METHODS AND ANALYSIS FOR RECOVERY LOGISTICS NETWORKS WITH UNCERTAINTY AND CHANNEL SELECTION CONSIDERATIONS

A Dissertation

by

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Submitted to the Office of Graduate and Professional Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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December 2014

Major Subject: Industrial Engineering

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ABSTRACT

In this dissertation, we develop models and methodologies for effective design and efficient operation of product recovery logistics networks. Recovery networks, employed for recycle-reuse-refurbish-remanufacture purposes, constitute an ever-expanding portion of supply chain networks. For such activities to make business-sense, it is important that the logistical decisions associated with designing and operating underlying networks are made carefully. With this main motivation, we focus on two fundamental problems.

First, we consider a generic Closed-Loop Supply Chain (CLSC) network setting under demand and return uncertainty and provide a new model and an efficient solution approach for the associated network design problem. Consideration of uncertainties and their impact on the CLSC network design is a largely ignored area in the literature, thus, this work contributes to closing this gap, in both modeling and solution methodology contexts, as well as in analysis.

Second, we consider the specific case of commercial returns, which is quite common in today's business climate, given the generous return policies provided by electronics and department stores as well as retail superstores. In this setting, for operational efficiency and financial effectiveness, it is important for providers to best determine appropriate return channels, i.e., the return channel selection, for commercial products whose values decrease over time. Return channel selection for commercial products is also a largely ignored area in the literature. We first address this problem from an operational efficiency perspective given an underlying network of facilities. In the related models and analysis, we introduce and capture the concepts of channel selection dependence on product and logistics network characteristics. Later, recognizing that the design of an underlying network may be under the control of the provider, we take an integrated design and operation perspective and incorporate the logistics network design into the model to further study dependence of channel selection on network characteristics. In addition to new models and analysis for commercial return logistics, our contributions also include the development of efficient solution algorithms with measurable solution quality.

We introduce the problems of interest and their context in today's business environment in the first chapter. In the second chapter of the dissertation, we develop a two-stage stochastic programming model for the generic CLSC network design problem under demand and return uncertainty, represented by a set of scenarios. For the model's solution, we develop a Benders Decomposition (BD) approach that significantly improves computational efficiency via surrogate constraints, strengthened Benders cuts, multiple Benders cuts, and mean value scenario based lower bounding inequalities. In the third chapter, we develop models for the channel selection problem for commercial products under time-value consideration. Based on this model, we analyze the optimal return channel selection strategies under varying underlying logistics network and product characteristics. For this purpose, we utilize real geographical data from the U.S. and product data for Hewlett Packard and Bosch. In the fourth chapter of this dissertation, we develop a Mixed Integer Linear Programming (MILP) model for integrated design and channel selection for commercial product returns under product time-value consideration. For the model's solution, we develop an efficient algorithm based on the Simulated Annealing (SA) approach, benchmarking the quality of solutions against the upper bound obtained by a Benders Decomposition approach. Using this model and the solution approach, we provide an extensive analysis of the relationship between recovery logistics network structure and product characteristics.

DEDICATION

I dedicate this dissertation to my parents.

ACKNOWLEDGMENTS

I would never have been able to finish my dissertation without the guidance of my committee members and support from my family and friends.

I would like to thank to my advisor, Dr. Üster, for his dedicated guidance during my Ph.D. studies. I have learned the field of supply chain network design as well as many optimization techniques from his lessons. I have also learned not only academic knowledge, but also research habits as a successful researcher. His enthusiastic attitude for research impressed and motivated me to be a better researcher.

I am also grateful to Dr. Çetinkaya, Dr. Curry, and Dr. Ketzenberg for serving as members of my advisory committee and for giving helpful advice. I obtained knowledge of inventory problems in a supply chain network via Dr. Çetinkaya's two courses (ISEN615 and ISEN617). Especially, Dr. Çetinkaya's previous research in the closed loop supply chain logistics area provided valuable knowledge for my dissertation work. Dr. Curry helped me to successfully adapt to the Ph.D. program. Dr. Curry suggested me to take the principles graduate courses which helped me become a more competitive Ph.D. student in industrial engineering major. I learned the principle of research methods from Dr. Ketzenberg's seminar class. In the INFO seminar class, Dr. Ketzenberg taught me how to develop and write a good research article. This lesson is very helpful for completing this dissertation. I also would like to thank to Dr. Daniel Guide, who is a faculty member at Pennsylvania State University, for providing me a valuable research idea that motivated me to start studying channel selection issues.

Lastly, I would like to thank to my family and friends. Without their enormous support, this dissertation would not be possible.

NOMENCLATURE

- BD Benders Decomposition
- B&C Branch and Cut
- CI Confidence Intervals
- CLSC Closed Loop Supply Chain
- DEP Deterministic Equivalent Problem
- EOL End Of Life
- EOU End Of Use
- EV Expected Value
- HP Hewlett Packard
- LP Linear Programming
- MILP Mixed Integer Linear Programming
- MIP Mixed Integer Programming
- MVT Marginal Value of Time
- RP Recourse Problem
- RSC Reverse Supply Chain
- SA Simulated Annealing
- SAA Sample Average Approximation
- VSS Value of Stochastic Solution

TABLE OF CONTENTS

	Pa	age
ABS	STRACT	ii
DEI	DICATION	iv
ACI	KNOWLEDGMENTS	v
NO	MENCLATURE	vi
TAI	BLE OF CONTENTS	vii
LIS'	T OF FIGURES	ix
LIS'	T OF TABLES	xi
1.	INTRODUCTION	1
-	 1.1 Motivations and Scope of the Dissertation	$\frac{2}{6}$
2.	CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN UNDER DE- MAND AND RETURN UNCERTAINTY	7
	 2.1 Literature Review	$ \begin{array}{r} 10 \\ 12 \\ 20 \\ 22 \\ 25 \\ 25 \\ 38 \\ 39 \\ 40 \\ 47 \\ 48 \\ 5. \end{array} $
4	2.4.4Analysis on Recovery Location and Rate2.5Conclusion	51 59
3.	CHANNEL SELECTION IN COMMERCIAL PRODUCT RECOVERY LOGISTICS NETWORK UNDER TIME-VALUE CONSIDERATIONS	62
	3.1Literature Review3.2Problem Definition and Assumptions	64 66

	3.3	Computational Analysis	75
		3.3.1 Preliminary Results	76
		3.3.2 Channel Selection Strategies based on Product Characteristics	80
		3.3.3 Channel Selection Strategies based on Logistics Network Char-	
		acteristics	84
		3.3.4 Channel Selection under Different Networks and Product Char-	
		acteristics	88
		3.3.5 Analysis of Presetting Channel Selections	91
	3.4	Conclusions	95
4.	NET	WORK DESIGN FOR COMMERCIAL PRODUCT RETURNS UN-	
	DEF	R TIME-VALUE CONSIDERATIONS	97
	4.1	Problem Definition and Assumptions	98
	4.2	Solution Methods	103
		4.2.1 Simulated Annealing Heuristic	104
		4.2.2 Benders Decomposition Method	109
	4.3	Computational Study	112
		4.3.1 Sensitivity Analysis of SA Parameters	113
		4.3.2 Performance of Heuristic Algorithm	115
	4.4	Case Study: HP Printer and Bosch Power Tool	117
		4.4.1 Sensitivity Analysis	125
	4.5	Conclusions	129
5.	CON	CLUSIONS AND FUTURE RESEARCH	131
	51	Conclusions and Future Research on a Generic CLSC Network Design	
	0.1	under Demand and Return Uncertainty	131
		5.1.1 Research Contributions	132
		5.1.2 Future Research	133
	5.2	Conclusions and Future Research on Channel Selection in Commercial	
	-	Product Returns Logistics Network under Time-Value Considerations	134
		5.2.1 Research Contributions	135
		5.2.2 Future Research	136
RF	TER	ENCES	137
- UL			

LIST OF FIGURES

FIGURE

Page

2.1	CLSC Network Structure	15
2.2	Geographical Distribution of 263 Largest Cities in the U.S	54
2.3	Bounds on Total CLSC Costs for 6 Case Settings for Varying Inspec- tion Costs	56
2.4	Active SF and CTR Locations for 6 Case Settings	58
3.1	Multi-channel Product Recovery Logistics Network Structure	71
3.2	Geographical Distribution of RFs, Centers, Retailers, and Customers in the U.S.	76
3.3	Channel Selection in HP Printer	77
3.4	Channel Selection in Bosch Power Tool	78
3.5	Channel Selections in Multi-products	80
3.6	Channel Selections under Different Disposal Rate	82
3.7	Channel Selections under Different Non-defective Rate	83
3.8	Channel Selection of HP and Bosch with less Travel Time \ldots .	84
3.9	Location of RFs, Centers, and Retailers under Different Geographic Scheme	85
3.10	Channel Selection of HP and Bosch under Different Networks	88
3.11	Optimal Solution Gap Percentage under Different Decay Value	93
3.12	Optimal Solution Gap under Different Non-defective and Disposal Rate	94
4.1	Multi-channel Product Recovery Logistics Network Structure with Lo- cation Decisions	99
4.2	A Sample Representation of the Feasible Solution	.04

4.3	Geographical Distribution of Potential RFs, Potential Centers, Retailers, and Customers in the U.S
4.4	Active RF and Center Locations in the HP printer 121
4.5	Active RF and Center Locations in the BOSCH Power Tool 122
4.6	Channel Selection for HP Printer (33% ND & 10% Disposal) under Different Location Costs
4.7	Channel Selection for Bosch Power Tool (0% ND & 10% Disposal) under Different Location Costs

LIST OF TABLES

TABLE

Page

2.1	Input Parameters for Test Classes
2.2	Distribution for Input Parameters
2.3	Computational Results for the Type 4 Benders Cut
2.4	Computational Results for the Type 1 - Type 3 Benders Cuts 43
2.5	Comparisons on Cut Strengthening with Type 4 Cuts 45
2.6	Runtimes for BD Algorithm with Type 4 Cuts and Varying Lower Bounding Inequalities
2.7	Comparison of Runtimes (seconds) for DEP (B&C) and BD $\ .$ 48
2.8	UB and LB Estimates (in 1000s) with 95% C.I. $\ldots \ldots \ldots \ldots \ldots 50$
3.1	Average Percentage of Selected Channel and Objective Value under Different Product and Network Characteristics
4.1	Problem Classes Used in Computational Testing
4.2	Distribution for Price, Demand, Non-defective and Disposal Rate Values.114
4.3	Solution Gaps and Algorithm Runtimes for Different SA Parameters . 115
4.4	Solution Gaps of CPLEX and Greedy Algorithm
4.5	Runtimes and Solution Gaps of SA and BD Algorithm
4.6	Number of Active RF and Center in the HP, Bosch, and Multi-productProblems.120
4.7	Number of Active RFs and Centers in HP and Bosch Solutions under Different Problem Settings
4.8	Average Percentage of Selected Channel and Objective Value under Different Location Costs (HP Printer)
4.9	Average Percentage of Selected Channel and Objective Value under Different Location Costs (Bosch Power Tool)

1. INTRODUCTION

Although recovery of used products has been implemented by companies over decades, only recently product recovery has received attention from practitioners and scholars. Of numerous reasons for growing attention on recovery activities, environmental and economical reasons are most essential. First, there is a growing concern about the environmental problems caused by industrial waste. According to Akçali et al. [1], more than 12 billion tons of industrial wastes are produced in the United States every year. Therefore, the disposal of industrial waste is becoming an important issue. Second, there is a significant economic potential in the product recovery business. On average, manufacturing companies generally spend 9% to 15%of total revenue on product return, which in turn may increase total sales by 5%, through implementing more effective recovery logistics [20]. Growing concern about the environmental problems and economic potential in the product recovery business have caused product recovery logistics to be an emerging issue in the logistics area and is, therefore, widely studied. Product recovery logistics can be classified into two categories: RSC and CLSC [1]. In the first part of the dissertation, we use a generic CLSC network structure under uncertainty to identify the performance of stochastic programming model.

Consumers generally return products for three main reasons, for which companies tend to have three different product recovery plans [22]. First is the commercial return which occurs right after purchase, mostly within 90 days of the purchase. Commercial return is the return of the product by product dissatisfaction or malfunction. Second, EOU return occurs if functional products can be replaced with newly upgraded products. Third, EOL return occurs when products are no longer used due to technical deterioration. Since product recovery activities are different based on the reasons for the return, product recovery strategies and recovery logistics networks may also vary depending on the reasons for product return. This variation in recovery strategies and recovery logistics networks are explored in the second and the third research problems.

In the first part of the dissertation, we study a CLSC network design under unknown demand and return as a manufacturer-supplier's problem. In the second and third part of the dissertation, we analyze channel selection strategy in the commercial product return logistics network under time-value consideration.

1.1 Motivations and Scope of the Dissertation

Characteristics of used products in RSC are far less predictable than supply resources in a traditional supply chain. Thus, RSC has more uncertainty, in terms of timing, quantity, and quality, than traditional supply chain. Although uncertainty in the RSC and CLSC are severe problems, only limited studies consider uncertainty. The studies on uncertainty in RSC and CLSC networks mostly use two stage stochastic programming. However, a stochastic programming model is difficult to compute, especially if stochastic parameters follow continuous distributions. Even if stochastic parameters follow discrete distributions, the stochastic problem is still complicated when large number of realizations of stochastic parameters are required. However, most literature that examines stochastic issues in the CLSC network design problem consider fewer scenarios in the model and use a commercial optimization solver.

The first research problem is based on two main purposes. First, on the modeling side, the research purpose is to develop a generic CLSC network design problem under demand and return uncertainty. The configuration of a CLSC network will vary with demand and return quantities. For example, if a company expects minimal returns, then only a limited number of remanufacturing facilities are needed in the CLSC network. Therefore, we consider uncertainty in the CLSC network design problem with respect to product demand/return quantities. To do so, we build on the deterministic CLSC network design setting studied by Easwaran and Üster [13] where manufacturing and remanufacturing locations are assumed to be co-located for operational efficiency. Similarly, the centers are assumed to be capable of handling both forward and reverse flows in the CLSC network.

In order to model uncertainty, we adopt a two-stage stochastic programming approach [7] in which the first stage corresponds to making optimal design decisions including the supply (manufacturing and remanufacturing) locations, the locations of distribution/collection centers, and their capacity levels. The second stage finds the best forward and reverse flow networks based on each scenario. Building on the developed two-stage stochastic programming model, we study whether the stochastic model performs better than the deterministic model. However, as the number of scenarios increases, the problem becomes more challenging to solve which opens room for the second research purpose.

The second research purpose, on the methodological side, is to develop an efficient solution approach based on the BD [5], which is also known as the L-shaped method in stochastic programming literature [48]. The BD approach typically requires enhancement in order to perform as an efficient convergent method, so we propose several techniques that improve the performance of the standard BD algorithm. The first approach is achieved by aggregating over scenario subgroups, rather than by generating a Benders cut for each scenario [7] or employing a traditional single Benders cut. The second approach is obtained as an extension of a cut strengthening technique previously developed for solving discrete deterministic models using BD [45, 46, 47]. The third approach initially utilizes the modification of the master problem based on mean value scenario as suggested by Batun et al. [3]. We suggest an alternative approach to generate lower bounding inequalities based on a dual subproblem under mean value scenarios. Lastly, the lower bound inequalities are also disaggregated in terms of scenario subgroups. For the BD algorithm, We experiment with different performance enhancement techniques and analyze how far performance of the BD algorithm improves.

Product recovery logistics network design varies not only by the quantities of demand/return, but also by the reasons for product return. In commercial product return, time value of the product is one of the factors that determines logistics network structure, since products are resold to consumers after repair and repackaging. A product with high decay value should be collected as quickly as possible so that total profit loss from product return is minimized.

In addition to time value of the product, the method for collecting the product is also important in commercial product return. A generous return policy is a strategy to increase the company's future profits via strengthening customer loyalty. Offering a multi-return channel is one of the generous policies that a company offers to consumers. It contributes to the company's need for operational flexibility, since various types of reverse flows network can be built based on characteristics of products. Based on the literature review, however, no studies have considered commercial product return and multi-channel issues in product recovery logistics network design, especially on a quantitative model.

In the second research problem, we study channel selection strategy in recovery logistics networks motivated by the commercial return process in industries that commonly handle both manufacturing and sales (e.g., electronics industry). In the commercial return process, customers generally return products to the manufacturer for product dissatisfaction or a functional failure. Product dissatisfaction is independent from product quality therefore, products are assumed to be non-defective. These non-defective products can be resold at the retailer after a minor inspection such as repackaging. On the other hand, functional failure returns relate to quality issues (i.e., defective products). Based on the degree of defectiveness, the product may be either repaired or disposed.

The developed CLSC network model consists of four entities: Repairing Facilities (RFs), centers, retailers, and customers. To collect returned products from customers, four different options of collection, called multi-channel, are used by the company: 1) RF via retailer, 2) RF via retailers and collection centers, 3) RF via collection centers, or 4) RF directly. We formulate the model as LP and introduce a time parameter to measure product residual value. The developed LP model determines return and redistribution channels to maximize the total profit from recovered products. We specifically analyze the best return channel selection strategy using real geographic and product data.

Next, we extend the developed commercial product return logistics network model by introducing location decisions: RF and center locations. The model is formulated as MILP and determines the best RF/center locations and return/redistribution channel to maximize the total profit. The developed MILP model, naturally, becomes more difficult as the problem size increases. The objective of the third research is to develop an efficient solution approach based on the SA heuristic algorithm. SA algorithm evaluates the goodness of a feasible solution multiple times. When optimization solver is used for the evaluation process, the SA algorithm takes an excessively long solution time. Therefore, we develop a greedy algorithm to evaluate a feasible solution. We test how the developed SA algorithm performs compared to an exact solution method, BD algorithm, based on randomly generated instances. After computational experiments of the SA algorithm, we solve the developed commercial product return logistics network design problem using real geographic and product data. In the computational results, we test the efficiency of the algorithms and illustrate how to determine recovery logistics network configuration along with return/redistribution channel selection strategy. We especially analyze characteristics of recovery logistics network configuration in terms of product characteristics, such as the non-defective and disposal rates.

1.2 Dissertation Organization

The rest of this dissertation is organized as follows: Chapter 2 develops a generic CLSC network design problem under uncertainty and proposes an exact solution method based on the BD algorithm. Chapter 3 considers channel selection in commercial product return logistics networks and analyzes the optimal channel selection strategy that maximizes total profit from recovered products using real product and geographic data. Chapter 4 extends the developed commercial product return logistics network model by considering location decisions and identifying the optimal logistics network configuration using the developed SA heuristic algorithm. Finally, chapter 5 presents conclusions, contributions and future research directions for the product recovery network design problems considered in the dissertation.

2. CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN UNDER DEMAND AND RETURN UNCERTAINTY

Design and operation of CLSC have attracted attention both in academia and in industry over the last couple of decades mainly because of two reasons. First, the interest can be attributed to the ever increasing environmental concerns and responsibility held by companies whose products are amenable to reuse via remanufacturing or refurbishing. It can easily be argued that recovering any remaining value in used products leads to less use of energy and resources in manufacturing new products while eliminating waste caused by the disposal of non-reused products. Hence, an overall less impact on the environment is made. In other situations, simply the legislation in a country/region necessitates planning and operation of reuse activities for the companies [6]. Second, reuse via remanufacturing/refurbishing presents significant economic potential [21, 22]. Whenever possible, satisfying new product demands by remanufactured/refurbished products is now a common practice. In fact, it is the business-sense that makes companies adopting voluntary reuse activities in the first place. The value that can be recovered after remanufacturing/refurbishing in otherwise-disposed products and, in many cases, value after simple inspection/repackaging of unused returned products can be staggeringly high. For example, the predictions at HP indicate that the total costs of returned product can amount up to 2% of total sales [23] and in large retailers, such as Home Depot, return rates of at least 10% is not uncommon (due to generous take-back policies) with a potential total value of return in the hundreds of millions of dollars [22]. The most typical examples of remanufacturing/refurbishing practices are found in the electronics and automotive industries represented by such companies as HP, Dell,

Xerox, and GM who extensively adopt remanufacturing practices [46].

While the traditional supply chain design refers to decisions for effective and efficient production and transportation of products from supply locations to demand locations through one or more intermediate facilities, the reverse flow for remanufacturing/refurbishing traces a similar path in reverse direction. Thus, it has been recognized that the design of CLSCs should incorporate decisions pertaining to both flow directions [e.g. 40]. Thus, our specific focus in this research is on the design of a general integrated CLSC network which is composed of two physical flow channels (as depicted in Figure 2.1). *Forward flow channel* refers to product flow from supply (manufacturing and remanufacturing/refurbishing) locations to demand (e.g., retailer) locations through distribution centers and the *reverse flow channel* refers to return flow from demand locations to supply (for remanufacturing/refurbishing) locations through collection centers. Observing that uncertainties are inevitable, especially for high technology electronics and automotive products, we explicitly consider *demand and return uncertainty* in CLSC network design.

The setting of our problem of interest builds on the deterministic CLSC design setting studied by Easwaran and Üster [13] where manufacturing and remanufacturing locations are assumed to be co-located for operational efficiencies and the centers are assumed to be capable of handling both forward and reverse flows in the CLSC network, i.e., centers act as both distribution and collection facilities. For modeling our problem of interest with uncertainty considerations, we adopt a twostage stochastic programming approach [7] in which the first stage corresponds to making optimal design decisions including the supply (manufacturing and remanufacturing/refurbishing) locations and their capacity levels, the locations of distribution and collection centers and their capacity levels while the second stage addresses the optimization of expected flow costs over a set of scenarios that capture uncertainty in demand and return quantities at retailer locations.

On the modeling side, our primary contributions include consideration of three non-trivial characteristics explicitly: 1) a scenario based representation of uncertainties in demand and return quantities; 2) capacity installment decisions at the sourcing facilities and the centers; and, 3) for operational flexibility and better capacity utilization, multi-sourcing of retailers, i.e., each demand location is not required to be assigned to a unique collection and/or distribution center. To this end, we also discuss how to modify our model to consider alternative return inspection locations.

On the methodological side, to solve our model with a given set of scenarios efficiently, we develop a solution approach based on BD [5] which is also known as Lshaped method in the stochastic programming literature [48]. The BD approach is a popular solution method due to its convenience as an algorithmic framework for solving two-stage stochastic programming models and relative ease of implementation. However, BD approach typically needs enhancements to perform as an efficiently convergent method. To this end, we provide enhancements facilitated via generation of multiple Benders cuts, strengthened Benders cuts, and lower bounding inequalities for the master problem. The first approach is achieved via aggregation over scenarios subgroups, rather than generating a Benders cut for each scenario [7] or employing a traditional single Benders cut; the second is obtained as an extension of a cut strengthening technique previously developed for solving discrete deterministic models using BD [45, 46, 47]; and the third approach initially utilizes the modification of the master problem based on mean value scenario as suggested by Batun et al. [3]. We also suggest an alternative approach to generate lower bounding inequalities based on dual subproblem under mean value scenarios as well as disaggregation of these lower bounding inequalities. We observe in our experimentations that these techniques help to tighten lower bounds and improve overall algorithmic performance

for solving our stochastic programming model. Later, in an extensive computational study, we utilize our algorithm to solve the stochastic programs with fixed sets of scenarios and determine high performing values for the number of samples and sample sizes to be employed in a SAA framework [26].

The rest of the chapter is organized as follows. In the following section, we provide a review of the related literature. In section 2.2, we introduce notation, a detailed problem definition and its mathematical formulation. In section 2.3, we propose efficient solution method based on Benders decomposition approach. In section 2.4, we present computational experiments on the performance of the proposed solution method. Finally, we provide our concluding remarks in section 2.5.

2.1 Literature Review

Literature on product recovery network is relatively new in the network design context. General reviews of product recovery logistics are given by [1, 16, 17, 18]. In the dissertation, our problem belongs to CLSC structure, so we focus on CLSC network design. Fleischmann et al. [19] consider a general quantitative model for the CLSC network design composed of hybrid manufacturing/remanufacturing facilities, distribution/collection centers, and customer locations. In the paper, authors propose an integrated design of forward and return flow network. Integrated forward and reverse flow network design has a cost advantage to sequential network design by sharing locations and resources for both manufacturing and remanufacturing operations. Although not every operation in manufacturing and repairing may be identical, both manufacturing and remanufacturing operations mostly require similar parts, machine, and work skills. Therefore, recent CLSC network design research adopt integrated forward and reverse network design [40]. Beamon and Fernandes [4] consider similar CLSC network structure given by Fleischmann et al. [19]. In the model, authors assume limited capacities at hybrid manufacturing/remanufacturing facility and collection center. Sim et al. [44] extend Fleischmann et al. [19] model by considering multi-product and limited capacity at all nodes in the CLSC network. Akçali et al. [1] point out that CLSC network design studies are limited and, more strikingly, only a few studies address the uncertainty involved in these logistical environments. In fact, the studies that incorporate uncertainty in demand and return are very limited in the CLSC and RSC network design area. In the reverse logistics context, Listes and Dekker [31] employ a three-stage stochastic programming approach for a sand recycling network. Salema et al. [41], building on the deterministic model given by [19], develop a model for a capacitated multi-product reverse logistics network with demand and return uncertainty. In above studies, as is commonly the case in recovery network design literature, the proposed models are solved using commercial optimization solver without algorithmic developments for efficient solution of large network instances. Lee and Dong [28] develop dynamic reverse supply chain network problem under demand and return uncertainty in which only the locations choices for intermediate facilities (centers in our case) are considered. The authors suggest a simulated annealing approach which utilizes SAA to handle the uncertainty.

In the context of CLSCs, Listes [30] present a stochastic model for network design in a setting where the new products are shipped directly to customer sites from manufacturing facilities, used products are shipped to collection centers for inspection and reusable return are then shipped to manufacturing facilities. A two-stage stochastic model and a solution algorithm based on branch-and-bound and L-shaped method are provided. Analysis includes only very small size problems based on 12 scenarios. Pishvaee and Jolai [37] also study a stochastic CLSC network design problem for which the solutions are obtained using a commercial software for small size problems with 4 scenarios. Finally, we note that in the multi-commodity flow type stochastic service network design context (i.e., routing specific origin-destination pairs rather than in a production-distribution system setting), recent studies include Crainic et al. [10], which uses progressive hedging heuristic approach, and Lium et al. [32], which solves deterministic equivalent problems on very small network for analysis.

2.2 Problem Definition and the Model

The underlying strategic and operational setting of our problem includes three types of facilities in a CLSC network, namely the Sourcing Facilities (SFs), Centers (CTRs), and demand (customers/retailer - RT) locations¹, as depicted in Figure 2.1. We assume that, since the designed system needs to satisfy the expected demand, the overall operations generate a certain revenue. Thus, we focus on logistical cost minimization in the design of the system. Product supply, both as new products and/or as recovered products, is provided by the SFs and flows through distribution centers to final demand locations, which face product return and are responsible for routing the flow through collection centers to recovery locations. Note that recovered products are assumed to be perfect substitutes for new products. Opening and operating sourcing facilities and centers imply associated fixed costs as well as variable processing costs for new and returned products. In addition, product flow between facilities implies variable transportation costs. In practice, mainly due to quality issues or the extent of problems in return, it is typically not possible to recover the whole returned flow through remanufacturing/refurbishing activities. Thus, we assume a certain recovery fraction to represent a percentage of returned products that are remanufactured. We further assume that, without loss of generality, for

¹Henceforth, we generically use the term Sourcing Facilities (SF) to refer to manufacturing and remanufacturing/refurbishing locations and Center (CTR) to refer to distribution and collection centers.

operational efficiencies and improved response times, a remanufacturing/refurbishing facility is located at a location only if a manufacturing facility is also available at the same location and that distribution and collection centers can be opened (but not required) at the same location but serve to flow in opposing channels. In our initial model and methodology development, we assume that the product inspection to identify recoverable return is conducted at the SF locations. Later, in Section 2.4.4, we show how this model should be modified to consider alternative network stages for inspection.

In this study, we consider three non-trivial extensions to the basic integrated model in [13] which span strategic and operational characteristics relevant in realistic settings as follows:

- The first one pertains to the description of the demand and return quantities which are assumed to be estimated and available a priori in [13]. We consider uncertainty in demand and return, and utilize a scenario approach to represent the uncertainties in these quantities. Specifically, a scenario is generated by randomly generating a demand and a return quantity for each retailer location. To generate return quantities randomly, we assume that a random fraction of the demand value of that location in that scenario will be faced as the return quantity. Furthermore, we explicitly consider scenarios that can differ in terms of the demand values they represent as high, medium, or low. In each such category of demand, we assume that multiple scenarios are utilized to capture the uncertainty in a more detailed manner. In turn, the uncertainties in return levels are also captured accordingly based on their relationship to the demand realizations.
- Secondly, as opposed to fixed capacity limitations at the CTRs in [13], we consider capacity installation decisions at both SFs and CTRs. We assume

that once a decision to open a facility is made, a base capacity (for supply/manufacturing at an SF or for processing at a CTR) is installed at a certain cost at that location. Any additional capacity can be installed at a cost proportional to the additional quantity handled. However, the addition can be done in a way that a maximum capacity limit, specified for each location, is not violated. Capacity expansion availability provides a good proxy about how big each facility should be planned to be and an opportunity to reduce overall supply chain costs by introducing flexibility to address trade-offs between fixed and variable costs in the system. Also recognizing the extensive focus on lean manufacturing practices and elimination of waste in today's manufacturing environment as well as the use of capacity as a buffer against variability, consideration of capacity limitation in design is even more important than ever.

• Thirdly, as opposed to forcing each demand location interact with only one CTR as in [13], we consider multi-sourcing for retailer locations for potentially improved operational efficiency and capacity utilization, i.e., a retailer is allowed to interact more than one distribution and/or collection center, as to be determined by the model. Consideration of multi-sourcing is also an important part of the model since 1) it generalizes single-sourcing, i.e., if a solution with single-sourcing is better, then our model would capture that but not vise-versa, 2) there are capacity limitations at the facilities that serve flows with uncertain amounts, and 3) in an application, large geographical regions are represented as retailer/customer nodes, e.g., in our case study presented in the last section using US data, RT locations are assumed to be large geographical areas such as a zip-code or a city as a whole, and, thus, they represent a large demand that may need to be satisfied from multiple locations.

