ESSAYS ON VOLUNTARY CONTRIBUTION WITH PRIVATE INFORMATION AND THRESHOLD UNCERTAINTY

A Dissertation

by

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ABSTRACT

This dissertation concerns individual voluntary contributions in the subscription game with three important model considerations: private information on public good valuations, threshold uncertainty and the timing of the contribution — simultaneous and sequential contribution.

In the first essay, we set up a simultaneous subscription game model and analyze how the contributions will be affected when individuals face different levels of threshold uncertainty. Comparative statics with respect to the changes in the cost distribution are derived. We find that when the cost of public good increases in the sense of first order stochastic dominance, individuals, on average, are more willing to contribute to the public good. But, when the cost distribution becomes more dispersed in the sense of mean-preserving spread, individuals, on average, are less willing to contribute to the public good.

The second essay introduces threshold uncertainty and private information on valuations for a discrete public good in a subscription game and analyzes how the players sequentially make their contribution decisions within this environment. I find that the earlier contributor's expected contribution is lower than the latter contributor's expected contribution. The result demonstrates that the earlier contributor can free ride off the latter contributor. Comparing the expected total contribution in the sequential contribution mechanism with that in the simultaneous contribution mechanism, this paper shows that the expected total contribution in the sequential model is lower.

The third essay provides the experimental evidence of comparative statics with respect to the changes in the cost distribution. I conduct a laboratory experiment to

test the theoretical predictions in the first essay. The experimental result supports the theoretical predictions of the comparative statics with respect to the threshold uncertainty: decreasing the degree of the threshold uncertainty in the sense of mean-preserving contraction, or increasing the mean of the threshold distribution in the sense of first-order stochastic dominance, individuals, on average, are more willing to contribute to the public good.

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1. INTRODUCTION

The mechanisms used to allocate public goods vary widely. The most common way to provide public goods in modern societies is for the government to levy taxes. Alternative institution, private contribution, plays a supplementary role to the tax-financed allocation mechanism that contributes to the provision on public goods. We can observe many private contribution processes such as donation, fundraising by non-profit organizations, and construction of activity centers by neighborhood associations. Private contributions, then, are not only attractive but also valuable to investigate.

According to the *Giving USA 2013 Report Highlights*, the total 2012 contribution was $ 316.23 billion[1]. Individual voluntary contribution accounted for 72% of this total giving amount. This empirical data shows that the individual contribution is the most important contribution source, thus, the issue explored in this dissertation concerns individual voluntary contribution behavior. I tackle the topic focused in particular on the subscription game with private information on valuation and threshold uncertainty.

This dissertation takes into account three important considerations that may affect individual contribution behavior. The first consideration is the valuation of the public good. Previous studies, for example, Palfrey and Rosenthal (1984), Bagnoli and Lipman (1989), Issac, Schmidtz and Walker (1989), have been performed in an extremely rich informational environment in which the valuation of the public good is commonly known. In contrast, in reality the individual hardly knows in advance the valuation of others. That means the valuation of the public good is private

[1] This report is available at http://www.givingusareports.org/

information.

Besides valuation, the individual also considers how much money is needed to provide the public good, which is called the threshold. However, it is possible that individuals do not know the exact provision cost and face cost uncertainty when contributing to the public good. Therefore, threshold uncertainty is another important factor that should be considered when investigating the private contribution behavior. Realizing that the threshold uncertainty may affect the player's strategic contribution behavior, the existing literature, Nitzan and Romano (1990), Suleiman (1997), and McBride (2006), have introduced threshold uncertainty into the discrete public good model. The first two papers find that the threshold uncertainty may result in an inefficient equilibrium. McBride (2006) shows that an increase of the threshold uncertainty in the sense of mean-preserving spread increases the player's contribution when the value of the public good is sufficiently high; otherwise, it decreases the player's contribution when the value of the public good is sufficiently low.

Another consideration is the timing of contribution. Some super markets announce checkstand donation campaigns to support the community. This contribution mechanism is called the simultaneous contribution mechanism since the cashier does not tell you how much money has been collected when you make the contribution decision. If you know the accumulated amounts when you contribute to the public good, this type of contribution mechanism is called the sequential contribution mechanism. For instance, churches may announce an organ fund campaign and report the updated contribution level frequently; local governments may announce the seed donations to future contributors when they launch new public good projects. These are examples of sequential contribution. Erev and Rapoport (1990) is the earliest experimental paper that studies simultaneous and sequential moves in a discrete public good game. Comparing the sequential institution to the simultaneous institution, they find that when subjects made their decisions simultaneously, the public good was provided 14.3% to 31.3% of the time. But when they made their decisions sequentially, the public good was provided 66.7% of the time.

Varian (1994) is an early theoretical study of sequential contribution to the public good. Varian finds that early contributors free ride off the later contributors in sequential contribution situations, thus total contribution under a sequential institution is lower than the total contribution under a simultaneous institution. Several later experimental and theoretical papers also focus on comparing voluntary contribution in simultaneous and sequential institution (see Andreoni (1998), Potters et al.(2005), Masclet and Willinger (2005), Levati *et al.* (2007), Vyrestekova and Garikipati (2008), Coats *et al.* (2009), Gächter *et al.* (2010) and Bracha *et al.*(2011)).

To the best of my knowledge, few papers consider these three model considerations at the same time when investigating private contribution to a discrete public good in the context of a subscription game. My dissertation addresses this unexplored setting.

The first essay, a joint work with Timothy J. Gronberg, studies how individual contributions are affected when facing different degrees of threshold uncertainty in a simultaneous subscription game. As far as we know, few papers consider both the threshold uncertainty and private information of public good's value in the subscription game. Thus, this paper complements earlier works on the contribution to the public good with threshold uncertainty and private information, and investigates the effect of changing the cost distribution on private contributions. By building a theoretical subscription game model, this paper demonstrates that if the cost increases in the sense of first order stochastic dominance, individuals, on average, are more willing to contribute to the public good. However, if the costs becomes more dispersed in the sense of mean-preserving spread, individuals, on average, are less willing to contribute to the public good.

The second essay develops a theoretical model and examines individual sequential contribution to the public good in a subscription game with threshold uncertainty and private information on valuation. This paper aims to analyze three questions: (1) How do individuals contribute to the public good in the environment with private information and the cost uncertainty? (2) Do individuals in different contribution orders contribute differently? (3) Comparing sequential and simultaneous contribution institutions, which institution produces higher total contributions? To the best of my knowledge, this paper is the first to investigate private contribution to a discrete public good under the sequential institution with private information and threshold uncertainty.

The theoretical result of the second essay shows that individual contributions are increasing with respect to the contributor's order. Earlier contributors contribute less than subsequent contributors. This result demonstrates that earlier contributors free ride off later contributor and enjoy first-mover advantage. Another important finding is that individuals contribute to the public good differently in a sequential contribution institution compared to a simultaneous contribution institution. Comparing the player's expected contribution in a sequential contribution mechanism to a simultaneous contribution mechanism, this paper finds that the expected total contribution in the sequential institution is lower than the expected total contribution in a simultaneous one. This result suggests that in an environment with private information on valuation and cost uncertainty, sequential institutions provide lower contribution than simultaneous institutions.

In the last essay, I conduct an experiment to test the theoretical predictions in

Gronberg and Peng (2014), and aim to analyze how individual contribution behavior is affected when faced with different degrees of cost uncertainty in a lab. The advantage of conducting a controlled laboratory experiment is that it helps us to explore how individuals make contribution decisions in a specific environment.

The experimental data strongly supports comparative statics with respect to threshold uncertainty, as predicted by Gronberg and Peng (2014). The main result of this paper is that decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction, the individual, on average, is more willing to contribute to the public good. Also, increasing the man of the threshold distribution in the sense of the first-order stochastic dominance, the individual, on average, is more willing to contribute to the public good. The results suggest that suppliers of public goods should consider what kind of information related to the cost uncertainty they should announce when collecting private contributions.

2. CHANGES IN THE THRESHOLD UNCERTAINTY IN A SIMULTANEOUS SUBSCRIPTION GAME*

2.1 Motivation and Related Literature

Fundraising by non-profit organizations, constructing new buildings by neighborhood associations, and donating to churches are some examples of voluntary contribution to public goods. The earliest literatures, Bergstrom, Blume, and Varian(1986), Bernheim (1986), and Andreoni (1989), investigate the situation under certainty. But in many real world examples of voluntary contribution uncertainty plays a critical role in individual contribution decisions. This uncertainty manifests in different ways. For example, individuals face a random distribution of their incomes, individuals are not familiar with production technology, or individuals do not know the cost of providing the public good. This paper considers a model of providing a discrete public good in a subscription game within an environment of threshold uncertainty and private information on public good valuations. The focus is on the comparative statics of a change in cost uncertainty on the private contribution equilibrium under a simultaneous institution.

A discrete public good, defined as a fixed quantity of a public good, is provided if the total contributions are large enough to cover its cost; otherwise, the public good is not provided. This kind of public good is also called a binary or threshold public good. Typical examples of discrete public goods are roads, parks, community libraries, local radio programs, school buildings, etc. In a subscription game, the players' contributions are refunded if the sum of the contributions are not large

*Reprinted with permission from "Changes in the Threshold Uncertainty in a Simultaneous Subscription Game" by Timothy J. Gronberg and Hui-Chun Peng, 2014. *Theoretical Economics Letters*, 4, 263-269, Copyright [2014] by authors, Timothy J. Gronberg and Hui-Chun Peng, and Scientific Research Publishing Inc.

enough to cover the cost of the public good.

Palfrey and Rosenthal (1984) and Bagnoli and Lipman (1989) are important papers that analyze private provision of discrete public good. Both papers assume players make their contribution strategies in the environment with complete information of the public good's value and a certain known threshold level of cost, but the types of contribution in these two papers are different. Palfrey and Rosenthal (1984) assumed the player to make a binary contribution — zero or a fixed amount of contribution, but Bagnoli and Lipman (1989) allowed the player to make a continuous contribution — the player can contribute any non-negative amount to the public good. These two papers both show that efficient provision of public goods in the subscription game may exist.

It is possible that the players do not face a certain threshold. For example, it might not be known how much money will be needed to build a community library or to complete a public project. Realizing that threshold uncertainty may affect the player's strategic contribution behavior, Nitzan and Romano (1990), Suleiman (1997), and McBride (2006) introduce threshold uncertainty into the discrete public good model. The first two papers find the possibility of inefficient equilibrium under threshold uncertainty. Inefficiency may exist because the *ex post* contribution exceed the required threshold quantity of contribution or the contributions are insufficient to cover the required threshold level. McBride (2006) investigates how the degree of threshold uncertainty affects the players' contributions and finds that instead of having a monotonic relationship between the threshold uncertainty and the contribution, the effect of changing the threshold uncertainty on the contributions depends on public good valuation. An increase of the threshold uncertainty in the sense of mean-preserving spread increases the player's contribution when the value of the public good is sufficiently high; otherwise, it decreases the player's contribution when the value of the public good is sufficiently low. In a follow-up paper, McBride (2010) designs an experiment to test the predictions in McBride (2006) and finds limited verification. His findings demonstrate that dispersing the threshold uncertainty is often, but not always, consistent with the predictions in McBride (2006).

Another branch of literature focuses on the private information on valuation for the public good. In reality, when an individual makes a contribution decision, he or she might not know how valuable the public good is to other. In other words, other's valuation of the public good is private information. Thus, private information of public good's value is a potential factor that affects the player's contribution to the public good. Menezes *et al.* (2001), Laussel and Palfrey (2003), and Barbieri and Malueg (2008) introduce private information of the public good's value into a subscription game with a discrete public good and examine the efficiency of the Bayesian Nash Equilibrium. Menezes *et al.* (2001) use the probability of provision given that it is socially desirable to provide the public good to measure the *ex post* efficiency. They show that the probability of provision is smaller than 1. Thus, the equilibrium in the subscription game is *ex post* inefficient. Moreover, they provide evidence that if the cost of the public good is high enough, the subscription game is better than the contribution game (the game without a refund rule). Laussel and Palfrey (2003) analyze interim incentive efficiency, defined by Holmström and Myerson (1983), in the subscription game. They find that the interim incentive efficient equilibrium may exist, and the efficient equilibrium must be a continuous equilibrium. Later, Barbieri and Malueg (2008) reexamine Laussel and Palfrey's work, but show the contrary result that there are no incentive efficient equilibria.

Barbieri and Malueg (2010) include both the threshold uncertainty and private information of public good's value in the subscription game. They assume that player's value is distributed with a common uniform distribution over $[0, 1]$ and discuss how the changes in intensity and dispersion of value affect individual expected contributions with holding the support of value in $[0, 1]$. They find that increasing player's value in the sense of first order stochastic dominance, or dispersed player's value distribution in the sense of mean-preserving spread increases the equilibrium contributions.

Although McBride (2006) analyzes how the change in the threshold distribution affects the contribution, he only considers the model with identical and known value of public good. Barbieri and Malueg (2010) assume the threshold uncertainty and private information in their model, but they only analyze the effect of changing the player's value distribution. This paper complements these existing papers by investigating the comparative statics effect of changing the cost distribution on private contributions within a Barbieri and Malueg setting with both threshold uncertainty and private information.

We show that if the cost distribution becomes more dispersed in the sense of mean-preserving spread, then the expected contributions will decrease. When the cost of public good increases in the sense of first order stochastic dominance, the expected contributions will increase.

Our results suggest that suppliers may increase contributions to the public good by reducing uncertainty over the cost distribution or increasing the mean of the cost distribution.

The rest of paper is organized as follows. Section 2 presents the model. Section 3 considers the comparative statics with respect to changes in cost distribution of the public good and characterizes the expected contribution. Section 4 is the conclusion.

2.2 Model

Consider a subscription game consists with $n \geq 2$ players. In order to provide a discrete public good, the players in this game simultaneously make contributions toward this public good of any non-negative amount. Let $x_i \in [0, v_i]$ be player i's contribution. Player i's value on the public good is v_i, $i = 1, ..., n$. And it is an independently distributed random variable with a continuous uniform distribution whose support is $[0, 1]$. With this assumption, each player's valuation for the public good is private information. That is, each player knows his/her own realized valuation of the public good but is uncertain about other players' valuations of the public good. Since each player's value follows the same distribution, this paper represents a symmetric case.

A discrete public good can be provided if and only if the total contributions are equal to or larger than the cost of the public good, c (also known as the cost threshold). Suppose c is unknown when the players contribute to the public good. However, all players believe that the cost is independent of all v_i's and distributed with a continuous uniform distribution, F, with support $[\bar{c} - z, \bar{c} + z]$, where $\bar{c}$ is the mean of the cost, z measures the degree of the cost uncertainty.

This paper considers the public good subscription game (Admati and Perry, 1991) where the player contributions will be fully refunded if the total contributions are less than the cost threshold. We also assume a zero rebate rule, which means that the excess contributions will be given to the producer of the public good.

Given $F(C) = \frac{1}{2z}\big[C - (\bar{c} - z)\big]$, the *ex ante* probability of providing the public good with total contributions, $\big[x_i + \sum_{j \neq i} E[x_j(v_j)]\big]$, is $F\big(x_i + \sum_{j \neq i} E[x_j(v_j)]\big) = \frac{1}{2z}\big[x_i + \sum_{j \neq i} E[x_j(v_j)] - (\bar{c} - z)\big]$. Since the player does not know other players' contributions when making the contribution decision, he/she needs to forecast the amounts other players will contribute, $\sum_{j \neq i} E[x_j(v_j)]$. Because the probability of provision should be between 0 and 1, we can get $(\bar{c}-z) \leq x_i+\sum_{j \neq i} E[x_j(v_j)] \leq (\bar{c}+z)$. Also, we have assumed that $v_j \sim U[0,1]$ and $x_i \in [0, v_i]$. This implies that an important constraint, $0 \leq (\bar{c}-z) < n \leq (\bar{c}+z)$, must be satisfied. This constraint demonstrates that the number of players in this subscription game needs to be larger than the lower bound of the possible cost, $(\bar{c}-z)$, but cannot be larger than the upper bound of the possible cost, $(\bar{c}+z)$.

The assumed objective for each player is to maximize his/her own expected payoff. If the public good is provided, then player i's payoff is $(v_i - x_i)$ with the probability of the provision, $F\big(x_i + \sum_{j \neq i} E[x_j(v_j)]\big) = \frac{1}{2z}\big[x_i + \sum_{j \neq i} E[x_j(v_j)] - (\bar{c}-z)\big]$; if the public good is not provided, then player i's payoff is 0 with the probability, $\big[1 - F\big(x_i + \sum_{j \neq i} E[x_j(v_j)]\big)\big]$. Thus, player i's expected payoff function can be written as:

$$U_i(x_i, v_i) = \frac{1}{2z}(v_i - x_i)\Big(x_i + \sum_{j \neq i} E[x_j(v_j)] - (\bar{c}-z)\Big) \tag{2.1}$$

Assume $K_j \equiv E[x_j(v_j)]$ is player j's expected contribution, thus, player i's expected utility function can be rewritten as:

$$U_i(x_i, v_i) = \frac{1}{2z}(v_i - x_i)\Big(x_i + \sum_{j \neq i} K_j - (\bar{c}-z)\Big) \tag{2.2}$$

Utilizing the maximizing Equation (2.2) with respect to x_i and taking the first order condition (F.O.C.) can yield player i's best response function:

$$x_i^*\big(v_i, \sum_{j \neq i} K_j\big) = \max\Big\{0, \frac{1}{2}\big[v_i - \sum_{j \neq i} K_j + (\bar{c}-z)\big]\Big\}, \quad \forall\ i \tag{2.3}$$

Under the assumption of a common uniform distribution in $[0, 1]$, the total expected contributions by other players, $\sum_{j \neq i} K_j$, can be written as $(n-1)K$ in symmetric equilibrium. Hence, Equation (2.3) can be rewritten as

$$x_i^*\big(v_i, (n-1)K\big) = \max\Big\{0, \frac{1}{2}\big[v_i - (n-1)K + (\bar{c} - z)\big]\Big\}, \quad \forall\, i \tag{2.4}$$

From the best response function, Equation (2.4), we can find that $\big[(n-1)K - (\bar{c} - z)\big]$ is the cutoff point for player i to begin contributing to the public good. In other words, player i is willing to contribute a positive amount to the public good when his/her valuation for the public good is equal to or larger than this cutoff point. The best response function also shows that once player i's contribution is positive, it is strictly increasing in a larger valuation of the public good and strictly decreasing in other players' expected contributions.

Using the definition of expected contribution, $K_i \equiv E[x_i(v_i)]$, and the best response function, Equation (2.4), we will have, in symmetric equilibrium,

$$K = E\bigg[\max\Big\{0, \frac{1}{2}\big[v_i - (n-1)K + (\bar{c} - z)\big]\Big\}\bigg] \tag{2.5}$$

With player values independently and uniformly distributed on $[0, 1]$, the total expected contributions by other players, $\sum_{j \neq i} K_j$, the expected equilibrium contribution is determined by

$$K = \int_0^{(n-1)K-(\bar{c}-z)} 0\, dv_i + \frac{1}{2}\int_{(n-1)K-(\bar{c}-z)}^{1} \big[v_i - (n-1)K + (\bar{c} - z)\big]\, dv_i$$

$$= \frac{1}{2}\Big[\frac{1}{2} - (n-1)K + (\bar{c} - z) + \frac{1}{2}[(n-1)K - (\bar{c} - z)]^{\frac{1}{2}}\Big] \tag{2.6}$$

We can solve Equation (2.6) for K which yields is

$$K^* = \frac{1+\bar{c}-z}{n-1} + \frac{2}{(n-1)^2}\Big\{1 - \Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{1}{2}}\Big\} \tag{2.7}$$

This solution concept is a symmetric Bayesian-Nash equilibrium.

Player i's equilibrium strategy, x_i^*, must satisfy Equation (2.4), and K^* is determined by Equation (2.7). Therefore, x_i^* can be written as:

$$x_i^*\big(v_i,(n-1)K^*\big) = \max\Big\{0, \frac{1}{2}\big[v_i - (n-1)K^* + (\bar{c}-z)\big]\Big\} \tag{2.8}$$

And the cutoff point in equilibrium for each player to begin contributing a positive amount to the public good, v_p, is

$$v_p = \big[(n-1)K^* - (\bar{c}-z)\big] = 1 + \frac{2}{n-1}\Big\{1 - \big[1+(n-1)(1+\bar{c}-z)\big]^{\frac{1}{2}}\Big\} \tag{2.9}$$

Since the lower bound of a player's value is 0, v_p has to be equal to or larger than 0. Thus, we can get a constraint that $(\bar{c}-z) \le \frac{n-1}{4}$.

Figure 2.1 depicts player i's equilibrium strategy.

2.3 Stochastic Dominance in Cost Threshold and Comparative Statics

Players may confront cost distributions with different levels of dispersion. If the cost distribution can be controlled or affected by supplier actions, the results in this paper suggest the benefits of changing information related to the cost of the public good measured in terms of increase expected total contributions to the public good. In this section we consider the effects of changes in the cost distribution on equilibrium contributions.

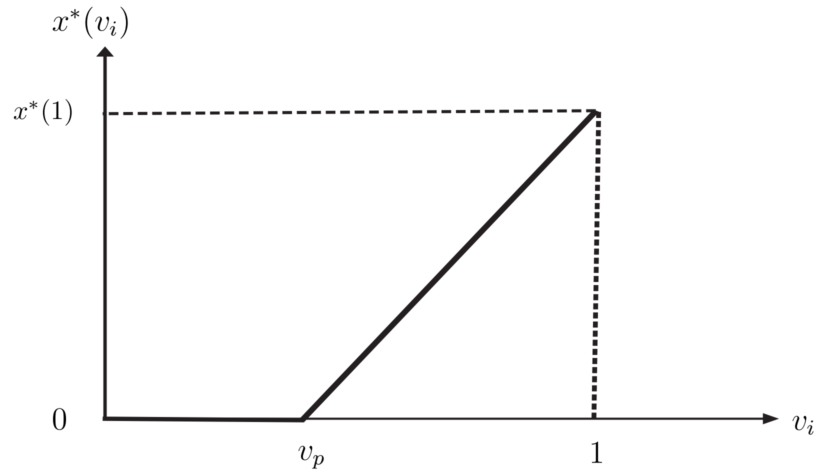

Figure 2.1: Cutpoint Equilibrium Strategy

2.3.1 Mean-Preserving Spread in Cost Threshold Distribution

Given each player's value distribution follows a common uniform distribution over $[0,1]$, assume the cost distribution becomes more uncertain in the sense of mean-preserving spread. For example, the new cost distribution is $c \in U \sim [\bar{c} - z^{'}, \bar{c} + z^{'}]$, where $z^{'} > z$.

Proposition 1. *A mean-preserving increase in the distribution of cost will* ***decrease*** *individual expected contribution and the total expected contributions.*

Proof.

