Erratum: Nonvanishing spin Hall currents in disordered spin-orbit coupling systems [Phys. Rev. B 71, 041304(R) (2005)]

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In Ref. 1 we reported on a numerical study of the influence of disorder on the spin Hall conductivity of the k-linear Rashba model, concluding incorrectly that it remains finite in the thermodynamic limit. Our original conclusion is at odds with new numerical findings[2] in which the extrapolation to infinite system size is carried out systematically. The new numerical results are consistent with the conclusion from perturbation theory calculations that the dc spin Hall conductivity of this model is zero.[3,4]

These numerical studies are complicated by strong frequency dependence and large fluctuations in the spin Hall conductivity values obtained in finite-size calculations. The earlier calculations erred by using the frequency dependence of the longitudinal conductivity, which has corrections that vary as $(\omega\tau)^2$ compared to the $(\omega\tau)^1$ dependence of the spin Hall conductivity discussed below, to judge whether or not the dc limit was reached. The new findings in Ref. 2 supersede the conclusions reached in Ref. 1 with regards to the thermodynamic dc limit of the k-linear Rashba model. The findings in Ref. 2 resolve the controversy that had been associated with the linear Rashba model.

The expression used for numerical evaluation of the spin Hall conductivity is

$$\sigma_{xy}^{z} = -\frac{i\hbar}{L^2}\sum_{\alpha,\alpha'}\frac{f(E_\alpha)-f(E_{\alpha'})}{E_\alpha - E_{\alpha'}}\frac{\langle\alpha|j_x^z|\alpha'\rangle\langle\alpha'|j_y|\alpha\rangle}{E_\alpha - E_{\alpha'} + i\eta}, \tag{1}$$

where $j_y = e\partial H/\partial p_y$, $j_x^z = \{\partial H/\partial p_x, s_z\}/2$, with s_z being $(\hbar/2)\sigma_z$ for electrons and $(3\hbar/2)\sigma_z$ for holes in the heavy hole band. $i\eta$ can be regarded as a complex frequency continued from the real axis to the imaginary axis and can be interpreted as an electric-field turn-on time. In metallic systems, like the ones considered here, η must exceed the simulation cell level spacing δE in order to obtain bulk values of the transport coefficients considered. At the same time, η must be smaller than all intensive energy scales including the Fermi energy E_F, the spin-orbit coupling splitting Δ_{SO}, and the disorder broadening $\hbar/\tau$, where τ is the scattering time. The finite value of η represents the coupling of a finite-subsystem of a macroscopic conducting sample to its environment, leading to lost resolution of the discrete individual energy levels of the subsystem. For a finite system with periodic boundary conditions, the spin-Hall conductivity is a function of $\delta E/E_F$, η/E_F, Δ_{SO}/E_F, and $\hbar/\tau E_F$. For the longitudinal conductivity, σ_{xx}, once η exceeds the level spacing, σ_{xx} hardly changes from a value that corresponds to the Drude conductivity with quantum weak localization corrections. In Ref. 1 we assumed a similar η dependence for the spin Hall conductivity. However, a more careful treatment showed that the finite-size behavior of $\sigma_{xy}^z(\eta)$ is qualitatively different from that of $\sigma_{xx}(\eta)$. The macroscopic dc spin Hall conductivity is obtained by extrapolating finite size results first to $\delta E \to 0$ $(L\to\infty)$ and then to $\eta\to 0$. Combining this scaling with a series of calculations at larger finite system sizes than considered previously in Ref. 1, the spin Hall conductivity behavior is consistent with it vanishing in the thermodynimic limit. This finding is also consistent with the suppression of the accumulated edge spin with increasing system size found in Ref. 5. The η-dependence of the spin Hall conductivity in the two-dimensional hole gas systems is quite different, as we reported in Ref. 2. In this case σ_{xy}^z and σ_{xx} scale similarly and the numerical results are inconsistent with a vanishing spin Hall conductivity, consistent with analytical perturbation theory results.[6] Bulk transport coefficients like the spin Hall conductivity and the anomalous Hall conductivity in ferromagnetic materials can be dominated from contributions due to induced interband coherence of states far from the Fermi energy.

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