

PRE-SERVICE TEACHERS' KNOWLEDGE OF ALGEBRA TEACHING FOR  
EQUITY

A Dissertation

by

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## ABSTRACT

This study examined validity and reliability of an instrument used to measure the cultural awareness beliefs and problem-solving strategies of pre-service mathematics teachers created by the Knowledge for Teaching Algebra Equitably (KATE) project team at Texas A&M University.

The dissertation was comprised of three journal articles. The first article synthesized literature pertaining to teacher's cultural awareness knowledge and beliefs for teaching mathematics equitably in the middle grades. A search of the Texas A&M Library database was used to find articles that matched criteria related to instruments that determined information from pre-service mathematics teachers pertaining to cultural awareness knowledge and beliefs, equity, and mathematical content knowledge. An exhaustive meta-synthesis showed that there were no current instruments that matched all of the above criteria.

The second article estimated the reliability and validity of the KATE instrument. Internal consistency reliability for the equity items was estimated to be .77 using Cronbach's alpha. An alpha value of .6 was used as the baseline for determining suitable internal consistency reliability. Content validity was estimated for the entire KATE instrument by discussing the appropriateness and wording of the items from the Knowledge for Teaching Algebra Equitably (KATE) instrument with a panel of experts reading responses PTs gave on the KATE instrument, and assessing feedback from PTs enrolled in the course. This resulted in the insertion, deletion, and rewording of items.

Construct validity was estimated by conducting an exploratory factor analysis of the equity items which estimated six factors.

Lastly, the third article revealed preliminary results from pre-service teachers who participated in the Knowledge for Teaching Algebra Equitably Project at Texas A&M University in the fall of 2011 and fall of 2012. An analysis of the test scores from the pre-service teachers (PTs) from the pre-test to the post-test was done comparing scores from PTs in both semesters. The two groups were not statistically different. The effects of the course on (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving was reported in confidence intervals that showed the equity items were not statistically significantly different, but the problem solving and teaching problem solving items were. A MANOVA was used to determine the difference in teaching problem solving scores was due to semester, race, and class by certification. The adjusted  $R^2$  values were reported to provide the correlation between the independent and dependent variables.

## DEDICATION

To my family, friends, and colleagues for being supportive, understanding, prayerful, and loving through this entire process.

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I would first like to thank God for giving me the strength to finish this race.

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## NOMENCLATURE

|      |  |
|------|--|
| KATE | Knowledge for Teaching Algebra Equitably     |
| KTA  | Knowledge for Teaching Algebra               |
| KTE  | Knowledge for Teaching Equity                |
| MASC | Integrated Mathematics & Science             |
| NCTM | National Council for Teachers of Mathematics |
| PT   | Pre-service teacher                          |

## TABLE OF CONTENTS

|   | Page |
|---|------|
| ABSTRACT .....  | ii   |
| DEDICATION .....  | iv   |
| ACKNOWLEDGEMENTS .....                                      | v    |
| NOMENCLATURE.....   | vi   |
| TABLE OF CONTENTS .....                                     | vii  |
| LIST OF FIGURES.....  | ix   |
| LIST OF TABLES .....  | xi   |
| CHAPTER I INTRODUCTION: THE IMPORTANCE OF RESEARCH .....    | 1    |
| Introduction .....  | 1    |
| Significance of Study .....                                 | 3    |
| Purpose of Study.....                                       | 4    |
| Research Questions .....                                    | 4    |
| Conclusion.....   | 5    |
| CHAPTER II A REVIEW OF RESEARCH ON THE KNOWLEDGE OF         |      |
| ALGEBRA TEACHING FOR EQUITY .....                           | 6    |
| Introduction .....  | 6    |
| Review of Literature.....                                   | 6    |
| Methods and Data Sources.....                               | 15   |
| Conclusion.....   | 21   |
| CHAPTER III DEVELOPMENT, VALIDATION, AND RELIABILITY OF THE |      |
| KNOWLEDGE OF TEACHING ALGEBRA FOR EQUITY TEST.....          | 24   |
| Review of Literature.....                                   | 24   |
| Methods and Data Sources .....                              | 34   |
| Data Analyses.....  | 41   |

|   | Page |
|---|------|
| Results .....   | 42   |
| Conclusion.....   | 49   |
| Limitations .....   | 51   |
| <br>  |      |
| CHAPTER IV EFFECTS OF A COURSE ON PRE-SERVICE TEACHERS TO |      |
| DEVELOP THE KNOWLEDGE TO TEACH ALGEBRA FOR EQUITY         | 52   |
| Review of Literature.....                                 | 52   |
| Methods and Data Sources .....                            | 56   |
| Data Analyses.....  | 71   |
| Results .....   | 72   |
| Conclusion.....   | 88   |
| Limitations .....   | 91   |
| <br>  |      |
| CHAPTER V SUMMARY AND CONCLUSIONS .....                   | 92   |
| <br>  |      |
| REFERENCES .....  | 98   |
| <br>  |      |
| APPENDIX .....  | 112  |

## LIST OF FIGURES

|             |   | Page |
|-------------|---|------|
| Figure 3.1  | Total participants by university.....   | 35   |
| Figure 3.2  | Gender .....  | 36   |
| Figure 3.3  | College classification level.....   | 37   |
| Figure 3.4  | Certification Level.....  | 38   |
| Figure 3.5  | Hispanic v. Non-Hispanic .....  | 39   |
| Figure 3.6  | Race.....   | 40   |
| Figure 4.1  | Gender total by semester .....  | 59   |
| Figure 4.2  | College classification level by semester.....   | 60   |
| Figure 4.3  | Certification level by semester .....   | 61   |
| Figure 4.4  | Hispanic v. Non-Hispanic by semester .....  | 62   |
| Figure 4.5  | Race by semester .....  | 63   |
| Figure 4.6  | Second Life Training Implementation .....   | 65   |
| Figure 4.7  | Second Life meeting space.....  | 66   |
| Figure 4.8  | Second Life classroom .....   | 67   |
| Figure 4.9  | Confidence intervals for combined Group 1 and Group 2 Equity<br>Total pretest and posttest..... | 79   |
| Figure 4.10 | Paired samples test for group 1 pre- and post- test problem<br>solving total .....              | 83   |
| Figure 4.11 | Paired samples test for group 1 pretest and posttest teaching<br>problem solving total.....     | 84   |
| Figure 4.12 | Paired samples test for Group 2 pretest and posttest problem<br>solving total .....             | 85   |

|   | Page |
|---|------|
| Figure 4.13 Paired samples test for group 2 pretest and posttest teaching<br>problem solving total..... | 86   |

## LIST OF TABLES

|           |  | Page |
|-----------|--|------|
| Table 3.1 | CABI Items Used in KATE Instrument, Factors Revealed from the Factor Analysis of the 20 CABI Items Used in the KATE Instrument, and Factors Revealed from the Factor Analysis of the 20 Items Used in the KATE Instrument..... | 44   |
| Table 3.2 | Factor pattern (P) and Structure (S) Matrices rotated for the Equity Items Post-test for Fall 2011 and Fall 2012.....  | 47   |
| Table 4.1 | Group Statistics for Group 1 and Group 2 Equity Total, Problem Solving Total, and Teaching Problem Solving Total Items .....   | 73   |
| Table 4.2 | Combined Group Statistics for Group 1 and Group 2 Pretest: Equity Total, Problem Solving Total, and Teaching Problem Solving Total Items .....   | 74   |
| Table 4.3 | Independent Samples Test for Group 1 and Group 2 Pretest for Equity Total, Problem Solving Total, and Teaching Problem Solving Total .....   | 75   |
| Table 4.4 | Statistics for Combined Group 1 and Group 2 Equity Total Pretest and Posttest.....   | 78   |
| Table 4.5 | Statistics for Pretests and Posttests for Group 1 and Group 2 Problem Solving Total and Teaching Problem Solving Total .....   | 80   |
| Table 4.6 | Mean and Cohen's <i>d</i> for the Pretest and Posttest scores for the Six CABI Factors .....   | 87   |

CHAPTER I  
INTRODUCTION: THE IMPORTANCE  
OF RESEARCH  
**Introduction**

In this dissertation, I reviewed and summarized the research literature necessary to build a framework for an instrument that assesses the cultural awareness beliefs and problem solving skills of pre-service teachers (PTs). Because many of the available instruments that assess PTs knowledge addressed only mathematics content or multicultural beliefs and attitudes. The Knowledge for Teaching Algebra for Equity (KATE) project team created a two part instrument. The first part was a Likert scale that addressed the cultural awareness and beliefs of mathematics PTs. The second part consisted of open-ended mathematics problems that addressed the cultural responsiveness, mathematical understanding, and problem solving skills of mathematics PTs in the middle grades.

The KATE research team discussed a need for an instrument that measured PTs' knowledge of teaching mathematics and doing it equitably. The KATE team considered all the available instruments in the literature. The reliability and validity of these instruments were then examined. Knowledge about these psychometrics will help researchers understand more about a PT's awareness of culture and mathematical knowledge.

Several instruments already exist in the literature. The current instruments that measured the knowledge needed to teach mathematics dealt with culture and diversity,

but not equity (Boykin, Tyler, Watkins-Lewis, & Kizzie, 2006; Larke, 1990; Love & Kruger, 2005; Peterson, Fennema, Carpenter, & Loef, 1989; Phuntsog, 2001; Roberts-Walter, 2007; Swartz & Bakari, 2005; Wayson, 1988). In general, the research reported on the content validity, but not the construct validity. The quality of the instrument cannot be measured if this statistic is not reported.

Diversity in the classroom is a perplexing reality. Variance in cultures, academic readiness, personalities, and skill levels amongst students in today's classrooms raises educational issues (Brookfield, 2006). As teachers learned about the students' backgrounds and abilities, they also needed to learn about the students' racial and ethnic identities (Brookfield, 2006; Payne, 2008). The National Council of Teachers of Mathematics (NCTM) created six principles for school mathematics. These overarching themes included equity, curriculum, teaching, learning, assessment, and technology (NCTM, 2000). The principle of equity stated, "Excellence in mathematics education requires equity - high expectations and strong support for all students" (NCTM, 2000, p. 11). It is therefore important that mathematics teacher preparation courses and programs assured that they are preparing PTs who are aware of and could apply strategies that helped every student successfully learn the concepts taught in the classroom. In order to improve mathematics teaching, more must be done to generate and share knowledge about teaching (Stigler & Hiebert, 1999) and diverse cultures. The focus of this dissertation was to determine the cultural awareness beliefs and mathematical knowledge of pre-service teachers that were enrolled in a junior-level Integrated Mathematics and Science course at a major university.

## **Significance of Study**

It is no secret that the field of education is in desperate need of effective and knowledgeable teachers. High turnover in schools exists in part because teachers do not know how to balance the three C's of content, students' cultures, and classroom environment. In addition, teachers that felt as though they could succeed were more likely to stay at a particular school (Darling-Hammond & Sykes, 2003). It was not uncommon that teachers were leaving the field of education after only a year or two of teaching. It was imperative that teacher preparation programs equip pre-service teachers with the tools needed to handle the three C's.

Many of the instruments that existed to measure the mathematical knowledge of pre-service teachers in preparation programs were multiple-choice, closed-ended, and did not connect equity to mathematical knowledge. The instruments for elementary (Hill et al., 2004), middle school (Saderholm, Ronau, Brown, & Collins, 2010) mathematics teaching, and multicultural knowledge (Larke, 1990; Roberts-Walter, 2007) were an example of this fact. Though these instruments had support on information about validity, there was a need to create and validate new instruments. The creation of the new instruments would provide more substantial insights to the connection between teachers' knowledge of mathematics and their beliefs about and knowledge of equity issues. There is only so much we can learn from a multiple-choice or Likert instrument. An instrument that assesses the problem solving skills of mathematics teachers through open-ended problems as well as their cultural awareness beliefs would add value to the knowledge base.

## **Purpose of the Study**

The principal purposes addressed through this dissertation were to (a) review the research on PTs' cultural awareness beliefs and problem solving strategies, (b) establish estimates of the validity and reliability of the scores of the KATE test, and (c) investigate the effect of presenting mathematics content through a culturally relevant pedagogical approach. The NCTM has made efforts to create, reinforce, and update the principles and standards for K-12 mathematics curriculum (NCTM, 1989, 2000). It was important that the same was done for the standards of knowledge needed to teach mathematics. In this dissertation I examined the knowledge needed to teach mathematics equitably.

## **Research Questions**

Specifically, eight research questions were addressed through this dissertation:

### **Chapter II**

1. What literature existed to support a unified theoretical framework for understanding teacher's cultural awareness knowledge and beliefs for teaching mathematics equitably in the middle grades?

### **Chapter III**

2. What was the construct validity of the Equity items from the Knowledge for Teaching Algebra Equitably (KATE) Instrument? What were the statistical measures from the factor analysis?
3. What was the content validity of the Knowledge for Teaching Algebra Equitably (KATE) Instrument? Did any items need to be revised, added, or deleted?

4. How was internal consistency reliability related to various subgroups within the sample?

#### **Chapter IV**

5. Was there a difference between fall 2011 and fall 2012 on (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?
6. What was the effect of the course on (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?
7. What was the relationship among (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?
8. What was the relationship among (one or more demographic variables) and (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?

#### **Conclusion**

The dissertation is a compilation of three journal articles. The first article is a synthesis of the literature pertaining to teachers' cultural awareness knowledge and beliefs for teaching mathematics equitably in the middle grades. The second article addresses estimates of the reliability and validity of the scores of the KATE instrument. Lastly, the third article reveals preliminary results from pre-service teachers who participated in the Knowledge for Teaching Algebra for Equity Project at Texas A&M University in the fall of 2011 and fall of 2012.

CHAPTER II  
A REVIEW OF RESEARCH ON THE KNOWLEDGE OF ALGEBRA TEACHING  
FOR EQUITY

**Introduction**

The equity component was missing from the instruments in the literature used to measure the knowledge needed to teach mathematics (Boykin, Tyler, Watkins-Lewis, & Kizzie, 2006; Larke, 1990; Love & Kruger, 2005; Peterson, Fennema, Carpenter, & Loef, 1989; Phuntsog, 2001; Roberts-Walter, 2007; Swartz & Bakari, 2005; Wayson, 1988). Some of these instruments made mention of culture and diversity, but not equity. Moreover, little has been done to address secondary teacher's knowledge of the function concept (Norman, 1992). The quality of the instruments that included culture and diversity could not be ascertained because there were no reports of the construct validity. In fact many of the instruments did not report on the construct validity, but the research reported on the content validity. There is a strong need to study teacher development using outcome measures that address the effects on classroom teaching (Kulm, 2008).

**Review of Literature**

**Knowledge Needed for Teaching**

Research showed that more than content knowledge needed to be possessed by the classroom teacher. What a teacher knows is one of the most important influences on what a student learns in a classroom (Fennema & Franke, 1992). Teacher preparation programs served as the starting point for developing highly qualified teachers for all students (Kulm et al, 2011). Shulman (1986) distinguished among three categories of

content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. In an expanded view of teacher knowledge, a minimum of eight constructs are included: 1) content knowledge, 2) general pedagogical knowledge, 3) curriculum knowledge, 4) pedagogical content knowledge, 5) knowledge of learners and their characteristics, 6) knowledge of educational contexts, 7) knowledge of educational ends, purposes, and values, 8) and their philosophical and historical grounds (Shulman, 1987). In addition, “academic ability, teachers’ professional knowledge and experience also make an important difference in student learning” (Darling-Hammond & Sykes, 2003, p. 8). The constructs of knowledge that teachers possess are the foundation of student learning.

The National Mathematics Advisory Board believed that all prepared students should have access to an authentic algebra course (National Mathematics Advisory Panel, 2008). Prepared students were those students entering into algebra having fulfilled all the necessary prerequisites. With a National Science Foundation funded grant, researchers at Texas A&M University were working to determine the knowledge needed to teach algebra equitably. Kulm et al. (2011) believed that PT’s knowledge of student’s learning and motivation is the basis for the PT’s knowledge for teaching algebra for equity. If teachers are prepared to teach students equitably then the teachers will address the needs of students in an algebra course better.

### **Cultural awareness knowledge**

Cultural awareness within a classroom was imperative for student motivation and understanding of the mathematics curriculum. Culture denoted the “deep structures of

knowing, understanding, acting, and being in the world” (Ladson-Billings, 1997, p. 700). In an observation that Ladson-Billings conducted with a sixth grade mathematics teacher, she observed many accounts of culturally relevant teaching (Ladson-Billings, 1995a). The teacher set the context of the algebra lesson by informing students of the African origins of the subject. Students asked and answered questions posed by the entire class as the teacher served as the facilitator. This instance illustrated that all students can be successful when the student’s understanding was linked to meaningful cultural contexts. Culture informs all human activity and thought (Ladson-Billings, 1997) and therefore, should not be separated from teaching.

The manner in which someone teaches influenced more than just the content and students. Culturally relevant teaching integrated students’ informal mathematical knowledge and their culture and experiences, critical mathematical thinking and critical approaches to knowledge in general, and empowerment orientations to culture and experience. Helping students develop tools to participate actively in society and work for social justice requires that teachers make connections between mathematics teaching and social activism. During a classroom observation it was observed that one teacher saw herself and her own experiences reflected in her students. However during other observations, other teachers were explicit about how they taught their mathematics and the relationships between producing leaders among students from marginalized groups (Gutstein, Lipman, Hernandez, & de los Reyes 1997). Also, it was critical that effective mathematics teachers developed knowledge and practice that will build on children’s

mathematical thinking, community, and culture (Turner et al., 2011). Community and culture played a vital role in the culturally relevant teaching experienced by students.

### **Constructivism**

Constructivist approaches can be linked to equitable teaching and learning environments which can lead to better student learning. Constructivist classrooms are a breeding ground for hands on learning related to the student's culture. The mathematics learner should be able to reconstruct some portion of the teacher's knowledge through instruction (Ernest, 1989). Constructivist classrooms help this happen (Campbel, 1996). Constructivism asserts two main principles: knowledge is not passively received but actively built up by the cognizing subject; the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (von Glasersfeld, 1989a; von Glasersfeld, 1989b). When mathematics is taught should be "understood as providing students with the opportunity and the stimulation to construct powerful mathematical ideas for themselves and to come to know their own power as mathematics thinkers and learners" (Simon & Schifter, p. 310). Constructivist classrooms allow for all students to produce knowledge themselves.

In order to determine successful teaching techniques, one must understand the development of concepts in the student's mind (National Mathematics Advisory Panel, 2008, Fennema & Franke, 1992; Vygotsky, 1962). In order to determine what and how a concept will be taught, a teacher must know the ways in which students think.

Vygotsky (1962) goes on to quote Leo Tolstoy:

When he has heard or read an unknown word in an otherwise comprehensible sentence, and another time in another sentence, he begins to have a hazy idea of the new concept; sooner or later he will... feel the need to use that word – and once he has used it, the word and the concept are his...” (p. 84)

Prior to this, Tolstoy mentioned that the meaning of a word cannot be taught.

The child needs “a chance to acquire new concepts and words from the general linguistic context” (Vygotsky, 1962, p. 83). Brown, Collins, and Duguid (1989) studied Vygotsky and his idea that the activity of knowledge “is not separable from or ancillary to learning and cognition” (p. 32). Knowledge therefore must be constructed by the learner.

Constructivism asserts two main principles: knowledge is not passively received but actively built up by the cognizing subject; the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (von Glasersfeld, 1989a; von Glasersfeld, 1989b). Moreover, knowledge cannot just be handed out giving a student a new concept and expecting that student to immediately comprehend it. Constructs are attributes of people (Cronbach & Meehl, 1955). Most researchers accept the view that “students actively construct their mathematical ways of knowing as they strive to be effective by restoring coherence to the worlds of their personal experience” (Cobb, 1994, p. 13). The way students think is adapted to their environment and knowledge is incurred when a student is actively engaged in a subject. The idea of teaching involves the exchange of ideas and the need for teachers to see the ideas from many different sides (Shulman, 1987). Telling does not equate to teaching

(Ladson-Billings, 2009). In other words, a student needs a teacher to represent objectives taught in terms of a context that can be understood by that particular student.

Constructivist in in-service programs can help teachers develop more powerful ideas with respect to mathematics and learning (Simon & Schifter, 1991). A classroom that is described as constructivist is likely to have teachers whose beliefs are to a large extent aligned with a problem-solving orientation to mathematics and with associated views of mathematics teaching and learning (Beswick, 2005).

Many constructivists believe that during psychological development signs and symbols are used as “a means by which students express and communicate their mathematical thinking” (Cobb, 1994, p. 13). The learning of mathematics should be viewed as a “process of active individual construction and a process of enculturation into the mathematical practices of wider society” (Cobb, 1994, p. 13). Priority is given to the individual’s senses and activity of conceptualizing. Here the teacher will be quite concerned with the quality of student’s interpretation of the mathematical activities and interaction with the social norms and mathematical processes. Thus, the student will actively construct knowledge (Bereiter, 1994; Beswick, 2007; Cobb, 1994; Ernest, 1989; Pirie & Kieren, 1992; Simon & Schifter, 1991). A constructivist classroom setting will help students perform better on more conceptually challenging problems (Cobb, Terry, Yackel, & Perlwitz, 1992). In a constructivist classroom students will be actively engaged in the learning process and working to construct knowledge themselves and with their classmates.

