

POWER SUPPRESSION IN D-BRANE INFLATION

An Undergraduate Research Scholars Thesis

by

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ABSTRACT

Power Suppression in D-Brane Inflation. (May 2014)

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Cosmological inflation is the period of rapid, accelerated expansion that occurred in the fraction of a second between the creation of spacetime and the Big Bang. Its proposal 35 years ago singlehandedly solved the three greatest issues of the Big Bang model . Now, researchers have found that certain inflation models may solve one more puzzle: the anomalous power-suppression at large angles in the cosmic microwave background (CMB). Despite consensus to the contrary, we argue that D-brane inflation is one such model. We show that there are initial conditions for which inflation deviates from “slow-roll inflation” dramatically enough to show adequate power-suppression in the CMB.

DEDICATION

For my parents.

ACKNOWLEDGEMENTS

I would like to thank Professor Bhaskar Dutta for his continuing support and guidance throughout this project. His willingness to stop whatever he is working on and answer my plethora of questions all of the times I have dropped by his office has made this project much easier.

I would also like to thank Sean Downes for the multiple times he sat with me and discussed this topic for literally hours, in spite of whatever work he actually had to do. I would never have come to understand this material half so well otherwise.

NOMENCLATURE

$a(t)$	Scale factor
N	Number of e-foldings
n_s	Spectral index
ϕ	Inflaton

CHAPTER I

INTRODUCTION

This research paper explores whether the theoretical model of D-brane inflation can account for the current discrepancy between theory and data that has been termed power suppression. Thus, in this paper, we will first define what inflation is. Second, we will narrow our focus to one specific model of inflation, called D-brane inflation, and survey its properties. Third, we will learn what power suppression means. Finally, we will use what we have learned to investigate whether D-brane inflation can truly account for power suppression, and we will find that it can.

What is inflation?

Inflation is the massive, accelerated expansion of the universe that occurred in the first 10^{-34} seconds. It was proposed in the early 1980's as a supplement to the Big Bang theory (BBT), and resolved three of BBT's greatest theoretical issues [1, 2]. Since then, it has acquired even more experimental support from the COBE, WMAP, and PLANCK collaborations, and also quite recently from the well-known BICEP2 results [3, 4, 5, 6]. Inflation has all but solidified itself, now, as a major part of our cosmological paradigm.

The Cosmic Microwave Background

Arguably the most important source of data for inflation researchers is the cosmic microwave background (CMB). It is shown in figure 1 [4]. This CMB is what COBE, WMAP, and PLANCK worked to observe at increasingly high resolutions. Simply put, it is a picture of the entire sky in microwave frequencies. Since this light is the oldest we can observe, it is perhaps our best window into the very early universe when inflation occurred.

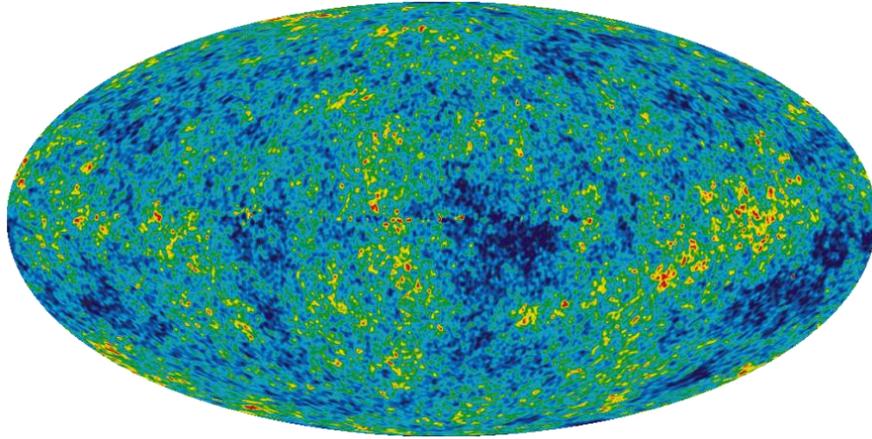


FIG. 1. Cosmic Microwave Background (CMB). The color differences indicate variation in temperature by 10^{-5} Kelvin. The average temperature of the CMB light is roughly 2.7 Kelvin.

One thing the CMB shows us is the minimum amount by which the universe had to expand during inflation. As you can see from figure 1, there are patches, or (temperature) anisotropies, in the CMB of all different scales. That is, some anisotropies are many degrees across in the sky while others are a small fraction of a degree across. All of these anisotropies originated as quantum fluctuations at the time of inflation. They were enlarged with the universe as inflation progressed. This means that the largest anisotropies were earlier quantum fluctuations, and the smaller ones originated as fluctuations closer to the end of inflation. Scientists observe that the largest (visible) anisotropies are so large that their original quantum fluctuations must have grown by a factor of roughly e^{60} during inflation. It is important to note that this places the *lower bound* on how large inflation's expansion was. We cannot establish an upper bound in the same way, simply because we cannot observe what the size of the largest anisotropy is – it is too large.

Quantizing the Growth

In the rest of this paper, we will frequently reference two parameters to describe the universe's size and growth during inflation. The first is the quantity “ $a(t)$,” called the scale factor, which

represents the size-scale of the universe as a function of time. We will normalize its value to $a_0 = 1$ at the beginning of inflation, so when $a(t) = 4$, every dimension of the universe will be four times longer (and therefore the universe will have expanded to 64 times its original volume). Scientists use the variable N_{total} to quantize how much the universe expanded over the course of inflation, where N_{total} is defined:

$$N_{\text{total}} \equiv \ln(a) \tag{1}$$

We can thus write the condition that the universe had to expand by a factor of at least e^{60} as:

$$N_{\text{total}} \geq 60 \tag{2}$$

We will return to this condition in the methods section when we discuss the constraints the data has placed upon our inflation models.

The Inflaton

We will also make extensive use of another quantity: the inflaton, which is represented by ϕ . In essence, the inflaton is the scalar field that caused inflation. In this paper, we are not concerned with the physical nature of ϕ . Rather, we will use it simply as an order parameter (or clock) to parameterize the time-evolution of the inflationary energy density.

What is important about the inflaton ϕ is that, for every value of the inflaton, we assign to it a potential energy. We can plot this potential energy as a function of ϕ as in figure 2 below [7].