Furthermore, Easwaran and Uster [13] consider multiple products and a single

manufacturing/remanufacturing facility for each product, thus, forcing each CTR to interact with only one SF per product. In this study, we consider single product or, rather, a product family, and we allow each CTR to interact with multiple SF locations where the products are remanufactured and/or manufactured. We note that while the extension of our base model to multiple products case with above considerations can be achieved in a straightforward fashion, this clearly results in larger problem sizes.

To develop a mathematical model, we first introduce the notation and the decision variables in the CLSC network to be designed and operated under above characteristics.



Figure 2.1: CLSC Network Structure

Sets and indices:

- \mathcal{I} set of candidate SF locations, $i \in \mathcal{I}$.
- \mathcal{J} set of candidate CTR locations, $j \in \mathcal{J}$.
- \mathcal{K} set of customers (RT locations), $k \in \mathcal{K}$.
- Ω set of scenarios, $\omega \in \Omega$.

Parameters:

 $D_{\omega k}$ demand at RT k under scenario ω $S_{\omega k}$ return at RT k under scenario ω G_{mn} unit transportation cost from a location m to a location n F_i^F fixed cost of opening a SF at location i F_i^R fixed cost of selecting SF i as remanufacturing site F_j^C fixed cost of opening a CTR at location j κ_i^F unit manufacturing cost at SF i κ_i^R unit remanufacturing cost at SF i η_i^F unit distribution processing cost at CTR j η_i^R unit collection processing cost at CTR j ψ_i^F unit manufacturing capacity expansion cost at SF i ψ_i^R unit remanufacturing capacity expansion cost at SF i ρ_i^F unit distribution capacity expansion cost at CTR j ρ_i^R unit collection capacity expansion cost at CTR j b_i^F base manufacturing capacity at SF i b_i^R base remanufacturing capacity at SF i l_i^F base distribution capacity at CTR j l_i^R base collection capacity at CTR j p_i^F allowed forward capacity expansion at SF i p_i^R allowed reverse capacity expansion at SF i q_i^F allowed distribution capacity expansion at CTR j q_i^R allowed collection capacity expansion at CTR j λ_i recovery fraction at SF iprobability of scenario ω H_{ω}

Decision Variables:

~. —		if SF i is open
$z_i =$) otherwise
au. —	$\int 1$	if SF i is selected for remanufacturing site
$g_i =$) otherwise
<i>m</i> —	$\int 1$	if CTR j is open
$x_j =$) otherwise
α_i^F	am	ount of manufacturing capacity expansion at SF i .

 α_i^R amount of remanufacturing capacity expansion at SF *i*.

 β_i^F amount of distribution capacity expansion at CTR *j*.

 β_j^R amount of collection capacity expansion at CTR *j*.

 $\mu_{\omega jk}$ fraction of customer k's demand satisfied by CTR j under scenario ω .

 $\nu_{\omega kj}$ fraction of customer k's return sent to CTR j under scenario ω .

 $\sigma_{\omega ij}$ amount of product flow from SF *i* to CTR *j* under scenario ω .

 $\tau_{\omega ji}$ amount of returned product flow from CTR j to SF i under scenario ω .

In our two-stage stochastic programming modeling, the first stage is concerned with design decisions to be made now while the second stage is concerned with decisions after uncertainties are resolved via realization of certain scenario. Thus, in the model, location and capacity decisions associated with SFs and CTRs belong to the first stage. Forward and reverse network flow decisions are belong to the second stage and they are to be determined after a demand and return scenario is realized. The purpose of two-stage stochastic programming model is to find a solution that performs well on average under all scenarios and this is achieved via minimization of a total cost given by first stage design costs and expected cost of transportation and processing over demand/return scenarios. The overall model is given as follows:

$$\begin{aligned}
\text{Min} \qquad \sum_{i \in \mathcal{I}} \left(F_i^F z_i + F_i^R y_i \right) + \sum_{j \in \mathcal{J}} F_j^C x_j + \sum_{i \in \mathcal{I}} \left(\psi_i^F \alpha_i^F + \psi_i^R \alpha_i^R \right) + \sum_{j \in \mathcal{J}} \left(\rho_j^F \beta_j^F + \rho_j^R \beta_j^R \right) \\
+ \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} H_\omega \left[\left(G_{ij} + \kappa_i^F \right) \sigma_{\omega ij} + \left(G_{ji} + \lambda_i \kappa_i^R - \lambda_i \kappa_i^F \right) \tau_{\omega ji} \right] \\
+ \sum_{\omega \in \Omega} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} H_\omega \left[\left(G_{jk} + \eta_j^F \right) D_{\omega k} \mu_{\omega j k} + \left(G_{kj} + \eta_j^R \right) S_{\omega k} \nu_{\omega k j} \right] \quad (2.1a)
\end{aligned}$$

subject to

$$y_i \le z_i \qquad \qquad \forall i \in \mathcal{I},$$
 (2.1b)

$$\alpha_i^F \le p_i^F z_i \qquad \qquad \forall i \in \mathcal{I}, \tag{2.1c}$$

$$\alpha_i^R \le p_i^R y_i \qquad \forall i \in \mathcal{I}, \qquad (2.1d)$$

$$\beta_i^F \le q_i^F x_i \qquad \forall j \in \mathcal{J}, \qquad (2.1e)$$

$$\beta_j^R \le q_j^R x_j \qquad \qquad \forall j \in \mathcal{J}, \qquad (2.1c)$$

$$\beta_j^R \le q_j^R x_j \qquad \qquad \forall j \in \mathcal{J}, \qquad (2.1f)$$

$$\sum_{j \in \mathcal{J}} \mu_{\omega j k} = 1 \qquad \qquad \forall k \in \mathcal{K}, \, \omega \in \Omega, \qquad (2.1g)$$

$$\sum_{j \in \mathcal{J}} \nu_{\omega k j} = 1 \qquad \qquad \forall k \in \mathcal{K}, \, \omega \in \Omega, \tag{2.1h}$$

$$\sum_{i\in\mathcal{I}}\sigma_{\omega ij} = \sum_{k\in\mathcal{K}} D_{\omega k}\,\mu_{\omega jk} \qquad \qquad \forall j\in\mathcal{J},\,\omega\in\Omega,\tag{2.1i}$$

$$\sum_{i\in\mathcal{I}}\tau_{\omega ji}=\sum_{k\in\mathcal{K}}S_{\omega k}\,\nu_{\omega kj}\qquad\qquad\forall j\in\mathcal{J},\,\omega\in\Omega,\qquad(2.1j)$$

$$\sum_{k \in \mathcal{K}} D_{\omega k} \, \mu_{\omega j k} \leq l_j^F \, x_j + \beta_j^F \qquad \forall j \in \mathcal{J}, \, \omega \in \Omega, \tag{2.1k}$$

$$\sum_{k \in \mathcal{K}} S_{\omega k} \nu_{\omega k j} \le l_j^R x_j + \beta_j^R \qquad \forall j \in \mathcal{J}, \, \omega \in \Omega,$$
(2.11)

$$\sum_{j \in \mathcal{J}} \sigma_{\omega i j} \le b_i^F z_i + \alpha_i^F \qquad \forall i \in \mathcal{I}, \, \omega \in \Omega,$$
 (2.1m)

$$\sum_{i \in \mathcal{I}} \tau_{\omega j i} \le b_i^R y_i + \alpha_i^R \qquad \qquad \forall i \in \mathcal{I}, \, \omega \in \Omega,$$
(2.1n)

$$\mu_{\omega jk}, \, \nu_{\omega kj}, \, \sigma_{\omega ij}, \, \tau_{\omega ji} \ge 0, \qquad \qquad \forall \, i \in \mathcal{I}, \, j \in \mathcal{J}, \, k \in \mathcal{K}, \, \omega \in \Omega \quad (2.10)$$

$$x_j, y_i, z_i \in \{0, 1\}, \ \alpha_i^F, \alpha_i^R, \beta_j^F, \beta_j^R \ge 0 \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}.$$
 (2.1p)

In the objective function, first two terms represent the fixed costs associated with locating the SFs and CTRs at their base capacity levels. The third term is associated with the forward and reverse capacity expansion costs at SFs and the fourth term is associated with the distribution (forward) and collection (reverse) capacity expansion costs at the CTRs. The fifth term is the total expected manufacturing/remanufacturing and transportation cost between SFs and CTRs in both forward and reverse directions. We note that, since new and remanufactured products are not distinguished, flow from SFs to CTRs, $\sigma_{\omega ij}$ include both new and remanufactured products. Thus, manufacturing cost at an SF $i \in \mathcal{I}$ is given by $\sum_{j\in\mathcal{J}} \{\kappa_i^F \sigma_{\omega ij} - \lambda_i \kappa_i^F \tau_{\omega ji}\}$ and the remanufacturing cost is $\sum_{j\in\mathcal{J}} \lambda_i \kappa_i^R \tau_{\omega ji}$. For a given scenario $\omega \in \Omega$, the cost expression given in brackets in the fifth term is obtained by combining these manufacturing and remanufacturing costs, and the related transportation costs. Finally, the sixth term represents total expected processing and transportation costs associated with CTRs and retailers.

Constraint set (2.1b) guarantees that only open SF can be selected as a remanufacturing site. Constraint set (2.1c) and (2.1d) make sure that SF capacities cannot be increased beyond the maximum capacity limit for manufacturing and remanufacturing, respectively. Similarly, constraint set (2.1e) and (2.1f) make sure that CTR capacities is not increased above the maximum limit for distribution and collection, respectively. Constraints (2.1g) and (2.1h) ensure that each demand location is assigned one CTR for receiving products and sending returned products. Constraints (2.1i) and (2.1j) represent the conservation of flow in forward and reverse directions at the CTRs, respectively. Constraints (2.1k) and (2.1l) guarantee that forward flow from a CTR and reverse flow to a CTR do not exceed CTR's respective assigned capacity and, similarly, constraints (2.1m) and (2.1n) guarantee that forward flow from an SF and reverse flow to an SF do not exceed SF's respective assigned manufacturing and remanufacturing capacities, respectively. Finally, constraints (2.1o) and (2.1p) are the restrictions on the decision variables.

2.3 Solution Approach

BD can provide an efficient framework to solve an MIP that is amenable to separation into two related problems (master problem and subproblem) with their associated objective function and constraints extracted from an overall formulation. The master problem typically contains only discrete variables, the subproblem contains only continuous variables, and the two problems relate via the use of a set of constraints and an auxiliary variable in the former. Our model is clearly a two-stage stochastic (binary) integer program in which the first-stage decisions are discrete design variables and the second-stage corresponds to a linear program to optimize expected variable costs. A commonly employed efficient framework to handle such programs is based on BD that is also known as L-shaped method in stochastic programming literature [7]. In the basic form of BD, we first obtain a reformulation of the overall minimization problem by explicitly stating its subproblem (a linear program with continuous variables) and then deriving the reformulation by using the dual subproblem solution and an auxiliary variable to define a set of cuts (known as Benders optimality cuts) that captures the optimum subproblem solution. Master problem is then obtained via consideration of only a subset of these cuts in the reformulation and, thus, its optimal solution provides a set of design decisions and its objective is a lower bound for the overall problem. Using the obtained discrete design variable values, the subproblem dual is well-defined and its optimal solution provides the necessary information to generate an upper bound for the overall problem as well as a Benders cut for the master problem. Master problem and the subproblem are solved in this delayed constraint generation fashion iteratively until a satisfactorily small gap between the bounds is achieved. Although the BD framework provides a very compelling approach to solve MIPs, it is not without issues which are mainly related to the strength of bounds it produces and, thus, the algorithmic convergence rate.

One of the issues faced in this process is that the solution of a master problem may provide a set of design variable values for which the subproblem is not feasible (or its dual is unbounded). In this case, a Benders feasibility cut based on extreme rays, rather than an optimality cut based on dual variable values, is generated and added to the master problem. If the feasibility of subproblem is always guaranteed for any solution provided by the master problem (first-stage decisions), then the stochastic program of interest is called to be one with relatively complete recourse and only an optimality cut is generated in each iteration. To ensure relatively complete recourse, induced constraints (or surrogate constraints) can be utilized in the master problem (§2.3.3.1). For our problem, we suggest induced constraints to ensure enough capacity availability at the SFs and CTRs while solving the second-stage problem, and, thus, resolve the convergence issues.

Another issue that can be faced is the goodness of the bounds, especially lower bounds, obtained throughout the iterations. Observing that the master problem solution provides lower bounds (since many of the Benders optimality cuts implied by the reformulation are relaxed), it becomes clear that, at each iteration, one needs to add strong optimality cuts that force the lower bounds to have higher values quickly. To this end, we suggest a strengthening technique for Benders optimality cuts (§2.3.3.3) along with the use of multiple cuts which are obtained by separating traditional single Benders cut for scenario groups categorized based on demand and return levels as well as forward and return channels (§2.3.3.2). Furthermore, we also develop disaggregated dual subproblem based mean value lower bounding cuts and add them into the master problem for improved lower bound values (§2.3.3.4).

2.3.1 Benders Subproblem and Its Dual

The primal subproblem, denoted by $SP(\sigma, \tau, \mu, \nu | \hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R)$, is obtained as follows for given values of design decisions involving locations and capacities of SFs and CTRs.

Min
$$Z_{SP} = \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} H_{\omega} \left[\left(G_{ij} + \kappa_i^F \right) \sigma_{\omega ij} + \left(G_{ji} + \lambda_i \kappa_i^R - \lambda_i \kappa_i^F \right) \tau_{\omega ji} \right] \\ + \sum_{\omega \in \Omega} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} H_{\omega} \left[\left(G_{jk} + \eta_j^F \right) D_{\omega k} \mu_{\omega jk} + \left(G_{kj} + \eta_j^R \right) S_{\omega k} \nu_{\omega kj} \right]$$
(2.2) subject to (2.1g) - (2.1o)

The optimal solution of $SP(\cdot)$ provides the forward $(\sigma_{\omega ij}, \tau_{\omega ji})$ and reverse flows $(\mu_{\omega jk}, \nu_{\omega kj})$ with minimum total expected processing and transportation cost for the scenario set Ω . As it is well-known within the L-Shaped approach context, the subproblem is separable for each scenario ω , thus, we represent a subproblem as $SP_{\omega}(\sigma, \tau, \mu, \nu | \hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R)$ for a scenario $\omega \in \Omega$. Then, the overall formulation (2.1) can be expressed as

$$\operatorname{Min} \qquad \sum_{i \in \mathcal{I}} \left(F_i^F z_i + F_i^R y_i \right) + \sum_{j \in \mathcal{J}} F_j^C x_j + \sum_{i \in \mathcal{I}} \left(\psi_i^F \alpha_i^F + \psi_i^R \alpha_i^R \right) + \sum_{j \in \mathcal{J}} \left(\rho_j^F \beta_j^F + \rho_j^R \beta_j^R \right) \\
+ \sum_{\omega \in \Omega} H_\omega SP_\omega \left(\sigma, \tau, \mu, \nu | \widehat{x}, \widehat{y}, \widehat{z}, \widehat{\alpha}^F, \widehat{\alpha}^R, \widehat{\beta}^F, \widehat{\beta}^R \right)$$
(2.3)

subject to (2.1b) - (2.1f), and (2.1p).

Observe that the subproblem $SP_{\omega}(\cdot)$ is also separable in terms of forward and reverse flow directions which are given, along with their duals, as follows.

Forward Subproblem for each scenario $\omega \in \Omega$, is obtained as

$$\begin{array}{ll}
\operatorname{Min} & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(G_{ij} + \kappa_i^F \right) \sigma_{\omega ij} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(G_{jk} + \eta_j^F \right) D_{\omega k} \, \mu_{\omega jk} \quad (2.4a)$$
subject to (2.1g), (2.1i), (2.1k), (2.1m)
$$\mu_{\omega jk}, \, \sigma_{\omega ij} \geq 0, \qquad \forall i \in \mathcal{I}, \, j \in \mathcal{J}, \, k \in \mathcal{K}. \quad (2.4b)$$

Defining the dual variables $\pi^1_{\omega k}$, $\pi^2_{\omega j}$, $\pi^3_{\omega j}$, and $\pi^4_{\omega i}$ for constraints (2.1g), (2.1i), (2.1k), and (2.1m), respectively, the **Forward Dual Subproblem** for $\omega \in \Omega$ (DSP-F ω) is obtained as

Max
$$Z_{\text{DSP-F}\omega} = \sum_{k \in \mathcal{K}} \pi_{\omega k}^1 + \sum_{j \in \mathcal{J}} \left(l_j^F \, \widehat{x}_j + \widehat{\beta}_j^F \right) \pi_{\omega j}^3 + \sum_{i \in \mathcal{I}} \left(b_i^F \, \widehat{z}_i + \widehat{\alpha}_i^F \right) \pi_{\omega i}^4 \qquad (2.5a)$$

subject to

$$\pi_{\omega k}^{1} - D_{\omega k} \pi_{\omega j}^{2} + D_{\omega k} \pi_{\omega j}^{3} \leq H_{\omega} \left(G_{jk} + \eta_{j}^{F} \right) D_{\omega k} \qquad \forall j \in \mathcal{J}, \ k \in \mathcal{K},$$
(2.5b)

$$\pi_{\omega j}^{2} + \pi_{\omega i}^{4} \leq H_{\omega} \left(G_{ij} + \kappa_{i}^{F} \right) \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, \qquad (2.5c)$$

$$\pi^1_{\omega k}, \, \pi^2_{\omega j} \, \text{unrestricted}, \, \pi^3_{\omega j}, \, \pi^4_{\omega i} \le 0 \qquad \qquad \forall \, i \in \mathcal{I}, \, j \in \mathcal{J}, \, k \in \mathcal{K}.$$
 (2.5d)

Reverse subproblem for each scenario $\omega \in \Omega$, is stated as

Min
$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(G_{ji} + \lambda_i \, \kappa_i^R - \lambda_i \, \kappa_i^F \right) \tau_{\omega j i} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(G_{kj} + \eta_j^R \right) S_{\omega k} \, \nu_{\omega k j}$$
(2.6a)

subject to (2.1h), (2.1j), (2.1l), (2.1n)

$$\nu_{\omega kj}, \, \tau_{\omega ji} \ge 0, \qquad \forall i \in \mathcal{I}, \, j \in \mathcal{J}, \, k \in \mathcal{K}.$$
(2.6b)

Defining the dual variables $\pi_{\omega k}^5$, $\pi_{\omega j}^6$, $\pi_{\omega j}^7$, and $\pi_{\omega i}^8$ for constraints (2.1h), (2.1j), (2.1l) and (2.1n), respectively, **Reverse Dual Subproblem** for $\omega \in \Omega$ (DSP-R ω) is derived as

Max
$$Z_{\text{DSP-R}\omega} = \sum_{k \in \mathcal{K}} \pi_{\omega k}^5 + \sum_{j \in \mathcal{J}} \left(l_j^R \, \widehat{x}_j + \widehat{\beta}_j^R \right) \pi_{\omega j}^7 + \sum_{i \in \mathcal{I}} \left(b_i^R \, \widehat{y}_i + \widehat{\alpha}_i^R \right) \pi_{\omega i}^8$$
 (2.7a)

subject to

$$\pi_{\omega k}^{5} - S_{\omega k} \pi_{\omega j}^{6} + S_{\omega k} \pi_{\omega j}^{7} \leq H_{\omega} \left(G_{kj} + \eta_{j}^{R} \right) S_{\omega k} \qquad \forall j \in \mathcal{J}, \, k \in \mathcal{K},$$
(2.7b)

$$\pi_{\omega j}^{6} + \pi_{\omega i}^{8} \leq H_{\omega} \left(G_{ji} + \lambda_{i} \kappa_{i}^{R} - \lambda_{i} \kappa_{i}^{F} \right) \qquad \forall i \in \mathcal{I}, j \in \mathcal{J},$$
(2.7c)

$$\pi_{\omega k}^5, \, \pi_{\omega j}^6 \, \text{unrestricted}, \, \pi_{\omega j}^7, \, \pi_{\omega i}^8 \le 0 \qquad \qquad \forall i \in \mathcal{I}, \, j \in \mathcal{J}, \, k \in \mathcal{K}.$$
 (2.7d)

2.3.2 Benders Master Problem

The master problem, denoted by $MP(x, y, z, \alpha^F, \alpha^R, \beta^F, \beta^R | \hat{\sigma}, \hat{\tau}, \hat{\mu}, \hat{\nu})$, can be obtained from the overall formulation given with objective (2.3). For this, we replace the last term representing second-stage objective with a function of the auxiliary variable(s) to be employed in constructing the Benders cut(s) in an iteration. In the following master problem formulation, we use the generic terms $\Theta(\text{CUTSET})$ and BENDERS CUTSET in place of this objective function term and the Benders cut(s) added in the course of iterations, respectively.

Min
$$Z_{MP} = \sum_{i \in \mathcal{I}} \left(F_i^F z_i + F_i^R y_i \right) + \sum_{j \in \mathcal{J}} F_j^C x_j + \sum_{i \in \mathcal{I}} \left(\psi_i^F \alpha_i^F + \psi_i^R \alpha_i^R \right)$$
$$+ \sum_{j \in \mathcal{J}} \left(\rho_j^F \beta_j^F + \rho_j^R \beta_j^R \right) + \Theta(\text{CUTSET})$$
(2.8)
subject to (2.1b) - (2.1f), and (2.1p)
BENDERS CUTSET

Next, we provide alternative approaches to replace $\Theta(\text{CUTSET})$ and BENDERS CUT-SET and other enhancement approaches we employ to obtain improved bounds and convergence.

2.3.3 Enhancing the Benders Algorithm

In order to accelerate overall BD convergence and reduce runtime with quality bounds for our problem, we employ three approaches including the introduction of so-called *induced constraints*, a multi-cut approach while populating the BENDERS CUTSET in each iteration, and strengthening of the Benders (optimality) cuts.
2.3.3.1 Induced Constraints

As noted before, in the case that the master problem does not provide an underlying network for the (primal) subproblem to have a feasible solution (or its dual is unbounded), then a feasibility cut based on extreme rays of the dual subproblem polyhedron is added to the master problem in the next iteration. The process, in general, has a hampering effect on the convergence properties of the algorithm. In the stochastic programming context, this leads to the case where the problem is called not to have *complete recourse* and a use of induced constraints is suggested to ensure complete recourse and, thus, generation of optimality cuts only. Induced constraints serve a similar purpose as the surrogate constraints employed in solving mixed integer programs using BD and since they are redundant they do not impact the original feasible domain of the problem.

In our context, the master problem determines the locations and capacity levels of the SFs and CTRs while the subproblems are solved for each scenario $\omega \in \Omega$ (and for forward and reverse flow channels separately) with their own demand and return realizations, respectively. Then, the master problem solution may not provide facility locations (SF and/or CTR) with enough available capacity to handle the overall flow for each scenario $\omega \in \Omega$ and flow channel. Observe that if total forward and reverse capacities at selected SFs and CTRs are larger than total quantities of demand and return, respectively, then their associated subproblems are always feasible. Therefore, as induced constraints, we introduce the following four constraints:

$$\sum_{i \in \mathcal{I}} \left(b_i^F z_i + \alpha_i^F \right) \geq \max_{\omega \in \Omega} \left\{ T_\omega : T_\omega = \sum_{k \in \mathcal{K}} D_{\omega k} \right\}$$
(2.9a)

$$\sum_{j \in \mathcal{J}} \left(l_j^F x_j + \beta_j^F \right) \geq \max_{\omega \in \Omega} \left\{ T_\omega : T_\omega = \sum_{k \in \mathcal{K}} D_{\omega k} \right\}$$
(2.9b)

$$\sum_{i \in \mathcal{I}} \left(b_i^R y_i + \alpha_i^R \right) \geq \max_{\omega \in \Omega} \left\{ T_\omega : T_\omega = \sum_{k \in \mathcal{K}} S_{\omega k} \right\}$$
(2.9c)

$$\sum_{j \in \mathcal{J}} \left(l_j^R x_j + \beta_j^R \right) \geq \max_{\omega \in \Omega} \left\{ T_\omega : T_\omega = \sum_{k \in \mathcal{K}} S_{\omega k} \right\}$$
(2.9d)

Constraint (2.9a) and (2.9b) relate to the forward channel and state that the total capacities installed at the manufacturing at SFs and CTRs, respectively, are at least as large to handle the scenario that has the maximum total demand at retail locations so that all subproblems are ensured to be feasible. Constraints (2.9c) and (2.9d) ensure capacity feasibility of all of the subproblems for the reverse channel in the similar way for remanufacturing at SFs and CTRs.

2.3.3.2 Multi-Cut Separation Schemes for Benders Cuts

The separation of the Benders subproblem for each scenario readily implies a potential to employ multiple Benders cuts, one for each scenario, to be added to the master problem in each iteration. This is known as the multi-cut approach [8]. As shown above, in our case, the subproblem is also separable for each flow channel, thus, there is potential to generate and add one Benders cut for each scenario and each channel at each iteration of the algorithm. Adding multiple cuts may strengthen the lower bounds so that total number of iterations and solution time are reduced. However, this amounts to adding $2 * |\Omega|$ cuts to master problem in each iteration. Accumulation of a large number of cuts in the master problem hinders its efficient solution and increases runtimes as noted in [7]. On the other hand, in the typical Benders framework, addition of one cut is suggested, and this one cut for our case can be obtained by simply combining the same $2 * |\Omega|$ cuts by addition. That way,

however, much valuable information about the solution space can be overlooked due to aggregation and the performance of the algorithm can be affected negatively due to weaker lower bounds provided by the master problem. Thus, it is reasonable to strive for a balance between these two extremes and consider forms of partial aggregation of Benders cuts. This is experimented before in the context of solving mixed integer programs with promising results [45, 46]. In addition to the above two types of cuts, we consider two additional types of disaggregated Benders cuts and outline four types of possible Benders cuts as follows:

Type 1 Benders Cut is the standard single Benders cut which is generated by employing all dual SP solutions, i.e., the solution to the dual of $SP(\sigma, \tau, \mu, \nu | \hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R)$, within one inequality given as $\Theta(\text{CUTSET})$ is replaced by Θ in (2.8) and the constraints BENDERS CUTSET include

$$\Theta \geq \sum_{\omega \in \Omega} \left[\sum_{k \in \mathcal{K}} \pi_{\omega k}^{1} + \sum_{j \in \mathcal{J}} \pi_{\omega j}^{3} \left(l_{j}^{F} x_{j} + \beta_{j}^{F} \right) + \sum_{i \in \mathcal{I}} \pi_{\omega i}^{4} \left(b_{i}^{F} z_{i} + \alpha_{i}^{F} \right) \right] + \sum_{\omega \in \Omega} \left[\sum_{k \in \mathcal{K}} \pi_{\omega k}^{5} + \sum_{j \in \mathcal{J}} \pi_{\omega j}^{7} \left(l_{j}^{R} x_{j} + \beta_{j}^{R} \right) + \sum_{i \in \mathcal{I}} \pi_{\omega i}^{8} \left(b_{i}^{R} y_{i} + \alpha_{i}^{R} \right) \right]$$

where $\Theta \geq 0$ is the auxiliary variable in Benders master problem.

Type 2 Benders Cut is the other end of the spectrum where, in each iteration, we add a total $2 \times |\Omega|$ cuts to BENDERS CUTSET, one for each flow channel and scenario, to the master problem given as

$$\Theta_{\omega}^{F} \geq \sum_{k \in \mathcal{K}} \pi_{\omega k}^{1} + \sum_{j \in \mathcal{J}} \pi_{\omega j}^{3} \left(l_{j}^{F} x_{j} + \beta_{j}^{F} \right) + \sum_{i \in \mathcal{I}} \pi_{\omega i}^{4} \left(b_{i}^{F} z_{i} + \alpha_{i}^{F} \right)$$

$$\Theta_{\omega}^{R} \geq \sum_{k \in \mathcal{K}} \pi_{\omega k}^{5} + \sum_{j \in \mathcal{J}} \pi_{\omega j}^{7} \left(l_{j}^{R} x_{j} + \beta_{j}^{R} \right) + \sum_{i \in \mathcal{I}} \pi_{\omega i}^{8} \left(b_{i}^{R} y_{i} + \alpha_{i}^{R} \right)$$

where $\Theta_{\omega}^{F} \geq 0$, Θ_{ω}^{R} unrestricted (due to the formation of the objective) are the auxiliary variables. Also, in (2.8), we replace the term $\Theta(\text{CUTSET})$ by $\sum_{\omega \in \Omega} (\Theta_{\omega}^{F} + \Theta_{\omega}^{R})$.

Type 3 Benders Cut are obtained after aggregation over scenarios in the previous type, thus, in each iteration, we add only two cuts to BENDERS CUTSET, one for each flow channel, given as

$$\Theta^{F} \geq \sum_{\omega \in \Omega} \left[\sum_{k \in \mathcal{K}} \pi^{1}_{\omega k} + \sum_{j \in \mathcal{J}} \pi^{3}_{\omega j} \left(l_{j}^{F} x_{j} + \beta_{j}^{F} \right) + \sum_{i \in \mathcal{I}} \pi^{4}_{\omega i} \left(b_{i}^{F} z_{i} + \alpha_{i}^{F} \right) \right]$$

$$\Theta^{R} \geq \sum_{\omega \in \Omega} \left[\sum_{k \in \mathcal{K}} \pi^{5}_{\omega k} + \sum_{j \in \mathcal{J}} \pi^{7}_{\omega j} \left(l_{j}^{R} x_{j} + \beta_{j}^{R} \right) + \sum_{i \in \mathcal{I}} \pi^{8}_{\omega i} \left(b_{i}^{R} y_{i} + \alpha_{i}^{R} \right) \right]$$

with auxiliary variables $\Theta^F \geq 0$, Θ^R urs and replacement of $\Theta(\text{CUTSET})$ by $(\Theta^F + \Theta^R)$ in (2.8).

