OPTIMAL SCREENING FOR PRECLINICAL DISEASES

A Dissertation

by

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ABSTRACT

Certain diseases comprise an initial asymptomatic period during which they can be identified only by a screening test. In many such cases, early detection translates into benefits of more treatment options and potentially better prognosis. In this dissertation, we consider the optimal policy to screen for a preclinical disease while under limited budget. Our objective is to place any given number of screening epochs over an individual's lifetime, such that the probability of identifying the disease while preclinical is maximized. We make mild assumptions about the sojourn times of the individual in the healthy and preclinical states, and we consider the possibility of fallible screening tests. We show that a unique optimal sequence of screening times exist for our model, and that it can be quickly found by any greedy-search algorithm. We further conduct numerical experimentations by which we identify sensitive model inputs. We lastly apply our model to breast cancer screening using practical information and we investigate additional characteristics of this model.

DEDICATION

To my loving parents, Li Xin-Guang and Liu Bao-Ling

ACKNOWLEDGEMENTS

I am grateful to my parents for their consistent support both financially and emotionally throughout my PhD study.

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1. INTRODUCTION

Certain chronic diseases are characterized by an initial phase with no outward symptoms on the patients (a.k.a. "preclinical" period). Screening tests are available to find a disease in its preclinical period. Once a preclinical disease is detected, benefits could be reaped in terms of less aggressive treatment options and/or improved prognosis. On the other hand, if the disease is left untreated and enters its symptomatic ("clinical") stage, then the treatment procedures may become much more involved and chances of long term survival much reduced. Examples of diseases that comprise such features include hypertension, diabetes, and a collection of cancers such as breast, cervical, colorectal and prostate cancers.

Despite the general acceptance on benefits of screening, different professional and governmental organizations are recommending different schedules for the public to go on screening tests. In the case of breast cancer for example, the American Cancer Society recommends all women at ages 40 and over to go on both mammography and clinical breast exam annually; the National Cancer Institute recommends only mammography every one or two years beginning at age 40; and the U.S. Preventive Services Task Force suggests mammography only for women between 50 to 75, and only biennial exams. International debates over screening policies in terms of their starting age and their frequency are ongoing.

It is clear that by scheduling many screenings over his/her life, one obtains an improved potential of detecting a preclinical disease, yet one also incurs higher cost. In the end, some of the capital spent on screening for one disease might have been better utilized to treat/screen for other diseases. On the other hand, certain screening exams are themselves risky, e.g. colonoscopy if not well performed may

cause perforation of the intestine, bleeding or incontinence, and mammography can be harmful if the device is not calibrated at its right radiation level. Also complicating the practice of screening are the side-effects associated with false-positive screening results. Usually if a screening tests positive, more accurate (and oftentimes more aggressive) follow-up procedures are required to confirm the case, e.g. biopsy for breast cancer. As such, a falsely-produced positive result by screening will translate into additional costs, medical risks, and negative emotions for the patient which are in fact unnecessary.

In this dissertation, we study the optimal policy to screen for a preclinical disease while considering cost. We use the term "screening policy" to mean a series of time points at which an individual is supposed to go on screening exams even if he/she seems healthy. Our goal is to develop a methodology that could be used by the public health community to provide informed recommendations for disease screening.

Our model aims to find the screening policy that detects the preclinical disease with maximal probability, and we consider cost in terms of a screening budget, which is defined as the maximum number of screenings allowed in a person's lifetime. In implementation, we may solve the model at a variety of budget levels and observe the performance of the optimal screening policy at each one. This process may be continued until the best trade-off is met between screening budget and the probability of detection in the eyes of a decision maker.

We assume known information about the evolvement of the disease and about the sensitivity of screening exams. In practice such information is not directly collectable and needs to be estimated by rather sophisticated statistical procedures. Specifically, we assume the population's healthy sojourn time has a density that is a log-concave function, and the preclinical sojourn time has a general density on

infinite support. We also model false-negative screening results by assuming a fixed sensitivity for all screenings.

The main result is that the model consists of a unique optimal screening policy, and that this policy can be quickly found by a greedy-search procedure due to the nice structure of the objective function.

We then numerically conduct a variety of sensitivity analyses on the model inputs. The results reveal that the variances of the distributions for both disease-free and preclinical sojourn times play a key role in the performance of the optimal solution.

We apply our model to the screening of breast cancer. With parameters assumed to the best of our knowledge, we observe that the equal-interval screening policy performs rather closely to the optimum. Additionally, we examine the expected number of screenings (for both disease-free and disease-affected populations) and the expected lead time (i.e. time gained in diagnosis due to screening for the disease-affected population). We show with practical evidence that the disease-free population actually receives many more screening examines compared with the disease-affected population who is our target. We also show for our particular breast cancer screening model that the screening policy which maximizes the probability of screening detection also maximizes the expected lead time to clinical detection.

We want to also point out that our model can be applied to other settings in which a system that comprises a non-self-announcing "incipient-failure" state and a self-announcing "hard-failure" state is maintained. One example as such is the infrastructure maintenance problem, in which invisible degradations could have occurred long before they become visible and dangerous.

This dissertation is organized as follows. In Section 2, we review the relevant literature on preclinical disease screening. In Section 3, we lay down our formulation

for the optimal screening policy model. The analytical solution to the model will be shown in Section 4. In Section 5, we present our sensitivity analysis results. In Section 6, we show our numerical results as we apply our model to breast cancer screening. Lastly, Section 7 will conclude our research and point out a few directions for future research.

2. LITERATURE REVIEW

Model-based studies of asymptomatic disease screening date back to Zelen and Feinleib (1969). In this pioneering study, the authors develop a statistical model to estimate the mean lead time for a public screening program. The lead time is defined to be the time gained in disease detection by screening than it normally would due to symptoms.

Under considerably strong assumptions, e.g. a single and perfectly sensitive screening conducted far from the time origin, and a constant prevalence level of preclinical samples in the population over time (so-called "stable disease model"), the authors derive a mean lead time estimator for the screen-detected population that is based solely on moments of the population's preclinical sojourn time distribution. Zelen and Feinleib (1969) further estimate these moments by information on the clinical incidence rates and preclinical prevalence level of the disease, which are collected from practice.

The model of Zelen and Feinleib (1969) is applied to data from the Health Insurance Plan for Greater New York (HIP) program, one of the earliest large-scale screening trials conducted in the US for better treating breast cancer. The outputs suggest that the preclinical sojourn time in this case is well-modeled by an exponential distribution.

Later, Albert et al. (1978a,b) and Louis et al. (1978) present a series of three reports, in which they mathematically define a large collection of traditional epidemiologic terms that are relevant to a preclinical disease. These terms include mean sojourn time, age-specific incidence rate, age-specific prevalence, lifetime attack rate and a variety of cohort effects. The definitions are based on a disease progression

model that consists of three disease states, namely, disease-free, preclinical and clinical states; and all quantities are expressed in terms of the joint-distribution of a population's age mix and its sojourn times in the various states. This disease progression model is so-called a "natural history model", for the reason that it considers only the progression of the disease undisturbed, without say, any early interventions due to results of screening.

As in Zelen and Feinleib (1969), evaluation of screening programs forms the goal of an early stream of studies of disease screening. Essential to this is knowledge about the population's experience while in the preclinical state, such as the sojourn time distribution and the sensitivity of screening tests. As these quantities are not directly observable from practice, estimation of them is the normally the first step of a program evaluation model.

Walter and Day (1983) and Day and Walter (1984) then adopt the stable disease model as proposed by Zelen and Feinleib (1969) and continue estimating the lead time of a screening program. As a generalization, their model considers multiple screenings and the possibility of false-negative screening results. In particular, the sensitivity of the screening test is treated as an unknown constant which is also to be estimated. In Walter and Day (1983), a few statistical distributions for the preclinical sojourn time are considered, and while applied to the HIP data, the exponential model again outperforms all others; thus in Day and Walter (1984), estimates on test sensitivity and the exponential density parameter are applied to derive the mean lead time. In doing this, each clinical incidence is assigned a zero lead time value, and the mean is taken over the entire screened population. The outcome is thus a program-wide mean lead time.

Relaxing the assumption of a stable disease, Lee and Zelen (1998) present the first work that estimates the time-dependent rates for the population's preclinical

incidences. By first recognizing the clinical incidence rates as convolution of the preclinical state's incidence rates and sojourn time density, and by assumed knowledge of clinical incidence rates and preclinical sojourn time distribution, the authors develop a de-convolution approach to infer preclinical incidence rates. As data of incidence rates are normally generated by age groups in practice, the output of the de-convolution procedure has the format of a step function.

With this update on the underlying disease progression model, Shen and Zelen (1999) consider again the estimation of mean screening program lead time yet with multiple screening modalities and possibly dependent test sensitivities among them. The statistical model developed is rather intricate, with many parameters to be estimated on test sensitivities and on the incidence rates and sojourn time distribution for the preclinical state.

In Parmigiani and Skates (2001), a generalized disease progression model is considered that allows for dependencies among the population's sojourn times in the disease-free, preclinical and clinical states, and that models the population's deaths due to other causes, i.e. competing risks, explicitly in each state. This model is first proposed in Parmigiani (1993) in which the cost-effectiveness of various screening strategies are compared. By assumed knowledge of clinical incidence rates, preclinical sojourn time distribution, and overall competing death rates for the population, the authors develop a de-convolution procedure to obtain preclinical incidence rates as well as competing death rates for the population while being preclinical. The latter rates are relevant to evaluating the over-diagnosis effects of a screening program.

Pinsky (2001) estimates preclinical incidence rates and sojourn time distributions, and tests sensitivity all at once while treating sensitivity as a linear function of sojourn time in the preclinical state at the time of screening. The de-convolution procedure consists also of a smoothing method to produce a continuous incidence

rate function.

Indeed, the lead time has been used by many studies as a performance measure to evaluate screening programs. Though it reflects the potential for better disease prognosis, for a total assessment of a screening program, one is more concerned with the program's benefit time. The benefit time is defined to be the additional time of survival a person gets as a result of early disease detection and treatment, and it should be measured relative to the situation in which the case is found and treated as a clinical incidence. To evaluate a screening program by its benefit time, a long follow-up period is required to generate the needed data. Ideally, data collection should last until all samples in the population die out.

Kafadar and Prorok (1994) develop a statistical model to simultaneously estimate the average lead time and benefit time of a screening program. The study adopts the stable disease model of Zelen and Feinleib (1969), and estimation is done by relating the screened and control populations' survival time distributions while having average lead time and benefit time treated as unknown constants. For each population, two distributions are considered that measure the survival times from the start of screening program and start of case treatments respectively. Quite many simplifications are made in Kafadar and Prorok (1994), such as perfect screening sensitivity, no competing death risks, and the independence of the survival distribution with respect to sojourns times in the healthy and preclinical states. Due to the lack of data from existing screening trials, the authors use simulation to evaluate the quality of the estimators.

Based on the same model, Kafadar et al. (1998) examine the variances of the two derived estimators. Later in Kafadar and Prorok (2003), various methods of categorizing the screening trial data are studied with the goal of minimizing biasness in estimation. Then in Kafadar and Prorok (2009), the effects of length-biased

sampling, i.e. the tendency of a screening to pick up samples with longer preclinical sojourn times, towards final estimation is investigated.

In addition to all the statistical models developed that evaluate screening programs, another type of model can be formulated that takes in the various characteristics of the system, such as sojourn time distributions and test sensitivities, as known inputs and generates an optimal schedule of screening. As we discussed, such model inputs are often by themselves the products of those screening program evaluation models, in which case the schedule of screening is treated as fixed and known.

To this end of optimal screening policy models, Zelen (1993) presents a pioneering model to place any given number of screening epochs over a population's lifetime, such that the probability of detecting the disease while preclinical is maximized. He adopts his earlier stable disease model, and solves the optimization problem by considering the first-order conditions. The main result of this work is the proof that the optimal solution has an equally-spaced structure if and only if the test sensitivity is one.

Another significant stream of models are due to Parmigiani and Kamlet (1993) and Parmigiani (1993, 1997). These models are all concerned with the overall cost-effectiveness of a screening program and seek the best screening schedule for it.

In Parmigiani and Kamlet (1993), the general disease progression model with competing death risks is first considered. Screening costs are assumed fixed for each exam and are considered also for the populations that die in any state due to competing risks. Treatment costs are treated alongside the Quality-Adjusted-Life-Years (QALY) as functions depending on sojourn times in the healthy and preclinical states. The model considers a baseline screening schedule and compares the marginal expected cost against the marginal expected QALY for several proposed screening

schedules.

Parmigiani (1993) presents a general optimization model to minimize the overall screening program cost. In the particular case of perfect screening, the author derives conditions on the input cost functions such that the optimal screening policy consists either of zero or of infinite number of screenings. In the latter situation, additional conditions are found to ensure a recursive algorithm to find the optimum. Indeed, this approach is reminiscent to a classical work in system reliability literature due to Barlow et al. (1963). Based on first-order optimality conditions, a set of equations are derived that can generate the screening schedule sequentially once the first epoch is fixed. In implementation, if either this first screening time is fixed before or after the optimum, particular faulty patterns will arise in the downstream schedule calculated. As such, a binary-search algorithm is in place to find the optimal schedule.

Then in Parmigiani (1997), the author takes a detour approach and approximates screening schedules by continuous intensity functions. The objective function remains at minimizing total costs and is also approximated. The optimal solution is searched from the space of intensity functions, and needs ultimately to be converted back into a discrete screening schedule. Optimality conditions are studied for this model.

Later, Lee and Zelen (1998) consider an alternative screening scheme based on their non-stable disease model. Under this scheme, screening times are placed in such a way that the prevalence of preclinical samples in the population is always bounded by a pre-specified upper threshold level. The prevalence function, which is unobservable, is derived based on assumed sojourn time distributions and test sensitivity. Once the schedule is derived by the scheme policy, its performance is measured by the overall probability of preclinical disease detection.

Parmigiani et al. (2002) study the optimal placement of a single screening time. Two objective functions are considered, which include the probability of screening detection and the expected life length for the disease-affected population. Dependent sojourn times are considered for the disease-free and preclinical states, and fallible tests were treated whose specificity depends on the sojourn times. The first-order conditions for optimality is derived. In a case study conducted on colorectal cancer, the authors obtain optimal solution by arbitrarily plotting out the objective function.

Ahern et al. (2011) consider two frameworks for an optimal screening policy. First, the policy is restricted to be equally spaced, and the authors seek the optimal number of planned screenings that minimizes the weighted cost between the number of screenings and the probability of screening detection. A sufficient condition for a unique optimum is derived, and the authors argue that the practical parameters for breast cancer will easily satisfy this condition. Secondly, the authors consider the optimal placement of any given number of screenings and prove the existence of optimal solution for this framework. Throughout the work the authors treat the disease-free duration with piece-wise linear densities, assuming an exponential distribution for the preclinical time, and consider independent and fallible screening tests.

3. OPTIMAL SCREENING POLICY MODEL FORMULATION

We adopt the natural history model of Parmigiani et al. (2002) as the framework for our decision making. As shown in Figure 3.1, we consider five states for the disease under screening. These are the "healthy", "preclinical" and "clinical" states as well as two "dead" states which correspond, respectively, to cases due to the disease and to competing risks. Transitions can occur as an individual progresses from healthy to preclinical, from preclinical to clinical, and from clinical to dead as a result of the disease; meanwhile, it is possible for one to die of other causes while s/he is in any state up to clinical.

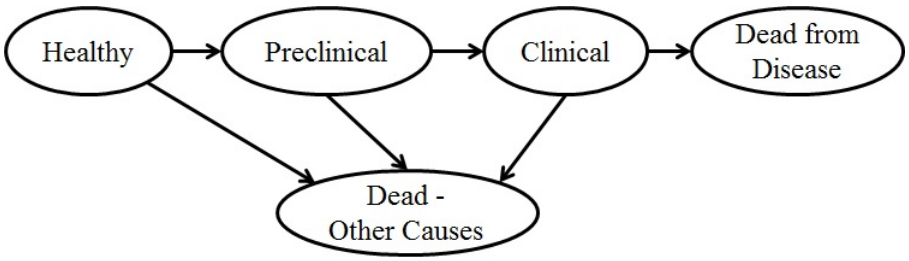

Figure 3.1: Disease Natural History Model

Our optimal screening policy model aims at finding the screening schedule that produces the maximal probability of preclinical disease detection. We model budget in terms of the total number of screenings allowed in a person's lifetime and in practice this can be easily converted to a dollar value. In our model, we confine our attention to samples that will develop clinical symptoms (before dying of a competing risk) if not screened. We acknowledge that in practice screening costs are also incurred for those who die without the disease and are thus actually irrelevant to

the screening program. We will discuss such implications of program-wide screening costs in our numerical analysis chapter.

Therefore, we assume that an individual will, with probability one, transit over three states: healthy, preclinical, and clinical. Let random variables X_0 and X_1 be the sojourn times, respectively, in the healthy and the preclinical states, and let f and g be the p.d.f.'s for X_0 and X_1. Figure 3.2 shows the simple disease progression model that we consider. Note that all individuals will eventually be "diagnosed", either by a screening, or due to clinical symptoms.

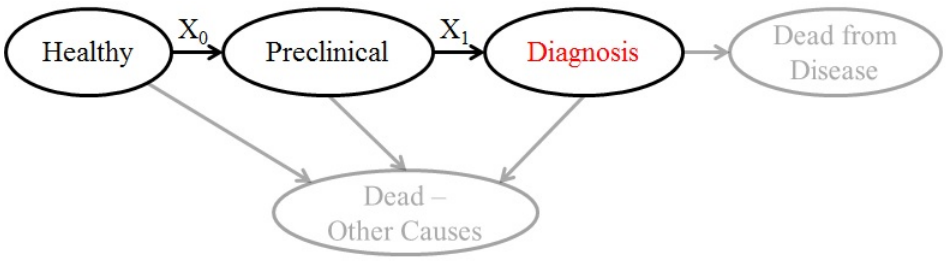

Figure 3.2: Scope of Our Optimal Screening Policy Model

We make the following assumptions:

- f is a log-concave function over (a, b) for $0 \leq a < b \leq \infty$,
- g is independent of f and is supported over $(0, \infty)$,
- all screenings are independent and have sensitivity β.

The class of log-concave density functions include a broad range of models such as all uniform, exponential and normal distributions as well as all Weibull and Gamma densities with shape parameter greater than one. Many interesting properties can be found in this class of functions (see Barlow et al. (1963)). In particular, all log-concave densities have increasing failure rates.

The other two assumptions are mild and can be found in many other studies (e.g. Day and Walter (1984), Zelen (1993), Lee and Zelen (1998), Shen and Zelen (1999), Ahern et al. (2011)).

Now, let $\mathcal{D} \subset \mathbb{R}^n := \{(\tau_1, \tau_2, ..., \tau_n) \mid 0 < \tau_1 < \tau_2 < ... < \tau_n < b\}$ be the set of all possible screening policies. Note that for each fixed policy, an individual will be missed detection either if his/her preclinical period covers no screening epoch on the schedule (see Figure 3.3); or, all the screening(s) performed during the preclinical period fail to report the truth. All other scenarios correspond to the event of successful detection. Figure 3.4 presents a few scenarios of possible successful detections. Note a detection only happens if at least one of the screenings during X_1 was accurate.

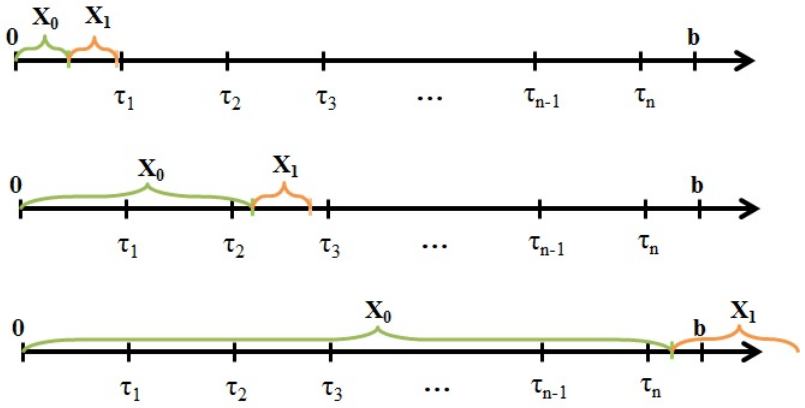

Figure 3.3: Examples of Missed Detections

We, therefore, derive our objective function as follows:

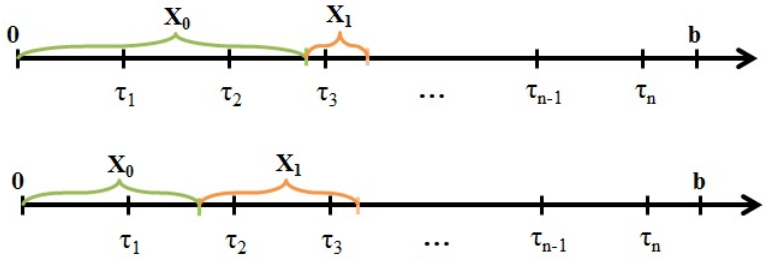

Figure 3.4: Examples of Potential Successful Detections

Proposition 1 *The objective function*

$$\begin{aligned} P(\boldsymbol{\tau}) &= Pr(\{\text{Preclinical Detection}\}) \\ &= 1 - \sum_{i=0}^{n}\sum_{j=i}^{n} \beta^{j-i} \int_{\tau_i}^{\tau_{i+1}} [G(\tau_{j+1}-u) - G(\tau_j - u)] f(u) du, \end{aligned}$$

where $\tau_0 = 0$ *and* $\tau_{n+1} = \infty$, β *is the false-negative rate for screenings, and where* F/f *and* G/g*are the distribution and density functions for the random sojourn times* X_0 *and* X_1*, respectively, of an individual in the healthy and preclinical states.*

Proof:

We have

$$\begin{aligned} P(\boldsymbol{\tau}) &:= Pr(\{\text{Preclinical Detection}\}) = 1 - Pr(\{\text{Missed Detection}\}) \\ &= 1 - \sum_{i=0}^{n}\sum_{j=i}^{n} Pr\Big(X_0 \in (\tau_i, \tau_{i+1}), X_0 + X_1 \in (\tau_j, \tau_{j+1}), \text{Missed Detection}\Big) \\ &= 1 - \sum_{i=0}^{n}\sum_{j=i}^{n} \int_{\tau_i}^{\tau_{i+1}} Pr\Big(X_0 \in (\tau_i, \tau_{i+1}), X_0 + X_1 \in (\tau_j, \tau_{j+1}), \\ &\qquad\qquad \text{Missed Detection} \mid X_0 = u\Big) f(u) du \\ &= 1 - \sum_{i=0}^{n}\sum_{j=i}^{n} \int_{\tau_i}^{\tau_{i+1}} Pr\Big(X_1 \in (\tau_j - u \vee 0, \tau_{j+1} - u), \\ &\qquad\qquad \text{Missed Detection} \mid X_0 = u\Big) f(u) du \\ &= 1 - \sum_{i=0}^{n}\sum_{j=i}^{n} \int_{\tau_i}^{\tau_{i+1}} \beta^{j-i} \cdot [G(\tau_{j+1}-u) - G(\tau_j - u)] f(u) du \end{aligned}$$

[by independence and the number of screenings while in preclinical state]

Note the above expression is regardless of b.

■

4. MODEL SOLUTION FOR PERFECT SCREENING CASE

We first consider the partial derivatives of the objective function.

Proposition 2 *For each* $k \in \{1, 2, \cdots, n\}$,

$$\begin{aligned}\frac{\partial}{\partial \tau_k} P(\boldsymbol{\tau}) =& (1-\beta)\Bigg\{ f(\tau_k)\Big[(1-\beta)\sum_{j=1}^{n-k}\beta^{j-1}G(\tau_{k+j}-\tau_k)+\beta^{n-k}\Big] \\ &- \sum_{i=0}^{k-2}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}} g(\tau_k-u)f(u)du - \int_{\tau_{k-1}}^{\tau_k} g(\tau_k-u)f(u)du\Bigg\}.\end{aligned}$$

Proof:

For each $k \in \{1, 2, \cdots, n\}$ and $\boldsymbol{\tau} \in \mathcal{D}$, it holds that:

$$\begin{aligned}\frac{\partial}{\partial \tau_k} P(\boldsymbol{\tau}) =& -\frac{\partial}{\partial \tau_k}\Bigg\{ \sum_{i=0}^{k-2}\int_{\tau_i}^{\tau_{i+1}} \beta^{k-i-1}[G(\tau_k-u)-G(\tau_{k-1}-u)]f(u)du \\ &+ \sum_{i=0}^{k-2}\int_{\tau_i}^{\tau_{i+1}} \beta^{k-i}[G(\tau_{k+1}-u)-G(\tau_k-u)]f(u)du \\ &+ \sum_{j=k-1}^{n}\int_{\tau_{k-1}}^{\tau_k} \beta^{j-k+1}[G(\tau_{j+1}-u)-G(\tau_j-u)]f(u)du \\ &+ \sum_{j=k}^{n}\int_{\tau_k}^{\tau_{k+1}} \beta^{j-k}[G(\tau_{j+1}-u)-G(\tau_j-u)]f(u)du\Bigg\} \\ =& -\frac{\partial}{\partial \tau_k}\Bigg\{ (1-\beta)\sum_{i=0}^{k-2}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}} G(\tau_k-u)f(u)du \\ &+ (1-\beta)\sum_{j=0}^{n-k}\beta^{j}\int_{\tau_{k-1}}^{\tau_k} G(\tau_{k+j}-u)f(u)du \\ &+ (1-\beta)\sum_{j=1}^{n-k}\beta^{j-1}\int_{\tau_k}^{\tau_{k+1}} G(\tau_{k+j}-u)f(u)du \\ &- (1-\beta)\beta^{n-k}F(\tau_k)\Bigg\}\end{aligned}$$

$$
\begin{aligned}
=& -(1-\beta)\Bigg\{\sum_{i=0}^{k-2}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}} g(\tau_k-u)f(u)du \\
&-(1-\beta)\sum_{j=1}^{n-k}\beta^{j-1}G(\tau_{k+j}-\tau_k)f(\tau_k) \\
&+\int_{\tau_{k-1}}^{\tau_k} g(\tau_k-u)f(u)du-\beta^{n-k}f(\tau_k)\Bigg\} \\
=&(1-\beta)\Bigg\{f(\tau_k)\Big[(1-\beta)\sum_{j=1}^{n-k}\beta^{j-1}G(\tau_{k+j}-\tau_k)+\beta^{n-k}\Big] \\
&-\sum_{i=0}^{k-2}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}} g(\tau_k-u)f(u)du-\int_{\tau_{k-1}}^{\tau_k} g(\tau_k-u)f(u)du\Bigg\}
\end{aligned}
$$

■

Our main result is the following.