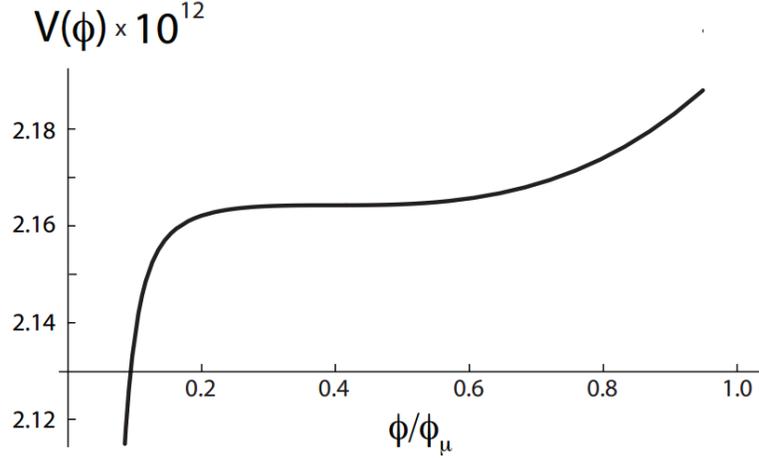


FIG. 2. Example potential energy graph. The potential energy here is plotted against the normalized inflaton. This is the potential energy graph associated with D-brane inflation.

Moreover, not only does the inflaton create this potential energy, but it also interacts with it in much the same way as a conventional particle in a common potential energy field. Quantum field theory tells us that this inflaton will interact with this potential with the equation of motion (3) [8].

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \quad (3)$$

Here dots indicate time derivatives and $H = (\frac{\dot{a}}{a})$, where a is, again, the scale factor. Studying (3) reveals that the second term acts as a drag force upon the inflaton as it tries to change with time. We will see in later sections that this drag force is highly relevant to the inflaton's time dependence, which affects inflation greatly.

What is D-brane inflation?

D-brane inflation is a model of inflation predicted by string theory IIB. Details about its physical nature can be found in Baumann et al. [7]. For our purposes, it suffices to extract from [7] their

main result: the potential $V(\phi)$, which we plotted in figure 2. This potential contains all of the relevant mathematical information about D-brane inflation. Indeed, the function $V(\phi)$ predicted by different inflation models is, in general, what distinguishes one inflation model from any other.

The equation $V(\phi)$ (that was plotted in figure 2) is printed in (4).

$$V(\phi) = \frac{a|A_0|^2}{3} \frac{e^{-2a\sigma(\phi)}}{U^2(\phi, \sigma(\phi))} g(\phi)^{2/n} [2a\sigma(\phi) + 6 - 6e^{a\sigma(\phi)}] \frac{W_0}{A_0} \left[\frac{1}{g(\phi)^{1/n}} + \frac{3c}{n} \frac{1}{g(\phi)^2} \frac{\phi}{\phi_\mu} - \frac{3}{n} \left(\frac{\phi}{\phi_\mu} \right)^{3/2} \frac{1}{g(\phi)} \right] + \frac{D[\phi]}{U^2(\phi, \sigma(\phi))} \quad (4)$$

The constituent equations and other parameters are printed in the Appendix.

How does inflation arise from the inflaton?

The size of the universe as a function of time can be calculated directly from $\phi(t)$. This can be seen clearly from (5) [9]. Note we have set all dimensional quantities equal to unity for simplicity.

$$H^2 = \frac{1}{3} \left(\frac{\dot{\phi}^2}{2} + V(\phi) \right) \quad (5)$$

Using (3), (4), (5), and any choice of initial conditions for ϕ , it is straightforward to calculate

$\phi(t)$. With $\phi(t)$ and (5), we can easily calculate N_{total} by noting that $N_{\text{total}} = \int_0^{\text{tend}} H dt$.

However, it is more convenient to parameterize time with N , where we take N to mean the number of e-foldings since the beginning of inflation to any time t (not just t_{end}). In such a case,

$N(t) = \ln(a(t))$ and thus $\dot{\phi} = H\phi'$ where primes denote derivatives with respect to N . In this variable, (5) becomes [10]:

$$H^2 = \frac{V}{3\left(1 - \frac{1}{6}\phi'^2\right)} \quad (6)$$

And (3) becomes

$$\phi'' = \frac{1}{2}(\phi' + \sqrt{6})(\phi' - \sqrt{6})(\phi' + \frac{V_\phi}{V}) \quad (7)$$

Where a subscript ϕ denotes a partial derivative with respect to the ϕ field. Equations (6) and (7) will be used throughout the methods section to simulate inflation.

It is highly important to note that not every size increase of the universe is considered inflation. Only the *accelerated* expansion of the universe is inflation. Indeed, we must be able to distinguish between accelerated expansion and otherwise. We do this by solving the general relativity equations that tell us how spacetime interacts with fields. The solution for the size of the universe as a function of time is [8]

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+\omega)}} \quad (8)$$

Where $a_0 = 1$ and $t = t_0$ at the beginning of inflation. Here, ω is the equation of state parameter, defined by

$$\omega \equiv \frac{p}{\rho} \quad (9)$$

Where ρ is the energy density of the field and p is the pressure of the field. Quantum field theory tells us that these quantities, for a scalar field, are given by [9]:

$$p = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad (10)$$

Equation (8) shows us that the universe will only accelerate (i.e. have a positive second derivative) if $\omega < -1/3$. In particular, it will expand perfectly exponentially if $\omega = -1$. (In such a case, (8) is manifestly no longer valid, and we must use (5) to determine $a(t)$). Equations (9) and (10) show that such an ω can be wrought if $V \gg \dot{\phi} \equiv \frac{d\phi}{dt}$. Therefore inflation occurs when $\frac{d\phi}{dt}$ is small. Again converting from time to N , we can find which values of $\frac{d\phi}{dN} \equiv \phi'$ are considered small enough for truly exponential expansion [10]:

$$|\phi'| \ll \sqrt{2} \quad (11)$$

In the future we will refer to this condition as “slow-roll inflation,” and we will heavily utilize the fact that inflation occurs if and only if the inflaton is changing this slowly.

We will also use two other well-known slow-roll parameters [11]:

$$(a) \epsilon(\phi) = \frac{1}{2} \left(\frac{V_\phi}{V} \right)^2$$

$$(b) \eta(\phi) = \frac{V_{\phi\phi}}{V} \quad (12)$$

For inflation to occur, the absolute value of each of these must be less than unity. These conditions tell us that inflation cannot happen for values of ϕ where $V(\phi)$ is too steep or too curved. This is why figure 2 has a manifest flat region – it is required for inflation. The end of the flat region marks where inflation ends. It is imperative to understand, however, that satisfying (12) is simply a

necessary constraint for inflation to occur. To have *sufficient* evidence that inflation is occurring at any value of ϕ , we must also satisfy (11)..

What is power suppression?

