ESTIMATING VEHICLE MILES TRAVELED

ON LOCAL ROADS

A Thesis

by

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ABSTRACT

This research presents a new method to estimate the local road vehicle miles traveled (VMT) with the concept of betweenness centrality. Betweenness centrality is a measure of a node's or link's centrality on a network that has been applied popularly in social science and we relate it to traffic volumes. We demonstrate that VMT on local roads exhibits a scale-free property: it follows two piecewise (double) power law distributions. In other words, the total local VMT can be obtained by properly connecting the two distributions at a breakpoint, each having a slope of the power law distribution. We show that the breakpoint can be predicted by using certain network topological measures, which indicates that the breakpoint may be an inherent property for a particular network. We also show that the highest betweenness centrality point can be estimated using network measures. Furthermore, we prove that the estimated VMT is not sensitive to the power of the power law distributions. This research highlights a potentially new direction of effort for local road VMT estimation.

DEDICATION

I would dedicate this thesis to my parents for their persistent love in my life.

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I would first like to thank my advisor, Dr. Bruce Wang for his guidance and support to my research.

I would also like to thank Dr. Zhang and Dr. Joh for serving on my committee.

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CHAPTER I

INTRODUCTION

Vehicle miles traveled (VMT) refers to the total miles traveled by vehicles on the roadway. It is often used for transportation design, planning, decision making, federal fund allocation, air quality control, and accident analysis. It also has a close relationship to gas tax receipts, the main source of funding for transportation projects. Every year, the state Departments of Transportation (DOT) report the VMT on all functional classes of roadways, both in urban and rural areas to the Federal Department of Transportation *(1)*.

Roadways are classified according to functions: interstate, other freeways and expressways, principal arterial road, minor arterial road, major collector road, minor collector road, and local road (Figure 1). Interstates are the highest class road and connect major cities of the 48 U.S. contiguous states. Arterial roads, classified as either urban or rural, include expressways without full control of access, U.S. numbered routes, and principal state routes. Collector roads serve as links between arterial roads and local roads. Local roads provide access to properties, and have factors such as low capacity and speed. Arterial roads focus on mobility, but local roads focus on accessibility. Collector roads serve in between the two goals. The local roads comprise approximately 60-70 percent of a state's road network as Table 1 shows, but the traffic on local roads is light compared to that on other classes of roads. Interstates, freeways, and major arterial roads are completely monitored by the Highway Performance Monitoring System (HPMS), a national inventory system which monitors nationwide highway travel performance. Collector roads are also covered by various traffic monitoring programs which are developed by DOTs. However, no detailed traffic data is collected on local roads.

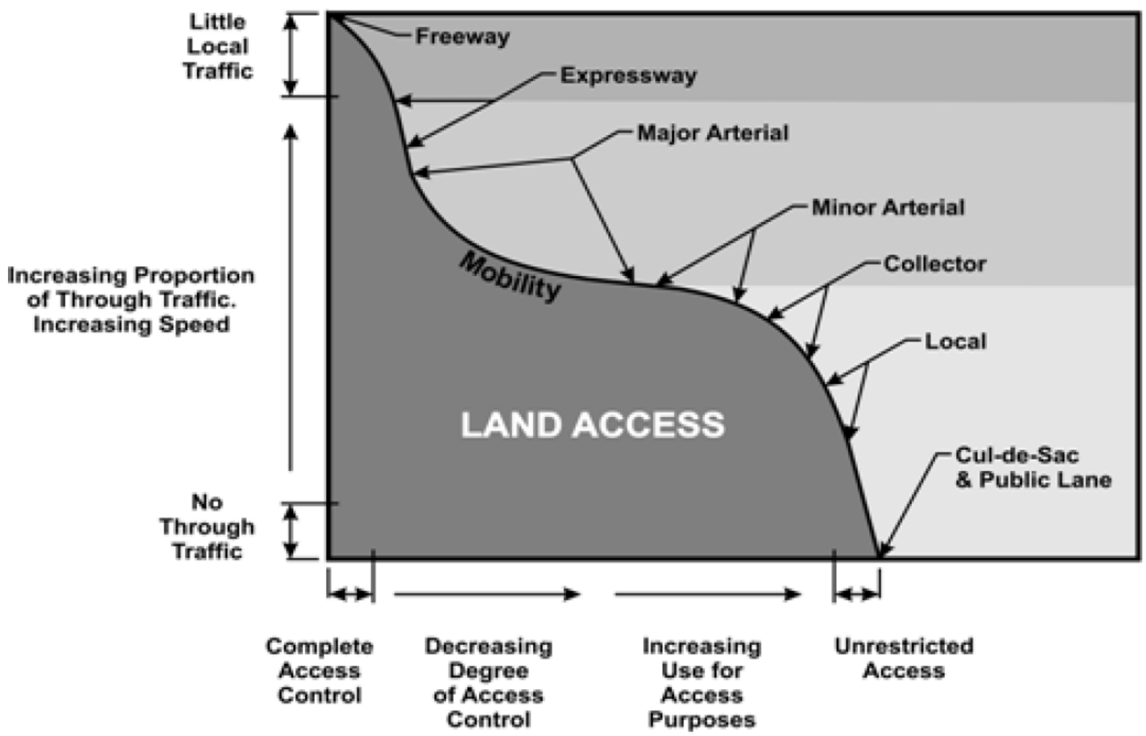


Figure 1. Diagram of land access mobility for each functional system. (Source: FHWA Functional Classification Guidance Update, 2011)

Table 1. VMT and mileage of each functional systems for urbanized areas. (Source: FHWA Functional Classification Guidelines, 2000)

	Range	**(percent)**
System	**VMT**	**Miles**
Principal arterial system	40-65	5-10
Principal arterial plus minor arterial street systems	65-80	15-25
Collector street system	5-10	5-10
Local street system	10-30	65-80

There have been many studies on VMT, and many methods to estimate VMT have been proposed in literature. The methods for collector roads and above are mature and consistent. However, there are issues with the VMT estimation on local roads. Most agencies estimate VMT by using ground count methods, such as the HPMS method. HPMS serves as a reliable data source for VMT, but it doesn't cover local roads because it is originally designed for high functional class roads. Besides, the Environmental Protection Agency (EPA) does not enforce the use of any particular method in the estimation of travel on local roads. Moreover, Federal Highway Administration (FHWA) did not develop schemes for local road VMT estimation. Instead, the estimating procedures are left to each state DOT. Therefore, currently no consistent method has been identified and adopted by all states. Furthermore, most state DOTs are reluctant to develop comprehensive programs for traffic data collection on local roads due to various reasons, such as the less important role of local roads in the state highway system, cost, and so on. All the reasons mentioned previously result in this situation: though local roads constitute a large portion of the total length of a road network, much fewer efforts have been made so far to estimate VMT on local roads than for other classifications of roads. Thus, the difficulty of estimating VMT on local roads lies on the lack of sufficient available traffic data on them. However, VMT on local roads has gained much more attention recently because of its importance for air quality control and traffic accident rate analysis.

Hence, it is essential to develop alternative methods to estimate local road VMT which are reliable, accurate, easy to use, and cost-effective. To do so, it is first necessary to analyze and take advantage of the existing data available on local roads. It is also necessary to deeply investigate the inherent characteristics of the local road VMT in terms of its distribution, patterns, and so forth.

1.1 Problem Statement

The purpose of this study is to estimate the VMT on local roads by using limited data available. In order to avoid dependence on traffic count data, as traditional methods do, we aim to explore mechanisms behind traffic distribution patterns on local road networks. We believe that trips on the local road community are determined and can be estimated by land use and road network structures. Our proposed approach is based on the concept of scale-free property (or power-law distribution) which has been found recently existing in most large-scale networks in sociology, computer science, finance, and other disciplines. Moreover, we take advantage of the concept of betweenness centrality, which is a useful tool to measure traffic on networks. Through simulation, we prove that betweenness centrality follows power-law distribution and the distribution is solely based on network properties. That said, we can develop a model which only requires limited network information to predict a whole picture of the traffic distribution pattern on certain local road network communities. This model is further validated and proved to be working well as a new direction for local road VMT estimation.

1.2 Research Objectives

The primary goal of this study is to estimate local road VMT using minimum information that is readily available. For this purpose, the following objectives are specified:

- To introduce and relate the concept of scale-free property with traffic pattern within typical a local road networks.
- To conduct simulations to generate data for analysis so as to develop VMT estimating models.
- To validate developed models.
- To give suggestions for practical application.

1.3 Thesis Organization

This thesis is composed of six chapters. Chapter 1 introduces the background, including the problem statement and research objectives. Chapter 2 provides a review of the past research on various methods for VMT estimation and other concepts regarding road network properties. Chapter 3 introduces the concept of power-law distribution and betweenness centrality and proves their relationship using simulations. Chapter 4 presents how the property of power-law distribution can be used to estimate VMT and proves that the distribution can be predicted based on network measures. Chapter 5 discusses the application of the proposed estimation model. Chapter 6, the conclusion of this thesis, includes the major findings of the study, limitations and some suggestions for further research.

CHAPTER II

LITERATURE REVIEW

This chapter reviews the previous studies regarding the methods of estimating VMT. First it introduces various existing methods and comments on their respective advantages and disadvantages. Then it reviews the past research with regard to the measures of the road network characteristics.

2.1 Reviews on VMT Estimation Methods

VMT refers to total miles traveled by all kinds of vehicles on a road network. It is of importance to transportation design, planning, decision making, fund allocation, air quality control, and accident analysis. Although various methods of estimating VMT have been proposed, they might generate different estimate results, relying on the availability of data sources. Nevertheless, all these estimates need to be evaluated and compared in order to obtain the reasonable VMT estimates. Moreover, due to various reasons, traditional methods do not pay enough attention to the VMT on local roads, which becomes increasingly important. The following is a brief review of current literature regarding VMT estimation. The related literature falls into three categories: traffic-count based approaches, non-traffic-count based approaches, and local road-specific approaches.

2.1.1 Traffic-Count Based VMT Estimation Approaches

The most common approach to estimate VMT is based on traffic counts. This type of approach directly takes advantage of actual traffic counts on major roads. After sampling procedures or conducting statistical model regression, the total VMT on a road network can be estimated.

The Highway Performance Monitoring System (HPMS) is a national inventory system which monitors nationwide highway travel performance. The HPMS method is a typical case of traffic-count-based methods, which use traffic counts on sampled road sections and road mileage to estimate VMT. Both the sampling and data collection procedures follow the HPMS manual *(2)* to determine the accuracy of final results. This method first obtains VMT in each volume group of each functional class, and then calculates the expansion factor for each group. After summing up the VMT of all groups, the total area VMT can be derived. As long as enough road sections are sampled, this method is highly accurate, since it is based on actual data and statistical principles. Moreover, it has proved to be mature and costless as well, since it has been developed for more than 30 years and current existing programs can be fully used. Robert K. Kumapley and Jon D. Fricker *(3)* reviewed this method and focused its localized version, which was proposed by INDOT (Indiana Department of Transportation). The INDOT procedure is also based on traffic counts and follows the HPMS manual. The difference from traditional method is that it uses its own inventory database, which is much more detailed. In spite of the improvement made by INDOT, this type of method still has two major shortcomings:

the unavailability of local road traffic data and its original designation for high functional class roads.

Despite its monitoring and recording function, the HPMS can serve as a reliable data source for other VMT estimating methods. Aikaterini Rentziou, Konstantina Gkritza, and Reginald R. Souleyrette *(4)* developed a method based on simultaneous equation models and panel data regression models to estimate VMT. They examined most of the well-established factors that would affect VMT, such as demographic and socioeconomic characteristics, fuel cost, land use, length of road network, and road capacity. Then, they found out the key variables related to VMT and analyzed the data from the HPMS to develop linear regressions. Using these models, the future VMT can be forecasted if predicted changes of influencing factors are given. Based on the results of estimation of rural, urban, and total VMT, it can be understood how the key variables affect VMT. According to their findings, state fuel tax and density have the most significant impact on VMT. The method developed from this paper may help policy makers make informed decisions to reduce energy consumption and emissions. Due to the fact that the VMT data are based on the HPMS, the method cannot predict local road VMT. However, the results obtained may also indicate possible relations between local VMT and influencing factors. Thus, it is possible to take advantage of the current available data sources of those factors to estimate or validate VMT on local roads.

Moreover, the report developed by the Fort Collins LUTRAQ Team *(5)* reviewed three methods to estimate VMT and its growth rate in the Fort Collins area. First, they use existing traffic demand forecasting models to estimate traffic on each road, taking into account housing patterns, employment patterns, and roadway capacity. The models are all calibrated by means of HPMS traffic count data. Total VMT is then calculated by adding up the VMT on each roadway. Second, while the HPMS method may be likely to under-estimate VMT, it can still be used to validate the VMT growth rate. Third, as for the fuel-use approach, it calculates the share that the Fort Collins area takes of statewide fuel use and then multiplies it by the fuel efficiency. According to the results, they conclude that the model-based method has the best accuracy. They also compare the VMT growth rate to the population growth rate, which is meaningful to city planning. The results show that VMT is growing faster than population. Moreover, they explain the significance of the finding to city planning and air quality control. The frequency of updating the traffic modeling and calibration is recommended as biennial. The drawbacks of using HPMS data, as described in the preceding paragraph, still exist in this method.

Additionally, HPMS data can also help researchers work together with other data sources. The report "TxLED VMT Estimation Project" by Cambridge Systematics Inc. *(6)* estimates truck VMT in order to evaluate the effectiveness of Texas' low-emission diesel (TxLED) fuel program. In this project, the truck VMT consists of three parts: pass-through truck VMT, internal-external/external-internal truck VMT, and internal truck VMT. Four data sources used in this method include the TxDOT Statewide Analysis Model (SAM), TxDOT Highway Performance Monitoring System (HPMS) vehicle classification data, Reebie TRANSEARCH freight flow data for State of Texas, and Metropolitan-level travel models of Houston and Dallas. The key part of this project is integrating the VMT estimates from various data sources. In terms of the characteristics of different data sources, the final estimate of VMT is developed by proportioning the trip type VMT estimates from the SAM to the VMT totals from the HPMS. As a result, VMT estimates are generated by multiplying the distances of a given OD pair by the number of trucks. However, some issues still exist in this method. For example, the fuel usage generated from a single survey in Houston's metropolitan area is not that accurate. More surveys are needed for various counties to obtain the generality of results. Also, there are significant amounts of data not used in the estimation, which means that the incorporating process should be modified and improved in the future.

2.1.2 Non-Traffic-Count Based Approaches

Non-traffic-count-based methods refer to those based on statistical analysis of factors such as demographical data, fuel sale, and network modeling. Normally, traffic-related data is not required.

Brian Stone, William Obermann and Stephanie Snyder *(7)* tried to analyze the relation between land use, demographics, and VMT, by which future VMT can be estimated according to the land use changes. The data sources used in the report include residential VMT data developed from the Nationwide Personal Transportation Survey (NPTS), commercial VMT data derived from the Freight Analysis Framework (FAF), and demographic data obtained from 1990 and 2000 censuses. This method derives VMT rates for each cluster of census tract based on demographical characteristics. It can obtain high-resolution graphics showing how VMT varies geographically. It can also estimate current and future VMT rates associated with land use and demographics. However, the detailed NPTS data is no longer open to the public as a result of privacy concern, so it is difficult for researchers to utilize. In addition, the Oak Ridge National Laboratory Transferability study used in this method is owned solely by the company and has not been publicly adopted.

A recent report, "Developing a Best Estimate of Annual Vehicle Mileage for 2009 NHTS Vehicles," *(8)* proposed a method called BESTMILE to estimate VMT based on single odometer readings. The data source is Version 3 of the 2009 NHTS (National Household Travel Survey) vehicles. The authors first analyzed the 2009 NHTS data quality and found that the single odometer reading with vehicle year data was the basis for the 2009 data method. Three regressions were conducted separately for three different types of vehicles – new, used, and all - in order to get the relation between vehicle age and annual miles driven. Then VMT for each year in the single odometer reading could be determined, given the vehicle age and accumulative miles.

Jon D. Fricker and Raymond K. Kumapley *(9)* also developed a non-count-based statewide VMT estimation model to supplement INDOT's traffic-count-based method and assist Indiana DOT in planning. The method uses a short-term cross-classification VMT forecasting model for INDOT, based on household and driver survey data such as population of licensed drivers, age, and gender, from NPTS data sources. This short-term VMT model developed for INDOT predicts the total vehicle miles driven by all licensed drivers for all vehicle types. However, the surveys are likely to contain inaccurate information provided by respondents and can only forecast a rough total VMT.

Most state DOTs estimate VMT using ground count methods. However, the local road network, which usually forms a majority of the total state road mileage, is biased in the data collection sampling process. Most state DOTs are also reluctant to develop comprehensive programs for traffic data. Thus, the difficulty to estimate VMT on local roads is due to the lack of traffic data on them. However, research with regard to estimating VMT on local roads has gained much more attention recently because of its importance for air quality control and traffic accident rate analysis. In the next section, limited literature for VMT estimation on local roads will be presented.

2.1.3 Local Road-Specific VMT Estimation Approaches

The work by Ming Zhong and Brody L. Hanson *(10)* is aimed at using travel demand models (TDM) to estimate traffic volumes on low-class roads. This method does not rely on traditional traffic monitoring systems, which cannot cover all roads in the networks. The authors conducted one case study to examine this approach on the York County and the Beresford area in the Province of New Brunswick. Major steps include building network and traffic analysis zones, trip generation, trip attraction, trip distribution, and trip assignment. Once the traffic volumes are obtained, it is easy to estimate VMTs for all road classes in the network. Their results demonstrate that the TDM method can serve as a practical and cost-effective way to estimate traffic volumes for low-class roads. Additionally, a number of Metropolitan Planning Organizations (MPOs) have relied on this kind of method *(11, 12)*. The advantage of this method, compared to the traditional method, is that it can capture the volume variation within each road group, since traffic volumes for each road can be estimated by this method. However, two major issues with this method exist. One issue is that the traffic volumes obtained from the model are always overestimated, especially for local and collector roads. The reason may be that too much traffic is distributed to the collector and local roads, due to the all-or-nothing assignment method. After incorporating traffic count data as the road capacity and the Stochastic User Equilibrium (SUE) method, the errors are reduced. Another issue is that reducing the study area might gain better accuracy. According to the author, boundaries encompassing only the urban influencing area, rather than the whole jurisdiction, should be chosen.

Fang Zhao and Soon Chung also proposed a method by using Geographic Information System (GIS) tools *(13)*. Different from Ming's method, this method is based on a multiple linear regression model of AADT. The study area is the entire Broward County in South Florida, a typical coastal area. The AADT data source is composed of traffic counts obtained from permanent count stations on state roads. In the regression model, AADT serves as the dependent variable and the predictors are roadway characteristics, socioeconomic characteristics, expressway accessibility, and accessibility to regional employment centers. The four variables can be obtained from GIS tools. Through analysis, they find that function class and number of lanes are the most significant predictors. This method can be used to estimate AADT on almost all road segments, as long as the variable data are provided. It can be used to estimate VMT on local roads, while there is insufficient validation for local roads due to the lack of AADT data on local roads. In addition, temporal stability and errors of the models need to be analyzed in the future.

Kelly Blume, et al. *(14)* propose to estimate local road VMT based on GIS data and regression modeling. The major difference is that they only use GIS data as stratifying tools and sample collected traffic data. This method takes advantage of census data and the correlation between travel and population density, job density, and roadway density. By stratification and sampling procedures, median AADT for each functional class and each stratum of road could be obtained. Then, the VMT can be estimated. According to the descriptions, the first important thing is to build a statewide GIS database containing all functional roads' information. After that, in order to define the most reasonable boundaries which are neither too small nor too large, a new concept called the ZIP code tabulation area (ZCTA) is introduced. Each ZCTA has the similar characteristics. The Florida state is then divided into ZCTAs by three categories – urban, rural, and mixed ZIP codes. The next step is to group these ZCTAs into strata. The sample size required for each stratum is calculated using the formula proposed by the HPMS Field Manual. As for count locations, they are randomly selected in the GIS database. The last step is to estimate VMT once the average or median ADT is obtained. After adjusting this value, a representative AADT for each classification road can be derived. Thus, VMT could be calculated by multiplying this AADT by the local road length. One feature of this method is that it is based on the correlation between travel, population density, job density, and roadway density, which is reasonable and intuitive. However, a complete local road database and accurate local road AADTs should be available. In addition, more work that should be done includes choosing better stratification variables, developing a more reliable and accurate GIS roadway database.