Type 4 Benders Cut is obtained via a slightly intermediate form of aggregation by taking advantage of the way the scenarios present themselves at varying levels of demand and return quantities. More specifically, we assume that the scenario for demand and return quantity can be categorized as a low level (l), a medium level (m) or a high level (h) scenario, for example, depending on the life-cycle of a product. We create the corresponding three sets of scenarios as Ω^l , Ω^m , and, Ω^h , respectively, so that collectively exhaustive and mutually exclusive. Then, for each channel and scenario group a Benders cut can be generated. This leads to a total of 6 cuts to be generated and added to the BENDERS CUTSET in each iteration given as follows

$$\Theta_s^F \geq \sum_{\omega \in \Omega^s} \left[\sum_{k \in \mathcal{K}} \pi_{\omega k}^1 + \sum_{j \in \mathcal{J}} \pi_{\omega j}^3 \left(l_j^F x_j + \beta_j^F \right) + \sum_{i \in \mathcal{I}} \pi_{\omega i}^4 \left(b_i^F z_i + \alpha_i^F \right) \right], \quad s \in \{l, m, h\}$$

$$\Theta_s^R \geq \sum_{\omega \in \Omega^s} \left[\sum_{k \in \mathcal{K}} \pi_{\omega k}^5 + \sum_{j \in \mathcal{J}} \pi_{\omega j}^7 \left(l_j^R x_j + \beta_j^R \right) + \sum_{i \in \mathcal{I}} \pi_{\omega i}^8 \left(b_i^R y_i + \alpha_i^R \right) \right], \quad s \in \{l, m, h\}$$

We define a total of six auxiliary variables as $\Theta_s^F \ge 0$ and Θ_s^R urs for $s \in \{l, m, h\}$ and the $\Theta(\text{CUTSET})$ is then obtained as $\sum_{s \in \{l, m, h\}} (\Theta_s^F + \Theta_s^R)$ to include in (2.8).

2.3.3.3 Strengthened Benders Cuts

Observe that, given master problem solution for location and capacity values for the SFs and CTRs, the subproblem is a network flow problem for which having multiple optimum solutions is not uncommon. Thus, the subproblem (and its dual) solution is one of many solutions with the same optimum objective value. Since each one of these solutions implies a different Benders cut(s), it is important that we generate and employ a stronger set of cuts. Generating strengthened Benders cuts is previously performed while solving deterministic mixed integer programs with good results [e.g. 12, 47]. Magnanti and Wong [34] define the strongness of a cut in an optimization problem $\min_{y \in Y, z \in R} \{z : z \ge f(u) + y g(u), \forall u \in U\}$ as follows: If $f(u^1) + y g(u^1) \ge f(u) + y g(u), \forall y \in Y$, then the cut $z \ge f(u^1) + y g(u^1)$ dominates or stronger than the cut $z \ge f(u) + y g(u)$. Then, following a two-phase approach similarly to the ones in [46, 47], we identify strengthened Benders cuts for each scenario and both forward and reverse channel subproblems.

In the first phase of solving DSP-F ω , we consider all $\pi^1_{\omega k}$ variables and only the $\pi^2_{\omega j}$, $\pi^3_{\omega j}$, and $\pi^4_{\omega i}$ variables whose respective \hat{x}_j and \hat{z}_i coefficients in (2.5a) are equal to one. Notice that, if an \hat{x}_j or a \hat{z}_i value is zero, then the corresponding $\hat{\beta}_j^F$ and $\hat{\alpha}_i^F$ values are also zero, respectively, and those terms are immaterial in the objective function of DSP-F ω . Accordingly, letting \mathcal{I}^O denote the set of open SFs for which $\hat{z}_i = 1$ and \mathcal{J}^O denote the set of open CTRs for which $\hat{x}_j = 1$, the first-phase forward dual subproblem is given as

Max
$$\sum_{k \in \mathcal{K}} \pi^{1}_{\omega k} + \sum_{j \in \mathcal{J}^{O}} \left(l_{j}^{F} \, \widehat{x}_{j} + \widehat{\beta}_{j}^{F} \right) \pi^{3}_{\omega j} + \sum_{i \in \mathcal{I}^{O}} \left(b_{i}^{F} \, \widehat{z}_{i} + \widehat{\alpha}_{i}^{F} \right) \pi^{4}_{\omega i}$$
(2.10a)

subject to

$$\pi^{1}_{\omega k} - D_{\omega k} \pi^{2}_{\omega j} + D_{\omega k} \pi^{3}_{\omega j} \le H_{\omega} \left(G_{jk} + \eta^{F}_{j} \right) D_{\omega k} \qquad \forall j \in \mathcal{J}^{O}, \, k \in \mathcal{K}$$
(2.10b)
$$\pi^{2}_{\omega k} + \pi^{4}_{\omega j} \le H_{\omega} \left(C_{\omega k} + \kappa^{F}_{j} \right) \qquad \forall i \in \mathcal{I}^{O}, \, i \in \mathcal{I}^{O}$$
(2.10c)

$$\pi_{\omega j}^{2} + \pi_{\omega i}^{4} \leq H_{\omega} \left(G_{ij} + \kappa_{i}^{F} \right) \qquad \forall i \in \mathcal{I}^{O}, \ j \in \mathcal{J}^{O} \qquad (2.10c)$$

$$\pi_{\omega k}^{1}, \ \pi_{\omega j}^{2} \text{ unrestricted}, \ \ \pi_{\omega j}^{3}, \ \pi_{\omega i}^{4} \leq 0 \qquad \forall i \in \mathcal{I}^{O}, \ j \in \mathcal{J}^{O}, \ k \in \mathcal{K} \qquad (2.10d)$$

In the second phase of solving DSP-F ω , we determine the values of remaining variables to obtain strengthened cuts. Specifically, after fixing the values of all the variables determined in the first phase, namely $\pi^1_{\omega k}$, $\forall k \in \mathcal{K}$, $\pi^2_{\omega j}$, $\pi^3_{\omega j}$, $\forall j \in \mathcal{J}^O$ and $\pi^4_{\omega i}$, $\forall i \in \mathcal{I}^O$, we solve a maximization problem given as

Max
$$\sum_{j \in \mathcal{J} \setminus \mathcal{J}^O} l_j^F \pi_{\omega j}^3 + \sum_{i \in \mathcal{I} \setminus \mathcal{I}^O} b_i^F \pi_{\omega i}^4$$
(2.11a)

subject to

$$-D_{\omega k}\pi_{\omega j}^{2}+D_{\omega k}\pi_{\omega j}^{3}\leq H_{\omega}\left(G_{jk}+\eta_{j}^{F}\right)D_{\omega k}-\pi_{\omega k}^{1}\qquad\forall j\in\mathcal{J}\backslash\mathcal{J}^{O},\,k\in\mathcal{K}\quad(2.11b)$$

$$\pi_{\omega j}^{2} + \pi_{\omega i}^{4} \le H_{\omega} \left(G_{ij} + \kappa_{i}^{F} \right) \qquad \forall i \in \mathcal{I}, j \in \mathcal{J} \qquad (2.11c)$$

$$\pi^2_{\omega j}$$
 unrestricted, $\pi^3_{\omega j}, \pi^4_{\omega i} \le 0$ $\forall i \in \mathcal{I}, j \in \mathcal{J}.$ (2.11d)

In the first phase of Solving DSP-R ω , we proceed similarly to solving DSP-F ω and first define the sets $\mathcal{I}^U = \{i : \hat{y}_i = 1\}$ and $\mathcal{J}^O = \{j : \hat{x}_j = 1\}$ based on master problem solution for x and y variables. Then, the first phase problem is

Max
$$\sum_{k \in \mathcal{K}} \pi_{\omega k}^{5} + \sum_{j \in \mathcal{J}^{O}} \left(l_{j}^{R} \, \widehat{x}_{j} + \widehat{\beta}_{j}^{R} \right) \pi_{\omega j}^{7} + \sum_{i \in \mathcal{I}^{U}} \left(b_{i}^{R} \, \widehat{y}_{i} + \widehat{\alpha}_{i}^{R} \right) \pi_{\omega i}^{8}$$
(2.12a)

subject to

$$\pi_{\omega k}^{5} - S_{\omega k} \pi_{\omega j}^{6} + S_{\omega k} \pi_{\omega j}^{7} \le H_{\omega} \left(G_{kj} + \eta_{j}^{R} \right) S_{\omega k} \qquad \forall j \in \mathcal{J}^{O}, \, k \in \mathcal{K}$$
(2.12b)

$$\pi_{\omega j}^{6} + \pi_{\omega i}^{8} \leq H_{\omega} \left(G_{ji} + \lambda_{i} \kappa_{i}^{R} - \lambda_{i} \kappa_{i}^{F} \right) \qquad \forall i \in \mathcal{I}^{U}, j \in \mathcal{J}^{O}$$
(2.12c)

$$\pi_{\omega k}^5, \pi_{\omega j}^6 \text{ unrestricted}, \ \pi_{\omega j}^7, \pi_{\omega i}^8 \le 0 \qquad \forall i \in \mathcal{I}^U, j \in \mathcal{J}^O, k \in \mathcal{K}.$$
 (2.12d)

In the second phase of solving DSP-R ω , fixing the values of $\pi_{\omega k}^5$, $k \in \mathcal{K}$, $\pi_{\omega j}^6$, $\pi_{\omega j}^7$, $j \in \mathcal{J}^O$, and $\pi_{\omega i}^8$, $i \in \mathcal{I}^U$ as obtained in the first phase, we solve the following maximization problem.

Max
$$\sum_{j \in \mathcal{J} \setminus \mathcal{J}^O} l_j^R \pi_{\omega j}^7 + \sum_{i \in \mathcal{I} \setminus \mathcal{I}^C} b_i^R \pi_{\omega i}^8$$
(2.13a)

subject to

$$-S_{\omega k}\pi_{\omega j}^{6}+S_{\omega k}\pi_{\omega j}^{7}\leq H_{\omega}\left(G_{k j}+\eta_{j}^{R}\right)S_{\omega k}-\pi_{\omega k}^{5}\qquad\forall j\in\mathcal{J}\backslash\mathcal{J}^{O},\ k\in\mathcal{K}\quad(2.13b)$$

$$\pi_{\omega j}^{6} + \pi_{\omega i}^{8} \leq H_{\omega} \left(G_{ji} + \lambda_{i} \kappa_{i}^{R} - \lambda_{i} \kappa_{i}^{F} \right) \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}$$
(2.13c)

$$\pi^6_{\omega j}$$
 unrestricted, $\pi^7_{\omega j}, \pi^8_{\omega i} \le 0$ $\forall i \in \mathcal{I}, j \in \mathcal{J}.$ (2.13d)

2.3.3.4 Mean Value Cuts

In order to improve the performance of the BD approach via generation of good lower bounds early in the iterations, Batun et al. [3] suggest a lower bounding inequality for the auxiliary variable Θ that are applicable in also solving a general two-stage stochastic programming model. The inequality they provide in Proposition 2 [3] relates Θ to the second stage cost obtained based on a feasible first-stage solution. Specifically, for our problem, we can state the following lower bounding inequality for Θ :

$$\Theta \ge Z_{SP_{\bar{\omega}}}(\widehat{x}, \widehat{y}, \widehat{z}, \widehat{\alpha}^F, \widehat{\alpha}^R, \widehat{\beta}^F, \widehat{\beta}^R, \overline{\omega})$$
(2.14)

where $(\hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R)$ and $Z_{SP_{\bar{\omega}}} = (\hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R, \bar{\omega})$ represent a feasible solution to the master problem and the optimum objective value of the subproblem solved for only the mean value scenario, respectively.

Batun et al. [3] suggest a generation of above cuts by utilizing primal subproblem under mean value scenario within the master problem. Below, we first show how this can be done for our problem. Later, we suggest an alternative approach that is based on the dual subproblem solution as in the generation of regular Benders cuts and that improves the solution performance even further than the one by [3].

Mean Value Cuts based on Primal Subproblem

In this approach, master problem defined in section 2.3.2 is appended with the set of primal subproblem constraints under mean value scenario $\bar{\omega}$ (a scenario with mean values of stochastic parameters) as well as the constraint that represents the relation between the auxiliary variable, Θ , and primal subproblem objective function. Since we use the mean value of stochastic parameters, additional parameters and auxiliary decision variables are needed to be defined as follows:

Parameters

- \overline{D}_k mean value of demand at customer $k \in K$
- \bar{S}_k mean value of return at customer $k \in K$

Auxiliary Decision Variables

- μ_{jk} –fraction of customer k 's demand satisfied by CTR j under $\bar{\omega}$
- ν_{kj} fraction of customer k's return sent to CTR j under $\bar{\omega}$
- σ_{ij} amount of product flow from SF *i* to CTR *j* under $\bar{\omega}$
- τ_{ji} amount of returned product flow from CTR j to SF i under $\bar{\omega}$

Therefore, master problem has following additional constraints (2.15a) - (2.15i) with one-to-one correspondence to (2.1g) - (2.1o):

$$\sum_{j \in \mathcal{J}} \mu_{jk} = 1 \qquad \forall k \in \mathcal{K}$$
(2.15a)

$$\sum_{j \in \mathcal{J}} \nu_{kj} = 1 \qquad \forall k \in \mathcal{K}$$
(2.15b)

$$\sum_{i\in\mathcal{I}}\sigma_{ij} = \sum_{k\in\mathcal{K}}\bar{D}_k\,\mu_{jk} \qquad \qquad \forall j\in\mathcal{J}$$
(2.15c)

$$\sum_{i\in\mathcal{I}}\tau_{ji} = \sum_{k\in\mathcal{K}}\bar{S}_k\,\nu_{kj} \qquad \forall j\in\mathcal{J}$$
(2.15d)

$$\sum_{k \in \mathcal{K}} \bar{D}_k \,\mu_{jk} \le l_j^F \, x_j + \beta_j^F \qquad \qquad \forall j \in \mathcal{J}$$
(2.15e)

$$\sum_{k \in \mathcal{K}} \bar{S}_k \,\nu_{kj} \le l_j^R \, x_j + \beta_j^R \qquad \qquad \forall j \in \mathcal{J}$$
(2.15f)

$$\sum_{j \in \mathcal{J}} \sigma_{ij} \le b_i^F z_i + \alpha_i^F \qquad \qquad \forall i \in \mathcal{I}$$
(2.15g)

$$\sum_{j \in \mathcal{J}} \tau_{ji} \le b_i^R y_i + \alpha_i^R \qquad \qquad \forall i \in \mathcal{I}$$
(2.15h)

$$\mu_{jk}, \nu_{kj}, \sigma_{ij}, \tau_{ji} \ge 0, \qquad \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K} \qquad (2.15i)$$

Noting that we employ multi-cut separation schema for regular Benders cuts as described in Section 2.3.3.2 and that the Type 4 cuts provide the best performing separation (as shown computationally later in Section 2.4.1.1), instead of a single auxiliary variable Θ , we consider auxiliary variables for demand and return categories and flow channels, i.e., Θ_s^F , Θ_s^R , $s \in \{l, m, h\}$. Thus, based on (2.14), we have the following lower bounding cut to also be added to the master problem:

$$\sum_{s \in \{l,m,h\}} \left(\Theta_s^F + \Theta_s^R \right) \geq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left[\left(G_{ij} + \kappa_i^F \right) \sigma_{ij} + \left(G_{ji} + \lambda_i \kappa_i^R - \lambda_i \kappa_i^F \right) \tau_{ji} \right] \\ + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left[\left(G_{jk} + \eta_j^F \right) \bar{D}_k \mu_{jk} + \left(G_{kj} + \eta_j^R \right) \bar{S}_k \nu_{kj} \right]$$
(2.16)

Then, similarly to the Benders cuts, (2.16) can be separated by flow directions and demand-return categories. Thus, we define Type A for a single, Type B for two (one reverse and one forward)), and Type C for six (forward/reverse channels and low/medium/high demand-return category) lower bounding valid inequalities as follows:

- **Type A** incorporates a single inequality (2.16) along with (2.15) in the master problem.
- **Type B** incorporates the following two inequalities below along with (2.15) in the master problem.

$$\sum_{s \in \{l,m,h\}} \Theta_s^F \ge \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(G_{ij} + \kappa_i^F \right) \sigma_{ij} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(G_{jk} + \eta_j^F \right) \bar{D}_k \mu_{jk}$$
$$\sum_{s \in \{l,m,h\}} \Theta_s^R \ge \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(G_{ji} + \lambda_i \kappa_i^R - \lambda_i \kappa_i^F \right) \tau_{ji} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(G_{kj} + \eta_j^R \right) \bar{S}_k \nu_{kj}$$

Type C incorporates the six inequalities along with (2.15) in the master problem.

To develop these inequalities we define the following notation in which mean value scenarios, denoted as $\bar{\omega}_s$, $s \in \{l, m, h\}$, are generated separately for each demand-return category.

Parameters

 \overline{D}_{sk} mean value of demand for category $s \in \{l, m, h\}$ at customer k.

 \bar{S}_{sk} mean value of return for category $s \in \{l, m, h\}$ at customer k.

Auxiliary Decision Variables

- μ_{sjk} fraction of customer k's demand satisfied by CTR j under $\bar{\omega}_s, s \in \{l, m, h\}$
- ν_{skj} fraction of customer k's return sent to CTR j under $\bar{\omega}_s, s \in \{l, m, h\}$
- σ_{sij} amount of product flow from SF *i* to CTR *j* under $\bar{\omega}_s, s \in \{l, m, h\}$
- τ_{sji} amount of returned product flow from CTR j to SF i under $\bar{\omega}_s, s \in \{l, m, h\}$

For this case, using the above notation, constraints (2.15) are extended to have three copies of each constraint for $s \in \{l, m, h\}$ and the lower bounding inequalities are separated as

$$\begin{split} \Theta_s^F &\geq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(G_{ij} + \kappa_i^F \right) \sigma_{sij} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(G_{jk} + \eta_j^F \right) \bar{D}_{sk} \, \mu_{sjk}, \qquad s \in \{l, m, h\} \\ \Theta_s^R &\geq \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(G_{ji} + \lambda_i \, \kappa_i^R - \lambda_i \, \kappa_i^F \right) \tau_{sji} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \left(G_{kj} + \eta_j^R \right) \bar{S}_{sk} \, \nu_{skj}, \qquad s \in \{l, m, h\} \end{split}$$

2.3.3.5 Mean Value Cuts based on Dual Subproblem and Separation Schemes

In order to generate alternative lower bounding cuts based on dual subproblem solution, we first identify a subproblem which can be seen as one of the subproblem for a scenario, defined as SP_{ω} earlier in Section 2.3.1 following (2.2). Specifically, for this subproblem $SP_{\bar{\omega}}$ in which we utilize the mean values of stochastic parameters as above, we have the constraint set given in (2.15) and the objective function as the right-hand-side of (2.16). The dual of this subproblem for a given set of feasible master problem solution $(\hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R)$ is well defined as before and can be particularly specified by employing the dual variables $\chi_k^1, \chi_j^5, \chi_j^2, \chi_i^6, \chi_k^3, \chi_j^7, \chi_j^4, \chi_i^8$ for constraints in (2.15), respectively. Based on the dual formulation solution for this specific subproblem for which a known value of χ_i^{\cdot} is denoted by $\hat{\chi}_i^{\cdot}$, we generate the following three types of lower bounding cuts with varying degrees of disaggregation.

Type A* generates and adds to the master problem the following single inequality after solving the dual of $SP_{\bar{\omega}}$

$$\sum_{s \in \{l,m,h\}} \left(\Theta_s^F + \Theta_s^R \right) \geq \sum_{k \in \mathcal{K}} \widehat{\chi}_k^1 + \sum_{j \in \mathcal{J}} \widehat{\chi}_j^3 \left(l_j^F x_j + \beta_j^F \right) + \sum_{i \in \mathcal{I}} \widehat{\chi}_i^4 \left(b_i^F z_i + \alpha_i^F \right) \\ + \sum_{k \in \mathcal{K}} \widehat{\chi}_k^5 + \sum_{j \in \mathcal{J}} \widehat{\chi}_j^7 \left(l_j^R x_j + \beta_j^R \right) + \sum_{i \in \mathcal{I}} \widehat{\chi}_i^8 \left(b_i^R y_i + \alpha_i^R \right)$$

Type B* generates and adds to the master problem the following two inequalities, one for each flow channel, after solving the dual of $SP_{\bar{\omega}}$

$$\sum_{s \in \{l,m,h\}} \Theta_s^F \geq \sum_{k \in \mathcal{K}} \widehat{\chi}_k^1 + \sum_{j \in \mathcal{J}} \widehat{\chi}_j^3 \left(l_j^F x_j + \beta_j^F \right) + \sum_{i \in \mathcal{I}} \widehat{\chi}_i^4 \left(b_i^F z_i + \alpha_i^F \right)$$
$$\sum_{s \in \{l,m,h\}} \Theta_s^R \geq \sum_{k \in \mathcal{K}} \widehat{\chi}_k^5 + \sum_{j \in \mathcal{J}} \widehat{\chi}_j^7 \left(l_j^R x_j + \beta_j^R \right) + \sum_{i \in \mathcal{I}} \widehat{\chi}_i^8 \left(b_i^R y_i + \alpha_i^R \right)$$

Type C* generates and adds the following six inequalities to the master problem after solving the dual of three subproblems $SP_{\bar{\omega}_s}$ which adopt mean value scenarios $\bar{\omega}_s$, $s \in \{l, m, h\}$ accordingly.

$$\Theta_s^F \geq \sum_{k \in \mathcal{K}} \widehat{\chi}_{sk}^1 + \sum_{j \in \mathcal{J}} \widehat{\chi}_{sj}^3 \left(l_j^F x_j + \beta_j^F \right) + \sum_{i \in \mathcal{I}} \widehat{\chi}_{si}^4 \left(b_i^F z_i + \alpha_i^F \right), \qquad s \in \{l, m, h\}$$

$$\Theta_s^R \geq \sum_{k \in \mathcal{K}} \widehat{\chi}_{sk}^5 + \sum_{j \in \mathcal{J}} \widehat{\chi}_{sj}^7 \left(l_j^R x_j + \beta_j^R \right) + \sum_{i \in \mathcal{I}} \widehat{\chi}_{si}^8 \left(b_i^R y_i + \alpha_i^R \right), \qquad s \in \{l, m, h\}$$

2.3.4 Overall Approach

Having the master $MP(x, y, z, \alpha^F, \alpha^R, \beta^F, \beta^R|\cdot)$ the subproblem $SP(\sigma, \tau, \mu, \nu|\cdot)$ as well as its dual $DSP(\pi^1, \pi^2, \pi^3, \pi^4, \pi^5, \pi^6, \pi^7, \pi^8|\hat{x}, \hat{y}, \hat{z}, \widehat{\alpha^F}, \widehat{\alpha^R}, \widehat{\beta^F}, \widehat{\beta^R})$ defined, we summarize the overall solution method in Algorithm 1. We first note that the overall subproblem solution to DSP is always obtained, regardless of the cut type employed, via the solutions to subproblems for each channel and scenario, i.e., DSP-F ω and DSP-R ω , and second, that if lower bounding inequalities are employed depending on their type, corresponding additional lines (3, 8, and 16) should be included.

Algorithm 1 BD Algorithm

- 1: Initialize Z^* , UB, Itr, MaxItr, gap values, and set BENDERS CUTSET empty
- 2: Solve $MP(x, y, z, \alpha^F, \alpha^R, \beta^F, \beta^R|\cdot)$
- 3: Embed corresponding (2.15) and (2.16) into $MP(\cdot)$ if lower bound inequalities are used
- 4: Set $LB = Z_{MP}$
- 5: while $(UB LB)/LB \ge gap \text{ AND } (Itr < MaxItr) do$
- 6: Itr=Itr + 1
- 7: Solve DSP-F ω and DSP-R ω to obtain $\hat{\sigma}, \hat{\tau}, \hat{\mu}, \hat{\nu}$
- 8: Solve the dual of $SP_{\bar{\omega}}$ or $SP_{\bar{\omega}_s}$ if lower bounding inequality is used.
- 9: Calculate Z_{SP} as $\sum_{\omega \in \Omega} (Z_{\text{DSP-F}\omega} + Z_{\text{DSP-R}\omega})$
- 10: Calculate UB= $(Z_{MP} \Theta(CUTSET)) + Z_{SP}$
- 11: **if** $(Z^* > UB)$ **then**
- 12: $Z^* = UB$
- 13: **end if**
- 14: Generate new cuts using $\hat{\sigma}, \hat{\tau}, \hat{\mu}, \hat{\nu}$ values and add to BENDERS CUTSET
- 15: Solve $MP(x, y, z, \alpha^F, \alpha^R, \beta^F, \beta^R|\cdot)$
- 16: Embed cuts Types A*, B*, or C* if our suggested mean value cuts approach is used
- 17: Set $LB = Z_{MP}$
- 18: end while
- 19: Solve $SP(\sigma, \tau, \mu, \nu | \hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R)$
- 20: **Report** $\hat{x}, \hat{y}, \hat{z}, \hat{\alpha}^F, \hat{\alpha}^R, \hat{\beta}^F, \hat{\beta}^R, \hat{\sigma}, \hat{\tau}, \hat{\mu}, \hat{\nu}$ and the corresponding value for (2.1a).

2.4 Computational Study

In this section, we present computational test and analysis results on two areas of interest for our study objectives including the following.

- 1. Testing of computational efficiency and effectiveness of the proposed solution approach: Specifically, we first examine the effects of algorithmic enhancements on the overall approach to solve a stochastic program with a given scenario set (§ 2.4.1). Then, we provide a computational comparison between the suggested BD approach and an alternative relying on the solution of DEP in § 2.4.2. Finally, using or suggested solution approach for each scenario set, we conduct a numerical study by implementing the SAA method of Kleywegt et al. [26] so as to obtain high quality solutions to our stochastic program (§ 2.4.3).
- 2. The effect of recovery rate parameter and recovery locations (Retailer, CTR or SF) on the network structure and the overall cost: In this context, we first present extended formulations of our problem to obtain network designs for alternative recovery locations and, then, using the SAA approach with our optimization algorithm embedded, we provide an analysis on design impacts of alternative recovery rates and locations. Using our results, we also provide an analysis on the value of the stochastic solution in the context of this case study (§ 2.4.4.3).

For the algorithmic performance tests, we randomly generate data following approach outlined in Üster et al. [46] and, for the analysis of recovery rate and locations, we use the data provided in Sahyouni et al. [40] for the continental US. The BD solution algorithm is run until an optimality gap of 2% (or better) is reached and it is implemented using C++ programming language and CPLEX Concert Technology. All implicit MIPs (master problems and dual subproblems) are solved using CPLEX

12.4 (64-bit) and runs are completed on multiple machines with a 3GHz Intel Core2 Quad processor and 8 GB RAM.

2.4.1 Performance of Algorithmic Enhancements

For computational testing of algorithmic enhancements, we generate 12 different problem classes by changing number of scenarios, SFs, CTRs and customers. Each class includes 10 different test instances. Table 2.1 shows detailed information of problem classes. The first and the second number in the bracket represent total number of binary and continuous variables in the first stage, respectively.

					First-stag	e	Second-stage		
Class	Scenario	\mathbf{SF}	CTR	Customer	Decision Vars (B/C)	Constraints	Decision Vars	Constraints	
C1	250	10	30	60	130(50/80)	90	1,050,000	65,000	
C2	250	10	30	90	130 (50/80)	90	1,500,000	80,000	
C3	250	10	30	120	130 (50/80)	90	1,950,000	95,000	
C4	500	10	30	60	$130 \ (50/80)$	90	2,100,000	130,000	
C5	500	10	30	90	$130 \ (50/80)$	90	3,000,000	160,000	
C6	500	10	30	120	130 (50/80)	90	3,900,000	190,000	
C7	250	20	60	60	$260 \ (100/160)$	180	2,400,000	100,000	
C8	250	20	60	90	$260 \ (100/160)$	180	3,300,000	115,000	
C9	250	20	60	120	$260 \ (100/160)$	180	4,200,000	130,000	
C10	500	20	60	60	$260 \ (100/160)$	180	4,800,000	200,000	
C11	500	20	60	90	260 (100/160)	180	6,600,000	230,000	
C12	500	20	60	120	260 (100/160)	180	8,400,000	260,000	

Table 2.1: Input Parameters for Test Classes

Generally, two-stage stochastic programming model is hard to solve when number of scenario is relatively large. Thus, Monte Carlo Sampling method is a common approach to reduce the problem size. In this section, we focus on improving the performance of BD algorithm for two-stage stochastic programming for a fixed number of scenarios, thus, we assume that generated scenarios cover all uncertainties. Since all test instances are solved within reasonable time using proposed solution method we do not consider any technique to reduce the problem size.

We use uniform distributions to generate problem parameters for low, medium, and high demand values $(D_{\omega^l k}, D_{\omega^m k}, D_{\omega^h k})$, return fractions (δ_k) , and recovery fractions (λ_i) . TD represents maximum of total demand among $|\Omega|$ scenarios (i.e. $TD = \max_{\omega \in \Omega} \{\sum_{k \in K} D_{\omega k}\}$) and TR represents maximum of total return among $|\Omega|$ scenarios (i.e. $TR = \max_{\omega \in \Omega} \{\sum_{k \in K} S_{\omega k}\}$). Based on TD and TR, we generate initial capacity at SF (b_i^F, b_i^R) and at CTR (l_j^F, l_j^R) , and capacity expansion limitation at SF (p_i^F, p_i^R) and CTR (q_j^F, q_j^R) . We note that, in general, our test instances generate optimum solution whose objective value is split between the first and second stage components roughly as 40% and 60% since we expect higher transportation costs with the inclusion of reverse flows under the presence of hybrid facilities which are utilized for both forward and reverse flows. Ranges for input parameters are provided in Table 2.2 and complete data sets for each instance can be found at http://ise.tamu.edu/LNS/clsc-data.html.