The cosmic microwave background (CMB) is plotted in figure 1. One of the CMB's main uses has been to construct the angular power spectrum. To do this, scientists measure the average temperature of the CMB's anisotropies as a function of the angular size of the anisotropies. They plot these measurements as in figure 3 [12]:

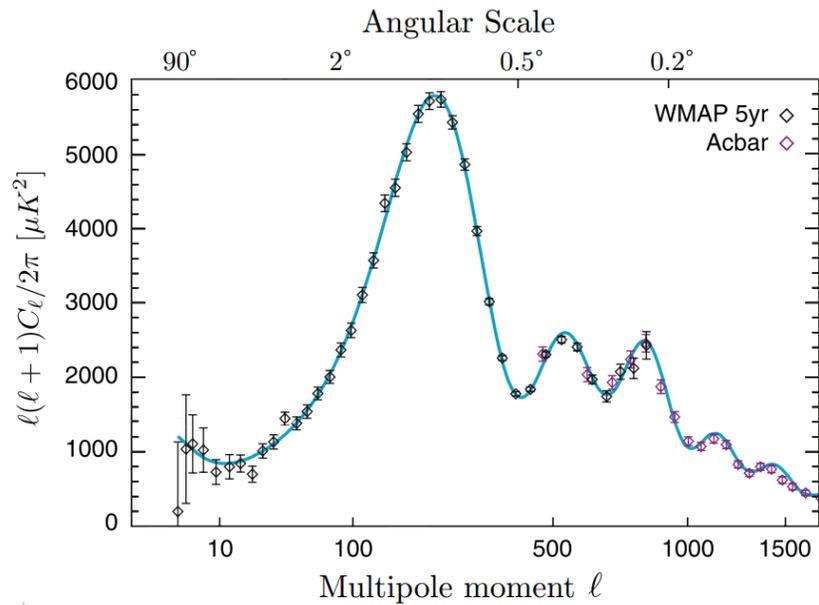


FIG. 3. The angular power spectrum. Our current best theoretical model matches the data extremely well, but arguably less well at low multipoles.

There are two takeaways from this plot. The first is that our current best theoretical model, Λ CDM (blue line), matches the data exceptionally, except at large angular scales (left side of plot). There, the data points are somewhat lower than predicted; we say they are suppressed. This

discrepancy between data and theory in the angular power spectrum is what we call “power suppression at large angular scales,” or “power suppression” for short.

Thus, when we argue that D-brane inflation can account for power suppression, we are arguing that, if our current cosmological model expanded to include D-brane inflation, then that blue line would dip at large angular scales and match the data better. Such an improved solution is plotted in figure 4 [13].

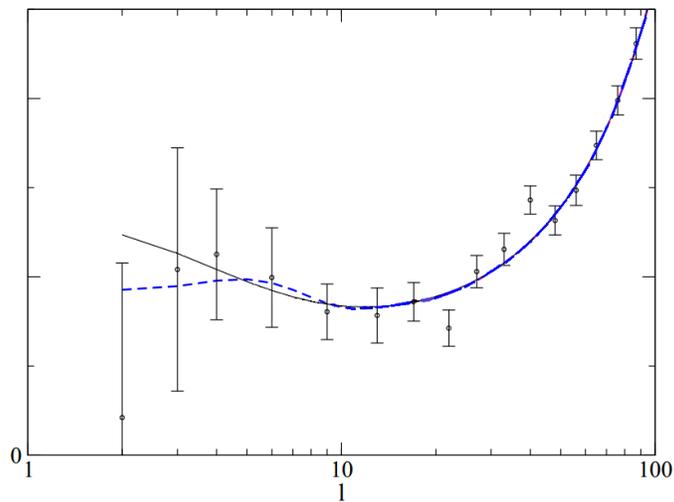


FIG. 4. Example of what an improvement to the angular power spectrum might look like. The solid black line depicts the current best theoretical model, Λ CDM. The dotted blue line is a qualitative example of what that current theoretical model might change to if (1) it included D-brane inflation and (2) D-brane inflation can account for power suppression.

The second takeaway from figure 4 is the spectral index n_s , which quantizes the “tilt” of the angular power spectrum. If n_s equaled 1, then the plot would be “un-tilted.” Since $n_s \approx 0.96$, the data values at large angular scales are slightly increased and those at small angular scales are slightly decreased, giving figure 4 a slight “clockwise-tilt.”

CHAPTER II

METHODS

Constraints placed by experimental data

We have seen that there are three primary constraints placed on our inflation model [5]:

- 1) $N \geq 60$
- 2) $n_s = 0.9603 \pm 0.0073$
- 3) Power suppression

It is our goal to simulate D-brane inflation and have it satisfy all three of these constraints. If we can do this, then we will have shown that D-brane inflation can account for power suppression. The methods in this section layout the procedure for simulating D-brane inflation and testing it against these constraints.

Revised constraints

Our situation is greatly simplified by the calculations in [14]. In that paper, Cicoli et al. show that power suppression occurs generally for any inflationary scenario in which both (1) inflation undergoes a shift from fast-roll to slow-roll and (2) $N \approx 60$. The principle they illustrate is that there is *always* a dip in the power spectrum when there is a fast-roll to slow-roll transition. However, if N is too large, then the anisotropy modes that were suppressed will have been pushed outside of the observable universe, and we would not be able to see them today. Therefore, for the largest *observable* modes to exhibit power suppression as they do, N must be near the minimal number.

Thus we constrain our inflation simulations with the following two constraints, in place of the original three:

- 1) $N \approx 60$ (within 2 or 3 e-foldings)
- 2) $n_s = 0.9603 \pm 0.0073$ (13)

These constraints' alleged mutual exclusivity

It is widely believed that D-brane inflation cannot account for power suppression precisely because the two constraints in (13) are thought to be mutually exclusive in D-brane inflation. The argument for their mutual exclusivity goes like the following: When simulating inflation, we can easily calculate the spectral index n_s as [12]:

$$n_s = 1 + 2\eta(\phi_{\text{cmb}}) - 6\epsilon(\phi_{\text{cmb}}) \tag{14}$$

Where η and ϵ are the slow-roll parameters from (12) and ϕ_{cmb} indicates we plug in the value of ϕ at which the CMB perturbations were created. Since the largest perturbations had to be created 60 e-foldings before the end of inflation, this value of ϕ occurred 60 e-foldings before the end of inflation. That is, $N(\phi_{\text{cmb}}) \equiv N_{\text{final}} - 60$.

