SIMULATING THE FATE OF AN EARTH-LIKE PLANET INCLINDED TO THE ECLIPTIC PLANE TO IMPROVE UNDERSTANDING OF PLANETARY SYSTEM FORMATION

An Undergraduate Research Scholars Thesis

by

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ABSTRACT

Simulating the Fate of an Earth-like Planet Inclined to the Ecliptic Plane to Improve Understanding of Planetary System Formation. (May 2013)

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The formation of our Earth and Solar System has befuddled humankind for centuries. Although there remain a number of peculiarities to be remedied by the currently held nebular theory of Solar System formation, there exists a widely held convergence on the basic components of planetary formation.

Interactions with the giant planets of our system, as well as heavy bombardment that occurred billions of years ago, played major roles in early Solar System formation and continue to shape its dynamics through huge gravitational perturbations. In order to better understand the effect that planetary giants have on bodies within our Solar System, this paper proposes to simulate the *n*-body problem for the Sun-Jupiter-Earth system so as to quantify the effect that a Jupiter giant would have on an Earth-like planet inclined to the ecliptic planet. Through iteration of the Earth-like planet's inclination, the maximum angle of inclination before ejection from the Solar System can be found.

Using only Newtonian forces for the three-body problem, the simulation runs using a Runge-Kutta 4 solver to plot each body's position, velocity, and acceleration against time. These results

give new insight into why our Solar System lies primarily in the ecliptic disc and how its dynamics will continue to vary over time. For the Sun-Earth-Jupiter system simulated in this paper (run over 119,000 years), orbits inclined to the ecliptic plane greater than 50° became unstable, with Earth ejection after 62,000 years (85°).

Furthermore, simulation of other solar systems leads to a more general theory on the impact of planetary formation and heavy bombardment on the fate of Earth-like planets elsewhere in the Universe. For the exoplanetary system simulated in this paper, which includes a hot Jupiter at 1.5 AU and an Earth-like planet at 1 AU (run over 94,000 years), orbits inclined to the ecliptic plane greater than 10° became unstable, with Earth ejection after 6,250 years (50°). Thus, as the Jupiter giant is moved inward, its influence over the Earth-like planet increases and the time to orbital decay of the Earth-like planet decreases. Overall, these results illustrate that the orbits of Earth-like planets in systems with Jupiter giants have restrictions on available orbital inclinations to remain stable.

NOMENCLATURE

$\boldsymbol{F}_G$	=	Force due to Gravity
G	=	Gravitational constant
m	=	Mass
r	=	Radius
$\hat{u}_r$	=	Unit vector in the radial direction
$\boldsymbol{F}_{AB}$	=	Force on body A due to body B
$\boldsymbol{R}_A$	=	Position vector of body A
$\boldsymbol{a}_A$	=	Acceleration vector of body A
$\boldsymbol{v}_A$	=	Velocity vector of body A
t	=	Time
r	=	Radius (Radial Distance from Sun)
$\boldsymbol{h}_A$	=	Angular momentum vector of body A
μ	=	Gravitational parameter
e	=	Eccentricity
θ	=	True Anomaly
a	=	Semimajor Axis
T	=	Period

CHAPTER I

INTRODUCTION

For thousands of years, the question of how Earth and our Solar System formed has captivated humans in cultures across the world. In 1778, Georges-Louis Leclerc proposed that the collision of a huge comet with the Sun caused the ejection of an accretion disk that eventually condensed to form the planets. Competing tidal theories proposed that as smaller bodies got too near the sun, they ripped material away from the Sun to become the planets. However, each of these theories does not adequately explain the difference in composition between the Sun and planets, or the insufficient amount of energy needed to produce such events.

Another set of theories dealt with the differing compositions between the Sun and planets by proposing that the Sun accreted material from the depths of outer space. However, this theory did not account for the difference in composition between the planets themselves. Yet another set of theories, the foundation of which our current models rest, proposed that the Sun and planets were formed simultaneously from a common collapsing nebula. Early activists for this nebular theory include Rene Descartes, Immanuel Kant, and Marquis de Laplace.[2]

While a significant number of problems with this theory still exist, most scientists would agree that there exists some sense of convergence today on the basic components of planetary system formation.[2] With our current understanding of how the Solar System came to be, questions as to how it will continue to evolve over time still intrigue the human imagination today.

This paper focuses on the large gravitational perturbations caused by giant planets of our Solar System, specifically Jupiter, and how these effects, along with large collisions, influenced the formation of our Solar System billions of years ago and will continue to influence its individual components, namely planets and large asteroids, in the future. By understanding the role that impacts and jovian planets have on Earth-like planets, the techniques of exoplanet detection and characterization may be improved.

Evolution of the Solar System[2]

From our current understanding of the solar system, the Sun was formed from the gravitational collapse of a solar nebula that most likely began as a large, roughly shaped cloud of very cold, very low-density gas. With the explosion of a nearby supernova, the nebula began to collapse until gravity took hold and accelerated the process. As the radius of the cloud decreased, its rate of spin increased to conserve angular momentum and an accretion disk was formed, with the Sun at the center where temperature and density were greatest.

Within this disk of material, a temperature gradient existed with respect to distance from the protosun such that rocks amalgamated throughout the disk and ices consolidated only in the areas beyond the outer asteroid belt. This frost line, as it is called, developed between the orbits of Mars and Jupiter and marks the transition between the warmer inner region and the cooler outer region of our Solar System. Collisions of small planetary chunks of material, called planetesimals, accreted to form the planets initially of electrostatic attraction and eventually of gravity once enough material accumulated.

While the terrestrial planets formed from the condensation of rock (condenses at 500-1300K) and metal (condenses at 1000-1600K) within the inner region, the larger jovian planets of Jupiter and Saturn were able to form from the condensation of hydrogen compounds such as ice (condenses at <150K), as well as from rock and metal, in the outer region.[1] The higher temperatures and lower masses found in the inner radius of the disk inhibited the accumulation of gases around those planets, while the cooler, more massive giants were able to gravitationally acquire massive primordial atmospheres of helium and hydrogen gas never condensed in the nebula. Figure 1 illustrates an artist's conception of what this might look like.

Figure 1 Accretion disk formed after collapse of solar nebula.[4]

As a result of these planetesimal collisions occurring within the accretion disk, most planets in the Solar System orbit the Sun in a nearly ecliptic plane, as defined by the Sun-Earth system. Table 1 gives the orbital data for the planets in our Solar System.

Table 1 Planetary Orbital Data.[2]

Planet	Mass ($M_{\oplus}$)	Semimajor Axis (AU)	Orbital Eccentricity	Sidereal Orbital Period (yr)	Orbital Inclination to Ecliptic (°)
Mercury	0.05528	0.3871	0.2056	0.2408	7.00
Venus	0.81500	0.7233	0.0067	0.6152	3.39
Earth	1.0000	1.0000	0.0167	1.0000	0.0000
Mars	0.10745	1.5236	0.0935	1.8808	1.850
Jupiter	317.83	5.2044	0.0489	11.8618	1.304
Saturn	95.159	9.5826	0.0565	29.4567	2.485
Uranus	14.536	19.2012	0.0457	84.0107	0.772
Neptune	17.147	30.0476	0.0113	164.79	1.769

As evident in Table 1, it is remarkable that all the planets in our Solar System have very low orbital inclinations to the ecliptic plane and very low orbital eccentricities as well. For instance, Mercury has the highest inclination and eccentricity at 7° and 0.2408 respectively.

Cratering of these bodies by remnant material continued throughout a time known as heavy bombardment. Collisions during this time further shaped our Solar System and even brought about the creation of many moons. Some moons were formed from local accretion disks during the formation of the giants, while others were formed when planetesimals and fragmented asteroids became captured by these massive planets. The Earth-Moon system was formed by a collision of our primitive Earth with a planetesimal the size of Mars. This impact tilted the Earth's axis and blasted rock from Earth's outer crust off to form the Moon. Pluto's moon Charon is thought to have been formed in much the same way. Other giant impacts are thought to have tilted Uranus on its side and stripped Mercury of its out crust, leaving the high density core we see today.

Icy objects that were not captured by the giant planets or destroyed by collisions had their orbits drastically altered by gravitational interaction with these massive planets. Some planetesimals were sent into highly elliptic orbits near Neptune and Pluto in the Oort Cloud or even ejected from the Solar System completely, while others were sent inward on a collision course with the Sun or another planet. These types of comet collisions are thought to have been the ones to bring water to Earth, ultimately enabling life to form under the right conditions.

The origins of life[1]

The key to finding life on exoplanets is first understanding how life came about on Earth and what enabled it to not only survive here, but thrive here. From observation of the diversity of life on Earth, there are three basic requirements for life to exist: a source of nutrients, energy to fuel the activities of life—such as the Sun or planetary thermal energy—and liquid water. While several planets within and outside of our Solar System have met the first two requirements, none have been found to possess life of any kind other than Earth. Thus it would seem that finding planets with habitable surfaces—those with temperatures and pressures which allow liquid water to exist—will be the driving factor in finding life in the Universe within the next century.

The current planet-detecting techniques use gravitational tugging, Doppler shifting, and transiting planets to indirectly locate exoplanets in our Universe, and have been successful in finding over 850 different planets.[7] However, these techniques create a selection bias based in the fundamental methods they use for detection. Most exoplanets found to date are considered to be *hot Jupiters*—very massive jovian planets orbiting very near their host stars. While these planets are exceedingly interesting to study, it is thought that these are unique systems where planetary migration and resonance has caused shifting within the system and are not the norm.

Furthermore, life is not thought to be possible on these types of planets. The planets of primary interest for supporting life are smaller and more terrestrial in nature.

In order to improve our techniques to find these more Earth-like planets, scientists must understand where to find them and under what conditions they can and cannot exist. Firstly, the host stars must be old enough to have allowed life time enough to develop. This rules out very massive stars which burn out relatively quickly (under a billion years). The host stars must also allow planets to orbit stably around them and have produced a large enough habitable zone for planets to exist in. This lessens the probably of finding habitable planets around binary and multi-star systems, as well as stars too small to produce an appreciable habitable zone. Stars on the order of our Sun (G) and slightly smaller (K, M) are the most likely candidates.

Also, some scientists believe there is a galactic habitable zone, analogous to stellar habitable zones, within which our own Milky Way Galaxy resides. Because the abundance of heavy elements (those composing terrestrial planets) decreases with distance from the galactic center, it is thought that the outer rim of the galactic disk may not produce many Earth-like planets. Furthermore, the occurrence of supernova increases within the more crowded inner regions of the galactic disk, making this area more radiation-intensive, which may be detrimental to biological life.

Some also believe that the tidal interactions of moons are important to the sustainment of life on a planet, as well as the planet's tilt and inclination out of plane in generating climate change and seasons. Thus it is of growing importance to understand the modes of solar system formation and how these modes affect the ability of life to develop under various conditions.

For example, while the importance of giant impacts is evident in our own Solar System, it is difficult to know whether the impact rate found in other systems would have decreased over time such as ours, or would have persisted longer. The Oort Cloud of our Solar System is a direct consequence of gravitational interactions between planetesimals and Jupiter, which pushed these bodies beyond the threatening impact region of Earth. From the observations presented here, the placement of Jupiter seems to be crucial to our developing life here on Earth—not only from the standpoint of sending water-bearing comets to impact with us over 4.5 billion years ago, but also from the view of sending many comets away from Earth into the Oort Cloud and preventing further fragmentation of our planet.

Planetary impacts play an impeccably large role in solar system formation and contribute significantly to the individual characteristics of a planet. Ultimately, these impacts and the gravitational interactions with jovian planets that result can have an effect on a planet's ability to support life. Understanding these types of interactions will be very beneficial in future endeavors to find life on other planets.

Simulating the vital role played by the giant planets in the formation of our Solar System is a logical place to start. This leads to the question of *How far out of the ecliptic plane must a planet be before it is kicked out of the Solar System by Jupiter's gravitational influence?* The *n*-body problem will be used in numerical simulation to attack this question.

CHAPTER II

METHODS

The *n*-body problem[3]

Consider *n* point masses in three dimensional space. Suppose that the forces between these points are exclusively Newtonian. If the initial positions and velocities are given for each particle at some instant in time, then the position and velocity of each particle at some later or earlier time can be found. Mathematically, the *n*-body problem asks for the global solution to the ordinary differential equations given by the initial value problem. The equations of motion for the *n*-body problem can be generalized using the three-body system given below in Figure 2.