### **Teaching mathematics equitably**

The Elementary and Secondary Education Act (ESEA) of 1965 was established to “offer equitable educational opportunities to the nation’s disadvantaged, this legislation provides financial resources to schools to enhance the learning experiences of underprivileged children” (Thomas & Brady, 2005, p. 51). Providing monies to schools that service underprivileged children will allow for an equitable learning environment across the nation. The National Council of Teachers of Mathematics’ *Principles and standards for school mathematics* (2000) provided six principles for school mathematics: equity, curriculum, teaching, learning, assessment, and technology. The equity principle included high expectations and support for all students. Integration of all six principles was necessary in mathematics classrooms and was imperative for an effective learning environment amongst students and the teacher. Mathematics must be relatable to students when taught. Nasir and Cobb (2002) believed equity partially depends on how students’ identities as learners are enabled as they engage in classroom mathematical practices. Mathematics must be made meaningful and accessible to all students (Ladson-Billings, 1995b).

The fact that mathematics was not made to be relevant to all students could be one reason for persistent existence of the mathematical achievement gap. The perpetual mathematical achievement gap that exists between races in the U.S. (NAEP, 2008) is one possible indicator that everyone is not receiving the same quality of education. An underlying assumption of the goal for teaching mathematics was that students developed

mathematical understanding (Ball, 1990a). A key to mathematical understanding for all students is the existence of equity in the classroom.

Equity pedagogy is defined as the “opportunities that *all* children have to benefit from classroom instruction” (Ladson-Billings, 1995b, p. 130) or the modified teaching done to increase the academic achievement of students from diverse groups (Banks, 2006). Researchers believed that the majority of students can be successful in mathematics (Escalante & Dirmann, 1990; Robinson, 1996) when their understanding was tied to meaningful cultural references (Ladson-Billings, 1995a; Ladson-Billings, 1997; Milner, 2011; Silver & Lane, 1995; Turner et al, 2011). As a result of increased attention to equity pedagogy, more classrooms were implementing cooperative and collaborative learning. This type of learning had the expectation that all students meet more rigorous educational challenges. Ladson-Billings (1995b) also discussed the need for context within mathematics. Students of different races would come to different conclusions because some situations have diverse meanings for students of varying races. Diversity is a part of students’ lives, cultures, and communities (Cobb & Hodge, 2002). Moreover, if the teachers are prepared for the classroom then all students can benefit from their educational experiences.

### **Content and pedagogical knowledge**

Content knowledge is one of the primary types of knowledge that teachers must possess in order for students to learn in the classroom (Ambrose, 2004). A teacher’s mathematical knowledge is comprised by a number of factors that include “its extent and depth; its structure and unifying concepts, knowledge of procedures and strategies; links

with other subjects; knowledge about mathematics as a whole and its history” (Ernest, 1989, p. 16). Much of this knowledge is acquired from early childhood onwards (Ernest, 1989; Ginsburg, Cannon, Eisenband, & Pappas, 2006). Teachers should have a masterful knowledge of arithmetical and algebraic problem solving strategies before carrying out complex instructional tasks, but there is evidence that a substantial number of teachers do not meet this requirement (Ball, 1990b; Kinach, 2002; Van Dooren, Verschaffel, & Onghena, 2002). In a teaching experiment with PTs, Kinach (2002) had the PTs explain and represent their concept of integers three times in different contexts. This showed the PTs’ understanding and non-understanding of the concept they were learning to teach. Ball (1990b) interviewed and gave questionnaires to 252 elementary and secondary pre-service mathematics teachers and found that their mathematical understandings brought to teacher education from precollege and college mathematics experiences were very thin and rule-bound. The study by Van Dooren, Verschaffel, & Onghena (2002) showed that PTs had trouble with problem solving. Pedagogical knowledge is the practical knowledge of teaching (Ernest, 1989). When teaching mathematics teachers should tune into what students are thinking in order to know what to teach next (Fennema & Franke, 1992; Herbel-Eisenmann & Phillips, 2005; Tirosh, 2000). Often times, because pedagogy and content are intertwined in teaching, the two are put together to form pedagogical content knowledge (Chinnappan & Thomas, 2001; An, Kulm, & Wu, 2004; Shulman, 1987).

## **Methods and Data Sources**

A meta-synthesis of the currently available research and instruments that related to teacher's cultural awareness of teaching mathematics and mathematical content knowledge in the middle grades was conducted. A list of key words and terms were developed to ensure a thorough search, which focused on research conducted from 1989 to 2012. Databases searched included ERIC (EBSCO), JSTOR, and Google Scholar, ProQuest. The purpose here was to determine what literature existed to support a unified theoretical framework for understanding teacher's cultural awareness knowledge and beliefs for teaching mathematics equitably in the middle grades.

### **Search Criteria**

The search engines that guided the investigation of instruments related to pre-service teachers' knowledge of mathematics and culture included ERIC (EBSCO), JSTOR, Google Scholar, and ProQuest. Because the NCTM *Curriculum & Evaluation Standards for School Mathematics* (1989) had major and lasting effects on mathematics education, as have the *Principles and Standards for School Mathematics* (2000) I searched for peer-reviewed articles published from 1989-2012. Key terms input into the Google Scholar search engine as Boolean phrases guiding this search included "diverse" & "equity" & "instrument" & "self-efficacy" & "culturally responsive" & "preservice teacher" & "multicultural education" & "psychometric".

### **Search Results**

This search produced 12 results. Articles were selected if they met the following criteria: 1) the article included information about an instrument dealing with (a) teacher

beliefs, (b) cultural beliefs, (c) self-efficacy, (d) mathematics teacher knowledge, or (e) equity beliefs; 2) psychometric information, and 3) sample demographics were provided. Of the 12 articles, only two met the criteria. Articles were also included that were identified through secondary sources (i.e. personal communications or reference list search). In total, 14 articles reviewed in more detail.

### **Review of Teacher Mathematics Knowledge and Cultural Instruments**

The construct of cultural beliefs and cultural awareness has taken on many forms throughout the years such as culturally relevant (Ladson-Billings, 1995a), culturally appropriate (Au & Jordan, 1981), culturally congruent (Mohatt & Erickson, 1981), culturally responsive (Cazden & Leggett, 1981; Erickson & Mohatt, 1982), cultural compatibility (Jordan, 1985; Vogt, Jordan & Tharp, 1987), and humanizing pedagogy (Bartolome, 1994) to name a few. Researchers have also developed instruments in hopes to measure beliefs, but have failed to agree upon a definition of the term beliefs throughout the years (McLeod & McLeod, 2002). It has been proposed that one of the reasons that it was so difficult to come to an agreement about the term, besides the various definitions that exist of the term, was the fact that there were various types of definitions of what a belief was. Types of definitions included informal, formal, and extended definitions (McLeod & McLeod, 2002). One definition of beliefs was “anything that an individual regards as true” (Beswick, 2007, p. 96). Culture can be defined as:

The deep structures of knowing, understanding, acting, and being in the world. It informs all human thought and activity and cannot be suspended as human beings

interact with particular subject matters or domains of learning. Its transmission is both explicit and implicit (Ladson-Billings, 1997).

### **Mathematics-oriented**

There were three articles that included instruments related to teacher's mathematical knowledge or beliefs. The Pre-service Mathematical Knowledge for Teaching (PMKT) instrument was developed to measure mathematical knowledge for teaching used for pre-service program assessment (Russell, 2011). The Mathematics Teachers Efficacy Beliefs Inventory (MTEBI) was developed to understand the efficacy beliefs of mathematics teachers (Enochs, Smith, & Huinker, 2000). The Mathematical Knowledge for Teaching (MKT) instrument created by Hill, Schilling, & Ball (2004) was intended to determine how and what mathematics knowledge was required for teaching. These were the instruments for which the psychometrics was available.

### **Cultural beliefs and equity**

There were 11 articles related to instruments on teacher's cultural beliefs and equity resulting in 12 instruments. Findings from Teachers' Pedagogical Content Beliefs in Mathematics instrument reported on teacher beliefs (Peterson, Fennema, Carpenter, & Loef, 1989). The Teaching in Urban Schools scale was developed to measure pre-service teachers' knowledge about effective teaching in urban schools (Swartz & Bakari, 2005). The items in the Teacher Beliefs and Student Achievement in Urban Schools emphasized teacher's cultural awareness beliefs (Love & Kruger, 2005). The Cultural Classroom Practices Questionnaire was created to "assess teachers' reported use of cultural-based classroom activities (Boykin, Tyler, Watkins-Lewis, &

Kizzie, 2006). The Multicultural Teaching Scale reflects content and activities that are important for professionals to possess that teach diverse learners (Wayson, 1988). The Culturally Responsive Teaching instrument ascertains “teachers’ perceptions of the importance of culturally responsive teaching practice” (Phuntsog, 2001, p. 52). The Cultural Awareness Beliefs Inventory (CABI) measures the perceptions and attitudes of urban teachers’ cultural awareness and beliefs (Roberts-Walter, 2007). The Multicultural Efficacy Scale (MES) was designed as a tool to measure multicultural teaching efficacy, “multicultural teacher education dimensions of intercultural experiences, minority group knowledge, attitudes about diversity, and knowledge of teaching skills in multicultural settings” (Guyton & Wesche, 2010, p. 23). The Culturally Responsive Teaching Self-Efficacy (CRTSE) and Culturally Responsive Teaching Outcome Expectancy Scales (CRTOES) include competencies that “reflect the essential skills and knowledge that are clearly identifiable among teachers who engage in culturally responsive teaching (Siwatu, 2007, p. 1089). The Cultural Diversity Awareness Inventory (CDAI) is composed of 28 Likert opinion statements that address cultural awareness (Henry, 1995). The Multicultural Teaching Competency Scale (Spanierman et al., 2011) is composed of 16 Likert items that measures teachers’ multicultural competencies. These twelve instruments were examined to understand the psychometric behaviors and the reporting in the literature.

### **Types of validity reported**

The five types used for this analysis of validity are predictive or concurrent validity, content validity, construct validity (Carmines & Zeller, 1979; Cronbach &

Meehl, 1955; Huck, 2004; Kumar, 2005) and criterion validity (Psychology dictionary, 2013). Content validity measures the degree to which different items cover the material the instrument is intended to cover (Huck, 2004; Sireci, 1998). Construct validity is “a term used to indicate that the test scores are to be interpreted as indicating the test taker’s standing on the psychological construct measured by the test” (AERA, APA, NCME, 1999, p. 174). Criterion validity predicts the level of adequate or inadequate criterion performance (AERA, APA, NCME, 1999), and concurrent validity is the degree of two measurements at the same time (Psychology dictionary, 2013).

Content validity was important to report because researchers should understand whether the instrument measures what it is intended to measure. Of the 15 instruments reviewed, five only reported content validity. Of the five instruments that only reported content validity, all of them used a panel of experts to report content validity.

Construct validity differs from content validity in that it measured a person’s proper level of a construct, but content validity does not. Of the 14 instruments reviewed, four only reported construct validity. Of the four instruments that only reported construct validity, exploratory or confirmatory factor analysis was used. A type of validity related to construct validity was criterion validity. Criterion validity was reported in only one article by looking at group differences.

Three of the instruments reported a mixture of types of validity. These studies included instruments which reported on construct and content validity. This was also established by using a panel of experts and conducting a factor analysis.

### **Reliability reported**

Of the eight studies that reported on reliability, internal consistency reliability or test-retest reliability was used. Internal consistency reliability measures the “consistency across the parts of a measuring instrument” (Huck, 2004, p. 78). Test-retest reliability occurs when a researcher measures a group of participants twice with the same instrument (Huck, 2004). Internal consistency reliability was reported as internal consistency reliability, Cronbach’s alpha, or an alpha coefficient. The highest reliability coefficient of 0.93 was reported for the Teachers’ Pedagogical Content Beliefs in Mathematics instrument (Peterson, Fennema, Carpenter, & Loef, 1989). Reporting reliability is important because the resulting reliability coefficient tells the researcher if the same test were given to the same group of students, then the order of scores by the students would not change.

### **Structure of instrument**

The 15 survey instruments were constructed in various ways. The instruments were Likert, open-ended, interview, multiple-choice, or a combination of these structures. The instrument created by Love & Kruger (2005) only included interview questions. The instrument created by Peterson, Fennema, Carpenter, & Loef (1989) was Likert and included interview questions. Nine instruments were solely Likert (Boykin, Tyler, Watkins-Lewis, & Kizzie, 2006; Enochs, Smith, & Huinker, 2000; Guyton & Wesche, 2010; Henry, 1995; Ponterotto, Baluch, Greig & Rivera, 1998; Roberts-Walter, 2007; Siwatu, 2007; Spanierman et al., 2011; Swartz & Bakari, 2005; Wayson, 1988). One instrument was multiple-choice (Hill, Schilling, & Ball, 2004). One instrument was

Likert and open-ended (Phuntsog, 2001). One instrument was Likert and multiple-choice (Russell, 2011).

### **Conclusion**

It is obvious that the construct of cultural awareness and cultural beliefs has taken on many different meanings in various arenas of thought throughout the years. Though one definition of culture or beliefs has not been agreed upon, researchers are still making valiant efforts to understand these constructs in the context of individuals' daily lives. The content and construct validity together were only reported on three of the instruments. The reliability was reported on eight of the instruments. More needs to be done to report the reliability and validity of such instruments. If not then the findings from projects and individual results will be meaningless. The intensive literature search showed that more must be done to report psychometric data on instruments and include instruments that address teachers' mathematical content knowledge and cultural awareness beliefs.

Through this intensive search of peer-reviewed articles and book chapters pertaining to instruments that measure teacher's cultural awareness and mathematical knowledge it was evident that there were no instruments that assessed both of these ideas together. In fact there were many more instruments available for review about teacher's cultural awareness knowledge than there were about teacher's mathematical knowledge. This was evident from the meta-synthesis and a search on the two topics individually. The KATE instrument measures both concepts. The research has shown that it is

imperative that mathematics teachers possess not only the math content knowledge, but they must also possess the knowledge of students' backgrounds and cultures.

Of the instruments reviewed, none measured PTs' knowledge of mathematics and awareness of culture. The instruments either measured one or the other. Understanding PTs' awareness of culture and equity has been a concern of educators for the past couple of decades. The wide difference in the articles that present instruments measuring PTs' awareness of culture and equity compared to the available instruments that measure PTs' knowledge of math could occur for various reasons. One reason may be that culture transcends all content areas of school, but mathematics is often times confined to its specific content area.

Also, many of the published instruments did not present psychometric information regarding the validity or reliability of the instrument. In order to decipher whether an instrument is "good" or not, the researcher must show that the scores are reliable and valid. Knowing that the results of an instrument can be replicated and that the instrument measures what it was intended to measure are key qualifications for an instrument that can be used by all researchers throughout the field of education.

An instrument was needed by the KATE team that measures the cultural awareness and mathematical problem solving skills of pre-service mathematics teachers. Such an instrument would produce valuable information that can assist researchers and in-service teachers to help PTs to become more effective in the classroom. The instruments that existed did not fill this need since they address cultural beliefs but not how those beliefs connected with teaching and learning mathematics.

It has been hard to produce evidence about the relationship of elementary and middle school teachers' mathematical knowledge in comparison to students' mathematics achievement because of the lack of valid and reliable measures of teachers' mathematical knowledge (National Mathematics Advisory Panel, 2008).

CHAPTER III  
DEVELOPMENT, VALIDATION, AND RELIABILITY OF THE KNOWLEDGE OF  
TEACHING ALGEBRA FOR EQUITY TEST

**Review of Literature**

Researchers can answer questions about data by determining the reliability and validity of instruments. The substance of science is a “process of formulating specific questions and then finding answers in order to gain a better understanding of nature” (Graziano & Raulin, 2000, p.1). When a researcher finds an answer to the problem, it is under the assumption that the answer is correct. An incorrect answer does no one any good when trying to develop a solution to move forward. Validity helps the researcher because it measures what was intended to be measured in an experiment (Carmines & Zeller, 1979; Thompson, 2003). Below the reader will find a literature review discussing the meaning of validity, importance of validity for instrument creation, types of validity and validity related to instrument design, ramifications for not considering validity prior to test administration of the target sample, cultural awareness and beliefs questionnaires, validity of cultural awareness and beliefs questionnaires, and reliability.

**Meaning of Validity**

Validity is the most important concept in developing and evaluating psychometrics (AERA, APA, NCME, 1999; Lissitz, 2009). In research, validity is the “result of the intersection of our *intent* with the *process* of its implementation” (Keller & Casadevall-Keller, 2010, p. 11). Stated another way, validity “refers to the interpretations or actions that are made on the basis of test scores” (Lissitz, 2009, p. 20).

The Joint Committee on Standards for Educational and Psychological Testing, which is comprised of members from the American Educational Research Association (AERA), American Psychological Association (APA), and the National Council on Measurement in Education (NCME) published *Standards for educational and psychological testing* (1999). The three organizations report that validity refers to the “degree to which evidence and theory support the interpretations of test scores entailed by proposed uses of tests” (AERA, APA, NCME, 1999, p. 9). When preparing for an experiment, researchers believe they know what they want to measure, but often times the available measures are a mixture of what they want to measure and something else. Randomly selecting participants for a study greatly decreases some of the external effects that are not intended to be measured, but it does not eliminate them.

The existence of complete validity is viewed differently by various researchers. Carmines and Zeller (1979) believe that complete validity can never exist while Keller and Casadevall-Keller (2010) believe it can exist if the perspective is very restricted. For instance, one might assume stating that a mummified body is dead would be a valid statement. The person that originally occupied the body might be dead, but there could be other organisms living within the chassis. This would mean that the body is not completely barren of life. Another example of this is affirming that a person is either male or female. Scientists have determined that there is a continuum that exists between males and females. Some males contain female traits and some females contain male traits. Therefore, a study that groups people into a category of male or female would not be completely valid. This notion is called the Uncertainty Principle, and allows

measurements to be very close to valid, but not completely (Keller & Casadevall-Keller, 2010).

When assessing most forms of validity three questions can be asked:

- To what extent is the measure (or the result) sufficiently valid for its intended purpose?
- How well can the answer to the first question be evidenced?
- Is the risk of the results being plausibly wrong or potentially harming others worth the information that is provided by the research? (Keller & Casadevall-Keller, 2010)

Validity is somewhat subjective, in that it relies on logic or judgment (Keller & Casadevall Keller, 2010; Piaget, 1983). It “logically begins with an explicit statement of the proposed interpretation of test scores, along with a rationale for the relevance of the interpretation to the proposed use (AERA, APA, NCME, 1999, p. 9). The proposed interpretation of test scores refers to concepts the test is intended to measure. The conceptual framework of the study is shaped by the ways in which the test scores will be used and interpreted.

Validation occurs when a scientifically sound validity argument is developed to support the envisioned interpretation of tests scores and the relevance of the scores to the proposed use of them (AERA, APA, NCME, 1999). The subjectivity of validity (Gorin, 2007) is evident with the evolution of the perfect score given to divers at the Olympics (Keller & Casadevall-Keller, 2010. Athletic performances that took gold medals years ago would fail to do so now. The pervasive conditioning of the judges made them fail to

believe that some of the dives, now performed safely, could by no means have been done years ago. This pervasive conditioning is the “process by which our lifelong experiences cause us to assume certain things are true, when in fact, they might not be (Keller & Casadevall-Keller, 2010, p. 13). When is something ever exactly what it should be? How should this be compared to something else that is exactly as it should be? The intended interpretation and use of test scores is left to the researcher.

### **Importance of Validity for Instrument Creation**

Many instruments that are created leave out elements that potential users think are important and include some elements that potential users think are not essential (AERA, APA, NCME, 1999). Construct underrepresentation and construct-irrelevant variance are measures of the above mentioned problems. The extent to which a test does not capture important parts of the construct is construct underrepresentation (AERA, APA, NCME, 1999, p. 10). The extent to which test scores are affected by methods external to its intended construct is construct-irrelevant variance (AERA, APA, NCME, 1999, p. 10). It is important that the construct is adequately represented and that the appropriate test format is used that will not limit the interpretation of test scores. An instrument can be totally valid for a particular phenomenon, but not for the use of the current study (Carmines & Zeller, 1979). It is therefore, very important that the test developer know the purpose for which the measuring instrument will be used. Aligning the purpose for instrument use with what the instrument is intended to measure will have positive results for determining the validity of an instrument.

## **Ramifications for Not Considering Validity (Prior To Test Administration Of Target Sample)**

The test developer and the test user jointly share the responsibility of validation (AERA, APA, NCME, 1999). Ultimately it is the responsibility of the test user to evaluate the “relevant evidence and a rationale in support of the intended test use” in the setting in which the test is to be administered (AERA, APA, NCME, 1999, p. 11). A sound validity argument relies on the use of” existing evidence and theory [to] support the intended interpretation of test scores for specific uses... evidence of careful test construction; adequate score reliability; appropriate test administration and scoring; accurate score scaling, equating, and standard setting; and careful attention to fairness for all examinees” (AERA, APA, NCME, 1999, p. 17). Addressing all of the validity concerns may show the researcher there is a need to refine the construct definition, revise other aspects of the testing process, and suggest further areas of study that are needed. Not accounting for the validity could distort the validity scores that are determined from the analysis of the data.

