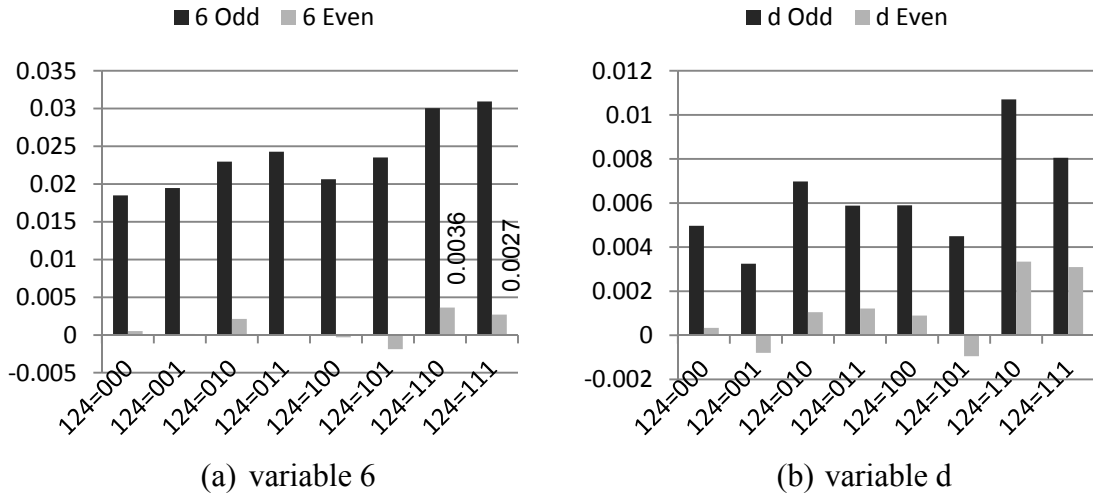


Figure 51. Integrated Odd and Even Parameters of 6d in Subtrees 124=xxx of 9b=11 of RFS

VI.2.2.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by each end of the subtree using equation 45. The values of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46, and are shown in Table 34. The resulting parameters in all subtrees are shown in Table 35. In total, 130 experiments were performed. The performance characteristic is projected in all subtrees, then the global model parameters are found. The highest thirty parameters are shown in Table 36 and Figure 52.

Table 34
Offset Errors in All Subtrees of RFS

| | | | | | | |
|------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| Subtree 9b1246d States | 00xx00x | 00xx01x | 00xx10x | 00xx11x | 01x000x | 01x001x |
| $\beta_{0,1,-1,Error}$ | -0.009238 | -0.006148 | -0.005093 | -0.001208 | -0.010313 | -0.007993 |
| Subtree 9b1246d States | 01x010x | 01x011x | 01x100x | 01x101x | 01x110x | 01x111x |
| $\beta_{0,1,-1,Error}$ | -0.002103 | 0.000628 | -0.006572 | -0.003617 | -0.000153 | 0.005387 |
| Subtree 9b1246d States | 10x000x | 10x001x | 10x010x | 10x011x | 10x100x | 10x101x |
| $\beta_{0,1,-1,Error}$ | -0.011628 | -0.007518 | -0.003997 | -0.000287 | -0.006103 | -0.002898 |
| Subtree 9b1246d States | 10x110x | 10x111x | 11000xx | 11001xx | 11010xx | 11011xx |
| $\beta_{0,1,-1,Error}$ | 0.002403 | 0.007743 | -0.011243 | -0.003390 | -0.000037 | 0.011865 |
| Subtree 9b1246d States | 11100xx | 11101xx | 1111000 | 1111001 | 1111010 | 1111011 |
| $\beta_{0,1,-1,Error}$ | -0.009385 | -0.002517 | -0.004438 | -0.000877 | -0.000273 | 0.007448 |
| Subtree 9b1246d States | 1111100 | 1111101 | 1111110 | 1111111 | | |
| $\beta_{0,Error}$ | 0.007822 | 0.015863 | 0.015093 | 0.021893 | | |

Table 35
Parameters in All Subtrees of RFS

| Index | 9b=00 46=xx | 9b=01 246=xxx | 9b=10 246=xxx | 9b=11 124=000 | 9b=11 124=001 | 9b=11 124=010 |
|-------|------------------|------------------|------------------|----------------------------|----------------------------|------------------|
| 1 | 0.000113 | 0.002033 | 0.004793 | | | |
| 2 | 0.002658 | | | | | |
| 3 | 0.001275 | 0.001275 | 0.001275 | 0.001275 | 0.001275 | 0.001275 |
| 4 | | | | | | |
| 5 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 6 | | | | 0.018500 | 0.019463 | 0.022963 |
| 7 | 0.001483 | 0.001483 | 0.001483 | 0.001483 | 0.001483 | 0.001483 |
| 8 | 0.002185 | 0.006118 | 0.005615 | 0.018220 | 0.018220 | 0.018220 |
| 9 | | | | | | |
| a | 0.005515 | 0.012270 | 0.010803 | 0.029195 | 0.029195 | 0.029195 |
| b | | | | | | |
| c | 0.002905 | 0.002905 | 0.002905 | 0.002905 | 0.002905 | 0.002905 |
| d | 0.000700 | 0.002400 | 0.003885 | 0.004965 | 0.003245 | 0.006978 |
| e | 0.004880 | 0.004880 | 0.004880 | 0.004880 | 0.004880 | 0.004880 |
| | 9b=11 124=011 | 9b=11 124=100 | 9b=11 124=101 | 9b=11 124=110, 6d=xx | 9b=11 124=111, 6d=xx | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | 0.001275 | 0.001275 | 0.001275 | 0.001275 | 0.001275 | |
| 4 | | | | | | |
| 5 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | |
| 6 | 0.024285 | 0.020638 | 0.023528 | | | |
| 7 | 0.001483 | 0.001483 | 0.001483 | 0.001483 | 0.001483 | |
| 8 | 0.018220 | 0.018220 | 0.018220 | 0.018220 | 0.018220 | |
| 9 | | | | | | |
| a | 0.029195 | 0.029195 | 0.029195 | 0.029195 | 0.029195 | |
| b | | | | | | |
| c | 0.002905 | 0.002905 | 0.002905 | 0.002905 | 0.002905 | |
| d | 0.005883 | 0.005895 | 0.004498 | | | |
| e | 0.004880 | 0.004880 | 0.004880 | 0.004880 | 0.004880 | |

Table 36
Highest Thirty Parameters of RFS

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | -0.100970 | 'd' | 0.003285 |
| 'b' | 0.096129 | '1b' | 0.003260 |
| '4' | 0.038886 | '69' | 0.003148 |
| '9b' | -0.032108 | '6b' | -0.003119 |
| '6' | -0.017665 | '9ab' | 0.002918 |
| 'a' | -0.014440 | 'c' | 0.002897 |
| '2' | 0.010563 | '19b' | -0.002295 |
| '49' | -0.008083 | '89b' | -0.002176 |
| '8' | 0.008029 | '24' | 0.001915 |
| '4b' | 0.007949 | '9d' | -0.001755 |
| 'ab' | -0.006301 | '7' | -0.001475 |
| '1' | 0.005694 | '26' | -0.001406 |
| '29' | -0.005648 | '49b' | -0.001404 |
| '2b' | 0.005590 | '3' | 0.001267 |
| '9a' | 0.005568 | '46' | -0.001243 |
| 'e' | 0.004872 | '249' | -0.001179 |
| '19' | -0.004641 | '24b' | 0.001086 |
| '8b' | 0.004148 | 'bd' | 0.001012 |
| '89' | -0.003898 | '269' | 0.000809 |
| '29b' | -0.003306 | '26b' | -0.000797 |

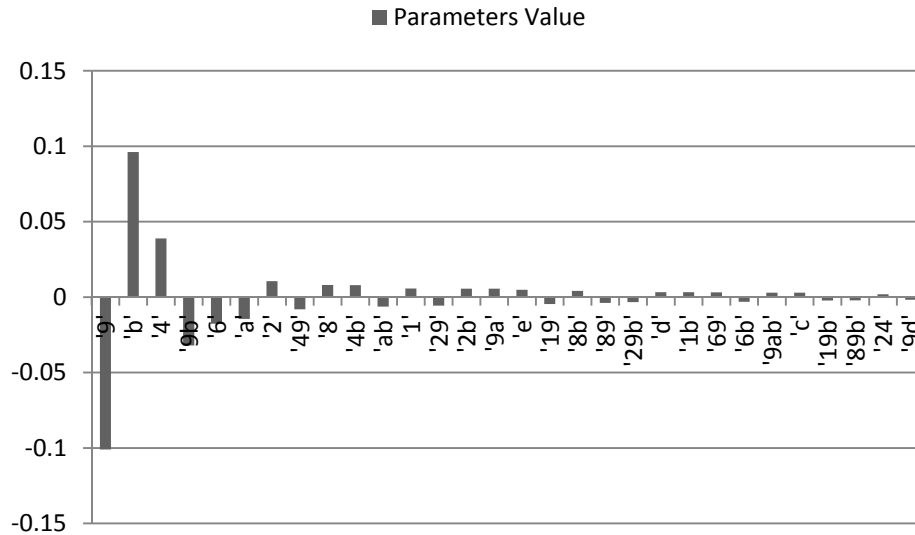


Figure 52. Highest Thirty Parameters of RFS

VI.2.2.7 Validation

In this section, it is desired to evaluate and compare the MHIV method results with a 128 Res IV FFD. The validation is performed by comparing each method projected results with the performed experiments of the other method. The errors of the projected RFS are shown in Figure 53. The RMSE of the MHIV projection was found as 0.008032, which is around 1.19% of RFS projected range and 2.97 times the threshold. The 128 FFD projection RMSE was found as 0.053884. Thus, the MHIV method seems to be significantly more accurate with a similar number of 130 experiments. The RC Method was tested on this performance characteristic too, and was found to produce a projection RMSE of 0.016898.

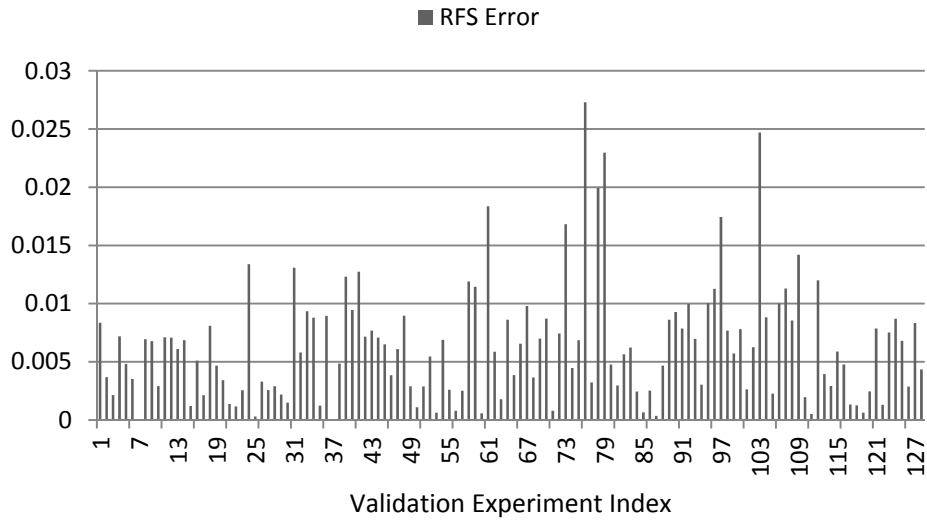


Figure 53. RFS’s Projection Error of the MSIV Method with Respect to the 128 Performed Experiments of the FFD

VI.2.3 Water Density Reactivity Coefficient Analysis

The density reactivity coefficient, α_d , is a measurement of the change of the reactivity due to the change of the density. The target for this coefficient is to be as high as possible in the positive direction. Thus, it is required that the reactivity decrease as the density drops. This coefficient was chosen as to combine the effect of water temperature and pressure in one performance characteristic. It is similar to the void coefficient of reactivity, but it will also include the effect of the moderator’s temperature coefficient of reactivity (see [85] for definitions).

VI.2.3.1 Performance Characteristic Directional Alignment

Since water represents a medium for moderation, it is expected that as the density is lost, the reactivity will reduce. As α_d approaches zero, the effect of water density change becomes very weak. It is, thus, expected that α_d is limited by a plateau at its low end. Since the saturation plateau is to be aligned with the bottom end of the performance characteristic, the performance characteristic is defined as:

$$\alpha_d = \rho_{ref} - \rho_{0.5Sat} = \frac{(k_{eff,ref} - 1)}{k_{eff,ref}} - \frac{(k_{eff,0.5Sat} - 1)}{k_{eff,0.5Sat}} \quad (60)$$

where ρ is the reactivity. The range of α_d that can be investigated is the full range of the density (0-100% void). However, since the uncertainty of this coefficient increase as the void increases [86], it was decided to investigate α_d from 0% void, representing normal operational state, to 50% void of the saturation state, assuming fixed pressure. Since density is a variable of the fourteen variables analyzed in the SMR design, the 50% quality of saturation density is different for each density state. The densities used are summarized in Table 37. The units used for α_d , from here onwards, is pcm, which is equivalent to 10^{-5} .

Table 37

Definition of Density's Binary States for 0% and 50% Void

| State\Void (%) | 0% void (Reference Case) | 50% void at Saturation |
|---|-----------------------------|-------------------------------|
| Density State Low (g/cm ³) | 0.716 (305C 15.5MPa) | 0.34815 (50% void at 15.5MPa) |
| Density State High (g/cm ³) | 0.813 (243C 7.2 MPa) | 0.38695 (50% void at 7.2MPa) |

VI.2.3.2 Variables Directional Alignment

The directional dependence of α_d on the variables is determined in this step. Variables are categorized into monotonic variables and variables with unknown direction of effect. Power is considered a passive variable. The energy spectrum was used as the criteria to determine the directions of effect of the variables. As the spectrum is thermalized, the effect of density changes on α_d increase. The variables with the highest monotonic behavior confidence are determined, and are shown in Table 38. The directional dependence is briefly justified in Table 39. According to the table, the maximum α_d occurs at the variables' states of 0000x010010111, thus the variables are aligned to all high at this combination.

Table 38Variables' Directions of Effect on α_d

| Variable Index | Variable | Direction of Effect on α_d |
|----------------|---------------------------------|-----------------------------------|
| 1 | No of Fuel Assemblies | - |
| 2 | Active Fuel Height | - |
| 3 | Core Radial Surrounding Water | - |
| 4 | Enrichments | - |
| 5 | Power | = |
| 6 | Moderator Density | - |
| 7 | Fuel Temperature Lib | + |
| 8 | No of Rods per Assembly | - |
| 9 | Rod Pitch | - |
| 10 (a) | No of DBARs per Assembly | + |
| 11 (b) | IFAB and Fuel Pellet Diameter | - * |
| 12 (c) | IFAB and Fuel Gap Thickness | + |
| 13 (d) | IFAB and Fuel Clad Thickness | + |
| 14 (e) | Discrete Burnable Absorber Area | + |

*Assumed negative, but found positive at the end results

Table 39Justification of Variables' Directions of Effect on α_d

| Index | Variable | Justification | Direction of Effect on α_d |
|--------|---------------------------------|---|-----------------------------------|
| 1 | No of Fuel Assemblies | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | - |
| 2 | Active Fuel Height | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | - |
| 3 | Core Radial Surrounding Water | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | - |
| 4 | Enrichments | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | - |
| 5 | Power | <ul style="list-style-type: none"> ▪ No effect | = |
| 6 | Moderator Density | <ul style="list-style-type: none"> ▪ More water increases the effect of its void ▪ Softer spectrum reduces the consequence of water void increase | ++(-) |
| 7 | Fuel Temperature Lib | <ul style="list-style-type: none"> ▪ Higher absorption in the epithermal energy region | + |
| 8 | No of Rods per Assembly | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | - |
| 9 | Rod Pitch | <ul style="list-style-type: none"> ▪ Softer spectrum reduces the consequence of water void increase | +(-) |
| 10 (a) | No of DBARs per Assembly | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | -(+) |
| 12 (c) | IFAB and Fuel Gap Thickness | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | -(+) |
| 13 (d) | IFAB and Fuel Clad Thickness | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | -(+) |
| 14 (e) | Discrete Burnable Absorber Area | <ul style="list-style-type: none"> ▪ Harder spectrum reduces the consequence of water void increase | -(+) |

VI.2.3.3 Variables Integrated Parameters Determination

Since thirteen variables are explored, 28 experiments are performed at this stage. The results of the performed experiments are shown in Appendix G. The corresponding integrated odd and even parameters of all variables are shown in Table 40.

Table 40
Integrated Odd and Even Parameters in the Main Tree of α_d

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 3293.2 | 1316.3 | 2304.8 | 988.4 |
| 2 | 2484.4 | 1595.2 | 2039.8 | 444.6 |
| 3 | 242.5 | 20.9 | 131.7 | 110.8 |
| 4 | 1123.4 | 166.3 | 644.8 | 478.6 |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6 | 1101.1 | 427.9 | 764.5 | 336.6 |
| 7 | 211.5 | 146.9 | 179.2 | 32.3 |
| 8 | 2265.9 | 491.1 | 1378.5 | 887.4 |
| 9 | 3964.6 | 2597.6 | 3281.1 | 683.5 |
| a | 1263.3 | -91.0 | 586.2 | 677.1 |
| b | 452.1 | -1813.0 | -681.0 | 1132.7 |
| c | 109.4 | 31.0 | 70.2 | 39.2 |
| d | 366.8 | 153.5 | 260.1 | 106.6 |
| e | 1148.2 | 141.7 | 645.0 | 503.3 |

VI.2.3.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of the integrated even parameters, and shown in Figure 54. It is concluded from the magnitudes of the integrated even parameters that variables are highly interacting. It is also concluded from the figure that it is not possible to clearly isolate the interacting

and non-interacting variables. If a threshold of integrated even parameters is set at 240 (three times the average error of the calculated α_d of 80), it is possible to categorize variables 18b as most interacting, variables 2469ae as interacting, and variables 357cd as non-interacting. The non-interacting variables are not further explored.

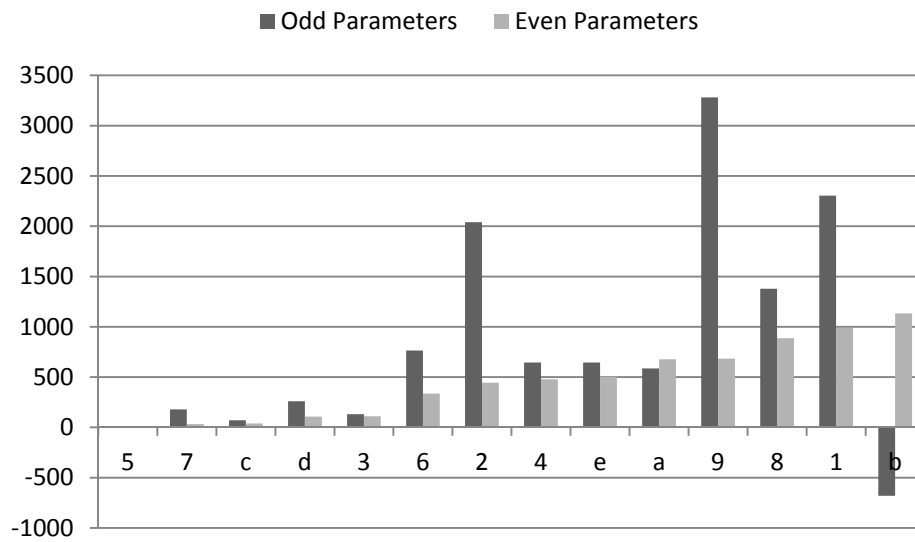


Figure 54. Integrated Odd and Even Parameters in the Main Tree of α_d

VI.2.3.5 Variables Interactions Exploration

Since three variables are selected as the most interacting variables, eight trees are to be established representing all binary states of 18b=xxx. Within each of these subtrees, the integrated odd and even parameters of each of 2469ae are found. In total twelve experiments per tree are performed. However, the top and bottom trees already have six experiments performed in each, thus the total number of experiments performed at this stage is 84. The resulting integrated odd and even parameters of variables 2469ae

are shown in Table 41, Table 42, and in Figure 55. Checking the integrated even parameters against the set threshold of 240, the variables highlighted in grey in Table 41 and Table 42 of subtrees $18b=xxx$ are not further explored and their integrated odd parameters are used as the first order parameters in $18b$ spanned subtrees.

As for subtree $18b=000$ and 100 , the unfolding of variable 4 is already performed, thus no further experiments are required, and subtrees $4=x$ of $18b=000,100$ are created. Subtrees $18b=001, 010,110$ have two interacting variables 4a, thus subtrees $4a=xx$ of $18b=001,010,110$ are created. Subtrees $18b=011,101,111$ have four interacting variables that need to be unfolded. Instead of unfolding the variables at one stage, which requires a large number of experiments, the highest two interacting variables 4a are unfolded, and the integrated odd and even parameters of the other two variables 9e are found in each of the new subtrees. The results of the unfolded subtrees are shown in Table 43 and Figure 56. It is concluded from the figure that variables 9 and e are not considered interacting in all combinations of $4a=xx$ of $18b=011,101,111$, thus no further subtree are to be created. The results of all performed experiments are shown in Appendix G.

Table 41Integrated Odd Parameters of 2469ae in Subtrees 18b=xxx of α_d

| Variable | 18b=000 | 18b=001 | 18b=010 | 18b=011 |
|----------|---------|---------|---------|---------|
| 2 | 1586.3 | 1904 | 1762.3 | 2089.8 |
| 4 | 759 | 267.7 | 762.8 | 436 |
| 6 | 319.5 | 625.3 | 340.3 | 729.7 |
| 9 | 2639.8 | 2987 | 2674 | 3258 |
| a | 88.5 | 374.7 | 199.2 | 593.8 |
| e | 188.8 | 476.5 | 261.3 | 694.8 |
| Variable | 18b=100 | 18b=101 | 18b=110 | 18b=111 |
| 2 | 1770.3 | 2135.5 | 1989 | 2311.8 |
| 4 | 764.5 | 521.5 | 921 | 604.3 |
| 6 | 459.8 | 871.3 | 560.5 | 980.8 |
| 9 | 2705.8 | 3437.3 | 2801.3 | 3615 |
| a | 144.3 | 553 | 230.3 | 692.3 |
| e | 160 | 545 | 234.3 | 755.8 |

Table 42Integrated Even Parameters of 2469ae in Subtrees 18b=xxx of α_d

| Variable | 18b=000 | 18b=001 | 18b=010 | 18b=011 |
|----------|---------|---------|---------|---------|
| 2 | -9.3 | 97.5 | -34.0 | 131.8 |
| 4 | 592.5 | 406.4 | 527.3 | 466.0 |
| 6 | -109.0 | -5.3 | -174.0 | -18.0 |
| 9 | 42.2 | 189.5 | -31.0 | 340.5 |
| a | 179.0 | 305.3 | 283.8 | 463.3 |
| e | 46.7 | 215.0 | 19.3 | 347.7 |
| Variable | 18b=100 | 18b=101 | 18b=110 | 18b=111 |
| 2 | -48.0 | 188.0 | 184.5 | 172.8 |
| 4 | 563.0 | 546.5 | 582.0 | 518.8 |
| 6 | -60.0 | 57.2 | -73.0 | 120.3 |
| 9 | -7.3 | 371.2 | 11.3 | 349.5 |
| a | 134.3 | 360.0 | 279.8 | 570.8 |
| e | 85.5 | 269.0 | 24.3 | 392.3 |

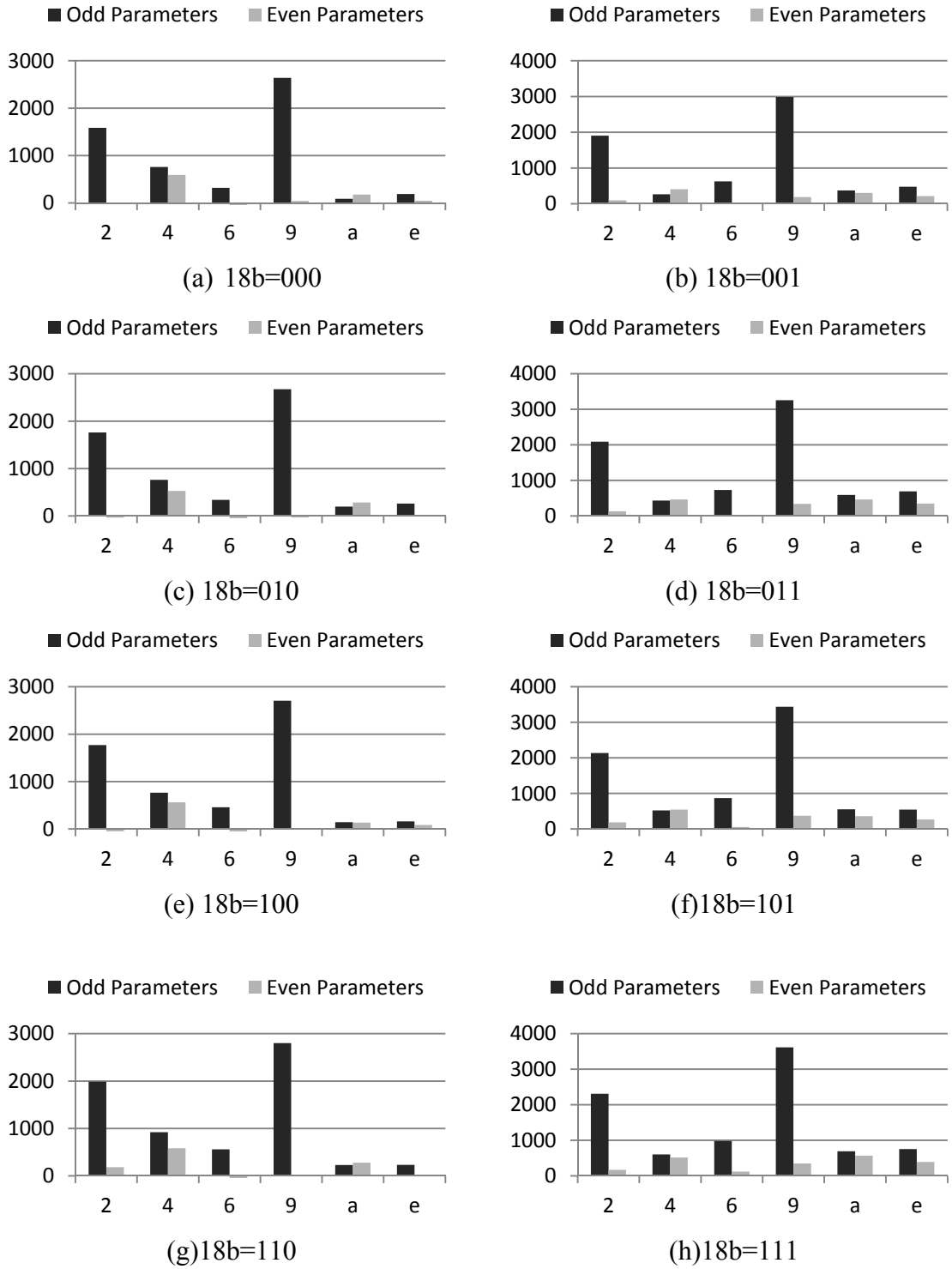


Figure 55. Integrated Odd and Even Parameters of 2469ae in $18b=xxx$ of α_d

Table 43Integrated Odd and Even Parameters of 9e in Subtrees 18b=011,101,111 of α_d

| Integrated Parameter | 4a=00 | 4a=01 | 4a=10 | 4a=11 |
|----------------------|---------|--------|--------|--------|
| | 18b=011 | | | |
| 9 Odd | 2878.5 | 3123.8 | 3387 | 3645 |
| 9 Even | -39 | -46 | -6 | -47 |
| e Odd | 380 | 847.8 | 400.3 | 1006.8 |
| e Even | 33 | 72.3 | 47.2 | 35.7 |
| | 18b=101 | | | |
| 9 Odd | 3039.8 | 3091.8 | 3496.8 | 3741 |
| 9 Even | -26 | -61 | 40.3 | 67.5 |
| e Odd | 279.3 | 559.5 | 337.8 | 785 |
| e Even | 3.25 | 18.5 | 19.8 | 29 |
| | 18b=111 | | | |
| 9 Odd | 3269.5 | 3634.0 | 3740.8 | 4002.5 |
| 9 Even | 4.0 | 3.5 | -4.8 | -38.0 |
| e Odd | 343.5 | 874.5 | 408.8 | 1122.8 |
| e Even | -20.0 | 12.0 | 40.2 | 25.3 |

VI.2.3.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by each end of the subtree using equation 45. The values of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46, and are shown in Table 44. The resulting parameters of all subtrees are shown in Table 45. In total, 168 experiments were performed. The performance characteristic is projected in all subtrees, then the global model parameters are found. The highest forty parameters are shown in Table 46 and Figure 57.

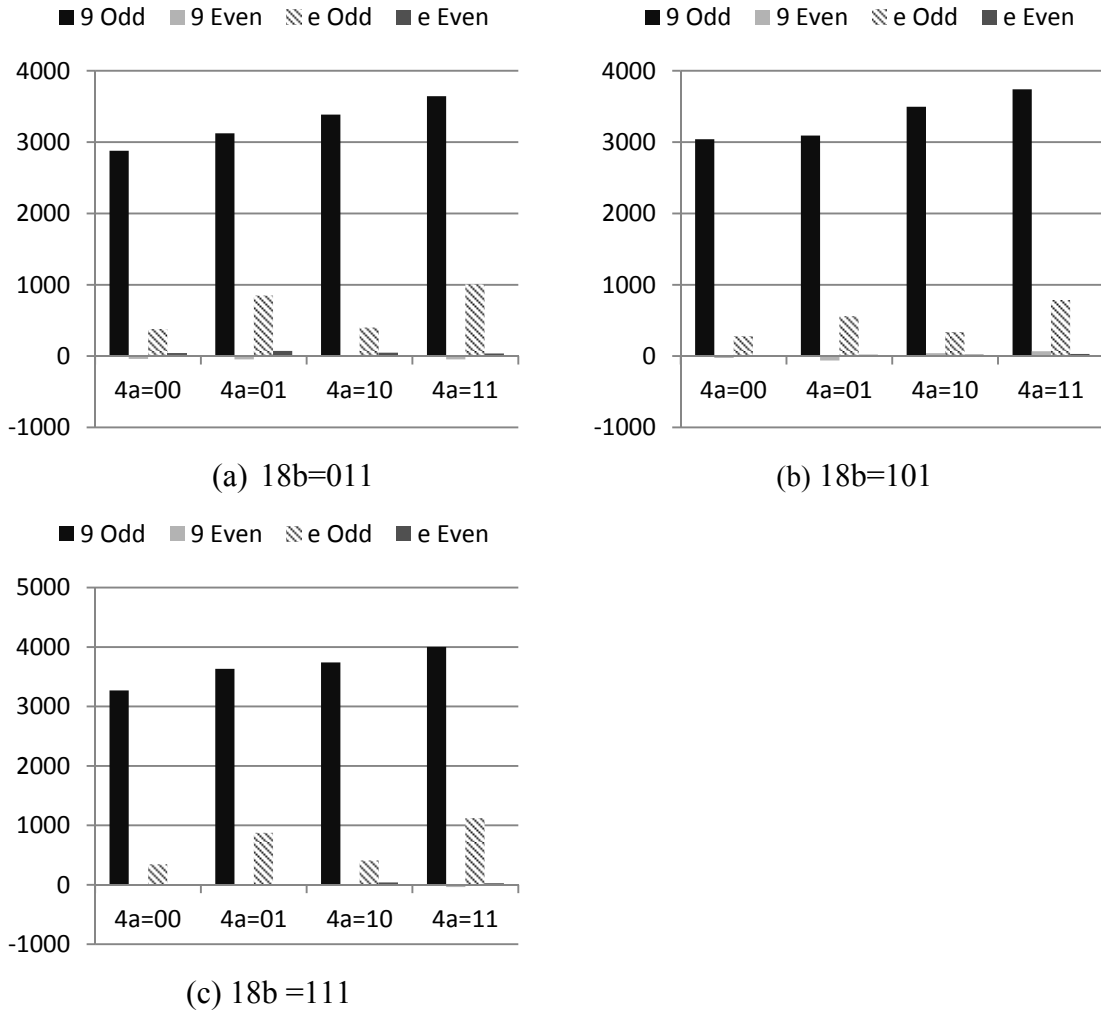


Figure 56. Integrated Odd and Even Parameters of $9e$ in Subtrees $18b=011, 101, 111$ of α_d

Table 44Offset Errors in All Subtrees of α_d

| | | | | | | | |
|------------------------|--------|-------|--------|--------|--------|-------|--------|
| Subtree 18b4a States | 0000x | 0001x | 00100 | 00101 | 00110 | 00111 | 01000 |
| $\beta_{0,1,-1,Error}$ | -476.5 | 708.5 | -651.6 | -105.4 | 96.6 | 643.9 | -594.5 |
| Subtree 18b4a States | 01001 | 01010 | 01011 | 01100 | 01101 | 01110 | 01111 |
| $\beta_{0,1,-1,Error}$ | -122.5 | 364.5 | 861.5 | -280.3 | -181.3 | 8.5 | 108.0 |
| Subtree 18b4a States | 1000x | 1001x | 10100 | 10101 | 10110 | 10111 | 11000 |
| $\beta_{0,1,-1,Error}$ | -225.8 | 900.2 | -255.0 | -93.8 | 96.0 | 194.0 | -514.8 |
| Subtree 18b4a States | 11001 | 11010 | 11011 | 11100 | 11101 | 11110 | 11111 |
| $\beta_{0,1,-1,Error}$ | -3.8 | 600.7 | 1083.8 | -165.8 | -108.3 | 146.7 | 353.5 |

Table 45Parameters in All Subtrees of α_d

| Parameter Index | 18b=000 4=x | 18b=001 4a=xx | 18b=010, 4a=xx | 18b=011, 4a=00 | 18b=011, 4a=01 | 18b=011, 4a=10 |
|-----------------|----------------|------------------|-------------------|-------------------|-------------------|-------------------|
| 1 | | | | | | |
| 2 | 1586.3 | 1904 | 1762.3 | 2089.8 | 2089.8 | 2089.8 |
| 3 | 131.8 | 131.8 | 131.8 | 131.8 | 131.8 | 131.8 |
| 4 | | | | | | |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6 | 319.5 | 625.3 | 340.3 | 729.7 | 729.7 | 729.7 |
| 7 | 179.3 | 179.3 | 179.3 | 179.3 | 179.3 | 179.3 |
| 8 | | | | | | |
| 9 | 2639.8 | 2987 | 2674 | 2878.5 | 3123.8 | 3387 |
| a | 88.5 | | | | | |
| b | | | | | | |
| c | 70.3 | 70.3 | 70.3 | 70.3 | 70.3 | 70.3 |
| d | 260 | 260 | 260 | 260 | 260 | 260 |
| e | 188.8 | 476.5 | 261.3 | 380 | 847.8 | 400.3 |

Table 45 Continued

| Parameter Index | 18b=011, 4a=11 | 18b=100, 4=x | 18b=101, 4a=00 | 18b=101, 4a=01 | 18b=101, 4a=10 | 18b=101, 4a=11 |
|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1 | | | | | | |
| 2 | 2089.8 | 1770.3 | 2135.5 | 2135.5 | 2135.5 | 2135.5 |
| 3 | 131.8 | 131.8 | 131.8 | 131.8 | 131.8 | 131.8 |
| 4 | | | | | | |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6 | 729.7 | 459.8 | 871.3 | 871.3 | 871.3 | 871.3 |
| 7 | 179.3 | 179.3 | 179.3 | 179.3 | 179.3 | 179.3 |
| 8 | | | | | | |
| 9 | 3645 | 2705.8 | 3039.8 | 3091.8 | 3496.8 | 3741 |
| a | | 144.3 | | | | |
| b | | | | | | |
| c | 70.3 | 70.3 | 70.3 | 70.3 | 70.3 | 70.3 |
| d | 260 | 260 | 260 | 260 | 260 | 260 |
| e | 1006.8 | 160 | 279.3 | 559.5 | 337.8 | 785 |
| Parameter Index | 18b=110, 4a=xx | 18b=111, 4a=00 | 18b=111, 4a=01 | 18b=111, 4a=10 | 18b=111, 4a=11 | |
| 1 | | | | | | |
| 2 | 1989 | 2311.8 | 2311.8 | 2311.8 | 2311.8 | |
| 3 | 131.8 | 131.8 | 131.8 | 131.8 | 131.8 | |
| 4 | | | | | | |
| 5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 6 | 560.5 | 980.8 | 980.8 | 980.8 | 980.8 | |
| 7 | 179.3 | 179.3 | 179.3 | 179.3 | 179.3 | |
| 8 | | | | | | |
| 9 | 2801.3 | 3269.5 | 3634 | 3740.8 | 4002.5 | |
| a | | | | | | |
| b | | | | | | |
| c | 70.3 | 70.3 | 70.3 | 70.3 | 70.3 | |
| d | 260 | 260 | 260 | 260 | 260 | |
| e | 234.3 | 343.5 | 874.5 | 408.8 | 1122.8 | |

Table 46
Highest Forty Parameters of α_d

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | -3009 | '3' | -132 |
| '1' | -2098 | '19' | 119.5 |
| '2' | -1944 | '12' | 108.7 |
| '8' | -1100 | '16' | 107.8 |
| 'b' | 1017.1 | 'ae' | 96.7 |
| '6' | -611 | '49' | 95.3 |
| '4' | -609 | '28' | 94.8 |
| 'e' | 395.2 | 'abe' | -95 |
| 'a' | 338.8 | '49b' | -92 |
| '9b' | 303.2 | '89' | 90.3 |
| '18' | 286.3 | 'c' | 71 |
| 'd' | 260.7 | '19b' | -70 |
| '8b' | 215.3 | '8a' | -67 |
| '4b' | -194 | '8e' | -66 |
| '6b' | 190.4 | '14' | 63.6 |
| 'be' | -183 | '89b' | -58 |
| '7' | 180 | '8ae' | -50 |
| '1b' | 174.5 | '8abe' | 49.3 |
| 'ab' | -172 | '48' | 49 |
| '2b' | 166.2 | '9a' | -46 |

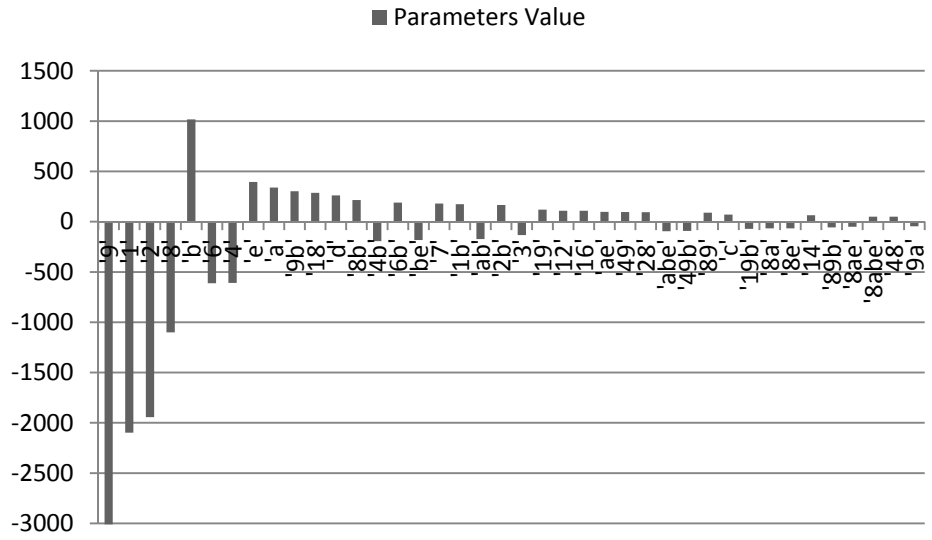


Figure 57. Highest Forty Parameters of α_d

VI.2.3.7 Validation

Since the MHIV method was compared earlier with the FFDs, the target of this section is to only evaluate the projected results with respect to a validation set. In order to ensure that the validation set is as spread as possible in the domain of experiments, the validation set selected is a FFD set of sixteen experiments. The results of the validation set are presented in Appendix G. Figure 58 shows the errors of the projected α_d . The RMSE with respect to the validation set is found as 462.3, which is around 1.8% of the α_d projected range and 1.92 times the threshold.

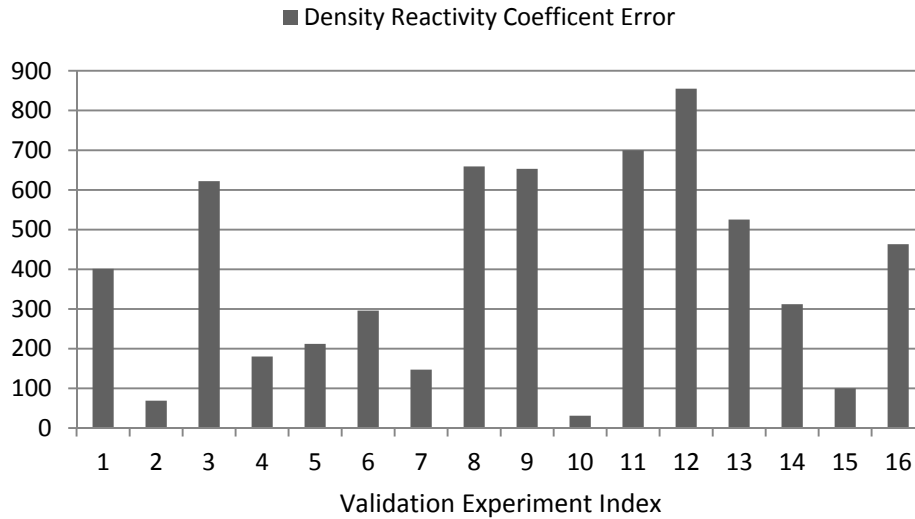


Figure 58. α_d 's Projection Error of the MHIV Method with Respect to the Sixteen Performed Experiments of the FFD

VI.2.4 Fuel Temperature Reactivity Coefficient Analysis

The fuel temperature reactivity coefficient, α_{FT} , is a measurement of the change of the reactivity due to the change of the fuel's temperature (see [85] for definition), and is one of the measures for reactors stability. The target for this coefficient is to be as high as possible in the negative direction, thus causing the reactivity to decrease as the temperature increase.

VI.2.4.1 Performance Characteristic Directional Alignment

The main cause of the negative effect of temperature on α_{FT} is the Doppler broadening of the resonance region in the energy spectrum of the cross sections. Since the fuel used in the SMR is Uranium Oxide, thermal expansion and other phenomenon

that could cause reactivity changes due to the change of temperature can be neglected due to their small magnitude [87]. The range of α_{FT} in this example is expected to vary from around zero to a high negative value. As a result, α_{FT} is expected to have a plateau at its high end of zero, and needs to be reversed. Since the saturation plateau is to be aligned with the bottom end of the performance characteristic, the performance characteristic is defined as:

$$\alpha_{FT} = -(\rho_{2500K} - \rho_{ref}) = (\rho_{ref} - \rho_{2500K}) = \frac{(k_{keff,ref} - 1)}{k_{keff,ref}} - \frac{(k_{keff,2500K} - 1)}{k_{keff,0.5Sat}} \quad (61)$$

where ρ is the reactivity. The low end of the range of α_{FT} is assumed to have a value between 600K and 900K. The fuel temperatures used are summarized in Table 47. The high end of the range of the fuel temperature was chosen based on the maximum temperature of cross section libraries in ENDF and MCNP. This was found as 2500K in [83, 88]. The units used for α_{FT} from here onwards is pcm, which is equivalent to 10^{-5} .

Table 47
Definition of Fuel Temperature's Binary States for Low and High Temperatures

| State\Temperature | Low (Reference Case) | High |
|----------------------------|----------------------|------|
| Temperature State Low (K) | 600 | 2500 |
| Temperature State High (K) | 900 | 2500 |

VI.2.4.2 Variables Directional Alignment

The directional dependence of α_{FT} on the variables is determined in this step. Variables are categorized into monotonic variables and variables with unknown direction of effect. The Doppler broadening of the resonance was used as the criterion to determine the directions of effect of the variables. As the spectrum is thermalized, the effect of Doppler broadening and temperature changes on α_{FT} reduce. The variables' directions of effect are shown in Table 48 . The directional dependence is briefly justified in Table 49. According to the table, the maximum α_{FT} occurs at the variables' states of 0000x001011111, thus the variables are aligned to all high at this combination.

VI.2.4.3 Variables Integrated Parameters Determination

Since thirteen variables are explored, 28 experiments are performed at this stage. The results of the performed experiments are shown in Appendix G. The corresponding integrated odd and even parameters of all variables are shown in Table 50.

Table 48
Variables' Directions of Effect on α_{FT}

| Variable Index | Variable | Direction of Effect on α_{FT} |
|----------------|---------------------------------|--------------------------------------|
| 1 | No of Fuel Assemblies | - |
| 2 | Active Fuel Height | - |
| 3 | Core Radial Surrounding Water | - |
| 4 | Enrichments | - |
| 5 | Power | = |
| 6 | Moderator Density | - |
| 7 | Fuel Temperature Lib | - |
| 8 | No of Rods per Assembly | + |
| 9 | Rod Pitch | - |
| 10 (a) | No of DBARs per Assembly | + |
| 11 (b) | IFAB and Fuel Pellet Diameter | + |
| 12 (c) | IFAB and Fuel Gap Thickness | + |
| 13 (d) | IFAB and Fuel Clad Thickness | + |
| 14 (e) | Discrete Burnable Absorber Area | + |

Table 49Justification of Variables' Directions of Effect on α_{FT}

| Index | Variable | Justification | Direction of Effect on α_{FT} |
|--------|---------------------------------|---|--------------------------------------|
| 1 | No of Fuel Assemblies | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | +(-) |
| 2 | Active Fuel Height | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | +(-) |
| 3 | Core Radial Surrounding Water | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | +(-) |
| 4 | Enrichments | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | +(-) |
| 5 | Power | <ul style="list-style-type: none"> ▪ No effect | = |
| 6 | Moderator Density | <ul style="list-style-type: none"> ▪ Softer spectrum reduces the effect of Doppler broadening | - |
| 7 | Fuel Temperature Lib | <ul style="list-style-type: none"> ▪ Softer spectrum reduces the effect of Doppler broadening | - |
| 8 | No of Rods per Assembly | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | + |
| 9 | Rod Pitch | <ul style="list-style-type: none"> ▪ Softer spectrum reduces the effect of Doppler broadening | - |
| 10 (a) | No of DBARs per Assembly | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | + |
| 11 (b) | IFAB and Fuel Pellet Diameter | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening ▪ More fuel increases the effect of temperature | ++ |
| 12 (c) | IFAB and Fuel Gap Thickness | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | + |
| 13 (d) | IFAB and Fuel Clad Thickness | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | + |
| 14 (e) | Discrete Burnable Absorber Area | <ul style="list-style-type: none"> ▪ Harder spectrum increases the effect of Doppler broadening | + |

Table 50Integrated Odd and Even Parameters in the Main Tree of α_{FT}

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 25.55 | 19.15 | 22.35 | 3.20 |
| 2 | 33.70 | 27.45 | 30.58 | 3.13 |
| 3 | 16.30 | -0.15 | 8.08 | 8.22 |
| 4 | 216.25 | 62.45 | 139.35 | 76.90 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 175.85 | 84.00 | 129.93 | 45.93 |
| 7 | 561.10 | 208.85 | 384.98 | 176.13 |
| 8 | 46.95 | -0.50 | 23.23 | 23.73 |
| 9 | 1001.20 | 410.00 | 705.60 | 295.60 |
| a | -10.15 | -34.50 | -22.32 | 12.18 |
| b | 713.45 | 252.20 | 482.83 | 230.63 |
| c | 29.55 | -1.70 | 13.93 | 15.63 |
| d | 102.80 | 11.65 | 57.23 | 45.58 |
| e | 157.70 | 17.80 | 87.75 | 69.95 |

VI.2.4.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of the integrated even parameters, and presented in Figure 59. If a threshold of integrated even parameters is set at 39 (39 is three times the average error of the calculated K_{eff} of 13), it is possible to categorize variables 79b as the most interacting variables, variables 46de as interacting variables, and variables 12358ac as non-interacting variables. The non-interacting variables are not further explored.

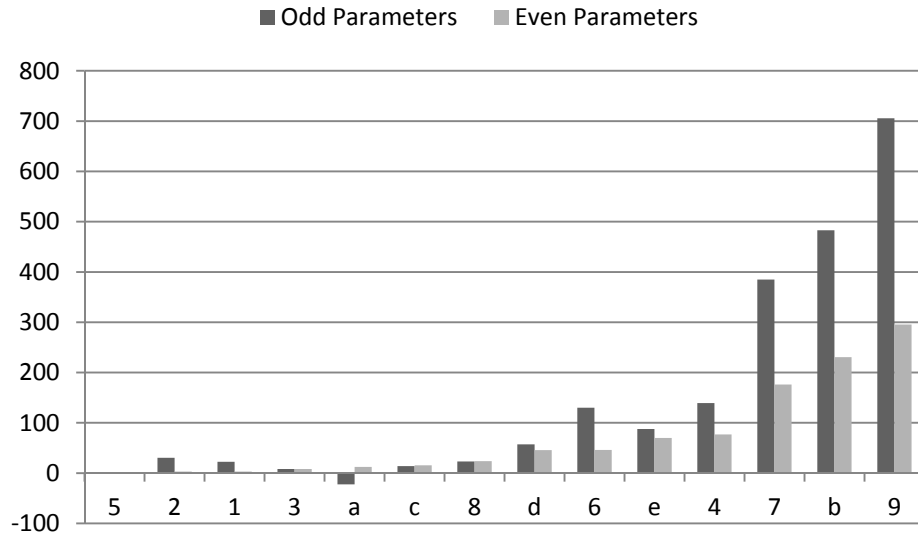


Figure 59. Integrated Odd and Even Parameters in the Main Tree of α_{FT}

VI.2.4.5 Variables Interactions Exploration

Since three variables are selected as the most interacting variables, eight subtrees are to be established representing all binary states of 79b. Within each of these subtrees, the integrated odd and even parameters of each of 46de are found. In total eight experiments per subtree are performed. However, the top and bottom subtrees already have four experiments performed in each. Thus, the total number of experiments performed at this stage is 56. The resulting integrated odd and even parameters of variables 46de are listed in Table 51 and Table 52, and shown in Figure 60. Checking the integrated even parameters against the threshold of 39, the variables highlighted in grey in Table 51 and Table 52 of subtrees 79b=xxx are not further explored, and their integrated odd parameters are used as the first order parameters in 79b spanned subtrees.

All variables of 79b subtrees fell below the threshold, and are considered as non-interacting. Thus, no further subtrees spanning is needed. The results of all performed experiments are shown in Appendix G.