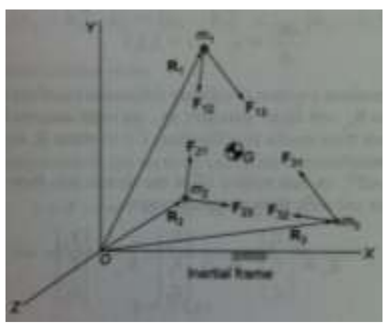


Figure 2 Three Body Problem.[3]

Each mass of the system experiences a gravitational attraction from the other masses of the system. The force of gravitational attraction between two bodies is given by

$$\boldsymbol{F}_G = \frac{Gm_1m_2}{r^2}\hat{\boldsymbol{u}}_r \qquad (2.1)$$

which acts along the line joining the mass centers of body 1 and 2.

For the three-body problem, the forces exerted on body 1 by bodies 2 and 3 are F_{12} and F_{13} respectively. Similarly, the forces experienced by body 2 are F_{21} and F_{23}, and the forces felt by body 3 are F_{31} and F_{32}.

$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21} = \frac{Gm_1m_2(\boldsymbol{R}_2-\boldsymbol{R}_1)}{\|\boldsymbol{R}_2-\boldsymbol{R}_1\|^3} \qquad (2.2a)$$

$$\boldsymbol{F}_{13} = -\boldsymbol{F}_{31} = \frac{Gm_1m_3(\boldsymbol{R}_3-\boldsymbol{R}_1)}{\|\boldsymbol{R}_3-\boldsymbol{R}_1\|^3} \qquad (2.2b)$$

$$\boldsymbol{F}_{23} = -\boldsymbol{F}_{32} = \frac{Gm_2m_3(\boldsymbol{R}_3-\boldsymbol{R}_2)}{\|\boldsymbol{R}_3-\boldsymbol{R}_2\|^3} \qquad (2.2c)$$

Relative to an inertial reference frame, the accelerations of the bodies are

$$\boldsymbol{a}_i = \ddot{\boldsymbol{R}}_i \qquad (2.3)$$

where $\boldsymbol{R_i}$ is the position vector of body *i*. Thus the equation of motion for body 1 is given by

$$\boldsymbol{F}_{12} + \boldsymbol{F}_{13} = m_1\boldsymbol{a}_1 \qquad (2.4)$$

Substituting in Equation 2, yields the acceleration for body 1

$$\boldsymbol{a}_1 = \frac{Gm_2(\boldsymbol{R}_2-\boldsymbol{R}_1)}{\|\boldsymbol{R}_2-\boldsymbol{R}_1\|^3} + \frac{Gm_3(\boldsymbol{R}_3-\boldsymbol{R}_1)}{\|\boldsymbol{R}_3-\boldsymbol{R}_1\|^3} \qquad (2.5a)$$

Likewise, the accelerations for bodies 2 and 3 are

$$\boldsymbol{a}_2 = \frac{Gm_1(\boldsymbol{R}_1-\boldsymbol{R}_2)}{\|\boldsymbol{R}_1-\boldsymbol{R}_2\|^3} + \frac{Gm_3(\boldsymbol{R}_3-\boldsymbol{R}_1)}{\|\boldsymbol{R}_3-\boldsymbol{R}_1\|^3} \qquad (2.5b)$$

$$\boldsymbol{a}_3 = \frac{Gm_1(\boldsymbol{R}_1-\boldsymbol{R}_3)}{\|\boldsymbol{R}_1-\boldsymbol{R}_3\|^3} + \frac{Gm_2(\boldsymbol{R}_2-\boldsymbol{R}_3)}{\|\boldsymbol{R}_2-\boldsymbol{R}_3\|^3} \quad (2.5c)$$

The velocities are related to the accelerations by

$$\frac{d\boldsymbol{v}_i}{dt} = \boldsymbol{a}_i \qquad (2.6)$$

Similarly, the positions are related to the velocities by

$$\frac{d\boldsymbol{R}_i}{dt} = \boldsymbol{v}_i \qquad (2.7)$$

These two equations can be used as a system of ordinary differential equations with respect to time. Given the initial positions and velocities of the bodies, numerical integration of Equations 6 and 7 will give the velocities and positions as functions of time.

First, each of the position and velocity vectors should be resolved into their components along the XYZ axes of the inertial reference frame.

$$\boldsymbol{R}_1 = \begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \end{Bmatrix} \quad \boldsymbol{R}_2 = \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \end{Bmatrix} \quad \boldsymbol{R}_3 = \begin{Bmatrix} X_3 \\ Y_3 \\ Z_3 \end{Bmatrix} \quad (2.8)$$

$$\boldsymbol{v}_1 = \begin{Bmatrix} \dot{X}_1 \\ \dot{Y}_1 \\ \dot{Z}_1 \end{Bmatrix} \quad \boldsymbol{v}_2 = \begin{Bmatrix} \dot{X}_2 \\ \dot{Y}_2 \\ \dot{Z}_2 \end{Bmatrix} \quad \boldsymbol{v}_3 = \begin{Bmatrix} \dot{X}_3 \\ \dot{Y}_3 \\ \dot{Z}_3 \end{Bmatrix} \quad (2.9)$$

Substituting into Equation 5

$$\boldsymbol{a}_1 = \begin{Bmatrix} \ddot{X}_1 \\ \ddot{Y}_1 \\ \ddot{Z}_1 \end{Bmatrix} = \begin{Bmatrix} \frac{Gm_2(\boldsymbol{X}_2-\boldsymbol{X}_1)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{X}_3-\boldsymbol{X}_1)}{\boldsymbol{R}_{13}^3} \\ \frac{Gm_2(\boldsymbol{Y}_2-\boldsymbol{Y}_1)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{Y}_3-\boldsymbol{Y}_1)}{\boldsymbol{R}_{13}^3} \\ \frac{Gm_2(\boldsymbol{Z}_2-\boldsymbol{Z}_1)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{Z}_3-\boldsymbol{Z}_1)}{\boldsymbol{R}_{13}^3} \end{Bmatrix} \quad (2.10a)$$

$$\boldsymbol{a}_2 = \begin{Bmatrix} \ddot{X}_2 \\ \ddot{Y}_2 \\ \ddot{Z}_2 \end{Bmatrix} = \begin{Bmatrix} \frac{Gm_1(\boldsymbol{X}_1-\boldsymbol{X}_2)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{X}_3-\boldsymbol{X}_2)}{\boldsymbol{R}_{13}^3} \\ \frac{Gm_1(\boldsymbol{Y}_1-\boldsymbol{Y}_2)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{Y}_3-\boldsymbol{Y}_2)}{\boldsymbol{R}_{13}^3} \\ \frac{Gm_1(\boldsymbol{Z}_1-\boldsymbol{Z}_2)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{Z}_3-\boldsymbol{Z}_2)}{\boldsymbol{R}_{13}^3} \end{Bmatrix} \quad (2.10b)$$

$$\boldsymbol{a}_3 = \begin{Bmatrix} \ddot{X}_3 \\ \ddot{Y}_3 \\ \ddot{Z}_3 \end{Bmatrix} = \begin{Bmatrix} \frac{Gm_2(\boldsymbol{X}_1-\boldsymbol{X}_3)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{X}_2-\boldsymbol{X}_3)}{\boldsymbol{R}_{23}^3} \\ \frac{Gm_2(\boldsymbol{Y}_1-\boldsymbol{Y}_3)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{Y}_2-\boldsymbol{Y}_3)}{\boldsymbol{R}_{23}^3} \\ \frac{Gm_2(\boldsymbol{Z}_1-\boldsymbol{Z}_3)}{\boldsymbol{R}_{12}^3} + \frac{Gm_3(\boldsymbol{Z}_2-\boldsymbol{Z}_3)}{\boldsymbol{R}_{23}^3} \end{Bmatrix} \quad (2.10c)$$

where $\boldsymbol{R}_{12} = \|\boldsymbol{R}_2 - \boldsymbol{R}_1\|$, $\boldsymbol{R}_{13} = \|\boldsymbol{R}_3 - \boldsymbol{R}_1\|$, $\boldsymbol{R}_{23} = \|\boldsymbol{R}_3 - \boldsymbol{R}_2\|$.

Next, the ode system vectors are formed from each of the components found above

$$\boldsymbol{y} = \{\boldsymbol{R}_1 \quad \boldsymbol{R}_2 \quad \boldsymbol{R}_3 \quad \boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3\}^T \quad (2.11)$$

$$\dot{\boldsymbol{y}} = \{\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3 \quad \boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \boldsymbol{a}_3\}^T \quad (2.12)$$

Because the accelerations are a function of the positions as shown in Equation 3, Equation 12 can be written as a function of position.

$$\dot{\boldsymbol{y}} = \boldsymbol{f}(t, \boldsymbol{y}) \qquad (2.13)$$

The system of ode's can be numerically integrated to find the bodies's positions at any given time.

Runge-Kutta integration[5]

The integration method used to determine the motion of bodies was a Runge-Kutta 4 solver. This method is a modified Euler method that uses a weighted average of four increments taken from the Taylor series. Given the value of a function y_n at a specific moment in time t_n, the value of the function y_{n+1} at any other time t_{n+1} can be found by taking the weighted average of the four Taylor series increments k_1, k_2, k_3, and k_4. The equations used in this method are given below. As illustrated, greater weight is given to the increments at the midpoints.

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \qquad (2.14)$$

$$t_{n+1} = t_n + h \qquad (2.15)$$

$$k_1 = f(t_n, y_n) \qquad (2.16)$$

$$k_2 = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1\right) \qquad (2.17)$$

$$k_3 = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2\right) \qquad (2.18)$$

$$k_4 = f\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_3\right) \qquad (2.19)$$

A fourth-order Runge-Kutta was chosen because of its wide use and efficiency—four evaluations of the function are required per step rather than one with the Euler method. The local error term is $O(h^5)$ and the global error term is $O(h^4)$. Higher-order (fifth, sixth, etc.) Runge-Kutta methods were not used because they require a greater number of function evaluations than the order of the method. Fourth-order and lower Runge-Kutta methods require the same number of evaluations as the order.

Simulation of the *n*-body problem employed the use of a build-up method to accurately test and represent the system. For this paper, the three bodies under consideration were the Sun, Jupiter, and an Earth-like planet. The three different cases of simulation are detailed below.

The Sun-Jupiter system

The first case simulated was the Sun-Jupiter system, modeled as a two-body problem. This simulation served to test that the code properly models the n-body problem and planetary systems in nature. The expected solution should match the Keplerian orbit satisfied by Newton's second law. Figure 3 illustrates the Sun-Jupiter system under consideration.

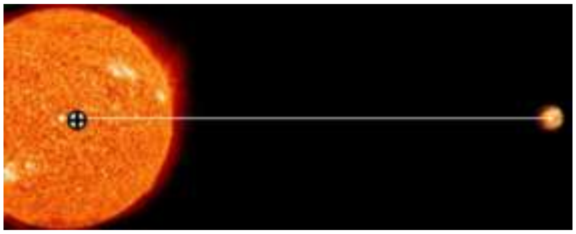


Figure 3 Case 1: Sun-Jupiter System (not to scale).

For verification of the code, the simulation data was plotted against Keplerian predictions. Kepler's first law states that a planet orbits the Sun in an ellipse, with the Sun at one focus of the ellipse. Equation 2.20 below defines the path of a planet around the Sun as an ellipse.

$$r = \frac{h^2}{\mu(1+e\cos\theta)} \qquad (2.20)$$

where r is the radius, h is the angular momentum, μ is the gravitational parameter, e is the eccentricity, and θ is the true anomaly. This equation assumes that the angular momentum, gravitational parameter, and eccentricity are constant. For the Keplerian prediction, this radial position of Jupiter was plotted over the true anomaly. For the simulation data, the position of Jupiter in the Cartesian coordinate system (x,y,z) was converted into polar coordinates (r, θ) and then plotted over the true anomaly to be compared with Kepler's data.

Kepler's second law states that a line connecting a planet to the Sun sweeps out equal areas in equal time intervals. This implies that the angular momentum of the system remains constant. Equation 2.21 gives the angular momentum of a planetary system.

$$h = \sqrt{\mu a(1 - e^2)} \qquad (2.21)$$

For the Keplerian prediction, this angular momentum was plotted as a constant over time. For the simulation data, the angular momentum was calculated using Jupiter's position and velocity.

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v} \qquad (2.22)$$

This angular momentum was also plotted over time and compared with Kepler's data.