## **Types of Validity**

The three basic types of validity are predictive or concurrent validity, content validity, and construct validity (Carmines & Zeller, 1979; Cronbach & Meehl, 1955; Huck, 2004; Kumar, 2005).

### **Predictive validity (criterion-related)**

The term predictive validity has taken on many forms since 1974 when it was explicitly introduced in the Standards for educational and psychological testing. In 1974 predictive validity was synonymous with criterion-related validity which occurs when a researcher wants to infer from a test score an individual's most probable standing on some other variable that is called a criterion (as cited in the 1999 standards). In the 1985 *Standards for educational and psychological testing* criterion-related validity was changed to criterion-related evidence which refers to one type of evidence that is found in a unitary conception of validity (AERA, APA, NCME, 1999). The 1999 *Standards for educational and psychological testing* evidence based on relations to other variables and test-criterion relationships. Evidence that is predictive "indicates how accurately test data can predict criterion scores that are obtained at a later time" (AERA, APA, NCME, 1999, p. 180). Nunnally (1978) refers to the usefulness of criterion-validity when "the purpose is to use an instrument to estimate some important form of behavior that is external to the measuring instrument itself, the latter being referred to as the criterion" (p. 87). The degree of criterion-related validity depends on the correlation between the test and the criterion (Carmines & Zeller, 1979). Concurrent validity correlates the measure and the criterion at the same point in time while predictive validity correlates a measure with a future criterion (Carmines & Zeller, 1979; Cronbach & Meehl, 1955; Kumar, 2005).

### **Content validity (face validity)**

Content validity has taken on different forms, as well, since it was explicitly introduced in the 1974 *Standards*. In 1974 content validity was a term used to refer to a *kind* or *aspect* of validity that was necessary when the researcher wished to estimate how a person performs in all the situations the test is intended to represent (as cited in the 1999 standards). In the 1985 *Standards* content validity was changed to content-related evidence which refers to “one type of evidence within a unitary conception of validity (AERA, APA, NCME, 1999, p. 174). The current *Standards* distinguished this type of evidence as “evidence based on test content” (AERA, APA, NCME, 1999, p. 174). Content validity can also be taken as the “degree to which various items collectively cover the material that the instrument is supposed to cover” (Huck, 2004, p. 89; Sireci, 1998). Kumar (2005) distinguished between face validity and content validity. Face validity established the link between the objectives of the study and the questions. Content validity goes a step further, making sure the instrument measures the intended issues, is balanced that the items cover the full range of issues or attitudes being measured.

### **Construct validity**

Construct validity is the most commonly reported type of validity (Zientek, Capraro, Capraro, 2008). The current *Standards* (1999) states that all test scores are measurements of some construct. This is redundant with the argument for validity set forth (AERA, APA, NCME, 1999). Therefore, this validity argument establishes the construct validity of a test. Construct validity is “a term used to indicate that the test

scores are to be interpreted as indicating the test taker's standing on the psychological construct measured by the test" (AERA, APA, NCME, 1999, p. 174). Construct validity is the "extent to which a particular measure relates to other measures consistent with theoretically derived hypotheses concerning the concepts (or constructs) that are being measured (Carmines & Zeller, 1979, p. 23). Huck (2004) stated that to establish the degree of construct validity associated with an instrument, the researcher developing the test will do one or a combination of three things:

- Provide correlational evidence showing that the construct has a strong relationship with certain measured variables *and* a weak relationship with other variables, with the strong and weak relationships conceptually tied to the new instrument's construct in a logical manner
- Show that certain groups obtain higher mean scores on the new instrument than other groups with the high- and low-scoring groups being determined on logical grounds *prior to* the administration of the new instrument; or
- Conduct a factor analysis on scores from the new instrument (p. 92)

Kline (2005) mentions that structural equation modeling can also be used to measure construct validity. Said more succinctly, "construct validation takes place when an investigator believes that his instrument reflects a particular construct, to which are attached certain meanings" (Cronbach & Meehl, p. 290). The goals in reporting construct validity make clear: what interpretation is proposed, how adequately the writer believes this interpretation is substantiated, and what evidence and reasoning lead him to this belief (Cronbach & Meehl, 1955). Though criterion-related validity and content

validity are limited regarding the applicability of generalizability in the social sciences, construct validity has generalized applicability in this field (Carmines & Zeller, 1979).

### **Types of Validity Related to Instrument Design**

Other types of validity include internal and external validity (Shadish, Cook, & Campbell, 2002; Thompson, 2006) and statistical conclusion validity (Shadish, Cook, & Campbell, 2002). One should not intertwine design validity with measurement validity (Thompson, 2006).

Internal design validity reports “about whether we can be certain that the intervention caused the observed effects” (Thompson, 2006, p. 26). That is, if an experiment or instrument is internally valid, the observed outcomes are a result of the intervention. Campbell (1957) defines internal validity in the form of the question “did in fact the experimental stimulus make some significant difference in this specific instance?” (p. 297). The experimental stimulus is whatever intervention placed upon the participants. This could take the form of a survey, medicine, or new curriculum, to name a few. Standard experimental designs should control for history, maturation, testing, instrument decay, regression, selection, and mortality (Campbell, 1957). These serve as threats to internal validity of an experiment.

Campbell (1957) defines external validity in the form of the question “to what populations, settings, and variables can this effect be generalized?” (p. 297).

Researchers should also be aware of reactive measurement effects, Hawthorne effects, John Henry effects, and double-blind design (Campbell, 1957). These serve as threats to external validity of an experiment.

The basic question that relates researcher's perspectives of reliability is "To what extent can we say the data are consistent?" (Huck, 2004, p. 76). A testing instrument only needs to be administered once to assess internal consistency reliability. Procedures that can be used to assess internal consistency reliability include Spearman-Brown, Kuder-Richardson #20, and Cronbach's alpha. Cronbach's alpha is the same as K-R 20 when the items are scored dichotomously. However, alpha is stronger than K-R 20 because it can be used with test instruments made up of items that contain three or more values. Alpha could measure the reliability of a survey that uses a 0-10 scale or a Likert-type questionnaire (Huck, 2004).

The current instruments used to measure the knowledge needed to teach mathematics failed to include equity (Boykin, Tyler, Watkins-Lewis, & Kizzie, 2006; Larke, 1990; Love & Kruger, 2005; Peterson, Fennema, Carpenter, & Loef, 1989; Phuntsog, 2001; Robert-Walter, 2007; Swartz & Bakari, 2005; Wayson, 1988). Some of these instruments make mention of culture and diversity, but not equity. In fact many of the instruments do not report on the construct validity, but the research reports on the content validity.

The research questions addressed were:

1. What was the construct validity of the Equity items from the Knowledge for Teaching Algebra Equitably (KATE) Instrument? What were the statistical measures from the factor analysis?
2. What was the content validity of the Knowledge for Teaching Algebra Equitably (KATE) Instrument? Did any items need to be revised, added, or deleted?

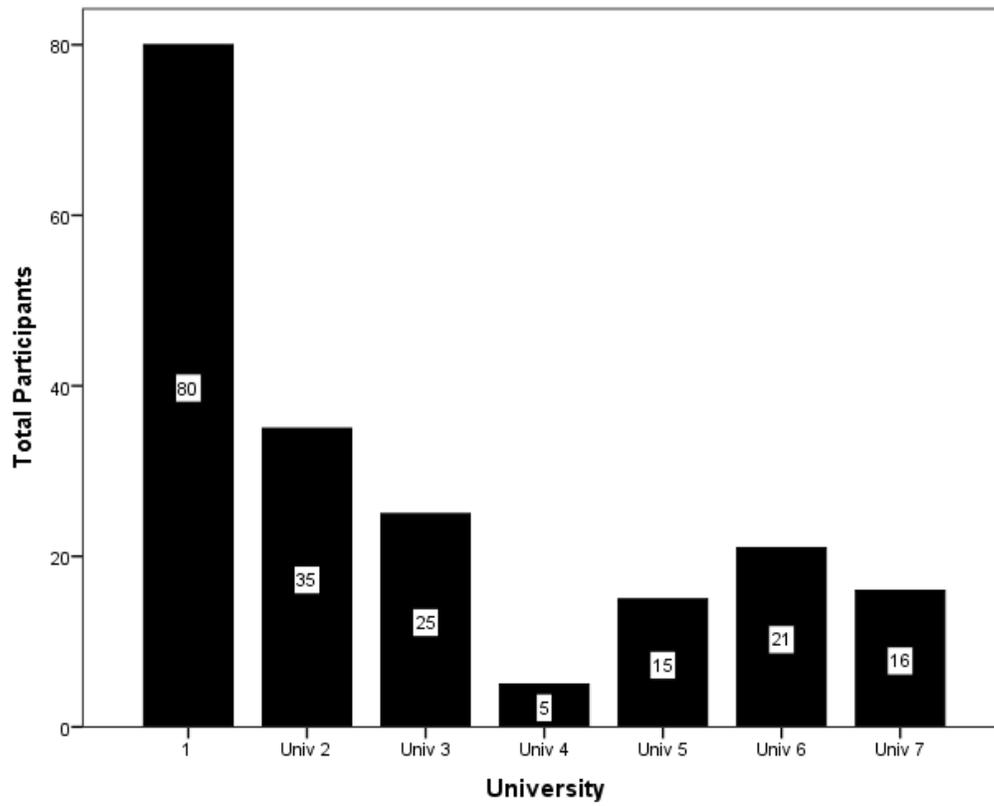
3. How was internal consistency reliability related to various subgroups within the sample?

### **Methods and Data Sources**

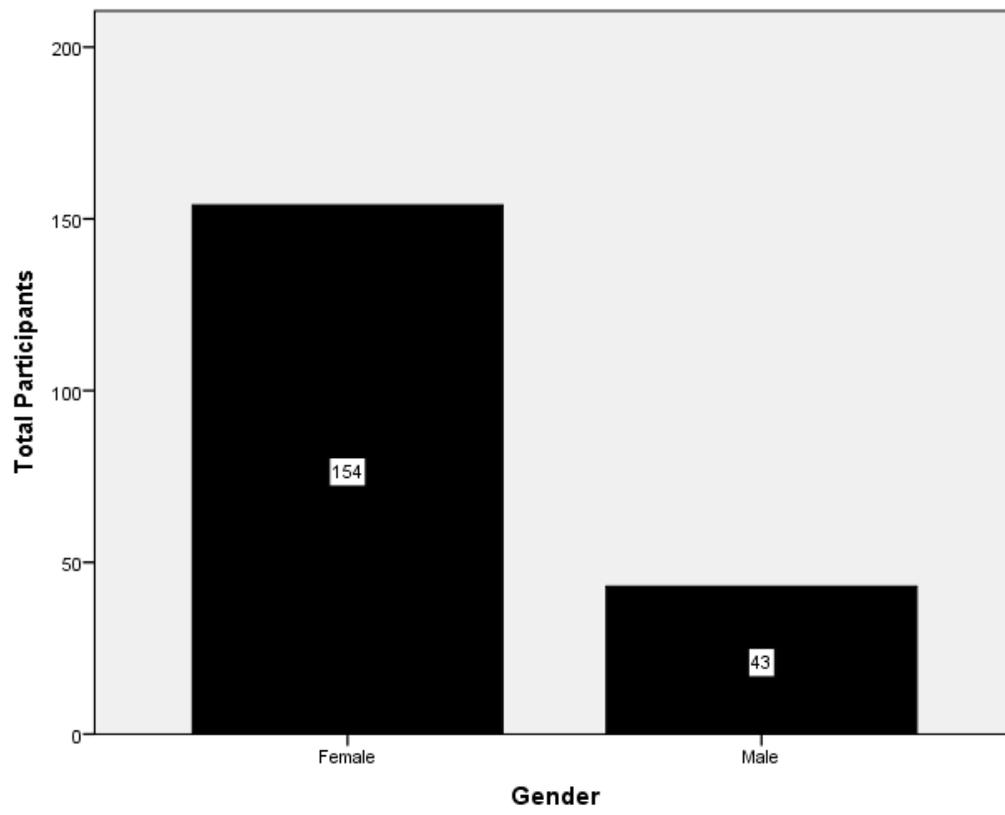
In the fall of 2012 and spring of 2013 I sent emails to mathematics education professors at universities throughout the U.S. The purpose of the correspondence was to attain professors that would be willing to distribute and administer the KATE instrument to their students.

#### **Participants**

There were 197 PTs from universities throughout the U.S. including Texas A&M University, the University of Kentucky, Bowling Green University, Texas Southern University, the University of Alabama, the University of South Florida, and North Carolina State University were willing to participate in the study and answer the survey. A total of 197 PTs (Figure 3.1) completed a consent form to participate in the study. Demographic information was reported by each PT that completed the KATE instrument. The students provided information on gender, college classification level, certification level, ethnic origin, and race. There were 154 female and 43 male students (see Figure 3.2). There were 7 sophomores, 65 juniors, and 105 seniors, and 20 post baccalaureate students (see Figure 3.3). The students classified themselves by certification level where 5 did not respond, 3 were elementary, 121 were middle school, 36 high school, 4 special education, and 28 dual certification (see Figure 3.4). There were 18 students that identified themselves as Hispanic, 178 that did not, and one that did not respond (see Figure 3.5). When asked about race 6 students did not respond, 172 were White, 12 were Black, 6 Asian, and one international (see Figure 3.6).



*Figure 3.1.* Total participants by university.



*Figure 3.2.* Gender.

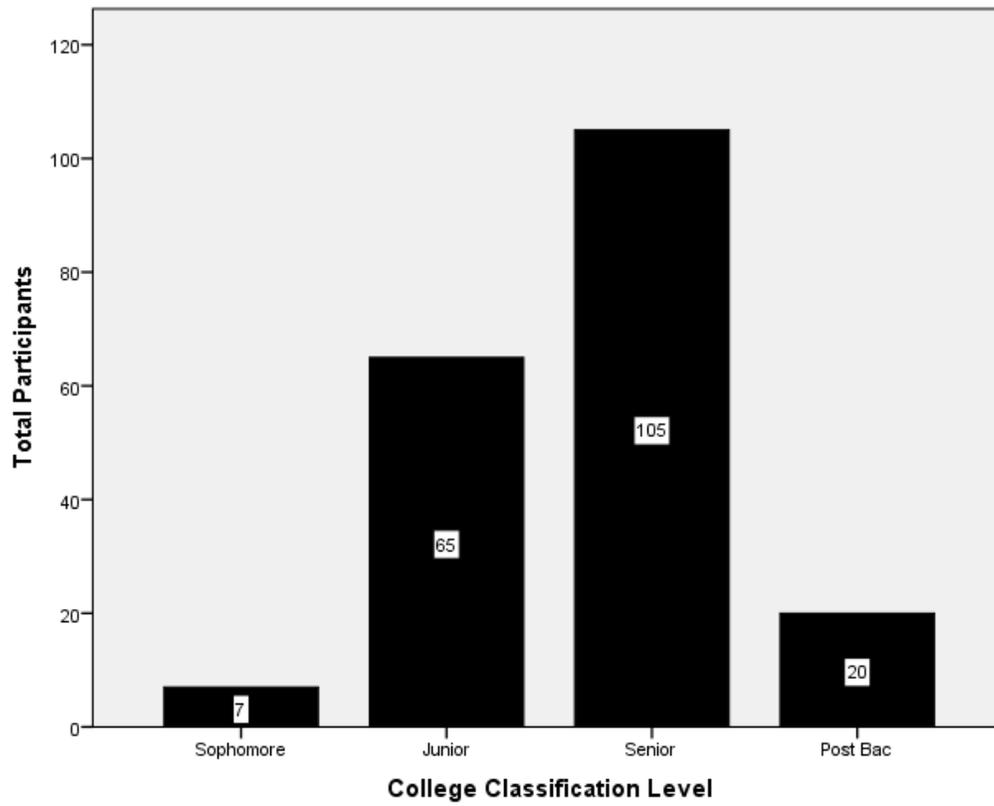


Figure 3.3. College classification level.

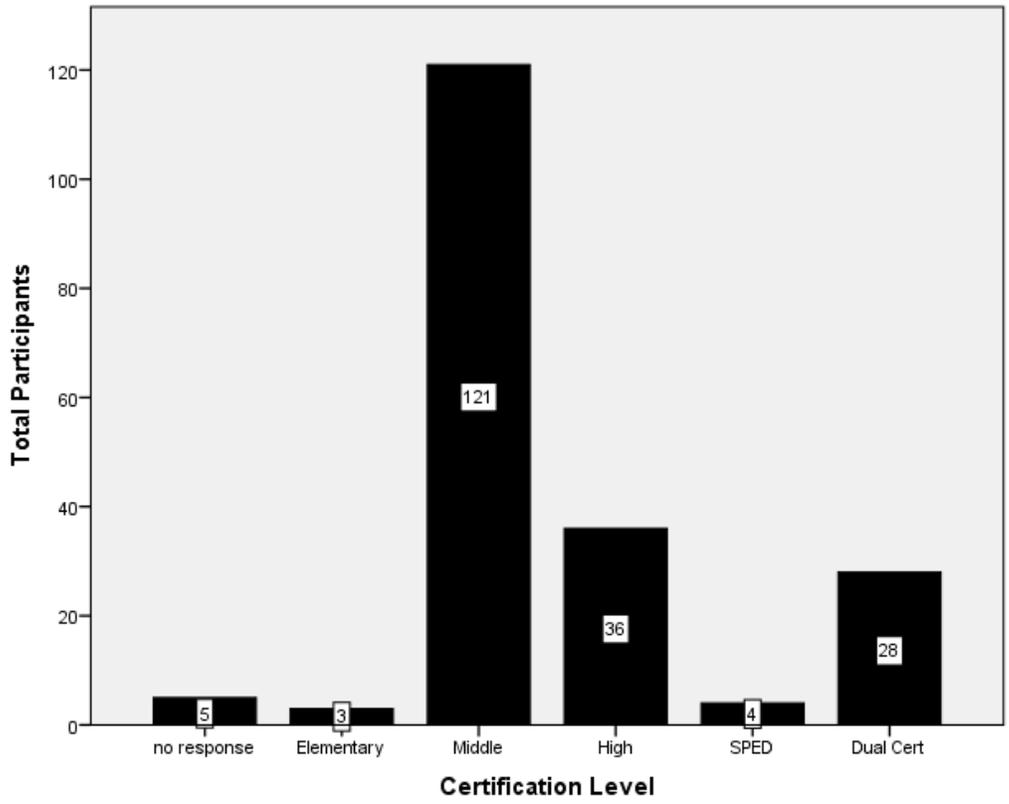


Figure 3.4. Certification level.

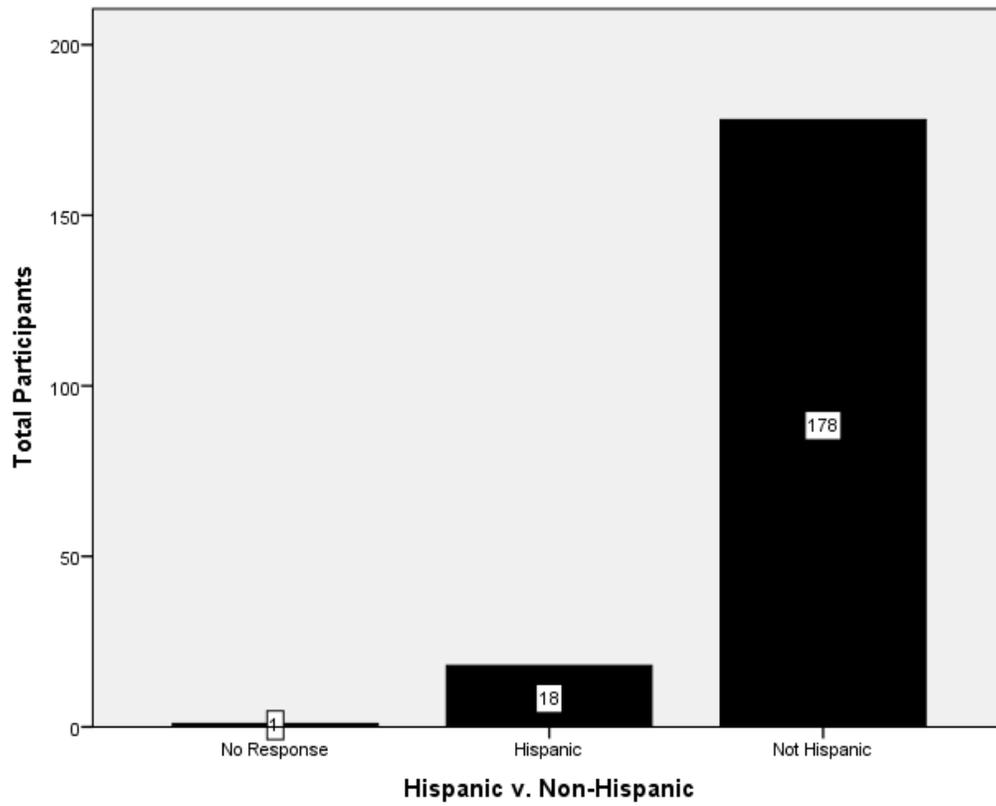


Figure 3.5. Hispanic v. Non-Hispanic.