Apart from GIS-based methods, William L. Seaver, Arun Chatterjee, and Mark Seaver *(15)* propose a mathematical model based on statistical analysis. It does not rely on traditional sampling procedures or traffic count programs. In order to develop the model, data from 80 counties in Georgia was selected for analysis. Differing from the traditional method, which tries to find the relationship between VMT and socioeconomic and geographic variables at the census tract level, the authors conducted the process at the county level. They tested 45 very general variables to derive models and adopted principal components out of 45 initial variables. Afterwards, the optimal multiple regression for ADT on rural local roads was derived. One feature of this method is the focus on identifying and selecting variables to predict ADT accurately. This method can also be applied to states which do not have traffic counts on local roads. As for limitations, the data - such as census data - on which the model relies does not update frequently, which results in a lag somehow. Additionally, relying solely on the demographic data may not be sufficient.

Furthermore, there is also a method that incorporates concepts from electrics. The paper proposed by Shengguo Wang et al. *(16)* developed a circuit network model and simulation to estimate VMT and AADT for local roads. This method assumes that the road network can be represented by circuit networks and that AADT is related directly to the households along each road. They found that there is a nearly linear relationship between the total entrance AADT with the number of community households. Then a circuit network was modeled among which resistor, current flow, and voltage were represented by road length, AADT, and VMT, respectively. Simply put, each entrance serves as a current source and each branch has a sink current source at its mid-point. Then, circuit models were developed and, with the help of software, AADT at each road segment and VMT of the whole area can be calculated automatically. This method can conduct the estimation without field data collection. However, there are still some issues. For example, it needs further validation when assuming that AADT is linearly related to the number of households. Moreover, some problems may arise if resistors are solely used to account for road lengths. For instance, houses and apartments along one road of the same length should be treated differently.

2.2 Measures of Road Network Characteristics

Networks are composed of a set of nodes and links. In transportation, nodes are generally the connection points of roads, such as intersections, while links are road sections. In two-dimensional networks, the number and spacing of nodes define the density and shape; the links between nodes define the level of connectivity. Different arrangement of nodes and links results in different network structures. There are a number of measures to quantitatively evaluate network structure so far, such as connectivity, heterogeneity, gamma index, compactness, etc. Since the 1970s, researchers have made efforts to investigate how traffic flows and travel pattern are related to different network structures. Several common measures of road network are reviewed below.

2.2.1 Connectivity

Connectivity, as one of the most common measures, is a measure of different ways to connect a pair of origins and destinations *(17)*. Generally, a high level of connectivity means one has more choices to make a trip from one point in the network to another point. Based on previous research, street connectivity plays an important role in defining traffic flow patterns. Moreover, data required to measure connectivity is easy to obtain and the concept is quite straightforward. Though connectivity can be defined in different ways, the most common and simplest form is dividing the number of links by the number of nodes (i.e. C=L/N, where L stands for number of links and N number of nodes). The following reviews partially examine previous research related to network topology and connectivity.

Carlos A. Alba and Edward Beimborn *(18)* examined the relation between connectivity of local residential streets and traffic volumes on nearby arterial roads. The study revealed that improved connectivity can reduce arterial traffic. The relative speed on the arterial vs. that on local roads defines the extent of the effect. It is proved that better connectivity of local streets can help spread out traffic volumes more efficiently throughout a network. Gil Tal, Susan Handy, and Marlon G. Boarnet *(19)* stated the connectivity changes in the evolvement of typical street network patterns in the U.S. They compared and commented on past studies on how connectivity will affect VMT and GHG (Greenhouse Gas) emissions. Dill Jennifer *(20)* evaluated various measures of network connectivity for the purposes of increasing walking and biking, based on a project in Portland. They selected street network density, connected node ratio, intersection density, and link-node ratio as variables. They found that the four measures are positively correlated, but they do not assign the same level of connectivity for an area. Mike Tresidder *(21)* examined the different measures for connectivity and evaluated their effectiveness and limitations. In the rest of their study, they proposed a method for measuring connectivity using GIS and analyzed limitations and issues of that method.

Pavithra Parthasarathi, Hartwig Hochmair, and David Levinson *(22)* incorporated land use and socio-demographic characteristics, in addition to connectivity, to analyze their impacts on VMT. Regression models were derived to show the influence. According to the results, the street network structure does influence the travel behavior of individuals. Meanwhile, connectivity, circuity, shape factor, and the population and employment density show a negative influence on VMT, after controlling other independent socio-demographic and land use variables.

2.2.2 Compactness

The measure "compactness" proposed by Courtat *(23)* is used to measure how a certain area is filled with roads. It is expressed as below:

$$\varphi = 1 - \frac{4A}{(l_T - 2\sqrt{A})^2} \tag{2-1}$$

where A is the area of a community of interest and l_T the total length of roads. The value of φ will be within the range of [0,1]. If there are no roads in the area, the value will be 0. It is very likely that the larger the value is, the more connected one network is.

2.2.3 Betweenness Centrality (BC)

Road networks have strongly heterogeneous functions. Some roads carry high traffic volumes and serve as a backbone for the whole network, while others only provide accessibility to neighborhoods. The underlying idea can be represented with a single measure — centrality. The study of centrality, however, originated from sociology. In the classic structural sociology network, nodes represent individuals and links represent the relationships between individuals. Freeman *(24)* proposed betweenness centrality to identify an individual's social status in terms of his social influence and connections. Recently, betweenness centrality has been extensively applied to other disciplines including computer communication networks *(25, 26)*, protein networks *(27)*, urban design *(28)*, and also transportation networks. Altshuler, Y., R. Puzis, Y. Elovici et al. *(29, 30)* proposed that applicability of BC, and certain augmented measures of it, can be used for the prediction of mobility patterns in transportation networks. Specifically, they found that there is a strong positive correlation between traffic flows through a node and its BC measures. Other literatures also pointed out the correlation between traffic flow patterns and betweenness centrality distribution *(31, 32)*.

When betweenness centrality applies to links in road networks, it is based on all shortest paths between nodes. In other words, it quantifies to what degree a link would separate the network into two parts, assuming that people choose to travel on their shortest paths. Mathematically, the betweenness centrality for a link is defined as the number of shortest paths from all vertices to all others that pass through that link, i.e.,

$$g(e) = \sum_{s,t \in V} \frac{\sigma_{st}(e)}{\sigma_{st}}, \tag{2-2}$$

where s and t represent nodes in the network, $\sigma_{st}(e)$ the number of shortest paths going from s to t through link e, σ_{st} and the total number of shortest paths between s and t within the network. A higher value of betweenness centrality indicates that the corresponding link likely rests on the edge between two parts of the network. From this perspective, links with high betweenness centrality are very likely to carry more traffic, as they play an important role in networks.

CHAPTER III

SCALE-FREE PROPERTY OF LOCAL TRAFFIC

A road network exhibits its specialty in terms of its geographical, historical, and social–economical characteristics compared to other networks. This section studies BC on local road networks and, via simulation, shows that traffic flow over a local neighborhood network has the scale-free property.

3.1 Introduction

Networks or graphs have been studied for a long time in many disciplines including mathematics, mathematical sociology, computer science, and quantitative geography. Since Albert and Barabasi *(33)* proposed the existence of degree heterogeneities in the real world, many researchers have been focusing on the properties of a spatial network, which is in contrast to random networks. That said, real networks are composed of nodes and edges, which are constrained by geometry and space. The combination of nodes and edges determines the topological properties of the network and affects the operating mechanism taking place on it.

Large-scale networks, either observed in reality or abstracted from real-world systems, have been found to exhibit two fundamental properties. The first is the small world property, which says that the shortest distance of any two random nodes among the network is usually short. The small world property also implies a high degree of clustering.

This property has been demonstrated in social networks, the Internet, and computer and mail networks *(34, 35, 36)*.

The second property of large-scale networks is the scale-free property. The scale-free property characterizes an insight in networks that a large portion of nodes have attained little connectivity and that a small portion of nodes have the most connectivity, when connectivity is measured as the number of links to a node *(37)*. If a network possesses such a property, the distribution of the connectivity follows a so-called power law distribution *(38)*. As indicated in Barabasi, A. L., and R. Albert. *(39)*, this property differentiates the real world networks from random networks since, in random networks, the distribution of connectivity may follow a bell-shaped exponential distribution. Although the scale-free property has been explored in some disciplines *(40, 41)*, recently it has also been discovered in urban street networks *(42, 43, 44)*. In a series of work *(32, 37, 45)*, Jiang and his colleagues demonstrated a small world property for the topologies of urban street networks, and a scale-free property for both street length and connectivity degree. Their findings also revealed the street hierarchy with different importance, which indicates that a minority of streets account for a majority of traffic flow *(45)*. The above sequential work indicates a strong connection between traffic patterns over the networks and the scale-free property.

3.2 Power Law

Scale-free property can be described by power law. That is why power law is also called scale-free distribution. And the graph possessing such a property is called a scale-free graph *(46)*. Power law is a functional relationship between two quantities, which can be expressed as p(x) = $Cx^{-\alpha}$. It is also noted as long tail distribution or scale-free distribution. Many societal entities have this property. For instance, the number of cities with a certain population varies according to a power of the population. In other words, those cities with a large population only account for a small percentage of all cities, which is shown in the left plot of Figure 2. Power law also applies to other areas such as income, word frequency, websites visited, and road segment lengths. One of the features of power law is that if we take log-log values for both sides of the equation, there will be a straight trend line overlaying the dots, as shown in the right plot of Figure 2. In this case, differing from Gaussian distribution, the average value may not be a good index to reflect the characteristics of the quantity, because of the heavy tail. The tail part of the distribution indicates variation, and is of interest and of importance to practice.

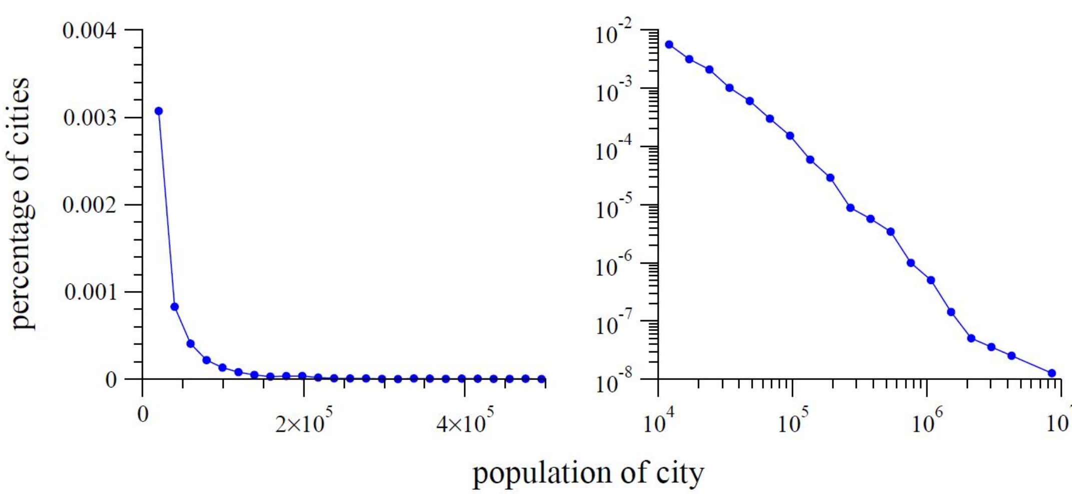


Figure 2. Left: histogram of the populations of all U.S. cities with population of 10,000 or more. Right: histogram of the same data, but plotted on logarithmic scales. (Source: Pareto distributions and Zipf's law, Newman, 2005)

However, there are some issues with this kind of expression in actual practice. One solution is using cumulative distribution function. The power law can be expressed by two different expressions in practice *(47)*. Pareto proposed a form of cumulative distribution when he investigated income distribution. He is interested in how many people have an income higher than x as the expression is defined as r $\sim x^{-\beta}$ (r = number of people with income higher than x, x = income). Meanwhile, he observed that around 80% of the land in Italy was owned by 20% of the population. This phenomenon also exists in income range. That is why this kind of distribution is known as the 80-20 rule or the law of the vital few. On the other hand, Zipf *(48)* proposed that the size of the nth largest occurrence of the event is inversely proportional to its rank, r. The expression is x $\sim r^{-\alpha}$ (r=rank, x=income). Actually Zipf's form is the transformation of Pareto's distribution. In reality, according to the expression, the best way to examine if one distribution follows power law is to see its log-log plot since the rank of a quantity can be easily obtained. If there is a straight line on the plot then there is enough evidence that the distribution follows power law. In this paper, Zipf's law for the modified betweenness centrality, which will be introduced below, is utilized to analyze traffic flow patterns on local road networks.

3.3 Modified Betweenness Centrality

VMT can be calculated by multiplying road length with the traffic volume on them. As the local road length is readily available from government database, estimation of traffic volumes via ADT (or AADT) on local roads is our major task.

A common feature in most local communities is that they are usually surrounded by collector roads. This kind of local community will be investigated in this study. From now on, the term 'local road network' is referred to as the local roads within a community surrounded by collector roads.

The variability of ADT across the local network reflects the movement intensity and, to some degree, the preference of travelers on local road networks. The two major factors that affect such preference are topology of road network and traffic demand distribution. Note that detailed demand information may be obtained from the metropolitan planning organization (MPO) and census data; thus, it is then the objective of this study to identify the statistical relationship between traffic flow pattern and road network properties in each local region. In order to achieve this goal, we first establish a relationship between the betweenness centrality measure and AADT values. Be aware that BC depends on the shortest paths. In a local neighborhood, the road traffic is normally low enough, and it is natural to assume people choose the shortest paths in their trips. Recall that betweenness centrality is calculated using the number of shortest paths through a link. If we incorporate the Origin-Destination (OD) matrix (in terms of daily volumes) into the calculation of betweenness centrality, then in an ideal local road network, betweenness centrality for a link will be sum of the ratio of traffic flow on a link from an OD pair to the total flow from this pair over the network. This modified betweenness centrality measure then becomes

$$g(e) = \sum_{s,t \in V} \frac{OD_{st}(e)}{OD_{st}} \tag{3-1}$$

where OD_{st} denotes the daily volume in OD matrix for OD pair $s - t$, and $OD_{st}(e)$ denotes the daily volume on link e from OD pair $s - t$. If the traffic flow from OD pair $s - t$, does not go through the link e, then $OD_{st}(e) = 0$.

The equation (3-1) can be further simplified because one attempt is to obtain VMT estimation in local community. Two assumptions are used here. First, the population density is uniformly distributed in a local neighborhood area. Accordingly, the origins of OD pairs are uniformly distributed. Second, trips generated from a local community have equal probability to all the destinations. Here, the destinations are represented by the exits of the community. Since there are only a limited number of exit points, the dimension of the OD matrix is manageable. The two assumptions give rise to a symmetric situation. Therefore, all OD_{st} can be regarded as equal. In this view, Equation (3-1) becomes

$$g(e) = \frac{\sum_{s,t \in V} OD_{st}(e)}{OD} = \frac{V(e)}{OD}, \tag{3-2}$$

where $V(e)$ represents the traffic volume on link e and OD represents the volume for each OD pair in the local community. Obviously, Equation (3-2) is a rescaled traffic volume on each link. However, the value of $g(e)$ depends on the number of OD pairs that goes through link e, which makes it hard to compare between different communities. Thus, we can further convert Equation (3-2) into Equation (3-3) by introducing the total traffic demand. In that case, $g(e)$ becomes rescaled so that its value falls within 0 and 1 and makes it possible to compare between different communities. Equation (3-3) is described as

$$g'(e) = \frac{\sum_{s,t \in V} OD_{st}(e)}{n * OD} = \frac{V(e)}{OD_{total}} \tag{3-3}$$

where $g'(e)$ is rescaled betweenness centrality, n is the number of OD pairs, and OD_{total} becomes the total traffic demand within each community.

3.4 Scale-Free Property for Local Traffic: Simulation Results

We have conducted extensive simulations to reveal the scale-free property of local traffic and to explore its possibility of estimating local road VMT. A fairly large array of neighborhood networks were generated, which are distinct from each other in terms of topology. Local trips regarding travel behaviors and population density were generated based on assumptions mentioned above. Then the modified measure of betweenness centrality was calculated according to Equation (3-3). Once the value of betweenness centrality is proved to follow power law and the associated parameters are shown to be determined by network properties, then it is possible to estimate local road VMT directly.

3.4.1 Simulation Network Settings

Though road networks exhibit numerous different shapes in reality, there are several patterns of layouts established for analysis. They represent dominating street design models for different periods in history. "American conceptions of the residential street network have changed dramatically from the interconnected rectilinear grid pattern of the turn-of-the-century, to the fragmented grid and warped parallel streets of the 1930s and 1940s and the discontinuous, insular patterns of cul-de-sacs and loops that have preferred since the 1950s" *(49)*. Figure 3 contains bird's-eye views from Google Maps, indicating different types of local road network layouts in the Houston Area. In these networks, local roads are marked by white lines and collector roads are marked by bold yellow lines. These figures support our earlier assumption that local roads are all surrounded by collector roads. Basically, these networks represent popular local community shapes in every urban area in the world.

Figure 3. Four examples of layout for local communities from the area of Houston, TX.

In order to make this study inclusive and results conclusive, we set up 30 types of local neighborhood networks for simulation. Their layouts vary in terms of the number of links, number of nodes, and the length of links, so as to represent the real-world local communities (Table 2). Figure 4 illustrates 15 examples of the local neighborhoods in our simulations, whose attributes are shown in Table 1. These configurations are also designed based on examples in Southworth and Ben-Joseph *(47)*. For simplicity and comparison purposes, we confined each local neighborhood network into a square area with the same length for side edges. All neighborhood layouts have their inner links as local roads and the surrounding links (i.e., the side edges of the square areas) as collector roads. Apart from the topology, the layouts of these communities are also different from the number of links connecting with local and collector roads.

As mentioned above, each network has the same area with a square shape. Specifically, the length of each side is 8 miles and thus the total area of each community is 64 square miles. The speed for local roads and collector roads is set at 15 mph and 40 mph, respectively. The reason for this setting is to make sure that collector roads have a higher priority for travelers than local roads. In reality, travelers do not like driving on local roads and prefer higher class roads because a collector road has higher speed and less resistance. As Figure 4 shows, road links are those segments between nodes. In order to reduce simulation errors, those extremely long links are split into multiple road segments.