Table 51
Integrated Odd Parameters of 46de in Subtrees 79b=xxx of α_{FT}

| Variable | 79b= | | | | | | | |
|----------|-------|--------|--------|--------|--------|--------|--------|--------|
| | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 4 | 76.17 | 92.85 | 97.43 | 164.50 | 93.45 | 126.05 | 128.25 | 185.25 |
| 6 | 86.87 | 121.30 | 105.30 | 142.55 | 109.98 | 150.83 | 139.95 | 171.48 |
| d | 22.75 | 37.93 | 28.55 | 79.20 | 16.65 | 50.50 | 48.90 | 90.38 |
| e | 44.92 | 45.50 | 54.88 | 97.78 | 44.08 | 68.10 | 78.65 | 120.53 |

Table 52
Integrated Even Parameters of 46de in Subtrees 79b=xxx of α_{FT}

| Variable | 79b= | | | | | | | |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 4 | 13.72 | 12.35 | 7.78 | 22.95 | 13.35 | 18.85 | 5.75 | 31.00 |
| 6 | 2.88 | 11.35 | 2.50 | 5.75 | 13.68 | 2.12 | -1.40 | 4.37 |
| d | 11.10 | 10.43 | -1.35 | 13.45 | 0.55 | -6.15 | -0.70 | 12.42 |
| e | 27.13 | 18.45 | 20.48 | 25.38 | 23.23 | 27.00 | 13.80 | 37.18 |

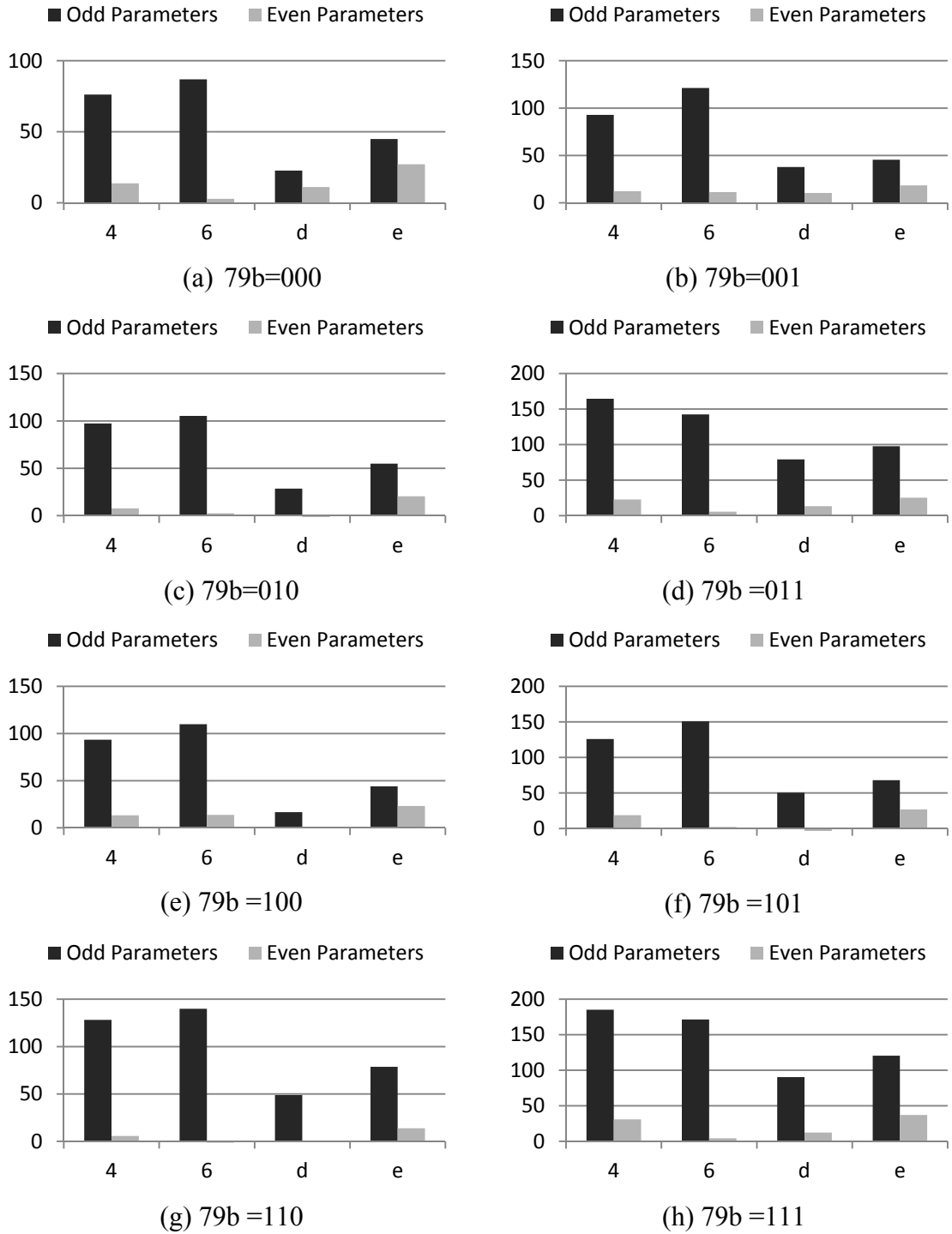


Figure 60. Integrated Odd and Even Parameters of 26de in $79b=xxx$ of α_{FT}

VI.2.4.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by each end of the subtree using equation 45. The values of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46, and are shown in Table 53. The parameters of all subtrees are shown in Table 54. In total 90, experiments were performed. The performance characteristic is projected in all subtrees, then the global model parameters are found. The highest thirty parameters are shown in Table 55 and Figure 61.

Table 53
Offset Errors in All Subtrees of α_{FT}

| Subtree 79b States | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
|--------------------------|---------|---------|--------|---------|---------|---------|--------|---------|
| $\beta_{0,1,-1,Error}$ | -50.650 | -65.150 | 15.525 | -66.900 | -31.025 | -71.150 | 18.775 | -52.800 |

Table 54
Parameters in All Subtrees of α_{FT}

| Index | 79b= | | | | | | | |
|-------|---------|---------|---------|---------|---------|--------|--------|--------|
| | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 1 | 22.35 | 22.35 | 22.35 | 22.35 | 22.35 | 22.35 | 22.35 | 22.35 |
| 2 | 30.575 | 30.575 | 30.575 | 30.575 | 30.575 | 30.58 | 30.58 | 30.58 |
| 3 | 8.08 | 8.08 | 8.08 | 8.08 | 8.08 | 8.08 | 8.08 | 8.08 |
| 4 | 76.175 | 92.85 | 97.425 | 164.5 | 93.45 | 126.05 | 128.25 | 185.25 |
| 5 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | 86.875 | 121.3 | 105.3 | 142.55 | 109.975 | 150.83 | 139.95 | 171.48 |
| 7 | | | | | | | | |
| 8 | 23.225 | 23.225 | 23.225 | 23.225 | 23.225 | 23.23 | 23.23 | 23.23 |
| 9 | | | | | | | | |
| a | -22.325 | -22.325 | -22.325 | -22.325 | -22.325 | -22.32 | -22.32 | -22.32 |
| b | | | | | | | | |
| c | 13.925 | 13.925 | 13.925 | 13.925 | 13.925 | 13.93 | 13.93 | 13.93 |
| d | 22.75 | 37.925 | 28.55 | 79.2 | 16.65 | 50.50 | 48.90 | 90.38 |
| e | 44.925 | 45.5 | 54.875 | 97.775 | 44.075 | 68.10 | 78.65 | 120.53 |

Table 55
Highest Thirty Parameters of α_{FT}

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | -670 | '4b' | -22 |
| 'b' | 453 | '9e' | -19 |
| '7' | -356 | '79b' | 18 |
| '9b' | -136 | '6b' | -18 |
| '6' | -128 | 'bd' | 18 |
| '4' | -120 | '9d' | -15 |
| '79' | 82 | '67' | 15 |
| 'e' | 69 | 'c' | 14 |
| '7b' | -57 | 'be' | 14 |
| 'd' | 47 | '47' | 13 |
| '2' | -30 | '69' | 11 |
| '49' | 23 | '49b' | 9 |
| '8' | 23 | '7e' | -9 |
| 'a' | -23 | '3' | -8 |
| '1' | -22 | '9be' | -7 |

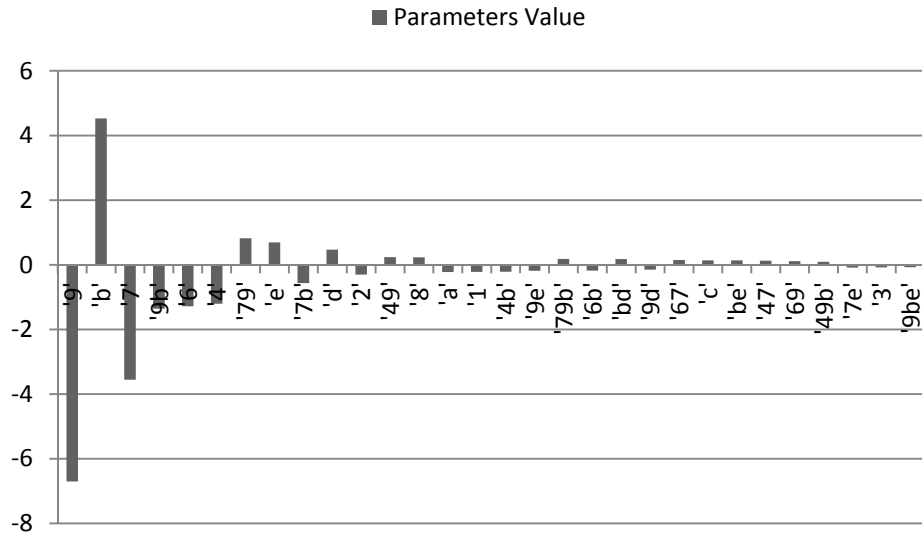


Figure 61. Highest Thirty Parameters of α_{FT}

VI.2.4.7 Validation

Since the MHIV method was compared earlier with the FFDs, the target of this section is to only evaluate the projected results with respect to a validation set. In order to ensure that the validation set is as spread as possible in the domain of experiments, the validation set selected is a FFD set of sixteen experiments. The results of the validation set are presented in Appendix G. Figure 62 shows the errors of the projected α_{FT} . The RMSE with respect to the validation model is found as 67.4, which is around 1.67% of the α_{FT} projected range and 1.72 times the threshold.

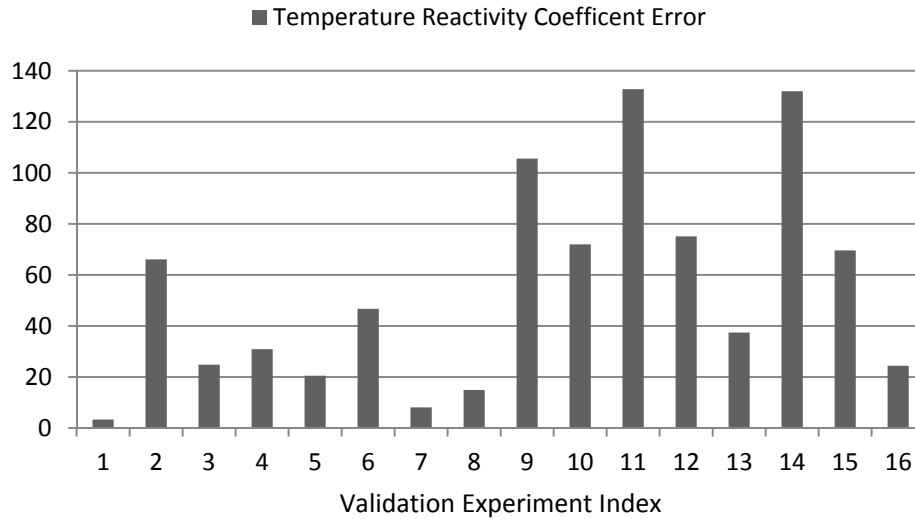


Figure 62. α_{FT} 's Projection Error of the MHIV Method with Respect to the Sixteen Performed Experiments of the FFD

VI.2.5 Power Peaking Factor Analysis

The power peaking factor, PPF, is an indication of the power distribution in the core. It is the ratio of the maximum power zone in the core to the average power of the core. In order to improve the thermohydraulic and materials aspects of the design, it is desired to have a power profile that is as flat as possible.

VI.2.5.1 Performance Characteristic Directional Alignment

As the power peaking is reduced, PPF approaches a value of one. This is a theoretical limit since only an infinitely large core has a PPF of one. On the other hand, as the core gets smaller, it is expected that PPF will increase due to a very steep power profile. Accordingly, PPF is limited by a plateau at its low end. Since the saturation

plateau is to be aligned with the bottom end of the performance characteristic, the performance characteristic is defined as:

$$PPF = \frac{Power_{Max}}{Power_{Avg}} \quad (62)$$

The core is split into 100 spatial elements consisting of ten axial elements and ten radial elements. PPF is the ratio of the power depositions in the highest element to the average of all 100 elements.

VI.2.5.2 Variables Directional Alignment

The directional dependence of PPF on the variables is determined in this step. Variables are categorized into monotonic variables and variables with unknown direction of effect. The criterion used is that it is expected that larger cores will have smaller PPF. It is also expected that as the heat source or fuel is more condensed, PPF increases. The variables' directions of effect are shown in Table 56. The directional dependence is briefly justified in Table 57. According to the table, the maximum PPF occurs at the variables' states of 1101x110101001, thus the variables are aligned to all high at this combination.

Table 56
Variables' Directions of Effect on PPF

| Variable Index | Variable | Direction of Effect on PPF |
|----------------|---------------------------------|----------------------------|
| 1 | No of Fuel Assemblies | + |
| 2 | Active Fuel Height | + |
| 3 | Core Radial Surrounding Water | - |
| 4 | Enrichments | + |
| 5 | Power | = |
| 6 | Moderator Density | + |
| 7 | Fuel Temperature Lib | + |
| 8 | No of Rods per Assembly | - |
| 9 | Rod Pitch | + |
| 10 (a) | No of DBARs per Assembly | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | + |
| 12 (c) | IFAB and Fuel Gap Thickness | - |
| 13 (d) | IFAB and Fuel Clad Thickness | - |
| 14 (e) | Discrete Burnable Absorber Area | + |

Table 57
Justification of Variables' Directions of Effect on PPF

| Index | Variable | Justification | Direction of Effect on PPF |
|--------|--|--|----------------------------|
| 1 | No of Fuel Assemblies | <ul style="list-style-type: none"> ▪ Larger core causes lower PPF | - (+) |
| 2 | Active Fuel Height | <ul style="list-style-type: none"> ▪ Larger core causes lower PPF | -(+) |
| 3 | Core Radial Surrounding Water | <ul style="list-style-type: none"> ▪ More water increases reflection at the edges | - |
| 4 | Enrichments | <ul style="list-style-type: none"> ▪ Higher enrichment causes higher heat production especially at the center | + |
| 5 | Power | <ul style="list-style-type: none"> ▪ No effect | = |
| 6 | Moderator Density | <ul style="list-style-type: none"> ▪ Softer spectrum results in higher heat production especially at the center | + |
| 7 | Fuel Temperature Lib | <ul style="list-style-type: none"> ▪ Higher temperature causes higher absorption especially at the center | + |
| 8 | No of Rods per Assembly | <ul style="list-style-type: none"> ▪ Larger core causes lower PPF ▪ Smaller burnable absorber to fuel ratio causes higher PPF | +-(-) |
| 9 | Rod Pitch | <ul style="list-style-type: none"> ▪ Softer spectrum causes higher heat production especially at the center ▪ Larger core causes lower PPF | +-(+) |
| 10 (a) | No of DBARs per Assembly | <ul style="list-style-type: none"> ▪ Higher absorption especially at the center | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | <ul style="list-style-type: none"> ▪ Harder spectrum especially at the center ▪ Higher heat production especially at the center | +-(+) |
| 12 (c) | IFAB and Fuel Gap Thickness | <ul style="list-style-type: none"> ▪ Harder spectrum especially at the center | - |
| 13 (d) | IFAB and Fuel Clad Thickness | <ul style="list-style-type: none"> ▪ Harder spectrum especially at the center | - |
| 14 (e) | Discrete Burnable Absorber Area (cm ²) | <ul style="list-style-type: none"> ▪ Higher absorption especially at the center | -(+) |

VI.2.5.3 Variables Integrated Parameters Determination

Since thirteen variables are explored, 28 experiments are performed at this stage. The results of the performed experiments are shown in Appendix G. The corresponding integrated odd and even effects of all variables are shown in Table 58. Since the range of the PPF variation is small, it is expressed in 10^{-2} from now onwards.

Table 58
Integrated Odd and Even Parameters in the Main Tree of PPF

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 10.58 | -2.69 | 3.95 | 6.63 |
| 2 | 16.20 | 2.04 | 9.12 | 7.08 |
| 3 | 8.79 | -0.26 | 4.26 | 4.52 |
| 4 | 15.47 | 1.88 | 8.67 | 6.80 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 6.15 | 2.28 | 4.21 | 1.94 |
| 7 | 6.75 | -2.59 | 2.08 | 4.67 |
| 8 | 2.24 | -8.25 | -3.00 | 5.25 |
| 9 | 13.38 | 7.38 | 10.38 | 3.00 |
| a | 21.27 | -6.60 | 7.33 | 13.94 |
| b | 4.72 | -10.40 | -2.84 | 7.56 |
| c | 5.66 | 0.64 | 3.15 | 2.51 |
| d | 4.59 | -1.12 | 1.74 | 2.86 |
| e | 7.26 | -6.43 | 0.42 | 6.85 |

VI.2.5.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of the integrated even parameters, and are shown in Figure 63. It is concluded from the magnitudes of the integrated odd and even parameters, that variables are highly

interacting. It is also clear that since the integrated odd parameters are smaller than the integrated even parameters for most of the variables, the variables are highly non-monotonic. As a result, the application of the dissertation's methods is not recommended for this performance characteristic. This, however, is not a limitation of the dissertation's methods. The fact that they can determine that the variables are interacting and non-monotonic is a major finding by itself. From this knowledge, the experimenter can know whether a very high Res FFD is needed for this performance characteristic.

This performance characteristic will be analyzed using the MHIV method and a FFD for demonstration. Starting with the MHIV method, if a threshold of integrated even parameters is set at 2, it is possible to categorize variables 2ab as most interacting, variables 134789cde as interacting, and variables 56 as non-interacting. The non-interacting variables are not further explored.

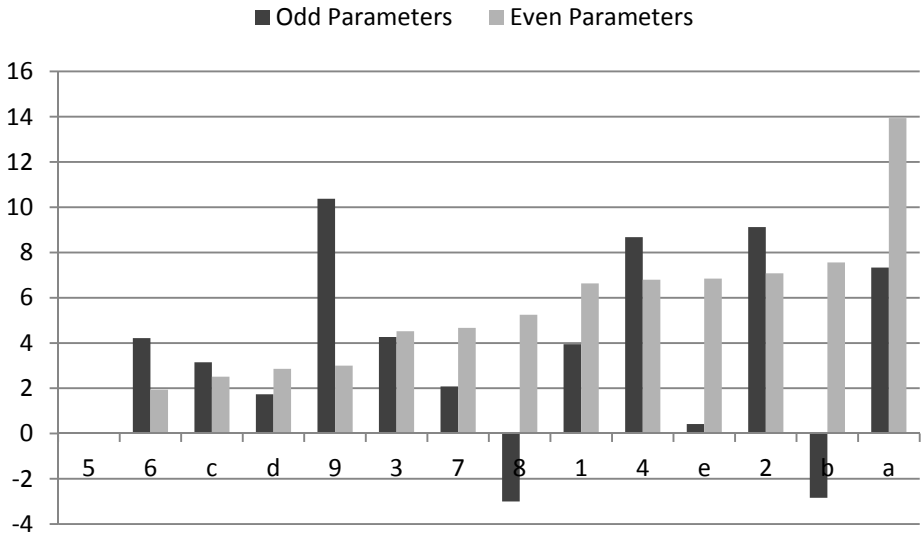


Figure 63. Integrated Odd and Even Parameters in the Main Tree of PPF

VI.2.5.5 Variables Interactions Exploration

Since three variables are selected as the most interacting variables, eight subtrees are to be established representing all binary states of $2ab=xxx$. Within each of these subtrees, the integrated odd and even parameters of each of $134789cde$ are found. In total eighteen experiments per subtree are performed. However, the top and bottom subtrees already have 9 experiments performed in each. Thus, the total number of experiments performed at this stage is 126. The resulting integrated odd and even parameters of variables $134789cde$ are listed in Table 59 and Table 60, and shown in Figure 64. Checking the integrated even parameters against the set threshold of 2, the variables highlighted in grey in Table 59 and Table 60 of subtrees $2ab=xxx$ are not further explored, and their integrated odd parameters are used as the first order parameters in $2ab$ spanned subtrees. The results of all performed experiments are shown in Appendix G.

Subtree $2ab=000$ has three interacting variables 148 , and they all are relatively far from the threshold of 2. As a result, it is decided to unfold all of them, thus creating subtrees $148=xxx$ of $2ab=000$. Subtree $2ab=001$ has two interacting variables, 19 , so they are also unfolded by creating subtrees $19=xx$ of $2ab=001$. Subtree $2ab=010$ has three interacting variables, $39d$. It is unfolded by creating subtrees $39d=xxx$ of $2ab=010$. However, since these variables are close to the threshold, it is possible to unfold them through multi-stages' exploration. Subtree $2ab=011$ has no interacting variables, so no further spanning is needed. Subtree $2ab=100$ has five interacting variables $189de$, thus a multistage spanning approach is applied. At the first stage, the two most interacting

variables, de, are unfolded and subtrees de=xx of 2ab=100 are created. Within every subtree, variables 189 integrated odd and even parameters are found. The results are shown in Table 61 and Figure 65. It is concluded from the results that some variables are still strongly interacting in all subtrees, thus subtrees 18=xx of de=00,01,10 of 2ab=100, and 9=x of de=11 of 2ab=100 are created.

Table 59
Integrated Odd Parameters of 134789cde in Subtrees 2ab=xxx of PPF

| Variable | 2ab= | | | | | | | |
|----------|-------|-------|-------|-------|--------|-------|-------|-------|
| | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 1 | 0.84 | 5.05 | 4.32 | 6.17 | 1.59 | 4.92 | 7.04 | 8.59 |
| 3 | -0.69 | -0.86 | 0.20 | 1.89 | -1.50 | 2.05 | 0.61 | 4.44 |
| 4 | 5.43 | 5.24 | 6.33 | 4.49 | 10.38 | 8.22 | 9.38 | 10.63 |
| 7 | -3.21 | -3.67 | -0.09 | 1.48 | -1.66 | 0.64 | -1.13 | 2.19 |
| 8 | -4.41 | -4.55 | -1.80 | -1.17 | -15.23 | -6.88 | 3.06 | -0.27 |
| 9 | 6.25 | 10.04 | 4.71 | 13.28 | 3.85 | 10.30 | 8.71 | 13.17 |
| c | -0.73 | -1.68 | -0.11 | 0.62 | 3.71 | -1.26 | 3.40 | 6.40 |
| d | -2.06 | 0.63 | 0.42 | -0.55 | -0.32 | 0.34 | 0.09 | 3.69 |
| e | -6.26 | -3.17 | -1.06 | -1.66 | -11.26 | -9.77 | 3.86 | 3.78 |

Table 60
Integrated Even Parameters of 134789cde in Subtrees 2ab=xxx of PPF

| Variable | 2ab= | | | | | | | |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 1 | 3.53 | 2.20 | 0.03 | 1.41 | 2.20 | 2.35 | -0.19 | 1.99 |
| 3 | -0.43 | -1.62 | -2.57 | 0.82 | -1.02 | 1.33 | -2.98 | 4.34 |
| 4 | 3.55 | 0.69 | 0.95 | 1.33 | -0.43 | 3.15 | 0.51 | 4.84 |
| 7 | -0.62 | -0.91 | -0.43 | 1.17 | 0.62 | -0.89 | -5.64 | 4.56 |
| 8 | 3.84 | -0.91 | -1.11 | -1.84 | -2.12 | -2.82 | 3.68 | 2.51 |
| 9 | -1.13 | -2.25 | -2.79 | 1.74 | -2.88 | -5.34 | -0.70 | 0.21 |
| c | -1.37 | -0.80 | -1.93 | 0.78 | 0.41 | -1.85 | -0.51 | -0.74 |
| d | -0.94 | 0.83 | -2.13 | -1.00 | -3.17 | 0.07 | -0.53 | 0.91 |
| e | 0.17 | -0.59 | -1.78 | -0.80 | -5.64 | -4.37 | -0.15 | 3.49 |

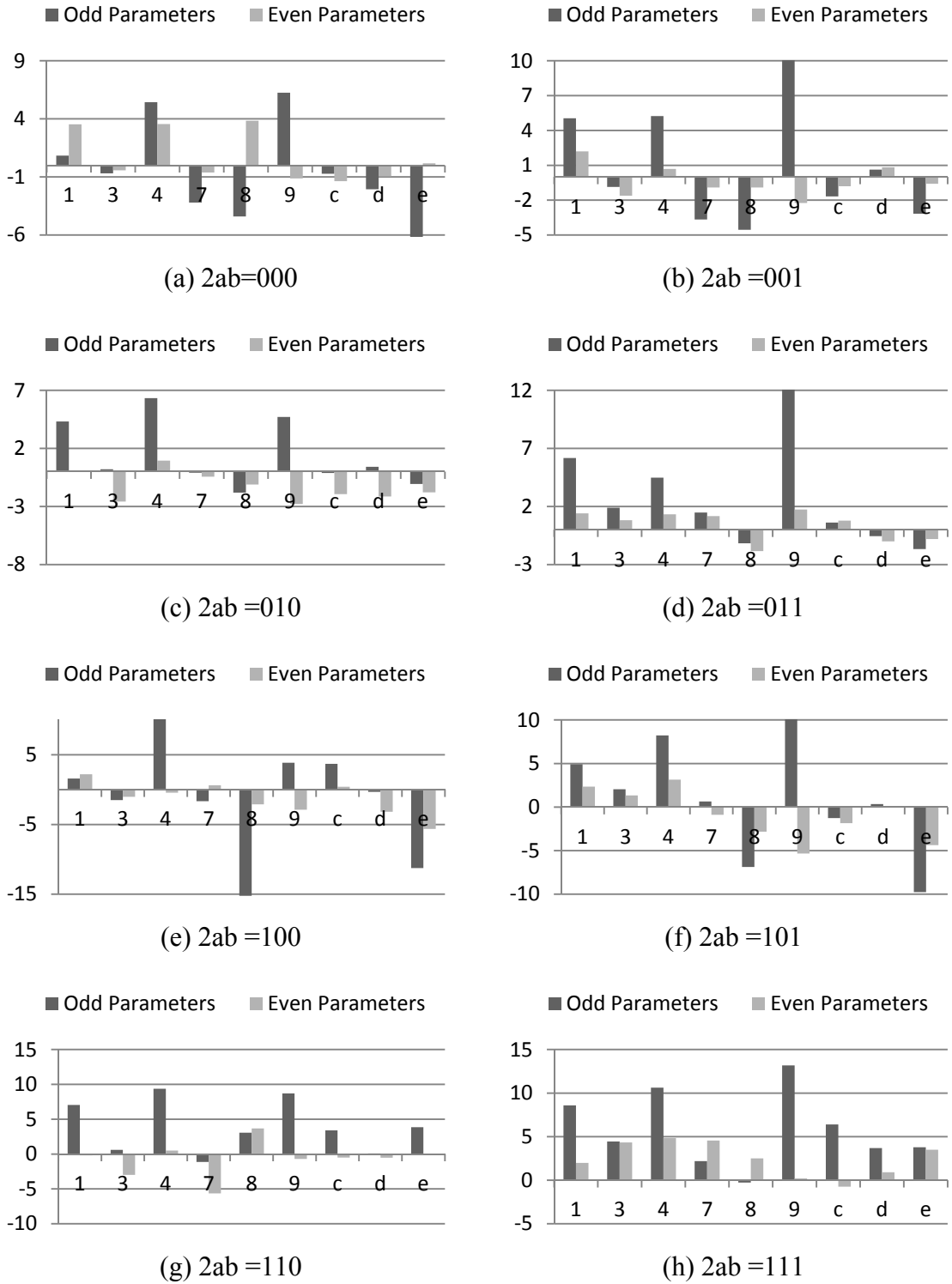


Figure 64. Integrated Odd and Even Parameters of 134789cde in Subtrees $2ab=xxx$ of PPF

Subtree 2ab=101 has five interacting variables 1489e. Variables 49e are unfolded at the first stage and subtrees 49e=xxx of 2ab=101 are created. Within every subtree, variables 18 integrated odd and even parameters are found. The results are shown in Table 62 and Figure 66. Based on the results is decided that no further unfolding is needed in subtree 49e=000,001,101 of 2ab=101. Variables 18 are interacting in subtrees 49e=010,011,110,111, so subtrees 18=xx of 49e=010,011,110,111 of 2ab=101 are created. Subtree 49e=100 has variable 1 as interacting, thus subtrees 1=x of 49e=100 of 2ab=101 are created.

Table 61
Integrated Odd and Even Parameters of 189 in Subtrees de=xx of PPF

| Odd Parameters Variable | de=00 | de=01 | de=10 | de=11 |
|-----------------------------|---------|----------|---------|----------|
| 1 | 4.8025 | 6.1600 | 2.2000 | 2.8850 |
| 8 | -7.5700 | -15.9975 | -9.6800 | -18.4375 |
| 9 | 8.2175 | 6.8825 | 5.2800 | 7.1750 |
| Even Parameters Variable | de=00 | de=01 | de=10 | de=11 |
| 1 | 5.4125 | 9.3350 | 3.7050 | 0.9050 |
| 8 | 5.5350 | 7.1925 | 8.9650 | 1.0875 |
| 9 | 1.4925 | 1.3125 | -0.1250 | -6.2100 |

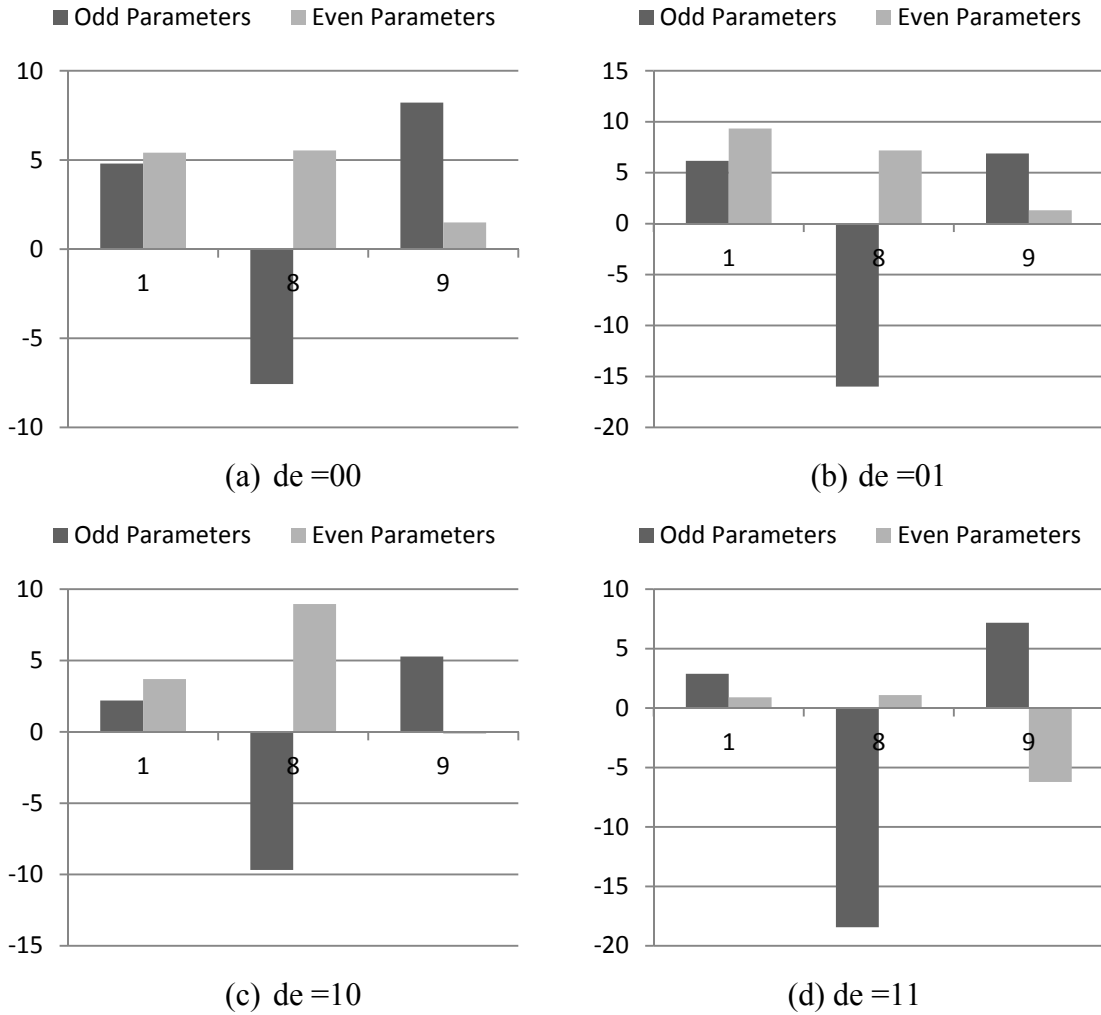


Figure 65. Integrated Odd and Even Parameters of 189 in Subtrees $de=xx$ of PPF

Table 62

Integrated Odd and Even Parameters of 18 in Subtrees $49e=xxx$ of $2ab=101$ of PPF

| Odd Parameters | | | | $49e=$ | | | | |
|----------------|-------|--------|--------|--------|-------|-------|-------|--------|
| Variable | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 1 | 4.41 | -0.45 | -1.56 | -3.07 | 7.90 | 10.04 | 7.98 | 4.64 |
| 8 | -5.95 | -12.74 | -11.29 | -15.27 | -3.31 | -6.37 | -7.00 | -13.96 |

| Even Parameters | | | | $49e=$ | | | | |
|-----------------|-------|-------|------|--------|-------|-------|------|------|
| Variable | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 1 | 1.84 | 0.48 | 2.91 | 7.04 | 4.02 | 1.48 | 5.91 | 2.63 |
| 8 | -1.89 | -1.78 | 3.89 | 5.96 | -1.29 | -0.57 | 6.05 | 4.26 |

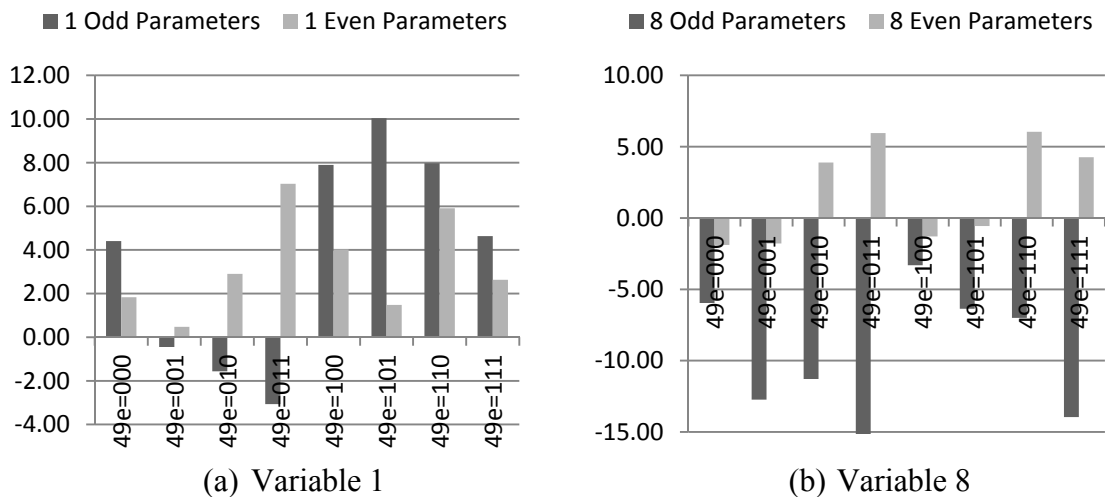


Figure 66. Integrated Odd and Even Parameters of 18 in Subtrees $49e=xxx$ of $2ab=101$ of PPF

Subtree $2ab=110$ has three interacting variables 378. They are unfolded in one stage by creating subtrees $378=xxx$ of $2ab=110$. Subtree $2ab=111$ has five interacting variables 3478e, so the most interacting variables, 347, are used to create the first stage

subtrees. The integrated odd and even parameters of variables 8e are found in each of the subtrees, and are shown in Table 63 and Figure 67. From the results, it is decided that subtrees 8e=xx of 347=000,111 of 2ab=111 are created. Subtrees 347=001 and 347=011 have variable e as interacting, so subtrees e=x of 347=001,011 of 2ab=111 are created. Subtree 347=010 has no interacting variables, thus no subtrees are spanned within this subtree. Subtrees 347=100,101 and 110 have variable 8 as an interacting variable, thus subtrees 8=x of 347=100,101,110 of 2ab=111 are created, and no further subtrees' spanning is needed.

Table 63
Integrated Odd and Even Parameters of 8e in Subtrees 347=xxx of 2ab=111 of PPF

| Odd Parameters | | 347= | | | | | | | |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| Variable | =000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 | |
| 8 | -0.61 | 4.04 | -2.19 | -1.60 | -1.59 | 1.96 | -3.22 | -2.00 | |
| e | -4.61 | -0.97 | -2.05 | -4.96 | -1.96 | -2.15 | -3.61 | 2.10 | |
| Even Parameters | | 347= | | | | | | | |
| Variable | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 | |
| 8 | 2.16 | 1.66 | 0.90 | 1.83 | -4.00 | 2.31 | 2.90 | 4.24 | |
| e | -4.89 | -3.41 | 1.61 | -3.82 | -0.85 | 0.58 | 1.43 | 5.16 | |

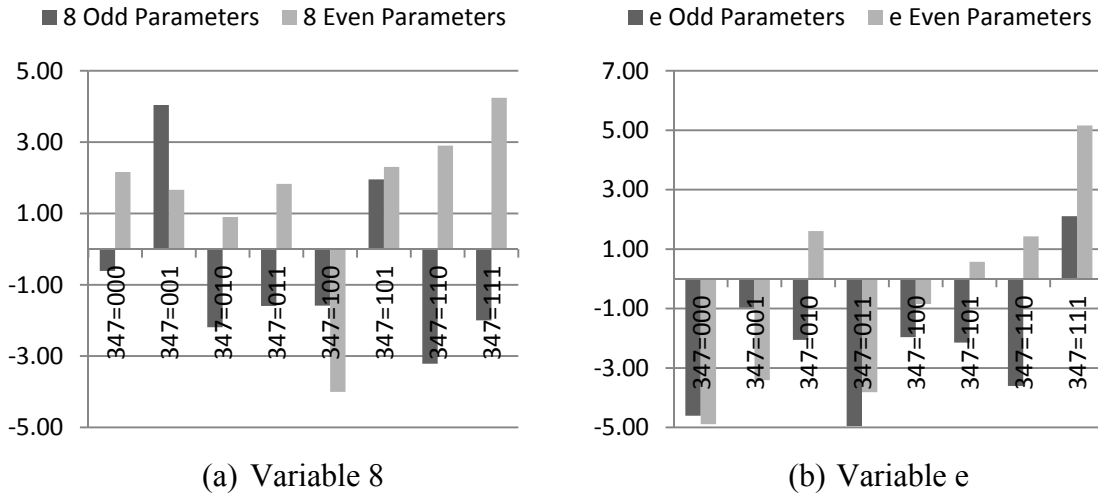


Figure 67. Integrated Odd and Even Parameters of 8e in Subtrees 347=xxx of 2ab=111 of PPF

VI.2.5.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by each end of the subtree using equation 45. The values of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46, and are shown in Table 64. The resulting parameters of all subtrees are shown in Table 65. In total, 292 experiments were performed. The performance characteristic is projected in all subtrees, then the global model parameters are found. The highest forty parameters are shown in Table 66 and Figure 68.

Table 64
Offset Errors in All Subtrees of PPF

| Subtree 2ab134789abde States | $\beta_{0,1,-1,Error}$ | Subtree 2ab134789abde States | $\beta_{0,1,-1,Error}$ | Subtree 2ab134789abde States | $\beta_{0,1,-1,Error}$ |
|------------------------------------|------------------------|------------------------------------|------------------------|------------------------------------|------------------------|
| 0000x0x0xxxxx | 9.33 | 1001xxx1xxx01 | -5.54 | 110x0x00xxxxx | -19.95 |
| 0000x0x1xxxxx | 7.59 | 1000xxx0xxx10 | -4.99 | 110x0x01xxxxx | -15.47 |
| 0000x1x0xxxxx | 17.96 | 1000xxx1xxx10 | 2.74 | 110x0x10xxxxx | -20.75 |
| 0000x1x1xxxxx | 13.27 | 1001xxx0xxx10 | -7.79 | 110x0x11xxxxx | -9.33 |
| 0001x0x0xxxxx | 4.16 | 1001xxx1xxx10 | -4.17 | 110x1x00xxxxx | -21.31 |
| 0001x0x1xxxxx | 3.26 | 100xxxxx0xx11 | 5.42 | 110x1x01xxxxx | -4.11 |
| 0001x1x0xxxxx | 6.35 | 100xxxxx1xx11 | -7.00 | 110x1x10xxxxx | -22.01 |
| 0001x1x1xxxxx | 4.75 | 101xx0xx0xxx0 | -5.61 | 110x1x11xxxxx | -12.96 |
| 0010xxxx0xxxx | 3.35 | 101xx0xx0xxx1 | 1.06 | 111x0000xxxx0 | -21.32 |
| 0010xxxx1xxxx | 2.82 | 1010x0x01xxx0 | -8.22 | 111x0000xxxx1 | -23.40 |
| 0011xxxx0xxxx | 11.72 | 1010x0x11xxx0 | -2.88 | 111x0001xxxx0 | -9.29 |
| 0011xxxx1xxxx | 7.44 | 1011x0x01xxx0 | -4.86 | 111x0001xxxx1 | -18.72 |
| 010x0xxx0xx0x | 3.03 | 1011x0x11xxx0 | -5.66 | 111x001xxxxx0 | -15.83 |
| 010x0xxx0xx1x | 2.87 | 1010x0x01xxx1 | -8.01 | 111x001xxxxx1 | -22.65 |
| 010x0xxx1xx0x | 2.97 | 1010x0x11xxx1 | -4.33 | 111x010xxxxxx | -14.36 |
| 010x0xxx1xx1x | -0.02 | 1011x0x01xxx1 | -2.18 | 111x011xxxxx0 | -10.59 |
| 010x1xxx0xx0x | -0.76 | 1011x0x11xxx1 | -5.11 | 111x011xxxxx1 | -18.22 |
| 010x1xxx0xx1x | 4.30 | 1010x1xx0xxx0 | -8.48 | 111x1000xxxxx | -15.85 |
| 010x1xxx1xx0x | -0.66 | 1011x1xx0xxx0 | -0.43 | 111x1001xxxxx | -23.85 |
| 010x1xxx1xx1x | 0.08 | 101xx1xx0xxx1 | -6.98 | 111x1010xxxxx | -26.67 |
| 011xxxxxxxxxxx | -3.50 | 1010x1x00xxx1 | -4.39 | 111x1011xxxxx | -22.06 |
| 1000xxx0xxx00 | -3.77 | 1010x1x10xxx1 | -6.67 | 111x1100xxxxx | -16.26 |
| 1000xxx1xxx00 | -9.63 | 1011x1x00xxx1 | -6.94 | 111x1101xxxxx | -10.46 |
| 1001xxx0xxx00 | -9.88 | 1011x1x10xxx1 | 1.16 | 111x1110xxxx0 | -19.07 |
| 1001xxx1xxx00 | -4.68 | 1010x1x01xxx1 | 3.97 | 111x1110xxxx1 | -10.72 |
| 1000xxx0xxx01 | -4.51 | 1010x1x11xxx1 | -2.14 | 111x1111xxxx0 | -12.56 |
| 1000xxx1xxx01 | -8.85 | 1011x1x01xxx1 | -5.40 | 111x1111xxxx1 | -7.05 |
| 1001xxx0xxx01 | -4.56 | 1011x1x11xxx1 | -9.68 | | |

Table 65
Parameters in All Subtrees of PPF

| Parameter Index | 2ab=000, 148=xxx | 2ab=001, 19=xx | 2ab=010, 39d=xxx | 2ab=011 |
|-----------------|---------------------|-------------------|---------------------|---------|
| 1 | | | 4.32 | 6.17 |
| 2 | | | | |
| 3 | -0.69 | -0.86 | | 1.89 |
| 4 | | 5.24 | 6.33 | 4.49 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 4.21 | 4.21 | 4.21 | 4.21 |
| 7 | -3.21 | -3.67 | -0.09 | 1.48 |
| 8 | | -4.55 | -1.80 | -1.17 |
| 9 | 6.25 | | | 13.28 |
| a | | | | |
| b | | | | |
| c | -0.73 | -1.68 | -0.11 | 0.62 |
| d | -2.06 | 0.63 | 0.00 | -0.55 |
| e | -6.26 | -3.17 | -1.06 | -1.66 |

| Parameter Index | 2ab=100, de=00, 18=xx | 2ab=100, de=01, 18=xx | 2ab=100, de=10, 18=xx | 2ab=100, de=11, 9=x |
|-----------------|-----------------------------|-----------------------------|-----------------------------|---------------------------|
| 1 | | | | 2.88 |
| 2 | | | | |
| 3 | -1.50 | -1.50 | -1.50 | -1.50 |
| 4 | 10.38 | 10.38 | 10.38 | 10.38 |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 4.21 | 4.21 | 4.21 | 4.21 |
| 7 | -1.66 | -1.66 | -1.66 | -1.66 |
| 8 | | | | -18.44 |
| 9 | 8.22 | 6.88 | 5.28 | |
| a | | | | |
| b | | | | |
| c | 3.71 | 3.71 | 3.71 | 3.71 |
| d | | | | |
| e | | | | |

Table 65 Continued

| Parameter Index | 2ab=101, 49e=000 | 2ab=101, 49e=001 | 2ab=101, 49e=010,011,110,111, 18=xx | 2ab=101, 49e=100, 1=x |
|-----------------|---------------------|---------------------|---|-----------------------------|
| 1 | 4.41 | -0.45 | | |
| 2 | | | | |
| 3 | 2.05 | 2.05 | 2.05 | 2.05 |
| 4 | | | | |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 4.21 | 4.21 | 4.21 | 4.21 |
| 7 | 0.64 | 0.64 | 0.64 | 0.64 |
| 8 | -5.95 | -12.74 | 0.00 | -3.31 |
| 9 | | | | |
| a | | | | |
| b | | | | |
| c | -1.26 | -1.26 | -1.26 | -1.26 |
| d | 0.34 | 0.34 | 0.34 | 0.34 |
| e | | | | |
| Parameter Index | 2ab=101, 49e=101 | 2ab=110 378=xxx | 2ab=111, 347=000, 8e=xx | 2ab=111, 347=001, e=x |
| 1 | 10.04 | 7.04 | 8.59 | 8.59 |
| 2 | | | | |
| 3 | 2.05 | | | |
| 4 | | 9.38 | | |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 4.21 | 4.21 | 4.21 | 4.21 |
| 7 | 0.64 | | | |
| 8 | -6.37 | | | 4.04 |
| 9 | | 8.71 | 13.17 | 13.17 |
| a | | | | |
| b | | | | |
| c | -1.26 | 3.40 | 6.40 | 6.40 |
| d | 0.34 | 0.09 | 3.69 | 3.69 |
| e | | 3.86 | | |

Table 65 Continued

| Parameter Index | 2ab=111, 347=010 | 2ab=111, 347=011, e=x | 2ab=111, 347=100, 8=x | 2ab=111, 347=101, 8=x |
|-----------------|---------------------|-----------------------------|-----------------------------|-----------------------------|
| 1 | 8.59 | 8.59 | 8.59 | 8.59 |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | 0.00 | 0.00 | 0.00 | 0.00 |
| 6 | 4.21 | 4.21 | 4.21 | 4.21 |
| 7 | | | | |
| 8 | -2.19 | -1.60 | | |
| 9 | 13.17 | 13.17 | 13.17 | 13.17 |
| a | | | | |
| b | | | | |
| c | 6.40 | 6.40 | 6.40 | 6.40 |
| d | 3.69 | 3.69 | 3.69 | 3.69 |
| e | -2.05 | | -1.96 | -2.15 |

| Parameter Index | 2ab=111, 347=110, 8=x | 2ab=111, 347=111, 8e=xx |
|-----------------|-----------------------------|-------------------------------|
| 1 | 8.59 | 8.59 |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | 0.00 | 0.00 |
| 6 | 4.21 | 4.21 |
| 7 | | |
| 8 | | |
| 9 | 13.17 | 13.17 |
| a | | |
| b | | |
| c | 6.40 | 6.40 |
| d | 3.69 | 3.69 |
| e | -3.61 | |

Table 66
Highest Forty Parameters of PPF Using the MHIV Method

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | 9.3388 | 'abe' | 1.4622 |
| '4' | 7.306 | '1a' | -1.3891 |
| '2' | 4.9845 | '28' | 1.3731 |
| '1' | 4.9745 | '2ae' | -1.3538 |
| 'a' | -4.7213 | 'ac' | 1.2098 |
| '8' | 4.4543 | '7a' | -1.2062 |
| '6' | 4.1129 | 'c' | -1.2023 |
| '8a' | 3.6607 | '2abd' | 1.1996 |
| 'e' | -3.2837 | 'abc' | 1.1753 |
| 'ae' | -2.9898 | '1b' | 1.0242 |
| '2a' | -2.9771 | '18b' | 0.98346 |
| 'b' | -2.968 | '18ab' | 0.97874 |
| '9b' | 2.3718 | '2d' | -0.90299 |
| '2b' | 2.1515 | '4b' | -0.85646 |
| '28a' | 2.0706 | '9a' | -0.82659 |
| 'ab' | 2.0116 | '27a' | 0.81844 |
| '2c' | -1.6417 | '28ae' | 0.80514 |
| '18' | -1.6255 | '8ae' | 0.80259 |
| '18a' | -1.601 | '8e' | 0.7984 |
| '24' | 1.5775 | '28e' | 0.78933 |

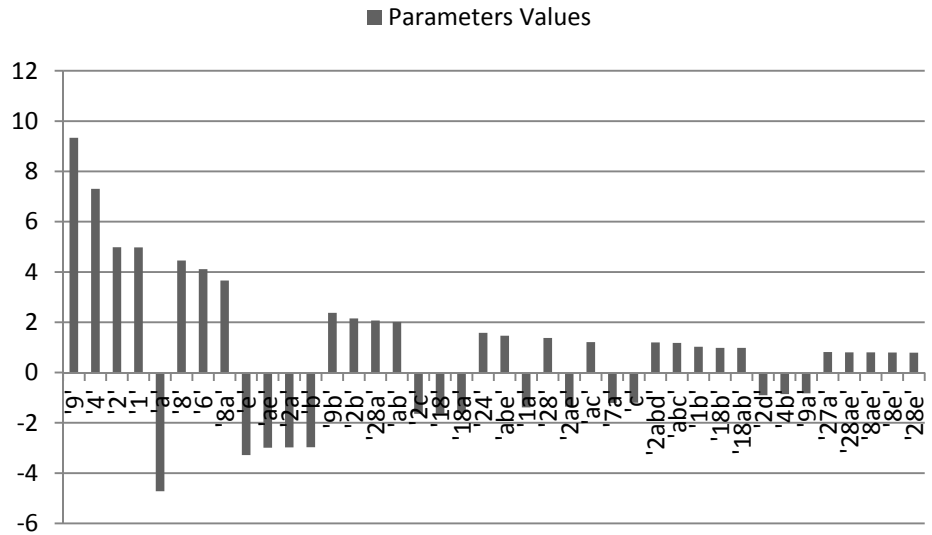


Figure 68. Highest Forty Parameters of PPF

VI.2.5.7 Validation

Since it was concluded earlier that the MHIV method is not expected to perform well on this performance characteristic due to the nature of the variables' interactions, a high Res FFD needs to be performed. A 256 Res V FFD set of experiments was performed. The results of the validation set are presented in Appendix G. The errors of the projected PPF using the MHIV method are shown Figure 69. The errors of the projected PPF using FFDs are shown in Figure 70. The RMSE of the MHIV method was found as 9.1, which is around 6.6% of the PPF projected range and 4.5 times the threshold. The RMSE of the 256 FFD was found as 6.52, which is around 30% lower than the RMSE of the MHIV method. The parameters of the 256 FFD are shown in Table 67 and Figure 71.