From (12a), we can see that ϵ will be negligibly small when V is flat. Moreover, we have already seen that inflation happens only in the flat region of V . Thus it is changes in η that primarily affect n_s . Specifically, from (12) we can see that $n_s > 1$ if $V(\phi_{\text{cmb}})$ is concave up, and $n_s < 1$ if $V(\phi_{\text{cmb}})$ is concave down. This situation is depicted graphically in figure 5 [12].

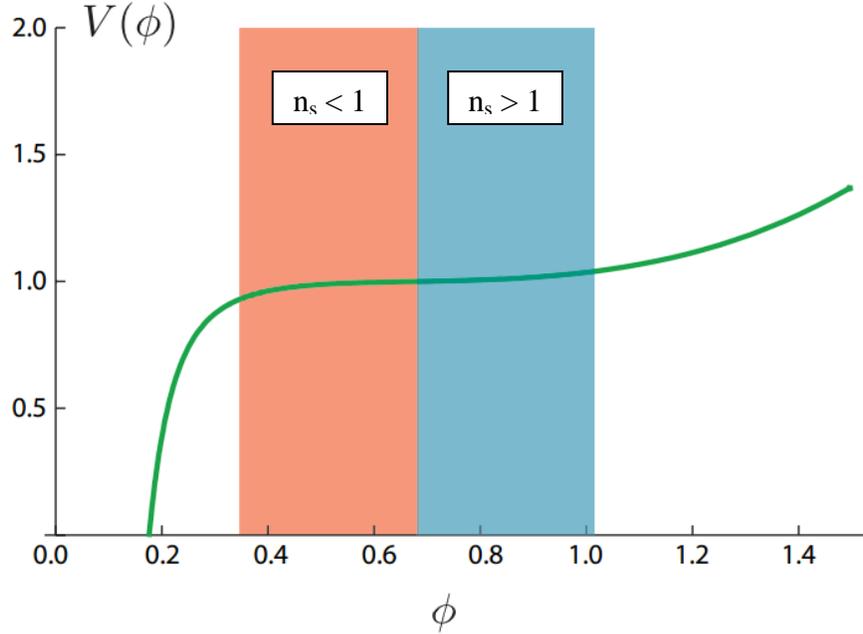


FIG. 5. A qualitative plot of the D-brane potential. The region suitable for inflation has been divided in half and colored either red (left) or blue (right). These colorings indicate the value of n_s if ϕ_{cmb} lands in those regions. The data tell us that ϕ_{cmb} must have occurred in the red region. Note that the region where inflation is possible is relatively symmetric about the inflection point, which divides the region in half. This means that an equal number of e-foldings will accumulate in both the blue and red regions, if the slow-roll conditions are satisfied as the inflaton enters the blue region [12].

A corollary is that, if the slow-roll conditions are satisfied over the entire flat region, then *any inflationary scenario with $N_{\text{total}} \approx 60$ will also have ϕ_{cmb} in the blue region.* It takes $N_{\text{total}} > 120$ for the last 60 e-foldings to accumulate in the red region. Therefore, it is impossible to have both $N_{\text{total}} \approx 60$ and $n_s \approx 0.96$ under these conditions. This argument is typically plotted like in figure 6 [7]:

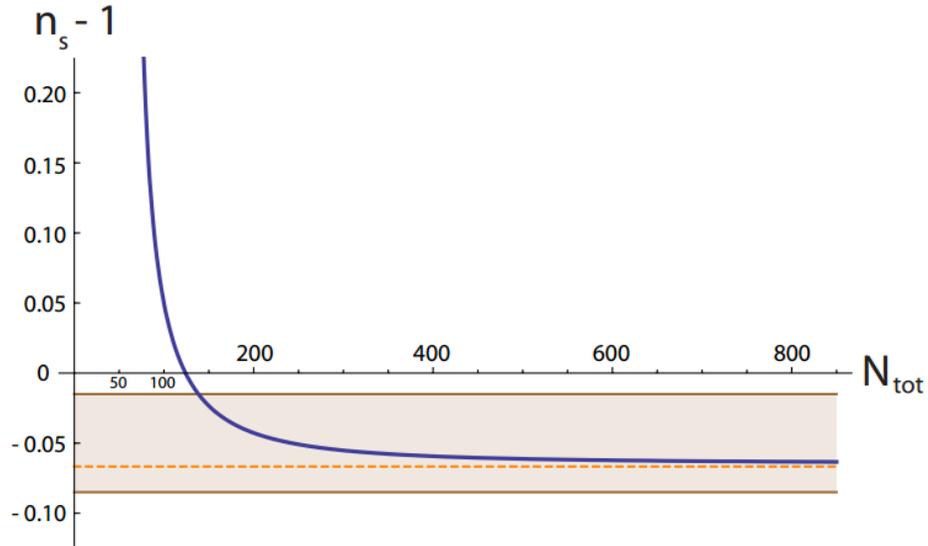


FIG. 6. The blue line depicts n_s versus N_{total} for D-brane inflation. We see that $n_s > 1$ for $N_{\text{total}} \approx 60$. The yellow dotted line indicates $n_s = 0.96$, and the shaded region is one standard deviation according to the WMAP data [4]. We must show that this plot was created with an assumption that is not always correct, or else (13) *are* mutually exclusive and D-brane inflation could not cause power suppression.

Their false assumption

It is our belief that the arguments made in the previous section used a non-general assumption. They explicitly assumed that the slow-roll conditions were satisfied at the beginning of the blue region in figure 6. It is our goal to illustrate that D-brane inflation allows scenarios where inflation actually begins in the red region. As we will show, both $N_{\text{total}} \approx 60$ and $n_s \approx 0.96$ can be satisfied in such a case.

CHAPTER III

RESULTS

This section will primarily serve as a sort of work flow for the theoretical tools we have developed. We will see that D-brane inflation can satisfy both $N_{\text{total}} \approx 60$ and $n_s \approx 0.96$ if the inflaton's initial velocity is tuned to certain values.

Working Potential

In the Appendix, we detail the full D-brane inflation potential that we work with here, and we specify all of the constituent parameters. It is important to note that we leave one parameter free: s . Varying s slightly allows us to control how many e-foldings the D-brane potential will allow.

In [15], Itzhaki and Kovetz illustrate that for inflection point inflation models (such as D-brane inflation), certain values of the parameters in the potential would *always produce the same value of N_{total}* , regardless of the inflaton's starting value. In plain English, they showed that if the potential is not too steep, then no matter how large ϕ_{initial} , the effective drag force will slow $\frac{d\phi}{dN}$ to slow-roll as the inflaton reaches the flat region where inflation begins. And of course, once slow-roll inflation has begun in the flat region, it will continue until the inflaton exits the flat region.