Theorem 1 *If screenings are perfectly sensitive (i.e. $\beta = 0$), then there exists a unique optimal policy $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \cdots, \sigma_n\}$, which is characterized by the following criterion:*

$$
\begin{cases}
(1)\ \int_0^{\sigma_1-\sigma_0}\frac{f(\sigma_1-s)}{f(\sigma_1)}g(s)ds & = & G(\sigma_2-\sigma_1) \\
\vdots & & \vdots \\
(i)\ \int_0^{\sigma_i-\sigma_{i-1}}\frac{f(\sigma_i-s)}{f(\sigma_i)}g(s)ds & = & G(\sigma_{i+1}-\sigma_i) \\
\vdots & & \vdots \\
(n)\int_0^{\sigma_n-\sigma_{n-1}}\frac{f(\sigma_n-s)}{f(\sigma_n)}g(s)ds \begin{cases} = & 1 \quad (\text{if } \sigma_n < b) \\ \leq & 1 \quad (\text{if } \sigma_n = b). \end{cases}
\end{cases}
$$

Proof:

If $\beta = 0$, the partial derivative for each τ_k is:

$$\begin{aligned}\frac{\partial}{\partial \tau_k} P(\boldsymbol{\tau}) &= f(\tau_k)G(\tau_{k+1} - \tau_k) - \int_{\tau_{k-1}}^{\tau_k} g(\tau_k - u)f(u)du \\ &= f(\tau_k)\Big[G(\tau_{k+1} - \tau_k) - \int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k - s)}{f(\tau_k)} g(s)ds\Big].\end{aligned}$$

Now notice that for τ_k over interval (τ_{k-1}, τ_{k+1}), $G(\tau_{k+1} - \tau_k)$ strictly decreases from $G(\tau_{k+1} - \tau_{k-1})$ to 0, and the term $\int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k - s)}{f(\tau_k)} g(s)ds$ strictly increases from 0 to $\int_0^{\tau_{k+1} - \tau_{k-1}} \frac{f(\tau_{k+1} - s)}{f(\tau_{k+1})} g(s)ds$. The latter monotonicity holds because for any $s \in (0, b)$, $\frac{f(\tau - s)}{f(\tau)}$ is non-decreasing in τ if f is logconcave (see Barlow et al. (1963)). As a result, for $\tau_{k-1} < \tau_k < \tau_k' < \tau_{k+1}$, we have:

$$\begin{aligned}\int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k - s)}{f(\tau_k)} g(s)ds &\leq \int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k' - s)}{f(\tau_k')} g(s)ds \\ &< \int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k' - s)}{f(\tau_k')} g(s)ds + \int_{\tau_k - \tau_{k-1}}^{\tau_k' - \tau_{k-1}} \frac{f(\tau_k' - s)}{f(\tau_k')} g(s)ds \\ &= \int_0^{\tau_k' - \tau_{k-1}} \frac{f(\tau_k' - s)}{f(\tau_k')} g(s)ds.\end{aligned}$$

Meanwhile, notice $f(\tau_k)$ is positive and continuous over (τ_{k-1}, τ_{k+1}).

Therefore, we know from the partials that for any chosen policy $\boldsymbol{\tau} \in \mathcal{D}$, as we vary each τ_k, the objective function will always turn from increasing to decreasing over (τ_{k-1}, τ_{k+1}), with the only exception that it could possibly never decrease in τ_n. As such, the optimal solution must have at least $n - 1$ zero partials, hence our criterion.

To show uniqueness, suppose two different solutions $\boldsymbol{\tau}$ and $\boldsymbol{\psi}$ both satisfy the optimal criterion. Let $i = \min\{k : \tau_k \neq \psi_k\}$ be the first element where the two

solutions differ, and without loss of generality, assume $\tau_i < \psi_i$. We have:

$$\begin{cases} (i) & \int_0^{\tau_i-\tau_{i-1}} \frac{f(\tau_i-s)}{f(\tau_i)} g(s)ds & = & G(\tau_{i+1}-\tau_i) \\ \vdots & & \vdots & \\ (n-1) & \int_0^{\tau_{n-1}-\tau_{n-2}} \frac{f(\tau_{n-1}-s)}{f(\tau_{n-1})} g(s)ds & = & G(\tau_n-\tau_{n-1}) \qquad \text{and} \\ (n) & \int_0^{\tau_n-\tau_{n-1}} \frac{f(\tau_n-s)}{f(\tau_n)} g(s)ds & \begin{cases} = & 1 \quad (\text{if } \tau_n < b) \\ \leq & 1 \quad (\text{if } \tau_n = b), \end{cases} \end{cases}$$

$$\begin{cases} (i)' & \int_0^{\psi_i-\psi_{i-1}} \frac{f(\psi_i-s)}{f(\psi_i)} g(s)ds & = & G(\psi_{i+1}-\psi_i) \\ \vdots & & \vdots & \\ (n-1)' & \int_0^{\psi_{n-1}-\psi_{n-2}} \frac{f(\psi_{n-1}-s)}{f(\psi_{n-1})} g(s)ds & = & G(\psi_n-\psi_{n-1}) \\ (n)' & \int_0^{\psi_n-\psi_{n-1}} \frac{f(\psi_n-s)}{f(\psi_n)} g(s)ds & \begin{cases} = & 1 \quad (\text{if } \psi_n < b) \\ \leq & 1 \quad (\text{if } \psi_n = b). \end{cases} \end{cases}$$

Now, sequentially for each $k \in \{i, \cdots, n-1\}$, it follows that:

$$\begin{aligned} (k) \Leftrightarrow \tau_{k+1}-\tau_k &= G^{-1}\Big(\int_0^{\tau_k-\tau_{k-1}} \frac{f(\tau_k-s)}{f(\tau_k)} g(s)ds \Big) \\ &< G^{-1}\Big(\int_0^{\psi_k-\psi_{k-1}} \frac{f(\psi_k-s)}{f(\psi_k)} g(s)ds \Big) = \psi_{k+1}-\psi_i \Leftrightarrow (k)', \end{aligned}$$

which gives $\tau_{k+1}-\tau_k < \psi_{k+1}-\psi_k$ and $\tau_{k+1} < \psi_{k+1}$.

Therefore, we have $\tau_n - \tau_{n-1} < \psi_n - \psi_{n-1}$ and $\tau_n < \psi_n$. But this is a contradiction, as:

$$(n) \Leftrightarrow 1 = \int_0^{\tau_n-\tau_{n-1}} \frac{f(\tau_n-s)}{f(\tau_n)} g(s)ds < \int_0^{\psi_n-\psi_{n-1}} \frac{f(\psi_n-s)}{f(\psi_n)} g(s)ds \leq 1 \Leftrightarrow (n)'.$$

As a result, a unique solution satisfies the optimality criterion.

■

In fact, for the general case where $\beta > 0$, note that:

$$\begin{aligned}\frac{\partial}{\partial \tau_k} P(\boldsymbol{\tau}) =&(1-\beta)\Bigg\{ f(\tau_k)\Big[(1-\beta)\sum_{j=1}^{n-k}\beta^{j-1}G(\tau_{k+j}-\tau_k)+\beta^{n-k}\Big] \\ &-\sum_{i=0}^{k-2}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}} g(\tau_k-u)f(u)du - \int_{\tau_{k-1}}^{\tau_k} g(\tau_k-u)f(u)du\Bigg\} \\ =&(1-\beta)f(\tau_k)\Bigg\{\Big[(1-\beta)\sum_{j=1}^{n-k}\beta^{j-1}G(\tau_{k+j}-\tau_k)+\beta^{n-k}\Big] \\ &-\int_0^{\tau_k-\tau_{k-1}}\frac{f(\tau_k-s)}{f(\tau_k)}g(s)ds\Bigg\} - (1-\beta)\sum_{i=0}^{k-2}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}} g(\tau_k-u)f(u)du,\end{aligned}$$

where the expression in $\Big\{\ \Big\}$ is strictly decreasing in τ_k from a positive quantity near τ_{k-1}. However, to guarantee uniqueness of optimal solution, more assumptions are required about densities f and g in order to leverage the last term.

In the following contents in this chapter, unless otherwise specified we assume that $\beta = 0$.

Now, notice that for any arbitrary τ_1 we may specify, the optimality criterion nicely allows us to calculate $\{\tau_2, \tau_3, \cdots, \tau_n\}$ sequentially through its first $n-1$ equations. And finally, the last condition is used to check for optimality of the solution generated. Consider the following algorithm.

Binary First Epoch Search Algorithm (BFESA)

Step 0: Let $L = 0$. If $b < \infty$, let $U = b$; otherwise, let U be a number big enough to contain σ_1, e.g. $U = 2F^{-1}(\frac{1}{n})$. Let ϵ be an arbitrary small number.

Step 1: Set $\tau_1 = \frac{L+U}{2}$.

Step 2: For each $k \in \{2, \cdots, n\}$, calculate $\tau_k = \tau_{k-1} + G^{-1}\Big(\int_0^{\tau_{k-1}-\tau_{k-2}} \frac{f(\tau_{k-1}-s)}{f(\tau_{k-1})} g(s)ds \Big)$.

Step 3: If $\int_0^{\tau_{k-1}-\tau_{k-2}} \frac{f(\tau_{k-1}-s)}{f(\tau_{k-1})} g(s)ds > 1$ for any $k \in \{2, \cdots, n\}$, or if $\tau_k \geq b$ for any $k \in \{2, \cdots, n-1\}$ or $\tau_n > b$, or if $\int_0^{\tau_n-\tau_{n-1}} \frac{f(\tau_n-s)}{f(\tau_n)} g(s)ds > 1+\epsilon$, then set $U = \tau_1$ and return to Step 1. Otherwise, if $\int_0^{\tau_n-\tau_{n-1}} \frac{f(\tau_n-s)}{f(\tau_n)} g(s)ds < 1-\epsilon$, then set $L = \tau_1$ and return to Step 1.

Step 4: The algorithm stops and the screening policy $\{\tau_1, \tau_2, \cdots, \tau_n\}$ is reported.

We prove that the BFESA finds the optimal solution.

Theorem 2 *The BFESA converges to the optimal solution.*

Proof:

We first prove that either τ_1 is chosen to be too large or too small, there will be one distinctive signal throughout calculating τ_2 through τ_n to report this.

Specifically, if $\tau_1 > \sigma_1$, we have:

$$\tau_2 - \tau_1 = G^{-1}\Big(\int_0^{\tau_1} \frac{f(\tau_1 - s)}{f(\tau_1)} g(s)ds \Big) > G^{-1}\Big(\int_0^{\sigma_1} \frac{f(\sigma_1 - s)}{f(\sigma_1)} g(s)ds \Big) = \sigma_2 - \sigma_1.$$

Note it is possible to have $\int_0^{\tau_1} \frac{f(\tau_1-s)}{f(\tau_1)} g(s)ds > 1$, or that $\tau_2 \geq b$. In either case, we obtain a signal that our chosen τ_1 is larger than σ_1.

If $\tau_2 < b$, we get $\tau_2 - \tau_1 > \sigma_2 - \sigma_1$ and $\tau_2 > \sigma_2$, and we proceed to equation (2). Now as each equation (k) is applied where $k \in \{2, \cdots, n-1\}$, we have:

$$\begin{aligned} \tau_{k+1} - \tau_k &= G^{-1}\Big(\int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k - s)}{f(\tau_k)} g(s)ds \Big) \\ &> G^{-1}\Big(\int_0^{\sigma_k - \sigma_{k-1}} \frac{f(\sigma_k - s)}{f(\sigma_k)} g(s)ds \Big) = \sigma_{k+1} - \sigma_k, \end{aligned}$$

under which it is possible that $\int_0^{\tau_k-\tau_{k-1}} \frac{f(\tau_k-s)}{f(\tau_k)} g(s)ds > 1$ or $\tau_{k+1} \geq b$, i.e. signals for

$\tau_1 > \sigma_1$. If not, we get $\tau_{k+1} - \tau_k > \sigma_{k+1} - \sigma_k$ and $\tau_{k+1} > \sigma_k$.

Now assume we have sequentially applied equations (1) through $(n-1)$ to find such $\{\tau_2, \cdots, \tau_n\}$ that$\tau_k - \tau_{k-1} > \sigma_k - \sigma_{k-1}$ and $\tau_k > \sigma_k$ for all $k \in \{2, \cdots, n\}$. Assume also that $\tau_n \leq b$. But from equation (n), we have:

$$\int_0^{\tau_n - \tau_{n-1}} \frac{f(\tau_n - s)}{f(\tau_n)} g(s) ds > \int_0^{\sigma_n - \sigma_{n-1}} \frac{f(\sigma_n - s)}{f(\sigma_n)} g(s) ds = 1,$$

which again indicates τ_1 to be too large.

On the other hand, if $\tau_1 < \sigma_1$, then by applying equations (1) through $(n-1)$ we will obtain such $\{\tau_2, \cdots, \tau_n\}$ that $\tau_k - \tau_{k-1} < \sigma_k - \sigma_{k-1}$ and $\tau_k < \sigma_k$ for all $k \in \{2, \cdots, n\}$. But in this case, equation (n) will give:

$$\int_0^{\tau_n - \tau_{n-1}} \frac{f(\tau_n - s)}{f(\tau_n)} g(s) ds < \int_0^{\sigma_n - \sigma_{n-1}} \frac{f(\sigma_n - s)}{f(\sigma_n)} g(s) ds = 1,$$

a signal that $\tau_1 < \sigma_1$.

In each iteration, notice the BFESA collects a signal and responds accordingly to cut off half of the search region for σ_1. It therefore converges to the optimal solution.

■

We next devote some effort to study the structure of the optimal screening policy. We have the following important result.

Theorem 3 *For each $\tau_1 \in (0, b)$, there exists a unique $\boldsymbol{\tau}^*(\tau_1) = \{\tau_2^*(\tau_1), \tau_3^*(\tau_1), \cdots, \tau_n^*(\tau_1)\}$ that maximizes $P(\tau_1, \cdot)$. Further, $\boldsymbol{\tau}^*$ behaves in such a way that each of its elements τ_k^* is strictly increasing and concave in τ_1 (with the only exception that τ_n^* could remain constant once it reaches b).*

Proof:

For fixed $\tau_1 \in (0, b)$ and each $k \in \{2, \cdots, n\}$, note the partial derivative of $P(\boldsymbol{\tau})$ with respect to τ_k can be treated as follows:

$$\begin{aligned}\frac{\partial}{\partial \tau_k} P(\boldsymbol{\tau}) =& f(\tau_k)\Big[G(\tau_{k+1} - \tau_k) - \int_0^{\tau_k - \tau_{k-1}} \frac{f(\tau_k - s)}{f(\tau_k)} g(s) ds\Big] \\ =& [1 - F(\tau_1)] \frac{f(\tau_k)}{1 - F(\tau_1)} \Big[G(\tau_{k+1} - \tau_k) - \int_0^{\tau_k - \tau_{k-1}} \frac{\frac{f(\tau_k - s)}{1 - F(\tau_1)}}{\frac{f(\tau_k)}{1 - F(\tau_1)}} g(s) ds\Big] \\ =& [1 - F(\tau_1)] f^{\tau_1}(\tau_k) \Big[G(\tau_{k+1} - \tau_k) - \int_0^{\tau_k - \tau_{k-1}} \frac{f^{\tau_1}(\tau_k - s)}{f^{\tau_1}(\tau_k)} g(s) ds\Big],\end{aligned}$$

where we define

$$f^{\tau_1}(s) := \frac{f(s)}{1 - F(\tau_1)} \quad \text{for } s \in (\tau_1, b)$$

to be the conditional p.d.f. for random variable X_0 given that it is greater than τ_1.

As the logarithm of f is concave, it is necessary that f^{τ_1}, which is f by a scalar, is log-concave also. Therefore, we may view the $n - 1$ partials as a full set of derivatives for the problem of $n - 1$ screenings, which has f^{τ_1} as the p.d.f. for X_0 and is scaled by a positive constant. Thus, by Theorem 1 there is a unique policy $\{\tau_2^*(\tau_1), \tau_3^*(\tau_1), \cdots, \tau_n^*(\tau_1)\}$ to maximize $P(\boldsymbol{\tau})$.

To show monotonicity and concavity of each τ_k^* with respect to τ_1, we pick $0 < \tau_1 < \tau_1' < b$, and let $\boldsymbol{\tau} = \{\tau_1, \tau_2, \cdots, \tau_n\}$ and $\boldsymbol{\tau}' = \{\tau_1', \tau_2', \cdots, \tau_n'\}$ be the policies to maximize $P(\tau_1, \cdot)$ and $P(\tau_1', \cdot)$ respectively. Also, let $\Delta_k = \tau_{k+1} - \tau_k$ and $\Delta_k' = \tau_{k+1}' - \tau_k'$ for each $k \in \{1, \cdots, n-1\}$.

We want to show that: (a) $\Delta_k' < \Delta_k$ for each $k \in \{1, \cdots, n-1\}$, and (b) $\tau_k' > \tau_k$ for each $k \in \{2, \cdots, n\}$ (except for possibly $\tau_n = \tau_n' = b$). Notice if (a) holds, then we will have $\tau_k' - \tau_k = (\tau_1' + \sum_{j=1}^{k-1} \Delta_j') - (\tau_1 + \sum_{j=1}^{k-1} \Delta_j) = (\tau_1' - \tau_1) + \sum_{j=1}^{k-1} (\Delta_j' -$

$\Delta_j) < \tau_1' - \tau_1$ for each $k \in \{2, \cdots, n\}$. As a result, each τ_k^* is increasing and concave in τ_1, and the theorem is thus proven.

By the optimality criterion from Theorem 1, we have the following:

$$\begin{cases} (2) \ \int_0^{\tau_2-\tau_1} \frac{f(\tau_2-s)}{f(\tau_2)} g(s)ds &= G(\tau_3-\tau_2) \\ \vdots & \vdots \\ (i) \ \int_0^{\tau_i-\tau_{i-1}} \frac{f(\tau_i-s)}{f(\tau_i)} g(s)ds &= G(\tau_{i+1}-\tau_i) \\ \vdots & \vdots \\ (n) \int_0^{\tau_n-\tau_{n-1}} \frac{f(\tau_n-s)}{f(\tau_n)} g(s)ds \begin{cases} = & 1 \quad (\text{if } \tau_n < b) \\ \leq & 1 \quad (\text{if } \tau_n = b) \end{cases} \end{cases}$$

$$\begin{cases} (2)' \ \int_0^{\tau_2'-\tau_1'} \frac{f(\tau_2'-s)}{f(\tau_2')} g(s)ds &= G(\tau_3'-\tau_2') \\ \vdots & \vdots \\ (i)' \ \int_0^{\tau_i'-\tau_{i-1}'} \frac{f(\tau_i'-s)}{f(\tau_i')} g(s)ds &= G(\tau_{i+1}'-\tau_i') \\ \vdots & \vdots \\ (n)' \int_0^{\tau_n'-\tau_{n-1}'} \frac{f(\tau_n'-s)}{f(\tau_n')} g(s)ds \begin{cases} = & 1 \quad (\text{if } \tau_n' < b) \\ \leq & 1 \quad (\text{if } \tau_n' = b) \end{cases} \end{cases}$$

We shall prove our claims in the order of $\Delta_1' < \Delta_1$, $\tau_2' > \tau_2$, $\Delta_2' < \Delta_2$, $\cdots$, $\Delta_{n-1}' < \Delta_{n-1}$, and $b \geq \tau_n' \geq \tau_n$. Consider the following *algorithmic* arguments:

For claim (a): suppose we have proven up to some $k \in \{1, \cdots, n-1\}$ that $\tau_j' > \tau_j \ \forall j \in \{1, \cdots, k\}$ and $\Delta_j' < \Delta_j \ \forall j \in \{1, \cdots, k-1\}$. We want to show $\Delta_k' < \Delta_k$.

Assume the claim is NOT true, i.e. $\Delta_k' \geq \Delta_k$. Let $i = k$, then:

(*) By assumption, it holds that $\Delta_i' \geq \Delta_i$ and $\tau_{i+1}' > \tau_{i+1}$.

If now $i = n-1$, we will have:

$$(n) \Leftrightarrow 1 = \int_0^{\Delta_{n-1}} \frac{f(\tau_n - s)}{f(\tau_n)} g(s)ds < \int_0^{\Delta'_{n-1}} \frac{f(\tau'_n - s)}{f(\tau'_n)} g(s)ds \leq 1 \Leftrightarrow (n)',$$

which is a contradiction.

Otherwise, we have:

$$\begin{aligned}(i+1) \Leftrightarrow \tau_{i+2} - \tau_{i+1} &= G^{-1}\Big(\int_0^{\Delta_i} \frac{f(\tau_{i+1} - s)}{f(\tau_{i+1})} g(s)ds\Big) \\ &< G^{-1}\Big(\int_0^{\Delta'_i} \frac{f(\tau'_{i+1} - s)}{f(\tau'_{i+1})} g(s)ds\Big) = \tau'_{i+2} - \tau'_{i+1} \Leftrightarrow (i+1)',\end{aligned}$$

which is $\Delta'_{i+1} > \Delta_{i+1}$.

Now let $i := i+1$, and go back to step (*). The same arguments will then go through iteratively until i reaches $n-1$, at which point we get a contradiction and conclude that $\Delta'_k < \Delta_k$.

For claim (b): suppose we have proven up to some $k \in \{1, \cdots, n-1\}$ that $\tau'_j > \tau_j$ and $\Delta'_j < \Delta_j$ $\forall j \in \{1, \cdots, k\}$. We want to show $\tau'_{k+1} > \tau_{k+1}$ if $k < n-1$, or that $b \geq \tau'_n \geq \tau_n$ if $k = n-1$.

Again assume the claim is NOT true, i.e. $\tau'_{k+1} \leq \tau_{k+1}$ if $k < n-1$ or $\tau'_n < \tau_n$ if $k = n-1$. Let $i = k$, then:

(*) By assumption, it holds that $\Delta'_i < \Delta_i$ and $\tau'_{i+1} \leq \tau_{i+1}$.

If $i = n-1$, then we have:

$$(n) \Leftrightarrow 1 \geq \int_0^{\Delta_{n-1}} \frac{f(\tau_n - s)}{f(\tau_n)} g(s)ds > \int_0^{\Delta'_{n-1}} \frac{f(\tau'_n - s)}{f(\tau'_n)} g(s)ds = 1 \Leftrightarrow (n)',$$

which is a contradiction.

Otherwise, we have:

$$(i+1) \Leftrightarrow \tau_{i+2} - \tau_{i+1} = G^{-1}\Big(\int_0^{\Delta_i} \frac{f(\tau_{i+1} - s)}{f(\tau_{i+1})} g(s) ds \Big)$$
$$> G^{-1}\Big(\int_0^{\Delta'_i} \frac{f(\tau'_{i+1} - s)}{f(\tau'_{i+1})} g(s) ds \Big) = \tau'_{i+2} - \tau'_{i+1} \Leftrightarrow (i+1)',$$

which is $\Delta'_{i+1} < \Delta_{i+1}$.

Now let $i := i + 1$, and go back to step (*). The same arguments will then go through iteratively until i reaches $n - 1$, at which point we get a contradiction and conclude that $\tau'_{k+1} > \tau_{k+1}$.

■

We have the following corollary.

Corollary 1 *Given event* $\{X_0 > \sigma_1\}$*, the policy* $\{\sigma_2, \sigma_3, \cdots, \sigma_n\}$ *is the optimal solution to the* $(n-1)$*-screening problem.*

Proof: Clear from proof to Theorem 3.

■

Practically, Theorem 3 is helpful for people who enter the screening program late. Indeed, provided a delayed first screening time, we now know that all the subsequent screenings shall be postponed for better probability of detection. On the other hand, Corollary 1 verifies that the optimal screening policy is self-consistent.

We have an additional result on the locations of the optimal screening epochs with respect to the optimal policy from the previous budget level.

Theorem 4 *Let $\boldsymbol{\phi} = \{\phi_1, \phi_2, \cdots, \phi_{n-1}\}$ denote the optimal policy for the $(n-1)$-screening problem. Define also that $\phi_0 = 0$ and $\phi_n = b$. Then the optimal n-screening policy $\boldsymbol{\sigma}$ is such that all its n screening epochs lie in the intervals created by $\boldsymbol{\phi}$, i.e. $\sigma_k \in (\phi_{k-1}, \phi_k)$ for $k \in \{1, \cdots, n-1\}$, and $\phi_{n-1} \leq \sigma_n \leq \phi_n$.*

Proof:

To show $\sigma_k > \phi_{k-1}$ for each $k \in \{1, \cdots, n-1\}$ and $\sigma_n \geq \phi_{n-1}$, we may treat $\boldsymbol{\phi}$ as the policy to optimize an n-screening problem, given that the first screening epoch is at time 0. Since $\sigma_1 > \phi_0$, the inequalities follow by Theorem 3.

And to show $\sigma_k < \phi_k$ for each $k \in \{1, \cdots, n-1\}$, notice $\boldsymbol{\sigma}$ is such that:

$$\begin{cases} (1) \ \int_0^{\sigma_1-\sigma_0} \frac{f(\sigma_1-s)}{f(\sigma_1)} g(s)ds & = & G(\sigma_2-\sigma_1) \\ \vdots & & \vdots \\ (i) \ \int_0^{\sigma_i-\sigma_{i-1}} \frac{f(\sigma_i-s)}{f(\sigma_i)} g(s)ds & = & G(\sigma_{i+1}-\sigma_i) \\ \vdots & & \vdots \\ (n) \int_0^{\sigma_n-\sigma_{n-1}} \frac{f(\sigma_n-s)}{f(\sigma_n)} g(s)ds \begin{cases} = & 1 \quad (\text{if } \sigma_n < b) \\ \leq & 1 \quad (\text{if } \sigma_n = b) \end{cases} \end{cases}$$

And $\boldsymbol{\phi}$ is such that:

$$\begin{cases} (1)' \quad \int_0^{\phi_1-\phi_0} \frac{f(\phi_1-s)}{f(\phi_1)} g(s)ds & = & G(\phi_2-\phi_1) \\ \vdots & & \vdots \\ (i)' \quad \int_0^{\phi_i-\phi_{i-1}} \frac{f(\phi_i-s)}{f(\phi_i)} g(s)ds & = & G(\phi_{i+1}-\phi_i) \\ \vdots & & \vdots \\ (n-1)' \int_0^{\phi_{n-1}-\phi_{n-2}} \frac{f(\phi_{n-1}-s)}{f(\phi_{n-1})} g(s)ds \begin{cases} = & 1 \quad (\text{if } \phi_{n-1} < b) \\ \leq & 1 \quad (\text{if } \phi_{n-1} = b) \end{cases} \end{cases}$$

Recall our proof to Theorem 1. If $\sigma_1 \geq \phi_1$, then for each $i \in \{1, \cdots, n-2\}$,

we have $\sigma_{i+1} - \sigma_i \geq \phi_{i+1} - \phi_i$ and $\sigma_{i+1} \geq \phi_{i+1}$. Further, it holds that:

$$G(\sigma_n - \sigma_{n-1}) = \int_0^{\sigma_{n-1}-\sigma_{n-2}} \frac{f(\sigma_{n-1} - s)}{f(\sigma_{n-1})} g(s)ds \geq \int_0^{\phi_{n-1}-\phi_{n-2}} \frac{f(\phi_{n-1} - s)}{f(\phi_{n-1})} g(s)ds = 1,$$

therefore, necessarily $\sigma_n = \infty$. But now, check that:

$$\lim_{\sigma_n \to \infty} \int_0^{\sigma_n - \sigma_{n-1}} \frac{f(\sigma_n - s)}{f(\sigma_n)} g(s)ds > \int_0^{\sigma_{n-1}-\sigma_{n-2}} \frac{f(\sigma_{n-1} - s)}{f(\sigma_{n-1})} g(s)ds = 1,$$

which is a contradiction by (n) in the optimality criterion. As a result, we have $\sigma_1 < \phi_1$.