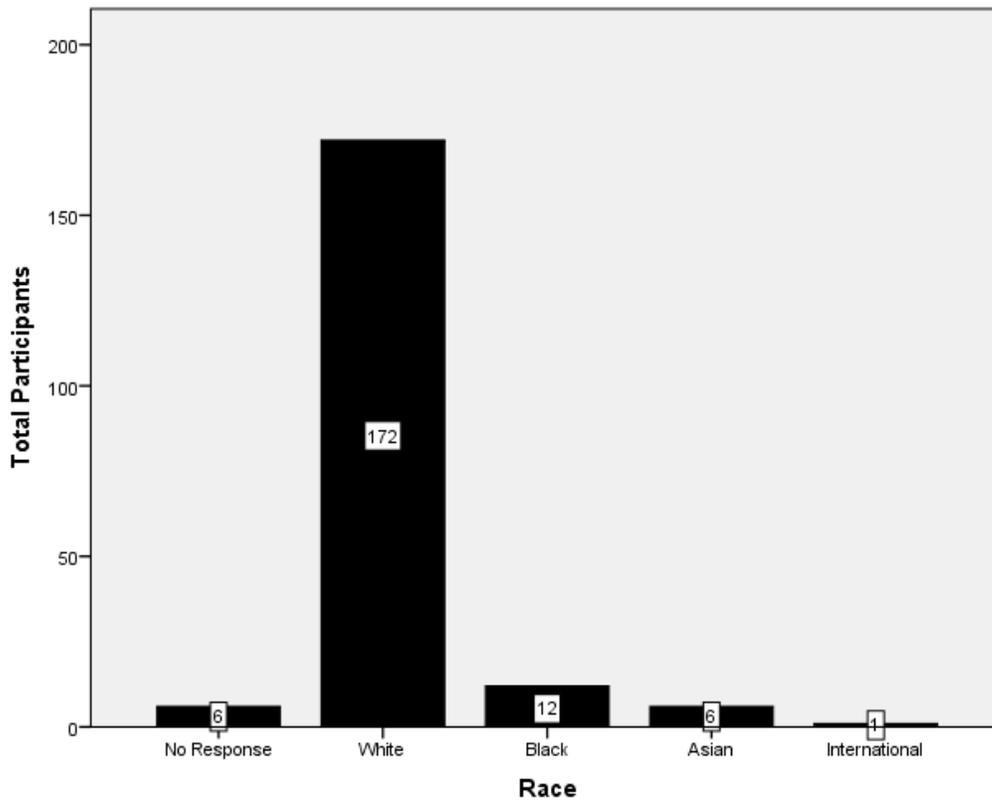


Figure 3.6. Race.

### Instrument

The KATE test consist of a) demographic background, b) twenty Likert items that address the cultural awareness beliefs of mathematics PTs and c) sixteen open-ended problem solving items that include problem solving, problem posing, and responding to a classroom scenario. The cultural awareness beliefs items were adapted from items in the Cultural Awareness Beliefs Inventory created by Carter and Webb-Johnson in 2005 (Roberts-Walter, 2007). The first fifteen open-ended problem solving items related to PTs' mathematical understanding and problem solving skills and were

adapted from You (2010) and the final classroom scenario problem was developed by the KATE project team.

### **Data Analyses**

Internal consistency reliability will be estimated using Cronbach's alpha, with the result being a number between 0 and 1 (Huck, 2004). An instrument is considered to be better "to the extent that the resulting coefficient is close to the upper limit of the continuum of possible results" (Huck, 2004, p. 79). An alpha value of .6 will be used as the baseline for determining suitable internal consistency reliability (Ashbaugh, Johnstone, & Warfield, 2002, p.130). Content validity will be estimated by discussing the appropriateness and wording of the items from the Knowledge for Teaching Algebra Equitably (KATE) instrument with a panel of experts. The experts will determine if each item can be measured by the rubric that has been created and whether the item is properly matched with its correct factor. The result will be the insertion, deletion, and rewording of items.

Construct validity will be estimated by conducting an exploratory factor analysis (Huck, 2004; Kline, 2005). Statistical Packages for the Social Sciences will be used to conduct the factor analysis. Exploratory factor analysis is used to generate theory while confirmatory factor analysis tests the theory (Henson & Roberts, 2006). An exploratory factor analysis will be conducted by using the most common matrix, the correlation matrix (Henson & Roberts, 2006) and rotated correlation matrix.

## Results

A few decision rules were assumed by the researcher. The number of factors found in the Exploratory Factor Analysis (EFA) were determined by the cumulative variance % in the extraction sums of squared loadings. If this percentage was  $> 70\%$ , then the number of factors (components) that made this percentage possible would be the number of factors assumed by the model. If the initial eigenvalue for the component was  $> 1$ , then that component was considered a factor. Any components that had eigenvalues  $< 1$  were not considered factors. To determine how many factors to retain in the EFA, a scree plot was used. The researcher started at the last component and drew a line to the previous component. Another line from the last component to two components over was drawn. If more components lied on this line than the original line, then that line was considered a factor. If the next line did not produce a greater amount of components lying on it than the previous line, then a new line was started from the last component of the previous line. This process continued until there was a jigsaw of straight lines drawn on the scree plot. The amount of lines drawn signified the number of factors resulting from the EFA. When determining the reliability of the instrument, if Cronbach's alpha increased more than .05 when an item was deleted, then it should be deleted from the instrument.

Content validity was established by a panel of experts from the KATE project team. The team read PTs' responses to the Problem Solving and Teaching Problem Solving items, listened to inquiries students had about the instrument, and feedback given on the instructor's end of course survey. This process was done multiple times

throughout the first two years of the project. Project team members discussed the wording of items from the entire instrument. As a result these equity and math items were revised to be more concise so students could better answer the question the way it was intended.

An exploratory factor analysis (EFA) was performed for the Equity items. Before conducting the EFA for the Equity items, I looked at the factor analysis conducted in the original article pertaining to the CABI. The Cronbach's alpha for the CABI was .83 (Foster-Walter, 2005). Of the 46-item CABI instrument, the KATE instrument contained 16 items closely related to the CABI. From the CABI, the items used to create the Equity items in the KATE instrument were 19, 20, 22, 23, 27, 28, 31, 32, 35, 37, 38, 39, 40, 46, 53, and 55. Items 5, 10, 11, and 16 from the KATE instrument were created by the KATE project team. The CABI items, factors related to each item from the CABI, and the factors related to each item on the KATE that matched the CABI were reported in Table 3.1. The CABI factors were (a) teachers beliefs (4 items in KATE) (b) school climate (0 items in KATE), (c) home and community (1 item in KATE), (d) teacher efficacy (3 items in KATE), (e) curriculum and instructional strategies (3 items in KATE), (f) teacher beliefs (2 items in KATE), (g) cultural awareness (1 item in KATE), and (h) behavior management (2 items in KATE). The KATE factor analysis revealed six factors. The KATE factors were (a) cultural awareness (b) teacher efficacy (c) cultural beliefs (d) cultural preferences (e) teacher perceptions (f) and racial differences. The remaining 30 items of the CABI instrument were not used because of their specificity to subject matter not related to mathematics.

Table 3.1

*CABI Items Used in KATE Instrument, Factors Revealed from the Factor Analysis of the 20 CABI Items Used in the KATE Instrument, and Factors Revealed from the Factor Analysis of the 20 Items Used in the KATE Instrument*

| Item # | Item Description  | CABI Factor                | KATE factor         |
|--------|---|----------------------------|---------------------|
| 1.     | I believe all middle school students are treated equitably regardless of their race, culture, disability, gender or social economic status. | Home and community         | Racial differences  |
| 2.     | I believe all families are supportive of teachers' work to effectively teach all middle school students.                                    | Home and community         | Teacher perceptions |
| 3.     | I believe teachers have strong support for academic excellence from the surrounding school community (civic, church, business).             | Home and community         | Teacher perceptions |
| 4.     | I believe some students do not want to learn.   | Teacher efficacy           | Cultural beliefs    |
| 5.     | I believe that poor teaching is the main factor that causes the gap in math achievement between White students and students of color.       | *                          | Racial differences  |
| 6.     | I believe I have the knowledge and skills I need to be a culturally responsive math teacher.  | Curriculum and instruction | Teacher efficacy    |
| 7.     | I believe I can implement cooperative learning effectively as an integral part of my math teaching  | Curriculum and instruction | Teacher efficacy    |

Table 3.1 Continued

| Item # | Item Description  | CABI Factor        | KATE factor        |
|--------|---|--------------------|--------------------|
|        | strategies.   |                    |                    |
| 8.     | I believe African American students have more behavior problems than other students.                                | Teacher beliefs    | Cultural beliefs   |
| 9.     | I believe most diverse students are not as eager to excel in math in comparison to their White peers.               | Teacher beliefs    | Cultural beliefs   |
| 10.    | I believe many middle school teachers engage in biased behavior toward students of color in the classroom.          | *                  | Racial differences |
| 11.    | I believe students who live in poverty are more difficult to teach.   | *                  | Cultural beliefs   |
| 12.    | I believe most diverse students do not bring as many strengths to the classroom as their White peers.               | Teacher beliefs    | Cultural awareness |
| 13.    | I believe it is important to identify with the racial groups of the students I serve.                               | Cultural awareness | Cultural awareness |
| 14.    | I believe I am comfortable with people who exhibit values or beliefs different from my own.                         | Cultural awareness | Cultural awareness |
| 15.    | I believe the cultural views of a diverse school community should be an integral component of my lesson planning.   | Cultural awareness | Cultural awareness |
| 16.    | I believe in asking families of diverse cultures how they wish to be identified (e.g., African American, Bi-racial, | *                  | Cultural awareness |

Table 3.1 Continued

| Item # | Item Description   | CABI Factor                                | KATE factor          |
|--------|--|--|----------------------|
| 17.    | Mexican.)<br>I believe that in a society with as many racial groups as the United States, I would accept the use of ethnic jokes or phrases by students. | Cultural sensitivity                       | Cultural perceptions |
| 18.    | I believe my teacher education courses focus too much on “multicultural” issues.   | Teacher efficacy                           | Cultural perceptions |
| 19.    | I believe I am able to effectively manage students from all racial groups.   | Culturally responsive classroom management | Teacher efficacy     |
| 20.    | I believe I would prefer to work with students and parents whose cultures are similar to mine.   | Teacher beliefs                            | Cultural perceptions |

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Note: (\*) Items were created by the KATE project team.

Results from the EFA on the Equity items were determined by assessing a pattern matrix and structure matrix (see Table 3.2). Six factors were determined and the comparison of these factors with those from the original CABI can be seen in Table 3.1. The cumulative % total variance from the extraction sum of squared loadings was 59.92%. This was the case for six factors. Six components had an eigenvalue > 1, suggesting the model had six factors as well. Two components had an eigenvalue between .9 and 1, but they were excluded because the criteria for being a factor meant that the eigenvalue from the component must be > 1. The same result was true from viewing the scree plot. The KATE factors were (a) cultural awareness (b) teacher

efficacy (c) cultural beliefs (d) cultural preferences (e) teacher perceptions (f) and racial differences.

Table 3.2

*Factor pattern (P) and Structure (S) Matrices rotated for the Equity Items Post-test for Fall 2011 and Fall 2012.*

| Var | <u>I</u> |       | <u>II</u> |             | <u>III</u> |             | <u>IV</u> |       | <u>V</u> |             | <u>VI</u> |             |
|-----|----------|-------|-----------|-------------|------------|-------------|-----------|-------|----------|-------------|-----------|-------------|
|     | P        | S     | P         | S           | P          | S           | P         | S     | P        | S           | P         | S           |
| E1  | .002     | -.067 | -.406     | -.152       | -.199      | .152        | .558      | .016  | -.044    | -.353       | -.045     | <u>.587</u> |
| E2  | .072     | -.055 | .554      | .116        | .231       | -.014       | -.040     | .003  | .488     | <u>.789</u> | .190      | -.049       |
| E3  | .214     | .068  | .477      | .165        | .187       | .100        | -.033     | <.001 | .408     | <u>.705</u> | .251      | -.025       |
| E4  | .235     | -.074 | -.062     | -.250       | -.503      | <u>.683</u> | .049      | .069  | .469     | .229        | .256      | .092        |
| E5  | .194     | -.023 | .053      | .129        | .145       | .033        | .734      | .093  | .237     | .240        | .024      | <u>.756</u> |
| E6  | .411     | .046  | .598      | <u>.816</u> | .124       | .025        | .218      | -.031 | -.340    | .167        | .022      | .088        |
| E7  | .533     | .059  | .578      | <u>.825</u> | .001       | .175        | .156      | .066  | -.312    | .140        | -.008     | .031        |
| E8  | .575     | .061  | -.001     | .181        | -.466      | <u>.692</u> | .177      | .228  | .120     | .001        | .080      | .182        |
| E9  | .710     | .333  | .051      | .313        | -.335      | <u>.658</u> | -.185     | .163  | -.067    | -.005       | .193      | -.164       |

Table 3.2 Continued

| Var | <u>I</u> |             | <u>II</u> |             | <u>III</u> |             | <u>IV</u> |             | <u>V</u> |       | <u>VI</u> |       |
|-----|----------|-------------|-----------|-------------|------------|-------------|-----------|-------------|----------|-------|-----------|-------|
|     | P        | S           | P         | S           | P          | S           | P         | S           | P        | S     | P         | S     |
|     | E10      | .229        | .354      | -.314       | .087       | .234        | -.037     | .500        | -.069    | -.179 | -.185     | .144  |
| E11 | .512     | .151        | -.054     | .136        | -.467      | <u>.684</u> | -.019     | .087        | .004     | -.075 | .196      | -.008 |
| E12 | .541     | <u>.569</u> | -.365     | .076        | -.033      | .322        | -.138     | .104        | -.214    | -.270 | .164      | -.026 |
| E13 | .342     | <u>.702</u> | -.446     | -.248       | .374       | .015        | -.142     | .069        | .142     | .079  | .275      | .073  |
| E14 | .660     | <u>.648</u> | -.162     | .235        | .164       | .221        | -.189     | .188        | -.149    | -.045 | .140      | -.076 |
| E15 | .632     | <u>.741</u> | -.166     | .246        | .344       | .105        | -.064     | .083        | -.152    | .032  | .247      | .070  |
| E16 | .479     | <u>.612</u> | -.228     | -.026       | .561       | -.181       | -.027     | .411        | .233     | .228  | -.074     | .166  |
| E17 | .539     | .195        | -.165     | .128        | -.125      | .232        | -.138     | <u>.591</u> | .015     | -.186 | -.380     | -.078 |
| E18 | .483     | .175        | -.244     | -.099       | .084       | .075        | .045      | <u>.781</u> | .388     | .055  | -.488     | .179  |
| E19 | .596     | .098        | .382      | <u>.644</u> | .060       | .083        | .013      | .441        | -.152    | .077  | -.326     | -.043 |
| E20 | .517     | .071        | .048      | .188        | -.076      | .190        | -.089     | <u>.637</u> | .172     | .044  | -.416     | -.055 |

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Note: P is coefficient from the pattern matrix and S is the coefficient from the structure matrix.

The obtained reliability from the KATE instrument related to the Equity items was .770. The Cronbach's alpha for the Item-Total statistics would increase if items 1 (Cronbach's alpha .783), 2 (Cronbach's alpha .777), 3 (Cronbach's alpha .771), or 4 (Cronbach's alpha .772) from the Equity items were deleted.

The obtained reliability from the KATE instrument related to the 16 Equity items that were adapted from the CABI was .741. The Cronbach's alpha for the Item-Total statistics increase if items 1 (Cronbach's alpha .765), 2 (Cronbach's alpha .751), 3 (Cronbach's alpha .742), or 4 (Cronbach's alpha .744) from the Equity items were deleted.

The obtained reliability from the KATE instrument related to the 4 Equity items created by the KATE project team was .347. The Cronbach's alpha for the Item-Total statistics would not increase if 3 of the Equity items were deleted. If item 11 were deleted, the Cronbach's alpha would increase to .411.

### **Conclusion**

The field of education speaks a lot on issues of culture, equity, and mathematical understanding amongst PTs. There are many instruments that measure these ideas amongst PTs, but of those that were reviewed, none related all three concepts. The KATE instrument is unique in this way because it measures the cultural awareness beliefs and mathematical understandings of PTs.

The EFA for the Equity items in the KATE instrument revealed six factors. The KATE factors were (a) cultural awareness (b) teacher efficacy (c) cultural beliefs (d) cultural preferences (e) teacher perceptions (f) and racial differences. These items are

closely related to the original CABI items and align somewhat with the original factors determined from the CABI. Items were clearly matched to factors and no items should have been deleted because of being closely related to other items. The reliability of the CABI related closely to the reliability of the Equity items in the KATE instrument.

Moreover, the EFA revealed that depending on the assumptions made by the researcher, there could have possibly been two more factors. The percentage variance for six factors was 59.92%. There were two components that have eigenvalues values between .9 and 1, suggesting that these components might have been factors. But because of the assumption made by the researcher, they were left out.

Content validity for the KATE instrument was established over a period of time by the KATE research team. Research members discussed the wording of items, student feedback given during the course evaluation, as well as item analysis from the responses the students gave on the KATE instrument. Items were thus re-worded over time. The original KATE instrument included 20 Equity items and 16 problem solving items. The final version used in this study contained the same number of items, but sub-questions were added to some of the problem solving items so students could explicitly give the information that the researcher was looking to attain.

Further research needs to be done with the KATE instrument to determine a good number of math problem solving items to use. An EFA for this instrument was done on a smaller population, but it produced 7 factors for 16 items. Possibly, more items should be added in the future. Also, increasing the population of PTs given the KATE instrument might strengthen the construct validity for this instrument. Though 197 PTs

took the survey and strong construct validity was attained, this might increase if N gets larger.

### **Limitations**

Limitations of this study were as follows:

1. Participants were asked voluntarily to participate in the administration of the KATE instrument. Therefore, the data in this study only represented those that took part in the study.
2. Participants from various universities were asked to participate. The guidelines of the study specifically stated that each participant must work on the instrument alone, with no outside help. This stipulation could not be monitored by the author.

## CHAPTER IV

### EFFECTS OF A COURSE ON PRE-SERVICE TEACHERS TO DEVELOP THE KNOWLEDGE TO TEACH ALGEBRA FOR EQUITY

#### **Review of Literature**

##### **Developing Teachers' Awareness of Equity**

Researchers have addressed attempts to develop PTs' knowledge of multicultural issues and perceived confidence in teaching. To account for the diversity in schools, educators must "possess the ability to communicate and interact effectively with those who are different from themselves" (Thompson, Bakken, & Wei-Cheng, 2009, p. 416). The teacher training program used by Thompson, Bakken, and Wei-Cheng (2009) allowed candidates to take prescribed courses over four semesters with courses addressing the present issues of diversity. A semester of student teaching concluded the program. A questionnaire was administered to the students and it was found that the program increased multicultural knowledge of the teachers as well as their confidence in teaching. In another study, the skills to teach in equitable environments were obtained by PTs by giving power and authority to the students (Lee & Anderson, 2009). This would help the learners understand their identity and make them more successful (Lee & Anderson, 2009).

Results of The Milwaukee Teacher Education Center Model, which has goals of multiculturalism and equity, brings forth characteristics that makes a teacher education program the proper preparation for teaching students in urban poverty schools (Haberman, 2000). After three years of on the job training a fully functioning urban

teacher should be able to remain in teaching, work with parents, motivate students, integrate several subjects into problem-solving units, and work in a team (Haberman, 2000). Urban teachers must be able to make “connections between the school curriculum and the students’ cultural backgrounds” (Haberman, 2000, p. 4). The activities and lessons taught show that urban teachers are aware of students’ diverse cultural values (Haberman, 2000). Differences possessed by children of color and White students are often times overlooked and serve as a reason for needing to prepare teachers for diversity amongst students (Ladson-Billings, 1999).

### **Developing Algebra Problem Solving Skills**

Algebra serves as the entryway into the future, higher level mathematics courses, and achievement in a career field. Algebra is a gatekeeper to high school graduation for many students, those that are college-bound and those that are not, simply because it is a requirement on degree plans (Moses, Kamii, Swap, & Howard, 1989). Algebraic thinking can be thought of in various ways because algebra is comprised of many features (Driscoll, 1999). Algebraic thinking can be used to focus on the abstract features that distinguish algebra from arithmetic, the capacity to represent quantitative situations so that relationships between variables become apparent, or use problem solving as the point of reference when referring to algebra (Driscoll, 1999). It is imperative that students have a firm foundation in algebra because the course contains important elements that will help students be successful in society.

As is the case with reading, writing, and arithmetic, lack of knowledge in algebra limits students’ opportunities (Usiskin, 1995). Algebra affords a general procedure that

is applicable to answer questions of the same type, but contain different numerical inputs (Usiskin, 1995). Instead of having to figure out how to do the same type of questions each time, the creation of a formula simplifies this matter. It is much easier to substitute into the formula, than recreating the process each time. Algebra allows students to develop multiple ways of solving a problem (Driscoll, 1999). It is very important that teachers allow students the time to discover various methods for solving a problem instead of always forcing them into a box of solving problems using a particular method. Since an introductory algebra class is the first major access point into secondary mathematics (Fennema & Romberg, 1999; Smith, 1996; Stacy & Chick, 2004), it is imperative that students excel in the course. Mathematics plays a major contributory role on the nation's economy. In order to upgrade students' level of algebraic understanding, increased attention must be given to problem solving and algebra skills (Usiskin, 1987).