Parameter	Value	Parameter	Value	Parameter	Value
$D_{\omega^l k}$	U[500, 1500]	$D_{\omega^m k}$	U[1500, 2500]	$D_{\omega^h k}$	U[2500, 3500]
δ_k	U[0.5, 0.8]	λ_i	U[0.6, 0.8]	b_i^F	$\mathrm{U}[0.1,0.2]\times\mathrm{TD}$
b_i^R	$\mathrm{U}[0.1,0.2]\times\mathrm{TR}$	l_j^F	$\mathrm{U}[0.1,0.2]\times\mathrm{TD}$	l_j^R	$\mathrm{U}[0.1,0.2]\times\mathrm{TR}$
p^F_i	$\mathrm{U}[0.1,0.2]\timesb_i^F$	p_i^R	$\mathrm{U}[0.1,0.2]\timesb_i^R$	q_j^F	$\mathrm{U}[0.1,0.2]\timesl_j^F$
q_j^R	$\mathrm{U}[0.1,0.2]\timesl_{j}^{R}$				

Table 2.2: Distribution for Input Parameters

2.4.1.1 Results of Multi-Cut Approach

In this section, we compare the performance of four types of Benders cuts defined in section 2.3.3.2. Our preliminary experiments showed that Type 4 Benders cut performs the best in terms of solution time to obtain a 2.0% optimality gap. Table 2.3 presents the average and maximum solution times and the number of iterations for BD algorithm with Type 4 Benders cut.

	Time (s	econds)	Itera	ation
	Ave	Max	Ave	Max
C1	157	306	31.9	57
C2	386	752	54.2	101
C3	412	760	42.6	73
C4	336	616	34.2	62
C5	583	970	40.1	68
C6	1185	2930	56.4	134
C7	268	520	19.8	32
C8	1051	4610	35.8	85
C9	812	2365	46.4	119
C10	593	961	22.2	35
C11	1402	2936	33.7	61
C12	1423	2188	40.2	60

Table 2.3: Computational Results for the Type 4 Benders Cut

Since the BD algorithm using the Type 1, 2, or 3 cuts fail to solve problem within a reasonable amount of time we employ an additional stopping criterion; We stop the iterations for an instance if the solution time exceeds the maximum solution time in its class when Type 4 cut was used or if a 2% gap is reached, whichever comes earlier. Table 2.4 presents the optimality gaps, solution time (in seconds),

		Ту	pe 1			Tyj	pe 2		Type 3				
	Opt g	gap(%)	Time	Iter	Opt g	ap(%)	Time	Iter	Opt g	Opt gap(%)		Time Iter	
	Ave	Max	Ave	Ave	Ave	Max	Ave	Ave	Ave	Max	Ave	Ave	
C1	4.02	7.89	302	33.8	2.62	4.39	255	19.3	2.02	2.39	189	40.3	
C2	4.86	6.82	755	46.7	3.54	5.74	743	26.8	2.05	2.78	530	75.1	
C3	6.46	8.64	768	32.8	3.08	4.91	736	25.4	2.06	3.29	612	64.7	
C4	6.89	8.89	628	18	3.25	4.49	589	17	2.15	3.03	462	48.1	
C5	8.01	11.65	999	18.9	3.57	5.5	992	19.7	2.05	2.37	765	52.5	
C6	6.9	9.7	3030	40.7	4.05	6.38	3319	25.4	2.06	2.87	1638	78.2	
C7	4.12	5.94	528	26.3	2.66	4	556	11.5	2.15	2.68	436	33.8	
C8	2.77	4.38	4256	105.8	2.52	4.4	3755	18.6	2.10	3.20	1946	68.6	
C9	4.63	9.09	2412	71.8	2.92	4.83	2018	25	2.08	3.23	1484	84	
C10	4.86	5.94	983	17.1	3.1	4.42	1081	9.8	2.27	3.02	853	33.2	
C11	4.28	5.48	3019	34	3.11	4.55	3432	13.9	2.24	3.25	2094	51.5	
C12	19.6	26.8	2289	6.3	4.34	7.25	2361	17.8	2.61	3.69	2036	56	

and number of iterations for the BD algorithm with Type 1, 2, and 3 Benders cuts.

Table 2.4: Computational Results for the Type 1 - Type 3 Benders Cuts

Since Type 1 approach aggregates over all scenarios, it allows the cut to carry only limited stochastic information. Therefore, lower bound increases slowly and this results in the large optimality gap compared to other approaches under same runtime limits. Type 2 approach generates total $2 \times |\Omega|$ number of Benders cuts which are the most disaggregated cuts since one cut is generated per scenario and flow channel. Consequently, the total number of auxiliary decision variables introduced to master problem are $2 \times |\Omega|$ and, in each iteration, $2 \times |\Omega|$ number of constraints are added to the master problem. As a result, the overall approach is comparatively slower, although not always, when compared to the use of Type 1 cuts, especially for large size instances. Type 3 cuts, on the other hand, aggregate the cuts over scenarios and, thus, consider only two cuts disaggregated based on flow channel allowing a very controlled increase in the size of master problem through the iteration but failing to capture the opportunity for disaggregation over scenario. This approach provides the best performance in terms solution quality when compared to using cut types 1 and 2. Finally, Type 4 cuts consider disaggregating scenarios into only three groups (high-medium-low demand/return categories) and, thus, generate a total of six Benders cuts in each iteration by also considering disaggregation for flow channels. This brings together the desirable properties of cut types 2 and 3 efficiently in which some level of disaggregation is achieved both in the scenario context as well as flow channels. With this approach, scenarios that belong to the same category present some level of uniformity so that their aggregation still captures probabilistic nature of the corresponding parameters, disaggregation over flow channels is still incorporated, and the number of additional auxiliary decision variables and the constraints do not burden the master problem to hamper its solution efficiency significantly.

2.4.1.2 Results of Two-phase Method for Strengthening Cuts

In the previous section, we employed strengthened Benders cuts while comparing the four types of cut disaggregation and concluded that Type 4 cuts provide the most desirable results. In order to actually measure the impact of strengthening cuts via the two-phase approach suggested in Section 2.3.3.3, we employ its effect on solution times when used in conjunction with Type 4 cuts. Specifically, we solve our test instances with and without cut-strengthening by adopting Type 4 Benders cuts. We initially observed in our testing that the runtimes without strengthening of the Benders cuts can be excessively long. Thus, for this group of runs, we again employ a termination rule with dual criteria (which ever comes first) that involves a 2.0% optimality gap or a time limit given by the maximum runtime with cut strengthening within a class. Table 2.5 shows comparison of both approaches. As we expected, algorithm with two-phase method performs well in terms of both solution quality and solution times on all problem classes. Two-phase method strengthens BD cuts so that the master problem can be solved easier and provide better lower bounds and it results in faster solution time although the average number of iterations is higher.

	With tw	o-phase	method	Withou	t two-phase method			
	Iteration	Time (gap $< 2\%$)		Iteration	Time	Opt $gap(\%)$		
	Ave	Ave	Max	Ave	Ave	Ave	Max	
C1	31.9	157	306	26.9	225	2.4	4.2	
C2	54.2	386	752	50.2	620	2.2	3.6	
C3	42.6	412	760	39.4	641	2.3	3.9	
C4	34.2	336	616	28.2	555	2.5	4.1	
C5	40.1	583	970	29.6	952	3.0	4.4	
C6	56.4	1185	2930	53.3	2304	2.2	3.9	
C7	19.8	268	520	23.9	469	2.1	2.8	
C8	35.8	1051	4610	44	1734	1.9	2.3	
C9	46.4	812	2365	53.1	1526	1.9	3.0	
C10	22.2	593	961	20.2	887	2.5	3.6	
C11	33.7	1402	2936	33.6	2289	2.1	3.1	
C12	40.2	1423	2188	29	2038	2.9	4.6	

Table 2.5: Comparisons on Cut Strengthening with Type 4 Cuts

2.4.1.3 Results of Mean Value Cut Approach

In this section, we test the performance of BD algorithm with (mean value scenario based) lower bounding inequalities presented in section 2.3.3.4. Again, we employ strengthened Type 4 BD cuts in the BD algorithm with 2% optimality gap as termination condition. For each combination use of lower bounding inequalities with the Type 4 cuts in the BD algorithm, we report the average and maximum runtime for our test classes in Table 2.6.

In Table 2.6, we first observe that the disaggregation of lower bounding inequalities is generally helpful in improving the solution times as indicated by higher runtimes with Type A or A^{*} cuts when compared to Types B or B^{*} or Type C or C^{*} cuts, respectively. Use of Type C* cuts provide the best performance overall as indicated by the bold entries in Table 2.6 and the use of Type C cuts performs especially better than the Type B or B^* cuts for larger instances. Secondly, we observe that the dual subproblem base lower bounding inequalities provide better performance than inequalities generated by employing a modified master problem. Specifically, Types A^{*}, B^{*}, and C^{*} inequalities provide better performance than their counterparts Types, A, B, and C, respectively. This is largely due to the fact that the master problem size is not extended in terms of both variables and constraints, but, rather, only additional (lower bounding) cuts are added without hindering its solution time significantly. Lastly, we also observe that adding valid inequalities improves the runtime efficiency of the BD algorithm more for the larger instances. For the Type 4 +Type C^{*} case, the average runtime improvements over the Type 4 case are 8.92%, 6.99%, 12.14%, 20.54%, 23.50%, 10.30% for classes C1-C6, respectively, while for larger instance classes, C7-C12, they are 25.00%, 62.04%, 24.51%, 29.68%, 49.64%, 26.26%, respectively.

	Тур	pe 4	Type	$4 + A^{*}$	Type	4 + A	Type 4	$4 + B^{*}$	Type	4 + B	Type 4	$1 + C^{*}$	Type	4 + C
	Ave	Max	Ave	Max	Ave	Max	Ave	Max	Ave	Max	Ave	Max	Ave	Max
C1	157	306	158	304	171	337	170	407	150	343	143	350	161	423
C2	386	752	403	795	425	845	405	821	430	883	359	628	419	797
C3	412	760	409	779	471	961	389	726	446	970	362	947	482	1248
C4	336	616	344	663	354	665	364	626	279	533	267	460	287	595
C5	583	970	600	1005	619	951	591	971	487	1019	446	882	502	1035
C6	1185	2930	1180	3,005	1290	3355	1212	3138	1359	3563	1063	2907	1224	3873
C7	268	520	264	491	353	950	261	516	265	475	201	559	234	583
C8	1051	4610	1064	5065	1131	5065	1069	5084	995	4719	399	1881	842	5653
C9	812	2365	762	1,887	959	3138	820	1977	976	3383	613	1318	847	2035
C10	593	961	578	916	611	1082	586	920	691	1729	417	893	537	$1,\!437$
C11	1402	2936	1411	3690	1040	2273	1192	2307	1610	5084	706	1707	990	3029
C12	1424	2188	1490	2433	1639	2378	1496	2380	1374	2722	1050	1937	1248	2248

Table 2.6: Runtimes for BD Algorithm with Type 4 Cuts and Varying Lower Bounding Inequalities

2.4.2 Comparison with Deterministic Equivalent Problem

As mentioned before, BD approach, more commonly known as L-Shaped Algorithm used to solve integer stochastic programs with a continuous second stage problem, is a widely used approach due to its efficiency since it allows decomposition of the overall MIP problem into a MIP (but with only one continuous variable) and a linear program, which is separable for each scenario. Perhaps, more importantly, BD approach provides an optimality gap on the solution it provides as opposed to a heuristic approach which produces a feasible solution. Clearly, an alternative to solving our stochastic program with a given scenario set is to use a branch-and-cut approach on the DEP. Thus, we examine the performance of solving our problem using branch-and-cut as implemented in CPLEX. For this, we attempt to solve the

C1	which	is	the	smallest	size	class	of	instances	with	varying	number	of	scenarios.
Th	e result	s a	are s	ummariz	ed in	Tabl	e 2	2.7.					

	50 scei	narios	100 sce	narios	150 scer	narios
	DEP	BD	DEP	BD	DEP	BD
Ave	61.71	5.53	247.28	11.29	1239.22	24.82
Max	75.50	9.31	326.06	18.03	1479.35	30.47

Table 2.7: Comparison of Runtimes (seconds) for DEP (B&C) and BD

As reported, in the alternative approach based on DEP, while we can obtain optimal solutions to instances with 50, 100, and 150 scenarios, the solution time increases drastically starting with 100 scenarios and, with 250 scenarios, no feasible solution is generated in the B&C tree within 2 hours of runtime. Thus, we are convinced strongly that an alternative efficient approach, as we develop employing a BD framework, is needed to obtain solution to our problem of interest.

2.4.3 SAA Implementation

An approach to solve our stochastic program is to generate enough number of scenarios in a scenario set and solve the corresponding stochastic program with this set. To ensure that the scenario set utilized in such an approach is a good representation of the underlying uncertainty in corresponding parameters, one needs to ensure that stability (both in-sample and out-of-sample) and (lack of) bias requirements are met as discussed by Kaut and Wallace [24]. However, for our problem, especially the out-of sample stability and bias requirements are difficult (if not impossible) to check due to the size and complexity of the formulation and, more importantly, the lack of overall population information. Thus, we solve our problem using the SAA approach [26, 29].

SAA method selects samples from discrete distributions and approximates expected value function using selected samples. Based on approximated expected value function, the problem is solved until stopping criterion based on optimality gap estimates is satisfied. The optimal value of approximated problem converges to the optimal value of the original problem as sample size increases. Kleywegt et al. [26] suggest the required number of samples to satisfy the defined solution quality. However, authors comment that the calculation of the required number of samples is not easy. Besides, obtained number may be large for practical problems. Therefore, a sample size is determined after performing preliminary computations as performed for our problem below.

For computational experiments on SAA, we generate 3 different size of CLSC network configuration (based on C1, C2, and C3) and seek required sample size that guarantees good quality solutions with high confidence level. Sampling approach obtains the optimal value of stochastic programming problem based on estimates of Upper Bounds (UB) and Lower Bounds (LB) on the optimal value. For LB estimates, we generate 50 independent samples with varying number of scenarios including 50, 100, 150, 200, and 250 scenarios. For UB estimates, we first choose the location and capacity level solutions with the lowest objective value in calculation of LB estimates. Based on selected location decisions, we estimate UB by generating 100 independent samples each with 2000 scenarios. Based on the results summarized in Table 2.8 that shows UB and LB estimates with their 95% CI, we conclude that 250 scenarios are enough to obtain the optimal value for our problem.

SF	CTR	Customer	Scenario	LB	UB
			50	21869.3 ± 293.4	22140.3 ± 59.2
			100	21980.9 ± 216.3	22045.7 ± 56.5
10	30	60	150	21532.2 ± 163.7	22057.1 ± 49.3
			200	21947.7 ± 130.9	22032.6 ± 44.3
(C1)			250	22050.0 ± 112.1	22002.6 ± 41.8
			50	31171.0 ± 388.1	31115.7 ± 54.4
			100	31116.8 ± 271.0	31039.3 ± 54.5
10	30	90	150	31044.4 ± 206.9	31096.3 ± 53.4
			200	31130.2 ± 181.9	31118.8 ± 53.3
(C2)			250	31165.1 ± 167.9	31099.3 ± 52.8
			50	33521.2 ± 298.3	33325.6 ± 50.7
			100	33234.4 ± 283.0	33305.1 ± 49.9
10	30	120	150	33457.4 ± 229.2	33325.7 ± 49.7
			200	33312.0 ± 220.1	33277.2 ± 49.7
(C3)			250	33393.1 ± 197.7	33265.2 ± 48.5

Table 2.8: UB and LB Estimates (in 1000s) with 95% C.I.

Since the focus of our study in the methodological context is the development of an efficient solution algorithm that can be applicable to general two-stage stochastic programming, we do not discuss detailed statistical analysis any further and refer the reader to the study by Linderoth et al. [29] for details on SAA implementation for large practical problems. Rather, we employ our sample size decisions and SAA implementation presented in this section in our case study that follows.

2.4.4 Analysis on Recovery Location and Rate

In our formulation (2.1), we assume that once the return products arrive at a SF, they are first inspected and, after the disposal of some of the returns (a constant per unit disposal cost is assumed regardless of the inspection stage), remanufacturing is performed on the λ_i fraction (recovery fraction) of the total returned products to the SF *i*. However, it is not uncommon that, depending on the product, reasons for return, or resource availabilities, inspections can be handled prior to the shipments of return to a remanufacturing center at either the retailer/customer (RT) locations or the collection centers (CTR).

In this section, we consider the adoption of alternative stages (SF, CTR, and RT) in the return channel for inspection operations to take place and examine their impact in terms of network design and total system costs. In doing so, we assume different inspection costs for each stage in such a way that per unit inspection cost is least costly at the SF stage, and most costly at RT stage. We demonstrate that our modeling approach captures the need to conduct return inspections earlier in the reverse chain, but not necessarily always at the RT level. In doing so, our approach explicitly takes into account trade-offs among input cost components to determine how early the inspections should be performed on return.

To this end, we first present the modifications to model (2.1) to indicate inspection stage considered as follows:

Inspection at SF is the case where all return are to be inspected before disposal or remanufacturing at the SF location and this case is already handled by the base model (2.1). Recall that a fraction λ_i of all returned products are remanufactured and the rest is disposed. We introduce a per unit inspection cost parameter, ζ_i^S for each SF $i \in \mathcal{I}$. The parameter ζ_i^S is then introduced into the objective function as follows:

$$\begin{aligned} \text{Min} \qquad & \sum_{i \in \mathcal{I}} \left(F_i^F \, z_i + F_i^R \, y_i \right) + \sum_{j \in \mathcal{J}} F_j^C \, x_j + \sum_{i \in \mathcal{I}} \left(\psi_i^F \, \alpha_i^F + \psi_i^R \, \alpha_i^R \right) + \sum_{j \in \mathcal{J}} \left(\rho_j^F \, \beta_j^F + \rho_j^R \, \beta_j^R \right) \\ & + \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} H_\omega \left[\left(G_{ij} + \kappa_i^F \right) \sigma_{\omega ij} + \left(G_{ji} + \lambda_i \, \kappa_i^R - \lambda_i \, \kappa_i^F + \zeta_i^S \right) \tau_{\omega ji} \right] \\ & + \sum_{\omega \in \Omega} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} H_\omega \left[\left(G_{jk} + \eta_j^F \right) D_{\omega k} \, \mu_{\omega j k} + \left(G_{kj} + \eta_j^R \right) S_{\omega k} \, \nu_{\omega kj} \right] \end{aligned}$$

Inspection at CTR is the case where the inspection takes place at CTR and only the recoverable returned items are shipped to SF locations for remanufacturing. We denote the fraction of recoverable return as λ_j^H for each CTR $j \in \mathcal{J}$ with High and Low values as given above, and set the recovery rate λ_i in (2.1) to one in the modified model. An inspection cost, ζ_j^H , at CTR $j \in \mathcal{J}$ is introduced for this case. Then, the following modifications are made in the objective function (2.1a)

$$\begin{aligned} \text{Min} \qquad & \sum_{i \in \mathcal{I}} \left(F_i^F z_i + F_i^R y_i \right) + \sum_{j \in \mathcal{J}} F_j^C x_j + \sum_{i \in \mathcal{I}} \left(\psi_i^F \alpha_i^F + \psi_i^R \alpha_i^R \right) + \sum_{j \in \mathcal{J}} \left(\rho_j^F \beta_j^F + \rho_j^R \beta_j^R \right) \\ & + \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} H_\omega \left[\left(G_{ij} + \kappa_i^F \right) \sigma_{\omega ij} + \left(G_{ji} + \kappa_i^R - \kappa_i^F \right) \tau_{\omega ji} \right] \\ & + \sum_{\omega \in \Omega} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} H_\omega \left[\left(G_{jk} + \eta_j^F \right) D_{\omega k} \mu_{\omega jk} + \left(G_{kj} + \eta_j^R + \zeta_j^H \right) S_{\omega k} \nu_{\omega kj} \right] \end{aligned}$$

and the flow balance constraint (2.1j)

$$\sum_{i \in \mathcal{I}} \tau_{\omega j i} = \lambda_j^H \sum_{k \in \mathcal{K}} S_{\omega k} \nu_{\omega k j} \qquad \forall \, \omega \in \Omega, \, j \in \mathcal{J}$$

Inspection at RT is the case where inspection takes place at the first stage before

the return are shipped at the retailer/customer level. Therefore, fraction of returned products, λ_k^C , are shipped to SF locations for remanufacturing via CTRs and as in the previous case, the original recovery rate λ_i is set to one. A per unit inspection cost ζ_k^C at RT $k \in \mathcal{K}$ is introduced. To obtain a modified model for this case, we make changes in the objective function, (2.1a) to become

$$\begin{split} \operatorname{Min} & \sum_{i \in \mathcal{I}} \left(F_i^F z_i + F_i^R y_i \right) + \sum_{j \in \mathcal{J}} F_j^C x_j + \sum_{i \in \mathcal{I}} \left(\psi_i^F \alpha_i^F + \psi_i^R \alpha_i^R \right) + \sum_{j \in \mathcal{J}} \left(\rho_j^F \beta_j^F + \rho_j^R \beta_j^R \right) \\ & + \sum_{\omega \in \Omega} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} H_\omega \left[\left(G_{ij} + \kappa_i^F \right) \sigma_{\omega ij} + \left(G_{ji} + \kappa_i^R - \kappa_i^F \right) \tau_{\omega ji} \right] \\ & + \sum_{\omega \in \Omega} \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} H_\omega \left[\left(G_{jk} + \eta_j^F \right) D_{\omega k} \mu_{\omega jk} + \left(G_{kj} + \eta_j^R \right) S_{\omega k} \nu_{\omega kj} \right] \\ & + \sum_{\omega \in \Omega} \sum_{k \in \mathcal{K}} H_\omega \zeta_k^C S_{\omega k} \end{split}$$

and reverse flow constraints (2.1j) and (2.1l) to become

$$\sum_{i \in \mathcal{I}} \tau_{\omega j i} = \sum_{k \in \mathcal{K}} \lambda_k^C S_{\omega k} \nu_{\omega k j} \qquad \forall \, \omega \in \Omega, \, j \in \mathcal{J}$$
$$\sum_{k \in \mathcal{K}} \lambda_k^C S_{\omega k} \nu_{\omega k j} \leq l_j^R x_j + \beta_j^R \qquad \forall \, \omega \in \Omega, \, j \in \mathcal{J}$$

In addition to three possible inspection stages representing SF, CTR or RT, we consider two levels of recovery rates as High (H) with 80% and Low (L) with 30%. Thus, we obtain a total of 6 different problem settings where each of the 6 settings is represented using a notation with entries given as "Recovery Rate (H or L) - Inspection Stage (one of SF, CTR, and RT)."

The instance for each setting is based on the real geographical data on 263 largest cities in the US given originally by Sahyouni et al. [40] (shown as RT locations in Figure 2.2). Based on this data, as also shown in Figure 2.2, we select 40 largest cities in the US and list them in their descending order of population. Next, we select the city with the largest population in this list and delete the cities from list within 250 miles of the selected city. As a result, we obtain total 16 potential SF locations shown in Figure 2.2. For the potential CTR locations, we first select the 100 largest cities in the US. Next, we randomly pick two cities per SF city in a way that each potential SF has at least two CTRs within 500 miles. Therefore, we have total 32 potential CTR locations as shown in Figure 2.2. Lastly, all of the 263 cities are selected as RT (demand and return) locations. Other problem parameters are set as defined in Table 2.2 except for the demand data which, as in [40], we assume that the demand at an RT location is proportional to its population and they are determined randomly within intervals with high, medium or low mean values. Specific input data for this case study is also reported online at http://ise.tamu.edu/LNS/clsc-data.html.



Figure 2.2: Geographical Distribution of 263 Largest Cities in the U.S.

In order to obtain the results in the following sections, similar to above computational experiments using SAA (§ 2.4.3), we solve each of our case settings with 250 scenarios for LB estimates and 2000 scenarios for UB estimates. We set the termination criterion to a 1.5% optimality gap when solving the stochastic programs for each scenario set with our algorithm.

2.4.4.1 Cost Comparisons

In order to examine the variation of inspection costs at the SF, CTR, and RT locations, we further generate three different inspection cost configurations as 2-8-14, 4-8-12, and 6-8-10 for SF-CTR-RT locations (i.e., $\zeta_i^S - \zeta_j^H - \zeta_k^C$), respectively. Upper and lower bounds on total costs with varying inspection cost configurations and recovery rates associated with a total of 18 settings (6 for each cost configuration) are given in Figure 2.3.

First, for the 4-8-12 cost configuration, we observe that when the recovery rate is high, where most of the return need to reach to SF locations, regardless of where the inspection is performed, we expect similar transportation and operation costs. This is because there is a high amount of reverse flow that needs to reach to SF for recovery operations and the trade-offs are mostly on the differences between transportation costs, inspection costs, and capacity installation (at SF and CTR locations) costs. We observe that inspection at CTRs appears to have cost advantages over the other two inspection location options. For this configuration, cost improvement by inspections at CTRs is about 1.7% and 2.4% over inspection at SFs and RTs, respectively.

On the other hand, if the recovery rate is low and the inspection is undertaken at RT locations, a relatively large number of returned products will be disposed at that level rather than being transported further back in the chain only to be inspected and disposed at these upper echelons. Therefore, inspection at RTs prevents unnecessary location, capacity installation, processing and transportation costs associated with reverse flows. This leads to much larger savings, 12% when compared inspection at SFs, but less savings at 1.4% when compared to inspection at CTRs.



Recovery Rate(H/L) - Inspection Stage (SF/CTR/RT)

Figure 2.3: Bounds on Total CLSC Costs for 6 Case Settings for Varying Inspection Costs

To further analyze with different sets of inspection costs, we examine the settings with 2-8-14 and 6-8-10 inspection cost configurations for which, as summarized in Figure 2.3, we observe relatively same outcomes in terms of overall costs. Specifically, as the difference between the inspection costs decreases (6-8-10), in the high recovery case, the overall cost associated with inspection at CTR is closer to the one with RT inspection (1.7% difference) as opposed to inspection at SF (2.4% difference). For the low recovery, inspection at RT is still better with even larger percentages from the CTR (by 2.2%) and SF (by 13.7%) locations.

On the other hand, when the inspection costs are highly dissimilar at different stages (i.e., as in the 2-8-14 configuration), inspection at CTRs is still more cost effective under high recovery rate with the improvements over inspections at RT and SF locations being 3.1% and 0.9%, respectively. The saving over RT inspection is largely due to high costs at the RTs and, over SF, it is due to elimination of some of transportation costs. For low recovery rates, we again observe lowest costs with inspection at RTs, however the savings over CTR and SF inspections are less at 0.7% and 10.4%, respectively. Reduced savings between the inspection sat RTs and CTRs can be attributed to the trade-off between increased inspection cost at RTs and the transportation to CTRs for less expensive inspection and disposal.

In summary, we can state that inspection at either the RT or the CTR locations are more preferable. In particular, under high recovery rates, it is beneficial to utilize CTRs for inspection especially if inspection costs vary significantly based on location (as in 2-8-14 and 4-8-12 cost configurations). For the low recovery rates, inspection at RTs is more beneficial to eliminate unnecessary transportation, processing, capacity installation costs by disposing non-recoverable return earlier although inspection at CTRs can still be attractive especially when significant difference in inspection costs exists (as in 2-8-14 and 4-8-12 configurations).

2.4.4.2 Comparison of Locations

We also compare the locations of active SF and CTR locations for 6 different settings under inspection cost setting of 4-8-12 for the SF, CTR, and RT locations, respectively. Although 263 cities are distributed all over the US, cities whose population is more than 500,000 are concentrated in 5 regions, West Coast (California), South (Texas), Southeast (Florida), Midwest (Illinois-Michigan), and East Coast (New York-Pennsylvania-Massachusetts) of the US. For our case study, as expected, we obtained active SF and CTR locations distributed to the five regions as shown in Figure 2.4. Although locations of active SF and CTR are similar in all 6 settings, character of CLSC network is different to recovery rate and inspection stage. For example, in the high recovery case, if the inspection takes place at the SFs, then all of the SF locations serve as hybrid locations (for both manufacturing and remanufacturing) which is not the case for cases with inspection CTRs and RTs. The difference is more striking in the low recovery case in which, with inspections at CTRs or RTs, only two SF locations are utilized for remanufacturing while the SF inspection produced all 6 SF locations as hybrid. Inspection at an earlier stage requires less number of hybrid SF locations, as a result relatively high location costs can be reduced. In the L-SF case, all active SFs operate but mainly for inspection purposes while the real remanufacturing may take place either at lower capacities at all six locations or with higher capacities in a subset of these six locations.



Recovery rate (H/L) - Inspection stage (SF/CTR/RT)

Active SF for manufacturing

 Active SF for manufacturing/remanufacturing
 Active CTR
 Figure 2.4: Active SF and CTR Locations for 6 Case Settings

2.4.4.3 Value of Stochastic Solution

VSS is a measure to represent the importance of two-stage stochastic model [7]. VSS can be computed by the difference between EEV, the expected results of using the EV solution, and RP, the solution from two-stage stochastic problem. For our problem, we compute relative VSS $\left(\frac{(EEV-RP)}{RP}\right)$ under different cost structures to check the effectiveness of stochastic solutions. For VSS test, we first generate 10 new instances from Class 1 to Class 6. Half of the instances represent the case that the first stage costs are higher than the second stage costs, whereas the others represent vice versa. The average relative VSS values are 5.40%, 8.44%, 5.61%, 4.82%, 4.26%, and 8.37% for Class 1 to Class 6, respectively. To test the sensitivity of the VSS to cost parameters, we also obtained solutions for instances with cost split roughly at 60%-40% for the first and second stages, respectively. For those instances, the VSS was calculated as 2.5% on average, i.e., the stochastic model is more useful when costs associated with second stage are relatively higher than cost associated with first stage.