From (2.7), we know that $K^* = \frac{1+\bar{c}-z}{n-1} + \frac{2}{(n-1)^2} - \frac{2}{(n-1)^2}\Big[1 + (n-1)(1+\bar{c}-z)\Big]^{\frac{1}{2}}$ in equilibrium. Differentiating (2.7) with respect to the degree of the cost uncertainty, z, we can get

$$\frac{dK^*}{dz} = \frac{1}{n-1}\Big\{ -1 + \Big[1 + (n-1)(1+\bar{c}-z)\Big]^{\frac{-1}{2}}\Big\} \tag{2.10}$$

Since $(\bar{c} - z)$ is the lower bound of the threshold and assume $(\bar{c} - z) \geq 0$, $\big[1 + (n -$

$1)(1+\bar{c}-z)\big]^{\frac{-1}{2}}$ will be less than 1. Thus, we can obtain $\frac{dK^*}{dz} < 0$.

Since $\frac{\partial(nK^*)}{\partial z} = n\frac{\partial K^*}{\partial z}$ and we have $\frac{\partial K^*}{\partial z} < 0$, $\frac{\partial nK^*}{\partial z} < 0$. Since the expected contribution per player decreases, so does the total expected contribution. ■

The proposition indicates that the players, on average, become less willing to contribute to the public good when the cost of the public good becomes more uncertain.

We have shown that the cutoff point in equilibrium is $v_p = 1 + \frac{2}{n-1}\big\{1 - \big[1+(n-1)(1+\bar{c}-z)\big]^{\frac{1}{2}}\big\}$. Since $\frac{\partial v_p}{\partial z} = \big[1+(n-1)(1+\bar{c}-z)\big]^{\frac{-1}{2}} > 0$, increasing z will increase the cutoff point. This indicates that the player will begin contributing a positive amount to the public good at a higher value as the variance of the cost increases.

The change in z also affects the player's best response function where $x^* > 0$. This effect can be divided into the direct and indirect effect using the player's best response function, x^*, to demonstrate these two effects.

$$\frac{\partial x^*}{\partial z} = \frac{-(n-1)}{2}\frac{\partial K^*}{\partial z} - \frac{1}{2} \tag{2.11}$$

$$= \left\{\frac{1}{2} - \frac{1}{2}\big[1+(n-1)(1+\bar{c}-z)\big]^{\frac{-1}{2}}\right\} - \frac{1}{2} \tag{2.12}$$

$$= \frac{-1}{2}\big[1+(n-1)(1+\bar{c}-z)\big]^{\frac{-1}{2}} < 0 \tag{2.13}$$

From Equation (2.11), we find that changing z will change x^* directly. Hence, the second term in Equation (2.11) represents the direct effect. Also, changing z will affect other players' expected contributions, K^*. The first term in Equation (2.11) measures this effect and is called the indirect effect. Equation (2.12) shows that increasing the degree of cost uncertainty induces the negative direct effect, the positive indirect effect. Notably, the direct effect dominates the indirect effect. Thus, increasing z decreases x^*.

We find that players start to contribute to the public goods at a higher cutoff point value and the contribution amounts at each possible value of public good weakly decrease. Hence, the expected contribution to the public good decreases with the degree of cost uncertainty, z.

Our proposition provides the policy implication that if the suppliers are able to reduce the uncertainty of the cost distribution, the private contribution to the public good will increase. The reduction in cost distribution uncertainty will encourage the players with low value to begin contributing to the public good and also increase contributions of inframarginal contributors.

2.3.2 First Order Stochastic Dominance in Cost Threshold Distribution

Given each player's value follows a common uniform distribution over $[0,1]$, assume the mean of the cost distribution becomes higher in the sense of first order stochastic dominance. For example, the new distribution of cost is $c \in U \sim [\bar{c}' - z, \bar{c}' + z]$, where $\bar{c}' > \bar{c}$.

Proposition 2. *A first order stochastic dominance increase in the distribution of cost will* ***increase*** *individual expected contribution and the total expected contributions.*

Proof.

From (2.7), we know that $K^* = \frac{1+\bar{c}-z}{n-1} + \frac{2}{(n-1)^2} - \frac{2}{(n-1)^2}\Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{1}{2}}$ in equilibrium. Differentiating (2.7) with respect to the mean of the cost uncertainty, c, we can get

$$\frac{dK^*}{d\bar{c}} = \frac{1}{n-1}\Big\{1 - \Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{-1}{2}}\Big\} \tag{2.14}$$

Since $(\bar{c}-z)$ is the lower bound of the threshold and assume $(\bar{c}-z) \geq 0$, $\big[1+(n-1)(1+\bar{c}-z)\big]^{\frac{-1}{2}}$ will be less than 1. Thus, we can obtain $\frac{dK^*}{d\bar{c}} > 0$.

Since $\frac{\partial(nK^*)}{\partial \bar{c}} = n\frac{\partial K^*}{\partial \bar{c}}$ and we have $\frac{\partial K^*}{\partial \bar{c}} > 0$, $\frac{\partial nK^*}{\partial \bar{c}} > 0$. Since the expected contribution per player increases, so does the total expected contribution. ∎

The proposition indicates that players, on average, become more willing to contribute to the public good when the mean cost of the public good becomes higher.

We have shown that the cutoff point in equilibrium is $v_p = 1 + \frac{2}{n-1}\{1 - [1 + (n-1)(1 + \bar{c} - z)]^{\frac{1}{2}}\}$. Since $\frac{\partial v_p}{\partial \bar{c}} = -[1 + (n-1)(1 + \bar{c} - z)]^{\frac{-1}{2}} < 0$, increasing $\bar{c}$ will decrease the cutoff point. This indicates that the player will begin contributing a positive amount to the public good at a lower value as the mean of the cost increases.

The change in $\bar{c}$ also affects the player's best response function where $x^* > 0$. This effect can be divided into the direct effect and the indirect effect using the player's best response function, x^*, to demonstrate these two effects.

$$\frac{\partial x^*}{\partial \bar{c}} = \frac{-(n-1)}{2}\frac{\partial K^*}{\partial \bar{c}} + \frac{1}{2} \tag{2.15}$$

$$= \left\{\frac{-1}{2} + \frac{1}{2}[1 + (n-1)(1 + \bar{c} - z)]^{\frac{-1}{2}}\right\} + \frac{1}{2} \tag{2.16}$$

$$= \frac{1}{2}[1 + (n-1)(1 + \bar{c} - z)]^{\frac{-1}{2}} > 0 \tag{2.17}$$

From Equation (2.15), we find that changing $\bar{c}$ will change x^* directly. Hence, the second term in Equation (2.15) represents the direct effect. Also, changing $\bar{c}$ will affect other players' expected contributions, K^*. The first term in Equation (2.15) measures this effect and represents the indirect effect. Equation (2.16) shows that increasing the mean of cost uncertainty induces the positive direct effect, the negative indirect effect, and that the direct effect dominates the indirect effect. Thus, increasing $\bar{c}$ results in the increase of x^*.

We find that players start to contribute to the public goods at a lower cutoff point value and the contribution amounts at each possible value of public good weakly increase. Hence, the expected contribution to the public good increases with the mean of cost uncertainty, $\bar{c}$.

This proposition provides the policy implication that if the suppliers are able to increase the mean of the cost distribution, the private contribution to the public good will increase. The increase in the mean cost will encourage the players with low value to begin contributing to the public good and also increase contributions of inframarginal contributors.

2.3.3 Numerical Example

In this subsection, we use two numerical examples to show that expected contribution, K^*, increases in $\bar{c}$, and decreases in z, respectively.

Example 1. (Mean-Preserving Spread)
In this example, we consider a subscription game with 5 players whose values are uniformly distributed in [0, 1]. We also assume that players do not know the cost of providing the public good but believe it follows a uniform distribution with support [1, 5], i.e. the initial $\bar{c} = 3$ and $z = 2$. If the cost distribution becomes more dispersed in the sense of mean-preserving spread, such as z increases from 2 to 2.2, Figure 2.2 shows that the expected contribution is decreasing in z.

Example 2. (First-Order Stochastic Dominance)
Consider another subscription game with 5 players whose values for the public good follow a common uniform distribution in [0, 1]. Players do not know the exact cost of providing the public good but they believe that it is uniformly distributed in [0, 4], that is, the initial $\bar{c} = 2$ and $z = 2$. If the cost of public good increases in the sense of first-order stochastic dominance, such as $\bar{c}$ increases from 2 to 2.2, Figure 2.3 shows that the expected contribution is increasing in $\bar{c}$.

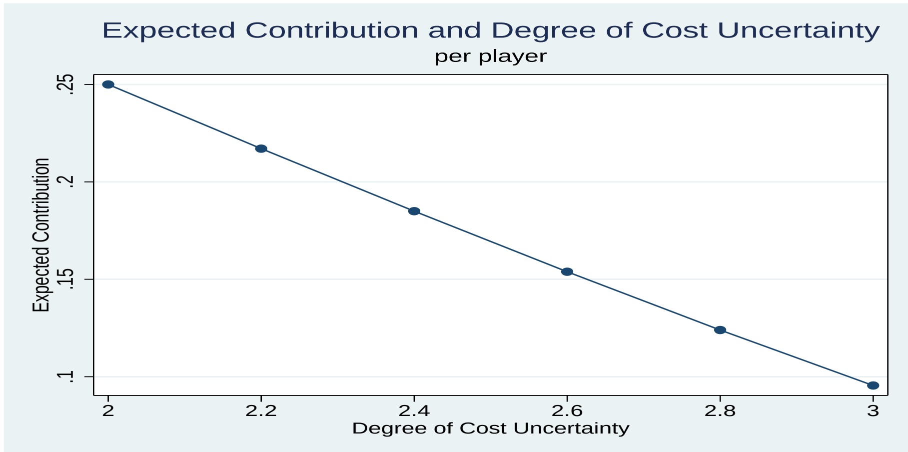

Figure 2.2: Expected Contributions in the Example of Mean-Preserving Spread

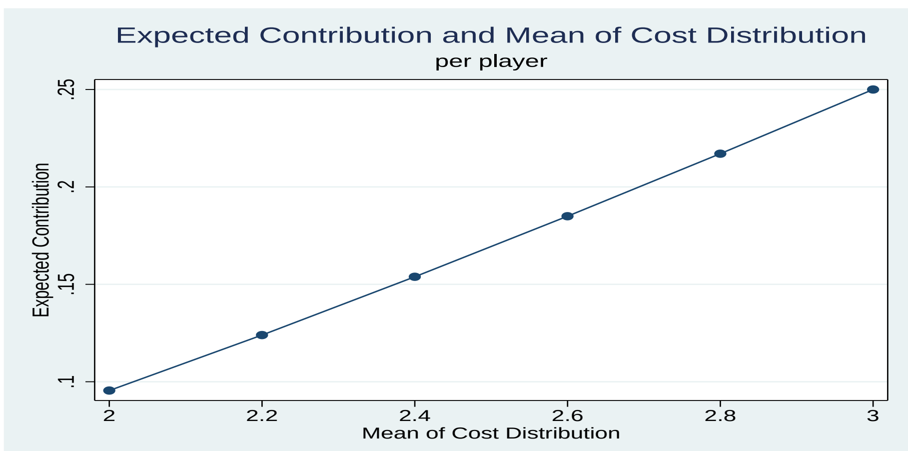

Figure 2.3: Expected Contributions in the Example of First-Order Stochastic Dominance

2.4 Conclusion

If the valuation for the public good exhibits complete information to all players and is identical for each player, McBride (2006) finds that the effect of increased cost uncertainty depends on the value of the public good. However, when we consider public good valuations as private information, then expected contributions are monotonic, and a more dispersed cost distribution always decreases the expected contributions. Moreover, we find that increasing the mean of the public good in the sense of first order stochastic dominance will increase the expected contribution.

From a policy perspective, we suggest that suppliers can increase the private contribution if they can either reduce the degree of uncertainty or increase the mean with respect to the cost distribution when there exists both threshold uncertainty and private information on public good valuations.

We offer two directions for future research. Many real-world private contribution institutions are not simultaneous, contributions are instead often collected sequentially. For example, churches may announce organ fund campaigns and report the updated contribution level frequently or local governments announce the seed donations to future contributors when they launch new public good projects. There is no published research that investigates how the sequential contribution would be affected by a change in the dispersion of the cost distribution or the value distribution in a subscription game under threshold uncertainty and private information of valuation for the public good. A second research direction is to test the hypothesis from our theoretical model using experimental methods in a laboratory environment. These future studies may result in a more complete understanding of behavior in mechanisms of private contribution to public goods.

3. SEQUENTIAL CONTRIBUTION TO A DISCRETE PUBLIC GOOD UNDER THRESHOLD UNCERTAINTY AND PRIVATE INFORMATION

3.1 Motivation

An important class of public good allocation problems involves discrete or fixed quantity public goods. Typical examples of discrete public goods include parks, local libraries, bridges, etc. Other interesting examples include: interest groups lobbying to get a bill through Congress, and non-profit organizations raising funds for non-profit agency events or projects. The cost of providing such a public good is often called the threshold. Individuals would like to know the amount of money needed to provide the public good when making contribution decisions; nevertheless, the cost threshold may be uncertain in many situations. Suppliers have limited resources to research the cost of completing a public goods project. For example, the exact cost of construction may be unknown because biddings among potential contractors has not been completed. Thus, threshold uncertainty is an important model consideration in this paper.

In addition to threshold uncertainty, contributors often do not know whether other contributors are willing to support the same project nor how much they value the public good. Therefore, it is interesting to analyze the contribution equilibrium assuming that valuation of the public good is private information.

Another important factor that may affect the contribution behavior is the timing of the contribution. Many real-world public/private contribution institutions do not receive contributions simultaneously. Contributions are instead made sequentially and early contributions are often announced publicly. For example, universities announce capital campaigns for new buildings and report the earlier contributions periodically on websites; churches may announce organ campaign targets and then update the donations to the organ funds in the weekly bulletin.

In this paper, I introduce cost threshold uncertainty and private information on public good valuations in a discrete public good subscription game and analyze the following three questions: (1) How do individuals contribute to the public good in the environment with private information and the cost uncertainty? (2) Do individuals operating in different contribution orders contribute differently? (3) Comparing sequential and simultaneous contribution institution, which institution produces higher expected total contributions?

In the sequential contribution institution considered in this paper, each player knows the total contributions made by the previous players before he/she makes his/her own contribution decision. I also assume that players make contributions in an exogenous sequence of mover and each player contributes only once. I derive the Bayesian equilibrium for this sequential contribution model and find that individual contribution increases with respect to the contribution order, that is, the earlier contributors contribute less to the public good than the subsequent contributors do. In addition, I offer results of comparing the expected contributions in the sequential and simultaneous contribution models. Results show that the expected total contribution in the sequential contribution institution is less than that in the simultaneous contribution institution.

In addition to considering both private information on valuation and the threshold uncertainty in the primary model, I briefly discuss cases considering only private information on valuation or threshold uncertainty in a sequential subscription game. I find that controlling for the mean valuation, each player's expected contribution with private information on valuation and the threshold uncertainty is higher than that with complete information on valuation and the threshold uncertainty. I also find that controlling for the mean of the threshold distribution, each player's expected contribution with private information on valuation and the threshold uncertainty is higher than that with private information on valuation and threshold certainty.

The rest of paper is organized as follows. Section 2 reviews the literatures related to threshold uncertainty, private information and sequential contribution. My model is presented in Section 3, followed by the comparisons of the expected contribution in the sequential and simultaneous institutions in Section 4. I briefly discuss the cases with complete information on valuation and threshold uncertainty, with private information on valuation and threshold certainty, and with no refund rule in Section 5. A conclusion is given in Section 6.

3.2 Literature Review

Palfrey and Rosenthal (1984), Bagnoli and Lipman (1989), investigate private provision of a discrete public good. Both papers assume players simultaneously make their contribution strategies in an environment with complete information on the public good valuation and a certain known threshold level of cost, but they consider different types of contribution. Palfrey and Rosenthal (1984) assume the player to make a binary contribution, whereas Bagnoli and Lipman (1989) allow the player to make a continuous contribution. These two papers show that if the full refund rule is introduced into the threshold public good game where players simultaneously make their contribution, no efficient Nash Equilibrium exists.

Admati and Perry (1991) investigate private provision of a discrete public good. Instead of considering simultaneous contribution, they assume players contribute "sequentially" in a subscription game (meaning that the contribution will be refunded if the total contribution to the public good is not large enough to cover the fixed cost of providing the public good). They analyze a 2-player subscription game with complete information where these two players make alternate contribution decisions until the total contribution is covered the cost threshold. They show that there is a unique, efficient subgame perfect equilibrium in the subscription game.

Nitzan and Romano (1990) extend Bagnoli and Lipman's game by introducing uncertainty regarding the cost of providing the public good and find different results. The equilibrium is inefficient because the uncertainty of the cost may cause the *ex post* contributions to exceed or to fall short of the required threshold.

McBride (2006) focuses on investigating how the level of threshold uncertainty affects the players' contributions. In his model, McBride assumes that each player makes a binary contribution decision simultaneously and that the contribution will not be refunded if the cost threshold is not met. He finds that instead of a monotonic relationship between the degree of threshold uncertainty and total contributions, the effect of changing the threshold uncertainty on the contributions depends on the value of the public good. An increase of threshold uncertainty in the sense of mean-preserving spread increases the player's contribution when the value of the public good is sufficiently high, but decreases the player's contribution when the value of the public good is sufficiently low. McBride (2010) designs an experiment to test his theory in a lab and finds limited verification.

Papers by Menezes *et al.* (2001), Laussel and Palfrey (2003), and Barbieri and Malueg (2008) introduce private information on the public good valuations into a subscription game with a discrete public good and focus on examining the efficiency of the Bayesian Nash equilibrium. Menezes *et al.* (2001) show that the equilibrium in the subscription game is *ex post* inefficient. Laussel and Palfrey (2003) analyze interim incentive efficiency, defined by Holmström and Myerson (1983), in the subscription game. They find that the interim incentive efficient equilibrium may exist. Later, Barbieri and Malueg (2008) reexamine Laussel and Palfrey's analysis and show the contrary result that there is no incentive efficient equilibrium.

Barbieri and Malueg (2010) include both the threshold uncertainty and private information on valuations for a public good in the subscription game. They focus on whether changing the intensity and dispersion of value distribution affect players' expected contributions. They find that increasing the value distribution in the sense of first order stochastic dominance or dispersing the value distribution in the sense of mean-preserving spread increases the expected contributions.

Barbieri and Malueg (2010) forms the basis for my model setting. The main difference between my model and and Barbieri and Malueg's is the timing of the contribution mechanism. I focus on the sequential contribution, while Barbieri and Malueg focus on the simultaneous contribution.

Gronberg and Peng (2014) consider both the threshold uncertainty and private information on public good's valuations in a subscription game and research the effects of changing the threshold distribution. They find that increasing the mean of the cost distribution in the sense of first order stochastic dominance increases individual expected contribution; while increasing the uncertainty level of the cost in the sense of mean-preserving spread decreases individual expected contribution.

An early analysis of sequential contribution to a public good is provided by Varian (1994). In a model with a continuous public good and complete information on public good valuation, he finds that sequential contribution enables the early contributor to free ride off the latter one and the total contribution under sequential institution is lower than the total contribution under simultaneous institution. This finding asserts that the leader in a sequential public good game tries to exploit the first mover advantage and leaves the burden of providing the public good to the following contributors.

Gächter *et al.* (2010) examines Varian's prediction via a laboratory experiment.

Their experimental result is consistent with the theoretical prediction that total contribution is lower under sequential mechanism than simultaneous alternative when contributors' preferences are sufficiently different (but not too different).

Bag and Roy (2011) extend Varian's model and treat donors' values of the public good as private information. They show that the expected total contribution generated in a perfect Bayesian equilibrium of the sequential contribution game is at least as large as that in a Bayesian-Nash equilibrium of the simultaneous contribution game. This occurs because when donors are uncertain about other players' values of the public good, the earlier donors may be cautious in free-riding on prospective donors.

Unlike Varian's model, Romano and Yildirim (2001) consider warm-glow effect noted by Andreoni (1989) as another contribution motivation in a sequential game. In their model, the contributors are not only concerned with total contributions, but also their own contribution level. With the warm-glow specification, they find that the level of the public good in the sequential-move mechanism is higher than in the simultaneous-move mechanism. Differing from Romano and Yildirim's (2001) model, I assume that individuals are only concerned with total contribution to a public good in this paper.

Cartwright and Patel (2010) suggest that the heterogeneity in individual behavior may affect the contribution in a sequential game. They find that the strategists, who behave strategically to maximize their own payoffs, will contribute to the public good if they are early enough in the sequence and if they believe there are enough imitated followers in a sequential game.

Andreoni (1998) focuses on the role of seed money in a discrete public good sequential game setting. He demonstrates that adopting a sequential fundraising strategy can increase the likelihood of providing the public good when a cost threshold exists. This occurs because when contributors are not willing to cover the cost single-handedly, simultaneous contribution may generate zero equilibrium. Sequential contribution can eliminate such inefficiency since announcing the previous player's donation can ensure that the latter player is willing to cover the remaining cost. Subsequently, Bracha *et al.*(2011) test Andreoni's theory experimentally and find that the experimental results are supportive of the theory when the cost threshold is sufficiently high.

Vesterlund (2003) considers a model in which the donors have common valuations but the quality of the charity is unknown. She shows that larger gifts from early donors prompt later donors to give higher donations. This motivates the high quality charities to announce the earlier contributions to the public. In contrast to Vestlund's paper, contributors in my paper have independent private information on a public good valuation. Thus, the results in this paper are not based on any informational advantage or signaling value of announcement of contributions.

There are several experimental studies on continuous public goods (see Potters et al.(2005), Masclet and Willinger (2005), Levati *et al.* (2007), Vyrestekova and Garikipati (2008)) and discrete public goods (see Coats *et al.* (2009)). These papers find the sequential contribution is significantly higher than the simultaneous contribution when players have complete information on the distribution of valuations of the public good.

3.3 Model

I consider a public good game with $n \geq 2$ players who sequentially contribute any non-negative amount to a discrete public good. Player i's value for the public good is shown by v_i, $i = 1, ..., n$, and is an independently distributed random variable with a continuous uniform distribution whose support is $[0, 1]$. With this assumption, each player's valuation for the public good is private information, meaning that each player knows his/her own realized value for the public good, but he/she is uncertain about other players' values for the public good. Let $x_i \in [0, v_i]$ be player i's contribution.

The discrete public good can be provided if and only if the total contributions are equal to or larger than the cost of the public good, c, known as the cost threshold. Suppose c is unknown when the players make their contribution decisions and all players believe that the cost is independent of all v_i's and distributed along a continuous uniform distribution, F, with support $[0, \tilde{c}]$, where $\tilde{c} \geq n$. Barbieri and Malueg (2010) show that in order to obtaining the unique equilibrium, the model requires the assumption that $\tilde{c} \geq n$. Since Barbieri and Malueg (2010) is the basis for my model, I use the same framework and maintain the same assumption in this paper.

In this paper I consider the subscription public good game (Admati and Perry, 1991). Accordingly, the player's contribution will be fully refunded if the total contributions are less than the cost threshold, c. I also assume a zero rebate rule, meaning the excess contributions will be given to the producer of the public good.