The mathematical tasks used to promote the learning of students and hypothesize about student's learning processes encompassed four schemes: direct translation, textbook four-step, generalized pattern, and heuristic (Brown et al., 2011). The direct translation scheme refers to solution methods, often times described in four or five steps that help students factor and simplify expressions and perform other operations of equivalence (NCTM, 2000). When a student is trying to find a solution, his/her way of looking at the problem will constantly change (Polya, 1957). The thoughts about the problem are likely incomplete when initially working on the problem. There are thus four phases that a student must follow when solving a problem: understand the problem,

devise a plan, carry out the plan, and look back (Polya, 1957). The generalized pattern scheme asserted the fact that algebra is a language of generalization (Usiskin, 1995). Lastly, Polya's four-step method was an example of a heuristic scheme. The intent of using the four schemes in a teacher preparation course was to increase the mathematical understanding of the PTs.

### **Culturally Relevant Teaching**

To use culturally relevant strategies teachers must adhere to the following behaviors: the conceptions of self and others through a sense of community and belief that all students are capable of academic success, the manner in which social relations are structured, and the conceptions of knowledge (Ladson-Billings, 1995). Past research encouraged PTs to take a stand for social justice by challenging the “oppressive structures and practices in the educational system” (Ahlquist, 1991, p. 158). Moreover, to teach in a culturally relevant manner the teacher preparation process needs to be rethought (Ladson-Billings, 2009).

One of the ways teachers support the instruction of their students is by attending to concerns for equity (Ball, Hill, & Bass, 2005). The KATE project team has developed three hypothetical learning trajectory schemes used in the MASC course that focus on the knowledge needed to teach equitably: situated learning, culturally relevant, and critical pedagogy (Brown, Davis, & Kulm, 2011). The situated perspective describes behavior “oriented toward practical activity and context” and cultural artifacts (Pellegrino, Chudowsky, & Glaser, 2001, p. 62). When students are learning they should actively construct knowledge rather than passively receive information from the

teacher (Bereiter, 1994; Beswick, 2007; Cobb, 1994; Ernest, 1989; Pirie & Kieren, 1992; Simon & Schifter, 1991). Culture informs all human activity and thought (Ladson-Billings, 1997) and therefore should not be separated from teaching. Ladson-Billings (1995) discussed the need for context within mathematics, which is important in the culturally relevant scheme. Critical pedagogy includes sensitivity and understanding for how certain everyday realities might differentially impact people based on culture. Schemes related to equity can support PTs when instructing students of various backgrounds.

### **Research Questions**

The research questions that were addressed:

1. Was there a difference between fall 2011 and fall 2012 on (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?
2. What was the effect of the course on (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?
3. What was the relationship among (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?
4. What was the relationship among (one or more demographic variables) and (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving?

### **Methods and Data Sources**

In this study I investigated the effects of a semester-long required course in Integrated Mathematics and Science (MASC) Problem Solving on middle grades mathematics PTs. This course normally focused solely on student problem solving

within the context of a science or mathematics classroom. For the purpose of the KATE project, the foundations of the course were altered. In addition to problem solving, the course instructors used culturally relevant pedagogy in an attempt to develop participants' cultural awareness beliefs, problem-solving skills, diversity, and preparation to apply these ideas in planning and presenting lessons for diverse learners. The main goal of this course was to strengthen the cultural awareness of pre-service mathematics teachers.

Classroom instruction was guided by the hypothetical learning trajectories (HLT) for Knowledge for Teaching for Equity (KTE) and Knowledge for Teaching Algebra (KTA) developed by Brown et al. (2011). The mathematical tasks used to promote the learning of students and hypotheses about student's learning processes encompassed the goal of HLTs (Simon, 1995). Each of these HLTs was used in the MASC classroom at Texas A&M with pre-service mathematics teachers. The schemes were used as a basis for daily classroom instruction and activities. The KTE HLTs were categorized as follows: situated learning, culturally relevant context, and critical pedagogy (Brown et al, 2011; Brown, Davis, & Kulm, 2011). Teachers must understand that learning is a process of social and individual construction (Simon, 1995). KTA HLTs were categorized as follows: direct translation, textbook four-step, generalized pattern, and heuristic (Brown et al., 2011). There was a clear gap between the knowledge of how teacher preparation affects classroom instruction and student achievement (Kulm, 2008). There is little research on the trajectories of developing deep algebraic thinking amongst pre-service mathematics teachers with algebra (Kulm et al., 2011). An instrument that

could measure the trajectory of PT learning through a course that addresses concepts of algebra and equity could strengthen the readiness of PT before officially teaching in the classroom full time.

### **Participants**

The participants were PTs enrolled in the fall 2011 and fall 2012 MASC 351 Problem Solving course at Texas A&M University. A total of 68 PTs (33 from the fall of 2011 and 35 from the fall of 2012) completed a consent form to participate in the study. Demographic information was reported by each PT that completed the KATE instrument. The students provided information on gender, college classification level, certification level, ethnic origin, and race. In the fall of 2011 there were 33 students, 29 female and 4 male (see Figure 4.1). There were 2 sophomores, 24 juniors, and 7 seniors (see Figure 4.2). The students classified themselves by certification level where 24 were middle school, 1 elementary school, 7 high school, and 1 special education (see Figure 4.3). There were 6 students that identified themselves as Hispanic and 27 that did not (see Figure 4.4). When asked about race 2 students did not respond, 29 were White, and 2 were Black (see Figure 4.5). In the fall of 2012 there were 35 students, 32 female and 3 male (see Figure 4.1). There were 23 juniors and 12 seniors (see Figure 4.2). The students classified themselves by certification level: 2 did not respond, 2 were elementary school, 23 middle school, 3 high school, 2 special education, and 3 dual focus (multiple levels) (see Figure 4.3). There were 5 students that identified themselves as Hispanic and 30 that did not (see Figure 4.4). When asked about race 2 students did not respond, 32 were White, and 1 was Black (see Figure 4.5).

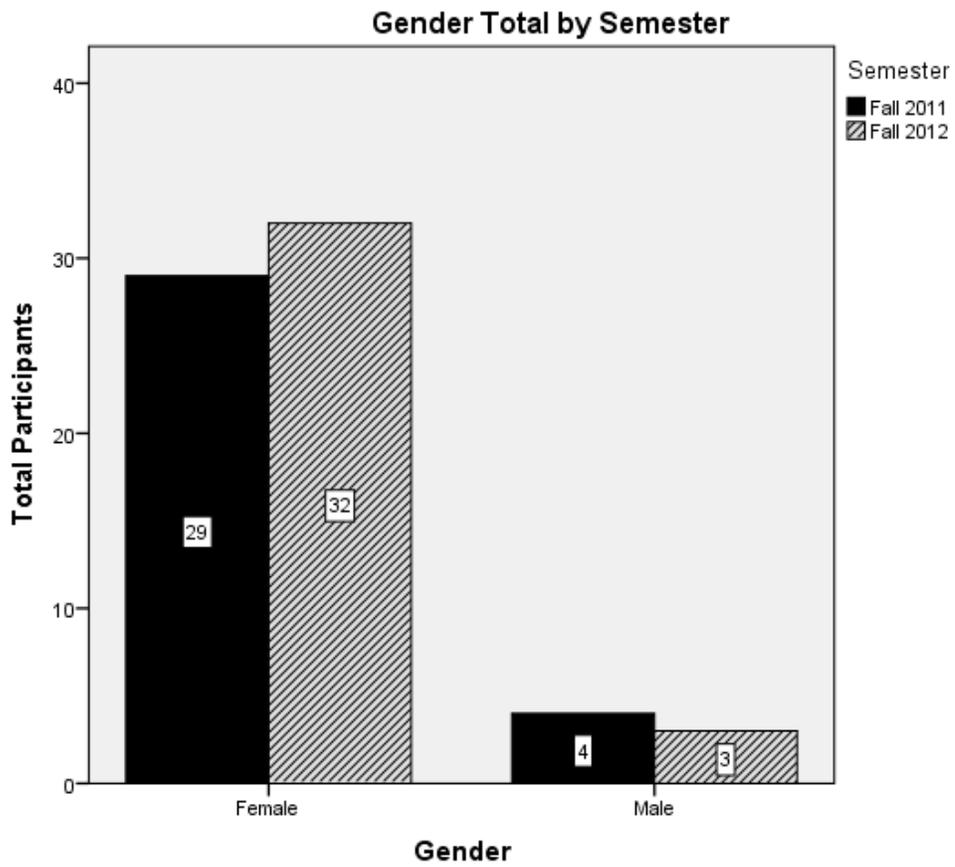


Figure 4.1. Gender total by semester.

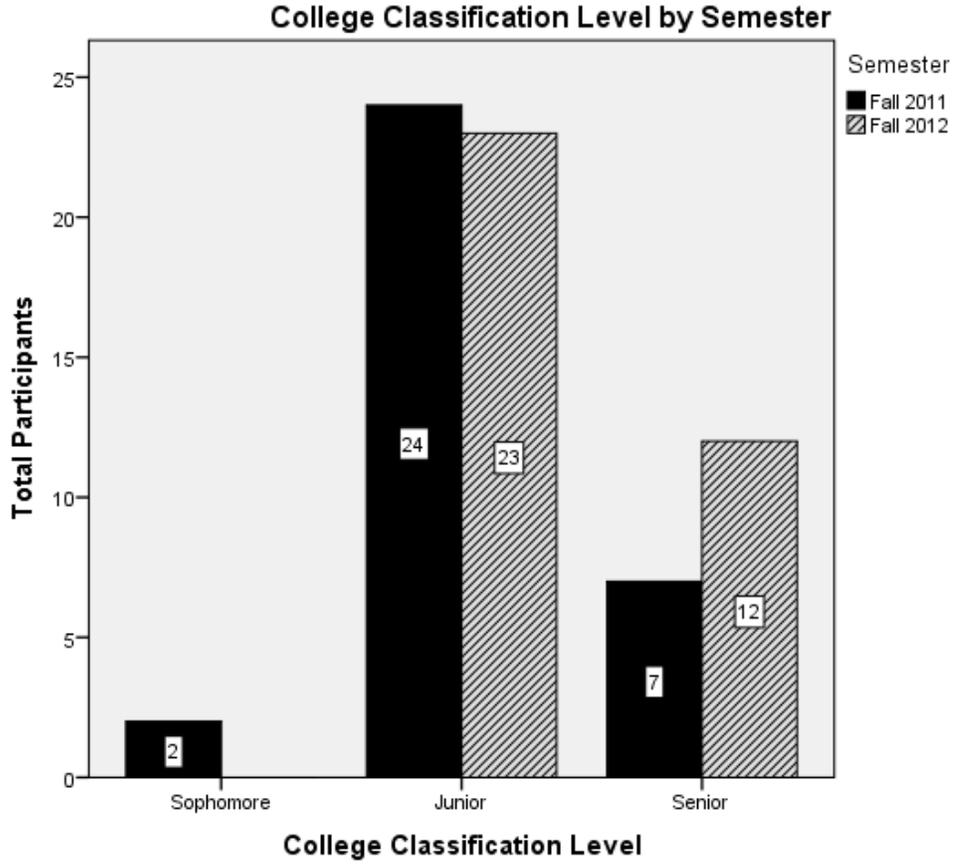


Figure 4.2. College classification level by semester.

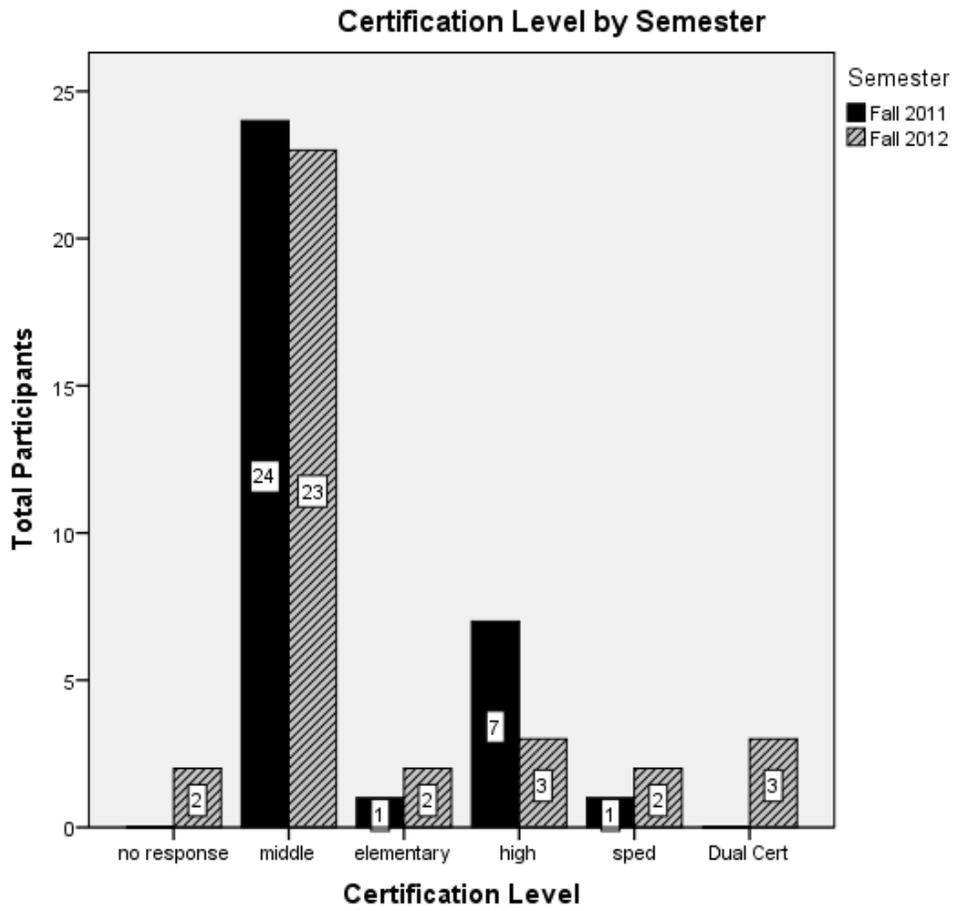


Figure 4.3. Certification level by semester.

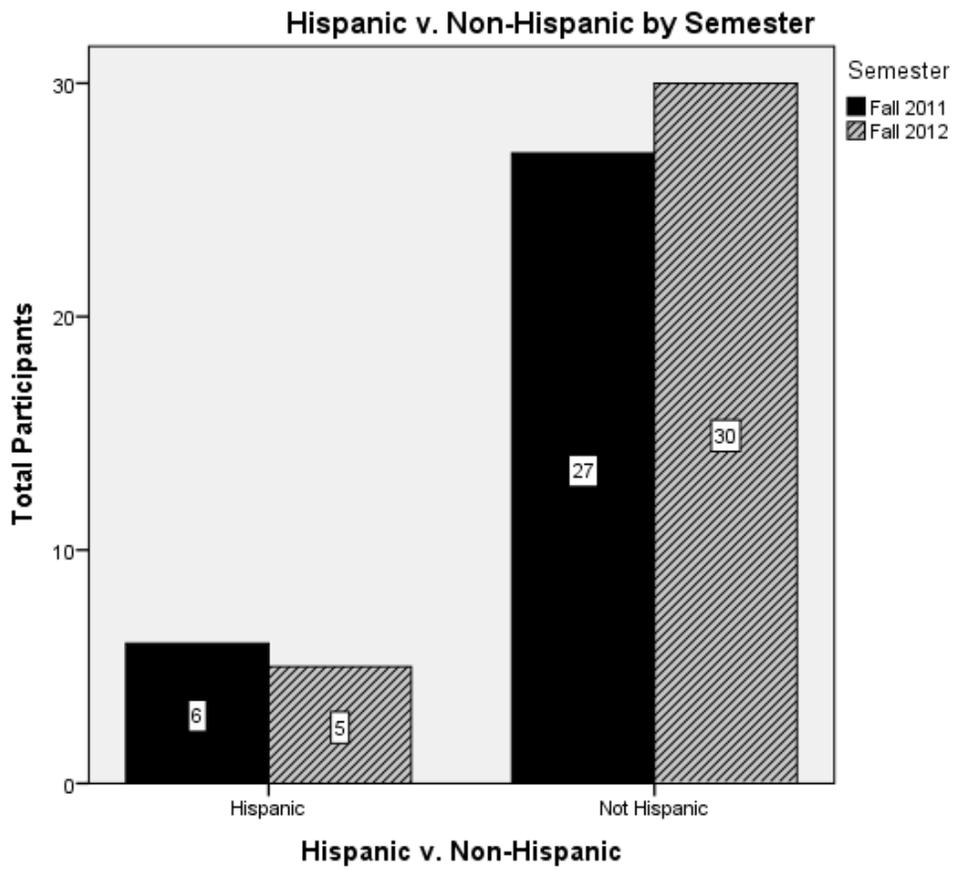


Figure 4.4. Hispanic v. Non-Hispanic by semester.

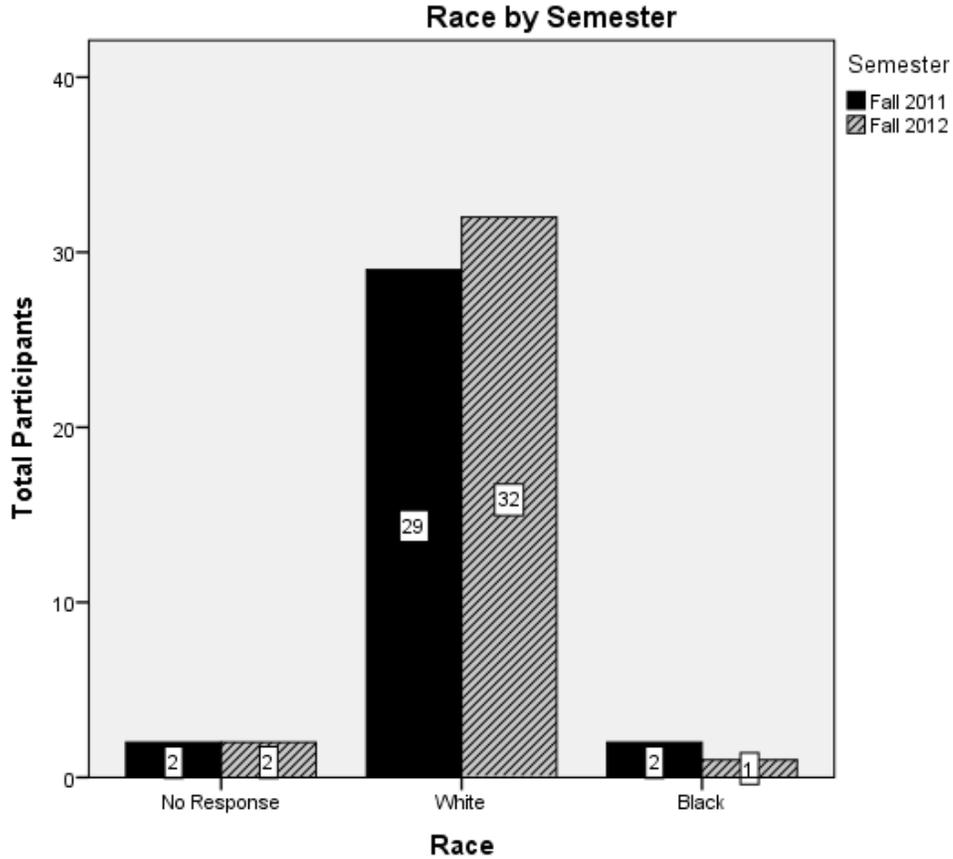


Figure 4.5. Race by semester.

### Treatment

Instruction on problem solving in the MASC 351 course included class presentations and reading assignments on heuristics (Polya, 2003) and problem posing. PTs were assigned five problem sets consisting of non-routine problems. The course also encompassed many lessons and activities that linked culture and equity to mathematics.

One key activity was the Equity challenge problems that were built on an “anchor” algebraic problem to solve. The PTs explained the relevance of the problem to the KTE schemes and suggested a context that would be relevant to diverse middle

grades students. They located the math content of the problem in the state TEKS standards, then answered questions on addressing common student misconceptions about the content. Finally, PTs created a new similar problem and developed a middle school mathematics lesson.

The second key activity in the course was the Second Life (SL) experiences that simulated teaching diverse middle grades students. SL is a 3-D browser that can promote mathematics learning and create specific learning activities (Caprotti & Seppala, 2007). The PTs their lesson to graduate students posing as middle school students in Second Life (SL) and learned problem solving and problem posing techniques.

The fall 2011 and fall 2012 courses had different instructors from the KATE project team but each followed the syllabus closely. Instruction for fall 2011 was led by a Post-doc who was an experienced teacher and had helped design the project proposal. Instruction included guest presentations on SL, one Equity Problem Challenge (The Dinner Problem), graduate research assistants developed avatar biographies and profiles to share with PTs, meeting and tutoring middle grade students posing as avatars in SL, and a SL lesson created by the PTs. The project team conducted pre- and post-interviews of PTs. The instructor implemented SL orientation (see Figure 4.6) “meet the middle grade students” and “tutor the middle grade students” (see Figure 4.7) activities that required the PTs to meet the graduate research assistants that were posing as middle school students in SL. The graduate research assistants also created an algebraic problem that addressed slope or rate of change and purposely solved it incorrectly.