Finally, we also note that, according to Maggioni and Wallace [33], even the VSS value is large, the EV may include useful information about the RP. Although we do not provide the details here, we also examined the upgradability of EV to an RP solution via resolving our stochastic program after adding constraint to ensure that the locations opened in the EV solution are forced to be active in the RP solution which can open additional locations. The comparison is then made between the first stage solutions of stochastic program solution with input from EV solution (RPwEV) and the stochastic solution (RP). Based on stochastic program solutions using the approach presented in this study and the SAA approach, we observed similarities in solution at varying levels. For example, under H-SF setting, five of the six active SFs and four of the seven active CTRs in RPwEV solutions also appear as active in the RP solution. On the other hand, in H-CTR setting, only three active SFs in RPwEV solutions appear in the RP solutions which indicate total six locations. In terms of CTR, we again observe only about half of the locations are the same for RpwEV and RP solutions. Therefore, we conclude that the EV solution provides some information about location selection although the resulting benefit may vary depending in an uncertain way on the problem parameters.

2.5 Conclusion

In this paper, we consider a single-product capacitated integrated CLSC network design problem under unknown demand and return. Model determines location of SFs and CTRs, capacity expansion level, forward&reverse flow network to satisfy customers' demand and return such that the total closed-loop supply chain is minimized.

Since we assume random demand and return, we build develop a two-stage stochastic integer program that captures the uncertainties via a set of scenarios. For the model solution, we develop an exact solution method based on an enhanced BD algorithm. In particular, we modify standard BD algorithm to accelerate algorithm convergence by introducing surrogate constraints, strengthened Benders cuts, the use of scenario-category based multiple Benders cuts, and mean value scenario based lower bounding inequalities obtained via disaggregated dual subproblem. In our computational tests, we verify the benefits of each enhancement approach and observe that the proposed solution method performs better than standard BD algorithm in terms of runtimes under 2% optimality gap criterion.

We utilize our model and the solution approach within an SAA framework to obtain solutions to the CLSC design problem using realistic geographical data and randomly generated other input parameters to examine the effects of varying return inspection locations and recovery rates on the overall design. Although early product inspection (but not necessarily always at the RT stage) has positive effects by saving unnecessary resources and costs, our analysis also indicate that parameters such as product type and reasons for return, expected recovery rates, inspection costs, and transportation costs can be instrumental in deciding where the return product inspection should take place and, in turn, dictating the overall cost as well as the structure of the CLSC network. In these experiments, we also observe that the value of the stochastic solution can be as high as 8.44%.

This work can be extended in various ways. For example, a multiple product view, rather than a product family or a uniform return channel, can be adopted to analyze trade-offs on shared resources and operational issues. In this context, inventory decisions and inspection decisions at varying locations in the network can be considered in conjunction with network design decisions addressed in our study.
3. CHANNEL SELECTION IN COMMERCIAL PRODUCT RECOVERY LOGISTICS NETWORK UNDER TIME-VALUE CONSIDERATIONS

Many retailers such as Best Buy and Walmart have online stores in addition to their traditional (offline) stores to increase sales. This business model is called as "bricks-and-clicks". Bricks-and-clicks model offers multiple sale channels to consumers who order products from online or offline stores. Also, this model allows consumers to decide delivery method either pick-up at a local store or direct delivery to their home. Because of the convenience of shopping, the combination of online and offline stores is a common business model and this has resulted in the fast growth of online business in recent years. According to Dinlersoz and Hernandez-Murillo [11], the quarterly growth rate of Internet sales is 8.6%, whereas that of retail sale is 1.3%in 2004, i.e., the growth rate of online sales overwhelms that of retail sales. The total amount of online sales in the U.S. is \$227 billion in 2008 [36]. Therefore, the online market becomes important as much as the traditional markets. Although establishing multiple sale channels requires more investments, multiple sale strategies provide benefits to retailers. Wallace et al. [49] point out that multiple channel strategies provide various purchase opportunities to customers and this improves customer loyalty by enhancing customer services. Customer loyalty is closely related to retailer's profits, because loyal customers purchase more and are less sensitive to price [39]. Thus, it is important to manage good relationships with customers.

Multiple sale channel strategy promotes not only product sales but also product returns since customers cannot experience products' characteristics from an online purchase. In particular, many returns occur in categories in which customers need to touch and feel the products [36]. Even though product returns give additional cost burdens to retailers, they provide an opportunity to improve customer relationships. Mollenkopf et al. [35] survey 464 customers from five internet retailers and find that generous return policies improve customer loyalty by minimizing dissatisfaction. Providing multiple return channels to customers is one way to enable easy returns. For example, Best Buy guarantees a 60 day return or exchange period and suggests two return methods: return to a retailer store or return to the return center via mail. Customers choose the method that they prefer. Therefore, multiple return channels provide various options to customers, and this improves customer loyalty similar to multiple sale channels.

In the literature on multiple channels strategies and return policies, most papers focus on customer service and no guidelines exist for designing product recovery networks in the presence of multiple return channels. In general, design of a product recovery network problem has been studied over the last couple of decades because of several reasons. Economic effect is one of the main drivers establishing a recovery network. Kodak and Xerox achieve financial success through remanufacturing singleuse cameras and refillable toner cartridges, respectively [46]. Also, HP saves half of total return costs by recycling operations [23]. Therefore, managing product returns is important due to economic potential as well as customer loyalty. Fleischmann et al. [19] provide comprehensive reviews of the RSC and CLSC research on return process. Blackburn et al. [9] discuss different perspective for designing a product recovery network and suggest that MVT should be considered to maximize profits from recovered products.

As we observe, most studies focus on cost-efficient recovery networks and only a few studies discuss return process in a business perspective such as maximizing profits from recovered products. Besides, product recovery networks problem with multiple channels are relatively new in the network design literature. Therefore, we consider multiple return and redistribution channels in a product recovery logistics network. We specifically analyze how to collect products to maximize profits using multiple return channels. The model is formulated as linear program and identifies appropriate return and redistribution channels to achieve maximization of total profit.

The rest of the chapter is organized as follows. In section 3.1, we provide a review of the related literature. In section 3.2, we describe the characteristics of commercial return network and model assumptions. Next, we introduce the notation and a mathematical formulation of the problem. In section 3.3, we provide computation analysis of channel selection strategy based on the characteristics of product and logistics network. We end the chapter with summary and conclusions.

3.1 Literature Review

Most studies on product recovery network design problems are focused on EOU or EOL return. In EOU and EOL return, cost efficiency is one of the most important issues and under the purpose of cost efficiency, a centralized reverse supply chain may be a desirable solution. Fisher [15] points out that the U.S food industry bears a cost of about \$30 billion every year because of inappropriate supply chain network. The author suggests that the ideal supply chain strategies should be based on the product characteristics which can be cast as functional product or innovative product. The efficient supply chains should be used for functional products (with low uncertainty) whereas responsive supply chains are more appropriate for innovative products (with high uncertainty). Blackburn et al. [9] study an appropriate RSC design strategy for commercial product return. Unlike the previous works, they emphasize the time value of product return in reverse supply chains network design. Therefore, they conclude that responsive reverse supply chains are appropriate for products with high MVT, whereas efficient reverse supply chains are appropriate for products with low MVT. Guide et al. [23] apply Blackburn's hypothesis to a CLSC network model by considering the residual value in commercial product return. They present a CLSC network model with time delay to identify major factors that have an impact on the design strategies. Based on analysis, a product decay parameter and a proportion of non-defective product return are the major drivers in reverse supply chain design. As a result, Guide et al. [23] lead to similar conclusions that centralized (i.e., efficient) reverse network are appropriate under low product decay rate and high proportion of non-defective product return while a decentralized (i.e., responsive) reverse network may be more important at high product decay rate. This is because time delays in handling returned products can lead to a significant loss of the products' value before they can be available for resale. In conclusion, it can be stated that the characteristics of returned products must be explicitly taken into account while the configuration of a recovery network is determined.

To this end, we observe that studies on quantitative models on multi-channels are quite limited in the product recovery logistics literature. Multi-sale channels strategy in the traditional supply chain is widely studied recently, since online business has been grown up over the past years. Wallace et al. [49] point out that multiple channels strategies provide various purchase opportunities to customers and this improves customer loyalty by enhancing customer services. Multiple sale channel strategy promotes not only product sales but also product return since customers cannot experience products' characteristics from an online purchase. In particular, many returns occur in categories in which customers need to touch and feel the products [36]. Even though product return gives additional cost burdens to retailers, they provide an opportunity to improve customer relationships. Mollenkopf et al. [35] survey 464 customers from five internet retailers and find that generous return policies improve customer loyalty by minimizing dissatisfaction. Customer loyalty is closely related to retailer's profits, because loyal customers purchase more and are less sensitive to price [39]. Thus, it is important to manage good relationships with customers. In the literature on multi-channels strategies and return policies, most papers focus on customer service and no guidelines exist for designing product recovery network in the presence of multiple return channels. Alptekinoglu and Tang [2] develop a two-stage multiple channel distribution model that includes multiple depots and sales locations with stochastic demand. They propose a near-optimal distribution policies (ordering and allocation decisions) using a decomposition approach. Savaskan et al. [42] study the appropriate reverse channel structure for the collection of used products. In the paper, three different collection channels are introduced: (1) manufacturer directly collects from customers, (2) retailer collects products from customers and delivers to manufacturer, and (3) contracted third party provider collects products. Characteristics of return channels are different based on the agent of collection activity and price structure. They show how the selection of the reverse channel affects the total profits.

3.2 Problem Definition and Assumptions

The product recovery network in this chapter is motivated by the commercial return process in industries that commonly handle both manufacturing and sale, e.g., electronics industry. Once the product return occurs, the company needs to decide how to collect products (i.e., what kind of return channel to be used in collection). Generally, customers return products for two reasons: product dissatisfaction or function failure. Product dissatisfaction return has nothing to do with quality issues, thus, products are assumed to be non-defective. These returned products can be resold at the retailer after a minor operation such as inspection and repackaging. On the other hand, function failure return relates to quality issues (i.e., defective products). Therefore, defective products will be remanufactured or disposed based on degree of defect.

The model consists of four entities: customer, retailer, center, and remanufacturing facility (RF) as depicted in Figure 3.1. The location of customer, retailer are assumed to be known a priori and the model determines the return/redistribution channels to maximize the total profit from the recovery of product returns. We consider the following potential return and redistribution channels between customers and RFs:

- Return channel from the customers to the RFs via the retailers and the centers.
- Return channel from the customers to the RFs via the retailers.
- Return channel from the customers to the RFs via the centers.
- Return channel from the customers to the RFs.
- Redistribution channel from the centers to the customers.
- Redistribution channel from the RFs to the customers.
- Redistribution channel from the Rfs to the customers via the centers.

We assume that a non-defective product collected by a retailer is put back on shelf at the corresponding retailer after minor processing. On the other hand, if non-defective products are collected by centers or RFs, then they are sent back to the retailers using one of the redistribution channels. Travel time and transportation costs are assumed to be different for each type of channel. For example, a return channel from the customers to RFs via the retailers and the centers has a low transportation cost, but it takes longer transportation time and increased operation costs in general. On the other hand, a return channel from the customers to the RFs directly has high transportation costs, but it takes less transportation time and operation costs. Similarly, we introduce multiple redistribution channels from RFs to the retailers.

In the model, we introduce two types of inspections: minor and major inspection. A minor inspection occurs when products are collected by retailers. This inspection checks whether a returned product is non-defective (by also utilizing cause-of-return). If a returned product is in good condition (i.e., it can be classified as non-defective product), then it is stored at the corresponding retailer for sale. Otherwise, products are sent back to a center or RF for a detailed major inspection, which dictates decisions on remanufacturing or disposal. This inspection is assumed to occur only at a center or an RF location while remanufacturing occurs only at an RF location. Therefore, defective products requiring remanufacturing operation are sent to RF locations. In the end, the remanufactured products are sent to the secondary market for sale. We assume that enough demand exist at the retailer and the second market, thus all non-defective products are sold at the retailers and all remanufactured products are sold at the second markets. Lastly, all activities have capacity limits. The capacities at the retailer, center, and RF can be shared by all products.

Recognizing that a product's price can change over time due to factors including depreciation, technological advancements, etc., in the model, we introduce a time parameter to express product's residual value over time. That is, we assume that products lose value over time period and we define a decay parameter to express product's residual value. In particular, the longer travel time spent in the network, the less profits we expect to have from the resale.

To develop a mathematical model, we first introduce the notation and the decision variables in the network.

Sets and indices:

- \mathcal{P} set of products, $p \in \mathcal{P}$.
- \mathcal{T} set of periods in the planning horizon, $t \in T = \{1, \dots, T_{max}\}$.
- \mathcal{I} set of customer locations, $i \in \mathcal{I}$.
- \mathcal{R} set of retailer locations, $r \in \mathcal{R}$.
- \mathcal{C} set of center locations, $c \in \mathcal{C}$.
- \mathcal{M} set of RF, $m \in \mathcal{M}$.

Parameters:

- R_p^G non-defective rate for a returned product $p \in \mathcal{P}$.
- R_p^D disposal rate for a defective product $p \in \mathcal{P}$.
- D_{pti} return of product $p \in \mathcal{P}$ at customer $i \in \mathcal{I}$ in time $t \in \mathcal{T}$.
- S_{pt}^1 selling price of a new product $p \in \mathcal{P}$ in time $t \in \mathcal{T}$.
- S_{pt}^2 selling price of a remanufactured product $p \in \mathcal{P}$ in time $t \in \mathcal{T}$.
- T_{ij} travel time between nodes *i* and *j*, *i*, *j* \in { $\mathcal{I}, \mathcal{R}, \mathcal{C}, \mathcal{M}$ }.
- P_{pr}^1 processing time of product $p \in \mathcal{P}$ at retailer $r \in \mathcal{R}$.
- P_{pc}^2 processing time of product $p \in \mathcal{P}$ at center $c \in \mathcal{C}$.
- P_{pm}^3 processing time of product $p \in \mathcal{P}$ at RF $m \in \mathcal{M}$.
- G_{ij} transportation cost per unit between node *i* and *j*, *i*, *j* \in {*I*, *R*, *C*, *M*}.
- Q_r^2 redistribution capacity at retailer $r \in \mathcal{R}$.
- Q_r^1 return capacity at retailer $r \in \mathcal{R}$.
- Q_c^2 redistribution capacity at center $c \in \mathcal{C}$.
- Q_c^1 return capacity at center $c \in \mathcal{C}$.
- Q_m return capacity at RF $m \in \mathcal{M}$.
- C_{pr}^1 return cost of product $p \in \mathcal{P}$ at retailer $r \in \mathcal{R}$.
- C_{pr}^2 redistribution cost of product $p \in \mathcal{P}$ at retailer $r \in \mathcal{R}$.

- C_{pc}^1 return cost of product $p \in \mathcal{P}$ at center $c \in \mathcal{C}$.
- C_{pc}^2 redistribution cost of product $p \in \mathcal{P}$ at center $c \in \mathcal{C}$.
- C_{pm}^1 return cost of product $p \in \mathcal{P}$ at RF $m \in \mathcal{M}$.
- RE_p remanufacturing cost of product $p \in \mathcal{P}$
- CD_p disposal cost of product $p \in \mathcal{P}$

Decision Variables:

 f_{ptir}^1 return quantity of product p from the customer i to the retailer r at time t f_{ptic}^2 return quantity of product p from the customer i to the center c at time t f_{ptim}^3 return quantity of product p from the customer i to the RF m at time t f_{ptrc}^4 return quantity of product p from the retailer r to the center c at time t f_{ptrm}^5 return quantity of product p from the retailer r to the RF m at time t f_{ptcm}^6 return quantity of product p from the center c to the RF m at time t f_{ptmc}^7 quantity of non-defective product p sent from the RF m to the center c at time t f_{ptmr}^8 quantity of non-defective product p sent from the RF m to the retailer r at time t f_{ptcr}^9 quantity of non-defective product p sent from the center c to the retailer r at time t

We next develop a LP model to determine return and redistribution flows to maximize the total profit over time periods. Since flow decisions include a time parameter, flow conservation constraints at the network stage are expressed using travel and processing times.



Figure 3.1: Multi-channel Product Recovery Logistics Network Structure

$$\begin{split} \operatorname{Max} & \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{ir}+P_{pr}^{1})}^{1} \cdot R_{p}^{G} \cdot f_{ptir}^{1} + \sum_{m \in \mathcal{M}} S_{p(t+T_{mr}+P_{pr}^{1})}^{1} \cdot f_{ptmr}^{8} \right) \\ &+ \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \left(\sum_{r \in \mathcal{R}} S_{p(t+T_{cr}+P_{pr}^{1})}^{1} \cdot f_{ptcr}^{9} + \sum_{m \in \mathcal{M}} S_{p(t+T_{cm}+P_{pm}^{3})}^{2} \cdot f_{ptcm}^{6} \right) \\ &+ \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} (1 - R_{p}^{D}) \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{im}+P_{pm}^{3})}^{2} \cdot (1 - R_{p}^{G}) \cdot f_{ptim}^{3} + \sum_{r \in \mathcal{R}} S_{p(t+T_{rm}+P_{pm}^{3})}^{2} \cdot f_{ptrm}^{5} \right) \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left(\sum_{r \in \mathcal{R}} G_{ir} \cdot f_{ptir}^{1} + \sum_{c \in \mathcal{C}} G_{ic} \cdot f_{ptic}^{2} + \sum_{m \in \mathcal{M}} G_{im} \cdot f_{ptim}^{3} \right) \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}} G_{rc} \cdot f_{ptrc}^{4} + \sum_{r \in \mathcal{R}} \sum_{m \in \mathcal{M}} G_{rm} \cdot f_{ptim}^{5} + \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} G_{cm} \cdot f_{ptcm}^{6} \right) \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} G_{mc} \cdot f_{ptmc}^{7} + \sum_{m \in \mathcal{M}} \sum_{r \in \mathcal{R}} G_{mr} \cdot f_{ptim}^{8} + \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} G_{cr} \cdot f_{ptcr}^{9} \right\} \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{i \in \mathcal{I}} \left(\sum_{r \in \mathcal{R}} C_{pr}^{1} \cdot f_{ptir}^{1} + \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptir}^{2} + \sum_{m \in \mathcal{M}} C_{pr}^{1} \cdot f_{ptim}^{3} \right) \right\} \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{i \in \mathcal{I}} \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptir}^{4} + \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptir}^{2} + \sum_{c \in \mathcal{C}} C_{pt}^{1} \cdot f_{ptim}^{3} \right) \right\} \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptirc}^{4} + \sum_{m \in \mathcal{M}} C_{pm}^{1} \left(\sum_{r \in \mathcal{R}} c_{pc}^{1} \cdot f_{ptim}^{3} \right) \right\} \\ &- \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{m \in \mathcal{M}} \sum_{c \in \mathcal{C}} C_{pc}^{2} \cdot f_{ptirc}^{7} + \sum_{r \in \mathcal{R}} C_{pr}^{2} \left(\sum_{m \in \mathcal{M}} f_{ptim}^{8} + \sum_{c \in \mathcal{C}} f_{pticm}^{3} \right) \right\} \end{aligned}$$

$$-\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{M}}RE_{p}\left\{\sum_{i\in\mathcal{I}}(1-R_{p}^{D})\cdot(1-R_{p}^{G})\cdot f_{ptim}^{3}+\sum_{r\in\mathcal{R}}(1-R_{p}^{D})\cdot f_{ptrm}^{5}+\sum_{c\in\mathcal{C}}f_{ptcm}^{6}\right\}$$
$$-\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}}CD_{p}\cdot R_{p}^{D}\left\{\sum_{i\in\mathcal{I}}\sum_{c\in\mathcal{C}}(1-R_{p}^{G})\cdot f_{ptic}^{2}+\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}(1-R_{p}^{G})\cdot f_{ptim}^{3}\right\}$$
$$-\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}CD_{p}\cdot R_{p}^{D}\left(\sum_{c\in\mathcal{C}}f_{ptrc}^{4}+\sum_{m\in\mathcal{M}}f_{ptrm}^{5}\right)$$
(3.1a)

subject to

$$\sum_{r \in \mathcal{R}} f_{ptir}^1 + \sum_{c \in \mathcal{C}} f_{ptic}^2 + \sum_{m \in \mathcal{M}} f_{ptim}^3 = D_{pti} \qquad \forall p \in \mathcal{P}, \ i \in \mathcal{I}, t \in \mathcal{T}$$
(3.1b)

$$(1 - R_p^G) \sum_{i \in \mathcal{I}} f_{p(t - T_{ir} - P_{pr}^1)ir}^1 = \sum_{c \in \mathcal{C}} f_{ptrc}^4 + \sum_{m \in \mathcal{M}} f_{ptrm}^5 \qquad \forall p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}$$

$$(3.1c)$$

$$(1 - R_p^G)(1 - R_p^D) \sum_{i \in \mathcal{I}} f_{p(t-T_{ic} - P_{pc}^2)ic}^2 + (1 - R_p^D) \sum_{r \in \mathcal{R}} f_{p(t-T_{rc} - P_{pc}^2)rc}^4 = \sum_{m \in \mathcal{M}} f_{ptcm}^6 \qquad \forall p \in \mathcal{P}, \ c \in \mathcal{C}, \ t \in \mathcal{T}$$

$$(3.1d)$$

$$R_p^G \sum_{i \in \mathcal{I}} f_{p(t-T_{im}-P_{pm}^3)im}^3 = \sum_{c \in \mathcal{C}} f_{ptmc}^7 + \sum_{r \in \mathcal{R}} f_{ptmr}^8 \qquad \forall p \in \mathcal{P}, \ m \in \mathcal{M}, \ t \in \mathcal{T}$$
(3.1e)

$$R_p^G \sum_{i \in \mathcal{I}} f_{p(t-T_{ic}-P_{pc}^2)ic}^2 + \sum_{m \in \mathcal{M}} f_{p(t-T_{mc}-P_{pc}^2)mc}^7 = \sum_{r \in \mathcal{R}} f_{ptcr}^9 \qquad \forall p \in \mathcal{P}, \ c \in \mathcal{C}, \ t \in \mathcal{T}$$

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}} f_{ptir}^1 \le Q_r^1 \qquad \qquad \forall r \in \mathcal{R}$$
(3.1g)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left(\sum_{i \in \mathcal{I}} f_{ptic}^2 + \sum_{r \in \mathcal{R}} f_{ptrc}^4 \right) \le Q_c^1 \qquad \forall c \in \mathcal{C}$$
(3.1h)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{m \in \mathcal{M}} f_{ptmc}^7 \le Q_c^2 \qquad \qquad \forall c \in \mathcal{C} \qquad (3.1i)$$

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left(\sum_{m \in \mathcal{M}} f_{ptmr}^8 + \sum_{c \in \mathcal{C}} f_{ptcr}^9 \right) \le Q_r^2 \qquad \forall r \in \mathcal{R}$$
(3.1j)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left(\sum_{i \in \mathcal{I}} f_{ptim}^3 + \sum_{r \in \mathcal{R}} f_{ptrm}^5 + \sum_{c \in \mathcal{C}} f_{ptcm}^6 \right) \le Q_m \qquad \forall m \in \mathcal{M}$$
(3.1k)

$$f_{ptir}^{1}, f_{ptic}^{2}, f_{ptim}^{3}, f_{ptrc}^{4}, f_{ptrm}^{5}, f_{ptcm}^{6}, f_{ptmc}^{7}, f_{ptmr}^{8}, f_{ptcr}^{9} \ge 0$$

$$\forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, r \in \mathcal{R}, c \in \mathcal{C}, m \in \mathcal{M} \quad (3.11)$$

Objective function (4.1a) represents the total profit as the difference between total revenue and the total costs including transportation, material handling, and remanufacturing/disposal costs over time periods. The first three terms represents the revenue from non-defective products and repaired products. The fourth, fifth, and sixth terms represent the transportation costs. The fourth and fifth terms are transportation costs associated with return flows, whereas the sixth term is transportation costs associated with redistribution flows. The seventh, eighth, and ninth terms are the product handling costs associated with return and redistribution. The tenth, eleventh, and twelfth terms represent repairing and disposal costs. Constraint set (3.1b) ensures that the returned products are collected by one of the retailers, the centers, or the RFs. Constraint sets (3.1c) and (3.1d) represent the conservation of return flows at the retailers and the centers, respectively, by also taking into account the time component (considering travel and processing times) in the model Specifically, suppose that travel time from the retailer r to center c is T_{rc} and processing time at center is P_{pc}^2 . If the product p leaves from the retailer r to center c at time $(t - T_{rc} - P_{pc}^2)$, in the amount $f_{p(t - T_{rc} - P_{pc}^2)rc}^4$, then product p will arrive at the center c at time $(t - P_{pc}^2)$. After operation at center c, for P_{pc}^2 time units, the product p leaves for RF *m* at time *t*, f_{ptcm}^6 . Therefore, the relation between f_{ptrc}^4 and f_{ptcm}^6 can be expressed as $\sum_{r \in \mathcal{R}} f_{p(t-T_{rc}-P_{pc}^2)rc}^4 = \sum_{m \in \mathcal{M}} f_{ptcm}^6$. Constraint sets (3.1e) and (3.1f) show that non-defective products are sent back to retailers after major inspection. Constraint sets (3.1g) - (3.1k) ensure that return and redistribution flows to a retailer, center, and RF do not exceed their respective assigned capacity. Lastly, Constraint set (3.1l) represents the restrictions on the decision variables.

In our model, there are essentially four return channels for selection and each channel possesses different characteristics. Return channel from customers to RFs via retailer and center denoted by I-R-C-M provides the lowest transportation costs, but this channel takes long travel time because of slow travel time and operations at retailer and center locations. Return channel from customer to RFs denoted by I-M provides the fastest travel time, but transportation cost of this channel is the most expensive due to fast travel time. Return channel channel from customer to RFs via retailer, I-R-M, and channel from customer to RFs via center, I-C-M, have the intermediate characteristics between I-R-C-M and I-C-M. Both I-R-M and I-C-M have less expensive transportation costs compared to I-M, but they have longer travel time. Similarly, both I-R-M and I-C-M have more expensive transportation costs than I-R-C-M, but they have faster travel time.

If products are initially collected by retailers and RFs, then the sale of nondefective and remanufactured products occurs at those retailer and second market locations. Therefore, collecting products by retailers and RFs can minimize loss of non-defective and remanufactured product value, respectively. For this reason, we regard both I-R-M and customer-RF (I-M) channels as the *responsive return channels*. Although the return channel customer-retailer-center-RF (I-R-C-M) can also re-shelve non-defective products at corresponding retailers, for the defective products, the delivery time is long from customers to RFs. Thus, the return channel I-R-C-M focuses more on minimizing transportation and product handling costs. For this reason, we regard the return channel I-R-C-M as the *cost efficient channel*.

3.3 Computational Analysis

In this section, we present computational analysis on channel selections described above for a product recovery network. We conduct our analysis on channel selection based on two main parameters that we assume to affect a channel selection strategy. These include

- product related characteristics including decay rate (rate of value loss), expected non-defective and disposal (non-recoverable) rates and
- logistics network characteristics including the number, spread, and proximity of center and RF facilities to customers and retailers.

For the purpose of accurate analysis, we use real geographical data of cities in the U.S. and product data from Guide et al. [23]. According to 2007 U.S. population data, there are 263 cities with the population larger than one million and those cities are located in 41 states. Thus, we first pick 41 cities from 41 states, one city per state. After selecting 41 cities, there are 72 cities with the population more than two million. Lastly, we add 7 more cities based on the population, so that we have total 120 customer locations. For the retailer locations, we again select 41 cities from 41 states and add 9 more cities from the populated area, such as California, Texas, New York, and Florida. For center locations, we select 10 cities from 7 regions, North-West (Washington-Oregon), West Coast (California), South (Texas), Midwest (Illinois-Michigan-Ohio), East Coast (New York-Pennsylvania), South-East (Georgia-Florida) and Central region (Colorado-Missouri). Lastly, for facility locations, we select 4 cities in West Coast (San Jose), Midwest (Chicago),

South (Dallas), and East Coast (New York). The sets of location in consideration are depicted in Figure 3.2.



Figure 3.2: Geographical Distribution of RFs, Centers, Retailers, and Customers in the U.S.

3.3.1 Preliminary Results

First, we study how channel selections are changed in terms of product characteristics, such as decay value, non-defective and disposal rate. Guide et al. [23] apply their theoretical results to actual data from HP inkjet printer and Bosch power tool. Similar to that study, we employ the same HP and Bosch data, summarized below, to our quantitative model for channel selection. We use the CPLEX 12.4 optimization solver to solve our model.

3.3.1.1 HP Printer Case

Guide et al. [23] expect that HP collects 1,668 units of printer per day in North America. Thus, we compute daily return quantities at customer locations multiplying 1,668 by corresponding population percentages. For example, population of New York City (NYC) is 8,323,732 in 2007, which is 14.29% of total population (120) customers). Therefore, daily printer return quantities at NYC is obtained by 1,668 times 0.1429. The price of HP printer is \$200 and 15% price discount is applied to the remanufactured printer, (i.e., \$170 is the price of remanufactured printer). The remanufacturing cost is defined as 7.5% of the price of a new printer and product handling costs at each stage lie in the range 1% - 3% of the product price. The decay parameter for both new and remanufactured printers are the same and we assume that a printer loses 1.0% of its value every week. Lastly, the percentages of nondefective and disposal rate are assumed to be 33% and 10%, respectively. Figure 3.3 shows the percentage of selected return channels in HP printer case over time period daily basis. In the graph, the x-axis represents the product's life length (365 days) and the y-axis represents the percentages of return channel selected for collecting products on each day.



Figure 3.3: Channel Selection in HP Printer

According to Figure 3.3, the I-R-M channel is the major return channel since percentage is almost 50% over all time period. We consider decay rate of HP printer relatively high, i.e., time is an important factor in their collection. As expected, the responsive return channels (I-R-M and I-M) are mainly used to collect HP printer; total percentages of selection of both channels are more than 70% over product's life.

3.3.1.2 Bosch Power Tool Case

Similar to HP printer, we calculate return quantities of Bosch power tool multiplying daily return, 750, by corresponding city's population percentages. The price of Bosch power tool is \$50 and 15% price discount is applied to the remanufactured power tool (\$42.5). The rest parameter values are the same as those in HP printer case. In Bosch case, we assume that both new and remanufactured power tool lose 1.0% of its value every month. The percentages of non-defective and disposal rates are assumed to be 0% and 10%, respectively.