Kepler's third law states that the square of the orbital period of a planet is directly proportional to the cube of its semimajor axis. This relation is given in Equation 2.23.

$$T^2 = \frac{2}{\mu} a^3 \qquad (2.23)$$

For the Keplerian prediction, a range of semimajor axes were used to calculate the variation in period. For the simulation data, the same range of semimajor axes were used to generate various orbital motions of Jupiter. For each case, the variation in period was found. For both sets of data, the log of the period was plotted against the log of the semimajor axis. A line with slope 2/3 was expected.

Once the two-body case was verified to produce Keplerian orbits, a third Earth-like body was added to the system.

The Sun-Jupiter-Earth system

The second case simulated was the Sun-Jupiter-Earth system, modeled as a three-body problem in the ecliptic plane. This simulation again tested that the code accurately models the *n*-body problem and planetary systems found in nature. The expected solution should match the Keplerian orbits of our own Solar System. While Kepler's equations only apply for the two-body system, the three-body simulation was analyzed using Kepler's laws for comparison. This is an acceptable analysis since the Sun and Jupiter contribute the majority of the mass to the three-body system. The Sun-Jupiter-Earth system is a simplified version (i.e. in plane) of the primary question under consideration (i.e. out of plane). The Sun-Jupiter-Earth system is illustrated in Figure 4 below.

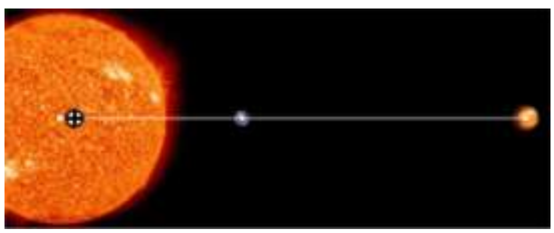


Figure 4 Case 2: Sun-Jupiter-Earth System in Plane (not to scale).

This second case was also used to determine the appropriate time step needed in simulation. The time step must be large enough so as not to take an infinite amount of time to compute, but also small enough so as not to distort the motion of the planets. In addition, the simulated orbits are expected to decay over time, as shown in Figure 5.

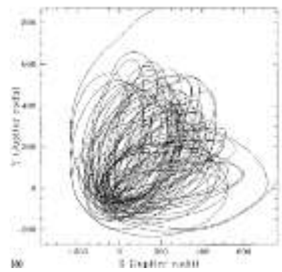


Figure 5 Orbital decay over time.[6]

This decay is simply a product of the numerical techniques used in solving the system. The time to decay will be noted, but will not negate orbital solutions up to that point in time.

The Sun-Jupiter-Earth inclined system

The third case simulated was the Sun-Jupiter-Earth system inclined, modeled as a three-body problem out of the ecliptic plane. The setup of this system is shown below in Figure 6.

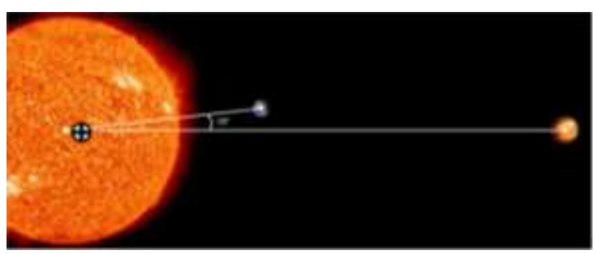


Figure 6 Case 3: Sun-Jupiter-Earth System Inclined (not to scale).

The simulation begins by positioning the Earth-like planet ten degrees out of the ecliptic plane as shown. Next, the code iterates the position of the Earth-like planet out of plane to see how its motion over time is affected. The orbital solutions of motion and times to orbital decay were measured for each iteration. In this manner, a minimum inclination out of plane was found for which an Earth-like planet would be ejected from the Solar System. It is expected that the rate of orbital decay will increase for increasing inclinations.

Exoplanetary system

The final cases simulated were other possible solar systems with different configurations of planets and host stars. The setup is the same as in the Sun-Jupiter-Earth Inclined System but with different masses and separation distances modeled after actual exoplanetary systems. In this paper, a hot Jupiter was simulated by moving a Jupiter-like planet into a Martian orbit at 1.5 AU with an eccentricity of 0.09. The setup of this system is shown below in Figure 7.

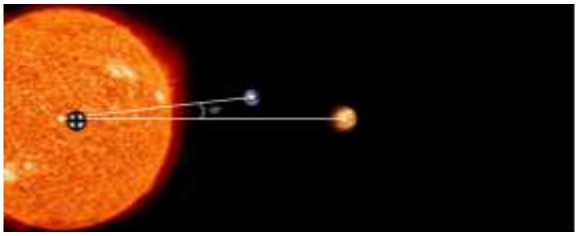


Figure 7 Case 3: Exoplanetary System Inclined (not to scale).

The iterative simulation again finds the maximum angle out of the ecliptic plane which the Earth-like planet can reside before it is ejected from the system. It is expected that the results of other exoplanetary systems will closely match those of our Solar System.

CHAPTER III

RESULTS

Using the equations defined above for the *n*- body problem, an *n*-body solver was developed in MATLAB. Input variables include orbital elements of each body, mass of each body, the number of orbits integrated (time), the number of data points per orbit, the number of orbits stored, and the number of orbits plotted. Appendix A shows a flowchart of the logic followed. Table 2 below tabulates the various input values for each case simulated.

Table 2 Simulation Cases with Parameters.

Variable	Sun-Jupiter System	Sun-Jupiter-Earth In plane System	Sun-Jupiter-Earth Inclined System	Exoplanetary System
Sun-Body	$m = 1.989 \times 10^{30}$ kg Initial Positions and Velocities from Origin			
Jupiter-Body	$m = 1.899 \times 10^{27}$ kg $a = 778.6 \times 10^{6}$ km $e = 0.0489$ Initial Positions and Velocities from Apoapsis ($\theta = \pi$)			$m = 1.899 \times 10^{27}$ kg $a = 227.9 \times 10^{6}$ km $e = 0.0933$ Initial Positions and Velocities from Apoapsis ($\theta = \pi$)
Earth-Body	n/a	$m = 5.974 \times 10^{24}$ kg $a = 149.6 \times 10^{6}$ km $e = 0.0167$ Initial Positions and Velocities from Apoapsis ($\theta = \pi$)		
Inclination	0°		0°, 50°, 85°	0°, 10°, 50°, 85°

The Jupiter and Earth were given initial positions at their apoapsi. Because the apoapsis is the farthest orbital point from the Sun for each planet, it has the greatest potential to add uncertainty to the motion calculations. Thus, by precisely calculating the conditions at this point and starting each planet there, the error is greatly reduced. The resolution of orbital data was chosen for each case such that the Earth would have at least 360 data points per orbit (a data point per degree at minimum). The time step for integration was chosen based on this resolution and the orbital period of the Jupiter planet. The standard unit of time in each simulation was based on the orbital period of the Jupiter planet, since its orbital period remained nearly constant throughout the simulation. The motion of each body was plotted from the Sun-Jupiter system center of mass as opposed to the Sun-Jupiter-Earth system of mass so that an Earth thrown from the system would not affect the system's translation through space.

The Sun-Jupiter system

The first case simulated was the Sun-Jupiter system, modeled using the two-body problem in the ecliptic plane. This simulation served to test that the code was properly modeling the *n*-body problem and giving results that follow Kepler's model. Plotting Jupiter's position over time in the inertial reference frame, Figure 8 shows the elliptical orbit of Jupiter about the Sun. The initial positions of the Sun and Jupiter are depicted as spheres. This Sun-Jupiter system remained stable for 10,000 Jupiter orbits, the longest time used in simulating various inclined orbits.

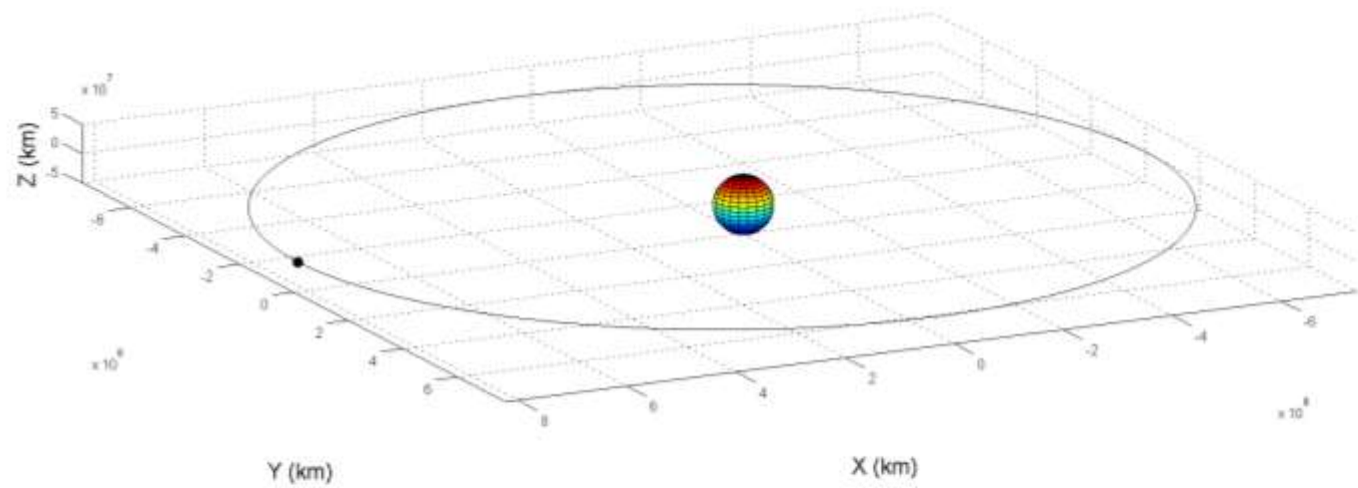


Figure 8 Orbit of Jupiter about the Sun.

To prove that the two-body code accurately models what is observed in nature, it was necessary to verify that Kepler's laws of planetary motion were upheld. For Kepler's first law, Figure 9 plots Jupiter's radius from the Sun versus rotation angle to show Jupiter's position varying in time. The Keplerian prediction is plotted in black, while the two-body simulation is plotted in red. The equation used to generate the Kepler prediction was based on Equation 2.20.

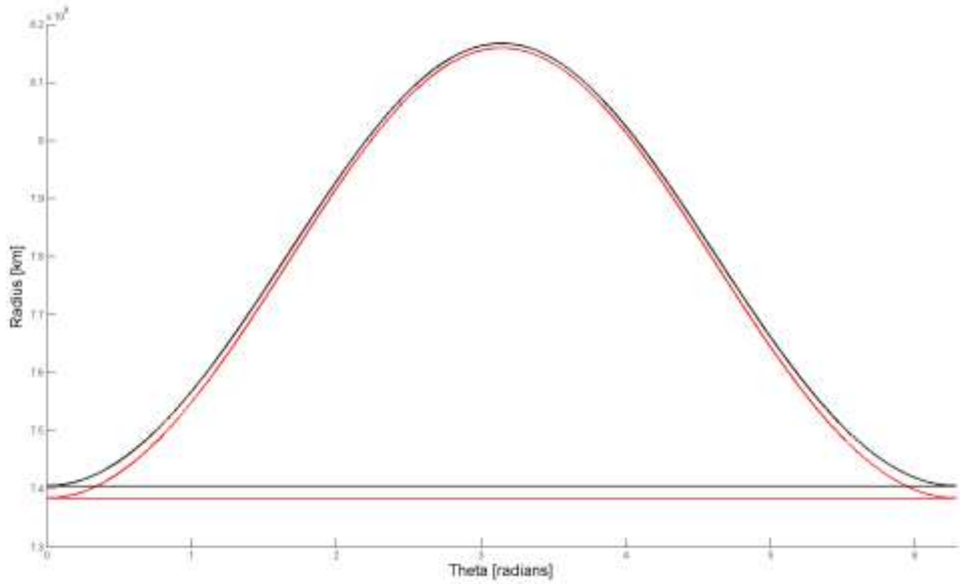


Figure 9 Verification of Kepler's First Law.

As seen in Figure 9, the simulation data and Keplerian predictions agree within 1% error. The slight shift seen between the lines is a product of the reference frame used in the simulation. While Kepler’s equations assume positions relative to the system center of mass, the simulation stores positions relative to the Sun’s center of mass at the origin. This accounts for the slight offset seen above.

For Kepler’s second law, Figure 10 plots the angular momentum of Jupiter over rotation angle. The Keplerian prediction is plotted in black, while the two-body simulation is plotted in red. The equation used to generate the Kepler prediction was based on Equation 2.21.