*Figure 4.6. Second Life Training Implementation.*

Note: Students enrolled in the MASC course would walk this route that contained information about SL. The students read Power Points that contained slides about the function of SL.



*Figure 4.7.* Second Life meeting space. Note: Middle grade avatars and PTs would meet for the first time at individual tables. This is also the space where PTs would tutor the middle grade avatars. The white lines separate the boundaries for audio. Conversations could only be heard by avatars sitting or standing near tables within the lines.

The PT met the graduate research assistant's SL avatar in the SL environment. The KATE team created Glasscock Island which contained orientation space, changing room, and indoor and outdoor classroom environments where the PT and avatar could speak about the problem and address misconceptions about it through audio and the chat box (see Figure 4.8 and 4.9). The PT developed a 15-minute problem solving lesson that was implemented in SL. This lesson was taught in a classroom using a SmartBoard in SL in a college classroom. Again, the graduate research assistants posed as middle grade student avatars and presented questions throughout the lesson that were addressed

through chat, on an easel, or on a classroom whiteboard. The PTs were interviewed after their SL teaching experience to get feedback from the assignment.



*Figure 4.8.* Second Life classroom. Note: View of Second Life classroom that contains middle grade avatars sitting at the desks and the MASC instructor walking around the room.

Conversations in SL were through chat and audio. Middle grade avatars had the ability to right with their mouse on a whiteboard in the room, place problems on an easel next to their desks that the PTs could answer while teaching their lesson. The middle grade avatars were also able to use gestures to let the PT know there was a question.

This was done through the raising of the hand or putting a question mark of the avatar's head.

In the fall of 2012, instruction was led by the Principal Investigator of the project via Skype, with a Grad Teaching assistant in the classroom. The intervention for the MASC 351 course was increased substantially. Adding to the original intervention, there was also a presentation from a project team member professor on algebraic misconceptions, teaching for equity and engagement, and culturally relevant math teaching (via Skype) where the professor had the students take to Twitter during the presentation to ask and answer questions immediately. Four Equity Problem Challenges were used: Dinner Problem, Human Graph Problem, Credit Card Problem, and the Basketball and Teachers Problem. A new textbook was added (Ellis, 2008) that included readings on teaching math for diversity and an assignment for PTs to analyze and reflect on their SL tutoring and teaching experiences. The project team developed and administered Diversity Impact Surveys during the semester to determine the effects of each activity on PTs diversity awareness, developed and used the Virtual Classroom Observation Instrument (VCOI) to evaluate PTs lessons in SL, and conducted in-depth pre- and post- interviews with 10 PSTs. This intervention was in addition to the intervention done in the fall of 2011.

### **Instrument**

The KATE instrument was divided into three sections: (1) 20 items pertaining to PTs' awareness of equity that were coded on a 4-point Likert scale from strongly agree, agree, disagree, to strongly disagree, (2) 15 algebraic items that assessed problem

solving (13 of the items were open-ended and 2 items were multiple choice) (3) and one open-ended multi-part item that assessed PTs' teaching of problem solving. Nine of the equity items (#1,4,8,9,11,12,17,18, and 20) were reverse coded due to the language of the question. The highest score the PTs could receive for each of the three items in the instrument: equity items was 80 points, problem solving items was 23 points, and the teaching problem solving item was 12 points. The KATE instrument is contained in the Appendix.

### **Description of the equity items**

There were 20 items that were adapted from the Cultural Awareness Beliefs Inventory (CABI) which measured the perceptions and attitudes of urban teachers' cultural awareness and beliefs (Roberts-Walter, 2007). The 46-item CABI used a Likert scale and included items based on eight factors: (a) teachers beliefs (b) school climate, (c) home and community, (d) teacher efficacy, (e) curriculum and instructional strategies, (f) teacher beliefs, (g) cultural awareness, and (h) behavior management. The KATE project team adapted 20 of the CABI items for the KATE instrument. The items either remained the same or were altered to address mathematics PTs in the middle grades. The KATE equity survey included seven factors: (a) teachers beliefs (4 items), (b) home and community (1 item), (c) teacher efficacy (3 items), (d) curriculum and instructional strategies (3 items), (e) teacher beliefs (2 items), (f) cultural awareness (1 item), and (g) behavior management (2 items). Items were coded from 1-4, where strongly agree=4, agree=3, disagree=2, strongly disagree=1.

### **Description of the problem solving items**

There were 15 items that were adapted from the dissertation of You (2006). There were 13 open-ended items and 2 multiple-choice items. All 15 of the items contained concepts of algebraic problem solving pertaining to rate of change. These items consisted of multiple representations including an equation, graph, table, and word problems. Items 1-9 were coded correct or incorrect, where 0=incorrect and 1=correct. Items 10-15 were coded correct, partially correct, or incorrect, where 0=incorrect, 1=partially correct, and 2=correct.

### **Description of the teaching problem solving items**

The teaching problem solving item was created by two members of the KATE research team. The content of the problem was systems of linear equations. This item includes a detailed summary of a culturally relevant classroom scenario. The PT is asked to determine how to respond to the class, how the student was correct, and how the PT would have solved this problem.

### **Reliability**

The obtained reliability of the KATE instrument Equity items was .758. The Cronbach's alpha for the Item-Total statistics would not increase if any of the 20 Equity items were deleted. The obtained reliability of the KATE instrument Problem Solving items was .522. The Cronbach's alpha for the Item-Total statistics would decrease if 12 of the 15 Problem Solving items were deleted. The obtained reliability from the KATE instrument related to the four Teaching Problem Solving items was .943. The

Cronbach's alpha for the Item-Total statistics would not increase if any of the 4 Teaching Problem Solving items were deleted.

### **Data Analyses**

The KATE pretest was administered to the PTs on the first day of class and the posttest was administered to the students at the end of the semester. Two semesters of data from implementation of the KATE pretest and posttest were examined. Analyses of the test scores from the PTs from the pretest to the posttest were done comparing scores from PTs in both semesters. The Statistical Package for Social Sciences (SPSS) and Excel were used for data analysis. The pre-analysis was used to determine if there was a difference between fall 2011 and fall 2012. If there was no statistically significant difference then the two samples could be combined for the rest of the analysis.

The effects of the course on (a) beliefs about equity, (b) problem solving, and (c) teaching problem solving was reported in confidence intervals (Capraro, 2004) and Cohen's *d* (Cohen, 1992), along with descriptive statistics. In the majority of the cases where the overlap of the confidence interval was .50 or greater,  $N=10$  or greater, and the variances were equal not differing by more than a standard error factor of 2, the *p*-value was between .04 and .05 (Cumming & Finch, 2005). Separate effects of the course on beliefs about equity clusters from a factor analysis were conducted to determine effects of the course on specific areas of beliefs (Henson, Capraro, & Capraro, 2004). The reliability was also determined.

A multiple regression analysis (MANOVA) was done to determine the relationship between (a) beliefs about equity, (b) problem solving, (c) teaching problem

solving, and d) demographic variables with  $R^2$  and adjusted  $R^2$  values being reported to provide the correlation between the independent and dependent variables (Huck, 2004).

A few decision rules were assumed by the researcher. If the  $p$ -value was  $> .05$  for the Levene's test for equality of variances then equal variances were assumed. If the  $p$ -value for the Levene's test for equality of variances was  $< .05$  equal variances were not assumed. When a  $p$ -value is  $> .05$  the sample means are assumed to not be different. If the sample means are not different, then neither are the variances. When a  $p$ -value is  $< .05$  the sample means are assumed to be different. If the sample means are different, then the variances are also different. Items were assumed to be the same if the coefficient in the correlation matrix was  $> .8$ . When determining the reliability of the instrument, if Cronbach's alpha increased more than  $.05$  when an item was deleted, then it should be deleted from the instrument.

## **Results**

Before the data were analyzed, the equality of the variances for Group 1 (fall 2011) and Group 2 (fall 2012) were checked as well as the statistical significance for the  $t$ -test comparison on the pre-test. This was done in order to determine if the two groups could be combined so an independent samples  $t$ -test was conducted. The means and standard deviations for the pretests and posttest for each variable and both semesters are shown in Table 4.1. The results for the combined groups on the pretest are presented in Table 4.2.

Table 4.1

*Group Statistics for Group 1 and Group 2 Equity Total, Problem Solving Total, and Teaching Problem Solving Total Items*

| Variable               | Semester  | N  | % Variance Explained |                    |
|------------------------|-----------|----|----------------------|--------------------|
|                        |           |    | Mean*                | Standard Deviation |
| Equity                 | Group 1   | 33 | 57.55(57.85)         | 4.37(5.59)         |
|                        | Group 2   | 35 | 57.91(60.43)         | 6.00(6.10)         |
| Prob. Solving          | Group 1   | 33 | 11.45(13.24)         | 2.85(2.48)         |
|                        | Group 2   | 35 | 9.77(12.14)          | 2.68(2.66)         |
| Teaching Prob. Solving | Group 1   | 33 | 3.03(3.91)           | 3.57(4.10)         |
|                        | Group 2   | 35 | 4.77(9.00)           | 3.46(2.24)         |
| Equity                 | Group 1 & | 68 | 57.74(57.85)         | 5.23(5.59)         |
|                        | Group 2   | -  | -                    | -                  |

Note: (\*) The mean and standard deviation for the pre-test is reported first and the mean and standard deviation for the post-test is reported in parentheses.

Table 4.2

*Combined Group Statistics for Group 1 and Group 2 Pretest: Equity Total, Problem Solving Total, and Teaching Problem Solving Total Items*

| Variable                          | Semester     | N  | % Variance Explained |                    |
|-----------------------------------|--------------|----|----------------------|--------------------|
|                                   |              |    | Mean                 | Standard Deviation |
| Equity (Pre-Test)                 | Fall '11&'12 | 68 | 57.74                | 5.23               |
| Prob. Solv. Tot. (Pre-test)       | Fall '11&'12 | 68 | 10.59                | 2.87               |
| Teac. Prob. Solv. Tot. (Pre-test) | Fall '11&'12 | 68 | 3.93                 | 3.59               |

The mean for the Equity Total (Pre-test) in Group 1 was 57.55 and 57.91 in Group 2, a difference of .36. The standard deviations were 4.37 and 6.00, a difference of 1.63, respectively. The mean and standard deviation for the combined groups was 57.74 and 5.23 respectively (see Table 4.2 and 4.3). The mean for the Problem Solving Total (Pre-test) in Group 1 was 11.46 and 9.77 in Group 2, a difference of 1.69. The standard deviations were 2.85 and 2.68, a difference of .17, respectively. The mean and standard deviation for the combined groups was 10.59 and 2.87 respectively (see Table 4.2 and 4.3). The mean for the Teaching Problem Solving Total (Pre-test) in Group 1 was 3.03

and 4.77 in Group 2, a difference of 1.74. The standard deviations were 3.57 and 3.46, a difference of .11, respectively. The mean and standard deviation for the combined groups was 3.93 and 3.59 respectively.

The results of the analyses of equality of variances and means for the three variables are shown in Table 4.3.

Table 4.3

*Independent Samples Test for Group 1 and Group 2 Pretest for Equity Total, Problem Solving Total, and Teaching Problem Solving Total*

| Variable                 | Levene's Test for Equality of Variances |      | t-test for Equality of Means |    |                 |                          |       |
|--------------------------|---|------|------------------------------|----|-----------------|--------------------------|-------|
|                          | F                                       | Sig. | t                            | df | Sig. (2-tailed) | 95% CI of the Difference |       |
|                          |   |      |                              |    |                 | Lower                    | Upper |
| Equity                   | 1.937                                   | .169 | -.288                        | 66 | .774            | -2.922                   | 2.184 |
| Problem Solving          | .141                                    | .709 | 2.509                        | 66 | .015            | .3494                    | 3.022 |
| Teaching Problem Solving | .017                                    | .896 | -2.044                       | 66 | .045            | -3.442                   | -.041 |

The Levene's test for equality of the variances for the Equity Total (Pre-test) indicated that the  $F$ -statistic was 1.937 and the  $p$ -value was .169. The  $t$ -test for equality of means of the Equity Total (Pre-test) indicated that the degrees of freedom ( $df$ ) was 66, the  $t$ -statistic was -.288, the  $p$ -value was .774, and Cohen's  $d$  was .067 (meaning the difference between the independent means was unimportant) for pre-test to pre-test for Group 1 and Group 2. Because it is always important to interpret practical effects, the Cohen's  $d$  was computed. The Cohen's  $d$  showed there was not a meaningful difference between the two groups. Therefore, the results of both tests for equal variances and equal means were not statistically significantly different so the groups were combined for all subsequent analyses dealing with Equity.

The Levene's test for equality of the variances for the Problem Solving Total (Pre-tests) indicated that the  $F$ -statistic was .141 and the  $p$ -value was .709. The  $t$ -test for equality of means of the Problem Solving Total (Pre-test) indicated that the  $df$  was 66, the  $t$ -statistic was 2.509, the  $p$ -value was .015, and Cohen's  $d$  was -.601 (meaning the difference between the independent means was practically important) for pre-test to pre-test for Group 1 and Group 2. The Cohen's  $d$  showed there was a meaningful difference between the two groups. Therefore, the results of the test for equal variances were not statistically significantly different, but the test for equal means was statistically significantly different so the groups should not be combined for all subsequent analyses dealing with Problem Solving.

The Levene's test for equality of the variances for the Teaching Problem Solving Total (Pre-test) indicated that the  $F$ -statistic was .017 and the  $p$ -value was .896. The  $t$ -

test for equality of means of the Teaching Problem Solving Total (Pre-test) indicated that the  $df$  was 66, the  $t$ -statistic was -2.044, the  $p$ -value was .045, and Cohen's  $d$  was .490 (meaning the difference between the independent means was important) for pre-test to pre-test for Group 1 and Group 2. The Cohen's  $d$  showed there was a meaningful difference between the two groups. Therefore, the results of the test for equal variances was not statistically significantly different and the test for equal means was statistically significantly different the groups should not be combined for all subsequent analyses dealing with Teaching Problem Solving. As a result of these analyses, the sample size for Equity was  $N=68$ , the sample size for Problem Solving and Teaching Problem Solving in Group 1 and Group 2 was  $N=33$  and  $N=35$ , respectively.

### **Effects of the Problem Solving Course**

In order to address the first research question, Cohen's  $d$  analyses were performed to determine the effects of the course on the pretest to posttest scores for each of the variables.

#### **Equity**

Table 4.4 shows the pretest to posttest means and standard deviations for the combined groups on the Equity Total score. The increase from pretest to post test was 1.44.

Table 4.4

*Statistics for Combined Group 1 and Group 2 Equity Total Pretest and Posttest*

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| Variable           | N  | Mean  | Standard Deviation |
|--------------------|----|-------|--------------------|
| Equity (Pre-test)  | 68 | 57.74 | 5.23               |
| Equity (Post-Test) | 68 | 59.18 | 5.96               |

---

Confidence intervals (CI) were reported to display the effects of the course for each group. CIs from a paired samples *t*-test indicated there was a statistically significant difference from pretest to posttest totals for the Equity Total score (see Figure 4.9).

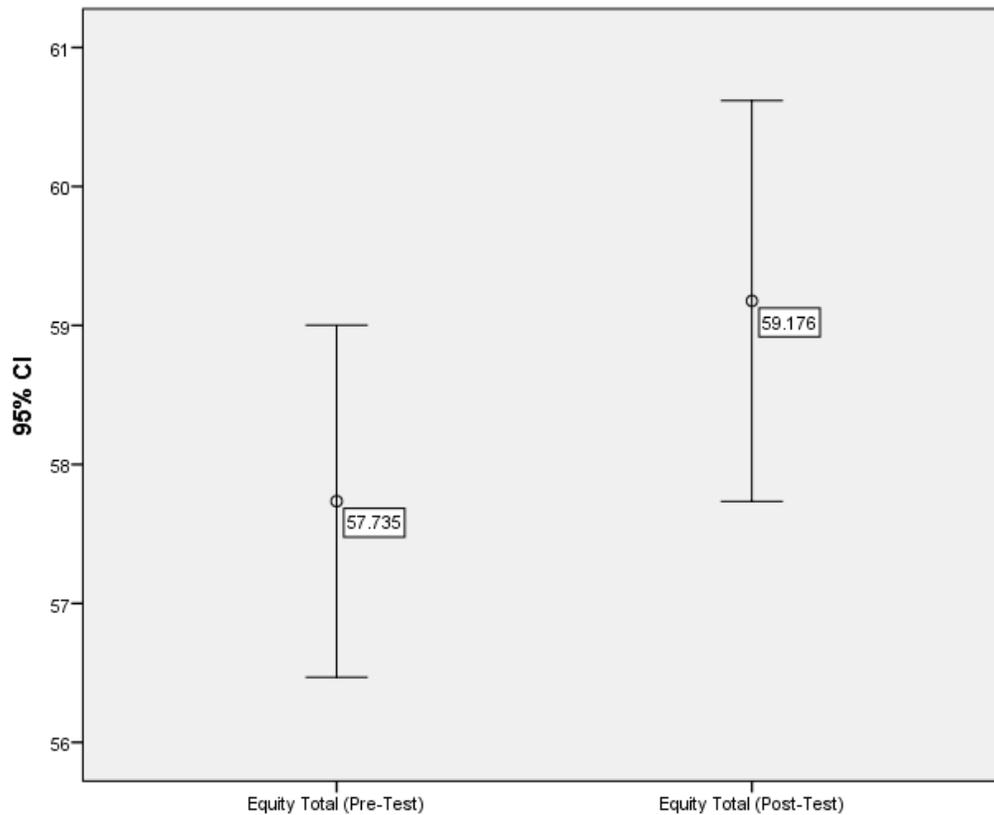


Figure 4.9. Confidence intervals for combined Group 1 and Group 2 Equity Total pretest and posttest.

The Cohen's  $d$  for the combined Group 1 and Group 2 pretest to posttest for Equity Total was  $-.020$ . This showed that there was no practical importance in the change in belief about equity before and after the course.

**Problem solving & teaching problem solving**

Analysis of the Problem Solving Total for Group 1 and separately for Group 2 and Teaching for Problem Solving Total items for Group 1 and separately for Group 2

was done (see Table 4.5). Cohen's *d*, sample means, standard deviation, and CIs from the paired samples were reported.

Table 4.5

*Statistics for Pretests and Posttests for Group 1 and Group 2 Problem Solving Total and Teaching Problem Solving Total*

| Variable                        | Semester | N  | Mean  | Standard Deviation |
|---------------------------------|----------|----|-------|--------------------|
| Problem Solving (Pre)           | Group 1  | 33 | 11.45 | 2.85               |
| Problem Solving (Post)          | Group 1  | 35 | 13.24 | 2.48               |
| Teaching Problem Solving (Pre)  | Group 1  | 33 | 3.03  | 3.57               |
| Teaching Problem Solving (Post) | Group 1  | 35 | 3.91  | 4.10               |
| Problem Solving (Pre)           | Group 2  | 33 | 9.77  | 2.68               |
| Problem Solving (Post)          | Group 2  | 35 | 12.14 | 2.66               |
| Teaching Problem Solving (Pre)  | Group 2  | 33 | 4.77  | 9.00               |
| Teaching Problem Solving (Post) | Group 2  | 35 | 3.46  | 2.24               |

A paired samples *t*-test comparing Group 1 reported a sample mean of 11.45 for the Problem Solving Total pretest and 13.24 for the Problem Solving Total posttest, an

increase of 1.78. The standard deviations were 2.85 and 2.48 respectively, a decrease of .37). The Cohen's  $d$  was .700 and  $p$ -value of .001, indicating practical importance. A paired samples  $t$ -test comparing Group 1 reported a sample mean of 3.03 for the Teaching Problem Solving Total pretest and 3.91 for the Teaching Problem Solving Total posttest. The standard deviations were 3.57 and 4.10 respectively, an increase of .53. The Cohen's  $d$  was .89 and  $p$ -value of .219, indicating practical importance.

A paired samples  $t$ -test comparing Group 2 reported a sample mean of 9.77 for the Problem Solving Total pretest and 12.14 for the Problem Solving Total posttest, an increase of 2.37. The standard deviations were 2.68 and 2.66 respectively, a decrease of .02. The Cohen's  $d$  was .23 and  $p$ -value of <.001, indicating the increase did not show practical importance. A paired samples  $t$ -test comparing Group 2 reported a sample mean of 4.77 for the Teaching Problem Solving Total pretest and 9.00 for the Problem Solving Total post-test, an increase of 4.23. The standard deviations were 3.46 and 2.24 respectively, a decrease of 1.22. The Cohen's  $d$  was 1.45 and  $p$ -value of <.001, indicating that the increase showed practical importance.

## Comparison of Groups

To address the second research question, the confidence intervals from a paired samples *t*-test for Group 1 and Group 2 on Problem Solving Total and Teaching Problem Solving Total were analyzed. The results showed the two groups were statistically significantly different from pretest to posttest. The Cohen's *d* for Group 1 pretest to posttest for Problem Solving Total and Teaching Problem Solving Total were .662 and .226, respectively. This shows that there was a practical importance between performance of Group 1 before and after the course for the Problem Solving Total, but not for the Teaching Problem Solving Total. The Cohen's *d* for Group 2 pretest to posttest for Problem Solving Total and Teaching Problem Solving Total were .878 and 1.435, respectively. This result shows that there was a practical importance between the performance of Group 2 at the beginning and end of the semester.