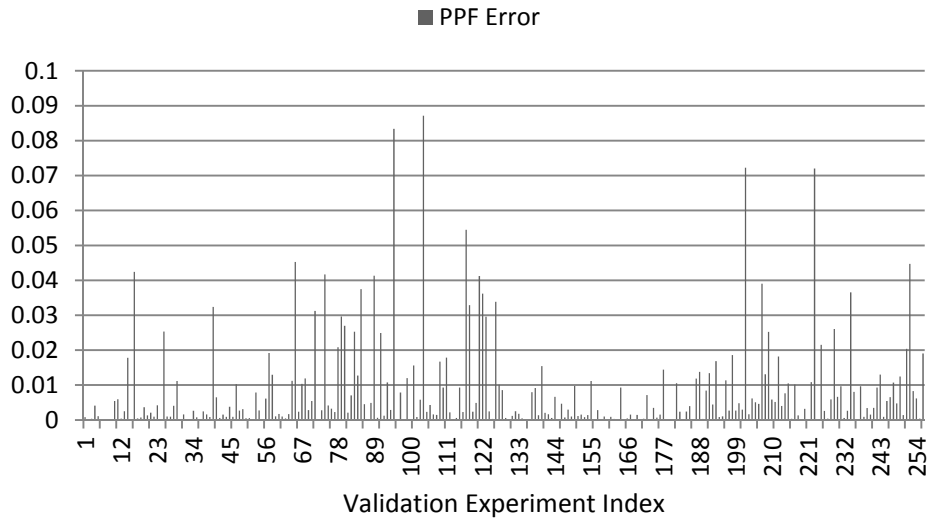


Figure 69. PPF’s Projection Error of the MHIV Method with Respect to the 256 Performed Experiments of the FFD

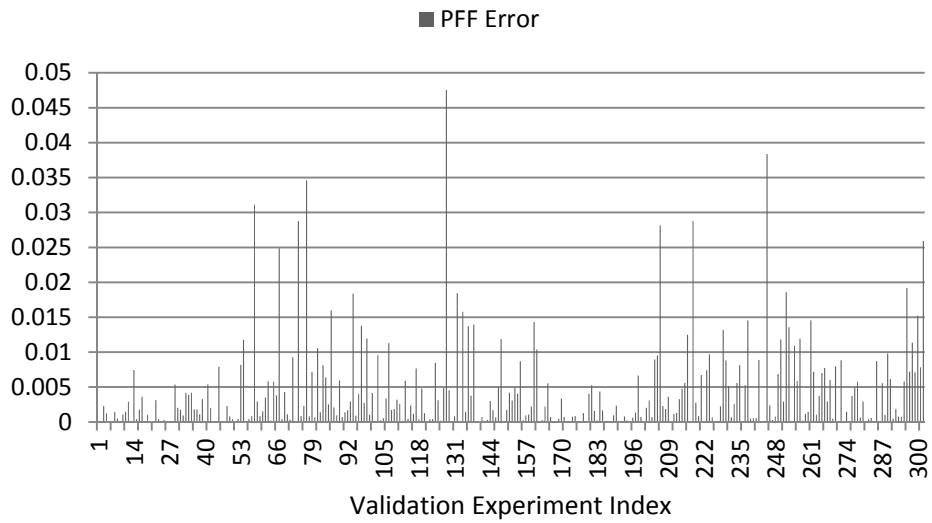


Figure 70. PPF’s Projection Error of the 256 FFD with Respect to the Performed Experiments of the MHIV Method

Table 67
Highest Forty Parameters of PPF Using a 256 Res V FFD

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '9' | 8.3027 | '18a' | -1.4306 |
| '4' | 7.5333 | '29' | -1.2766 |
| '2' | 5.7365 | '49' | 1.1405 |
| '8' | 5.4852 | 'ab' | 1.0974 |
| '1' | 4.9249 | '189' | -1.0513 |
| 'a' | -4.6061 | '169' | -0.94178 |
| 'e' | -4.1716 | '7e' | 0.91107 |
| '2a' | -3.936 | '46' | 0.90051 |
| '18' | -3.1934 | '2ae' | -0.88414 |
| '8a' | 3.0531 | '1ae' | -0.86767 |
| 'ae' | -2.6135 | '68' | 0.8675 |
| 'b' | -2.4644 | '7d' | 0.86518 |
| '6' | 2.2713 | '8e' | 0.84945 |
| '9b' | 2.1151 | '4a' | 0.82802 |
| '2e' | -2.1077 | '16' | -0.79712 |
| '2b' | 2.0579 | '7bc' | -0.79184 |
| '28' | 1.7501 | '28b' | -0.78641 |
| '14' | 1.6095 | '9ab' | -0.78148 |
| '24' | 1.5694 | '28a' | 0.76893 |
| '19' | -1.4958 | '18b' | 0.76105 |

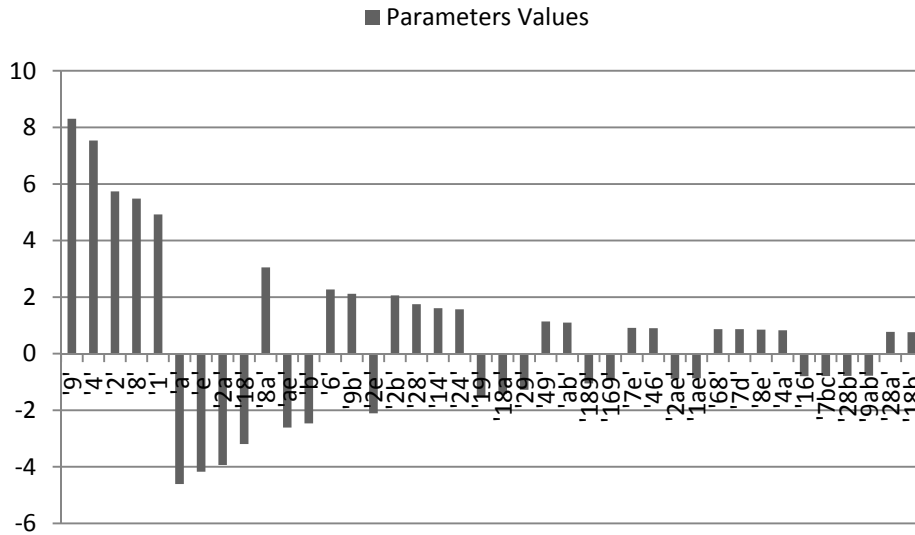


Figure 71. Highest Forty Parameters of PPF Using a 256 Res V FFD

VI.2.6 Multiplication Factor Depletion Analysis

In this section, K_{eff} is analyzed with respect to time. As the reactor core is depleted, K_{eff} is expected to drop due to fissile materials depletion (see [85] for definition). The aim for the effective multiplication factor is to not drop below one at the End of Life (EOL). The drop of K_{eff} (ΔK_{eff}) is defined as the difference between K_{eff} at the BOL and K_{eff} at the EOL, which is 5 years from BOL.

VI.2.6.1 Performance Characteristic Directional Alignment

Since the SMR is a water cooled thermal reactor and has a lower than one conversion ratio, ΔK_{eff} after five years cannot be negative, thus ΔK_{eff} is expected to

have a plateau at its low end. Since the saturation plateau is to be aligned with the low end of the performance characteristic, the performance characteristic is defined as:

$$\Delta K_{eff} = K_{eff,BOL} - K_{eff,5y} \quad (63)$$

VI.2.6.2 Variables Directional Alignment

The directional dependence of ΔK_{eff} on the variables is determined at this stage. Variables are categorized into monotonic variables and variables with unknown direction of effect. The criterion used is that as the core fuel mass increase or the power decrease, ΔK_{eff} is expected to decrease. Power is an active variable on this performance characteristic, and is considered as a monotonic variable. The variables' directions of effect are shown in Table 68. The directional dependence is briefly justified in Table 69. According to the table, the maximum ΔK_{eff} occurs at the variables' states of 00011100100000, thus the variables are aligned to all high at this combination.

Table 68Variables' Directions of Effect on ΔK_{eff}

| Variable Index | Variable | Direction of Effect on ΔK_{eff} |
|----------------|---------------------------------|---|
| 1 | No of Fuel Assemblies | - |
| 2 | Active Fuel Height | - |
| 3 | Core Radial Surrounding Water | - |
| 4 | Enrichments | + |
| 5 | Power | + |
| 6 | Moderator Density | + |
| 7 | Fuel Temperature Lib | - |
| 8 | No of Rods per Assembly | - |
| 9 | Rod Pitch | + |
| 10 (a) | No of DBARs per Assembly | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | - |
| 12 (c) | IFAB and Fuel Gap Thickness | - |
| 13 (d) | IFAB and Fuel Clad Thickness | - |
| 14 (e) | Discrete Burnable Absorber Area | - |

Table 69Justification of Variables' Directions of Effect on ΔK_{eff}

| Variable Index | Variable | Justification | Direction of Effect on ΔK_{eff} |
|----------------|---------------------------------|---|---|
| 1 | No of Fuel Assemblies | ▪ Higher fuel to power ratio | - |
| 2 | Active Fuel Height | ▪ Higher fuel to power ratio | - |
| 3 | Core Radial Surrounding Water | ▪ Unknown effect | ?(-) |
| 4 | Enrichments | ▪ More fissile Material | -(+) |
| 5 | Power | ▪ Faster fuel depletion | + |
| 6 | Moderator Density | ▪ Thermalizing spectrum reduces breeding | + |
| 7 | Fuel Temperature Lib | ▪ Unknown effect | ?(-) |
| 8 | No of Rods per Assembly | ▪ Higher fuel to power ratio | - |
| 9 | Rod Pitch | ▪ Thermalizing spectrum reduces breeding | + |
| 10 (a) | No of DBARs per Assembly | ▪ Absorber depletes with time, thus adds reactivity | - |
| 11 (b) | IFAB and Fuel Pellet Diameter | ▪ Higher fuel to power ratio | - |
| 12 (c) | IFAB and Fuel Gap Thickness | ▪ Hardening spectrum increases breeding | - |
| 13 (d) | IFAB and Fuel Clad Thickness | ▪ Hardening spectrum increases breeding | - |
| 14 (e) | Discrete Burnable Absorber Area | ▪ Absorber depletes with time, thus adds reactivity | - |

VI.2.6.3 Variables Integrated Parameters Determination

Since fourteen variables are explored, thirty experiments are performed at this stage. The results of the performed experiments are shown in Appendix G. The

corresponding integrated odd and even parameters of all variables are shown in Table 70. The units used for ΔK_{eff} are in 10^{-3} from here onwards.

Table 70
Integrated Odd and Even Parameters in the Main Tree of ΔK_{eff}

| Variable | Top End Drop | Bottom End Drop | Integrated Odd Parameters | Integrated Even Parameters |
|----------|--------------|-----------------|---------------------------|----------------------------|
| 1 | 40.215 | 20.74 | 30.478 | 9.737 |
| 2 | 58.805 | 41.535 | 50.17 | 8.635 |
| 3 | 0.56 | -0.29 | 0.135 | 0.425 |
| 4 | 39.325 | 12.425 | 25.875 | 13.45 |
| 5 | 66.66 | 40.225 | 53.443 | 13.218 |
| 6 | 19.165 | 1.2 | 10.183 | 8.982 |
| 7 | 5.345 | -0.56 | 2.39 | 2.955 |
| 8 | 20.69 | -1.84 | 9.423 | 11.268 |
| 9 | 76.425 | 6.405 | 41.415 | 35.01 |
| a | -2.59 | 18.43 | 7.923 | -10.51 |
| b | 80.425 | 5.99 | 43.208 | 37.218 |
| c | 1.295 | -0.95 | 0.173 | 1.122 |
| d | 3.61 | 0.515 | 2.063 | 1.547 |
| e | 8.58 | 10.23 | 9.405 | -0.82 |

VI.2.6.4 Variables Categorization

The results of the integrated odd and even parameters are sorted according to the strength of integrated even parameters and presented in Figure 72. It is clear from the magnitudes of the integrated even parameters that most of the variables are interacting. If a threshold of integrated even parameters is set at 8.5, it is possible to categorize variables 9b as most interacting, variables 124568a as interacting (a is slightly non-

monotonic), variables e as important and non-interacting and variables 37cd as non-important and non-interacting. The non-interacting variables are not further explored.

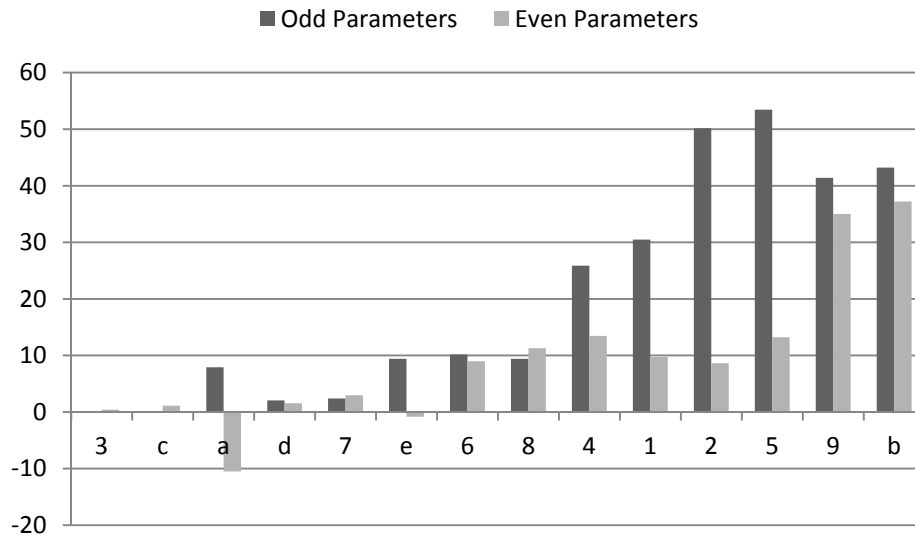


Figure 72. Integrated Odd and Even Parameters in the Main Tree of ΔK_{eff}

VI.2.6.5 Variables Interactions Exploration

Since two variables are selected as the most interacting variables, four subtrees are to be established representing all binary states of 9b=xx. Within each of these subtrees, the integrated odd and even parameters of each of 124568a are found. In total fourteen experiments per subtree is performed. However, the top and bottom subtrees already have seven experiments performed in each, thus the total number of experiments performed at this stage is 42. The resulting integrated odd and even parameters of variables 124568a are listed in Table 71 and Table 72, and shown in Figure 73.

Checking the integrated even parameters against the set threshold of 8.5, the variables highlighted in grey in Table 71 and Table 72 of subtrees 9b=xx are not further explored, and their integrated odd parameters are used as the first order parameters in 9b spanned subtrees. The results of all performed experiments are shown in Appendix G.

Table 71

Integrated Odd Parameters of 124568a in Subtrees 9b=xx of ΔK_{eff}

| Variable | 9b=00 | 9b=01 | 9b=10 | 9b=11 |
|----------|--------|--------|--------|---------|
| 1 | 16.398 | 41.500 | 31.265 | 62.818 |
| 2 | 32.875 | 67.605 | 58.118 | 95.448 |
| 4 | 20.470 | 22.755 | 20.418 | 20.188 |
| 5 | 33.498 | 69.065 | 61.125 | 101.115 |
| 6 | 8.125 | 12.458 | 9.643 | 13.233 |
| 8 | -0.145 | 12.413 | 7.200 | 26.695 |
| a | 7.857 | 10.940 | 9.455 | 12.033 |

Table 72

Integrated Even Parameters of 124568a in Subtrees 9b=xx of ΔK_{eff}

| Variable | 9b=00 | 9b=01 | 9b=10 | 9b=11 |
|----------|---------|---------|---------|---------|
| 1 | -4.343 | -8.305 | -12.290 | -22.603 |
| 2 | -8.660 | -17.440 | -28.708 | -36.643 |
| 4 | 8.045 | 12.090 | 12.518 | 19.138 |
| 5 | -6.728 | -15.445 | -25.995 | -34.455 |
| 6 | 6.925 | 6.943 | 6.943 | 5.932 |
| 8 | 1.700 | 1.443 | -1.000 | -6.005 |
| a | -10.573 | -17.975 | -11.845 | -14.618 |

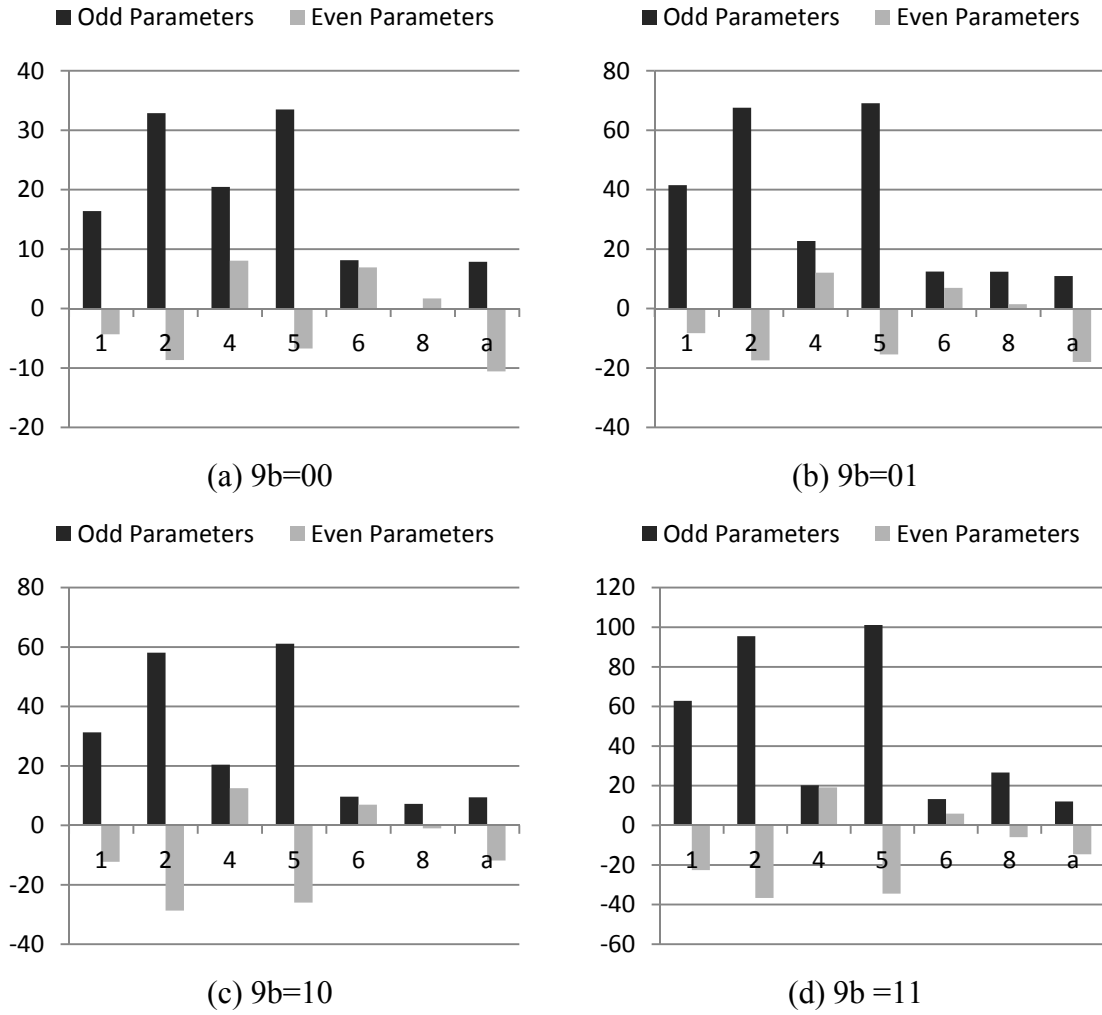


Figure 73. Integrated Odd and Even Parameters of 124568a in Subtrees 9b=xx of ΔK_{eff}

In Subtree 9b=00, variables 2 and a are the only interacting variables, thus 2a are unfolded by creating subtrees 2a=xx of 9b=00. Subtree 9b=01 has four interacting variables 245a. The two most interacting variables of 9b=01 are unfolded first, thus subtrees 2a=xx of 9b=01 are created. In every subtree, the integrated odd and even parameters of variables 45 are determined. The results are shown in Table 73 and Figure 74. It is clear that except in subtree 2a=10, variables 45 are interacting in all subtrees of 2a=xx of 9b=01. As a result, subtrees 45=xx are created in 2a=00,01,11 of 9b=01 and subtrees 5=x are created in 2a=10 of 9b=01.

Table 73

Integrated Odd and Even Parameters of 45 in Subtrees 2a=xx of 9b=01 of ΔK_{eff}

| Odd Parameters | | | | |
|-----------------|---------|--------|--------|--------|
| Variable | 2a=00 | 2a=01 | 2a=10 | 2a=11 |
| 4 | 25.158 | 21.583 | 29.948 | 26.143 |
| 5 | 68.245 | 72.978 | 53.238 | 43.738 |
| Even Parameters | | | | |
| Variable | 2a=00 | 2a=01 | 2a=10 | 2a=11 |
| 4 | 14.493 | 12.933 | 7.492 | 8.703 |
| 5 | -16.265 | -9.728 | 19.563 | 9.883 |

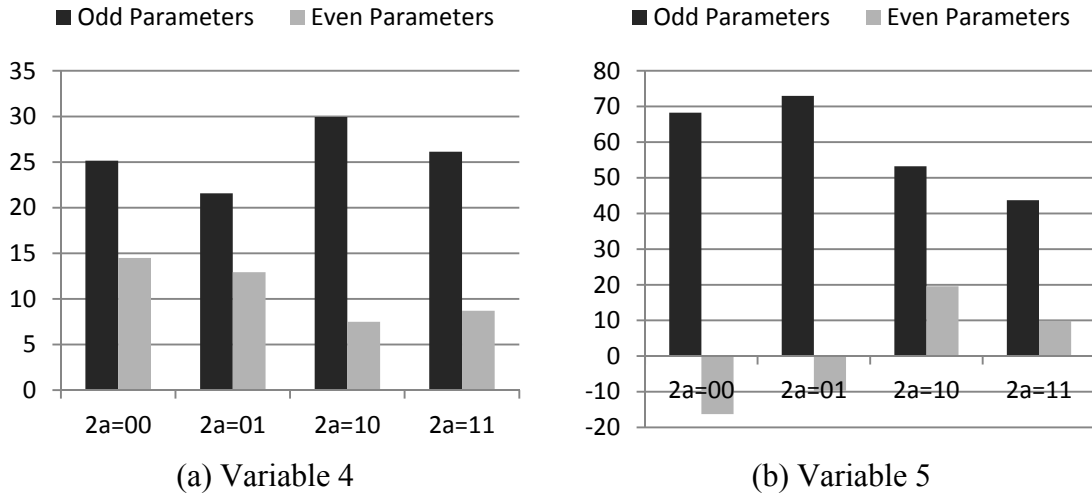


Figure 74. Integrated Odd and Even Parameters of 45 in Subtrees 2a=xx of 9b=01 of ΔK_{eff}

Subtree 9b=10 has variables 1245a as interacting variables. The two most interacting variables 25 are selected, and subtrees 25=xx of 9b=10 are created. Within every subtree of 25=xx, the integrated odd and even interactions of variables 14a are found. The results are shown in Table 74 and Figure 75. As a result, subtrees 1a=xx are created in 25=00 of 9b=10, subtrees 4=x are created in 25=01,10 of 9b=10, and subtrees a=x are created in 25=11 of 9b=10.

Subtree 9b=11 is similar to subtree 9b=10, since it has variables 1245a as interacting. Since the values of the integrated even parameters of these variables are high, three variables, 125, are unfolded. Within every subtree of 125=xxx, the integrated odd and even interactions of variables 4a are found. The results are shown in Table 75 and Figure 76. Subtrees 125=001,010,011,101,110 of 9b=11 have no interacting variables, and require no subtrees spanning. Subtrees 125=000,100,111 have one interacting variable, a, thus subtrees a=x of 125=000,100,111 of 9b=11 are created. The results of all performed experiments are shown in Appendix G.

Table 74

Integrated Odd and Even Parameters of 14a in Subtrees 25=xx of 9b=10 of ΔK_{eff}

| Odd Parameters | | | | |
|-----------------|---------|--------|--------|--------|
| Variable | 25=00 | 25=01 | 25=10 | 25=11 |
| 1 | 53.623 | 41.865 | 40.370 | 17.445 |
| 4 | 4.080 | 22.890 | 22.630 | 35.305 |
| a | 8.945 | 11.340 | 9.763 | 7.600 |
| Even Parameters | | | | |
| Variable | 25=00 | 25=01 | 25=10 | 25=11 |
| 1 | 10.068 | -0.145 | 0.140 | 1.530 |
| 4 | -3.820 | 13.305 | 11.660 | -2.370 |
| a | -12.355 | -5.065 | -4.328 | -9.990 |

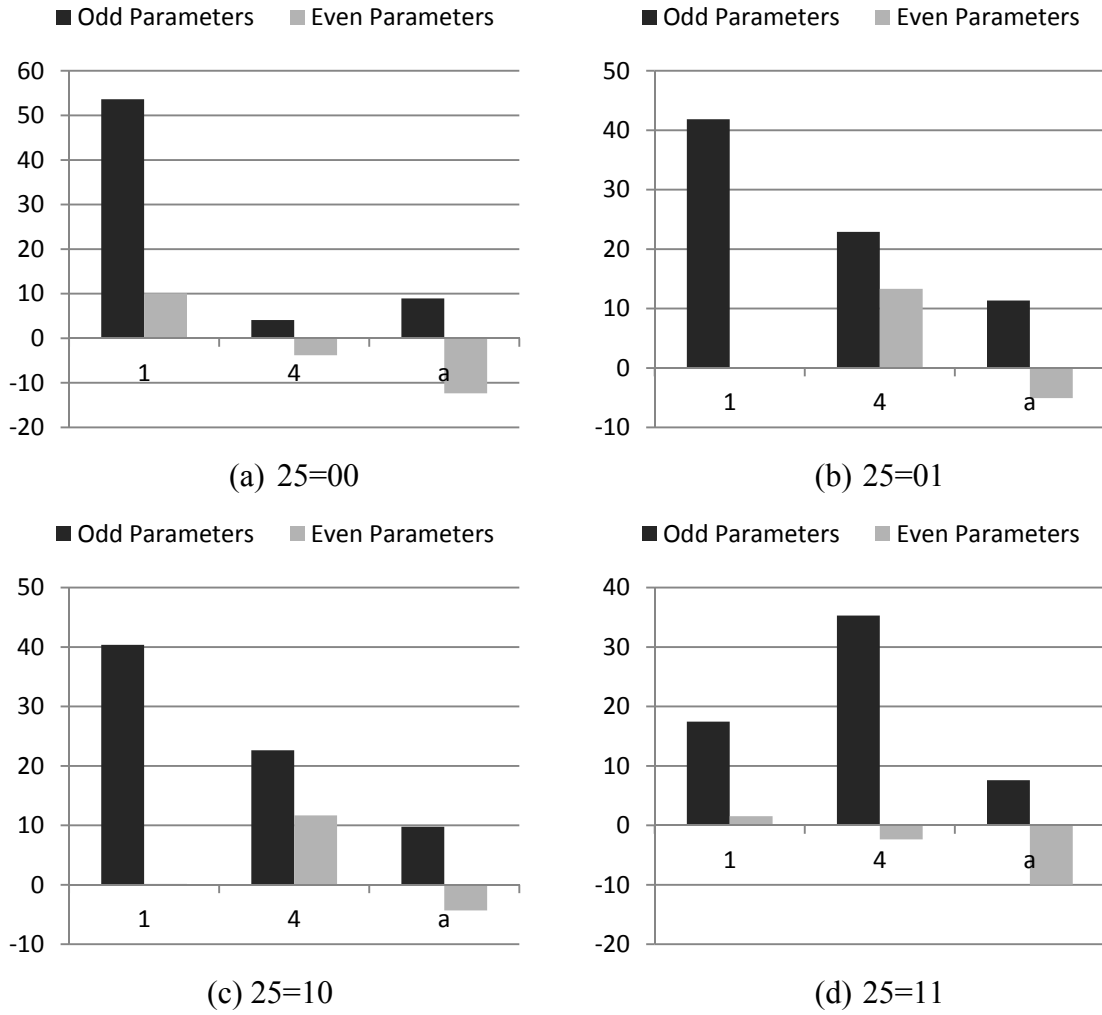


Figure 75. Integrated Odd and Even Parameters of 14a in Subtrees 25=xx of 9b=01 of ΔK_{eff}

Table 75

Integrated Odd and Even Parameters of 4a in Subtrees $125=xxx$ of $9b=11$ of ΔK_{eff}

| Odd Parameters | | 125= | | | | | | |
|-----------------|---------|--------|--------|--------|---------|--------|--------|---------|
| Variable | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 4 | -4.320 | 36.865 | 36.568 | 42.778 | 9.558 | 43.338 | 41.515 | 39.513 |
| a | 13.818 | 18.380 | 17.425 | 17.738 | 7.818 | 21.128 | 19.740 | 10.463 |
| Even Parameters | | 125= | | | | | | |
| Variable | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 4 | -5.370 | 6.465 | 7.538 | 0.422 | 8.403 | -0.728 | -0.445 | -0.188 |
| a | -12.833 | -6.610 | -4.530 | -8.253 | -11.718 | -5.683 | -6.275 | -13.048 |

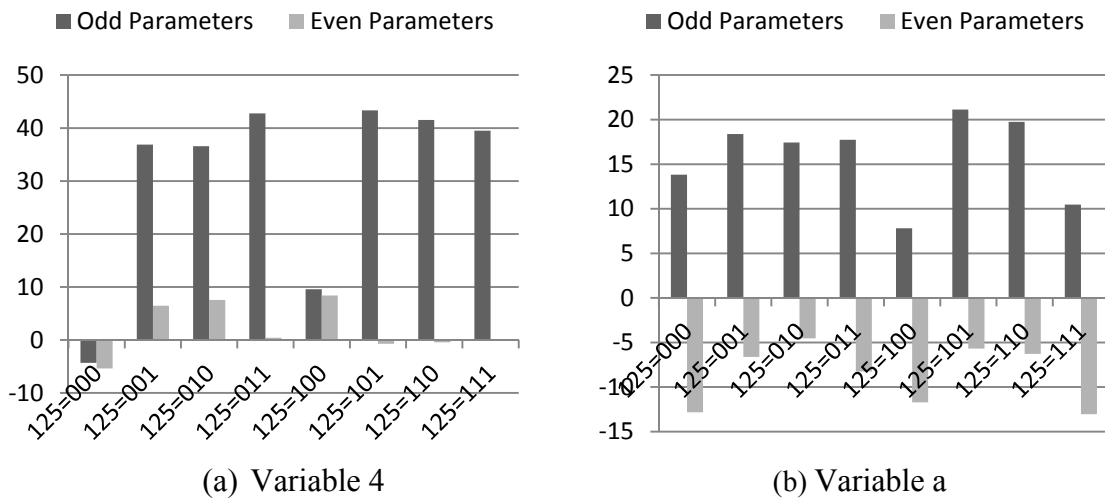


Figure 76. Integrated Odd and Even Parameters of 4a in Subtrees $125=xxx$ of $9b=11$ of ΔK_{eff}

VI.2.6.6 Projection of Subtrees and Results Generation

The offset parameter of every subtree is found by averaging the offset determined by each end of the subtree using equation 45. The values of $\beta_{0,1,-1,Error}$ in every subtree is found using equation 46, and are shown in Table 76. The resulting parameters of all subtrees are shown in Table 77. In total, 154 experiments were performed. The performance characteristic is projected in all subtrees, then the global model parameters are found. The highest forty parameters are shown in Table 78 and Figure 77.

VI.2.6.7 Validation

Since the MHIV method was compared earlier with FFDs, the target of this section is to only evaluate the projected results with respect to a validation set. In order to ensure that the validation set is as spread as possible in the domain of experiments, the validation set selected is a FFD set of sixteen experiments. The results of the validation set are presented in Appendix G. Figure 78 shows the errors of the projected ΔK_{eff} . The RMSE was found as 24.4, which is around 3.67% of the projected range, and 2.87 times the threshold. The error ratio of RMSE to threshold is slightly higher in this example than with the other performance characteristics. This is because the threshold was selected slightly below the integrated even parameters of many variables, to limit the number of experiment.

Table 76Offset Errors in All Subtrees of ΔK_{eff}

| | | | | | | | |
|------------------------------|---------|---------|---------|---------|---------|---------|---------|
| Subtree 9b1245a States | 00x0xx0 | 00x0xx1 | 00x1xx0 | 00x1xx1 | 01x0000 | 01x0010 | 01x0100 |
| $\beta_{0,1,-1,Error}$ | 46.755 | 30.505 | 34.330 | 15.105 | 37.570 | -13.920 | 47.595 |
| Subtree 9b1245a States | 01x0110 | 01x0001 | 01x0011 | 01x0101 | 01x0111 | 01x1001 | 01x1011 |
| $\beta_{0,1,-1,Error}$ | 2.840 | 15.580 | -25.935 | 19.385 | -10.005 | -10.393 | 28.733 |
| Subtree 9b1245a States | 01x1101 | 01x1111 | 01x1x00 | 01x1x10 | 1000x00 | 1000x01 | 1010x00 |
| $\beta_{0,1,-1,Error}$ | -27.360 | -7.220 | -9.580 | -6.600 | 3.762 | -10.488 | 34.358 |
| Subtree 9b1245a States | 1010x01 | 10x001x | 10x011x | 10x100x | 10x110x | 10x1x10 | 10x1x11 |
| $\beta_{0,1,-1,Error}$ | 11.883 | 9.547 | 36.158 | 9.100 | 32.420 | 24.388 | 4.407 |
| Subtree 9b1245a States | 1100x00 | 1100x01 | 1100x1x | 1101x0x | 1101x1x | 1110x00 | 1110x01 |
| $\beta_{0,1,-1,Error}$ | 17.993 | -7.672 | -5.713 | -7.365 | -11.278 | 42.600 | 19.165 |
| Subtree 9b1245a States | 1110x1x | 1111x0x | 1111x1x | 1111x1x | | | |
| $\beta_{0,1,-1,Error}$ | -22.078 | -22.368 | 25.260 | -0.835 | | | |

Table 77Parameters in All Subtrees of ΔK_{eff}

| Parameter Index | 9b=00 2a=xx | 9b=01,2a=00,01,11 45=xx | 9b=01,2a=10 5=x | |
|--------------------|----------------|----------------------------|--------------------|--|
| 1 | 16.398 | 41.500 | 41.500 | |
| 2 | | | | |
| 3 | 0.135 | 0.135 | 0.135 | |
| 4 | 20.470 | | 29.948 | |
| 5 | 33.498 | | | |
| 6 | 8.125 | 12.458 | 12.458 | |
| 7 | 2.390 | 2.390 | 2.390 | |
| 8 | -0.145 | 12.413 | 12.413 | |
| 9 | | | | |
| a | | | | |
| b | | | | |
| c | 0.172 | 0.172 | 0.172 | |
| d | 2.063 | 2.063 | 2.063 | |
| e | 9.405 | 9.405 | 9.405 | |

| Parameter Index | 9b=10,25=00 1a=xx | 9b=10,25=01 4=x | 9b=10,25=10 4=x | 9b=10,25=11 a=x |
|--------------------|----------------------|--------------------|--------------------|--------------------|
| 1 | 0.000 | 41.865 | 40.370 | 17.445 |
| 2 | | | | |
| 3 | 0.135 | 0.135 | 0.135 | 0.135 |
| 4 | 4.080 | | | 35.305 |
| 5 | | | | |
| 6 | 9.643 | 9.643 | 9.643 | 9.643 |
| 7 | 2.390 | 2.390 | 2.390 | 2.390 |
| 8 | 7.200 | 7.200 | 7.200 | 7.200 |
| 9 | | | | |
| a | | 11.340 | 9.763 | |
| b | | | | |
| c | 0.172 | 0.172 | 0.172 | 0.172 |
| d | 2.063 | 2.063 | 2.063 | 2.063 |
| e | 9.405 | 9.405 | 9.405 | 9.405 |

Table 77 Continued

| Parameter Index | 9b=11,125=000 a=x | 9b=11,125=001 | 9b=11,125=010 | 9b=11,125=011 |
|-----------------|----------------------|---------------|---------------|---------------|
| 1 | | | | |
| 2 | | | | |
| 3 | 0.135 | 0.135 | 0.135 | 0.135 |
| 4 | -4.320 | 36.865 | 36.568 | 42.778 |
| 5 | | | | |
| 6 | 13.233 | 13.233 | 13.233 | 13.233 |
| 7 | 2.390 | 2.390 | 2.390 | 2.390 |
| 8 | 26.695 | 26.695 | 26.695 | 26.695 |
| 9 | | | | |
| a | | 18.380 | 17.425 | 17.738 |
| b | | | | |
| c | 0.172 | 0.172 | 0.172 | 0.172 |
| d | 2.063 | 2.063 | 2.063 | 2.063 |
| e | 9.405 | 9.405 | 9.405 | 9.405 |

| Parameter Index | 9b=11,125=100 a=x | 9b=11,125=101 | 9b=11,125=110 | 9b=11,125=111 a=x |
|-----------------|----------------------|---------------|---------------|----------------------|
| 1 | | | | |
| 2 | | | | |
| 3 | 0.135 | 0.135 | 0.135 | 0.135 |
| 4 | 9.558 | 43.338 | 41.515 | 39.513 |
| 5 | | | | |
| 6 | 13.233 | 13.233 | 13.233 | 13.233 |
| 7 | 2.390 | 2.390 | 2.390 | 2.390 |
| 8 | 26.695 | 26.695 | 26.695 | 26.695 |
| 9 | | | | |
| a | | 21.128 | 19.740 | |
| b | | | | |
| c | 0.172 | 0.172 | 0.172 | 0.172 |
| d | 2.063 | 2.063 | 2.063 | 2.063 |
| e | 9.405 | 9.405 | 9.405 | 9.405 |

Table 78
Highest Forty Parameters of ΔK_{eff}

| Parameter Index | Parameter Value | Parameter Index | Parameter Value |
|-----------------|-----------------|-----------------|-----------------|
| '5' | 59.987 | '1b' | 6.6345 |
| '2' | -57.84 | '5b' | -6.55 |
| 'b' | -57.507 | '12' | -6.4417 |
| '9' | 55.426 | '59b' | 6.3531 |
| '1' | -34.369 | '129' | -6.3493 |
| '4' | 24.572 | '15' | 6.1055 |
| '25' | 13.595 | '159' | 6.0202 |
| '59' | 13.42 | '19b' | -5.8498 |
| 'a' | -12.138 | '45' | 5.6557 |
| '29' | -11.709 | '2b' | 5.4537 |
| '8' | -11.616 | '24' | -5.3857 |
| '6' | 10.94 | '89' | -5.3829 |
| 'e' | -9.4888 | '19' | -5.3203 |
| '259' | 9.1181 | '9b' | -5.1359 |
| '125b' | -8.1978 | '25b' | -4.5785 |
| '1259b' | -8.1743 | '4b' | -3.6028 |
| '8b' | 7.9368 | '245' | 3.2864 |
| '125' | 7.086 | '459' | 3.2113 |
| '1259' | 6.9876 | '249' | -3.1485 |
| '29b' | -6.924 | 'ab' | 2.7747 |

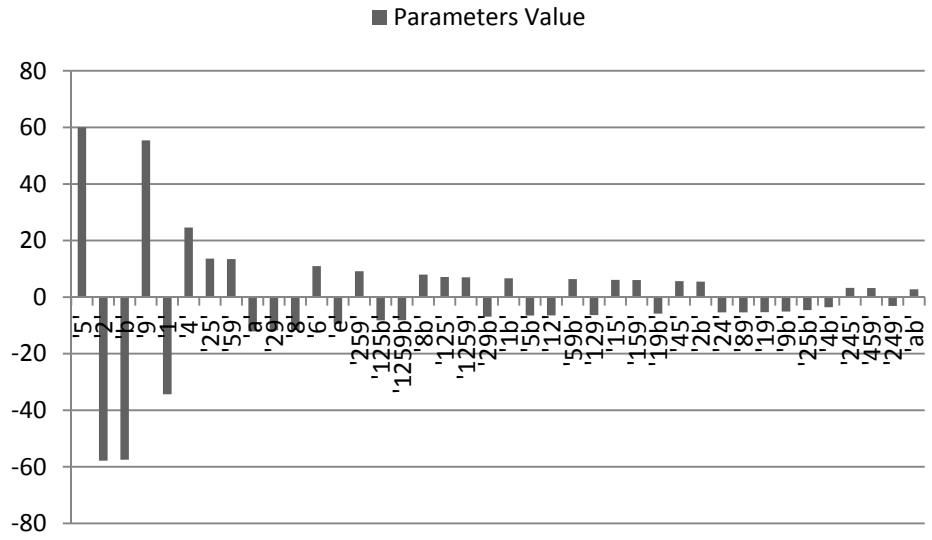


Figure 77. Highest Forty Parameters of ΔK_{eff}

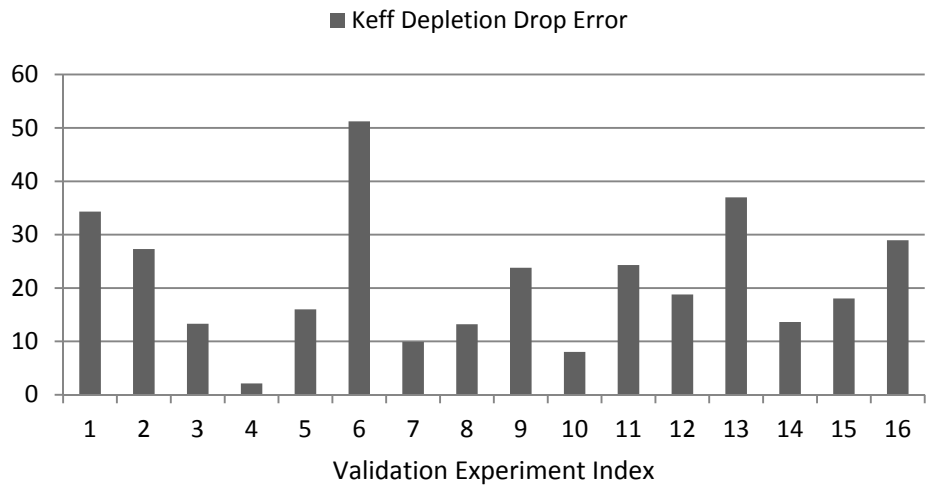


Figure 78. ΔK_{eff} 's Projection Error of the MHIV Method with Respect to the Sixteen Performed Experiments of the FFD

VII. CONCLUSIONS

VII.1 Methods

Four new methods to utilize the knowledge of variables' directions of effect and monotonic behavior were developed in this dissertation. Three of the four developed methods succeeded in reducing the projection's error from the most common existing methods, and generated a more accurate approximation of the model's parameters. The fourth method is an improvement and expansion to an existing method, but failed to produce accurate results.

The methods' first few steps, which introduced the variables' directional alignment to cotter's definition of summed odd and even parameters, and utilized common system regularities, represent a significant contribution in determining variables' strength of interactions.

Three of the four methods achieved the method's objectives of the dissertation, since they:

- Utilized various degrees of knowledge of the directions of variables' effects on the performance characteristic
- Developed an efficient process of selection of experiments in a system with large number of interacting variables
- Achieved adaptive modeling and identification of significant performance-relevant main effects and interactions with a quantified estimated response uncertainty

In summary, the dissertation introduced “Novel Evaluation Methods for Complex Systems via Adaptive Sequential Exploration of Variables Interactions”. The methods used in the application were coded to automate and simplify the experiments’ selection process. Since the two methods of MSIV and MHIV were mainly applied in the application, the findings of the two methods are summarized in the following sections.

VII.1.1 MSIV Method Specific Findings

- The application of the method on K_{eff} demonstrated that, in comparison to the 128 FFD, the MSIV method used 75% the number of experiments and resulted in 25% error reduction. The RMSE was found as 2.1% of the response range
- The uncertainty found using the difference between the integrated even parameters of the first order effects and the summation of integrated odd parameters of the second order interactions was used to predict parameters accuracy and set a stop condition. This criterion was found to perform well if the monotonic behavior is significantly present.
- If it is the interest of the experimenter to explore a certain variable, such as Variable 6, it is possible to perform a FFD on the non-interacting variables’ subtree at the low state of the variable, such as 469be=10111, to unfold its interactions with non-interacting variables.

VII.1.2 MHIV Method Specific Findings

- The application of the method on RFS demonstrated that, in comparison to the 128 FFD, the MHIV method used 101% the number of experiments, and resulted in 85% error reduction. The application of the method on K_{eff} used 55% the number of experiments, and resulted in a similar error to the 128 FFD.
- The error of the projected response was found in the same order of magnitude as the integrated even parameters threshold.
- Except for the PPF, the application of the method on all performance characteristics resulted in an RMSE that is lower than 4% of the response range.
- The PPF example demonstrated how assuming highly non-monotonic variables as monotonic resulted in 30% error increase in comparison to a FFD. This poor performance was expected due to the invalidity of the method's main assumption.
- Since two nodes are performed in every subtree, the difference of the end nodes offsets, in the subtree, was used as an indication of the validity of assuming that interactions are negligible within the subtree. Ideally, if all interactions are indeed negligible, the two found values by each end are similar.
- If it is the interest of the experimenter to explore a certain variable, such as Variable 6, it is possible to use a different threshold for the variable's integrated even parameters.

VII.1.3 General Findings

- If a variable has confounded interactions, it is likely that they will be captured, since the other parent variables of the interactions are explored to an extent that they do not interact, thus unfolding the confounded interactions.
- If a new variable, such as water boron concentration, is added to the set of variables to be evaluated, the variable is simply explored at the main tree level. It can be explored in a similar manner as if the variables were 15 instead of 14. Adding variables in the methods is easy, and does not affect the path of selected experiments as long as the threshold is kept the same.
- The number of experiments needed was driven mainly by the strength of variables' interactions. This is observed in Table 79 and Table 80. In addition, the variables' monotonic behavior had a significant effect on the convergence of interactions exploration.

Table 79
Number of Performed Experiments for the Six Performance Characteristics

| Performance Characteristic | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------------------|-----------|-----|------------|---------------|-----|------------------|
| No of Performed Experiments | 98 | 130 | 168 | 90 | 292 | 154 |

Table 80

Number and Order of Parameters in the Highest Thirty Parameters of the Six Performance Characteristics

| Parameter Order | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------|-----------|-----|------------|---------------|-----|------------------|
| 1st | 12 | 11 | 12 | 13 | 9 | 10 |
| 2nd | 13 | 15 | 16 | 14 | 16 | 10 |
| 3rd | 3 | 4 | 2 | 3 | 5 | 7 |
| 4th | 1 | 0 | 0 | 0 | 0 | 2 |
| 5th | 1 | 0 | 0 | 0 | 0 | 1 |

The main features of the dissertation's methods over FFD can be summarized as:

- Monotonic behavior is a broader physical behavior than weak interaction. A system with no interactions is a special case of systems with monotonically behaving variables.
- Monotonic behavior is a significantly present physical behavior. The behavior of weak interactions is not as significant.
- It is easier for experts to judge the significance of monotonic behavior of variables, while it is not as easy to judge the significance of interactions.
- The introduced methods enable the experimenter to determine the strength of variables' interactions on a performance characteristic and choose an appropriate resolution.
- The introduced methods bias the experiments toward important variables and interactions only.
- The introduced methods enable the experimenter to unfold a certain interaction with a small number of experiments.

- The introduced methods do not bind by a number of experiments, thus enabling the experimenter to decide on the number of experiments to invest
- The introduced methods overcome the limitation of resolution change of FFDs.
- The introduced methods allow the addition of variables after the method application without the need to reevaluate the already evaluated variables and interactions.

VII.2 Application

In this section, the results of the application of the dissertation's methods on the design of the SMR version of the advanced pressurized water reactor are analyzed. The findings are summarized as:

- An approximation model can be setup from the determined parameters, and can be used to estimate any of the six performance characteristics of any design with variables falling in the defined ranges. The errors of the performance characteristics are known from the performed validation. If validation was not performed, the error order of magnitude can be estimated from the defined threshold or from the methods' projection errors.
- The mathematical model used assumes linear behavior of single variable. This means that if all variables are fixed and one variable is varied, the behavior is expected to be close to linear. The small magnitude of the first order parameters indicates that the error due to this assumption is not expected to be significant.

- The variables were found to be weakly interacting in their effects on K_{eff} and α_{FT} , moderately interacting on their effect on the RFS and α_d , and relatively strongly interacting on their effect on the PPF and ΔK_{eff} . The list of the highest 100 parameters of the six performance characteristics are shown in Appendix H. The percentage main effects of the fourteen variables on the six performance characteristics are shown in Table 81. The percentage main effect is calculated using

$$ME_i(\%) = 100 \frac{2\beta_i}{(Y_{max} - Y_{min})}, \quad (64)$$

and is a measurement of the effect of the variable on the response in comparison to the span of the projected response. The variables are classified into levels of strength in Table 82. The same is applied to the strongest interactions, and is shown in Table 83 and Table 84.

- Variables 3,c and d, representing radial surrounding water, gap thickness and clad thickness, have weak to very weak effects on the performance characteristics, thus can be omitted from the list of variable in the optimization process. The weak effect of the radial surrounding water indicates that the radial surrounding water is sufficient to sustain good neutrons economy and small fluence on the vessel.