Consequently, again by applying the first $n-1$ equations in the optimality criterion, we obtain $\sigma_i < \phi_i$ for all $i \in \{1, \cdots, n-1\}$.

■

Next, we investigate some models with more specific assumptions. In the class of logconcave densities, note that the uniform and the exponential models respectively have a constant and a linear logarithm which are special cases of concave functions. We have the following result for the case of uniform disease-free duration.

Proposition 3 *If $X_0 \sim$ Unif$(0, b)$ for some $b > 0$, then the optimal screening policy is equally spaced, i.e., $\boldsymbol{\sigma} = \{\frac{b}{n}, \frac{2b}{n}, \cdots, \frac{(n-1)b}{n}, b\}$, if and only if $\beta = 0$.*

Proof:

When $X_0 \sim \text{Unif}(0, b)$ and $\beta = 0$, for each $k \in \{1, 2 \cdots, n\}$, we have:

$$\begin{aligned} G(\sigma_{k+1} - \sigma_k) &= \int_0^{\sigma_k - \sigma_{k-1}} \frac{f(\sigma_k - s)}{f(\sigma_k)} g(s)ds \\ &= \int_0^{\sigma_k - \sigma_{k-1}} g(s)ds \quad [f = \frac{1}{b}] \\ &= G(\sigma_k - \sigma_{k-1}). \end{aligned}$$

Therefore $\sigma_k - \sigma_{k-1} = \frac{b}{n}$ for all $k \in \{1, 2, \cdots, n\}$.

Conversely, if $\beta > 0$, assume that $\boldsymbol{\tau} = \{\frac{b}{n}, \frac{2b}{n}, \cdots, \frac{(n-1)b}{n}, b\}$ is the optimal policy. We have:

$$\begin{aligned}
\frac{\partial}{\partial \tau_1} P(\boldsymbol{\tau}) =& (1-\beta)\left\{ f(\tau_1)\Big[(1-\beta)\sum_{j=1}^{n-1}\beta^{j-1}G(\tau_{1+j}-\tau_1) + \beta^{n-1}\Big] - \int_0^{\tau_1} g(\tau_1 - u) f(u) du \right\} \\
=& \frac{1-\beta}{b}\Big[(1-\beta)\sum_{j=1}^{n-1}\beta^{j-1}G(\tau_{1+j}-\tau_1) + \beta^{n-1} - G(\tau_1)\Big] \\
=& \frac{1-\beta}{b}\Big[(1-\beta)\sum_{j=1}^{n-1}\beta^{j-1}G(\frac{jb}{n}) + \beta^{n-1} - G(\frac{b}{n})\Big] \\
=& \frac{1-\beta}{b}\left\{ \sum_{j=1}^{n-2}\beta^{j}\Big[G\Big(\frac{(j+1)b}{n}\Big) - G(\frac{jb}{n})\Big] + \beta^{n-1}\Big[1 - G\Big(\frac{(n-1)b}{n}\Big)\Big] \right\} > 0,
\end{aligned}$$

which is a contradiction to the assumption that $\boldsymbol{\tau}$ is optimal.

■

Notice, however, that in the single-screening case where:

$$\begin{aligned}
\frac{d}{d\tau} P(\tau) =& (1-\beta)\Big[f(\tau) - \int_0^{\tau} g(\tau - u) f(u) du\Big] \\
=& \frac{1-\beta}{b}[1 - G(\tau)] > 0 \quad \forall \tau \in (0, b),
\end{aligned}$$

the optimal policy is to wait until time b to screen, regardless of β.

We have yet another interesting result about the structure of optimal policy for the case of exponential X_0 with perfect screenings.

Proposition 4 *If $X_0 \sim Exp(\lambda)$ for some $\lambda > 0$, and $\beta = 0$, then the optimal policy $\boldsymbol{\sigma}$ is such that $\sigma_{i+1} - \sigma_i > \sigma_i - \sigma_{i-1} \quad \forall i \in \{1, \cdots, n-1\}$, i.e., the screening interval gets wider as the person gets older.*

Proof:

When $X_0 \sim \text{Exp}(\lambda)$ and $\beta = 0$, for each $k \in \{1, 2 \cdots, n\}$, we have:

$$\begin{aligned} G(\sigma_{k+1} - \sigma_k) &= \int_0^{\sigma_k - \sigma_{k-1}} \frac{f(\sigma_k - s)}{f(\sigma_k)} g(s) ds \\ &= \int_0^{\sigma_k - \sigma_{k-1}} \frac{e^{-\lambda(\sigma_k - s)}}{e^{-\lambda \sigma_k}} g(s) ds \\ &= \int_0^{\sigma_k - \sigma_{k-1}} e^{\lambda s} g(s) ds \\ &> \int_0^{\sigma_k - \sigma_{k-1}} g(s) ds \quad [e^{\lambda s} > 1 \ \forall s > 0] \\ &= G(\sigma_k - \sigma_{k-1}). \end{aligned}$$

■

We hereby highlight that in the literature, Barlow et al. (1963) and Yang and Klutke (2000) have shown for various inspection problems with an exponential system lifetime that the optimal schedules have equal intervals. However, with the additional preclinical state in our model, and with our particular objective to capture the disease while in that state, the structure of the optimal solution is different.

Lastly, on the sideline, we prove that all logconcave densities are bounded.

Proposition 5 *If f is logconcave over $(0, b)$, where $b \leq \infty$, then it is bounded.*

Proof:

Since f is continuous, it suffices to show $\lim_{x \to 0} f(x)$ and $\lim_{x \to b} f(x)$ are finite. We will prove for the end of $x \to b$ and the other side will follow in the same way.

As $logf$ is concave, $(logf)'$ decreases over $(0, b)$. There are then two possibilities:

One, there is some $x^* \in (0, b)$ for which $(logf)'(x) \leq 0 \quad \forall x \in [x^*, b)$. In this case, $logf$ decreases over $[x^*, b)$, and so does $f = e^{logf}$. As $f > 0$, $\lim\limits_{x \to b} f(x)$ must be finite.

Otherwise, if $\lim\limits_{x \to b}(logf)'(x) \geq 0$, let $x^* := \frac{b}{2}$. It then holds that $(logf)'(x^*) \geq (logf)'(x)$ $\forall x \in [x^*, b)$, and so $logf(x) \leq logf(x^*) + (x - x^*)(logf)'(x^*) \leq logf(x^*) + \frac{b}{2}(logf)'(x^*)$, which is a constant. As a result, $f = e^{logf}$ is bounded over $[\frac{b}{2}, b)$ and $\lim\limits_{x \to b} f(x)$ is finite.

■

5. SENSITIVITY ANALYSIS

In the previous chapter we have shown that a unique optimal solution exists for our preclinical disease screening model once the screening sensitivity and the distribution functions for the healthy and the preclinical durations are specified. In this chapter, we investigate the effects of changing these model inputs on the optimal screening policy and its performance.

We solve all our optimization instances by Matlab's "constrained optimization" (fmincon) routine. Indeed, the only constraints involved are those that ensure the increasing order of screening epochs.

5.1 Effect of Screening Sensitivity

We have proven in Proposition 3 that if the healthy duration X_0 follows a uniform distribution and if screening sensitivity is one, then the optimal screening policy is equally spaced. Our first investigation is then the effect of screening sensitivity on a uniform X_0 model.

We assume X_0 follows a uniform distribution with a range of 24 years. One could interpret this with arbitrary starting and ending ages, e.g. from 40 to 64, or from 50 to 74 years old. In all our numerical results to follow, we assume all X_0 densities start at age 40. On the other hand, we assume that the preclinical duration X_1 has an exponential distribution with mean of 3 years. We consider the screening budget to be from 1 to 24 times in a person's lifetime. The 24-screening scenario corresponds to holding an average of one screening per year over the support of X_0.

We consider four false-negative rates of screening (β-errors), namely, 0, 0.4, 0.8, and 0.99. The computation time for a typical case across all 24 budget levels is about 10 seconds on a computer with Intel(R) Core(TM)2 Duo CPUs each running

at 3.16GHz and with 4.00G RAM.

Figure 5.1 shows the performances of both optimal and equal-interval policies across budget levels.

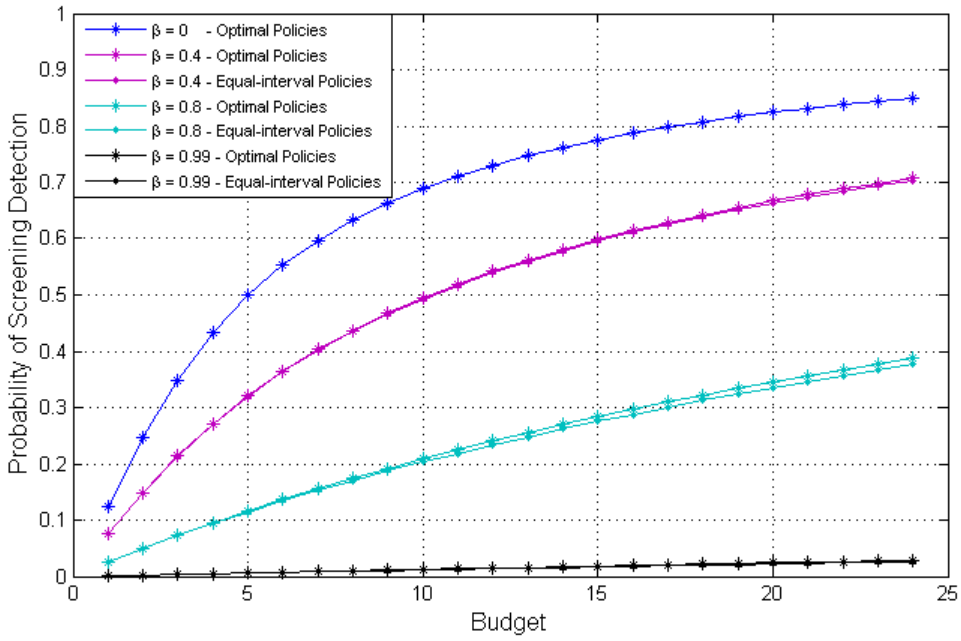

Figure 5.1: Effect of β-error on Policy Performances - Uniform X_0 Model

As expected, the performance of optimal screening policies decreases as β-error increases. Meanwhile, Figure 5.1 shows that even with 99% false-negative screenings, there is no distinctive difference between optimal and equal-interval policies' performances.

We then take a closer look at the change of optimal policy itself as we vary β. In Figure 5.2 we plot the optimal policies for the 12-screening scenario over various levels of β-error.

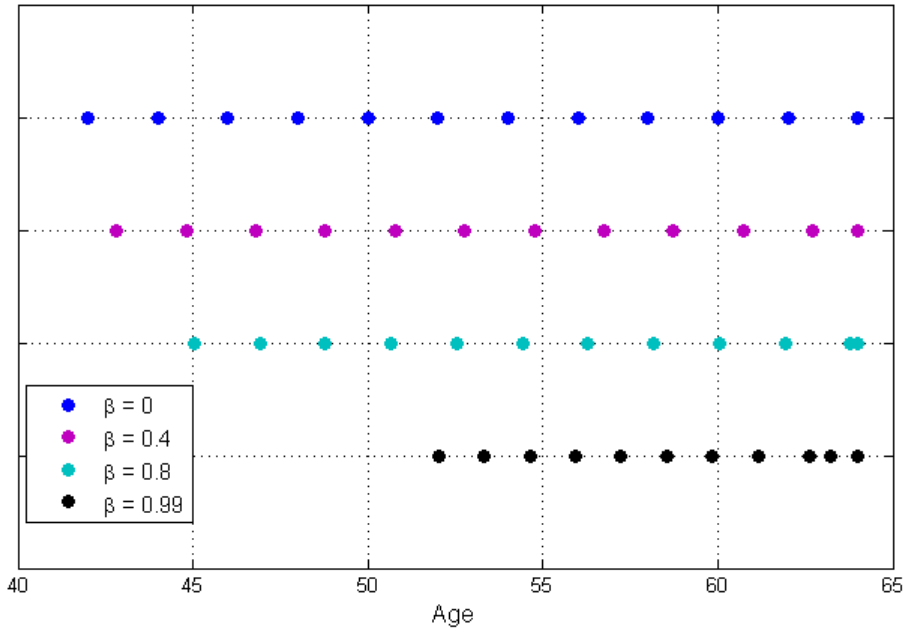

Figure 5.2: Effect of β-error on Optimal Policy Structure - Uniform X_0 Model

As shown, as β-error increases, the screening epochs appear postponed in the optimal policy.

We next investigate the effect of β-error for another model in which there exists considerable difference between the optimal and equal-interval policies' performances. We assume that both X_0 and X_1 follow gamma distributions, with respective means of 12 and 3 years, and respective variances of 16 and 3. We experiment with three levels of β-error respectively at 0.2, 0.6, and 0.9. Figure 5.3 plots the performances of both optimal and equal-interval policies across β-error levels.

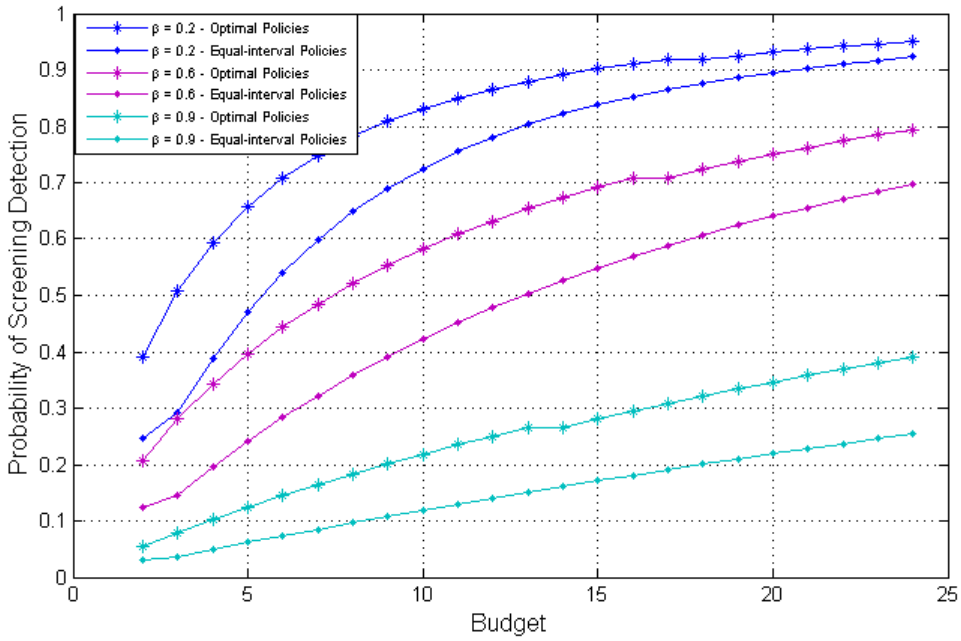

Figure 5.3: Effect of β-error on Policy Performances - Gamma X_0 Model

As depicted, by increasing β there is no significant change on the difference between optimal and equal-interval policies' performances. This result is consistent with that of the uniform X_0 model.

We further plot the structure of optimal policies for the 12-screening case:

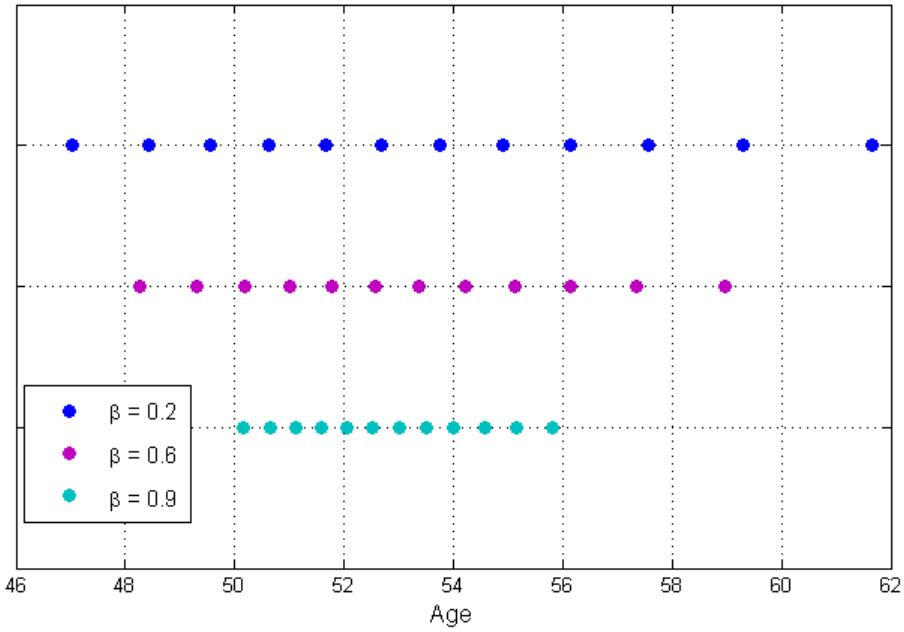

Figure 5.4: Effect of β-error on Optimal Policy Structure - Gamma X_0 Model

As shown in Figure 5.4, as β-error increases the optimal screening epochs tend to tighten up as opposed to getting postponed as in the uniform X_0 case.

5.2 Effect of X_0 Distribution

We next consider two alternative log-concave models for X_0, namely, the gamma model and the 2-parameter Weibull model each with shape parameter no less than one. Since both models comprise two parameters, we may match their first two moments to that of the uniform distribution we considered in the earlier section.

Table 5.1 summarizes the parameters of our double-moment-matching alternative models for X_0. Their p.d.f.'s are plotted in Figure 5.5. Note that all three density functions consist of a mean of 12 years and a variance of 48. We assume that X_1 follows exponential distribution with mean of 3 years and that screening sensitivity is 0.8 in this section.

Table 5.1: Parameters for Double-Moment-Matching Models for X_0

Model	Parameters
Uniform	$a = 0, b = 24$
Gamma	$k = 3, \theta = 4$
Weibull	$\lambda = 13.4908, k = 1.7915$

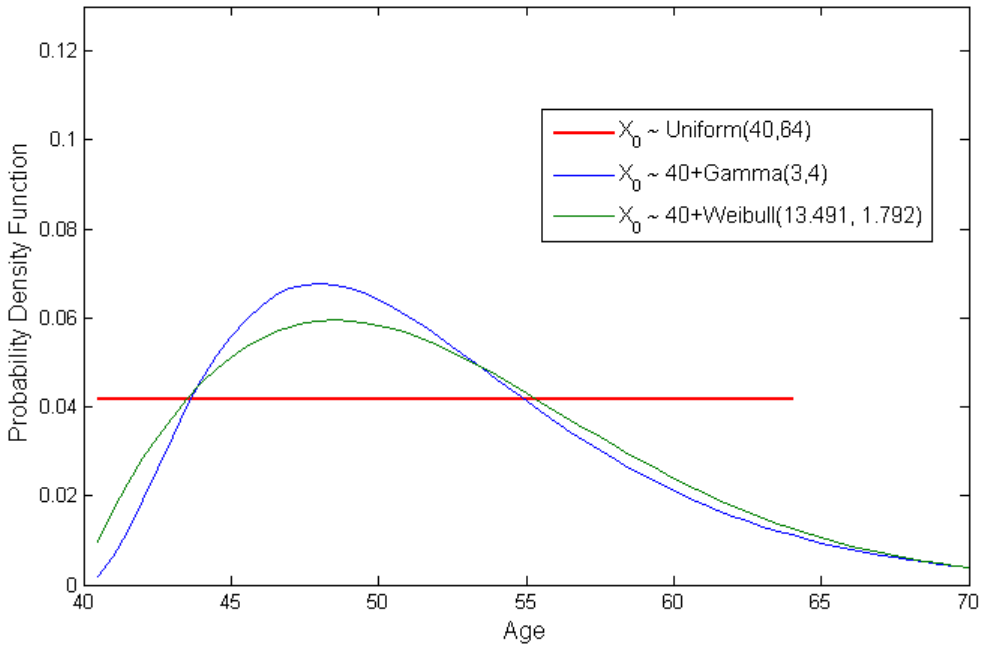

Figure 5.5: Plot of Double-Moment-Matching p.d.f.'s for X_0

The performances of these models are shown in Figures 5.6 and 5.7. A typical instance with gamma X_0 takes about 2 hours to solve, and with Weibull X_0 1.5 hours.

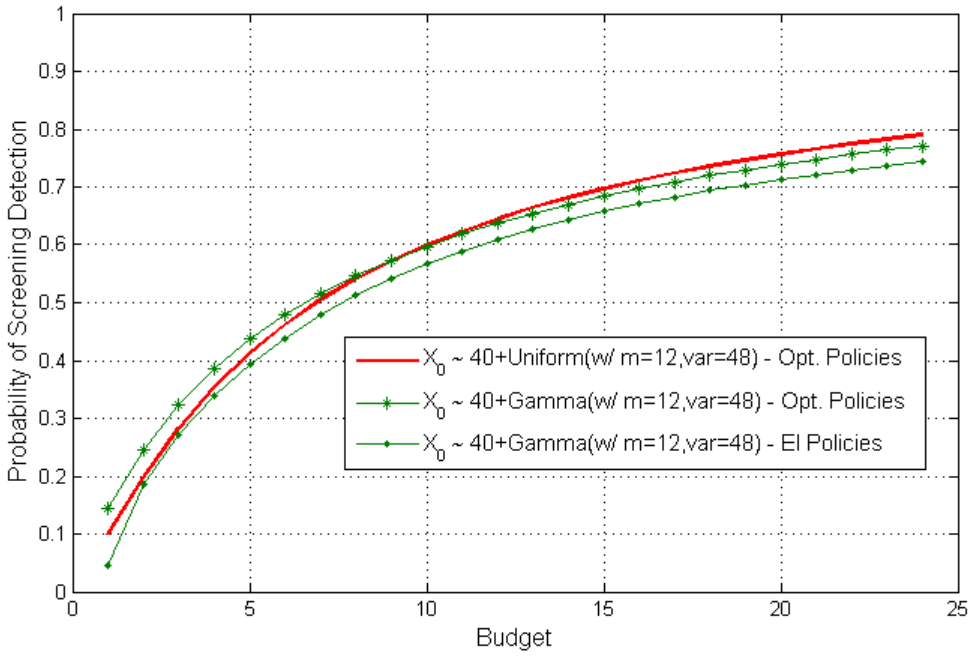

Figure 5.6: Sensitivity Analysis - Double-Moment-Matching Gamma X_0

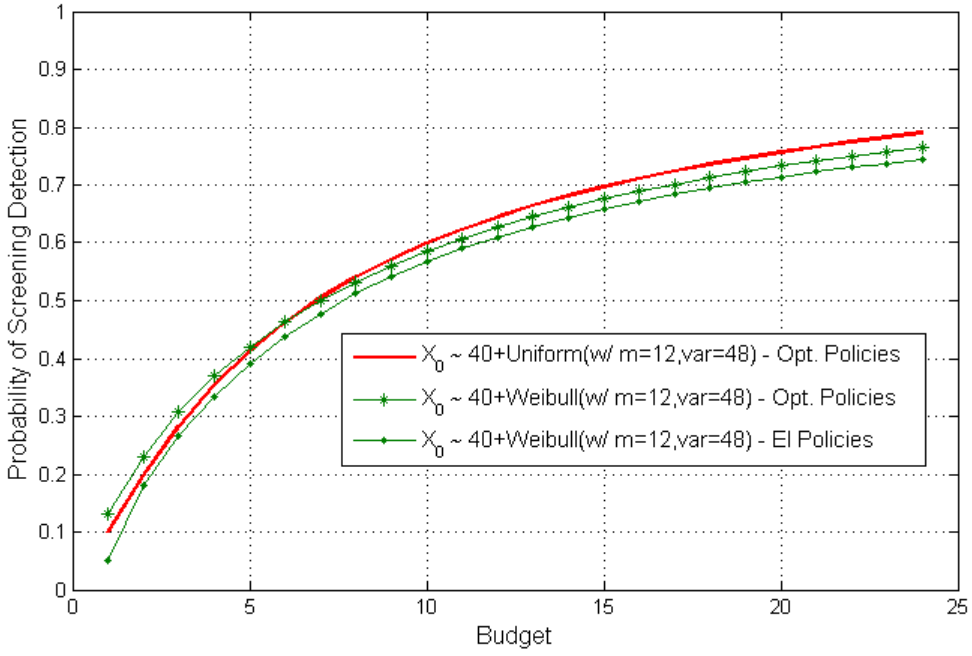

Figure 5.7: Sensitivity Analysis - Double-Moment-Matching Weibull X_0

As shown, for both gamma and Weibull X_0 models, the performances of the optimal policies are slightly worse than in the uniform model. Also, not much probability of screening-detection will be sacrificed even if we simply apply the equal-interval policies. These results suggest that when the first two moments are fixed for X_0, the exact density function would not impact the optimal policy performance greatly.

We next investigate cases where only the mean of X_0 is fixed. Specifically, for each alternative model, we consider two additional values for its variance, namely, 16 and 144. Notice that in both gamma and Weibull families, the only density that has mean of 12 and variance of 144 is the exponential distribution with rate 1/12, which is the special case having shape parameter one in each family.

Table 5.2 summarizes the parameters of the three gamma densities we consider. Their p.d.f.'s are plotted in Figure 5.8, and the performances of optimal policies for the three models are shown in Figure 5.9.