CIs from a paired samples *t*-test indicated there was a statistically significant difference from pretest to posttest for the Problem Solving Total score. The increase in means for Group 1 and Group 2 were 1.79 and 2.37, respectively. For Teaching Problem Solving, the increase in means for Group 1 and Group 2 was .88 and 4.23, respectively. (See Figure 4.10, Figure 4.11, Figure 4.12, and Figure 4.13).

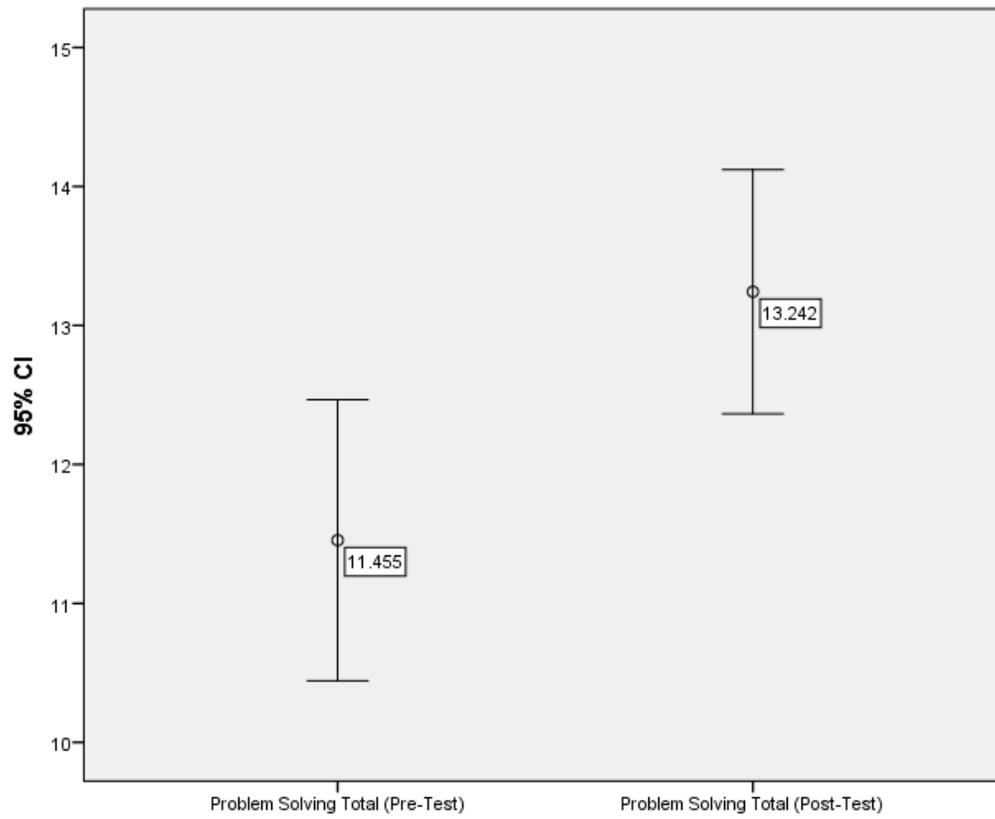


Figure 4.10. Paired samples test for group 1 pre- and post- test problem solving total.

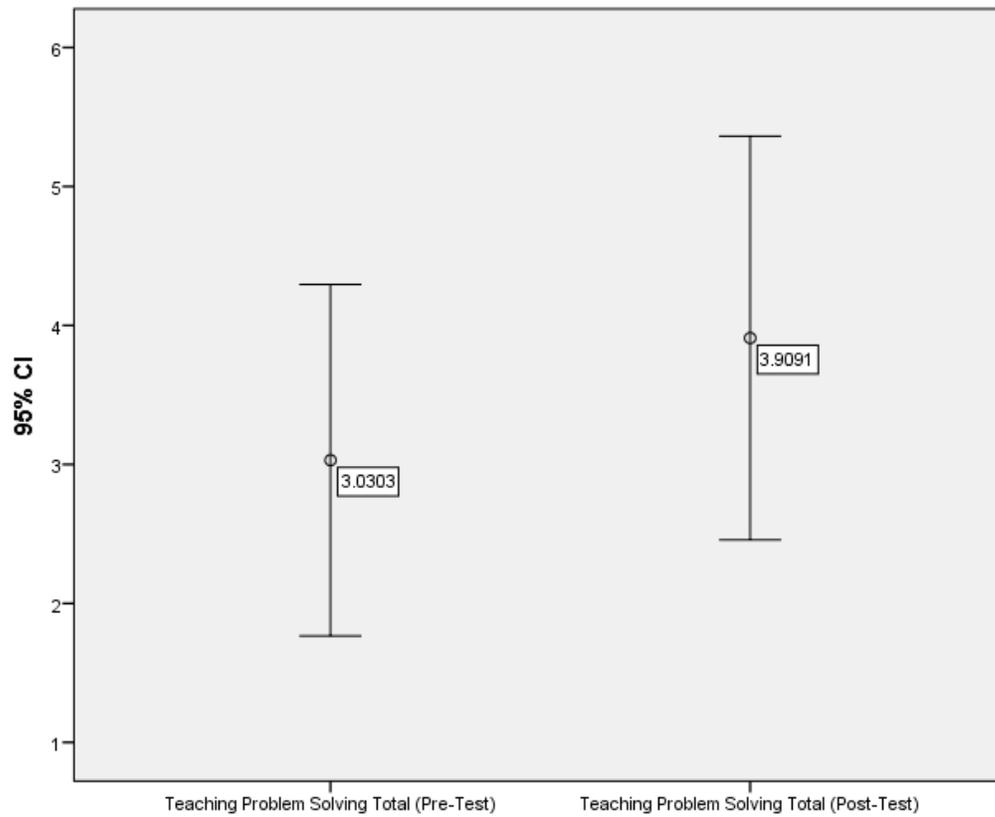


Figure 4.11. Paired samples test for group 1 pretest and posttest teaching problem solving total.

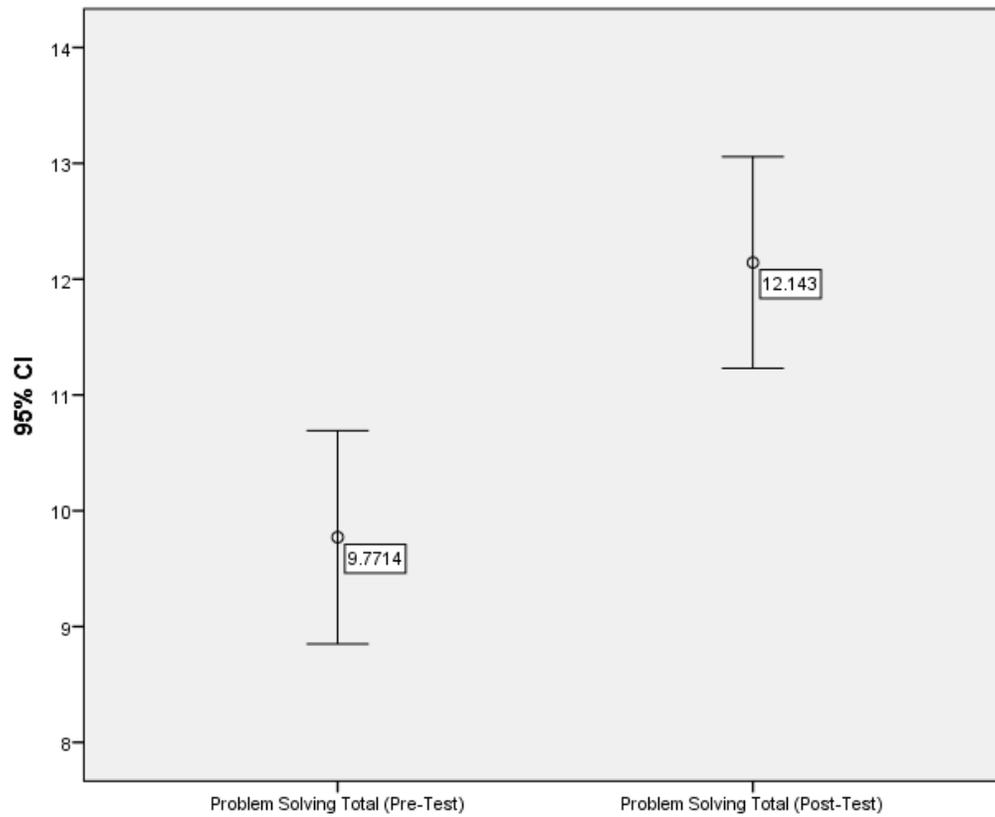


Figure 4.12. Paired samples test for Group 2 pretest and posttest problem solving total.

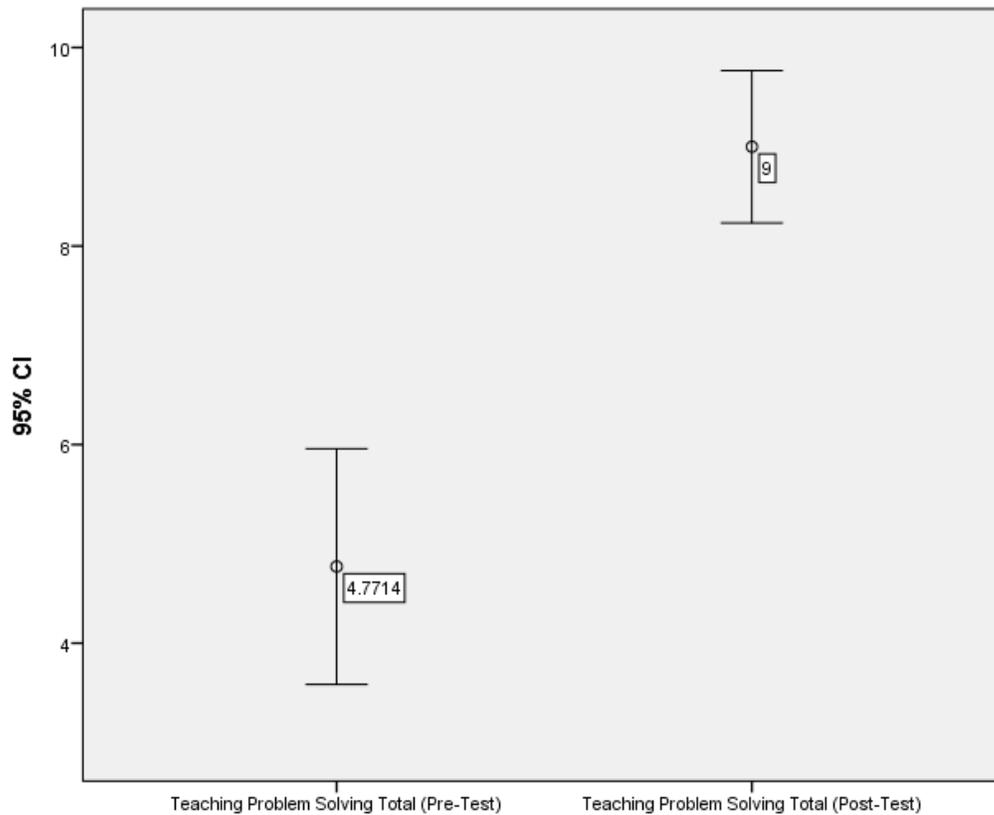


Figure 4.13. Paired samples test for group 2 pretest and posttest teaching problem solving total.

### Effects of the Course on Equity Belief Factors

Further analyses were performed to explore the first research question. The results of an exploratory factor analysis (EFA) of the CABI produced six factors. The six factors were used to compare the pretest to posttest responses of the combined Groups 1 and 2. The factors were (a) cultural awareness (items 12-16), (b) teacher efficacy (items 6, 7, and 19), (c) cultural beliefs (items 4, 8, 9, and 11), (d) cultural

preferences (items 17, 18, 20), (e) teacher perceptions (items 2 and 3), and (f) racial differences (items 1, 5, and 10). The means and Cohen's *d* for each factor are shown in Table 4.6.

Table 4.6

*Means and Cohen's d for the Pretest and Posttest Scores for Six CABI Factors*

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| <u>Factor</u>        | <u>Mean#</u> | <u>Cohen's <i>d</i></u> |
|----------------------|--------------|-------------------------|
| Cultural Awareness   | 16.13(16.37) | .100                    |
| Teacher Efficacy     | 9.54(10.18)  | .405*                   |
| Cultural Beliefs     | 10.72(11.35) | .347*                   |
| Cultural Preferences | 9.15(8.81)   | -.216                   |
| Teacher Perceptions  | 4.31(4.79)   | .389*                   |
| Racial Differences   | 7.59(8.12)   | .324*                   |

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Note: The mean for the pre-test is reported first and the mean for the post-test is reported in parentheses. A (\*) indicates practical importance.

## **Relationships of Variables to Demographics**

To address the final research question, a multivariate analysis of variance (MANOVA) was conducted to examine the influence of independent variables on the three dependent variables (Equity Total posttest, Problem Solving Total posttest, and the Teaching Problem Solving Total posttest). Unlike a univariate analysis, a multivariate ANOVA takes into account the intercorrelations between the dependent variables. The MANOVA allows for multiple dependent variables, while the ANOVA allows for a single dependent variable (Haase & Ellis, 1987). For the MANOVA only specified interactions were of research interest, therefore a partial factorial model (class by certification level) was used. The results from the MANOVA indicated that Teaching Problem Solving was statistically significant ( $p < .001$ ) while neither Equity nor Problem Solving were statistically significant. Teaching Problem Solving was statistically significant by semester ( $N=35$  and mean = 9.00 for Group 2) and race ( $N=61$  for White and mean = 6.77), and class by certification level ( $N=36$  and mean = 6.42 for middle level certification seeking juniors). Simply stated, White PTs, junior PTs, and PTs in fall 2012 accounted for most of the increase amongst scores on the Teaching Problem Solving items. The adjusted  $R^2$  for Equity was .20, Problem Solving was .17, and Teaching Problem Solving was .43.

## **Conclusion**

The purpose of this study was to determine the effects of a problem solving course that included activities intended to build awareness and knowledge of teaching algebra for equity. Two semesters of the course, with differing emphases and assignment

on diversity were studied and compared. The three main dependent variables were measures of equity, problem solving ability, and knowledge of teaching problem solving. Finally, the study examined what relationship or effects the selected demographic variables had with the dependent variables.

In comparing the two semesters, the results showed the equity items were independent by semester; that is, the effects of the course on beliefs about teaching for equity did not differ despite the significant differences in the amount and emphases on diversity in the two semesters. Huck (2004) describes the use of a MANOVA to estimate the effect between independent and dependent variables. The semester, race, and class by certification level showed interaction with the teaching problem solving score but not with the equity or problem solving scores. Though the course focused on equity, problem solving, and teaching problem solving, the latter had interaction with other variables. This may have resulted because the PTs' final assignment was to teach an algebraic lesson in SL that was sensitive to student's culture. Since this is the last memory the students had from the course, this may be a reason for the correlation found between the teaching problem solving items and demographic variables.

The analysis of the six factors of the CABI revealed that the mean scores of the items related to each factor related to the equity items increased except for items related to the factor of cultural preferences. Though the decrease of the mean score of items related to this factor was less than a point, it should still be noted. The KATE factors were (a) cultural awareness (b) teacher efficacy (c) cultural beliefs (d) cultural preferences (e) teacher perceptions (f) and racial differences. The course taught did not

emphasize PT's cultural preferences. It served as a resource of knowledge to show students how to incorporate culture, equity, and math knowledge into the classroom. Because cultural preferences were not a focal point of the course, they may have changed in a slightly negative way because the course related material students may not have been willing to embrace because it was different from their thinking.

The problem solving and teaching problem solving mean scores were statistically significantly different by semester. The overall reason for the difference can be attributed to the different interventions in fall 2011 and fall 2012. The intervention in fall 2012 had added features such as more challenge problems, guest lecturers that reported on culture and equity, as well as lectures on problem posing and students' algebraic misconceptions. It is interesting that the greatest difference between the two groups was on the teaching problem solving score. It is not possible to differentiate among the specific activities that might have contributed most to improving the participants' responses to the teaching problem solving items. However, the time they spent on the additional Equity Challenge problems in which they planned lessons and anticipated responses to student classroom questions might have contributed. The added assignment in which participants reflected on their Second Life tutoring and teaching might also have been important. In these assignments, participants reviewed and identified situations where they could have asked students questions about their understanding of a problem, rather than focusing on the procedure the student used. It is worth noting that activities such as these that simulate classroom teaching might help develop pre-service teachers' knowledge about teaching for equity.

The participants come from various backgrounds, cultures, and environments. It is therefore imperative that the content, culture, and classroom environment are linked together in order to maximize the needed knowledge base of PTs before entering the classroom to teach for the first time. One cannot assume that a PT previously possesses awareness of various cultures or even has a solid foundation in their knowledge of mathematics (Ball, 1990b; Kinach, 2002; Van Dooren, Verschaffel, & Onghena, 2002). Teacher preparation programs need to do a better job in increasing PTs' mathematical, cultural, and equity knowledge upon graduation. Activities and simulated teaching such as the ones used in the course described here have the potential to provide this essential knowledge base.

### **Limitations**

Limitations of this study were as follows:

1. Participants were asked voluntarily to participate in the administration of the KATE instrument. Therefore, the data in this study only represented those that took part in the study.

## CHAPTER V

### SUMMARY AND CONCLUSIONS

The validity of the KATE instrument was aligned through both content and construct validity. Content validity was done on the entire KATE instrument by a panel of experts on the KATE project team. The instrument was edited by re-wording CABI items to be more relatable to mathematics and the mathematics problem solving items were re-worded for student clarity so PTs would answer questions with the information researchers were looking to analyze. Construct validity was estimated by conducting an exploratory factor analysis. An EFA was conducted on just the equity items for a large data set. The mutual adherence of the content and the construct validity for the ideas at hand were positively related. PTs' scores on items for the KATE instrument increased on average from pre- to post- test. Along with the reliability that was equally high at .77 for the Cronbach's alpha for Equity items on the large data set (the reliability from the original CABI was .83) indicates that the KATE and the CABI provided dependable and reliable data for the subject population which was mostly junior level, female PTs.

The EFA conducted on the large data set of 197 PTs across the U.S. revealed six factors. The KATE factors were (a) cultural awareness (b) teacher efficacy (c) cultural beliefs (d) cultural preferences (e) teacher perceptions (f) and racial differences. The PTs' mean sum of item scores within each factor increased on all factors except for the cultural preferences factor. Though the decrease was less than a point, it should still be noted.

This evidence indicates that the KATE and CABI were suitable for exploring the phenomenon at hand which was the intervention of culturally relevant mathematics. This intervention was conducted in a junior level integrated science and mathematics course at Texas A&M University. PTs were presented with information related to culture, equity, problem solving, middle grade student algebraic misconceptions, and teaching algebra. Many of the activities were guided by technology such as Second Life. Therefore, the indication of the test results from this intervention shows the equity, problem solving, and teaching problem solving are dependable and reliable. So therefore, the changes in scores from fall 2011 to fall 2012 were positive and strongly related and we know that because the reliability and validity were reasonably high that those scores are accurate and robust. Changes to the intervention which took place from time 1 to time 2 showed merit because of the increase in test scores over equity, problems solving, and teaching problem solving.

Therefore the KATE instrument provided important understandings for the use of the intervention in this study. The intervention that changed from fall 2011 to fall 2012 changed in a meaningful way so that it produced measurable differences for equity, problem solving, and teaching problem solving. This is important because in order to consider the findings robust you must consider that the use of the instrument or the use of the intervention as positive would have to have evidence that supports their interactions for math learning. If this were used in more university classrooms the potential would be to have better student teachers for diverse learners.

The meta-synthesis conducted exemplified the non-existence and niche to create an instrument that measured PTs' mathematical knowledge and understandings of equity and cultural awareness beliefs. The current literature that exists was heavily saturated with instruments related to cultural beliefs, but lacked sufficient markings on measuring mathematical content knowledge. Moreover, the psychometric information that was available to review on these instruments was limited. This information made it possible for the KATE project to create an instrument that measured both equity and cultural awareness beliefs and mathematical knowledge amongst PTs.

The above research is important for a few reasons. First of all, it showcases the increased number of instruments that focus on culture and equity and the lack thereof of the instruments that pertain to mathematical understanding. Culture has been of increased importance over the couple of decades in the education research. It is therefore understandable why there would be a plethora of instruments in the literature related to this topic. Moreover, lack of mathematical achievement amongst students in the U.S. has also been of importance. This fact showcases the need to continue to find ways to help students with this problem. It is my opinion that the link of culture understanding with teachers for their students and increased attention to the need for a strong mathematical background to be possessed by teachers will increase student achievement with diverse learners tremendously. More psychometrics need to be reported when developing an instrument to be used in academia.

Research must continue being conducted to determine what helps the PTs make the gains reported in this study. When conducting the MANOVA it was found that the

gains were related to the semester, race, and class by certification. More research must be done to see why particular groups revealed in these categories accounted for most of the gains. Lastly, construct and content validity was determined for the entire KATE instrument, while construct validity was determined for the Equity items in the KATE instrument. This adds value to the literature because it reports the important psychometrics that will allow researchers to successfully use the instrument in the future. Though, more must be done to strengthen the Problem Solving and Teaching problem Solving items. This is a result of the small number of items in the section of the KATE instrument.