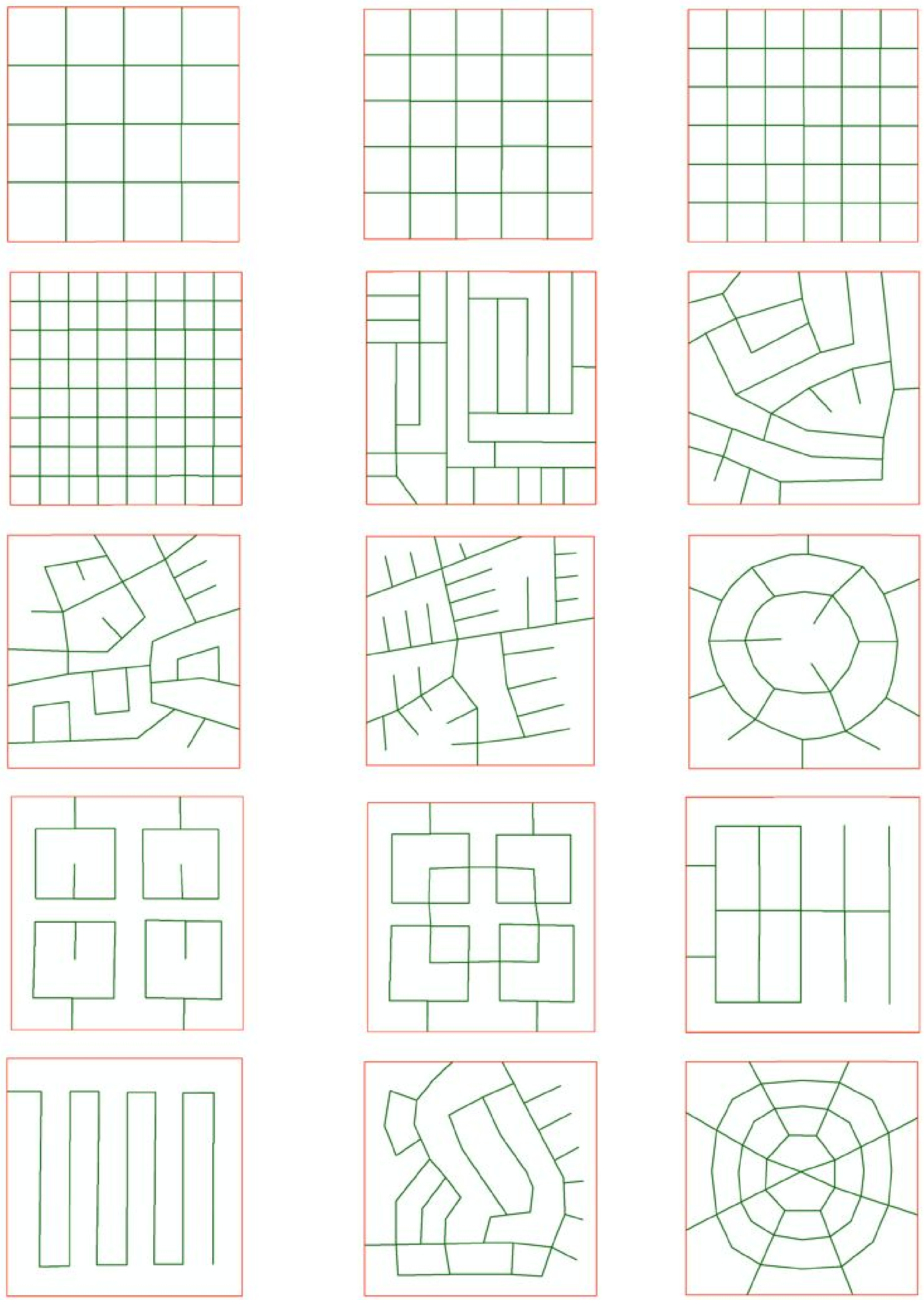


Figure 4. Layouts of 30 simulation networks

Table 2. Characteristics of simulation networks

ID	Edge	Node	Average Degree	Total Length	Avg_Link Length	Link/Node
1	24	9	5.33	47.99	2.00	2.67
2	40	16	5	63.98	1.60	2.50
3	60	25	4.8	79.98	1.33	2.40
4	112	49	4.57	111.97	1.00	2.29
5	55	33	3.33	75.77	1.38	1.67
6	45	30	3	62.46	1.39	1.50
7	47	33	2.85	63.46	1.35	1.42
8	59	51	2.31	63.92	1.08	1.16
9	35	23	3.04	52.80	1.51	1.52
10	40	36	2.22	50.86	1.27	1.11
11	48	36	2.67	59.94	1.25	1.33
12	31	25	2.48	46.12	1.49	1.24
13	28	28	2	47.82	1.71	1.00
14	50	39	2.56	59.20	1.18	1.28
15	68	41	3.32	73.17	1.08	1.66
16	88	41	4.29	88.11	1.00	2.15
17	96	41	4.68	96.04	1.00	2.34
18	104	41	5.07	104.01	1.00	2.54
19	105	41	5.12	105.01	1.00	2.56
20	98	41	4.78	97.94	1.00	2.39
21	91	41	4.44	91.16	1.00	2.22
22	89	41	4.34	89.16	1.00	2.17
23	98	41	4.78	98.11	1.00	2.39
24	96	41	4.68	87.16	0.91	2.34
25	85	41	4.15	85.16	1.00	2.07
26	94	41	4.59	94.13	1.00	2.29
27	90	41	4.39	90.13	1.00	2.20
28	86	41	4.2	86.14	1.00	2.10
29	92	41	4.49	92.02	1.00	2.24
30	77	41	3.76	76.97	1.00	1.88

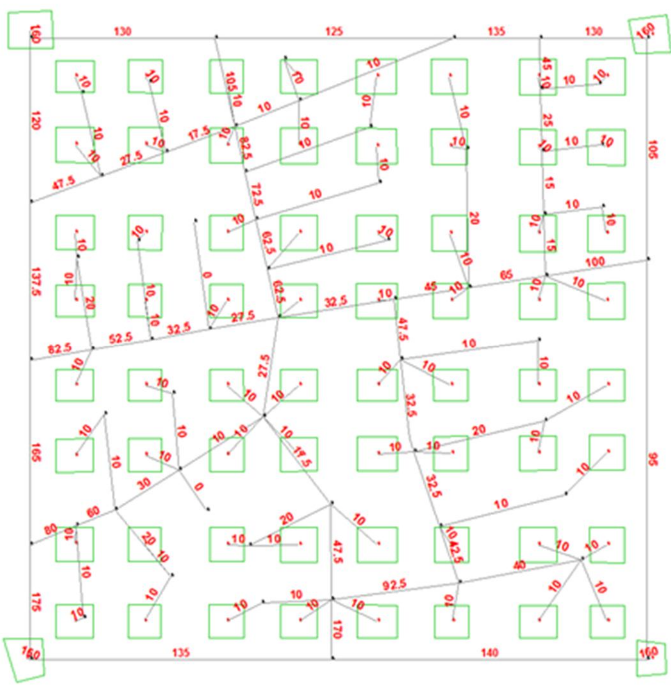
3.4.2 Trip Generation

Since the simulation involves traffic demand analysis, it is necessary to clarify influencing factors. Briefly speaking, two elements must be considered: land use mix and activity intensity. First, traffic is derived from land use, as land use determines where people live, work, and shop. In our simulation, we avoid land use mix by assuming all traffic originates from the local network community and goes outside. Otherwise, the existence of land use mix will cause traffic to be absorbed internally and it will be impossible to analyze network property individually. Second, activity intensity, or travel demand intensity, determines the total number of trips between each OD pair. Intuitively, traffic flow will change proportionally with travel demand. So travel demand is set equal for each simulation network.

In our simulation setting coded in TransCAD software (version 4.5), there are 64 traffic analysis zones (TAZs) that represent the traffic demand sources, and all travelers originate from the center of each TAZ. A certain number of TAZs (from one to four) at the corners of each community are constructed to make travelers go along collector roads through those corners, so as to reach other places. The volumes attracted by these four TAZs are equal. As mentioned above, we assume that traffic demand is uniformly distributed within each community. It is important to note that this assumption is reasonable for general local communities and serves as a comparison purpose only. It does not affect the scale-free property of the local traffic. We set the total trip production for each community as 640 demand units, and the trips attracted by each corner TAZs as 160 demand units. Such setup enables us to generate the trips similar to the scale of observations in the available data from a Texas MPO. However, one can rescale the demand to any real situations according to the local monitoring data. Figure 5 (a) shows the 64 TAZs (in blue) within the layout of example from Figure 4.

The volumes on links of each local network are obtained through simulation output. As the daily volume on each link in a local neighborhood network is usually low, we assume that no congestion occurred on each link. Therefore, the simulation is conducted based on an all-or-nothing approach that would assign travelers to the shortest path in terms of travel time, which is consistent with the observations in the local neighborhood. Figure 5 (b) shows the resulting traffic volume on each link and in each TAZ for the example (with four TAZs at corners) in Figure 5 (a). One thing to note is that though centroid connectors are loaded with traffic, they are not counted as local roads.

(a)



(b)

Figure 5. Example for traffic demand settings: (a) 64 TAZs (in blue) within an example network; (b) Resulting traffic volume on each link and in each TAZ.

3.4.3 Scale-Free Property: Two-Piecewise Power Law Distributions

As explained already, traffic flows can be expressed with betweenness centrality. In our case, the betweenness centrality for each link is calculated according to Equation (3-3), based on the results from simulation for each local community. Then, all the values of betweenness centrality are sorted from the largest to the smallest, and the associated links are assigned a number (starting from 1) to represent their rank in the sorted results. In order to examine Zipf's law for traffic patterns, the logarithmic values are used to find the relationship between the rank and betweenness centrality. We denote the logarithmic value of each link rank by r, and the logarithmic value of betweenness centrality by y. The results show that the link betweenness centrality exhibits two piecewise power law distributions, i.e.,

$$y = \gamma_i - \beta_i r \tag{3-4}$$

where γ_i and β_i are constants, $i = 1$ for $r \leq b$ and $i = 2$ for $r > b$. The value of r starts from 1 to the total number of links in the network. We call the value of y at $r = b$ as the breakpoint between two pieces. The first piece includes the first 20% of the links and the second piece involves the other 80% of the links. The linear regression result for r and y shows a significant high R-square value for each piece.

For illustrative purposes, Figure 7 shows the results of six selected local communities respectively, which are shown in Figure 6. Results for the rest of the networks are attached in Appendix 1. Note that the breakpoints between two pieces of power law distributions may vary for different topological communities, because the total number of links vary. However, as we require r to start from 1, the point with the largest value of y always lies on the y-axis. Obviously, the relatively flat top part (red regression line) represents the links with the higher betweenness centrality, and the second part (blue regression line) represents links with lower betweenness centrality. We will further utilize this feature to estimate local road VMT.

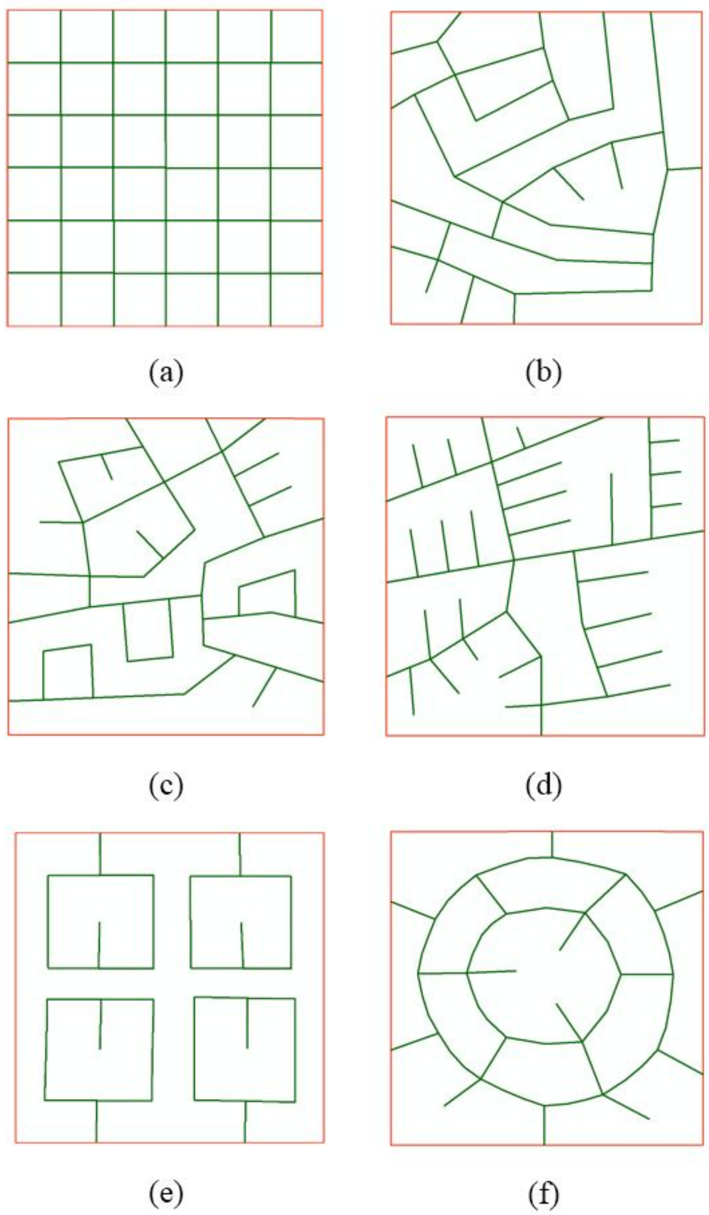
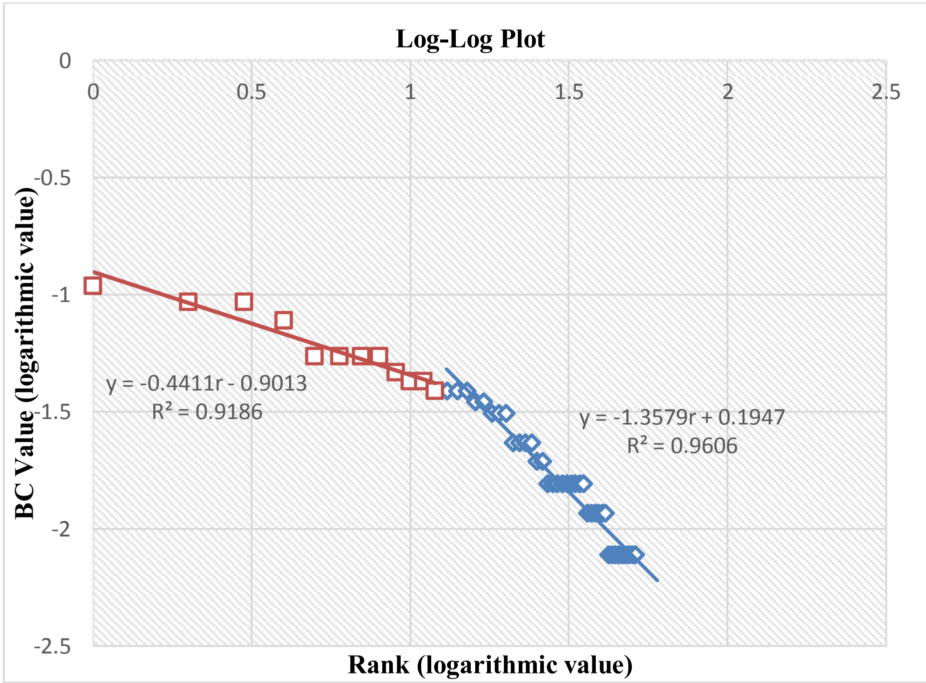
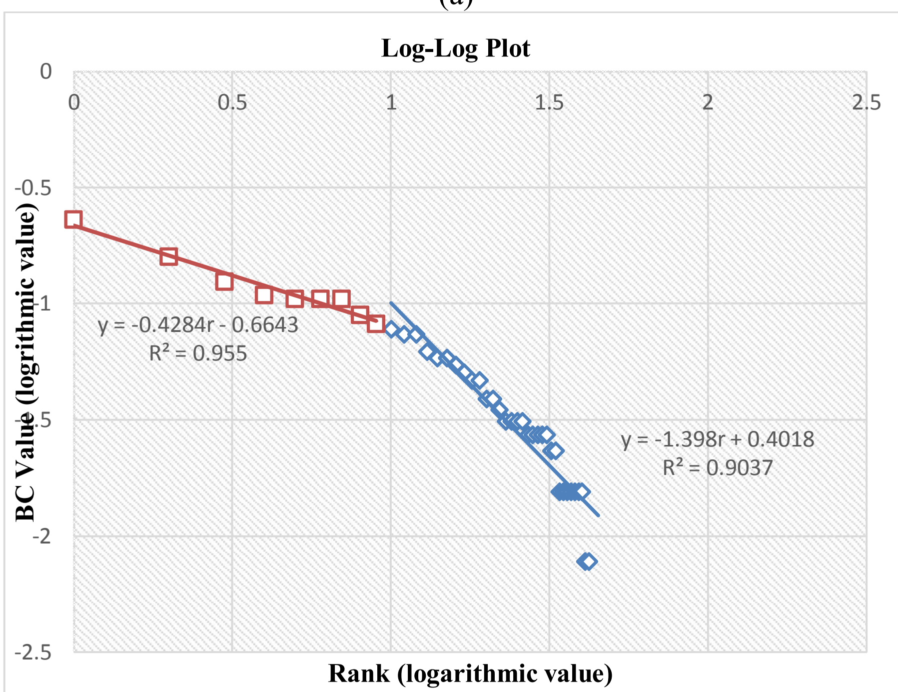


Figure 6. Selected 6 examples for typical layouts of 30 simulation networks.

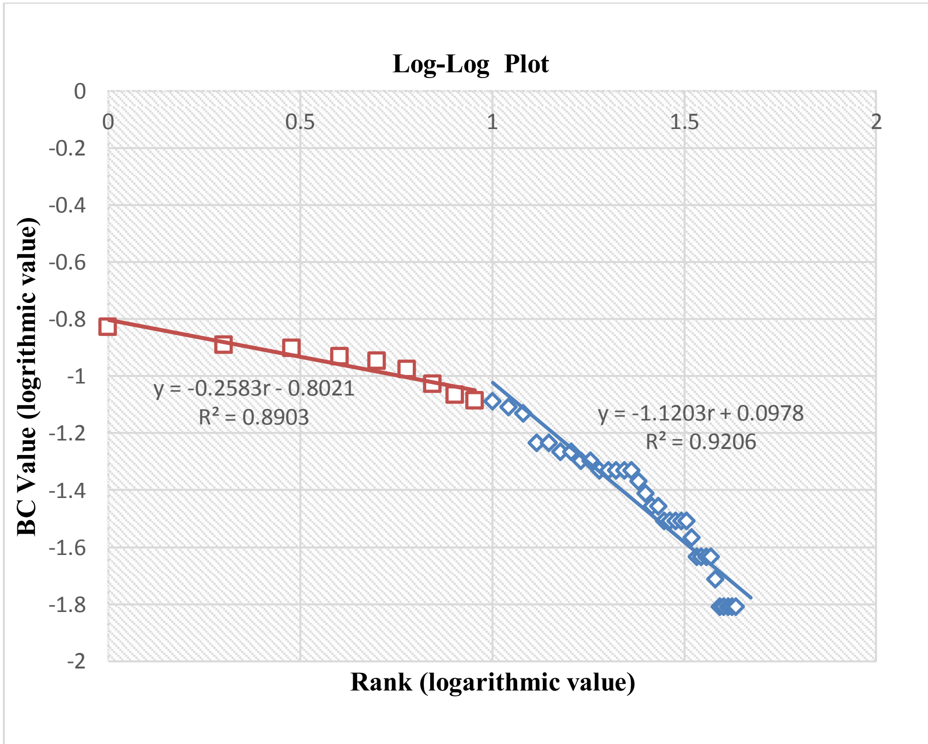


(a)

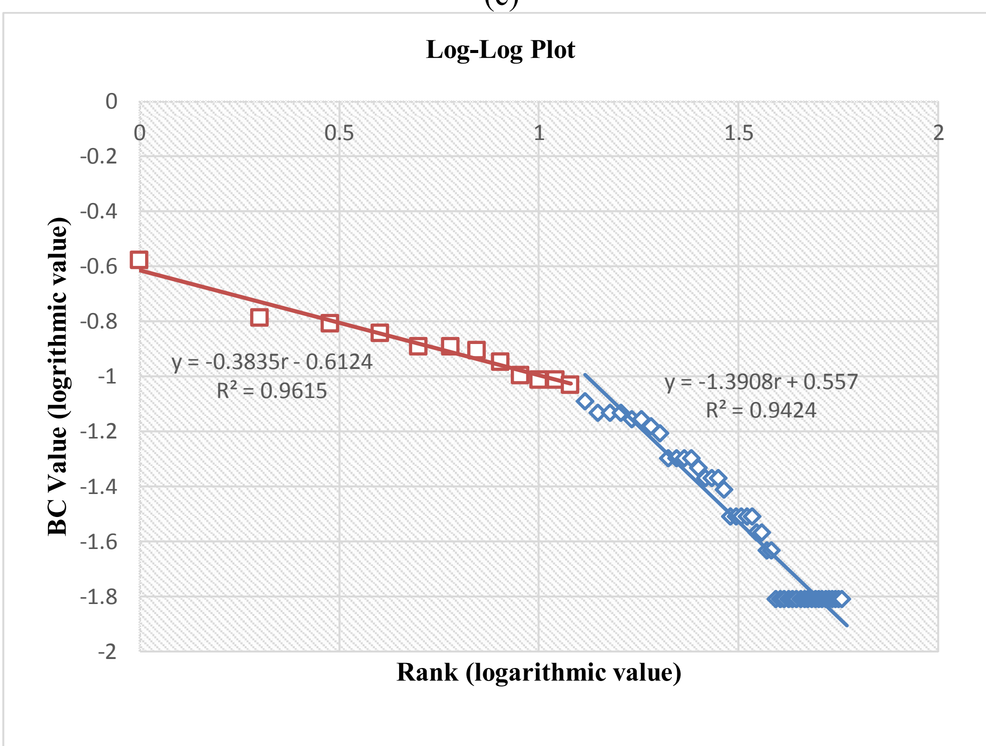


(b)

Figure 7. Betweenness centrality exhibits two piecewise power law distributions for the local communities in Figure 6, respectively. The logarithmic value of the link rank (Rank) by r, and the logarithmic value of betweenness centrality (BC value) by y. The first piece (red) includes 20% total links and the second (blue) involves 80% links.