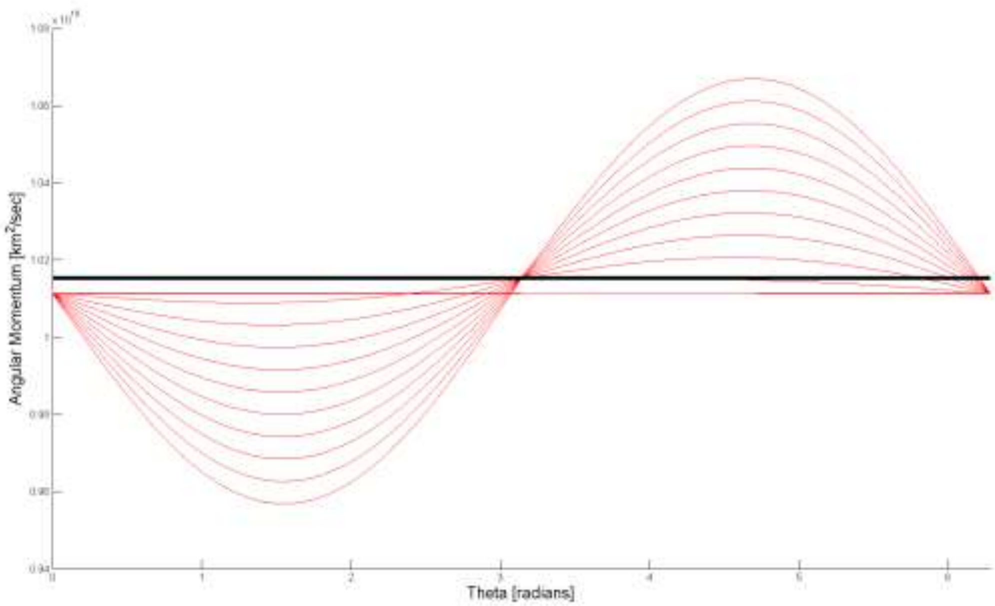


Figure 10 Verification of Kepler's Second Law.

As seen in Figure 10, the angular momentum of Jupiter is relatively constant and only varies from Kepler’s prediction by 0.3%. The sinusoidal variation over time is due to the exchange of momenta between Jupiter and the Sun. Because neither of these bodies is at rest—both orbit around the system center of gravity—this exchange of momenta is expected so long as the overall angular momentum of the Sun-Jupiter system is conserved.

For Kepler’s third law, Figure 11 plots the log of Jupiter’s period versus the log of Jupiter’s semi-major axis. The Keplerian prediction is shown in black with circular points, while the two-body simulation is plotted in red. The equation used to generate the Kepler prediction was based on Equation 2.23.

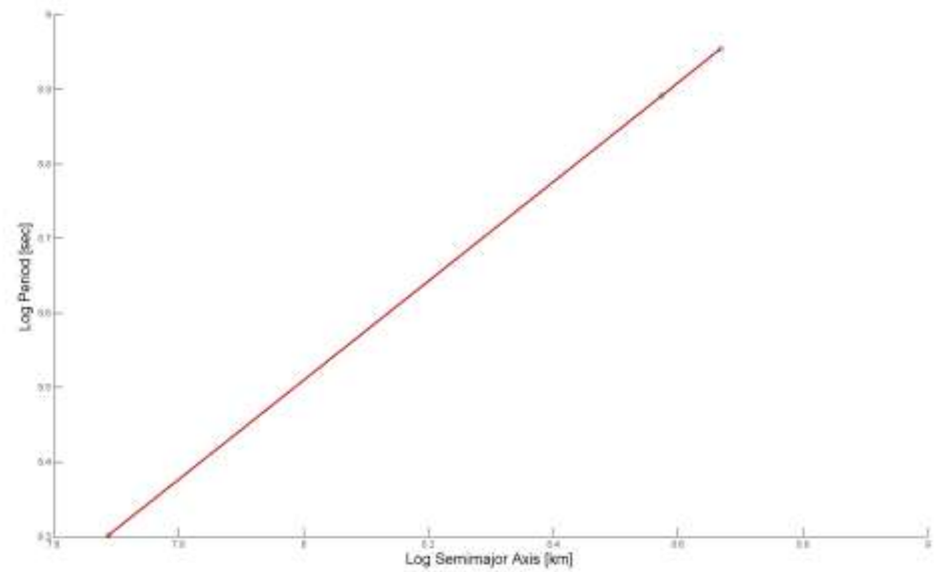


Figure 11 Verification of Kepler's Third Law.

As seen in Figure 11, the resulting lines both have a slope of 2/3, which serves as verification of Kepler's third law. Thus, the two-body simulation has been shown to satisfy all three of Kepler's laws and can be considered an accurate representation of our Solar System.

The Sun-Jupiter-Earth system

The second case simulated was the Sun-Jupiter-Earth system, modeled as a three-body problem in the ecliptic plane. This simulation again tested that the code accurately models planetary systems in nature. Plotting Jupiter and Earth's positions over time in the inertial frame, Figure 12 shows their elliptical orbits about the Sun. The initial positions of the Sun, Jupiter, and Earth are depicted as spheres. This Sun-Jupiter-Earth system remained stable for 10,000 Jupiter orbits, the longest time used in simulating various inclined orbits. One Jupiter period is equivalent to roughly 11.86 Earth years.

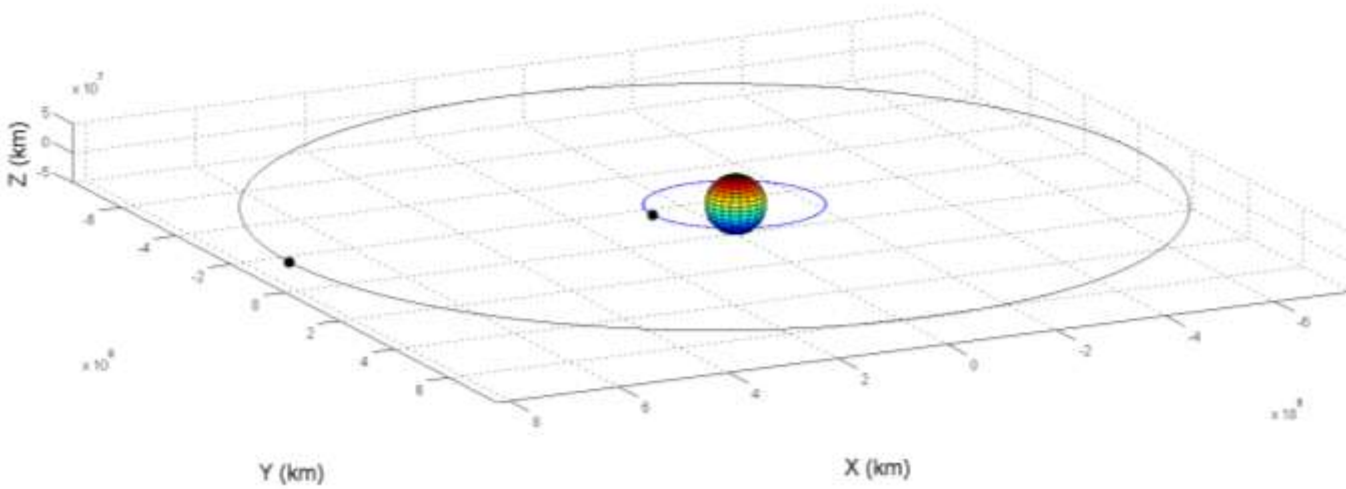


Figure 12 Orbits of Jupiter and Earth about the Sun.

While Kepler's laws are only able to predict motion for two planetary bodies, the three-body simulation data was analyzed using Kepler's equations for comparison. Because the Sun and Jupiter are the primary contributing masses to the system, the Keplerian predictions for Jupiter in the three-body system were expected to match those of the two-body system. The Keplerian predictions for Earth in the three-body system were not expected to agree since Earth is greatly influenced by Jupiter, which is not accounted for by Kepler's equations. For Kepler's first law, Figure 13 plots the planets' radii from the Sun versus rotation angle to show their positions varying in time. The Keplerian prediction is plotted in black, while the simulated Jupiter (top) is plotted in red and the simulated Earth (bottom) is plotted in blue.

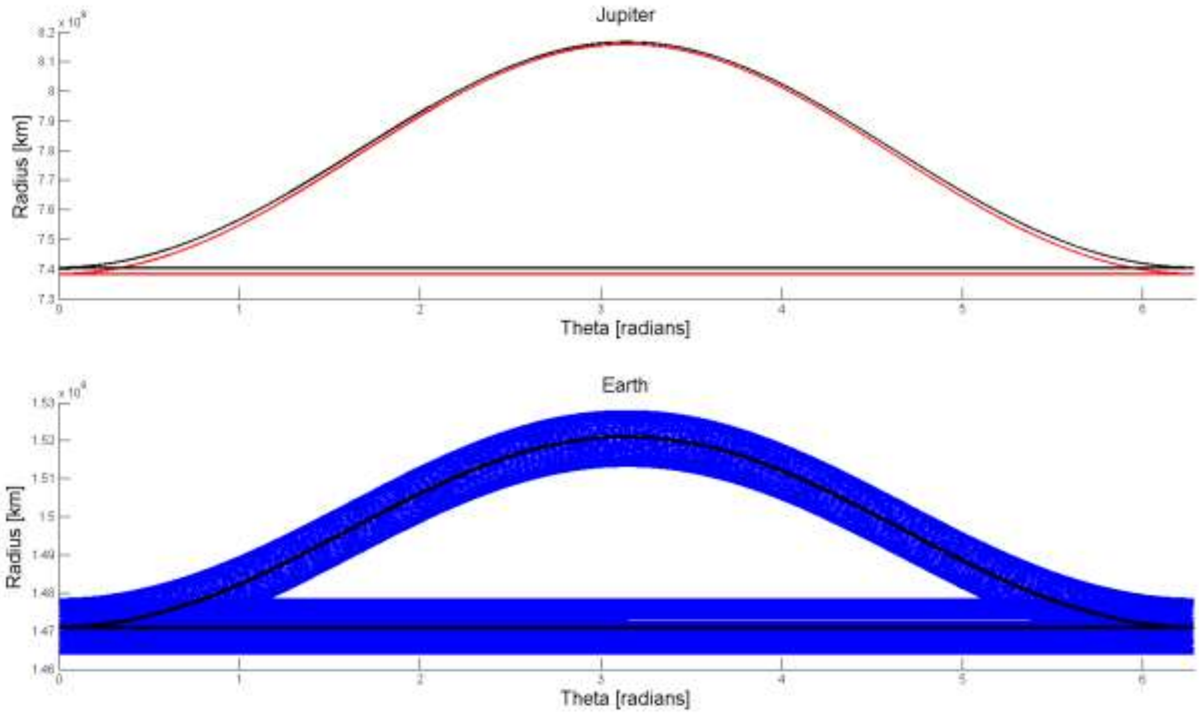


Figure 13 Verification of Kepler's First Law.

As seen in Figure 13, Jupiter's radial position agrees within 0.03% error of that predicted by Kepler's equations. In addition, these three-body results match almost perfectly with the two-body results given in Figure 9. The simulated Earth's radial position varies noticeably on either side of the Keplerian prediction when compared with Jupiter's, but still agrees within 0.07% error of Kepler's equations. As evidenced by the variance in its radius, Earth is very affected by the motion of Jupiter, which Kepler's equations cannot account for due to the two-body limitation. As stated earlier, the slight offset between the simulation data and the Keplerian predictions is a product of the reference frame used in the simulation.

For Kepler's second law, Figure 14 plots the angular momenta of Jupiter and Earth over rotation angle. The Keplerian prediction is plotted in black, while the simulated Jupiter (top) is plotted in red and the simulated Earth (bottom) is plotted in blue.