Figure 3.4: Channel Selection in Bosch Power Tool

Figure 3.4 shows the percentage of return channel selected by customers in Bosch power tool case. Unlike HP case, all return channels are similarly selected to collect power tools, i.e. there is no dominant return channel. In Bosch power tool case, all returned products are assumed to be defective, so RF looks a favorable location to collect products initially. However, according to Figure 3.4, the percentages of the return channel I-R-C-M is more than 30%, the highest selection percentages among the four channels. On the other hand, the return channel I-M, which regards as the responsive return channel, has the lowest selection percentages. Decay rate of Bosch power tool is relatively low, so we expect that time has less impact on the channel selection. For this reason, the cost efficient channels (I-R-C-M) are expected to be more appropriate to Bosch power tool case. Besides the channel selections are mostly consistent with time period in this result compared to HP printer case because of low decay value. In other words, products residual value is consistent with time, so once return channel is determined, decisions is rarely changed.

3.3.1.3 Consideration of Multiple Products

One of our objectives is to analyze the channel selection strategy in the presence of multiple products with different characteristics. Therefore, using the same input data as above, in our model, we consider the collection of both HP printers and Bosch power tools which are high and low decay rate products, respectively. Figure 3.5 shows the channel selections for HP and Bosch individually, after solving the model for both simultaneously. Since capacities at the retailers, centers, and RFs are shared by both products, we expect that a responsive return channel with faster travel time may be appropriate for a product with high decay rate, while a cost efficient return channel may be better for a product with low decay rate.

Comparing to the results in Figure 3.5a and Figure 3.3 for the HP case, the average percentage of channel I-M and I-R-M increases, from 46% to 47% and 24% to 26%, respectively, whereas the percentages of channels I-R-C-M, and I-C-M decrease, from 13% to 11% and from 17% to 16%. For the Bosch power tool case, i.e., Figure 3.4 vs Figure 3.5b, we observe completely the opposite results; the use of channel I-

R-C-M and I-C-M increases, whereas the use of channel I-M and I-R-M decreases. Therefore, as we expected, we conclude that if multiple products need to be collected simultaneously, then the more responsive return channels are assigned to the product with high decay rate.



Figure 3.5: Channel Selections in Multi-products

3.3.2 Channel Selection Strategies based on Product Characteristics

To obtain general insights on the sensitivity to input parameters in determining a channel selection strategy, we solve our model individually for each product with varying non-defective rates, disposal rates, and travel time.

3.3.2.1 Analysis on Disposal Rate

In the model, we assume that disposal decisions are made only after major inspection at centers or RFs. Thus, if remanufacturing or disposal decisions are made earlier, then unnecessary transportation and product handling costs can be saved. For this reason, we expect that the centers or RFs are the favorable locations to collect products for which the disposal rate is typically high. For HP printer case, we fix the non-defective rate (pr_p^G) as 33% and examine the disposal rates (pr_p^D) of 10%, 30%, and 50%. Figure 3.6a shows the channel selections of HP printer case under different disposal rate. The use of return channel I-R-C-M increases and the use of return channel I-R-M decreases with disposal rate. The return channel I-R-C-M and I-C-M can save unnecessary costs through determining disposal at the center locations. Especially, the return channel I-R-C-M pursues both maximizing profits by non-defective products and minimizing transportation costs. Although selection of I-R-M is still the highest percentages, return channel I-R-C-M becomes popular option as disposal rate increases. Similarly, we fix the non-defective rate as 0% and change the disposal rates to 10%, 30%, and 50% in Bosch case. By the same reason in HP case, the percentage of return channel I-R-C-M and I-C-M increases as disposal rate increases. Both I-R-C-M and I-C-M channels save transportation and handling costs by shipping products from customers to RFs via intermediate locations, i.e. both channels pursue cost-efficient. According to Figure 3.6b, more than half of customers select both channels for product returns and their selection percentages increase with disposal rate. Therefore, we again conclude that cost-efficient return channel is appropriate to Bosch power tool case.

3.3.2.2 Analysis on Non-defective Rate

A returned product is resold at the retailer, after minor processing, if it is identified as non-defective. Therefore, if non-defective rate is high, it is expected that retailers will initially collect returns to avoid unnecessary costs. To analyze impact of non-defective rate in channel selections, we fix the disposal rate (R_p^D) as 10% and consider non-defective rates (R_p^G) of 10%, 33%, and 50% in HP printer case.

Figure 3.7a shows the channel selection in HP printer under different non-defective rates. Low non-defective rate means that most products are defective and the major inspection is required. Thus, if we set the non-defective rate to 10%, the return channel I-M is a major channel in collection. On the other hand, high non-defective rate



Figure 3.6: Channel Selections under Different Disposal Rate

means that most returned products are non-defective products and these products are resold at the retailers. Thus the return channel I-R-M dominates other channels, especially if non-defective rate is relatively high. For Bosch power tool case, we fix the disposal rate as 10% and consider the non-defective rates as 0%, 30%, and 50%. According to Figure 3.7b, all four different return channels are used at relatively significantly throughout product life-cycle for collection in 0% non-defective rate. Unlike HP printer case, if non-defective rate is high, more than 30%, then the return channel I-R-C-M is the major return channel in Bosch power tool case. Decay rate of Bosch power tool is low, so time parameter is not a critical factor in channel selection. According to results, the percentage of I-R-C-M is much higher than that of I-R-C-M in HP case as expected.



Figure 3.7: Channel Selections under Different Non-defective Rate

3.3.2.3 Analysis on Time

In our modelling, we assume that product loses value over time. Therefore, if product's decay rate is relatively high, then the return channel with less travel time is an appropriate channel to minimize product's residual value loss. In this section, we analyze the impact of time in channel selection of both HP printer and Bosch power tool by changing processing time at retailers, centers, and RFs. Previously, processing time at retailer (P_{pr}^1) , center (P_{pc}^2) , and RF (P_{pm}^3) are defined as 7, 10, and 21 days, respectively and we change processing time to 5, 7, and 15 days (i.e., decrease by 30%).

Figure 3.8 shows the channel selections of HP and Bosch with less travel time which is obtained via decreasing the sojourn time of returned products in return process. For the HP case, in this new setting, the return channel I-C-M can handle both non-defective and defective products more quickly. Besides, the unnecessary transportation costs can be saved by disposing returned products earlier. By the same reason, the return channel I-R-C-M handles products more quickly. Thus, the average percentage of return channels I-R-C-M and I-C-M increases, from 13% to 17% and from 17% to 19%, respectively, whereas the percentage of return channel I-R-M and I-M decreases. On the other hand, channel selections are not changed for Bosch power tool case when compared to Figure 3.4. Therefore, we conclude that time parameter is one of the important factors in HP case as a determinant of return channel selection, while it has little impact in the Bosch power tool case, mainly due to its low decay value.



(a) HP Printer (b) Bosch Power Tool

Figure 3.8: Channel Selection of HP and Bosch with less Travel Time

3.3.3 Channel Selection Strategies based on Logistics Network Characteristics

We analyze the return channel selection strategy based on the product characteristics in the previous section. We conclude that channel selection decisions are changed with product characteristics. However, channel selection decisions are also affected by product recovery networks. For example, customers in New York, generally return products to RF directly, since RF is located in their close vicinity, i.e., the return channel I-M is selected. On the contrary, customers in Phoenix, return products to their closest retailers, since neither RFs nor centers do not exist in close proximity. Thus, they send products to RF via intermediate locations, either using retailers (I-R-M) or using retailers and centers (I-R-C-M). In this section we examine how a product recovery network affects return channel selections. For problem data, we use the same value from HP printer case, except for RF, center, and retailer locations. For comparing channel selection strategies with different product recovery networks, we define three different network configurations by changing number RFs, centers, and retailers.



Figure 3.9: Location of RFs, Centers, and Retailers under Different Geographic Scheme

Previously, we use geographical data with 4 RFs, 10 centers, 50 retailers, and 120 customers. We regard this network configuration as Network-M (N-M). Next, we decrease number of RFs, centers, and retailers to 3, 8, and 20, respectively, and obtain a smaller test network called as N-S. Lastly, we increase number of RFs, and centers to 5 and 12, respectively, to generate a larger test network called as N-L. The sets of locations in these three networks are depicted in Figure 3.9. The grey small dots represent the locations of RFs, centers, and retailers in N-S. The blue larger dots are added to N-S to obtain the set of RF, center, and retailer locations in N-M. Lastly, the largest red dots are included in N-M to obtain the locations in N-L.

First, we analyze HP printer case (30% Non-defective / 10% Disposal rate) with three different networks, N-S, N-M and N-L and obtain the results summarized in Figure 3.10a. Based on results, we observe that channel selections are different to network configurations. As the network becomes larger, the percentage of channel I-R-C-M decreases while the percentages of I-R-M and I-M increase. For example, more than 20% of customers return products use the return channel I-R-C-M under N-S. Customers in Dallas do not have RF and retailer under N-S, so they return their products to a close retailer located in Oklahoma City. However, they return products directly to RFs under N-M and N-L since both retailers and RFs are located in Dallas. In other words, return channel can be affected by the logistical network configuration, specifically by the locations of retailers, centers or RFs. Generally, if customers are close to RFs, then the return channel I-M is likely to be selected due to savings in transportation and handling costs. On the other hand, if retailers are close to customers but RFs are less scattered and far from the customer locations, then products are first returned to retailers. Also, return channel choice between I-R-C-M and I-R-M for a retailer largely depends on whether the retailer has a close by RF or not. In summary, we observe that the return channel can be affected by the network structure which dictate the proximity of RFs, centers, and retailers to customer locations. As customers have more retailers and RFs in their vicinity, the channels I-R-M and I-M become cost efficient channels and are selected more frequently. On the other hand, we also recognize that, since HP printer is assumed to have a high decay rate, choice of channels I-R-M and I-M are also responsive channels (as mentioned in the previous section) and , thus also desired due to product characteristics.

Similar to the HP printer case, we analyze Bosch power tool case (at 0% Nondefective and 10% Disposal rates) under three different networks with results summarized in Figure 3.10b. As we have observed earlier, all four channels are selected similarly in all three networks. The percentage of channel I-R-C-M decreases as the percentages of I-R-M increase with increasing network size. Under network N-S, almost 40% of customers use return channel I-R-C-M, but, in the network N-L, only 20% of customers select return channel I-R-C-M. As more RFs and retailers are included in the network, more customers select return channel I-R-M instead of return channel I-R-C-M. For example, customers in Houston return power tools via channel I-R-C-M under network N-S, since they do not have RFs or centers in their vicinity. However, once RFs and retailers are located in Dallas, they change channel from I-R-C-M to I-R-M for return. That is, return channel I-R-M becomes more costefficient channel for customers in Houston under network N-L. Unlike return channel I-R-M, selection percentage of return channel I-M is similar in all three networks. The responsive return channel (I-M) is utilized less frequently for Bosch power tool case because a highly responsive channel is not needed due to low decay rate for the product value, only for customers who reside near RFs I-M is chosen as the return channel. Observing that the selection of return channel is affected by both product characteristics and recovery networks configuration, next we analyze channel selection by incorporating the interaction of these main return channel determinants by considering them simultaneously.



Figure 3.10: Channel Selection of HP and Bosch under Different Networks

3.3.4 Channel Selection under Different Networks and Product Characteristics

To analyze channel selections under general product characteristics (including decay, non-defective, and disposal rates) in conjunction with recovery network characteristics (including the locations of retailers, centers, and RFs), we use the same three recovery network settings as shown in Figure 3.9. Furthermore, we define two levels for decay, non-defective and disposal rates as being high and low. For decay rate, we consider a product value loss of 1.0% in a day (high) or in a week (low). Similarly, we consider 10% (low) or 50% (high) for both non-defective and disposal rates.

Table 3.1 shows the average percentages of selected return channels and per-

centage increases in objective values (total profit) as the network becomes larger (Small-to-Medium and Medium-to-Large) under different product and network characteristics.

Decay (H/L)	Low Non-defective - Low Disposal					Low Non-defective - High Disposal				
Network (L/M/S)	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %
L-L	0.0%	22.2%	28.7%	49.1%	0.67	0.0%	15.6%	43.5%	40.9%	1.15
L-M	4.1%	27.0%	23.2%	45.7%	1.24	8.8%	23.3%	32.0%	35.9%	1.48
L-S	3.3%	30.3%	32.0%	34.5%	-	14.0%	19.3%	37.6%	29.2%	-
H-L	0.0%	16.6%	23.4%	60.0%	14.88	0.5%	14.7%	38.9%	45.9%	105.93
H-M	3.4%	24.4%	18.4%	53.7%	5.44	7.8%	20.8%	31.2%	40.1%	31.15
H-S	7.1%	21.5%	27.2%	44.2%	-	11.5%	15.1%	40.5%	32.9%	-
Decay (H/L)	High Non-defective - Low Disposal					High Non-defective - High Disposal				
Network (L/M/S)	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %
L-L	16.9%	70.4%	0.9%	11.8%	0.19	46.9%	44.8%	1.8%	6.4%	0.21
L-M	26.3%	58.7%	2.1%	12.9%	0.51	52.2%	35.4%	3.6%	8.7%	0.36
L-S	42.5%	48.3%	2.8%	6.4%	-	61.7%	29.3%	5.8%	3.1%	-
H-L	17.5%	71.4%	4.0%	7.2%	1.16	38.2%	52.5%	3.1%	6.1%	1.19
H-M	24.5%	62.4%	4.1%	9.0%	4.84	44.8%	43.7%	4.0%	7.5%	5.11
H-S	30.3%	52.7%	7.2%	9.9%	-	46.2%	37.9%	7.3%	8.5%	-

Table 3.1: Average Percentage of Selected Channel and Objective Value under Different Product and Network Characteristics

3.3.4.1 Observations under High Non-defective Rates

1. We first notice that, when the non-defective rate is high, the channels I-R-M and I-R-C-M are heavily utilized. That is, the returned product mostly reach to a retailer location first and the non-defective ones (which are large in number) are put back on the shelf after minor processing. Further, if disposal rate of defective products is low, I-R-C-M is utilized significantly less than I-M since it unnecessarily introduces one extra stop (at a center) before processing at an RF location. On the other hand, in addition to non-defective rate, if disposal rate is high as well, then channel I-R-C-M and I-R-M are both significantly

utilized.

- 2. In both of the high non-defective cases, regardless of the product value decay rate, as the network size increases, the channel I-R-M use increases while the I-R-C-M use decreases. This is because the increase network size improves proximity of RFs to retailer and renders I-R-M as a more cost-efficient channel when compared to I-R-C-M. A similar trend is observed when I-C-M and I-M are compared as well.
- 3. We further observe that, within low decay rate groups, regardless of the disposal rate level, the objective function values improve only slightly as the network becomes larger. For example, the total profit increases by 0.19% and 0.51% as the network size changes small-to-medium and medium-to-large, respectively. We observe larger improvements under high decay rate, that is, larger spread of RF and center locations induces responsiveness and help to improve profits more significantly than the low decay rate products.
- 3.3.4.2 Observations under Low Non-defective Rates
 - 1. In this case where there is a high number of returned products requiring significant rework at RF locations (more so when disposal rate is low), the channel I-R-C-M is least utilized and the channels I-M and I-C-M are the most significantly employed ones. If the disposal rate is low, I-M is use is significantly more than I-C-M and I-R-M, especially in larger networks, due to benefits of direct shipment to RFs for remanufacturing. This is more pronounced in the high decay rate case where a responsive channel such as I-M is more beneficial. On the other hand, when the disposal rate is high, use of I-C-M increases significantly due to the opportunities to dispose early without bearing additional transportation and handling costs. We also observe that, for small and medium

networks, use of I-R-C-M also increases due to its cost efficiency in the lack of proximity to RF locations.

- 2. In both of the low non-defective cases, as the network size increases, the use of channel I-M increases while the use of I-R-C-M and I-C-M decreases. This is because the increased network size improves proximity to RFs thus making I-M a cost-efficient channel when compared to I-C-M and I-R-C-M. This holds regardless of the product value decay rate, perhaps an exception is in low decay rate with high disposal case in which the use of I-C-M increase with larger networks due to small amounts of time-insensitive returned products needing rework.
- 3. Furthermore, in terms of the objective value (profit) changes, we observe that, in the low decay rate case, slight improvements are obtained as the network size increases. This is because while most of the returned products are defective and need to be worked on, they do not lose much value in time and, thus, do not require extensive networks for realizing their value in logistically cost efficient manner. On the other hand, if the decay rate is high, profit improvements can be quite substantial when the network size is increased due to the fact that a large network (with many center and RF locations) provides the ability to process returns both faster and cheaper.

3.3.5 Analysis of Presetting Channel Selections

From the perspective of company's own operations, it may appear desirable to fix the return channel to one of the channels that we formulate above in our model. As observed in the above analysis, this may lead to very high profit losses depending on the product and network characteristics. Thus, in this section, we specifically examine cases where a company decides the return method versus a company allows customers to decide the return method. For this analysis, we employ the medium sized network N-M introduced above.

Our original model represents the first case when a company determines the return channel based on model selection as in previous sections. For the second case, we modify original model by restricting channel selection i.e., we force the model to collect products using only a single return channel, I-R-C-M, I-R-M, I-C-M, or I-M. For the purposes of comparison under varying product characteristics, we define four decay rates including product value loss of 1% per day (1D), per week (1W), per two weeks (2W), and per three weeks (3W). We consider three different non-defective and disposal rates as 10%, 30%, and 50%, with other data (demand, cost, price, etc.) based on HP printer case. For each combination of these decay, non-defective, and disposal rates, we first find the optimal solution and return channel selections based on our original model. Next, we obtain the optimal objective values by restricting the return channel to one of four channels and examine their closeness to the objective value of the original model. In Figure 3.11, OptGap (%) shows the percentage decrease in the optimal solution (profit decrease) if the corresponding specific channel is employed over optimum solution of our original model. Our observation can be summarized as follows:

- 1. In Figure 3.11, notice that the profit decrease values are significantly larger for the high decay rate (1D) case (note the the differences in scale in y-axis). It is clear that the average profit decrease due to return channel fixing is the largest for high decay rate case in which it is particularly large for low non-defective rates.
- 2. As an overall trend, we observe that profit decrease values with I-R-M and I-R-C-M decrease with increased non-defective and disposal rates, i.e., the forcing of the returns to retailer locations for immediate re-shelving and quick access

to disposal at a center or RF (if not re-shelved) provide the least profit loss (over the optimum solution. I-M and I-C-M appear to be behaving in a similar fashion, however, their profit decrease values are typically much higher than the ones with I-R-C-M and I-C-M and an improvement is observed only when the non-defective rates increase from 10% to 30%.

3. For each fixed non-defective rate group, we observe that profit decrease increases as the disposal rate increases. The reason for this is also related to the impact of disposal on the revenue, i.e., high disposal rate of returned products leads to lost revenues, and thus to lower profits.

In summary, ad hoc choice of a channel for returns always introduces high profit loss which is very significant especially for the products with high value decay rate. Next, in Figure 3.12, we present profit decrease (over the optimum channel selection)



Figure 3.11: Optimal Solution Gap Percentage under Different Decay Value

due to channel presetting with respect to varying value decay rates including 1% decay rate per 3 days (3D) and per 5 days (5D). Again noting the scale differences in y-axis, we observe that the profit decrease is generally lower with channel fixing when the non-defective rate is high with presetting to e channels I-R-M and I-M providing smaller profit losses. As mentioned above, these channels provide quick re-shelving of non-defective products if not disposed and, otherwise, cost efficient disposal at centers and RFs. On the other hand, when the non-defective rates are low, most of the products need to be re-worked, if not disposed, channel presetting causes large losses in profit, especially for high decay rate products.

Overall, we clearly observe that optimal channel selection, rather than an ad hoc presetting, is critical for high value decay rate products, e.g., 1D case, but not insignificant, in terms of profit losses even for low decay rate products, although the rate of profit decrease diminishes quickly with increasingly low value decay rates.



Figure 3.12: Optimal Solution Gap under Different Non-defective and Disposal Rate

3.4 Conclusions

When products are collected, product and recovery network characteristics should be considered in order to maximize their residual value which has direct impact on overall profit.

In the previous sections, we analyze the return channel selection strategy based on product and network characteristics. First, we analyze how product characteristics affect the channel selection. We find that the product decay parameter is one of the important factors in channel selection. The product with high decay rate (HP printer) requires return channel with faster travel time so that the product can be sold with relatively high price. On the other hand, the product with low decay rate (Bosch power tool) prefers less costly return channels. Also, the non-defective and disposal rates have impact on the return channel selections. Generally the final destination of the returned products is determined based on product's condition and we can expect the condition of product based on information on non-defective and disposal rates. For example, if product's non-defective rate is relatively high, then retailer should initially collect product so that products can can be resold at the retailer right away thereby avoiding unnecessary costs. We observe that our model captures this trade-off between time and cost involved in effective value recovery from returned products.

Next, we study return channel selection strategy with different product recovery networks and observe relation between customer locations and return channel selection. Although percentage of selected return channel varies with product characteristics, the products are generally collected at locations (retailer, center or RF) in close proximity to customers. Product with high non-defective rate should be returned to retailers, so considering extra retailers is more effective, whereas RF locations are more importance in the collection of product with low non-defective rate.

Lastly, we observe certain situations that a company should determine the return channel decision. First, if returned products have high decay value, then a company should manage return channel for minimizing profit loss. Also, if product's nondefective rate is low, then a company should take care of products for minimizing profit loss from defective products.

4. NETWORK DESIGN FOR COMMERCIAL PRODUCT RETURNS UNDER TIME-VALUE CONSIDERATIONS

In the previous chapter, we considered commercial product returns in recovery logistics networks to determine the best channel selection strategy for maximizing profit from recovered products. As expected, both the product characteristics and recovery logistics network configurations affect channel selection decisions. We observeed that total profit and channel selections vary depending on the recovery network configuration. The next natural question, then, is to ask "what is the best recovery logistics configuration for profit maximization?" To answer this question, we extend the model for return channel selection in commercial product recovery logistics networks by introducing location decisions associated with RFs and centers. As reviewed in section 3.1, most product recovery network design problems do not consider the commercial product return case. In other words, product residual value is largely ignored in models and analysis in the previous studies.

The objective of this chapter is to study the return channel selection and network design problems in an integrated fashion for the commercial product return case. We formulate the model as MILP and in order to solve large size instances efficiently, we develop a solution approach based on the SA heuristic algorithm. The SA heuristic algorithm is a well-known approach for solving optimization problems. In the SA algorithm, a feasible solution is evaluated many times to check the goodness of solution. However, if an optimization solver, such as CPLEX, is used for evaluation, it takes an excessively long solution time. Therefore, we propose an alternative evaluation method, called the greedy algorithm. We compare performance of the developed SA algorithm against an exact solution method, BD, using randomly generated in-
stances. In our experiments, we observe that the developed SA algorithm solves the problem efficiently in terms of solution time and solution quality as benchmarked against the upper bound information from the BD solution.

The rest of this chapter is organized as follows. In section 4.1, we introduce the notation and mathematical formulation of the problem. In section 4.2, we propose heuristic solution methodology along with the solution representation, the greedy algorithm for evaluating the objective function value, a construction heuristic, and an improvement heuristic. We end the section with an outline of the BD reformulation. In section 4.3, we provide computational results of the heuristic solution and in section 4.4, we analyze recovery logistics network design and channel selection strategy based on the real product data from HP and Bosch. We end this chapter with summary and conclusions.

4.1 Problem Definition and Assumptions

The problem definition and assumptions are covered in section 3.2, except for location decisions. The model consists of four entities: customer, retailer, center, and remanufacturing/repairing facility (RF) as depicted in Figure 4.1. The locations of customers and retailers are assumed to be known a priori and the model determines RF/center locations and the return/redistribution channels to maximize the total profit from the recovery of product returns.



Figure 4.1: Multi-channel Product Recovery Logistics Network Structure with Location Decisions

To develop a mathematical model, we first explain newly introduced problem parameters and decision variables in the model.

Additional Parameters:

- F_c fixed cost of opening a center at location $c \in \mathcal{C}$.
- F_m fixed cost of opening a RF at location $m \in \mathcal{M}$

Additional Decision Variables:

- $x_c = 1$ if center c is open, 0 otherwise.
- y_m 1 if RF *m* is open, 0 otherwise.

We develop a MILP model for the problem. The model determines locations of RFs/centers and redistribution/return flows to maximize the total profits over a product's life cycle.

$$\begin{aligned} \operatorname{Max} \qquad \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{T}} \sum_{r \in \mathcal{R}} \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{ir}+P_{pr}^{1})}^{1} \cdot R_{p}^{G} \cdot f_{ptrr}^{1} + \sum_{m \in \mathcal{M}} S_{p(t+T_{mr}+P_{pr}^{1})}^{1} \cdot f_{ptmr}^{8} \right) \\ + \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \left(\sum_{r \in \mathcal{R}} S_{p(t+T_{cr}+P_{pr}^{1})}^{1} \cdot f_{ptcr}^{9} + \sum_{m \in \mathcal{M}} S_{p(t+T_{cm}+P_{pn}^{3})}^{1} \cdot f_{ptcm}^{6} \right) \\ + \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} (1 - R_{p}^{D}) \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{irr}+P_{pm}^{3})}^{2} \cdot (1 - R_{p}^{G}) \cdot f_{ptim}^{3} + \sum_{r \in \mathcal{R}} S_{p(t+T_{rm}+P_{pm}^{3})}^{2} \cdot f_{ptrm}^{6} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \left(1 - R_{p}^{D} \right) \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{irr}+P_{pm}^{3})}^{2} \cdot (1 - R_{p}^{G}) \cdot f_{ptim}^{3} + \sum_{r \in \mathcal{R}} S_{p(t+T_{rm}+P_{pm}^{3})}^{2} \cdot f_{ptrm}^{6} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left(\sum_{r \in \mathcal{R}} G_{ir} \cdot f_{ptir}^{1} + \sum_{c \in \mathcal{C}} G_{ic} \cdot f_{ptir}^{2} + \sum_{m \in \mathcal{M}} G_{im} \cdot f_{ptim}^{3} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}} G_{rc} \cdot f_{ptirc}^{1} + \sum_{r \in \mathcal{R}} \sum_{m \in \mathcal{M}} G_{rm} \cdot f_{ptim}^{8} + \sum_{c \in \mathcal{C}} \sum_{r \in \mathcal{R}} G_{cr} \cdot f_{ptic}^{9} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{i \in \mathcal{I}} \sum_{r \in \mathcal{R}} G_{mc} \cdot f_{ptirc}^{1} + \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptirc}^{1} + \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptirc}^{1} + \sum_{c \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptirc}^{3} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{i \in \mathcal{I}} \sum_{r \in \mathcal{C}} C_{pc}^{1} \cdot f_{ptirc}^{4} + \sum_{m \in \mathcal{M}} C_{pr}^{2} \left(\sum_{r \in \mathcal{R}} C_{pm}^{1} \cdot f_{ptirc}^{3} \right) \right\} \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} R_{cc} C_{pc}^{2} \cdot f_{ptirc}^{2} + \sum_{r \in \mathcal{R}} C_{pr}^{2} \left(\sum_{m \in \mathcal{M}} S_{ptirm}^{9} \right) + \sum_{c \in \mathcal{C}} S_{ptirm}^{9} + \sum_{c \in \mathcal{C}} S_{ptirm}^{9} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} R_{cc} C_{pc}^{2} \cdot f_{ptirc}^{7} + \sum_{m \in \mathcal{M}} C_{pc}^{2} \left(\sum_{m \in \mathcal{M}} S_{ptirm}^{9} \right) + \sum_{c \in \mathcal{C}} S_{ptirm}^{9} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} C_{p} \cdot R_{p}^{D} \left\{ \sum_{i \in \mathcal{I}} \sum_{c \in \mathcal{C}} (1 - R_{p}^{0}) \cdot f_{ptirc}^{3} + \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} (1 - R_{p}^{0}) \cdot f_{ptirm}^{3} \right) \\ - \sum_{p \in \mathcal{P}} \sum_{t \in$$

subject to

$$\sum_{r \in \mathcal{R}} f_{ptir}^1 + \sum_{c \in \mathcal{C}} f_{ptic}^2 + \sum_{m \in \mathcal{M}} f_{ptim}^3 = D_{pti} \qquad \forall p \in \mathcal{P}, \, i \in \mathcal{I}, t \in \mathcal{T}$$

$$(4.1b)$$

$$(1 - R_p^G) \sum_{i \in \mathcal{I}} f_{p(t - T_{ir} - P_{pr}^1)ir}^1 = \sum_{c \in \mathcal{C}} f_{ptrc}^4 + \sum_{m \in \mathcal{M}} f_{ptrm}^5 \qquad \forall p \in \mathcal{P}, r \in \mathcal{R}, t \in \mathcal{T}$$

$$(4.1c)$$

$$(1 - R_p^G)(1 - R_p^D) \sum_{i \in \mathcal{I}} f_{p(t - T_{ic} - P_{pc}^2)ic}^2 + (1 - R_p^D) \sum_{r \in \mathcal{R}} f_{p(t - T_{rc} - P_{pc}^2)rc}^4 = \sum_{m \in \mathcal{M}} f_{ptcm}^6 \qquad \forall p \in \mathcal{P}, \, c \in \mathcal{C}, \, t \in \mathcal{T}$$

$$(4.1d)$$

$$R_p^G \sum_{i \in \mathcal{I}} f_{p(t-T_{im}-P_{pm}^3)im}^3 = \sum_{c \in \mathcal{C}} f_{ptmc}^7 + \sum_{r \in \mathcal{R}} f_{ptmr}^8 \qquad \forall p \in \mathcal{P}, m \in \mathcal{M}, t \in \mathcal{T}$$

$$(4.1e)$$

$$R_p^G \sum_{i \in \mathcal{I}} f_{p(t-T_{ic}-P_{pc}^2)ic}^2 + \sum_{m \in \mathcal{M}} f_{p(t-T_{mc}-P_{pc}^2)mc}^7 = \sum_{r \in \mathcal{R}} f_{ptcr}^9 \qquad \forall p \in \mathcal{P}, \ c \in \mathcal{C}, \ t \in \mathcal{T}$$

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}} f_{ptir}^1 \le Q_r^1 \qquad \qquad \forall r \in \mathcal{R}$$
(4.1g)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left(\sum_{i \in \mathcal{I}} f_{ptic}^2 + \sum_{r \in \mathcal{R}} f_{ptrc}^4 \right) \le Q_c^1 \cdot x_c \qquad \forall c \in \mathcal{C}$$
(4.1h)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \sum_{m \in \mathcal{M}} f_{ptmc}^7 \leq Q_c^2 \cdot x_c \qquad \forall c \in \mathcal{C}$$
(4.1i)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left(\sum_{m \in \mathcal{M}} f_{ptmr}^8 + \sum_{c \in \mathcal{C}} f_{ptcr}^9 \right) \le Q_r^2 \qquad \forall r \in \mathcal{R}$$
(4.1j)

$$\sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left(\sum_{i \in \mathcal{I}} f_{ptim}^3 + \sum_{r \in \mathcal{R}} f_{ptrm}^5 + \sum_{c \in \mathcal{C}} f_{ptcm}^6 \right) \le Q_m \cdot y_m \qquad \forall m \in \mathcal{M}$$
(4.1k)

$$x_c, y_m \in \{0, 1\},\$$

$$f_{ptir}^{1}, f_{ptic}^{2}, f_{ptim}^{3}, f_{ptrc}^{4}, f_{ptrm}^{5}, f_{ptcm}^{6}, f_{ptmc}^{7}, f_{ptmr}^{8}, f_{ptcr}^{9} \ge 0$$

$$\forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, r \in \mathcal{R}, c \in \mathcal{C}, m \in \mathcal{M} \quad (4.11)$$

Objective function, (4.1a), consists of total profit, total location, transportation, material handling, and repair/disposal costs over time periods. The first three terms represent the profit from the sale of non-defective and recovered products. The fourth, fifth, and sixth terms represent transportation costs. The fourth and fifth terms are transportation costs associated with return flows, whereas the sixth term is transportation costs associated with redistribution flows. The seventh, eighth, and ninth terms are the product handling costs associated with return and redistribution. The tenth and eleventh terms represent repair and disposal costs. Finally, the last two terms represent the fixed costs associated with opening the centers and the RFs. Constraint set (4.1b) ensure that products will be collected by one of the retailers, the centers, or the RFs. Constraint set (4.1c) and (4.1d) represent the conservation of return flows at the retailers and the centers, respectively. Flow conservation constraints are expressed using travel and processing time. Constraint set (4.1e) and (4.1f) show that non-defective products are sent back to retailers. Constraint sets (4.1g) - (4.1k) ensure that return and redistribution flows to a retailer and center do not exceed their respective assigned capacity. Lastly, constraint set (4.11) represents restrictions on the decision variables.