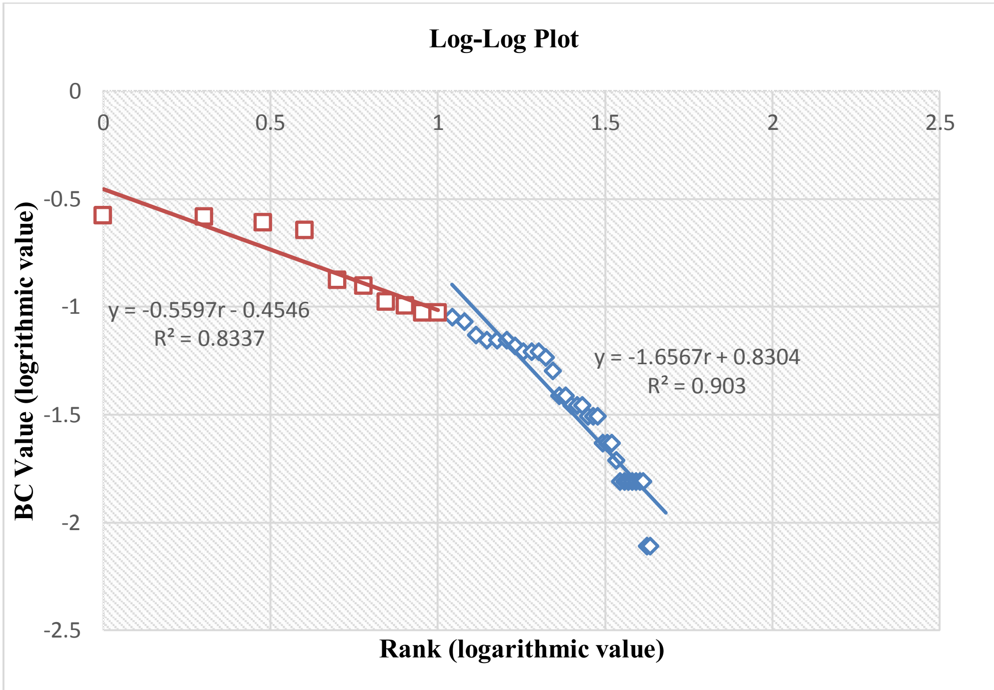


(c)

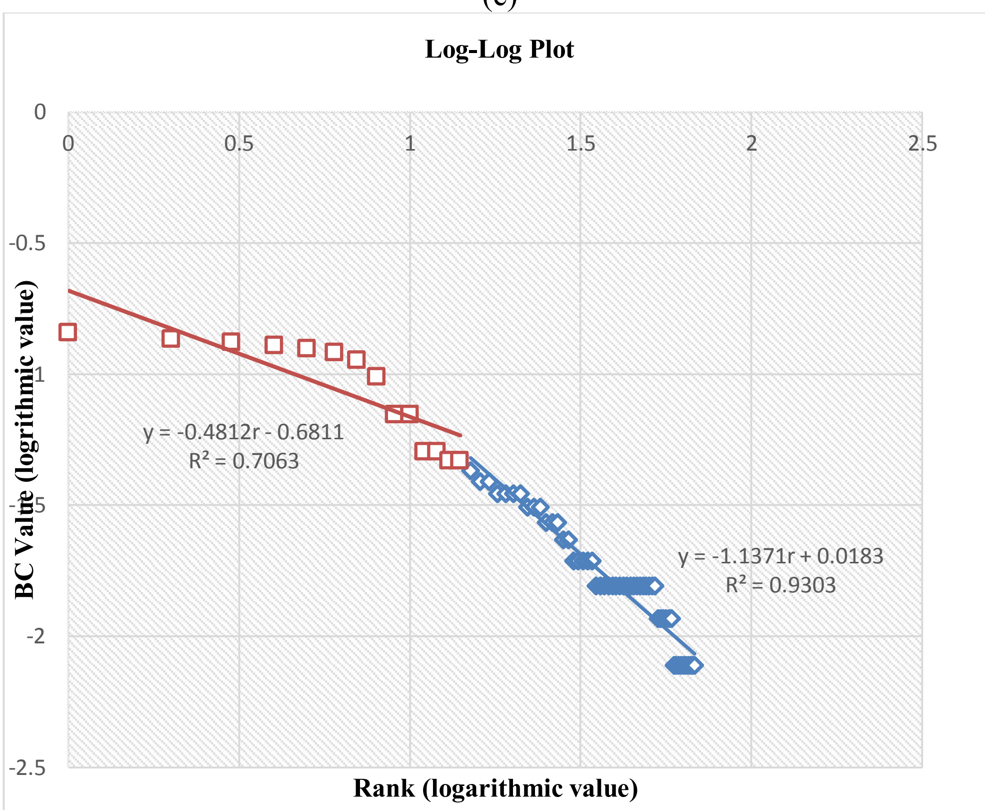


(d)

Figure 7. continued



(e)



(f)

Figure 7. continued

CHAPTER IV

LOCAL VMT ESTIMATION BASED ON SCALE-FREE PROPERTY

As we illustrated above, the converted betweenness centrality versus rank plots do not follow a straight line. Instead, they can be partitioned into two parts – the top 20% part and the bottom 80% part. Each part can be approximately overlaid by a straight line. This phenomenon illustrates deviation from traditional power law distribution, which only has one line. In fact, this kind of double exponent power law distribution has been revealed by other researchers *(31)*, but the reason behind it is still unclear. However, we can still take advantage of its characteristics to estimate local road VMT.

4.1 Illustrative Framework

For illustrative purposes, we assume that the ADT values are given on the links with the first 20% rank, and the slope β_2 (in Equation (3-4)) as well as the breakpoint of power law distributions for the remaining 80% links are also known. Our objective is to predict VMT on the remaining 80% roads so as to obtain the VMT over the entire local neighborhood, based on the above known values. However, we will further demonstrate that the actual results are not quite sensitive to the value of β_2 , and the breakpoint between two power law distributions can be predicted by the related topological measures without observed traffic count data. Therefore, our framework suggests an applicable way for VMT estimation that we will demonstrate later.

Table 3 shows the results for VMT estimation on 30 local networks based on the slope β_2 and the breakpoint. In the table, ID denotes the number of network in simulation.

We use the simulated volume from multiple runs on each link to represent ADT. Hence, the actual total local VMT can be obtained by summing up all the products of the link length and its associated ADT. The average link length is computed given the profile of the local network, and the value is scaled relative to the unit length in the simulation. The ADTs for the links with lower 80% ranks are predicted by using the Equation (3-3), where the betweenness centrality is viewed as the rescaled ADT. We remark here that in order to estimate 80% road ADTs, one only needs to obtain the total number of links over the entire local networks, since one does not need to specify the value y in Equation (3-4) on a particular road. Then, the estimated VMT is computed by multiplying the estimated ADT and average link length. We can see from the last column (Diff %) in Table 3 that most estimates of VMT are accurate within the 15% gap. Only 3 networks show the errors larger than 20%. The results imply that the VMT can be estimated precisely, based on the power law.

Table 3. Results for VMT estimation on 30 local networks based on the slope $\boldsymbol{\beta_2}$ and the known breakpoint.

ID	Actual Total Local VMT	Avg link Length	β_2	Estimated VMT (80%)	Actual VMT (20%)	Estimated Total VMT	Diff. (%)
1	1596.90	2.00	1.1007	785.92	656.18	1442.11	9.69
2	1468.07	1.60	1.248	755.60	636.39	1391.99	5.18
3	1252.86	1.33	1.3579	529.28	651.16	1180.43	5.78
4	1044.92	1.00	1.7478	437.47	719.07	1156.54	10.68
5	1537.15	1.38	1.6482	544.60	712.30	1256.90	18.23
6	1705.91	1.39	1.398	825.14	756.36	1581.49	7.29
7	1875.50	1.35	1.1203	1043.30	805.84	1849.14	1.41
8	2006.39	1.08	1.3908	876.82	932.73	1809.55	9.81
9	1933.45	1.51	1.6388	649.99	805.99	1455.99	24.69
10	2256.31	1.27	1.508	761.33	1181.36	1942.68	13.90
11	2295.96	1.25	1.6567	785.58	1262.79	2048.37	10.78
12	3994.24	1.49	2.5578	1257.79	2705.87	3963.65	0.77
13	14608.41	1.71	1.145	6979.37	4504.31	11483.68	21.39
14	2613.75	1.18	1.5134	790.54	1549.29	2339.83	10.48
15	1596.70	1.08	1.1371	623.58	897.10	1520.68	4.76
16	1908.87	1.00	1.87	499.27	1290.71	1789.97	6.23
17	1388.79	1.00	1.6877	493.62	842.94	1336.57	3.76
18	1199.35	1.00	1.7808	508.43	714.47	1222.90	1.96
19	1313.84	1.00	2.0801	428.31	889.59	1317.91	0.31
20	1832.15	1.00	2.4069	411.93	1315.32	1727.25	5.73
21	2462.78	1.00	1.8211	1038.75	1821.17	2859.92	16.13
22	2778.49	1.00	1.494	1267.18	1858.39	3125.57	12.49
23	1559.06	1.00	1.2372	1108.37	1049.20	2157.58	38.39
24	3100.33	0.91	1.4737	1118.91	2100.33	3219.24	3.84
25	3661.29	1.00	1.4398	1250.90	1959.18	3210.08	12.32
26	1876.07	1.00	1.4482	916.47	1065.64	1982.12	5.65
27	2196.07	1.00	1.3567	931.20	1335.21	2266.41	3.20
28	2756.47	1.00	1.4694	1026.71	1834.33	2861.04	3.79
29	1142.77	1.00	1.6866	376.98	744.22	1121.20	1.89
30	1350.51	1.00	1.4254	566.65	749.56	1316.21	2.54

Note: $Diff\% = \frac{|Estimated\ VMT - Actual\ VMT|}{Actual\ VMT}$

4.2 Sensitivity of Slope in VMT Estimation

Now we will show the results in Table 3 are not sensitive to the value of β_2 . By analyzing the distribution of all beta values, we find that the mean value of β_2 is -1.561536667 with the 95% confidence interval between -1.4327 and -1.6904. Assuming breakpoint is known, we utilize the linear relationship, i.e., Equation (3-4), and -1.56 for β_2 to estimate the local VMT. Table 4 shows how results change compared to Table 3 and Figure 8 illustrates the comparison between estimated results and actual VMT. It is obvious that the results by using the average value of β_2 are close to the actual results with an average error 12%. The largest error occurs at the network with ID 13, which has extreme conditions (only one exit to collector road).

Table 4. Results for VMT estimation on 30 local networks based on the average slope and the known breakpoint.

ID	Actual Total Local VMT	Avg link Length	β_2	Estimated VMT (80%)	Actual VMT (20%)	Estimated Total VMT	Diff. (%)
1	1596.90	2.00	1.56	578.08	656.18	1234.26	22.71
2	1468.07	1.60	1.56	617.67	636.39	1254.06	14.58
3	1252.86	1.33	1.56	463.16	651.16	1114.31	11.06
4	1044.92	1.00	1.56	493.24	719.07	1212.31	16.02
5	1537.15	1.38	1.56	574.76	712.30	1287.06	16.27
6	1705.91	1.39	1.56	743.68	756.36	1500.03	12.07
7	1875.50	1.35	1.56	772.49	805.84	1578.33	15.84
8	2006.39	1.08	1.56	785.30	932.73	1718.02	14.37
9	1933.45	1.51	1.56	680.58	805.99	1486.57	23.11
10	2256.31	1.27	1.56	736.58	1181.36	1917.94	15.00
11	2295.96	1.25	1.56	831.91	1262.79	2094.70	8.77
12	3994.24	1.49	1.56	2072.32	2705.87	4778.18	19.63
13	14608.41	1.71	1.56	5304.68	4504.31	9808.99	32.85
14	2613.75	1.18	1.56	766.92	1549.29	2316.21	11.38
15	1596.70	1.08	1.56	468.63	897.10	1365.73	14.47

Table 4. continued

ID	Actual Total Local VMT	Avg link Length	β_2	Estimated VMT (80%)	Actual VMT (20%)	Estimated Total VMT	Diff. (%)
16	1908.87	1.00	1.56	603.22	1290.71	1893.93	0.78
17	1388.79	1.00	1.56	535.38	842.94	1378.32	0.75
18	1199.35	1.00	1.56	583.82	714.47	1298.29	8.25
19	1313.84	1.00	1.56	586.05	889.59	1475.64	12.31
20	1832.15	1.00	1.56	665.68	1315.32	1980.99	8.12
21	2462.78	1.00	1.56	1222.87	1821.17	3044.04	23.60
22	2778.49	1.00	1.56	1212.45	1858.39	3070.84	10.52
23	1559.06	1.00	1.56	889.14	1049.20	1938.35	24.33
24	3100.33	0.91	1.56	1055.30	2100.33	3155.63	1.78
25	3661.29	1.00	1.56	1154.14	1959.18	3113.32	14.97
26	1876.07	1.00	1.56	850.39	1065.64	1916.03	2.13
27	2196.07	1.00	1.56	811.49	1335.21	2146.70	2.25
28	2756.47	1.00	1.56	965.90	1834.33	2800.24	1.59
29	1142.77	1.00	1.56	408.69	744.22	1152.91	0.89
30	1350.51	1.00	1.56	517.40	749.56	1266.96	6.19

Note: $Diff\% = \frac{|Estimated\ VMT - Actual\ VMT|}{Actual\ VMT}$

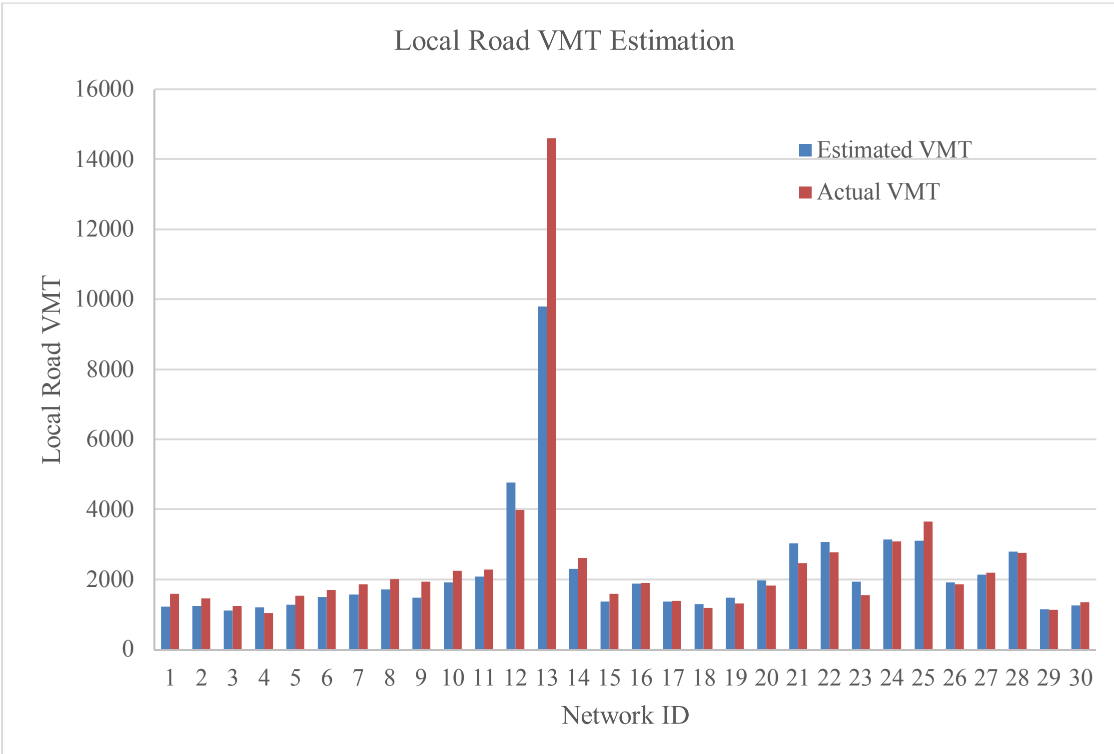


Figure 8. VMT estimation by assuming the average slope ($\boldsymbol{\beta_2}$) for the second power law distribution. The average error is as low as 12%.

4.3 The Breakpoint Prediction

Since breakpoint plays a critical role in power law distribution, now we will show the breakpoint (i.e., the value of betweenness centrality at $r = b$ in Equation (3-4)) can be predicted by a certain number of network measures. We tried to use common linear regression to estimate betweenness centrality at breakpoint. The form is provided as follows.

$$BC = \beta_0 + \beta_1 X_1 + \cdots\cdots + \beta_i X_i + \varepsilon, \tag{4-1}$$

where X_i represents an independent variable, which is selected network property measurement. By trial and error, the independent variables chosen in our regression model include:

1. Compactness

The index has been introduced before. It measures how a certain area is filled with roads and is expressed as below:

$$\varphi = 1 - \frac{4A}{(l_T - 2\sqrt{A})^2} \tag{4-2}$$

where A is the area of a city or community and l_T the total length of roads.

2. Accessible Sides

This index refers to how many side of the community are connected by local road networks. That said, if each side of a surrounding collector road square is accessible to local roads, then the index is 4. Obviously, the value varies from 1 to 4.

3. Completeness Index

This index is a measurement for degree distribution since, in planar networks, the degree distribution is peaked around 3 or 4. We proposed the index as

$$r_N = \frac{N(3)+N(4)}{\sum_{k=1,2,3,4} N(k)} \tag{4-3}$$

where $N(k)$ means the number of nodes with the degree of k. $N(4)$ and $N(3)$ express the number of four-leg and T intersections, which can be considered complete crossings compared to cul-de-sacs. If r_N is large, that means the regular crossings are dominant and the network is well organized.

We obtained a linear regression with R-squared value of 0.8171. The form is given as follows:

$$BC = \frac{(-1.7485 - 8.9102 * r_N * \frac{1}{L} + 31.2834 * \frac{1}{L} + 1.6943 * \varphi)}{S} \quad (4\text{-}4)$$

where

BC = breakpoint;

r_N = completeness index;

L = total number of links;

φ = compactness;

S = number of accessible sides.

Figure 9 shows the closeness between the predicted values from regression Equation (4-4) and the actual values. The prediction of breakpoint based on the network measures has both theoretical and practical meanings. From the theoretical perspective, it indicates the breakpoint as an inherent feature for the network with a certain topology. From the practical point of view, it enables the proposed VMT estimation to be independent from field observations and readily applicable.

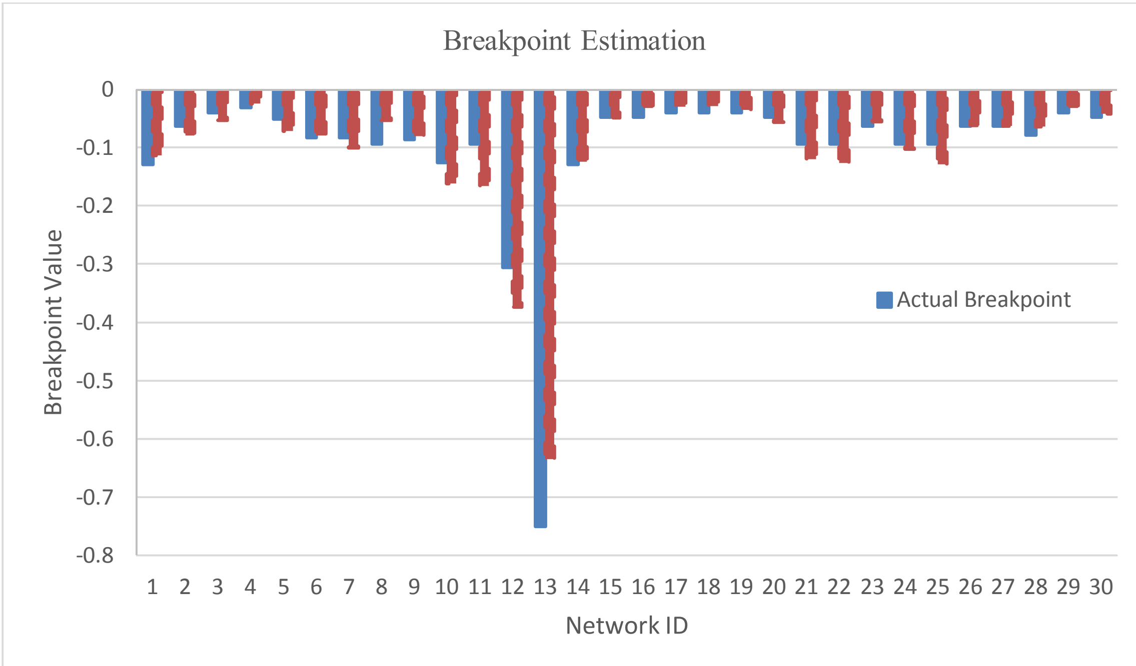


Figure 9. The prediction of breakpoint by the measures of network topologies for each network.