Table 5.2: Parameters for Gamma Densities for X_0

Choice	Parameters
Uniform	$k = 9, \theta = 4/3$ (Mean= 12, Var= 16)
Gamma	$k = 3, \theta = 4$ (Mean= 12, Var= 48)
Weibull	$k = 1, \theta = 12$ (Mean= 12, Var= 144)

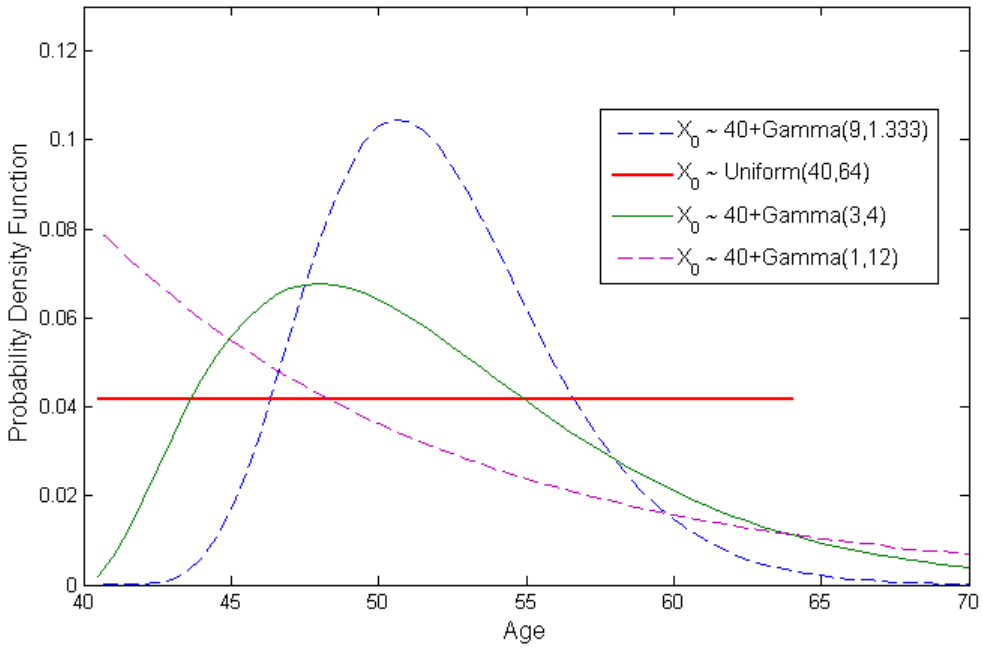

Figure 5.8: Plot of Gamma p.d.f.'s for X_0

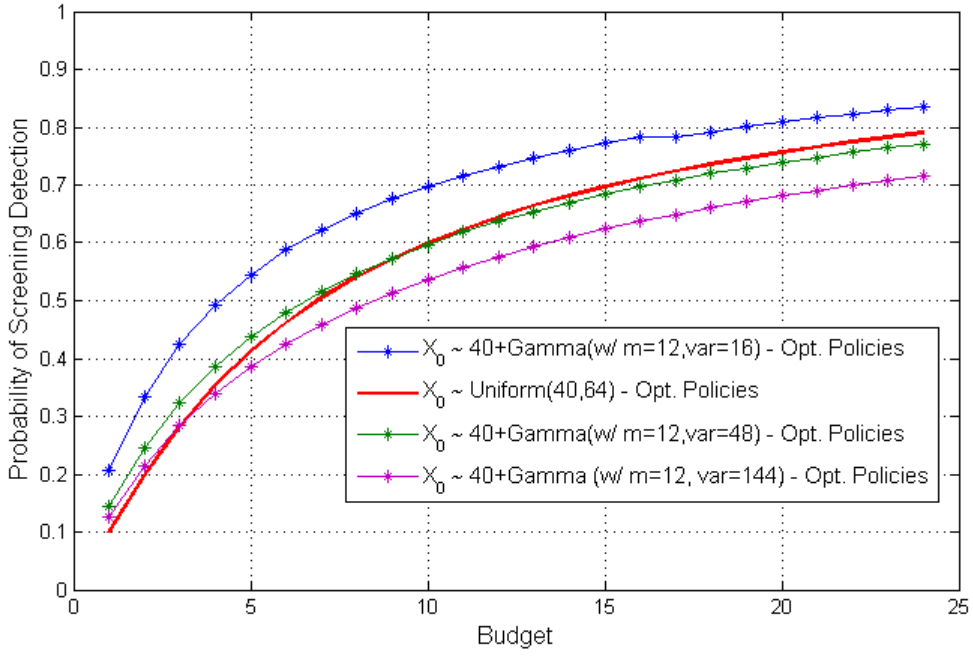

Figure 5.9: Sensitivity Analysis - Variance of Gamma X_0

Additionally, Table 5.3 summarizes the parameters of the three Weibull densities we consider. The p.d.f.'s are plotted in Figure 5.10, and the performances of optimal policies for the three models are shown in Figure 5.11.

Table 5.3: Parameters for Weibull Densities for X_0

Choice	Parameters
1	$\lambda = 13.3770, k = 3.3035$ (Mean= 12, Var= 16)
2	$\lambda = 13.4908, k = 1.7915$ (Mean= 12, Var= 48)
3	$\lambda = 12.0000, k = 1.0000$ (Mean= 12, Var= 144)

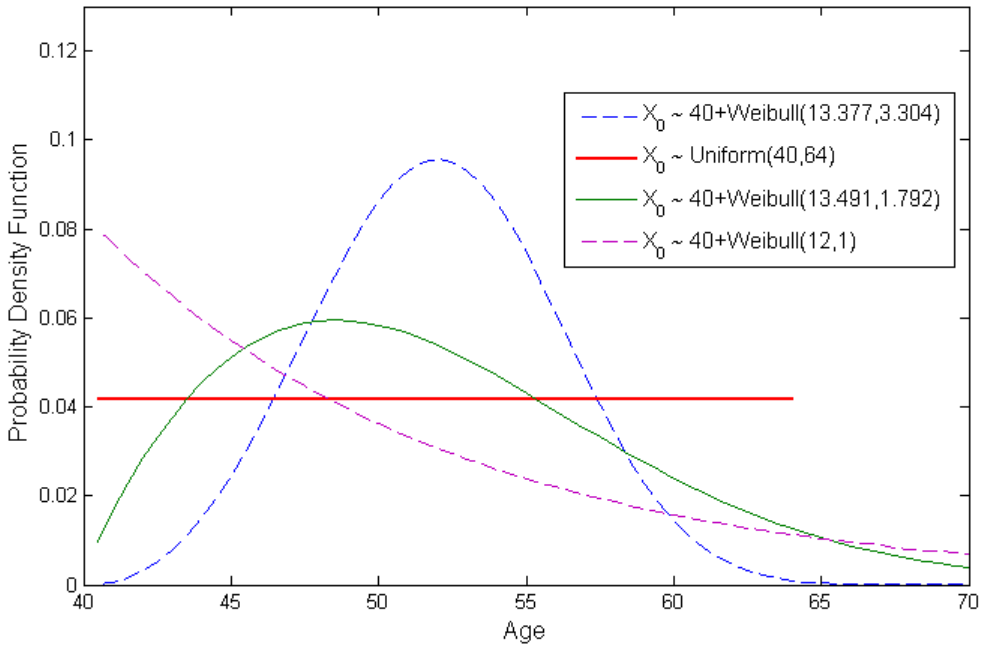

Figure 5.10: Plot of Weibull p.d.f.'s for X_0

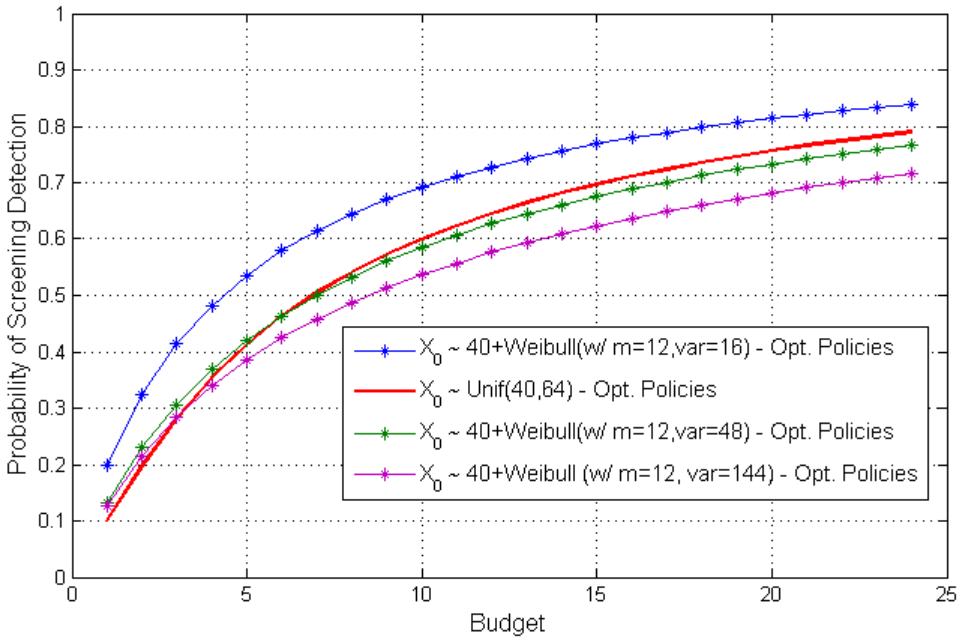

Figure 5.11: Sensitivity Analysis - Variance of Weibull X_0

As shown in Figures 5.9 and 5.11, the variance of X_0 does play a key role on the optimal policy's performance. In the 24-screening scenario, the probability of detection ranges from around 70% to 85% at optimality as variance decreases, for both gamma and Weibull X_0 models.

5.3 Effect of Distribution of X_1

We now consider alterative models for X_1. We assume that X_0 follows an exponential distribution with mean of 12 years and that screening sensitivity is 0.8 in this section.

We first consider gamma densities for X_1. Again, we fix the first moment, at 3 years, and we consider three versions of distribution that respectively have variances of 3, 9, and 27. Notice the second choice is the exponential distribution.

The parameters considered for the gamma densities for X_1 are summarized

in Table 5.4, and their corresponding p.d.f.'s are plotted in Figure 5.12.

Table 5.4: Parameters for Gamma Densities for X_1

Choice	Parameters
default	$a = 1, b = 3$
1	$a = 3, b = 1$
2	$a = 1/3, b = 9$

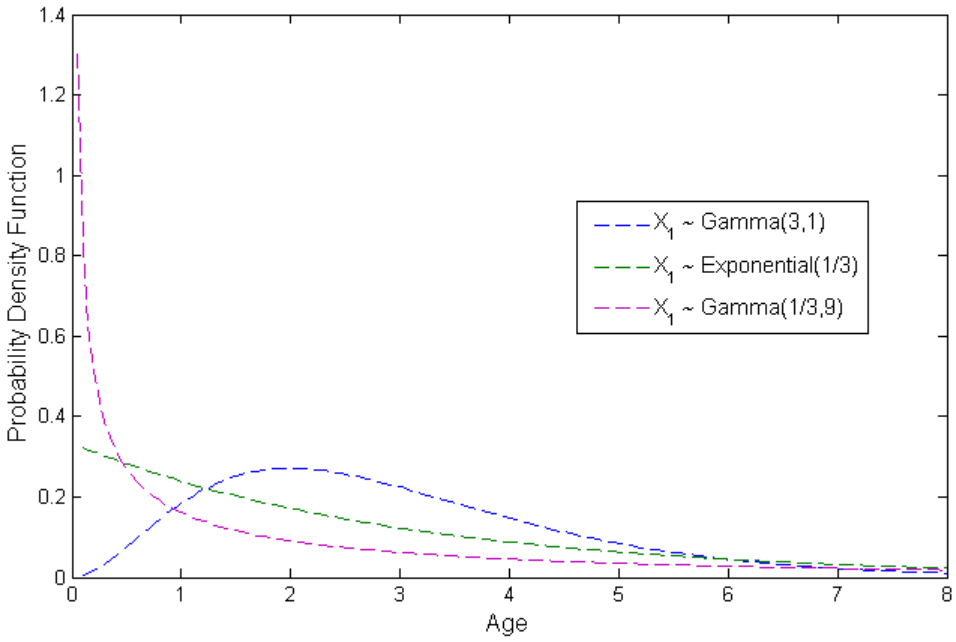

Figure 5.12: Plot of Gamma p.d.f.'s for X_1

The model outputs are presented in Figure 5.13. The computation time for a typical instance up to 24 screenings is 1.5 hours.

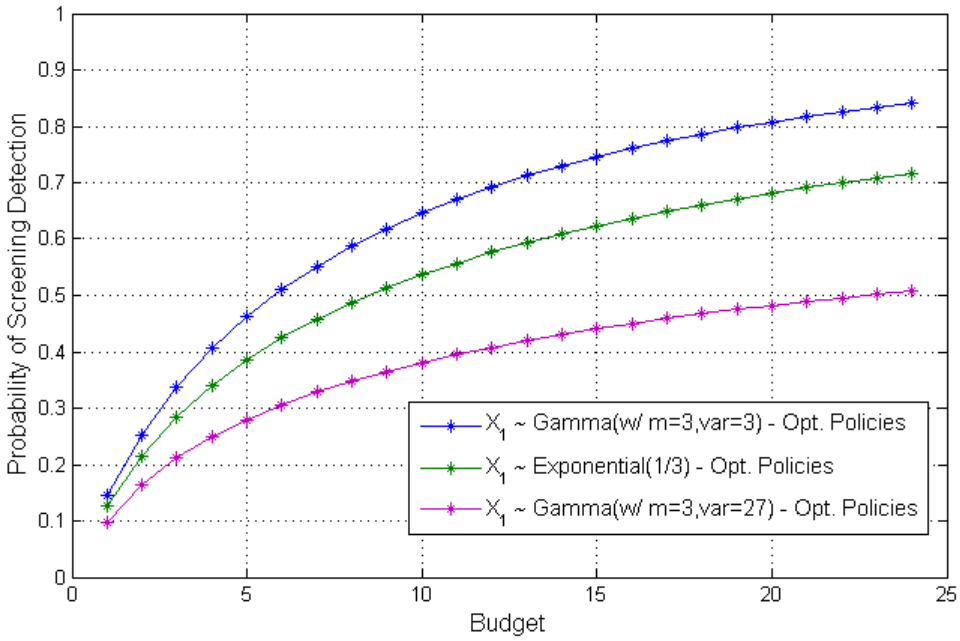

Figure 5.13: Sensitivity Analysis - Variance of Gamma X_1

As in the case of X_0, the performance of optimal policy is sensitive to the variance of X_1. At the 24-screening budget level, the probability of screen-detection can reach as high as 0.85 when X_1 has variance 3, and as low as 0.40 when X_1 has variance 27. From Figure 5.12, one would not reject the small variance gamma density as a realistic representation for a disease. If this is the underlying truth, then promising screening performance can be achieved at optimality.

We next consider a lognormal model for X_1. The parameters considered are summarized in Table 5.5, and the p.d.f.'s plotted in Figure 5.14.

Table 5.5: Parameters for Lognormal Densities for X_1

Choice	Parameters
1	$\mu = 0.9548, \sigma = 0.5364$ (Mean= 3, Var= 3)
2	$\mu = 0.7520, \sigma = 0.8326$ (Mean= 3, Var= 9)
3	$\mu = 0.4055, \sigma = 1.1774$ (Mean= 3, Var= 27)

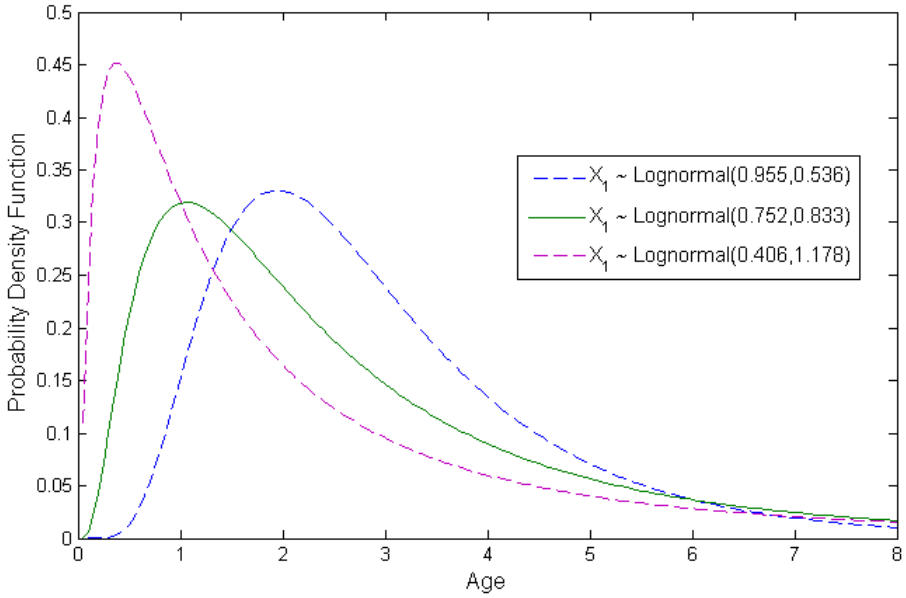

Figure 5.14: Plot of Lognormal p.d.f.'s for X_1

The optimal policy performances are shown in Figure 5.15. A typical case takes about 1.5 hours to solve.

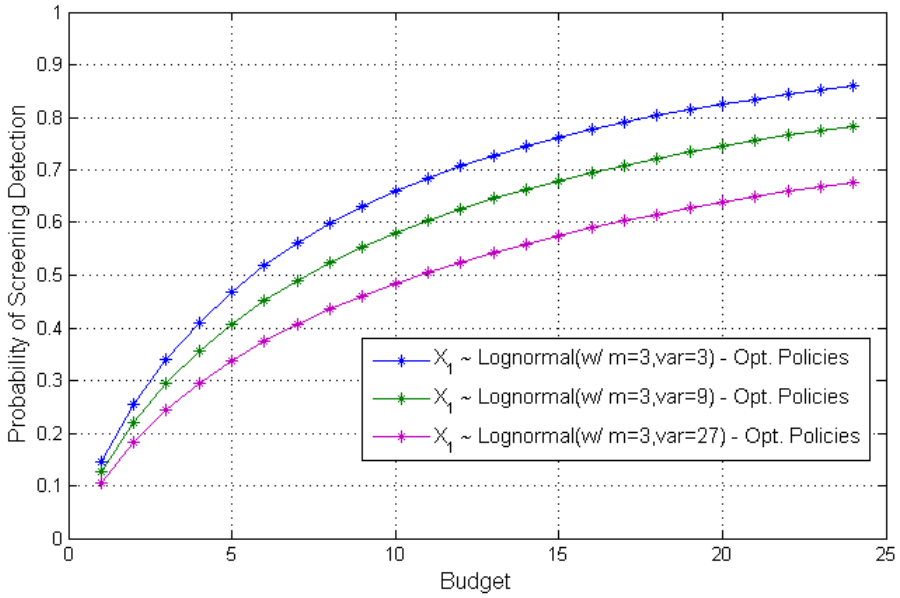

Figure 5.15: Sensitivity Analysis - Variance of Lognormal X_1

Similar results hold on the strong sensitivity of X_1's variance over the performance of optimal policy. In addition, at all three variance levels, the optimal policy from the lognormal X_1 model outperforms that of the gamma X_1 model.

6. A CASE STUDY USING BREAST CANCER DATA

In this chapter, we present the results we obtain as we apply our model to the screening for breast cancer.

The practice of breast cancer screening started as early as in the 1960s. Several screening modalities are currently in place to detect initial stage of breast cancers, including mammogram, clinical breast exam, breast self-exam, and in some cases, ultrasound and magnetic resonance imaging (MRI). However, the very questions of when the screening should start and how often it should be conducted have been debated over the decades.

In the literature, it is often considered that the population progresses into the preclinical breast cancer stage at a constant rate (see Zelen and Feinleib (1969), Zelen (1993), Walter and Day (1983), Day and Walter (1984)). Such models are called "stable disease models", and they essentially assume that the healthy duration follows a uniform distribution. Furthermore, based on stable disease models, statistical works have been done that found the exponential model for X_1 to best represent data collected from the actual screening trials (see Zelen and Feinleib (1969), Walter and Day (1983)). Thus, we set up our case-study model by assuming uniform and exponential distributions for the two random times.

We first set up our model by specifying its objective function and by extracting parameters from the literature for the two density functions and the sensitivity of screening exams. We then solve the model to optimality and consider its robustness. We last investigate issues of average number of screenings and the expected lead time under optimal policies.

6.1 Model Setup

We first analytically specify the objective function for our case-study model. This takes away unnecessarily numerical integrations in running the optimization routine.

Proposition 6 *If* $X_0 \sim$ *Unif* $(0, b)$ *for some* $b < \infty$ *and* $X_1 \sim$ *Exp*(λ) *for* $\lambda > 0$, *then the objective function is*

$$\max_{\boldsymbol{\tau} \in \mathcal{D}} \frac{1-\beta}{\lambda b} \sum_{i=0}^{n-1} \sum_{j=i+1}^{n} \beta^{j-i-1} \Big[e^{-\lambda(\tau_j - \tau_{i+1})} - e^{-\lambda(\tau_j - \tau_i)} \Big]$$

Proof:

$$\begin{aligned}
\max_{\boldsymbol{\tau} \in \mathcal{D}} \quad & 1 - \sum_{i=0}^{n} \sum_{j=i}^{n} \beta^{j-i} \int_{\tau_i}^{\tau_{i+1}} [G(\tau_{j+1} - u) - G(\tau_j - u)] f(u) du \\
= \; & 1 - \Big\{ \sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \beta^{j-i} \frac{1}{b} \int_{\tau_i}^{\tau_{i+1}} e^{-\lambda(\tau_j - u)} - e^{-\lambda(\tau_{j+1} - u)} du \\
& + \sum_{i=0}^{n-1} \frac{1}{b} \int_{\tau_i}^{\tau_{i+1}} [1 - e^{-\lambda(\tau_{i+1} - u)}] du + \sum_{i=0}^{n-1} \beta^{n-i} \frac{1}{b} \int_{\tau_i}^{\tau_{i+1}} e^{-\lambda(\tau_n - u)} du + \frac{1}{b} \int_{\tau_n}^{b} 1 \; du \Big\} \\
= \; & 1 - \Big\{ \sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \beta^{j-i} \frac{1}{b} (e^{-\lambda \tau_j} - e^{-\lambda \tau_{j+1}}) \int_{\tau_i}^{\tau_{i+1}} e^{\lambda u} du \\
& + \sum_{i=0}^{n-1} \frac{1}{b} \Big[(\tau_{i+1} - \tau_i) - e^{-\lambda \tau_{i+1}} \int_{\tau_i}^{\tau_{i+1}} e^{\lambda u} du \Big] + \sum_{i=0}^{n-1} \beta^{n-i} \frac{1}{b} e^{-\lambda \tau_n} \int_{\tau_i}^{\tau_{i+1}} e^{\lambda u} du \\
& + \frac{1}{b} (b - \tau_n) \Big\} \\
= \; & 1 - \Big\{ \sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \beta^{j-i} \frac{1}{\lambda b} \Big[e^{-\lambda(\tau_j - \tau_{i+1})} + e^{-\lambda(\tau_{j+1} - \tau_i)} - e^{-\lambda(\tau_{j+1} - \tau_{i+1})} - e^{-\lambda(\tau_j - \tau_i)} \Big] \\
& - \sum_{i=0}^{n-1} \frac{1}{\lambda b} \Big[1 - e^{-\lambda(\tau_{i+1} - \tau_i)} \Big] + \sum_{i=0}^{n-1} \beta^{n-i} \frac{1}{\lambda b} \Big[e^{-\lambda(\tau_n - \tau_{i+1})} - e^{-\lambda(\tau_n - \tau_i)} \Big] + 1 \Big\}
\end{aligned}$$

$$
\begin{aligned}
&= 1 - \frac{1}{\lambda b} \sum_{i=0}^{n-1} \Bigg\{ (1-\beta) \sum_{j=i+2}^{n} \beta^{j-i-1} \Big[e^{-\lambda(\tau_j - \tau_i)} - e^{-\lambda(\tau_j - \tau_{i+1})} \Big] \\
&\quad + (1-\beta) \Big[e^{-\lambda(\tau_{i+1} - \tau_i)} - 1 \Big] \Bigg\} - 1 \\
&= \frac{1-\beta}{\lambda b} \sum_{i=0}^{n-1} \sum_{j=i+1}^{n} \beta^{j-i-1} \Big[e^{-\lambda(\tau_j - \tau_{i+1})} - e^{-\lambda(\tau_j - \tau_i)} \Big]
\end{aligned}
$$

■

We parameterize our model as follows. The uniform distribution of X_0 is assumed to have a range of 24 years. We take this number based on U.S. Preventive Services Task Force's recommendation that women should be screened between ages 50 and 75. We choose 24 years as the actual range as we can then easily refer to a collection of periodic policies (e.g. yearly, biennial , 3-yearly and 4-yearly screenings) which all divide the support exactly. We further assume that the risk of preclinical breast cancer begins at age 40, according to American Cancer Society. As a result, we treat random variable X_0 with a uniform $(40, 64)$ distribution.

The exponential distribution for X_1 is assumed to have mean length of 3 years, and the rate of β-error for screening exams is assumed to be 0.2. We take these numbers off various works in the literature (see Walter and Day (1983), Shen and Zelen (1999), Shen and Parmigiani (New York: Springer, 2006)), and from surveying domain experts.

In Table 6.1, we summarize our choices of distributions and parameters in our breast cancer screening model.

Table 6.1: Breast Cancer Screening Model Parameters

Input	Distribution/Parameter
r.v. X_0	Unif$(40, 64)$
r.v. X_1	Exp$(1/3)$
β-error	0.2

6.2 Optimal Solution and Its Robustness

In Figure 6.1 we plot the performance of optimal screening polices across budget levels 3 through 8, which we consider practical. Recall our objective function is the probability of detection by screening. Recall also that in Figure 5.2 we had shown for a model with the same setup that the equal-interval policy performs almost as greatly as optimal.

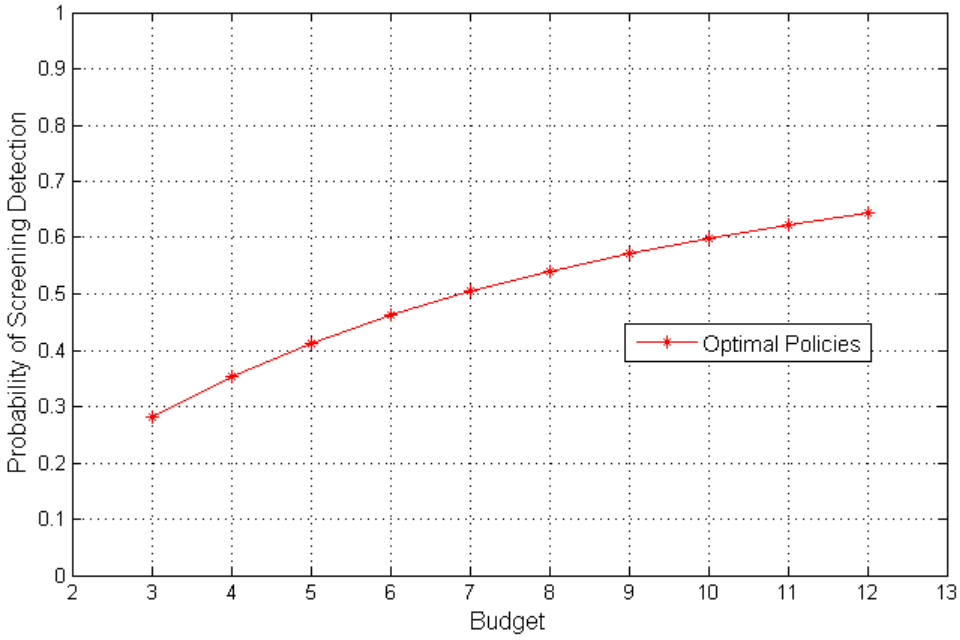

Figure 6.1: Breast Cancer Model - Optimal Policy Performance

As shown, with 12 screenings planed, at optimality about 65% preclinical cases can be found by screening.

Next we investigate the robustness of the above result.