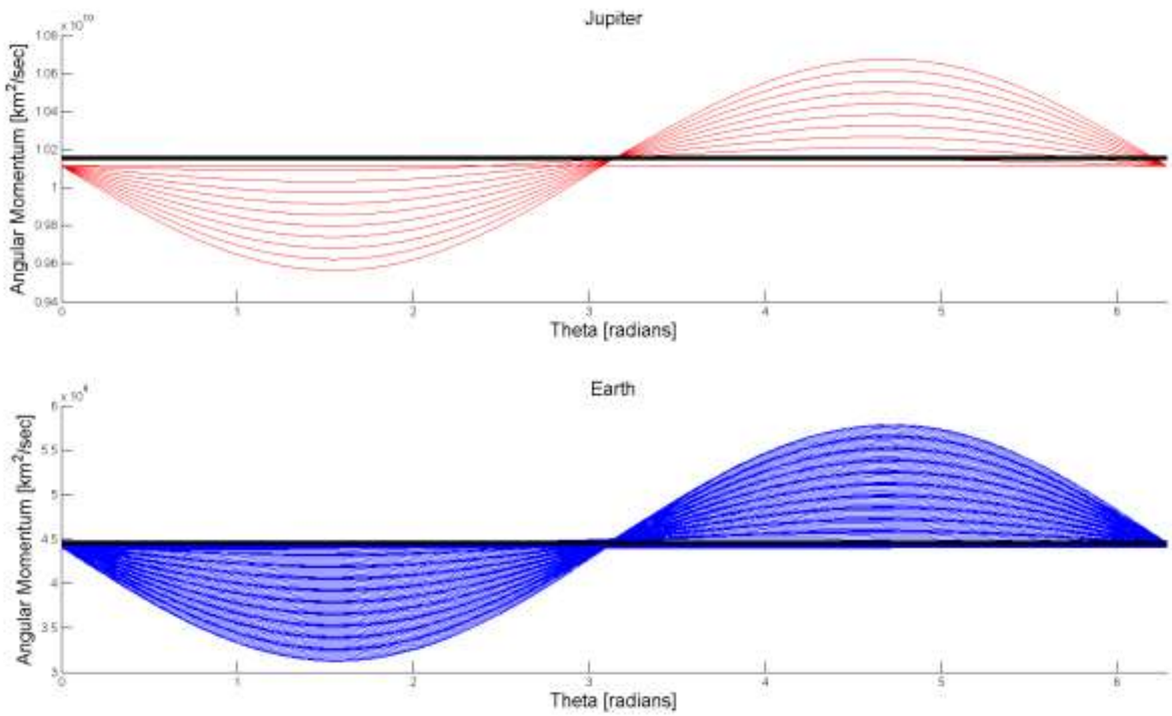


Figure 14 Verification of Kepler's Second Law.

As seen in Figure 14, the angular momentum of Jupiter remains relatively constant over the time period and matches within 0.3% of that predicted by Kepler’s equations. In addition, these three-body results match almost perfectly with the two-body results given in Figure 10. This is expected since the Sun and Jupiter are the primary contributing masses to the system. The Earth’s angular momentum agrees to Keplerian predictions within 0.04% error. As stated earlier, the sinusoidal variation over time is due to the exchange of momenta between the planets and the Sun. This exchange of momenta is expected so long as overall angular momentum of the Sun-Jupiter-Earth system is conserved.

For Kepler’s third law, Figure 15 plots the log of each planet’s period versus the log of its semimajor axis. The Keplerian prediction is plotted in black with circular points, while the simulated Jupiter (left) is plotted in red and the simulated Earth (right) is plotted in blue.

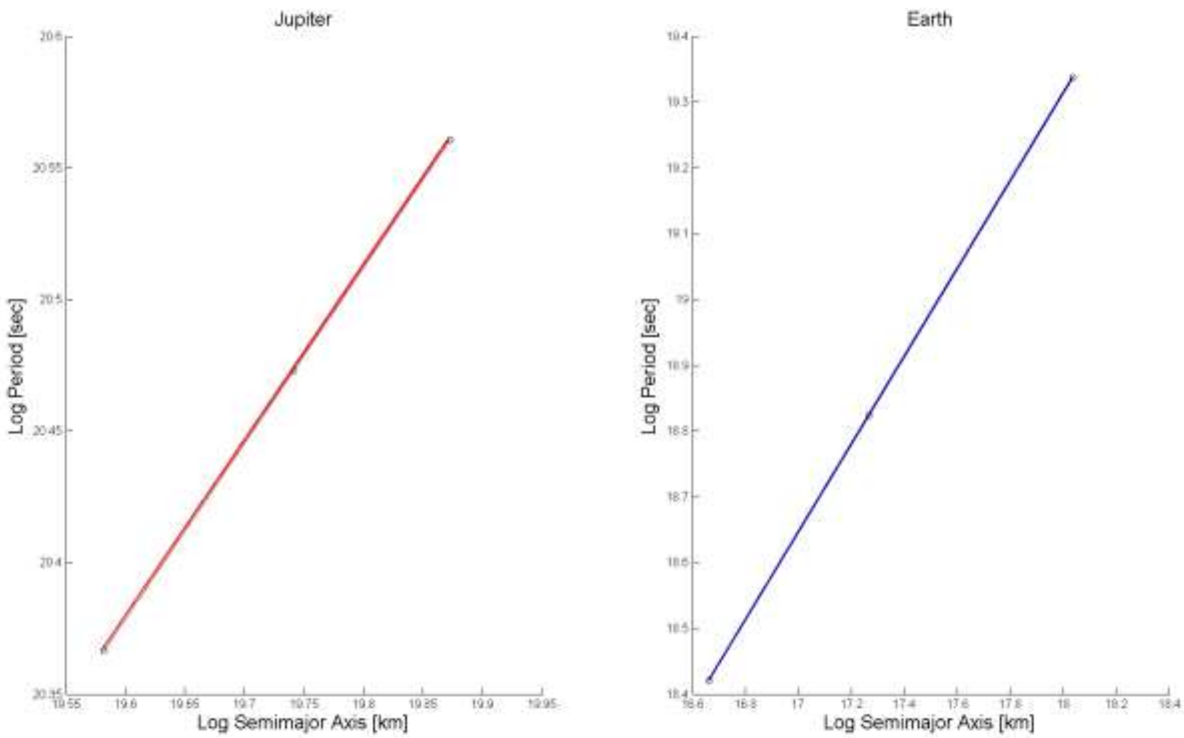


Figure 15 Verification of Kepler's Third Law.

As seen in Figure 15, the resulting lines each have a slope of 2/3, which serves as verification of Kepler's third law. Thus, the three-body simulation has been shown to satisfy all three of Kepler's laws when compared to the two-body simulation and to be stable over an adequate range of time for simulation.

The Sun-Jupiter-Earth inclined system

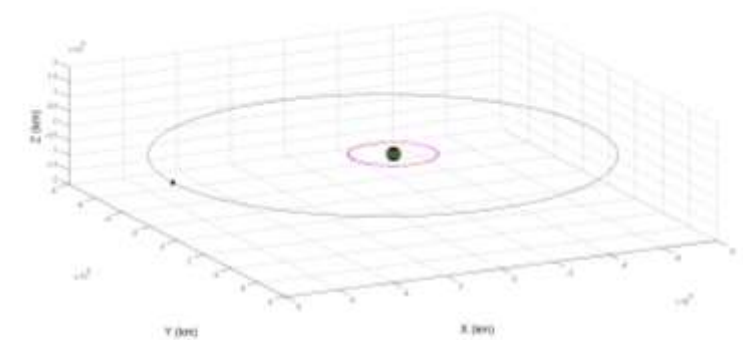
The third case simulated was the Sun-Jupiter-Earth system inclined, modeled as a three-body problem out of the ecliptic plane. Because the inclined orbits were expected to go chaotic, each block of orbits was plotted in a different color so that orbital evolution was visible. The orbits change in color from magenta to red to dark blue over time. Each iteration was run over 10, 000

Jupiter orbits (119,000 Earth Years) and the time to ejection of the Earth from the system was noted. Table 3 shows the initial conditions used for this simulation.

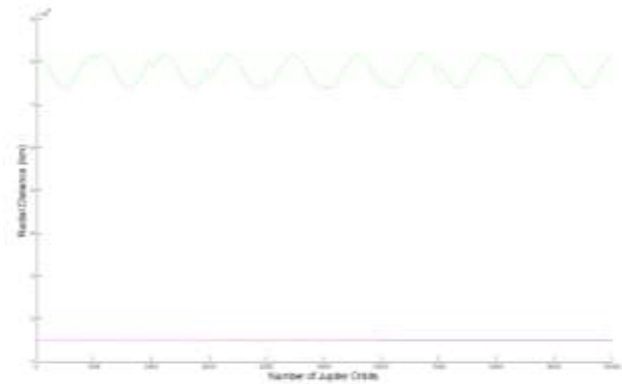
Table 3 Initial Conditions for Sun-Jupiter-Earth Inclined System.

Variable	Sun-Body	Jupiter-Body	Earth-Body
Mass	1.989×10^{30} kg	1.899×10^{27} kg	5.974×10^{24} kg
Semimajor Axis	--	778.6×10^{6} km	149.6×10^{6} km
Eccentricity	--	0.0489	0.0167
Initial Position and Velocity	From Origin	From Apoapsis ($\theta = \pi$)	From Apoapsis ($\theta = \pi$)
Inclination	0°	0°	0°, 50°, 85°

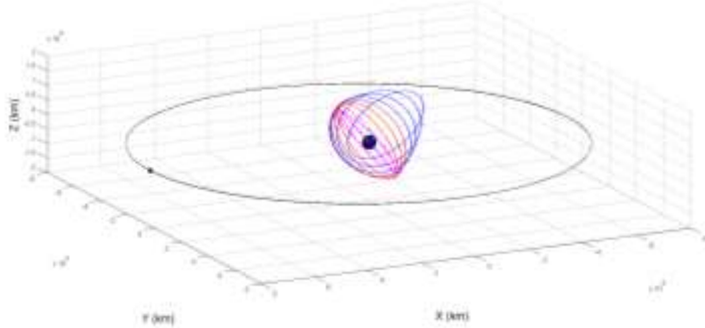
The three-body solver iterated through inclinations of 0°, 50°, and 85° for the Earth-like planet. The results for each simulation are shown in Figure 16. The plots in the left column show the orbits of Jupiter (black) and Earth (multi) about the Sun over time. The plots in the right column show the radial distance of Jupiter (green) and Earth (multi) from the Sun over time.



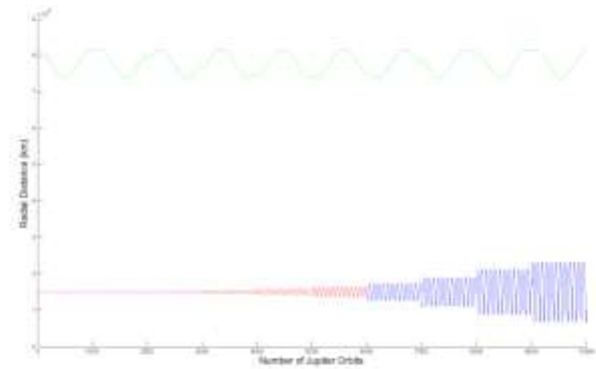
(a) Orbit of Jupiter and Earth inclined 0° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



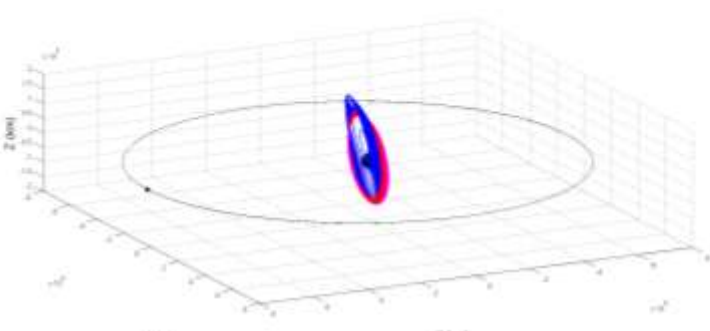
(b) Radial distance from the Sun. Earth inclined 0° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



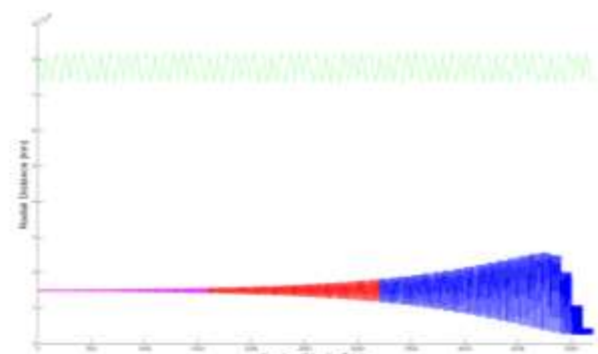
(c) Orbit of Jupiter and Earth inclined 50° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



(b) Radial distance from the Sun. Earth inclined 50° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



(e) Orbit of Jupiter and Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system.



(f) Radial distance from the Sun. Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system around 5,200 Jupiter orbits.

Figure 16 Earth-Jupiter-Sun System Inclined 0°-50°-85° Results.

As shown, the first case (a, b) inclined 0° to the ecliptic is stable over the 10,000 Jupiter orbit regime. The radial distances of Jupiter and Earth oscillate and correctly model elliptical orbit motion about the Sun.