4.4 Highest Betweenness Centrality Prediction

In this study, we prove that the highest betweenness centrality can also be predicted according to network properties. As Equation (3-4) shows, the betweenness centrality distribution includes two parts. Due to the fact that we can reliably predict a breakpoint, and that the slope of the second part can be proved to be insensitive to network layouts, we state that the second part of betweenness centrality distribution has been solved. Now we focus on the prediction of the first part of the distribution. Since the slope of that part is determined by the highest point and breakpoint, the remaining work is about accurately predicting the highest point which indicates the link with the highest traffic volume. Here we choose to use the linear regression model as well. The newly incorporated variables include:

1. Connectivity

This index has been widely used in the transportation planning area. Here, it is measured as the number of edges divided by the nodes. The nodes include traditional intersections, cul-de-sacs, and any node connecting two street links. The higher the connectivity index is, the more connected the road network will be. A reasonable connectivity index is considered to be at least 1.4.

2. Total Number of Exits

This index refers to the total number of exits connecting local roads to surrounding collector roads. The reason we resemble those links as exits or outlets is because we assume all traffic originates from inside the local road community and gets outside through those exits. It is obvious that the more exits there are, the less distance drivers travel.

3. Average Number of Exits

This index is calculated by simply dividing the total number of exits by the number of sides, which has been covered above. This index can indicate whether those exits are scattered compactly or loosely.

By trial and error, based on the highest R squared value of 0.8014, the final model selected is written as follows:

$$\mathrm{BC} = \frac{(0.0238 + 1.334 * \frac{1}{\alpha} + 0.3845 * log_{10} * \beta - 0.1145 * \gamma)}{s} \tag{4-5}$$

where

α = average number of exits;

β = total number of exits;

γ = connectivity index;

S = number of accessible sides.

Figure 10 illustrates how the estimated highest betweenness centrality is compared to its actual value. The results prove that the highest BC point is also dependent upon network property measures. We can easily find out that the more exits there are, the higher the BC value will be. In the extreme case where there is only one exit, of course the highest BC value is 1, meaning all traffic has to go through that link. With the highest point and breakpoint predictable, we get a better picture of traffic patterns on local roads. It also means that it is now possible to estimate VMT without knowing traffic count data.

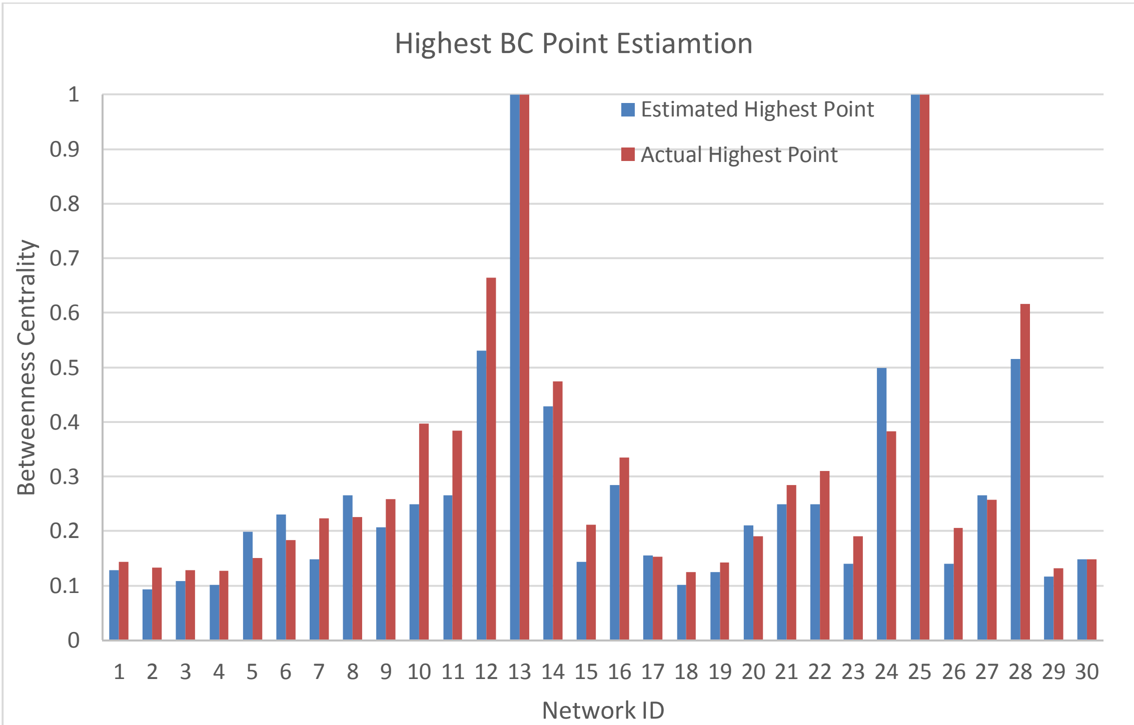


Figure 10. The prediction of highest betweenness centrality by the measures of network topologies for each network.

CHAPTER V

VMT ESTIMATION PROCEDURES AND VALIDATION

Considering the results so far, predicting VMT according to betweenness centrality distribution seems viable. In practice, if our assumption stands, traffic distribution on local road networks should follow the pattern as illustrated above. Therefore, if one can estimate the total trips generated from a local community, one can estimate the rescaled breakpoint and highest BC point (in terms of modified betweenness centrality) by Equation (3-4) and draw a line to approximately estimate ADTs on 20% of links which have higher traffic on them. Then the breakpoint serves as the highest value (betweenness centrality) for the remaining 80% of local roads. The ADTs on 80% of local roads can be easily obtained by the average β_2 value and the given total number of roads in the local network.

We remark here that the value β_1 can be obtained by the breakpoint and the largest betweenness centrality (rescaled ADT). To summarize, we propose a general procedure for local road VMT estimation in the following section.

5.1 Estimating Procedures

The proposed procedures include 5 steps, as listed as follows.

Assumption: The trips are generated uniformly in the local network.

Step 1: Collect information about local communities including demographic data and local road network topological measures;

Step 2: Given total trips generated in the local network, estimate the breakpoint and highest BC value by using Equations (3-3) and (3-4);

Step 3: Estimate the value β_1 based on the highest BC value (rescaled) and the breakpoint, and estimate ADTs on the 20% of local roads;

Step 4: Obtain ADTs on the remaining 80% of local roads based on a constant β_2 value and the given total number of roads in the local network;

Step 5: The total VMT is computed by multiplying the estimated total ADT on the local network and the average link length.

The procedures above are based on the simulation results, which illustrate that betweenness centrality follows two-piecewise power-law distribution among most local road networks. In order to validate this method and involved regression models, more simulations are necessary.

5.2 Validation

In this section, our proposed models for VMT prediction are checked for validity. The model validation was conducted through 10 more simulations. The simulations are based on 10 newly-set networks. The 10 new networks have different layouts and are different from the previously-set 30 networks (Figure 11). However, settings for simulation remain the same. The results are used to evaluate the capability of the proposed model. The predicted values of VMT are calculated by following steps in the procedures above. The estimated and actual values of the VMT are provided in Table 5. The differences in percentage between them are also presented. We can observe that the average difference is around 15%. Even the highest difference is below 30%. In order to provide a closer look at the results, the comparison of the two values is shown in Figure

12. From the table and figure we can see that the results are not seriously biased. The results indicate that the model is able to estimate VMT reliably.

One thing that is worth noting is about the cause of errors. From the results we find that the significance of VMT estimation errors largely depends on the accuracy of breakpoint prediction. As long as the breakpoint is predicted accurately, the errors of VMT estimation will be much reduced.

Table 5. Validation results of proposed VMT estimating procedures.

ID	Actual VMT	Estimated VMT	Difference (%)
1	1475.975	1477.93133	0.132545
2	2549.425	2208.70251	13.36468
3	1914.25	1691.17054	11.65362
4	2060.45	1528.84734	25.80032
5	2032.475	1583.83419	22.07362
6	1892.325	1634.63112	13.61784
7	2290.45	2292.12712	0.073222
8	2245.925	1738.08174	22.61176
9	1954.5	1435.05817	26.57671
10	1873.075	1624.70974	13.25976

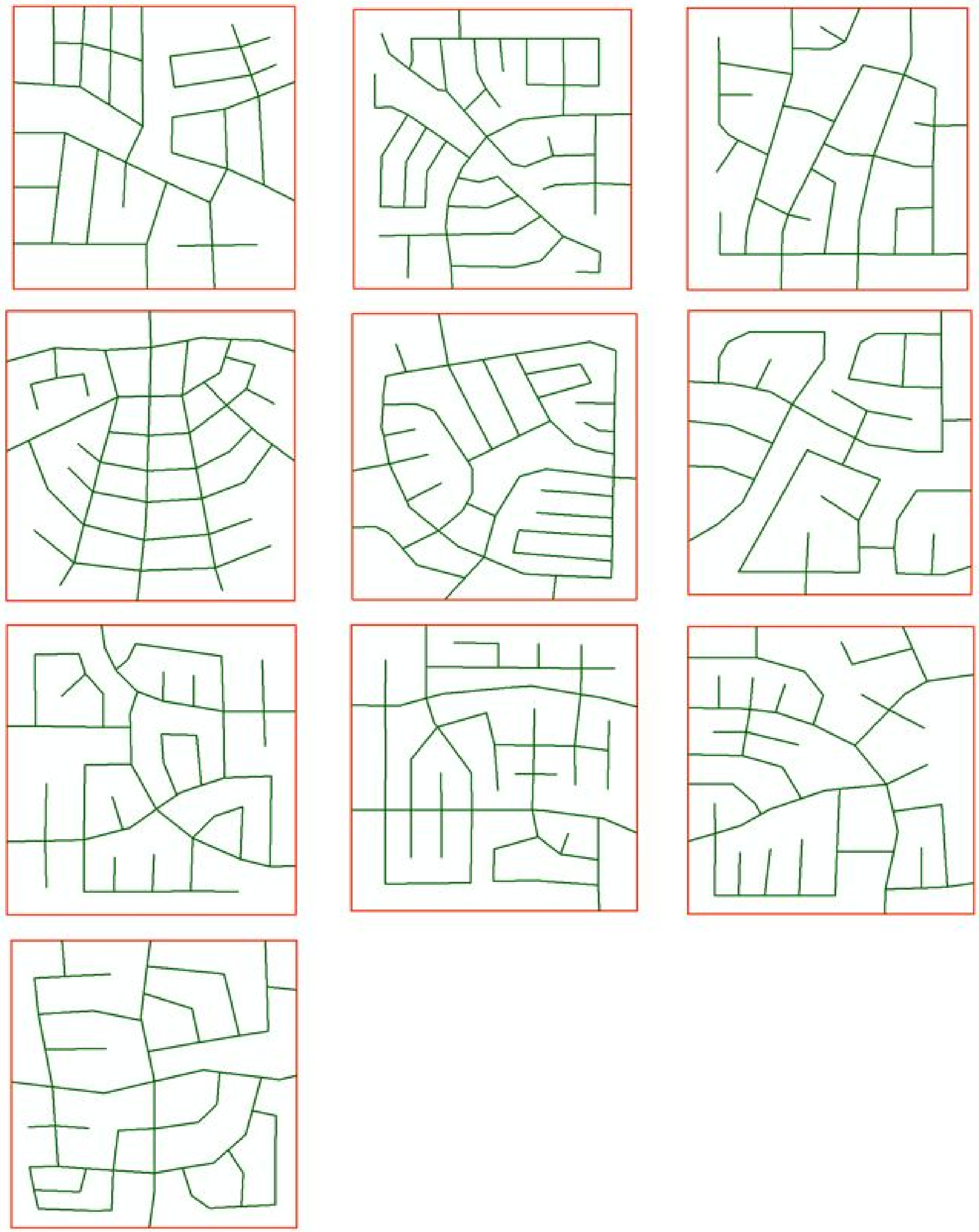


Figure 11. Layouts of 10 validation networks

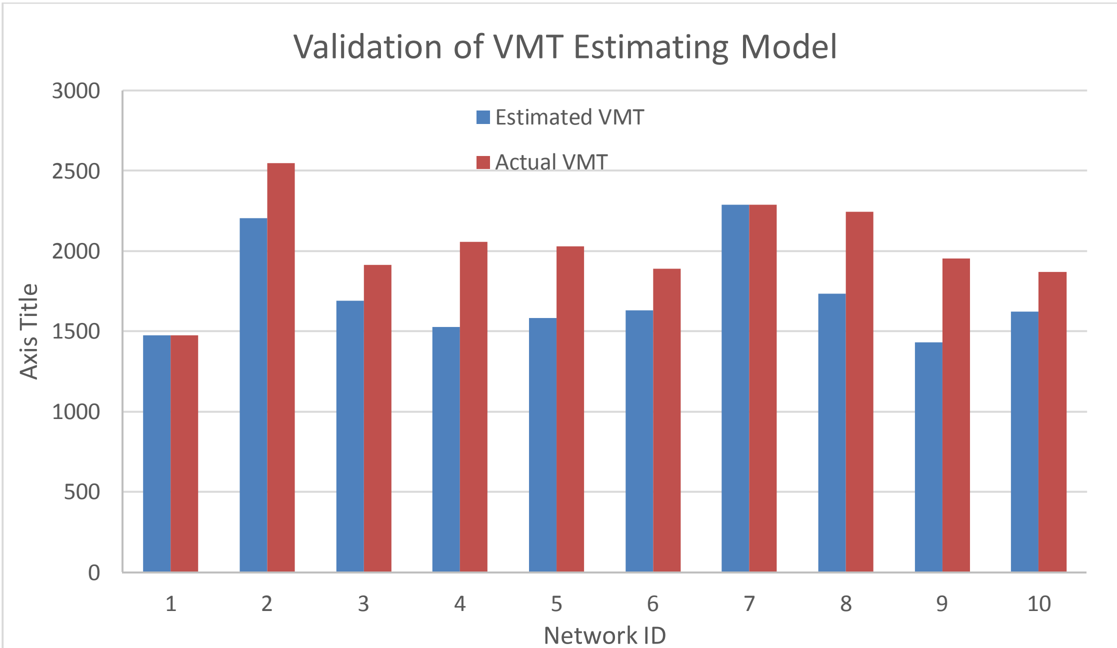


Figure 12. Validation results of VMT estimating model

CHAPTER VI

CONCLUSIONS AND FUTURE WORK

This study develops a close collaboration with experimental observation of and application to local road VMT estimation. We have demonstrated that the trips, in terms of betweenness centrality (rescaled ADTs) on the local networks, exhibit two piece-wise power law distributions – scale-free property. Once the total travel demand within a local area is known, the total VMT can be obtained by the knowledge of distribution of betweenness centrality. We have also showed that the breakpoint and highest point can be predicted from a certain number of network measures, and that the estimated VMT is not sensitive to the slope value of the falling part of power law distributions. These facts enable the proposed VMT estimation to be independent from field observations and readily applicable.

Furthermore, our findings suggest a promising approach in practice for local road VMT estimation: the transportation agencies can solely rely on demographic and geographic information, which are much more readily available than field traffic count data. A certain number of network properties can help determine the parameters in two piecewise power law distributions for rescaled ADTs. Then, the agency can use scale-free property of the volumes to obtain the VMT estimation over the entire local roads.

There are still several unsolved problems in our general framework. For example, we find that in some typical networks (e.g. a network with just one road, dominating branch road, or ring road in it) the power-law distribution does not hold perfectly. Besides, we are unclear if the model can also be applied in local road communities without uniformly distributed traffic demand. For example, in rural area, traffic generations will be sparsely scattered within a large area. In that case, the assumption of uniform traffic demand distribution may not hold anymore. This fact indicates one of the possible limitations of our proposed method. Further work is required to analyze such scenario. But considering that traffic demand in rural area is much lighter than that in urban area, the errors may not be significant. Moreover, we need to test if the results will change when the speed settings are different from those in our simulation. Last, but not the least, the results should be compared to actual traffic count data in similar scenarios. Only by doing this can we truly prove the feasibility and practicableness of our proposed method. In addition, some steps in our research process demand further improvements. For example, we may try to choose nonlinear curves to fit the power law distribution. Also the selection of test network is somewhat arbitrary. The more reasonable method is to categorize various types of layouts into groups according to topological measures. Then representative networks layouts are analyzed in order to obtain more reliable results. These problems will be dealt with in our future work. Ultimately, we hope our findings here could expand VMT estimation and theoretical inquiry in new directions.

REFERENCES

1. Puentes, R., and A. Tomer. "The road less traveled: an analysis of vehicle miles traveled trends." U.S. Brookings Institution. http://www.brookings.edu/~/media/research/files/blogs/2011/3/03%20transportation%20puentes%20tomer/vehicle_miles_traveled_report.pdf , Accessed Jul. 24, 2013.

2. FHWA. "Highway performance monitoring system field manual." U.S. Department of Transportation, Feb. 1995.

3. R. K. Kumapley and J. D. Fricker. "Review of methods for estimating vehicle miles traveled", *Transportation Research Record*, vol. 1551, p.59-66, 1996.

4. Aikaterini Rentziou, Konstantina Gkritza, Reginald R. Souleyrette. "VMT, energy consumption, and GHG emissions forecasting for passenger transportation." *Transportation Research Part A: Policy and Practice*, Volume 46, Issue 3, March 2012, p.487–500.

5. Fort Collins LUTRAQ Team. "Estimation of VMT and VMT growth rate." http://www.fcgov.com/climateprotection/pdf/ctf-res-lutraq.pdf , Accessed June 2001.

6. Cambridge Systematics, Inc.. "TxLED VMT estimation project." http://www.tceq.texas.gov/assets/public/implementation/air/am/contracts/reports/mob/TxLED_VMT_Estimation-csi.pdf , 2004.

7. Stone, Brian; Obermann, William; Snyder, Stephanie. "A new method for estimating and projecting vehicle miles of travel − linkages to landscape change and ozone impacts to northern forests." *Gen. Tech. Rep. NC-253.* St. Paul, U.S. Department of Agriculture, Forest Service, North Central Research Station. p. 24, 2005.

8. Oak Ridge National Laboratory. "Developing a best estimate of annual vehicle mileage for 2009 NHTS vehicles." FHWA, http://nhts.ornl.gov/2009/pub/BESTMILE.pdf , 2011.

9. J. Fricker and R. Kumapley. “Updating procedure to estimate and forecast vehicle miles traveled.” *Joint Transportation Research Program*, Final report, project No: C-36-17DDD, File No: 8-4-56, 2002.

10. Zhong, Ming, and Brody Hanson. “GIS-based travel demand modeling for estimating traffic on low-class roads.” *Transportation Planning and Technology,* 32.5 (2009): 423-39. Print.

11. Rogue Valley MPO. “2008-2011 Transportation improvement program.” https://scholarsbank.uoregon.edu/xmlui/bitstream/handle/1794/9406/RVMPO_MTIP.pdf?sequence=1, Accessed Jul. 29, 2013.

12. ICF Consulting. “Sample methodologies for regional emissions analysis in small urban and rural areas.” http://www.fhwa.dot.gov/environment/air_quality/conformity/research/methodologies.pdf, Accessed Jul. 29, 2013.

13. Fang Zhao and Soon Chung. “Estimation of annual average daily traffic in a Florida county using GIS and regression.” *Transportation Research Board*, 1660(1999): 32-40. Print.

14. Blume, K., et al.. “Cost-effective reporting of travel on local roads.” *Transportation Research Record* 1917. (2005): 1-10. Print.

15. Seaver, William, Arun Chatterjee, and Mark Seaver. “Estimation of traffic volume on rural local roads.” *Transportation Research Record* 1719.1 (2000): 121-28. Print.