This result is important to ours, because it will allow us to find parameter values that produce enough inflation without having to fine-tune the initial conditions too much. It turns out that s is a parameter that gives us control over N_{total} in the way described above, and our calculations below will utilize this to study only potentials that give the appropriate number for N_{total} .

Slow-Roll Parameters and the Spectral Index

Since (12) and (7) depend only $V(\phi)$ and not on the dynamics of any specific inflation trajectory, we can plot η , ϵ , and n_s without reference to any initial conditions. We do this in figure 7 and figure 8.

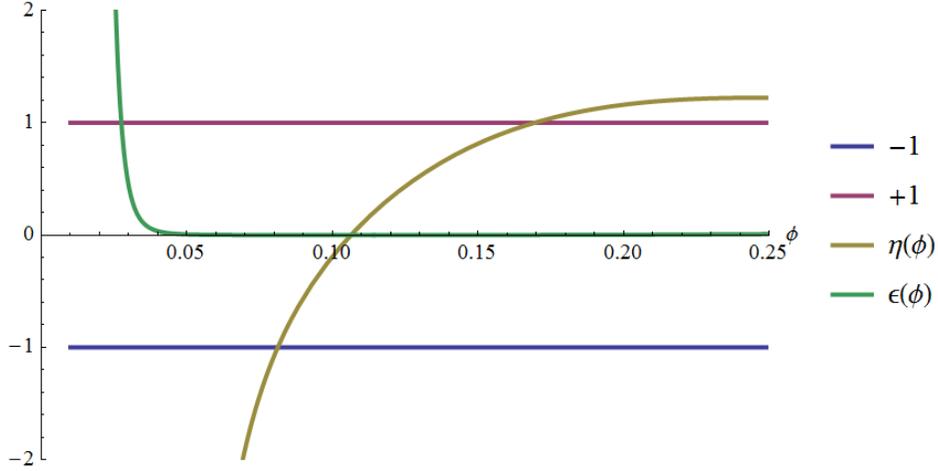


FIG. 7. Plots of $\eta(\phi)$ and $\epsilon(\phi)$ for the potential in the Appendix. We see that inflation can only happen between $\phi \approx 0.08$ and $\phi \approx 0.17$. Thus in our simulations we end inflation around $\phi \approx 0.08$, though our results are not sensitive to the exact ending point: the majority of inflation happens when both η and ϵ are small, around $\phi \approx 0.11$.

Note that, while figure 7 shows where the potential is shallow enough that inflation *can* occur, it is not sufficient to tell us that inflation *does* happen there. This is because these plots, again, only reference the potential and are blind to the actual speed of the inflaton throughout. To discover where inflation *does* occur, we must utilize the dynamical slow-roll parameter (15):

$$|\phi'| \ll \sqrt{2} \quad (15)$$

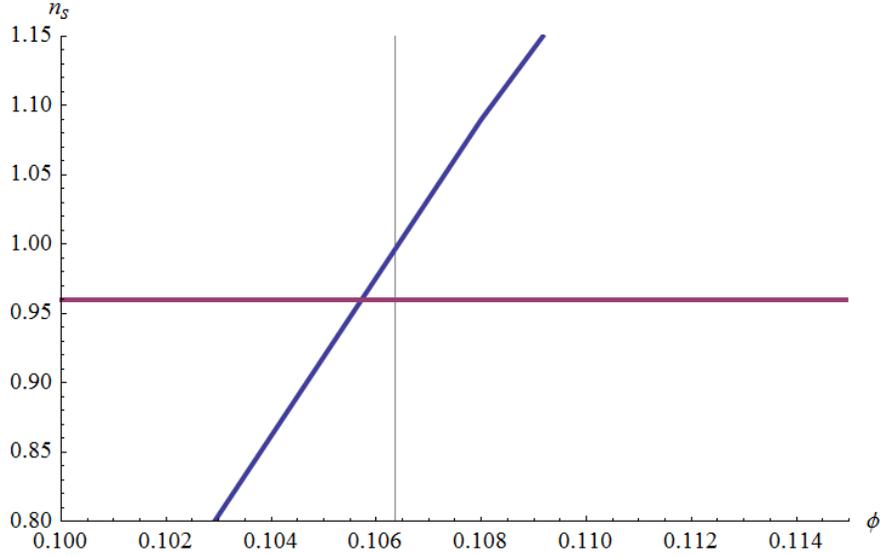


FIG. 8. The blue, diagonal line is $n_s(\phi)$. The value of ϕ at the inflection point, ϕ_* , is indicated by the vertical, gray line. Note that $n_s(\phi_*) = 1$, as we should expect since the curvature at ϕ_* is necessarily zero. The horizontal purple line indicates $n_s = 0.96$. It intersects $n_s(\phi)$ at $\phi \approx 0.10564$, so that is where we need to make inflation begin. Note that this ϕ is left of ϕ_* , so the inflaton will need to pass ϕ_* before inflation begins, as we thought.

We will use figures 7 and 8 as well as equation (15) to assess the results in the following sections.

Generic Inflation Trajectory

We begin our search for inflation trajectories that satisfy both $N \approx 60$ and $n_s \approx 0.96$ with the most widely-used initial conditions: initial $\frac{d\phi}{dN} = 0$, $\phi_0 > \phi_*$. Here, ϕ_* is the value of ϕ at the inflection point. In figure 9, you see these initial conditions give us too great an N_{total} .

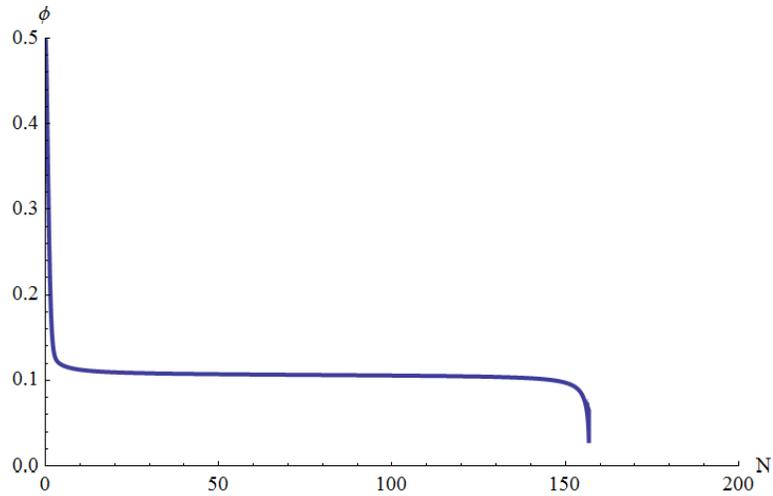


FIG. 9. $\phi(N)$ plotted with $\phi_0 = 0.5$ and initial $\frac{d\phi}{dN} = 0$. Inflation ends at roughly $N = 150$, which is too large.