The field of education is a place where teachers do the greatest work on earth. Research is done at the college level and in the classroom with the hopes that someday it will be related to teachers in a way that will be useful and successful in their classrooms. Teachers stop teaching at times because they cannot relate to or form a relationship with their students or they just cannot find a successful way to present the content in the lesson plans to the students effectively. If researchers can determine ways that will measure PTs' mathematical content knowledge and awareness of culture and equity whether this be through the use of an intervention or just to get a baseline of where PTs stand, it will be easier to figure how to increase these understandings and awarenesses. Determining why an intervention was successful is key to getting positive results. Such a generalizability as to the reason for an occurrence would be masterful in helping to ultimately improve student understandings in the classroom. This whole process starts at the college level in the PTs' college classroom though.

The KATE project team sought forth to find ways to implement cultural awareness and equitable classroom activities and mathematical problem solving techniques into a PT course. The intent was to create interventions that would allow for increased understanding and awareness with these topics. This idea is important because the focus of classrooms today cannot simply be to teach a concept at a procedural level. Students need lessons to be tailored to their specific needs. These needs can be understood by addressing the culture inhibited by the student through equitable processes. An intervention was implemented through this project and an instrument was validated. The KATE instrument can therefore be used by other researchers to gain an understanding of this phenomenon. Moreover, the successful implementation of this intervention can be implemented in PTs' classrooms.

I personally, was on the team that helped to write the proposal seeking the grant to fund the KATE project. After the grant was awarded to our project team at Texas A&M University, I came on part time as a graduate research associate. In this capacity I served as a teaching assistant within the MASC 351 course and a middle grade avatar working with the PTs in Second Life. As a high school mathematics teacher I would like to see Second Life implemented into classrooms for students to use as a venue for learning. All students do not function at their highest abilities in the classroom. Creating a SL environment would provide an innovative outlet for learning to occur that is applicable to students' technological underpinnings. SL could be used as a forum for teaching and tutoring, whether this is facilitated by the teacher or by other students.

The field of education is in dire need of a face lift. I see teachers leaving the profession or current school they are at in the middle or end of the year at alarming rates. It is my belief that students want to learn in an environment that is culturally relevant to them and with a teacher that can relate the material to class in a meaningful manner. The research is being done at the university level and some K-12 administrations are working hard to link the research to the practice. The link though, must become stronger. Partnerships should be created by every school district with a university that focuses on student achievement and teacher preparation. This partnership will allow for more communication amongst teachers, administrators, and professors.

In the broader scheme of things, the KATE instrument has the potential to help the masses in a myriad of ways. This instrument is intended to measure the cultural awareness, equity, and mathematical problem solving ability of PTs. If PTs are made aware of their beliefs base and knowledge of math, then the weaknesses can be exposed and worked on while still at the university level. Once the problem is diagnosed, PTs can get the proper support needed to be successful once stepping foot into the classroom full-time. Moreover, the data from the pre- to post- test from the intervention that was implemented in the MASC 351 classroom at Texas A&M University shows that linking topics of equity and culture to mathematical problem solving can strongly increase mathematical problem solving and cultural and equity awareness amongst PTs. These are qualities that effective mathematics PTs should possess.

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APPENDIX

**Knowledge for Algebra Teaching for Equity**

Student Research Number – AC1 \_\_\_ \_\_\_ \_\_\_

Sex: \_\_\_ Female \_\_\_ Male

Academic Classification:

\_\_\_ Freshman \_\_\_ Sophomore \_\_\_ Junior \_\_\_ Senior \_\_\_ Post-Bac

Mathematics Teacher Certification Level:

\_\_\_ Elementary \_\_\_ Middle School \_\_\_ High School

Ethnic Origin: Are you of Hispanic or Latino origin?

\_\_\_ Hispanic or Latino origin

\_\_\_ Not Hispanic or Latino origin

Race: Select one or more choices indicating your race.

\_\_\_ White

\_\_\_ Black or African-American

\_\_\_ Asian

\_\_\_ American Indian or Alaskan Native

\_\_\_ International

\_\_\_ Native Hawaiian or Other Pacific Islander

\_\_\_ Unknown

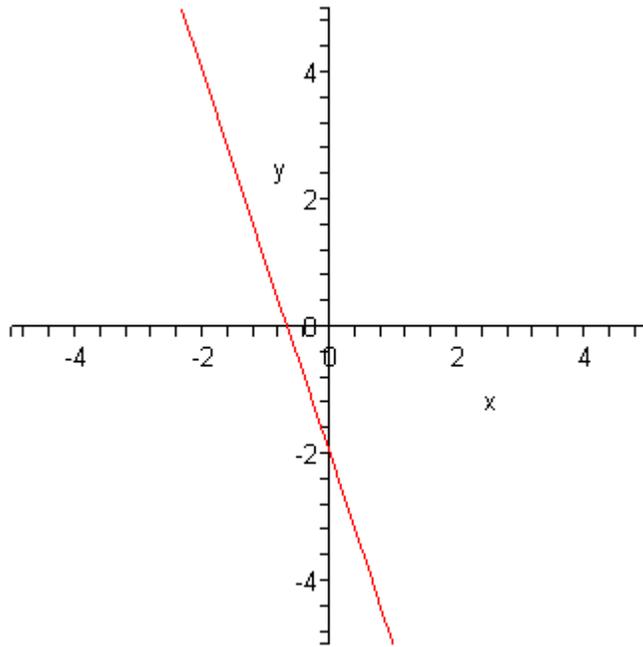
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At Speedy Delivery Service, the cost to deliver a package is made up of a fixed cost plus an additional cost per pound.

1) Fill in the missing values in the table below and write an equation that represents the relationship between the number of pounds and the total cost.

| <b>Number of Pounds</b> | <b>Total Cost</b> |
|-------------------------|-------------------|
| 2                       | ?                 |
| <b>3.5</b>              | \$20.00           |
| 4                       | ?                 |
| <b>5.5</b>              | \$27.00           |

The graph below represents the equation:  $Ax + 3y = -6$  (We do not know the value of the coefficient of  $x$ ).

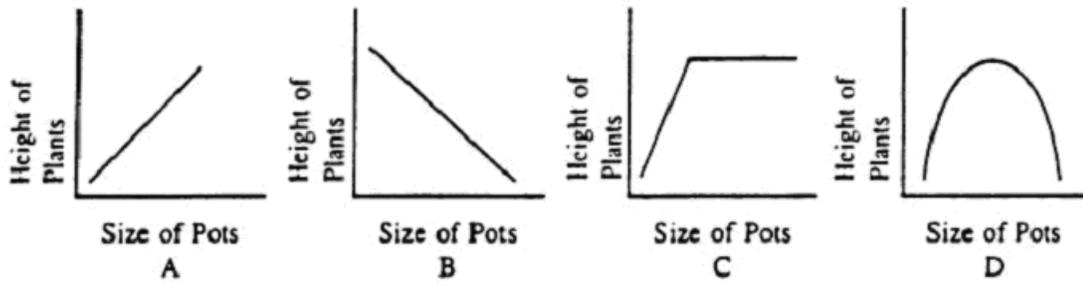


2) Is it possible to find the missing value?

3) If yes, what is the missing value; if no, why is it not possible?

4) Describe a real life situation that represents the equation  $y = 6x + 2$ ;

Which graph is best described by each of the following statements?



5) As the pot size increases, the plant height decreases.

6) As the pot size increases the plant height increases up to a certain pot size. With larger pots, plant height remains the same.

A truck is loaded with boxes; assume each box weighs 20 pounds. If the empty truck weighs 4500 pounds, find the following:

7) The total weight of the truck if it contains 75 boxes. \_\_\_\_\_

8) The number of boxes if the total weight of the truck is 6710 pounds. \_\_\_\_\_

9) Using  $W$  for the total weight of the truck and  $x$  for the number of boxes, write a symbolic rule (or equation) that expresses the weight as a function of the number of boxes.

Suppose that the following table gives the value ( $V$ ), in dollars, of a car for different numbers of years ( $t$ ) after it is purchased.

| $t$ | $V$      |
|-----|----------|
| 0   | \$16,800 |
| 2   | \$13,600 |
| 5   | \$8,800  |
| 8   | \$4,000  |
| 10  | ?        |

Write a symbolic rule expressing  $V$  as a function of  $t$ .

One student's response was as follows:

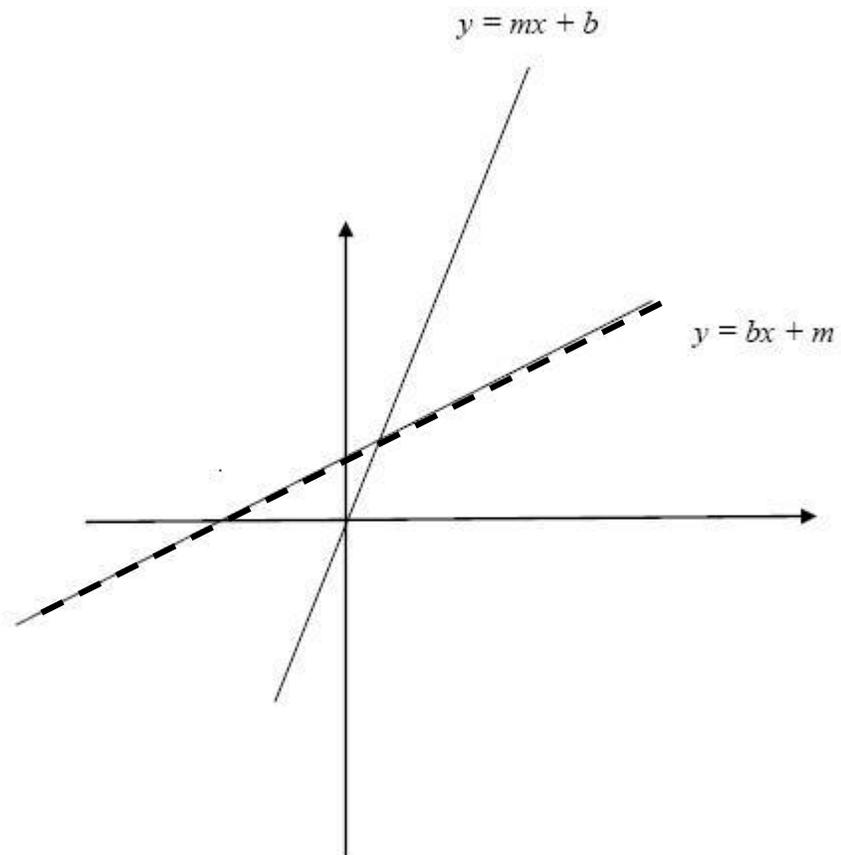
"It can't be done, because the value goes down by different amounts each year."

10) Is the answer correct?

11) If you think the student has misconceptions with respect to the problem, how would you assist this student?

The graph of the equation  $y = mx + b$  is shown in figure below. Draw a graph that represents  $y = bx + m$ .

One student's response (dashed line) was as follows:



12) Is the answer correct?

13) If you think the student has misconceptions with respect to the problem, how would you assist this student?

Simplify  $2x + 7 + 3x - 9$ .

One student's response was as follows:

$$2x + 7 + 3x - 9 = 0$$

$$5x - 2 = 0$$

$$5x = 2$$

$$x = 2/5$$

14) Is the answer correct?

15) If you think the student has misconceptions with respect to the problem, how would you assist this student?

Your third period algebra class ended yesterday with you in a quandary about what to do. You were introducing the solution of systems of two linear equations. You had written the following problem on the board to illustrate setting up two equations.

Shawna took some shirts and pants to the dry cleaner for her mom. There were 9 items and the clerk said that the total charge would be \$30.00. How many shirts and how many pants did she take?

| <b>Price List</b> |        |
|-------------------|--------|
| Shirts            | \$2.00 |
| Pants             | \$5.00 |
| Coats             | \$7.00 |

One of your students, Jaquann, seldom volunteers answers in class and is mostly quiet and introspective. It was quite a surprise to you that he spoke up. In less than 2 minutes after you wrote the problem on the board, and just as you were about to lead a discussion on writing two equations, Jaquann raised his hand and said that he had an answer.

When you asked for his answer, Jaquann responded quietly but confidently, “There were 5 shirts and 4 pants.”

When you asked how he knew, he explained: “Well, if they were all shirts, it would cost \$18. The extra \$12 would be for 4 pants, because they cost \$3 more than shirts. That means there were 5 shirts.”

Thankfully, this occurred at the very end of the class and you were literally “saved by the bell.”

16) How will you address your third period algebra class today?

17) Explain why Jaquann’s answer is correct.

18) Show how you would solve this problem.

**Answer the questions using the following scale:**

**(A) = Strongly Agree (B) = Agree (C) = Disagree (D) Strongly Disagree**

- |  |                  |
|--|------------------|
| 1. I believe all middle school students are treated equitably regardless of their race, culture, disability, gender or social economic status. | A<br>B<br>C<br>D |
| 2. I believe all families are supportive of teachers' work to effectively teach all middle school students.                                    | A<br>B<br>C<br>D |
| 3. I believe teachers have strong support for academic excellence from the surrounding school community (civic, church, business).             | A<br>B<br>C<br>D |
| 4. I believe some students do not want to learn  | A<br>B<br>C<br>D |
| 5. I believe that poor teaching is the main factor that causes the gap in math achievement between White students and students of color.       | A<br>B<br>C<br>D |
| 6. I believe I have the knowledge and skills I need to be a culturally responsive math teacher.  | A<br>B<br>C<br>D |
| 7. I believe I can implement cooperative learning effectively as an integral part of my math teaching strategies.                              | A<br>B<br>C<br>D |
| 8. I believe African American students have more behavior problems than other students.  | A<br>B<br>C<br>D |
| 9. I believe most diverse students are not as eager to excel in math in comparison to their White peers.                                       | A<br>B<br>C<br>D |
| 10. I believe many middle school teachers engage in biased behavior toward students of color in the classroom.                                 | A<br>B<br>C<br>D |
| 11. I believe students who live in poverty are more difficult to teach.  | A                |

- |   |   |
|---|---|
|   | B |
|   | C |
|   | D |
| 12. I believe most diverse students do not bring as many strengths to the classroom as their White peers.                                       | A |
|   | B |
|   | C |
|   | D |
| 13. I believe it is important to identify with the racial groups of the students I serve.   | A |
|   | B |
|   | C |
|   | D |
| 14. I believe I am comfortable with people who exhibit values or beliefs different from my own.   | A |
|   | B |
|   | C |
|   | D |
| 15. I believe the cultural views of a diverse school community should be an integral component of my lesson planning.                           | A |
|   | B |
|   | C |
|   | D |
| 16. I believe in asking families of diverse cultures how they wish to be identified (e.g., African American, Bi-racial, Mexican).               | A |
|   | B |
|   | C |
|   | D |
| 17. I believe that in a society with as many racial groups as the United States, I would accept the use of ethnic jokes or phrases by students. | A |
|   | B |
|   | C |
|   | D |
| 18. I believe my teacher education courses focus too much on “multicultural” issues.  | A |
|   | B |
|   | C |
|   | D |
| 19. I believe I am able to effectively manage students from all racial groups.  | A |
|   | B |
|   | C |
|   | D |
| 20. I believe I would prefer to work with students and parents whose cultures are similar to mine.  | A |
|   | B |
|   | C |
|   | D |

## CONSENT FORM

### **Test of Preservice Teachers' Knowledge for Teaching Algebra for Equity in the Middle Grades**

#### **Introduction**

The purpose of this form is to provide you information that may affect your decision as to whether or not to participate in this research study. If you decide to participate in this study, this form will also be used to record your consent.

You have been asked to participate in a research project studying how preservice teachers learn to teach algebra to diverse students. The purpose of this study is to find how effective virtual simulations are in developing preservice teachers' knowledge and skill in teaching algebra to diverse students.

You were selected to be a possible participant because you are taking a junior level course in middle grades mathematics teacher preparation. This study is being sponsored/funded by the National Science Foundation.

#### **What will I be asked to do?**

If you agree to participate in this study, you will be asked to: **You will complete a written test of your mathematics knowledge and cultural awareness knowledge.**

#### **What are the risks involved in this study?**

The risks associated in this study are minimal, and are not greater than risks ordinarily encountered in daily life as a student.

#### **What are the possible benefits of this study?**

The possible benefits of participation are enhanced awareness about how to teach diverse middle grade students. You will receive no direct benefit from participating in this study; however, what we learn may help to better prepare teachers and close the achievement gap that exists now between students of different ethnicities.

#### **Do I have to participate?**

No. Your participation is voluntary. You may decide not to participate or to withdraw at any time without your current or future relations with your university being affected.

#### **Who will know about my participation in this research study?**

This study is confidential. The records of this study will be kept private. No identifiers linking you to this study will be included in any sort of report that might be published. Research records will be stored securely and only Dr. Kulm's research team will have access to the records.

**Is there anything else I should consider?**

No.

**Whom do I contact with questions about the research?**

If you have questions regarding this study, you may contact Dr. Gerald Kulm, 979-255-5385, [gkulm@tamu.edu](mailto:gkulm@tamu.edu)

**Whom do I contact about my rights as a research participant?**

This research study has been reviewed by the Human Subjects' Protection Program and/or the Institutional Review Board at Texas A&M University. For research-related problems or questions regarding your rights as a research participant, you can contact these offices at (979)458-4067 or [irb@tamu.edu](mailto:irb@tamu.edu).

**Signature**

Please be sure you have read the above information, asked questions and received answers to your satisfaction. You will be given a copy of the consent form for your records. By signing this document, you consent to participate in this study.

**Signature of Participant:** \_\_\_\_\_

**Date:** \_\_\_\_\_

**Printed Name:**

\_\_\_\_\_

**Signature of Person Obtaining Consent:** \_\_\_\_\_

**Date:** \_\_\_\_\_

**Printed Name:**

\_\_\_\_\_

## KATE ver. 2.0 - Scoring problems 10 – 16

- Problems 10 – 15 exist as coupled pairs of questions based on a sample middle grade student problem and solution.
- The first question of the pair asks, “Is the answer correct?” and is used to give the PST the opportunity to judge the correctness of the middle grade student’s response. **The PST should receive a score of “1” if they properly ascertain the correctness of the MGS’ response otherwise they should receive a grade of “0”.**
- The second question asks the PST, “If you think the student has misconceptions with respect to the problem, how would you assist this student?” and is used to give the PST an opportunity to explain a strategy they might use to help the MGS. **The PST should receive a score of “0”, “1” or “2” based on the following rubric:**
  - 0 - No response, completely incorrect, irrelevant or incoherent.
  - 1 - The response provides a partial or incomplete description of strategies for addressing students’ misconceptions. However, the strategies reveal factual or procedural nature, and entail some conceptual nature.
  - 2 - The response provides a complete description of strategies for addressing students’ misconceptions. Furthermore, the response entails accurate and complete conceptual strategies.
- **Problem 16** should be scored based on a different rubric. **The PST should receive a score of “0”, “1”, “2” or “3” based on the following rubric:**

| Objective/Criteria                                    | Performance Indicators  |  |   |  |
|---|---|--|---|--|
|   | 3 points  | 2 points   | 1 point   | 0 points   |
| <b>Mathematical understanding of the problem</b>      | Evidence of understanding the problem and underlying mathematics concepts; correct use of data and information related to real world (pre-service teacher addresses the “correctness” of Jaquan’s response) | Some gaps in understanding the relevant mathematical knowledge that could lead to a solution. (pre-service teacher tries but does not adequately address Jaquan’s response or only partially addresses the “correctness” of Jaquan’s response) | Significant gaps or lack of understanding of the conditions of the problem or underlying mathematics concepts; major errors in use of data and information (pre-service teacher does not solve problem or solves it incorrectly and does not properly address the “correctness” of Jaquan’s response) | Complete lack of understanding the problem. (pre-service teacher solves problem incorrectly or not at all and improperly addresses the “correctness” of Jaquan’s response or does not address it at all) |
| <b>Problem solving strategies</b>                     | Correct use of process and mathematical ideas that could lead to a solution (pre-service teacher uses correct problem solving strategy (i.e system of two equations, substitution, etc.)                    | Uses guess and check, or a strategy that might work, but only for a particular problem.  | Unworkable approach or lack of direction and reason (pre-service teacher uses an incorrect problem solving strategy)  | Not present (no problem solving strategy is present)   |
| <b>Clarity in the solution process of the problem</b> | Uses clear and correct written and mathematical language and symbols effectively and accurately explains reasons for solution attempts and approaches   | Presentation is not completely clear; word descriptions or mathematical language (diagrams) are not always easy to read or understand. Unclear explanation of attempts or reasoning  | Unclear, confusing explanation of attempts or solutions (whether there is a correct answer or not); difficult to follow line of thinking using either mathematical language or words describing the solution process  | No explanation or presentation   |
| <b>Completeness of the problem</b>                    | Presentation is complete; all necessary steps present (whether correct or not).   | Some key missing steps in the solution or incomplete reasoning.  | Significant important steps missing. Only the answer is given.  | No indication of the steps that led to the solution.   |
| <b>Total</b>  |   |  |   |  |