Table 81

Heat Map of the Percentage Main Effects on the Six Performance Characteristics

| Parameter Index | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------|-----------|--------|------------|---------------|--------|------------------|
| '0' | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |
| '1' | 8.25 | 1.69 | -16.75 | -1.10 | 7.17 | -10.18 |
| '2' | 8.66 | 3.13 | -15.52 | -1.51 | 8.35 | -17.13 |
| '3' | 0.00 | 0.38 | -1.06 | -0.39 | -0.32 | -0.06 |
| '4' | 15.10 | 11.53 | -4.86 | -5.97 | 10.96 | 7.28 |
| '5' | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 17.76 |
| '6' | 6.12 | -5.24 | -4.88 | -6.37 | 3.31 | 3.24 |
| '7' | -1.47 | -0.44 | 1.44 | -17.63 | -0.20 | -0.73 |
| '8' | 9.17 | 2.38 | -8.78 | 1.14 | 7.98 | -3.44 |
| '9' | 23.79 | -29.95 | -24.03 | -33.23 | 12.08 | 16.41 |
| 'a' | -11.42 | -4.28 | 2.71 | -1.12 | -6.70 | -3.59 |
| 'b' | -8.76 | 28.51 | 8.12 | 22.45 | -3.59 | -17.03 |
| 'c' | -0.44 | 0.86 | 0.57 | 0.68 | -0.22 | -0.08 |
| 'd' | -1.19 | 0.97 | 2.08 | 2.32 | -0.88 | -0.64 |
| 'e' | -7.80 | 1.45 | 3.15 | 3.43 | -6.07 | -2.81 |

Table 82

Strength of the Percentage Main Effects on the Six Performance Characteristics

| Index | Variable | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-------|-------------------------------|-----------|-----|------------|---------------|-----|------------------|
| '1' | No of Active Core Assemblies | M | W | S | W | M | M |
| '2' | Active Fuel Height | M | W | S | W | M | S |
| '3' | Core Radial Surrounding Water | | | W | | | |
| '4' | Enrichments | S | M | W | M | M | M |
| '5' | Power | | | | | | S |
| '6' | Moderator Density | M | M | W | M | W | W |
| '7' | Fuel Temperature | W | | W | S | | |
| '8' | No of Rods per Assembly | M | W | M | W | M | W |
| '9' | Rod Pitch | VS | VS | VS | VS | M | S |
| 'a' | No of DBAR per Assembly | M | W | W | W | M | W |
| 'b' | IFAB and Fuel Pellet Diameter | M | VS | M | VS | W | S |
| 'c' | IFAB and Fuel Gap Thickness | | | | | | |
| 'd' | IFAB and Fuel Clad Thickness | W | | W | W | | |
| 'e' | Discrete Burnable Ab. Area | M | W | W | W | M | W |

VS Very Strong(>20)

S Strong (>15)

M Moderate (>5)

W Weak (>1)

Blank Very Weak (<1)

Table 83

Heat Map of the Most Important Percentage Interactions of the Six Performance Characteristics.

| Parameter Index | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------|-----------|-------|------------|---------------|-------|------------------|
| '9b' | 7.04 | -9.52 | 2.42 | -6.73 | 3.08 | -1.52 |
| '2a' | 0.00 | -0.01 | -0.01 | 0.01 | -5.73 | -0.42 |
| '18' | -1.20 | 0.00 | 2.29 | 0.01 | -4.65 | 0.00 |
| '8a' | 0.00 | 0.00 | -0.53 | 0.00 | 4.44 | -0.03 |
| '79' | 0.00 | 0.00 | -0.01 | 4.05 | 0.10 | 0.01 |
| '25' | 0.00 | 0.01 | -0.01 | -0.01 | 0.00 | 4.03 |
| '59' | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 3.97 |
| 'ae' | -2.39 | 0.00 | 0.77 | 0.00 | -3.80 | -0.03 |
| '29' | 0.00 | -1.68 | 0.00 | 0.00 | -1.86 | -3.47 |
| '2e' | -0.04 | 0.01 | -0.01 | 0.00 | -3.07 | -0.02 |
| '2b' | -0.02 | 1.66 | 1.33 | 0.01 | 3.00 | 1.61 |
| '7b' | -0.20 | 0.00 | 0.01 | -2.81 | -0.12 | -0.02 |
| '259' | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 2.70 |
| '28' | 0.00 | 0.01 | 0.76 | 0.01 | 2.55 | -0.02 |
| '125b' | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -2.43 |
| '1259b' | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -2.42 |
| '49' | 1.21 | -2.40 | 0.76 | 1.16 | 1.66 | 0.43 |
| '4b' | -1.79 | 2.36 | -1.55 | -1.07 | -0.54 | -1.07 |
| '8b' | -1.00 | 1.23 | 1.72 | -0.01 | -0.82 | 2.35 |
| '14' | 0.00 | 0.18 | 0.51 | 0.00 | 2.34 | -0.20 |
| 'ab' | 2.34 | -1.87 | -1.38 | -0.01 | 1.60 | 0.82 |
| '24' | 0.00 | 0.57 | 0.02 | 0.00 | 2.28 | -1.59 |
| '19' | 0.00 | -1.38 | 0.95 | 0.00 | -2.18 | -1.58 |
| '125' | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 2.10 |
| '18a' | 0.00 | 0.00 | -0.04 | 0.01 | -2.08 | 0.00 |
| '1259' | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.07 |
| '29b' | 0.00 | -0.98 | 0.00 | 0.00 | -0.40 | -2.05 |
| '1b' | -0.12 | 0.97 | 1.39 | 0.01 | 0.32 | 1.96 |
| '5b' | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | -1.94 |
| '12' | 1.08 | 0.05 | 0.87 | -0.01 | 0.99 | -1.91 |
| '59b' | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 1.88 |
| '129' | 0.00 | -0.04 | 0.00 | 0.00 | -0.54 | -1.88 |
| '15' | 0.00 | 0.01 | -0.01 | -0.01 | 0.00 | 1.81 |
| '159' | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.78 |
| '19b' | 0.00 | -0.68 | -0.56 | 0.00 | 0.25 | -1.73 |
| '45' | 0.00 | 0.01 | -0.03 | 0.00 | 0.00 | 1.67 |

Table 83 Continued

| Parameter Index | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------|-----------|-------|------------|---------------|-------|------------------|
| '9a' | 0.00 | 1.65 | -0.37 | 0.00 | -0.34 | -0.23 |
| '89' | 0.00 | -1.16 | 0.72 | 0.00 | 0.53 | -1.59 |
| '189' | 0.00 | 0.00 | -0.11 | 0.00 | -1.53 | 0.01 |
| '6b' | 0.42 | -0.93 | 1.52 | -0.89 | 1.04 | -0.56 |
| 'be' | 0.49 | 0.00 | -1.46 | 0.67 | -0.26 | -0.02 |
| '169' | 0.00 | 0.18 | 0.00 | 0.00 | -1.37 | -0.01 |
| '69' | -1.37 | 0.93 | 0.00 | 0.56 | -0.52 | 0.16 |
| '25b' | 0.00 | 0.01 | -0.01 | 0.00 | 0.00 | -1.36 |
| '7e' | -0.01 | 0.00 | 0.01 | -0.42 | 1.33 | -0.01 |
| '46' | 0.16 | -0.37 | 0.02 | 0.00 | 1.31 | 0.01 |
| '2ae' | 0.00 | 0.00 | 0.00 | 0.01 | -1.29 | -0.01 |
| '1ae' | 0.05 | 0.00 | -0.22 | 0.01 | -1.26 | -0.01 |
| '68' | 0.00 | -0.01 | 0.34 | 0.00 | 1.26 | 0.00 |
| '7d' | 0.00 | -0.01 | 0.01 | -0.24 | 1.26 | -0.01 |
| '8e' | 0.71 | 0.00 | -0.53 | 0.00 | 1.24 | 0.00 |
| '4a' | 0.00 | -0.01 | -0.22 | 0.00 | 1.21 | 0.10 |
| '16' | 0.00 | -0.18 | 0.86 | 0.00 | -1.16 | 0.00 |
| '7bc' | 0.00 | 0.00 | 0.01 | 0.00 | -1.15 | 0.00 |
| '28b' | 0.00 | 0.01 | 0.03 | 0.00 | -1.14 | 0.00 |
| '9ab' | 0.00 | 0.87 | 0.35 | 0.00 | -1.14 | 0.04 |
| '28a' | 0.00 | 0.00 | 0.00 | 0.01 | 1.12 | -0.01 |
| '18b' | 0.09 | 0.00 | -0.13 | 0.00 | 1.11 | 0.00 |
| '1d' | 0.01 | 0.07 | -0.01 | 0.00 | -1.09 | 0.00 |
| '9e' | 1.03 | 0.00 | 0.00 | -0.93 | 0.63 | 0.01 |

Table 84
Strength of the Most Important Percentage Interactions of the Six Performance Characteristics

| Parameter Index | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------|-----------|-----|------------|---------------|-----|------------------|
| '9b' | M | M | W | M | W | W |
| '2a' | | | | | M | |
| '18' | W | | W | | W | |
| '8a' | | | | | W | |
| '79' | | | | W | | |
| '25' | | | | | | W |
| '59' | | | | | | W |
| 'ae' | W | | | | W | |
| '29' | | W | | | W | W |
| '2e' | | | | | W | |
| '2b' | | W | W | | W | W |
| '7b' | | | | W | | |
| '259' | | | | | | W |
| '28' | | | | | W | |
| '125b' | | | | | | W |
| '1259b' | | | | | | W |
| '49' | W | W | | W | W | |
| '4b' | W | W | W | W | | W |
| '8b' | W | W | W | | | W |
| '14' | | | | | W | |
| 'ab' | W | W | W | | W | |
| '24' | | | | | W | W |
| '19' | | W | | | W | W |
| '125' | | | | | | W |
| '18a' | | | | | W | |
| '1259' | | | | | | W |
| '29b' | | | | | | W |
| '1b' | | | W | | | W |
| '5b' | | | | | | W |
| '12' | W | | | | | W |
| '59b' | | | | | | W |
| '129' | | | | | | W |
| '15' | | | | | | W |
| '159' | | | | | | W |

Table 84 Continued

| Parameter Index | K_{eff} | RFS | α_d | α_{FT} | PPF | ΔK_{eff} |
|-----------------|-----------|-----|------------|---------------|-----|------------------|
| '19b' | | | | | | W |
| '45' | | | | | | W |
| '9a' | | W | | | | |
| '89' | | W | | | | W |
| '189' | | | | | W | |
| '6b' | | | W | | W | |
| 'be' | | | W | | | |
| '169' | | | | | W | |
| '69' | W | | | | | |
| '25b' | | | | | | W |
| '7e' | | | | | W | |
| '46' | | | | | W | |
| '2ae' | | | | | W | |
| '1ae' | | | | | W | |
| '68' | | | | | W | |
| '7d' | | | | | W | |
| '8e' | | | | | W | |
| '4a' | | | | | W | |
| '16' | | | | | W | |
| '7bc' | | | | | W | |
| '28b' | | | | | W | |
| '9ab' | | | | | W | |
| '28a' | | | | | W | |
| '18b' | | | | | W | |
| '1d' | | | | | W | |
| '9e' | W | | | | | |

- Since the current system has fourteen variables and six performance characteristics, it is currently underdetermined, and additional performance characteristics can be introduced. However, three of the fourteen variables were found unimportant to any

of the six performance characteristics. As a result, the current actual degree of freedom of the system is $14-6-3=5$. The dependence of the performance characteristics on the variables can be summarized as:

$$K_{eff} = f(1,2,4,6,8,9, a, b, e) \quad (65)$$

$$RFS = f(4,6,9, b) \quad (66)$$

$$\alpha_d = f(1,2,8,9, b) \quad (67)$$

$$\alpha_{FT} = f(4,6,7,9, b) \quad (68)$$

$$PFF = f(1,2,4,8,9, a, e) \quad (69)$$

$$\Delta K_{eff} = f(1,2,4,5,9, b) \quad (70)$$

- As the size of the SMR is varied, the signs of the effects of variables on the performance characteristics establish a balance. For example, K_{eff} response to variables 1 and 2 can be balanced by variables b and a. This balance enables the designer to evaluate and mitigate the consequences of changing a variable by glancing at the table.
- Except for ΔK_{eff} , variables 1 and 2, representing the number of fuel assemblies and active fuel height, have almost identical main effects on all performance characteristics. This behavior enables the designer to increase either variable and balance it by reducing the other. This, however, is not the case of their interactions.
- Variable 9, representing the rod pitch, is the most important variable that significantly affects all the considered performance characteristics. In order to change its effect, it is possible to change b, representing pellet diameter, since it will increase the effect of 9 in three performance characteristics while reducing the effect

of 9 in the other three. It is also possible to change the effect of 9 on ΔK_{eff} by using 59, on α_{FT} by using 79, or other interactions as shown in Table 83.

- Variable b is the second most important variable. Its effect is moderately coupled with 9 and weakly coupled with other variables.
- Interaction 9b, representing rod pitch and pellet diameter, is the most important interaction. This is consistent with the common design consideration of pitch and diameter ratio. The main effects and interaction of pitch and diameter indicate that the reactor design domain fall completely in the under-moderation region, since both variables' effects are monotonically positive and negative consequently.
- ΔK_{eff} dependence on 9 and b is as strong as its dependence on the mass of fuel in the core. This is probably due to the effect of these variable on the spectrum, thus on the fissile materials conversion of the reactor.

VIII. FUTURE WORK

Various topics for methods performance improvement can be investigated. These can be summarized as:

- Introduction of variables' first order nonlinearity: A mean to detect and adaptively introduce nonlinearity need to be developed. This addition will improve the methods' projection accuracies and expand its domain of applicability.
- Expanding the methods to enable evaluating combined performance characteristics: In this dissertation, each performance characteristic was handled as a separate problem. However, extracting more than one performance characteristic from the same experiment might be necessary, and an approach to handle such a scenario needs to be developed.
- Quantification of the probability of variables' monotonic behavior: A quantitative representation of the variables' monotonic behavior could be introduced to enable belief dependent categorization of projected results. A survey of the means to quantify the variables' monotonic behavior is available in Appendix I.

As for the application, the future work can be summarized as:

- Application of additional performance characteristics, thus reducing the degree of freedom of the system and narrowing down the design domain into a smaller domain.
- Application of additional variables to include water boron concentration, fuel composition and other design variables. This will not require reevaluating the variables and interactions that has already been evaluated.

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APPENDIX A

1. Example Graphical Representation of Confounding Effects

This section explains how the strength of confounding effects can hide active variables if they are group screened. Figure 79 shows surfaces generated from two variables, x_1, x_2 . If group screening is applied, it is probable that the end states of the surface will be explored first. These are $x_1 x_2 = 00, 11$. The surface of Figure 79 (a) has a very small degree of interaction's nonlinearity. The surface of Figure 79 (b) has slightly stronger interaction's nonlinearity. In both cases, the importance of both variables is detectable. The surfaces of Figure 79 (c) and (d) demonstrate strong nonlinearity. The surfaces hide the importance of the two variables due to the nonlinearity in variables or due to their interaction. Case (c) is very unlikely in physical systems. It indicates a confounding effect that is as significant as the main effect and that opposes the direction of both main effects. This behavior is not consistent with the ASIS and hierarchy behaviors observed in physical systems in [36]. Case (d) is usually caused by composite variables or responses. This means that the nonlinearity is due to x_1 being composed of two or more variables $x_{1,1}$ and $x_{1,2}$ that are hidden within this variable. These variables interact in a manner to cause the change of direction. This nonlinearity can also be due to the response, y , being composed of two or more responses, $y_{1,1}$ and $y_{1,2}$, that are linked to x_1 in a semi-linear manner. The combination of the two responses in one response can cause the response nonlinearity. This explains the need to decompose variables, and

sometimes responses, to their basic components before evaluation to reduce nonlinearity. For example, instead of using the fuel cell pitch to fuel pellet diameter ratio (p/d) in a reactor design, the pitch and the diameter can be considered as two separate variables. It is also possible to use the p/d ratio along with the diameter or the pitch as an additional variable. The addition of a decomposition variable is expensive, since adding one variable to N adds 2^N variables' combinations. Instead of variables' decomposition, case (d) nonlinearity can also be expressed by adding a square term to the potential nonlinear variables in the approximation model.

In regression analysis, the nonlinearity problem is usually handled through stepwise selection. Variables sometimes appear like noise till other confounding variables or interactions are introduced causing the original variable behavior to develop a pattern, and its significance to be revealed. This is applied through testing whether the approximation model better explains the response if more variables or interactions are added [27].

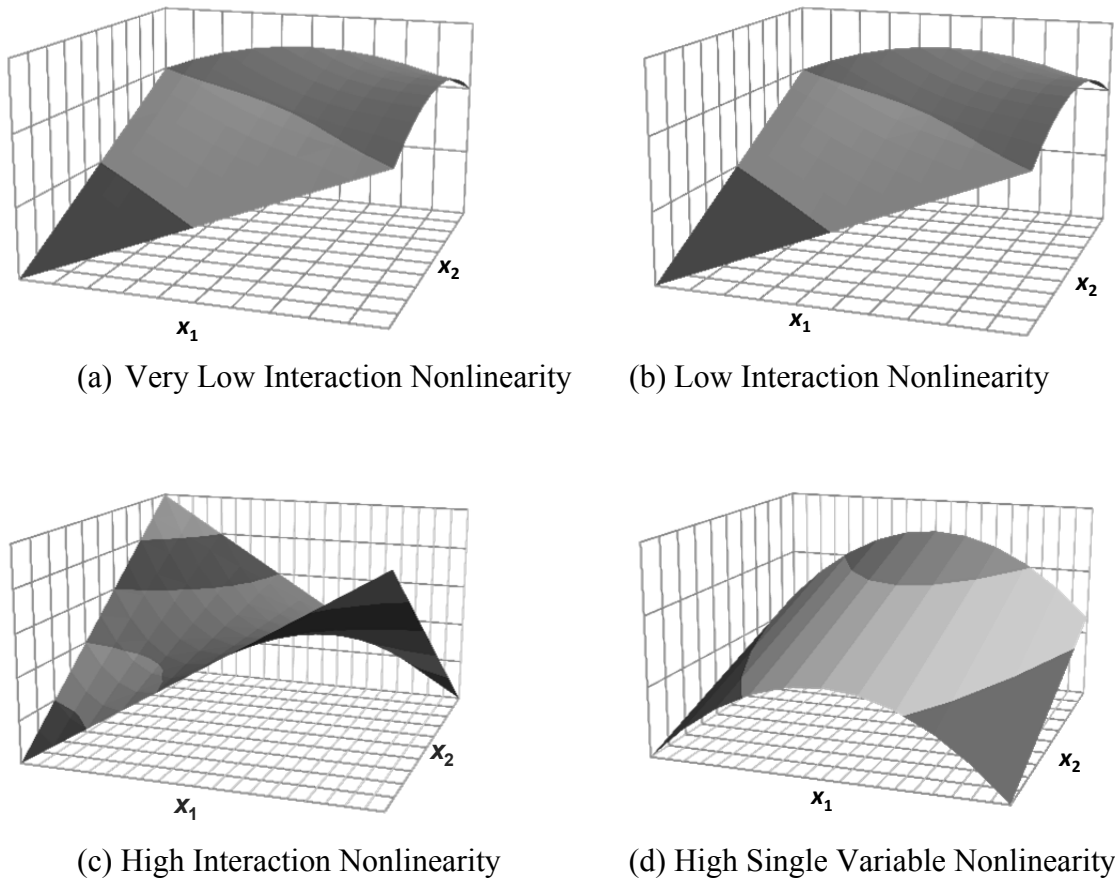


Figure 79. Scenarios of Two Confounding Variables

2. Example Application of PB

In this example, the model developed by Dittus-Boelter for determining the heat transfer coefficient in heated or cooled pipes is analyzed. This is assuming that the model is unknown for demonstration purposes. The most common approximation metric model used is [89, 90]:

$$Nu = \frac{hD}{k} = 0.023 \left(\frac{\rho VD}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.333} \quad (71)$$

This model can be used for turbulent flow with a Reynolds number:

$$\text{Re} = \frac{\rho V D}{\mu} \quad (72)$$

exceeding 10,000. The model shows that six variables affect the heat transfer coefficient h . As a result, a full factorial set of 2^6 experiments are needed to determine all main effects and interactions. In order to reduce the number of variables to a small number, the parameters related to water are combined. Water is assumed to have two states of 40C and 90C at 1 MPa. The properties of water at the two states are shown in Table 85. Due to the integration of water variables in two temperature states, only three variables' effects are considered on the heat transfer coefficient. The ranges of the three variables are shown in Table 86. The full factorial and Res III PB designs to screen the variables are shown in Table 87 and Table 88 along with the results for every combination. The full factorial and Res III PB designs' parameters are shown in Table 89. PB is found to severely underestimate the temperature especially when the compared to the diameter. This is due to the fact this Res III PB design confounds second order interactions in main effects. The confounding of the velocity and diameter interaction in the temperature causes the temperature first order parameters to appear as:

$$T = T + V \times D = 316.65 - 306.32 = 10.33 \approx 10$$

It is, thus, shown in this example that confounded effects can cause a significant error in the experimenter evaluation of variables. As the resolution increases, this error decreases, since higher order interactions are then confound in the main effects.

Table 85

Water's Relevant Properties at the Two States of the PB Example

| State | Temperature T (C) | Density ρ (kg/m ³) | Viscosity μ (Pa.s) | Specific Heat C_p (J/Kg.K) | Fluid Conductivity k (W/m.K) |
|-------|----------------------|--|------------------------------|------------------------------------|------------------------------------|
| -1 | 40 | 992.6 | 6.53E-04 | 4071 | 0.631 |
| 1 | 90 | 965.7 | 3.15E-04 | 3819 | 0.676 |

Table 86

Variables' States of the PB Example

| State | Temperature T (Cel.) | Mean Velocity V (m/s) | Pipe Diameter D (m) |
|-------|-------------------------|--------------------------|------------------------|
| 1 | 90 | 1 | 1 |
| -1 | 40 | 0.1 | 0.1 |

Table 87

Results of the Full Factorial Design's Application to the PB Example

| Exp. Index | Temp. T State | Mean Velocity V State | Pipe Diameter D State | Temp. T (Cel.) | Mean Velocity V (m/s) | Pipe Diameter D (m) | Heat Transfer Coeff. h (W/m ² .K) |
|---------------|---------------------|--------------------------------|--------------------------------|----------------------|--------------------------------|------------------------------|--|
| 1 | -1 | -1 | -1 | 40 | 0.1 | 0.1 | 519 |
| 2 | -1 | -1 | 1 | 40 | 0.1 | 1 | 327 |
| 3 | -1 | 1 | -1 | 40 | 1 | 0.1 | 3275 |
| 4 | -1 | 1 | 1 | 40 | 1 | 1 | 2066 |
| 5 | 1 | -1 | -1 | 90 | 0.1 | 0.1 | 731 |
| 6 | 1 | -1 | 1 | 90 | 0.1 | 1 | 461 |
| 7 | 1 | 1 | -1 | 90 | 1 | 0.1 | 4615 |
| 8 | 1 | 1 | 1 | 90 | 1 | 1 | 2912 |

Table 88
Results of the Res III PB's Application to the PB Example

| Exp. Index | Temp. T | Mean Velocity V | Pipe Diameter D | Temp T | Mean Velocity V | Pipe Diameter D | Heat Transfer Coeff. h |
|------------|---------|-----------------|-----------------|--------|-----------------|-----------------|------------------------|
| | State | State | State | (Cel.) | (m/s) | (m) | (W/m ² .K) |
| 1 | 1 | 1 | 1 | 90 | 1 | 1 | 2912. |
| 2 | 1 | -1 | -1 | 90 | 0.1 | 0.1 | 731 |
| 3 | -1 | 1 | -1 | 40 | 1 | 0.1 | 3275 |
| 4 | -1 | -1 | 1 | 40 | 0.1 | 1 | 327 |

Table 89
PB and Full Factorial Design's Parameters of the PB Example

| Parameter | PB | Full Factorial |
|----------------------------|---------|----------------|
| Offset | 1811 | 1863.71 |
| Temp | 10 | 316 |
| Velocity | 1282.13 | 1353 |
| Diameter | -191 | -421 |
| Temp x Velocity | --- | 230 |
| Temp x Diameter | --- | -71 |
| Velocity x Diameter | --- | -306 |
| Temp x Velocity x Diameter | --- | -52 |

APPENDIX B

1. Conversion Between the Design Matrix Domains of -1,1 and 0,1:

This section explains the relationship between the two common domains of the design matrix using a three variables' example. The conversion from the 0,1 domain parameters ($\beta_{1,0}$ s) to the -1,1 domain parameters ($\beta_{1,-1}$ s) is found using:

$$X_{1,-1}^{-1} X_{1,0} = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.5 & 0.25 & 0.25 & 0.25 & 0.125 \\ 0 & 0.5 & 0 & 0 & 0.25 & 0.25 & 0 & 0.125 \\ 0 & 0 & 0.5 & 0 & 0.25 & 0 & 0.25 & 0.125 \\ 0 & 0 & 0 & 0.5 & 0 & 0.25 & 0.25 & 0.125 \\ 0 & 0 & 0 & 0 & 0.25 & 0 & 0 & 0.125 \\ 0 & 0 & 0 & 0 & 0 & 0.25 & 0 & 0.125 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0.125 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.125 \end{bmatrix} \quad (73)$$

If the approach of [23] is used to find the main effect for any variable x_i in the 0,1, domain of three variables, then:

$$ME(x_i) = 2 \cdot \frac{1}{8} \text{Exp} \left[\begin{array}{l} (x_{i,1}x_{j,1}x_{k,1} - x_{i,0}x_{j,1}x_{k,1}) + (x_{i,1}x_{j,0}x_{k,0} - x_{i,0}x_{j,0}x_{k,0}) \\ + (x_{i,1}x_{j,1}x_{k,0} - x_{i,0}x_{j,1}x_{k,0}) + (x_{i,1}x_{j,0}x_{k,1} - x_{i,0}x_{j,0}x_{k,1}) \end{array} \right] \quad (74)$$

Exp is defined here as the experiment performed at the three variables' states $x_i x_j x_k$. Each of the terms in the round brackets present a fixed state of the variables, jk, at different i states. This means that the main effect is an average effect of changing i at every state of j and k. Using the three variables' model in the 0,1 domain:

$$\begin{aligned}
y(x_i, x_j, x_k) &= \beta_{0,1,0} + \beta_{i,1,0}x_i + \beta_{j,1,0}x_j + \beta_{k,1,0}x_k + \beta_{ij,1,0}x_i x_j + \beta_{ik,1,0}x_i x_k \\
&+ \beta_{jk,1,0}x_j x_k + \beta_{ijk,1,0}x_i x_j x_k
\end{aligned} \tag{75}$$

the main effect, which is in the -1,1 domain, is found as

$$\begin{aligned}
ME(x_i) &= 2\beta_i = 2 \frac{1}{8} \left[\begin{aligned} & \left(\left[\beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \beta_{k,1,0} + \beta_{ij,1,0} + \beta_{ik,1,0} + \beta_{jk,1,0} + \beta_{ijk,1,0} \right] \right) \\ & - \left[\beta_{0,1,0} + \beta_{j,1,0} + \beta_{k,1,0} + \beta_{jk,1,0} \right] \\ & + \left(\left[\beta_{0,1,0} + \beta_{i,1,0} \right] - \left[\beta_{0,1,0} \right] \right) \\ & + \left(\left[\beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \beta_{ij,1,0} \right] - \left[\beta_{0,1,0} + \beta_{j,1,0} \right] \right) \\ & + \left(\left[\beta_{0,1,0} + \beta_{i,1,0} + \beta_{k,1,0} + \beta_{ik,1,0} \right] - \left[\beta_{0,1,0} + \beta_{k,1,0} \right] \right) \end{aligned} \right] \\
&= 2 \cdot \left[\frac{1}{2} \beta_{i,1,0} + \frac{1}{4} \beta_{ij,1,0} + \frac{1}{4} \beta_{ik,1,0} + \frac{1}{8} \beta_{ijk,1,0} \right]
\end{aligned} \tag{76}$$

This indicates that the main effect for any variable is a combination of all variable's parameters in the 0,1 domain. Each order has half the weight of the lower order. Thus, the main effects are biased towards the lower orders of the 0,1 domain parameters by definition. The second order interaction in the -1,1 domain is defined as:

$$\begin{aligned}
IE(x_i x_j) &= 2 \cdot \frac{1}{8} \text{Exp} \left[\begin{aligned} & \left((x_{i,1} x_{j,1} x_{k,1} - x_{i,0} x_{j,1} x_{k,1}) - (x_{i,1} x_{j,0} x_{k,1} - x_{i,0} x_{j,0} x_{k,1}) \right) \\ & + \left((x_{i,1} x_{j,1} x_{k,0} - x_{i,0} x_{j,1} x_{k,0}) - (x_{i,1} x_{j,0} x_{k,0} - x_{i,0} x_{j,0} x_{k,0}) \right) \end{aligned} \right]
\end{aligned} \tag{77}$$

Substituting the parameters' model:

$$\begin{aligned}
IE(x_i x_j) &= 2 \cdot \frac{1}{8} \left[\begin{aligned} & \left(\left[\beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \beta_{k,1,0} + \beta_{ij,1,0} + \beta_{ik,1,0} + \beta_{jk,1,0} + \beta_{ijk,1,0} \right] \right) \\ & - \left[\beta_{0,1,0} + \beta_{j,1,0} + \beta_{k,1,0} + \beta_{jk,1,0} \right] \\ & - \left(\left[\beta_{0,1,0} + \beta_{i,1,0} + \beta_{k,1,0} + \beta_{ik,1,0} \right] - \left[\beta_{0,1,0} + \beta_{k,1,0} \right] \right) \\ & + \left(\left[\beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \beta_{ij,1,0} \right] - \left[\beta_{0,1,0} + \beta_{j,1,0} \right] \right) \\ & - \left(\left[\beta_{0,1,0} + \beta_{i,1,0} \right] - \left[\beta_{0,1,0} \right] \right) \end{aligned} \right] \\
&= 2 \cdot \frac{1}{8} \left[2\beta_{ij,1,0} + \beta_{ijk,1,0} \right] = 2 \cdot \left[\frac{1}{4} \beta_{ij,1,0} + \frac{1}{8} \beta_{ijk,1,0} \right]
\end{aligned} \tag{78}$$

Thus, the interaction is a portion of the i and j main affects' parameters with the same weights as the main effects' parameters. The three variables' interaction is defined as:

$$IE(x_i x_j x_k) = 2 \cdot \frac{1}{8} \text{Exp} \left[\begin{array}{l} (x_{i,1} x_{j,1} x_{k,1} - x_{i,0} x_{j,1} x_{k,1}) - (x_{i,1} x_{j,0} x_{k,1} - x_{i,0} x_{j,0} x_{k,1}) \\ - (x_{i,1} x_{j,1} x_{k,0} - x_{i,0} x_{j,1} x_{k,0}) - (x_{i,0} x_{j,0} x_{k,0} - x_{i,1} x_{j,0} x_{k,0}) \end{array} \right] \quad (79)$$

Substituting the parameters' model:

$$IE(x_i x_j x_k) = 2 \cdot \frac{1}{8} \left[\begin{array}{l} [\beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \beta_{k,1,0} + \beta_{ij,1,0} + \beta_{ik,1,0} + \beta_{jk,1,0} + \beta_{ijk,1,0}] \\ - [\beta_{0,1,0} + \beta_{j,1,0} + \beta_{k,1,0} + \beta_{jk,1,0}] \\ - ([\beta_{0,1,0} + \beta_{i,1,0} + \beta_{k,1,0} + \beta_{ik,1,0}] - [\beta_{0,1,0} + \beta_{k,1,0}]) \\ - ([\beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \beta_{ij,1,0}] - [\beta_{0,1,0} + \beta_{j,1,0}]) \\ - ([\beta_{0,1,0}] - [\beta_{0,1,0} + \beta_{i,1,0}]) \end{array} \right] \quad (80)$$

$$= 2 \cdot \frac{1}{8} \beta_{ijk,1,0}$$

Table 90 shows all main effects and interactions of the three variables. For every variables' combination, the parameters' contribution, in the 0,1 domain, vary from offset contribution to all parameters contribution as shown in Table 91. As the order of the parameter in the 0,1 domain increases, its importance decreases. The weight of the parameters can be also found using the variables' middle point in the 0,1 domain:

$$y(0.5,0.5,0.5) = \beta_{0,1,0} + 0.5\beta_{i,1,0} + 0.5\beta_{j,1,0} + 0.5\beta_{k,1,0} + 0.25\beta_{ij,1,0} + 0.25\beta_{ik,1,0} + 0.25\beta_{jk,1,0} + 0.125\beta_{ijk,1,0} \quad (81)$$

Table 90
 Definition of Main Effects and Interactions in the 0,1 Domain of the Design Matrix

| Index | Effect | 0,1 Parameters |
|-------|-----------------|--|
| 1 | $ME(x_1)$ | $2 \left[\frac{1}{2} \beta_{1,1,0} + \frac{1}{4} \beta_{12,1,0} + \frac{1}{4} \beta_{13,1,0} + \frac{1}{8} \beta_{123,1,0} \right]$ |
| 2 | $ME(x_2)$ | $2 \left[\frac{1}{2} \beta_{2,1,0} + \frac{1}{4} \beta_{12,1,0} + \frac{1}{4} \beta_{23,1,0} + \frac{1}{8} \beta_{123,1,0} \right]$ |
| 3 | $ME(x_3)$ | $2 \left[\frac{1}{2} \beta_{3,1,0} + \frac{1}{4} \beta_{13,1,0} + \frac{1}{4} \beta_{23,1,0} + \frac{1}{8} \beta_{123,1,0} \right]$ |
| 4 | $IE(x_1x_2)$ | $2 \left[\frac{1}{4} \beta_{12,1,0} + \frac{1}{8} \beta_{123,1,0} \right]$ |
| 5 | $IE(x_1x_3)$ | $2 \left[\frac{1}{4} \beta_{13,1,0} + \frac{1}{8} \beta_{123,1,0} \right]$ |
| 6 | $IE(x_2x_3)$ | $2 \left[\frac{1}{4} \beta_{23,1,0} + \frac{1}{8} \beta_{123,1,0} \right]$ |
| 7 | $IE(x_1x_2x_3)$ | $2 \left[\frac{1}{8} \beta_{123,1,0} \right]$ |

Table 91
Response of a Three Variables' Example in Parameters of the 0,1 Domain

| Index | x_3 | x_2 | x_1 | Y |
|-------|-------|-------|-------|--|
| 1 | 0 | 0 | 0 | $\beta_{0,1,0}$ |
| 2 | 0 | 0 | 1 | $\beta_{0,1,0} + \beta_{1,1,0}$ |
| 3 | 0 | 1 | 0 | $\beta_{0,1,0} + \beta_{2,1,0}$ |
| 4 | 0 | 1 | 1 | $\beta_{0,1,0} + \beta_{1,1,0} + \beta_{2,1,0} + \beta_{12,1,0}$ |
| 5 | 1 | 0 | 0 | $\beta_{0,1,0} + \beta_{3,1,0}$ |
| 6 | 1 | 0 | 1 | $\beta_{0,1,0} + \beta_{1,1,0} + \beta_{3,1,0} + \beta_{13,1,0}$ |
| 7 | 1 | 1 | 0 | $\beta_{0,1,0} + \beta_{2,1,0} + \beta_{3,1,0} + \beta_{23,1,0}$ |
| 8 | 1 | 1 | 1 | $\beta_{0,1,0} + \beta_{1,1,0} + \beta_{2,1,0} + \beta_{3,1,0} + \beta_{12,1,0} + \beta_{13,1,0} + \beta_{23,1,0} + \beta_{123,1,0}$ |

2. Response Variance Propagation to Parameters Variance

The target of this section is to determine the effects of response values' uncertainty on the calculated parameters' uncertainty. The variance of the parameters is equal to:

$$\text{var}(\beta) = X^{-1} \text{var}(Y)(X^{-1})^T \quad (82)$$

X^{-1} and $(X^{-1})^T$ can be found from the design matrix. The matrix of $\text{var}(Y)$ is the only unknown that is needed to find $\text{var}(\beta)$. It is defined as:

$$\text{var}(Y) = \begin{bmatrix} \text{var}(y_1) & \text{Cov}(y_1, y_2) & \dots & \text{Cov}(y_1, y_n) \\ \text{Cov}(y_2, y_1) & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \text{Cov}(y_n, y_1) & \text{Cov}(y_n, y_2) & \dots & \text{var}(y_n) \end{bmatrix} \quad (83)$$

In a two variables scenario, x_1x_2 where $N=2$ and $n=4$, the response variance is:

$$\text{var}(Y) = \begin{bmatrix} \text{var}(y_1) & \text{Cov}(y_1, y_2) & \text{Cov}(y_1, y_3) & \text{Cov}(y_1, y_4) \\ \text{Cov}(y_2, y_1) & \text{var}(y_2) & \text{Cov}(y_2, y_3) & \text{Cov}(y_2, y_4) \\ \text{Cov}(y_3, y_1) & \text{Cov}(y_3, y_2) & \text{var}(y_3) & \text{Cov}(y_3, y_4) \\ \text{Cov}(y_4, y_1) & \text{Cov}(y_4, y_2) & \text{Cov}(y_4, y_3) & \text{var}(y_4) \end{bmatrix} \quad (84)$$

The covariance between the experiments is not zero if the variables are assumed monotonic. In monotonic variables, the result of one experiment will provide information on other experiments. The two variables' design matrix of the 0,1 domain is defined as:

$$X_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad (85)$$

and its inverse is:

$$X_{1,0}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 \end{bmatrix}, \quad (86)$$

thus:

$$\left(X_{1,0}^{-1}\right)^T = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (87)$$

Substituting these matrices in the variance propagation equation for the 0,1 domain:

$$\text{var}(\beta_{1,0}) = X_{1,0}^{-1} \text{var}(Y) (X_{1,0}^{-1})^T \quad (88)$$

$$X_{1,0}^{-1} \text{var}(y) = \begin{bmatrix} \text{var}(y_1) & \text{Cov}(y_1, y_2) & \text{Cov}(y_1, y_3) & \text{Cov}(y_1, y_4) \\ -\text{var}(y_1) & -\text{Cov}(y_1, y_2) & -\text{Cov}(y_1, y_3) & -\text{Cov}(y_1, y_4) \\ +\text{Cov}(y_2, y_1) & +\text{var}(y_2) & +\text{Cov}(y_2, y_3) & +\text{Cov}(y_2, y_4) \\ -\text{var}(y_1) & -\text{Cov}(y_1, y_2) & -\text{Cov}(y_1, y_3) & -\text{Cov}(y_1, y_4) \\ +\text{Cov}(y_3, y_1) & +\text{Cov}(y_3, y_2) & +\text{var}(y_3) & +\text{Cov}(y_3, y_4) \\ \text{var}(y_1) & \text{Cov}(y_1, y_2) & \text{Cov}(y_1, y_3) & \text{Cov}(y_1, y_4) \\ -\text{Cov}(y_2, y_1) & -\text{var}(y_2) & -\text{Cov}(y_2, y_3) & -\text{Cov}(y_2, y_4) \\ -\text{Cov}(y_3, y_1) & -\text{Cov}(y_3, y_2) & -\text{var}(y_3) & -\text{Cov}(y_3, y_4) \\ +\text{Cov}(y_4, y_1) & +\text{Cov}(y_4, y_2) & +\text{Cov}(y_4, y_3) & +\text{var}(y_4) \end{bmatrix} \quad (89)$$

$$\text{var}(\beta_{1,0}) = \begin{bmatrix}
\text{var}(y_1) & -\text{var}(y_1) & -\text{var}(y_1) & \text{var}(y_1) \\
& +\text{Cov}(y_1, y_2) & +\text{Cov}(y_1, y_3) & -\text{Cov}(y_1, y_2) \\
& & & -\text{Cov}(y_1, y_3) \\
& & & +\text{Cov}(y_1, y_4) \\
& & & -\text{var}(y_1) \\
& & & +\text{Cov}(y_2, y_1) \\
& & & +\text{Cov}(y_1, y_2) \\
-\text{var}(y_1) & +\text{var}(y_1) & \text{var}(y_1) & -\text{var}(y_2) \\
+\text{Cov}(y_2, y_1) & -\text{Cov}(y_2, y_1) & -\text{Cov}(y_2, y_1) & +\text{Cov}(y_1, y_3) \\
& -\text{Cov}(y_1, y_2) & -\text{Cov}(y_1, y_3) & -\text{Cov}(y_2, y_3) \\
& +\text{var}(y_2) & +\text{Cov}(y_2, y_3) & -\text{Cov}(y_1, y_4) \\
& & & +\text{Cov}(y_2, y_4) \\
& & & -\text{var}(y_1) \\
& & & +\text{Cov}(y_3, y_1) \\
& & & +\text{Cov}(y_1, y_2) \\
-\text{var}(y_1) & +\text{var}(y_1) & \text{var}(y_1) & -\text{Cov}(y_3, y_2) \\
+\text{Cov}(y_3, y_1) & -\text{Cov}(y_3, y_1) & -\text{Cov}(y_3, y_1) & +\text{Cov}(y_1, y_3) \\
& -\text{Cov}(y_1, y_2) & -\text{Cov}(y_1, y_3) & -\text{var}(y_3) \\
& +\text{Cov}(y_3, y_2) & +\text{var}(y_3) & -\text{Cov}(y_1, y_4) \\
& & & +\text{Cov}(y_3, y_4) \\
& & & \text{var}(y_1) \\
& & & -\text{Cov}(y_2, y_1) \\
& & & -\text{Cov}(y_3, y_1) \\
& & & +\text{Cov}(y_4, y_1) \\
& & & -\text{Cov}(y_1, y_2) \\
& & -\text{var}(y_1) & -\text{var}(y_1) & -\text{Cov}(y_1, y_2) \\
& & +\text{Cov}(y_2, y_1) & +\text{Cov}(y_2, y_1) & +\text{var}(y_2) \\
\text{var}(y_1) & +\text{Cov}(y_3, y_1) & +\text{Cov}(y_3, y_1) & +\text{Cov}(y_3, y_2) \\
-\text{Cov}(y_2, y_1) & -\text{Cov}(y_4, y_1) & -\text{Cov}(y_4, y_1) & -\text{Cov}(y_4, y_2) \\
-\text{Cov}(y_3, y_1) & \text{Cov}(y_1, y_2) & \text{Cov}(y_1, y_3) & -\text{Cov}(y_1, y_3) \\
+\text{Cov}(y_4, y_1) & -\text{var}(y_2) & -\text{Cov}(y_2, y_3) & +\text{Cov}(y_2, y_3) \\
& -\text{Cov}(y_3, y_2) & -\text{var}(y_3) & +\text{var}(y_3) \\
& +\text{Cov}(y_4, y_2) & +\text{Cov}(y_4, y_3) & -\text{Cov}(y_4, y_3) \\
& & & \text{Cov}(y_1, y_4) \\
& & & -\text{Cov}(y_2, y_4) \\
& & & -\text{Cov}(y_3, y_4) \\
& & & +\text{var}(y_4)
\end{bmatrix} \tag{90}$$

In the 1,-1 domain, the variance propagation equation is defined as:

$$\text{var}(\beta_{1,-1}) = X_{1,-1}^{-1} \text{var}(Y) (X_{1,-1}^{-1})^T \quad (91)$$

$$\text{var}(\beta_{1,-1}) = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{bmatrix} \text{var}(Y) \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \quad (92)$$

Multiplying the first two matrices then multiplying the result by the third matrix results

in:

$$\begin{aligned}
X_{1,-1}^{-1} \text{var}(Y) = & \left[\begin{array}{cccc}
\frac{1}{4} \text{var}(y_1) & \frac{1}{4} \text{Cov}(y_1, y_2) & \frac{1}{4} \text{Cov}(y_1, y_3) & \frac{1}{4} \text{Cov}(y_1, y_4) \\
+\frac{1}{4} \text{Cov}(y_2, y_1) & +\frac{1}{4} \text{var}(y_2) & +\frac{1}{4} \text{Cov}(y_2, y_3) & +\frac{1}{4} \text{Cov}(y_2, y_4) \\
+\frac{1}{4} \text{Cov}(y_3, y_1) & +\frac{1}{4} \text{Cov}(y_3, y_2) & +\frac{1}{4} \text{var}(y_3) & +\frac{1}{4} \text{Cov}(y_3, y_4) \\
+\frac{1}{4} \text{Cov}(y_4, y_1) & +\frac{1}{4} \text{Cov}(y_4, y_2) & +\frac{1}{4} \text{Cov}(y_4, y_3) & +\frac{1}{4} \text{var}(y_4) \\
-\frac{1}{4} \text{var}(y_1) & -\frac{1}{4} \text{Cov}(y_1, y_2) & -\frac{1}{4} \text{Cov}(y_1, y_3) & -\frac{1}{4} \text{Cov}(y_1, y_4) \\
+\frac{1}{4} \text{Cov}(y_2, y_1) & +\frac{1}{4} \text{var}(y_2) & +\frac{1}{4} \text{Cov}(y_2, y_3) & +\frac{1}{4} \text{Cov}(y_2, y_4) \\
-\frac{1}{4} \text{Cov}(y_3, y_1) & -\frac{1}{4} \text{Cov}(y_3, y_2) & -\frac{1}{4} \text{var}(y_3) & -\frac{1}{4} \text{Cov}(y_3, y_4) \\
+\frac{1}{4} \text{Cov}(y_4, y_1) & +\frac{1}{4} \text{Cov}(y_4, y_2) & +\frac{1}{4} \text{Cov}(y_4, y_3) & +\frac{1}{4} \text{var}(y_4) \\
-\frac{1}{4} \text{var}(y_1) & -\frac{1}{4} \text{Cov}(y_1, y_2) & -\frac{1}{4} \text{Cov}(y_1, y_3) & -\frac{1}{4} \text{Cov}(y_1, y_4) \\
-\frac{1}{4} \text{Cov}(y_2, y_1) & -\frac{1}{4} \text{var}(y_2) & -\frac{1}{4} \text{Cov}(y_2, y_3) & -\frac{1}{4} \text{Cov}(y_2, y_4) \\
+\frac{1}{4} \text{Cov}(y_3, y_1) & +\frac{1}{4} \text{Cov}(y_3, y_2) & +\frac{1}{4} \text{var}(y_3) & +\frac{1}{4} \text{Cov}(y_3, y_4) \\
+\frac{1}{4} \text{Cov}(y_4, y_1) & +\frac{1}{4} \text{Cov}(y_4, y_2) & +\frac{1}{4} \text{Cov}(y_4, y_3) & +\frac{1}{4} \text{var}(y_4) \\
\frac{1}{4} \text{var}(y_1) & \frac{1}{4} \text{Cov}(y_1, y_2) & \frac{1}{4} \text{Cov}(y_1, y_3) & \frac{1}{4} \text{Cov}(y_1, y_4) \\
-\frac{1}{4} \text{Cov}(y_2, y_1) & -\frac{1}{4} \text{var}(y_2) & -\frac{1}{4} \text{Cov}(y_2, y_3) & -\frac{1}{4} \text{Cov}(y_2, y_4) \\
-\frac{1}{4} \text{Cov}(y_3, y_1) & -\frac{1}{4} \text{Cov}(y_3, y_2) & -\frac{1}{4} \text{var}(y_3) & -\frac{1}{4} \text{Cov}(y_3, y_4) \\
+\frac{1}{4} \text{Cov}(y_4, y_1) & +\frac{1}{4} \text{Cov}(y_4, y_2) & +\frac{1}{4} \text{Cov}(y_4, y_3) & +\frac{1}{4} \text{var}(y_4)
\end{array} \right] \quad (93)
\end{aligned}$$

$$\begin{aligned}
\text{var}(\beta_{1,-1}) = & \left[\begin{array}{l}
+\frac{1}{4^2} \text{var}(y_1) \\
+\frac{2}{4^2} \text{Cov}(y_2, y_1) \\
+\frac{2}{4^2} \text{Cov}(y_3, y_1) \quad -\frac{1}{4^2} \text{var}(y_1) \quad -\frac{1}{4^2} \text{var}(y_1) \\
+\frac{2}{4^2} \text{Cov}(y_4, y_1) \quad -\frac{2}{4^2} \text{Cov}(y_3, y_1) \quad -\frac{2}{4^2} \text{Cov}(y_2, y_1) \\
+\frac{1}{4^2} \text{var}(y_2) \quad +\frac{1}{4^2} \text{var}(y_2) \quad -\frac{1}{4^2} \text{var}(y_2) \\
+\frac{2}{4^2} \text{Cov}(y_3, y_2) \quad +\frac{2}{4^2} \text{Cov}(y_4, y_2) \quad +\frac{1}{4^2} \text{var}(y_3) \\
+\frac{2}{4^2} \text{Cov}(y_4, y_2) \quad -\frac{1}{4^2} \text{var}(y_3) \quad +2 \cdot \frac{1}{4^2} \text{Cov}(y_4, y_3) \\
+\frac{1}{4^2} \text{var}(y_3) \quad +\frac{1}{4^2} \text{var}(y_4) \quad +\frac{1}{4^2} \text{var}(y_4) \\
+\frac{2}{4^2} \text{Cov}(y_4, y_3) \\
+\frac{1}{4^2} \text{var}(y_4)
\end{array} \right] \begin{array}{l}
+\frac{1}{4^2} \text{var}(y_1) \\
+\frac{2}{4^2} \text{Cov}(y_4, y_1) \\
-\frac{1}{4^2} \text{var}(y_2) \\
-\frac{2}{4^2} \text{Cov}(y_3, y_2) \\
-\frac{1}{4^2} \text{var}(y_3) \\
+\frac{1}{4^2} \text{var}(y_4)
\end{array} \\
& + \frac{1}{4^2} \text{var}(y_1) \\
& -\frac{2}{4^2} \text{Cov}(y_2, y_1) \\
& -\frac{1}{4^2} \text{var}(y_1) \quad +\frac{2}{4^2} \text{Cov}(y_3, y_1) \quad +\frac{1}{4^2} \text{var}(y_1) \quad -\frac{1}{4^2} \text{var}(y_1) \\
& -2 \cdot \text{Cov}(y_3, y_1) \quad -\frac{2}{4^2} \text{Cov}(y_4, y_1) \quad -2 \cdot \frac{1}{4^2} \text{Cov}(y_4, y_1) \quad +\frac{2}{4^2} \text{Cov}(y_2, y_1) \\
& +\frac{1}{4^2} \text{var}(y_2) \quad +\frac{1}{4^2} \text{var}(y_2) \quad -0.25^2 \text{var}(y_2) \quad -\frac{1}{4^2} \text{var}(y_2) \\
& +\frac{2}{4^2} \text{Cov}(y_4, y_2) \quad -\frac{2}{4^2} \text{Cov}(y_3, y_2) \quad +2 \cdot \frac{1}{4^2} \text{Cov}(y_3, y_2) \quad +\frac{1}{4^2} \text{var}(y_3) \\
& -\frac{1}{4^2} \text{var}(y_3) \quad +\frac{2}{4^2} \text{Cov}(y_4, y_2) \quad -\frac{1}{4^2} \text{var}(y_3) \quad -\frac{2}{4^2} \text{Cov}(y_4, y_3) \\
& +\frac{1}{4^2} \text{var}(y_4) \quad +\frac{1}{4^2} \text{var}(y_3) \quad +\frac{1}{4^2} \text{var}(y_4) \quad +\frac{1}{4^2} \text{var}(y_4) \\
& -\frac{2}{4^2} \text{Cov}(y_4, y_3) \\
& +\frac{1}{4^2} \text{var}(y_4)
\end{array} \right] \tag{94}
\end{aligned}$$

$$\begin{array}{cccc}
& & & + \frac{1}{4^2} \text{var}(y_1) \\
& & & + \frac{2}{4^2} \text{Cov}(y_2, y_1) \\
- \frac{1}{4^2} \text{var}(y_1) & + \frac{1}{4^2} \text{var}(y_1) & - \frac{2}{4^2} \text{Cov}(y_3, y_1) & - \frac{1}{4^2} \text{var}(y_1) \\
- \frac{2}{4^2} \text{Cov}(y_2, y_1) & - \frac{2}{4^2} \text{Cov}(y_4, y_1) & - \frac{2}{4^2} \text{Cov}(y_4, y_1) & + \frac{2}{4^2} \text{Cov}(y_3, y_1) \\
- \frac{1}{4^2} \text{var}(y_2) & - \frac{1}{4^2} \text{var}(y_2) & + \frac{1}{4^2} \text{var}(y_2) & + \frac{1}{4^2} \text{var}(y_2) \\
+ \frac{1}{4^2} \text{var}(y_3) & + \frac{2}{4^2} \text{Cov}(y_3, y_2) & - \frac{2}{4^2} \text{Cov}(y_3, y_2) & - \frac{2}{4^2} \text{Cov}(y_4, y_2) \\
+ \frac{2}{4^2} \text{Cov}(y_4, y_3) & - \frac{1}{4^2} \text{var}(y_3) & - \frac{2}{4^2} \text{Cov}(y_4, y_2) & - \frac{1}{4^2} \text{var}(y_3) \\
+ \frac{1}{4^2} \text{var}(y_4) & + \frac{1}{4^2} \text{var}(y_4) & + \frac{1}{4^2} \text{var}(y_3) & + \frac{1}{4^2} \text{var}(y_4) \\
& & + \frac{2}{4^2} \text{Cov}(y_4, y_3) & \\
& & + \frac{1}{4^2} \text{var}(y_4) & \\
& & & \\
& & & \frac{1}{4^2} \text{var}(y_1) \\
& & & - \frac{2}{4^2} \text{Cov}(y_2, y_1) \\
\frac{1}{4^2} \text{var}(y_1) & - \frac{1}{4^2} \text{var}(y_1) & - \frac{1}{4^2} \text{var}(y_1) & - \frac{2}{4^2} \text{Cov}(y_3, y_1) \\
+ \frac{2}{4^2} \text{Cov}(y_4, y_1) & + \frac{2}{4^2} \text{Cov}(y_2, y_1) & + \frac{2}{4^2} \text{Cov}(y_3, y_1) & + \frac{2}{4^2} \text{Cov}(y_4, y_1) \\
- \frac{1}{4^2} \text{var}(y_2) & - \frac{1}{4^2} \text{var}(y_2) & + \frac{1}{4^2} \text{var}(y_2) & + \frac{1}{4^2} \text{var}(y_2) \\
- \frac{2}{4^2} \text{Cov}(y_3, y_2) & + \frac{1}{4^2} \text{var}(y_3) & - \frac{2}{4^2} \text{Cov}(y_4, y_2) & + \frac{2}{4^2} \text{Cov}(y_3, y_2) \\
- \frac{1}{4^2} \text{var}(y_3) & - \frac{2}{4^2} \text{Cov}(y_4, y_3) & - 0 \frac{1}{4^2} \text{var}(y_3) & - \frac{2}{4^2} \text{Cov}(y_4, y_2) \\
+ \frac{1}{4^2} \text{var}(y_4) & + \frac{1}{4^2} \text{var}(y_4) & + \frac{1}{4^2} \text{var}(y_4) & + \frac{1}{4^2} \text{var}(y_3) \\
& & & - \frac{2}{4^2} \text{Cov}(y_4, y_3) \\
& & & + \frac{1}{4^2} \text{var}(y_4)
\end{array}$$

This result can also be found if the conversion between the two domains is used:

$$\beta_{1,-1} = X_{1,-1}^{-1}Y = X_{1,-1}^{-1} \underbrace{X_{1,0}X_{1,0}^{-1}}_I Y = X_{1,-1}^{-1}X_{1,0}\beta_{1,0} \quad (95)$$

$$\text{var}(\beta_{1,-1}) = X_{1,-1}^{-1}X_{1,0} \text{var}(\beta_{1,0})(X_{1,-1}^{-1}X_{1,0})^T \quad (96)$$

The conclusion from equations 90 and 94 is that in the 0,1 domain, the parameters' variance increase as the parameters' interaction order increase. However, in the -1,1 domain, the parameters' variances are of the same order of magnitude. This is due to the reducing transformation weight of high order interactions' parameters from the 0,1 domain to the -1,1 domain, as was explained in the previous section. As the parameters' interaction order increases in the -1,1 domain, the variance increase of the 0,1 domain is balanced by the transformation weight reduction.