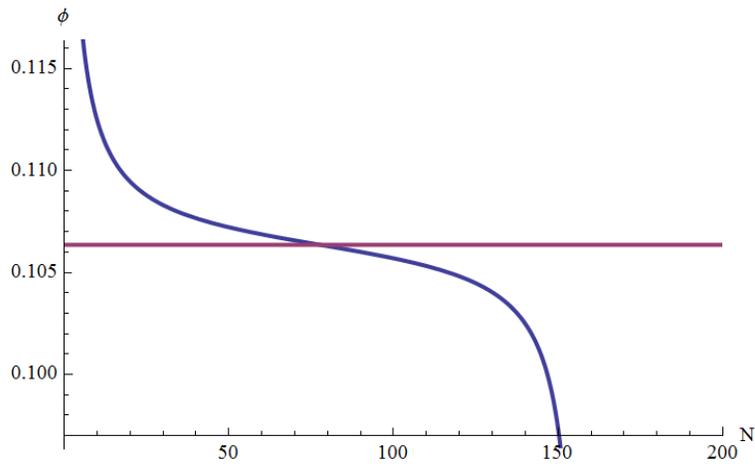


FIG. 10. A zoomed in version of figure 9. Here we emphasize the change in ϕ over time, and how it symmetrically straddles ϕ_* (the inflection point, purple line). This shows that roughly equal inflation is occurring on the left side of ϕ_* as on the right side.

Furthermore, we can combine figures 8 and 10 to show us n_s versus N , in figure 11.

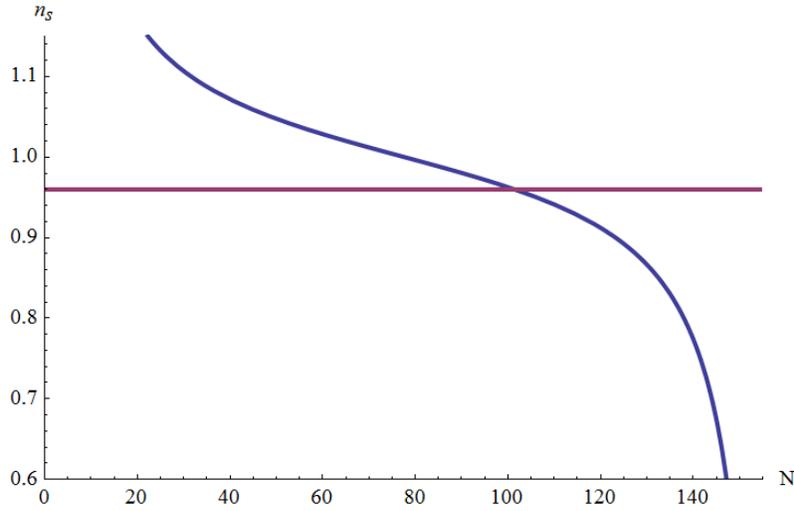


FIG. 11. The blue line shows $n_s(N)$ for $\phi_0 = 0.5$ and initial $\frac{d\phi}{dN} = 0$. The purple line depicts $n_s = 0.96$. The ideal value of n_s occurs around $N = 100$.

Since inflation ended at $N \approx 150$, the n_s value that this predicts is $n_s(90)$ from figure 11. That is, $n_s \approx 0.98$, which is close to the desired range. This run serves as an example for how inflation is typically analyzed. If we so desired, we could tune the parameters in the Appendix to lessen N_{total} to roughly $N_{\text{total}} \approx 60$ for these same initial conditions. However, we would find that this pushes n_s *farther* from the needed range. To attain both of them concurrently, we will have to alter the inflaton's initial conditions.

Speeding up the Initial Conditions

We will now try to solve this mutual-exclusivity issue by starting initial $\frac{d\phi}{dN} \neq 0$, while still placing initial ϕ at the value it was above. For a relatively high initial $\frac{d\phi}{dN}$, we find:

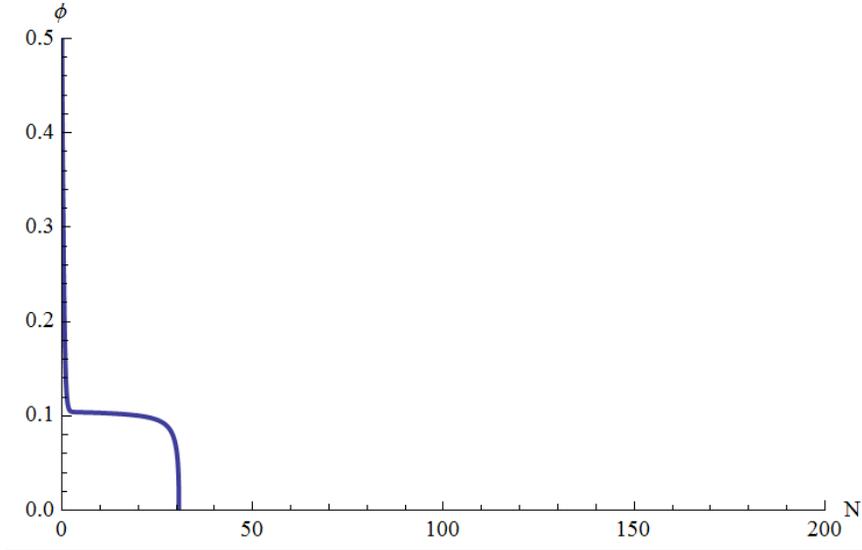


FIG. 12. $\phi(N)$ for $\phi_0 = 0.5$, initial $\frac{d\phi}{dN} = -0.7$. Only 30 e-foldings occur.

The inflaton plummeted down the potential graph far too quickly. These initial conditions give us too little inflation. Unfortunately, this type of trajectory is easy to create: it occurs any time we select an initial $\frac{d\phi}{dN}$ that is too great for the effective drag force to slow before it passes up too much of the flat region. This tells us that any initial $\frac{d\phi}{dN}$ that will serve our purposes must fall in some ideal window. Too slow, and the drag force will make sure inflation still begins to the right of the inflection point. Too fast, as it was here, and the drag force will not be able to slow it *enough*. Thus the game we play is to find the middle ground, where the drag force happens to stop it somewhere left of the inflection point, but not so far left that it cannot attain $N_{\text{total}} \approx 60$.