Players in this model make their contribution decisions sequentially. I assume that the order of the move is exogenous. Each player contributes to the pubic good only once. The total contribution is updated and announced to the public after any player make his/her contribution decision. Thus, when a player makes his/her contribution decision, he/she can observe the total accumulated contribution made by the earlier players and must anticipate the contributions of prospective players.

Assume each player has a linear utility function

$$z_i = w_i - x_i + g_i(x_i + x_{-i}) \tag{3.1}$$

where, w_i is player i's wealth and

$$g_i(x_i + x_{-i}) = \begin{cases} v_i & \text{if } x_i + x_{-i} \geq c \\ x_i & \text{if } x_i + x_{-i} < c \end{cases} \tag{3.2}$$

If total contribution is larger than or equal to the cost threshold, c, the public good is provided and player i receives v_i with provision probability, p_i; otherwise, the public good is not provided and player i receives his/her own refunded contribution, x_i.

Thus, player i's expected utility function can be written as

$$\begin{aligned} Z_i = E[z_i] &= \big[w_i - x_i + v_i\big] \times p_i + w_i \times (1 - p_i) \\ &= w_i + (v_i - x_i) \times p_i \\ &= w_i + U_i(x_i|v_i) \end{aligned} \tag{3.3}$$

where $U_i(x_i|v_i)$ is player i's expected payoff from the public good.

Each player's objective is to maximize his/her expected utility. However, in the linear utility framework, maximizing the expected utility is the same as maximizing the expected payoff from the public good. Thus, in the following model, I assume that each player's objective is to maximize his/her expected payoff from the public good, $U_i(x_i|v_i)$.

Since the player contributes to the public good sequentially, he/she does not know what contribution the subsequent players will make and has to calculate their expected contributions. Because the contribution order is different for each player, the number of remaining players is different for each player. For example, the first player in a 3-player game needs to expect the second and the third player's contributions; but, the second player can observe the first player's contribution and only needs to expect the third player's contribution. Hence, the probability of providing the public good each player faces, p_i, is different. Depending on the sequence, players' expected payoff functions can be written as:

Player 1's expected payoff function is:

$$U_1(x_1|v_1) = (v_1 - x_1)\Big[\frac{x_1 + E_1[x_2] + E_1[x_3] + ... + E_1[x_n]}{\tilde{c}}\Big] \tag{3.4}$$

where $E_1[x_j]$, $j = 2..., n$ is the contribution Player 1 expects Player j to make.

Player 2's expected payoff function is:

$$U_2(x_2|v_2) = (v_2 - x_2)\Big[\frac{x_1 + x_2 + E_2[x_3] + E_2[x_4] + ... + E_2[x_n]}{\tilde{c}}\Big] \tag{3.5}$$

where $E_2[x_k]$, $k = 3..., n$ is the contribution Player 2 expects Player k to make.

⋮

⋮

Player n's expected payoff function is:

$$U_n(x_n|v_n) = (v_n - x_n)\Big[\frac{x_1 + x_2 + ... + x_{n-1} + x_n}{\tilde{c}}\Big] \tag{3.6}$$

Since Player n is the last player in an n-player game, he/she does not anticipate any other player's contribution.

From Equation (3.4) to Equation (3.6), I arrive at the expected payoff function of Player i (the i-th player in the game) is:

$$U_i(x_i|v_i) = (v_i - x_i)\Big[\frac{\sum_{l=1}^{i-1} x_l + x_i + \sum_{m=i+1}^{n} E_i[x_m]}{\tilde{c}}\Big] \tag{3.7}$$

where $\sum_{l=1}^{i-1} x_l$ is the total contribution made by the player who acts before Player i, and $E_i[x_m]$ is the contribution that Player i expects Player m to make.

3.3.1 Best Response Function

To derive each player's best response function in the sequential game, I use the method of backward induction. For simplicity, I start with a 2-player case.

2-player case

There are two players, Player 1 and Player 2. Player 1 makes his/her contribution decision first and Player 2 makes his/her contribution decision subsequently. Thus, Player 1's expected payoff function is

$$U_1(x_1|v_1) = (v_1 - x_1)\Big[\frac{x_1 + E_1[x_2]}{\tilde{c}}\Big] \tag{3.8}$$

and Player 2's expected payoff function is

$$U_2(x_2|v_2) = (v_2 - x_2)\Big[\frac{x_1 + x_2}{\tilde{c}}\Big] \tag{3.9}$$

By the method of backward induction, I discuss Player 2's behavior first.

Maximizing Equation (3.9) with respect to x_2 and taking the first order condition (F.O.C), I get:

$$\frac{\partial U_2}{\partial x_2} = \frac{-1}{\tilde{c}}(x_1 + x_2) + \frac{1}{\tilde{c}}(v_2 - x_2) = 0$$

$$\Longrightarrow x_2 = \frac{v_2 - x_1}{2} \tag{3.10}$$

It yields Player 2's best response function:

$$x_2 = \begin{cases} 0 & \text{if } 0 \leq v_2 \leq x_1 \\ \frac{v_2 - x_1}{2} & \text{if } x_1 \leq v_2 \leq 1 \end{cases} \tag{3.11}$$

From Equation (3.11), I find that $v_2 = x_1$ is the cutoff point for Player 2 to begin contributing to the public good in the 2-player case. In other words, Player 2 is willing to contribute a positive amount to the public good when his/her value is equal to or larger than the cutoff point, x_1. Also, Equation (3.11) shows that once Player 2's contribution is positive, it is strictly increasing in his/her value of the public good and strictly decreasing in Player 1's contribution.

Next, I solve for Player 1's behavior. Since Player 1 cannot observe Player 2's contribution when he/she makes his/her contribution decision, he/she has to calculate Player 2's expected contribution, namely $E_1[x_2]$ in Equation (3.8). Consider Player 2's value, v_2, which follows a uniform distribution with $[0, 1]$ and his/her best response function, Equation (3.11), Player 2's expected contribution Player 1 expects is

$$E_1\big[x_2\big] = \int_0^{x_1} 0 dv_2 + \int_{x_1}^{1} \big[\frac{v_2 - x_1}{2}\big] dv_2$$

$$E_1\big[x_2\big] = \frac{1}{4} - \frac{x_1}{2} + \frac{x_1^2}{4} \tag{3.12}$$

Substitute Equation (3.12) into Equation (3.8), Player 1's expected payoff function can be rewritten as

$$U_1(x_1|v_1) = \Big(v_1 - x_1\Big)\Big(\frac{\frac{1}{4} + \frac{x_1}{2} + \frac{x_1^2}{4}}{\tilde{c}}\Big) \tag{3.13}$$

Maximizing Player 1's expected payoff function, Equation (3.13), and taking the first order condition (F.O.C), I get:

$$\frac{\partial U_1}{\partial x_1} = \frac{-1}{\tilde{c}}\Big(\frac{1}{4} + \frac{x_1}{2} + \frac{x_1^2}{4}\Big) + \frac{1}{\tilde{c}}(v_1 - x_1)(\frac{1}{2} + \frac{x_1}{2}) = 0$$

$$\Longrightarrow x_1 = \frac{2v_1 - 1}{3} \tag{3.14}$$

It yields Player 1's best response function:

$$x_1 = \begin{cases} 0 & \text{if } 0 \leq v_L < \frac{1}{2} \\ \frac{2v_1-1}{3} & \text{if } \frac{1}{2} \leq v_L \leq 1 \end{cases} \tag{3.15}$$

From Equation (3.15), I find that $v_1 = \frac{1}{2}$ is the cutoff point for Player 1 to begin contributing to the public good in the 2-player case. In other words, Player 1 is willing to contribute a positive amount to the public good when his/her value is equal to or larger than the cutoff point, $\frac{1}{2}$. Also, Equation (3.15) shows that once Player 1's contribution is positive, it is strictly increasing in his/her value of the public good.

3-player case

Assume there are three players in a subscription game. Player 1 makes his/her contribution decision first and is followed by Player 2. Finally, Player 3 makes his/her contribution decision.

Player 1's expected payoff function is

$$U_1(x_1|v_1) = (v_1 - x_1)\left[\frac{x_1 + E_1[x_2] + E_1[x_3]}{\tilde{c}}\right] \tag{3.16}$$

Player 2's expected payoff function is

$$U_2(x_2|v_2) = (v_2 - x_2)\left[\frac{x_1 + x_2 + E_2[x_3]}{\tilde{c}}\right] \tag{3.17}$$

and Player 3's expected function is

$$U_3(x_3|v_3) = (v_3 - x_3)\left[\frac{x_1 + x_2 + x_3}{\tilde{c}}\right] \tag{3.18}$$

Using the same basic method I use to solve the 2-player case, I get each player's best response function:

Player 3's best response function is

$$x_3 = \begin{cases} 0 & \text{if } 0 \leq v_3 \leq (x_1 + x_2), \\ \frac{v_3 - (x_1 + x_2)}{2} & \text{if } (x_1 + x_2) \leq v_3 \leq 1 \end{cases} \tag{3.19}$$

Player 2's best response function is

$$x_2 = \begin{cases} 0 & \text{if } 0 \leq v_2 \leq \frac{1+x_1}{2}, \\ \frac{2v_2 - x_1 - 1}{3} & \text{if } \frac{1+x_1}{2} \leq v_2 \leq 1 \end{cases} \tag{3.20}$$

Player 1's best response function is

$$x_1 = \begin{cases} 0 & \text{if } 0 \leq v_1 < \frac{2}{3}, \\ \frac{3v_1 - 2}{4} & \text{if } \frac{2}{3} \leq v_1 \leq 1 \end{cases} \tag{3.21}$$

Following the same procedure in the 2-player and the 3-player case and using the method of induction, I derive each player's best response function in a n-player case. The best response function of Player i, who is the i-th player in the n-player case, can be written as

$$x_i = \begin{cases} 0 & \text{if } 0 \leq v_i \leq \frac{(n-i) + \sum_{p<i} x_p}{(n+1-i)}, \\ \frac{(n+1-i)v_i - (n-i) - \sum_{p<i} x_p}{n+2-i} & \text{if } \frac{(n-i) + \sum_{p<i} x_p}{(n+1-i)} \leq v_i \leq 1 \end{cases} \tag{3.22}$$

Player i's best response function, Equation (3.22), shows that the higher the player's valuation for the public good is, the higher the contribution Player i will make. But, the higher the total contribution made by the earlier players, the lower the contribution Player i will make.

3.3.2 Bayesian Equilibrium

Since players have independent private information on valuations for a public good, the solution concept is Bayesian equilibrium. In this subsection, I explain how to arrive at the Bayesian Equilibrium.

2-player case

In the 2-player case I mentioned in the previous subsection, Equation (3.11) and Equation (3.15) represent Player 1 and Player 2 best response functions, respectively. Using these best response functions, I find each player's equilibrium strategy. From Equation (3.15), Player 1's equilibrium strategies are

$$x_1^* = \begin{cases} 0 & \text{if } 0 \leq v_L < \frac{1}{2} \\ \frac{2v_1-1}{3} & \text{if } \frac{1}{2} \leq v_L \leq 1 \end{cases} \tag{3.23}$$

Thus, Player 1's Bayesian equilibrium, also known as the *ex ante* expected contribution, is:

$$E[x_1^*] = \int_0^{\frac{1}{2}} 0 \; dv_1 + \int_{\frac{1}{2}}^1 \frac{2v_1 - 1}{3} dv_1$$

$$\Longrightarrow E[x_1] = \frac{1}{12} \tag{3.24}$$

Using Equation (3.23) and Player 2's best response function, Equation (3.11), yields Player 2's equilibrium strategies:

- 1. if Player 1's equilibrium is $x_1^* = 0$

$$x_2^* = \frac{v_2}{2} \quad \forall v_2 \tag{3.25}$$

- 2. if Player 1's equilibrium is $x_1^* = \frac{2v_1-1}{3}$

$$x_2 = \begin{cases} 0 & \text{if } 0 \leq v_2 \leq \frac{2v_1-1}{3} \\ \frac{3v_2-2v_1+1}{6} & \text{if } \frac{2v_1-1}{3} \leq v_2 \leq 1 \end{cases} \tag{3.26}$$

According to Player 2' equilibrium strategies, Equation (3.25) and Equation (3.26), his/her Bayesian equilibrium is

$$E[x_2^*] = \int_0^{\frac{1}{2}} \int_0^1 \frac{v_2}{2} dv_2 dv_1 + \int_{\frac{1}{2}}^1 \int_{\frac{2v_1-1}{3}}^1 \frac{3v_2 - 2v_1 + 1}{3} dv_2 dv_1$$

$$\Longrightarrow E[x_2^*] = \frac{1}{8} + \frac{23}{108} = \frac{8}{27} \tag{3.27}$$

3-player case

Using the best response functions I find in the last subsection, Equation (3.19) to Equation (3.21), I calculate each player's equilibrium strategy and his/her expected contribution.

Based on Player 1's best response function, Equation (3.21), his/her equilibrium strategies are

$$x_1^* = \begin{cases} 0 & \text{if } 0 \leq v_1 < \frac{2}{3}, \\ \frac{3v_1-2}{4} & \text{if } \frac{2}{3} \leq v_1 \leq 1 \end{cases} \tag{3.28}$$

Thus, Player 1's *ex ante* expected contribution is calculated from Equation (3.28), and can be written as

$$E[x_1^*] = \int_0^{\frac{2}{3}} 0 \ dv_1 + \int_{\frac{2}{3}}^1 \frac{3v_1 - 2}{4} dv_1$$

$$\Longrightarrow E[x_1] = \frac{1}{24} \tag{3.29}$$

Using Equation (3.28) and Player 2's best response function, Equation (3.20), yields Player 2's equilibrium strategies:

- 1. if Player 1's equilibrium is $x_1^* = 0$

$$x_2 = \begin{cases} 0 & \text{if } 0 \leq v_2 \leq \frac{1}{2} \\ \frac{2v_2-1}{3} & \text{if } \frac{1}{2} \leq v_2 \leq 1 \end{cases} \tag{3.30}$$

- 2. if Player 1's equilibrium is $x_1^* = \frac{3v_1-2}{4}$

$$x_2 = \begin{cases} 0 & \text{if } 0 \leq v_2 \leq \frac{3v_1+2}{8} \\ \frac{8v_2-3v_1-2}{12} & \text{if } \frac{3v_1+2}{8} \leq v_2 \leq 1 \end{cases} \tag{3.31}$$

Using Equation (3.30) and Equation (3.31), I get Player 2's Bayesian equilibrium:

$$E[x_2^*] = \int_0^{\frac{2}{3}} \int_{\frac{1}{2}}^{1} \frac{2v_2 - 1}{3} dv_2 dv_1 + \int_{\frac{2}{3}}^{1} \int_{\frac{3v_1+2}{8}}^{1} \frac{8v_2 - 3v_1 - 2}{12} dv_2 dv_1$$

$$\implies E[x_2^*] = \frac{1}{18} + \frac{37}{1728} = \frac{133}{1728} \tag{3.32}$$

Based on Player 1 and Player 2's strategies and Player 3's best response function, Equation (3.19), I get Player 3's equilibrium strategies:

- 1. if Player 1's equilibrium is $x_1^* = 0$ and if Player 2's equilibrium is $x_2^* = 0$:

$$x_3^* = \frac{v_3}{2} \quad \forall v_3 \tag{3.33}$$

- 2. if Player 1's equilibrium is $x_1^* = 0$ and if Player 2's equilibrium is $x_2^* = \frac{2v_2-1}{3}$:

$$x_3^* = \begin{cases} 0 & \text{if } 0 \leq v_3 \leq \frac{2v_2-1}{3}, \\ \frac{3v_3-v_2+1}{6} & \text{if } \frac{2v_2-1}{3} \leq v_3 \leq 1 \end{cases} \tag{3.34}$$

- 3. if Player 1's equilibrium is $x_1^* = \frac{3v_1-2}{4}$ and if Player 2's equilibrium is $x_2^* = 0$:

$$x_3^* = \begin{cases} 0 & \text{if } 0 \leq v_3 \leq \frac{3v_1-2}{4}, \\ \frac{4v_3-3v_1+2}{8} & \text{if } \frac{3v_1-2}{4} \leq v_3 \leq 1 \end{cases} \tag{3.35}$$

- 4. if Player 1's equilibrium is $x_1^* = \frac{3v_1-2}{4}$ and if Player 2's equilibrium is $x_2^* = \frac{2v_2-1}{3}$:

$$x_3^* = \begin{cases} 0 & \text{if } 0 \leq v_3 \leq \frac{3v_1+4v_2-4}{6}, \\ \frac{3v_1+4v_2-4}{6}\frac{6v_3-3v_1-4v_2+4}{12} & \text{if } \leq v_3 \leq 1 \end{cases} \tag{3.36}$$

Using Equation (3.33) to Equation (3.36), I get Player 3's Bayesian equilibrium:

$$E[x_3^*] = \int_0^{\frac{2}{3}} \int_0^{\frac{1}{2}} \int_0^1 \frac{v_3}{2} dv_3 dv_2 dv_1 + \int_0^{\frac{2}{3}} \int_{\frac{1}{2}}^1 \int_{\frac{2v_2-1}{3}}^1 \frac{3v_3 - v_2 + 1}{6} dv_3 dv_2 dv_1$$

$$+ \int_{\frac{2}{3}}^1 \int_0^{\frac{3v_1+2}{8}} \int_{\frac{3v_1-2}{4}}^1 \frac{4v_3 - 3v_1 + 2}{8} dv_3 dv_2 dv_1$$

$$+ \int_{\frac{2}{3}}^1 \int_{\frac{3v_1+2}{8}}^1 \int_{\frac{3v_1+4v_2-4}{6}}^1 \frac{6v_3 - 3v_1 - 4v_2 + 4}{12} dv_3 dv_2 dv_1$$

$$\Longrightarrow E[x_3^*] = \frac{1}{12} + \frac{19}{324} + \frac{659}{18432} + \frac{3325}{165888} = \frac{4101}{20736} \tag{3.37}$$

To solve the n-player case, I adopt the method used in the 2-player and the 3-player cases; however, it is complicated and difficult to solve for the Bayesian equilibrium in a case with large number of players. This is because a player's equilibrium strategies I need to derive increase with the number of players in the subscription game.

For example, to solve the Bayesian equilibrium of the last player in an n-player game, I need to derive his/her equilibrium strategies first. According to $(n-1)$ earlier players' strategy combinations, there are 2^{n-1} different groups of Player n's equilibrium strategies. Next, I have to calculate the multiple integrals over $v_1, ..., v_n$ and to get the expected contributions in these 2^{n-1} different groups. The last step to get the nth player's Bayesian equilibrium is to sum the expected contributions in these 2^{n-1} different groups. Due to the complication of deriving the Bayesian equilibrium directly, I use the method of simulation to analyze the Bayesian equilibrium in a game with a larger number of players.

In the following paragraphs, I describe how to simulate the Bayesian equilibrium in this paper explicitly. I divide each player's valuation into 1001 "units" from 0 to 1. That is, a player's value may be $0, 0.001, 0.002, ..., 1$. Player 1 makes his/her contribution depending on his/her best response function and his/her valuation for the public good. Since there are 1001 possible values for Player 1, I get 1001 x_1^*. Then, summing these 1001 x_1^* and dividing by 1001, I will get Player 1's expected contribution.

When Player 2 makes his/her contribution, he/she will not only consider his/her own best response function and his/her valuation for the public good but also considers the contribution made by Player 1. Since there are 1001 different Player 2's values for the public good and 1001 x_1^*, I get 1001^2 x_2^*. I sum these 1001^2 x_2^* and divide by 1001^2, arriving at Player 2's expected contribution.

Thus, to calculate the i-th player's expected contribution in an n-player game, I, first, get 1001^i x_i^* according to Player i's possible valuation for the public good and the contribution made by the players who contribute earlier. Then, Player i's expected contribution is calculated by summing up these 1001^i x_i^* and dividing by 1001^i.

Figure 3.1 shows the simulation results of each player's expected contribution in the 2-player to 7-player sequential contribution cases. I find that the expected contribution increases in the order of movement in these 4 cases.

This result demonstrates that the earlier contributors can free ride off the subsequent contributors in the sequential contribution institution and they enjoy the first-mover advantage by contributing smaller amounts to the public good, relying on other contributors to provide the public good on their own. Thus, the order of contributing to the public good plays an important role to the contributors.

Another finding is that the gap between the first player's and the last player's expected contribution increases in the number of players in the game. For example, in the 2-player case, the first player's expected contribution is one third of the last player's expected contribution; but, in the 5-player case, the first player's expected contribution is only one tenth of the last player's expected contribution. This result shows that the free-riding problem becomes more serious when the number of players contribute to the public good in a subscription game gets larger.

3.4 Comparison of the Expected Contribution

Except for the institution of contribution, the model setting in this paper is similar to that in Barbieri and Malueg (2010). My approach in this section is to compare the expected contribution under sequential contribution with the expected contribution under simultaneous contribution.