We first consider varying the range of X_0 and the mean duration of X_1. To this end, we show a simple yet insightful result for the special case in which screening is perfectly sensitive. As we know that the optimal policy is equally spaced in this case, we may derive a closed-form expression for the optimal objective value.

Proposition 7 *If $X_0 \sim$ Unif $(0, b)$, $X_1 \sim$ Exp(λ) and $\beta = 0$, then the optimal objective value is $P(\boldsymbol{\sigma}^{(n)}) = \frac{n}{b\lambda}(1-e^{-\frac{b\lambda}{n}})$, which increases in n and $1/\lambda$, and decreases in b.*

Proof:

If $\beta = 0$, then

$$\begin{aligned}
P(\boldsymbol{\sigma}^{(n)}) &= \sum_{i=0}^{n-1} \int_{\sigma_i^{(n)}}^{\sigma_{i+1}^{(n)}} f(s)[1 - G(\sigma_{i+1}^{(n)} - s)]ds \\
&= \sum_{i=0}^{n-1} \int_{\frac{b}{n}(i)}^{\frac{b}{n}(i+1)} \frac{1}{b} e^{-\lambda[\frac{b}{n}(i+1)-s]} ds \\
&= \frac{1}{b} \sum_{i=0}^{n-1} e^{-\frac{b\lambda}{n}(i+1)} \int_{\frac{b}{n}(i)}^{\frac{b}{n}(i+1)} e^{\lambda s} ds \\
&= \frac{1}{b\lambda} \sum_{i=0}^{n-1} e^{-\frac{b\lambda}{n}(i+1)} [e^{\frac{b\lambda}{n}(i+1)} - e^{\frac{b\lambda}{n}i}] \\
&= \frac{1}{b\lambda} \sum_{i=0}^{n-1} (1 - e^{-\frac{b\lambda}{n}}) \\
&= \frac{n}{b\lambda}(1 - e^{-\frac{b\lambda}{n}})
\end{aligned}$$

Now, let $x = \frac{b\lambda}{n}$, and consider $P(\boldsymbol{\sigma}^{(n)})$ as $P(x) = \frac{1-e^{-x}}{x}$. Note as $x \to 0$, both

numerator $1-e^{-x}$ and denominator x tend to 0; but for all $x > 0$, $(1-e^{-x})' = e^{-x} < 1$ while $x' = 1$. We therefore have $P(x)$ is a decreasing function for positive x.

■

The above result suggests that the longer the preclinical sojourn time is in comparison to the disease-free time, the easier it will be for screenings to capture the disease.

We next consider a few alternative configurations on the distributions of X_0 and X_1. Specifically, we hold the means of the two random variables respectively at 12 years and 3 years, and we consider Weibull and gamma models for X_0 with a variance of 16 (original being 48), and also a gamma model for X_1 with a variance of 3 (original being 9). We hold β at 0.2 throughout this investigation.

Additionally, for all alternative models, we consider Quantile-Based Inspection (QBI) policies as follows: at each budget level n, we schedule the screening epochs at the $1/(n+1)$ through $n/(n+1)$ quantiles of the X_0 distribution. The QBI policies were initially considered in the context of replaceable system inspection by Yang and Klutke (2000).

We compare the performances of optimal, QBI and EI policies for our four alternative models and we present the results in Figures 6.2 through 6.5.

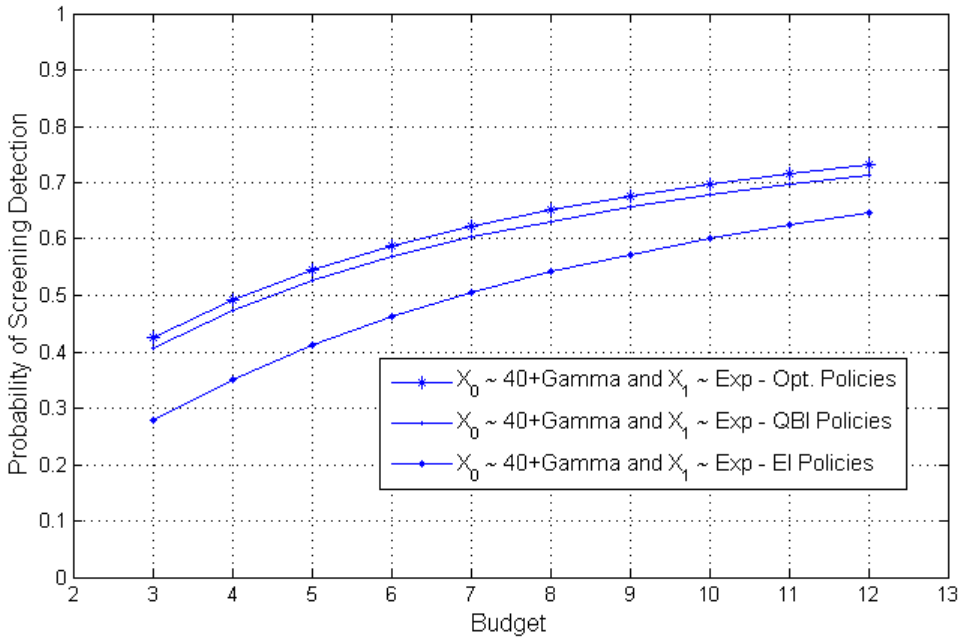

Figure 6.2: Breast Cancer Model Robustness - Gamma X_0 and Exponential X_1

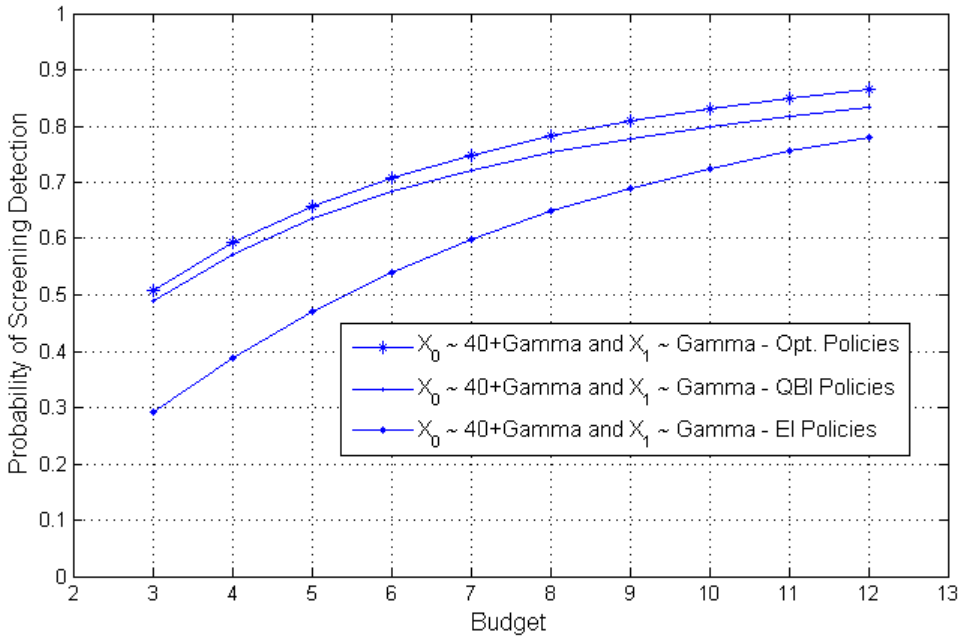

Figure 6.3: Breast Cancer Model Robustness - Gamma X_0 and Gamma X_1

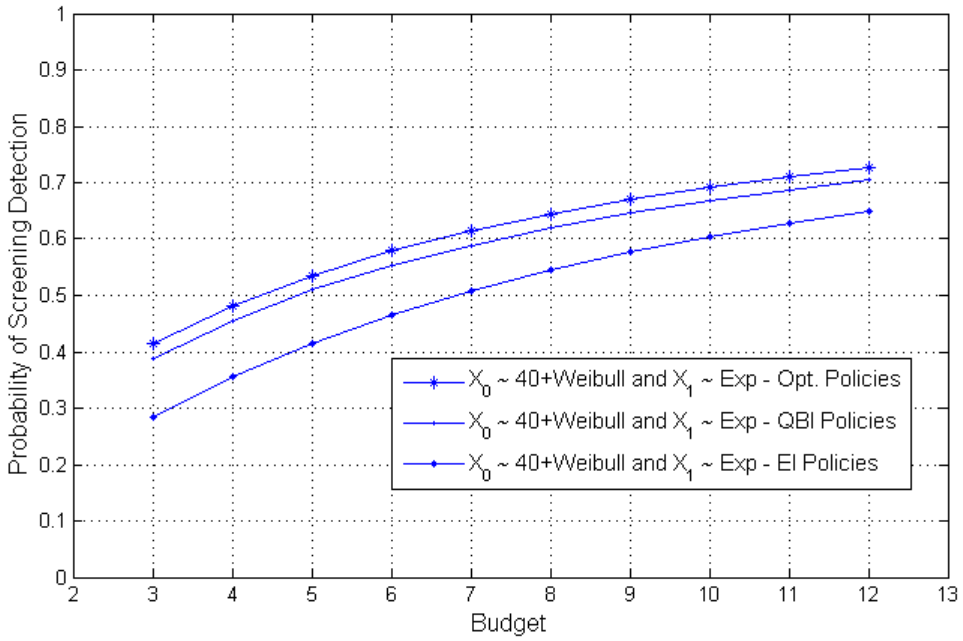

Figure 6.4: Breast Cancer Model Robustness - Weibull X_0 and Exponential X_1

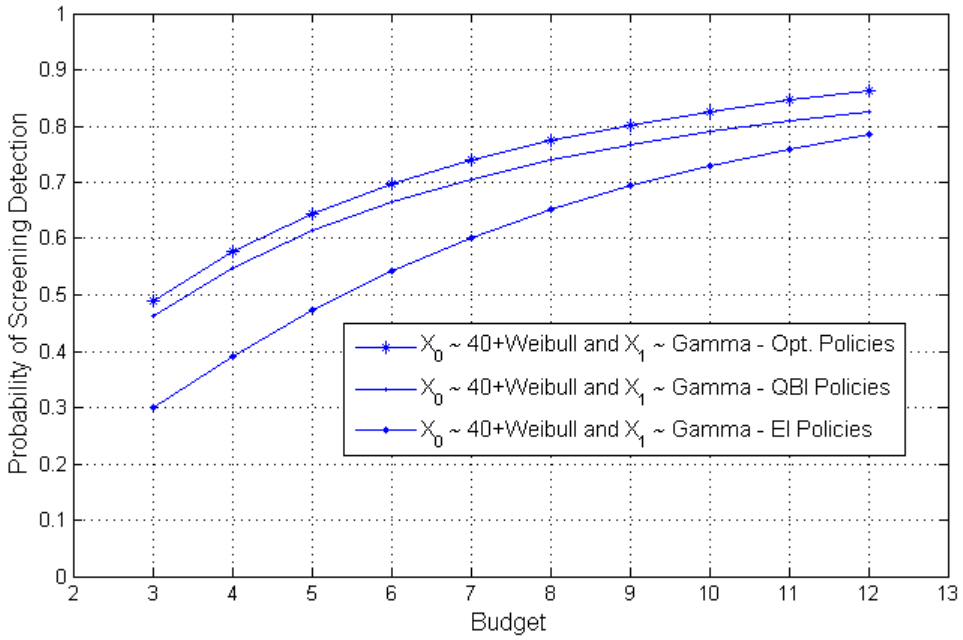

Figure 6.5: Breast Cancer Model Robustness - Weibull X_0 and Gamma X_1

As shown, in all four cases, the EI policies perform much worse than optimal. We raise our concern about this observation as in practice most screening recommendations are equally-spaced in nature. Although many studies assume a uniform X_0, there has been rather limited empirical evidence to support this treatment. Our results reveal that when the underlying preclinical incidence is not stable, the EI policy can act far off the mark.

On the other hand, the QBI policies perform rather closely to optimal in all four cases. We consider the reason for this as that the QBI in its nature exploits the information about X_0 distribution. In practice, QBI may serve as a good heuristic to compute high-quality screening policies without running the time-consuming optimization routines.

Furthermore, as we had seen from the previous chapter, when variance is not so high for either X_0 or X_1's underlying distribution, the performance of optimal policy can be much enhanced. In cases of both small variances on X_0 and X_1 (see Figures 6.3 and 6.5), close to 90% probability of detection is achievable with only 12 screenings planned.

6.3 Additional Evaluations

We last study a few additional issues in our disease screening model and we evaluate these numerically for our breast cancer screening case.

6.3.1 Expected Number of Screenings

First, we note that throughout our analysis, the "screening budget" is defined as the "maximum number of screenings allowed" in a person's lifetime. The actual number of screenings for an individual is indeed a random variable which depends not only on internal factors such as the disease-free and preclinical sojourn times, but also external ones like screening sensitivity and the screening policy itself.

We therefore first derive an expression for the expected number of screenings for an individual in the disease-affected population.

Proposition 8 For screening policy $\boldsymbol{\tau} = \{\tau_1, \tau_2, ..., \tau_n\}$, and for a disease-affected population that has f and g respectively as its disease-free and preclinical sojourn time densities and β as screening sensitivity, the average number of screenings per person (denote this by "N^D") is:

$$N^D(\boldsymbol{\tau}) = \sum_{i=1}^{n} i\Big\{ \int_{\tau_i}^{\tau_{i+1}} G(\tau_{i+1}-u)f(u)du + (1-\beta)\sum_{j=0}^{i-1}\int_{\tau_j}^{\tau_{j+1}} [1-G(\tau_i-u)]\beta^{i-j-1}f(u)du\Big\},$$

where $\tau_{n+1} := \infty$.

Proof:

$$\begin{aligned}
N^D(\boldsymbol{\tau}) &= \sum_{i=1}^{n} i \cdot Pr\Big(\text{Number of Screenings} = i\Big) \\
&= \sum_{i=1}^{n} i\Big\{ \sum_{j=0}^{i-1}\int_{\tau_j}^{\tau_{j+1}} Pr(\text{Number of Screenings} = i \mid X_0 = u)f(u)du \\
&\quad + \int_{\tau_i}^{\tau_{i+1}} Pr(\text{Number of Screenings} = i \mid X_0 = u)f(u)du\Big\} \\
&= \sum_{i=1}^{n} i\Big\{ \sum_{j=0}^{i-1}\int_{\tau_j}^{\tau_{j+1}} [1-G(\tau_i-u)]\beta^{i-j-1}(1-\beta)f(u)du \\
&\quad \text{[event happens when } X_1 \text{ survives at least } \tau_i - u \text{ amount of time, first} \\
&\quad i-j-1 \text{ screenings are all false-negative, and the } (i-j)^{th} \text{ screening} \\
&\quad \text{is successful]} \\
&\quad + \int_{\tau_i}^{\tau_{i+1}} G(\tau_{i+1}-u)f(u)du\Big\} \\
&\quad \text{[event happens when } X_1 \text{ does not survive till } \tau_i\text{]} \\
&\quad = \sum_{i=1}^{n} i\Big\{ \int_{\tau_i}^{\tau_{i+1}} G(\tau_{i+1}-u)f(u)du
\end{aligned}$$

$$+(1-\beta)\sum_{j=0}^{i-1}\int_{\tau_j}^{\tau_{j+1}}[1-G(\tau_i-u)]\beta^{i-j-1}f(u)du\Big\}$$

■

The above expression is derived by assigning the right probabilities to each number of screenings while considering the two sojourn times and the β-errors. We omit the details as these are similar to our derivation for the probability of screening detection in Proposition 1.

In particular, under our basic breast cancer model assumptions, we have the following.

Corollary 2 If $X_0 \sim Unif(0,b)$ and $X_1 \sim Exp(\lambda)$, then:

$$\begin{aligned} N^D(\boldsymbol{\tau}) &= \sum_{i=1}^{n-1}\frac{i}{b}\Big\{(\tau_{i+1}-\tau_i)-\frac{1}{\lambda}[1-e^{-\lambda(\tau_{i+1}-\tau_i)}] \\ &\quad +\frac{1-\beta}{\lambda}\sum_{j=0}^{i-1}\beta^{i-j-1}[e^{-\lambda(\tau_i-\tau_{j+1})}-e^{-\lambda(\tau_i-\tau_j)}]\Big\} \\ &\quad +\frac{n}{b}\Big\{(b-\tau_n)+\frac{1-\beta}{\lambda}\sum_{j=0}^{n-1}\beta^{n-j-1}[e^{-\lambda(\tau_n-\tau_{j+1})}-e^{-\lambda(\tau_n-\tau_j)}]\Big\}. \end{aligned}$$

Proof:

$$\begin{aligned} N^D(\boldsymbol{\tau}) &= \sum_{i=1}^{n-1}\frac{i}{b}\Big\{\int_{\tau_i}^{\tau_{i+1}}[1-e^{-\lambda(\tau_{i+1}-u)}]du+(1-\beta)\sum_{j=0}^{i-1}\beta^{i-j-1}\int_{\tau_j}^{\tau_{j+1}}e^{-\lambda(\tau_i-u)}du\Big\} \\ &\quad +\frac{n}{b}\Big\{\int_{\tau_n}^{b}1du+(1-\beta)\sum_{j=0}^{n-1}\beta^{n-j-1}\int_{\tau_j}^{\tau_{j+1}}e^{-\lambda(\tau_n-u)}du\Big\} \end{aligned}$$

$$
\begin{aligned}
= & \sum_{i=1}^{n-1} \frac{i}{b}\Big\{(\tau_{i+1}-\tau_i) - e^{-\lambda\tau_{i+1}} \int_{\tau_i}^{\tau_{i+1}} e^{\lambda u} du \\
& + (1-\beta)\sum_{j=0}^{i-1} \beta^{i-j-1} e^{-\lambda\tau_i} \int_{\tau_j}^{\tau_{j+1}} e^{\lambda u} du\Big\} \\
& + \frac{n}{b}\Big\{(b-\tau_n) + (1-\beta)\sum_{j=0}^{n-1} \beta^{n-j-1} e^{-\lambda\tau_n} \int_{\tau_j}^{\tau_{j+1}} e^{\lambda u} du\Big\} \\
= & \sum_{i=1}^{n-1} \frac{i}{b}\Big\{(\tau_{i+1}-\tau_i) - \frac{1}{\lambda}[1 - e^{-\lambda(\tau_{i+1}-\tau_i)}] \\
& + \frac{1-\beta}{\lambda}\sum_{j=0}^{i-1} \beta^{i-j-1}[e^{-\lambda(\tau_i-\tau_{j+1})} - e^{-\lambda(\tau_i-\tau_j)}]\Big\} \\
& + \frac{n}{b}\Big\{(b-\tau_n) + \frac{1-\beta}{\lambda}\sum_{j=0}^{n-1} \beta^{n-j-1}[e^{-\lambda(\tau_n-\tau_{j+1})} - e^{-\lambda(\tau_n-\tau_j)}]\Big\}
\end{aligned}
$$

■

Next, we examine the expected number of screenings per person in a population that never develops the disease in its lifetime. This measure reflects the impact of a screening programme to people who are not benefitted yet who follow the same recommendation to screen.

The following is clear.

Proposition 9 For screening policy $\boldsymbol{\tau} = \{\tau_1, \tau_2, ..., \tau_n\}$, and for a disease-free population with lifetime distribution function H, the average number of screenings per person (denote this by "N^F") is:

$$N^F(\boldsymbol{\tau}) = \sum_{i=1}^{n} i[H(\tau_{i+1}) - H(\tau_i)],$$

where $H(\tau_{n+1}) := 1$.

We then apply the above two definitions to data from breast cancer screening. Specifically, for the disease-affected population, we assume the model with parameters in Table 6.1, and for the disease-free population, we take the 2008 American females life table (National Vital Statistics System (2012)) for its lifetime distribution. In doing so, we assume that the proportion of disease-affected samples is rather small in the overall population. We calculate the expected number of screenings for both populations as the *optimal* policy is applied at each budget level. The results are shown in Table 6.2 and plotted in Figure 6.6.

Table 6.2: Expected Number of Screenings under Optimal Policies

Budget	Disease-affected Group	Disease-free Group
1	0.1	0.9
2	1.6	1.8
3	2.1	2.8
4	2.6	3.7
5	3.1	4.7
6	3.6	5.6
7	4.0	6.6
8	4.5	7.5
9	5.0	8.5
10	5.5	9.4
11	5.9	10.3
12	6.4	11.3
13	6.9	12.2
14	7.4	13.2
15	7.9	14.1
16	8.4	15.1
17	8.8	16.0
18	9.3	16.9
19	9.8	17.9
20	10.3	18.8
21	10.8	19.8
22	11.3	20.7
23	11.8	21.7
24	12.2	22.6

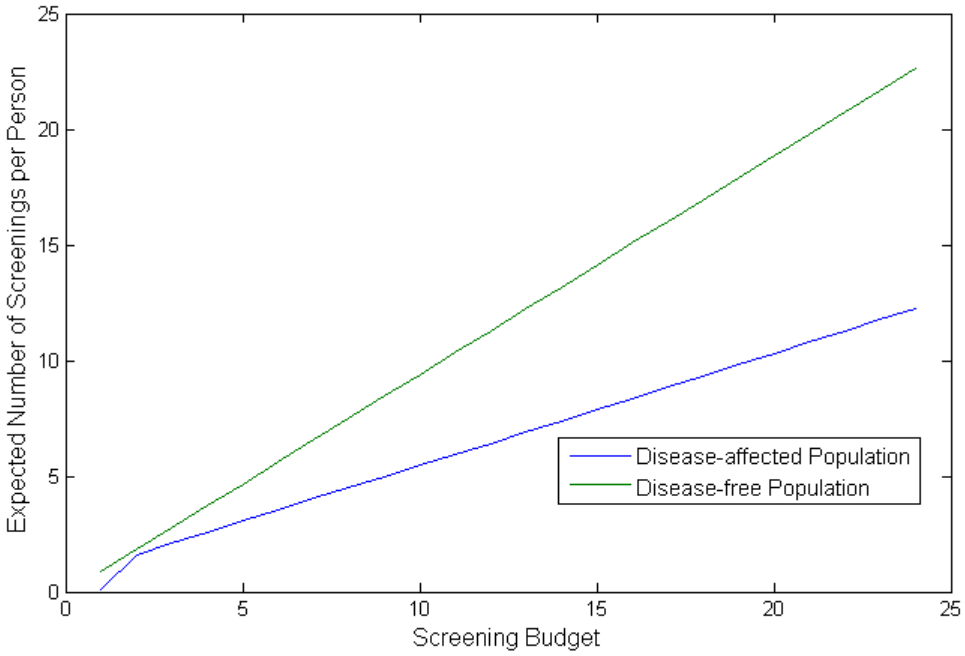

Figure 6.6: Expected Number of Screenings under Optimal Policies

As depicted, both populations appear to consist of a fairly linear relationship between screening budget and the average number of screenings their individuals will experience. For the disease-free population, screening budget is almost identical to the actual number of screenings. This is because in reality not many women will die between ages 40 and 64 (our assumed age range for the preclinical breast cancer incidence) and as a result, almost every woman ends up going on all the planned screening exams. As for the disease-affected population, the ratio between expected number of screenings and screening budget is approximately 1 to 2. Recall an individual may stop screening at any time due to occurrence of clinical symptoms or due to screen-detection of the disease.

As a result, in evaluating the real benefits of a screening programme, one needs not only to beware of the very big number of redundant screenings performed

to people who are never to develop the disease, but also, to the "diluted" number of screenings even for the relevant group, thanks to the drastic ratio between the "nominal" screening budget and the "effective" number of screenings per person.

6.3.2 Expected Lead Time

Our last investigation is on the expected lead time of a screening policy for the disease-affected population. Indeed, the expected lead time (denote this by "ELT") has been used in numerous studies as performance measure of screening policies (e.g. Zelen and Feinleib (1969), Walter and Day (1983), Parmigiani (1993), Shen and Zelen (1999), Kafadar and Prorok (2009)). We first derive this quantity as follows.

Proposition 10 For screening policy $\boldsymbol{\tau} = \{\tau_1, \tau_2, ..., \tau_n\}$, and for a disease-affected population with f and g respectively as disease-free and preclinical sojourn time densities and β as screening sensitivity, the expected lead time is:

$$ELT(\boldsymbol{\tau}) = (1-\beta)\sum_{i=0}^{n-1}\sum_{j=i+1}^{n}\sum_{k=i+1}^{j}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}}\int_{\tau_j-u}^{\tau_{j+1}-u}(u+s-\tau_k)g(s)f(u)dsdu.$$

Proof:

$$\begin{aligned}
ELT(\boldsymbol{\tau}) &= \sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}} E[\text{Lead Time} \mid X_0 = u]f(u)du \\
&= \sum_{i=0}^{n-1}\sum_{j=i+1}^{n}\int_{\tau_i}^{\tau_{i+1}}\int_{\tau_j-u}^{\tau_{j+1}-u} E[\text{Lead Time} \mid X_0 = u, X_1 = s]g(s)f(u)dsdu \\
&= \sum_{i=0}^{n-1}\sum_{j=i+1}^{n}\int_{\tau_i}^{\tau_{i+1}}\int_{\tau_j-u}^{\tau_{j+1}-u}\sum_{k=i+1}^{j}(1-\beta)\beta^{k-i-1}(u+s-\tau_k)g(s)f(u)dsdu \\
&= (1-\beta)\sum_{i=0}^{n-1}\sum_{j=i+1}^{n}\sum_{k=i+1}^{j}\beta^{k-i-1}\int_{\tau_i}^{\tau_{i+1}}\int_{\tau_j-u}^{\tau_{j+1}-u}(u+s-\tau_k)g(s)f(u)dsdu.
\end{aligned}$$

■

In particular, when X_1 is exponential and $\beta = 0$, we have the following result.

Proposition 11 If $X_1 \sim Exp(\lambda)$ and $\beta = 0$, then $ELT(\boldsymbol{\tau})$ equals $\frac{1}{\lambda}P(\boldsymbol{\tau})$ and is therefore also maximized by $\boldsymbol{\sigma}$.

Proof:

$$\begin{aligned}
ELT(\boldsymbol{\tau}) &= \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} \int_{\tau_{i+1}-u}^{\infty} (s + u - \tau_{i+1}) g(s) ds f(u) du \\
&= \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} \left\{ (u - \tau_{i+1}) e^{-\lambda(\tau_{i+1}-u)} - \int_{\tau_{i+1}-u}^{\infty} s(e^{-\lambda s})' ds \right\} f(u) du \\
&= \frac{1}{\lambda} \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} \int_{\tau_{i+1}-u}^{\infty} \lambda e^{-\lambda s} ds f(u) du \\
&= \frac{1}{\lambda} \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} e^{-\lambda(\tau_{i+1}-u)} f(u) du.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
P(\boldsymbol{\tau}) &= 1 - \sum_{i=0}^{n} \int_{\tau_i}^{\tau_{i+1}} G(\tau_{i+1} - u) f(u) du \\
&= \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} [1 - G(\tau_{i+1} - u)] f(u) du \\
&= \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} e^{-\lambda(\tau_{i+1}-u)} f(u) du \\
&= \lambda \cdot ELT(\boldsymbol{\tau}).
\end{aligned}$$

■

The above result follows from the memoryless property of exponential distribution. Indeed, knowing that the disease is captured by a screening, the lead time, i.e. the remaining time in the preclinical state, is but a new exponential quantity.