In the Middle Ground

After probing the space of initial conditions, we find that one suitable initial condition is $\frac{d\phi}{dN} = -0.685$. This gives us the following plots:

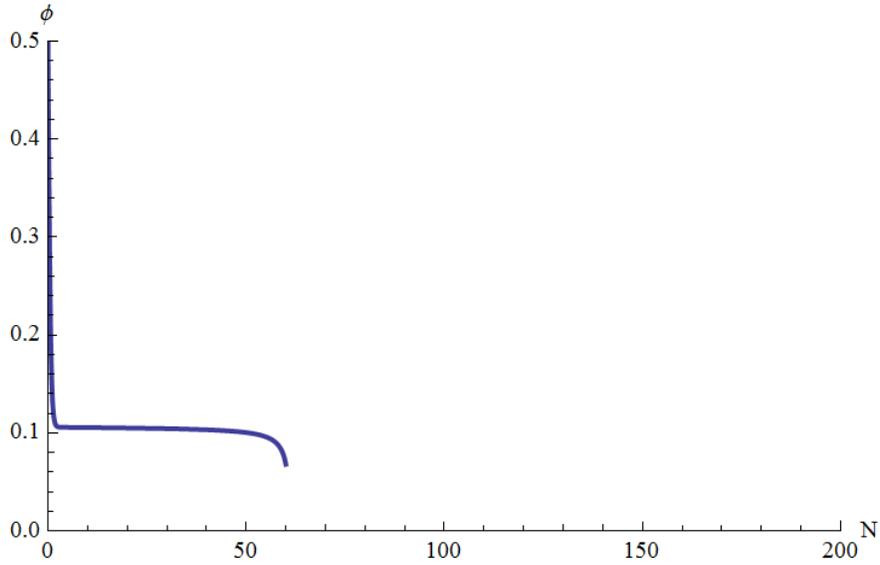


FIG. 13. $\phi(N)$ for $\phi_0 = 0.5$, initial $\frac{d\phi}{dN} = -0.685$. We find $N_{\text{total}} = 60$: exactly our target.

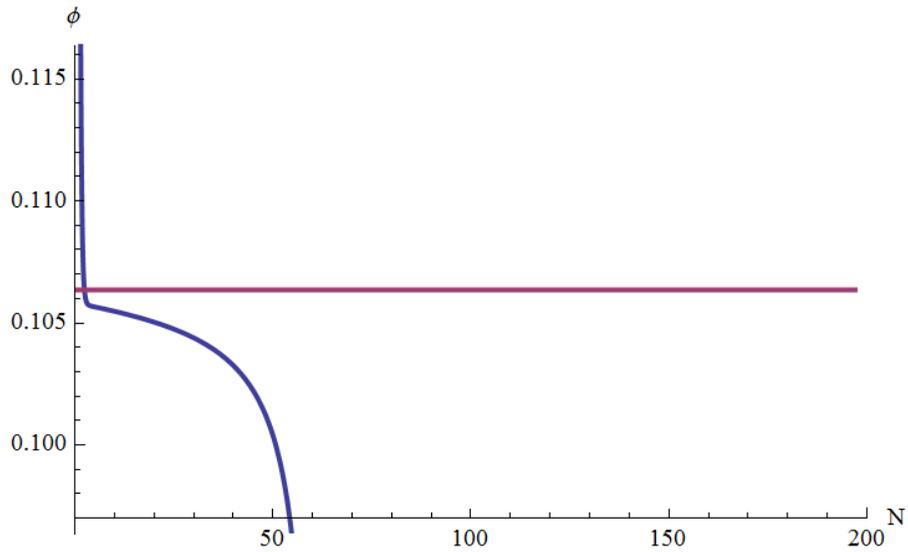


FIG. 14. A zoomed in version of figure 13. Here we emphasize the change in ϕ over time, and that essentially *no inflation occurs before the inflaton passes ϕ_** (purple line). All of the inflation is happening *left* of the inflection point.

Which is *exactly* the type of trajectory we were searching for. Not only does inflation end with $N_{\text{total}} = 60$, but the inflaton spends all of its time inflating *left* of the inflection point. As we expect, this produces a graph of n_s versus N that shows we have satisfied both constraints:

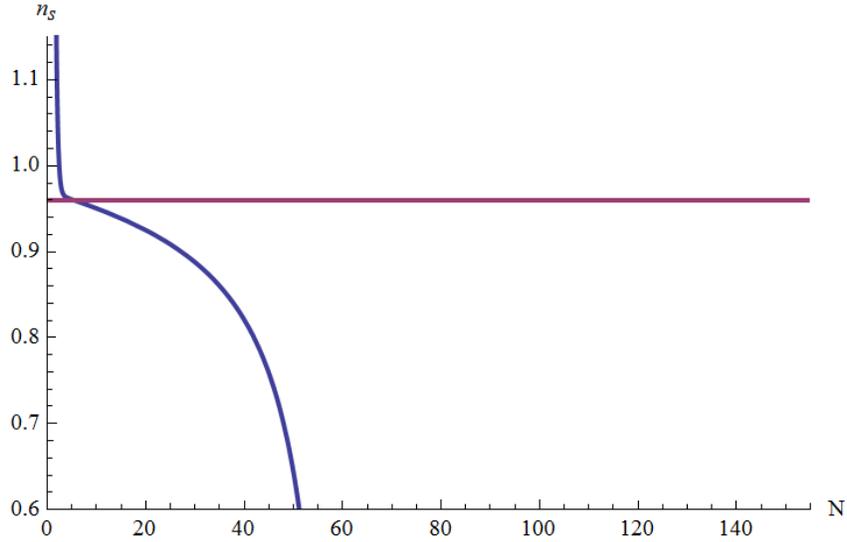


FIG. 15. We see that n_s plummets to the correct value before inflation begins. This is because of how rapidly the inflaton passes up the unsuitable blue region from figure 5. Since $N \approx 60$, we can see that $N(\phi_{\text{cmb}})$ is of order 1, which has the correct value of n_s . Thus, for this trajectory of

$$N \approx 60, \text{ we also have } n_s \approx 0.96.$$

For thoroughness, before we can claim success, we must make sure that inflation is really happening where we think it is. We check this by plotting ϕ' as a function of N , and making sure that it is satisfied while the inflaton is on the left side of the flat region.