Before comparing the results of these two contribution institutions, I briefly introduce Barbieri and Malueg's model and equilibrium result. Their model setting is

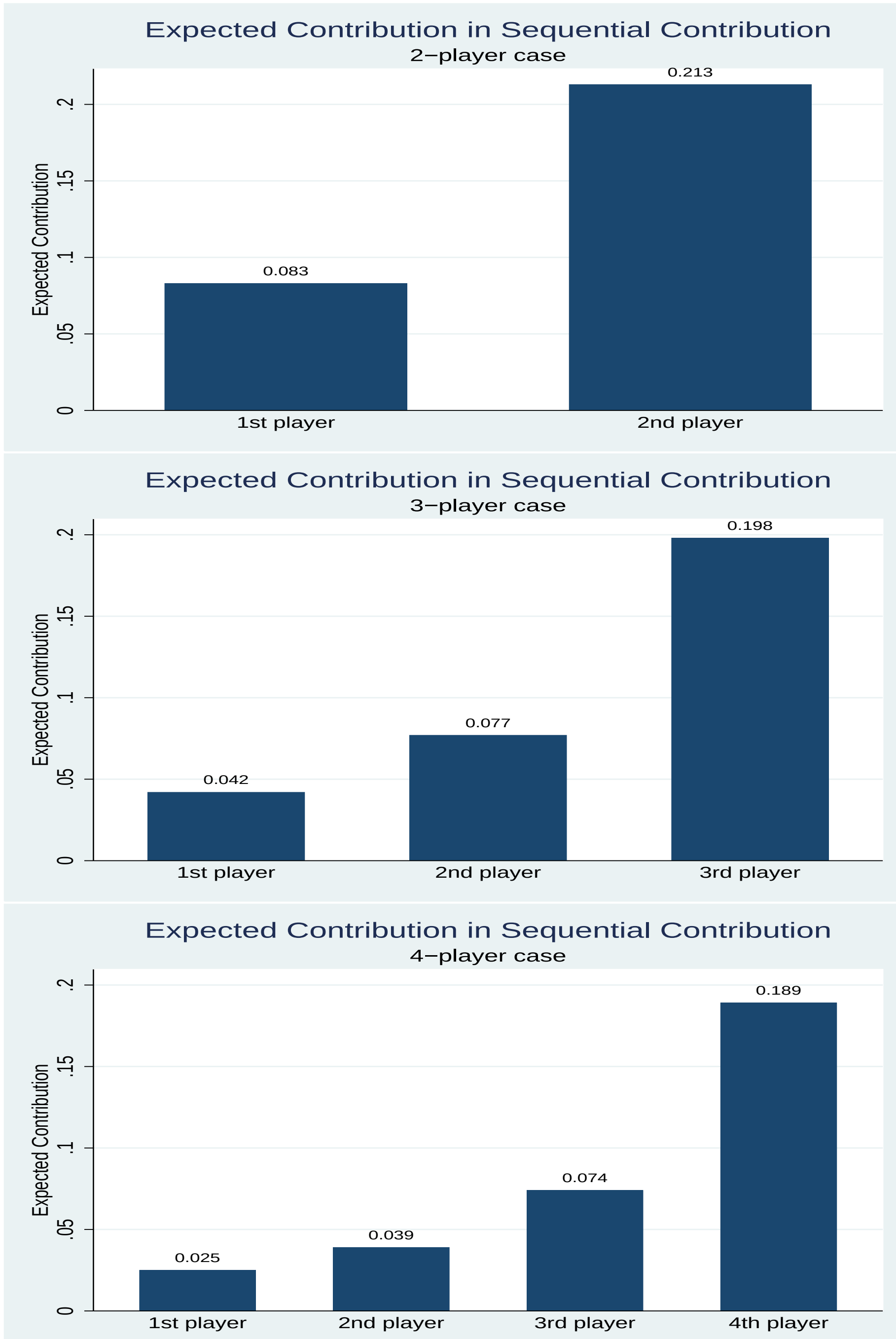

Figure 3.1: Expected Contribution per Player in the Sequential Contribution

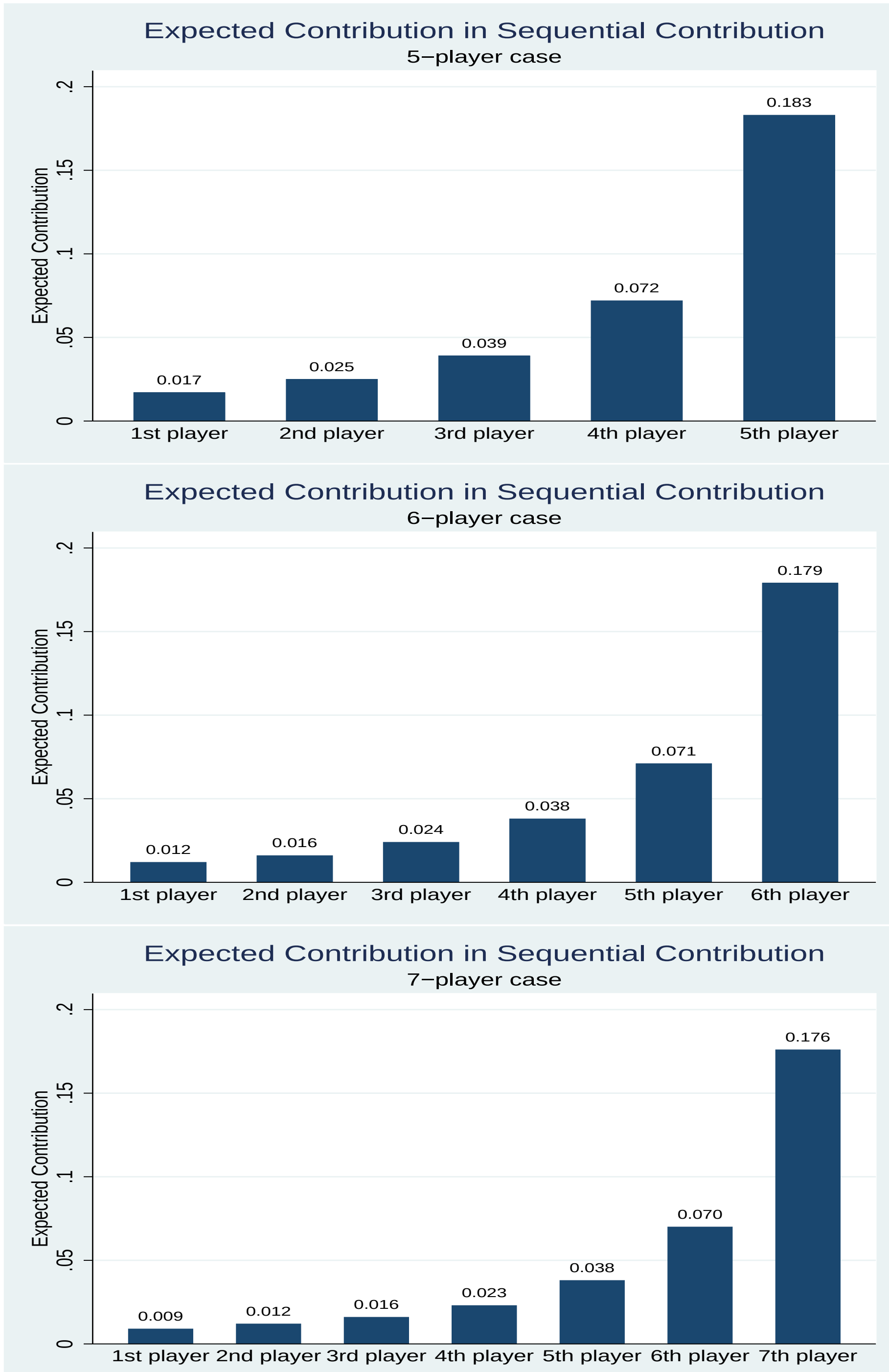

Figure 3.1 continued.

the same as the model in this paper: player's value is private information and follows a common uniform distribution with support $[0, 1]$; the cost threshold of providing the public good is uncertain and follows an uniform distribution with support $[0, \tilde{c}]$, where $\tilde{c} \geq n$. The only difference between Barbieri and Malueg (2010) and this paper is that players make their contribution decisions simultaneously in Barbieri and Malueg (2010), while players make their contribution decisions sequentially in this paper. Player's expected contribution in Barbieri and Malueg (2010) can be written as

$$K = E\left[\max\left\{0, \frac{1}{2}\left[v_i - (n-1)K\right]\right\}\right]$$

$$\Longrightarrow K^* = \frac{1}{n-1} + \frac{2}{(n-1)^2}\left[1 - n^{\frac{1}{2}}\right] \tag{3.38}$$

Thus, the expected contribution in Barbieri and Malueg's simultaneous model is a symmetric Bayesian-Nash equilibrium.

Figure 3.2 shows each player's sequential and simultaneous expected contributions in the 2-payer to 7-player case. I find that the first $(n-1)$ players' expected contributions in the sequential contribution model are lower than that in the simultaneous contribution model, but the last player's expected contribution in the sequential contribution model is higher than that in the simultaneous contribution model.

Figure 3.3 displays expected total contribution in the sequential and simultaneous contribution models. It shows that the expected total contribution in the sequential model is lower than that in the simultaneous contribution model when 2 to 7 players participate in the subscription game. It also shows that the gap between the expected total contribution in these two models is increasing in the number of players. This is because the increment of the expected total contribution in the sequential model is smaller than that in the simultaneous model.

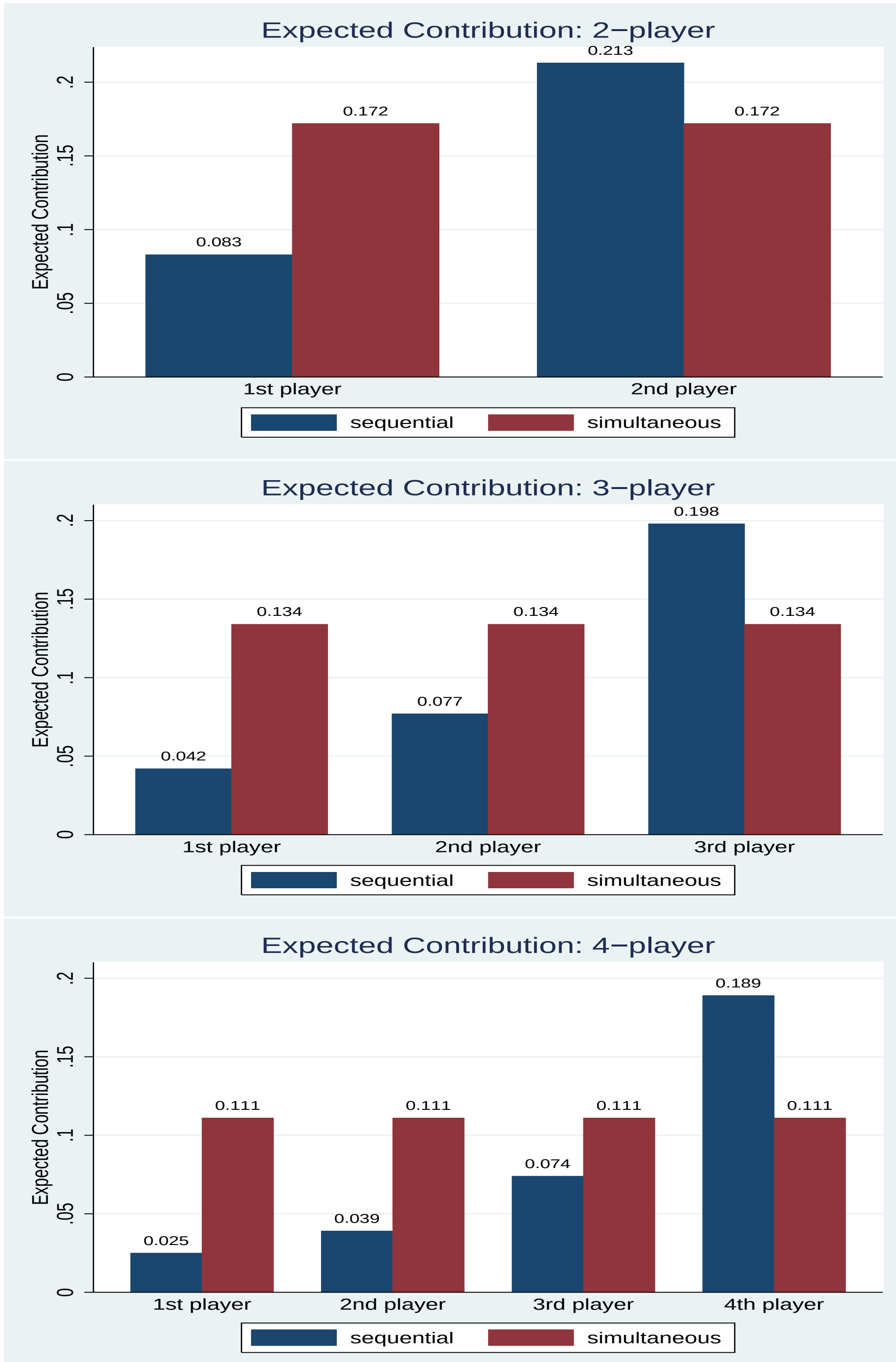

Figure 3.2: Each Player's Expected Contribution in the Sequential and Simultaneous Contribution Model

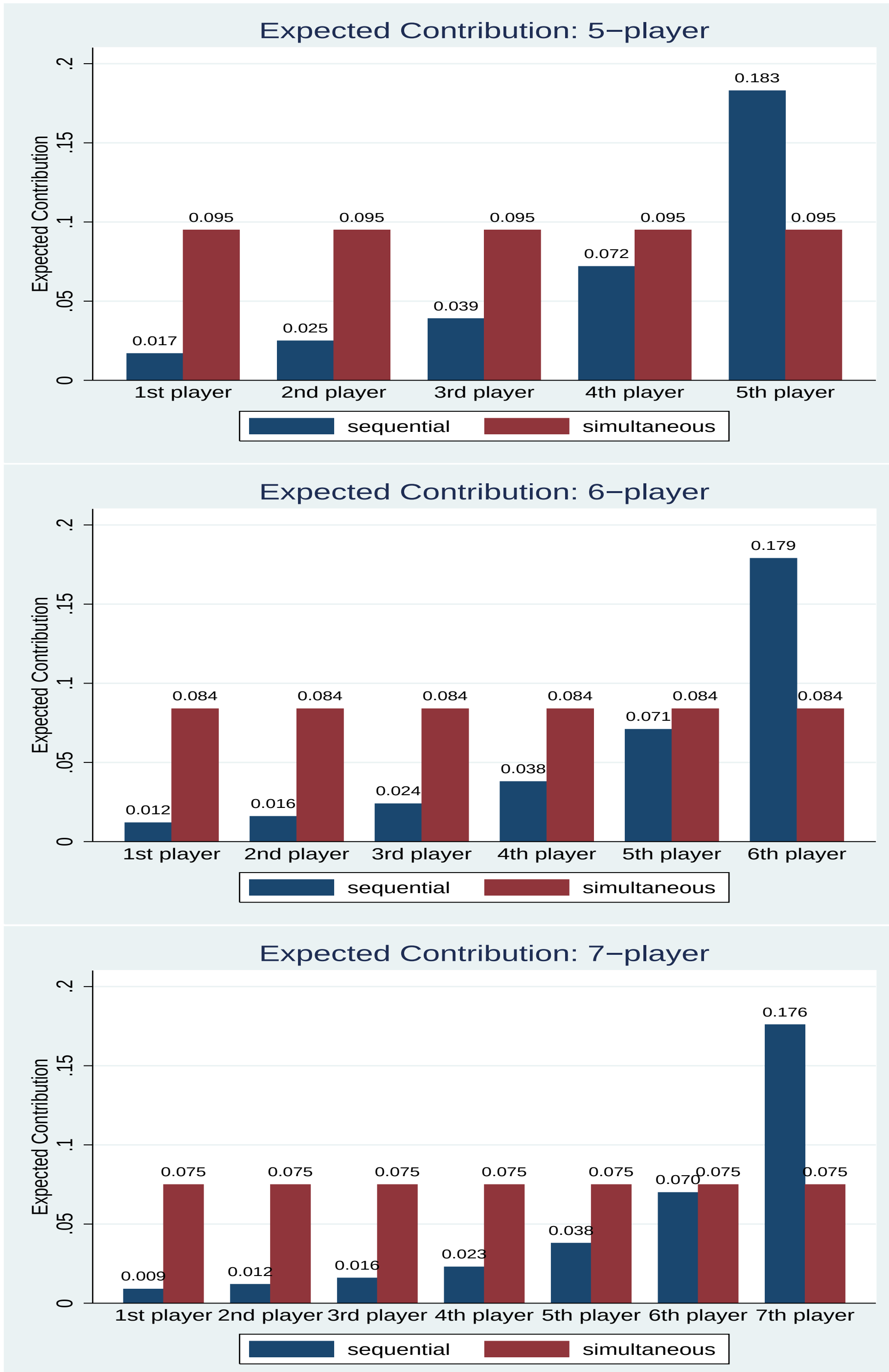

Figure 3.2 continued.

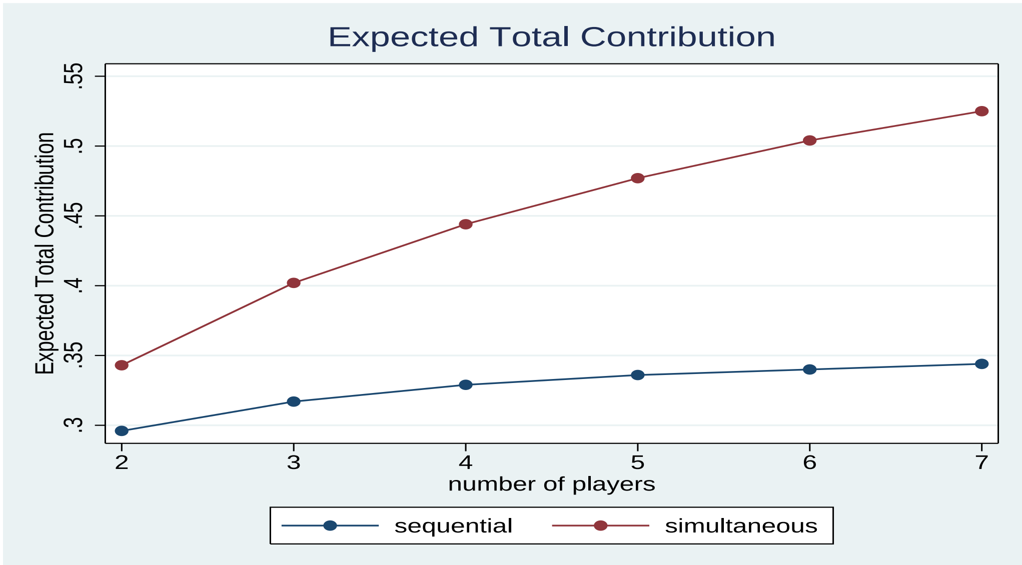

Figure 3.3: Expected Total Contribution in the Sequential and Simultaneous Contribution Model

Figures 3.2 and Figure 3.3 show that although the last player's expected contribution in the sequential model is higher than that in the simultaneous model, it is not large enough to cover the decrease of the first $(n-1)$ players' expected contributions. Thus the expected total contribution in the sequential model is lower.

Due to the model setting, $\tilde{c} \geq n$, increasing the number of players increases the upper bound of threshold distribution. To compare the *ex ante* probability of providing the public good, I assume that $\tilde{c} = n$ in an n-player case. With this assumption, the cost of public good provision per player is unchanged, which is equal to 1. Figure 3.4 illustrates the *ex ante* probability of providing the public good in a 2-player case to a 7-player case. It shows that the *ex ante* probability in the sequential model is lower than that in the simultaneous model.

Figure 3.4 also shows that whether players contribute to a public good in the sequential institution or in the simultaneous institution, the *ex ante* probability is

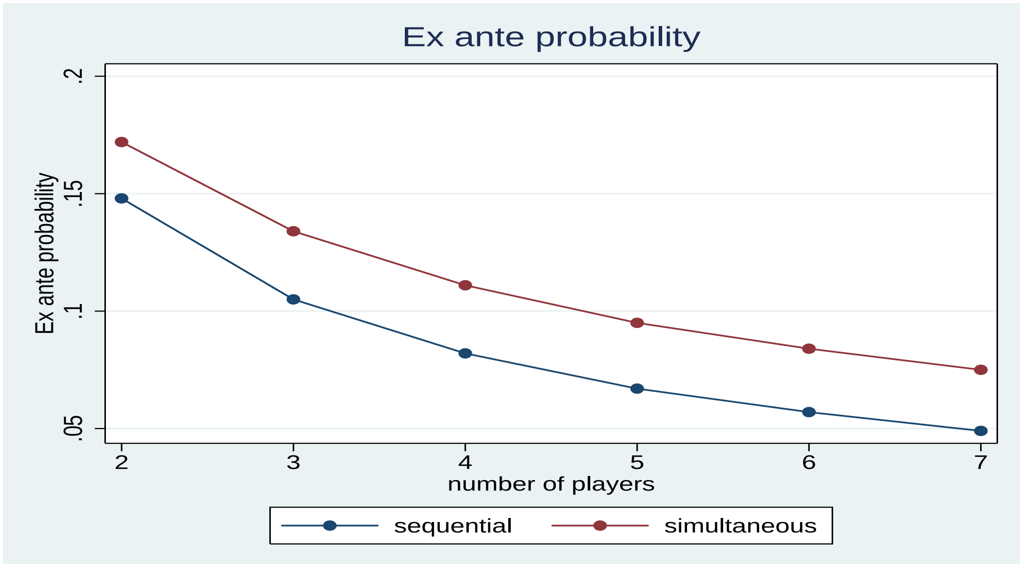

Figure 3.4: *Ex Ante* Probability in the Sequential and Simultaneous Contribution Model

decreasing in the number of players even if the cost of public good provision per player is unchanged.

3.5 Other Considerations

In this section, I briefly discuss the equilibria in the different theoretical environments. The first case I will discuss is based on the assumption that players have complete information but still face the threshold uncertainty when making contribution decisions. The second case that I focus on assumes that players have private information on valuation but they know the exact cost of providing the public good when contributing to the public good. Besides private information on valuation and threshold uncertainty, an important assumption in the primary model is the full refunded rule. Another special case discussed in this section centers on the assumption that there is no refund rule rather than the full refund rule if the total contribution does not cover the cost of providing the public good.

3.5.1 *Complete Information on Valuation and Threshold Uncertainty*

I consider a 4-player case and assume that the valuation of the public good, v, is common knowledge and is the same for each player. As to the cost of providing the public good, I assume it is unknown when the players make their contribution decisions, independent of v and follows the uniform distribution, $c \sim U[0, \tilde{c}]$, where $\tilde{c} \geq n$. This assumption is the same as the assumption in the primary model. In this case, the i-th player's expected payoff function can be written as

$$U_i(x_i|v) = \big(v - x_i\big)\Big[\frac{\sum_{j=1}^{4} x_j}{\tilde{c}}\Big] \tag{3.39}$$

Using the method of backward induction, I derive the Subgame Perfect Equilibrium. For Player 4, his/her expected payoff function is

$$U_4(x_4|v) = \big(v - x_4\big)\Big[\frac{x_1 + x_2 + x_3 + x_4}{\tilde{c}}\Big] \tag{3.40}$$

To maximize Player 4's expected payoff, I get his/her best response function

$$x_4 = \begin{cases} 0 & \text{if } v \leq x_1 + x_2 + x_3 \\ \frac{v - x_1 - x_2 - x_3}{2} & \text{if } v \geq x_1 + x_2 + x_3 \end{cases} \tag{3.41}$$

If $x_4 = \frac{v - x_1 - x_2 - x_3}{2}$ (this implies $v \geq x_1 + x_2 + x_3$), then Player 3's expected payoff function can be written as

$$U_3(x_3|v) = \big(v - x_3\big)\Big[\frac{v + x_1 + x_2 + x_3}{2\tilde{c}}\Big] \tag{3.42}$$

From Equation (3.42)

$$\frac{\partial U_3}{\partial x_3} = \frac{-x_1 - x_2 - 2x_3}{2\tilde{c}} < 0 \tag{3.43}$$

This indicates that the smaller x_3 is, the higher U_3 is. Thus, Player 3's best response function is $x_3 = 0$.

If $x_3 = 0$ and $x_4 = \frac{v-x_1-x_2-x_3}{2}$, Player 2' expected payoff function can be written as

$$U_2(x_2|v) = \big(v - x_2\big)\left[\frac{v + x_1 + x_2}{2\tilde{c}}\right] \tag{3.44}$$

From Equation (3.44)

$$\frac{\partial U_2}{\partial x_2} = \frac{-x_1 - 2x_2}{2\tilde{c}} < 0 \tag{3.45}$$

This indicates that the smaller x_2 is, the higher U_2 is. Thus, Player 2's best response function is $x_2 = 0$.