In our model, there are essentially four return channels for selection and each channel possesses different characteristics, I-R-C-M, I-R-M, I-C-M, and I-M. If we, however, consider return channel together with redistribution channel, we have a total of five channels. Based on model assumptions, there are three redistribution channels, from centers to retailers, C-R, from RFs to retailers via centers, M-C-R, and from RFs to retailers, M-R. Redistribution channels M-C-R and M-R send nondefective products from return channel I-M and redistribution channel C-R sends non-defective products from return channel I-C-M. For non-defective products from return channels, I-R-C-M and I-R-M, we do not consider additional redistribution flows, since those products are re-shelved at the corresponding retailer locations. Therefore, by considering both return and redistribution channels simultaneously, we have a total of five channels: I-R-C-M, I-R-M, I-C-M/C-R, I-M/M-C-R, and I-M/M-R.

4.2 Solution Methods

In this section, we propose two methods to solve the developed problem: a heuristic solution method and an exact solution method. First, we describe the heuristic method including the initial solution construction, the method for evaluating the goodness of a solution, and the neighborhood solution search. The proposed heuristic method is based on the SA algorithm, which is a well-known heuristic method for solving optimization problems. The SA was originally proposed by Kirkpatrick et al. [25] and has been widely used to solve optimization problems since the 1980s. The idea of SA originates from the annealing process in the production of metal. Kirkpatrick et al. [25] show the similarities between annealing process and the combinatorial optimization problem that searches for a global optimum. In the annealing process, a substance is heated first and then is slowly cooled down, so that it becomes a stronger structure. Similarly, the SA seeks for better solutions through an annealing process algorithm. In the annealing process, the SA uses a stochastic approach to allow for the degradation of a solution. For example, the SA accepts moves that improve solutions during the solution search. The SA also accepts moves that degrade solutions with a probability calculated with current temperature and the amount of degradation of solutions. This solution degradation helps solutions to escape from local optimum. The annealing process continues until the number of iterations reaches predefined iteration termination number and the algorithm terminates, if current temperature is less than predefined final temperature. SA solution

quality generally depends on the algorithm's cooling rate and the number of iterations in the algorithm.

RSC/CLSC network design problems are well-known NP-hard problems, thus some research has proposed a heuristic approach as their solution method [12, 27, 43, 44]. The SA has advantages against other heuristic methods, since the SA algorithm is relatively easy to implement and obtains a good quality solution quickly [14]. Pishvaee et al. [38] consider multi-stage reverse logistics network with limited capacities. They propos using a SA heuristic algorithm with priority-based encoding method and special neighborhood search mechanisms. Lee and Dong [28] develop a two-stage stochastic programming model for dynamic reverse logistics network design. To enhance solution performance of a SAA method, the authors use the SA algorithm.

4.2.1 Simulated Annealing Heuristic

Before implementing the SA algorithm, we define a solution vector representing the location choices in a solution. The solution has total of m potential RFs and c potential centers in the model. Thus, a solution vector is represented by an array of size (m + c). If certain locations of the RF and centers are open, then the corresponding element in the solution vector has value 1; otherwise, it has value 0.

•	m]	RFs —		c centers				
1	0	••••	0	0	••••	1	0	

Figure 4.2: A Sample Representation of the Feasible Solution.

Once the RF/center locations are determined, the original MILP model becomes a LP model. The resulting LP problem can be solved using an optimization solver, such as CPLEX. However SA evaluates the goodness of solutions many times, so using an optimization solver may take an excessively long solution time. Therefore, we propose a greedy algorithm to solve the LP model. The LP model determines return and redistribution network flows that maximize total profit from the recovered products. We expect that among the five channels (I-R-C-M, I-R-M, I-C-M/C-R, I-M/M-C-R, and I-M/M-R), the most profitable channel is used for product return and redistribution as long as capacity constraints are satisfied. In order to maximize profits, we need to increase total revenue or decrease total costs. However, it is not easy to determine whether increasing total revenue or decreasing total cost contributes more to the objective function. For example, decreasing total cost may contribute more to the objective function if a product's decay value is relatively low. Therefore, we propose using a greedy algorithm to calculate the most profitable channel. In this greedy algorithm, we first compute the minimum total cost flows for all five channels based on available customers, retailers, centers and RFs in the current network. Because, under the same channel type, less transportation cost generally indicates less travel time. Thus, we can minimize a product's value loss at the retailer and the second market locations. For the each channel type, we compute the minimum total cost flow using Dijkstra's algorithm based on the current logistics network. Once five candidate channels are obtained, one for each channel type, we calculate the unit profit of products for five candidate channels and select the most profitable channel. The flow amounts in the selected channel are set by the minimum of return amounts at the retailer, or capacities in the channel.

The procedure for greedy algorithm is shown in Algorithm 2.

Algorithm 2 Greedy Algorithm

1: Obtain a solution vector and initialize the parameters. 2: for each $r \in \mathcal{R}$ do $\operatorname{CAP}_{r}^{F} = Q_{r}^{F}, \operatorname{CAP}_{r}^{R} = Q_{r}^{R} \text{ and } \mathcal{S}^{R} = \{r\} \cup \mathcal{S}^{R}.$ 3: 4: end for 5: if center $c \in \mathcal{C}$ (RF $m \in \mathcal{M}$) is open then $\operatorname{CAP}_{c}^{F} = Q_{c}^{F}, \operatorname{CAP}_{c}^{R} = Q_{c}^{R} (\operatorname{CAP}_{m} = Q_{m}) \text{ and } \mathcal{S}^{C} = \{c\} \cup \mathcal{S}^{C} (\mathcal{S}^{M} = \{m\} \cup \mathcal{S}^{M}).$ 6: 7: else $\operatorname{CAP}_{c}^{F} = 0$ and $\operatorname{CAP}_{c}^{R} = 0$ ($\operatorname{CAP}_{m} = 0$). 8: 9: end if 10: Create set $\mathcal{P}' = \mathcal{P}$, and label elements in the decreasing order of price. 11: while $\mathcal{P}' \neq \emptyset$ do Select $p \in \mathcal{P}'$ with the lowest label. 12:13:for each $t \in \mathcal{T}$ do for each $i \in \mathcal{I}$ do 14: $\operatorname{RT}_i = D_{pti} \text{ and } \mathcal{S}^I = \{i\} \cup \mathcal{S}^I.$ 15:16:end for while $\mathcal{S}^{I} \neq \emptyset$ do 17:for each $s \in S^Q$ do 18:19:Calculate MTCF_s Based on $MTCF_s$, $PROFIT_s \leftarrow (Revenue_s - MTCF_s)$. 20: end for 21: 22: $s^* = \arg \max_{s \in \mathcal{S}^Q} \{ s | \text{PROFIT}_s \}.$ Denote i^*, r^*, c^* , and m^* as the customer, retailer, center, and RF in the channel 23: s^* , respectively. $\text{flow}_{s^*} = \min \left[\text{RT}_{i^*}, \text{CAP}_{r^*}^F, \text{CAP}_{r^*}^R, \text{CAP}_{c^*}^F, \text{CAP}_{c^*}^R, \text{CAP}_{m^*} \right].$ 24:Update $Z = Z + P_{best} \times \text{flow}_{s^*}$. 25:Update $\operatorname{RT}_{i^*} = \operatorname{RT}_{i^*} - \operatorname{flow}_{s^*}, \operatorname{CAP}_k^F = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, \operatorname{CAP}_k^R = \operatorname{CAP}_k^F - \operatorname{flow}_{s^*}, k \in \{r^*, c^*\}, k \in \{r^*, c^*\},$ 26: $\operatorname{CAP}_{k}^{R} - \operatorname{flow}_{s^{*}}, k \in \{r^{*}, c^{*}\}, \text{ and } \operatorname{CAP}_{m^{*}} = \operatorname{CAP}_{m^{*}} - \operatorname{flow}_{s^{*}}.$ if $\operatorname{CAP}_{r^*}^F = 0$ OR $\operatorname{CAP}_{r^*}^R = 0$ then 27: $\mathcal{S}^R = \mathcal{S}^R \setminus \{r^*\}$. Repeat the same process for c^* and m^* . 28:29:end if if updated $RT_{i^*} = 0$ then 30: $\mathcal{S}^I = \mathcal{S}^I \setminus \{i^*\}.$ 31: 32: end if 33: end while 34:end for 35: $\mathcal{P}' = \mathcal{P}' \setminus \{p\}.$ 36: end while 37: **Report** results.

Before explaining the SA algorithm, we introduce the basic parameters of the SA algorithm as follows.

SA Algorithm Parameters:

T_0	: Initial temperature.
Т	: Current temperature.
T_f	: Freezing temperature.
CS	: Cooling rate of current temperature.
k	: Number of iteration at each temperature.
K^m	: Iteration termination number at each temperature.
X_0	: Initial solution.
X	: Current solution.
X_{nh}	: Neighborhood of current solution.
X_{best}	: Best solution obtained from algorithm.
C(X)	: objective function value of solution X.

The proposed SA heuristic consists of two parts: constructive heuristic and improvement heuristic.

4.2.1.1 Constructive Heuristic

We construct the initial solution (X_0) in this stage. First, we randomly select RFs until total capacities of selected RFs are larger than total return quantities over a product's life cycle. Next, we randomly select two centers for open locations.

4.2.1.2 Improvement Heuristic

In this stage, we improve the current solution by modifying locations of RFs and centers. For both RF and center locations, we use three different moves: **Move1**, **Move2**, and **Move3**. We randomly select a candidate move among the three moves to find the neighborhood of the current solution, X_{nh} .

Move1: Two randomly selected open locations are closed and two randomly selected closed locations are opened.

Move2: A randomly selected closed location is opened.

Move3: A randomly selected open location is closed.

If total capacities of RF in X_{nh} are less than the total number of returned products, the algorithm opens additional RF until capacity conditions are satisfied. Similarly, if the current solution does not include any active center, then **Move3** is automatically excluded from selection. Once a neighborhood solution, X_{nh} , is obtained, the goodness of solution is evaluated based on the greedy algorithm shown in Algorithm 2. The overall SA algorithm outlines are given in Algorithm 3.

Algorithm 3 Procedure for SA Algorithm 1: Initialize algorithm parameters (T, T_f, CS, K^m, k) and construct X_0 . 2: Compute $C(X_0)$ and set $X_{best} = X_0$, $X = X_0$. 3: while $T < T_f$ do Generate X_{nh} using Move1, Move2, or Move3. 4: Evaluate $C(X_{nh})$ using Greedy algorithm. 5:Let $\Delta C = C(X_{nh}) - C(X)$ 6: if $\Delta C \geq 0$ then 7: $X = X_{nh}$. If $C(X_{nh}) > C(X_{best})$, $X_{best} = X_{nh}$. 8: 9: else $y \leftarrow \text{UNIF}(0,1), z = e^{-\frac{\Delta C}{T}}$. If $y < z, X = X_{nh}$ 10: end if 11: Update current iteration, k = k + 1. 12:if $k \leq K^m$ then 13:Go to line 4. 14:15:else 16:Go to line 18 17:end if 18:Initialize iteration k = 0. $T = CS \times T$ 19:20: end while 21: **Report** results.

4.2.2 Benders Decomposition Method

For benchmarking the proposed SA heuristic algorithm, we solve the problem using BD algorithm. We first describe the BD subproblem, the associated dual subproblem, and the master problem.

4.2.2.1 Benders Subproblem and Its Dual

The primal subproblem, denoted by $SP(f^1, f^2, \ldots, f^9 | \hat{x}, \hat{y})$, is obtained based on determined location decisions.

$$\begin{aligned} \operatorname{Max} \quad & Z_{SP} \quad = \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}} \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{ir}+P_{pr}^{1})}^{1} \cdot R_{p}^{G} \cdot f_{ptir}^{1} + \sum_{m \in \mathcal{M}} S_{p(t+T_{mr}+P_{pr}^{1})}^{1} \cdot f_{ptmr}^{8} \right) \\ & + \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}} \left(\sum_{r \in \mathcal{R}} S_{p(t+T_{cr}+P_{pr}^{1})}^{1} \cdot f_{ptcr}^{9} + \sum_{m \in \mathcal{M}} S_{p(t+T_{cm}+P_{pm}^{3})}^{2} \cdot f_{ptcm}^{6} \right) \\ & + \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} (1 - R_{p}^{D}) \left(\sum_{i \in \mathcal{I}} S_{p(t+T_{im}+P_{pm}^{3})}^{2} \cdot (1 - R_{p}^{G}) \cdot f_{ptim}^{3} + \sum_{r \in \mathcal{R}} S_{p(t+T_{rm}+P_{pm}^{3})}^{2} \cdot f_{ptrm}^{5} \right) \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} \left(\sum_{r \in \mathcal{R}} G_{ir} \cdot f_{ptir}^{1} + \sum_{c \in \mathcal{C}} G_{ic} \cdot f_{ptic}^{2} + \sum_{m \in \mathcal{M}} G_{im} \cdot f_{ptim}^{3} \right) \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}} G_{rc} \cdot f_{ptrc}^{4} + \sum_{r \in \mathcal{R}} \sum_{m \in \mathcal{M}} G_{rm} \cdot f_{ptrm}^{5} + \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} G_{cm} \cdot f_{ptcn}^{6} \right) \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} G_{mc} \cdot f_{ptirc}^{1} + \sum_{r \in \mathcal{R}} \sum_{m \in \mathcal{M}} G_{mr} \cdot f_{ptirm}^{8} + \sum_{c \in \mathcal{C}} \sum_{m \in \mathcal{M}} G_{cm} \cdot f_{ptcr}^{9} \right) \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{i \in \mathcal{I}} \left(\sum_{r \in \mathcal{R}} C_{i}^{1} \cdot f_{ptir}^{1} + \sum_{c \in \mathcal{C}} C_{i}^{1} \cdot f_{ptir}^{2} + \sum_{m \in \mathcal{M}} C_{i}^{1} \cdot f_{ptir}^{3} \right) \right\} \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \left\{ \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}} C_{i}^{1} \cdot f_{ptir}^{4} + \sum_{m \in \mathcal{M}} C_{i}^{1} \left(\sum_{r \in \mathcal{R}} c_{i}^{1} \cdot f_{ptir}^{4} \right) \right\} \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} \sum_{c \in \mathcal{C}} C_{i}^{2} \cdot f_{ptirc}^{7} + \sum_{r \in \mathcal{R}} C_{i}^{2} \left(\sum_{m \in \mathcal{M}} c_{i}^{8} - c_{i}^{6} - c_{i}^{6} \right) \right\} \\ & - \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{m \in \mathcal{M}} R_{E_{p}} \left\{ \sum_{i \in \mathcal{I}} (1 - R_{p}^{D}) \cdot (1 - R_{p}^{O}) \cdot f_{ptim}^{3} + \sum_{c \in \mathcal{C}} (1 - R_{p}^{O}) \cdot f_{ptrm}^{5} + \sum_{c \in \mathcal{C}} f_{ptcm}^{6} \right\} \right\}$$

$$-\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}}CD_{p}\cdot R_{p}^{D}\left\{\sum_{i\in\mathcal{I}}\sum_{c\in\mathcal{C}}(1-R_{p}^{G})\cdot f_{ptic}^{2}+\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}(1-R_{p}^{G})\cdot f_{ptim}^{3}\right\}$$
$$-\sum_{p\in\mathcal{P}}\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}CD_{p}\cdot R_{p}^{D}\left(\sum_{c\in\mathcal{C}}f_{ptrc}^{4}+\sum_{m\in\mathcal{M}}f_{ptrm}^{5}\right)$$
(4.2)

subject to
$$(4.1b) - (4.1k)$$

 $f_{ptir}^1, f_{ptic}^2, f_{ptim}^3, f_{ptrc}^4, f_{ptrm}^5, f_{ptcm}^6, f_{ptmc}^7, f_{ptmr}^8, f_{ptcr}^9 \ge 0$
 $\forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, r \in \mathcal{R}, c \in \mathcal{C}, m \in \mathcal{M}$

The optimal solution of $SP(\cdot)$ provides return channels and redistribution channels with maximum total profit. Therefore, the original problem can be expressed as follows.

Max
$$SP(f^1, f^2, \dots, f^9 | \hat{x}, \hat{y}) - \sum_{c \in \mathcal{C}} F_c \cdot x_c - \sum_{m \in \mathcal{M}} F_m \cdot y_m$$
 (4.3)
subject to $x_c, y_m \in \{0, 1\}, \forall c \in \mathcal{C}, m \in \mathcal{M}$

We define dual variables $\pi_{pti}^1, \pi_{ptr}^2, \pi_{ptc}^3, \pi_{ptm}^4, \pi_{ptc}^5, \pi_r^6, \pi_c^7, \pi_c^8, \pi_r^9$, and π_m^{10} for constraints (4.1b)-(4.1k), respectively. Then **Dual subproblem** is obtained as

$$\operatorname{Min} \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} D_{pti} \cdot \pi_{pti}^{1} + \sum_{r \in \mathcal{R}} \left(Q_{r}^{1} \cdot \pi_{r}^{6} + Q_{r}^{2} \cdot \pi_{r}^{9} \right) + \sum_{c \in \mathcal{C}} \hat{x}_{c} \left(Q_{c}^{1} \cdot \pi_{c}^{7} + Q_{c}^{2} \cdot \pi_{c}^{8} \right) \\
+ \sum_{m \in \mathcal{M}} Q_{m} \cdot \hat{y}_{m} \cdot \pi_{m}^{10}$$
(4.4a)

subject to

$$\pi_{pti}^{1} + (1 - R_{p}^{G}) \cdot \pi_{p(t+T_{ir}+P_{pr}^{1})r}^{2} + \pi_{r}^{6} \ge R_{p}^{G} \cdot S_{p(t+T_{ir}+P_{pr}^{1})}^{1} - G_{ir} - C_{pr}^{1}$$
$$\forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, r \in \mathcal{R} \qquad (4.4b)$$

$$\pi_{pti}^{1} + (1 - R_{p}^{G}) \cdot (1 - R_{p}^{D}) \cdot \pi_{p(t+T_{ic}+P_{pc}^{2})c}^{3} + R_{p}^{G} \cdot \pi_{p(t+T_{ic}+P_{pc}^{2})c}^{5} \ge +\pi_{c}^{7}$$

$$- G_{ic} - C_{pc}^{1} - (1 - R_{p}^{G}) \cdot R_{p}^{D} \qquad \forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, c \in \mathcal{C}$$
(4.4c)

$$\pi_{pti}^{1} + R_{p}^{G} \cdot \pi_{p(t+T_{im}+P_{pm}^{3})m}^{4} + \pi_{m}^{10} \ge (1 - R_{p}^{G}) \cdot (1 - R_{p}^{D}) \cdot \left(S_{p(t+T_{im}+P_{pm}^{3})}^{2} - RE_{p}\right) - (1 - R_{p}^{G}) \cdot R_{p}^{D} \cdot CD_{p} - G_{im} - C_{pm}^{1} \qquad \forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, m \in \mathcal{M}$$
(4.4d)

$$-\pi_{ptr}^{2} + (1 - R_{p}^{D}) \cdot \pi_{p(t+T_{rc}+P_{pc}^{2})c}^{3} + \pi_{c}^{7} \ge -G_{rc} - C_{pc}^{1} - R_{p}^{D} \cdot CD_{p}$$
$$\forall p \in \mathcal{P}, t \in \mathcal{T}, r \in \mathcal{R}, c \in \mathcal{C} \qquad (4.4e)$$

$$-\pi_{ptr}^{2} + \pi_{m}^{10} \ge (1 - R_{p}^{D}) \cdot (S_{p(t+T_{rm}+P_{pm}^{3})}^{2} - RE_{p}) - G_{rm} - C_{pm}^{1} - R_{p}^{D} \cdot CD_{p}$$
$$\forall p \in \mathcal{P}, t \in \mathcal{T}, r \in \mathcal{R}, m \in \mathcal{M} \quad (4.4f)$$

$$-\pi_{ptc}^{3} + \pi_{m}^{10} \ge S_{p(t+T_{cm}+P_{pm}^{3})}^{2} - G_{cm} - C_{pm}^{1} - RE_{p}$$
$$\forall p \in \mathcal{P}, t \in \mathcal{T}, c \in \mathcal{C}, m \in \mathcal{M} \qquad (4.4g)$$

$$-\pi_{ptm}^4 + \pi_{p(t+T_{mc}+P_{pc}^2)c}^5 + \pi_c^8 \ge -G_{mc} - C_{pc}^2 \quad \forall p \in \mathcal{P}, t \in \mathcal{T}, m \in \mathcal{M}, c \in \mathcal{C} \quad (4.4h)$$

$$-\pi_{ptm}^4 + \pi_r^9 \ge S_{p(t+T_{mr}+P_{pr}^1)}^1 - G_{mr} - C_{pr}^2 \qquad \forall p \in \mathcal{P}, t \in \mathcal{T}, r \in \mathcal{R}, m \in \mathcal{M}$$
(4.4i)

$$-\pi_{ptc}^{5} + \pi_{r}^{9} \ge S_{p(t+T_{cr}+P_{pr}^{1})}^{1} - G_{cr} - C_{pr}^{2} \qquad \forall p \in \mathcal{P}, t \in \mathcal{T}, r \in \mathcal{R}, c \in \mathcal{C}$$
(4.4j)

$$\pi_{pti}^{1}, \pi_{ptr}^{2}, \pi_{ptc}^{3}, \pi_{ptm}^{4}, \pi_{ptc}^{5} \text{ unrestricted, } \pi_{r}^{6}, \pi_{c}^{7}, \pi_{c}^{8}, \pi_{r}^{9}, \pi_{m}^{10} \ge 0$$
$$\forall p \in \mathcal{P}, t \in \mathcal{T}, i \in \mathcal{I}, r \in \mathcal{R}, c \in \mathcal{C}, m \in \mathcal{M}$$
(4.4k)

4.2.2.2 Benders Master Problem

The master problem, denoted by $MP(x, y | \hat{\pi}^1, \hat{\pi}^2, \dots, \hat{\pi}^{10})$, can be obtained from the overall formulation given with objective (4.3). For this, we replace the first term representing the subproblem objective with a function of the auxiliary variable (Θ) to be employed in constructing the Benders cut in an iteration. The following master problem includes auxiliary variable and BENDERS CUTSET.

Max
$$Z_{MP} = \Theta - \sum_{c \in \mathcal{C}} F_c \cdot x_c - \sum_{m \in \mathcal{M}} F_m \cdot y_m$$
 (4.5a)
subject to $x_c, y_m \in \{0, 1\}, \forall c \in \mathcal{C}, m \in \mathcal{M}$

$$\sum_{m \in \mathcal{M}} Q_m \cdot y_m \ge \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{I}} D_{pti}$$
(4.5b)

BENDERS CUTSET

Constraint set (4.5b) is a surrogate constraint to ensure that the recovery logistics network design from the master problem has enough RF capacity to handle all returned products. As a result, the feasibility of the $SP(\cdot)$ is always guaranteed. We summarize the overall BD algorithm in Algorithm 4.

Algorithm 4 BD Algorithm

1: Initialize Z^* , UB, Itr, gap values, and set BENDERS CUTSET empty 2: Solve $MP(x, y|\cdot)$ 3: Set $UB = Z_{MP}$ 4: while $(UB - LB)/UB \ge gap do$ 5: Itr=Itr + 1Solve DSP to obtain $\pi_{pti}^1, \pi_{ptr}^2, \pi_{ptc}^3, \pi_{ptm}^4, \pi_{ptc}^5, \pi_r^6, \pi_c^7, \pi_c^8, \pi_r^9, \pi_m^{10}$ 6:Calculate $LB = Z_{MP} - \Theta + Z_{SP}$ 7: if $(LB > Z^*)$ then 8: $Z^* = \mathsf{LB}$ 9: end if 10: Update BENDERS CUTSET in the master problem and Solve $MP(x, y|\cdot)$ 11: Set $UB = Z_{MP}$ 12:13: end while 14: Solve $SP(f^1, f^2, \dots, f^9 | \hat{x}, \hat{y})$ 15: Report results

4.3 Computational Study

In this section, we present computational experiments of the SA algorithm. To evaluate performance of the SA algorithm, we compare SA solutions to exact solutions (BD) in terms of solution time and quality. The BD algorithm is run until a solution gap of 2% (or better) is reached. The BD algorithm is implemented using C++ programming language and CPLEX Concert Technology. All implicit MIPs (master problem and dual subproblem) are solved using CPLEX 12.4 (64-bit). Similarly, the SA algorithm is implemented using C++ programming language. Both algorithms are run on a computer with a 3.4GHz Intel i7-3770 CPU and 32 GB RAM.

We first develop a testbed of random data instances to analyze the performance of the proposed SA algorithm. Test instances are generated under two data settings (Set H: High location costs, and Set L: Low location costs) by altering the number of products $|\mathcal{P}|$, the number of potential RFs $|\mathcal{M}|$, the number of potential centers $|\mathcal{C}|$, the number of retailers $|\mathcal{R}|$, and the number of customer locations $|\mathcal{I}|$, as shown in Table 4.1. Under Set H, location costs of RFs are assumed to be 5000 times more expensive than the highest product's price, while location costs of centers are 50% of RF location costs. Under Set L, location costs of RFs are assumed to be 1000 times more expensive than the highest product's price, while location costs of centers are 50% of RF location costs. We create 10 random instances for each class.

Next, we use uniform distributions to randomly generate a product's selling price (S_{pt}^N) , return quantities (D_{pti}) , non-defective rate (R_p^G) , and disposal rate (R_p^D) outlined in Table 4.2. The rest of the problem parameters are generated based on the HP printer case shown in section 4.4.

4.3.1 Sensitivity Analysis of SA Parameters

In order to implement the proposed SA algorithm efficiently, we first conduct a sensitivity analysis of SA parameters. Based on the preliminary experiment, 110,000 (T) and 10,000 (T_f) are proper values of the initial and freezing temperatures, re-

Class	$ \mathcal{T} $	$ \mathcal{P} $	$ \mathcal{I} $	$ \mathcal{R} $	$ \mathcal{C} $	$ \mathcal{M} $	Location Costs
C1	180	2	5	10	40	80	Н
C2	180	2	5	10	80	160	Н
C3	180	4	5	10	40	80	Н
C4	180	4	5	10	80	160	Н
C5	180	2	10	20	40	80	Н
C6	180	2	10	20	80	160	Н
C7	180	2	5	10	40	80	L
C8	180	2	5	10	80	160	\mathbf{L}
C9	180	4	5	10	40	80	L
C10	180	4	5	10	80	160	\mathbf{L}
C11	180	2	10	20	40	80	\mathbf{L}
C12	180	2	10	20	80	160	L

Table 4.1: Problem Classes Used in Computational Testing

Parameter	Distribution
S_{pt}^N	Uniform[50, 200]
D_{pti}	Uniform[500, 1,000]
R_p^G	Uniform[0.2, 0.6]
R_p^D	Uniform[0.2, 0.6]

Table 4.2: Distribution for Price, Demand, Non-defective and Disposal Rate Values.

spectively. We test the number of iterations (IT) and the cooling rate (CS) in the following analysis by solving the sample problems: Classes 1 and 2 in Table 4.1. Test values of the iteration number (IT) are set as 5, 10, and 15, while the test values of the cooling rate (CS) are set at 0.95, 0.9, and 0.85. Table 4.3 shows the performance of the SA algorithm under different IT and CS values. Before analyzing results, if the solution gap, (BD upper bound - SA solution)/(BD upper bound)*100, is less than or close to 2.5%, then the obtained SA solution is viewed as good. According to the results, the SA algorithm provides good quality solutions under six settings. We observe that among the six settings, SET-5, when CS and IT value are set as 0.9 and 10, results are the best in terms of run times. Therefore we set the values of IT

	CS	IT	Ave Run	times (Seconds)	Ave Opt gap $(\%)$		
	05	11	Class 1	Class 2	Class 1	Class 2	
SET-1		5	697	4715	2.43	2.41	
SET-2	0.95	10	1330	8812	2.31	2.35	
SET-3		15	2542	17083	2.21	2.25	
SET-4		5	407	2517	3.21	3.52	
SET-5	0.9	10	654	4281	2.48	2.51	
SET-6		15	1292	8431	2.41	2.42	
SET-7		5	235	1513	4.12	4.31	
SET-8	0.85	10	438	2925	3.11	3.32	
SET-9		15	884	5829	2.50	2.47	

and CS as 0.9 and 10, respectively, for further experiments.