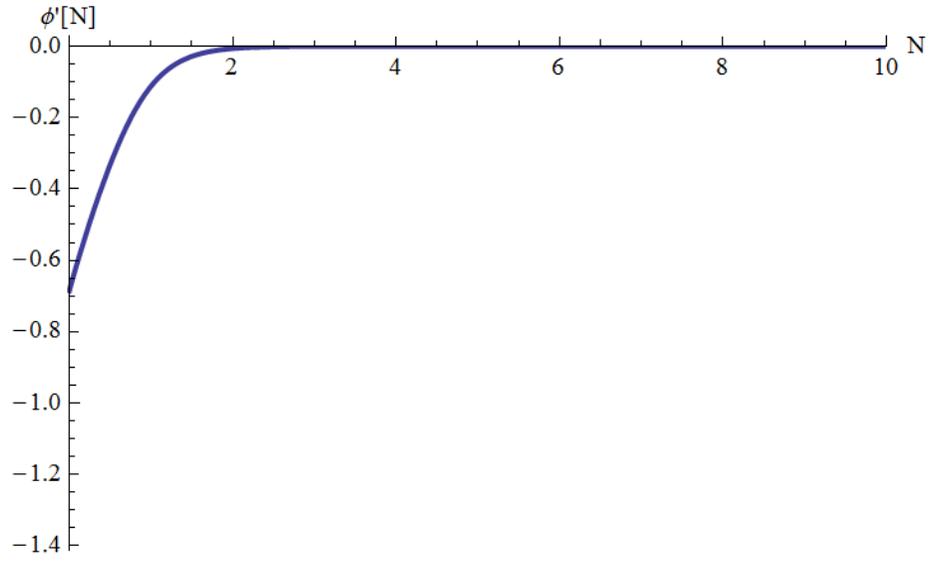


FIG. 16. We see that $\phi' \ll \sqrt{2}$ around $N = 2$. Afterwards, slow-roll inflation is underway.

Figure 16 assures us that we *are* inflating once we have reached the left side.

CHAPTER IV

SUMMARY AND CONCLUSIONS

We can finally claim success. We have found initial conditions in D-brane inflation where $N_{\text{total}} \approx 60$ and $n_s \approx 0.96$. From [14], we know that this is sufficient to show that D-brane inflation can cause power suppression.

However, since this work was simply a proof of concept, we have left many questions to be answered. These questions include:

- 1) Are these initial conditions too finely tuned to be natural?
- 2) Is the power suppression caused by D-brane inflation *enough* or *of the correct shape* to significantly improve the angular power spectrum's fit to the data?
- 3) Are there other ways for D-brane inflation to exhibit power suppression (besides initial velocity implementation)?
- 4) Does D-brane inflation hold up to other constraints from the data?

Question 1 is tricky to address, because we simply do not fully understand the physical principles that selected the real initial conditions. This is why we are not too troubled by finely tuning the potential's parameters and inflaton's initial conditions. We might discover that there is a physical principle that favors initial inflaton velocities in the range we found above. Nonetheless, there could easily not be, so we cannot be too bold answering this question.

Question 2, on the other hand, would not be difficult to explore. To answer it, we would simply need to solve the Mukhanov-Sasaki equation to find the detailed form of the power spectrum for different parameter values.

Question 3 can almost certainly be answered with a “yes.” The results of this paper imply we could similarly answer the mutual-exclusivity problem by steepening the potential to the right of the flat region. In this way, when the inflaton reaches the flat region, it will be moving so quickly that it will not have time to slow down before passing up the right-half of the flat region. This is promising future work.

Question 4 has been the topic of many prior papers [7, 16]. However, it does warrant mentioning that the recent BICEP2 results all but rule-out D-brane inflation [6]. BICEP2 observed a relatively large tensor-to-scalar perturbation ratio, and D-brane inflation has been shown to be unable to account for that [7]. This is an unfortunate blow to this model, but one that perhaps D-brane inflation can recover from. After all, D-brane inflation was also believed to be unable to account for power suppression.

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APPENDIX

This Appendix is devoted to listing the potential used in this paper. In doing so, we will define values for the parameters found in the potential, and briefly explain why those parameters have been chosen.

The potential that we use comes from [7].

$$V(\phi) = \frac{a|A_0|^2}{3} \frac{e^{-2a\sigma(\phi)}}{U^2(\phi, \sigma(\phi))} g(\phi)^{2/n} \left[2a\sigma(\phi) + 6 - 6e^{a\sigma(\phi)} \left| \frac{W_0}{A_0} \right| \frac{1}{g(\phi)^{1/n}} + \frac{3c}{n} \frac{1}{g(\phi)^2} \frac{\phi}{\phi\mu} - \frac{3}{n} \left(\frac{\phi}{\phi\mu} \right)^{3/2} \frac{1}{g(\phi)} \right] + \frac{D[\phi]}{U^2(\phi, \sigma(\phi))} \quad (\text{A1})$$

As [7] also lists, the constituent functions are defined as

$$U[\phi, \sigma[\phi]] = 2\sigma[\phi] - \frac{\sigma_0}{3} \phi^2$$

$$g[\phi] = 1 + \left(\frac{\phi}{\phi\mu} \right)^{\frac{3}{2}}$$

$$D[\phi] = D \left(1 - \frac{3D}{16\pi^2} \frac{1}{\left(\frac{2}{3} \phi^2 \right)^2} \right)$$

The function $\sigma[\phi]$ must be calculated numerically with $\frac{\partial V}{\partial \sigma} |_{\sigma[\phi]} = 0$. The parameters are

$$n = 8$$

$$a = \frac{2\pi}{n}$$

$$A_0 = 1$$

$$W_0 = -3.432 * 10^{-4}$$

$$\phi\mu = \frac{1}{4}$$

$$c = \frac{9}{4 n a \sigma_0 \phi\mu^2}$$

$$\sigma_0 = \frac{10.1}{a}$$

$$D = \frac{s | V_F(0, \sigma_F) |}{U^{-2}(0, \sigma_F)}$$

$$\sigma_F = \frac{10}{a}$$

And V_F is (A1) without the last term, $\frac{D[\phi]}{U^2(\phi, \sigma(\phi))}$. Note we have left one parameter undefined: s .

This parameter only appears in D . We use it as a useful method for adjusting D . As [7] discusses, only certain values of s produce a $V(\phi)$ with a region flat enough for prolonged inflation. For the parameters listed above, $s \approx 1.1$ for such inflation.

In this paper, we use $s = 1.1215$. We decided on this value because it provides a flat region suitable for inflation that is also steep enough for a $N_{\max} \approx 150$, given reasonable initial conditions. Had we chosen a smaller s , we would have found a smaller value for N_{\max} . Conversely, larger s implies larger N_{\max} . This is because, to a limited extent, s varies the steepness of the flat region where inflation occurs.