If $x_2 = 0$, $x_3 = 0$ and $x_4 = \frac{v-x_1-x_2-x_3}{2}$, Player 1' expected payoff function can be written as

$$U_1(x_1|v) = \big(v - x_1\big)\left[\frac{v + x_1}{2\tilde{c}}\right] \tag{3.46}$$

From Equation (3.46)

$$\frac{\partial U_1}{\partial x_1} = \frac{-x_1}{\tilde{c}} < 0 \tag{3.47}$$

This indicates that the smaller x_1 is, the higher U_1 is. Thus, Player 1's best response function is $x_1 = 0$. Based on each player's best response function, I derive the Subgame Perfect Equilibrium: when $v \geq 0$, $x_1^* = x_2^* = x_3^* = 0$ and $x_4^* = \frac{v}{2}$. This Subgame Perfect Equilibrium shows that the higher the valuation of the public good is the higher the last player's contribution is; however, the first $n - 1$ player's contribution is independent of the valuation. That is, the first $n - 1$ player always contribute zero to the public good. Using the same process I get a trivial Subgame Perfect Equilibrium: when $v \leq 0$, $x_1^* = x_2^* = x_3^* = x_4^* = 0$.

In order to compare the result in this case with the results in the primary model (public good's valuation is private information and followed a uniform distribution with support $[0, 1]$), I let $v = \frac{1}{2}$ in this new case. That is, v equals to the mean of value distribution in the main model. I assume the threshold distribution is $c \sim U[0, 4]$. Thus, the Subgame Perfect Equilibrium in this 4-player case is $x_1^* = x_2^* = x_3^* = 0$ and $x_4^* = \frac{1}{4}$. From the equilibrium, I observe that when the public good's valuation is known and the threshold is uncertain, the first three players are not willing to contribute to the public good and the last player will contribute half of his/her valuation to the public good. This result suggests that the first three players rely on the last player to contribute to the public good and enjoy the first-mover advantage. Thus, there exists the free rider problem.

Table 3.1 shows the comparison of equilibria under different value information settings.

Table 3.1: Comparison of Equilibria under Different Value Information

	complete information $v = \frac{1}{2}$	incomplete information $v \sim U[0, 1]$
threshold uncertainty $c \sim U[0, 4]$	$x_1 = 0$ $x_2 = 0$ $x_3 = 0$ $x_4 = 0.25$ $total = 0.25$	$E^*[x_1] = 0.025$ $E^*[x_2] = 0.039$ $E^*[x_3] = 0.074$ $E^*[x_4] = 0.189$ $total = 0.327$

In the primary model, which considers private information on valuation and threshold uncertainty, the Bayesian Equilibrium in a 4-player case is $E^*[x_1] = 0.025$, $E^*[x_2] = 0.039$, $E^*[x_3] = 0.074$, and $E^*[x_4] = 0.189$. Comparing this initial result with the result in this subsection, I find that whether the public goods's valuation is private information or complete information, the last player contributes the most to the public good and a free rider problem exists in both cases.

Since the valuation in the case of this subsection equals to the mean of valuation distribution in the main model, the contribution equilibrium results show that keeping the mean of valuation the same, when the degree of valuation dispersion shrinks to be zero, the free rider problem becomes more serious. This is because the earlier contributors can predict the latter contributors' contributions more accurately when the valuation of the public good is complete information. This result suggests that the early contributors enjoy a stronger first-mover advantage and rely more on the last player to provide the public good.

3.5.2 Private Information on Valuation and Threshold Certainty

Next, I discuss another case when the cost of providing the public good, c, is certain and known when players make their contribution decisions. As to the valuation of the public good, I assume each player's valuation of public good, v, is private information and followed a common uniform distribution, $v \sim U[0,1]$. I discuss this particular theoretical setting via a 4-player case.

The i-th player's expected payoff function in this game can be written

$$U_i(x_i|v_i) = (v_i - x_i)Q_i \tag{3.48}$$

Q_i represents the probability of providing the public good for Player i and it is equal to

$$Q_i = \begin{cases} 0 & \text{if } \sum_{l=1}^{i-1} x_l + x_i + \sum_{m=i+1}^{n} E_i[x_m] \leq c \\ 1 & \text{if } \sum_{l=1}^{i-1} x_l + x_i + \sum_{m=i+1}^{n} E_i[x_m] \geq c \end{cases}$$

where $\sum_{l=1}^{i-1} x_l$ represents the total contributions made by the players whose contribution sequence is prior to Player i's and $E_i[x_m]$ is the subsequent Player m's expected contribution that Player i expects. If the total (expected) contribution is equal to or larger than the provision cost, c, the public good can be provided. That is, $Q_i = 1$ if the provision cost is met; otherwise, the public good cannot be provided, or $Q_i = 0$.

From Equation (3.48), Player 4's expected payoff function can be written

$$U_4(x_4|v_4) = \big(v_4 - x_4\big)Q_4 \tag{3.49}$$

$$Q_4 = \begin{cases} 0 & \text{if } x_1 + x_2 + x_3 + x_4 \leq c \\ 1 & \text{if } x_1 + x_2 + x_3 + x_4 \geq c \end{cases}$$

To maximize Player 4's expected payoff, I get his/her best response function

$$x_4 = \begin{cases} 0 & \text{if } v_4 \leq c - x_1 - x_2 - x_3 \\ c - x_1 - x_2 - x_3 & \text{if } v_4 \geq c - x_1 - x_2 - x_3 \end{cases} \tag{3.50}$$

Assume $c - x_1 - x_2 - x_3 \geq 1$, I get the contribution Player 3 expects Player 4 to make, seen below:

$$E_3[x_4] = 0 \tag{3.51}$$

For Player 3, his/her expected payoff function can be written

$$U_3(x_3|v_3) = \big(v_3 - x_3\big)Q_3 \tag{3.52}$$

$$Q_3 = \begin{cases} 0 & \text{if } x_1 + x_2 + x_3 + E_3[x_4] \leq c \\ 1 & \text{if } x_1 + x_2 + x_3 + E_3[x_4] \geq c \end{cases}$$

Using Player 3's expected payoff function, I derive his/her best response function

$$x_3 = \begin{cases} 0 & \text{if } v_3 \leq c - x_1 - x_2 \\ c - x_1 - x_2 & \text{if } v_3 \geq c - x_1 - x_2 \end{cases}$$

If $x_3 = c - x_1 - x_2$, then $c - x_1 - x_2 - x_3$ becomes $c - x_1 - x_2 - (c - x_1 - x_2) = 0 < 1$ which contradicts the condition $c - x_1 - x_2 - x_3 \geq 1$. Thus, $x_3 = c - x_1 - x_2$ cannot be Player 3's best response function given the case with the condition $c - x_1 - x_2 - x_3 \geq 1$. Therefore, Player 3's best response function should be

$$x_3 = 0 \quad \forall v_3 \tag{3.53}$$

Based on Player 3 and 4's best response function, the contributions Player 2 expects Player 3 and 4 to make are $E_2[x_3] = 0$ and $E_2[x_4] = 0$.

Similarly, I get Player 2's expected payoff function

$$U_2(x_2|v_2) = \big(v_2 - x_2\big)Q_2 \tag{3.54}$$

$$Q_2 = \begin{cases} 0 & \text{if } x_1 + x_2 + E_2[x_3] + E_2[x_4] \leq c \\ 1 & \text{if } x_1 + x_2 + E_2[x_3] + E_2[x_4] \geq c \end{cases}$$

Using the same procedure, I derive Player 2's best response function, which in this case is:

$$x_2 = 0 \quad \forall v_2 \tag{3.55}$$

According to Equation (3.50), (3.53) and (3.55) and the condition, $c - x_1 - x_2 - x_3 \geq 1$, I get what Player 1 expects Player 2, Player 3, and Player 4's contribution are $E_1[x_2]$, $E_1[x_3] = 0$ and $E_1[x_4] = 0$. Thus, Player 1's expected payoff function is

$$U_1(x_1|v_1) = \big(v_1 - x_1\big)Q_1 \tag{3.56}$$

$$Q_1 = \begin{cases} 0 & \text{if } x_1 + E_1[x_2] + E_1[x_3] + E_1[x_4] \leq c \\ 1 & \text{if } x_1 + E_1[x_2] + E_1[x_3] + E_1[x_4] \geq c \end{cases}$$

and his/her best response function should be

$$x_1 = 0 \ \ \forall v_1 \tag{3.57}$$

Using each player's best response function, I derive the Bayesian Equilibrium $E^*[x_1] = E^*[x_2] = E^*[x_3] = E^*[x_4] = 0$, when $c \geq 1$.

To compare the result in this case with the results in the primary model, which assumes the provision cost is unknown when players make contribution decisions and followed a uniform distribution, $c \sim U[0, \tilde{c}]$, $\tilde{c} \geq n$, I assume the provision cost in this new case equals to $\frac{\tilde{c}}{2}$. Also, I assume the upper bound of the possible cost, $\tilde{c}$, is 4. Thus, the main differences between the case in this subsection and the case in the primary model are that the provision cost in the previous one equals to 2 and the provision cost in the latter one follows the distribution, $c \sim U[0, 4]$.

Table 3.2 shows the comparison of equilibria under different cost distribution settings. Based on this numerical setting, the Bayesian Equilibrium when the provision cost is certain, known and equals to 2 is $E^*[x_1] = E^*[x_2] = E^*[x_3] = E^*[x_4] = 0$ since the provision cost is larger than 1. As to the Bayesian Equilibrium in a 4-player case when assuming private information on valuation and threshold uncertainty, it can be shown that $E^*[x_1] = 0.025$, $E^*[x_2] = 0.039$, $E^*[x_3] = 0.074$, and $E^*[x_4] = 0.189$.

Table 3.2: Comparison of Equilibria in Different Threshold Uncertainty

	no threshold uncertainty ($c = 2$)	threshold uncertainty ($c \sim U[0, 4]$)
incomplete information $v \sim U[0, 1]$	$E^*[x_1] = 0$ $E^*[x_2] = 0$ $E^*[x_3] = 0$ $E^*[x_4] = 0$ $total = 0$	$E^*[x_1] = 0.025$ $E^*[x_2] = 0.039$ $E^*[x_3] = 0.074$ $E^*[x_4] = 0.189$ $total = 0.327$

Comparing the case in this subsection with the case in the primary model, I find that keeping the mean of the cost threshold the same, if the degree of threshold dispersion shrinks to be zero, each player's expected contribution decreases to zero. One explanation for this result is that the threshold in this particular numerical case may be too high, so the players, on average, are not willing to contribute to the public good. Each player's average valuation is $\frac{1}{2}$ and the threshold is 2 in this new case. If each player is burdened with the same share of the threshold, this means the players should, on average, contribute $\frac{1}{2}$ to provide the public good. However, the player whose valuation is lower than $\frac{1}{2}$ must contribute the amount lower than $\frac{1}{2}$; and the player whose valuation is higher than $\frac{1}{2}$ will not contribute the amount equals to his/her valuation. Thus, each player's contribution, on average, is lower than $\frac{1}{2}$. This result suggests that the public good may not be provided, so each player's expected contribution is equal to zero.

Another reason the expected contribution is higher in the primary model is that if the true cost is low and the public good is provided, the players get a positive payoff. If the true cost is high and the public good cannot be provided, the contributions will be fully refunded. Thus, the contributors know that contributing to the public good will not make them worse off and they may get a positive payoff when threshold uncertainty exists. Thus, whether the provision order is, the player's expected contribution under the threshold uncertainty is higher than that under the threshold certainty.

3.5.3 No Refunded Rule

In this subsection, I will discuss another particular model setting — a no refund rule in a 3 player case. Keeping other assumptions in the model unchanged but considering the no refunded rule, Player 3's expected payoff can be written as

$$U_3(x_3|v_3) = v_3\Big[\frac{x_1 + x_2 + x_3}{\tilde{c}}\Big] - x_3 \tag{3.58}$$

From Equation (3.58)

$$\frac{\partial U_3}{\partial x_3} = \frac{v_3}{\tilde{c}} - 1 \tag{3.59}$$

Since I assume that $\tilde{c} \geq n(= 3$ in this case), and $0 \leq v_3 \leq 1$, I can get $\frac{\partial U_3}{\partial x_3} < 0$. This means that the more Player 3 contributes, the less his/her expected payoff is. Thus, Player 3's best response function is

$$x_3 = 0 \quad \forall v_3 \tag{3.60}$$

and Player 3's expected contribution is

$$E[x_3] = \int_0^1 0 \; dv_3 = 0 \tag{3.61}$$

For Player 2, his/her expected utility function is

$$\begin{aligned} U_2(x_2|v_2) &= v_2\Big[\frac{x_1 + x_2 + E_2[x_3]}{\tilde{c}}\Big] - x_2 \\ &= v_2\Big[\frac{x_1 + x_2}{\tilde{c}}\Big] - x_2 \end{aligned} \tag{3.62}$$

then, Player 2's best response function is

$$x_2 = 0 \quad \forall v_2 \tag{3.63}$$

and Player 2's expected contribution is

$$E[x_2] = \int_0^1 0 \; dv_2 = 0 \tag{3.64}$$

Similarly, Player 1's expected payoff function can be written as

$$U_1(x_1|v_1) = v_1\left[\frac{x_1 + E_1[x_2] + E_1[x_3]}{\tilde{c}}\right] - x_1$$

$$= v_1\left[\frac{x_1}{\tilde{c}}\right] - x_1 \tag{3.65}$$

then his/her best response function is

$$x_1 = 0 \quad \forall v_1 \tag{3.66}$$

and his/her expected contribution is

$$E[x_1] = \int_0^1 0 \; dv_1 = 0 \tag{3.67}$$

From this example, I find that if the player's contribution cannot be refunded when the public good is not provided, his/her expected contribution is zero regardless of the order he holds in the sequence. Thus, with the zero refund rule, contributors are afraid that they cannot make a high enough contribution to cover the cost of the public good and will thus suffer a loss from contributing. This results in zero contribution to the public good.

Comparing the results in the full refund case to that in the no refund case, I find that using the full refund rule can encourage individuals to contribute to the public good. This is because contributors know that making a contribution will not make them worse off with the full refund rule.

3.6 Conclusion

This paper considers private information on public good valuations and threshold uncertainty in a sequential contribution mechanism and derives the Bayesian equilibrium. I find that expected contributions are increasing with respect to contributor order in this sequential contribution institution. The pattern of earlier movers free ride off later mover and enjoy the first-mover advantage in the Bayesian equilibrium to the private information game mirrors the predicted pattern of play under subgame perfect Nash equilibrium and complete information.

This paper compares the player's expected contribution in the sequential contribution mechanism and that in Barbieri and Malueg's (2010) simultaneous contribution mechanism. I find that the expected total contribution in the sequential institution is lower than that in the simultaneous institution. Also, the *ex ante* provision probability in the sequential institution is lower than that in the simultaneous institution.

Using the simple 4-player case, I find that if the degree of valuation dispersion shrinks to zero, the total contribution will becomes lower and the free rider problem becomes more serious. This is because the earlier contributors rely more on the last player's contribution and enjoy stronger first-mover advantage. I also find that if the degree of threshold dispersion shrinks to zero, the total expected is lower than that with threshold uncertainty.

Future researchers could analyze changes in the threshold distribution. I have investigated how the changing of the threshold distribution affects player's expected contribution in the simultaneous contribution model. I am interested in this same question in the sequential contribution model. However, if I adopt the same model setting I use in the simultaneous model may result in numerous and complicated cases. It will be very difficult to analyze the same question. I am still looking for some more clever methods to investigate this issue.

Another direction is to test the theoretical private contribution predictions using a lab experiment. I propose from the theoretical model, for example, that the expected total contribution in a sequential contribution institution is low than that in the simultaneous institution and early contributors may free ride off of prospective contributors. Future research could test this theoretical prediction in a laboratory setting. This could help us to understand how different institution affect the behavior of making private contribution decisions.

4. AN EXPERIMENTAL STUDY ON SIMULTANEOUS CONTRIBUTION AND THRESHOLD UNCERTAINTY

4.1 Motivation

In this paper, I seek to re-analyze individual contribution behaviors in a simultaneous subscription game with private information on valuations and threshold uncertainty. Because it is difficult to test individual contribution behavior using empirical data, I conduct an experiment to investigate how individual contribution behavior is affected when the contributor is given different cost uncertainties and test the theoretical predictions in Gronberg and Peng (2014). The advantage of conducting a controlled laboratory experiment is that it helps us explore how individuals make their contribution decisions in a specific environment (in this paper, I consider private information on valuations and cost uncertainty). If subject's behavior is indeed affected by the factors I consider, it suggests that suppliers of public goods should take into account private information on valuations and/or cost uncertainty when they collect private contributions. If subject's behavior is not affected by these factors, it helps us explore other possible factors that may affect individual private contribution behavior.

To test the theoretical comparative statics with respect to the threshold uncertainty, I find that decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction or increasing the mean of the threshold distribution in the sense of the first-order stochastic dominance, the individual, on average, is more willing to contribute to the public good. As to the success rates of providing the public good, the experimental results suggest that the empirical probability of providing the public good is higher than the *ex ante* probabilities in all treatments.

The empirical results also show that the success rates in the Baseline Treatment and the Mean-Preserving Treatment are significantly higher than that in the Variance-Preserving Treatment.

According to the results of the random effect Tobit regression, individual contributions increase with the valuation of the public good. I also find that individual contributions decrease with the period of the experiment. The individual contributes less to the public good in the latter periods. As to the individual characteristic variables, the estimation results show that females contribute higher amounts to the public good. The results also indicate that individuals who are more risk-loving contribute smaller amounts to the public good, although this result is not statistically significant.

The paper is structured as follows, Section 2 is the literature review. Section 3 summarizes the theoretical model that serves as the basis for the experimental design. Section 4 introduces the experimental design and the procedures. Section 5 describes the hypotheses. Section 6 discusses the experimental results and Section 7 includes a conclusion.

4.2 Literature Review

Earlier studies investigate the contribution behavior in the complete informational environment. Palfrey and Rosenthal (1984) and Bagnoli and Lipman (1989) both assume that players make contribution decisions in the simultaneous contribution institution with complete information on the public good valuation and a certain known threshold level of cost. These two papers show that if the full refund rule is introduced into the threshold public good game, the Nash equilibria are efficient. Issac, Schmidtz and Walker (1989) conduct an experiment to test the efficacy of a threshold public good mechanism where the valuation of the public good is common knowledge and the cost threshold is known. Their result shows that the full refund rule dramatically improves the provision of the public good in the high and medium provision cost environments.

In a real world environment, valuation of the public good is generally private rather than public information. Some experimental literature considers the effect of private information on contribution. Marks and Croson (1999) conduct a discrete public good experiment where subjects have incomplete information about the valuations of others. They find no significant differences in the rate of successful provisions or level of group contributions when the subjects have limited information about others' valuations than when they have complete information.

Levati *et al.* (2009) move a step further and suppose that one does not know his/her own marginal benefit from the public good, but he/she is informed that it can take one of two values with the same probability. They examine the effect of imperfect information on contributions by a two-person linear voluntary contribution mechanism with stochastic marginal benefits from a public good. They show that limited information about the value of the public good significantly lowers average contributions.

Some studies instead focus on threshold uncertainty. Nitzan and Romano (1990) extend Bagnoli and Lipman's game by introducing uncertainty regarding the cost of providing the public good and find different results. They find that the equilibrium is inefficient because the uncertainty in cost may cause the *ex post* contributions to exceed or fall short of the required threshold.

Wit and Wilke (1998) investigate the effects of provision threshold uncertainty on contribution to the discrete public good. They assume two different threshold uncertainty levels; under the low uncertainty case, the provision threshold is randomly sampled from a uniform distribution over the range $[800, 1000]$, while under the high uncertainty case, the provision threshold is randomly sampled from a uniform distribution over the range $[400, 1400]$. The main finding in this paper is that threshold uncertainty decreases the level of cooperation only under the high uncertainty case, not under the low uncertainty case.

Gustafsson *et al.* (1999) conduct two experiments to compare the voluntary contribution amount to public goods with the same expected provision threshold but different variances. They find that subjects contribute more than the expected provision threshold, but the average contribution is smaller in the high variance group.

Analyzing a similar question, Suleiman *et al.* (2001) show that the effect of threshold uncertainty is moderated by the threshold mean: contribution to the public good increases as a function of uncertainty for the lower threshold mean, and decreases (though not significantly) for the higher threshold mean.

McBride (2006) investigates how the level of threshold uncertainty affects the players' contributions. In his model, McBride assumes that each player makes a binary contribution decision simultaneously and the contribution will not be refunded if the cost threshold is not met. He finds that instead of a monotonic relationship between the degree of threshold uncertainty and total contributions, the effect of changing the threshold uncertainty on the contributions depends on the value of the public good. An increase of the threshold uncertainty in the sense of mean-preserving spread increases the player's contribution when the value of the public good is sufficiently high, but decreases the player's contribution when the value of the public good is sufficiently low. In a recent literature, McBride (2010) designs an experiment to test his theory in a lab and finds limited verification.

Barbieri and Malueg (2010) include both the threshold uncertainty and private information on valuations for a public good in a subscription game. They focus on whether changing the intensity and dispersion of value distribution affects players' equilibrium contributions. They find that increasing the value distribution in the sense of first order stochastic dominance, or dispersing the value distribution in the sense of mean-preserving spread increases the equilibrium contributions.

Gronberg and Peng (2014) consider both the threshold uncertainty and private information on public good's valuation in a subscription game but focus on the effect of changing the threshold distribution. They find that increasing the mean of the cost in the sense of first order stochastic dominance increases individual contribution, while increasing the uncertainty level of the cost in the sense of mean preserve spread decreases individual contribution. Gronberg and Peng (2014) forms the theoretical basis for this paper. This experiment tests the theoretical predictions found in Gronberg and Peng (2014).

4.3 Theoretical Model

The main objective of this paper is to test the theoretical predictions in Gronberg and Peng (2014). To demonstrate the focus for this paper, I start by summarizing their model. The theoretical model and theoretical equilibrium will serve as the basis for the experimental design.

4.3.1 Basic Setup

Assume $n \geq 2$ players simultaneously contribute any non-negative amounts to the public good in a subscription game (Admati and Perry, 1991). Let $x_i \in [0, v_i]$ be player i's contribution. Player's valuation for the public good, v_i, $i = 1, ..., n$, is private information. That is, each player knows only his/her own realized valuation for the public good. Each player believes that other players true valuations independently follow a uniform distribution with support $[0, 1]$. Since each player's value follows the same distribution, this is a symmetric case.

To provide the public good, the total contribution should equal or exceed the provision cost, c. Suppose c is unknown when the players contribute to the public good. However, all players believe that the cost is independent of all v_i's and distributed along a uniform distribution, G, with support $[\bar{c}-z, \bar{c}+z]$, where $\bar{c}$ is the mean of the cost, z measures the degree of the cost uncertainty. In order to obtain the unique equilibrium, the model should have the constraint that $0 \leqslant (\bar{c}-z) \leq \frac{n-1}{4} < \frac{n}{2} \leqslant (\bar{c}+z)$.

Gronberg and Peng consider a subscription game. Thus, the players' contributions will be fully refunded if the total contributions are less than the realized cost threshold. Also, they assume a zero rebate rule, which means that the excess contributions will be given to the producer of the good.