16. Sheng-Guo Wang, Libin Bai, Xun Bao, Yue Liu, Guangyi Cao. “Novel estimation method of community AADT and VMT via circuit network models and simulation.” *Decision and Control and European Control Conference (CDC-ECC),* 2011 50th IEEE Conference, p. 5305–5310, 2011.

17. Division of Planning. “Street connectivity zoning and subdivision model ordinance.” Kentucky Transportation Cabinet, 2009.

18. Carlos A. Alba and Edward Beimborn. "Analysis of the effects of local street connectivity on arterial traffic." *Transportation Research Board*, Washington D.C. 2005.

19. Gil Tal, Susan Handy, and Marlon G. Boarnet. "Network connectivity." University of California, Davis, http://www.des.ucdavis.edu/faculty/handy/TTP220/NETWORK_CONNECTIVITY_5_10.pdf, Accessed 2010.

20. Dill Jennifer. "Measuring network connectivity for bicycling and walking." In Joint Congress of ACSP-AESOP. Leuven, Belgium, 2003.

21. Tresidder, Mike. "Using GIS to measure connectivity: an exploration of issues." Portland: Portland State University, School of Urban Studies and Planning; 2005.

22. Parthasarathi, Pavithra, Hochmair, Hartwig and Levinson, David Matthew. "The influence of network structure on travel distance." Available at SSRN: http://ssrn.com/abstract=1736326, Accessed July, 2009.

23. Courtat, T., C. Gloaguen, and S. Douady. "Mathematics and morphogenesis of the city: a geometrical approach." *Phys. Rev. E.* Vol. 83, No. 3, 2011, Paper 036106.

24. Freeman, L. C. "Centrality in social networks: conceptual clarification." *Social Networks*, Vol. 1, No. 3, 1979, pp. 215–239.

25. Faloutsos, M., P. Faloutsos, and C. Faloutsos. "On power-law relationships of the internet topology." *SIGCOMM Comput. Comm*. Rev., 29(4):251–262., 1999.

26. Yook, S.H., H. Jeong, and A.-L. Barabasi. "Modeling the internet's large-scale topology." *Proceedings of the National Academy of Science*, 99(21):13382–13386., 2002.

27. P. Bork, L. J. Jensen, C. von Mering, A. K. Ramani, I. Lee, and E. M. Marcotte. "Protein interaction networks from yeast to human." Curr. Opin. Struct. Biol., 14(3):292–299, 2004.

28. Porta S, Crucitti P and Latora V. “The network analysis of urban streets: a primal approach.” *Environment and Planning B: Planning and Design,* 33(5) 705 – 725, 2006.

29. Altshuler, Y., R. Puzis, Y. Elovici, S. Bekhor, and A. Pentland. “Augmented betweenness centrality for mobility prediction in transportation networks.” *International Workshop on Finding Patterns of Human Behaviors in Network and Mobility Data (NEMO)*, Athens, Greece, September, 9th, 2011.

30. Puzis, R., Y. Altshuler, Y. Elovici, S. Bekhor, Y. Shiftan, and A. Pentland. “Augmented betweenness centrality for environmentally aware traffic monitoring in transportation networks.” *Journal of Intelligent Transportation Systems*, Vol. 17, No. 1, 2013, pp. 91-105.

31. Jiang, B. “A topological pattern of urban street networks: universality and peculiarity.” *Physica A: Statistical Mechanics and its Applications*, 384: 647 – 655. 2007.

32. Jiang, B. and C. Liu. “Street-based topological representations and analyses for predicting traffic flow in GIS.” *International Journal of Geographical Information Science,* 23.9: 1119–1137. 2009.

33. Albert, R., H. Jeong, and A. L. Barabasi. “Diameter of the World-Wide-Web.” *Nature,* 401: 130-131. 1999.

34. Milgram, S.. “The small world problem.” *Psychology Today* 2: 60-67. 1967.

35. Kleinfel J.S.. “The small world problem.” *Society* 39(2): 61-66. 2002.

36. Watts D. J.. “Six degrees: the science of a connected age.” W. W. Norton & Company, Boston, MA. 2003.

37. Jiang, B. and C. Claramunt. “Topological analysis of urban street networks.” *Environment and Planning B: Planning and Design*, 31:151-162. 2004.

38. Barabasi, A. L. "Linked: is about how everything is connected to everything else and what it means for business, science, and everyday life." Penguin Group (USA) Inc., New York, 2003.

39. Barabasi, A. L., and R. Albert. "Emergence of scaling in random networks." *Science*, Vol. 286, 1999, pp. 509-512.

40. Newman, M. E. J.. "The structure of scientific collaboration networks." *Proceedings of the National Academy of Science*, Vol. 98, 2001, pp. 404-409.

41. Jeong, H., B. Tombor, R. Albert, Z. N. Oltvai, and A.-L. Barabasi. "The large-scale organization of metabolic networks." *Nature*, Vol. 407, 2000, pp. 651-654.

42. Carvalho, R., and A. Penn. "Scaling and universality in the micro-structure of urban space." *Physica A*, Vol. 332, 2004, pp. 539-547.

43. Benguigui, L., and E. Blumenfeld-Lieberthal. "Beyond the power law − a new approach to analyze city size distribution." *Computers, Environment and Urban Systems*, Vol. 31, 2007, pp. 648-666.

44. Kalapala, V., V. Sanwalani, A. Clauset, and C. Moore. "Scale invariance in road networks." *Physical Review E*, Vol. 73, 2006, Paper 026130.

45. Jiang, B. "Street hierarchies: a minority of streets account for a majority of traffic flow." *International Journal of Geographical Information Science*, Vol. 23, No. 8, 2009, pp. 1033-1048.

46. Durrett, R. "Random graph dynamics." Cambridge University Press, New York, NY, 2007.

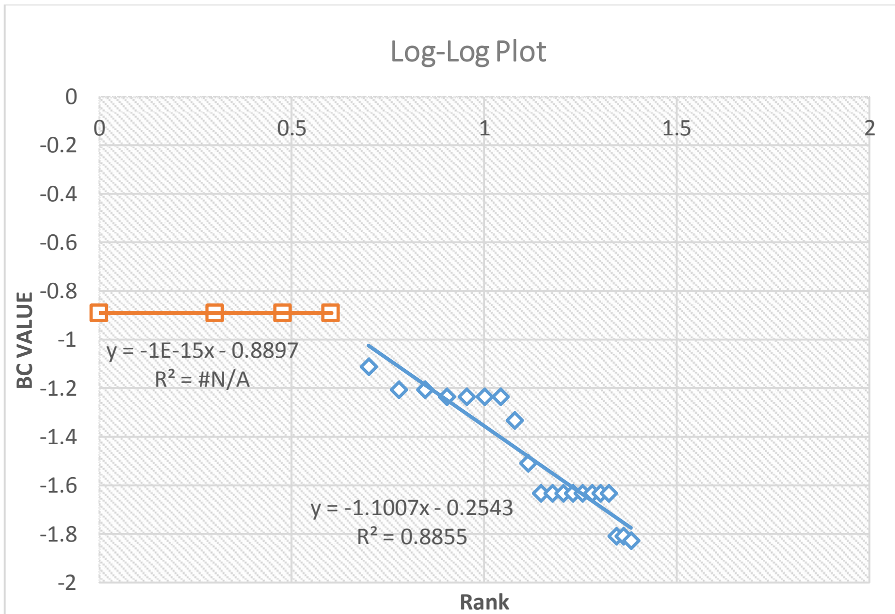
47. MEJ Newman, "Pareto distributions and Zipf's law." *Contemporary Physics,* Vol. 46, 2005, pp. 323–351.

48. Zipf, G. K. “Human behavior and the principles of least effort.” Addison Wesley, Cambridge, MA, 1949.

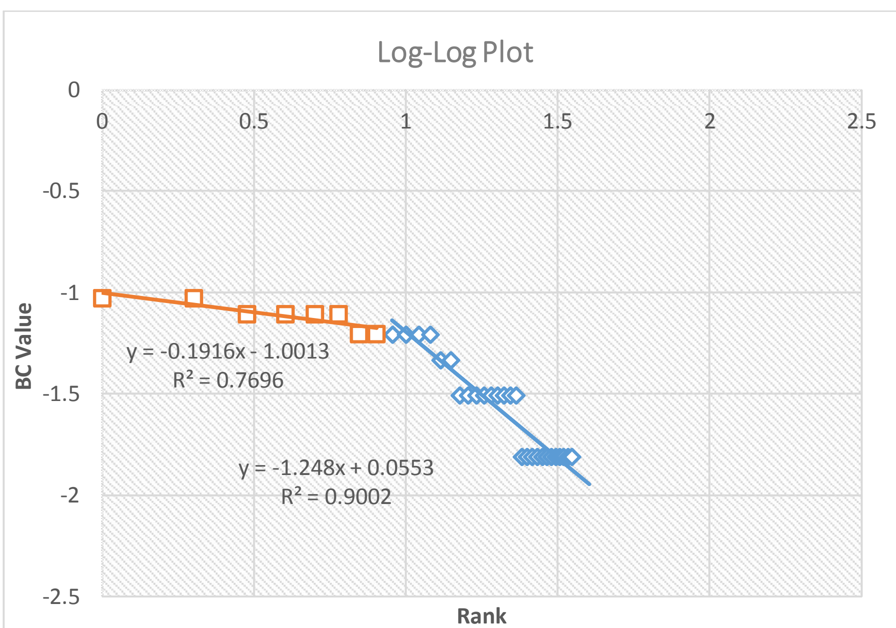
49. Southworth, M., and E. Ben-Joseph. “Streets and the shaping of towns and cities.” New York: McGraw-Hill, 1997.

APPENDIX

Illustration of Piecewise Power Law Distributions for Rest Networks



1



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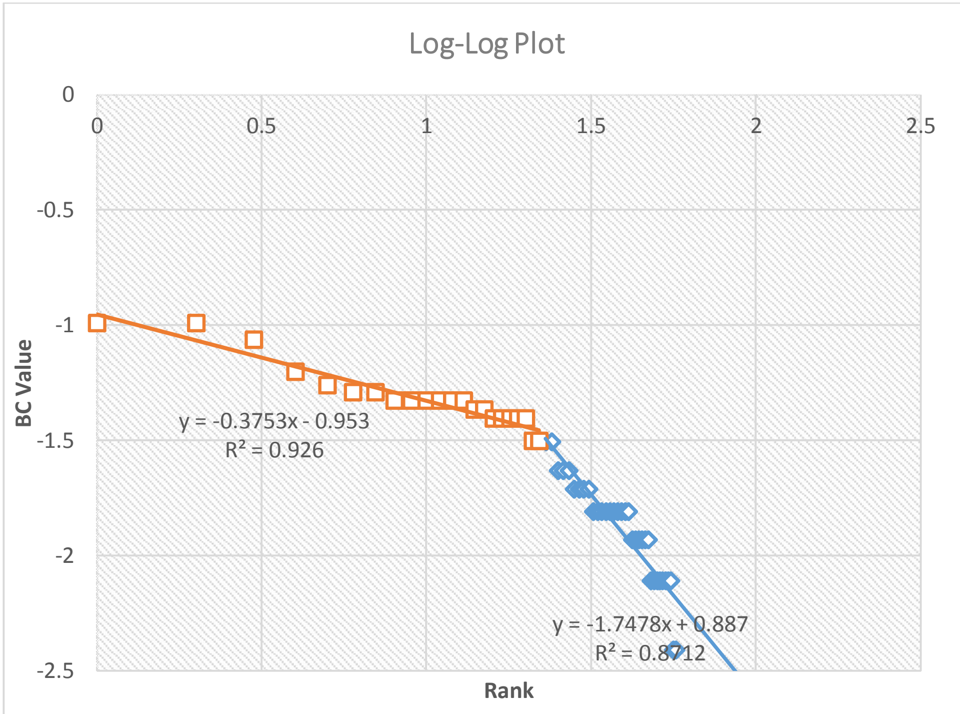
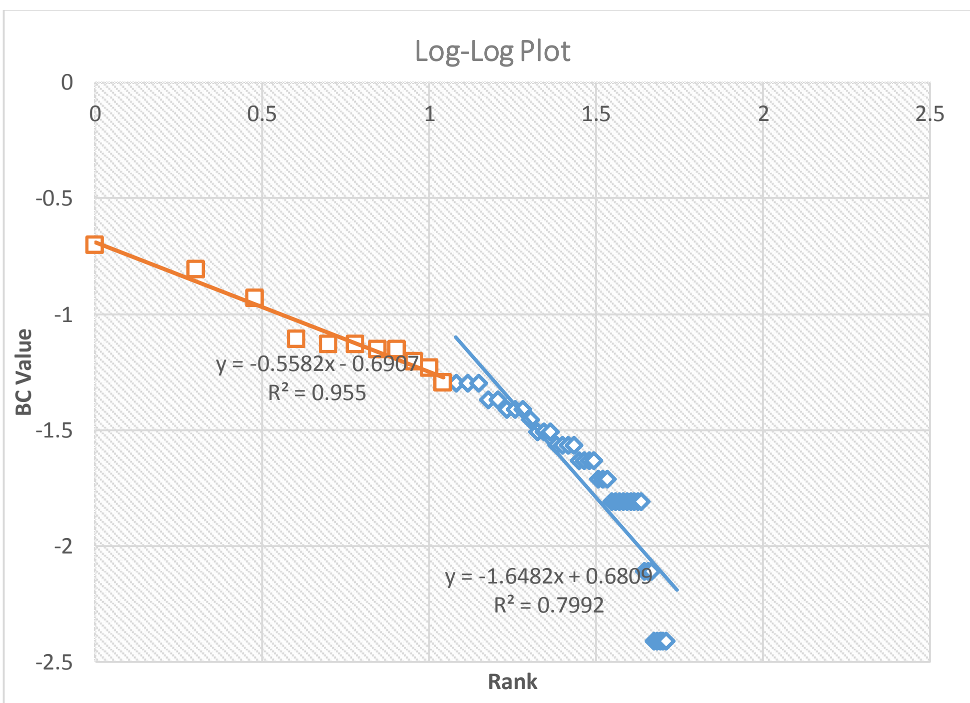

Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.3753x - 0.953
R² = 0.926
y = -1.7478x + 0.887
R² = 0.8712

4


Log-Log Plot
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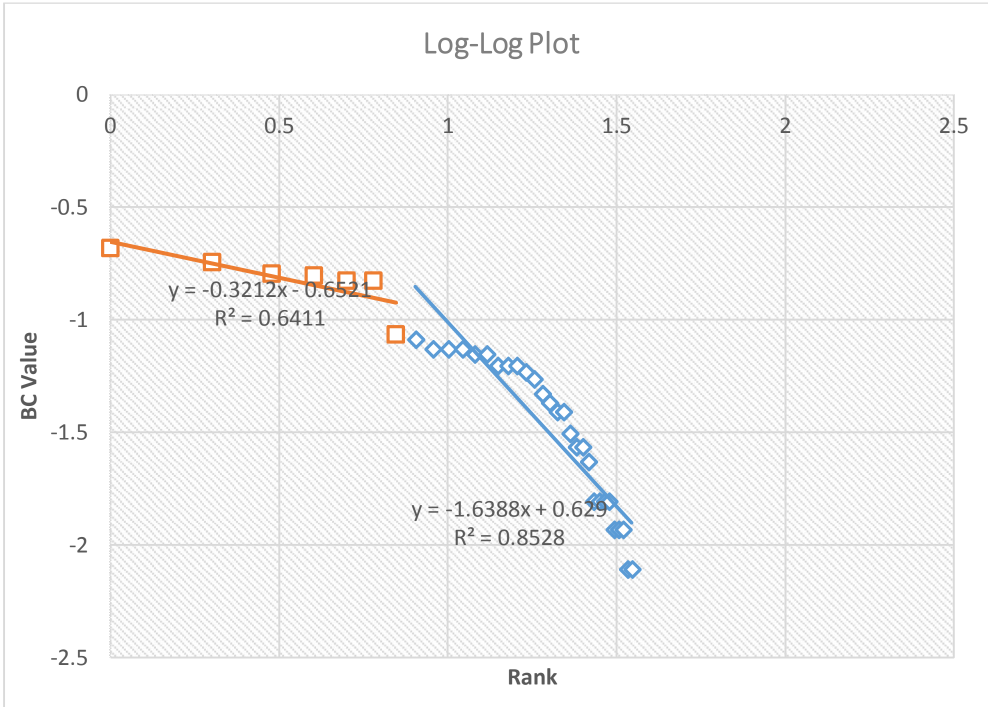
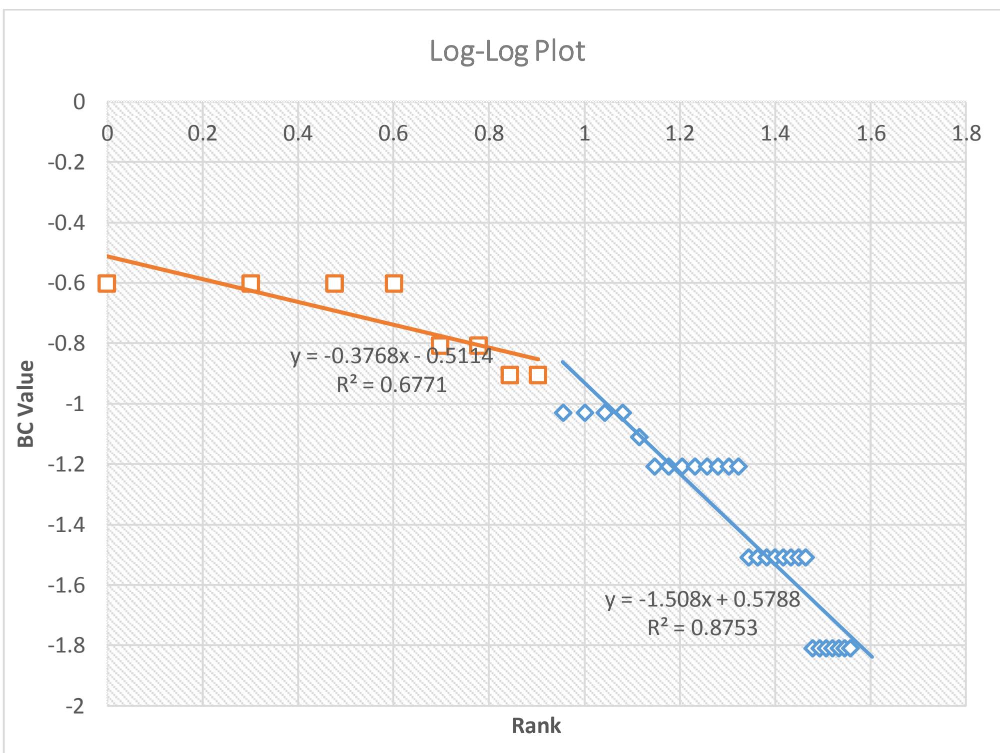
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.5582x - 0.6907
R² = 0.955
y = -1.6482x + 0.6809
R² = 0.7992

5


Log-Log Plot
BC Value
Rank
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
y = -0.3212x - 0.6521
R² = 0.6411
y = -1.6388x + 0.629
R² = 0.8528