The second case (c, d) inclined 50° to the ecliptic begins to show signs of instability over the 10,000 Jupiter orbit regime. The Earth's orbit begins to precess about the Sun and its eccentricity increases, as well as its radial distance from the Sun. While this system shows the potential to go highly chaotic, this case should be run over a longer time period to see how Earth's orbit evolves.

The final case (e, f) inclined 85° to the ecliptic produces strong enough gravitational perturbations to throw the Earth out of the Solar System around 5,200 Jupiter orbits. The Earth's orbit becomes more eccentric over time. It's radial distance from the Sun first increases, and then rapidly diminishes beginning around 4,800 Jupiter orbits until it is ejected from the system completely. Because the Earth appears to be pulled in very close to the Sun over time, it is possible that it was sent inward by its interactions with Jupiter and collided with the Sun. However, this simulation modeled the Sun as a point mass and more analysis will be needed to determine whether it was ejected from the system or sent to collide with the Sun. Figure 17 shows the Earth's ejection as a function of radial distance from the Sun after only 5,200 Jupiter orbits (62,000 Earth years). Additional plots for the Sun-Jupiter-Earth inclined systems are given in Appendix B.

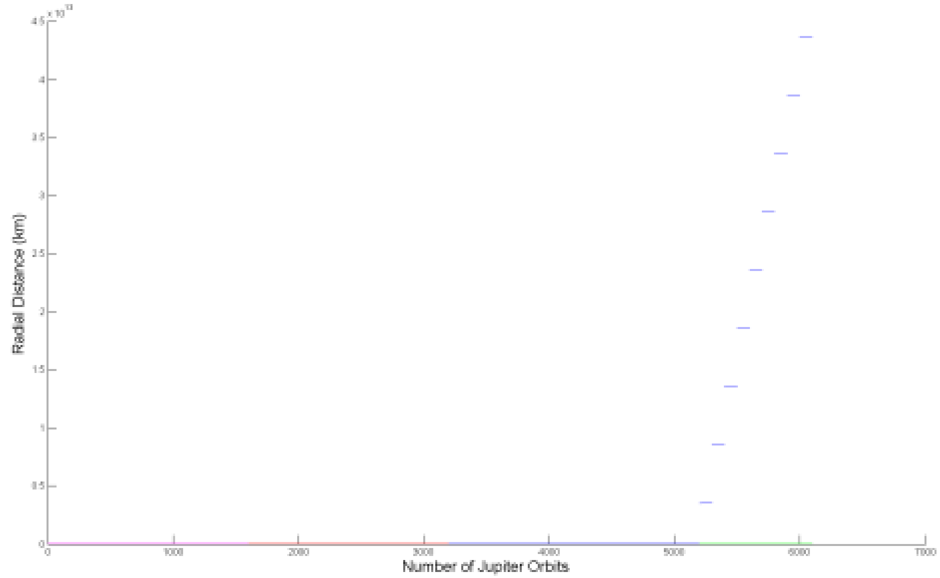


Figure 17 Earth ejected from the Sun-Jupiter-Earth System inclined 85° to the ecliptic. Ejection occurred at 5,200 Jupiter orbits (62,000 Earth years).

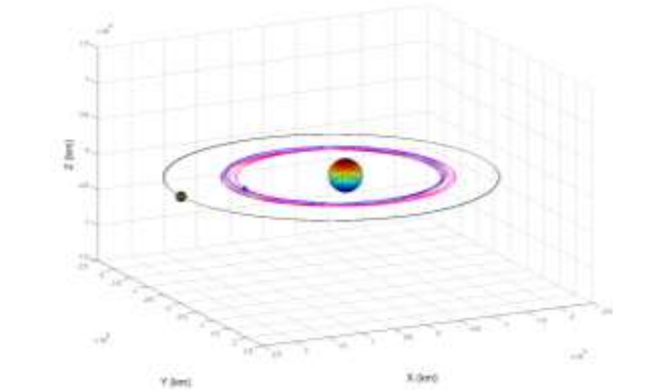
Exoplanetary system

The final cases simulated focused on exoplanetary systems. Because hot Jupiters are one of the most prevalent types of exoplanets found to date, the following cases brought Jupiter from a 5.2 AU orbit about the Sun into an equivalent Martian orbit 1.5 AU about the Sun. Because the inclined orbits were again expected to go highly chaotic, each block of orbits was plotted in a different color so that the orbital evolution was visible. The first block of orbits is plotted in magenta, the second in red, and the third in blue. Each iteration was run over 50, 000 Martian orbits (94,000 Earth Years). Table 4 shows the initial conditions used for this simulation.

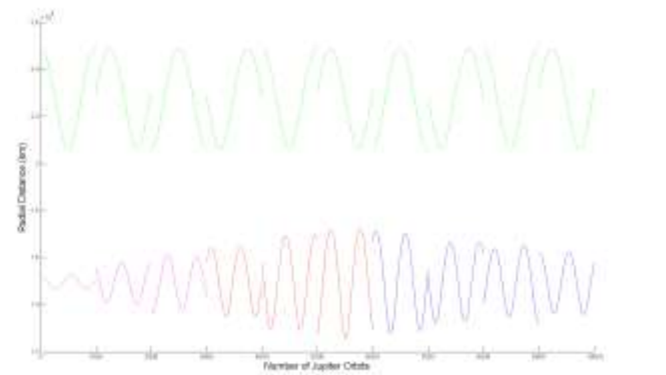
Table 4 Initial Conditions for the Exoplanetary System.

Variable	Sun-Body	Jupiter-Body	Earth-Body
Mass	1.989×10^{30} kg	1.899×10^{27} kg	5.974×10^{24} kg
Semimajor Axis	--	227.9×10^{6} km	149.6×10^{6} km
Eccentricity	--	0.0933	0.0167
Initial Position and Velocity	From Origin	From Apoapsis ($\theta = \pi$)	From Apoapsis ($\theta = \pi$)
Inclination	$0°$	$0°$	$0°$, $10°$, $50°$, $85°$

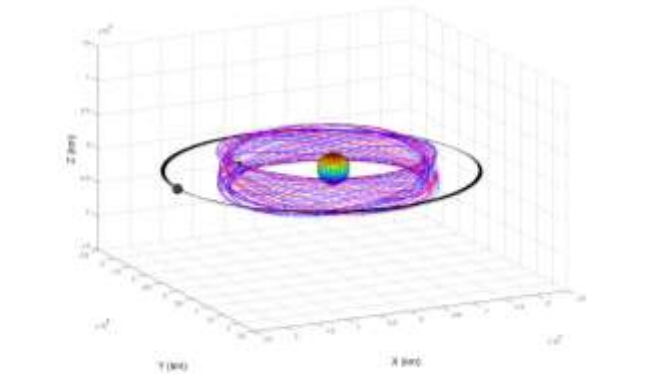
The three-body solver iterated through inclinations of 0°,10°, 50°, and 85° for the Earth-like planet. The results for each simulation are shown in Figure 18. The plots in the left column show the orbits of Jupiter (black) and Earth (multi) about the Sun over time. The plots in the right column show the radial distance of Jupiter (green) and Earth (multi) from the Sun over time.



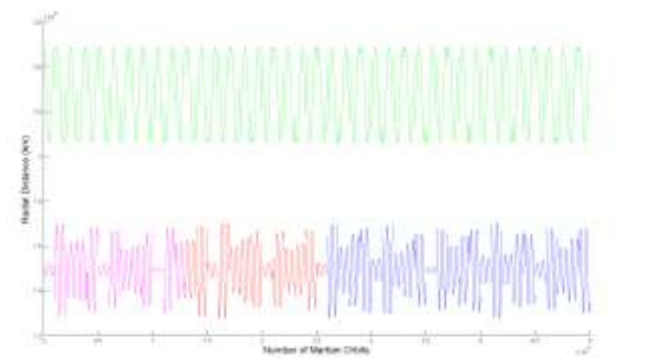
(g) Orbit of Hot Jupiter and Earth inclined 0° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



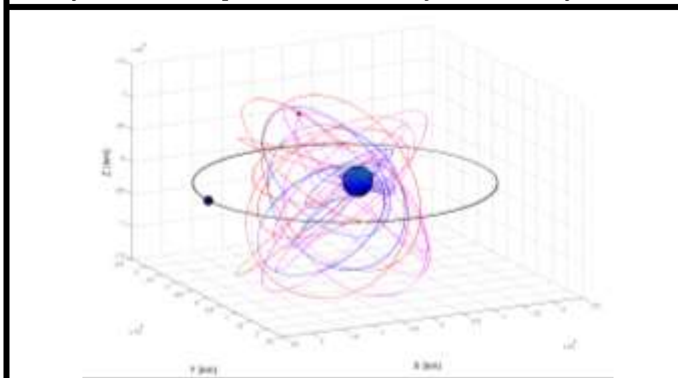
(h) Radial distance from the Sun. Earth inclined 0° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



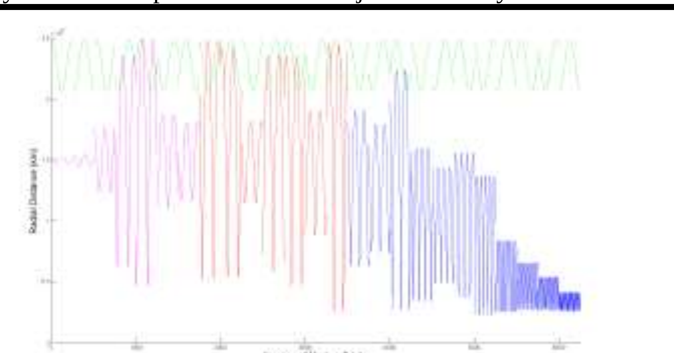
(i) Orbit of Hot Jupiter and Earth inclined 10° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



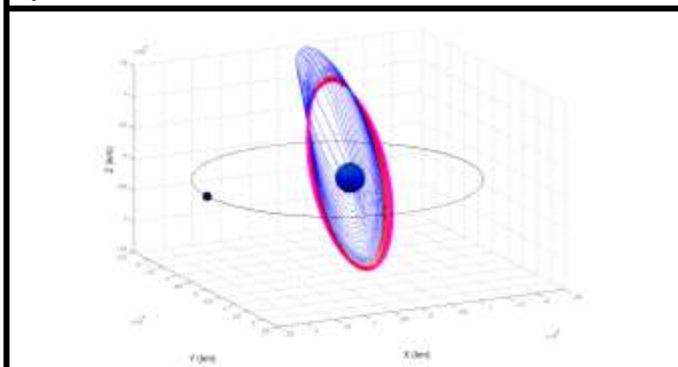
(j) Radial distance from the Sun. Earth inclined 10° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



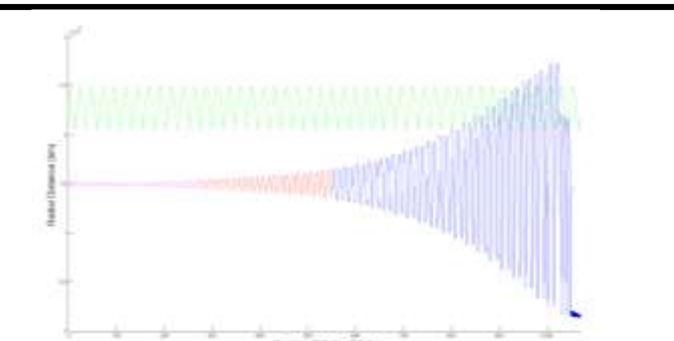
(k) Orbit of Hot Jupiter and Earth inclined 50° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system.



(l) Radial distance from the Sun. Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system around 6,250 Martian orbits.



(m) Orbit of Hot Jupiter and Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system.



(n) Radial distance from the Sun. Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system around 1,050 Martian orbits.

Figure 18 Exoplanetary System Inclined 0°-10°-50°-85° Results.

As shown, the first case (g, h) inclined 0° to the ecliptic is stable over the 50,000 Martian orbit regime. The radial distances of Jupiter and Earth oscillate and correctly model elliptical orbit motion about the Sun. The motion of Earth is more easily perturbed by the hot Jupiter than in the Sun-Jupiter-Earth system because it is much closer to the Earth by 2.7AU. This is evident in the radial distance plot as Earth's motion oscillates at a lower frequency in addition to a higher frequency. The higher frequency motion is a product of the elliptical orbit of Earth, while the lower frequency is a consequence of the periodic close encounters with the hot Jupiter.