The objective to each player is to maximize his/her expected payoff. Based on the assumptions above, player i's expected payoff function can be written as:

$$U_i(x_i, v_i) = \frac{1}{2z}\big(v_i - x_i\big)\Big(x_i + \sum_{j \neq i} E[x_j(v_j)] - (\bar{c} - z)\Big) \tag{4.1}$$

Assume $K_j \equiv E[x_j(v_j)]$ is player j's expected contribution. In a symmetric equilibrium, $x_j(v_j)$ is independently and identically distributed. Thus, each player's expected contribution in this model should be identical and K_j can be replaced by K. Therefore, the total expected contribution by $(n-1)$ other contributors can be represented by $(n-1)K$ and player i's expected payoff function can be rewritten as:

$$U_i(x_i, v_i) = \frac{1}{2z}\big(v_i - x_i\big)\Big(x_i + (n-1)K - (\bar{c} - z)\Big) \tag{4.2}$$

4.3.2 Bayesian Nash Equilibrium and Decision Rule

Maximizing Equation (4.2) with respect to x_i and taking the first order condition (F.O.C) yields player i's best respond function:

$$x_i\big(v_i, (n-1)K\big) = \max\Big\{0, \frac{1}{2}\big[v_i - (n-1)K + (\bar{c} - z)\big]\Big\}, \quad \forall\ i \tag{4.3}$$

Using the definition of expected contribution, $K_i \equiv E[x_i(v_i)]$, and the best response function, Equation (4.3), in symmetric equilibrium,

$$K = E\Big[\max\Big\{0, \frac{1}{2}\big[v_i - (n-1)K + (\bar{c} - z)\big]\Big\}\Big] \tag{4.4}$$

Assuming that players' values are independently and uniformly distributed on $[0, 1]$, the expected contribution, in equilibrium, is

$$K^* = \frac{1 + \bar{c} - z}{n-1} + \frac{2}{(n-1)^2}\Big\{1 - \Big[1 + (n-1)(1 + \bar{c} - z)\Big]^{\frac{1}{2}}\Big\} \tag{4.5}$$

This solution concept is a symmetric Bayesian-Nash equilibrium.

Player i's equilibrium strategy, x_i^*, must satisfy Equation (4.3) and K^*, therefore, x_i^* can be written as

$$x_i^*\big(v_i, (n-1)K^*\big) = \max\Big\{0, \frac{1}{2}\big[v_i - (n-1)K^* + (\bar{c} - z)\big]\Big\} \tag{4.6}$$

where K^* should be equal to Equation (4.5).

4.3.3 Comparative Statics

Since K^* is a function of $\bar{c}$ and z, the cost distribution may affect player's expected contribution. Gronberg and Peng (2014) consider the changes in the uniform cost distribution through mean-preserving spread and first order stochastic dominance, and try to analyze how these changes affect player's expected contribution.

First, I demonstrate the effect of changing threshold uncertainty in the sense of mean-preserving spread. From Equation (4.5), $K^* = \frac{1+\bar{c}-z}{n-1} + \frac{2}{(n-1)^2} - \frac{2}{(n-1)^2}\Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{1}{2}}$ in equilibrium. Differentiating Equation (4.5) with respect to the variance of the cost distribution, z, it is shown that

$$\frac{dK^*}{dz} = \frac{1}{n-1}\Big\{-1+\Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{-1}{2}}\Big\} \tag{4.7}$$

Since $(\bar{c}-z)$ is the lower bound of the threshold and assume $(\bar{c}-z) \geq 0$, $\frac{dK^*}{dz} < 0$, this result shows that when the cost distribution becomes more dispersed, the expected contributions will decrease.

Second, I demonstrate the effect of increasing the mean of threshold distribution in the sense of first order stochastic dominance. From Equation (4.5), $K^* = \frac{1+\bar{c}-z}{n-1} + \frac{2}{(n-1)^2} - \frac{2}{(n-1)^2}\Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{1}{2}}$ in equilibrium. Differentiating Equation (4.5) with respect to the mean of the cost distribution, $\bar{c}$, it is shown that

$$\frac{dK^*}{d\bar{c}} = \frac{1}{n-1}\Big\{1-\Big[1+(n-1)(1+\bar{c}-z)\Big]^{\frac{-1}{2}}\Big\} \tag{4.8}$$

Since $(\bar{c}-z)$ is the lower bound of the threshold and assume $(\bar{c}-z) \geq 0$, $\frac{dK^*}{d\bar{c}} > 0$, this result shows that increasing the mean of the public good in the sense of first order stochastic dominance will increase the expected contribution.

4.4 Experimental Design and Procedures

4.4.1 Experimental Design

I am interested in examining the effects of changing the cost distribution on individual contribution equilibrium. In order to test these effects, I conduct a simple between-subject experiment, which compares the contribution behaviors with different cost distributions. Since this experiment is a between-subject design, each subject participates in only one session and treatment.

There are 3 treatments in this experiment. The differences among the treatments are the list of possible provision costs. One of them is called the Baseline Treatment. In the Baseline Treatment, there are 14 possible provision costs: 10, 30, 50, 70, 90, 110, 130, 150, 170, 190, 210, 230, 250, or 270. The second Treatment is called the Mean-Preserving Treatment. In the Mean-Preserving Treatment, I decrease the dispersion degree of the cost distribution but keep the mean of cost distribution the same as that in the Baseline Treatment. In the Mean-preserving Treatment, 8 possible provision costs are listed: 70, 90, 110, 130, 150, 170, 190, or 210. The third treatment is called the Variance-Preserving Treatment. In this treatment, I increase the mean of the cost distribution but keep the variance the same as that in the Baseline Treatment. There are 14 possible provision costs in the Variance-Preserving Treatment: 70, 90, 110, 130, 150, 170, 190, 210, 230, 250, 270, 290, 310, or 330. Each treatment has 2 sessions.

I assume the value is private information for each subject. There are 6 possible values and they are the same in each treatment. These possible values are 0, 20, 40, 60, 80, or 100. Each subject knows that his/her group members' values are independently and randomly drawn from these 6 values. His/her own value, which is only known by himself/herself, is also one of these values.

The experiment is based on the following four-player game. Each subject is given 100 tokens in each period, and has to decide how to use his/her endowment. The player has to decide how many tokens he/she wants to contribute to a project and how many tokens to keep for himself/herself. Subjects make decisions simultaneously. At the beginning of each period, each subject is given the following information:

1. His/her own valuation of the project. If the project will be implemented, he/she receives his/her own valuation of the project, taking one of the following values: 0, 20, 40, 60, 80, or 100. The value changes every period.

2. A list of possible valuations of the project his/her group members may have. Each group member's valuation of the project is independently and randomly drawn from 6 possible values: 0, 20, 40, 60, 80, or 100.

3. A list of possible provision costs. There is the cost of providing the project. To receive the valuation of the project, the total contribution of the group must equal or exceed the provision cost. However, the provision cost is not disclosed until each subject in the same group makes his/her decision. Each participant is informed of the different list of possible provision costs depending on the treatment the subject is assigned to. At the end of each period, the provision cost is independently and randomly drawn from the announced possible costs by the computer.

Figure 4.1 shows an example of the computer interface in the Baseline Treatment the subject may face when making his/her contribution decision.

After making the decision, subjects are informed about the realized provision cost, total contribution of his/her group, whether the contribution is refunded or not and the income he/she receives in the current period. Each subject's income in each period consists of three parts:

1. Income from tokens kept: the tokens which the subject has kept for himself/herself.

2. Income from the project: Whether the subject will get the payoff from the project depends on whether the total contribution is equal to or larger than

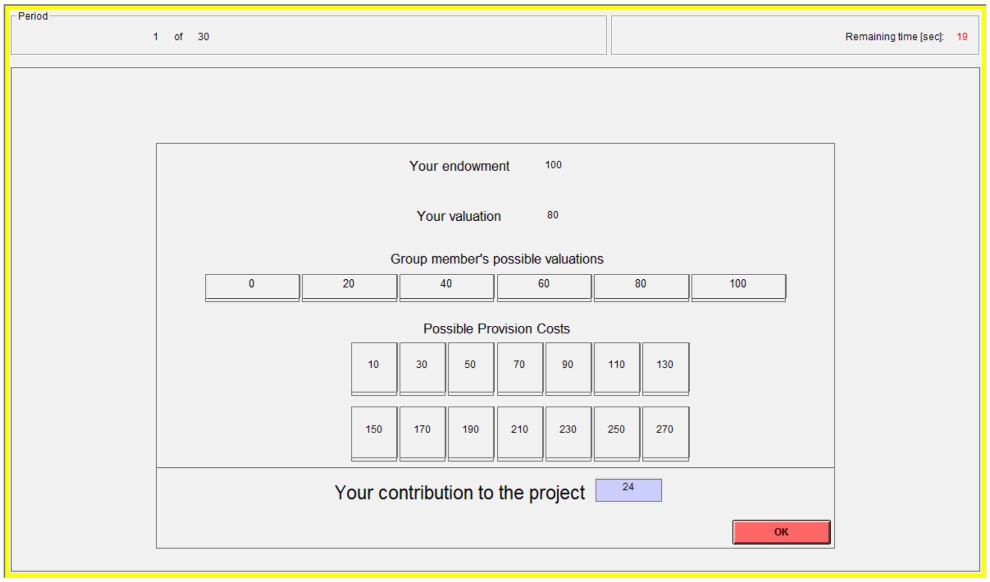

Figure 4.1: Example of Decision Screen

the realized provision cost. Income from the project is determined as follows:

- If the provision cost is met: Income from the project = The subject's valuation of the project.
- If the provision cost is NOT met: Income from the project = 0.

3. Income from the refund rule: the amount the subject invests into the project will be fully refunded to him/her if the total contribution in his/her group is smaller than the realized provision cost. The income from the refund rule is determined as follows:

   - If the provision cost is NOT met & you contribute X tokens to the project: Income from the refund rule = X.
   - If the provision cost is met: Income from the refund rule = 0.

Thus, each subject's income in each period can be represented by the following equation:

$$\pi_i = 100 - x_i + G_i(\sum_{j=1}^{4} x_j) + R_i \tag{4.9}$$

where

$$G_i(\sum_{j=1}^{4} x_j) = \begin{cases} v_i & \text{if } \sum_{j=1}^{4} x_j \geq c \\ 0 & \text{if } \sum_{j=1}^{4} x_j < c \end{cases}$$

$$R_i = \begin{cases} x_i & \text{if } \sum_{j=1}^{4} x_j < c \;\&\; x_i > 0 \\ 0 & \text{o/w} \end{cases}$$

Figure 4.2 is the example of payment screen in the Baseline Treatment.

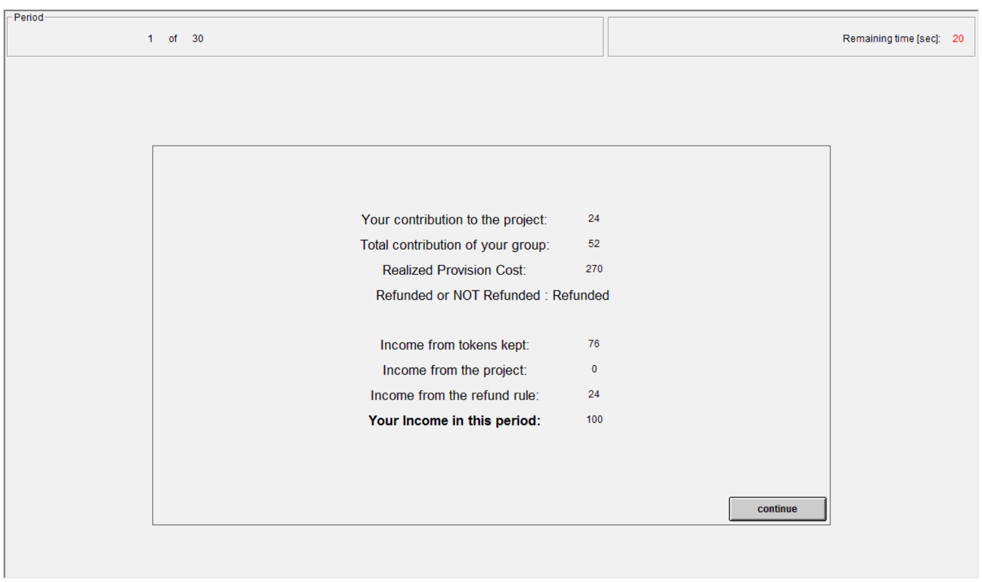

Figure 4.2: Example of Payment Screen

4.4.2 Experimental Procedures

This experiment was conducted at the Economics Research Lab (ERL) at Texas A&M University. 72 subjects were recruited from the university-wide pool of students by the online system ORSEE (Greiner, 2004). Six sessions were conducted (two per treatment) with 12 participants per session. Each session was conducted using z-tree software (Fischbacher, 2007).

Upon arrival, each subject picked a chip to decide his/her seat with a privacy partition. Subjects were then given the instructions (shown on the computer in front of them). The experimenter read the instructions aloud. A short quiz was given to gauge the subject's understanding of the instructions and all subjects were given the same quiz questions. Subjects had to answer all questions correctly for the experiment to continue.

There were 12 participants in each session. The session consisted 30 periods of the subscription game. Subjects were randomly re-matched with 3 other participants in each session. Subjects were not informed of the identities of other participants they were matched with, neither during nor after the experiment so that subjects' decisions were not associated with ID numbers which could be used to establish reputations. I randomly reassigned groups every period in an attempt to minimize repeated game effect and approximate the theoretical environment of a one-shot game, making it much harder for a group effect to develop.

At the end of the 30th period, each subject should drew two numbers randomly to determine his/her own payment periods. Each subject got paid based on the income he/she made in the two chosen periods. Since the subjects do not know which period the payment will be based on, they should do their best in every period. This payment method avoids the wealth effect. At the end of the experiment, subjects were asked to complete a short survey asking for basic demographic information and were then privately paid according to their incomes in the two periods which had been randomly selected at the end of the 30th period. The conversion rate for the experiment is one token = 5 cents. Subjects earned approximately $15.83, including

$5 show-up payment. Average session length was about one hour.

4.5 Hypotheses

As mentioned above, this experimental design represents a subscription game with private information on valuation and threshold uncertainty. Thus, the equilibrium is a Bayesian Nash Equilibrium, also known as the expected contribution. Based on the specific parameters chosen for the study, the Bayesian Nash Equilibrium prediction in each treatment is shown in Table 4.1. In the Baseline Treatment, the Bayesian Nash Equilibrium is 10.89. Mean-Preserving Treatment and Variance-Preserving Treatment have the same Bayesian Nash Equilibrium, 19.88.

Table 4.1: Bayesian Nash Equilibrium Prediction

	Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
Bayesian Nash Equilibrium	10.89	19.88	19.88

From these Bayesian Nash Equilibria, I have the following three Hypotheses, which are the comparative statics with respect to the threshold uncertainty.

Hypothesis 1a:

Keeping the mean of the cost distribution unchanged but decreasing the dispersion of the cost distribution will **increase** individual expected contribution (Comparison between the Baseline Treatment and the Mean-Preserving Treatment).

Hypothesis 1b:

Keeping the variance of the cost distribution unchanged but increasing the variance of the cost distribution will **increase** individual expected contribution (Comparison between the Baseline Treatment and the Variance-Preserving Treatment).

Hypothesis 1c:

With the same lower bound of the cost distribution, the expected contributions in the Mean-Preserving Treatment and the Variance-Preserving Treatment should be **the same**.

In this paper I also focus on the success rate (that is, the probability of providing the public good successfully). According to the experimental parameters in this paper, I calculate the *ex ante* probability of providing the public good in each treatment, shown in Table 4.2. From Table 4.2, the success rate in the Baseline, Mean-Preserving and Variance-Preserving Treatment are 18.13%, 19.86% and 11.35%, respectively. Comparing the success rates in different treatments, I propose the following hypotheses:

Table 4.2: *Ex Ante* Probability

	Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
Ex Ante Probability	18.13%	19.86%	11.35%

Hypothesis 2a:

The *ex ante* probability in the Baseline Treatment is **lower** than the *ex ante* probability in the Mean-Preserving Treatment.

Hypothesis 2b: The *ex ante* probability in the Baseline Treatment is **higher** than the *ex ante* probability in the Variance-Preserving Treatment.

Hypothesis 2c:

The *ex ante* probability in the Mean-Preserving Treatment is **higher** than the *ex ante* probability in the Variance-Preserving Treatment.

4.6 Results

4.6.1 Average Contribution

Figure 4.3 tracks the average contribution to the public good over the 30 periods in each treatment. From this figure, I find that although the average contribution in each treatment fluctuates over the 30 period, the average contribution in the Baseline Treatment is the lowest and the average contribution in the Variance-Preserving Treatment is the highest in the most periods.

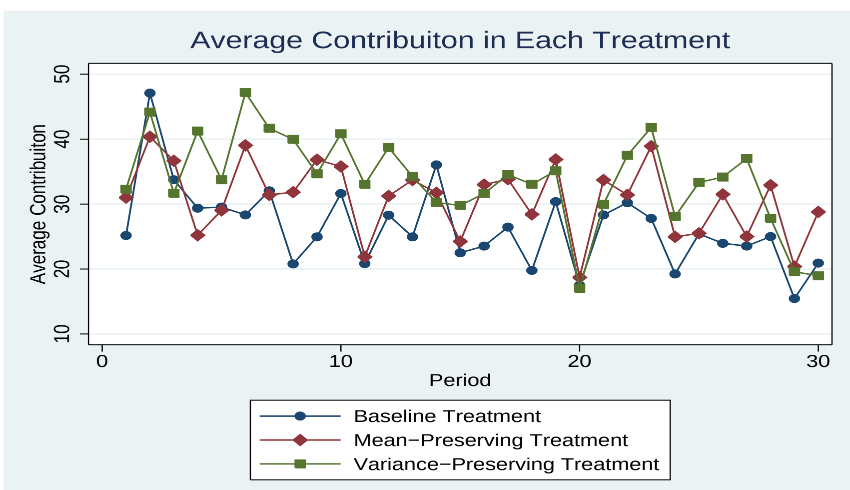

Figure 4.3: Average Contribution over the 30 Periods

Table 4.3 presents the empirical average contribution in three treatments. The average contributions in the Baseline, Mean-Preserving and Variance-Preserving Treatments are 26.43, 30.80, and 33.77, respectively. Comparing the average contribution with the Bayesian Nash Equilibrium prediction, shown in Table 4.1, I find that the average contribution is significantly higher than the Bayesian Nash Equilibrium prediction in every treatment ($p = 0.00$ in each treatment, t-test).

Table 4.3: Average Contribution in Three Treatments

	Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
Average Contribution (standard error)	26.43 (1.00)	30.80 (0.94)	33.77 (1.10)

Result 1: The average contribution in all treatments are significantly larger than the Bayesian Nash Equilibrium, which derives from maximizing individual expected payoff.

Next, I compare the average contribution in different treatments: Baseline Treatment vs. Mean-Preserving Treatment, Baseline Treatment vs. Variance-Preserving Treatment, and Mean-Preserving Treatment vs. Variance-Preserving Treatment. Comparing the average contribution between the Baseline Treatment and the Mean-Preserving Treatment, I find that the average contribution in the former treatment is significantly smaller than the average contribution in the latter treatment ($p = 0.00$, t-test). Thus, the experimental result supports Hypothesis 1a: Decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction causes individuals, on average, to be more willing to contribute to the public good.

Comparing the average contribution between the Baseline Treatment and the Variance-Preserving Treatment, I show that the average contribution in the Baseline

Treatment is significantly smaller than the average contribution in the Variance-Preserving Treatment ($p = 0.00$, t-test). Thus, the experimental result also supports Hypothesis 1b: Increasing the mean of the cost distribution in the sense of the first order stochastic dominance causes individuals, on average, to be more willing to contribute to the public good.

Comparing the Mean-Preserving and the Variance-Preserving Treatment, I find that the average contribution in the Mean-Preserving Treatment is significantly smaller than the average contribution in the Variance-Preserving Treatment ($p = 0.01$, t-test). This experimental result rejects Hypothesis 1c that the average contribution in these two treatments should be the same.

> **Result 2:** I find empirical supports for Hypothesis 1a and Hypothesis 1b. I find that decreasing the degree of threshold uncertainty in the sense of the mean-preserving contraction increases average individual contribution. I also find that increasing the mean of the threshold distribution in the sense of first-order stochastic dominance increases average individual contribution.

The empirical results above show that although some hypotheses are significantly supported, the level of the average contribution is significantly higher than the theoretical prediction in each treatment. I try to analyze which value of the public good an individual has will result in contributing higher amounts to the public good. Figure 4.4 depicts the distribution of value and contribution by treatment. The top figure illustrates the Baseline Treatment, followed by the Mean-Preserving Treatment and the Variance-Preserving Treatment.

The red triangle in the figure represents the equilibrium strategy prediction for each value. For example, in the Baseline Treatment, the equilibrium strategy prediction of an individual with value 0 is 0 tokens and the equilibrium strategy prediction

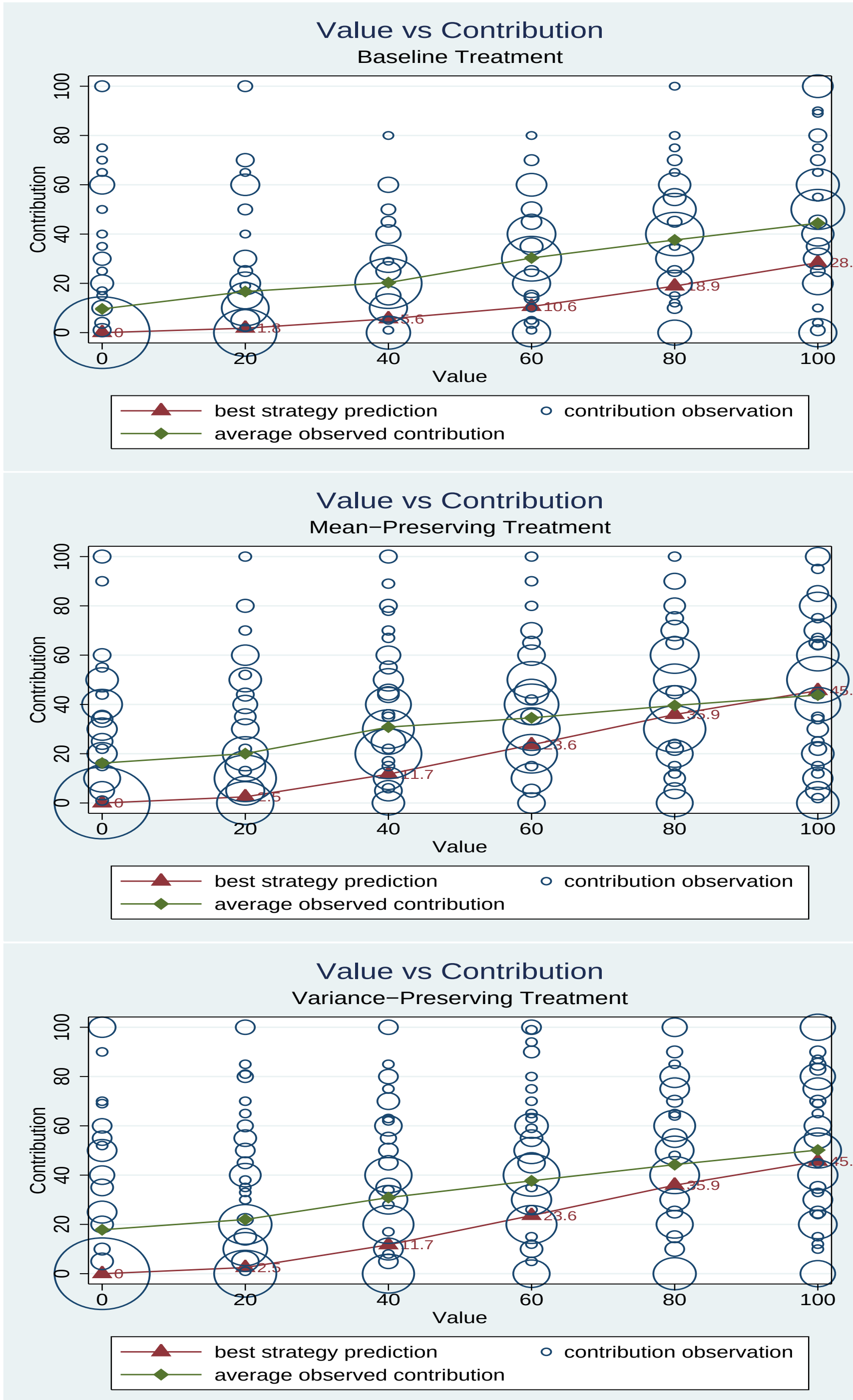

Figure 4.4: Value and Contribution in Three Treatments

of an individual with value 60 is 10.6 tokens. The circle represents the individual contribution observations in the experiment. The larger the circle is, the higher the frequency this contribution level made by the individual is observed. For example, when value is 0, the contribution amount, 0, has the highest frequency. The green diamond in the figure represents the average contribution level for each value. For example, in the Baseline Treatment, the average contribution level of an individual with value = 0 is 9.62 tokens and the average contribution level under value = 60 is 30.30 tokens.