9


Log-Log Plot
BC Value
Rank
0
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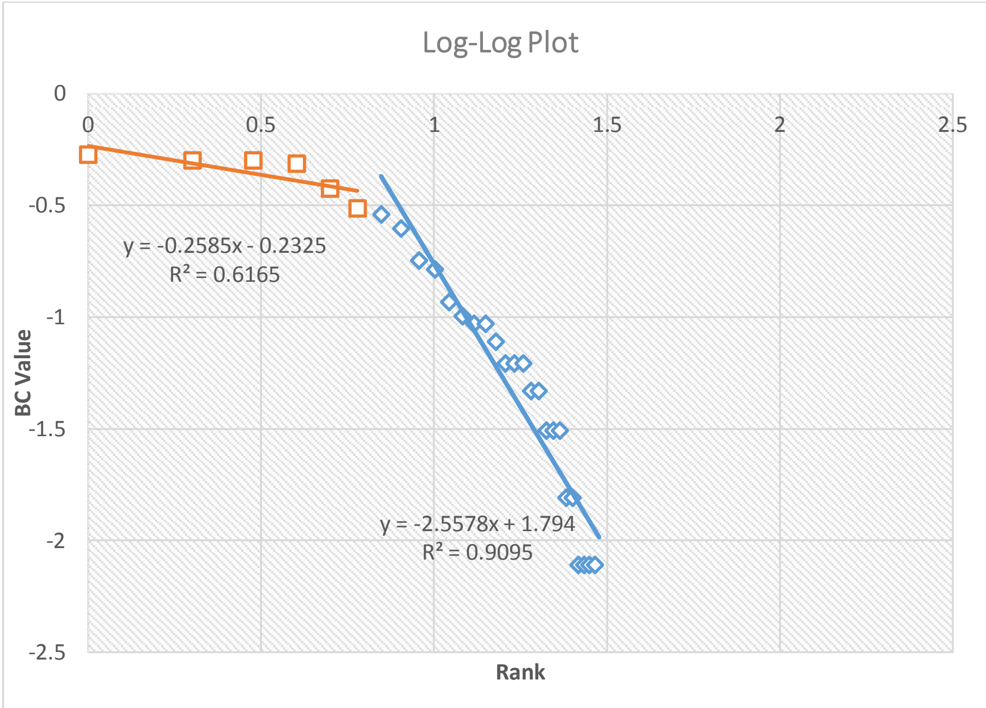
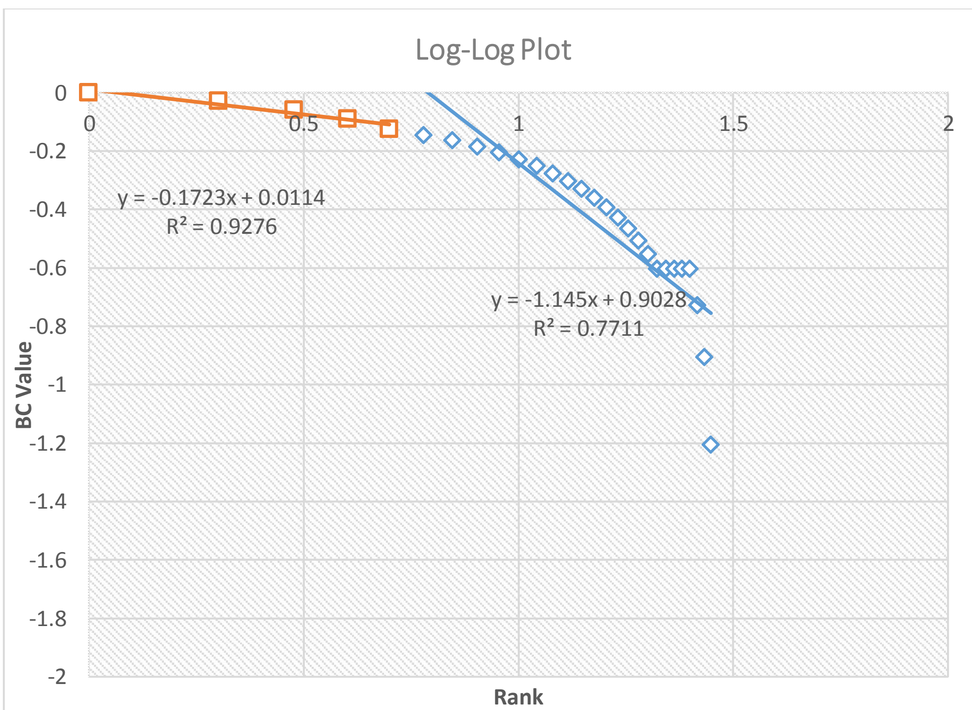
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
y = -0.3768x - 0.5114
R² = 0.6771
y = -1.508x + 0.5788
R² = 0.8753

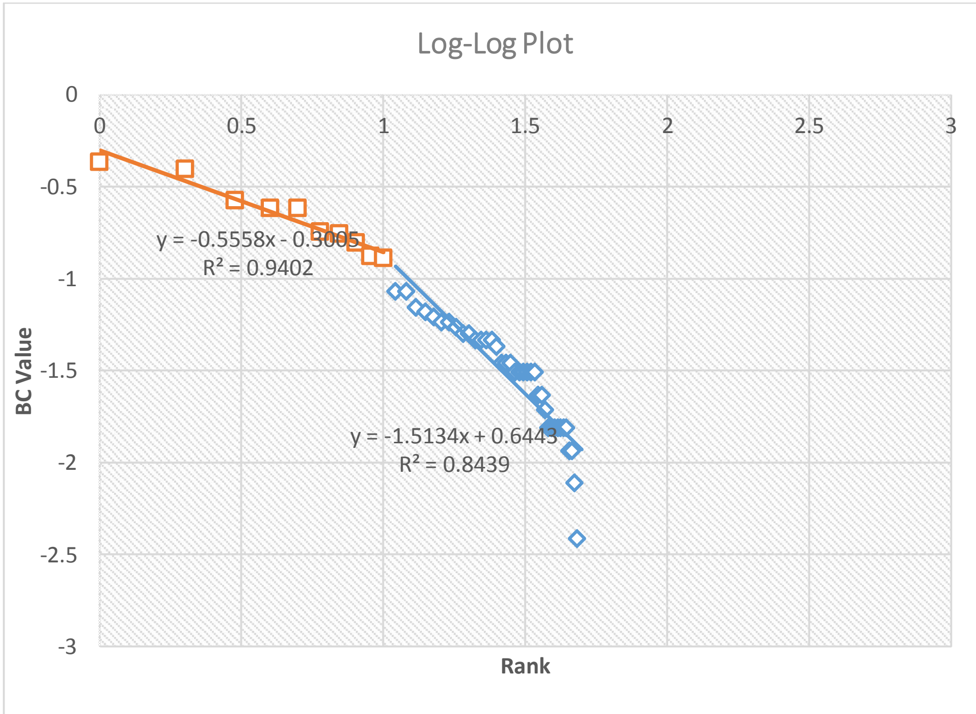
10


Log-Log Plot
0
0
0.5
1
1.5
2
2.5
-0.5
-1
-1.5
-2
-2.5
y = -0.2585x - 0.2325
R² = 0.6165
y = -2.5578x + 1.794
R² = 0.9095
BC Value
Rank

12

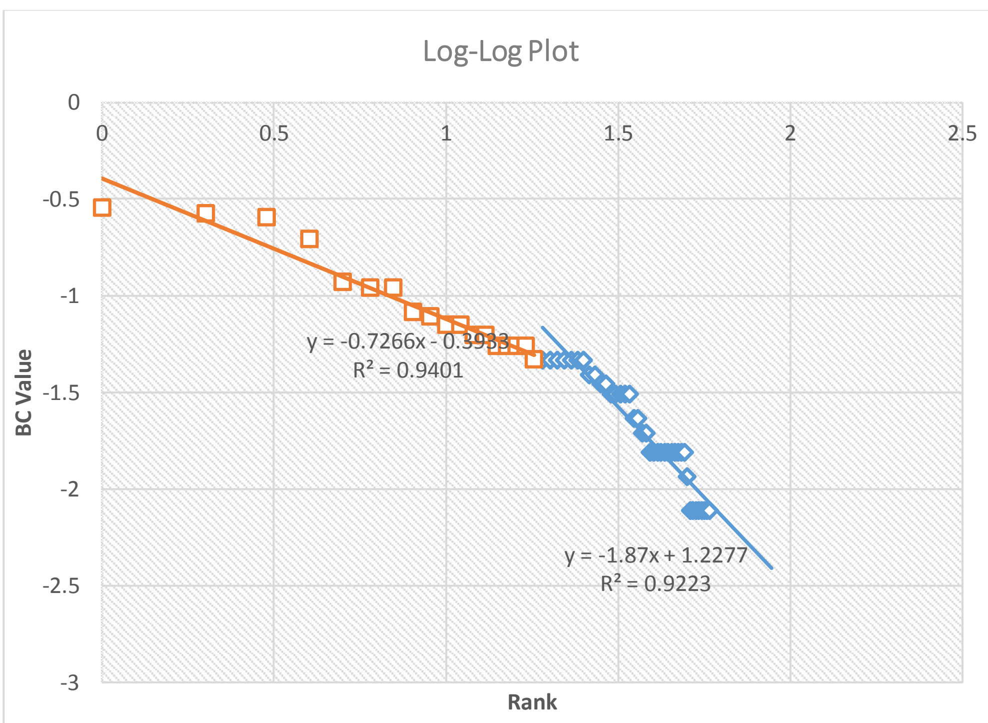

Log-Log Plot
0
0
0.5
1
1.5
2
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
y = -0.1723x + 0.0114
R² = 0.9276
y = -1.145x + 0.9028
R² = 0.7711
BC Value
Rank

13



Log-Log Plot
BC Value
Rank
y = -0.5558x - 0.3005
R² = 0.9402
y = -1.5134x + 0.6443
R² = 0.8439

14



Log-Log Plot
BC Value
Rank
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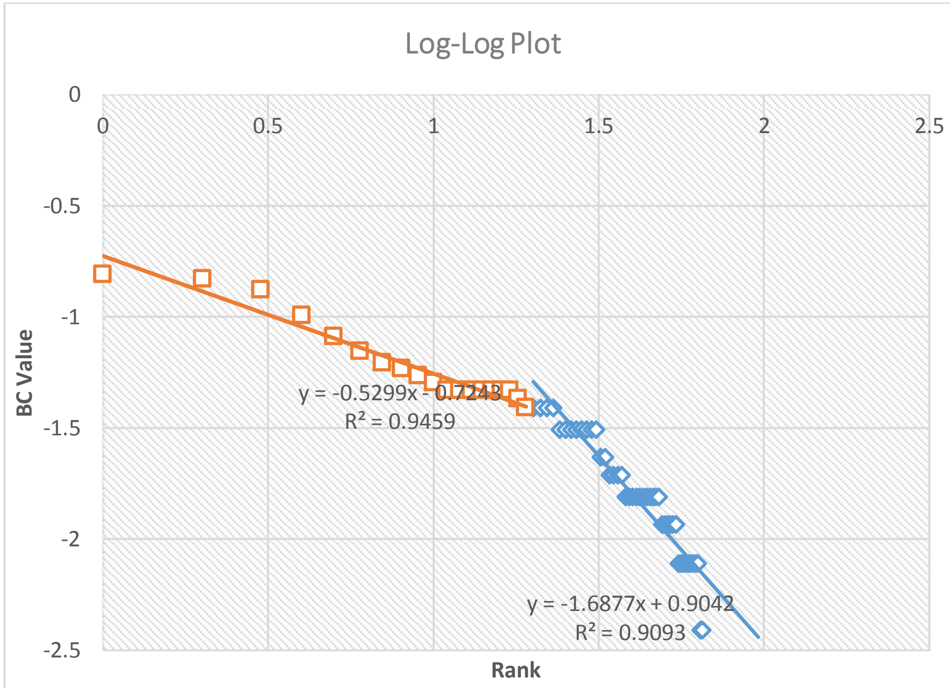
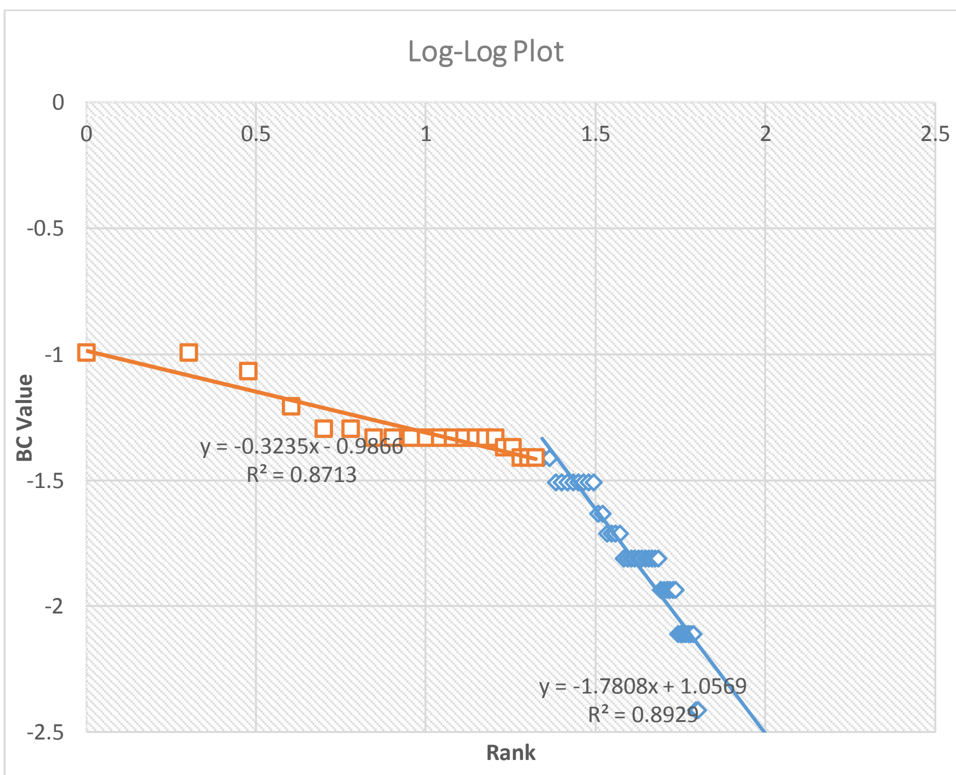
R² = 0.9401
y = -1.87x + 1.2277
R² = 0.9223

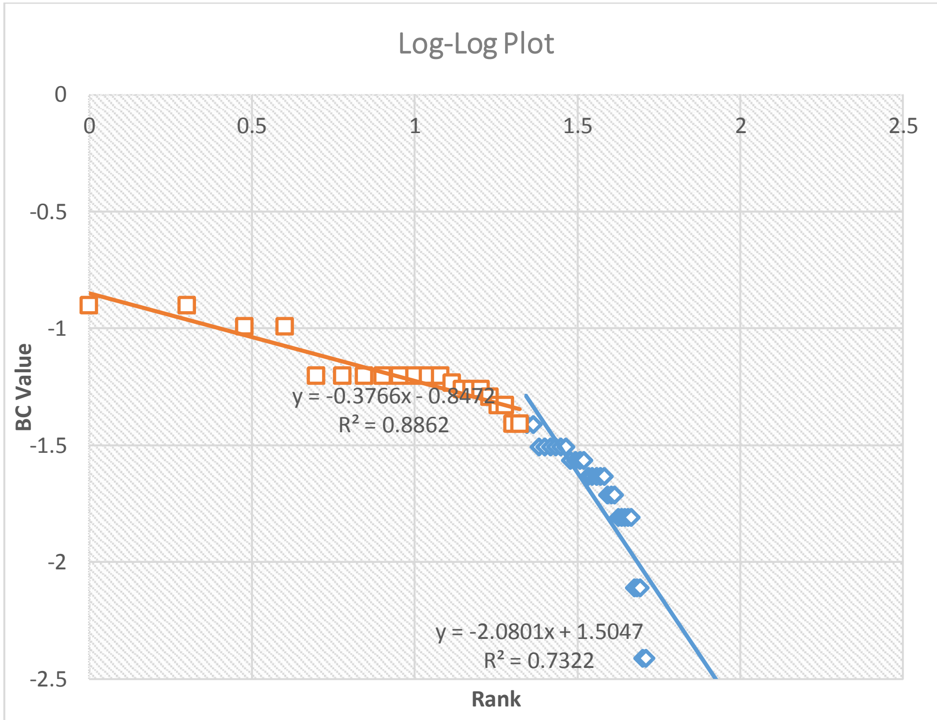
16


Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.5299x - 0.7243
R² = 0.9459
y = -1.6877x + 0.9042
R² = 0.9093

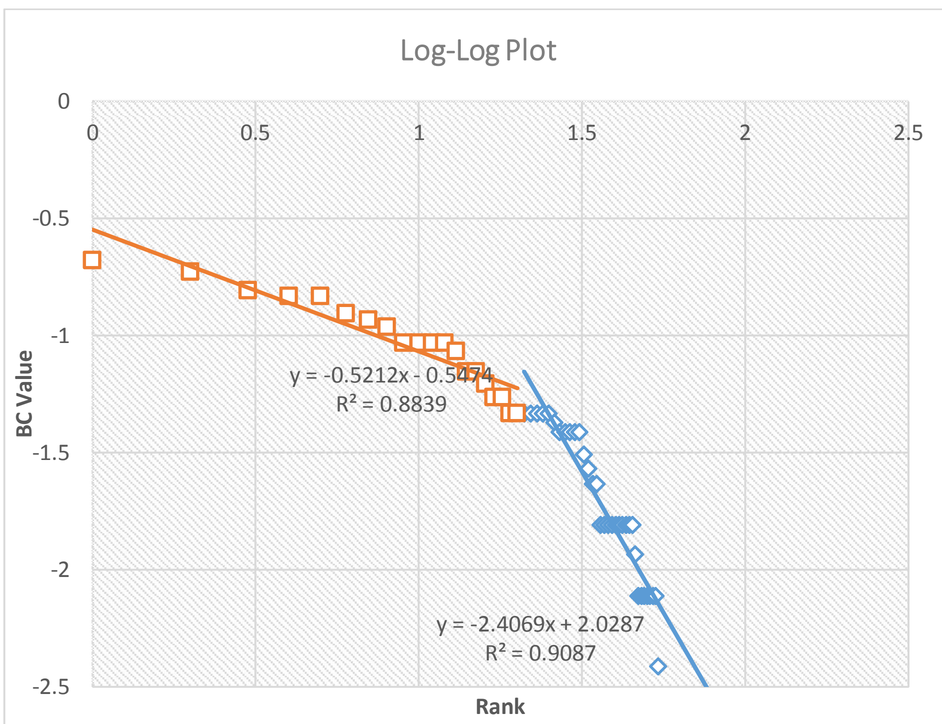
17


Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.3235x - 0.9866
R² = 0.8713
y = -1.7808x + 1.0569
R² = 0.8929

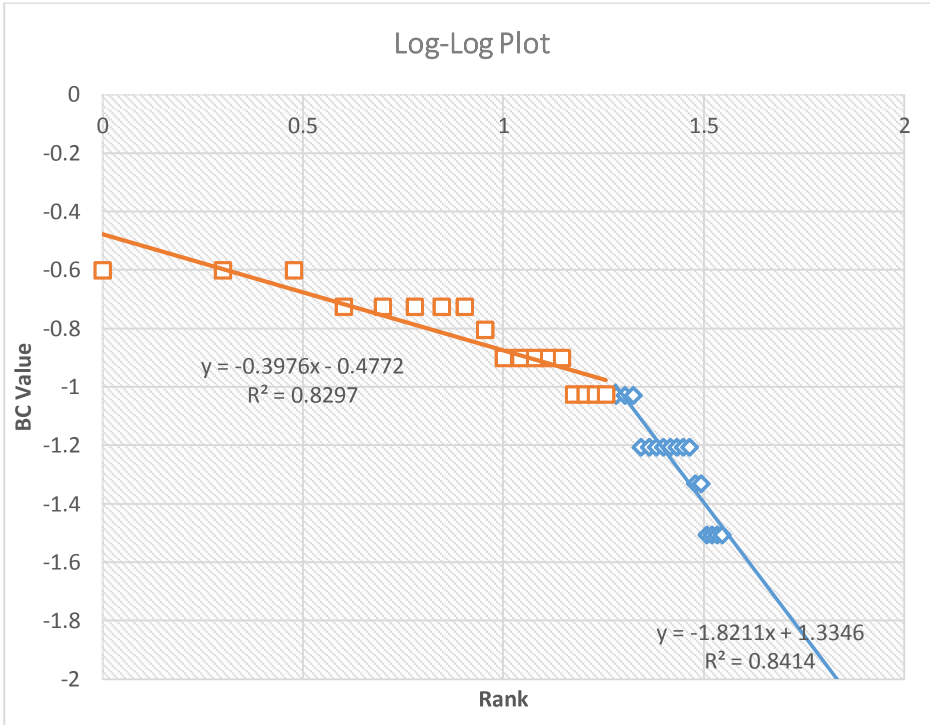
18


Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.3766x - 0.8472
R² = 0.8862
y = -2.0801x + 1.5047
R² = 0.7322