The second case (i, j) inclined 10° to the ecliptic appears to be stable over the 50,00 Martian orbit regime. While Earth's orbit remains out of the plane, its instability is bound overtime and does not grow to become chaotic. It's orbital eccentricity remains constant. The Earth's orbit has the potential to go chaotic only over very large time scales that would not be relevant to systems in existence today.

The third case (k, l) inclined 50° to the ecliptic produces strong enough gravitational perturbations to throw the Earth out of the system around 6,250 Martian orbits. The Earth's orbit becomes more eccentric over time and exhibits rapid precession about the Sun. The Earth's radial distance from the Sun immediately increases to that of Jupiter's and oscillates between this orbit and a lower orbit for approximately 3,000 Martian orbits. This lower frequency oscillation in Earth's motion is again a sign of its periodic close encounters with the hot Jupiter. Around 4000 Martian orbits, the Earth's orbit begins to decay until it is ejected from the system completely. Figure 19 shows the Earth's ejection as a function of radial distance from the Sun after 6,250 Martian orbits (12,000 Earth years).

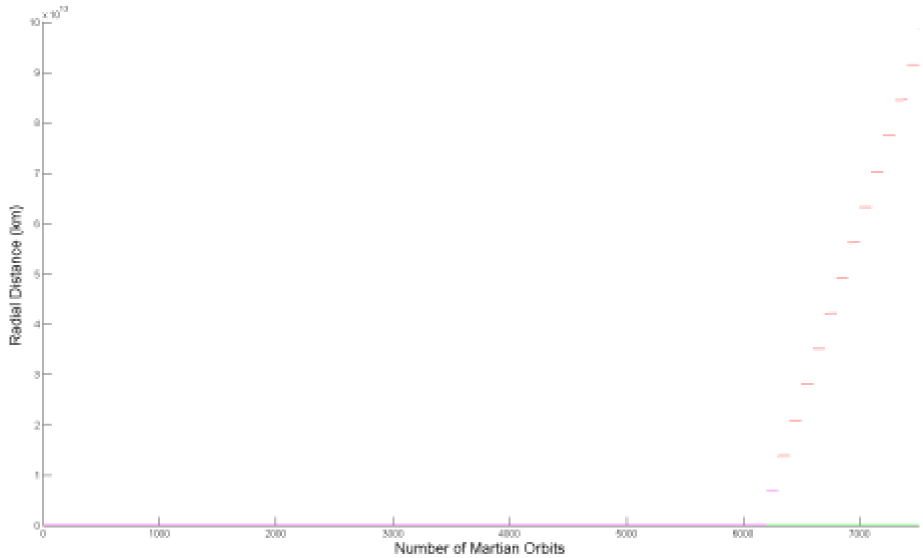


Figure 19 Earth ejected from the Exoplanetary System inclined 50° to the ecliptic. Ejection occurred at 6,250 Martian orbits (12,000 Earth years).

The final case (m, n) inclined 85° to the ecliptic also produces adequate gravitational interactions to eject Earth from the system. In this instance, however, Earth's orbit does not become chaotic. Instead, its eccentricity increases slowly over time as its radial distance from the Sun increases exponentially. This pattern continues over approximately 1,000 Martian orbits. After such, the Earth experiences a massive interaction with Jupiter and is sent hurtling toward the Sun. Its radial distance changes from an one beyond Jupiter's to one inside Mercury's in a matter of Martian orbits. As said previously, this simulation modeled the Sun as a point mass and more analysis is needed to determine whether the Earth was ejected from the system or sent to collide with the Sun. However, the time to Earth's orbital decay in the hot Jupiter system was much less than that in the Solar Jupiter system. Figure 20 shows the Earth's ejection as a function of radial distance from the Sun after only 1,050 Martian orbits (2,000 Earth years). Additional plots for the Exoplanetary inclined systems are given in Appendix C.

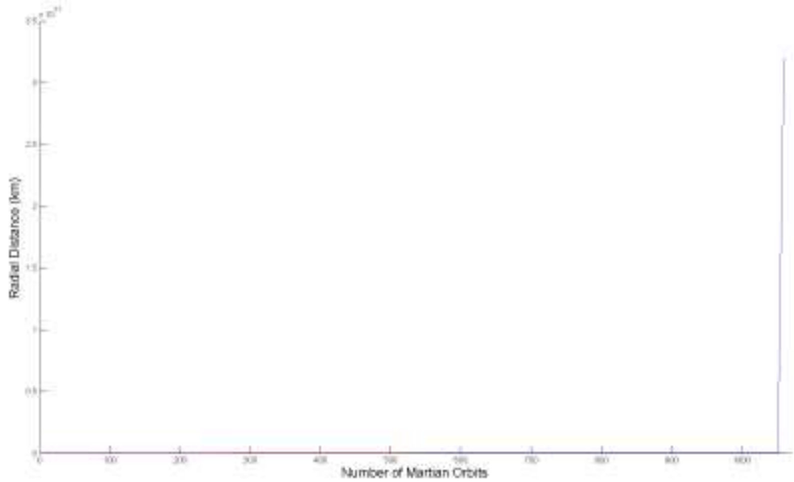


Figure 20 Earth ejected from the Exoplanetary System inclined 85° to the ecliptic. Ejection occurred at 1,050 Martian orbits (2,000 Earth years)

Simulation summary

In summary, these results have shown that inclining an Earth-like planet to the ecliptic plane increases the system's dynamic instability. In addition, moving the Jupiter planet inward increases the magnitude of influence on the Earth-like planet and decreases the time to decay for Earth's orbit. Table 5 below tabulates the time to Earth ejection for each of the cases presented above.

Table 5 Time to Earth Ejection Comparisons.

Inclination	Sun-Jupiter-Earth Inclined System	Exoplanetary System
0°	**Not Ejected** **119,000+ Earth Years** (10,000+ Jupiter Orbits)	**Not Ejected** **94,000+ Earth Years** (50,000+ Martian Orbits)
10°	--	**Not Ejected** **94,000+ Earth Years** (50,000+ Martian Orbits)
50°	**Not Ejected** **119,000+ Earth Years** (10,000+ Jupiter Orbits)	**Ejected** **12,000 Earth Years** (~6,250 Martian Orbits)
85°	**Ejected** **62,000 Earth Years** (~5,200 Jupiter Orbits)	**Ejected** **2,000 Earth Years** (~1,050 Martian Orbits)

Future work will include longer run times for low inclinations and intermediate inclinations not simulated above. These cases will serve to complete the picture of planetary system dynamics presented above.

CHAPTER IV

CONCLUSIONS

In conclusion, Jupiter has a huge gravitational influence on the orbits of smaller bodies in the Solar System. By simulating the *n*-body problem, how this influence acts on an Earth-like planet was quantified. The conditions for which an Earth-like planet would be ejected from the Solar System were found through iteration of its inclination to the ecliptic plane. For the Sun-Earth-Jupiter system simulated in this paper (run over 119,000 years), orbits inclined to the ecliptic plane greater than 50° became unstable, with Earth ejection after 62,000 years (85°).

Furthermore, simulation of other solar systems leads to a more general theory on the impact of planetary formation and heavy bombardment on the fate of Earth-like planets elsewhere in the Universe. For the exoplanetary system simulated in this paper, which includes a hot Jupiter in a Martian orbit and an Earth-like planet at 1 AU (run over 94,000 years), orbits inclined to the ecliptic plane greater than 10° became unstable, with Earth ejection after 6,250 years (50°). Thus, as the Jupiter giant is moved inward, its influence over the Earth-like planet increases and the time to orbital decay for the Earth-like planet decreases.

For several of the results, the Earth-like planet migrated out towards Jupiter and was then sent violently inward to circuit the Sun in a close, highly eccentric orbit. From there, most were sent to collide with the Sun or were ejected from the system completely. Further analysis and research will show how to differentiate between these two possibilities and what happens to these planets if they are thrown from the system. Overall, these results illustrate that the orbits of Earth-like planets in systems with Jupiter giants have restrictions on available orbital inclinations to remain stable. In the simulations of this paper, highly inclined planets (greater than 50°) tended to not be stable and led to planetary ejections or collisions, while planets with small inclinations (less than 10°) tended to remain stable over long periods of time. All of these results lead to explanations of why our Solar System primarily lies in the ecliptic plane and how it will continue to evolve over time.

In addition, it has been observed that gravitational interactions can affect a planet's ability to support life. This is evident in the presence of water on Earth, thought to have been brought by comets, and in the existence of the Oort Cloud, believed to have been formed by Jupiter's influence over comets and asteroids. Future work will focus on modeling different exoplanetary systems with variations in host star type, planet mass, semimajor axis, eccentricity, and inclination. Further analysis on how the habitable zone of the star and habitable surface of the planet are affected by changes in these parameters (and resulting system dynamics) should be evaluated as well. It is expected that the results of other exoplanetary systems will closely match those of our Solar System with respect to inclination and eccentricity for stable configurations. Regardless, the results should hold important clues as to whether the formation of our Solar System was unique, along with the life that was created here, or if other systems form in much the same way and life more common than we previously thought.

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APPENDIX A

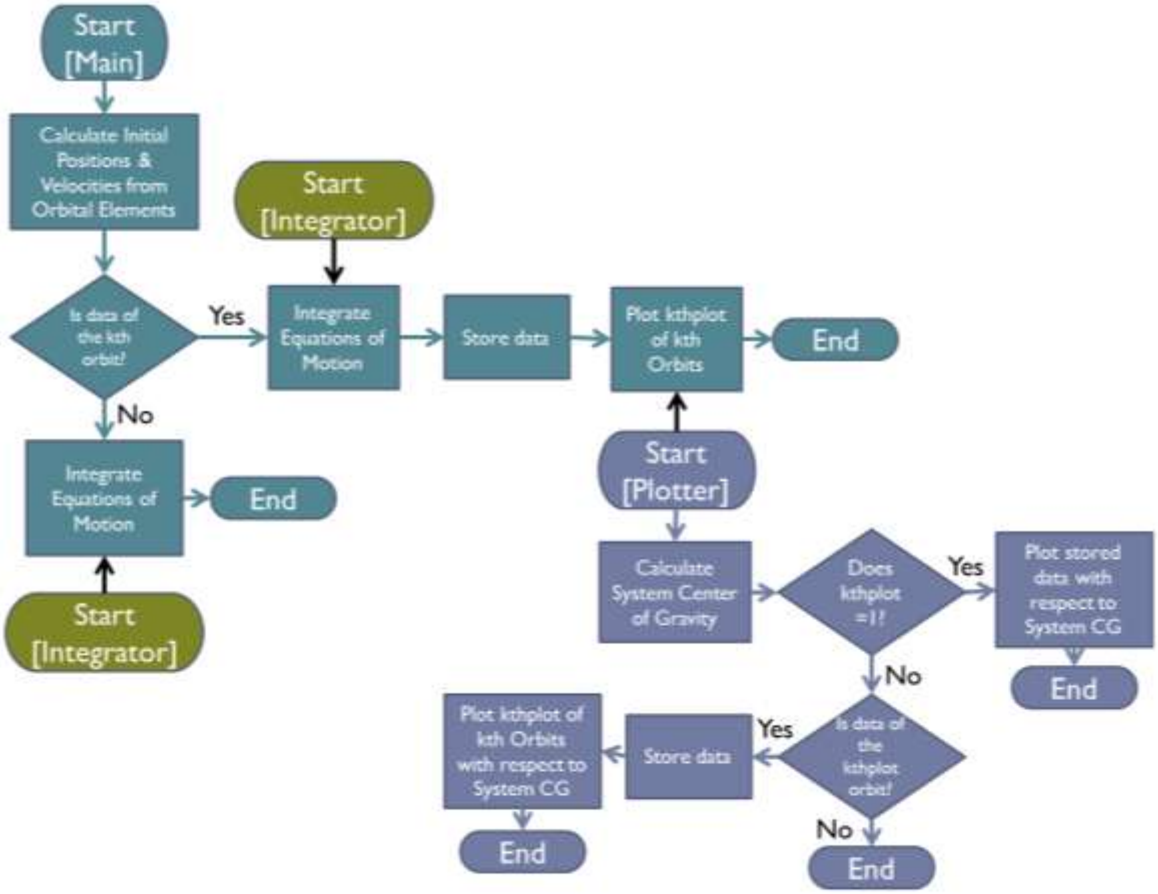
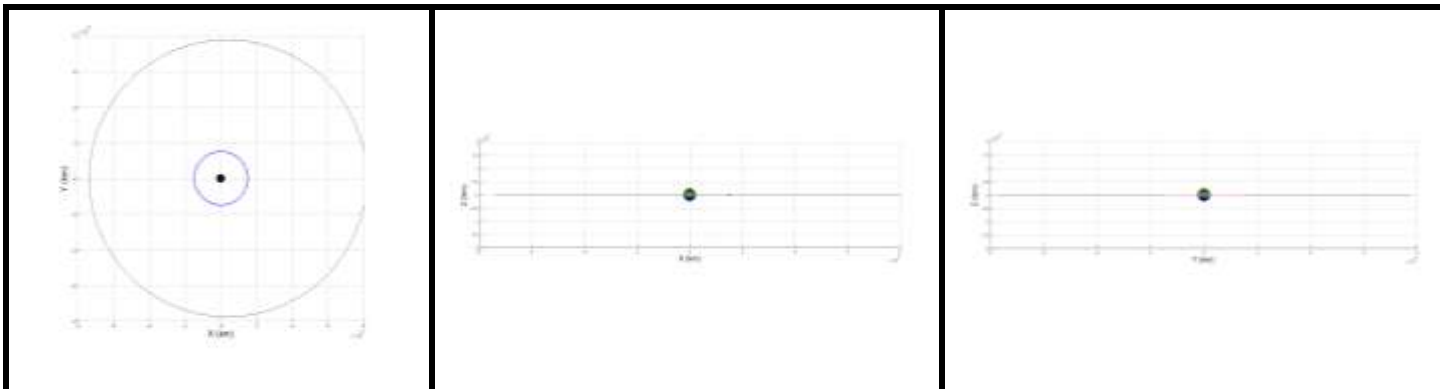
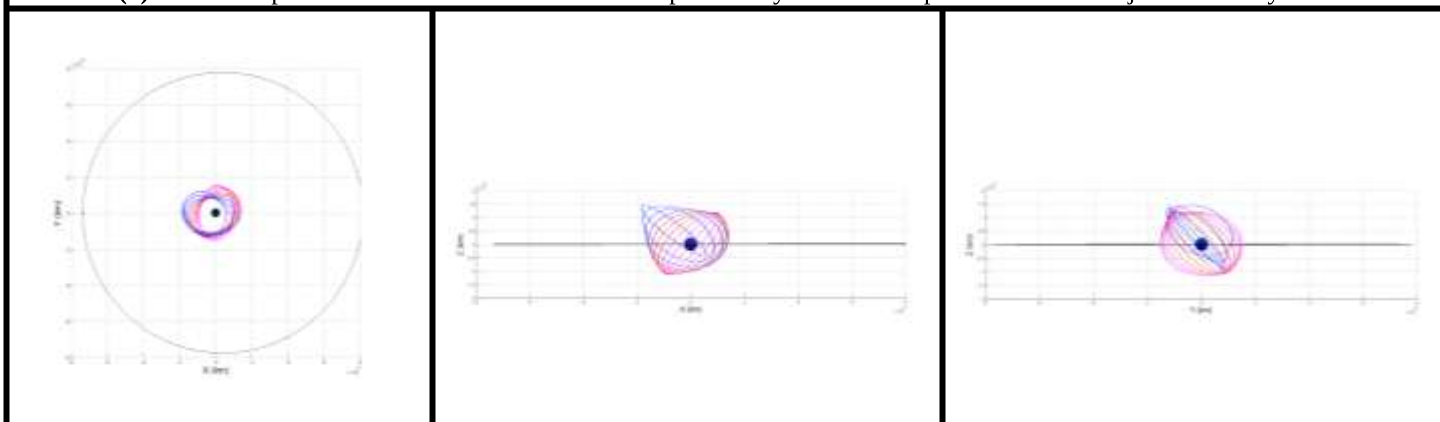