APPENDIX C

1. Derivation of the Mathematical Representation of Monotonic Variables

Behavior in the -1,1 Design Domain

The mathematical representation of positive monotonic variables in the 0,1 domain requires that the response increase as more variables are set to the high state, thus:

$$\underbrace{\left(\begin{array}{l} \beta_{0,1,0} + \beta_{i,1,0} + \beta_{j,1,0} + \\ \beta_{k,1,0} + \beta_{l,1,0} + \dots + \\ \beta_{ij,1,0} + \beta_{ik,1,0} + \beta_{il,1,0} + \\ \beta_{jk,1,0} + \beta_{jl,1,0} + \beta_{kl,1,0} + \dots \\ + \beta_{ijk,1,0} + \beta_{ijl,1,0} + \beta_{ikl,1,0} \\ \beta_{jkl,1,0} + \dots \end{array} \right)}_{i,j,k,l,\dots=1} > \underbrace{\left(\begin{array}{l} \beta_{0,1,0} + \beta_{j,1,0} + \\ \beta_{k,1,0} + \beta_{l,1,0} + \dots + \\ \beta_{jk,1,0} + \beta_{jl,1,0} + \\ \beta_{kl,1,0} + \dots \\ + \beta_{jkl,1,0} + \dots \end{array} \right)}_{i=0 \text{ And } j,k,l,\dots=1} > \underbrace{\left(\begin{array}{l} \beta_{0,1,0} + \beta_{k,1,0} + \\ \beta_{l,1,0} + \dots + \\ \beta_{kl,1,0} + \dots \end{array} \right)}_{i,j=0 \text{ And } k,l,\dots=1} > \dots > \beta_{0,1,0} \quad (97)$$

If every inequality is written separately and simplified then:

$$\left(\begin{array}{l} \beta_{i,1,0} + \beta_{ij,1,0} + \\ \beta_{ik,1,0} + \beta_{il,1,0} + \\ \beta_{ijk,1,0} + \beta_{ijl,1,0} + \\ \beta_{ikl,1,0} + \dots \end{array} \right) > 0, \quad \left(\begin{array}{l} \beta_{j,1,0} + \beta_{jk,1,0} + \\ \beta_{jl,1,0} + \beta_{jkl,1,0} + \dots \end{array} \right) > 0, \dots \beta_{n,1,0} > 0, \quad (98)$$

where ijkl represent every possibility of variables' indices. The right most inequality $\beta_{n,0,1} > 0$ means that all first order terms must be positive if a monotonic behavior is present. Starting with the inequality to the left, the parameters of the inequality can be expanded to:

$$\left(\begin{aligned}
&0.5\beta_{i,1,0} + 0.5\beta_{i,1,0} + \\
&0.25\beta_{ij,1,0} + 0.25\beta_{ij,1,0} + 0.25\beta_{ij,1,0} + 0.25\beta_{ij,1,0} \\
&0.25\beta_{ik,1,0} + 0.25\beta_{ik,1,0} + 0.25\beta_{ik,1,0} + 0.25\beta_{ik,1,0} + \\
&0.25\beta_{il,1,0} + 0.25\beta_{il,1,0} + 0.25\beta_{il,1,0} + 0.25\beta_{il,1,0} \\
&0.125\beta_{ijk,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijk,1,0} + \\
&0.125\beta_{ijk,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijk,1,0} + \\
&0.125\beta_{ijl,1,0} + 0.125\beta_{ijl,1,0} + 0.125\beta_{ijl,1,0} + 0.125\beta_{ijl,1,0} + \\
&0.125\beta_{ijl,1,0} + 0.125\beta_{ijl,1,0} + 0.125\beta_{ijl,1,0} + 0.125\beta_{ijl,1,0} + \\
&0.125\beta_{ikl,1,0} + 0.125\beta_{ikl,1,0} + 0.125\beta_{ikl,1,0} + 0.125\beta_{ikl,1,0} + \\
&0.125\beta_{ikl,1,0} + 0.125\beta_{ikl,1,0} + 0.125\beta_{ikl,1,0} + 0.125\beta_{ikl,1,0} + \dots
\end{aligned} \right) > 0 \tag{99}$$

This can be rearranged to:

$$\left(\begin{aligned}
&0.5\beta_{i,1,0} + 0.25\beta_{ij,1,0} + 0.25\beta_{ik,1,0} + 0.25\beta_{il,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijl,1,0} \\
&+ 0.125\beta_{ikl,1,0} + \dots + \\
&0.5\beta_{i,1,0} + 0.25\beta_{ij,1,0} + 0.25\beta_{ik,1,0} + 0.25\beta_{il,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijl,1,0} \\
&+ 0.125\beta_{ikl,1,0} + \dots + \\
&0.25\beta_{ij,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijl,1,0} + \dots + \\
&0.25\beta_{ij,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ijl,1,0} + \dots + \\
&0.25\beta_{ik,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ikl,1,0} + \dots + \\
&0.25\beta_{ik,1,0} + 0.125\beta_{ijk,1,0} + 0.125\beta_{ikl,1,0} + \dots + \\
&0.25\beta_{il,1,0} + 0.125\beta_{ijl,1,0} + 0.125\beta_{ikl,1,0} + \dots + \\
&0.25\beta_{il,1,0} + 0.125\beta_{ijl,1,0} + 0.125\beta_{ikl,1,0} + \dots + \\
&0.125\beta_{ijk,1,0} + \dots + \\
&0.125\beta_{ijk,1,0} + \dots + \\
&0.125\beta_{ijl,1,0} + \dots + \\
&0.125\beta_{ijl,1,0} + \dots + \\
&0.125\beta_{ikl,1,0} + \dots + \\
&0.125\beta_{ikl,1,0} + \dots + \\
&+ \dots
\end{aligned} \right) > 0 \tag{100}$$

Each of the rows represents half a main effect or interaction as was explained in the previous Appendix. The inequality can be abbreviated then rewritten as

$0.5[M(i) + I(i, j) + I(i, k) + I(i, l) + \dots + I(i, j, k) + I(i, j, l) + I(i, k, l) + \dots] > 0$, where ijkl represent every possibility of variables' indices. This can be expressed as $[M(i) + \sum I(i, \dots)] > 0$. The second monotonic behavior inequality of the inequalities 98

requires that

$$\left(\begin{array}{l} \beta_{j,1,0} + \beta_{jk,1,0} + \\ \beta_{jl,1,0} + \beta_{jkl,1,0} + \dots \end{array} \right) > 0 \quad (101)$$

This can be expanded then rearranged to:

$$\left(\begin{array}{l} 0.5\beta_{j,1,0} + 0.25\beta_{jk,1,0} + 0.25\beta_{jl,1,0} + 0.125\beta_{jkl,1,0} \\ 0.5\beta_{j,1,0} + 0.25\beta_{jk,1,0} + 0.25\beta_{jl,1,0} + 0.125\beta_{jkl,1,0} + \dots \\ 0.25\beta_{jk,1,0} + 0.125\beta_{jkl,1,0} \\ 0.25\beta_{jk,1,0} + 0.125\beta_{jkl,1,0} \\ 0.25\beta_{jl,1,0} + 0.125\beta_{jkl,1,0} \\ 0.25\beta_{jl,1,0} + 0.125\beta_{jkl,1,0} \\ 0.125\beta_{jkl,1,0} \\ 0.125\beta_{jkl,1,0} \end{array} \right) > 0 \quad (102)$$

Each of the rows represents half a main effect or interaction and can be abbreviated

as $0.5[M(j) - I(i, j) + I(j, k) + I(j, l) + \dots - I(i, j, k) - I(i, j, l) - \dots + I(j, k, l) + \dots] > 0$,

where ijkl represent every possibility of variables' indices. This can be expressed as

$[M(j) + \sum I(j, \dots) - \sum I(i, j, \dots)] > 0$. This is a similar inequality to inequality 100.

However, it is for any variable j after dropping all i terms by setting its state to 0 in the

0,1 domain or -1 in the -1,1 domain. The inequalities will drop one variable after the other till the last one. Combining all the findings in the -1,1 domain:

$$\begin{aligned}
 & \left[\begin{array}{l} \beta_{i,1,-1} + \beta_{ij,1,-1} + \beta_{ik,1,-1} + \beta_{il,1,-1} + \dots \\ + \beta_{ijk,1,-1} + \beta_{ijl,1,-1} + \beta_{ikl,1,-1} + \dots + \beta_{ijkl,1,-1} + \dots \end{array} \right] > 0 \\
 & \left[\begin{array}{l} \beta_{i,1,-1} - \beta_{ij,1,-1} + \beta_{ik,1,-1} + \beta_{il,1,-1} + \dots \\ - \beta_{ijk,1,-1} - \beta_{ijl,1,-1} + \beta_{ikl,1,-1} + \dots - \beta_{ijkl,1,-1} + \dots \end{array} \right]_{j=-1} > 0 \\
 & \left[\begin{array}{l} \beta_{i,1,-1} - \beta_{ij,1,-1} - \beta_{ik,1,-1} + \beta_{il,1,-1} + \dots \\ + \beta_{ijk,1,-1} - \beta_{ijl,1,-1} - \beta_{ikl,1,-1} + \dots + \beta_{ijkl,1,-1} + \dots \end{array} \right]_{j,k=-1} > 0 \\
 & \left[\begin{array}{l} \beta_{i,1,-1} - \beta_{ij,1,-1} - \beta_{ik,1,-1} - \beta_{il,1,-1} + \dots \\ + \beta_{ijk,1,-1} + \beta_{ijl,1,-1} + \beta_{ikl,1,-1} + \dots - \beta_{ijkl,1,-1} + \dots \end{array} \right]_{j,k,l=-1} > 0 \\
 & \dots
 \end{aligned} \tag{103}$$

This can be generalized as:

$$\beta_{i,1,-1} + \sum_{j=1, j \neq i}^N s_j \beta_{ij,1,-1} + \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N s_{jk} \beta_{ijk,1,-1} + \sum_{l>k, l \neq i}^N \sum_{k>j, k \neq i}^N \sum_{j=1, j \neq i}^N s_{jkl} \beta_{ijkl,1,-1} + \dots > 0 \tag{104}$$

for all combinations of $s_j, s_{jk}, s_{jkl}, \dots$, where $s_j = x_j, s_{jk} = x_j x_k, s_{jkl} = x_j x_k x_l, \dots$.

Accordingly, inequality 104 is the mathematical representation of the monotonic behavior condition for any variable i in the -1,1 domain.

APPENDIX D

1. Example of Parameters Domain Conversion to Cotter's Integrated Parameters

This section explains the conversion of the -1,1 domain parameters to the integrated parameters' domain, then determines the new domain's design matrix. This is demonstrated through a four variables' example. The conversion matrix C between the parameters' domain, β , and the integrated parameters' domain, α , for this example is a combination of two conversion matrices, one for the integrated odd parameters and another for the integrated even parameters. The α domain parameters' vectors for four variables are defined as:

$$\alpha_{Odd} = \begin{bmatrix} \alpha_{0,O} \\ \alpha_{i,O} \\ \vdots \\ \alpha_{ij,O} \\ \vdots \\ \alpha_{ijk,O} \\ \vdots \\ \vdots \\ \alpha_{ijk\dots,O} \end{bmatrix}, \quad \alpha_{Even} = \begin{bmatrix} \alpha_{0,E} \\ \alpha_{i,E} \\ \vdots \\ \alpha_{ij,E} \\ \vdots \\ \alpha_{ijk,E} \\ \vdots \\ \vdots \\ \alpha_{ijk\dots,E} \end{bmatrix}, \quad (105, 106)$$

The conversion matrices for the integrated odd and even parameters are based on:

$$\alpha_{R,O} = \beta_{R,1,-1} + \sum_{i=1}^u \beta_{R+2,1,-1} + \dots \quad (107)$$

$$\alpha_{R,E} = \sum_{j=1}^v \beta_{R+1,1,-1} + \sum_{k=1}^w \beta_{R+3,1,-1} + \dots, \quad (108)$$

where R is an interaction order, and u , v and w are the number of interactions containing the R interaction of the order $R+2$, $R+1$, and $R+3$ consequently. Thus, the conversion matrices for the integrated odd parameters and the integrated even parameters are:

$$C_{odd} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (109)$$

$$C_{Even} = \begin{bmatrix} 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (110)$$

The matrices of C_{Odd} and C_{Even} are combined in one matrix C . However, $n/2$ of the rows of C are dependent, and are eliminated. The resulting combined conversion matrix is:

$$C = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
\tag{111}$$

and the associated α vector is:

$$\alpha = \begin{bmatrix} \alpha_{0,O} \\ \alpha_{0,E} \\ \alpha_{1,O} \\ \alpha_{1,E} \\ \alpha_{2,O} \\ \alpha_{2,E} \\ \alpha_{3,O} \\ \alpha_{3,E} \\ \alpha_{4,O} \\ \alpha_{4,E} \\ \alpha_{12,O} \\ \alpha_{12,E} \\ \alpha_{13,O} \\ \alpha_{13,E} \\ \alpha_{14,O} \\ \alpha_{14,E} \end{bmatrix} \quad (112)$$

The integrated odd and even parameters are found using:

$$Y = X\beta = XC^{-1}\alpha \Rightarrow \alpha = (XC^{-1})^{-1}Y \quad (113)$$

The matrix $(XC^{-1})^{-1}$ can be found, since X and C are already known. Substituting

$(XC^{-1})^{-1}$ in equation 113:

$$\alpha = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 1/4 \\ 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 1/4 \\ -1/4 & 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 0 & 0 & 1/4 \\ 1/4 & 0 & 0 & 0 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 0 & 0 & 1/4 \\ -1/4 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 1/4 \\ 1/4 & 0 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 0 & 1/4 \\ -1/4 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 1/4 \\ 1/4 & -1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/4 & 1/4 \\ 1/8 & 0 & 0 & 1/8 & -1/8 & 0 & 0 & -1/8 & -1/8 & 0 & 0 & -1/8 & 1/8 & 0 & 0 & 1/8 \\ -1/8 & 0 & 0 & 1/8 & 1/8 & 0 & 0 & -1/8 & 1/8 & 0 & 0 & -1/8 & -1/8 & 0 & 0 & 1/8 \\ 1/8 & 0 & -1/8 & 0 & 0 & 1/8 & 0 & -1/8 & -1/8 & 0 & 1/8 & 0 & 0 & -1/8 & 0 & 1/8 \\ -1/8 & 0 & 1/8 & 0 & 0 & 1/8 & 0 & -1/8 & 1/8 & 0 & -1/8 & 0 & 0 & -1/8 & 0 & 1/8 \\ 1/8 & -1/8 & 0 & 0 & 0 & 0 & 1/8 & -1/8 & -1/8 & 1/8 & 0 & 0 & 0 & 0 & -1/8 & 1/8 \\ -1/8 & 1/8 & 0 & 0 & 0 & 0 & 1/8 & -1/8 & 1/8 & -1/8 & 0 & 0 & 0 & 0 & -1/8 & 1/8 \end{bmatrix} \quad (114)$$

This equation demonstrates that as the order of the integrated odd and even parameters increase, the number of contributing response nodes increase. If the last six integrated parameters are assumed negligible:

$$\alpha = \begin{bmatrix} \alpha_{0,O} \\ \alpha_{0,E} \\ \alpha_{1,O} \\ \alpha_{1,E} \\ \alpha_{2,O} \\ \alpha_{2,E} \\ \alpha_{3,O} \\ \alpha_{3,E} \\ \alpha_{4,O} \\ \alpha_{4,E} \\ \alpha_{12,O} \\ \alpha_{12,E} \\ \alpha_{13,O} \\ \alpha_{13,E} \\ \alpha_{14,O} \\ \alpha_{14,E} \end{bmatrix} = \begin{bmatrix} \alpha_{0,O} \\ \alpha_{0,E} \\ \alpha_{1,O} \\ \alpha_{1,E} \\ \alpha_{2,O} \\ \alpha_{2,E} \\ \alpha_{3,O} \\ \alpha_{3,E} \\ \alpha_{4,O} \\ \alpha_{4,E} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (115)$$

then the experiments to find y_3 , y_5 , y_6 , y_9 , y_{10} , and y_{12} are not needed, as they do not contribute to any of the integrated parameters. The relationship of the response and the integrated parameters is found as:

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & -2 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 1 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 4 & 4 \\ 1 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 4 & -4 \\ 0 & 1 & 2 & -2 & 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 & 4 & -4 & 0 & 0 \\ 1 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -2 & 2 & -2 & 0 & 0 & 0 & 0 & 4 & -4 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \alpha \quad (116)$$

The magnification of the neglected high order integrated parameters in the middle region of the y vector causes a significant error. This is analogous to the projection of a spline using the two end points and slopes. This magnification demonstrates why the HIP method is inaccurate in the projection of the response and the estimation of -1,1 domain parameters.

APPENDIX E

1. Experiments Path Selection Approaches for the RC Method

In this section, several approaches are explored to determine the best experiments' selection approach. After that successful alignment of variables was performed and the importance of variables is found, the following approaches are explored for path selection.

Nodes with the Maximum Number of Connection

This approach targets the nodes with the maximum number of connections to nodes with high ranges. Thus, the node with highest potential impact is targeted first. This is done by summing the ratios of the ranges of all upper and lower connected nodes to the total range span of the tree. The index of the next experiment to conduct is of the node with the highest E_i where:

$$E_i = \sum_{j=1}^n e_j \delta_{ij} \text{ where } \delta_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{if } i \text{ and } j \text{ are not connected} \end{cases} \quad (117)$$

$$e_j = \frac{(y_{j,\max} - y_{j,\min})}{(y_{\max} - y_{\min})_{\text{Current}}} \quad (118)$$

$(y_{\max} - y_{\min})_{\text{Current}}$ is the current span of the tree. The problem with this approach is that it does not take into account the expected value of the node, and the impact of this value on the rest of the tree.

Nodes with the Highest Median Impact

In this approach, the median value for every node's range is used, and the impact of such assumption on the total reduction of nodes' ranges is found. The node that causes the maximum range reduction is selected. The index of the next experiment to conduct is of the node with the lowest E_i where:

$$E_i = \sum_{j=1}^n e_j \delta_{ij} \text{ where } \delta_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{if } i \text{ and } j \text{ are not connected} \end{cases} \quad (119)$$

$$e_j = \frac{(y_{j,\max} - y_{j,\min})_{R_i = \text{Median}(y_i)}}{(y_{j,\max} - y_{j,\min})_{\text{Current}}} \quad (120)$$

R_i is the assumed result of i . $(y_{j,\max} - y_{j,\min})_{\text{Current}}$ is the range of the j node before assuming a value for i . This approach predicts the value of every node, but does not account for the location of the node in the tree. It is, thus, predicted that adjacent nodes in the tree will have close results.

Nodes with the Highest Absolute Linear Interpolation Impact

In this approach, the results of the nodes are assumed to follow a linear distribution from the lowest node to the highest node. The effect of such assumption in the total reduction of nodes ranges is found for every node. The node that causes the maximum ranges reduction is selected. The index of the next experiment to conduct is of the node with the lowest E_i where:

$$E_i = \sum_{j=1}^n e_j \delta_{ij} \text{ where } \delta_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{if } i \text{ and } j \text{ are not connected} \end{cases} \quad (121)$$

$$e_j = \frac{(y_{j,\max} - y_{j,\min})_{R_i=L_i}}{(y_{j,\max} - y_{j,\min})_{Current}}, L_i = y_{\min} + T_i(y_{\max} - y_{\min}) \quad (122)$$

T_i is the level ratio of the node in the tree, 0 for the bottom node, and 1 for the top node.

This approach assumes results linearity, but it is rarely the case that nodes behave in a linear manner due to sparsity and interactions.

Nodes with the Highest Local Linear Interpolation Impact

In this approach, the results of the nodes are assumed to follow a linear distribution. However, the interpolation is from the closest lower performed and connected node to the closest upper performed and connected node. The effect of such assumption in the total reduction of nodes ranges is found for every node. The node that causes the maximum ranges reduction is selected. The index of the next experiment to conduct is of the node with the lowest E_i where

$$E_i = \sum_{j=1}^n e_j \delta_{ij} \text{ where } \delta_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{if } i \text{ and } j \text{ are not connected} \end{cases} \quad (123)$$

$$e_j = \frac{(y_{j,\max} - y_{j,\min})_{R_i=L_i}}{(y_{j,\max} - y_{j,\min})_{Current}}, L_i = y_{\min,Local} + T_i(y_{\max,Local} - y_{\min,Local}) \quad (124)$$

$y_{\min,Local}$ is result of the closest lower performed and connected node and $y_{\max,Local}$ is the index of the closest upper performed and connected node. T_i is the level ratio of the node in the tree, 0 for the closest connected lower performed node, and 1 for the closest connected upper performed node.

This approach is better than the absolute linear approach, but it still assumes that nodes behave in a local linear manner.

Nodes with the Highest Local Linear Interpolation Weighted Impact

This approach is the same as the local linear interpolation impact. However, a weight is added to every node with the weight proportional to the range of the node. This biases the path selection toward nodes with the highest ranges. The index of the next experiment to conduct is of the node with the lowest E_i where

$$E_i = \sum_{j=1}^n e_j \delta_{ij} \text{ where } \delta_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{if } i \text{ and } j \text{ are not connected} \end{cases} \quad (125)$$

$$e_j = w_j \frac{(y_{j,\max} - y_{j,\min})_{R_i=L_i}}{(y_{j,\max} - y_{j,\min})_{Current}} = \frac{(y_{j,\max} - y_{j,\min})_{Current}}{(y_{\max} - y_{\min})_{Current}} \frac{(y_{j,\max} - y_{j,\min})_{R_i=L_i}}{(y_{j,\max} - y_{j,\min})_{Current}} = \frac{(y_{j,\max} - y_{j,\min})_{R_i=L_i}}{(y_{\max} - y_{\min})_{Current}} \quad (126)$$

$$L_i = y_{\min,Local} + T(y_{\max,Local} - y_{\min,Local}) \quad (127)$$

This approach provides better results than the non-weighted approach. However, it also assumes local linearity and ignores interactions effects.

Nodes with the Highest Span Impact

In this approach, the effect of changing every node from its range lower limit to its range higher limit is found. The node with the span change that causes the maximum impact is selected. The index of the next experiment to conduct is of the node with the highest ΔE_i where

$$\Delta E_i = \left[\sum_{j=1}^n |e_{j,\max} - e_{j,\min}| \right] \delta_{ij} \text{ where } \delta_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{if } i \text{ and } j \text{ are not connected} \end{cases} \quad (128)$$

$$e_{j,\min} = \frac{(y_{j,\max} - y_{j,\min})_{R_i = \min(y_i)}}{(y_{j,\max} - y_{j,\min})_{Current}}, e_{j,\max} = \frac{(y_{j,\max} - y_{j,\min})_{R_i = \max(y_i)}}{(y_{j,\max} - y_{j,\min})_{Current}} \quad (129)$$

This approach biases the path selection towards nodes that the tree is most sensitive to, and does not take into account the estimated value of the node.

Nodes that Reveal Link Directional Belief

This approach will target the links with high uncertainty in the directions of effect. As these links are explored, the belief in these links' directions of effect is changed. The links monotonic behavior is either further confirmed or disconfirmed. This approach biases the experiment's path selection toward inactive variables, since they are the ones that will demonstrate weak monotonic behavior. Thus, the approach fails to effectively reduce the nodes ranges.

Nodes Based on a Probability Distribution

In this approach, every node will have a probability distribution with a mean value and variance. Every node range is split into M points. For every point, the number of allowed combinations of other nodes is found. This number will reflect the probability that the node will have a value at this point. To illustrate this approach, two nodes in a five variables example are used. If after performing a number of experiments, node 00010 is found to have a range of 5-10, and node 00011 is found to have a range of 6-11, and if M is set at 6, then the possible results are 5, 6, 7, 8, 9 and 10 for 00010 and 6, 7, 8, 9, 10 and 11 for 00011.

A combinations matrix is found by testing all states of 00010 and 00011 that satisfy 00011>00010. The results of all combinations, without the 00011>00010 condition, and with the condition, are shown in Table 92. The histograms for y_1 and y_2 of both cases, with and without the monotonic behavior condition, are shown in Figure 80. The figure shows that y_2 is skewed with the highest probability of occurrence at 10 or 11. This is due to the fact that when y_1 is at the lowest value, 5, y_2 will vary freely. As y_1 increases, the range of allowable values of y_2 is reduced up to the point when y_1 is 10 and y_2 is 10 or 11. Once a distribution is developed for every node, the mean and variance of every node is found. The covariance of two nodes is found by fixing the two nodes to every possible combination and determining the probability distribution of every combination of other nodes. The equations used to find the variance and covariance, when the condition is applied, are:

$$Var(y_1) = \frac{1}{S-1} \left[\sum_{i=1}^u (y_{1,i} - \bar{y}_1)^2 \delta(\text{if } y_{2,i} > y_{1,i}) \right] \quad (130)$$

$$Var(y_2) = \frac{1}{S-1} \left[\sum_{i=1}^u (y_{2,i} - \bar{y}_2)^2 \delta(\text{if } y_{2,i} > y_{1,i}) \right] \quad (131)$$

$$Cov(y_1, y_2) = \frac{1}{S-1} \left[\sum_{i=1}^{M^u} (y_{1,i} - \bar{y}_1)(y_{2,i} - \bar{y}_2) \delta(\text{if } y_{2,i} > y_{1,i}) \right] \quad (132)$$

$$S = \sum_{i=1}^{M^u} \delta(\text{if } y_{2,i} > y_{1,i}) \quad (133)$$

where u is the number of unperformed nodes. Another way to determine the covariance is to develop a correlation between the two responses and from that find the covariance through:

$$Cov(y_1, y_2) = Cor(y_1, y_2) \sqrt{Var(y_1)} \sqrt{Var(y_2)} \quad (134)$$

The variance is known, so the correlation factor is to be found to determine the covariance.

The correlation's accuracy is dependent on M. In the example above, using M=10, 50 and 100 generated, through regression, the correlation factors of 0.4173, 0.4270 and 0.4285. The combinations are shown in Figure 81. In order to improve the accuracy of the covariance and the corresponding probability distribution, M needs to be high enough. The problem is that as M increases, M^n increases exponentially. This represents 6^2 in the example. In a ten variables problem, the number of nodes is $2^{10} = 1024$ and if ten increments per node is desired, the number of combinations is 10^{1024} . Since the number of combinations needed to apply this approach makes it computational expensive, an approximation could be used by determining the combinations of every two nodes separately, then combining the overall probability by multiplying the connected nodes probabilities. This is, however, still computationally demanding when a large number of nodes and increments are considered. Thus, other approaches were investigated to approximate the correlation without the need to discretize the ranges.

Table 92
Discretized Results of the Combinations of Two Overlapping Nodes

| Index | No Condition | | 00011>00010 | |
|-----------------------|--------------|-------|-------------|--------------|
| | 00010 | 00011 | 00010 | 00011 |
| 1 | 5 | 6 | 5 | 6 |
| 2 | 5 | 7 | 5 | 7 |
| 3 | 5 | 8 | 5 | 8 |
| 4 | 5 | 9 | 5 | 9 |
| 5 | 5 | 10 | 5 | 10 |
| 6 | 5 | 11 | 5 | 11 |
| 7 | 6 | 6 | 6 | 6 |
| 8 | 6 | 7 | 6 | 7 |
| 9 | 6 | 8 | 6 | 8 |
| 10 | 6 | 9 | 6 | 9 |
| 11 | 6 | 10 | 6 | 10 |
| 12 | 6 | 11 | 6 | 11 |
| 13 | 7 | 6 | --- | --- |
| 14 | 7 | 7 | 7 | 7 |
| 15 | 7 | 8 | 7 | 8 |
| 16 | 7 | 9 | 7 | 9 |
| 17 | 7 | 10 | 7 | 10 |
| 18 | 7 | 11 | 7 | 11 |
| 19 | 8 | 6 | --- | --- |
| 20 | 8 | 7 | --- | --- |
| 21 | 8 | 8 | 8 | 8 |
| 22 | 8 | 9 | 8 | 9 |
| 23 | 8 | 10 | 8 | 10 |
| 24 | 8 | 11 | 8 | 11 |
| 25 | 9 | 6 | --- | --- |
| 26 | 9 | 7 | --- | --- |
| 27 | 9 | 8 | --- | --- |
| 28 | 9 | 9 | 9 | 9 |
| 29 | 9 | 10 | 9 | 10 |
| 30 | 9 | 11 | 9 | 11 |
| 31 | 10 | 6 | --- | --- |
| 32 | 10 | 7 | --- | --- |
| 33 | 10 | 8 | --- | --- |
| 34 | 10 | 9 | --- | --- |
| 35 | 10 | 10 | 10 | 10 |
| 36 | 10 | 11 | 10 | 11 |
| Variance, Covariance: | | 3,0 | | 2.553, 1.046 |

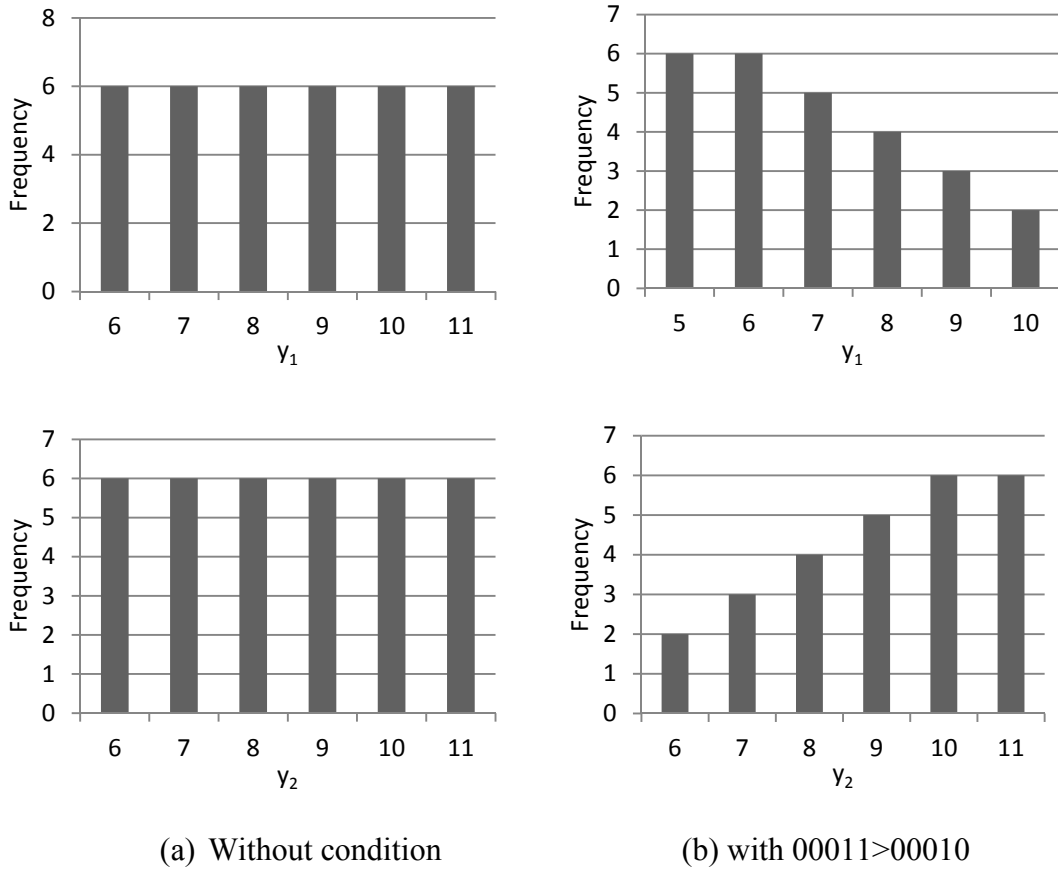
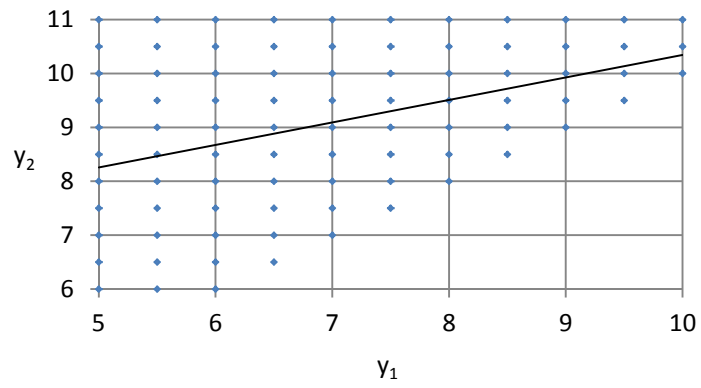
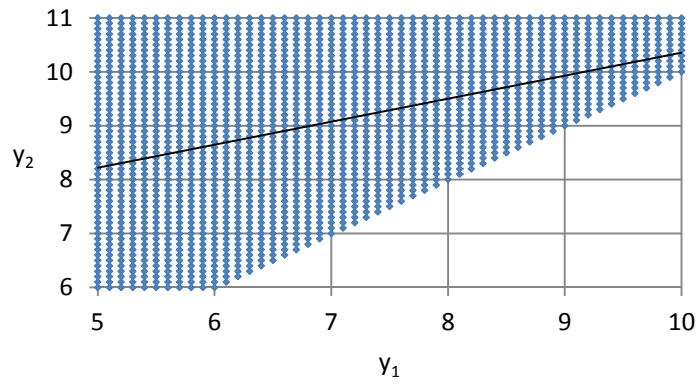


Figure 80. Node's Potential Value Distribution by Other Nodes' Effects

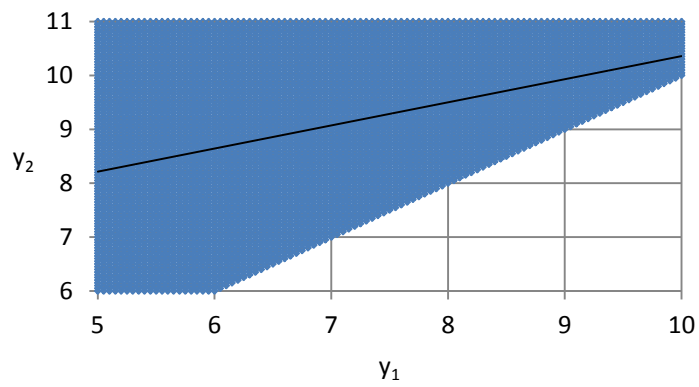
One approach that was investigated is to consider the fact that the shape shown in Figure 81 consist of a rectangle with an area of 5 and a trapezoid with an area of $(10-6)(1+5)/2=12$. The slope of the line in the rectangle is zero and in the trapezoid is $((10-6)/(10-6) + 0)/2=1/2$. A weighted average gives $(0 \times 5 + 1/2 \times 12) / (5 + 12) = 0.35$. This value underestimates the correlation factor. Another approach is to sum vectors that compose the shown line. The vectors summation is: $1i + 0j + (10-6)i + (10.5 - 8.5)j = 5i + 2j$. The slope of the vector is $2/5=0.4$. This value was found to underestimate the correlation factor too.



(a) $M=10$



(b) $M=50$



(c) $M=100$

Figure 81. M Effect on the Accuracy of the Correlation of Two Nodes

The best approach, to determine the two responses correlation, was developed using the overlapping interval of the two ranges. The overlapping interval's points are set to equal the probability defined as:

If $y_{i,\min} \leq y_{j,\min}$

$$P_{i,m,j} = \frac{y_{j,\max} - y_{i,m}}{y_{j,\max} - y_{j,\min}} \text{ if } y_{i,m} \geq y_{j,\min} \quad (135)$$

$$P_{i,m,j} = 1 \text{ if } y_{i,m} < y_{j,\min} \quad (136)$$

If $y_{i,\min} \geq y_{j,\min}$

$$P_{i,m,j} = \frac{y_{i,m} - y_{j,\min}}{y_{j,\max} - y_{j,\min}} \text{ if } y_{i,m} \leq y_{j,\max} \quad (137)$$

$$P_{i,m,j} = 1 \text{ if } y_{i,m} > y_{j,\max} \quad (138)$$

$P_{i,m,j}$ is the probability of increment m occurrence in node i with respect to node j . These equations are applied to the two nodes example, and the results are shown in Table 93. The table shows a distribution that is similar to the histogram of Figure 80. The probability for every point m in node i is combined using:

$$P_{i,m} = \prod P_{i,m,j} \quad (139)$$

This probability is an approximation since it ignores the correlation between the j s and just considers the effect of every node i on j separately. To include the j correlations, M^{u-1} combinations, for every point m in node i , are needed, which is computationally very expensive.

Table 93

Probabilities of Increments Based on the Overlapping Proportions of Ranges

| i=00010 | $P_{1,m,2}$ | i=00011 | $P_{2,m,1}$ |
|---------|-------------|---------|-------------|
| 5 | 1 | 6 | 1/5 |
| 6 | 1 | 7 | 2/5 |
| 7 | 4/5 | 8 | 3/5 |
| 8 | 3/5 | 9 | 4/5 |
| 9 | 2/5 | 10 | 1 |
| 10 | 1/5 | 11 | 1 |

The probability distribution approach was found good for low number of variables, but as the number of variables increase, the portability distribution converged on a step shape with the mean at the high limit of the range for the top tree nodes, and at the low limit of the range for the bottom tree nodes. This is logical, since it is expected that the nodes in the upper part of the tree are close to the top node, and the ones in the lower part of the tree are close to the bottom node. However, the severity of the bias of this approach makes it useless in the experiments path selection process especially for large trees.

Nodes Selection Based on System Regularities

In this approach, the effects of physical systems' regularities on the experiments' tree are considered. As explained in the dissertation, studies have surveyed physical systems for behavioral patterns, and some patterns were proven significant. This approach was the one selected for the RC method, and is explained as part of the dissertation.

Convergence Test Criterion

This approach was developed as a stopping criterion for the experiments path selection. The parameters' convergence is used as the criterion in this approach. After every experiment is preformed, and results are populated, Lenth method [75] is applied to the average values of the projected variables' parameters, $\beta_{i,-1}$ s. These values will converge in a dropping exponential manner to the correct values.

Lenth method is used to establish three parameters regions defined by two thresholds. Parameters with an absolute value that fall in the region lower than the Margin of Error (referred to as MEr in this section) are confirmed as inactive parameters. Parameters with an absolute value that fall in the region above the Simultaneous Margin of Error (referred to as SMER in this section) are confirmed as active parameters. Parameters with an absolute value that fall in the region between the MEr and SMER are questionable parameters. The path selection continues, as long as there are questionable parameters. Once there are no parameters in the questionable zoon, the experimentation stops and the parameters are classified. The stopping criteria can, thus, be expressed as

$$|\beta_{i,1,-1}|, |\beta_{ij,1,-1}|, |\beta_{ijk,1,-1}|, \dots < SMER \quad \text{or} \quad > MEr, \quad i=1:N, j>i, k>j, \dots \quad (140)$$

Modified Lenth's MEr and SMER values were suggested in [91] to provide less conservative results, and can be used in this criterion.

APPENDIX F

1. Example Application of the RC Method on a Regular System

This section demonstrates the application of the RC method on the response of a regular system from literature [81]. The first steps of the MHIV method application, on the same example, aligned the performance characteristic and variables. Instead of finding the integrated odd and even parameters, it is desired to determine the importance of each variable, thus $N+1$ experiments are only needed. The results of these experiments are shown in the first eight rows of Table 94. As a result, variables 1235 were categorized as important. Assuming the total number of experiments allowed is sixteen experiments, out of which, eight ($N+2$) experiments are already performed, eight experiments are remaining. The experiments tree is shown in Figure 82.

If two subtrees are established using the two most important variables, 15, experiments 100000,000010,100010 and 011101 are performed to bound the subtrees. The results of these experiments are shown in Table 94. The results are reflected into the nodes' ranges of every subtree. The resulting subtrees are shown in Figure 83. Out the four subtrees of $15=xx$, the spans of the last subtrees $15=00,10$ are relatively small. The subtree $15=11$ has one additional level of performed nodes. The total span of nodes in the subtree is 1139 Units. This is compared to the total span of subtree $15=01$, which is 793 units. As a result, it is decided to explore the first subtree further. This subtree is further spanned into subtrees based on the two local most important variables, 23, thus establishing subtrees $2=x$ of $15=11$ and $3=x$ of $15=11$. Since the two upper nodes of the

subtrees have already been performed, the lower nodes of these subtrees are performed. The results are reflected into the nodes' ranges of every subtree. The resulting subtree is shown in Figure 84. The total span of the current subtree is 454.3. Since this is smaller than the total span of the 15=01 subtree, the 15=01 subtree is spanned. Variable 2 is the most important variable in this subtree, thus subtrees 2=x of 15=10 are created. The results are reflected into the nodes' ranges of every subtree. The results are shown in Table 94.

Sixteen experiments have been performed. In summary, the steps that were followed are:

- Align the variables to point toward one direction of effect on the performance characteristics
- Perform N+2 experiments of the bottom node, the top node and the layer of nodes below the top node
- Generate subtrees of the most important nodes
- Find the subtrees with highest total span and target them next
- Within the subtrees, generate subtrees of the next most important nodes
- Keep track of total span in subtrees at all levels to determine when to step out of a subtree and move to the next one

After all experiments have been performed, the average value for every node's range is used to estimate of the response, then the overall model parameters are found. The normalized span, after every experiment was performed, is shown in Table 94, and is exponentially decreasing. The RMSE of the developed model is 13.75 which 4% of the

projected response's range. To validate the performance of this method, a Res IV FFD was performed. The FFD results are shown in Table 95. The RMSE of the FFD application is 40.02. Thus, the RMSE of the RC method is 34% the RMSE of the FFD. The parameters of both methods, in addition the full factorial design, are shown in Table 96. This example shows that the RC method can outperform FFDs when applied to a system with highly monotonic variables.

Table 94
Production's Results and Parameters' Convergence Using the RC Method

| Exp. Index | Variables States* | Result | Parameters Range (Response Unit) | Current Normalized Span |
|------------|-------------------|--------|----------------------------------|-------------------------|
| 1 | 0 0 0 0 0 0 | 39.2 | --- | --- |
| 2 | 1 1 1 1 1 1 | 373.1 | 323.47 | 100.00 |
| 3 | 0 1 1 1 1 1 | 179.6 | 227.55 | 70.35 |
| 4 | 1 0 1 1 1 1 | 226.1 | 187.88 | 58.08 |
| 5 | 1 1 0 1 1 1 | 242 | 168.32 | 52.04 |
| 6 | 1 1 1 0 1 1 | 375.9 | 163.10 | 50.42 |
| 7 | 1 1 1 1 0 1 | 124.2 | 113.13 | 34.98 |
| 8 | 1 1 1 1 1 0 | 372.4 | 107.90 | 33.36 |
| 9 | 0 0 0 0 1 0 | 118.6 | 74.70 | 23.09 |
| 10 | 1 0 0 0 0 0 | 103 | 59.41 | 18.37 |
| 11 | 1 0 0 0 1 0 | 131.6 | 55.70 | 17.22 |
| 12 | 0 1 1 1 0 1 | 44.2 | 36.87 | 11.40 |
| 13 | 1 0 1 0 1 0 | 218.5 | 31.32 | 9.68 |
| 14 | 1 1 0 0 1 0 | 233.6 | 26.17 | 8.09 |
| 15 | 0 0 1 1 1 1 | 153.2 | 22.75 | 7.03 |
| 16 | 0 1 0 0 1 0 | 152.2 | 18.64 | 5.76 |

*Even though the parameters found are in the -1,1 domain, the 0,1 notation was used for variables states in this method to follow the dissertation's tree convention.