As lead time is 0 for cases that are missed by screening, the expected lead time is simply the probability of detection times the mean of the exponential X_1. The next corollary follows.

Corollary 3 If $X_0 \sim Unif(0, b)$, $X_1 \sim Exp(\lambda)$, and $\beta = 0$, then the expected lead time is maximized by the equally spaced policy $\boldsymbol{\sigma}$, and $ELT(\boldsymbol{\sigma}) = \frac{n}{b\lambda^2}(1 - e^{-\frac{b\lambda}{n}})$.

Proof: Clear from Propositions 7 and 11.

■

7. CONCLUSIONS

In this dissertation, we have studied the problem of how to schedule a sequence of screening times over a person's lifetime in order to maximize the chance of capturing a disease while preclinical. Our main result is the proof of uni-modality of the objective function, by which any problem instance in practice is guaranteed to be solved optimally with a greedy-search algorithm.

In our numerical experiments we have found that the variances of both the disease-free and the preclinical sojourn times have large impacts on the performance of the optimal screening policy. The application of our model to breast cancer screening further reveals that the equally spaced screenings policies can perform far from optimal, when the preclinical incidence is non-uniform and when the two sojourn time distributions have small variances. We further found with our breast cancer screening model that the disease-free population in practice is screened many more times than the disease-affected population. We argue that without convincing practical evidence about the underlying disease progression and screening sensitivity, we should remain alert about the effectiveness of our current guidelines.

We consider several directions of future work valuable. First, from the modelling's perspective, it will be beneficial to relax the independence assumptions (between disease-free and preclinical sojourn times, and between screening sensitivity and preclinical duration at the time of screening), in order to handle more general cases. We note that such assumptions played a crucial role for our proofs, and we expect more specific distributions and/or models to be assumed in order to attain good analytical results.

Also, it may be interesting to model the disease development (e.g. tumor

growth) in the preclinical state by some stochastic process models as opposed to a simple sojourn time as in our approach. The challenge to this end is on the one hand that the model will become much more intricate to handle, while on the other, there has been rather limited data from practice to validate/parameterize such models.

Thirdly, other optimization criteria may as well be considered. In this work, we have shown that maximizing the expected lead time is equivalent to maximizing the probability of detection for our basic breast cancer model. However, this result is not easily generalized to cases with different distributions for the two sojourn times. Analytically, it will be more challenging to handle objective functions that comprise higher orders of integration such as the expected lead time. Additional techniques must be developed to tackle such harder problems.

Last but not least, in light of the drastic difference on the expected number of screenings between the disease-free and the disease-affected populations, it will be of great economic value to consider tailored screening policies for populations with varying risk factors. Take breast cancer again for example, certain genetic markers (e.g. BRAC1, BRAC2) are known to distinguish women's risk profiles significantly. The question remains on how we can effectively collect data to characterize the various risk groups and how to communicate any tailored yet distinctive screening policies to the public.

In short, despite the almost half-century history of quantitative research on preclinical disease screening, many more significant and interesting results are yet to be reaped.

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APPENDIX A. MODEL OUTPUTS

Table A.1: Model Outputs - $X_0 \sim Unif(40, 64), X_1 \sim Exp(1/3), \beta = 0$

Scr.	Perf.	Optimal Screening Policies																							
Bud.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	12%	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	25%	52.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	35%	48.0	56.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	43%	46.0	52.0	58.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	50%	44.8	49.6	54.4	59.2	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	55%	44.0	48.0	52.0	56.0	60.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	60%	43.4	46.9	50.3	53.7	57.1	60.6	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	63%	43.0	46.0	49.0	52.0	55.0	58.0	61.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	66%	42.7	45.3	48.0	50.7	53.3	56.0	58.7	61.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	69%	42.4	44.8	47.2	49.6	52.0	54.4	56.8	59.2	61.6	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	71%	42.2	44.4	46.5	48.7	50.9	53.1	55.3	57.5	59.6	61.8	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	73%	42.0	44.0	46.0	48.0	50.0	52.0	54.0	56.0	58.0	60.0	62.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-
13	75%	41.8	43.7	45.5	47.4	49.2	51.1	52.9	54.8	56.6	58.5	60.3	62.2	64.0	-	-	-	-	-	-	-	-	-	-	-
14	76%	41.7	43.4	45.1	46.9	48.6	50.3	52.0	53.7	55.4	57.1	58.9	60.6	62.3	64.0	-	-	-	-	-	-	-	-	-	-
15	78%	41.6	43.2	44.8	46.4	48.0	49.6	51.2	52.8	54.4	56.0	57.6	59.2	60.8	62.4	64.0	-	-	-	-	-	-	-	-	-
16	79%	41.5	43.0	44.5	46.0	47.5	49.0	50.5	52.0	53.5	55.0	56.5	58.0	59.5	61.0	62.5	64.0	-	-	-	-	-	-	-	-
17	80%	41.4	42.8	44.2	45.6	47.1	48.5	49.9	51.3	52.7	54.1	55.5	56.9	58.4	59.8	61.2	62.6	64.0	-	-	-	-	-	-	-
18	81%	41.3	42.7	44.0	45.3	46.7	48.0	49.3	50.7	52.0	53.3	54.7	56.0	57.3	58.7	60.0	61.3	62.7	64.0	-	-	-	-	-	-
19	82%	41.3	42.5	43.8	45.1	46.3	47.6	48.8	50.1	51.4	52.6	53.9	55.2	56.4	57.7	58.9	60.2	61.5	62.7	64.0	-	-	-	-	-
20	82%	41.2	42.4	43.6	44.8	46.0	47.2	48.4	49.6	50.8	52.0	53.2	54.4	55.6	56.8	58.0	59.2	60.4	61.6	62.8	64.0	-	-	-	-
21	83%	41.1	42.3	43.4	44.6	45.7	46.9	48.0	49.1	50.3	51.4	52.6	53.7	54.9	56.0	57.1	58.3	59.4	60.6	61.7	62.9	64.0	-	-	-
22	84%	41.1	42.2	43.3	44.4	45.5	46.5	47.6	48.7	49.8	50.9	52.0	53.1	54.2	55.3	56.4	57.5	58.5	59.6	60.7	61.8	62.9	64.0	-	-
23	84%	41.0	42.1	43.1	44.2	45.2	46.3	47.3	48.3	49.4	50.4	51.5	52.5	53.6	54.6	55.7	56.7	57.7	58.8	59.8	60.9	61.9	63.0	64.0	-
24	85%	41.0	42.0	43.0	44.0	45.0	46.0	47.0	48.0	49.0	50.0	51.0	52.0	53.0	54.0	55.0	56.0	57.0	58.0	59.0	60.0	61.0	62.0	63.0	64.0

Table A.2: Model Outputs - $X_0 \sim Unif(40, 64), X_1 \sim Exp(1/3), \beta = 0.2$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	10%	10%	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	20%	20%	52.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	28%	28%	48.4	56.2	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	35%	35%	46.5	52.3	58.2	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	41%	41%	45.3	50.0	54.7	59.4	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	46%	46%	44.4	48.4	52.3	56.3	60.2	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	50%	50%	43.9	47.2	50.6	54.0	57.4	60.8	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	54%	54%	43.4	46.4	49.4	52.3	55.3	58.3	61.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	57%	57%	43.1	45.7	48.4	51.0	53.7	56.3	59.0	61.6	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	60%	60%	42.8	45.2	47.6	49.9	52.3	54.7	57.1	59.5	61.9	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	62%	62%	42.5	44.7	46.9	49.1	51.2	53.4	55.6	57.8	60.0	62.1	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	64%	65%	42.3	44.3	46.3	48.3	50.3	52.3	54.3	56.3	58.3	60.3	62.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-
13	66%	66%	42.2	44.0	45.9	47.7	49.6	51.4	53.3	55.1	57.0	58.8	60.6	62.5	64.0	-	-	-	-	-	-	-	-	-	-	-
14	68%	68%	42.0	43.7	45.5	47.2	48.9	50.6	52.3	54.1	55.8	57.5	59.2	60.9	62.6	64.0	-	-	-	-	-	-	-	-	-	-
15	70%	70%	41.9	43.5	45.1	46.7	48.3	49.9	51.5	53.1	54.7	56.3	58.0	59.6	61.2	62.8	64.0	-	-	-	-	-	-	-	-	-
16	71%	71%	41.8	43.3	44.8	46.3	47.8	49.3	50.8	52.3	53.8	55.3	56.9	58.4	59.9	61.4	62.9	64.0	-	-	-	-	-	-	-	-
17	72%	72%	41.7	43.1	44.5	45.9	47.4	48.8	50.2	51.6	53.0	54.5	55.9	57.3	58.7	60.1	61.6	63.0	64.0	-	-	-	-	-	-	-
18	73%	74%	41.6	42.9	44.3	45.6	47.0	48.3	49.7	51.0	52.3	53.7	55.0	56.4	57.7	59.0	60.4	61.7	63.1	64.0	-	-	-	-	-	-
19	75%	75%	41.5	42.8	44.1	45.3	46.6	47.9	49.2	50.4	51.7	53.0	54.2	55.5	56.8	58.1	59.3	60.6	61.9	63.1	64.0	-	-	-	-	-
20	76%	76%	41.4	42.7	43.9	45.1	46.3	47.5	48.7	49.9	51.1	52.3	53.5	54.8	56.0	57.2	58.4	59.6	60.8	62.0	63.2	64.0	-	-	-	-
21	76%	77%	41.4	42.5	43.7	44.8	46.0	47.1	48.3	49.5	50.6	51.8	52.9	54.1	55.2	56.4	57.5	58.7	59.8	61.0	62.1	63.3	64.0	-	-	-
22	77%	77%	41.3	42.4	43.5	44.6	45.7	46.8	47.9	49.0	50.1	51.2	52.3	53.4	54.5	55.6	56.7	57.8	58.9	60.0	61.1	62.2	63.3	64.0	-	-
23	78%	78%	41.3	42.3	43.4	44.4	45.5	46.5	47.6	48.6	49.7	50.8	51.8	52.9	53.9	55.0	56.0	57.1	58.1	59.2	60.2	61.3	62.3	63.4	64.0	-
24	79%	79%	41.2	42.2	43.2	44.2	45.3	46.3	47.3	48.3	49.3	50.3	51.3	52.3	53.3	54.4	55.4	56.4	57.4	58.4	59.4	60.4	61.4	62.4	63.5	64.0

Table A.3: Model Outputs - $X_0 \sim Unif(40, 64), X_1 \sim Exp(1/3), \beta = 0.4$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	7%	7%	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	15%	15%	52.8	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	21%	21%	49.0	56.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	27%	27%	47.1	52.8	58.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	32%	32%	45.9	50.5	55.1	59.7	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	36%	36%	45.0	48.9	52.8	56.6	60.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	40%	40%	44.4	47.8	51.1	54.4	57.8	61.1	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	44%	44%	44.0	46.9	49.8	52.8	55.7	58.6	61.6	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	47%	47%	43.6	46.2	48.8	51.5	54.1	56.7	59.3	61.9	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	49%	49%	43.3	45.7	48.0	50.4	52.8	55.1	57.5	59.9	62.2	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	52%	52%	43.0	45.2	47.4	49.5	51.7	53.8	56.0	58.2	60.3	62.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	54%	54%	42.8	44.8	46.8	48.8	50.8	52.8	54.8	56.7	58.7	60.7	62.7	64.0	-	-	-	-	-	-	-	-	-	-	-	-
13	56%	56%	42.6	44.5	46.3	48.2	50.0	51.8	53.7	55.5	57.4	59.2	61.1	62.9	64.0	-	-	-	-	-	-	-	-	-	-	-
14	58%	58%	42.5	44.2	45.9	47.6	49.3	51.1	52.8	54.5	56.2	57.9	59.6	61.3	63.1	64.0	-	-	-	-	-	-	-	-	-	-
15	60%	60%	42.3	43.9	45.5	47.1	48.8	50.4	52.0	53.6	55.2	56.8	58.4	60.0	61.6	63.2	64.0	-	-	-	-	-	-	-	-	-
16	61%	61%	42.2	43.7	45.2	46.7	48.2	49.8	51.3	52.8	54.3	55.8	57.3	58.8	60.3	61.8	63.3	64.0	-	-	-	-	-	-	-	-
17	63%	63%	42.1	43.5	44.9	46.4	47.8	49.2	50.6	52.1	53.5	54.9	56.3	57.7	59.2	60.6	62.0	63.4	64.0	-	-	-	-	-	-	-
18	64%	64%	42.0	43.3	44.7	46.0	47.4	48.7	50.1	51.4	52.8	54.1	55.5	56.8	58.2	59.5	60.8	62.2	63.5	64.0	-	-	-	-	-	-
19	65%	66%	41.9	43.2	44.5	45.7	47.0	48.3	49.6	50.8	52.1	53.4	54.7	56.0	57.2	58.5	59.8	61.1	62.4	63.6	64.0	-	-	-	-	-
20	66%	67%	41.8	43.0	44.3	45.5	46.7	47.9	49.1	50.3	51.6	52.8	54.0	55.2	56.4	57.6	58.8	60.1	61.3	62.5	63.7	64.0	-	-	-	-
21	67%	68%	41.7	42.9	44.1	45.2	46.4	47.5	48.7	49.9	51.0	52.2	53.3	54.5	55.7	56.8	58.0	59.1	60.3	61.5	62.6	63.8	64.0	-	-	-
22	68%	69%	41.7	42.8	43.9	45.0	46.1	47.2	48.3	49.4	50.5	51.7	52.8	53.9	55.0	56.1	57.2	58.3	59.4	60.5	61.6	62.8	63.9	64.0	-	-
23	69%	70%	41.6	42.7	43.7	44.8	45.9	46.9	48.0	49.0	50.1	51.2	52.2	53.3	54.4	55.4	56.5	57.5	58.6	59.7	60.7	61.8	62.9	63.9	64.0	-
24	70%	71%	41.5	42.6	43.6	44.6	45.6	46.6	47.7	48.7	49.7	50.7	51.7	52.8	53.8	54.8	55.8	56.8	57.9	58.9	59.9	60.9	61.9	63.0	64.0	64.0

Table A.4: Model Outputs - $X_0 \sim Unif(40, 64), X_1 \sim Exp(1/3), \beta = 0.8$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	2%	2%	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	5%	5%	54.4	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	7%	7%	51.1	57.7	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	9%	10%	49.4	54.4	59.4	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	11%	12%	48.3	52.4	56.5	60.6	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	13%	14%	47.4	50.9	54.4	57.9	61.4	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	15%	16%	46.8	49.9	52.9	55.9	59.0	62.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	17%	17%	46.3	49.0	51.7	54.4	57.1	59.8	62.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	19%	19%	45.9	48.4	50.8	53.2	55.6	58.0	60.5	62.9	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	20%	21%	45.6	47.8	50.0	52.2	54.4	56.6	58.8	61.0	63.2	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	22%	22%	45.3	47.3	49.3	51.4	53.4	55.4	57.5	59.5	61.5	63.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	23%	24%	45.0	46.9	48.8	50.7	52.5	54.4	56.3	58.2	60.0	61.9	63.8	64.0	-	-	-	-	-	-	-	-	-	-	-	-
13	25%	26%	44.8	46.5	48.3	50.0	51.8	53.5	55.3	57.0	58.8	60.5	62.3	64.0	64.0	-	-	-	-	-	-	-	-	-	-	-
14	26%	27%	44.6	46.2	47.8	49.5	51.1	52.7	54.3	56.0	57.6	59.2	60.9	62.5	64.0	64.0	-	-	-	-	-	-	-	-	-	-
15	27%	28%	44.4	45.9	47.4	49.0	50.5	52.0	53.5	55.1	56.6	58.1	59.6	61.2	62.7	64.0	64.0	-	-	-	-	-	-	-	-	-
16	29%	30%	44.2	45.6	47.1	48.5	49.9	51.4	52.8	54.2	55.7	57.1	58.6	60.0	61.4	62.9	64.0	64.0	-	-	-	-	-	-	-	-
17	30%	31%	44.0	45.4	46.8	48.1	49.5	50.8	52.2	53.5	54.9	56.2	57.6	59.0	60.3	61.7	63.0	64.0	64.0	-	-	-	-	-	-	-
18	31%	32%	43.9	45.2	46.5	47.8	49.0	50.3	51.6	52.9	54.2	55.5	56.7	58.0	59.3	60.6	61.9	63.2	64.0	64.0	-	-	-	-	-	-
19	32%	33%	43.8	45.0	46.2	47.4	48.6	49.9	51.1	52.3	53.5	54.8	56.0	57.2	58.4	59.6	60.9	62.1	63.3	64.0	64.0	-	-	-	-	-
20	33%	35%	43.6	44.8	46.0	47.1	48.3	49.5	50.6	51.8	53.0	54.1	55.3	56.4	57.6	58.8	59.9	61.1	62.3	63.4	64.0	64.0	-	-	-	-
21	35%	36%	43.5	44.6	45.8	46.9	48.0	49.1	50.2	51.3	52.4	53.5	54.6	55.8	56.9	58.0	59.1	60.2	61.3	62.4	63.5	64.0	64.0	-	-	-
22	36%	37%	43.4	44.5	45.6	46.6	47.7	48.7	49.8	50.9	51.9	53.0	54.1	55.1	56.2	57.3	58.3	59.4	60.5	61.5	62.6	63.6	64.0	64.0	-	-
23	37%	38%	43.3	44.3	45.4	46.4	47.4	48.4	49.4	50.5	51.5	52.5	53.5	54.6	55.6	56.6	57.6	58.6	59.7	60.7	61.7	62.7	63.7	64.0	64.0	-
24	38%	39%	43.2	44.2	45.2	46.2	47.2	48.1	49.1	50.1	51.1	52.1	53.0	54.0	55.0	56.0	57.0	57.9	58.9	59.9	60.9	61.9	62.9	63.8	64.0	64.0

Table A.5: Model Outputs - $X_0 \sim Unif(40, 64), X_1 \sim Exp(1/3), \beta = 0.99$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0%	0%	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	0%	0%	52.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	0%	0%	55.2	56.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	0%	0%	54.9	57.8	58.9	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	1%	1%	54.6	57.0	59.8	60.8	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	1%	1%	54.2	56.4	58.7	61.3	62.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	1%	1%	53.3	55.0	56.9	59.1	59.1	61.6	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	1%	1%	53.1	54.8	56.5	58.2	60.4	60.4	62.5	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	1%	1%	52.8	54.4	56.1	57.7	59.3	61.5	61.5	63.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	1%	1%	52.5	54.0	55.5	56.9	58.4	59.9	61.4	62.3	64.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	1%	1%	52.3	53.7	55.1	56.5	57.9	59.3	60.8	62.4	62.9	64.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	1%	1%	52.0	53.3	54.6	55.9	57.2	58.5	59.8	61.2	62.6	63.2	64.0	64.0	-	-	-	-	-	-	-	-	-	-	-	-
13	1%	2%	51.8	53.0	54.2	55.4	56.7	57.9	59.1	60.3	61.5	62.9	63.8	64.0	64.0	-	-	-	-	-	-	-	-	-	-	-
14	2%	2%	51.6	52.7	53.9	55.0	56.1	57.3	58.4	59.6	60.8	62.2	62.4	63.5	64.0	64.0	-	-	-	-	-	-	-	-	-	-
15	2%	2%	51.4	52.4	53.5	54.6	55.7	56.8	57.8	58.9	60.0	61.1	62.5	62.9	64.0	64.0	64.0	-	-	-	-	-	-	-	-	-
16	2%	2%	51.2	52.2	53.2	54.2	55.2	56.2	57.2	58.2	59.3	60.3	61.3	62.4	62.9	64.0	64.0	64.0	-	-	-	-	-	-	-	-
17	2%	2%	51.0	52.0	52.9	53.9	54.9	55.8	56.8	57.8	58.7	59.7	60.7	61.6	62.7	63.4	64.0	64.0	64.0	-	-	-	-	-	-	-
18	2%	2%	50.8	51.8	52.7	53.6	54.5	55.5	56.4	57.3	58.2	59.2	60.1	61.1	62.3	62.5	63.4	64.0	64.0	64.0	-	-	-	-	-	-
19	2%	2%	50.7	51.6	52.4	53.3	54.2	55.1	56.0	56.8	57.7	58.6	59.5	60.4	61.2	62.1	63.0	63.8	64.0	64.0	64.0	-	-	-	-	-
20	2%	2%	50.5	51.4	52.2	53.1	53.9	54.8	55.6	56.5	57.3	58.2	59.0	59.8	60.9	62.0	62.1	62.9	63.7	64.0	64.0	64.0	-	-	-	-
21	2%	3%	50.4	51.2	52.0	52.8	53.6	54.4	55.2	56.0	56.8	57.6	58.4	59.2	60.1	60.9	61.7	62.4	63.2	64.0	64.0	64.0	64.0	-	-	-
22	2%	3%	50.2	51.0	51.8	52.5	53.3	54.1	54.9	55.6	56.4	57.2	58.0	58.7	59.5	60.2	61.3	61.8	62.5	63.2	64.0	64.0	64.0	64.0	-	-
23	3%	3%	50.1	50.9	51.6	52.4	53.1	53.9	54.6	55.4	56.1	56.9	57.6	58.4	59.1	59.9	60.6	61.4	62.3	62.7	63.5	64.0	64.0	64.0	64.0	-
24	3%	3%	50.0	50.8	51.5	52.2	52.9	53.7	54.4	55.1	55.8	56.6	57.3	58.0	58.7	59.5	60.2	61.1	61.9	62.2	62.8	63.5	64.0	64.0	64.0	64.0

Table A.6: Model Outputs - $X_0 \sim 40 + Gamma(9, 4/3), X_1 \sim Gamma(3, 1), \beta = 0.2$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	2%	23%	52.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	25%	39%	51.1	54.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	29%	51%	50.2	52.9	55.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	39%	59%	49.5	51.8	54.1	56.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	47%	66%	49.0	51.1	53.0	55.1	57.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	54%	71%	48.6	50.5	52.2	53.9	55.9	58.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	60%	75%	48.2	50.0	51.5	53.1	54.7	56.6	59.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	65%	78%	47.9	49.6	51.0	52.4	53.8	55.4	57.3	59.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	69%	81%	47.7	49.2	50.6	51.8	53.1	54.5	56.0	57.8	60.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	73%	83%	47.4	48.9	50.2	51.4	52.6	53.8	55.1	56.6	58.4	60.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	76%	85%	47.2	48.7	49.9	51.0	52.1	53.2	54.4	55.7	57.1	58.8	61.2	-	-	-	-	-	-	-	-	-	-	-	-	-
12	78%	87%	47.0	48.4	49.6	50.6	51.7	52.7	53.8	54.9	56.2	57.6	59.3	61.7	-	-	-	-	-	-	-	-	-	-	-	-
13	80%	88%	46.9	48.2	49.3	50.3	51.3	52.3	53.2	54.3	55.4	56.6	58.0	59.7	62.1	-	-	-	-	-	-	-	-	-	-	-
14	82%	89%	46.7	48.0	49.1	50.0	51.0	51.9	52.8	53.8	54.8	55.8	57.0	58.4	60.1	62.4	-	-	-	-	-	-	-	-	-	-
15	84%	90%	46.6	47.8	48.8	49.8	50.7	51.5	52.4	53.3	54.2	55.2	56.3	57.4	58.8	60.5	62.8	-	-	-	-	-	-	-	-	-
16	85%	91%	46.5	47.7	48.7	49.5	50.4	51.2	52.0	52.9	53.7	54.6	55.6	56.7	57.8	59.2	60.8	63.1	-	-	-	-	-	-	-	-
17	87%	92%	46.3	47.5	48.5	49.3	50.1	50.9	51.7	52.5	53.3	54.2	55.0	56.0	57.0	58.2	59.5	61.1	63.4	-	-	-	-	-	-	-
18	88%	92%	41.4	46.3	47.5	48.5	49.3	50.1	50.9	51.7	52.5	53.3	54.2	55.0	56.0	57.0	58.2	59.5	61.1	63.4	-	-	-	-	-	-
19	89%	93%	41.3	46.2	47.4	48.3	49.1	49.9	50.7	51.4	52.2	53.0	53.7	54.6	55.4	56.4	57.4	58.5	59.8	61.4	63.7	-	-	-	-	-
20	90%	93%	41.2	46.1	47.2	48.1	49.0	49.7	50.4	51.2	51.9	52.6	53.4	54.1	54.9	55.8	56.7	57.7	58.8	60.1	61.7	64.0	-	-	-	-
21	90%	94%	41.2	46.0	47.1	48.0	48.8	49.5	50.2	50.9	51.6	52.3	53.0	53.7	54.5	55.3	56.1	57.0	58.0	59.1	60.4	62.0	64.3	-	-	-
22	91%	94%	41.1	45.9	47.0	47.9	48.6	49.3	50.0	50.7	51.4	52.0	52.7	53.4	54.1	54.8	55.6	56.4	57.3	58.3	59.4	60.7	62.3	64.5	-	-
23	92%	95%	41.1	45.8	46.9	47.7	48.5	49.2	49.8	50.5	51.1	51.8	52.4	53.1	53.7	54.4	55.2	55.9	56.7	57.6	58.6	59.7	61.0	62.5	64.8	-
24	92%	95%	41.0	45.7	46.8	47.6	48.3	49.0	49.7	50.3	50.9	51.5	52.2	52.8	53.4	54.1	54.8	55.5	56.2	57.0	57.9	58.9	60.0	61.2	62.8	65.0