Table 4.4 illustrates the equilibrium strategy prediction, average contribution level and the test result for each value in each treatment.

Table 4.4: Equilibrium Strategy and Average Contribution for Each Value

		Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
	Equilibrium	0.0	0.0	0.0
$v = 0$	Average Contribution	9.6***	16.2***	17.8***
	(standard error)	(1.9)	(2.0)	(2.5)
	Equilibrium	1.8	2.5	2.5
$v = 20$	Average Contribution	16.7***	20.0***	22.0***
	(standard error)	(2.1)	(1.9)	(2.3)
	Equilibrium	5.6	11.7	11.7
$v = 40$	Average Contribution	20.3***	30.8***	30.9***
	(standard error)	(1.4)	(1.9)	(2.3)
	Equilibrium	10.6	23.6	23.6
$v = 60$	Average Contribution	30.3***	34.6***	37.7***
	(standard error)	(1.6)	(1.7)	(2.1)
	Equilibrium	18.9	35.9	35.9
$v = 80$	Average Contribution	37.6***	39.6**	44.3***
	(standard error)	(1.8)	(2.1)	(2.5)
	Equilibrium	28.4	45.6	45.6
$v = 100$	Average Contribution	44.4***	43.8	50.2**
	(standard error)	(2.5)	(2.5)	(2.6)

Focusing on the Baseline Treatment first, I find that no matter what value the individual places on the public good (based on his/her assignment), his/her contribution, on average, is significantly higher than the equilibrium strategy. ($p = 0.00$, t-test). This result notes that the average contribution is higher than Nash Bayesian Equilibrium because the individual contributes a higher amount than the equilibrium strategy. Mean-Preserving Treatment has similar results to the Baseline Treatment when the value of public good is 0, 20, 40, 60 or 80. That is, individual contribution, on average, is significantly higher than the theoretical equilibrium strategy when the value of the public good is 0, 20, 40, 60 ($p = 0.00$, t-test) or 80 ($p = 0.04$, t-test). Therefore, the higher average contribution occurs because individuals with value 0, 20, 40, 60 or 80 contribute a higher amount to the public good in the Mean-Preserving Treatment. The individual with value 100 in the Mean-Preserving Treatment contributes lower amounts than the theoretical equilibrium strategy, but it is not statistically significant($p = 0.24$, t-test). As to the Variance-Preserving Treatment, I find that individuals with value 0, 20, 40, 60, 80 ($p = 0.00$, t-test) or 100 ($p = 0.04$, t-test) also contribute higher amounts than the theoretical prediction.

> **Result 3:** The average contribution is higher than the Bayesian-Nash Equilibrium prediction from individuals assigned a valuation of the public good of 0, 20, 40, 60 and 80 for all treatments and with a valuation of 100 in the Baseline Treatment and the Variance-Preserving Treatment. These individuals are willing to contribute higher amounts to the public good than that predicted by the equilibrium model.

Three reasons could explain why the average contribution is higher than that predicted by the Bayesian Nash Equilibrium model. First, individuals do not follow the objective of maximizing expected payoff. When an individual makes his/her contribution decision, he/she may take into account factors such as risk attitude, altruism, or cooperation above expected payoff maximization. The second reason is that private information on valuation and threshold uncertainty make decision making more complicated. Thus, individuals might have trouble arriving at the equilibrium strategy. Another reason could be that participants in the experiment do not understand fully the rules of this game. Figure 4.4 shows that many observations are above the diagonal, meaning that the contribution is higher than the valuation. In this subscription game, the subject may be worse off when his/her contribution is higher than his/her value and the public project is implemented. For example, a subject with value 40 contributes 50 tokens to the public project and the public project is provided, his/her payoff is 90 tokens. This amount is lower than his/her initial endowment, 100 tokens. Thus, if the participant realizes that he/she may be worse off when his/her contribution is larger than this value, he/she should avoid this situation happened.

I draw the frequency of contribution that is higher than the value in each treatment and ascertain whether the participants contribute more than his/her own value occurs because the participant is not familiar with the subscription game. From Figure 4.5, I observe that regardless of the treatment the subjects participate in, the frequency of contribution that is higher than the value is highest in Period 1. Although the frequency does not decrease dramatically, I find that the frequency in the last 10 periods is relatively low. Therefore, I will use the observations in the last 10 periods to analyze the results of comparative statics again.

Table 4.5 illustrates the empirical average contribution in the last 10 periods in each treatment. The average contribution in the Baseline Treatment is 23.99. Although it is lower than the average contribution in all 30 periods, 26.43, it is still significantly higher than the Bayesian Nash Equilibrium, 10.89 (p = 0.00, t-

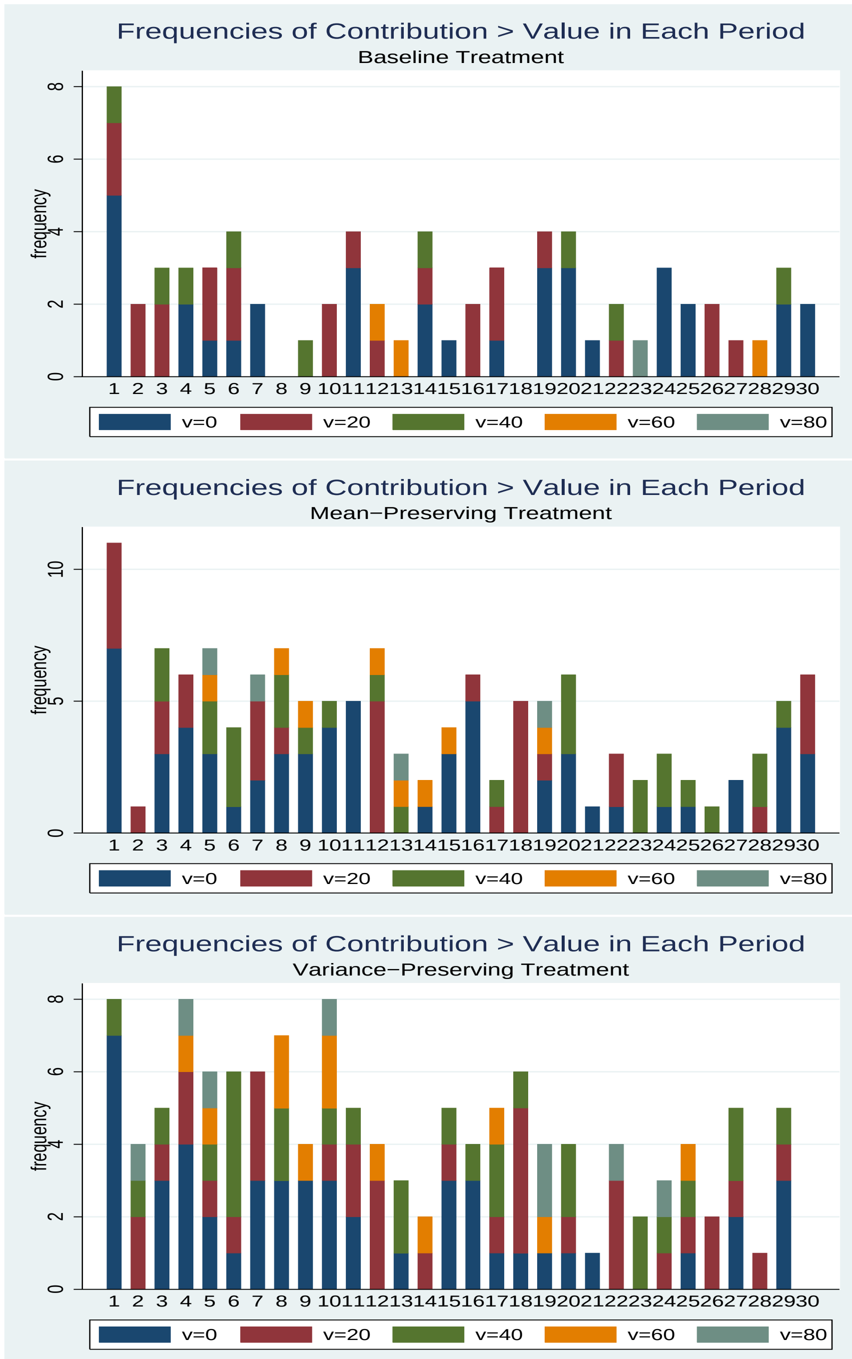

Figure 4.5: Frequencies of Contribution that is Higher Than the Value in Three Treatments

test). As to the Mean-Preserving Treatment, the average contribution of the last 10 periods , 29.31, is very close to the average contribution of all periods, 30.80, and it is significantly higher than the Bayesian Nash Equilibrium, 19.88 ($p = 0.00$, t-test). The average contribution in the Variance-Preserving Treatment is 30.82, which is lower than the average contribution of 30 periods, 33.77, is significantly higher than the Bayesian Nash Equilibrium, 19.88 ($p = 0.00$, t-test).

Table 4.5: Average Contribution of Period 21∼30 in Three Treatments

	Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
Average Contribution (standard error)	23.99 (1.44)	29.31 (1.65)	30.82 (1.92)

Result 4: Using the experimental data in Period 21∼30, the average contribution in all treatments is still significantly larger than the Bayesian-Nash equilibria, which derive from maximizing individual expected payoff.

I also test the comparative statics with respect to the uncertainty using the last 10 period data. To compare the average contribution between the Baseline Treatment and the Mean-Preserving Treatment, I find that the average contribution in the Baseline Treatment is significantly smaller than the average contribution in the Mean-Preserving Treatment ($p = 0.00$, t-test). Thus, the experimental data in the last 10 periods also supports Hypothesis 1a: Decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction causes individuals, on average, to be more willing to contribute to the public good.

Comparing the average contribution between the Baseline Treatment and the Variance-Preserving Treatment, I show that the average contribution in the Baseline

Treatment is significantly smaller than the average contribution in the Variance-Preserving Treatment ($p = 0.00$, t-test). The experimental data in the last 10 periods, again, supports Hypothesis 1b: Increasing the mean of the cost distribution in the sense of first order stochastic dominance causes individuals, on average, to be more willing to contribute to the public good.

Comparing the Mean-Preserving and the Variance-Preserving Treatment, I cannot reject the null hypothesis that the average contribution in the Mean-Preserving Treatment is equal to the average contribution in the Variance-Preserving Treatment using only the last 10 periods of experimental data ($p = 0.51$, t-test). This experimental result is consistent with Hypothesis 1c: With the same lower bound of the cost distribution, the average contribution in the Mean-Preserving Treatment and the Variance-Preserving Treatment should be the same.

To summarize the results of comparative statics with respect to the threshold, I find that whether I use all periods data or the last 10 periods data, the empirical results are consistent with Hypothesis 1a and Hypothesis 1b: Decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction causes individuals, on average, to be more willing to contribute to the public good. Increasing the mean of the cost distribution in the sense of first order stochastic dominance also causes individuals, on average, to be more willing to contribute to the public good. However, the empirical results partly support Hypothesis 1c: the average contribution in the Mean-Preserving Treatment and the Variance-Preserving Treatment should be the same. Another important experimental result is that although the empirical data supports hypotheses of comparative statics, the level of average contribution to the public project is far higher than the Bayesian Nash Equilibrium.

Although the frequency with which the contribution is higher than the value is relative small in the last 10 periods, the average contribution in the last 10 periods is still significantly higher than the Bayesian Nash Equilibrium. In the following paragraphs, I will discuss how an individual with different values contributes to the public good project using the last 10 periods of observations. Figure 4.6 illustrates the relation between the value and the contribution using the last 10 periods of data. The red triangles represent the equilibrium strategy prediction, which derives from maximizing the expected payoff. The green diamonds represent the average contribution of the last 10 periods under each value. The circles are the contribution observations.

Figure 4.6 shows that the average contribution increases with the value assigned to participants. That is, the higher the value is, the higher the average contribution is. Thus, the relationship between the value and the contribution is consistent with the theoretical prediction. However, the levels of average contribution differ from the equilibrium strategies.

Table 4.6 shows the average contribution of Period 21$\sim$30 and the equilibrium strategy under each value in three treatments. In the Baseline Treatment, the average contribution is significantly higher than the equilibrium strategy under all values ($p = 0.00$, t-test). This result shows that the average contribution in the Baseline Treatment is higher than the Bayesian Nash Equilibrium projection, consistent with the result using all periods of data.

In the Mean-Preserving Treatment, the individual, on average, contributes significantly higher amounts to the public good when his/her value is 0, 20, 40, or 60 ($p = 0.00$, t-test). As to the Variance-Preserving Treatment, the individual, on average, contributes significantly higher amounts to the public good under all values except 80 ($p = 0.00$ when value is 0, 20, 40, or 60 and $p = 0.07$ when value is 100, t-test). The results in these two treatment are similar to the results using all periods of data.

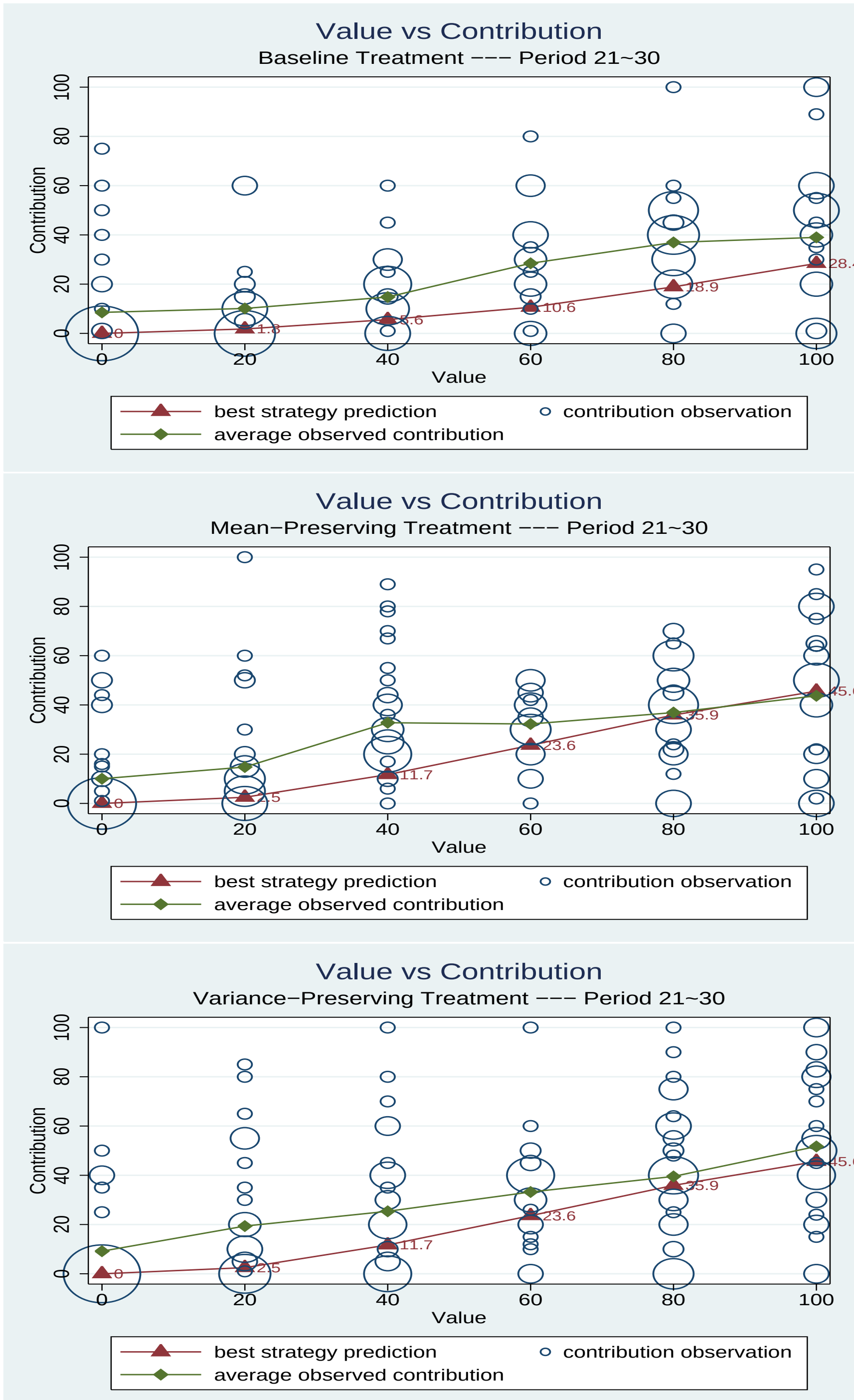

Figure 4.6: Value and Contribution in Period 21∼30 in Three Treatments

Table 4.6: Equilibrium Strategy and Average Contribution for Each Value in Period 21∼30

		Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
	Equilibrium	0.0	0.0	0.0
$v = 0$	Average Contribution	8.5***	10.0***	9.2***
	(standard error)	(3.15)	(3.0)	(3.6)
	Equilibrium	1.8	2.5	2.5
$v = 20$	Average Contribution	10.1***	17.8***	19.4***
	(standard error)	(2.6)	(3.4)	(4.0)
	Equilibrium	5.6	11.7	11.7
$v = 40$	Average Contribution	14.8***	32.8***	25.4***
	(standard error)	(2.1)	(3.2)	(4.0)
	Equilibrium	10.6	23.6	23.6
$v = 60$	Average Contribution	28.5***	32.3***	33.2***
	(standard error)	(3.7)	(2.3)	(3.4)
	Equilibrium	18.9	35.9	35.9
$v = 80$	Average Contribution	36.4***	36.9	39.5
	(standard error)	(2.4)	(2.8)	(3.7)
	Equilibrium	28.4	45.6	45.6
$v = 100$	Average Contribution	39.0***	43.7	51.7*
	(standard error)	(4.3)	(4.3)	(4.1)

According to the data of Period 21∼30, I find that although the frequency of the observations that the contribution is higher than the value is relative low among the whole periods, the individual still contributes higher amounts than the equilibrium strategy under most values. This results suggest that many other factors exist that affect individual contribution decisions in a subscription game.

4.6.2 Success Rate

This paper shows that the average contribution in all treatments is higher than the Bayesian Nash Equilibrium. Now, I analyze whether the success rates of proving the public good are consistent with the theoretical *ex ante* probabilities using the Binomial probability test. I calculate the success rates using all the data and with only the last 10 periods of data, calling these two success rates P_{all} and P_{21-30}, respectively. When I use all-period data to calculate the success rate, P_{all}, there are 180 group observations in each treatment — 3 groups per period × 30 periods per session × 2 sessions per treatment. When I use the last 10 periods data to calculate the success rate, P_{21-30}, there are 60 group observations — 3 groups per period × 10 periods per session × 2 sessions per treatment. Table 4.7 reports the two observed success rates for each treatment.

Table 4.7: Observed Success Rate

	Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
P_{all}	37.22%	39.44%	27.78%
(standard error)	(0.04)	(0.04)	(0.03)
P_{21-30}	35.00%	38.33%	21.67%
(standard error)	(0.06)	(0.06)	(0.05)

I first discuss the results related to P_{all} in each treatment. Among 180 group observations in the Baseline Treatment, 67 observations successfully provide the public good. The P_{all} in the Baseline Treatment is 37.22%, and it is statistically significant higher than the theoretical predicted probability, 18.13% ($p = 0.00$). The Mean-Preserving Treatment has the similar result. 71 group observations succeed in providing the public good. The P_{all} in the Mean-Preserving Treatment, 39.44%, is also statistically significant higher than the theoretical prediction, 19.86%, ($p = 0.00$). In the Variance-Preserving Treatment, 50 of 180 group observations successfully provide the public good. The P_{all} in the Variance-Preserving Treatment, 27.78%, is statistically significant higher than the theoretical prediction, 11.35% ($p = 0.00$).

To calculate P_{21-30}, I use 60 group observations in each treatment. Among these

60 group observations in the Baseline Treatment, 21 observations successfully provide the public good. Thus, P_{21-30} in the Baseline Treatment is 35%. This is significantly higher than the theoretical prediction ($p = 0.00$). In the Mean-Preserving Treatment, 23 observations provide the public good successfully, thus, P_{21-30} in the Mean-Preserving Treatment is 38.33%. It is also significantly higher than the theoretical prediction ($p = 0.00$). As to the Variance-Preserving Treatment, 13 observations successfully provide the public good, and the P_{21-30} in the Variance-Preserving Treatment is 21.67%. This is still significantly higher than the theoretical prediction ($p = 0.00$).

In order to test Hypothesis 2a -2c, I conduct the two-sample test of proportion. I first compare the success rates in any two treatments using all periods of data. That is, I compare P_{all} across different treatments. To test the P_{all} in the Baseline Treatment and the Mean-Preserving Treatment, I cannot reject the null hypothesis that the P_{all} in these two treatments are equal (two-tailed p-value $p = 0.66$, two-sample test of proportion). However, the P_{all} in the Baseline Treatment is significantly higher than the P_{all} in the Variance-Preserving Treatment (one-tailed p-value $p = 0.03$, two-sample test of proportion). Comparing the P_{all} in the Mean-Preserving Treatment and the Variance-Preserving Treatment, I find that the P_{all} in the Mean-Preserving Treatment is significantly higher than the P_{all} in the Variance-Preserving Treatment (one-tailed p-value $p = 0.01$, two-sample test of proportion).