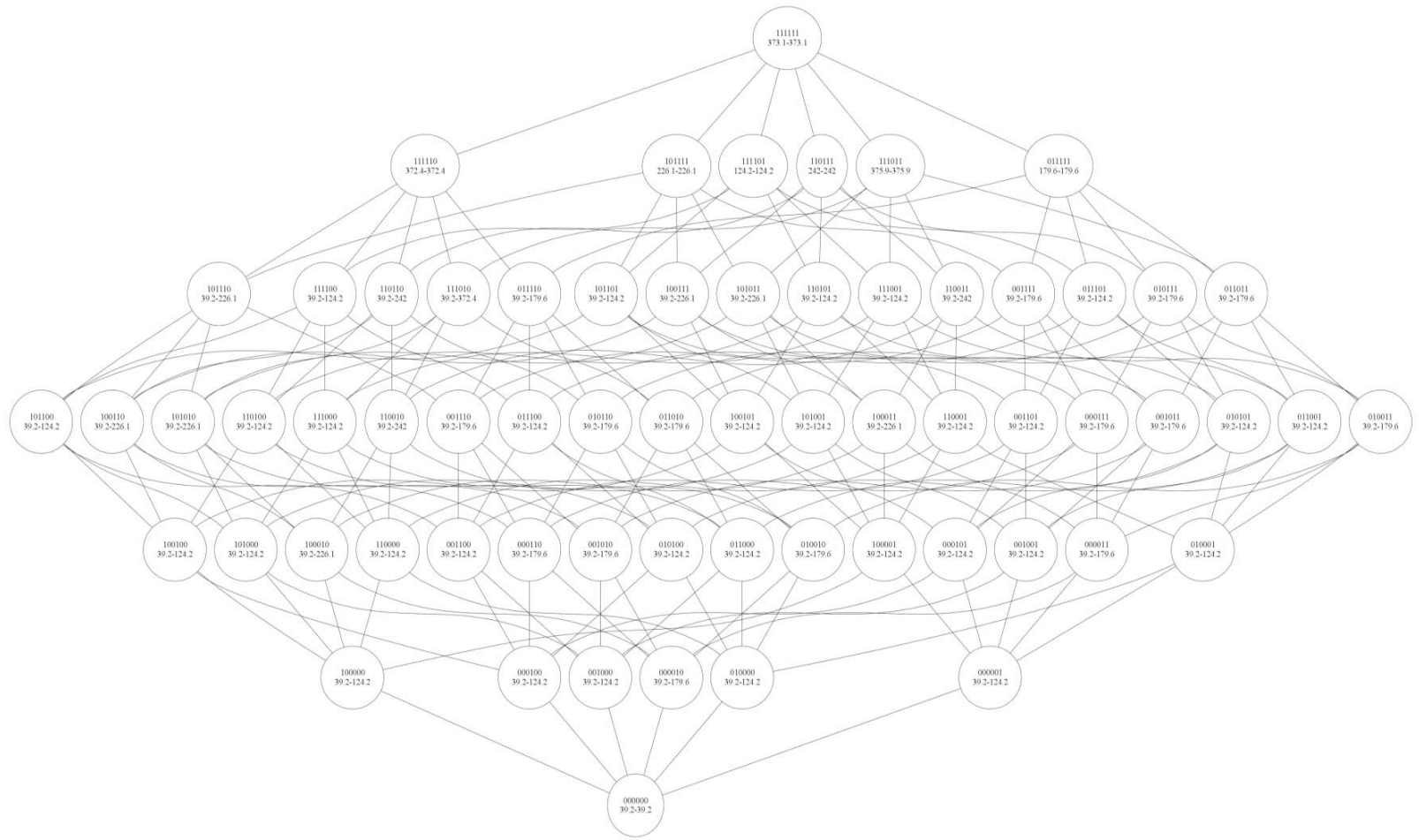


Figure 82. Production’s Diamond Tree after Eight Experiments of the RC Method Application

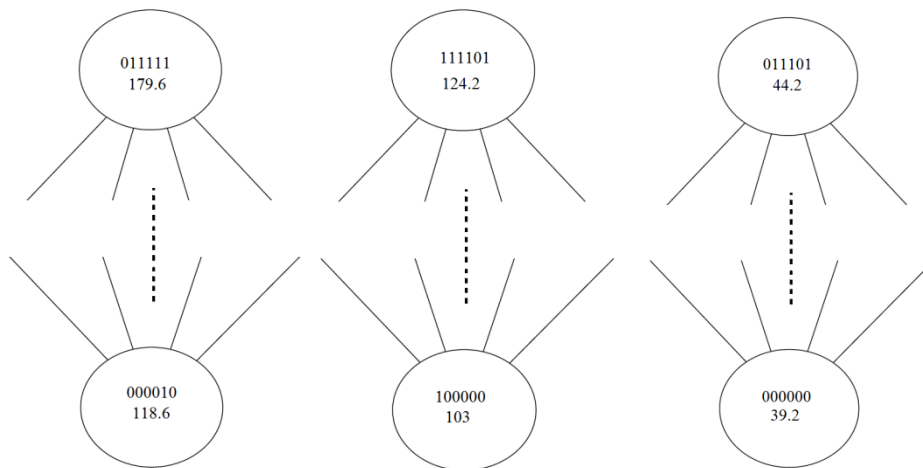
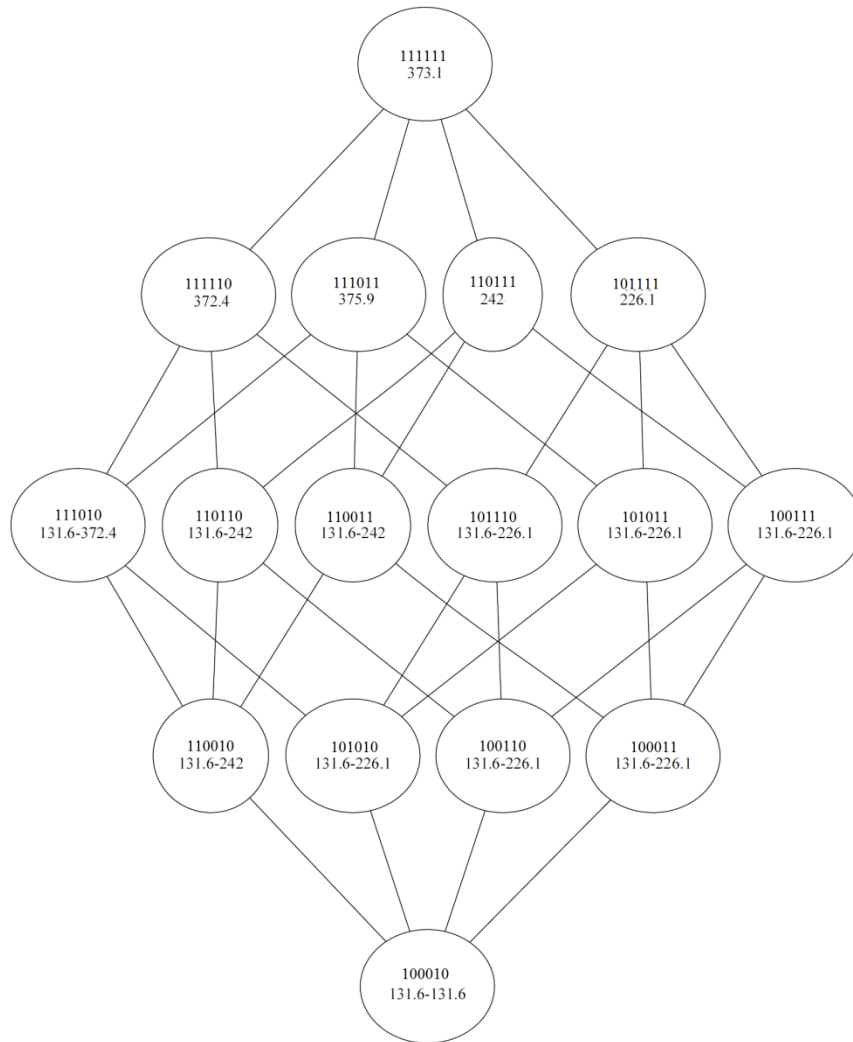


Figure 83. Production's Four Subtrees of the RC Method's Application

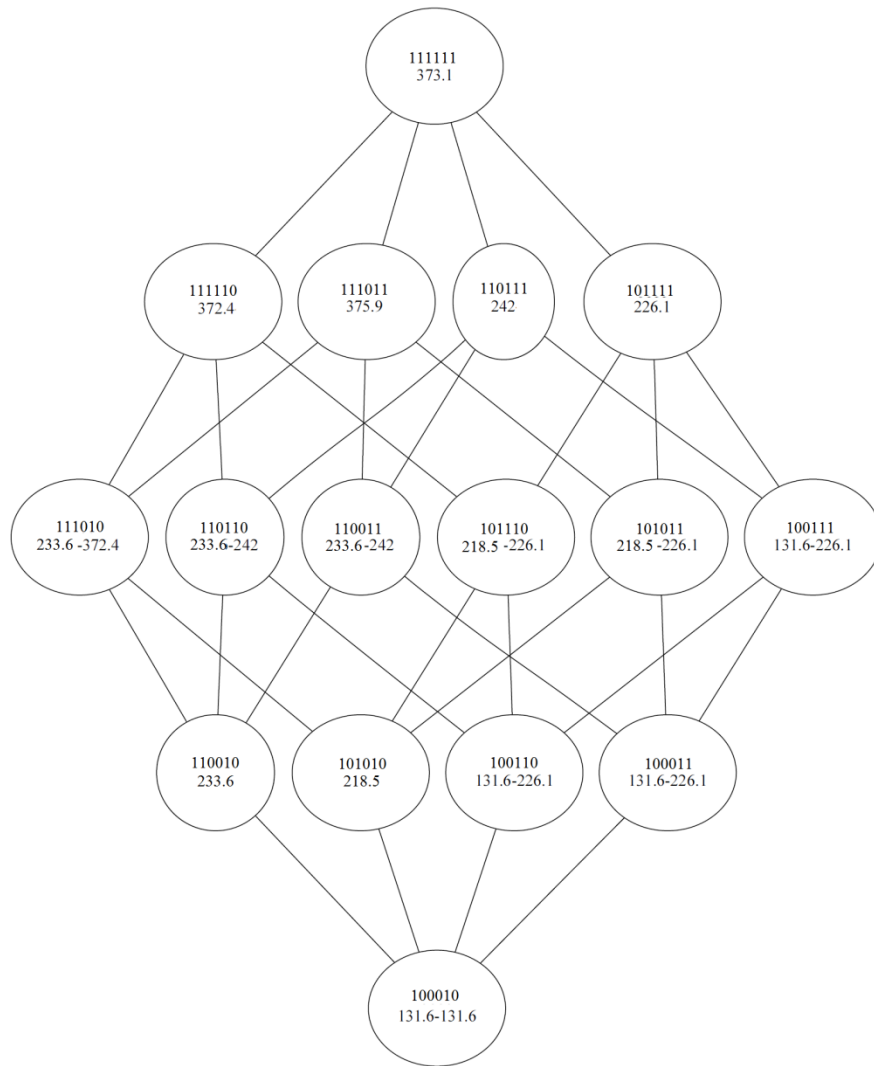


Figure 84. Production's First Subtree of the RC Method's Application

Table 95
Production's Results of the Res IV FFD

| Exp. Index | Variables States | First Response (Production) Result |
|------------|-------------------|------------------------------------|
| 1 | -1 -1 -1 -1 -1 -1 | 39.2 |
| 2 | -1 -1 -1 1 -1 1 | 373.1 |
| 3 | -1 -1 1 -1 1 1 | 179.6 |
| 4 | -1 -1 1 1 1 -1 | 226.1 |
| 5 | -1 1 -1 -1 1 1 | 242 |
| 6 | -1 1 -1 1 1 -1 | 375.9 |
| 7 | -1 1 1 -1 -1 -1 | 124.2 |
| 8 | -1 1 1 1 -1 1 | 372.4 |
| 9 | 1 -1 -1 -1 1 -1 | 118.6 |
| 10 | 1 -1 -1 1 1 1 | 103 |
| 11 | 1 -1 1 -1 -1 1 | 131.6 |
| 12 | 1 -1 1 1 -1 -1 | 44.2 |
| 13 | 1 1 -1 -1 -1 1 | 218.5 |
| 14 | 1 1 -1 1 -1 -1 | 233.6 |
| 15 | 1 1 1 -1 1 -1 | 153.2 |
| 16 | 1 1 1 1 1 1 | 152.2 |

Table 96
 Production's Estimated Parameters Using the RC Method and a FFD

| Parameter Index | RC Method Value | FFD Value | Full Factorial Value |
|-----------------|-----------------|-----------|----------------------|
| 0 | 138.00 | 140.71 | 137.39 |
| 1 | 41.70 | 43.14 | 40.92 |
| 2 | 16.94 | 30.29 | 21.10 |
| 3 | 12.23 | 29.20 | 18.45 |
| 4 | 3.41 | 0.55 | 1.04 |
| 5 | 60.35 | 61.23 | 58.65 |
| 6 | 3.52 | 1.59 | 0.94 |
| 12 | 9.29 | 28.51 | 13.00 |
| 13 | 10.13 | 28.73 | 11.28 |
| 14 | 1.31 | 0.93 | 0.58 |
| 15 | 5.75 | 5.30 | 4.16 |
| 16 | 1.42 | 0.96 | 0.08 |
| 23 | 3.83 | 0.00 | 1.72 |
| 24 | 0.20 | 0.55 | 0.00 |
| 25 | 16.13 | 0.00 | 17.94 |
| 26 | 0.31 | 0.46 | 0.27 |
| 34 | 0.29 | 0.00 | -0.11 |
| 35 | 11.41 | 0.00 | 16.00 |
| 36 | 0.40 | 0.00 | 0.41 |
| 45 | 2.59 | 0.00 | 0.88 |
| 46 | -1.87 | 0.00 | -0.38 |
| 56 | 2.70 | 0.00 | 0.39 |
| 123 | 4.05 | 0.00 | 2.95 |
| 124 | 0.43 | 0.33 | 0.08 |
| 125 | 8.78 | 0.00 | 11.08 |
| 126 | 0.54 | 0.64 | 0.13 |
| 134 | 0.29 | 0.00 | -0.03 |
| 135 | 9.63 | 0.00 | 9.76 |
| 136 | 0.40 | 0.00 | 0.06 |
| 145 | 0.81 | 0.00 | 0.48 |
| 146 | -1.87 | 0.00 | -0.13 |
| 156 | 0.92 | 0.00 | -0.38 |
| 234 | 1.94 | 0.00 | -0.07 |
| 235 | 3.83 | 0.00 | 2.66 |

Table 96 Continued

| Variables Index | Estimated Parameter | FFD Parameter | Full Factorial Parameter |
|-----------------|---------------------|---------------|--------------------------|
| 236 | 2.05 | 0.00 | 0.24 |
| 245 | 0.20 | 0.00 | 0.10 |
| 246 | 0.02 | 0.00 | -0.20 |
| 256 | 0.31 | 0.00 | 0.34 |
| 345 | 0.29 | 0.00 | -0.15 |
| 346 | 0.99 | 0.00 | -0.40 |
| 356 | 0.40 | 0.00 | 0.43 |
| 456 | -1.87 | 0.00 | -0.25 |
| 1234 | 1.78 | 0.00 | -0.16 |
| 1235 | 4.05 | 0.00 | 3.58 |
| 1236 | 1.89 | 0.00 | 0.20 |
| 1245 | 0.43 | 0.00 | 0.06 |
| 1246 | -0.14 | 0.00 | -0.21 |
| 1256 | 0.54 | 0.00 | 0.12 |
| 1345 | 0.29 | 0.00 | 0.03 |
| 1346 | -1.11 | 0.00 | -0.18 |
| 1356 | 0.40 | 0.00 | 0.24 |
| 1456 | -1.87 | 0.00 | -0.15 |
| 2345 | 1.12 | 0.00 | -0.25 |
| 2346 | -1.98 | 0.00 | -0.11 |
| 2356 | 1.23 | 0.00 | 0.17 |
| 2456 | -0.80 | 0.00 | -0.30 |
| 3456 | 0.17 | 0.00 | -0.29 |
| 12345 | 1.28 | 0.00 | -0.35 |
| 12346 | -1.75 | 0.00 | -0.19 |
| 12356 | 1.39 | 0.00 | 0.24 |
| 12456 | -0.64 | 0.00 | -0.17 |
| 13456 | -1.61 | 0.00 | -0.12 |
| 23456 | -1.98 | 0.00 | -0.25 |
| 123456 | -1.75 | 0.00 | -0.19 |

2. Example Application of the RC Method on a Non-Regular System

In this section, the RC method is applied to the study of the “Parameters Contributing to Power Loss in Disengaged Wet Clutches” [82]. This is a highly interacting non-regular system. In addition to the RC method, a sample of existing methods is applied for comparison. The methods applied in this example are Res III Plackett-Burman (PB) designs [28], Res IV mirror foldover of Plackett-Burman (PB) designs, Res III Tagushi orthogonal arrays [29], Res IV mirror foldover of Tagushi orthogonal arrays, Res III FFDs [25], Res IV FFDs, a no-interactions estimation utilizing OFAT and Adaptive OFAT (AOFAT) experiments selection process [51, 52], SB’s method [46], and Cotter’s method [34]. The full factorial design that is performed in the study will be used for validating the results of each method.

Applying the RC method, the same steps that were followed in the previous section are followed here. Variables are assumed monotonic, though this assumption is not always valid in this example. The results of all performed experiment are shown in Table 97. The importance of variables is found from the first nine rows in Table 97. The variables, sorted by importance, are 3165742. Assuming sixteen experiments are targeted in this example, variables 13 are selected to create subtrees $13=xx$. Experiments one, three, five and ten through thirteen are the end points of these subtrees. Instead of a graphical representation of the subtrees, a table is used in this example for illustration.

The performed nodes in every subtree are shown in Table 98. The results are reflected into nodes’ ranges of every subtree. Within subtree $13=10$, the bottom node result is below the minimum of the main tree indicating non-monotonic behavior. As a

result, the lower limit of subtree 13=10 ranges are set to 1.4 and only the bottom is kept as 0.9. The summation of spans in every subtree is shown in Table 99. Accordingly, it is decided to target subtrees 13=10 and 13=01 next. Since only three experiments remain to be performed, nodes from the second top level of each subtree are only selected. This is shown as experiments fourteen and fifteen in Table 97. The updated total spans are shown in Table 100. Since subtree 13=11 and 13=01 are close in span, it is possible to perform one node in either subtree. Subtree 13=11 was selected, and the result of the last experiment, sixteen, is shown in Table 97.

After all experiments have been performed, the average value for every node's range is used to estimate of the response, then the overall model parameters are found. The first order parameters of the resulting model are shown in the 17th rows of Table 97. The results of applying existing methods are listed in Table 101 to Table 110. The summary of first order parameters in addition to their RMSEs from the full factorial results are shown in Table 111. It is concluded from the summary table that the Res III PB method outperformed all used methods except for the RC method and the three Res IV orthogonal arrays' methods. The RC method, Res IV mirror foldover of PB, Res IV mirror foldover of Tagushi and Res IV FFD produced the lowest RMSE of first order parameters' projection. This is expected since Res III methods will require a smaller number of experiments, but produce less accurate results, especially if second order interactions are significant.

The RC method produced the most accurate estimation of first order parameters. It also produced the most accurate estimation of second order interactions as shown in

Table 112. This example demonstrates that even in a system with moderately non-monotonic variables, the RC method produced very accurate results when compared to a sample of evaluation methods.

Table 97
Experiments and Parameters of Parasitic Drag Torque Using the RC Method

| Exp. Index | Variables States* | | | | | | | y |
|------------------|-------------------|-------|-------|-------|-------|-------|-------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.4 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| 3 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 8 |
| 4 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 11 |
| 5 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 6.4 |
| 6 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 10.8 |
| 7 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 9.6 |
| 8 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 9 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 10.5 |
| 10 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 3.6 |
| 11 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 2.6 |
| 12 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9 |
| 13 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 4.9 |
| 14 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 2.6 |
| 15 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 5 |
| 16 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 10.2 |
| Estimated 1st | | | | | | | | |
| Order Parameters | 1.029 | 0.264 | 1.840 | 0.229 | 0.476 | 0.640 | 0.236 | |

*Even though the parameters found are in the -1,1 domain, the 0,1 notation was used for variables states in this method to follow the dissertation's tree convention.

Table 98
 Performed Experiments in the Four Subtrees of Parasitic Drag Torque

| Exp. Index | Variables States* | | | | | | | y |
|----------------|-------------------|---|---|---|---|---|---|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| Subtree 13= 00 | | | | | | | | |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1.4 |
| 10 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 3.6 |
| Subtree 13=01 | | | | | | | | |
| 11 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 2.6 |
| 3 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 8 |
| Subtree 13=10 | | | | | | | | |
| 12 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9 |
| 5 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 6.4 |
| Subtree 13=11 | | | | | | | | |
| 13 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 4.9 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| 4 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 11 |
| 6 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 10.8 |
| 7 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 9.6 |
| 8 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 9 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 10.5 |

*Even though the parameters found are in the -1,1 domain, the 0,1 notation was used for variables states in this method to follow the dissertation's tree convention.

Table 99
 Total Span of the Four Subtrees after Experiment No 13 of Parasitic Drag Torque

| | | | | |
|-------------|----|-----|-----|-----|
| Subtree 13= | 00 | 10 | 01 | 11 |
| Span | 66 | 150 | 162 | 113 |

Table 100
 Total Span of the Four Subtrees after Experiment No 15 of Parasitic Drag Torque

| | | | | |
|-------------|----|------|-------|-----|
| Subtree 13= | 00 | 10 | 01 | 11 |
| Span | 51 | 91.8 | 114.6 | 113 |

Table 101
Experiments and Parameters of Parasitic Drag Torque Using Res III PB

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|------|-------|------|----|-------|-------|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 8 |
| 2 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 4.1 |
| 3 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 4 |
| 4 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 3.6 |
| 5 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 2.4 |
| 6 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 9.4 |
| 7 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 3.1 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| Estimated 1st Order Parameters | 1.525 | -0.1 | 1.875 | -0.8 | 0 | 0.375 | 0.225 | |

Table 102
Experiments and Parameters of Parasitic Drag Torque Using Res IV Mirror Foldover of PB

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 8 |
| 2 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 4.1 |
| 3 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 4 |
| 4 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 3.6 |
| 5 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 2.4 |
| 6 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 9.4 |
| 7 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 3.1 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 9 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 3 |
| 10 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 6.8 |
| 11 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 4.2 |
| 12 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 3.2 |
| 13 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | 6 |
| 14 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 2.1 |
| 15 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 4 |
| 16 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| Estimated 1st Order Parameters | 2.244 | 0.494 | 1.944 | 0.344 | 0.969 | 0.944 | 1.094 | |

Table 103
Experiments and Parameters of Parasitic Drag Torque Using Res III Tagushi

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|----|------|-------|-----|-----|-------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 2 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 4 |
| 3 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 3 |
| 4 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 4.6 |
| 5 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 11.4 |
| 6 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 7.8 |
| 7 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 7.4 |
| 8 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1.6 |
| Estimated 1st Order Parameters | 1.9 | -1 | 1.55 | -0.65 | 1.7 | 0.4 | -0.15 | |

Table 104
Experiments and Parameters of Parasitic Drag Torque Using Res IV Mirror Foldover of Tagushi

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|-------|-------|-------|-------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 2 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 4 |
| 3 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 3 |
| 4 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 4.6 |
| 5 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 11.4 |
| 6 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 7.8 |
| 7 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 7.4 |
| 8 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1.6 |
| 9 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| 10 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 4.8 |
| 11 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1.2 |
| 12 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 4.4 |
| 13 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 2.8 |
| 14 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 3.1 |
| 15 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 3.8 |
| 16 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 4 |
| Estimated 1st Order Parameters | 1.931 | 0.519 | 2.031 | 0.331 | 1.569 | 1.281 | 1.019 | |

Table 105
Experiments and Parameters of Parasitic Drag Torque Using Res III FFD

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 3.5 |
| 2 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 4.4 |
| 3 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 3.1 |
| 4 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 2.1 |
| 5 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 3.8 |
| 6 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 9.8 |
| 7 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 3.2 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| Estimated 1st Order Parameters | 2.712 | 0.737 | 2.837 | 0.987 | 2.737 | 1.412 | 1.462 | |

Table 106
Experiments and Parameters of Parasitic Drag Torque Using Res IV FFD

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 2 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 6 |
| 3 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | 2.2 |
| 4 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 5.2 |
| 5 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 4 |
| 6 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 6.4 |
| 7 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 3.2 |
| 8 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 8 |
| 9 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 3 |
| 10 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | 3.6 |
| 11 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 2.6 |
| 12 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1.6 |
| 13 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 4 |
| 14 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 8.4 |
| 15 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 4.1 |
| 16 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| Estimated 1st Order Parameters | 2.106 | 0.569 | 1.969 | 0.619 | 1.006 | 0.769 | 1.256 | |

Table 107

Experiments and Parameters of Parasitic Drag Torque Using OFAT Experiments
Selection Process

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|--------|-------|-------|-------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 2 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0.9 |
| 3 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 4 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 4.8 |
| 5 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 4.4 |
| 6 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 4.4 |
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 10.5 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| Estimated 1st Order Parameters | -0.250 | 0.250 | 1.700 | -0.200 | 0.000 | 3.050 | 4.250 | |

Table 108

Experiments and Parameters of Parasitic Drag Torque Using AOFAT Experiments
Selection Process

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|-------|-------|-------|-------|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| 3 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 |
| 4 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 6 |
| 5 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 4 |
| 6 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 3.9 |
| 7 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 2 |
| 8 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1.6 |
| Estimated 1st Order Parameters | 5.500 | 1.000 | 1.000 | 0.050 | 0.950 | 0.200 | 0.100 | |

Table 109

Experiments and Parameters of Parasitic Drag Torque Using Sequence Bifurcation's Method

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|--------|-------|-------|-------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| 3 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 4.4 |
| 4 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 3.9 |
| 5 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 6 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 3.4 |
| 7 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 4.8 |
| 8 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1.6 |
| 9 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0.9 |
| 10 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1.5 |
| 11 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 10.5 |
| 12 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 6.8 |
| 13 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 9.6 |
| 14 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 9 |
| Estimated 1st Order Parameters | -0.150 | 0.150 | 1.550 | -0.050 | 2.900 | 3.200 | 2.725 | |

Table 110
Experiments and Parameters of Parasitic Drag Torque Using Cotter's Method

| Exp. Index | Variables States | | | | | | | y |
|-----------------------------------|------------------|-------|-------|-------|-------|-------|-------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 19 |
| 2 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 |
| 3 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 11 |
| 4 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | 6.4 |
| 5 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 10.8 |
| 6 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 9.6 |
| 7 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 9 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 10.5 |
| 9 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1.4 |
| 10 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 0.9 |
| 11 | -1 | 1 | -1 | -1 | -1 | -1 | -1 | 1.5 |
| 12 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 2.6 |
| 13 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1.8 |
| 14 | -1 | -1 | -1 | -1 | 1 | -1 | -1 | 2.8 |
| 15 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | 1.8 |
| 16 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1.6 |
| Estimated 1st Order Parameters | 1.313 | 1.013 | 1.725 | 1.075 | 1.350 | 1.300 | 1.088 | |

Table 111

Estimated and Full Factorial Design's 1st Order Parameters of Parasitic Drag Torque with the Associated RMSEs

| Method | 1st Order Parameter | | | | | | | RMSE |
|-------------------------|---------------------|--------|-------|--------|-------|-------|--------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
| RC Method | 1.029 | 0.264 | 1.840 | 0.229 | 0.476 | 0.640 | 0.236 | 0.333 |
| Res III PB | 1.525 | -0.100 | 1.875 | -0.800 | 0.000 | 0.375 | 0.225 | 0.513 |
| PB Mirror Foldover | 2.244 | 0.494 | 1.944 | 0.344 | 0.969 | 0.944 | 1.094 | 0.411 |
| Res III Tagushi | 1.900 | -1.000 | 1.550 | -0.650 | 1.700 | 0.400 | -0.150 | 0.630 |
| Tagushi Mirror Foldover | 1.931 | 0.519 | 2.031 | 0.331 | 1.569 | 1.281 | 1.019 | 0.463 |
| Res III FFD | 2.713 | 0.738 | 2.838 | 0.988 | 2.738 | 1.413 | 1.463 | 1.079 |
| Res IV FFD | 2.106 | 0.569 | 1.969 | 0.619 | 1.006 | 0.769 | 1.256 | 0.455 |
| OFAT | -0.250 | 0.250 | 1.700 | -0.200 | 0.000 | 3.050 | 4.250 | 1.663 |
| AOFAT | 5.500 | 1.000 | 1.000 | 0.050 | 0.950 | 0.200 | 0.100 | 1.487 |
| SB | -0.150 | 0.150 | 1.550 | -0.050 | 2.900 | 3.200 | 2.725 | 1.453 |
| Cotter | 1.313 | 1.013 | 1.725 | 1.075 | 1.350 | 1.300 | 1.088 | 0.593 |
| Full Factorial | 1.504 | 0.046 | 1.404 | 0.034 | 0.904 | 0.774 | 0.663 | |

Table 112
RMSEs of the Second Order Parameters of Parasitic Drag Torque

| Method | 2nd Order Parameters RMSE |
|-----------------------------------|---------------------------|
| RC Method | 0.22 |
| Res IV Mirror Foldover of PB | 0.32 |
| Res IV Mirror Foldover of Tagushi | 0.35 |
| Res IV FFD | 0.34 |

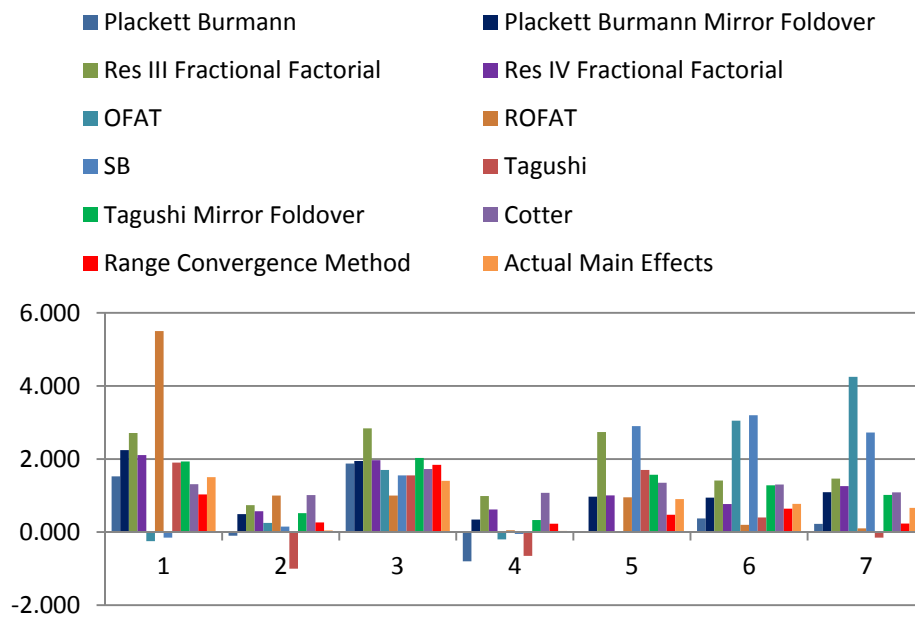


Figure 85. Parasitic Drag Torque's Estimated and Actual First Order Parameters

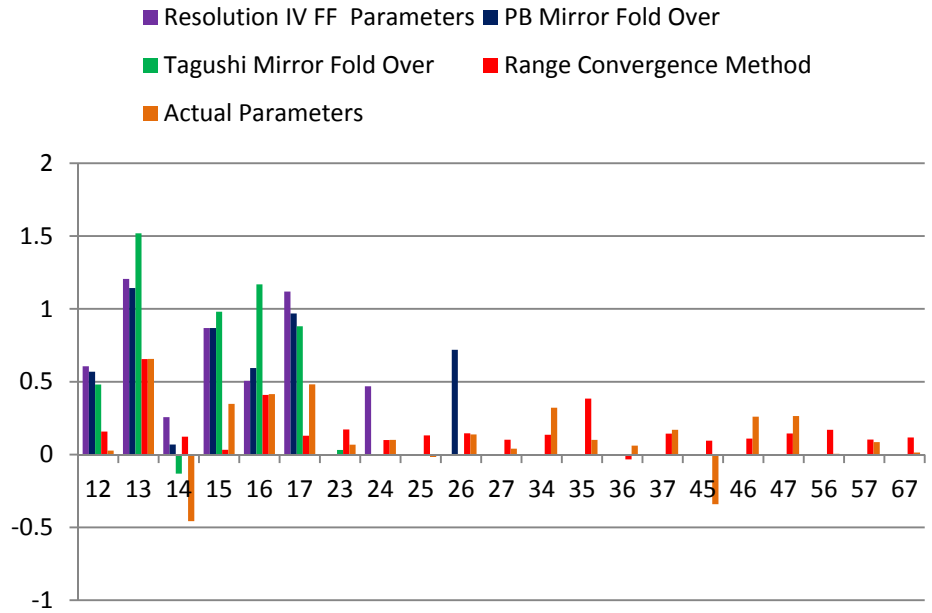


Figure 86. Parasitic Drag Torque's Estimated and Actual Second Order Parameters

3. Example Application of Artificial Neural Networks on Keff

The concept of Artificial Neural Networks (ANNs) was introduced in 1943 [92], and is widely used in literature, so it will not be explained in this section. In this section, a brute force approach was followed to choose the number of hidden layers of the neural network. For every number of hidden layers, an ANN was developed and the validation error was checked. The ANN with the lowest error was selected. The set of experiments used for the model development is the 128 FFD set of experiments of K_{eff} . The ratio of development, testing and validation sets are 70:15:15 and 90:5:5. In both cases, the most accurate ANN was found to be with three hidden layers. Higher number of hidden layers over fits the multidimensional response shape, and reduces the accuracy of the model. The validations of both models with the 98 performed experiments of the MSIV are shown in Table 87 and Table 88. The RMSE of both cases are 32.90776 and 31.718 consequently. This is almost three times the error of the 128 FFD. The main reason of the ANN poor performance is that it assumes the same number of layers for all variables, and does not account for sparsity. It also uses a portion of the experiments to develop the model and allocates the other portion for testing and validation.

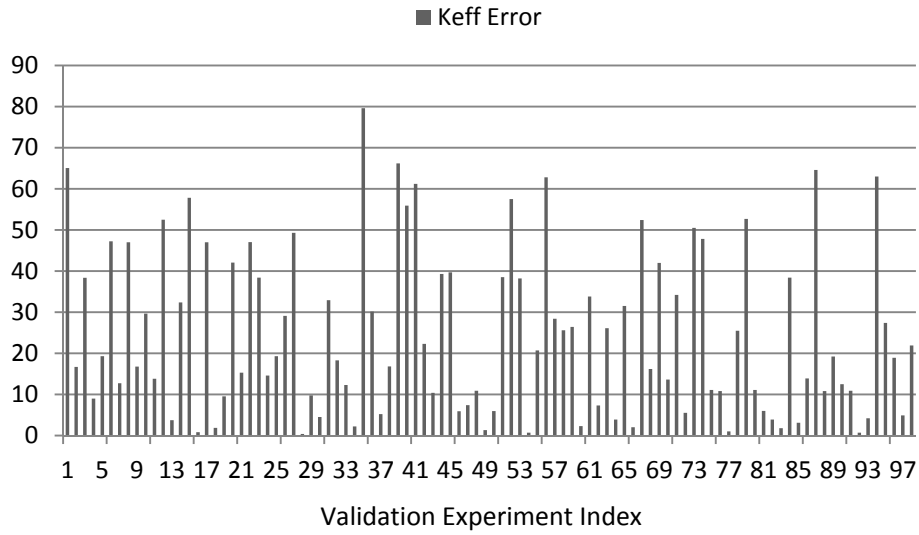


Figure 87. K_{eff} 's 70:15:15 Projection Error of the ANN with Respect to the Performed Experiments of the MSIV Method

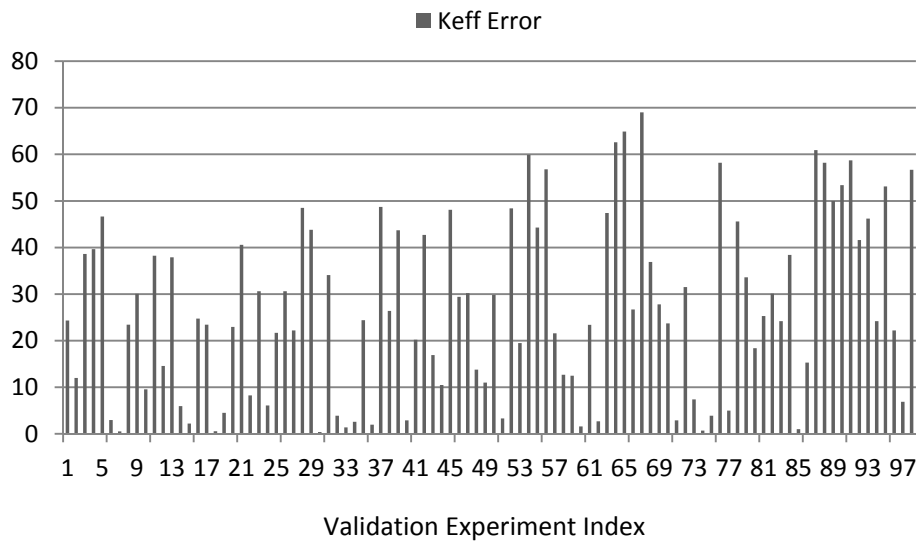


Figure 88. K_{eff} 's 90:5:5 Projection Error of the ANN with Respect to the Performed Experiments of the MSIV Method

APPENDIX G

1. Multiplication Factor Experiments: MHIV Method Results (71)

Table 113

Application of the MHIV method to K_{eff}

| | Low End Variables State | K_{eff} | | High End Variables State | K_{eff} |
|------------|----------------------------|-----------|----------|-----------------------------|-----------|
| | '00000010011111' | 0.8159 | | '11111101100000' | 1.3079 |
| 1 Up | '10000010011111' | 0.86386 | 1 Down | '01111101100000' | 1.2714 |
| 2 Up | '01000010011111' | 0.85005 | 2 Down | '10111101100000' | 1.262 |
| 3 Up | '00100010011111' | 0.8152 | 3 Down | '11011101100000' | 1.3079 |
| 4 Up | '00010010011111' | 0.87231 | 4 Down | '11101101100000' | 1.2204 |
| 5 Up | '00001010011111' | 0.8159 | 5 Down | '11110101100000' | 1.3079 |
| 6 Up | '00000110011111' | 0.84808 | 6 Down | '11111001100000' | 1.292 |
| 7 Up | '00000000111111' | 0.82285 | 7 Down | '11111111100000' | 1.2998 |
| 8 Up | '00000011011111' | 0.86214 | 8 Down | '11111100100000' | 1.2692 |
| 9 Up | '00000010111111' | 0.96608 | 9 Down | '11111101000000' | 1.235 |
| a Up | '00000010001111' | 0.86433 | a Down | '11111101110000' | 1.254 |
| b Up | '00000010010111' | 0.86676 | b Down | '11111101101000' | 1.2835 |
| c Up | '00000010011011' | 0.82242 | c Down | '11111101100100' | 1.3068 |
| d Up | '00000010011101' | 0.83213 | d Down | '11111101100010' | 1.3031 |
| e Up | '00000010011110' | 0.8683 | e Down | '11111101100001' | 1.2936 |
| 49 Up | 00010010111111' | 1.0349 | 49 Down | 11101101000000' | 1.1634 |
| 4e Up | 000100100111110' | 0.92537 | 4e Down | 11101101100001' | 1.2058 |
| 9e Up | 000000101111110' | 1.0092 | 9e Down | 11111101000001' | 1.2173 |
| 49e Up | 000100101111110' | 1.0794 | 49e Down | 11101101000001' | 1.1464 |
| 49e=111-6b | | | | | |
| | 000100101111110' | Performed | | 11111101100000' | Performed |
| 6 Up | 000101101111110' | 1.1132 | 6 Down | 11111001100000' | 1.292 |
| b Up | 000100101101110' | 1.084 | b Down | 11111101101000' | 1.2835 |
| 6b Up | 000101101101110' | 1.1115 | 6b Down | 11111001101000' | 1.2574 |
| 49e=110-6b | | | | | |
| | 000100101111111' | Performed | | 11111101100001' | Performed |
| 6 Up | 000101101111111' | 1.0684 | 6 Down | 11111001100001' | 1.2758 |
| b Up | 000100101101111' | 1.0361 | b Down | 11111101101001' | 1.2699 |
| 6b Up | 000101101101111' | 1.0647 | 6b Down | 11111001101001' | 1.2443 |

Table 113 Continued

| Low End Variables State | | K_{eff} | High End Variables State | | K_{eff} |
|-------------------------|-----------------|-----------|--------------------------|-----------------|-----------|
| 49e=101-6b | | | | | |
| | 00010010011110' | Performed | | 1111101000000' | Performed |
| 6 Up | 00010110011110' | 0.96206 | 6 Down | 11111001000000' | 1.206 |
| b Up | 00010010010110' | 0.99135 | b Down | 1111101001000' | 1.1325 |
| 49e=100-6b | | | | | |
| | 00010010011111' | Performed | | 1111101000001' | Performed |
| 6 Up | 00010110011111' | 0.90748 | 6 Down | 11111001000001' | 1.1876 |
| b Up | 00010010010111' | 0.93535 | b Down | 1111101001001' | 1.1162 |
| 49e=011-6b | | | | | |
| | 00000010111110' | Performed | | 11101101100000' | Performed |
| 6 Up | 00000110111110' | 1.0405 | 6 Down | 11101001100000' | 1.2093 |
| b Up | 00000010110110' | 1.0017 | b Down | 11101101101000' | 1.2123 |
| 6b Up | 00000110110110' | 1.0239 | 6b Down | 11101001101000' | 1.1884 |
| 49e=010-6b | | | | | |
| | 00000010111111' | Performed | | 11101101100001' | Performed |
| 6 Up | 00000110111111' | 0.99527 | 6 Down | 11101001100001' | 1.1929 |
| b Up | 00000010110111' | 0.95519 | b Down | 11101101101001' | 1.1992 |
| 6b Up | 00000110110111' | 0.97787 | 6b Down | 11101001101001' | 1.1758 |
| 49e=001-6b | | | | | |
| | 00000010011110' | Performed | | 11101101000000' | Performed |
| 6 Up | 00000110011110' | 0.90203 | 6 Down | 11101001000000' | 1.138 |
| b Up | 00000010010110' | 0.92222 | b Down | 11101101001000' | 1.0753 |
| 49e=000-6b | | | | | |
| | 00000010011111' | Performed | | 11101101000001' | Performed |
| 6 Up | 00000110011111' | 0.84808 | 6 Down | 11101001000001' | 1.1191 |
| b Up | 00000010010111' | 0.86676 | b Down | 11101101001001' | 1.0548 |

2. Multiplication Factor Experiments: MSIV Method Results (98)

Table 114

First Order Effects' Exploration of the MSIV method's Application to K_{eff}

| Main Tree Node | Variables State | K_{eff} | Main Tree Node | Variables State | K_{eff} |
|----------------|------------------|-----------|----------------|------------------|-----------|
| Top End | '11111101100000' | 1.3079 | Bottom End | '00000010011111' | 0.8159 |
| 1 Down | '01111101100000' | 1.2714 | 1 Up | '10000010011111' | 0.86386 |
| 2 Down | '10111101100000' | 1.262 | 2 Up | '01000010011111' | 0.85005 |
| 3 Down | '11011101100000' | 1.3079 | 3 Up | '00100010011111' | 0.8152 |
| 4 Down | '11101101100000' | 1.2204 | 4 Up | '00010010011111' | 0.87231 |
| 5 Down | '11110101100000' | 1.3079 | 5 Up | '00001010011111' | 0.8159 |
| 6 Down | '11111001100000' | 1.292 | 6 Up | '00000110011111' | 0.84808 |
| 7 Down | '11111111100000' | 1.2998 | 7 Up | '00000000111111' | 0.82285 |
| 8 Down | '11111100100000' | 1.2692 | 8 Up | '00000011011111' | 0.86214 |
| 9 Down | '11111101000000' | 1.235 | 9 Up | '00000010111111' | 0.96608 |
| a Down | '11111101110000' | 1.254 | a Up | '00000010001111' | 0.86433 |
| b Down | '11111101101000' | 1.2835 | b Up | '00000010010111' | 0.86676 |
| c Down | '11111101100100' | 1.3068 | c Up | '00000010011011' | 0.82242 |
| d Down | '11111101100010' | 1.3031 | d Up | '00000010011101' | 0.83213 |
| e Down | '11111101100001' | 1.2936 | e Up | '00000010011110' | 0.8683 |

Table 115

Second Order Interactions' Exploration and One Node per Tree of the MSIV method's Application to K_{eff}

| Tree Index | Top Node | Top Node's Value | Bottom Node | Bottom Node's Value |
|------------|---------------------|------------------|---------------------|---------------------|
| 1 | '11111101100000' | 1.3079 | '00010110110110' | Projected |
| 2 | '11101101100000' 4 | 1.2204 | '00010110110110' 4 | Projected |
| 3 | '11111001100000' 6 | 1.292 | '00010110110110' 6 | Projected |
| 4 | '11111101000000' 9 | 1.235 | '00010110110110' 9 | Projected |
| 5 | '11111101101000' b | 1.2835 | '00010110110110' b | Projected |
| 6 | '11111101100001' e | 1.2936 | '00010110110110' e | Projected |
| 7 | '11101001100000' 46 | 1.2093 | '00010110110110' 46 | Projected |
| 8 | '11101101000000' 49 | 1.1634 | '00010110110110' 49 | Projected |
| 9 | '11101101101000' 4b | 1.2123 | '00010110110110' 4b | Projected |
| 10 | '11101101100001' 4e | 1.2058 | '00010110110110' 4e | Projected |
| 11 | '11111001000000' 69 | 1.206 | '00010110110110' 69 | Projected |
| 12 | '11111001101000' 6b | 1.2574 | '00010110110110' 6b | Projected |
| 13 | '11111001100001' 6e | 1.2758 | '00010110110110' 6e | Projected |
| 14 | '11111101001000' 9b | 1.1325 | '00010110110110' 9b | Projected |
| 15 | '11111101000001' 9e | 1.2173 | '00010110110110' 9e | Projected |
| 16 | '11111101101001' be | 1.2699 | '00010110110110' be | Projected |
| 17 | '11101001001001' | Projected | '00000010011111' | 0.8159 |
| 18 | '11101001001001' 4 | Projected | '00010010011111' 4 | 0.87231 |
| 19 | '11101001001001' 6 | Projected | '00000110011111' 6 | 0.84808 |
| 20 | '11101001001001' 9 | Projected | '00000010111111' 9 | 0.96608 |
| 21 | '11101001001001' b | Projected | '00000010010111' b | 0.86676 |
| 22 | '11101001001001' e | Projected | '00000010011110' e | 0.8683 |
| 23 | '11101001001001' 46 | Projected | '00010110011111' 46 | 0.90792 |
| 24 | '11101001001001' 49 | Projected | '00010010111111' 49 | 1.036 |
| 25 | '11101001001001' 4b | Projected | '00010010010111' 4b | 0.93516 |
| 26 | '11101001001001' 4e | Projected | '00010010011110' 4e | 0.92483 |
| 27 | '11101001001001' 69 | Projected | '00000110111111' 69 | 0.99527 |
| 28 | '11101001001001' 6b | Projected | '00000110010111' 6b | 0.89914 |
| 29 | '11101001001001' 6e | Projected | '00000110011110' 6e | 0.90155 |
| 30 | '11101001001001' 9b | Projected | '00000010110111' 9b | 0.95525 |
| 31 | '11101001001001' 9e | Projected | '00000010111110' 9e | 1.0088 |
| 32 | '11101001001001' be | Projected | '00000010010110' be | 0.92288 |

Table 116Application of a FFD to the Non-Interacting Variables of K_{eff}

| Top End | | b down | | e down | |
|------------------|-----------|------------------|-----------|------------------|-----------|
| Variables State | K_{eff} | Variables State | K_{eff} | Variables State | K_{eff} |
| '00011100100000' | 1.1808 | '00011100101000' | 1.1649 | '00011100100001' | 1.1644 |
| '10011100110110' | 1.1619 | '10011100111110' | 1.1655 | '10011100110111' | 1.1169 |
| '01011101100110' | 1.2667 | '01011101101110' | 1.2366 | '01011101100111' | 1.2518 |
| '11011101110000' | 1.2544 | '11011101111000' | 1.2507 | '11011101110001' | 1.2204 |
| '00111101110100' | 1.1746 | '00111101111100' | 1.1694 | '00111101110101' | 1.1399 |
| '10111101100010' | 1.2581 | '10111101101010' | 1.2303 | '10111101100011' | 1.2438 |
| '01111100110010' | 1.1571 | '01111100111010' | 1.1638 | '01111100110011' | 1.1123 |
| '11111100100100' | 1.2692 | '11111100101100' | 1.2504 | '11111100100101' | 1.2499 |
| '00011111110010' | 1.1659 | '00011111111010' | 1.1576 | '00011111110011' | 1.1323 |
| '10011111100100' | 1.2554 | '10011111101100' | 1.2274 | '10011111100101' | 1.2401 |
| '01011110110100' | 1.1543 | '01011110111100' | 1.1595 | '01011110110101' | 1.1088 |
| '11011110100010' | 1.2588 | '11011110101010' | 1.2352 | '11011110100011' | 1.2409 |
| '00111110100110' | 1.1678 | '00111110101110' | 1.1459 | '00111110100111' | 1.1504 |
| '10111110110000' | 1.1617 | '10111110111000' | 1.1667 | '10111110110001' | 1.1165 |
| '01111111100000' | 1.265 | '01111111101000' | 1.2398 | '01111111100001' | 1.2509 |
| '11111111110110' | 1.2442 | '11111111111110' | 1.2337 | '11111111110111' | 1.2094 |

3. Multiplication Factor Experiments: 128 Res IV FFD Results (128)

Table 117

Application of a 128 Res IV FFD to K_{eff}

| Variables State | K_{eff} | Variables State | | Variables State | K_{eff} |
|------------------|-----------|------------------|---------|------------------|-----------|
| '00000000000000' | 0.99717 | '01010011010110' | 1.0886 | '10100110010110' | 1.0077 |
| '00000001111111' | 1.027 | '01010100101001' | 1.1885 | '10100111101001' | 1.1478 |
| '00000010111111' | 0.96608 | '01010101010110' | 1.1283 | '10110000101010' | 1.1693 |
| '00000011000000' | 1.0397 | '01010110010110' | 1.0655 | '10110001010101' | 1.0697 |
| '00000100111111' | 1.0021 | '01010111101001' | 1.2263 | '10110010010101' | 1.0033 |
| '00000101000000' | 1.0816 | '01100000011001' | 0.87993 | '10110011101010' | 1.1907 |
| '00000110000000' | 1.0238 | '01100001100110' | 1.1659 | '10110100010101' | 1.0437 |
| '00000111111111' | 1.0498 | '01100010100110' | 1.108 | '10110101101010' | 1.2303 |
| '00010000111100' | 1.0936 | '01100011011001' | 0.92344 | '10110110101010' | 1.1925 |
| '00010001000011' | 1.0814 | '01100100100110' | 1.1333 | '10110111010101' | 1.0929 |
| '00010010000011' | 1.0181 | '01100101011001' | 0.96482 | '11000000001111' | 0.96256 |
| '00010011111100' | 1.1288 | '01100110011001' | 0.90416 | '11000001110000' | 1.1513 |
| '00010100000011' | 1.0636 | '01100111100110' | 1.1782 | '11000010110000' | 1.0989 |
| '00010101111100' | 1.1693 | '01110000100101' | 1.1778 | '11000011001111' | 0.98322 |
| '00010110111100' | 1.1203 | '01110001011010' | 1.017 | '11000100110000' | 1.1155 |
| '00010111000011' | 1.1101 | '01110010011010' | 0.97107 | '11000101001111' | 1.0283 |
| '00100000110011' | 0.96191 | '01110011100101' | 1.2262 | '11000110001111' | 0.98548 |
| '00100001001100' | 0.95046 | '01110100011010' | 1.0141 | '11000111110000' | 1.1562 |
| '00100010001100' | 0.90347 | '01110101100101' | 1.2556 | '11010000110011' | 1.1426 |
| '00100011110011' | 1.024 | '01110110100101' | 1.1965 | '11010001001100' | 1.091 |
| '00100100001100' | 0.9495 | '01110111011010' | 1.0412 | '11010010001100' | 1.0508 |
| '00100101110011' | 1.0514 | '10000000100101' | 1.1017 | '11010011110011' | 1.1937 |
| '00100110110011' | 0.9808 | '10000001011010' | 0.95983 | '11010100001100' | 1.097 |
| '00100111001100' | 0.97868 | '10000010011010' | 0.92547 | '11010101110011' | 1.2173 |
| '00110000001111' | 0.92513 | '10000011100101' | 1.1373 | '11010110110011' | 1.155 |
| '00110001110000' | 1.1529 | '10000100011010' | 0.96621 | '11010111001100' | 1.114 |
| '00110010110000' | 1.0905 | '10000101100101' | 1.1627 | '11100000111100' | 1.1147 |
| '00110011001111' | 0.95697 | '10000110100101' | 1.1165 | '11100001000011' | 1.1083 |
| '00110100110000' | 1.122 | '10000111011010' | 0.98323 | '11100010000011' | 1.0533 |
| '00110101001111' | 1.0039 | '10010000011001' | 0.95762 | '11100011111100' | 1.1423 |
| '00110110001111' | 0.95348 | '10010001100110' | 1.2329 | '11100100000011' | 1.0913 |
| '00110111110000' | 1.1714 | '10010010100110' | 1.1894 | '11100101111100' | 1.1716 |
| '01000000101010' | 1.0998 | '10010011011001' | 0.98856 | '11100110111100' | 1.1282 |

Table 117 Continued

| Variables State | K_{eff} | Variables State | | Variables State | K_{eff} |
|------------------|-----------|------------------|---------|------------------|-----------|
| '01000001010101' | 0.99248 | '10010100100110' | 1.2204 | '11100111000011' | 1.1269 |
| '01000010010101' | 0.91594 | '10010101011001' | 1.0324 | '11110000000000' | 1.1667 |
| '01000011101010' | 1.1352 | '10010110011001' | 0.98309 | '11110001111111' | 1.1837 |
| '01000100010101' | 0.95251 | '10010111100110' | 1.2497 | '11110010111111' | 1.131 |
| '01000101101010' | 1.1719 | '10100000010110' | 0.98626 | '11110011000000' | 1.1959 |
| '01000110101010' | 1.1198 | '10100001101001' | 1.128 | '11110100111111' | 1.1649 |
| '01000111010101' | 1.0135 | '10100010101001' | 1.0846 | '11110101000000' | 1.2362 |
| '01010000010110' | 1.0401 | '10100011010110' | 1.0235 | '11110110000000' | 1.1877 |
| '01010001101001' | 1.2054 | '10100100101001' | 1.1208 | '11110111111111' | 1.2003 |
| '01010010101001' | 1.1475 | '10100101010110' | 1.0597 | | |