Table A.7: Model Outputs - $X_0 \sim 40 + Gamma(9, 4/3), X_1 \sim Gamma(3, 1), \beta = 0.6$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1%	12%	52.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	12%	21%	51.5	54.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	15%	28%	50.8	52.8	55.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	20%	34%	50.3	52.0	53.7	55.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	24%	40%	49.9	51.4	52.8	54.4	56.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	28%	44%	49.5	50.9	52.2	53.5	54.9	56.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	32%	48%	49.3	50.6	51.7	52.9	54.1	55.5	57.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	36%	52%	49.0	50.2	51.3	52.4	53.4	54.6	55.9	57.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	39%	55%	48.8	50.0	51.0	52.0	52.9	53.9	55.0	56.3	58.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	42%	58%	48.6	49.7	50.7	51.6	52.5	53.4	54.4	55.4	56.7	58.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	45%	61%	48.4	49.5	50.4	51.3	52.1	53.0	53.8	54.8	55.8	57.0	58.7	-	-	-	-	-	-	-	-	-	-	-	-	-
12	48%	63%	48.3	49.3	50.2	51.0	51.8	52.6	53.4	54.2	55.1	56.1	57.4	59.0	-	-	-	-	-	-	-	-	-	-	-	-
13	50%	65%	48.1	49.1	50.0	50.7	51.5	52.2	53.0	53.8	54.6	55.5	56.5	57.7	59.3	-	-	-	-	-	-	-	-	-	-	-
14	53%	67%	48.0	49.0	49.8	50.5	51.2	51.9	52.6	53.4	54.1	54.9	55.8	56.8	57.9	59.5	-	-	-	-	-	-	-	-	-	-
15	55%	69%	47.9	48.8	49.6	50.3	51.0	51.7	52.3	53.0	53.7	54.4	55.2	56.1	57.1	58.2	59.8	-	-	-	-	-	-	-	-	-
16	57%	71%	47.8	48.7	49.4	50.1	50.8	51.4	52.1	52.7	53.4	54.0	54.8	55.5	56.4	57.3	58.5	60.0	-	-	-	-	-	-	-	-
17	59%	71%	41.4	47.8	48.7	49.4	50.1	50.8	51.4	52.1	52.7	53.4	54.0	54.8	55.5	56.4	57.3	58.5	60.0	-	-	-	-	-	-	-
18	61%	72%	41.3	47.6	48.5	49.3	49.9	50.6	51.2	51.8	52.4	53.0	53.7	54.3	55.0	55.8	56.6	57.6	58.7	60.2	-	-	-	-	-	-
19	62%	74%	41.3	47.5	48.4	49.1	49.8	50.4	51.0	51.6	52.2	52.8	53.4	54.0	54.6	55.3	56.1	56.9	57.8	58.9	60.5	-	-	-	-	-
20	64%	75%	41.2	47.4	48.3	49.0	49.6	50.2	50.8	51.4	51.9	52.5	53.1	53.7	54.3	54.9	55.6	56.3	57.1	58.0	59.2	60.7	-	-	-	-
21	66%	76%	41.2	47.3	48.2	48.9	49.5	50.1	50.6	51.2	51.7	52.3	52.8	53.4	53.9	54.5	55.2	55.8	56.6	57.4	58.3	59.4	60.9	-	-	-
22	67%	77%	41.1	47.3	48.1	48.7	49.3	49.9	50.5	51.0	51.5	52.0	52.6	53.1	53.6	54.2	54.8	55.4	56.1	56.8	57.6	58.5	59.6	61.1	-	-
23	68%	78%	41.1	47.2	48.0	48.6	49.2	49.8	50.3	50.8	51.3	51.8	52.3	52.8	53.4	53.9	54.5	55.0	55.6	56.3	57.0	57.8	58.7	59.8	61.3	-
24	70%	79%	41.0	47.1	47.9	48.5	49.1	49.6	50.2	50.7	51.1	51.6	52.1	52.6	53.1	53.6	54.1	54.7	55.3	55.9	56.5	57.2	58.0	58.9	60.0	61.4

Table A.8: Model Outputs - $X_0 \sim 40 + Gamma(9, 4/3), X_1 \sim Gamma(3, 1), \beta = 0.9$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0%	3%	52.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	3%	6%	52.2	53.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	4%	8%	51.8	52.7	53.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	5%	10%	51.5	52.3	53.1	54.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	6%	12%	51.3	52.0	52.7	53.5	54.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	7%	15%	51.0	51.7	52.4	53.1	53.8	54.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	9%	16%	50.9	51.5	52.1	52.7	53.4	54.1	54.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	10%	18%	50.7	51.3	51.8	52.4	53.0	53.7	54.4	55.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	11%	20%	50.5	51.1	51.6	52.2	52.7	53.3	53.9	54.6	55.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	12%	22%	50.4	50.9	51.4	52.0	52.5	53.0	53.6	54.2	54.8	55.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	13%	23%	50.3	50.8	51.3	51.8	52.3	52.8	53.3	53.8	54.4	55.0	55.6	-	-	-	-	-	-	-	-	-	-	-	-	-
12	14%	25%	50.2	50.6	51.1	51.6	52.1	52.5	53.0	53.5	54.0	54.6	55.2	55.8	-	-	-	-	-	-	-	-	-	-	-	-
13	15%	27%	50.1	50.5	51.0	51.4	51.9	52.3	52.8	53.2	53.7	54.2	54.7	55.3	56.0	-	-	-	-	-	-	-	-	-	-	-
14	16%	27%	41.7	50.1	50.5	51.0	51.4	51.9	52.3	52.8	53.2	53.7	54.2	54.7	55.3	56.0	-	-	-	-	-	-	-	-	-	-
15	17%	28%	41.6	50.0	50.4	50.8	51.3	51.7	52.1	52.6	53.0	53.4	53.9	54.4	54.9	55.5	56.1	-	-	-	-	-	-	-	-	-
16	18%	29%	41.5	49.9	50.3	50.7	51.1	51.6	52.0	52.4	52.8	53.2	53.6	54.1	54.6	55.1	55.7	56.3	-	-	-	-	-	-	-	-
17	19%	31%	41.4	49.8	50.2	50.6	51.0	51.4	51.8	52.2	52.6	53.0	53.4	53.8	54.3	54.7	55.2	55.8	56.4	-	-	-	-	-	-	-
18	20%	32%	41.4	49.7	50.1	50.5	50.9	51.3	51.7	52.0	52.4	52.8	53.2	53.6	54.0	54.4	54.9	55.4	55.9	56.5	-	-	-	-	-	-
19	21%	33%	41.3	49.6	50.0	50.4	50.8	51.2	51.5	51.9	52.3	52.6	53.0	53.4	53.7	54.2	54.6	55.0	55.5	56.1	56.7	-	-	-	-	-
20	22%	35%	41.2	49.5	49.9	50.3	50.7	51.0	51.4	51.8	52.1	52.5	52.8	53.2	53.5	53.9	54.3	54.7	55.2	55.7	56.2	56.8	-	-	-	-
21	23%	36%	41.2	49.4	49.8	50.2	50.6	50.9	51.3	51.6	52.0	52.3	52.6	53.0	53.3	53.7	54.1	54.5	54.9	55.3	55.8	56.3	56.9	-	-	-
22	24%	37%	41.1	49.4	49.8	50.1	50.5	50.8	51.2	51.5	51.8	52.2	52.5	52.8	53.2	53.5	53.9	54.2	54.6	55.0	55.5	55.9	56.5	57.0	-	-
23	25%	38%	41.1	49.3	49.7	50.0	50.4	50.7	51.1	51.4	51.7	52.0	52.3	52.7	53.0	53.3	53.7	54.0	54.4	54.7	55.1	55.6	56.1	56.6	57.1	-
24	25%	39%	41.0	49.2	49.6	50.0	50.3	50.6	51.0	51.3	51.6	51.9	52.2	52.5	52.8	53.1	53.5	53.8	54.1	54.5	54.9	55.3	55.7	56.2	56.7	57.2

Table A.9: Model Outputs - $X_0 \sim 40 + Gamma(9, 4/3), X_1 \sim Exp(1/3), \beta = 0.2$

Scr.	Perf.	Perf.	Perf.	Optimal Screening Policies																						
Bud.	EI Pol.	QBI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	4%	19%	21%	53.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	23%	32%	33%	51.4	55.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	28%	41%	43%	50.4	53.1	56.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	35%	47%	49%	49.7	51.9	54.2	57.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	41%	53%	54%	49.2	51.1	53.0	55.1	58.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	46%	57%	59%	48.8	50.5	52.1	53.8	55.8	58.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	51%	60%	62%	48.4	50.0	51.5	52.9	54.5	56.4	59.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	54%	63%	65%	48.1	49.6	51.0	52.2	53.6	55.1	57.0	59.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	57%	66%	68%	47.9	49.3	50.5	51.7	52.9	54.2	55.6	57.4	60.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	60%	68%	70%	47.7	49.0	50.2	51.2	52.3	53.4	54.7	56.1	57.9	60.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	62%	70%	72%	47.5	48.8	49.8	50.9	51.8	52.9	53.9	55.1	56.5	58.3	60.8	-	-	-	-	-	-	-	-	-	-	-	-	-
12	65%	71%	73%	47.3	48.5	49.6	50.5	51.4	52.4	53.3	54.4	55.5	56.9	58.6	61.1	-	-	-	-	-	-	-	-	-	-	-	-
13	66%	73%	75%	47.2	48.3	49.3	50.2	51.1	51.9	52.8	53.8	54.8	55.9	57.2	58.9	61.4	-	-	-	-	-	-	-	-	-	-	-
14	68%	74%	76%	47.0	48.2	49.1	50.0	50.8	51.6	52.4	53.2	54.1	55.1	56.2	57.6	59.2	61.7	-	-	-	-	-	-	-	-	-	-
15	70%	75%	77%	46.9	48.0	48.9	49.7	50.5	51.3	52.0	52.8	53.6	54.5	55.5	56.6	57.9	59.5	62.0	-	-	-	-	-	-	-	-	-
16	71%	76%	78%	46.8	47.9	48.7	49.5	50.3	51.0	51.7	52.4	53.2	54.0	54.8	55.8	56.9	58.1	59.8	62.2	-	-	-	-	-	-	-	-
17	72%	77%	78%	41.4	46.8	47.9	48.7	49.5	50.3	51.0	51.7	52.4	53.2	54.0	54.8	55.8	56.9	58.1	59.8	62.2	-	-	-	-	-	-	-
18	74%	78%	79%	41.4	46.7	47.7	48.6	49.3	50.0	50.7	51.4	52.1	52.8	53.5	54.3	55.1	56.1	57.1	58.4	60.0	62.5	-	-	-	-	-	-
19	75%	79%	80%	41.3	46.6	47.6	48.4	49.1	49.8	50.5	51.1	51.8	52.4	53.1	53.8	54.6	55.4	56.3	57.4	58.6	60.3	62.7	-	-	-	-	-
20	76%	80%	81%	41.2	46.5	47.5	48.3	49.0	49.6	50.3	50.9	51.5	52.1	52.8	53.4	54.1	54.9	55.7	56.6	57.6	58.9	60.5	62.9	-	-	-	-
21	76%	81%	82%	41.2	46.4	47.4	48.1	48.8	49.5	50.1	50.7	51.3	51.8	52.4	53.1	53.7	54.4	55.1	55.9	56.8	57.9	59.1	60.7	63.1	-	-	-
22	77%	81%	82%	41.1	46.3	47.3	48.0	48.7	49.3	49.9	50.5	51.0	51.6	52.2	52.7	53.4	54.0	54.7	55.4	56.2	57.1	58.1	59.3	60.9	63.3	-	-
23	78%	82%	83%	41.0	46.2	47.2	47.9	48.6	49.2	49.7	50.3	50.8	51.4	51.9	52.5	53.0	53.6	54.2	54.9	55.6	56.4	57.3	58.3	59.5	61.1	63.4	-
24	79%	82%	84%	41.0	46.1	47.1	47.8	48.4	49.0	49.6	50.1	50.6	51.1	51.7	52.2	52.7	53.3	53.9	54.5	55.1	55.8	56.6	57.5	58.5	59.7	61.3	63.6

Table A.10: Model Outputs - $X_0 \sim 40 + Gamma(3,4), X_1 \sim Exp(1/3), \beta = 0.2$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	5%	14%	51.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	19%	25%	49.1	54.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	27%	32%	47.9	51.5	56.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	34%	39%	47.1	50.1	53.3	57.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	39%	44%	46.5	49.1	51.8	54.8	58.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	44%	48%	46.1	48.4	50.7	53.1	56.0	60.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	48%	52%	45.7	47.8	49.8	51.9	54.2	57.0	60.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	51%	55%	45.4	47.4	49.2	51.0	53.0	55.3	58.0	61.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	54%	57%	45.1	47.0	48.6	50.3	52.1	54.0	56.1	58.8	62.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	57%	60%	44.9	46.6	48.2	49.7	51.3	53.0	54.8	57.0	59.6	63.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	59%	62%	44.7	46.3	47.8	49.2	50.6	52.2	53.8	55.6	57.7	60.3	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	61%	64%	44.5	46.1	47.5	48.8	50.1	51.5	52.9	54.5	56.3	58.4	61.0	64.7	-	-	-	-	-	-	-	-	-	-	-	-
13	63%	65%	44.3	45.8	47.1	48.4	49.6	50.9	52.2	53.7	55.2	57.0	59.0	61.6	65.3	-	-	-	-	-	-	-	-	-	-	-
14	64%	67%	44.2	45.6	46.9	48.1	49.2	50.4	51.6	52.9	54.3	55.8	57.6	59.6	62.1	65.8	-	-	-	-	-	-	-	-	-	-
15	66%	68%	44.1	45.4	46.6	47.8	48.9	50.0	51.1	52.3	53.6	54.9	56.4	58.1	60.2	62.7	66.3	-	-	-	-	-	-	-	-	-
16	67%	70%	43.9	45.3	46.4	47.5	48.5	49.6	50.7	51.8	52.9	54.2	55.5	57.0	58.7	60.7	63.2	66.8	-	-	-	-	-	-	-	-
17	68%	71%	43.8	45.1	46.2	47.3	48.3	49.2	50.3	51.3	52.4	53.5	54.7	56.0	57.5	59.2	61.2	63.7	67.3	-	-	-	-	-	-	-
18	69%	72%	43.7	45.0	46.0	47.0	48.0	48.9	49.9	50.9	51.9	52.9	54.0	55.2	56.6	58.0	59.7	61.6	64.1	67.7	-	-	-	-	-	-
19	70%	73%	43.6	44.9	45.9	46.8	47.8	48.7	49.6	50.5	51.4	52.4	53.5	54.6	55.7	57.0	58.5	60.1	62.1	64.6	68.1	-	-	-	-	-
20	71%	74%	43.6	44.7	45.7	46.6	47.5	48.4	49.3	50.1	51.0	52.0	52.9	54.0	55.0	56.2	57.5	58.9	60.6	62.5	65.0	68.6	-	-	-	-
21	72%	75%	43.5	44.6	45.6	46.5	47.3	48.2	49.0	49.8	50.7	51.6	52.5	53.4	54.4	55.5	56.7	57.9	59.3	61.0	62.9	65.4	68.9	-	-	-
22	73%	76%	43.4	44.5	45.5	46.3	47.1	47.9	48.7	49.5	50.4	51.2	52.0	52.9	53.9	54.9	55.9	57.1	58.3	59.7	61.4	63.3	65.7	69.3	-	-
23	74%	76%	43.3	44.4	45.3	46.2	47.0	47.7	48.5	49.3	50.1	50.9	51.7	52.5	53.4	54.3	55.3	56.3	57.5	58.7	60.1	61.7	63.7	66.1	69.7	-
24	74%	77%	43.3	44.3	45.2	46.0	46.8	47.6	48.3	49.0	49.8	50.5	51.3	52.1	52.9	53.8	54.7	55.7	56.7	57.9	59.1	60.5	62.1	64.0	66.5	70.0

Table A.11: Model Outputs - $X_0 \sim 40 + Exp(1/12), X_1 \sim Exp(1/3), \beta = 0.2$

Scr.	Perf.	Perf.	Perf.	Optimal Screening Policies																						
Bud.	EI Pol.	QBI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	4%	12%	13%	45.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	13%	20%	22%	44.0	49.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	21%	27%	28%	43.3	46.9	51.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	28%	33%	34%	42.8	45.7	49.2	54.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	34%	38%	39%	42.5	44.9	47.8	51.3	56.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	39%	42%	42%	42.2	44.3	46.7	49.6	53.2	58.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	43%	45%	46%	42.0	43.9	46.0	48.4	51.3	54.9	59.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	46%	48%	49%	41.8	43.5	45.4	47.5	50.0	52.8	56.4	61.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	49%	51%	51%	41.7	43.3	45.0	46.9	49.0	51.4	54.3	57.8	62.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	51%	53%	54%	41.6	43.0	44.6	46.3	48.2	50.3	52.7	55.6	59.2	64.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	54%	55%	56%	41.5	42.8	44.3	45.8	47.5	49.4	51.5	53.9	56.8	60.4	65.4	-	-	-	-	-	-	-	-	-	-	-	-	-
12	56%	57%	58%	41.4	42.7	44.0	45.4	47.0	48.7	50.6	52.7	55.1	58.0	61.5	66.6	-	-	-	-	-	-	-	-	-	-	-	-
13	57%	59%	59%	41.3	42.5	43.8	45.1	46.5	48.1	49.8	51.7	53.8	56.2	59.1	62.6	67.6	-	-	-	-	-	-	-	-	-	-	-
14	59%	60%	61%	41.3	42.4	43.5	44.8	46.1	47.6	49.1	50.8	52.7	54.8	57.2	60.1	63.7	68.7	-	-	-	-	-	-	-	-	-	-
15	60%	62%	62%	41.2	42.3	43.4	44.5	45.8	47.1	48.6	50.1	51.8	53.7	55.8	58.2	61.1	64.7	69.7	-	-	-	-	-	-	-	-	-
16	61%	63%	64%	41.1	42.1	43.2	44.3	45.5	46.7	48.1	49.5	51.1	52.8	54.6	56.8	59.2	62.0	65.6	70.6	-	-	-	-	-	-	-	-
17	63%	64%	65%	41.1	42.0	43.0	44.1	45.2	46.4	47.6	49.0	50.4	52.0	53.7	55.5	57.7	60.1	62.9	66.5	71.5	-	-	-	-	-	-	-
18	64%	65%	66%	41.1	42.0	42.9	43.9	45.0	46.1	47.2	48.5	49.8	51.3	52.8	54.5	56.4	58.5	60.9	63.8	67.4	72.4	-	-	-	-	-	-
19	65%	66%	67%	41.0	41.9	42.8	43.7	44.7	45.8	46.9	48.1	49.3	50.7	52.1	53.6	55.4	57.2	59.3	61.8	64.6	68.2	73.2	-	-	-	-	-
20	65%	67%	68%	41.0	41.8	42.7	43.6	44.5	45.5	46.6	47.7	48.9	50.1	51.5	52.9	54.4	56.1	58.0	60.1	62.6	65.4	69.0	74.0	-	-	-	-
21	66%	68%	69%	40.9	41.7	42.6	43.4	44.4	45.3	46.3	47.4	48.5	49.6	50.9	52.2	53.7	55.2	56.9	58.8	60.9	63.3	66.2	69.8	74.8	-	-	-
22	67%	69%	70%	40.9	41.7	42.5	43.3	44.2	45.1	46.0	47.0	48.1	49.2	50.4	51.6	53.0	54.4	56.0	57.7	59.5	61.7	64.1	66.9	70.5	75.5	-	-
23	68%	70%	71%	40.9	41.6	42.4	43.2	44.0	44.9	45.8	46.8	47.8	48.8	49.9	51.1	52.3	53.7	55.1	56.7	58.4	60.3	62.4	64.8	67.7	71.2	76.2	-
24	68%	71%	72%	40.8	41.6	42.3	43.1	43.9	44.7	45.6	46.5	47.5	48.4	49.5	50.6	51.8	53.0	54.4	55.8	57.4	59.1	60.9	63.1	65.5	68.3	71.9	76.9

Table A.12: Model Outputs - $X_0 \sim 40 + Weibull(13.3770, 3.3035)(mean = 12, var = 16), X_1 \sim Exp(1/3), \beta = 0.2$

Scr.	Perf.	Perf.	Perf.	Optimal Screening Policies																						
Bud.	EI Pol.	QBI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	3%	17%	20%	54.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	21%	30%	32%	52.2	56.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	28%	39%	41%	50.9	53.9	57.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	36%	46%	48%	50.0	52.5	54.9	57.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	42%	51%	54%	49.3	51.6	53.6	55.7	58.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	47%	55%	58%	48.7	50.8	52.6	54.4	56.3	58.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	51%	59%	61%	48.2	50.2	51.8	53.4	55.0	56.8	59.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	54%	62%	64%	47.8	49.7	51.2	52.6	54.0	55.5	57.2	59.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	58%	65%	67%	47.5	49.3	50.7	52.0	53.2	54.5	55.9	57.6	59.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	60%	67%	69%	47.2	48.9	50.2	51.4	52.6	53.8	55.0	56.3	57.9	60.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	63%	69%	71%	46.9	48.6	49.8	51.0	52.1	53.1	54.2	55.4	56.6	58.2	60.3	-	-	-	-	-	-	-	-	-	-	-	-	-
12	65%	70%	73%	46.7	48.3	49.5	50.6	51.6	52.6	53.6	54.6	55.7	56.9	58.4	60.6	-	-	-	-	-	-	-	-	-	-	-	-
13	67%	72%	74%	46.5	48.0	49.2	50.2	51.2	52.1	53.0	54.0	55.0	56.0	57.2	58.7	60.8	-	-	-	-	-	-	-	-	-	-	-
14	69%	73%	76%	46.3	47.8	48.9	49.9	50.8	51.7	52.6	53.4	54.3	55.3	56.3	57.5	58.9	60.9	-	-	-	-	-	-	-	-	-	-
15	70%	75%	77%	46.1	47.5	48.7	49.6	50.5	51.3	52.2	53.0	53.8	54.7	55.6	56.5	57.7	59.1	61.1	-	-	-	-	-	-	-	-	-
16	72%	76%	78%	46.0	47.3	48.4	49.4	50.2	51.0	51.8	52.6	53.3	54.1	54.9	55.8	56.8	57.9	59.3	61.3	-	-	-	-	-	-	-	-
17	73%	77%	79%	45.8	47.2	48.2	49.1	49.9	50.7	51.5	52.2	52.9	53.7	54.4	55.2	56.1	57.0	58.1	59.4	61.4	-	-	-	-	-	-	-
18	74%	78%	80%	45.7	47.0	48.0	48.9	49.7	50.4	51.2	51.9	52.5	53.2	53.9	54.7	55.4	56.3	57.2	58.3	59.6	61.5	-	-	-	-	-	-
19	75%	79%	81%	45.5	46.8	47.8	48.7	49.5	50.2	50.9	51.6	52.2	52.9	53.5	54.2	54.9	55.7	56.5	57.4	58.4	59.8	61.7	-	-	-	-	-
20	76%	79%	81%	45.4	46.7	47.7	48.5	49.2	50.0	50.6	51.3	51.9	52.5	53.2	53.8	54.5	55.1	55.9	56.7	57.6	58.6	59.9	61.8	-	-	-	-
21	77%	80%	82%	45.3	46.5	47.5	48.3	49.1	49.7	50.4	51.0	51.6	52.2	52.8	53.4	54.1	54.7	55.4	56.1	56.8	57.7	58.7	60.0	61.9	-	-	-
22	78%	81%	83%	45.2	46.4	47.3	48.1	48.9	49.5	50.2	50.8	51.4	51.9	52.5	53.1	53.7	54.3	54.9	55.6	56.3	57.0	57.9	58.9	60.2	62.0	-	-
23	79%	81%	83%	45.1	46.3	47.2	48.0	48.7	49.3	50.0	50.5	51.1	51.7	52.2	52.8	53.3	53.9	54.5	55.1	55.7	56.4	57.2	58.0	59.0	60.3	62.1	-
24	79%	82%	84%	45.0	46.2	47.1	47.8	48.5	49.2	49.8	50.3	50.9	51.4	52.0	52.5	53.0	53.6	54.1	54.7	55.3	55.9	56.6	57.3	58.2	59.1	60.4	62.2

Table A.13: Model Outputs - $X_0 \sim 40 + Weibull(13.4908, 1.7915)(mean = 12, var = 48), X_1 \sim Exp(1/3), \beta = 0.2$

Scr.	Perf.	Perf.	Optimal Screening Policies																							
Bud.	EI Pol.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	5%	13%	51.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	18%	23%	49.3	54.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	27%	31%	47.9	52.0	56.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	33%	37%	46.9	50.4	53.9	58.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	39%	42%	46.2	49.2	52.2	55.4	59.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	44%	46%	45.7	48.4	50.9	53.6	56.6	60.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	48%	50%	45.2	47.7	50.0	52.3	54.8	57.7	61.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	51%	53%	44.9	47.2	49.2	51.3	53.4	55.8	58.6	62.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	54%	56%	44.6	46.7	48.6	50.5	52.4	54.4	56.7	59.4	63.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	57%	59%	44.3	46.3	48.1	49.8	51.5	53.3	55.3	57.5	60.2	63.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	59%	61%	44.1	46.0	47.6	49.2	50.8	52.4	54.2	56.1	58.2	60.8	64.5	-	-	-	-	-	-	-	-	-	-	-	-	-
12	61%	63%	43.9	45.7	47.2	48.7	50.2	51.7	53.3	54.9	56.8	58.9	61.4	65.0	-	-	-	-	-	-	-	-	-	-	-	-
13	63%	64%	43.7	45.4	46.9	48.3	49.7	51.1	52.5	54.0	55.6	57.4	59.5	62.0	65.6	-	-	-	-	-	-	-	-	-	-	-
14	64%	66%	43.5	45.2	46.6	47.9	49.2	50.5	51.8	53.2	54.7	56.3	58.0	60.0	62.5	66.0	-	-	-	-	-	-	-	-	-	-
15	66%	68%	43.4	44.9	46.3	47.6	48.8	50.0	51.3	52.5	53.9	55.3	56.8	58.6	60.5	63.0	66.5	-	-	-	-	-	-	-	-	-
16	67%	69%	43.3	44.7	46.0	47.3	48.4	49.6	50.8	52.0	53.2	54.5	55.9	57.4	59.1	61.0	63.5	66.9	-	-	-	-	-	-	-	-
17	68%	70%	43.1	44.6	45.8	47.0	48.1	49.2	50.3	51.4	52.6	53.8	55.0	56.4	57.9	59.5	61.5	63.9	67.3	-	-	-	-	-	-	-
18	69%	71%	43.0	44.4	45.6	46.7	47.8	48.9	49.9	51.0	52.1	53.2	54.3	55.6	56.9	58.4	60.0	61.9	64.3	67.7	-	-	-	-	-	-
19	70%	72%	42.9	44.3	45.4	46.5	47.5	48.5	49.5	50.5	51.6	52.6	53.7	54.9	56.1	57.4	58.8	60.4	62.3	64.7	68.0	-	-	-	-	-
20	71%	73%	42.8	44.1	45.2	46.3	47.3	48.2	49.2	50.2	51.1	52.1	53.2	54.2	55.3	56.5	57.8	59.2	60.8	62.7	65.0	68.4	-	-	-	-
21	72%	74%	42.8	44.0	45.1	46.1	47.0	48.0	48.9	49.8	50.7	51.7	52.7	53.7	54.7	55.8	57.0	58.2	59.6	61.2	63.1	65.4	68.7	-	-	-
22	73%	75%	42.7	43.9	44.9	45.9	46.8	47.7	48.6	49.5	50.4	51.3	52.2	53.1	54.1	55.1	56.2	57.4	58.6	60.0	61.6	63.4	65.7	69.0	-	-
23	74%	76%	42.6	43.8	44.8	45.7	46.6	47.5	48.4	49.2	50.1	50.9	51.8	52.7	53.6	54.6	55.6	56.6	57.8	59.0	60.4	61.9	63.7	66.0	69.3	-
24	74%	77%	42.5	43.7	44.7	45.6	46.4	47.3	48.1	48.9	49.8	50.6	51.4	52.3	53.1	54.0	55.0	56.0	57.0	58.1	59.4	60.7	62.2	64.0	66.3	69.6

Table A.14: Model Outputs - $X_0 \sim 40 + Exp(1/12), X_1 \sim Gamma(3,1), \beta = 0.2$