19

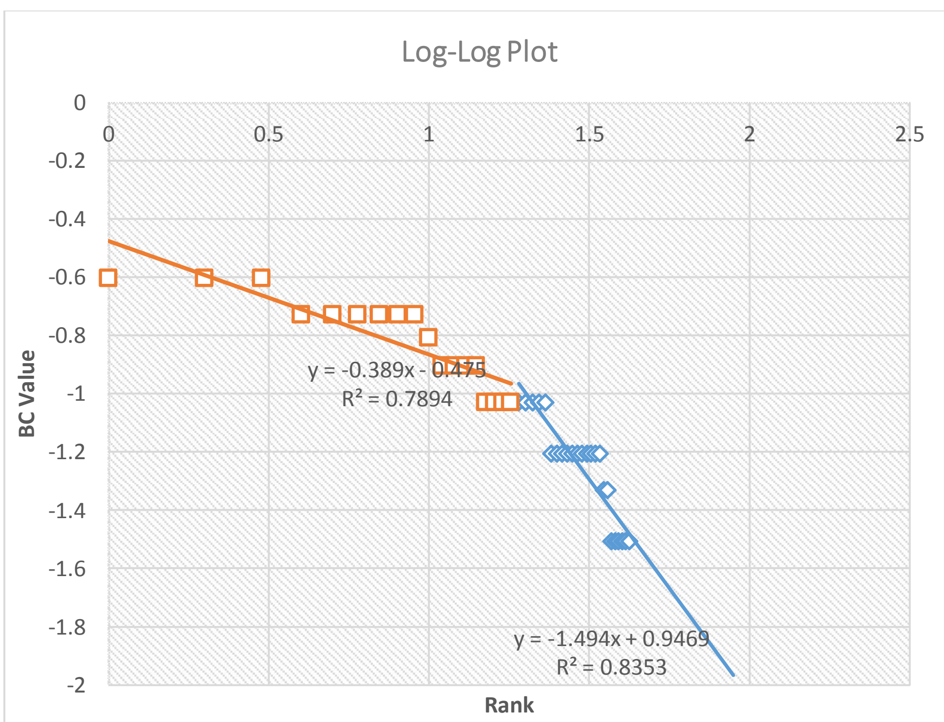

Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.5212x - 0.5474
R² = 0.8839
y = -2.4069x + 2.0287
R² = 0.9087

20



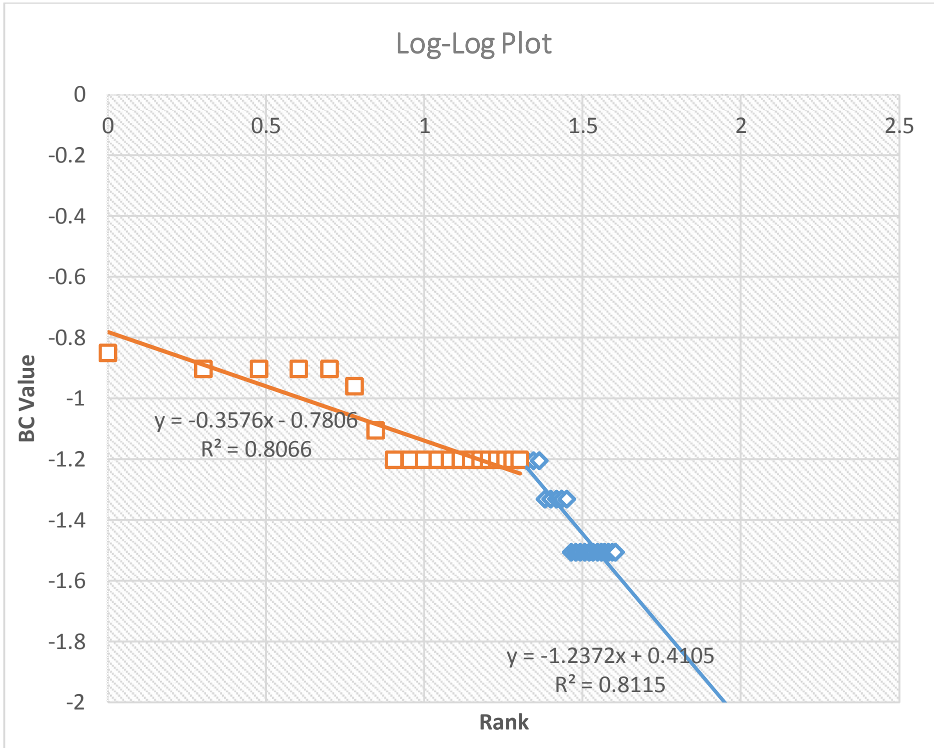
Log-Log Plot
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
0
0.5
1
1.5
2
BC Value
Rank
y = -0.3976x - 0.4772
R² = 0.8297
y = -1.8211x + 1.3346
R² = 0.8414

21



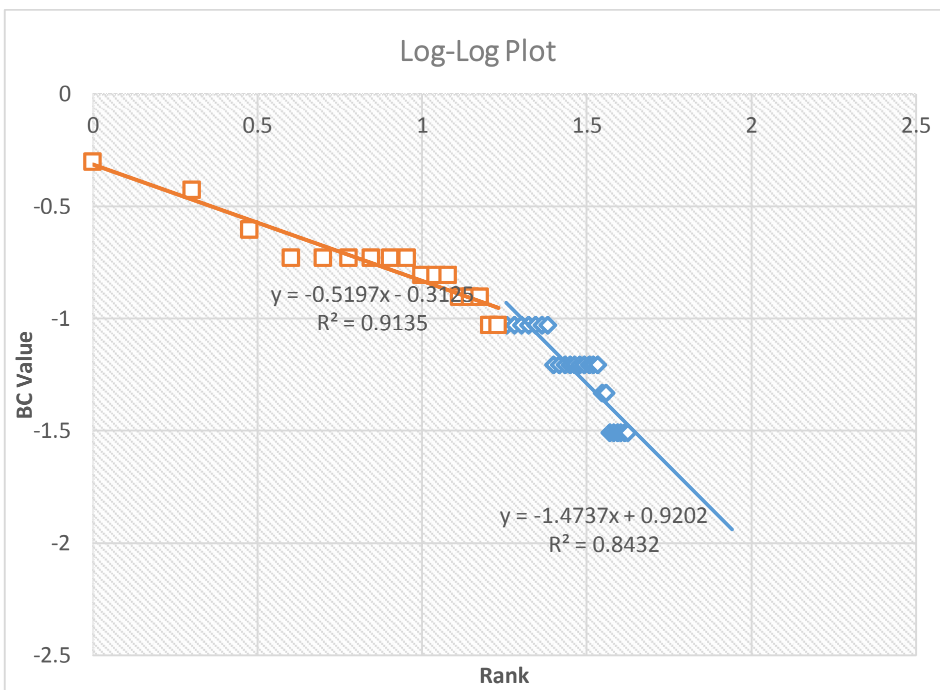
Log-Log Plot
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.389x - 0.475
R² = 0.7894
y = -1.494x + 0.9469
R² = 0.8353

22



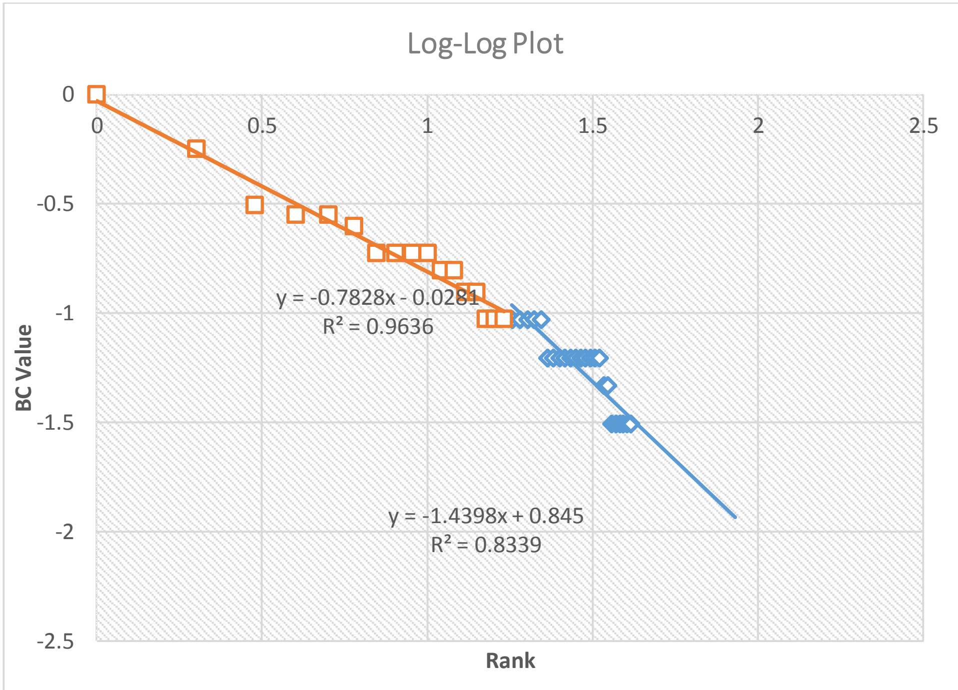
Log-Log Plot
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.3576x - 0.7806
R² = 0.8066
y = -1.2372x + 0.4105
R² = 0.8115

23

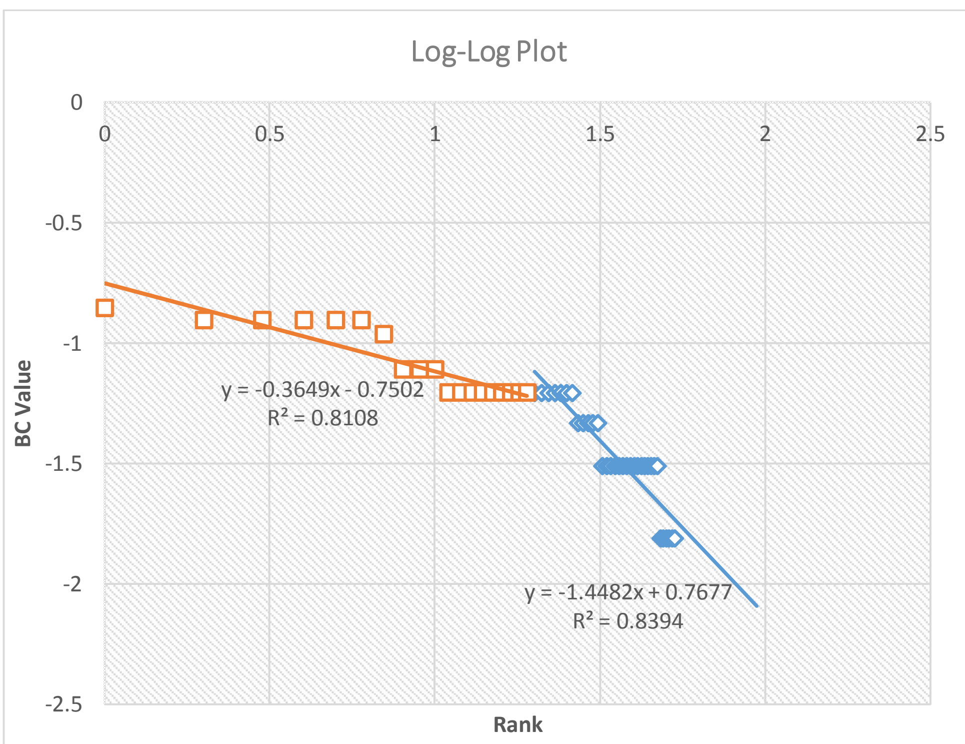


Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.5197x - 0.3125
R² = 0.9135
y = -1.4737x + 0.9202
R² = 0.8432

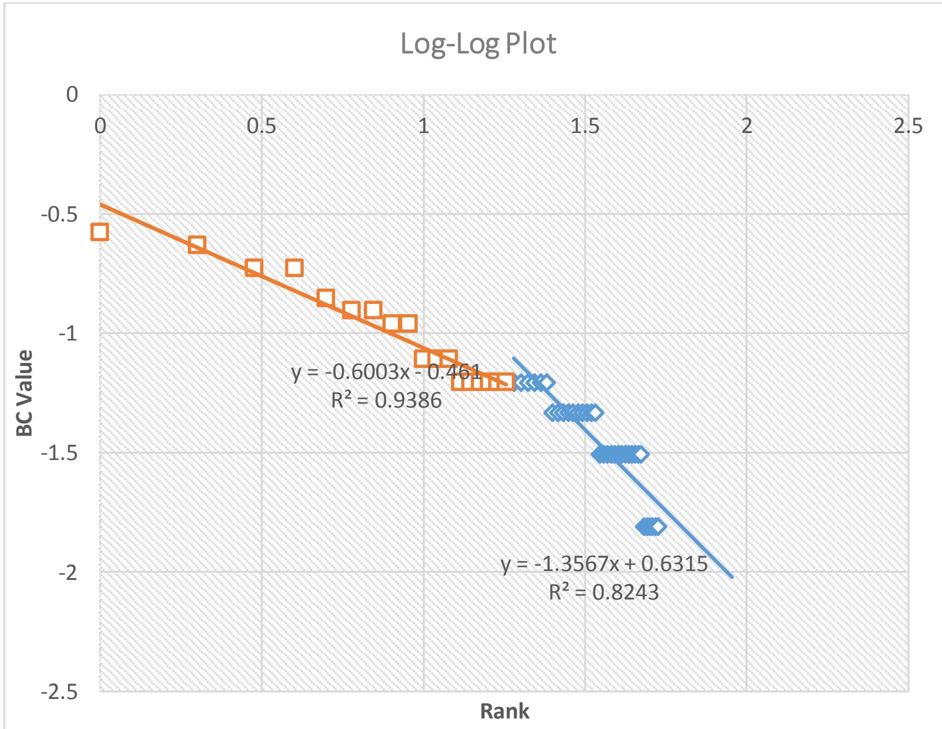
24


Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.7828x - 0.0281
R² = 0.9636
y = -1.4398x + 0.845
R² = 0.8339

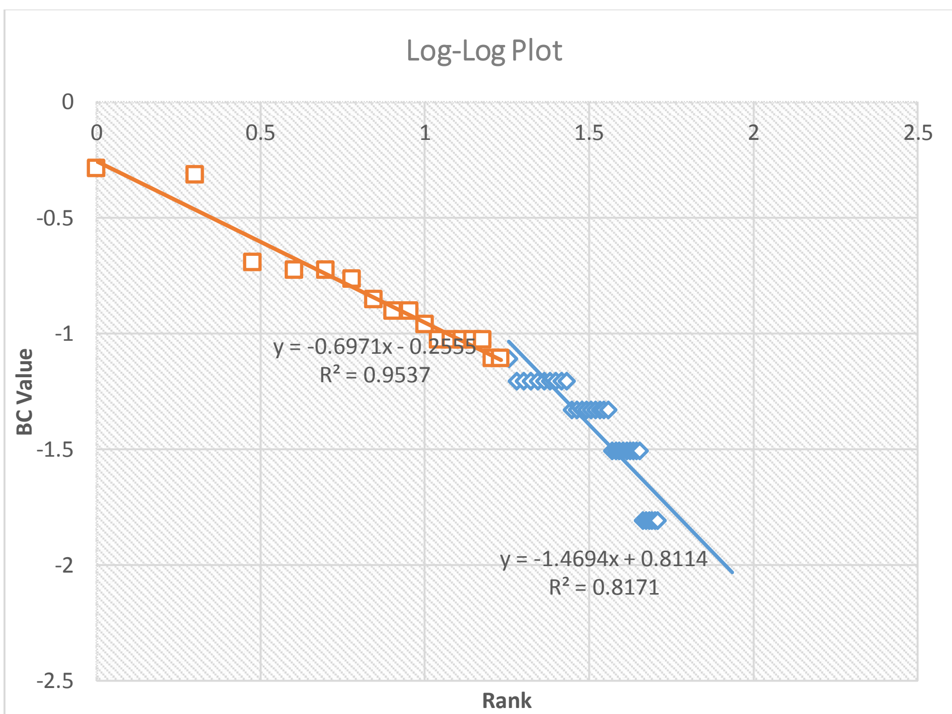
25


Log-Log Plot
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
BC Value
Rank
y = -0.3649x - 0.7502
R² = 0.8108
y = -1.4482x + 0.7677
R² = 0.8394

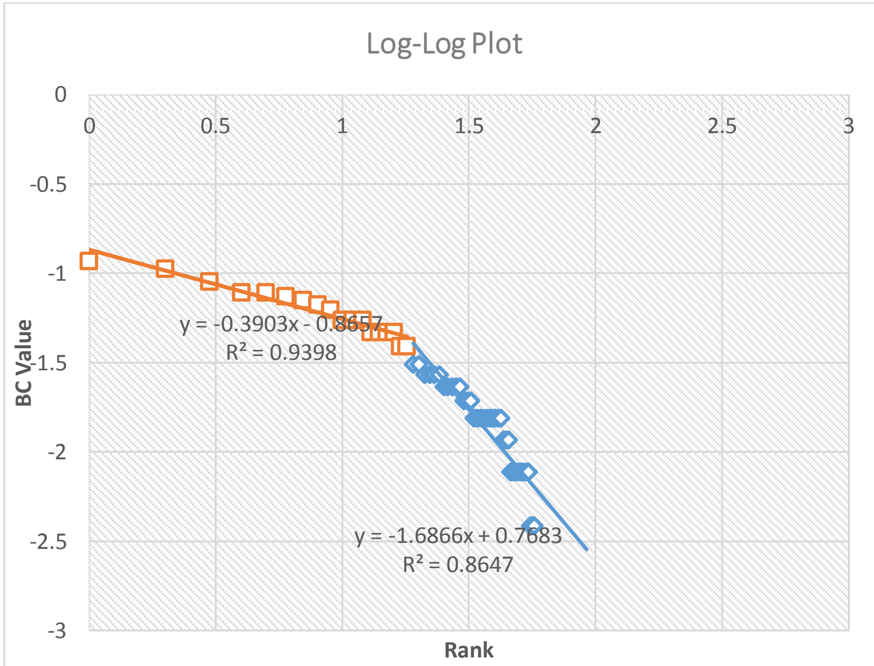
26


Log-Log Plot
0
0
0.5
1
1.5
2
2.5
-0.5
-1
-1.5
-2
-2.5
BC Value
Rank
y = -0.6003x - 0.461
R² = 0.9386
y = -1.3567x + 0.6315
R² = 0.8243

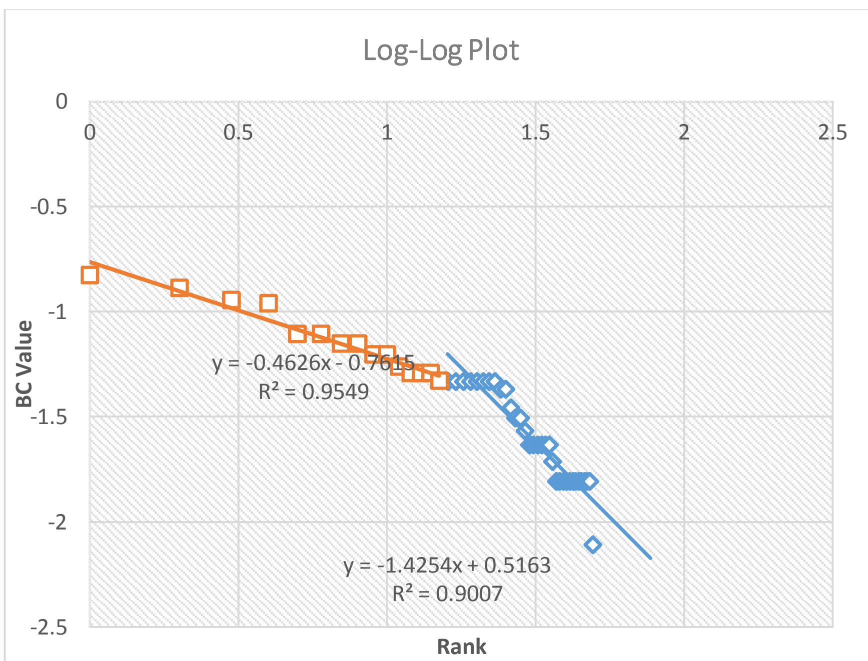
27


Log-Log Plot
0
0
0.5
1
1.5
2
2.5
-0.5
-1
-1.5
-2
-2.5
BC Value
Rank
y = -0.6971x - 0.2555
R² = 0.9537
y = -1.4694x + 0.8114
R² = 0.8171

28


Log-Log Plot
y = -0.3903x - 0.8657
R² = 0.9398
y = -1.6866x + 0.7683
R² = 0.8647
BC Value
Rank
0
0.5
1
1.5
2
2.5
3
-0.5
-1
-1.5
-2
-2.5
-3

29


Log-Log Plot
y = -0.4626x - 0.7615
R² = 0.9549
y = -1.4254x + 0.5163
R² = 0.9007
BC Value
Rank
0
0.5
1
1.5
2
2.5
-0.5
-1
-1.5
-2
-2.5

30