4. Flux Spectrum Experiments: MHIV Method Results (130)

Table 118
Application of the MHIV Method to RFS

| | Low End Variables State | RFS | | High End Variables State | RFS |
|--------------|----------------------------|----------|-------------|-----------------------------|----------|
| | '00000110110000' | 0.508673 | | '11111001001111' | 1.1739 |
| 1 Up | '10000110110000' | 0.508518 | 1 Down | '01111001001111' | 1.1164 |
| 2 Up | '01000110110000' | 0.509814 | 2 Down | '10111001001111' | 1.0771 |
| 3 Up | '00100110110000' | 0.507769 | 3 Down | '11011001001111' | 1.1679 |
| 4 Up | '00010110110000' | 0.551146 | 4 Down | '11101001001111' | 1.0322 |
| 5 Up | '00001110110000' | 0.508673 | 5 Down | '11110001001111' | 1.1739 |
| 6 Up | '00000010110000' | 0.526067 | 6 Down | '11111101001111' | 1.1066 |
| 7 Up | '00000100110000' | 0.508001 | 7 Down | '11111011001111' | 1.1673 |
| 8 Up | '00000111110000' | 0.515172 | 8 Down | '11111000001111' | 1.1397 |
| 9 Up | '00000110010000' | 0.610874 | 9 Down | '11111001101111' | 0.7891 |
| a Up | '00000110100000' | 0.520291 | a Down | '11111001011111' | 1.1133 |
| b Up | '00000110111000' | 0.602156 | b Down | '11111001000111' | 0.8039 |
| c Up | '00000110110100' | 0.509087 | c Down | '11111001001011' | 1.1627 |
| d Up | '00000110110010' | 0.509684 | d Down | '11111001001101' | 1.1516 |
| e Up | '00000110110001' | 0.515092 | e Down | '11111001001110' | 1.1608 |
| 9b Up | '00000110011000' | 0.7893 | 9b Down | '11111001100111' | 0.612632 |
| 9B=11-12468a | | | | | |
| 1 Up | Performed | | 1 Down | '10000110011000' | 0.8114 |
| 2 Up | Performed | | 2 Down | '01000110011000' | 0.8128 |
| 4 Up | Performed | | 4 Down | '00010110011000' | 0.8821 |
| 6 Up | Performed | | 6 Down | '00000010011000' | 0.8252 |
| 8 Up | Performed | | 8 Down | '00000111011000' | 0.8280 |
| a Up | Performed | | a Down | '00000110001000' | 0.8455 |
| d Up | Performed | | d Down | '00000110011010' | 0.7985 |
| 12 UP | '00111001001111' | 1.03671 | 12 Down | '11000110011000' | 0.8359 |
| 14 UP | '01101001001111' | 0.988533 | 14 Down | '10010110011000' | 0.9102 |
| 24 UP | '10101001001111' | 0.961631 | 24 Down | '01010110011000' | 0.9164 |
| 124 UP | '00101001001111' | 0.929627 | 124 Down | '11010110011000' | 0.9490 |

Table 118 Continued

| Low End Variables | | RFS | High End Variables | | RFS |
|-------------------|------------------|----------|--------------------|------------------|--------|
| State | | | State | | |
| 9B=01-12468ad | | | | | |
| 1 UP | '01111001101111' | 0.78235 | 1 Down | '10000110111000' | 0.6035 |
| 2 UP | '10111001101111' | 0.764409 | 2 Down | '01000110111000' | 0.6080 |
| 4 UP | '11101001101111' | 0.699203 | 4 Down | '00010110111000' | 0.6632 |
| 6 UP | '11111101101111' | 0.742225 | 6 Down | '00000010111000' | 0.6306 |
| 8 UP | '11111000101111' | 0.779302 | 8 Down | '00000111111000' | 0.6168 |
| a UP | '11111001111111' | 0.765755 | a Down | '00000110101000' | 0.6279 |
| d UP | '11111001101101' | 0.783883 | d Down | '00000110111010' | 0.6065 |
| 24 UP | '10101001101111' | 0.681338 | 24 Down | '01010110111000' | 0.6758 |
| 26 UP | '10111101101111' | 0.725689 | 26 Down | '01000010111000' | 0.6397 |
| 46 UP | '11101101101111' | 0.661594 | 46 Down | '00010010111000' | 0.6964 |
| 246 UP | '10101101101111' | 0.648256 | 246 Down | '01010010111000' | 0.7116 |
| 9B=10-12468ad | | | | | |
| 1 UP | '01111001000111' | 0.790952 | 1 Down | '10000110010000' | 0.6171 |
| 2 UP | '10111001000111' | 0.775675 | 2 Down | '01000110010000' | 0.6143 |
| 4 UP | '11101001000111' | 0.711845 | 4 Down | '00010110010000' | 0.6722 |
| 6 UP | '11111101000111' | 0.757461 | 6 Down | '00000010010000' | 0.6371 |
| 8 UP | '11111000000111' | 0.794913 | 8 Down | '00000111010000' | 0.6244 |
| a UP | '11111001010111' | 0.781006 | a Down | '00000110000000' | 0.6312 |
| d UP | '11111001000101' | 0.792079 | d Down | '00000110010010' | 0.6146 |
| 24 UP | '10101001000111' | 0.693385 | 24 Down | '01010110010000' | 0.6814 |
| 26 UP | '10111101000111' | 0.735456 | 26 Down | '01000010010000' | 0.6464 |
| 46 UP | '11101101000111' | 0.673355 | 46 Down | '00010010010000' | 0.7050 |
| 246 UP | '10101101000111' | 0.658892 | 246 Down | '01010010010000' | 0.7171 |
| 9B=00-12468ad | | | | | |
| 1 UP | '01111001100111' | 0.612033 | 1 Down | Performed | |
| 2 UP | '10111001100111' | 0.603136 | 2 Down | Performed | |
| 4 UP | '11101001100111' | 0.557196 | 4 Down | Performed | |
| 6 UP | '11111101100111' | 0.584385 | 6 Down | Performed | |
| 8 UP | '11111000100111' | 0.610389 | 8 Down | Performed | |
| a UP | '11111001110111' | 0.602192 | a Down | Performed | |
| d UP | '11111001100101' | 0.610836 | d Down | Performed | |
| 46 UP | '11101101100111' | 0.533618 | 46 Down | '00010010110000' | 0.5716 |

Table 118 Continued

| Low End Variables | | RFS | High End Variables | | RFS |
|-------------------|------------------|----------|--------------------|------------------|--------|
| State | | | State | | |
| 9B=11-124= 111 | | | | | |
| 6 UP | '11010010011000' | 1.00551 | 6 Down | Performed | |
| d UP | '11010110011010' | 0.958957 | d Down | Performed | |
| 6d UP | 11010010011010' | 1.0142 | 6d Down | '11111101001101' | 1.0806 |
| 9B=11-124= 011 | | | | | |
| 6 UP | '01111101001111' | 1.067885 | 6 Down | '01010010011000' | 0.9651 |
| d UP | '01111001001101' | 1.102244 | d Down | '01010110011010' | 0.9258 |
| 9B=11-124= 101 | | | | | |
| 6 UP | '10111101001111' | 1.033805 | 6 Down | '10010010011000' | 0.9610 |
| d UP | '10111001001101' | 1.070034 | d Down | '10010110011010' | 0.9211 |
| 9B=11-124= 110 | | | | | |
| 6 UP | '11101101001111' | 0.964785 | 6 Down | '11000010011000' | 0.8887 |
| d UP | '11101001001101' | 1.004097 | d Down | '11000110011010' | 0.8506 |
| 6d UP | '11101101001101' | 0.942951 | 6d Down | '11000010011010' | 0.9014 |
| 9B=11-124= 001 | | | | | |
| 6 UP | '00111101001111' | 0.997506 | 6 Down | '00010010011000' | 0.9208 |
| d UP | '00111001001101' | 1.031768 | d Down | '00010110011010' | 0.8902 |
| 9B=11-124= 010 | | | | | |
| 6 UP | '01101101001111' | 0.93835 | 6 Down | '01000010011000' | 0.8545 |
| d UP | '01101001001101' | 0.972479 | d Down | '01000110011010' | 0.8247 |
| 9B=11-124= 100 | | | | | |
| 6 UP | '10101101001111' | 0.92098 | 6 Down | '10000010011000' | 0.8533 |
| d UP | '10101001001101' | 0.948047 | d Down | '10000110011010' | 0.8214 |
| 9B=11-124= 000 | | | | | |
| 6 UP | '00101101001111' | 0.891583 | 6 Down | Performed | |
| d UP | '00101001001101' | 0.919033 | d Down | Performed | |

5. Flux Spectrum Experiments: 128 FFD Results (128)

Table 119
Application of a 128 Res IV FFD to RFS

| Variables State | RFS | Variables State | RFS | Variables State | RFS |
|------------------|---------|------------------|---------|------------------|---------|
| '0000000000000' | 0.66256 | '01010011010110' | 0.75672 | '10100110010110' | 0.62461 |
| '0000001111111' | 0.66186 | '01010100101001' | 0.71942 | '10100111101001' | 0.64033 |
| '0000001011111' | 0.64512 | '01010101010110' | 0.71541 | '10110000101010' | 0.7446 |
| '0000001100000' | 0.66966 | '01010110010110' | 0.69128 | '10110001010101' | 0.75586 |
| '0000010011111' | 0.62 | '01010111101001' | 0.73115 | '10110010010101' | 0.74025 |
| '0000010100000' | 0.63857 | '01100000011001' | 0.87108 | '10110011101010' | 0.75523 |
| '0000011000000' | 0.63123 | '01100001100110' | 0.55227 | '10110100010101' | 0.70582 |
| '0000011111111' | 0.63139 | '01100010100110' | 0.54891 | '10110101101010' | 0.71526 |
| '00010000111100' | 0.69906 | '01100011011001' | 0.91291 | '10110110101010' | 0.70706 |
| '00010001000011' | 0.76092 | '01100100100110' | 0.52913 | '10110111010101' | 0.71839 |
| '00010010000011' | 0.75092 | '01100101011001' | 0.87138 | '11000000001111' | 1.0032 |
| '00010011111100' | 0.72218 | '01100110011001' | 0.82768 | '11000001110000' | 0.54048 |
| '00010100000011' | 0.71638 | '01100111100110' | 0.5307 | '11000010110000' | 0.53118 |
| '00010101111100' | 0.68894 | '01110000100101' | 0.60533 | '11000011001111' | 1.027 |
| '00010110111100' | 0.66809 | '01110001011010' | 1.0425 | '11000100110000' | 0.51057 |
| '00010111000011' | 0.72364 | '01110010011010' | 0.97972 | '11000101001111' | 0.96937 |
| '00100000110011' | 0.53447 | '01110011100101' | 0.60983 | '11000110001111' | 0.94841 |
| '00100001001100' | 0.91308 | '01110100011010' | 0.92721 | '11000111110000' | 0.51816 |
| '00100010001100' | 0.88645 | '01110101100101' | 0.582 | '11010000110011' | 0.58782 |
| '00100011110011' | 0.54142 | '01110110100101' | 0.57874 | '11010001001100' | 1.1342 |
| '00100100001100' | 0.84911 | '01110111011010' | 0.98765 | '11010010001100' | 1.0961 |
| '00100101110011' | 0.52165 | '10000000100101' | 0.54585 | '11010011110011' | 0.60002 |
| '00100110110011' | 0.5157 | '10000001011010' | 0.90106 | '11010100001100' | 1.0382 |
| '00100111001100' | 0.86783 | '10000010011010' | 0.8649 | '11010101110011' | 0.57205 |
| '00110000001111' | 1.0039 | '10000011100101' | 0.54747 | '11010110110011' | 0.56224 |
| '00110001110000' | 0.5836 | '10000100011010' | 0.82379 | '11010111001100' | 1.0695 |
| '00110010110000' | 0.57039 | '10000101100101' | 0.52488 | '11100000111100' | 0.64754 |
| '00110011001111' | 1.0342 | '10000110100101' | 0.52546 | '11100001000011' | 0.70602 |
| '00110100110000' | 0.54939 | '10000111011010' | 0.85771 | '11100010000011' | 0.70062 |
| '00110101001111' | 1.0027 | '10010000011001' | 0.98425 | '11100011111100' | 0.66551 |
| '00110110001111' | 0.97485 | '10010001100110' | 0.60067 | '11100100000011' | 0.66375 |
| '00110111110000' | 0.55878 | '10010010100110' | 0.59573 | '11100101111100' | 0.63032 |
| '01000000101010' | 0.67774 | '10010011011001' | 1.0282 | '11100110111100' | 0.61323 |

Table 119 Continued

| | | | | | |
|------------------|---------|------------------|---------|------------------|---------|
| '01000001010101' | 0.68204 | '10010100100110' | 0.57071 | '11100111000011' | 0.66823 |
| '01000010010101' | 0.66028 | '10010101011001' | 0.97675 | '11110000000000' | 0.76941 |
| '01000011101010' | 0.68662 | '10010110011001' | 0.94038 | '11110001111111' | 0.76051 |
| '01000100010101' | 0.62672 | '10010111100110' | 0.57366 | '11110010111111' | 0.73649 |
| '01000101101010' | 0.65083 | '10100000010110' | 0.65535 | '11110011000000' | 0.77851 |
| '01000110101010' | 0.64309 | '10100001101001' | 0.67308 | '11110100111111' | 0.69881 |
| '01000111010101' | 0.64445 | '10100010101001' | 0.66975 | '11110101000000' | 0.73459 |
| '01010000010110' | 0.72865 | '10100011010110' | 0.66903 | '11110110000000' | 0.72585 |
| '01010001101001' | 0.77369 | '10100100101001' | 0.63638 | '11110111111111' | 0.71922 |
| '01010010101001' | 0.75901 | '10100101010110' | 0.63662 | | |

6. Density Reactivity Coefficient: MHIV Method Results (168 Experiments)

Table 120

Application of the MHIV Method to α_d

| | Low End Variables State | α_d | | High End Variables State | α_d |
|----------------|----------------------------|-------------|-------------|-----------------------------|------------|
| | '11110101101000' | 0.132042136 | | '00001010010111' | 0.339774 |
| 1 Up | '01110101101000' | 0.158369056 | 1 Down | '10001010010111' | 0.27391 |
| 2 Up | '10110101101000' | 0.163946134 | 2 Down | '01001010010111' | 0.290085 |
| 3 Up | '11010101101000' | 0.132460298 | 3 Down | '00101010010111' | 0.334924 |
| 4 Up | '11100101101000' | 0.135367492 | 4 Down | '00011010010111' | 0.317306 |
| 5 Up | '11111101101000' | 0.132042136 | 5 Down | '00000010010111' | 0.339774 |
| 6 Up | '11110001101000' | 0.140599564 | 6 Down | '00001110010111' | 0.317752 |
| 7 Up | '11110111101000' | 0.134980368 | 7 Down | '00001000010111' | 0.335544 |
| 8 Up | '11110100101000' | 0.14186363 | 8 Down | '00001011010111' | 0.294456 |
| 9 Up | '11110101001000' | 0.183994517 | 9 Down | '00001010110111' | 0.260482 |
| a Up | '11110101111000' | 0.130225321 | a Down | '00001010000111' | 0.314508 |
| b Up | '11110101100000' | 0.095778625 | b Down | '00001010011111' | 0.330731 |
| c Up | '11110101101100' | 0.132662819 | c Down | '00001010010011' | 0.337585 |
| d Up | '11110101101010' | 0.1351116 | d Down | '00001010010101' | 0.332439 |
| e Up | '11110101101001' | 0.134875472 | e Down | '00001010010110' | 0.316809 |
| 18' Up | '01110100101000' | 0.177207466 | 18 Down | '10001011010111' | 0.241224 |
| 1b Up | '01110101100000' | 0.125125604 | 1b Down | '10001010011111' | 0.277105 |
| 8b Up | '11110100100000' | 0.108239758 | 8b Down | '00001011011111' | 0.298034 |
| 18b Up | '01110100100000' | 0.148200065 | 18b Down | '10001011011111' | 0.258824 |
| 18b=111-2469ae | | | | | |
| | '01110100100000' | Performed | | '00001010010111' | Performed |
| 2 Up | '00110100100000' | 0.190980509 | 2 Down | '01001010010111' | Performed |
| 4 Up | '01100100100000' | 0.14991092 | 4 Down | '00011010010111' | Performed |
| 6 Up | '01110000100000' | 0.16541123 | 6 Down | '00001110010111' | Performed |
| 9 Up | '01110100000000' | 0.213506302 | 9 Down | '00001010110111' | Performed |
| a Up | '01110100110000' | 0.150627601 | a Down | '00001010000111' | Performed |
| e Up | '01110100100001' | 0.155471452 | e Down | '00001010010110' | Performed |
| 4a Up | '01100100110000' | 0.151518 | 4a Down | '00011010000111' | 0.295824 |
| 18b=111-4a=01 | | | | | |
| 9 Up | '01100100000000' | 0.224820822 | 9 Down | '00011010110111' | 0.244559 |

Table 120 Continued

| | Low End Variables State | α_d | | High End Variables State | α_d |
|----------|------------------------------------|-------------|---------|-----------------------------|------------|
| e Up | 01100100100001''' 18b=111-4a=10 | 0.157281268 | e Down | '00011010010110' | 0.299576 |
| 9 Up | '01110100010000' | 0.223244579 | 9 Down | '00001010100111' | 0.23979 |
| e Up | '01110100110001' 18b=111-4a=00 | 0.167881963 | e Down | '00001010000110' | 0.305527 |
| 9 Up | '01100100010000' | 0.232327227 | 9 Down | '00011010100111' | 0.230345 |
| e Up | '01100100110001' 18b=011-2469ae | 0.1734716 | e Down | '00011010000110' | 0.289347 |
| 1 Up | '11110100100000' | Performed | 1 Down | '10001010010111' | Performed |
| 2 Up | '10110100100000' | 0.147401555 | 2 Down | '11001010010111' | 0.229476 |
| 4 Up | '11100100100000' | 0.107642248 | 4 Down | '10011010010111' | 0.255865 |
| 6 Up | '11110000100000' | 0.123191632 | 6 Down | '10001110010111' | 0.259674 |
| 9 Up | '11110100000000' | 0.166585965 | 9 Down | '10001010110111' | 0.201942 |
| a Up | '11110100110000' | 0.110848885 | a Down | '10001010000111' | 0.252774 |
| e Up | '11110100100001' | 0.115178383 | e Down | '10001010010110' | 0.253059 |
| 4a Up | '11100100110000' 18b=011-4a=01 | 0.109496268 | 4a Down | '10011010000111' | 0.237024 |
| 9 Up | '11100100000000' | 0.175497999 | 9 Down | '10011010110111' | 0.194308 |
| e Up | '11100100100001' 18b=011-4a=10 | 0.114698126 | e Down | '10011010010110' | 0.237473 |
| 9 Up | '11110100010000' | 0.174238989 | 9 Down | '10001010100111' | 0.185146 |
| e Up | '11110100110001' 18b=011-4a=00 | 0.126355925 | e Down | '10001010000110' | 0.24382 |
| 9 Up | '11100100010000' | 0.183328942 | 9 Down | '10011010100111' | 0.180228 |
| e Up | '11100100110001' 18b=101-2469ae | 0.128921977 | e Down | '10011010000110' | 0.228756 |
| | '01110101100000' | Performed | | '00001011010111' | Performed |
| 2 Up | '00110101100000' | 0.164075015 | 2 Down | '01001011010111' | 0.247985 |
| 4 Up | '01100101100000' | 0.124625927 | 4 Down | '00011011010111' | 0.273096 |
| 6 Up | '01110001100000' | 0.141407261 | 6 Down | '00001111010111' | 0.27589 |
| 9 Up | '01110101000000' | 0.186445502 | 9 Down | '00001011110111' | 0.218294 |
| a Up | '01110101110000' | 0.128988713 | a Down | '00001011000111' | 0.276201 |
| e Up | '01110101100001' | 0.130652319 | e Down | '00001011010110' | 0.278176 |
| 4a Up | '01100101110000' | 0.127103832 | 4a Down | '00011011000111' | 0.259374 |

Table 120 Continued

| Low End Variables State | | α_d | High End Variables State | | α_d |
|----------------------------|------------------|-------------|-----------------------------|------------------|------------|
| 18b=101-4a=01 | | | | | |
| 6 Up | '01100101000000' | 0.193762833 | 6 Down | '00011011110111' | 0.212486 |
| e Up | '01100101100001' | 0.130988194 | e Down | '00011011010110' | 0.261542 |
| 18b=101-4a=10 | | | | | |
| 6 Up | '01110101010000' | 0.19205033 | 6 Down | '00001011100111' | 0.205463 |
| e Up | '01110101110001' | 0.139810748 | e Down | '00001011000110' | 0.269054 |
| 18b=101-4a=00 | | | | | |
| 6 Up | '01100101010000' | 0.200574943 | 6 Down | '00011011100111' | 0.199101 |
| e Up | '01100101110001' | 0.142217806 | e Down | '00011011000110' | 0.25372 |
| 18b=110-2469ae- | | | | | |
| | '01110100101000' | Performed | | '00001010011111' | Performed |
| 2 Up | '00110100101000' | 0.213302531 | 2 Down | '01001010011111' | 0.287256 |
| 4 Up | '01100100101000' | 0.183988344 | 4 Down | '00011010011111' | 0.300671 |
| 6 Up | '01110000101000' | 0.189887183 | 6 Down | '00001110011111' | 0.320993 |
| 9 Up | '01110100001000' | 0.233008574 | 9 Down | '00001010111111' | 0.274475 |
| a Up | '01110100111000' | 0.176224286 | a Down | '00001010001111' | 0.320532 |
| e Up | '01110100101001' | 0.181405557 | e Down | '00001010011110' | 0.32556 |
| 4a Up | '01100100111000' | 0.184531697 | 4a Down | '00011010001111' | 0.291442 |
| 18b=001-2469ae | | | | | |
| | '11110101100000' | Performed | | '10001011010111' | Performed |
| 2 Up | '10110101100000' | 0.131906816 | 2 Down | '11001011010111' | 0.201195 |
| 4 Up | '11100101100000' | 0.093005285 | 4 Down | '10011011010111' | 0.227741 |
| 6 Up | '11110001100000' | 0.108385085 | 6 Down | '10001111010111' | 0.228819 |
| 9 Up | '11110101000000' | 0.151729416 | 9 Down | '10001011110111' | 0.177695 |
| a Up | '11110101110000' | 0.097166255 | a Down | '10001011000111' | 0.227623 |
| e Up | '11110101100001' | 0.101012984 | e Down | '10001011010110' | 0.227392 |
| 4a Up | '11100101110000' | 0.095659871 | 4a Down | '10011011000111' | 0.21543 |
| 18b=010-2469ae | | | | | |
| | '11110100101000' | Performed | | '10001010011111' | Performed |
| 2 Up | '10110100101000' | 0.177786445 | 2 Down | '11001010011111' | 0.242543 |
| 4 Up | '11100100101000' | 0.146568991 | 4 Down | '10011010011111' | 0.251304 |
| 6 Up | '11110000101000' | 0.152154461 | 6 Down | '10001110011111' | 0.273784 |
| 9 Up | '11110100001000' | 0.195961726 | 9 Down | '10001010111111' | 0.22424 |
| a Up | '11110100111000' | 0.140165735 | a Down | '10001010001111' | 0.267437 |

Table 120 Continued

| | Low End Variables State | α_d | | High End Variables State | α_d |
|----------------|----------------------------|-------------|---------|-----------------------------|------------|
| e Up | '11110100101001' | 0.146699539 | e Down | '10001010011110' | 0.271489 |
| 4a Up | '11100100111000' | 0.146287571 | 4a Down | '10011010001111' | 0.243548 |
| 18b=100-2469ae | | | | | |
| | '01110101101000' | Performed | | '00001011011111' | Performed |
| 2 Up | '00110101101000' | 0.194730124 | 2 Down | '01001011011111' | 0.263579 |
| 4 Up | '01100101101000' | 0.162395049 | 4 Down | '00011011011111' | 0.271482 |
| 6 Up | '01110001101000' | 0.168756386 | 6 Down | '00001111011111' | 0.29003 |
| 9 Up | '01110101001000' | 0.212625859 | 9 Down | '00001011111111' | 0.244059 |
| a Up | '01110101111000' | 0.158566403 | a Down | '00001011001111' | 0.292458 |
| e Up | '01110101101001' | 0.159859097 | e Down | '00001011011110' | 0.293118 |
| 18b=000-2469ae | | | | | |
| | '11110101101000' | Performed | | '10001011011111' | Performed |
| 2 Up | '10110101101000' | Performed | 2 Down | '11001011011111' | 0.227283 |
| 4 Up | '11100101101000' | Performed | 4 Down | '10011011011111' | 0.231792 |
| 6 Up | '11110001101000' | Performed | 6 Down | '10001111011111' | 0.254596 |
| 9 Up | '11110101001000' | Performed | 9 Down | '10001011111111' | 0.205176 |
| a Up | '11110101111000' | Performed | a Down | '10001011001111' | 0.253467 |
| e Up | '11110101101001' | Performed | e Down | '10001011011110' | 0.254111 |

Table 121Application of a Sixteen FFD to α_d

| Variables State | α_d | Variables State | α_d | Variables State | α_d |
|------------------|------------|------------------|------------|------------------|------------|
| '00000100101101' | 0.23449 | '01100101010101' | 0.21949 | '10110001100100' | 0.1498 |
| '00010011010001' | 0.26674 | '01110010101001' | 0.19813 | '11000111100000' | 0.094937 |
| '00100011001110' | 0.28653 | '10000001111011' | 0.20048 | '11010000011100' | 0.20639 |
| '00110100110010' | 0.19918 | '10010110000111' | 0.22554 | '11100000000011' | 0.20226 |
| '01000010110110' | 0.18537 | '10100110011000' | 0.25422 | '11110111111111' | 0.14764 |
| '01010101001010' | 0.22102 | | | | |

7. Fuel Temperature Reactivity Coefficient: MHIV Method Results (90)

Table 122

Application of the MHIV Method to α_{FT}

| Low End Variables State | | α_{FT} | High End Variables State | | α_{FT} |
|----------------------------|------------------|---------------|-----------------------------|------------------|---------------|
| | '11111110100000' | -0.01631 | | '00000001011111' | -0.05471 |
| 1 Up | '01111110100000' | -0.0167 | 1 Down | '10000001011111' | -0.0542 |
| 2 Up | '10111110100000' | -0.01686 | 2 Down | '01000001011111' | -0.05404 |
| 3 Up | '11011110100000' | -0.01631 | 3 Down | '00100001011111' | -0.05439 |
| 4 Up | '11101110100000' | -0.01756 | 4 Down | '00010001011111' | -0.05039 |
| 5 Up | '11110110100000' | -0.01631 | 5 Down | '00001001011111' | -0.05471 |
| 6 Up | '11111010100000' | -0.01799 | 6 Down | '00000101011111' | -0.05119 |
| 7 Up | '11111100100000' | -0.02049 | 7 Down | '00000011011111' | -0.04349 |
| 8 Up | '11111111100000' | -0.0163 | 8 Down | '00000000111111' | -0.05377 |
| 9 Up | '11111110000000' | -0.02451 | 9 Down | '00000001111111' | -0.03469 |
| a Up | '11111110110000' | -0.01562 | a Down | '00000001001111' | -0.05491 |
| b Up | '11111110101000' | -0.02136 | b Down | '00000001010111' | -0.04044 |
| c Up | '11111110100100' | -0.01628 | c Down | '00000001011011' | -0.05412 |
| d Up | '11111110100010' | -0.01655 | d Down | '00000001011101' | -0.05266 |
| e Up | '11111110100001' | -0.01667 | e Down | '00000001011110' | -0.05156 |
| 79 Up | '11111100000000' | -0.03064 | 79 Down | '00000011111111' | -0.02752 |
| 7b Up | '11111100101000' | -0.02668 | 7b Down | '00000011010111' | -0.03206 |
| 9b Up | '11111110001000' | -0.03363 | 9b Down | '00000001110111' | -0.02667 |
| 79b Up | '11111100001000' | -0.0429 | 79b Down | '00000011110111' | -0.02143 |
| 79b=111-46de | | | | | |
| | '11111100001000' | Performed | | '00000001011111' | Performed |
| 4 Up | '11101100001000' | -0.04598 | 4 Down | '00010001011111' | Performed |
| 6 Up | '11111000001000' | -0.04624 | 6 Down | '00000101011111' | Performed |
| d Up | '11111100001010' | -0.04446 | d Down | '00000001011101' | Performed |
| e Up | '11111100001001' | -0.04456 | e Down | '00000001011110' | Performed |
| 79b=110-46de | | | | | |
| | '11111100000000' | Performed | | '00000001010111' | Performed |
| 4 Up | '11101100000000' | -0.03308 | 4 Down | '00010001010111' | -0.03776 |
| 6 Up | '11111000000000' | -0.03346 | 6 Down | '00000101010111' | -0.03767 |

Table 122 Continued

| | Low End Variables State | α_{FT} | | High End Variables State | α_{FT} |
|--------------|----------------------------|---------------|--------|-----------------------------|---------------|
| d Up | '11111100000010' | -0.03163 | d Down | '00000001010101' | -0.03948 |
| e Up | '111111000000001' | -0.03193 | e Down | '00000001010110' | -0.03859 |
| 79b=101-46de | | | | | |
| | '11111100101000' | Performed | | '00000001111111' | Performed |
| 4 Up | '11101100101000' | -0.02883 | 4 Down | '00010001111111' | -0.03179 |
| 6 Up | '11111000101000' | -0.02966 | 6 Down | '00000101111111' | -0.03163 |
| d Up | '11111100101010' | -0.02782 | d Down | '00000001111101' | -0.0338 |
| e Up | '11111100101001' | -0.02751 | e Down | '00000001111110' | -0.03278 |
| 79b=100-46de | | | | | |
| | '11111100100000' | Performed | | '00000001110111' | Performed |
| 4 Up | '11101100100000' | -0.02209 | 4 Down | '00010001110111' | -0.02453 |
| 6 Up | '11111000100000' | -0.02242 | 6 Down | '00000101110111' | -0.0242 |
| d Up | '11111100100010' | -0.02081 | d Down | '00000001110101' | -0.02632 |
| e Up | '11111100100001' | -0.02091 | e Down | '00000001110110' | -0.02532 |
| 79b=011-46de | | | | | |
| | '11111110001000' | Performed | | '00000011011111' | Performed |
| 4 Up | '11101110001000' | -0.03646 | 4 Down | '00010011011111' | -0.03974 |
| 6 Up | '11111010001000' | -0.03637 | 6 Down | '00000111011111' | -0.04052 |
| d Up | '11111110001010' | -0.03495 | d Down | '00000011011101' | -0.04164 |
| e Up | '11111110001001' | -0.03508 | e Down | '00000011011110' | -0.04103 |
| 79b=010-46de | | | | | |
| | '11111110000000' | Performed | | '00000011010111' | Performed |
| 4 Up | '11101110000000' | -0.02631 | 4 Down | '00010011010111' | -0.02996 |
| 6 Up | '11111010000000' | -0.02657 | 6 Down | '00000111010111' | -0.02991 |
| d Up | '11111110000010' | -0.02511 | d Down | '00000011010101' | -0.03152 |
| e Up | '11111110000001' | -0.0252 | e Down | '00000011010110' | -0.03056 |
| 79b=001-46de | | | | | |
| | '11111110101000' | Performed | | '00000011111111' | Performed |
| 4 Up | '11101110101000' | -0.02297 | 4 Down | '00010011111111' | -0.02542 |
| 6 Up | '11111010101000' | -0.02356 | 6 Down | '00000111111111' | -0.02487 |
| d Up | '11111110101010' | -0.02191 | d Down | '00000011111101' | -0.02655 |
| e Up | '11111110101001' | -0.0219 | e Down | '00000011111110' | -0.02624 |
| 79b=000-46de | | | | | |
| | '11111110100000' | Performed | | '00000011110111' | Performed |
| 4 Up | '11101110100000' | Performed | 4 Down | '00010011110111' | -0.01963 |
| 6 Up | '11111010100000' | Performed | 6 Down | '00000111110111' | -0.01964 |
| d Up | '11111110100010' | Performed | d Down | '00000011110101' | -0.02075 |
| e Up | '11111110100001' | Performed | e Down | '00000011110110' | -0.01999 |

Table 123Application of a Sixteen FFD to α_{FT}

| Variables State | α_{FT} | Variables State | α_{FT} |
|------------------|---------------|------------------|---------------|
| '00000100101101' | 0.031321 | '10010110000111' | 0.026668 |
| '00010011010001' | 0.028449 | '10100110011000' | 0.035088 |
| '00100011001110' | 0.042735 | '01010101001010' | 0.045717 |
| '00110100110010' | 0.020816 | '10110001100100' | 0.022983 |
| '01000010110110' | 0.019758 | '11000111100000' | 0.017582 |
| '01100101010101' | 0.035623 | '11010000011100' | 0.044473 |
| '01110010101001' | 0.02434 | '11100000000011' | 0.038042 |
| '10000001111011' | 0.033617 | '11110111111111' | 0.022427 |

8. Power Peaking Factor: MHIV Method Results (292)

Table 124

Application of the MHIV Method to PPF

| | Low End Variables State | PPF | | High End Variables State | PPF |
|-------------------|----------------------------|-----------|----------|-----------------------------|-----------|
| | '00101001010110' | 2.2828 | | '11010110101001' | 2.8326 |
| 1 Up | '10101001010110' | 2.2291 | 1 Down | '01010110101001' | 2.621 |
| 2 Up | '01101001010110' | 2.3236 | 2 Down | '10010110101001' | 2.5086 |
| 3 Up | '00001001010110' | 2.2776 | 3 Down | '11110110101001' | 2.6569 |
| 4 Up | '00111001010110' | 2.3204 | 4 Down | '11000110101001' | 2.5232 |
| 5 Up | '00100001010110' | 2.2828 | 5 Down | '11011110101001' | 2.8326 |
| 6 Up | '00101101010110' | 2.3283 | 6 Down | '11010010101001' | 2.7096 |
| 7 Up | '00101011010110' | 2.231 | 7 Down | '11010100101001' | 2.6976 |
| 8 Up | '00101000010110' | 2.1178 | 8 Down | '11010111101001' | 2.7877 |
| 9 Up | '00101001110110' | 2.4303 | 9 Down | '11010110001001' | 2.565 |
| a Up | '00101001000110' | 2.1508 | a Down | '11010110111001' | 2.4072 |
| b Up | '00101001011110' | 2.0748 | b Down | '11010110100001' | 2.7381 |
| c Up | '00101001010010' | 2.2955 | c Down | '11010110101101' | 2.7194 |
| d Up | '00101001010100' | 2.2604 | d Down | '11010110101011' | 2.7407 |
| e Up | '00101001010111' | 2.1542 | e Down | '11010110101000' | 2.6873 |
| 2a Up | '01101001000110' | 2.3142 | 2a Down | '10010110111001' | 2.445 |
| 2b Up | '01101001011110' | 2.2527 | 2b Down | '10010110100001' | 2.5381 |
| ab Up | '00101001001110' | 2.0034 | ab Down | '11010110110001' | 2.2512 |
| 2ab Up | '01101001001110' | 2.2077 | 2ab Down | 10010110110001' | 2.445 |
| 2ab=111,134789cde | | | | | |
| | '01101001001110' | Performed | | '11010110101001' | Performed |
| 1 Up | '11101001001110' | 2.3398 | 1 Down | '01010110101001' | Performed |
| 3 Up | '01001001001110' | 2.2097 | 3 Down | '11110110101001' | Performed |
| 4 Up | '01111001001110' | 2.3235 | 4 Down | '11000110101001' | Performed |
| 7 Up | '01101011001110' | 2.1603 | 7 Down | '11010100101001' | Performed |
| 8 Up | '01101000001110' | 2.1522 | 8 Down | '11010111101001' | Performed |
| 9 Up | '01101001101110' | 2.467 | 9 Down | '11010110001001' | Performed |
| c Up | '01101001001010' | 2.3506 | c Down | '11010110101101' | Performed |
| d Up | '01101001001100' | 2.2634 | d Down | '11010110101011' | Performed |
| e Up | '01101001001111' | 2.2134 | e Down | '11010110101000' | Performed |
| 34 Up | '01011001001110' | 2.3798 | 34 Down | '11100110101001' | 2.5583 |
| 37 Up | '01001011001110' | 2.2928 | 37 Down | '11110100101001' | 2.6729 |

Table 124 Continued

| | Low End Variables State | PPF | | High End Variables State | PPF |
|-------------------|----------------------------|-----------|----------|-----------------------------|-----------|
| 47 Up | '01111011001110' | 2.3545 | 47 Down | '11000100101001' | 2.4631 |
| 347 Up | '01011011001110' | 2.3418 | 347 Down | '11100100101001' | 2.4978 |
| 2ab=011,134789cde | | | | | |
| | '00101001001110' | Performed | | '10010110101001' | Performed |
| 1 Up | '10101001001110' | 2.0986 | 1 Down | '00010110101001' | 2.3571 |
| 3 Up | '00001001001110' | 2.0248 | 3 Down | '10110110101001' | 2.4544 |
| 4 Up | '00111001001110' | 2.0666 | 4 Down | '10000110101001' | 2.3921 |
| 7 Up | '00101011001110' | 2.0095 | 7 Down | '10010100101001' | 2.4556 |
| 8 Up | '00101000001110' | 2.0168 | 8 Down | '10010111101001' | 2.5688 |
| 9 Up | '00101001101110' | 2.2342 | 9 Down | '10010110001001' | 2.2082 |
| c Up | '00101001001010' | 2.0001 | c Down | '10010110101101' | 2.4806 |
| d Up | '00101001001100' | 2.0123 | d Down | '10010110101011' | 2.5396 |
| e Up | '00101001001111' | 1.9863 | e Down | '10010110101000' | 2.5578 |
| 2ab=110,134789cde | | | | | |
| | '01101001000110' | Performed | | '11010110100001' | Performed |
| 1 Up | '11101001000110' | 2.4587 | 1 Down | '01010110100001' | 2.6011 |
| 3 Up | '01001001000110' | 2.386 | 3 Down | '11110110100001' | 2.7856 |
| 4 Up | '01111001000110' | 2.4917 | 4 Down | '11000110100001' | 2.5403 |
| 7 Up | '01101011000110' | 2.4045 | 7 Down | '11010100100001' | 2.8734 |
| 8 Up | '01101000000110' | 2.3018 | 8 Down | '11010111100001' | 2.6032 |
| 9 Up | '01101001100110' | 2.5024 | 9 Down | '11010110000001' | 2.5778 |
| c Up | '01101001000010' | 2.3924 | c Down | '11010110100101' | 2.6803 |
| d Up | '01101001000100' | 2.3267 | d Down | '11010110100011' | 2.747 |
| e Up | '01101001000111' | 2.3945 | e Down | '11010110100000' | 2.664 |
| 37 Up | '01001011000110' | 2.3095 | 37 Down | '11110100100001' | 2.7263 |
| 38 Up | '01001000000110' | 2.2216 | 38 Down | '11110111100001' | 2.7234 |
| 78 Up | '01101010000110' | 2.2382 | 78 Down | '11010101100001' | 2.6938 |
| 378 Up | '01001010000110' | 2.2633 | 378 Down | '11110101100001' | 2.6492 |
| 2ab=101,134789cde | | | | | |
| | '01101001011110' | Performed | | '11010110111001' | Performed |
| 1 Up | '11101001011110' | 2.3041 | 1 Down | '01010110111001' | 2.2619 |
| 3 Up | '01001001011110' | 2.267 | 3 Down | '11110110111001' | 2.3396 |
| 4 Up | '01111001011110' | 2.3542 | 4 Down | '11000110111001' | 2.1798 |

Table 124 Continued

| | Low End Variables State | PPF | | High End Variables State | PPF |
|-------------------|----------------------------|-----------|----------|-----------------------------|-----------|
| 7 Up | '01101011011110' | 2.2832 | 7 Down | '11010100111001' | 2.4122 |
| 8 Up | '01101000011110' | 2.1714 | 8 Down | 11010111111001''' | 2.6011 |
| 9 Up | '01101001111110' | 2.5654 | 9 Down | '11010110011001' | 2.308 |
| c Up | '01101001011010' | 2.2645 | c Down | '11010110111101' | 2.4693 |
| d Up | '01101001011100' | 2.2581 | d Down | '11010110111011' | 2.399 |
| e Up | '01101001011111' | 2.1448 | e Down | 11010110111000''' | 2.69 |
| 2ab=100,134789cde | | | | | |
| | '01101001010110' | Performed | | '11010110110001' | Performed |
| 1 Up | '11101001010110' | 2.3114 | 1 Down | '01010110110001' | 2.1754 |
| 3 Up | '01001001010110' | 2.314 | 3 Down | '11110110110001' | 2.3014 |
| 4 Up | '01111001010110' | 2.5398 | 4 Down | '11000110110001' | 2.0521 |
| 7 Up | '01101011010110' | 2.278 | 7 Down | '11010100110001' | 2.272 |
| 8 Up | '01101000010110' | 2.0615 | 8 Down | '11010111110001' | 2.5982 |
| 9 Up | '01101001110110' | 2.4581 | 9 Down | '11010110010001' | 2.2319 |
| c Up | '01101001010010' | 2.3894 | c Down | '11010110110101' | 2.1688 |
| d Up | '01101001010100' | 2.3807 | d Down | '11010110110011' | 2.321 |
| e Up | '01101001010111' | 2.2113 | e Down | '11010110110000' | 2.5891 |
| ed Up | '01101001010101' | 2.1316 | ed Down | 11010110110010''' | 2.5404 |
| 2ab=010,134789cde | | | | | |
| | '00101001000110' | Performed | | '10010110100001' | Performed |
| 1 Up | '10101001000110' | 2.2365 | 1 Down | '00010110100001' | 2.451 |
| 3 Up | '00001001000110' | 2.2062 | 3 Down | '10110110100001' | 2.5854 |
| 4 Up | '00111001000110' | 2.2584 | 4 Down | '10000110100001' | 2.3926 |
| 7 Up | '00101011000110' | 2.1576 | 7 Down | '10010100100001' | 2.5484 |
| 8 Up | '00101000000110' | 2.1369 | 8 Down | '100101111100001' | 2.5963 |
| 9 Up | '00101001100110' | 2.3007 | 9 Down | '10010110000001' | 2.4998 |
| c Up | '00101001000010' | 2.1871 | c Down | '10010110100101' | 2.5788 |
| d Up | '00101001000100' | 2.2017 | d Down | '10010110100011' | 2.5724 |
| e Up | '00101001000111' | 2.1653 | e Down | '10010110100000' | 2.5949 |
| 39 Up | '00001001100110' | 2.3495 | 39 Down | '10110110000001' | 2.4951 |
| 3d Up | '00001001000100' | 2.1778 | 3d Down | '10110110100011' | 2.5961 |
| 9d Up | '00101001100100' | 2.3497 | 9d Down | '10010110000011' | 2.427 |
| 39d Up | '00001001100100' | 2.3005 | 39d Down | '10110110000011' | 2.4474 |

Table 124 Continued

| Low End Variables State | | PPF | High End Variables State | | PPF |
|-------------------------|------------------|-----------|--------------------------|------------------|-----------|
| 2ab=001,134789cde | | | | | |
| | '00101001011110' | Performed | | '10010110111001' | |
| 1 Up | '10101001011110' | 2.1317 | 1 Down | '00010110111001' | 2.3 |
| 3 Up | '00001001011110' | 2.0899 | 3 Down | '10110110111001' | 2.4945 |
| 4 Up | '00111001011110' | 2.1657 | 4 Down | '10000110111001' | 2.3265 |
| 7 Up | '00101011011110' | 2.0198 | 7 Down | '10010100111001' | 2.5366 |
| 8 Up | '00101000011110' | 2.0021 | 8 Down | '10010111111001' | 2.5542 |
| 9 Up | '00101001111110' | 2.3205 | 9 Down | '10010110011001' | 2.2892 |
| c Up | '00101001011010' | 2.0573 | c Down | '10010110111101' | 2.4945 |
| d Up | '00101001011100' | 2.0708 | d Down | '10010110111011' | 2.4159 |
| e Up | '00101001011111' | 2.0233 | e Down | '10010110111000' | 2.5202 |
| 19 Up | '00010110011001' | 2.0649 | 19 Down | '10101001111110' | 2.3731 |
| 2ab=000,134789cde | | | | | |
| | '00101001010110' | Performed | | '10010110110001' | |
| 1 Up | '10101001010110' | Performed | 1 Down | '00010110110001' | 2.3576 |
| 3 Up | '00001001010110' | Performed | 3 Down | '10110110110001' | 2.4672 |
| 4 Up | '00111001010110' | Performed | 4 Down | '10000110110001' | 2.2654 |
| 7 Up | '00101011010110' | Performed | 7 Down | '10010100110001' | 2.5215 |
| 8 Up | '00101000010110' | Performed | 8 Down | '10010111110001' | 2.4565 |
| 9 Up | '00101001110110' | Performed | 9 Down | '10010110010001' | 2.3426 |
| c Up | '00101001010010' | Performed | c Down | '10010110110101' | 2.4871 |
| d Up | '00101001010100' | Performed | d Down | '10010110110011' | 2.5048 |
| e Up | '00101001010111' | Performed | e Down | '10010110110000' | 2.5666 |
| 14 Up | '10111001010110' | 2.3791 | 14 Down | '00000110110001' | 2.22 |
| 18 Up | '10101000010110' | 2.2497 | 18 Down | '00010111110001' | 2.63 |
| 48 Up | '00111000010110' | 2.1418 | 48 Down | '10000111110001' | 2.2628 |
| 148 Up | '10111000010110' | 2.3996 | 148 Down | '00000111110001' | 2.4199 |
| 2ab=111,347-e | | | | | |
| | '11010110101001' | Performed | | '11010110101001' | Performed |
| 3 Up | '11110110101000' | 2.8323 | 3 Down | '11110111101001' | 2.6522 |
| 4 Up | '11000110101000' | 2.5546 | 4 Down | '11000111101001' | 2.438 |
| 7 Up | '11010100101000' | 2.7412 | 7 Down | '11010101101001' | 2.7039 |
| 34 Up | '11100110101000' | 2.646 | 34 Down | '11100111101001' | 2.4442 |
| 37 Up | '11110100101000' | 2.6818 | 37 Down | '11110101101001' | 2.6987 |
| 47 Up | '11000100101000' | 2.5193 | 47 Down | '11000101101001' | 2.5749 |
| 347 Up | '11100100101000' | 2.6878 | 347 Down | '11100101101001' | 2.4668 |

Table 124 Continued

| Low End Variables State | | PPF | High End Variables State | | PPF |
|-------------------------|------------------|-----------|--------------------------|------------------|-----------|
| 2ab=111,347-8 | | | | | |
| 3 Up | '01001000001110' | 2.258 | 3 Down | '01001001001111' | 2.1874 |
| 4 Up | '01111000001110' | 2.2617 | 4 Down | '01111001001111' | 2.2503 |
| 7 Up | '01101010001110' | 2.2079 | 7 Down | '01101011001111' | 2.209 |
| 34 Up | '01011000001110' | 2.2575 | 34 Down | '01011001001111' | 2.2791 |
| 37 Up | '01001010001110' | 2.2858 | 37 Down | '01001011001111' | 2.2384 |
| 47 Up | '01111010001110' | 2.286 | 47 Down | '01111011001111' | 2.3317 |
| 347 Up | '01011010001110' | 2.217 | 347 Down | '01011011001111' | 2.2807 |
| 347-8e | | | | | |
| Up | '11100101101000' | 2.5028 | Down | '01011010001111' | 2.2523 |
| 2ab=101,49e=111 | | | | | |
| | '01111001111110' | 2.5495 | | '11010110111001' | Performed |
| 1 Up | '11111001111110' | 2.5896 | 1 Down | '01010110111001' | Performed |
| 8 Up | '01111000111110' | 2.1851 | 8 Down | '11010111111001' | Performed |
| 18 Up | '11111000111110' | 2.4811 | 18 Down | '01010111111001' | 2.7484 |
| 2ab=101,49e=110 | | | | | |
| | '01111001111110' | 2.6869 | | '11010110111000' | Performed |
| 1 Up | '11111001111110' | 2.7282 | 1 Down | '01010110111000' | Performed |
| 8 Up | '01111000111110' | 2.4259 | 8 Down | '11010111111000' | 2.7091 |
| 18 Up | '11111000111110' | 2.5472 | 18 Down | '01010111111000' | 2.7187 |
| 2ab=101,49e=101 | | | | | |
| | '01111001011111' | 2.2544 | | '11010110011001' | Performed |
| 1 Up | '11111001011111' | 2.4256 | 1 Down | '01010110011001' | 2.0776 |
| 8 Up | '01111000011111' | 2.1384 | 8 Down | '11010111011001' | 2.4466 |
| 18 Up | '11111000011111' | 2.3591 | 18 Down | '01010111011001' | 2.4167 |
| 2ab=101,49e=100 | | | | | |
| | '01111001011110' | Performed | | '11010110011000' | 2.4764 |
| 1 Up | '11111001011110' | 2.4317 | 1 Down | '01010110011000' | 2.238 |
| 8 Up | '01111000011110' | 2.3138 | 8 Down | '11010111011000' | 2.5685 |

Table 124 Continued

| Low End Variables State | | PPF | High End Variables State | | PPF |
|----------------------------|-------------------|-----------|-----------------------------|------------------|-----------|
| 2ab=101,49e=011 | | | | | |
| | '01101001111111' | 2.492 | | '11000110111001' | Performed |
| 1 Up | '11101001111111' | 2.29 | 1 Down | '01000110111001' | 2.1004 |
| 8 Up | '01101000111111' | 2.0674 | 8 Down | '11000111111001' | 2.366 |
| 18 Up | '11101000111111' | 2.1624 | 18 Down | '01000111111001' | 2.4514 |
| 2ab=101,49e=010 | | | | | |
| | '011010011111110' | Performed | | '11000110111000' | Performed |
| 1 Up | '111010011111110' | 2.476 | 1 Down | '01000110111000' | 2.3238 |
| 8 Up | '011010001111110' | 2.2618 | 8 Down | '11000111111000' | 2.4985 |
| 18 Up | '111010001111110' | 2.3441 | 18 Down | '01000111111000' | 2.5205 |
| 2ab=101,49e=001 | | | | | |
| | '01101001011111' | Performed | | '11000110011001' | 2.0219 |
| 1 Up | '11101001011111' | 2.1263 | 1 Down | '01000110011001' | 2.0214 |
| 8 Up | '01101000011111' | 1.9257 | 8 Down | '11000111011001' | 2.3122 |
| 2ab=101,49e=000 | | | | | |
| | '011010010111110' | Performed | | '11000110011000' | 2.2292 |
| 1 Up | '111010010111110' | Performed | 1 Down | '01000110011000' | 2.1044 |
| 8 Up | '011010000111110' | Performed | 8 Down | '11000111011000' | 2.3861 |
| 2ab=100,de=11,1 | | | | | |
| 1 Up | '11101001010101' | 2.1712 | 1 Down | '01010110110001' | Performed |
| 8 Up | '01101000010101' | 1.7411 | 8 Down | '11010111110001' | Performed |
| 9 Up | '01101001110101' | 2.3993 | 9 Down | '11010110010001' | Performed |
| 2ab=100,de=01,1 | | | | | |
| 1 Up | '11101001010111' | 2.1478 | 1 Down | '01010110110011' | 2.0111 |
| 8 Up | '01101000010111' | 1.7475 | 8 Down | '11010111110011' | 2.4971 |
| 9 Up | '01101001110111' | 2.3227 | 9 Down | '11010110010011' | 2.1571 |
| 18 Up | '11101000010111' | 1.9913 | 18 Down | '01010111110011' | 2.5616 |
| 2ab=100,de=10 | | | | | |
| 1 Up | '11101001010100' | Performed | 1 Down | '01010110110000' | 2.471 |
| 8 Up | '01101000010100' | 2.0078 | 8 Down | '11010111110000' | 2.6034 |
| 9 Up | '01101001110100' | 2.4888 | 9 Down | '11010110010000' | 2.486 |
| 18 Up | '11101000010100' | 2.264 | 18 Down | '01010111110000' | 2.6895 |
| 2ab=100,de=00 | | | | | |
| 1 Up | '11101001010110' | Performed | 1 Down | '01010110110010' | 2.3361 |
| 8 Up | '01101000010110' | Performed | 8 Down | '11010111110010' | 2.5811 |
| 9 Up | '01101001110110' | Performed | 9 Down | '11010110010010' | 2.3462 |
| 18 Up | '11101000010110' | 2.1667 | 18 Down | '01010111110010' | 2.7155 |