Table 4.3: Solution Gaps and Algorithm Runtimes for Different SA Parameters

4.3.2 Performance of Heuristic Algorithm

We summarize the computational results of the proposed greedy algorithm in terms of solution quality and SA algorithms in terms of solution time and quality. The performance of the greedy algorithm is benchmarked against the solution from CPLEX. We use total of 60 instances from C1 to C6 classes as seen in Table 4.1. For each instance, we construct a flow networks problem based on randomly fixed RF and center locations. Obtained flow networks problem is a LP problem and we solve this problem using CPLEX and the greedy algorithm. Table 4.4 shows the solution gap between greedy algorithm and CPLEX, calculated as (CPLEX solution - greedy solution)/(CPLEX solution)*100. According to the results, the greedy algorithm provides close solution to CPLEX solution. Under all six classes, on average, the solution gap is less than 1%. Even in the worst case, the solution gap is close to 1%. In short, Table 4.4 demonstrates the effectiveness of the greedy algorithm, so we use the greedy algorithm for a solution evaluation.

Class	Ave Sol Gap (%)	Max Sol Gap (%)
C1	0.80	1.01
C2	0.66	0.84
C3	0.87	1.22
C4	0.82	1.13
C5	0.74	1.10
C6	0.85	1.17

Table 4.4: Solution Gaps of CPLEX and Greedy Algorithm

Next, we conduct experiments the performance of the SA algorithm. Once the SA algorithm terminates, we build a flow networks problem using locations of RFs/centers from the best SA solution. Again, we solve constructed flow networks problem using CPLEX to improve solution quality and update total profits considering location costs. Table 4.5 shows the solution gap and solution times (seconds) of the SA and the BD algorithms. The solution gap of the SA algorithm is calculated as (BD upper bound - SA solution)/(BD upper bound)*100 and solution gap of the BD algorithm is calculated as (BD upper bound - BD lower bound)/(BD lower bound)*100.

We observe that the proposed SA algorithm is efficient, since it provides good solutions within a reasonable time. On average, the solution gap of SA algorithm is under 3% or close to 3%. For classes C5, C6, C11, and C12, the BD algorithm fails to solve the problems within a reasonable amount of time. Thus, we employ an additional stopping criterion; We terminate the BD algorithm, if the solution time exceeds 8 hours (28800 seconds) or if a 2% solution gap is reached, whichever comes first. In these four classes, the SA algorithm finds better solutions in a short period of time, when compared to the BD algorithm. Although the solution time of the SA algorithm does not improve much in comparison to the solution time of BD on the

average, SA always has shorter solution times than the BD algorithm. Moreover, the worst-case of SA performs much better than the worst-case of BD. For example, in class C4, the BD algorithm takes 29278 seconds to obtain a solution in the worst-case. On the other hand, the SA algorithm only takes 9764 seconds to solve the same problem in the worst-case. In short, the computational results justify the use of the proposed SA algorithm to solve the developed model.

	Ave Ru	n Times (seconds)	Max Ru	n Times (seconds)	Ave C	Opt Gap (%)	Max Opt Gap (%)	
	SA	BD	SA	BD	SA	BD	SA	BD
C1	693.7	1032.2	772.9	1854.7	1.85	1.58	2.23	1.96
C2	5143.9	8191.8	6368.7	18323.0	1.94	1.52	2.26	1.96
C3	1877.0	3927.7	4262.2	18404.2	1.95	1.62	2.84	1.96
C4	9238.0	11952.2	9763.8	29278.9	1.84	1.63	2.10	1.98
C5	904.3	28800 Limit	935.4	-	2.81	3.73	3.77	5.82
C6	5192.6	28800 Limit	6099.6	-	2.98	4.29	3.69	6.43
C7	691.2	1780.6	755.7	9333.5	1.77	1.66	2.01	1.97
C8	5141.7	7073.5	5585.8	26181.7	1.82	1.63	2.40	1.97
C9	2011.7	4374.2	2361.6	11604.5	1.92	1.59	2.75	1.95
C10	8859.3	10991.8	9161.3	13513.7	1.95	1.67	2.67	1.96
C11	1134.6	28800 Limit	1904.1	-	3.02	4.43	4.82	7.53
C12	6190.4	28800 Limit	6502.8	-	3.01	5.21	4.18	7.57

Table 4.5: Runtimes and Solution Gaps of SA and BD Algorithm

4.4 Case Study: HP Printer and Bosch Power Tool

For the purpose of accurate analysis, we use the same geographical data of cities in the U.S and the real data from HP and Bosch shown in section 3.3. The location sets in consideration are depicted in Figure 4.3: 5 potential RFs, 12 potential centers, 50 retailers, and 120 customers.



(c) Retailer Locations (d) Customer Locations

Figure 4.3: Geographical Distribution of Potential RFs, Potential Centers, Retailers, and Customers in the U.S.

In this section, we study how active RF/center locations and return channel selections change with different location costs. As previously observed, return channel selection is affected by logistics networks as well as by product characteristics. In this research, we consider location decisions for RF and center to identify the best recovery logistics networks configuration and return channel selection strategies. Location decisions vary depending on fixed location cost and product profit. Therefore we introduce four different location cost settings based on HP printer prices, LC-A, LC-B, LC-C, and LC-D. For example, fixed RF costs under the LC-A setting are assumed to be 5000 times more than the price of a HP printer. Similarly, we define fixed RF costs in LC-B, LC-C, and LC-D settings as being 2500, 1250, and 500 times more expensive than the HP printer price, respectively. We also assume that fixed center costs are 50% of fixed RF costs in their location costs setting. The objective function of this model is to maximize total profit from recovered products. In order to achieve this objective, we must increase revenue or decrease costs. However, product characteristics and logistics network largely determine whether increasing revenue or decreasing costs is more beneficial to the objective function. Therefore, considering the location cost settings, we expect to identify the importance of revenue and costs in the optimal objective function value.

We first analyze the number of active RFs and centers in HP, Bosch, and multiproduct solutions. Table 4.6 shows the number of active RFs and centers in the HP printer, Bosch power tool and multi-product (HP & Bosch) solutions, respectively. Not surprisingly, we have more active RFs and centers in the network as location costs decrease. If location costs are relatively high, a minimum number of RFs that can handle total products are open. However as locations cost decrease, more RFs are shown in the network.

Recovery logistics network design is different from product characteristics, such as price and decay value. For example, in the Bosch power tool solutions, generally fewer number of active RFs are located in the network when compared to HP printer solutions. We expect the recovery logistics network design between the HP printer and Bosch power tool to be different. The decay value of the Bosch power tool is much lower than the decay value of the HP printer. Thus, more active RFs are required in the HP printer case, so that the residual value of the recovered HP printer is as high as possible. However the value of the Bosch power tool change little over time, so minimizing total system costs is more important than maximizing revenue. For this reason, if location costs are relatively high, then only a minimum required number of RFs are shown in the solution of the Bosch power tool case. In this problem, we use the real data value with the HP printers and Bosch power tools, to make an accurate analysis of the return channel selection strategy. The characteristics of printers and power tools are totally different, so sharing the same RF location and resources for repair may not be realistic. However, in real business situations, many companies provide a wide range of product lines to increase sales. For example, Apple produces various computers ranging from low-price laptops to high-end desktops. Characteristics of these products, such as decay value and price, are different, but the required resources for repairing these products may be the same. Thus, we expect that our assumption about sharing the same center and RF location for multi-products can be applied to a real business model. Unlike active RF locations, the number of active centers in the Bosch power tool solutions is larger than the number of active centers in the HP printer solutions. Return channels that include center locations, provide lower total costs by disposing of defective products at an early stage along with low transportation costs. Therefore, using return channels with center locations are appropriate in order to collect products with low decay value, such as the Bosch power tool.

	LC-A		LC-B		LC-C		LC-D	
	\mathbf{RF}	Center	\mathbf{RF}	Center	\mathbf{RF}	Center	\mathbf{RF}	Center
HP	3	0	4	1	5	1	5	3
BOSCH	2	1	3	1	4	2	5	3
Multi-Product	3	0	4	1	4	2	5	3

Table 4.6: Number of Active RF and Center in the HP, Bosch, and Multi-product Problems.

Figure 4.4 shows active RF and center locations in the HP printer solutions under four different location settings, while Figure 4.5 shows active RF and center locations in the Bosch power tool solutions with different location costs. Among potential RF locations, New York, Chicago, San Jose, Dallas, and Jacksonville, RFs are located in the most populated area first. For example, under LC-A, the highest location cost setting, RFs are located in New York and San Jose in the Bosch power tool case. Similarly RFs are located in New York, Dallas, and San Jose in the HP printer case. As location costs decrease, RFs are located in the following order: Dallas, Chicago, then Jacksonville. None of the centers are active in certain situations, since return channels I-R-M and I-M do not require an active center in their return process. According to the results, centers are also located in the populated area similar to RF locations. In the Bosch power tool case, centers are widely spread across the map, while centers are located close to RFs in the HP printer case. Minimizing total costs is a major concern in the Bosch power tool case due to low decay value, so widely spread centers enable decreased transportation costs. On the other hand, minimizing a product's value loss, i.e. minimizing product's sojourn time in the network, is the most important objective in the HP printer case. Centers located close to RFs decrease not only transportation costs, but also travel time from customer to RF.



Figure 4.4: Active RF and Center Locations in the HP printer



Figure 4.5: Active RF and Center Locations in the BOSCH Power Tool

Next, we analyze return channel selections of the HP printer and Bosch power tool cases. Figure 4.6 and Figure 4.7 show the percentage of selected return channel in the HP and Bosch cases over a daily time period. In the graph, the X-axis represents product life length (365 days) and the Y-axis represents the percentages of return channels selected by customers. According to results, return channel selections of HP printers are similar in four different location cost settings. Although selection percentages of return channels I-R-C-M and I-C-M vary depending on the existence of center locations in the networks, their selection percentages are always under 10% in all four location cost settings. In other words, return channel selection strategies for the HP printer are rarely affected by location costs. For the HP printer return process, maximizing revenue is more important than minimizing costs due to high selling price and decay value. Thus, return channel I-R-M and I-M are most commonly selected by customers to increase revenue. According to numerical results of the HP printer case, total handling, transportation, and location costs are relatively lower than revenue from the recovered printer. For example, transportation and location costs are 3.4% and 3.7% of total revenue under the LC-A setting,

respectively. As location costs decrease, the portion of the transportation and locations costs become smaller: 2.3% and 1.7% of total revenue under the LC-D setting. On the other hand, total revenue from the recovered printer is similar in all four location cost settings, because of the similarity of the return channel selections. As a result, more RFs and centers are located in the networks under the LC-D setting, leading to an increase in total profit of 4.8% when compared to total profit under the LC-A setting.

In the Bosch power tool case, although I-R-M is a major return channel in all four settings, the selection percentage gap between return channels becomes closer as location cost decreases. For example, under the LC-A location cost setting, the selection percentage of return channel I-R-M is more than 60%, while the selection percentages of the other three channels are under 20%. However, selection percentages of all four channels are under 50% in the LC-C and LC-D location cost settings. Thus, return channel selection strategy varies depending on the locations of RFs and centers in the Bosch power tool case. Unlike the HP printer case, minimizing total cost is the important issue in regard to total profit, because of low selling price and decay value. According to numerical results of the Bosch power tool, total transportation and location costs are about 17% and 35% of total revenue under the LC-A Setting. As location cost decreases, more RFs and centers are dispersed in the recovery logistics network, leading to a decrease in transportation cost. For example, total transportation cost is only 7% of total revenue under the LC-D location cost setting. Since total cost is relatively high in the optimal value of the Bosch power tool case, we observe substantial profit improvement, as location cost decreases. Total profit under the LC-D setting is 37% higher than total profits under the LC-A setting in the Bosch power tool case.

Next, we consider both the HP printer and Bosch power tool cases simultaneously



Figure 4.6: Channel Selection for HP Printer (33% ND & 10% Disposal) under Different Location Costs

in the problem. It is hard to compare the channel selection strategies of multi-product and single product problems, since location and number of active RFs and centers in both a single product and multi-product systems are the same. Thus, we compare the return channel selection of the HP printer and Bosch power tool based on LC-D setting. Generally selection percentages of the return channel for HP printer and Bosch power tool in a multi-product problem are similar to the selection percentages in a single product problem. A slightly more responsive return channel is assigned to customers with HP printer, while a more cost-efficient return channel is selected by customers with the Bosch power tool. For example, in the HP printer case, the selection percentage of return channel I-M increases from 22% to 25%, while the selection percentage of return channel I-R-C-M increases from 16.7% to 18% in the Bosch power tool case. In the multi-product problem, both the HP printer and Bosch power tool's revenue are aggregated. Since revenue from recovered HP printers is



Figure 4.7: Channel Selection for Bosch Power Tool (0% ND & 10% Disposal) under Different Location Costs

much higher than revenue from Bosch power tool, we observe similar revenue and cost improvements in a multi-product problem compared to the HP printer case.

4.4.1 Sensitivity Analysis

In the previous experiments, non-defective and disposal rates are assumed to be 33% and 10% for the HP printer and 0% and 10% for Bosch power tool. We expect the return channel selections and location decisions to differ with product characteristics as well as location costs. Therefore, for the detailed analysis of channel selection strategy with product characteristics and location decisions, we vary the non-defective and disposal rates. Again, we conduct experiments under four different location cost settings, LC-A, LC-B, LC-C, and LC-D. Table 4.7 shows the number of active RFs and centers in the HP and Bosch cases under different problem settings (location cost, non-defective rate, and disposal rate). First, the number of active RFs and centers are different in non-defective and disposal rates. We observe that more RFs are shown in the high non-defective rate case, while more centers are shown in the high disposal rate case. Low non-defective means that most returned products are defective and these defective products require repair for resale in the second market. Therefore, more RFs are required to handle defective products quickly. On the other hand, high disposal means that most defective products are disposed of at the center or RF location. Early inspection in the return process has advantages, since unnecessary transportation and handling costs can be saved through early disposal. For this reason, more centers are shown in the recovery logistics network to decrease total costs under the high disposal rate case.

		HP Inkjet Printer											
	Low N	D - Low Disposal	Low N	D - High Disposal	High 1	ND - Low Disposal	High ND - High Disposal						
Location Costs	RF	Center	RF	Center	RF	Center	RF	Center					
LC-A	3	0	3	1	3	0	2	1					
LC-B	4	1	4	2	3	1	3	1					
LC-C	5	1	4	4	4	1	4	2					
LC-D	5	2	5	6	5	2	5	6					
	Bosch power tool												
				Bosch p	ower too	ol							
	Low N	D - Low Disposal	Low N	Bosch p D - High Disposal	ower too High I	ol ND - Low Disposal	High	ND - High Disposal					
Location Costs	Low N RF	D - Low Disposal Center	Low N RF	Bosch p D - High Disposal Center	ower too High I RF	ol ND - Low Disposal Center	High RF	ND - High Disposal Center					
Location Costs LC-A	Low N RF 2	D - Low Disposal Center 1	Low N RF 2	Bosch p D - High Disposal Center 2	ower too High I RF 2	ol ND - Low Disposal Center 1	High RF 2	ND - High Disposal Center 2					
Location Costs LC-A LC-B	Low N RF 2 3	D - Low Disposal Center 1 2	Low N RF 2 3	Bosch p D - High Disposal Center 2 3	ower too High I RF 2 2	ol ND - Low Disposal Center 1 2	High RF 2 2	ND - High Disposal Center 2 3					
Location Costs LC-A LC-B LC-C	Low N RF 2 3 4	D - Low Disposal Center 1 2 2	Low N RF 2 3 4	Bosch p D - High Disposal Center 2 3 3 3	ower too High I RF 2 2 4	ol ND - Low Disposal Center 1 2 3	High RF 2 2 3	ND - High Disposal Center 2 3 4					

Table 4.7: Number of Active RFs and Centers in HP and Bosch Solutions under Different Problem Settings.

Table 4.8 shows the average percentages of selected return channels and percentage increases in objective values (total profit) in the HP printer case. Table 4.9 shows the average percentages of selected return channels and percentage increases in objective values (total profit) in the Bosch power tool case.

Generally, the change in channel selection percentages with location cost is relatively small in the HP printer case compared to the Bosch power tool case. Trans-

HP Printer	Printer Low ND - Low Disposal Low ND - High Disposal									
Location Costs	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %
LC-A	0%	49.8%	0%	50.2%	-	5%	48.3%	10.3%	36%	-
LC-B	3%	37.6%	8.3%	51.1%	2.78	6.7%	38.9%	14%	40.5%	5.51
LC-C	3%	33.6%	8.3%	55.1%	1.95	5.2%	36.8%	19.4%	38.5%	3.61
LC-D	1.3%	32.7%	13.6%	52.4%	1.37	0%	31.8%	25.6%	42.6%	3.18
HP Printer		High N	ID - Low	Disposal			High N	D - High	Disposal	
Location Costs	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %
LC-A	0%	93.1%	0%	6.9%	-	38%	56%	0%	5.9%	-
LC-B	21.2%	71.9%	0%	6.9%	1.91	26.3%	67.8%	0%	6%	2.08
LC-C	8.8%	79.6%	0%	11.6%	1.76	39.5%	51%	0%	9.5%	1.87
LC-D	8.1%	79.4%	0.6%	11.8%	0.98	37.8%	53.2%	2%	7%	1.27

Table 4.8: Average Percentage of Selected Channel and Objective Value under Different Location Costs (HP Printer)

Bosch power tool		Low N	D - Low I	Disposal			Low N	D - High	Disposal	
Location Costs	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %
LC-A	19.1%	67.5%	0%	13.3%	-	35.2%	47.8%	2.8%	14.2%	-
LC-B	28.9%	46.9%	6.7%	17.5%	13.42	49.6%	26%	7.7%	16.7%	24.33
LC-C	32.1%	39.5%	7.5%	20.8%	6.81	39.6%	33.1%	8.3%	18.9%	14.32
LC-D	21.7%	45.3%	10.8%	22.2%	4.69	37.3%	24.3%	20.8%	17.5%	9.81
Bosch power tool		High N	D - Low	Disposal		High ND - High Disposal				
Location Costs	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %	I-R-C-M	I-R-M	I-C-M	I-M	ObjInc %
LC-A	36.6%	57.7%	0%	5.7%	-	71.3%	20.1%	2.7%	5.9%	-
LC-B	71.5%	22.4%	2.7%	3.4%	7.25	77.1%	14.2%	2.7%	5.9%	8.2
LC-C	67.3%	23.3%	2.7%	6.7%	3.51	70.6%	20%	2.7%	6.7%	5.08
LC-D	67.3%	20.8%	5.2%	6.7%	3.41	74.8%	14.7%	4.4%	6.2%	4.27

Table 4.9: Average Percentage of Selected Channel and Objective Value under Different Location Costs (Bosch Power Tool)

portation costs decrease immensely when more RFs and centers are shown in the recovery logistics network. As observed, decreasing transportation costs contribute more to the objective function value in the Bosch power tool case. Therefore, we observe that return channel selection is more sensitive to location costs in the Bosch power tool case. For a similar reason, objective function (total profit) improvement of the Bosch power tool is much higher than profit improvement of the HP printer in all cases based on results.

4.4.1.1 Observations on Non-defective Rates

We notice that, when the non-defective rate is high, the channels I-R-M and I-R-C-M are heavily utilized in all location cost settings. That is, the returned products reach a retailer first and the non-defective ones (which are large in number) are put back on the shelf after minor processing. Since retailer locations are priori given, thus only return channel R-M in I-R-M and return channel R-C-M in I-R-C-M vary depending on the locations of active RFs and centers. Although more RFs and centers are located in the network as location costs decrease, total profit improves slightly in the high non-defective rate case when compared to low non-defective rate. On the other hand, a low non-defective rate represents the fact that there is a high number of returned products that require significant rework at RF locations. Thus, locations of RF are important in the return process. If both non-defective and disposal rates are low, among five potential RF locations, only three RFs are active in HP printer and only two RFs are active in Bosch power tool case under LC-A location cost setting, while all RFs are active in HP printer and Bosch power tool case under LC-D location costs setting. Thus, we observe substantial revenue and transportation cost improvement in the low non-defective rate when location costs decrease.

4.4.1.2 Observations on Disposal Rates

The role of centers in the recovery logistics network is to decrease costs by providing early inspection and lowering unit transportation costs. Thus, the locations of centers are more important when the disposal rate is high and product's decay value is low. Low disposal rate shows that most defective products can be resold on the second market after the repair process at the RFs. Therefore, an opening center has less impact on total profit unless a product's decay value is relatively low. On the other hand, as disposal rate increases, more defective products will be disposed. Therefore, in a high disposal rate case, centers play an important role by handling defective products in the early stage, especially for products with a low decay value. For example, under the LC-A cost setting, only two active centers are shown in the Bosch case with a low non-defective and high disposal rate case. However, when the location cost is at its lowest, the number of active centers increases by eight. Therefore, we observe handling and transportation cost improvement as location cost decreases. Also, improvement in costs from opening centers can be more beneficial when a product's decay value is low.

4.5 Conclusions

In this chapter, we consider channel selection in commercial product recovery logistics network with location decisions. The model determines locations of RFs/centers and return/redistribution channels to maximize profits from recovered products. For the solution methodology, we develop a heuristic solution method based on the SA algorithm. Moreover, to evaluate the objective function value of a feasible solution, we use a greedy algorithm based on Dijkstra's shortest path algorithm. Solution time of the heuristic method can be reduced significantly via using the greedy algorithm. We first test the proposed heuristic solution method on a testbed that we developed under two data settings, high and low location costs. Computational results show that the proposed heuristic solution method performs well in terms of solution quality and time. Next, we analyze return channel selection strategy and recovery logistics network design using HP printer and Bosch power tool data. For a detailed analysis of channel selection strategy and recovery logistics network design, we solve the model under four different location cost settings. Generally, more active RFs are required for the product with high decay value to minimize the product's value loss, whereas more active centers are needed for the product with low decay value to minimize total costs. For return channel selection strategy, products with high decay value require a return channel with faster travel time. On the other hand, products with low decay value prefer less costly return channels. This study can be extended in several directions. On the problem domain side, the current model could be extended by considering stochastic parameters, such as uncertainties regarding product return quantities, non-defective and disposal rates. On the methodology side, we develop a new solution method by integrating proposed SA heuristics within a Benders framework.

5. CONCLUSIONS AND FUTURE RESEARCH

This dissertation provides effective approaches to designing a product recovery logistics network. It achieves research purposes in two related parts. First, we consider stochastic issues in the generic CLSC network design problem and analyze how the stochastic programming model performs against the deterministic model. Then, we consider channel selection strategy in commercial product return logistics networks and analyze the optimal network configuration along with channel selection strategy based on product characteristics.

5.1 Conclusions and Future Research on a Generic CLSC Network Design under Demand and Return Uncertainty

In the first research problem, we develop a generic CLSC network design problem under demand and return uncertainty. In the model, we determine the best location of SFs, CTRs, capacity expansion level, and forward/reverse flow network to minimize total system costs. The model is formulated as a two-stage stochastic model, and uncertainty is handled through a scenario approach. For the solution of the model for large scale instances, we propose an exact solution method based on a multi-cut version of the BD algorithm. In the solution algorithm, we introduce induced constraints, strengthened Benders cuts, scenario-category based multiple Benders cuts, and lower bounding inequalities based on mean value scenarios. In the end, we analyze appropriate inspection locations in the developed CLSC problem using real geographical data of the U.S. cities. We modify the developed model by varying inspection location, SFs, CTRs, and retailers and solve it using the proposed solution method within a SAA framework.

5.1.1 Research Contributions

We obtain the following research contributions from the first research problem. First, our model accurately reflects demand/return uncertainty, and the developed stochastic programming model performs well in terms of solution quality. According to relative VSS results, the objective value of the stochastic model always provides better solutions than that of the deterministic model based on mean value. When the portion of the second stage costs are large, in the optimal solution, VSS is relatively high, which allow us to observe that the stochastic programming model should be studied in the CLSC network design problem.

Second, we use the stochastic approach to handle uncertainty and categorize scenarios based on low, medium, and high demand/return quantities. This type of categorization reflects better product demand/return patterns, since typical product life cycle belongs to periods with low, medium, and high designations. Moreover, dual subproblems in the BD algorithm can be aggregated based on scenario categorizations, so corresponding Benders cuts are also aggregated. This aggregation scheme improves solution performance.

Third, we obtain a general idea of an inspection strategy based on product characteristics. Similar to the postponement strategy in the traditional forward flow network, the early inspection strategy in the reverse flow network generally provides better solutions via saving unnecessary transportation and operational costs. The early inspection strategy performs best, especially if the disposal rate of the return product is relatively high. Since we use randomly generated input data for the experiments, obtained results do not provide accurate insight related to the inspection strategy. However, we still observe preferable inspection locations based on product characteristics. Lastly, we propose an efficient solution method that enhances the performance of the BD algorithm. In the proposed BD algorithm, we develop several solution techniques to enhance performance of the algorithm. Enhancement includes additions such as strengthened Benders cuts, multiple Benders cuts based on scenario categorization, and valid inequalities for Benders cuts using mean value scenarios. These solution techniques can be applied to a typical two-stage stochastic programming model. Therefore, we expect that the proposed solution method provides good insight to a reader who wants to use the BD algorithm in the stochastic programming model.

5.1.2 Future Research

The first research problem can be extended in several ways. First, the value of the integrated CLSC network design problem can be analyzed by considering multiproducts. As expected, sharing common resources and locations provides a better solution under a single product problem. However, if multi-products with different characteristics are considered in the CLSC network, then integrated network design may not provide a better solution due to operational issues. Therefore, the tradeoff between saving location costs and increasing operational costs could be studied in the multi-products CLSC network design problem. Second, the developed model could be extended by considering both strategic and tactical decisions simultaneously. Recently, studies have considered strategic and tactical level decisions together. The developed CLSC network problem includes strategic level, location and capacity expansion decisions, tactical level, flow decisions. Therefore, if we include operational decisions, such as routing decisions or inventory decisions, the model may provide better logistics solutions based on an integrated view. Last, more study is required to enhance the performance of the solution algorithm. In the two-stage stochastic
programming model, sub-problems can be decomposed into several problems in terms of scenarios. The use of a parallel algorithm enables us to solve each separated subproblem at the same time, so the performance of the proposed algorithm will improve significantly.

5.2 Conclusions and Future Research on Channel Selection in Commercial Product Returns Logistics Network under Time-Value Considerations

In the second research problem, we study channel selection strategy in a commercial product return logistics network problem. The developed recovery logistics network consists of RFs, centers, retailers, and customers. Additionally, multi-return and multi-redistribution channels are introduced for operational flexibility. Since the time value of a product is the most important factor when commercial product return is considered, we measure product residual value with time in the model. We first analyze the optimal channel selection strategy based on product and logistics network characteristics. According to the analysis, the responsive channel is appropriate for products with high decay value, while the cost-efficient channel is good for collection of products with low decay value. On the logistics view, customer location also has effects on channel selection decisions. For example, as customers are located close to RFs, the return channel I-M, known as the responsive return channel, is mostly used regardless of product type. Last, we extend the commercial product return logistics network problem by introducing location decisions to identify the optimal logistics network configuration. Generally, RF and center locations are different, based on product characteristics. In products with a high decay value, more RFs are located in the recovery network to facilitate repair operations. On the other hand, more centers are located in the solution for products with a low decay value, in order to minimize transportation costs. In short, we obtain managerial insights for designing logistics networks for commercial product return via this research.

5.2.1 Research Contributions

We obtain the following research contributions from the network design for the commercial return problem. On the modeling side, we consider the commercial product return case in the recovery logistics network design problem. As reviewed, there is no study that considers commercial return in the context of network design. One characteristic of commercial return is that returned products can be resold to customers through re-packaging or repair operation. Therefore, in order to maximize profit, the value of returned product should be maintained as high as possible. For measuring product residual value, we introduce a time parameter and compute the product's residual value based on the product's sojourn time in the network. We expect that the proposed modeling approach provides a good starting point to a person interested product recovery logistics network design associated with commercial return. Next, a multi-channel issue is considered in the developed model. Multi-return and multi-redistribution channels enable a company to establish various logistics plans based on product characteristics. Therefore, a multi-channel model not only strengthens customer loyalty, but also improves operational flexibility.

On the methodology side, we develop the SA heuristic algorithm. The proposed heuristic algorithm is very effective in terms of finding good quality solutions and is also efficient in terms of computational time. Moreover, to evaluate the objective function value of a feasible solution, we develop a greedy algorithm. The use of the developed greedy algorithm significantly reduces the computational time.

5.2.2 Future Research

This study can be extended in several directions. Uncertainty, such as quantities associated with return or quality of product, is one difficulty in recovery logistics network problems. Therefore, considering uncertainty in the developed model may be fruitful for future research. On the methodology side, use of the developed heuristic algorithm with an exact solution method (BD), called a hybrid algorithm, may offer interesting exploration. This hybrid algorithm enables the BD algorithm to start with a good quality solution and this may significantly reduce the solution time for the BD algorithm.

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