Figure 21 Logic for *n*-body Solver.

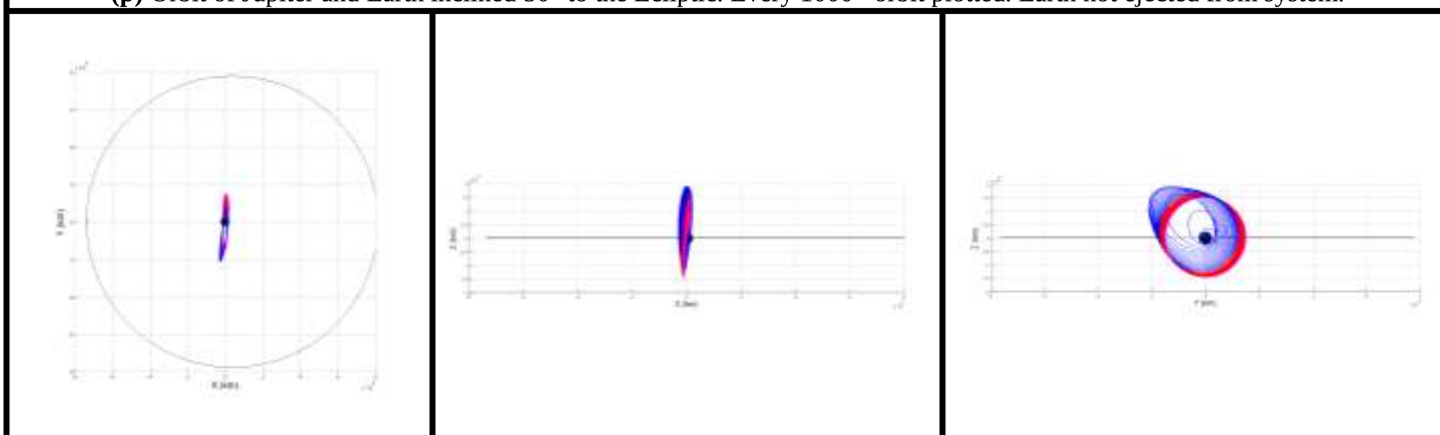
APPENDIX B



(o) Orbit of Jupiter and Earth inclined 0° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



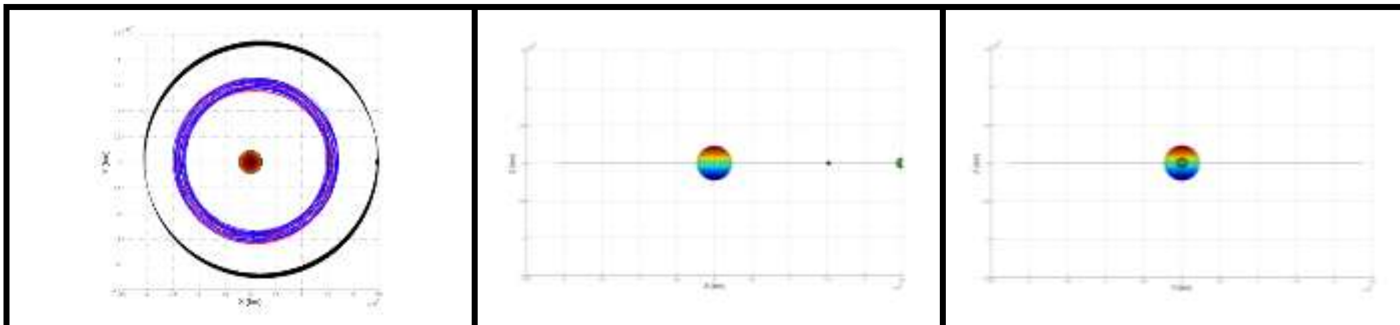
(p) Orbit of Jupiter and Earth inclined 50° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



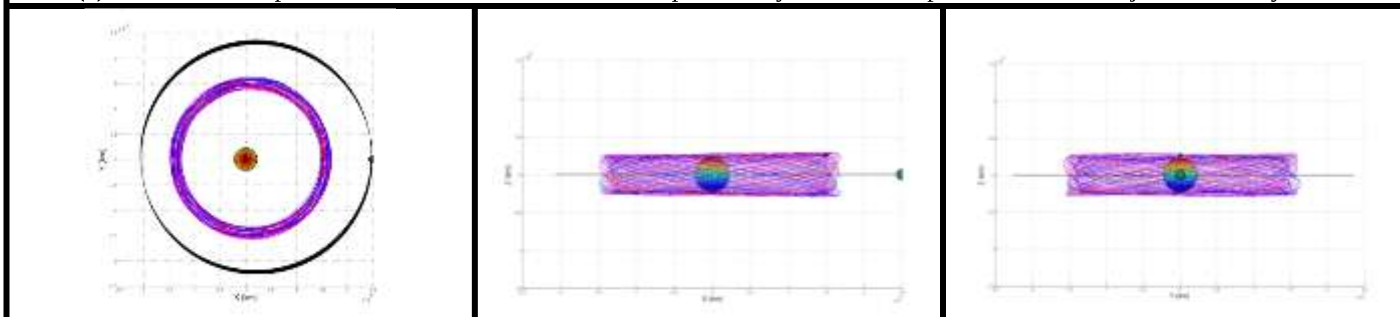
(q) Orbit of Jupiter and Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system.

Figure 22 Sun-Jupiter-Earth System Inclined 0°-50°-85° Results. XY-, XZ-, YZ-planes.

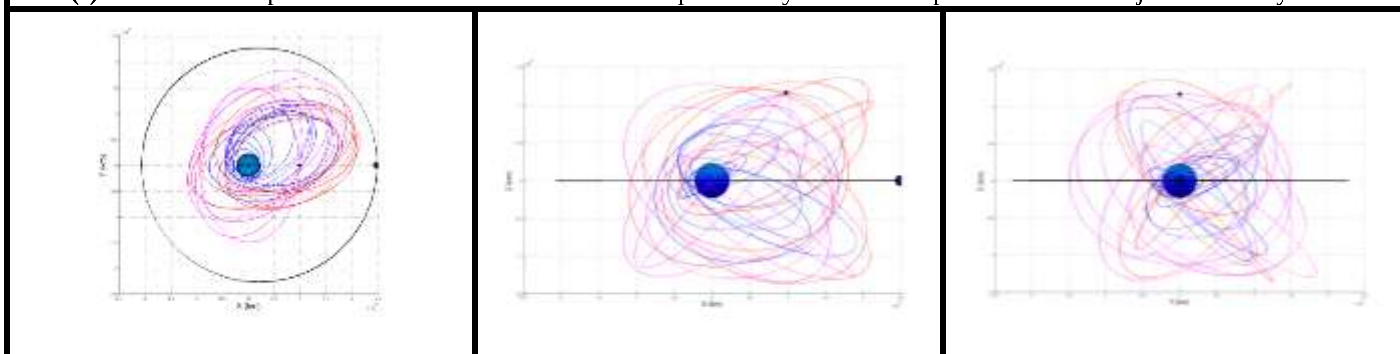
APPENDIX C



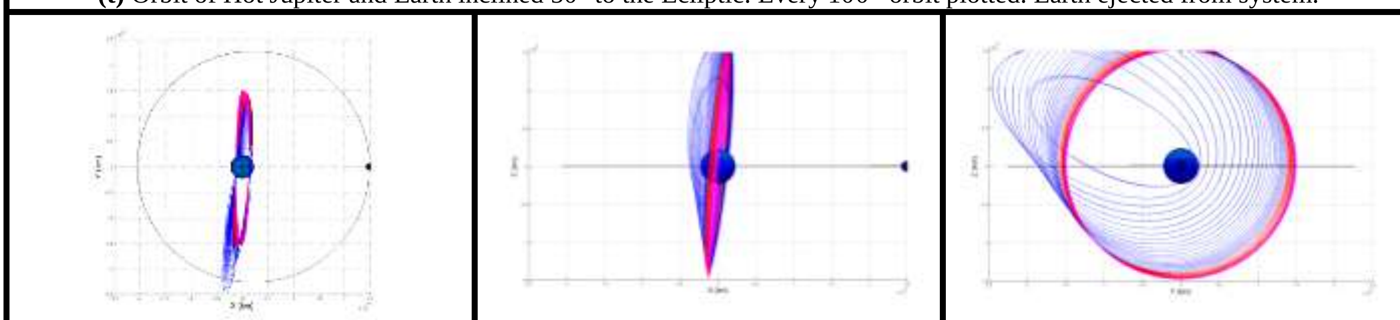
(r) Orbit of Hot Jupiter and Earth inclined 0° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



(s) Orbit of Hot Jupiter and Earth inclined 10° to the Ecliptic. Every 1000th orbit plotted. Earth not ejected from system.



(t) Orbit of Hot Jupiter and Earth inclined 50° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system.



(u) Orbit of Hot Jupiter and Earth inclined 85° to the Ecliptic. Every 100th orbit plotted. Earth ejected from system.

Figure 23 Exoplanetary System Inclined 0°-10°-50°-85° Results. XY-, XZ-, YZ-planes.