9. Power Peaking Factor: 256 Res V FFD Results (256)

Table 125
Application of a 128 Res IV FFD to PPF

| Variables State | PPF | Variables State | PPF |
|------------------|--------|------------------|--------|
| '0000000010001' | 2.0279 | '1000000000011' | 2.2501 |
| '00000000101110' | 2.2404 | '10000000111100' | 2.3395 |
| '00000001001110' | 2.0248 | '10000001011100' | 2.2096 |
| '00000001110001' | 2.3705 | '10000001100011' | 2.3558 |
| '00000010001101' | 2.0013 | '10000010011111' | 1.9618 |
| '00000010110010' | 2.2245 | '10000010100000' | 2.3685 |
| '00000011010010' | 2.2673 | '10000011000000' | 2.2995 |
| '00000011101101' | 2.2569 | '10000011111111' | 2.2891 |
| '00000100001011' | 2.0491 | '10000100011001' | 2.0514 |
| '00000100110100' | 2.2743 | '10000100100110' | 2.3819 |
| '00000101010100' | 2.3285 | '10000101000110' | 2.3326 |
| '00000101101011' | 2.3551 | '10000101111001' | 2.3059 |
| '00000110010111' | 2.0249 | '10000110000101' | 2.2874 |
| '00000110101000' | 2.2301 | '10000110111010' | 2.3574 |
| '00000111001000' | 2.1184 | '10000111011010' | 2.1846 |
| '00000111110111' | 2.5884 | '10000111100101' | 2.3676 |
| '00010000001000' | 2.0951 | '10010000011010' | 2.2249 |
| '00010000110111' | 2.2335 | '10010000100101' | 2.5254 |
| '00010001010111' | 2.3007 | '10010001000101' | 2.3855 |
| '00010001101000' | 2.3994 | '10010001111010' | 2.5559 |
| '00010010010100' | 2.1099 | '10010010000110' | 2.3419 |
| '00010010101011' | 2.316 | '10010010111001' | 2.3852 |
| '00010011001011' | 2.0398 | '10010011011001' | 2.2125 |
| '00010011110100' | 2.5141 | '10010011100110' | 2.5148 |
| '00010100010010' | 2.1712 | '10010100000000' | 2.4516 |
| '00010100101101' | 2.4098 | '10010100111111' | 2.4274 |
| '00010101001101' | 2.1672 | '10010101011111' | 2.2515 |
| '00010101110010' | 2.6172 | '10010101100000' | 2.5826 |
| '00010110001110' | 2.0809 | '10010110011100' | 2.3222 |
| '00010110110001' | 2.3576 | '10010110100011' | 2.5724 |
| '00010111010001' | 2.3656 | '10010111000011' | 2.4392 |
| '00010111101110' | 2.4791 | '10010111111100' | 2.5746 |
| '00100000000111' | 2.1192 | '10100000010101' | 2.1459 |
| '00100000111000' | 2.1851 | '10100000101010' | 2.3641 |

Table 125 Continued

| Variables State | PPF | Variables State | PPF |
|------------------|--------|------------------|--------|
| '00100001011000' | 2.1147 | '10100001001010' | 2.1059 |
| '00100001100111' | 2.3071 | '10100001110101' | 2.3258 |
| '00100010011011' | 1.8968 | '10100010001001' | 2.0792 |
| '00100010100100' | 2.2717 | '10100010110110' | 2.3647 |
| '00100011000100' | 2.1992 | '10100011010110' | 2.2459 |
| '00100011111011' | 2.3584 | '10100011101001' | 2.367 |
| '00100100011101' | 1.9702 | '10100100001111' | 2.0816 |
| '00100100100010' | 2.3524 | '10100100110000' | 2.4312 |
| '00100101000010' | 2.2237 | '10100101010000' | 2.2803 |
| '00100101111101' | 2.4123 | '10100101101111' | 2.3239 |
| '00100110000001' | 2.1918 | '10100110010011' | 2.1717 |
| '00100110111110' | 2.2387 | '10100110101100' | 2.3776 |
| '00100111011110' | 2.2205 | '10100111001100' | 2.1688 |
| '00100111100001' | 2.3567 | '10100111110011' | 2.2364 |
| '00110000011110' | 2.0664 | '10110000001100' | 2.2672 |
| '00110000100001' | 2.3942 | '10110000110011' | 2.4294 |
| '00110001000001' | 2.3101 | '10110001010011' | 2.3053 |
| '00110001111110' | 2.47 | '10110001101100' | 2.4518 |
| '00110010000010' | 2.1789 | '10110010010000' | 2.4298 |
| '00110010111101' | 2.1859 | '10110010101111' | 2.4689 |
| '00110011011101' | 2.0959 | '10110011001111' | 2.1906 |
| '00110011100010' | 2.3746 | '10110011110000' | 2.4966 |
| '00110100000100' | 2.2665 | '10110100010110' | 2.4682 |
| '00110100111011' | 2.236 | '10110100101001' | 2.4799 |
| '00110101011011' | 2.2329 | '10110101001001' | 2.3428 |
| '00110101100100' | 2.5189 | '10110101110110' | 2.5344 |
| '00110110011000' | 2.1432 | '10110110001010' | 2.267 |
| '00110110100111' | 2.5031 | '10110110110101' | 2.5176 |
| '00110111000111' | 2.328 | '10110111010101' | 2.4124 |
| '00110111111000' | 2.5492 | '10110111101010' | 2.5067 |
| '0100000000100' | 2.3951 | '11000000010110' | 2.2198 |
| '01000000111011' | 1.991 | '11000000101001' | 2.588 |
| '01000001011011' | 2.0696 | '11000001001001' | 2.3533 |
| '01000001100100' | 2.458 | '11000001110110' | 2.4536 |
| '01000010011000' | 2.1662 | '11000010001010' | 2.3132 |
| '01000010100111' | 2.3113 | '11000010110101' | 2.0915 |
| '01000011000111' | 2.3434 | '11000011010101' | 2.2324 |
| '01000011111000' | 2.5186 | '11000011101010' | 2.5202 |

Table 125 Continued

| Variables State | PPF | Variables State | PPF |
|------------------|--------|------------------|--------|
| '01000100011110' | 2.1502 | '11000100001100' | 2.4039 |
| '01000100100001' | 2.3819 | '11000100110011' | 1.9332 |
| '01000101000001' | 2.3396 | '11000101010011' | 2.19 |
| '01000101111110' | 2.5514 | '11000101101100' | 2.566 |
| '01000110000010' | 2.3338 | '11000110010000' | 2.2463 |
| '01000110111101' | 2.0245 | '11000110101111' | 2.4738 |
| '01000111011101' | 2.128 | '11000111001111' | 2.4369 |
| '01000111100010' | 2.5802 | '11000111110000' | 2.3591 |
| '01010000011101' | 2.0592 | '11010000001111' | 2.4114 |
| '01010000100010' | 2.5266 | '11010000110000' | 2.6563 |
| '01010001000010' | 2.5876 | '11010001010000' | 2.5965 |
| '01010001111101' | 2.5241 | '11010001101111' | 2.7153 |
| '01010010000001' | 2.2748 | '11010010010011' | 2.2178 |
| '01010010111110' | 2.3458 | '11010010101100' | 2.688 |
| '01010011011110' | 2.3223 | '11010011001100' | 2.499 |
| '01010011100001' | 2.6682 | '11010011110011' | 2.4779 |
| '01010100000111' | 2.3264 | '11010100010101' | 2.2377 |
| '01010100111000' | 2.4908 | '11010100101010' | 2.7642 |
| '01010101011000' | 2.5341 | '11010101001010' | 2.618 |
| '01010101100111' | 2.6616 | '11010101110101' | 2.5924 |
| '01010110011011' | 2.0666 | '11010110001001' | 2.565 |
| '01010110100100' | 2.5282 | '11010110110110' | 2.6173 |
| '01010111000100' | 2.6519 | '11010111010110' | 2.7098 |
| '01010111111011' | 2.6915 | '11010111101001' | 2.7877 |
| '01100000010010' | 2.128 | '11100000000000' | 2.5044 |
| '01100000101101' | 2.2883 | '11100000111111' | 2.1624 |
| '01100001001101' | 2.1627 | '11100001011111' | 2.1263 |
| '01100001110010' | 2.5223 | '11100001100000' | 2.5763 |
| '01100010001110' | 2.2079 | '11100010011100' | 2.3467 |
| '01100010110001' | 1.9722 | '11100010100011' | 2.5236 |
| '01100011010001' | 2.1505 | '11100011000011' | 2.3847 |
| '01100011101110' | 2.5607 | '11100011111100' | 2.4227 |
| '01100100001000' | 2.2972 | '11100100011010' | 2.276 |
| '01100100110111' | 1.9464 | '11100100100101' | 2.4342 |
| '01100101010111' | 2.2107 | '11100101000101' | 2.5085 |
| '01100101101000' | 2.5058 | '11100101111010' | 2.4745 |
| '01100110010100' | 1.957 | '11100110000110' | 2.4682 |
| '01100110101011' | 2.4427 | '11100110111001' | 2.1597 |

Table 125 Continued

| Variables State | PPF | Variables State | PPF |
|------------------|--------|------------------|--------|
| '01100111001011' | 2.3284 | '11100111011001' | 2.2835 |
| '01100111110100' | 2.5344 | '11100111100110' | 2.557 |
| '01110000001011' | 2.2191 | '11110000011001' | 2.2988 |
| '01110000110100' | 2.3247 | '11110000100110' | 2.7463 |
| '01110001010100' | 2.4421 | '11110001000110' | 2.5571 |
| '01110001101011' | 2.578 | '11110001111001' | 2.487 |
| '01110010010111' | 2.0669 | '11110010000101' | 2.6262 |
| '01110010101000' | 2.5007 | '11110010111010' | 2.6273 |
| '01110011001000' | 2.4018 | '11110011011010' | 2.5185 |
| '01110011110111' | 2.5483 | '11110011100101' | 2.7098 |
| '01110100010001' | 1.98 | '11110100000011' | 2.5933 |
| '01110100101110' | 2.674 | '11110100111100' | 2.6122 |
| '01110101001110' | 2.4362 | '11110101011100' | 2.6611 |
| '01110101110001' | 2.6459 | '11110101100011' | 2.6353 |
| '01110110001101' | 2.2931 | '11110110011111' | 2.2643 |
| '01110110110010' | 2.2783 | '11110110100000' | 2.723 |
| '01110111010010' | 2.5181 | '11110111000000' | 2.6598 |
| '01110111101101' | 2.6636 | '11110111111111' | 2.4945 |

10. Multiplication Factor Depletion: Experiments Results (154 experiments)

Table 126

Application of the MHIV Method to ΔK_{eff}

| | Low End Variables State | ΔK_{eff} | | High End Variables State | ΔK_{eff} |
|--------------|----------------------------|------------------|-------------|-----------------------------|------------------|
| | '11100011011111' | 0.02029 | | '00011100100000' | 0.68692 |
| 1 Up | '01100011011111' | 0.06177 | 1 Down | '10011100100000' | 0.60649 |
| 2 Up | '10100011011111' | 0.10336 | 2 Down | '01011100100000' | 0.56931 |
| 3 Up | '11000011011111' | 0.01971 | 3 Down | '00111100100000' | 0.6858 |
| 4 Up | '11110011011111' | 0.04514 | 4 Down | '00001100100000' | 0.60827 |
| 5 Up | '11101011011111' | 0.10074 | 5 Down | '00010100100000' | 0.5536 |
| 6 Up | '11100111011111' | 0.02269 | 6 Down | '00011000100000' | 0.64859 |
| 7 Up | '11100001011111' | 0.01916 | 7 Down | '00011110100000' | 0.67623 |
| 8 Up | '11100010011111' | 0.0166 | 8 Down | '00011101100000' | 0.64554 |
| 9 Up | '11100011111111' | 0.0331 | 9 Down | '00011100000000' | 0.53407 |
| a Up | '11100011001111' | 0.05715 | a Down | '00011100110000' | 0.69209 |
| b Up | '11100011010111' | 0.03227 | b Down | '00011100101000' | 0.52607 |
| c Up | '11100011011011' | 0.01839 | c Down | '00011100100100' | 0.68433 |
| d Up | '11100011011101' | 0.02132 | d Down | '00011100100010' | 0.6797 |
| e Up | '11100011011110' | 0.04075 | e Down | '00011100100001' | 0.66976 |
| 9b Up | '11100011110111' | 0.0769 | 9b Down | '00011100001000' | 0.35161 |
| 9b=11 12468a | | | | | |
| | '11100011110111' | Performed | | '00011100100000' | Performed |
| 1 Up | '01100011110111' | 0.24774 | 1 Down | '10011100100000' | Performed |
| 2 Up | '10100011110111' | 0.34108 | 2 Down | '01011100100000' | Performed |
| 4 Up | '11110011110111' | 0.079 | 4 Down | '00001100100000' | Performed |
| 5 Up | '11101011110111' | 0.34804 | 5 Down | '00010100100000' | Performed |
| 6 Up | '11100111110111' | 0.0915 | 6 Down | '00011000100000' | Performed |
| 8 Up | '11100010110111' | 0.1423 | 8 Down | '00011101100000' | Performed |
| a Up | '11100011100111' | 0.1302 | a Down | '00011100110000' | Performed |
| 12 Up | '00100011110111' | 0.36764 | 12 Down | '11011100100000' | 0.55529 |
| 15 Up | '01101011110111' | 0.37635 | 15 Down | '10010100100000' | 0.54252 |
| 25 Up | '10101011110111' | 0.39983 | 25 Down | '01010100100000' | 0.45244 |
| 125 Up | '00101011110111' | 0.45436 | 125 Down | '11010100100000' | 0.2144 |

Table 126 Continued

| | Low End Variables State | ΔK_{eff} | | High End Variables State | ΔK_{eff} |
|---------------|----------------------------|------------------|------------|-----------------------------|------------------|
| 9b=10 12468a | | | | | |
| | '11100011111111' | Performed | | '00011100101000' | Performed |
| 1 Up | '01100011111111' | 0.12021 | 1 Down | '10011100101000' | 0.48812 |
| 2 Up | '10100011111111' | 0.20675 | 2 Down | '01011100101000' | 0.46725 |
| 4 Up | '11110011111111' | 0.0489 | 4 Down | '00001100101000' | 0.4602 |
| 5 Up | '11101011111111' | 0.20734 | 5 Down | '00010100101000' | 0.45581 |
| 6 Up | '11100111111111' | 0.0385 | 6 Down | '00011000101000' | 0.4929 |
| 8 Up | '11100010111111' | 0.0495 | 8 Down | '00011101101000' | 0.51367 |
| a Up | '11100011101111' | 0.0757 | a Down | '00011100111000' | 0.53085 |
| 25 Up | '10101011111111' | 0.31456 | 25 Down | '01010100101000' | 0.25228 |
| 9b=01 12468a | | | | | |
| | '11100011010111' | Performed | | '00011100000000' | Performed |
| 1 Up | '01100011010111' | 0.13188 | 1 Down | '10011100000000' | 0.46768 |
| 2 Up | '10100011010111' | 0.20236 | 2 Down | '01011100000000' | 0.43374 |
| 4 Up | '11110011010111' | 0.0536 | 4 Down | '00001100000000' | 0.46438 |
| 5 Up | '11101011010111' | 0.20129 | 5 Down | '00010100000000' | 0.42683 |
| 6 Up | '11100111010111' | 0.0433 | 6 Down | '00011000000000' | 0.49527 |
| 8 Up | '11100010010111' | 0.05421 | 8 Down | '00011101000000' | 0.50636 |
| a Up | '11100011000111' | 0.0901 | a Down | '00011100010000' | 0.54814 |
| 2a Up | '10100011000111' | 0.25004 | 2a Down | '01011100010000' | 0.41382 |
| 9b=00 12468a | | | | | |
| | '11100011011111' | Performed | | '00011100001000' | Performed |
| 1 Up | '01100011011111' | Performed | 1 Down | '10011100001000' | 0.3275 |
| 2 Up | '10100011011111' | Performed | 2 Down | '01011100001000' | 0.30318 |
| 4 Up | '11110011011111' | Performed | 4 Down | '00001100001000' | 0.29458 |
| 5 Up | '11101011011111' | Performed | 5 Down | '00010100001000' | 0.29807 |
| 6 Up | '11100111011111' | Performed | 6 Down | '00011000001000' | 0.32151 |
| 8 Up | '11100010011111' | Performed | 8 Down | '00011101001000' | 0.3485 |
| a Up | '11100011001111' | Performed | a Down | '00011100011000' | 0.35704 |
| 2a Up | '10100011001111' | 0.13638 | 2a Down | '01011100011000' | 0.29882 |
| 9b=11 125=111 | | | | | |
| | '00101011110111' | Performed | | '00011100100000' | Performed |
| 4 Up | '00111011110111' | 0.53376 | 4 Down | '00001100100000' | Performed |
| a Up | '00101011100111' | 0.50138 | a Down | '00011100110000' | Performed |

Table 126 Continued

| Low End Variables State | | ΔK_{eff} | High End Variables State | | ΔK_{eff} |
|----------------------------|------------------|------------------|-----------------------------|------------------|------------------|
| 9b=11 125=011 | | | | | |
| | '10101011110111' | Performed | | '10011100100000' | Performed |
| 4 Up | '10111011110111' | 0.48454 | 4 Down | '10001100100000' | 0.52009 |
| a Up | '10101011100111' | 0.45181 | a Down | '10011100110000' | 0.58752 |
| 9b=11 125=101 | | | | | |
| | '01101011110111' | Performed | | '01011100100000' | Performed |
| 4 Up | '01111011110111' | 0.46448 | 4 Down | '01001100100000' | 0.48409 |
| a Up | '01101011100111' | 0.42997 | a Down | '01011100110000' | 0.53842 |
| 9b=11 125=110 | | | | | |
| | '00100011110111' | Performed | | '00010100100000' | Performed |
| 4 Up | '00110011110111' | 0.45156 | 4 Down | '00000100100000' | 0.47146 |
| a Up | '00100011100111' | 0.41967 | a Down | '00010100110000' | 0.52667 |
| 9b=11 125=001 | | | | | |
| | '11101011110111' | Performed | | '11011100100000' | Performed |
| 4 Up | '11111011110111' | 0.40884 | 4 Down | '11001100100000' | 0.46863 |
| a Up | '11101011100111' | 0.39802 | a Down | '11011100110000' | 0.53175 |
| 9b=11 125=010 | | | | | |
| | '10100011110111' | Performed | | '10010100100000' | Performed |
| 4 Up | '10110011110111' | 0.39914 | 4 Down | '10000100100000' | 0.45431 |
| a Up | '10100011100111' | 0.38499 | a Down | '10010100110000' | 0.51673 |
| 9b=11 125=100 | | | | | |
| | '01100011110111' | Performed | | '01010100100000' | Performed |
| 4 Up | '01110011110111' | 0.25005 | 4 Down | '01000100100000' | 0.41652 |
| a Up | '01100011100111' | 0.28681 | a Down | '01010100110000' | 0.46024 |
| 9b=11 125=000 | | | | | |
| | '11100011110111' | Performed | | '11010100100000' | Performed |
| 4 Up | '11110011110111' | Performed | 4 Down | '11000100100000' | 0.23378 |
| a Up | '11100011100111' | Performed | a Down | '11010100110000' | 0.21243 |
| 9b=10 25=11 | | | | | |
| | '10101011111111' | Performed | | '00011100101000' | Performed |
| 4 Up | '10111011111111' | 0.38991 | 4 Down | '00001100101000' | Performed |
| a Up | '10101011101111' | 0.34974 | a Down | '00011100111000' | Performed |
| 1 Up | '00101011111111' | 0.34639 | 1 Down | '10011100101000' | Performed |

Table 126 Continued

| Low End Variables State | | ΔK_{eff} | High End Variables State | | ΔK_{eff} |
|----------------------------|------------------|------------------|-----------------------------|------------------|------------------|
| 9b=10 25=01 | | | | | |
| | '11101011111111' | Performed | | '01011100101000' | Performed |
| 4 Up | '11111011111111' | 0.22651 | 4 Down | '01001100101000' | 0.39486 |
| a Up | '11101011101111' | 0.24015 | a Down | '01011100111000' | 0.4547 |
| 1 Up | '01101011111111' | 0.29136 | 1 Down | '11011100101000' | 0.38381 |
| 9b=10 25=10 | | | | | |
| | '10100011111111' | Performed | | '00010100101000' | Performed |
| 4 Up | '10110011111111' | 0.22869 | 4 Down | '00000100101000' | 0.38723 |
| a Up | '10100011101111' | 0.23493 | a Down | '00010100111000' | 0.44494 |
| 1 Up | '00100011111111' | 0.28721 | 1 Down | '10010100101000' | 0.37479 |
| 9b=10 25=00 | | | | | |
| | '11100011111111' | Performed | | '01010100101000' | Performed |
| 4 Up | '11110011111111' | Performed | 4 Down | '01000100101000' | 0.25176 |
| a Up | '11100011101111' | Performed | a Down | '01010100111000' | 0.2591 |
| 1 Up | '01100011111111' | Performed | 1 Down | '11010100101000' | 0.1249 |
| 1a Up | '01100011101111' | 0.15834 | 1a Down | '11010100111000' | 0.1108 |
| 9b=01 2a=11 | | | | | |
| | '10100011000111' | Performed | | '00011100000000' | Performed |
| 4 Up | '10110011000111' | 0.28492 | 4 Down | '00001100000000' | Performed |
| 5 Up | '10101011000111' | 0.31775 | 5 Down | '00010100000000' | Performed |
| 45 Up | '10111011000111' | 0.3862 | 45 Down | '00000100000000' | 0.35639 |
| 9b=01 2a=01 | | | | | |
| | '11100011000111' | Performed | | '01011100000000' | Performed |
| 4 Up | '11110011000111' | 0.1074 | 4 Down | '01001100000000' | 0.36471 |
| 5 Up | '11101011000111' | 0.25551 | 5 Down | '01010100000000' | 0.30724 |
| 45 Up | '11111011000111' | 0.29268 | 45 Down | '01000100000000' | 0.28233 |
| 9b=01 2a=10 | | | | | |
| | '10100011010111' | Performed | | '00011100010000' | Performed |
| 4 Up | '10110011010111' | 0.24727 | 4 Down | '00001100010000' | 0.47326 |
| 5 Up | '10101011010111' | 0.26971 | 5 Down | '00010100010000' | 0.40254 |
| 9b=01 2a=00 | | | | | |
| | '11100011010111' | Performed | | '01011100010000' | Performed |
| 4 Up | '11110011010111' | Performed | 4 Down | '01001100010000' | 0.33452 |
| 5 Up | '11101011010111' | Performed | 5 Down | '01010100010000' | 0.30986 |
| 45 Up | '11111011010111' | 0.24707 | 45 Down | '01000100010000' | 0.26848 |

Table 127Application of a Sixteen FFD to ΔK_{eff}

| Variables State | ΔK_{eff} | Variables State | ΔK_{eff} | Variables State | ΔK_{eff} |
|------------------|------------------|------------------|------------------|------------------|------------------|
| '00001001011011' | 0.20857 | '01101010101010' | 0.34693 | '10110011001001' | 0.17811 |
| '00010110100011' | 0.52095 | '01110101010010' | 0.21982 | '11001111000000' | 0.31502 |
| '00100110011100' | 0.22268 | '10000011110110' | 0.37658 | '11010000111000' | 0.1014 |
| '00111001100100' | 0.60942 | '10011100001110' | 0.30728 | '11100000000111' | 0.11722 |
| '01000101101101' | 0.18267 | '10101100110001' | 0.45336 | '11111111111111' | 0.25298 |
| '01011010010101' | 0.3109 | | | | |

APPENDIX H

1. Parameters of the Six Performance Characteristics Sorted by Their Maximum

Effect

Table 128

Highest 100 Parameters of the Six Performance Characteristics

| Index | K_{eff} | Index | RFS | Index | α_d |
|---------|-----------|-------|-----------|-------|------------|
| '0' | 1.079800 | '0' | 0.720680 | '0' | 0.204740 |
| '9' | 0.058541 | '9' | -0.100970 | '9' | -0.030093 |
| '4' | 0.037155 | 'b' | 0.096129 | '1' | -0.020978 |
| 'a' | -0.028103 | '4' | 0.038886 | '2' | -0.019441 |
| '8' | 0.022566 | '9b' | -0.032108 | '8' | -0.010997 |
| 'b' | -0.021556 | '6' | -0.017665 | 'b' | 0.010171 |
| '2' | 0.021316 | 'a' | -0.014440 | '6' | -0.006114 |
| '1' | 0.020297 | '2' | 0.010563 | '4' | -0.006089 |
| 'e' | -0.019189 | '49' | -0.008083 | 'e' | 0.003952 |
| '9b' | 0.017328 | '8' | 0.008029 | 'a' | 0.003388 |
| '6' | 0.015057 | '4b' | 0.007949 | '9b' | 0.003032 |
| 'ae' | -0.005875 | 'ab' | -0.006301 | '18' | 0.002863 |
| 'ab' | 0.005753 | '1' | 0.005694 | 'd' | 0.002607 |
| '4b' | -0.004397 | '29' | -0.005648 | '8b' | 0.002153 |
| '7' | -0.003622 | '2b' | 0.005590 | '4b' | -0.001939 |
| '69' | -0.003361 | '9a' | 0.005568 | '6b' | 0.001904 |
| '49' | 0.002970 | 'e' | 0.004872 | 'be' | -0.001832 |
| '18' | -0.002953 | '19' | -0.004641 | '7' | 0.001800 |
| 'd' | -0.002916 | '8b' | 0.004148 | '1b' | 0.001745 |
| '12' | 0.002647 | '89' | -0.003898 | 'ab' | -0.001723 |
| '9e' | 0.002539 | '29b' | -0.003306 | '2b' | 0.001662 |
| '8b' | -0.002472 | 'd' | 0.003285 | '3' | -0.001325 |
| '69b' | 0.002282 | '1b' | 0.003260 | '19' | 0.001195 |
| '8e' | 0.001744 | '69' | 0.003148 | '12' | 0.001087 |
| '469be' | 0.001656 | '6b' | -0.003119 | '16' | 0.001078 |
| 'be' | 0.001216 | '9ab' | 0.002918 | 'ae' | 0.000967 |
| '46e' | 0.001100 | 'c' | 0.002897 | '49' | 0.000953 |
| 'c' | -0.001091 | '19b' | -0.002295 | '28' | 0.000948 |

Table 128 Continued

| Index | K_{eff} | Index | RFS | Index | α_d |
|--------|-----------|---------|-----------|---------|------------|
| '469e' | -0.001084 | '89b' | -0.002176 | 'abe' | -0.000946 |
| '6b' | 0.001033 | '24' | 0.001915 | '49b' | -0.000920 |
| '12e' | 0.000969 | '9d' | -0.001755 | '89' | 0.000903 |
| '6e' | 0.000936 | '7' | -0.001475 | 'c' | 0.000710 |
| '6be' | -0.000823 | '26' | -0.001406 | '19b' | -0.000705 |
| 'bd' | -0.000791 | '49b' | -0.001404 | '8a' | -0.000670 |
| '49b' | 0.000741 | '3' | 0.001267 | '8e' | -0.000660 |
| '1a' | 0.000716 | '46' | -0.001243 | '14' | 0.000636 |
| '9be' | -0.000684 | '249' | -0.001179 | '89b' | -0.000576 |
| '12b' | -0.000541 | '24b' | 0.001086 | '8ae' | -0.000498 |
| '469' | -0.000534 | 'bd' | 0.001012 | '8abe' | 0.000493 |
| '4be' | -0.000526 | '269' | 0.000809 | '48' | 0.000490 |
| '7b' | -0.000503 | '26b' | -0.000797 | '9a' | -0.000460 |
| '46be' | -0.000483 | '16' | -0.000622 | '9ab' | 0.000440 |
| '49be' | -0.000412 | '14' | 0.000616 | '68' | 0.000421 |
| 'bc' | -0.000403 | '169' | 0.000605 | '1489b' | 0.000403 |
| '46' | 0.000394 | '16b' | -0.000605 | '1489' | -0.000401 |
| '1c' | 0.000391 | '169b' | 0.000605 | '1a' | -0.000359 |
| '1b' | -0.000284 | '14b' | 0.000595 | '8be' | 0.000290 |
| '69be' | 0.000232 | '149' | -0.000595 | '149b' | 0.000286 |
| '13' | 0.000222 | '149b' | -0.000590 | '149' | -0.000283 |
| '4e' | 0.000211 | '2d' | 0.000411 | '1abe' | 0.000281 |
| '18b' | 0.000209 | '29d' | -0.000397 | '1ae' | -0.000277 |
| 'ce' | -0.000206 | '2bd' | 0.000393 | '4a' | -0.000270 |
| '1bc' | 0.000178 | '29bd' | -0.000392 | '4e' | -0.000267 |
| '46b' | -0.000149 | '249b' | -0.000371 | '89a' | 0.000262 |
| '17b' | -0.000147 | '469b' | -0.000262 | '89ab' | -0.000256 |
| '1e' | -0.000138 | '1d' | 0.000243 | '148b' | -0.000243 |
| '13b' | -0.000122 | '19bd' | -0.000239 | '489' | -0.000238 |
| '1ae' | 0.000113 | '19d' | -0.000236 | '489b' | 0.000236 |
| '2e' | -0.000106 | '1bd' | 0.000236 | '4be' | 0.000232 |
| '69e' | 0.000104 | '126b' | -0.000221 | '14b' | -0.000187 |
| '3b' | 0.000097 | '1269' | 0.000221 | '8ab' | 0.000175 |
| '49e' | 0.000083 | '269b' | 0.000220 | '18abe' | 0.000171 |
| '18e' | 0.000069 | '1269b' | 0.000216 | '18ae' | -0.000171 |
| '3e' | -0.000063 | '126' | -0.000215 | '16b' | -0.000170 |
| 'de' | 0.000056 | '69b' | -0.000206 | '148' | -0.000168 |
| '1de' | 0.000056 | '49bd' | 0.000168 | '18b' | -0.000158 |

Table 128 Continued

| Index | K_{eff} | Index | RFS | Index | α_d |
|---------|-----------|---------|-----------|----------|------------|
| '2b' | -0.000047 | '4bd' | -0.000164 | '4ae' | -0.000154 |
| '1ab' | 0.000034 | '49d' | 0.000163 | '4abe' | 0.000153 |
| '469b' | -0.000034 | '12' | 0.000157 | '189' | -0.000137 |
| '13e' | -0.000025 | '9bd' | -0.000156 | '19ab' | -0.000131 |
| '17e' | -0.000025 | '1249' | -0.000153 | '14be' | -0.000130 |
| '1ce' | 0.000019 | '124b' | 0.000151 | '14e' | 0.000128 |
| '1d' | 0.000016 | '1249b' | -0.000148 | '19a' | 0.000127 |
| '7e' | -0.000013 | '124' | 0.000146 | '1be' | 0.000121 |
| '1bd' | -0.000009 | '4d' | -0.000142 | '68b' | -0.000115 |
| '3' | 0.000003 | '129' | -0.000141 | '48ab' | 0.000109 |
| '17' | -0.000003 | '12b' | 0.000139 | '168' | -0.000108 |
| '234' | 0.000000 | '129b' | -0.000137 | '1ab' | 0.000103 |
| '124' | 0.000000 | '129d' | -0.000102 | '1489a' | -0.000096 |
| '34' | 0.000000 | '12bd' | 0.000102 | '1489ab' | 0.000094 |
| '1245' | 0.000000 | '146b' | -0.000098 | '168b' | -0.000089 |
| '235' | 0.000000 | '1469' | 0.000098 | '489ab' | -0.000089 |
| '356' | 0.000000 | '12d' | 0.000095 | '48b' | -0.000087 |
| '125' | 0.000000 | '129bd' | -0.000095 | '489a' | 0.000086 |
| '236' | 0.000000 | '1469b' | 0.000091 | '48a' | 0.000070 |
| '145' | 0.000000 | '146' | -0.000090 | '14ae' | 0.000069 |
| '23' | 0.000000 | '246b' | -0.000058 | '14abe' | -0.000066 |
| '2346' | 0.000000 | '2469' | -0.000056 | '48e' | 0.000065 |
| '24' | 0.000000 | '2c' | 0.000046 | '48be' | -0.000063 |
| '14' | 0.000000 | '27' | -0.000045 | '128b' | -0.000061 |
| '134' | 0.000000 | '4c' | 0.000045 | '4ab' | 0.000054 |
| '1235' | 0.000000 | '2e' | 0.000045 | '12b' | -0.000053 |
| '12346' | 0.000000 | '23' | 0.000045 | '189ab' | -0.000052 |
| '45' | 0.000000 | '47' | -0.000045 | '48ae' | 0.000051 |
| '156' | 0.000000 | '4e' | 0.000045 | '189a' | 0.000050 |
| '3457' | 0.000000 | '34' | 0.000045 | '48abe' | -0.000048 |
| '2567' | 0.000000 | '25' | 0.000044 | '18a' | -0.000047 |
| '1346' | 0.000000 | '45' | 0.000044 | '14ab' | -0.000044 |
| '1256' | 0.000000 | '2469b' | -0.000040 | '148be' | -0.000044 |
| '0' | 0.031095 | '0' | 2.3547 | '0' | 0.311170 |
| '9' | -0.006701 | '9' | 0.083027 | '5' | 0.059987 |
| 'b' | 0.004526 | '4' | 0.075333 | '2' | -0.057840 |

Table 128 Continued

| Index | K_{eff} | Index | RFS | Index | α_d |
|--------|-----------|-------|----------|---------|------------|
| '7' | -0.003556 | '2' | 0.057365 | 'b' | -0.057507 |
| '9b' | -0.001358 | '8' | 0.054852 | '9' | 0.055426 |
| '6' | -0.001284 | '1' | 0.049249 | '1' | -0.034369 |
| '4' | -0.001203 | 'a' | -0.04606 | '4' | 0.024572 |
| '79' | 0.000818 | 'e' | -0.04172 | '25' | 0.013595 |
| 'e' | 0.000691 | '2a' | -0.03936 | '59' | 0.013420 |
| '7b' | -0.000566 | '18' | -0.03193 | 'a' | -0.012138 |
| 'd' | 0.000467 | '8a' | 0.030531 | '29' | -0.011709 |
| '2' | -0.000304 | 'ae' | -0.02614 | '8' | -0.011616 |
| '49' | 0.000234 | 'b' | -0.02464 | '6' | 0.010940 |
| '8' | 0.000230 | '6' | 0.022713 | 'e' | -0.009489 |
| 'a' | -0.000225 | '9b' | 0.021151 | '259' | 0.009118 |
| '1' | -0.000221 | '2e' | -0.02108 | '125b' | -0.008198 |
| '4b' | -0.000216 | '2b' | 0.020579 | '1259b' | -0.008174 |
| '9e' | -0.000187 | '28' | 0.017501 | '8b' | 0.007937 |
| '79b' | 0.000182 | '14' | 0.016095 | '125' | 0.007086 |
| '6b' | -0.000179 | '24' | 0.015694 | '1259' | 0.006988 |
| 'bd' | 0.000176 | '19' | -0.01496 | '29b' | -0.006924 |
| '9d' | -0.000149 | '18a' | -0.01431 | '1b' | 0.006635 |
| '67' | 0.000145 | '29' | -0.01277 | '5b' | -0.006550 |
| 'c' | 0.000137 | '49' | 0.011405 | '12' | -0.006442 |
| 'be' | 0.000136 | 'ab' | 0.010974 | '59b' | 0.006353 |
| '47' | 0.000128 | '189' | -0.01051 | '129' | -0.006349 |
| '69' | 0.000113 | '169' | -0.00942 | '15' | 0.006106 |
| '49b' | 0.000093 | '7e' | 0.009111 | '159' | 0.006020 |
| '7e' | -0.000085 | '46' | 0.009005 | '19b' | -0.005850 |
| '3' | -0.000079 | '2ae' | -0.00884 | '45' | 0.005656 |
| '9be' | -0.000074 | '1ae' | -0.00868 | '2b' | 0.005454 |
| '9bd' | -0.000053 | '68' | 0.008675 | '24' | -0.005386 |
| '7d' | -0.000048 | '7d' | 0.008652 | '89' | -0.005383 |
| '79bd' | -0.000035 | '8e' | 0.008495 | '19' | -0.005320 |
| '479b' | 0.000033 | '4a' | 0.00828 | '9b' | -0.005136 |
| '79d' | 0.000031 | '16' | -0.00797 | '25b' | -0.004579 |
| '79be' | -0.000031 | '7bc' | -0.00792 | '4b' | -0.003603 |
| '79e' | 0.000031 | '28b' | -0.00786 | '245' | 0.003286 |
| '7be' | -0.000028 | '9ab' | -0.00781 | '459' | 0.003211 |
| '679b' | 0.000015 | '28a' | 0.007689 | '249' | -0.003149 |
| '679' | -0.000013 | '18b' | 0.007611 | 'ab' | 0.002775 |

Table 128 Continued

| Index | K_{eff} | Index | RFS | Index | α_d |
|-------|-----------|--------|----------|---------|------------|
| '7bd' | -0.000011 | '1d' | -0.00751 | '245b' | -0.002552 |
| '69b' | -0.000009 | '6b' | 0.007143 | '7' | -0.002474 |
| '47b' | 0.000007 | '38' | -0.00684 | 'd' | -0.002147 |
| '5' | 0.000002 | '12' | 0.00677 | '6b' | -0.001905 |
| '125' | 0.000002 | '79d' | 0.006448 | '2459' | 0.001882 |
| '12a' | -0.000002 | '1a' | -0.0064 | '45b' | -0.001758 |
| '12c' | -0.000002 | '9bd' | 0.006386 | '89b' | 0.001725 |
| '135' | 0.000002 | '138' | -0.00607 | '24b' | 0.001563 |
| '158' | -0.000002 | '268' | 0.006034 | '25a' | -0.001494 |
| '15a' | -0.000002 | 'd' | -0.00603 | '159b' | -0.001479 |
| '15c' | -0.000002 | '7de' | 0.005916 | '15b' | -0.001469 |
| '1ac' | 0.000002 | '26' | -0.00576 | '129b' | 0.001440 |
| '235' | 0.000002 | '349' | -0.00575 | '49' | 0.001440 |
| '23a' | -0.000002 | '8b' | -0.0056 | '12b' | 0.001422 |
| '23c' | -0.000002 | '6c' | 0.005583 | '2a' | -0.001410 |
| '258' | -0.000002 | '4e' | 0.005477 | '25ab' | 0.001223 |
| '25a' | -0.000002 | '13' | 0.00547 | '49b' | -0.001107 |
| '25c' | -0.000002 | '48c' | -0.0054 | '2459b' | -0.001073 |
| '28c' | 0.000002 | '3be' | -0.00527 | '29a' | 0.001038 |
| '2ac' | 0.000002 | '9d' | 0.005222 | '125a' | 0.000794 |
| '358' | -0.000002 | 'bc' | -0.00509 | '9a' | -0.000761 |
| '35a' | -0.000002 | '38b' | 0.005066 | '249b' | -0.000761 |
| '35c' | -0.000002 | '13d' | -0.005 | '459b' | 0.000757 |
| '3ac' | 0.000002 | '19a' | -0.00497 | '29ab' | -0.000755 |
| '58a' | 0.000002 | '148' | -0.00467 | '1259a' | 0.000722 |
| '58c' | 0.000002 | '67' | 0.004628 | '149' | -0.000718 |
| '5ac' | 0.000002 | '8ab' | -0.0046 | '14b' | 0.000690 |
| '123' | 0.000002 | '12e' | 0.004588 | '14' | -0.000686 |
| '128' | -0.000002 | '234' | -0.00454 | '149b' | 0.000656 |
| '138' | -0.000002 | '19e' | -0.00454 | '1249' | -0.000601 |
| '13a' | -0.000002 | '14d' | 0.004529 | '124b' | 0.000601 |
| '13c' | -0.000002 | '178' | -0.00448 | '124' | -0.000568 |
| '18a' | 0.000002 | '9e' | 0.004358 | '1249b' | 0.000559 |
| '18c' | 0.000002 | '7ac' | -0.00433 | '69' | 0.000549 |
| '238' | -0.000002 | '1268' | 0.004281 | '1a' | -0.000520 |
| '28a' | 0.000002 | '78e' | -0.00424 | '1459' | 0.000516 |
| '38a' | 0.000002 | '6ce' | -0.00417 | '145b' | -0.000512 |
| '38c' | 0.000002 | 'cd' | 0.004101 | '259a' | -0.000481 |

Table 128 Continued

| Index | K_{eff} | Index | RFS | Index | α_d |
|---------|-----------|--------|----------|----------|------------|
| '8ac' | -0.000002 | '39a' | -0.00402 | '59ab' | 0.000477 |
| '238ac' | -0.000002 | '467' | 0.004016 | '145' | 0.000461 |
| '12358' | -0.000002 | '8bc' | -0.00401 | '1459b' | -0.000460 |
| '1235a' | -0.000002 | '39b' | -0.00394 | '19a' | -0.000412 |
| '1235c' | -0.000002 | '68a' | 0.003909 | '1259ab' | -0.000386 |
| '1238a' | 0.000002 | '148b' | 0.003856 | '24a' | 0.000372 |
| '123ac' | 0.000002 | '128c' | 0.003794 | '249a' | -0.000365 |
| '1258a' | 0.000002 | '1e' | -0.00377 | '125ab' | -0.000364 |
| '1258c' | 0.000002 | '78a' | -0.00372 | '4a' | 0.000327 |
| '125ac' | 0.000002 | '129' | -0.00371 | '249ab' | 0.000319 |
| '128ac' | -0.000002 | '4b' | -0.0037 | '24ab' | -0.000314 |
| '1358a' | 0.000002 | '23d' | 0.00368 | '59a' | -0.000312 |
| '1358c' | 0.000002 | '89' | 0.003629 | '45a' | -0.000292 |
| '135ac' | 0.000002 | '69' | -0.00356 | '49a' | -0.000291 |
| '138ac' | -0.000002 | '149b' | -0.00354 | '5ab' | 0.000290 |
| '158ac' | -0.000002 | '48d' | -0.00353 | '459a' | 0.000286 |
| '2358a' | 0.000002 | '14e' | -0.0035 | '12ab' | -0.000274 |
| '2358c' | 0.000002 | '139' | -0.00349 | '4ab' | -0.000269 |
| '235ac' | 0.000002 | '26a' | -0.00348 | '49ab' | 0.000260 |
| '258ac' | -0.000002 | '16e' | -0.00346 | '245ab' | -0.000257 |

APPENDIX I

1. Quantification of Monotonic Knowledge Buildup

The methods used in the dissertation assumed monotonic behavior of variables. This assumption was valid often, but since it is not always the case, methods to quantify the monotonic knowledge was investigated in this section. Every node of the tree of experiments combines knowledge from other nodes with an unknown probability of knowledge validity. This validity depends on the combined degree of confidence in the monotonic behavior of the link between every node and all other nodes. A survey of potential means of probabilities aggregation to quantify the monotonic behavior is introduced in this section.

Fisher method is one of the common approaches used to combine probability distributions [93]. However, it is biased toward the rejection hypothesis as described in [94]. This implies that if two nodes provide consistent or supporting information on a range limit with two different probabilities, p-values, e.g. 1 and 0.2 p-values, the final combined probability will be biased toward the lower probability, 0.33 for the example. Stouffer's method sums the Z-scores of contributing nodes instead [93], and is defined as:

$$Z_S = \sum_{i=1}^k Z_i / \sqrt{k} \quad (141)$$

where k is the number of combined nodes probabilities. This was modified by adding weights:

$$Z_w = \sum_{i=1}^k w_i Z_i / \sqrt{\sum_{i=1}^k w_i^2} \quad (142)$$

A comparison of the weighted Z method and Fisher's method is performed in [94], and concluded that the weighted Z method is more powerful and precise. As a result, the Stouffer's method is evaluated next.

To illustrate Stouffer's method, a three nodes example is used in the dissertation context. Assuming the upper range limit of node A is set by a value of 120, and the lower range limit is set by two connected lower nodes, B and C, then if B sets A's lower limit by $y_B=40$ with a probability of 0.99865, and C sets A's lower limit by $y_C=30$ with a probability of 0.97725, the constructed two normal distributions will overlap as shown in Figure 89. Assuming that the point of interest is 40 in this example, the probability that the lower limit will fall between 40 and 120 is 0.99865 from B and 0.84134 from C. The Z scores for B and C are 3 and 1 respectively. Using Stouffer's non-weighted formula, the resulting Z score is 2.828427. This results in a lower limit of 40 with a combined probability of 0.99766 . This is lower than the B resulting probability of 0.99865, which indicates that confidence was lost as the two data were considered conflicting rather than consistent. This behavior is not desired in the quantification of monotonic behavior knowledge. If both curves were identical, the combined Z score of any point will consist of $2/\sqrt{2}$, which is higher than the individual Z, and results in a smaller probability. This again demonstrates that this approach is not designed for consistent results. As the number of identical contributing nodes increases, Stouffer's method Z value increases as shown in Figure 90. Using weights was found to reduce the severity of the unwanted confidence loss, but will not result in the logically expected increase of probability.

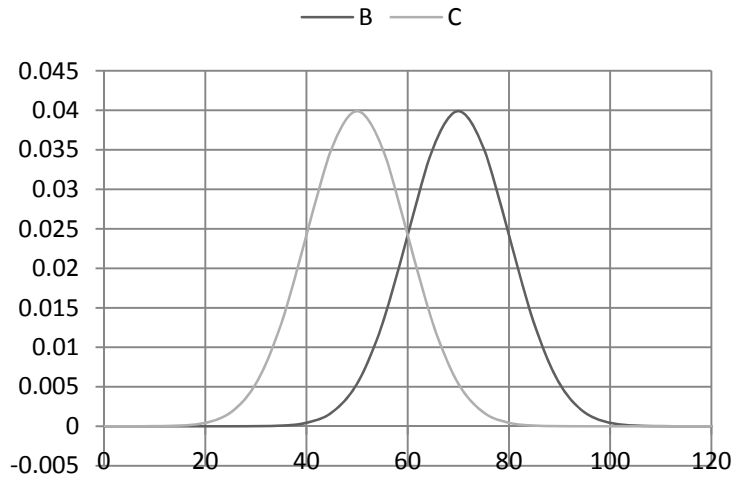


Figure 89. Example of Two Normal Distributions of a Range's Lower Limit

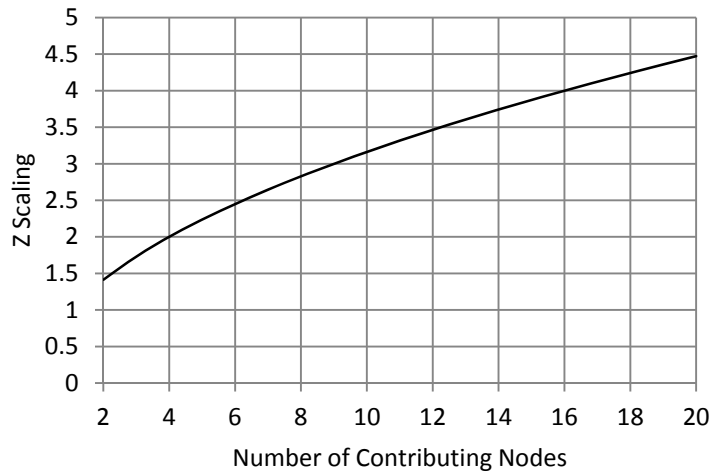


Figure 90. Effect of the Number of Contributing Nodes on the Combined Z Value

The use of probabilities indicate certain knowledge while in some cases, knowledge does not exist. For example, a probability of zero indicates that an event will never occur, while a knowledge value of zero indicates that the probability is unknown. It

could be zero and it could be any value up to one. If the knowledge of some behavior is 0.5, this indicates that 50% of the time this event will occur and there is no information on the other 50%. This implies a probability $P \geq 0.5$. Knowledge is aggregated while probability is combined. Knowledge behaves in a similar manner to an OR operator of probabilities. It is enough to have one sufficient knowledge source out of a number of knowledge sources to have sufficient knowledge. Knowledge (K) can be aggregated for two knowledge sources, A and B, using:

$$K(A, B) = K(A) + K(B) - K(A)K(B) \quad (143)$$

The search for the concept of knowledge led to the introduced and well-defined theory of belief and evidence. Evidence theory is a science that has been formalized in a mathematical approach by Dempster and Shafer in the 60s and 70s (see [95] for a summary). The most important addition of this concept is that belief splits the probability range to confirmatory, dis-confirmatory and uncertain regions, instead of the Bayesian approach of confirmatory and non-confirmatory regions. The example of Fisher or Stouffer methods is a Bayesian approach to two independent events and not an evidence driven approach. It deals with portions of samples that an event occurred, and is usually associated with randomness behavior of sampling.

The method of the Dempster Shafer is widely explained in literature. One of its main disadvantages is that the computational requirements increase exponentially as the number of evidence contributors increases. [96] addressed this issue, and reduced the exponential increase of computational power to a linear increase through introducing an assumption. The study of [97] used an assumption to shrink the tree into categories of

interest. For example, if out of A, B, C and D variables effects and interactions, AB and CD, their children, A, B, C and D, and their grandparent, ABCD, are of interest, the new tree will consist of three levels: The first level will have ABCD, the second level will have AB and CD, and the third level will have A, B, C and D. The method generates the total belief of the new tree and its nodes. This approach is useful in categorical problems, in which a tree search is needed with certain interest in some nodes of the tree. In the dissertation's methods, it is desired to know if all nodes' links are monotonic, thus aggregating the belief in subsets of the complete variables' tree is not desired.

Methods other than Dempster and Shafer were explored. Entropy [98] can be used to address the same issue, and is widely addressed in literature. This concept utilized the historical results in prediction of future results. The Certainty Factor, CF is another approach that was introduced to address a similar problem in medical diagnostics applications. This assumes that the increase of belief in one variable should be reflected in a decrease of belief in other variables as described in [97]. Several combinations rules for Dempster Shafer are surveyed in in [99]. From this section, it is concluded that the Dempster Shafer's method using the Dempster rule should be used for the quantification of the monotonic behavior of the dissertation's methods.