I also compare the success rate across different treatments by the last 10 periods data, which is called P_{21-30}. I get a similar results that the Baseline Treatment and the Mean-Preserving Treatment both have significantly higher success rates than the Variance-Preserving Treatment (one-tailed p-value $p = 0.05$ and 0.02, respectively, two-sample test of proportion). As for comparing the Baseline Treatment with the Mean-Preserving Treatment, I find that the public good is less often provided in the

Baseline Treatment compared to in the Mean-Preserving Treatment, but it is not statistically significant (one-tailed p-value $p = 0.33$, two-sample test of proportion).

Using a two-sample test of proportion, the empirical data supports Hypothesis 2b and 2c that the success rates in the Baseline Treatment and the Mean-Preserving Treatment is significantly higher than in the Variance-Preserving Treatment. This holds true for the last 10 periods and the all periods together. However, comparing the Baseline Treatment with the Mean-Preserving Treatment, I do not find a significant difference for the last 10 periods and all periods together.

> **Result 5:** The empirical probabilities of providing the public good are higher than the *ex ante* probabilities all treatment. But the empirical data only supports Hypothesis 2b and Hypothesis 2c that the success rates in the Baseline Treatment and the Mean-Preserving Treatment are higher than that in the Variance-Preserving Treatment.

4.6.3 Average Payoff

Suppliers of the public good care about the amount of total contribution they can receive from contributors. Contributors care about the payoff they may receive after making a contribution to a public good. In this subsection, I compare the individual payoff in the three treatments.

Table 4.8 shows the expected payoff and the average payoff in each treatment. Average Payoff_{all} is measured using all periods of data and Average Payoff_{21-30} is measured using the last 10 periods of data.

Although the average payoff in each treatment is close to the expected payoff, this does not happen as a result of individuals making contribution decisions based on maximizing their expected payoff. Figure 4.7 shows the average payoff under each value in the three treatments.

Table 4.8: Expected Payoff and Average Payoff in Three Treatments

	Baseline Treatment	Mean-Preserving Treatment	Variance-Preserving Treatment
Expected Payoff	108.24	108.05	104.60
Average Payoff_{all} (standard error)	108.73 (0.89)	106.91 (0.86)	102.90 (0.73)
Average Payoff_{21-30} (standard error)	109.06 (1.26)	109.35 (1.52)	103.76 (1.16)

I find that individuals with specific values are worse off when contributing to the public good. For example, I show that an individual with value = 0, on average, contributes 9 tokens to the public good. Since the participant's contribution is higher than his/her value, he/she will suffer from loss when the public good is provided. From Figure 4.7, I observe that the average payoffs of the subject with value 0, 20, 40, and 60 are lower than the theoretical prediction derived from maximizing the expected payoff. This result demonstrates that although these subjects make contributions higher than the Bayesian Nash Equilibrium, they can increase the probability of providing the public good. However, the increased payoff from the higher provision probability is not large enough to cover the decreased payoff from contributing higher than the optimal amount to maximize expected payoff.

The subject with values 80 and 100 receives higher average payoff than the theoretical prediction. For these subjects, the increased payoff from higher provision probability is larger than the decreased payoff from higher contribution, therefore, they can enjoy higher average payoff even if their average contributions are higher than the Bayesian Nash Equilibrium.

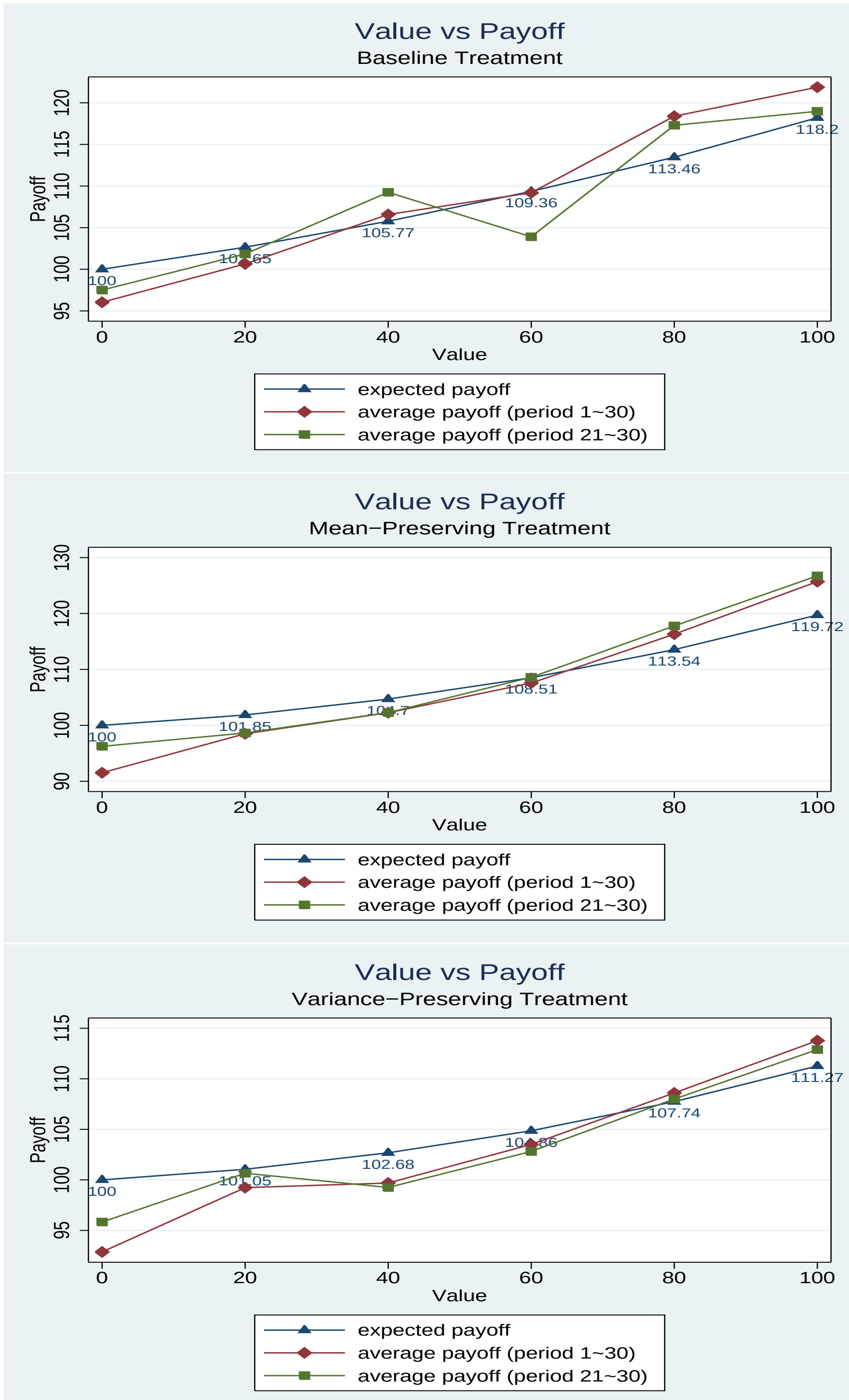

Figure 4.7: Expected Payoff and Average Payoff in Three Treatments

4.6.4 Individual Results

In this subsection, I report a random effect Tobit regression using individual contribution data. The dependent variable is each subject's contribution, which is naturally censored to lie between 0 and 100. The independent variables included in the model are described below:

- **Value**: measures the value of the public good
- **Mean-Preserving**: treatment dummy variable. It equals 1 if the treatment is Mean-Preserving Treatment; 0 otherwise.
- **Variance-Preserving**: treatment dummy variable. It equals 1 if the treatment is Variance-Preserving Treatment; 0 otherwise.
- **Period**: dummy variables that capture time fixed effect.
- **Female**: equals 1 if the subject is female; 0 otherwise
- **Risk**: measures subject's risk preference. I use Eckel and Grossman's (2008) model to measure risk preference. Each subject choose one of 5 choices he prefers:

  1. 50% chance of receiving $10 and 50% chance of receiving $10.
  2. 50% chance of receiving $18 and 50% chance of receiving $6.
  3. 50% chance of receiving $26 and 50% chance of receiving $2.
  4. 50% chance of receiving $34 and 50% chance of receiving -$2.
  5. 50% chance of receiving $42 and 50% chance of receiving -$6.

  The higher number of choice means the subject is more risk-loving.

- **Success**$_{t-1}$: equals 1 if the public good was successfully provided in the previous period; 0, otherwise.

- **Total Contribution**$_{t-1}$: total contribution in the previous period.

- **Mean-Preserving** $*$ **Value**: interactions of **Value** with treatment variables **Mean-Preserving**.

- **Variance-Preserving** $*$ **Value**: interactions of **Value** with treatment variables **Variance-Preserving**.

- **Mean-Preserving** $*$ **Success**$_{t-1}$: interactions of **Success**$_{t-1}$ with treatment variables **Mean-Preserving**.

- **Variance-Preserving** $*$ **Success**$_{t-1}$: interactions of **Success**$_{t-1}$ with treatment variables **Variance-Preserving**.

- **Mean-Preserving** $*$ **Total Contribution**$_{t-1}$: interactions of treatment variables **Mean-Preserving** with **Total Contribution**$_{t-1}$.

- **Variance-Preserving** $*$ **Total Contribution**$_{t-1}$: interactions of treatment variables **Variance-Preserving** with **Total Contribution**$_{t-1}$.

Table 4.9 reports the results of all observations (72 subjects, 30 periods) and Table 4.10 reports the results of the last 10-period observations (72 subjects, 10 periods).

Table 4.9: Estimation of Random Effect Tobit Regression — All Observations

Random effect Tobit regression	(A-1)	(A-2)	(A-3)	(A-4)
Dependent variable: contribution	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)
Value	0.427***	0.435***	0.435***	0.490***
	(0.00)	(0.00)	(0.00)	(0.00)
Mean-Preserving	8.481	8.425	8.375	15.387***
	(0.11)	(0.12)	(0.12)	(0.01)
Variance-Preserving	10.001*	9.567*	9.800*	12.94**
	(0.06)	(0.08)	(0.07)	(0.02)
Period	−0.488***	−0.487***	−0.462***	−0.489***
	(0.00)	(0.00)	(0.00)	(0.00)
Female	8.303*	8.432*	8.482*	8.349*
	(0.07)	(0.07)	(0.07)	(0.07)
Risk	−0.051*	−2.717	−2.716	−2.796*
	(0.10)	(0.12)	(0.11)	(0.10)
Success_{t-1}		−3.755***		
		(0.00)		
$\text{Total Contribution}_{t-1}$			0.005	
			(0.61)	
Mean-Preserving ∗ Value				−0.131***
				(0.00)
Variance-Preserving ∗ Value				−0.054
				(0.16)
Constant	10.487	11.260	8.810	7.150
	(0.13)	(0.11)	(0.22)	(0.31)
Log likelihood	−7933.70	−7638.67	−7644.27	−7927.66
Left-censored	469	456	456	469
Uncensored	1636	1578	1578	1636
Right-censored	55	54	54	55

*** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 4.9 continued.

Random effect Tobit regression	(A-5)	(A-6)
Dependent variable: contribution	Coefficient (p-value)	Coefficient (p-value)
Value	0.497***	0.498***
	(0.00)	(0.00)
Mean-Preserving	17.789***	22.596***
	(0.00)	(0.00)
Variance-Preserving	13.618**	11.920*
	(0.06)	(0.07)
Period	−0.495***	−0.461***
	(0.00)	(0.00)
Female	8.454*	8.293*
	(0.07)	(0.08)
Risk	−2.774	−2.788
	(0.11)	(0.11)
Success_{t-1}	−0.728	
	(0.70)	
Total $\text{Contribution}_{t-1}$		0.019
		(0.30)
Mean-Preserving ∗ Value	−0.134***	−0.135***
	(0.00)	(0.00)
Variance-Preserving ∗ Value	−0.049	−0.057
	(0.21)	(0.15)
Mean-Preserving ∗ Success_{t-1}	−5.619**	
	(0.03)	
Variance-Preserving ∗ Success_{t-1}	−3.664	
	(0.18)	
Mean-Preserving ∗ Total $\text{Contribution}_{t-1}$		−0.059**
		(0.03)
Variance-Preserving ∗ Total $\text{Contribution}_{t-1}$		0.004
		(0.87)
Constant	6.930	4.067
	(0.34)	(0.59)
Log likelihood	−7630.21	−7634.47
Left-censored	456	456
Uncensored	1578	1578
Right-censored	54	54

*** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 4.10: Estimation of Random Effect Tobit Regression — Last 10-Period Observations

Random effect Tobit regression	(L-1)	(L-2)	(L-3)	(L-4)
Dependent variable: contribution	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)
Value	0.479***	0.479***	0.489***	0.467***
	(0.00)	(0.00)	(0.00)	(0.00)
Mean-Preserving	9.570**	9.591**	9.277*	11.298*
	(0.09)	(0.09)	(0.10)	(0.09)
Variance-Preserving	9.543*	9.596*	9.097	5.499
	(0.09)	(0.09)	(0.11)	(0.42)
Period	−0.575**	−0.577**	−0.556**	−0.575**
	(0.04)	(0.04)	(0.05)	(0.04)
Female	11.482**	11.490**	11.354**	11.428**
	(0.02)	(0.02)	(0.02)	(0.02)
Risk	−2.460	−2.458	−2.428	−2.440
	(0.17)	(0.17)	(0.17)	(0.18)
Success_{t-1}		0.324		
		(0.86)		
$\text{Total Contribution}_{t-1}$			0.016	
			(0.34)	
Mean-Preserving $*$ Value				−0.032
				(0.62)
Variance-Preserving $*$ Value				0.070
				(0.28)
Constant	−3.548	−3.662	−5.175	−2.882
	(0.64)	(0.63)	(0.51)	(0.72)
Log likelihood	−2527.68	−2527.67	−2527.23	−2526.33
Left-censored	184	184	184	184
Uncensored	524	524	524	524
Right-censored	12	12	12	12

*** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

Table 4.10 continued.

Random effect Tobit regression	(L-5)	(L-6)
Dependent variable: contribution	Coefficient (p-value)	Coefficient (p-value)
Value	0.466***	0.468***
	(0.00)	(0.00)
Mean-Preserving	13.153*	16.625**
	(0.06)	(0.04)
Variance-Preserving	6.758	5.584
	(0.33)	(0.48)
Period	−0.575**	−0.561**
	(0.04)	(0.05)
Female	11.264**	11.063**
	(0.02)	(0.02)
Risk	−2.448	−2.406
	(0.18)	(0.18)
Success_{t-1}	3.040	
	(0.31)	
$\text{Total Contribution}_{t-1}$		0.031
		(0.30)
Mean-Preserving ∗ Value	−0.033	−0.034
	(0.60)	(0.59)
Variance-Preserving ∗ Value	0.073	0.065
	(0.26)	(0.32)
Mean-Preserving ∗ Success_{t-1}	−5.099	
	(0.24)	
Variance-Preserving ∗ Success_{t-1}	−4.444	
	(0.35)	
Mean-Preserving ∗ $\text{Total Contribution}_{t-1}$		−0.051
		(0.23)
Variance-Preserving ∗ $\text{Total Contribution}_{t-1}$		−0.006
		(0.89)
Constant	−3.852	−5.907
	(0.63)	(0.49)
Log likelihood	−2525.53	−2525.10
Left-censored	184	184
Uncensored	524	524
Right-censored	12	12

*** denotes 1% significance; ** denotes 5% significance; * denotes 10% significance.

I first concentrate on the variable **Value**. Whether I use all observations or the last 10-period observation, I show that **Value** has significantly positive effect on the contribution. This means that the higher the individual values the public good, the higher contribution that participant makes. This result supports the theoretical contribution strategy that the contribution is increasing in the value of the public good. When I include the interactions of **Value** with treatment variables **Mean-Preserving** and **Variance-Preserving** (Model (A-4) to Model (A-6) in Table 4.9 and Model (L-4) to Model (L-6) in Table 4.10), I find that while increasing the value of the public goods increases the contribution, the increase is significantly smaller in the Mean-Preserving Treatment by all observations.

Second, I focus on the treatment dummy variables, **Mean-Preserving** and **Variance-Preserving**, and analyze the differences in contributions depending on the treatment the individual participates in. Table 4.9 and 4.10 show that **Mean-Preserving** is significantly positive. Thus, an individual who participates in the Mean-Preserving Treatment contributes significantly higher amounts to the public good than in the Baseline Treatment. This result provides additional support for Hypothesis 1a. The other treatment variable, **Variance-Preserving**, is marginally significantly positive with all observations and in models (L-1) and (L-2) with the last 10-period observations. Although I get limited verification that the individual who takes part in the Variance-Preserving Treatment contributes significantly higher amounts to the public good than in the Baseline Treatment, the result provides supplementary support for Hypothesis 1b.

The variable **Period** is significantly negative. This indicates that an individual contributes significantly smaller amounts to the public good project in latter periods of the experiment. This result is consistent with the assertion that the frequency with which the contribution is higher than the valuation becomes lower and the contribution amount decreases in latter periods.

I also include the time invariant variables: gender, **Female**, and risk preference, **Risk**. I find that females contribute higher amounts to the public good project than males. The risk preference variable shows that the individual who is more risk-loving contributes smaller amounts to the public project. However, this result is not statistically significant.

When considering whether the public good was provided in the previous period, **Success**$_{t-1}$, I find that if the public good was provided in the previous period, the individual will decrease his/her contribution (Model (A-3)). However, considering the interaction terms, **Mean-Preserving** $*$ **Success**$_{t-1}$ and **Variance-Preserving** $*$ **Success**$_{t-1}$, together, whether the public good was provided in the previous period or not does not affect individual contributions. But, the individuals contribute less in the Mean-Preserving Treatment when the public good was successfully provided (Model (A-5)).

When I include the previous total contribution and the interaction terms with value as an independent variable, **Total Contribution**$_{t-1}$, **Mean-Preserving** $*$ **Total Contribution**$_{t-1}$ and **Variance-Preserving** $*$ **Total Contribution**$_{t-1}$, I find that these variables do not significantly affect the contribution. However, individuals contribute less in the Mean-Preserving Treatment when the total contribution in the group is higher in the previous period.

4.7 Conclusion

In this paper, I conduct an experiment to analyze individual voluntary contribution behavior in an environment with private information on valuation and threshold uncertainty. I also test the comparative statics with respect to the threshold uncertainty.

From the experimental data, I show that whatever threshold uncertainty the individuals face, the average contribution is significantly higher than the Bayesian Nash equilibrium, which is derived from maximizing individual expected payoff. This result implies that making contribution decisions consistent with the Bayesian Nash Equilibrium condition of maximizing individual expected payoff seems difficult in an environment with private information on valuation and threshold uncertainty. This result also leads me to believe that other important factors affect individual voluntary contribution behavior.

Although the level of the average contribution is higher than the Bayesian Nash Equilibrium, the experimental data supports the theoretical predictions of the comparative statics in Gronberg and Peng (2014). I find that decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction makes individuals, on average, more willing to contribute to a public good. Also, I find that increasing the mean of the threshold distribution in the sense of first-order stochastic dominance makes individuals, on average, more willing to contribute to a public good.

The success rate of providing the public good represents a strong performance measure for the supplier of a public good. Keeping the variance unchanged, I find that the success rate in the case of smaller mean of cost distribution is significantly higher than that in the case of larger mean of cost distribution. However, keeping the mean unchanged, the success rate in the case of small dispersion of cost distribution and in the case of large dispersion of cost dispersion is not significantly different.

From a policy perspective, I suggest that supplier should try to reduce the degree of uncertainty with respect to the cost distribution where there exists both threshold uncertainty and private information on public good valuations. In doing so, I predict they would receive higher contributions and higher success rates.

I also get limited verification that there is a tradoff between contribution and success rate when the supplier considers whether they should increase the provision cost in the sense of first order stochastic dominance. The supplier can receive higher contribution by increasing each possible provision cost. However, they might suffer a lower probability of providing the public good.

In this paper, I have shown that the different degrees of the threshold uncertainty indeed affect individual voluntary contribution decisions. Another factor, private information on valuation, is worth investigating. Future researchers could conduct another laboratory experiment to test whether individual contribution behavior is affected by participants being given different information on valuation.

5. CONCLUSIONS

This dissertation investigates individual voluntary contribution behavior from three different, but related, perspectives. In the entire work, I place emphasis on the subscription game with private information on valuation and threshold uncertainty, both theoretically and experimentally. The final goal is to provide a set of theoretical frameworks and experimental tools which can be used to understand private contribution to the public goods. On the one hand, I theoretically show that the contribution decisions are different when individuals face different levels of threshold uncertainty; they are also different when individuals are in different types of contribution institution. On the other hand, I investigate the subscription game from an experimental point of view and provide an experimental evidence that individual contribution are affected when individuals are informed different levels of threshold uncertainty.

The first essay builds a theoretical subscription game model and studies the effects of comparative statics with respect to threshold uncertainty. We find that if the costs becomes more dispersed in the sense of mean-preserving spread, individuals, on average, are less willing to contribute to the public good when there exists both private information on public good valuations and threshold uncertainty. But, if the cost increases in the sense of first order stochastic dominance, individuals, on average, are more willing to contribute to the public good. This theoretical result provides a policy implication that suppliers can increase the private contribution if they can either reduce the degree of uncertainty or increase the mean with respect to the cost distribution in a simultaneous subscription with threshold uncertainty and private information on public good valuations.

The second essay develops a theoretical model and examines individual sequential contribution to the public good in a subscription game with threshold uncertainty and private information on valuation. This essay shows that individual contribution is increasing with respect to the contributor's order. Earlier contributors contribute less than subsequent contributors. This result implies that earlier contributors can free ride off later contributor and enjoy first-mover advantage. Comparing the individual expected contribution in a sequential contribution institution to a simultaneous contribution institution, I find that the expected total contribution in the sequential institution is lower than the expected total contribution in a simultaneous one. I also find that the *ex ante* probability of providing the public good in the sequential institution is lower than that in the simultaneous institution.

In the last essay, I conduct an experiment to analyze individual voluntary contribution in an environment with private information on valuation and threshold uncertainty and test the comparative statics with respect to the threshold uncertainty. This essay shows that no matter what threshold uncertainty the individuals face, the average contribution is significantly higher than the Bayesian Nash equilibrium. This result demonstrates that individual contribution behavior may not be followed the objective of maximizing his/her own expected payoff. Individual may consider other factors when contributing to the public good. As to the comparative statics with respect to threshold uncertainty, I find that decreasing the degree of threshold uncertainty in the sense of mean-preserving contraction makes individuals, on average, more willing to contribute to a public good and receives higher success rate. I also find that increasing the mean of the threshold distribution in the sense of first-order stochastic dominance makes individuals, on average, more willing to contribute to a public good. According to the experimental results, I would like to suggest that suppliers should try to reduce the degree of uncertainty with respect to the cost distribution when there exists both threshold uncertainty and private information on public good valuations. In doing so, I predict they would receive higher total contributions and higher success rates.

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