Scr.	Perf.	Optimal Screening Policies																							
Bud.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	15%	44.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	25%	43.5	47.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	34%	43.1	46.2	50.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	41%	42.7	45.4	48.6	52.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	46%	42.5	44.8	47.5	50.7	54.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	51%	42.3	44.4	46.8	49.4	52.6	56.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	55%	42.1	44.1	46.2	48.5	51.2	54.4	58.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	59%	42.0	43.8	45.7	47.8	50.2	52.9	56.1	60.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	62%	41.9	43.5	45.3	47.3	49.4	51.7	54.4	57.6	61.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	65%	41.8	43.3	45.0	46.8	48.7	50.8	53.2	55.9	59.1	63.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	67%	41.7	43.2	44.7	46.4	48.2	50.1	52.2	54.6	57.3	60.4	64.5	-	-	-	-	-	-	-	-	-	-	-	-	-
12	69%	41.6	43.0	44.5	46.0	47.7	49.5	51.4	53.5	55.9	58.6	61.8	65.8	-	-	-	-	-	-	-	-	-	-	-	-
13	71%	41.6	42.9	44.3	45.7	47.3	48.9	50.7	52.7	54.8	57.1	59.8	63.0	67.1	-	-	-	-	-	-	-	-	-	-	-
14	73%	41.5	42.8	44.1	45.5	46.9	48.5	50.1	51.9	53.9	56.0	58.3	61.0	64.2	68.3	-	-	-	-	-	-	-	-	-	-
15	75%	41.5	42.6	43.9	45.2	46.6	48.1	49.6	51.3	53.1	55.0	57.1	59.5	62.2	65.4	69.4	-	-	-	-	-	-	-	-	-
16	76%	41.4	42.5	43.8	45.0	46.3	47.7	49.2	50.7	52.4	54.2	56.1	58.2	60.6	63.3	66.5	70.5	-	-	-	-	-	-	-	-
17	77%	41.4	42.5	43.6	44.8	46.1	47.4	48.8	50.2	51.8	53.5	55.2	57.2	59.3	61.7	64.3	67.5	71.6	-	-	-	-	-	-	-
18	79%	41.3	42.4	43.5	44.6	45.8	47.1	48.4	49.8	51.3	52.8	54.5	56.3	58.2	60.3	62.7	65.4	68.6	72.6	-	-	-	-	-	-
19	80%	41.3	42.3	43.4	44.5	45.6	46.8	48.1	49.4	50.8	52.3	53.8	55.5	57.3	59.2	61.3	63.7	66.4	69.6	73.6	-	-	-	-	-
20	81%	41.2	42.2	43.3	44.3	45.4	46.6	47.8	49.1	50.4	51.8	53.2	54.8	56.4	58.2	60.2	62.3	64.6	67.3	70.5	74.6	-	-	-	-
21	82%	41.2	42.2	43.2	44.2	45.3	46.4	47.5	48.7	50.0	51.3	52.7	54.2	55.7	57.4	59.2	61.1	63.2	65.6	68.3	71.4	75.5	-	-	-
22	83%	41.2	42.1	43.1	44.1	45.1	46.2	47.3	48.4	49.6	50.9	52.2	53.6	55.1	56.6	58.3	60.1	62.0	64.1	66.5	69.2	72.4	76.4	-	-
23	83%	41.1	42.0	43.0	43.9	44.9	46.0	47.0	48.2	49.3	50.5	51.8	53.1	54.5	56.0	57.5	59.2	61.0	62.9	65.0	67.4	70.1	73.2	77.3	-
24	84%	41.1	42.0	42.9	43.8	44.8	45.8	46.8	47.9	49.0	50.2	51.4	52.6	54.0	55.3	56.8	58.4	60.0	61.8	63.7	65.9	68.2	70.9	74.1	78.2

Table A.15: Model Outputs - $X_0 \sim 40 + Exp(1/12), X_1 \sim Gamma(1/3, 9), \beta = 0.2$

Scr.	Perf.	Optimal Screening Policies																							
Bud.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	10%	47.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	16%	44.9	51.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	21%	43.8	48.1	54.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	25%	43.1	46.4	50.7	57.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	28%	42.6	45.3	48.5	52.9	59.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	31%	42.3	44.5	47.2	50.4	54.8	61.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	33%	42.0	43.9	46.2	48.8	52.1	56.4	63.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	35%	41.8	43.5	45.5	47.7	50.3	53.6	57.9	64.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	37%	41.6	43.2	44.9	46.8	49.1	51.7	55.0	59.3	66.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	38%	41.5	42.9	44.4	46.1	48.1	50.3	53.0	56.2	60.6	67.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	40%	41.4	42.7	44.1	45.6	47.3	49.2	51.5	54.1	57.4	61.7	68.4	-	-	-	-	-	-	-	-	-	-	-	-	-
12	41%	41.3	42.5	43.7	45.1	46.7	48.4	50.3	52.5	55.2	58.5	62.8	69.4	-	-	-	-	-	-	-	-	-	-	-	-
13	42%	41.2	42.3	43.5	44.7	46.1	47.7	49.4	51.3	53.5	56.2	59.5	63.8	70.4	-	-	-	-	-	-	-	-	-	-	-
14	43%	41.1	42.1	43.2	44.4	45.7	47.1	48.6	50.3	52.3	54.5	57.1	60.4	64.7	71.4	-	-	-	-	-	-	-	-	-	-
15	44%	41.1	42.0	43.0	44.1	45.3	46.6	48.0	49.5	51.2	53.1	55.4	58.0	61.3	65.6	72.3	-	-	-	-	-	-	-	-	-
16	45%	41.0	41.9	42.9	43.9	45.0	46.1	47.4	48.8	50.3	52.0	54.0	56.2	58.8	62.1	66.4	73.1	-	-	-	-	-	-	-	-
17	46%	41.0	41.8	42.7	43.7	44.7	45.8	46.9	48.2	49.6	51.1	52.8	54.8	57.0	59.7	62.9	67.3	73.9	-	-	-	-	-	-	-
18	47%	40.9	41.7	42.6	43.5	44.4	45.4	46.5	47.7	49.0	50.4	51.9	53.6	55.5	57.8	60.4	63.7	68.0	74.7	-	-	-	-	-	-
19	47%	40.9	41.6	42.4	43.3	44.2	45.1	46.1	47.2	48.4	49.7	51.1	52.6	54.3	56.3	58.5	61.1	64.4	68.8	75.4	-	-	-	-	-
20	48%	40.8	41.6	42.3	43.1	44.0	44.9	45.8	46.8	47.9	49.1	50.4	51.8	53.3	55.0	56.9	59.2	61.8	65.1	69.4	76.1	-	-	-	-
21	49%	40.8	41.5	42.2	43.0	43.8	44.6	45.5	46.5	47.5	48.6	49.8	51.0	52.4	54.0	55.7	57.6	59.8	62.5	65.8	70.1	76.7	-	-	-
22	50%	40.8	41.4	42.1	42.8	43.6	44.4	45.3	46.2	47.1	48.1	49.2	50.4	51.7	53.1	54.6	56.3	58.2	60.5	63.1	66.4	70.7	77.4	-	-
23	50%	40.7	41.4	42.0	42.7	43.5	44.2	45.0	45.9	46.8	47.7	48.7	49.8	51.0	52.3	53.7	55.2	56.9	58.8	61.1	63.7	67.0	71.3	78.0	-
24	51%	40.7	41.3	42.0	42.6	43.3	44.0	44.8	45.6	46.5	47.4	48.3	49.3	50.4	51.6	52.9	54.3	55.8	57.5	59.4	61.7	64.3	67.6	71.9	78.6

Table A.16: Model Outputs - $X_0 \sim 40 + Exp(1/12), X_1 \sim Lognormal(0.9548, 0.5364)(mean = 3, var = 3), \beta = 0.2$

Scr.	Perf.	Optimal Screening Policies																							
Bud.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	14%	44.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	25%	43.4	47.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	34%	42.9	46.0	49.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	41%	42.6	45.2	48.3	52.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	47%	42.4	44.7	47.3	50.4	54.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	52%	42.2	44.3	46.6	49.2	52.3	56.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	56%	42.1	44.0	46.1	48.4	51.0	54.0	57.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	60%	42.0	43.7	45.7	47.7	50.0	52.6	55.7	59.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	63%	41.9	43.5	45.3	47.2	49.3	51.6	54.2	57.3	61.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	66%	41.8	43.3	45.0	46.8	48.7	50.8	53.1	55.7	58.7	62.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	68%	41.7	43.2	44.7	46.4	48.2	50.1	52.2	54.5	57.1	60.1	64.0	-	-	-	-	-	-	-	-	-	-	-	-	-
12	71%	41.7	43.0	44.5	46.1	47.7	49.5	51.4	53.5	55.8	58.4	61.5	65.4	-	-	-	-	-	-	-	-	-	-	-	-
13	73%	41.6	42.9	44.3	45.8	47.4	49.0	50.8	52.7	54.8	57.1	59.7	62.7	66.6	-	-	-	-	-	-	-	-	-	-	-
14	75%	41.5	42.8	44.1	45.5	47.0	48.6	50.2	52.0	53.9	56.0	58.3	60.9	64.0	67.9	-	-	-	-	-	-	-	-	-	-
15	76%	41.5	42.7	44.0	45.3	46.7	48.2	49.8	51.4	53.2	55.1	57.2	59.5	62.1	65.1	69.0	-	-	-	-	-	-	-	-	-
16	78%	41.5	42.6	43.8	45.1	46.5	47.9	49.3	50.9	52.5	54.3	56.2	58.3	60.6	63.2	66.3	70.2	-	-	-	-	-	-	-	-
17	79%	41.4	42.5	43.7	44.9	46.2	47.5	48.9	50.4	52.0	53.6	55.4	57.3	59.4	61.7	64.3	67.4	71.3	-	-	-	-	-	-	-
18	80%	41.4	42.4	43.6	44.8	46.0	47.3	48.6	50.0	51.5	53.0	54.7	56.5	58.4	60.5	62.8	65.4	68.4	72.3	-	-	-	-	-	-
19	81%	41.3	42.4	43.5	44.6	45.8	47.0	48.3	49.6	51.0	52.5	54.1	55.7	57.5	59.4	61.5	63.8	66.4	69.5	73.4	-	-	-	-	-
20	82%	41.3	42.3	43.4	44.5	45.6	46.8	48.0	49.3	50.6	52.0	53.5	55.1	56.7	58.5	60.4	62.5	64.8	67.4	70.4	74.4	-	-	-	-
21	83%	41.3	42.2	43.3	44.3	45.4	46.6	47.8	49.0	50.3	51.6	53.0	54.5	56.0	57.7	59.5	61.4	63.5	65.8	68.4	71.4	75.3	-	-	-
22	84%	41.3	42.2	43.2	44.2	45.3	46.4	47.5	48.7	49.9	51.2	52.5	53.9	55.4	57.0	58.6	60.4	62.3	64.4	66.7	69.3	72.4	76.3	-	-
23	85%	41.2	42.1	43.1	44.1	45.1	46.2	47.3	48.4	49.6	50.8	52.1	53.4	54.8	56.3	57.9	59.5	61.3	63.2	65.3	67.6	70.2	73.3	77.2	-
24	86%	41.2	42.1	43.0	44.0	45.0	46.0	47.1	48.2	49.3	50.5	51.7	53.0	54.3	55.7	57.2	58.8	60.4	62.2	64.1	66.2	68.5	71.1	74.2	78.1

Table A.17: Model Outputs - $X_0 \sim 40 + Exp(1/12), X_1 \sim Lognormal(0.7520, 0.8326)(mean = 3, var = 9), \beta = 0.2$

Scr.	Perf.	Optimal Screening Policies																							
Bud.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	13%	44.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	22%	43.6	48.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	30%	42.9	46.1	50.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	36%	42.5	45.1	48.3	52.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	41%	42.3	44.5	47.1	50.3	54.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	45%	42.1	44.0	46.2	48.8	52.0	56.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	49%	41.9	43.7	45.6	47.8	50.4	53.6	58.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	52%	41.8	43.4	45.1	47.1	49.3	51.9	55.1	59.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	55%	41.6	43.1	44.8	46.5	48.5	50.7	53.3	56.5	60.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	58%	41.5	42.9	44.4	46.1	47.8	49.8	52.0	54.6	57.8	62.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	60%	41.5	42.8	44.2	45.7	47.3	49.1	51.0	53.3	55.8	59.0	63.5	-	-	-	-	-	-	-	-	-	-	-	-	-
12	63%	41.4	42.6	43.9	45.3	46.8	48.5	50.2	52.2	54.4	57.0	60.2	64.6	-	-	-	-	-	-	-	-	-	-	-	-
13	65%	41.3	42.5	43.7	45.0	46.4	47.9	49.6	51.3	53.3	55.5	58.1	61.3	65.7	-	-	-	-	-	-	-	-	-	-	-
14	66%	41.3	42.4	43.6	44.8	46.1	47.5	49.0	50.6	52.4	54.4	56.6	59.2	62.4	66.8	-	-	-	-	-	-	-	-	-	-
15	68%	41.2	42.3	43.4	44.6	45.8	47.1	48.5	50.0	51.6	53.4	55.4	57.6	60.2	63.4	67.8	-	-	-	-	-	-	-	-	-
16	70%	41.2	42.2	43.3	44.4	45.5	46.8	48.1	49.5	51.0	52.6	54.4	56.3	58.6	61.2	64.4	68.8	-	-	-	-	-	-	-	-
17	71%	41.1	42.1	43.1	44.2	45.3	46.5	47.7	49.0	50.4	51.9	53.5	55.3	57.3	59.5	62.1	65.3	69.7	-	-	-	-	-	-	-
18	72%	41.1	42.0	43.0	44.0	45.1	46.2	47.4	48.6	49.9	51.3	52.8	54.4	56.2	58.2	60.4	63.0	66.2	70.6	-	-	-	-	-	-
19	73%	41.1	42.0	42.9	43.9	44.9	46.0	47.1	48.3	49.5	50.8	52.2	53.7	55.3	57.1	59.1	61.3	63.9	67.1	71.5	-	-	-	-	-
20	75%	41.0	41.9	42.8	43.8	44.7	45.8	46.8	47.9	49.1	50.3	51.6	53.0	54.5	56.2	57.9	59.9	62.1	64.7	67.9	72.3	-	-	-	-
21	76%	41.0	41.8	42.7	43.6	44.6	45.5	46.6	47.6	48.7	49.9	51.2	52.5	53.9	55.4	57.0	58.7	60.7	62.9	65.5	68.7	73.2	-	-	-
22	77%	41.0	41.8	42.6	43.5	44.4	45.4	46.3	47.4	48.4	49.5	50.7	51.9	53.3	54.7	56.1	57.8	59.5	61.5	63.7	66.3	69.5	73.9	-	-
23	78%	41.0	41.7	42.6	43.4	44.3	45.2	46.1	47.1	48.1	49.2	50.3	51.5	52.7	54.0	55.4	56.9	58.5	60.3	62.3	64.5	67.1	70.3	74.7	-
24	78%	40.9	41.7	42.5	43.3	44.2	45.0	45.9	46.9	47.9	48.9	49.9	51.1	52.2	53.5	54.8	56.2	57.7	59.3	61.1	63.0	65.2	67.8	71.0	75.5

Table A.18: Model Outputs - $X_0 \sim 40 + Exp(1/12), X_1 \sim Lognormal(0.4055, 1.1774)(mean = 3, var = 27), \beta = 0.2$

Scr.	Perf.	Optimal Screening Policies																							
Bud.	Opt. Pol.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	11%	45.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	18%	43.8	48.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	24%	43.0	46.3	51.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	29%	42.5	45.1	48.5	53.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	34%	42.1	44.3	46.9	50.3	55.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	37%	41.9	43.8	46.0	48.6	51.9	56.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	41%	41.7	43.4	45.3	47.4	50.0	53.4	58.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	44%	41.6	43.1	44.7	46.6	48.8	51.4	54.7	59.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	46%	41.5	42.8	44.3	46.0	47.8	50.0	52.6	56.0	61.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	48%	41.4	42.6	44.0	45.5	47.1	49.0	51.2	53.8	57.1	62.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	51%	41.3	42.4	43.7	45.1	46.6	48.2	50.1	52.3	54.9	58.2	63.2	-	-	-	-	-	-	-	-	-	-	-	-	-
12	53%	41.2	42.3	43.5	44.7	46.1	47.6	49.2	51.1	53.3	55.9	59.2	64.3	-	-	-	-	-	-	-	-	-	-	-	-
13	54%	41.1	42.2	43.3	44.4	45.7	47.0	48.5	50.2	52.1	54.2	56.8	60.2	65.2	-	-	-	-	-	-	-	-	-	-	-
14	56%	41.1	42.1	43.1	44.2	45.3	46.6	48.0	49.5	51.1	53.0	55.2	57.8	61.1	66.1	-	-	-	-	-	-	-	-	-	-
15	58%	41.0	42.0	42.9	44.0	45.1	46.2	47.5	48.8	50.3	52.0	53.9	56.0	58.6	62.0	67.0	-	-	-	-	-	-	-	-	-
16	59%	41.0	41.9	42.8	43.8	44.8	45.9	47.1	48.3	49.7	51.2	52.8	54.7	56.9	59.5	62.8	67.8	-	-	-	-	-	-	-	-
17	60%	41.0	41.8	42.7	43.6	44.6	45.6	46.7	47.8	49.1	50.5	52.0	53.6	55.5	57.7	60.3	63.6	68.6	-	-	-	-	-	-	-
18	62%	40.9	41.7	42.6	43.4	44.4	45.3	46.4	47.4	48.6	49.9	51.2	52.7	54.4	56.3	58.4	61.0	64.4	69.4	-	-	-	-	-	-
19	63%	40.9	41.7	42.5	43.3	44.2	45.1	46.1	47.1	48.2	49.4	50.6	52.0	53.5	55.1	57.0	59.2	61.8	65.1	70.1	-	-	-	-	-
20	64%	40.8	41.6	42.4	43.2	44.0	44.9	45.8	46.8	47.8	48.9	50.1	51.3	52.7	54.2	55.8	57.7	59.9	62.5	65.8	70.9	-	-	-	-
21	65%	40.8	41.5	42.3	43.0	43.9	44.7	45.6	46.5	47.5	48.5	49.6	50.7	52.0	53.4	54.9	56.5	58.4	60.6	63.2	66.5	71.5	-	-	-
22	66%	40.8	41.5	42.2	42.9	43.7	44.5	45.4	46.2	47.2	48.1	49.2	50.2	51.4	52.7	54.0	55.5	57.2	59.0	61.2	63.8	67.2	72.2	-	-
23	67%	40.8	41.4	42.1	42.8	43.6	44.4	45.2	46.0	46.9	47.8	48.8	49.8	50.9	52.1	53.3	54.7	56.2	57.8	59.7	61.9	64.5	67.8	72.8	-
24	68%	40.7	41.4	42.1	42.7	43.5	44.2	45.0	45.8	46.6	47.5	48.4	49.4	50.4	51.5	52.7	53.9	55.3	56.8	58.4	60.3	62.5	65.1	68.5	73.5

APPENDIX B. LIFE TABLE FOR FEMALES: UNITED STATES, 2008

Table 2. Life table for males: United States, 2008

Spreadsheet version available from: ftp://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/61_03/Table02.xls.

Age (years)	Probability of dying between ages x and $x+1$ q_x	Number surviving to age x l_x	Number dying between ages x and $x+1$ d_x	Person-years lived between ages x and $x+1$ L_x	Total number of person-years lived above age x T_x	Expectation of life at age x e_x
0–1	0.007195	100,000	720	99,374	7,559,612	75.6
1–2	0.000504	99,280	50	99,255	7,460,238	75.1
2–3	0.000319	99,230	32	99,215	7,360,983	74.2
3–4	0.000248	99,199	25	99,187	7,261,768	73.2
4–5	0.000189	99,174	19	99,165	7,162,581	72.2
5–6	0.000172	99,155	17	99,147	7,063,417	71.2
6–7	0.000156	99,138	15	99,131	6,964,270	70.2
7–8	0.000140	99,123	14	99,116	6,865,139	69.3
8–9	0.000119	99,109	12	99,103	6,766,023	68.3
9–10	0.000093	99,097	9	99,093	6,666,920	67.3
10–11	0.000074	99,088	7	99,084	6,567,827	66.3
11–12	0.000080	99,081	8	99,077	6,468,743	65.3
12–13	0.000132	99,073	13	99,066	6,369,666	64.3
13–14	0.000239	99,060	24	99,048	6,270,600	63.3
14–15	0.000384	99,036	38	99,017	6,171,552	62.3
15–16	0.000535	98,998	53	98,972	6,072,535	61.3
16–17	0.000675	98,945	67	98,912	5,973,563	60.4
17–18	0.000813	98,878	80	98,838	5,874,651	59.4
18–19	0.000949	98,798	94	98,751	5,775,813	58.5
19–20	0.001081	98,704	107	98,651	5,677,062	57.5
20–21	0.001225	98,598	121	98,537	5,578,411	56.6
21–22	0.001359	98,477	134	98,410	5,479,874	55.6
22–23	0.001447	98,343	142	98,272	5,381,464	54.7
23–24	0.001470	98,201	144	98,129	5,283,192	53.8
24–25	0.001444	98,056	142	97,986	5,185,064	52.9
25–26	0.001403	97,915	137	97,846	5,087,078	52.0
26–27	0.001372	97,777	134	97,710	4,989,232	51.0
27–28	0.001352	97,643	132	97,577	4,891,522	50.1
28–29	0.001353	97,511	132	97,445	4,793,945	49.2
29–30	0.001371	97,379	134	97,313	4,696,499	48.2
30–31	0.001399	97,246	136	97,178	4,599,187	47.3
31–32	0.001427	97,110	139	97,040	4,502,009	46.4
32–33	0.001461	96,971	142	96,900	4,404,969	45.4
33–34	0.001504	96,829	146	96,757	4,308,068	44.5
34–35	0.001551	96,684	150	96,609	4,211,312	43.6
35–36	0.001611	96,534	156	96,456	4,114,703	42.6
36–37	0.001688	96,378	163	96,297	4,018,247	41.7
37–38	0.001782	96,216	171	96,130	3,921,950	40.8
38–39	0.001899	96,044	182	95,953	3,825,820	39.8
39–40	0.002042	95,862	196	95,764	3,729,866	38.9
40–41	0.002203	95,666	211	95,561	3,634,102	38.0
41–42	0.002388	95,455	228	95,342	3,538,542	37.1
42–43	0.002614	95,228	249	95,103	3,443,200	36.2
43–44	0.002881	94,979	274	94,842	3,348,097	35.3
44–45	0.003176	94,705	301	94,555	3,253,255	34.4
45–46	0.003478	94,404	328	94,240	3,158,701	33.5
46–47	0.003789	94,076	356	93,898	3,064,460	32.6
47–48	0.004132	93,719	387	93,526	2,970,563	31.7
48–49	0.004522	93,332	422	93,121	2,877,037	30.8
49–50	0.004958	92,910	461	92,680	2,783,916	30.0
50–51	0.005431	92,449	502	92,198	2,691,236	29.1
51–52	0.005922	91,947	545	91,675	2,599,038	28.3
52–53	0.006423	91,403	587	91,109	2,507,362	27.4
53–54	0.006925	90,816	629	90,501	2,416,253	26.6
54–55	0.007436	90,187	671	89,852	2,325,752	25.8
55–56	0.007983	89,516	715	89,159	2,235,900	25.0
56–57	0.008581	88,802	762	88,421	2,146,741	24.2
57–58	0.009219	88,040	812	87,634	2,058,321	23.4
58–59	0.009899	87,228	863	86,796	1,970,687	22.6
59–60	0.010626	86,364	918	85,906	1,883,891	21.8
60–61	0.011414	85,447	975	84,959	1,797,985	21.0
61–62	0.012274	84,471	1,037	83,953	1,713,026	20.3
62–63	0.013209	83,435	1,102	82,884	1,629,073	19.5

See footnote at end of table.

Table 2. Life table for males: United States, 2008—Con.

Spreadsheet version available from: ftp://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/61_03/Table02.xls.

Age (years)	Probability of dying between ages x and $x + 1$	Number surviving to age x	Number dying between ages x and $x + 1$	Person-years lived between ages x and $x + 1$	Total number of person-years lived above age x	Expectation of life at age x
	q_x	l_x	d_x	L_x	T_x	e_x
63–64	0.014236	82,333	1,172	81,747	1,546,189	18.8
64–65	0.015382	81,160	1,248	80,536	1,464,443	18.0
65–66	0.016699	79,912	1,334	79,245	1,383,907	17.3
66–67	0.018184	78,578	1,429	77,863	1,304,662	16.6
67–68	0.019793	77,149	1,527	76,385	1,226,799	15.9
68–69	0.021473	75,622	1,624	74,810	1,150,414	15.2
69–70	0.023251	73,998	1,720	73,138	1,075,604	14.5
70–71	0.025139	72,277	1,817	71,369	1,002,466	13.9
71–72	0.027310	70,460	1,924	69,498	931,097	13.2
72–73	0.029927	68,536	2,051	67,511	861,599	12.6
73–74	0.032876	66,485	2,186	65,392	794,088	11.9
74–75	0.036072	64,299	2,319	63,140	728,696	11.3
75–76	0.039506	61,980	2,449	60,756	665,557	10.7
76–77	0.043153	59,531	2,569	58,247	604,801	10.2
77–78	0.047308	56,962	2,695	55,615	546,554	9.6
78–79	0.052154	54,268	2,830	52,852	490,940	9.0
70–80	0.057697	51,437	2,968	49,953	438,087	8.5
80–81	0.063533	48,469	3,079	46,930	388,134	8.0
81–82	0.069684	45,390	3,163	43,809	341,204	7.5
82–83	0.076575	42,227	3,234	40,610	297,395	7.0
83–84	0.084612	38,994	3,299	37,344	256,785	6.6
84–85	0.093410	35,694	3,334	34,027	219,441	6.1
85–86	0.103950	32,360	3,364	30,678	185,414	5.7
86–87	0.115393	28,996	3,346	27,323	154,736	5.3
87–88	0.127809	25,650	3,278	24,011	127,412	5.0
88–89	0.141219	22,372	3,159	20,792	103,401	4.6
89–90	0.155630	19,213	2,990	17,718	82,609	4.3
90–91	0.171033	16,223	2,775	14,835	64,891	4.0
91–92	0.187401	13,448	2,520	12,188	50,056	3.7
92–93	0.204688	10,928	2,237	9,809	37,868	3.5
93–94	0.222829	8,691	1,937	7,723	28,059	3.2
94–95	0.241737	6,754	1,633	5,938	20,336	3.0
95–96	0.261304	5,122	1,338	4,452	14,398	2.8
96–97	0.281406	3,783	1,065	3,251	9,946	2.6
97–98	0.301903	2,719	821	2,308	6,695	2.5
98–99	0.322643	1,898	612	1,592	4,387	2.3
99–100	0.343465	1,286	442	1,065	2,795	2.2
100 and over	1.000000	844	844	1,730	1,730	2.0

SOURCE: CDC/NCHS, National Vital Statistics System.